ISOLATED ROTOR NOISE DUE TO INLET DISTORTION OR TURBULENCE

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The report presents the theoretical formulation, analysis, and results necessary to analyze quadrupole noise generated from a loaded, isolated, subsonic rotor because of its interaction with an inflow distortion or inlet turbulence. The ratio of quadrupole to dipole noise is largely a function of the axial flow Mach number, wheel tip Mach number, rotor solidity, and total pressure ratio across the rotor. It is relatively independent of the specific form of the inflow distortion or inlet turbulence. Comparisons with experimental data only succeed in predicting gross levels at a given speed and fail to predict the variation of noise at fixed speed with flow and pressure ratio. Likely sources of this discrepancy are discussed.
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SUMMARY

The present report presents the theoretical formulation, analysis and results necessary to analyze quadrupole noise generated from a loaded, isolated, subsonic rotor due to its interaction with an inflow distortion or inlet turbulence. Programs to calculate these interactions are given in an appendix. These programs also yield dipole noise contributions from these two mechanisms. The ratio of quadrupole to dipole noise is largely a function of the axial flow Mach number, wheel tip Mach number, rotor solidity, total pressure ratio across the rotor. It is relatively independent of the specific form of the inflow distortion or inlet turbulence. Comparisons with experimental data only succeed in predicting gross levels at a given speed and fail to predict the variation of noise at fixed speed with flow and pressure ratio. Looking at the separate variations of the theoretical predictions of dipole and quadrupole noise, however, one notices that if these levels were closer together, a suitable composite of them would yield the trends of the experimental data. This leads to a suggestion that the calculation may be overestimating the ratio of dipole to quadrupole noise by overestimating the dipole noise. Likely sources of this overestimate are suggested along with corrective procedures.
INTRODUCTION

The purpose of the present study is to quantify a quadrupole noise source which was first proposed in (2) as a likely contributor to fan noise. The source arises from the fluctuating Reynolds stresses introduced by a combination of two unsteady velocities, one being due to the potential flow field of the rotor and the other due to a solenoidal velocity field such as an inlet distortion or inlet turbulence.

The study proceeds as follows. Since interest is in fan/compressor noise where substantial duct axial velocities are involved, a pertinent Lighthill equation including uniform axial flow is derived first for the study of quadrupole noise. Secondly the potential flow fields of a subsonic rotor are derived as a function of the pressure ratio across it, the wheel tip and axial flow Mach numbers. Thirdly a systematic approach to the acoustic problem of estimating the noise from this source is worked out based on repeated use of Fourier exponential transforms.

We next present calculations under several constraints of the relative magnitudes of the quadrupole versus the dipole noise source. Comparisons are also carried out with some available data on the influence of pressure ratio on fan noise. In an appendix, computer programs for calculations of quadrupole and dipole noise due to both inlet distortion and inlet turbulence are given.

Ivan H. Edelfelt of the General Electric Research and Development Center helped considerably in programming these calculations. Thomas F. Gelder and Marvin E. Goldstein of the NASA-Lewis Research Center provided several helpful discussions.
The full Lighthill equation for aerodynamic noise for an inviscid gas, in the absence of mass, energy or force sources (and assuming the isentropic relation \( p = a_0^2 \rho \)) may be written as:

\[
\frac{\partial^2 \rho}{\partial t^2} - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j} (\rho u_i u_j) \ldots \tag{1}
\]

Now let \( u_{ij} = U \delta_{ij} + u_{ij} \) where \( U \) is uniform and steady and \( 1 \) corresponds to the axial direction. Then we may show rigorously that (1) reduces to:

\[
\left[ \frac{\partial}{\partial t} + U \frac{\partial}{\partial x_i} \right]^2 \rho - a_0^2 \nabla^2 \rho = \frac{\partial^2}{\partial x_i \partial x_j} (\rho u_{ij} u_{ij}) \ldots \tag{2}
\]

Equation (2) is the desired form of the Lighthill equation for blade loading noise from an isolated rotor. The form of \( \rho u_{ij} u_{ij} \), correct to second order, would be \((\rho_0 u_i u_j)\). For an axial flow device, two important sources of velocity fluctuations are solenoidal velocity fluctuations associated with inlet distortion or inlet turbulence and secondly irrotational velocity fluctuations associated with the flow field of a loaded rotor. Thus we may write \( u_{ij} = u_{ij}s + u_{ij}p \) where \( u_{ij}s \) denotes the solenoidal velocity fluctuation associated with inlet distortion or turbulence and \( u_{ij}p \) that associated with the rotor potential flow field. The calculation of the quadrupole noise field due to interaction between the rotor potential flow field and an inlet distortion or turbulence field thus boils down to solving (2) with a source term of type,

\[
2 \frac{\partial^2}{\partial x_i \partial x_j} (\rho_0 u_{ip} u_{jp}).
\]

The factