ISOLATED ROTOR NOISE DUE TO
INLET DISTORTION OR TURBULENCE

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The report presents the theoretical formulation, analysis, and results necessary to analyze quadrupole noise generated from a loaded, isolated, subsonic rotor because of its interaction with an inflow distortion or inlet turbulence. The ratio of quadrupole to dipole noise is largely a function of the axial flow Mach number, wheel tip Mach number, rotor solidity, and total pressure ratio across the rotor. It is relatively independent of the specific form of the inflow distortion or inlet turbulence. Comparisons with experimental data only succeed in predicting gross levels at a given speed and fail to predict the variation of noise at fixed speed with flow and pressure ratio. Likely sources of this discrepancy are discussed.
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SUMMARY

The present report presents the theoretical formulation, analysis and results necessary to analyze quadrupole noise generated from a loaded, isolated, subsonic rotor due to its interaction with an inflow distortion or inlet turbulence. Programs to calculate these interactions are given in an appendix. These programs also yield dipole noise contributions from these two mechanisms. The ratio of quadrupole to dipole noise is largely a function of the axial flow Mach number, wheel tip Mach number, rotor solidity, total pressure ratio across the rotor. It is relatively independent of the specific form of the inflow distortion or inlet turbulence. Comparisons with experimental data only succeed in predicting gross levels at a given speed and fail to predict the variation of noise at fixed speed with flow and pressure ratio. Looking at the separate variations of the theoretical predictions of dipole and quadrupole noise, however, one notices that if these levels were closer together, a suitable composite of them would yield the trends of the experimental data. This leads to a suggestion that the calculation may be overestimating the ratio of dipole to quadrupole noise by overestimating the dipole noise. Likely sources of this overestimate are suggested along with corrective procedures.
INTRODUCTION

The purpose of the present study is to quantify a quadrupole noise source which was first proposed in (2) as a likely contributor to fan noise. The source arises from the fluctuating Reynolds stresses introduced by a combination of two unsteady velocities, one being due to the potential flow field of the rotor and the other due to a solenoidal velocity field such as an inlet distortion or inlet turbulence.

The study proceeds as follows. Since interest is in fan/compressor noise where substantial duct axial velocities are involved, a pertinent Lighthill equation including uniform axial flow is derived first for the study of quadrupole noise. Secondly the potential flow fields of a sub-sonic rotor are derived as a function of the pressure ratio across it, the wheel tip and axial flow Mach numbers.Thirdly a systematic approach to the acoustic problem of estimating the noise from this source is worked out based on repeated use of Fourier exponential transforms.

We next present calculations under several constraints of the relative magnitudes of the quadrupole versus the dipole noise source. Comparisons are also carried out with some available data on the influence of pressure ratio on fan noise. In an appendix, computer programs for calculations of quadrupole and dipole noise due to both inlet distortion and inlet turbulence are given.

Ivan H. Edelfelt of the General Electric Research and Development Center helped considerably in programming these calculations. Thomas F. Gelder and Marvin E. Goldstein of the NASA-Lewis Research Center provided several helpful discussions.
DEVELOPMENT OF PERTINENT LIGHTHILL EQUATION FOR BLADE LOADING NOISE

The full Lighthill equation for aerodynamic noise for an inviscid gas, in the absence of mass, energy or force sources (and assuming the isentropic relation $p = a^2_0 \rho$) may be written as:

$$\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{3}{a_1^2 \partial x_i \partial x_j} (\rho \ u_i \ u_j). \quad (1)$$

Now let $u_j = U \delta_{1j} + u_j^i$ where $U$ is uniform and steady and 1 corresponds to the axial direction. Then we may show rigorously that (1) reduces to:

$$\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \rho - a_0^2 \frac{\partial^2 \rho}{\partial x_i \partial x_j} = \frac{3}{a_1^2 \partial x_i \partial x_j} (\rho \ u_i^i \ u_j^i). \quad (2)$$

Equation (2) is the desired form of the Lighthill equation for blade loading noise from an isolated rotor. The form of $\rho \ u_i^i \ u_j^j$, correct to second order, would be $(p_0^0 u_i^i u_j^j)$. For an axial flow device, two important sources of velocity fluctuations are solenoidal velocity fluctuations associated with inlet distortion or inlet turbulence and secondly irrotational velocity fluctuations associated with the flow field of a loaded rotor. Thus we may write $u_j^i = u_{js}^i + u_{ip}^i$ where $u_{js}^i$ denotes the solenoidal velocity fluctuation associated with inlet distortion or turbulence and $u_{ip}^i$ that associated with the rotor potential flow field. The calculation of the quadrupole noise field due to interaction between the rotor potential flow field and an inlet distortion or turbulence field thus boils down to solving (2) with a source term of type,

$$2 \frac{3}{a_1^2 \partial x_i \partial x_j} (\rho_0 \ u_{ip}^i u_{js}^j).$$

The factor of 2 arises from the symmetry of the quadrupole term with respect to interchange of $i, j$.

DETERMINATION OF FLUCTUATING VELOCITIES DUE TO THE ROTOR POTENTIAL FLOW FIELD

For the determination of the fluctuating velocity field of the rotor, the most convenient quantity is the (nondimensional) lift coefficient of the steady lift exerted by the rotor blades on the fluid. However the quantities most conveniently available in practical terms are the pressure ratio across the rotor, its wheel tip and axial flow Mach numbers, etc. So we first evaluate $C_L$ in terms of these quantities.

The work per lbm of the flow is

$$C_L \ = \ \frac{\rho \ \frac{W^2 r}{2} \ c \ \cos(\alpha_r) V}{2 \rho \ U \ d} = \frac{\sigma \ C_L}{2} \ \frac{U^2 \ \sin(\alpha_r)}{\cos^2(\alpha_r)}$$

$$= C_p \ \frac{\ T_{02}}{T_{01}} \ (\frac{T_{02}}{T_{01}} - 1)$$

2
\[
\begin{align*}
Y - 1 & = \frac{\gamma RT_{1}(1 + \frac{\gamma - 1}{2} M_{a}^{2})}{(\gamma - 1)} \left( \frac{p_{02}}{p_{01}} \right)^{\gamma - 1} \\
\sigma C_{L} \sin(\alpha_{r}) & = \frac{\gamma - 1}{\cos^{2}(\alpha_{r}) \left( \frac{2}{(\gamma - 1) M_{a}^{2}} + 1 \right)} = \frac{p_{02}}{p_{01}} \left( \frac{\gamma - 1}{\gamma - 1} \right)
\end{align*}
\]

This gives the desired expression for \( C_{L} \) as:

\[
C_{L} = \frac{\cos^{2}(\alpha_{r})}{\sigma \sin(\alpha_{r})} \left( 1 + \frac{2}{(\gamma - 1) M_{a}^{2}} \right) \left( \frac{p_{02}}{p_{01}} \right)^{\gamma - 1}
\]

To calculate the \( u' \), \( v' \) (x and y components of velocity) requires a linearized, compressible analysis of the flow field of the isolated rotor.

Let \( W_{r} = a M_{r} \). We first find the solution for velocity components \( u', v' \), parallel to the \( x' - y' \) coordinate system of Figure 1, due to equally spaced concentrated unit forces at the origin and its corresponding points as shown in Figure 1. (Note that the blade exerts a force on the fluid equal and opposite to the force by the fluid on the blade.) We use a frame of reference fixed w.r.t. the translating blade row so that we have a steady state problem. We have to consider the effect of a sum of forces:

\[
\frac{1}{d} \delta(x') \sum_{n = -\infty}^{\infty} \delta[y' - 2n \frac{d}{2}]
\]

where \( \delta \) denotes a unit force vector, and \( \delta \) stands for the Dirac delta function. By using a result on page 68 of (1) concerning the sum of an infinite row of equally spaced delta functions, clearly the above is equal to:

\[
\frac{1}{d} \delta(x') \sum_{n = -\infty}^{\infty} \exp[j \frac{2\pi ny'}{d}]
\]

The linearized equations of motion and continuity are:

\[
\begin{align*}
\rho \left[ \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} \right] + W_{r} \left[ \cos(\alpha_{r}) \frac{\partial p'}{\partial x'} + \sin(\alpha_{r}) \frac{\partial p'}{\partial y'} \right] & = 0 \quad (1a) \\
\cos(\alpha_{r}) \frac{\partial u'}{\partial x'} + \sin(\alpha_{r}) \frac{\partial u'}{\partial y'} & = -\frac{1}{\rho W_{r}} \frac{\partial p'}{\partial x'} + \frac{\sin(\alpha_{r}) \delta(x')}{\rho d W_{r}}
\end{align*}
\]

and
\[
\cos(\alpha_r) \frac{\partial v'}{\partial x'} + \sin(\alpha_r) \frac{\partial v'}{\partial y'} = -\frac{1}{\rho \cdot W_r} \frac{\partial p'}{\partial y'}
\]

Eliminating \(p'\) from (1b, c) we derive that:

\[
- \cos(\alpha_r) \delta(x') \int_{-\infty}^{\infty} \frac{\delta(x')}{\rho \cdot W_r} \sum_{n=-\infty}^{\infty} \exp(j \frac{2\pi ny'}{d}) = 0
\]

and since \(u', v'\) and the delta function term vanish far from the blade row

\[
\frac{\partial u'}{\partial y'} - \frac{\partial v'}{\partial x'} - \frac{\delta(x')}{\rho \cdot W_r} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \exp(j \frac{2\pi ny'}{d}) = 0
\]

[Kutta Joukowski Law]

Next we eliminate the force terms in (1b, c) and assuming an isentropic relation between \(p'\) and \(\rho'\) one obtains:

\[
\frac{\partial u'}{\partial x'} \left[1 - M^2_r \cos^2(\alpha_r)\right] + \frac{\partial v'}{\partial y'} \left[1 - M^2_r \sin^2(\alpha_r)\right] = M^2_r \sin(\alpha_r) \cos(\alpha_r) \left[\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}\right]
\]

(modified continuity equation)

Using (2), (3) single equations for \(u', v'\) may be obtained which may be solved by requiring that \(u', v'\) vanish as \(x' \to \pm \infty\). We omit the details and give the result.

\[
u' \text{ for } x' > 0 = \frac{1}{2\rho \cdot d \cdot W_r} \left[\frac{\text{sgn}(n) \cdot j \pi - M^2_r \pm M^2_r \sin(\alpha_r) \cos(\alpha_r)}{(1 - M^2_r \cos^2 \alpha_r)}\right] \int_{-\infty}^{\infty} \exp(j \frac{2\pi ny'}{d}) \exp(-\frac{2\pi nx'}{d(1 - M^2_r \cos^2 \alpha_r)})
\]

Similarly:

\[
v' \text{ for } x' < 0 = \frac{1}{2\rho \cdot d \cdot W_r} \sum_{n=-\infty}^{\infty} \exp(j \frac{2\pi ny'}{d}) \exp(-\frac{2\pi nx'}{d(1 - M^2_r \cos^2 \alpha_r)})
\]

*\(\text{sgn}(n) = 1\) if \(n > 0\), \(-1\) if \(n < 0\) and \(0\) if \(n = 0\).
When viewed in a frame of reference fixed with respect to the casing of the machine, these \( u' \), \( v' \) velocities given by (4a), (4b) appear as:

\[
\begin{align*}
\text{\( u' \) for } x & \geq 0 = -\frac{C_L \sigma W_r}{4(1 - M^2_a)} \sum_{n} \left\{ \pm M_a M_t + \text{sgn}(n) j \sqrt{1 - M^2_r} \right\} \\
& \exp(j \frac{2\pi nx}{d}) \exp\left[ \frac{2\pi nx^2 \text{sgn}(n) \sqrt{1 - M^2_r} + j M_a M_t}{d(1 - M^2_x)} \right] \\
& \exp(j \frac{2\pi nx}{d} V_t) \ldots \ (5a)
\end{align*}
\]

\[
\begin{align*}
\text{\( v' \) for } x & < 0 = \frac{C_L \sigma W_r}{4} \sum_{n} \exp(j \frac{2\pi nx}{d}) \exp\left( \frac{2\pi nx^2 \text{sgn}(n) \sqrt{1 - M^2_r} + j M_a M_t}{d(1 - M^2_x)} \right) \\
& \exp(j \frac{2\pi nx}{d} V_t) \ldots \ (5b)
\end{align*}
\]

\[
\begin{align*}
& \text{THE AXIAL VELOCITY OF THE INLET DISTORTION}
\end{align*}
\]

Far upstream from the rotor, the inlet distortion appears as regions of axial velocity defect superposed on a uniform inlet velocity.

Some assumption regarding the form of the inlet distortion is necessary in order to analyze its effect. The form assumed is sketched in Figure 2.

Let the axial velocity, in Mach number, associated with the uniform total pressure region be \( M_a \). The axial velocity defect \( \Delta M_a \) associated with the inlet distortion may be deduced as follows from the condition that far upstream there is no variation of static pressure.

\[
\frac{p_t - \Delta p_t}{p_t} = \left( 1 + \frac{\gamma - 1}{2} (M_a - \Delta M_a)^2 \right) \frac{\gamma}{\gamma - 1} \ldots \ (6)
\]

(\( \gamma \) is the specific heat ratio of the gas)

Now

\[
\frac{p_t}{p_t} = \left( 1 + \frac{\gamma - 1}{2} M_a^2 \right) \frac{\gamma}{\gamma - 1} \ldots \ (7)
\]

Thus one may solve for \( \Delta M_a \) in (6) as:
The form of Eq. (8) indicates that $\Delta p_t/p_t$ and $M_a$ are subject to a restriction that

$$\left(1 - \frac{\Delta p_t}{p_t}\right)^{\frac{\gamma - 1}{\gamma}} \left(1 + \frac{\gamma - 1}{2} M_a^2\right) > 1$$

(9)

(Other than the above restriction, $M_a$ and $\Delta p_t/p_t$ may be specified arbitrarily.)

The restriction of (9) follows from the assumption that the distortion consists of regions of constant static pressure and varying axial velocity. For such a distortion, (9) simply expresses the requirement that the static pressure not exceed the total pressure anywhere. For small $\Delta p_t/p_t$, and small $M_a^2$, (9) may be expressed approximately as

$$\frac{\Delta p_t}{p_t} \leq \frac{\gamma}{2} M_a^2$$

(9)

The interaction of the inlet distortion with the isolated rotor may be analyzed (as sketched in Figure 2) as the interaction of a shear wave, convecting along the fan inlet duct at an axial Mach number $M_a$, with an isolated rotor.

Let the inlet distortion be Fourier analyzed in an x-y coordinate system at the mean radius of analysis "a" as follows. The basic periodicity of the distortion in the y-direction is "$2\pi a$/S" (for an $S$ lobed distortion). Let the distorted velocity profile be expressed as:

$$\sum_{i=-\infty}^{\infty} \exp(-j i S_y a) C_i$$

(10)

Here $j = \sqrt{-1}$, $i$ = dummy index of summation.

$$C_i = \frac{-S}{2\pi a} \int_0^{2\pi a/S} \Delta M_a(y) e^{-j i S_y a} dy$$

(11)

THEORY OF QUADRUPOLE NOISE GENERATION BY INLET DISTORTION

Let $A = \frac{C_L \sigma \omega M_a M_t}{4(1 - M_a^2)}$, $A' = 2A$
B = \frac{C_L \sigma W r \sqrt{1 - M^2}}{4(1 - M^2) r}, \quad B' = 2B \quad (\text{Sometimes } B \text{ is also used to denote the number of rotor blades and the context of its use makes this fairly clear.})

C = \frac{C_L \sigma W}{4}, \quad C' = 2C

\alpha_n = \frac{2\pi n |n| \sqrt{1 - M^2}}{d(1 - M^2) a} \quad (-\infty < n < \infty)

\beta_n = \frac{2\pi n d}{a} \quad (-\infty < n < \infty)

\gamma_n = \beta_n M_t \quad (-\infty < n < \infty)

\delta_n = \frac{2\pi n M_a M_t}{d(1 - M^2) a} \quad (-\infty < n < \infty)

\epsilon_n = \beta_n S/nB \quad (-\infty < n < \infty)

Only an axial quadrupole of type \( \rho_o \), \( u_x \), \( u_y \) and a transverse one of type \( \rho_o \), \( v_x \), \( v_y \) where \( u_x \), \( v_x \) are the axial and tangential velocity distortions associated with the rotor potential flow field and \( u_y \), the axial velocity distortion associated with the inlet distortion contributes to the noise (at blade passing frequency and its multiples).

Exploiting the three facts that (a) there is no interest in the zeroth harmonic of blade passing frequency (which is a dc field) (b) that the rotor is subsonic \( (M < 1) \) and (c) we are only interested in propagating noise fields, we may show that the noise generation for a particular harmonic \( \nu \) \((\nu > 0)\) of blade passing frequency noise is governed by:

\[
\left( \frac{\partial}{\partial t} + M_a \frac{\partial}{\partial x} \right)^2 p - \nabla^2 p
\]

\[
= \int \frac{\partial^2}{\partial x^2} \rho_o \left( (\pm A' + j B') \sum_{i=1}^{\infty} C_i' \exp(-\alpha_n |x|) \exp(j \delta_n x) \exp(j \beta_n (1 - \frac{i \pi}{nB}) y) \right)
\]

\[
\int \frac{\partial^2}{\partial x \partial y} \rho_o C_i' \sum_{i=1}^{\infty} C_i' \exp(-\alpha_n |x|) \exp(j \delta_n x) \exp(j \beta_n (1 - \frac{i \pi}{nB}) y)
\]
\[
\exp[j \beta_n M_t a_0 t)] \ldots \tag{1}
\]

(where \(C_i' = a_0 C_i\))

Clearly \(p\) depends on \(y, t\) (given \(i\)) as

\[
\exp[j \beta_n (1 - \frac{iS}{nB}) y] \exp[j \beta_n M_t a_0 t].
\]

Thus we deduce that the axial part of \(p\) depends on \(x\) as:

\[
\frac{d^2 p}{dx^2} (1 - M^2_a) - 2j M_a M_t \beta_n \frac{dp}{dx} \\
+ \beta_n^2 (M^2_t - (1 - \frac{iS}{nB})^2) p \\
= + \rho_o \frac{\partial^2}{\partial x^2} (\pm A' + j B') C_i' \exp(-\alpha_n |x|) \exp(j \delta_n x) \\
\pm \frac{\partial}{\partial x} \rho_o j C' \beta_n (1 - \frac{1}{nB}) \exp(-\alpha_n |x|) \exp(j \delta_n x) \tag{2}
\]

To solve (2) introduce the Fourier exponential transform \(P(z)\) of \(p\) by:

\[
P(z) = \int_{-\infty}^{\infty} p(x) e^{-jzx} \, dx
\]

so that

\[
p = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(z) e^{jzx} \, dz
\]

Then:

\[
(z - z_+)(z - z_-)P(z) \\
= \frac{-2j z[A'(z - \delta_n)z - \alpha_n z B' + C'(z - \delta_n)\beta_n (1 - \frac{iS}{nB}) C_i' \rho_o}{[\alpha_n^2 + (z - \delta_n^2)(1 - M^2_a)]}
\tag{3}
\]

where

\[
z_{\pm} = \frac{\beta_n}{(1 - M^2_a)} \{M_a M_t \mp \sqrt{M^2_a t - (1 - M^2_a)(1 - \frac{iS}{nB})^2}
\]

Let

\[
G(z) = \frac{-2j z C_i' [A'(z - \delta_n)z - \alpha_n z B' + C'(z - \delta_n)\beta_n (1 - \frac{iS}{nB}) \rho_o}{[\alpha_n^2 + (z - \delta_n^2)(1 - M^2_a)]}
\tag{4}
\]

By the method of residues, the downstream/upstream waves have amplitudes given by:
Independent waves (above cut-off) for each \( n \) will be produced over a range of \( i \) given by:

\[
\frac{nB}{S}(1 - \frac{M_t}{\sqrt{1 - M_a^2}}) < i < \frac{nB}{S}(1 + \frac{M_t}{\sqrt{1 - M_a^2}})
\]

For each of these \( i \), amplitudes of upstream and downstream waves are computed from (5). It is demonstrable that the rms values of the axial components of the intensity are (power/cross sectional area of annulus):

\[
\frac{|A_{\pm}^2| \Delta M_t (1 - M_a^2)^2}{2 \rho_o a_o (M_t + \Delta M_t)^2}
\]

where

\[
\Delta = [M_t^2 - (1 - M_a^2)(1 - \frac{iS}{nB})^2]^{1/2}.
\]

This completes the theory of inlet distortion quadrupole noise generation.

**NOISE GENERATION BY INLET TURBULENCE**

We represent the turbulent velocity components by:

\[
(u_t, v_t, w_t) = \int dZ_{u,v,w} (\kappa) e^{j(\kappa \cdot r)} \delta(kx) Ut
\]

adopting the usual Fourier Stieltjes form where

\[
\overline{dZ_u(\kappa') dZ_v(\kappa'')} = \delta(\kappa' - \kappa'') \phi_{uv}(\kappa') d^3\kappa' d^3\kappa''
\]

and \( \phi_{uv}(\kappa) \) is the usual spectral density tensor.

The noise generation problem is given by:

\[
\frac{\partial}{\partial t} + M_a \frac{\partial}{\partial x} \right)^2 p - \nu^2 p = 2\rho_o (\frac{\partial^2}{\partial x^2} (u_p u_t) + \frac{\partial^2}{\partial x \partial y} (u_p v_t) + \frac{\partial^2}{\partial y^2} (v_p w_t)).
\]

A typical term of the RHS of (7) may be written as:
\[-\rho_o \frac{\partial^2}{\partial x^2} ((\pm A + sgn(n)j B)dz_u(...))\]

\[-\rho_o \frac{\partial^2}{\partial x \partial y} (((\pm A + j sgn(n)B)dz_v(...))\]

\[\pm C dz_u(...) \mp \rho_o \frac{\partial^2}{\partial y^2} C dz_v(...)\]  \hspace{1cm}  (8)

where (...) stands for:

\[\exp\{-\alpha_n |x|\} \exp[j(\delta_n + k_x)x] \]

\[\exp[j(\beta_n + k_y)y] \exp[j(\beta_n M_t - M_a k_x)ao t] \]  \hspace{1cm}  (9)

A word of explanation is in order here concerning the model of turbulence employed in the analysis. The model is the same as was employed in (3). A three dimensional model of turbulence is employed but the spectrum functions are integrated over the third coordinate (z) so that nowhere does the z dependence appear explicitly.

Substituting for the y dependence from (9), the source term (8) is effectively:

\[-\rho_o \frac{\partial^2}{\partial x^2} [((\pm A + j sgn(n)B)dz_u(...))\]

\[-\rho_o \frac{\partial}{\partial x} j(\beta_n + k_y)((\pm A + j sgn(n)B)dz_v \pm C dz_u)(...)\]

\[\pm (\beta_n + k_y)^2 \rho_o C dz_v(...)\]  \hspace{1cm}  (10)

As before the Fourier exponential transform of the term in (10) may be written as:

\[-2j \rho_o \frac{\alpha_n^2}{\alpha_n^2 + [z - (\delta_n + k_x)]^2} \{z[(z - (\delta_n + k_x))A - sgn(n)B \alpha_n]\}

+ C(z - \delta_n - k_x)(\beta_n + k_y)dz_u\]

\[+ (\beta_n + k_y)(z[\delta_n - k_x)]) \exp[j(\beta_n M_t - M_a k_x)ao t] \]  \hspace{1cm}  (11)

As before the downstream and upstream waves will have form:

(i) if \((\beta_n M_t - k_x M_a) > 0\), then with
the waves are given by
\[ \exp[j z_\pm x] \exp[j(\beta_n x + k_y) y] \]
\[ \exp[j(\beta_n M_t - k_x M_a) a_0 t] \] \tag{13}

(ii) If \((\beta_n M_t - k_x M_a) < 0\), then
\[ z_\pm = \frac{1}{(1 - M_a^2)} \{ (\beta_n M_t - k_x M_a) M_a \}
\pm [(\beta_n M_t - k_x M_a)^2 - (1 - M_a^2)(\beta_n + k_y)^2]^{1/2} \tag{14} \]

and the rest is as in (13).

Let \(\Delta = [(\beta_n M_t - k_x M_a)^2 - (1 - M_a^2)(\beta_n + k_y)^2]^{1/2} \tag{15a}\)

and
\[ f = |\beta_n M_t - k_x M_a| \tag{15b} \]

Then the acoustic energy produced downstream and upstream by such waves is given by taking the mean square of (13) and multiplying the resulting quantity by:
\[ \frac{1}{2\rho_0 a_0} \frac{\Delta f(1 - M_a^2)^2}{(f + M_a \Delta)^2} \tag{16} \]

The remaining procedure is more or less mechanical. Let us say that the interest is in the acoustic energy between wave numbers lying between \(\chi\) and \(\chi + \Delta \chi\). (Wave number = frequency in radians/sec. ÷ speed of sound). This determines that for each \(n\) from \(-\infty\) to \(\infty\), a range of \(k_x\) lying between:
\[ (\beta_n \tan(\alpha_r) - \frac{X}{M_a}) \text{ to } (\beta_n \tan \alpha_r - \frac{X}{M_a} - \frac{\Delta X}{M_a}) \]  

is of interest. This determines (for each \( n \)) a specific value of \( \chi \). A range of \( k_y \) from
\[ \pm \frac{X}{\sqrt{1 - M_a^2}} - \beta_n \leq k_y \leq \pm \frac{X}{\sqrt{1 - M_a^2}} - \beta_n \]  

is to be considered (depending on whether \( \chi \) is positive or negative). The quantity
\[ G^2(z_\pm) \phi_{uu} + 2G_u(z_\pm)G_v(z_\pm) \phi_{uv} + G_v^2(z_\pm) \phi_{vv} \]
times the factor in (16) is to be integrated over the range of \( k \) indicated in (18), given \( \chi, \beta_n \) (and hence \( k \) by (17)) for each \( n \) in range \(-\infty \) to \( \infty \). For given physical wave number \( \chi \) both energy corresponding to \( +\chi \) and \( -\chi \) need to be considered. By adding up the infinite series of contributions from \( n = -\infty \) to \( n = \infty \) for \( +\chi \) and \( -\chi \), taking note of the fact that the width in \( k \) is \( (\Delta X/M_a) \), etc., the required spectral density \( dI^2/d\chi \) may be deduced. \( \phi_{uu}, \phi_{uv} \) and \( \phi_{vv} \) should first be integrated over \( k \) from \(-\infty \) to \( \infty \), in keeping with two dimensional or plane nature of present analysis. Actually it turns out to be more pertinent to think in terms of \( \chi dI^2/d\chi \) as this quantity has the units of power and is related directly to measurements obtained by constant percent bandwidth or constant octave filters. If we assume a specific form of the longitudinal velocity correlation function for the turbulence of type \( \exp(-r/L) \), \( \phi_{uu}, \phi_{uv} \) and \( \phi_{vv} \) can be easily written down for given turbulence intensity.

**DISCUSSION OF RESULTS**

Previously developed analyses (3, 4) of dipole noise were employed to estimate the dipole contribution to inlet distortion and inlet turbulence noise.

We show in Figures 3(a), (b), (c) first the nondimensional constant percent filter spectra due to inlet turbulence rotor interaction noise. We study a fixed operating line characterized by a steady lift coefficient of unity associated with a rotor whose solidity is unity. In turbulence noise calculations, one parameter that enters the calculation is the ratio of the integral length scale of turbulence to the blade spacing designated herein as (L/D). We have independently verified that the specific value of the ratio (L/D) does not materially affect the ratio of dipole/quadrupole noise contribution. For rotors with axial Mach numbers of order two-thirds the wheel tip Mach number, at wheel tip Mach numbers of order 0.8, the quadrupole noise contribution does indeed begin to exceed the dipole contribution especially at the higher frequencies. The low frequency end of the spectrum (frequencies less than half the blade passing frequency) is still dominated by dipole noise. Similarly calculations were carried out for a rotor with \( (M_a/M_t) = 0.5 \) and \( C_L = 0.5 \) and are shown in Figures 4(a), (b) and (c). Similar trends are evident insofar as the fact that the ratio of quadrupole/dipole contribution increases
with $M_t$ and also, at a given $M_t$, increases with frequency. However one finds that the ratio of quadrupole/dipole noise, at given $M_t$, increases with the ratio ($M_a/M_t$). To bring this out in somewhat sharper focus, a calculation is shown in Figure 4(d) of a rotor with ($M_a/M_t$) being unity, at the same relative Mach number and lift coefficient, etc., as the rotor in Figure 4(c). The ratio of quadrupole/dipole noise is much greater in Figure 4(d) than in Figure 4(c). In Figures 5(a), (b), (c) and (d) are shown calculations of noise spectra at a fixed axial Mach number and total pressure ratio but with varying tip speed. Again as might be expected, higher tip speeds (and associated higher frequencies) do increase the relative ratio of quadrupole/dipole noise. However no evidence is available in these results to support the notion that fan noise could be minimized for constant work by choice of high tip speed and low loading. For other specific designs over some limited tip speed ranges, some such result may be true but it is certainly not a generally valid principle. Indeed the results of Figure 5 support the notion that for subsonic fans, to minimize noise for a given flow and total pressure ratio one should employ high blade loading and low tip speed. This conclusion applies of course only to the physical generation process and not to aspects such as effectiveness of treatment or perceived noise considerations.

In Figures 6 and 7, calculations corresponding to Figure 5 and Figure 3 are shown for 1st and 2nd harmonic pure tone (blade passing frequency) noise generated by an inlet distortion - rotor interaction. In both cases a maximum total pressure defect of 2% of the inlet total pressure was assumed. A four lobed distortion is assumed to impinge on a 36 bladed rotor. Other details are given in Figures 6, 7. A fairly smooth profile of the distortion as shown in Figures 6, 7 was assumed. The results of Figures 6, 7 are similar to those of Figures 5, 3 in that the ratio of quadrupole/dipole noise is greater for second harmonic rather than first harmonic noise and also increases with tip Mach number when a fixed operating line study is considered (ratio of ($M_a/M_t$) fixed, fixed $C_L$).

Finally an attempt was made to compare the results of the current analysis with experimental data from (5). The data given in (5) pertains to 50 Hz bandwidth forward radiated power at the blade passing frequency measured at several speeds and several pressure ratios at each speed. The data for two rotors (designated rotor 1 and rotor 2 in (5)) were employed.

Figure 8 indicates the results for rotor 1. This case has also been analyzed by the authors of (6) and, as assumed by them, a four lobed distortion with maximum velocity defect equal to 1% of the axial velocity is assumed to impinge on the rotor. The shape of the distortion was assumed to be a triangular pulse of width 10% of the extent of the lobe. This width corresponds roughly to the width of each of four struts placed upstream of the rotor in the experiments reported in (6).

Shown in Figure 8 are the actual measured data at 50, 60, 70 and 80% speeds and predictions by the current analysis using the results of the current study and those of (4). The dipole and quadrupole contributions are shown separately (as predicted by the theory). The dipole noise levels predicted by the theory are roughly in the ballpark of the results observed in (5) by Gelder and Soltis but in one major respect the theory fails completely. The present calculations employ the two dimensional Sears gust formula for estimation of unsteady forces needed to obtain the dipole noise. According to these results, the dipole noise greatly
exceeds the quadrupole noise (by at least 10 dB) over the operating range of rotor 1 of (5). Since dipole noise is independent of blade loading and is proportional to the rotor relative velocity (which decrease as the loading is raised at constant wheel tip speed), the inlet noise is always predicted to decrease with increasing pressure ratio (and associated lower weight flow and rotor relative velocities). The data of (5) of course fail to show this trend at all.

In case of rotor 2, an attempt was made to predict the noise (in a 50 Hz bandwidth) with the aid of the turbulence noise prediction procedures. Measurements of scale and intensity of turbulence are pretty scarce but a good set of measurements for one flow condition and one inlet is available in (9) and these were crudely scaled to the conditions of the inlet in (5). A 4% intensity of turbulence effective over the outer 10% of the rotor blade span and an (L/D) of about 0.5 was assumed for these calculations. Again one notices a fair ability to account for the gross noise levels by the inlet turbulence dipole noise theoretical estimates but a total failure to predict the variation with pressure ratio and flow at fixed speed largely due to the extreme predominance of dipole noise according to the theoretical estimates.

To put these results in perspective, we start by remarking that the quadrupole noise mechanism pointed out in (2) is a second order effect while dipole noise is a first order effect. Thus, with a dipole mechanism, the acoustic pressures scale roughly as \((\rho_0 W_r u')\) where \(\rho_0\) = mean fluid density, \(W_r\) = steady relative velocity through the rotor and \(u'\) the fluctuating solenoidal velocity (whether due to distortion or turbulence). With a quadrupole noise mechanism, they scale as \((\rho_0 u_p u')\) where \(u_p\) is the potential flow field induced by the rotor and generally \(u_p << W_r\).

Indeed the very procedure adopted to estimate \(u_p\) is a small perturbation calculation assuming that \(u_p \ll W_r\).

Again the full quadrupole source term \(\frac{\partial^2}{\partial x_i \partial x_j} (\rho_0 u_i u_j)\) may be expanded as \((\theta^2 + 2u_i \frac{\partial \theta}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j})\) where \(\theta\) is the divergence of the velocity. So far as the inflow distortion or inflow turbulence is concerned, these velocity fields have zero divergence. Even the potential flow field of the rotor does not have a significant divergence at modest axial and wheel tip Mach numbers. For \(M_a = M_t\), for example, the divergence of the rotor velocity field is \(M_a^2 (\theta + 2(\partial u/\partial y))\) and thus of second order in the Mach number. Hence only the term
\[
\rho_0 \frac{\partial u_i}{\partial x_i} \frac{\partial u_j}{\partial x_j} \frac{\partial^2}{\partial x_i \partial x_j} (\rho_0 u_i u_j)
\]
may be expected to contribute significantly.

Finally the quadrupole source is an extended source subject to extensive phase cancellation while the dipole source is generally assumed to be compact (as is the case in the present model). In this regard it is interesting to note that the authors of (6) considered only the potential flow field upstream of the rotor plane as contributing to the upstream noise and it is possible that such an approach will involve less phase cancellation.

Rao and Chu (8) concur with the results of the current calculations in obtaining estimates of the ratio of dipole/quadrupole noise of the order of 20 dB. Morfey (7) has calculated noise from the axial quadrupole \((\rho_0 u_p u_s)\) but it is difficult to see why he did not include the contribution of the x-y quadrupole \((\rho_0 v_p u_s)\). Thus the term that Morfey calculates is
\[ \rho_0 \frac{\partial^2}{\partial x^2}(u_p u_s) \]. Actually if one includes the term \( \frac{\partial^2}{\partial x \partial y}(\rho_0 \nu_p u_s) \) the quadrupole noise source term can be written as

\[ \rho_0 \frac{\partial u_s}{\partial y} \frac{\partial v}{\partial x} + \rho_0 u_s \frac{\partial \theta}{\partial x} \]

where \( \theta = \) divergence of rotor potential flow field = \( \frac{\partial u_p}{\partial x} + \frac{\partial v}{\partial y} \). For the low Mach number applications considered by Morfey (he has \( 0 < M_s < M_a < 0.7 \)) and especially for the Mach numbers \( M \approx 0.25 \) where he claims the quadrupole mechanism overtakes the dipole mechanism, \( \theta_p \) was probably negligible and thus the term he should have dealt with was really

\[ \rho_0 \frac{\partial u_s}{\partial x} \frac{\partial v}{\partial x} \]. The term that he really employed viz. \( \rho_0 \frac{\partial^2}{\partial x^2}(u_p u_s) \) is actually more or less cancelled by a part of \( \frac{\partial^2}{\partial x \partial y}(\rho_0 \nu_p u_s) \). Thus a serious question arises with regard to the correctness of his relative estimates of the quadrupole/dipole mechanism.

We are still left however with the question of the total failure of the present results in Figures 8, 9 to agree with the data of (5). Two remarks are in order here. First of all, absolute prediction is really much less of a goal than the prediction of relative trends. This is because absolute prediction requires a whole host of good inputs other than the acoustic data such as distortion, turbulence data which have only been estimated in this case. Paradoxically of course the present predictions have done somewhat better on absolute prediction than on the relative trends. We notice consistently from Figures 8, 9 that if the dipole-quadrupole levels were more closely matched, the relative trends in the data would be much better explained. In other words, the actual data in Figures 8, 9 often exhibit trends intermediate between those of the dipole and quadrupole noise trends considered separately. Needless to say, since the dipole noise is predicted to be so much higher than the quadrupole source, any attempt to simply add them algebraically would of course fail to explain the data trends. This brings us to the second remark. It appears that the present calculations may be considerably overestimating the ratio of dipole/quadrupole energy. The quadrupole noise calculation is, in many respects, the more exact calculation. The dipole noise estimate employs the Sears gust formula and there are indeed several reasons for believing that it leads to overestimation of the unsteady force. Note that the Sears gust formula applies only to plane, incompressible flows involving isolated, flat plate airfoils at zero angle of attack. Firstly accounting for longitudinal or chordwise gusts generally leads to reduced values of gust loads (7). For inlet distortion noise, the reduction may be estimated as \( (\tan(\alpha_p) - C_L/\pi)/\tan(\alpha_p) \). Secondly suggestions have been made that the Sears function could be revised to accommodate real fluid effects by multiplying it by the steady value of \( (dC_L/da) \) at finite loading to the steady value of \( (dC_L/da) \) at zero loading. This correction would have to be determined experimentally but \( (dC_L/da) \) generally decreases with increasing loading (or \( C_L \)) becoming zero before the airfoil stalls. Similarly inclusion of finite aspect ratio effects or of gust obliquity would also lead to...
lowered estimates of unsteady loads (see (6) for suggestions on including the aspect ratio effect - the suggested procedure is very similar to the one concerning real fluid effects). Finally compressibility and cascade effects also generally tend to lower estimates of the gust loads. The work of Timman (10) on compressibility effects at high Mach numbers for isolated airfoils does predict a reduced gust load (at fixed reduced frequency). The source compactness assumed in the present calculations for dipole noise may again be a source of overestimation in view of the tendency of phase cancellation to occur with a distributed source.

One final remark with regard to Morfey's study (7) is that by choice of a distortion with period equal to the blade spacing (one that is not likely to occur in practice), he starts, a priori, with a situation of low gust loads due to the very high reduced frequencies involved. Also he considered a rotor of unusually low tip solidity = 0.5, a model which again inherently favors the quadrupole mechanism.

CONCLUSIONS

1. A systematic theory of quadrupole noise generation in fans/compressors has been worked out both for inlet distortion and inlet turbulence - rotor noise. The calculations along with previously developed analyses of dipole noise due to these agencies have been coded into working programs included in the present report.

2. Parametric calculations with hypothetical constraints indicate that:

   a) At any fixed operating point, the ratio quadrupole/dipole noise increases with frequency whether one considers inlet turbulence or inlet distortion to be the source of noise.

   b) Along a fixed operating line, i.e. for fixed \( \left( \frac{M_a}{M_t} \right) \) and for given \( C_L \), the ratio of quadrupole to dipole noise increases with tip Mach number.

   c) At fixed \( M_a \) and given total pressure ratio, the variation of ratio of quadrupole to dipole noise with tip Mach number is not very simple. This is because with increasing tip speed and fixed pressure ratio, the loading (or \( C_L \)) decreases with tip speed but the frequencies increase. These features are illustrated in Figure 5 where the ratio of quadrupole to dipole noise is minimum in Figure 5(c) at \( M_t = .6 \).

   d) The ratio of quadrupole/dipole noise is largely a function of \( M_a, M_t \), rotor solidity and pressure ratio (or \( C_L \)) and largely independent of the specific form of the distortion or turbulence (within broad limits).

   e) The analysis shows the quadrupole noise to be independent of rotor solidity (at fixed \( M_a, M_t \) and total pressure ratio) but dipole noise increases with rotor solidity and hence the ratio of quadrupole to dipole noise decreases with increasing rotor solidity.

3. No general support is obtained for the idea that for given total pressure ratio, fan noise can be lowered by operating at high tip speed and low loading based on present calculations (see Figure 5).
4. Comparisons with data from two rotors as reported in (5) reveal that the analysis does fairly well in predicting the gross PWL levels at given speeds but fails substantially in predicting the trends of the variation of PWL at fixed speed with flow and pressure ratio. When this deficiency is examined in detail, it is found that if one looks at the separate theoretical predictions of the trends of variation of PWL due to dipole and quadrupole mechanisms, the data of (5) often exhibit trends intermediate to those of the dipole and quadrupole predictions. However, the theory consistently places the dipole noise levels at values much higher than the quadrupole levels for the design parameters of the two rotors studied in (5) and hence a composite of the two theoretical predictions does not exhibit the trend of the real data of (5) at all. A possible inference from this is that the ratio of dipole/quadrupole noise is overestimated by the theory and it is argued that this may be chiefly due to overestimate of the gust loads by relying on the Sears gust formula to predict them. Modifications to the gust formula to allow for real fluid, chordwise gust, aspect ratio, spanwise variation, compressibility and cascade effects are all suggested refinements likely to bring the predicted levels of dipole and quadrupole noise much closer together. If this happens, it is possible that a composite of the two will result in predicted trends in much better agreement with the data of (5).
APPENDIX 1

COMPUTER PROGRAMS FOR CALCULATION OF DIPOLE AND QUADRUPOLE NOISE DUE TO INLET DISTORTION OR TURBULENCE

INLET DISTORTION NOISE

Inputs needed are:

TPD  total pressure defect, maximum value of \((\Delta p_t)/p_t\).
IS   number of lobes in distortion.
EMA  \(M_a\)
EMT  \(M_t\)
NDP  number of points used to specify the distortion shape (see Figure A1), should be less than 51.
GAM  \(\gamma\)
RHØ  gas density in lbm/cft.
C    speed of sound in fps.
A    area of annulus being studied in square feet.
N    harmonic of blade passing noise of interest.
IB   number of rotor blades.
SIGR \(\sigma\)
TPR  \((p_{02}/p_{01})\)
TPDS(I) NDP values specifying relative shape of total pressure distortion (max. value would be unity: \(0 \leq TPDS(I) \leq 1\)). See Figure A1.

Outputs given are:

1) All inputs except NDP, TPDS(I).

2) Sound power in dB re: \(10^{-13}\) watts in each lobe number \(M\) where \(M = N \cdot IB - I \cdot (IS)\). \(M, I\) are also printed. The dipole and quadrupole noise upstream and downstream are given as PUDDB, PDDDB, PUQDB, PDQDB. The sum over all propagating \(M\) is given as

SPUDDB (dipole noise upstream),
SPDDDB (dipole noise downstream),
SPUQDB (quadrupole noise upstream) and
SPDQDB (quadrupole noise downstream).
APPENDIX 1

INLET DISTORTION NOISE

Inputs needed are:

TPD \hspace{1cm} \text{total pressure defect, maximum value of } (\Delta p_t)/p_t\text{.}

IS \hspace{1cm} \text{number of lobes in distortion.}

EMA \hspace{1cm} M_a

EMT \hspace{1cm} M_t

NDP \hspace{1cm} \text{number of points used to specify the distortion shape (see Figure A1), should be less than 51.}

GAM \hspace{1cm} \gamma

RHØ \hspace{1cm} \text{gas density in lbm/ft}^3.

C \hspace{1cm} \text{speed of sound in fps.}

A \hspace{1cm} \text{area of annulus being studied in square feet.}

N \hspace{1cm} \text{harmonic of blade passing noise of interest.}

IB \hspace{1cm} \text{number of rotor blades.}

SIGR \hspace{1cm} \sigma

TPR \hspace{1cm} (p_{02}/p_{01})

TPDS(I) \hspace{1cm} \text{NDP values specifying relative shape of total pressure distortion (max. value would be unity: } 0 \leq \text{TPDS(I)} \leq 1\text{). See Figure A1.}

Outputs given are:

1) All inputs except NDP, TPDS(I).

2) Sound power in dB re: $10^{-13}$ watts in each lobe number $M$ where $M = N \cdot IB - I \cdot (IS)$. $M$, $I$ are also printed. The dipole and quadrupole noise upstream and downstream are given as PUUDB, PDDDB, PUQDB, PDQDB. The sum over all propagating $M$ is given as

SPUDDB \hspace{1cm} \text{(dipole noise upstream)}

SPDDB \hspace{1cm} \text{(dipole noise downstream)}

SPUQDB \hspace{1cm} \text{(quadrupole noise upstream)}

SPDQDB \hspace{1cm} \text{(quadrupole noise downstream).}
INLET TURBULENCE NOISE

Inputs needed are:

- \( EMA \)  
- \( EMT \)
- \( TPR \)  
- \( NELMAX \) number of \((L/D)\) needed to be investigated.
- \( M \) number of frequencies at which \( f \frac{dI}{df} \) is desired.
- \( TI \) turbulence intensity \(<u'^2> \div U^2\)
- \( GAM \) \( \gamma \)
- \( RH\) \( \rho \) in lbm/cft.
- \( CS \) speed of sound in fps.
- \( AI \) cross sectional area of annulus of interest, sq. ft.
- \( SIGR \) \( \sigma \)

\[ F(I)(I = 1, M): M \text{ values of frequency expressed as } \left(\frac{f}{f_b}\right) \]
where \( f_b \) = blade passing frequency.

\[ ELIN(I)(I = 1, NELMAX): NELMAX \text{ values of } (L/D) \text{ of interest.} \]

Outputs are:

1) All inputs except \( NELMAX \).

2) For each \( L/D \) (written as \( EL \)) and \( F \), 10 log \( 10 \left[ f \frac{dI}{df} \div \frac{5}{2} \rho U<u'^2>\right] \)
for both dipole and quadrupole sources as \( SDVDBR \), \( SDDDBR \), \( SQVDBR \), \( SQDDBR \). Also sum of dipole and quadrupole contributions to \( f \frac{dI}{df} \), upstream and downstream re: \( 10^{-13} \) watts as \( PVDB \) and \( PDDB \).
INLET TURBULENCE NOISE

09999 FILENAME OUTPUT
10000 DIMENSION AEW(46), AED(46), ZMM(46), ZPP(46), DCV1(91), DCD1(91),
       & F(20), DCV2(91), DCD2(91), STH0SR(91), AEWLOC(2, 91),
10002 & AEDET(2, 91), ELIN(25)
10010 OUTPUT="ITOUT"
10020 BEGIN FILE OUTPUT
10030 END FILE OUTPUT
10100 100 FORMAT(V)
10200 READ ("ITINPUT", 100) LN, EMA, EMT, TPR
10205 READ ("ITINPUT", 100) LN, NELMAX, M, TI
10210 READ ("ITINPUT", 100) LN, GAM, RH0, CS, AI, SL GR
10220 IBEG=1
10230 IEND=10
10240 240 IF (IEND .GT. M) IEND=M
10250 READ ("ITINPUT", 100) LN, (F(I), I=1 BEG, I END)
10260 IF (IEND .EQ. M) GOTO 300
10270 IBEG=1 IEND=50
10280 IEND=IEND 10
10290 GOTO 240
10300 300 IBEG=1
10310 IEND=10
10320 320 IF (IEND .GT. NELMAX) IEND=NELMAX
10330 READ ("ITINPUT", 100) LN, (ELIN(I), I=1 BEG, I END)
10340 IF (IEND .EQ. NELMAX) GOTO 500
10350 IBEG=1 IEND=IEND 10
10360 IEND=IEND 10
10370 GOTO 320
10500 500 WRITE (OUTPUT, 510)
10510 510 FORMAT(/32H EMA EMT M TI)
10520 WRITE (OUTPUT, 530) EMA, EMT, M, TI
10530 530 FORMAT(2F9.3, I6, F9.2)
10550 550 FORMAT(/53H GAM RH0 C A SL GR
10551 & TPR)
10560 WRITE (OUTPUT, 570) GAM, RH0, C, A, SI GR, TPR
10600 600 DBL=130.4*3.42945*AL0G*(105*RH0*(C*EMA)**3*TI**2*A)
10610 PI=3.1415926
10615 TPI=2.*PI
10620 G10V2=(GAM-1.)/2.
10630 G10VG=(GAM-1.)/GAM
10640 EMR=SQRT(EMA**2+EMT**2)
10650 T11=TPR**G10VG-1.
10660 T12=1.+1./(G10V2*EMA**2)
10670 SR1MM2=SQRT(1.-EMA**2)
10680 SR1MR2=SQRT(1.-EMR**2)
10690 CR=EMA/EMR
10695 CR2=CR*CR
10700 SR=EMT/EMR
10705 SR2=SR*SR
10710 SRCR=SR*CR

21
10715 TR=EMT/EMA
10720 EMRC=EM/(S.*CR)
10730 CL=CR²*T12*T11/(SI GR*SR)
10740 AA=1.*-EMA**2
10750 A=CL*SI GR*EMA*EMT/((A.*AA)
10760 B=CL*SI GR*SR*MP2/(4.*AA)
10770 C=CL*SI GR/4.
10780 ALG=TP1*SR*MP2
10790 BETC=TP1
10800 CHIC=BETC*EMT
10810 DELC=TP1*EMA*EMT
10840 CDP=PI*SIG/R/2.
10845 CDP²=CDP*CDP
10850 CAE=1./(EMA*SR*MM2)
10900 NINC02=10
10910 NINC=2*NINC02
10920 FNINC=NINC
10930 DELTH=PI/FNINC
10940 THETA=PI/2.
10950 IMAX=NINC+1
1100 D0 1230 I =1,IMAX
11010 CTH=COS(THETA)
11020 EMAMC=EMA-CTH
11030 EMAPC=EMA+CTH
11040 IF (I .GT. (NINC02+1)) G0 T0 1090
11050 ZPP(I)=EMAMC
11060 ZPP(I)=EMAPC
11070 AEV(I)=CAE/(1.+EMA*CTH)**2
11080 AED(I)=CAE/(1.-EMA*CTH)**2
11090 1090 STH=SIN(THETA)
11100 TERM=SR/MM2*STH*CR
11110 DCV1(I)=(EMAPC*SR-TERM)**2
11120 DCV2(I)=(EMAPC*SR+TERM)**2
11130 DCD1(I)=(EMAPC*SR-TERM)**2
11140 DCD2(I)=(EMAPC*SR+TERM)**2
11150 STH0SR(I)=STH/SR/MM2
11160 INDEX=I
11170 IF (I .GT. (NINC02+1)) INDEX=IMAX+1-I
11180 AEVETC(I,1)=AEV(INDEX)*CDP²*DCV1(I)
11190 AEVETC(2,I)=AEV(INDEX)*CDP²*DCV2(I)
11200 AEDETC(I,1)=AED(INDEX)*CDP²*DCD1(I)
11210 AEDETC(2,I)=AED(INDEX)*CDP²*DCD2(I)
11220 THETA=THETA+DELTH
11230 1230 CONTINUE
11250 D0 3010 NEL=1,NELMAX
11255 EL=ELIN(NEL)
11260 WRITE (OUTPUT,1261) EL
11261 1261 FORMAT(/11H ***** EL=1P1E12.3,6H *****)
11270 CPHI=1./(4.*PI*EL)
11275 ELC=1./(EL*EL)
11300 D0 3000 J=1,M
11310 WRITE (OUTPUT,1320) F(J)
11320 1320 FORMAT(/7H F(J)=E12.3)
\[ CHI = F(J) \times CHIC \]
\[ CHI2 = CHI \times CHI \]
\[ SDV = 0. \]
\[ SDD = 0. \]
\[ SQV = 0. \]
\[ SQD = 0. \]
\[ NVAL = 8 \]
\[ NVALP1 = NVAL + 1 \]
\[ D0 2800 NN = 1, NVALP1 \]
\[ N = NN - 1 \]
\[ IF (N) 1530, 1550, 1570 \]
\[ 1530 1530 SGN = -1. \]
\[ 1540 G0 T0 1580 \]
\[ 1550 1550 SGN = 0. \]
\[ 1560 G0 T0 1580 \]
\[ 1570 1570 SGN = 1. \]
\[ 1580 1580 EN = N \]
\[ 1590 AL = ABS(EN) \times ALC \]
\[ 1595 AL2 = AL \times AL \]
\[ 1600 ALSGNB = AL \times SGN \times B \]
\[ 1605 BET = EN \times BETC \]
\[ 1610 DEL = DELC \times EN \]
\[ 1630 SUMDV = 0. \]
\[ 1640 SUMDD = 0. \]
\[ 1650 SUMQV = 0. \]
\[ 1660 SUMQD = 0. \]
\[ 1670 THETA = -\pi / 2. \]
\[ 1800 D0 2600 I = 1, IMAX \]
\[ 1805 AKX = BET \times TR - CHI / EMA \]
\[ 1810 AKY = -BET + CHI \times STH0SR(I) \]
\[ 1820 AKY2 = AKY \times AKY \]
\[ 1830 INDEX = I \]
\[ 1840 IF (I GT (NINC02 + 1)) INDEX = IMAX + 1 - I \]
\[ 1850 ZM = ZMM(INDEX) \times CHI \]
\[ 1860 ZP = ZPP(INDEX) \times CHI \]
\[ 1900 D0 2310 L1 = 1, 2 \]
\[ 1910 AKX2 = AKX \times AKX \]
\[ 1920 ELAKX2 = ELC \times AKX2 \]
\[ 1930 CPHI1 = CPHI / (ELAKX2 + AKY2) ** 2.5 \]
\[ 1940 PHIXX = (4 * AKY2 + ELAKX2) \times CPHI1 \]
\[ 1950 PHIXY = -3 * AKY \times AKX \times CPHI1 \]
\[ 1960 PHIYY = (4 * AKX2 + ELC + AKY2) \times CPHI1 \]
\[ 1965 TPHIXY = 2 * PHIXY \]
\[ 1970 \Delta MR = SI GR \times (AKX \times CR + AKY \times SR) / 2. \]
\[ 1980 SRF = 1. / (1 + TPI \times ABS(\Delta MR)) \]
\[ 1990 PHIT = PHIXX \times SR2 - TPHIXY \times SRCR + PHIYY \times CR2 \]
\[ 1995 PHITS = PHIT \times SRF \]
\[ 2000 IF (L1 EQ. 2) G0 T0 2040 \]
\[ 2010 FNDVP = AEVETC(1, I) \times PHITS \]
\[ 2020 FNDDP = AEDETC(1, I) \times PHITS \]
\[ 2030 G0 T0 2060 \]
\[ 2040 2040 FNDVM = AEVETC(2, I) \times PHITS \]
\[ 2050 FNDDM = AEDETC(2, I) \times PHITS \]
12060  2060  ZTERM=ZF
12070  AKXA=AKX*AA
12080  D0  2200  L2=1,2
12090  ZDELAK=ZTERM-DEL-AKXA
12100  DEN=(AL2+ZDELAK**2)**2
12110  PART=ZTERM*(ZDELAK*A-AL SGNE)+C*ZDELAK*CHI*STH0 SR(I)*AA
12120  GX=ZTERM*PART
12130  GY=CHI*STH0 SR(I)*PART*AA
12140  TQ=(GX**2*PHIXX+GY**2*PHIYY+GX*GY*TPHIXY)/DEN
12150  IF (L2. EQ.  2) G0 T0 2180
12160  FNQV=TQ*AEV(INDEX)
12170  G0 T0 2190
12180  2180 FNQD=TQ*AES(INDEX)
12190  2190 ZTERM=ZM
12200  2200 CONTINUE
12210  IF (L1 . EQ.  2) G0 T0 2250
12220  FNQVP=FNQV
12230  FNQDP=FNQD
12232  AKX=BET*TR+CHI/EMA
12234  ZP=-ZP
12236  ZM=-ZM
12240  G0 T0 2310
12250  2250 FNQVM=FNQV
12260  FNQDM=FNQD
12300  2310 CONTINUE
12400  FDV=FNQDP+FNDVM
12410  FDD=FNDDP+FNDDM
12420  FQV=FNQVP+FNQVM
12430  FQD=FNQDP+FNQDM
12440  IF (((I . NE. 1) .AND. (I . NE. IMAX)) G0 T0 2490
12450  FDV=FDV/2.
12460  FDD=FDD/2.
12470  FQV=FQV/2.
12480  FQD=FQD/2.
12490  2490 SUMDV=SUMDV+FDV
12500  SUMDD=SUMDD+FDD
12510  SUMQV=SUMQV+FQV
12520  SUMQD=SUMQD+FQD
12530  THETA=THETA+DELTH
12600  2600 CONTINUE
12610  SNDV=CHI*SUMDV*DELTH
12620  SNDD=CHI*SUMDD*DELTH
12630  SNQV=SUMQV*DELTH
12640  SNQD=SUMQD*DELTH
12650  IF (N . GT.  0) G0 T0 2750
12700  SDV=SDV+SNDV
12710  SDD=SDD+SNDD
12720  SQV=SQV+SNQV
12730  SQD=SQD+SNQD
12740  G0 T0 2800
12750  2750 SDV=SDV+2.*SNDV
12760  SDD=SDD+2.*SNDD
12770  SQV=SQV+2.*SNQV
12780  SQD=SQD+2.*SNQD
12800 2800 CONTINUE
12802  SDV=SDV*EMRC
12803  SDD=SDD*EMRC
12804  SQV=SQV*EMRC
12805  SQD=SQD*EMRC
12810  SDVDBR=4.342945*ALOG(SDV)
12820  SDDDBR=4.342945*ALOG(SDD)
12830  SQVDBR=4.342945*ALOG(SQV)
12840  SQDDBR=4.342945*ALOG(SQD)
12850  PV=SDV+SQV
12860  PD=SDD+SQD
12870  PVDB=4.342945*ALOG(PV)+DBL
12880  PDDB=4.342945*ALOG(PD)+DBL
12900  WRITE (OUTPUT, 2910)
12910  2910 FORMAT(/40H  SDVDBR  SDDDBR  SQVDBR  SQDDBR)
12920  WRITE (OUTPUT, 2930) SDVDBR, SDDDBR, SQVDBR, SQDDBR
12930  2930 FORMAT(/F10.1)
12940  WRITE (OUTPUT, 2950)
12950  2950 FORMAT(/20H  PVDB  PDDB)
12960  WRITE (OUTPUT, 2970) PVDB, PDDB
12970  2970 FORMAT(2F10.1)
13000  3000 CONTINUE
13010  3010 CONTINUE
13100  STOP
13110  END
INLET DISTORTION NOISE

FILENAME OUTPUT
0990 FORMAT (V)
1000 DIMENSION SIGN(2), ETA(2), TPD(S), DELM(50), PD(2), PQ(2)
1010 OUTPUT="IDOUT"
1010 BEGIN FILE OUTPUT
1030 END FILE OUTPUT
1040 FORMAT(V)
1050 READ ("IDINPUT", 100) LN, TPD, IS, EMA, EMT, NDP
1020 READ ("IDINPUT", 100) LN, GAM, PHI, C, A, N, IB, SIGR, TPR
1020 IBEG=2
1020 IEND=11
1020 IBEG=I END+10
1020 G0 T0 500
1050 500 S=15
1050 EN=N
1050 B=IB
1050 WRITE (OUTPUT, 610)
1060 FORMAT(///36H TPD IS EMA EMT)
1070 WRITE (OUTPUT, 630) TPD, IS, EMA, EMT
1080 FORMAT(F10.3, I7/2F10.3)
1090 WRITE (OUTPUT, 650)
1050 FORMAT(///65H GAM PHI C A N IB
1065 & SIGR TPR)
1060 WRITE (OUTPUT, 665) GAM, PHI, C, A, N, IB, SIGR, TPR
1060 PI=3.1415926
1060 IMAX=500
1060 JMAX=NDP+2
1060 JMAX1=JMAX-1
1060 TPD(S(1))=0.
1060 TPD(S(JMAX))=0.
1060 DELM(1)=0.
1060 DELM(IMAX)=0.
1060 G10V2=(GAM-1.)/2.
1060 G10VG=(GAM-1.)/GAM
1070 T1=1.+G10V2*EMA**2
1070 T11=TPR**G10VG-1.
1070 T12=1.+1./(G10V2*EMA**2)
1070 D0 800 I=2,IMAX1
1070 FI=I
1070 TERM1=(FI-1.)/FIMAX1
1070 J1=TERM1*FIMAX1+1.
1070 J2=J1+1
1070 FJ1=J1
1070 FJ2=J2
1070 TERMJ1=(FJ1-1.)/FJMAX1
1070 TERMJ2=(FJ2-1.)/FJMAX1
10778 TPDI = TPDS(J1) + (TPDS(J2) - TPDS(J1)) * (TERM1 - TERMJ1) / (TERMJ2 - TERMJ1)
10780 TPDD = TPDI * TPDD
10785 T2 = (TPD + TPDD) * GIO VG
10790 DELM1 = EMA - SQRT((T1 * T2 - 1.) / GIO V2)
10800 800 CONTINUE
10810 EMR = SQRT(EMA**2 + EMT**2)
10820 SR1MM2 = SORT(1. - EMA**2)
10830 SR1MR2 = SORT(1. - EMT**2)
10840 IMIN = (1. - EMT / SR1MM2) * ENB / S + .999
10850 IMAX = (1. + EMT / SR1MM2) * ENB / S
10860 ALSDR = ATAN(EMT / EMA)
10870 VR = C * EMA / COS(ALSRR)
10880 CONST = (VR * VR * SIGR / 2.)**2 * RH0 * 746. / (C*32. * 550.)
10890 SR = SIN(ALSRR)
10900 CX1 = SR
10910 TXI = COS(ALSRR) / SR
10920 CLT = C0S(ALSRR) ** 2 * T12 * T11 / (SIGR * SR)
10930 AA = 1. - EMA**2
10940 AP = CLT * EMA * EMT / AA
10950 BP = CLT * SR1MR2 / AA
10955 CP = CLT
10960 BB = ENB * EMT
10970 BB2 = BB * BB
10980 CC = EMA * BB
10990 DD = EMA / BB
11000 ALPHA = ENB * BR1MR2 / AA
11010 SPUD = 0.
11020 SPDD = 0.
11030 SPUD = 0.
11040 SPDQ = 0.
11050 SIGN(1) = 1.
11060 SIGN(2) = -1.
11070 CRN = PI * SR * SIGR * S / B
11080 CCLC = PI * SIN(2. * ALSRR) / EMA
11090 WRITE (OUTPUT, 1800)
11100 D0 1900 I = IMIN, IMAX
11110 FI = I
11120 M = N * I - I * S
11130 EM = M
11140 DELTH = 2. * PI / (S * Fimax1)
11150 THETA = DELTH
11160 SUM1 = 0.
11170 SUM2 = 0.
11180 DO 400 K = 2, IMAX1
11190 TERM = FI * S * THETA
11200 F1 = DELM(K) * COS(TERM)
11210 F2 = DELM(K) * SIN(TEPM)
11220 SUM1 = SUM1 + F1
11230 SUM2 = SUM2 + F2
11240 THETA = THETA + DELTH
11250 400 CONTINUE
11260 TER1 = DELTH * S / (2. * PI)
11270 CIR = TERR1 * SUM1
CII=TERM1*SUM2
CI=SQR(T(CIR**2+CII**2)
DEL=SQR(T(BB2-AA*EM**2)
T0PD=(CXI/DEL)*CCLC*CI
EMTXI=EM*TXI
T0 PQ=CI/((ALPH**2+(DEL/AA)**2)*EMR)
0M=FI*0M
APP=1811
SEARS=SQR(T((APP+0M)/(APP+(PI*APP+1.)*0M+2.*PI*0M*0M))
D0 1700 K=1,2
CAY=(SIGN(K)*CC+DEL)/AA
A0VFQ=0T PD*CAY*(1+EMTXI*SIGN(K)/CAY)*SEARS
F1=CAY/SORT(CAY**2+EM**2)
F2=1.+SIGN(K)*CAY*DD
ETA(K)=5*(F1-SIGN(K)*EMK)/F2
A0VFQ=CAY*(AP*DEL*CAY/AA-ALPH*BP*CAY*SIGN(K)+CP*SIGN(K)*DEL*
& EM/AA+T0 PQ/DEL
PD(K)=ETA(K)*A0VFQ**2
PQ(K)=ETA(K)*A0VFQ**2
1700 1700 CONTINUE
1710 PUD=PD(1)*CONST
1720 PDD=PD(2)*CONST
1730 PU0=P0(1)*CONST
1740 PDQ=PQ(2)*CONST
1750 PUDDB=130.+4.342945*ALOG(PUD)
1760 PDDDB=130.+4.342945*ALOG(PDD)
1770 PUQDB=130.+4.342945*ALOG(PUQ)
1780 PDQDB=130.+4.342945*ALOG(PDQ)
1800 1800 FORMAT (/50H I M PUDDB PDDDB PUQDB PDQDB)
1801 & PDQDB)
1810 WRITE (OUTPUT, 1820) I,M,PUDDB,PDDBB,PUQDB,PDQDB
1820 1820 FORMAT (2I5/4F10.1)
1830 SPUD=SPUD+PUD
1840 SPDD=SPDD+PDD
1850 SPQ=SPQ+PUQ
1860 SPDQ=SPDQ+PDQ
1890 1900 CONTINUE
1910 SPUDDB=130.+4.342945*ALOG(SPUD)
1920 SPDDDB=130.+4.342945*ALOG(SPDD)
1930 SPQDB=130.+4.342945*ALOG(SPUQ)
1940 SPQDB=130.+4.342945*ALOG(SPDQ)
1950 WRITE (OUTPUT, 1960)
1960 1960 FORMAT (/4H SPUDDB SPQDBB SPQDBB SPQDBB SPQDBB)
1970 WRITE (OUTPUT, 1980) SPUDDB,SPQDBB,SPQDBB,SPQDBB,SPQDBB
1980 1980 FORMAT (4F10.1)
20000 ST0P
20010 END
LIST OF SYMBOLS

- $a$, speed of sound.
- $a$, sometimes used to denote radius of fan annulus being studied.
- $B$, number of blades in the rotor.
- $C_L$, lift coefficient of the rotor.
- $C$, rotor chord.
- $C_p$, specific heat at constant pressure.
- $C_i$, coefficients of inlet distortion.
- $d$, pitch of rotor.
- $j$, integral length scale of turbulence/rotor pitch.
- $L/D$, integral length scale of turbulence/rotor pitch.
- $M_a$, $M_r$, and $M_t$, axial, relative and wheel tip Mach numbers.
- $p$, pressure.
- $p_0$, $p_0'$, local total pressure.
- $R$, gas constant.
- $S$, number of lobes in distortion.
- $T_0$, $T_0'$, $T_1$, total temperature upstream and downstream of rotor, temperature ahead of rotor.
- $U$, axial velocity entering rotor.
- $V$, wheel velocity.
- $W_r$, relative velocity through rotor.
- $\alpha_r$, stagger angle of rotor.
- $\delta_{ij}$, Kronecker delta.
- $\delta$, delta function.
- $\rho$, density.
- $\sigma$, rotor solidity.
REFERENCES


FIGURE 1

CALCULATION OF ROTOR POTENTIAL FLOW FIELD
\[ p_t(y) \]

\[ 2\pi a \quad a = \text{mean radius of annulus} \]

**FIGURE 2**

VARIATION OF TOTAL PRESSURE AROUND THE ANNULUS
FIGURE 3. INLET TURBULENCE - ROTOR NOISE

- DIPOLE NOISE
- QUADRUPOLE NOISE

\[ f_b \]: blade passing frequency

\[ C_L = 1.00, \sigma = 1.00 \]

\[ L/D = 1.8 \]

\[ 10 \log_{10} \left( \frac{d}{dP} \right) \frac{1}{5} \left( \frac{U}{w} \right)^2 \]

\[ .2 \ 4 \ 1 \ 2 \ 3 \ f/f_b \]

\[ M_a = 0.2, M_t = 0.3 \]

\[ .2 \ 4 \ 1 \ 2 \ 3 \ f/f_b \]

\[ M_a = 0.4, M_t = 0.6 \]

Rest as in Fig. 3(a)
$M_a = 0.55, M_t = 0.825$, Rest as in Figure 3(a)
FIGURE 4. INLET TURBULENCE - ROTOR NOISE (Contd.)

\[ \sigma = 1.00, \ CL = 0.5, \ L/D = 0.5 \]

(a) \[ M_a = 0.2, \ M_t = 0.4 \]

(b) \[ M_a = 0.35, \ M_t = 0.7, \ \text{Rest as in 4(a)} \]
FIGURE 4 (Concluded)

\[ M_a = 0.425, M_t = 0.85, \text{ Rest as in 4(a)} \]

\[ M_a = 0.7, M_t = 0.7, \text{ Rest as in 4(a)} \]
FIGURE 5. EFFECT OF TIP SPEED AT CONSTANT PRESSURE RATIO

- - - - - - - DIPOLE
- - - - - - - QUADRUPOLE

\[ M_t = 0.4, \sigma = 1.00, \text{ pressure ratio } = 1.4, \]
\[ L/D = 1.8 \]

\[ 10 \log_{10} \left[ \frac{5}{2} \frac{d}{df} \rho_0 \langle U^2 \rangle \right] \]

\[ .2 \quad .4 \quad 1 \quad 2 \quad 3 \quad f/f_b \]

(a) \quad M_t = 0.4

\[ .2 \quad .4 \quad 1 \quad 2 \quad 3 \quad f/f_b \]

(b) \quad M_t = 0.5, \text{ Rest as in 5(a)}
\( M_t = 0.6 \), Rest as in 5(a)

\( M_t = 0.9 \), Rest as in 5(a)
\[ \Delta p_t / p_t = 0.02, M_a = 0.4, B = 38, S = 4, \sigma = 1.00, \]
\[ C_L = 1.00 \]

\[ M_a = \frac{2}{3} M_t, \Delta p_t / p_t = 0.02, C_L = 1.00, S = 4, \]
\[ B = 38, \sigma = 1.00 \]
FIGURE 8. COMPARISON WITH (5): ROTOR 1.
\[ \frac{\Delta p_t}{(\Delta p_t)_{\text{max}}} = \text{TPDS} \]

**FIGURE A1**

DEFINITION SKETCH FOR INPUT FUNCTION TPDS
"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."
—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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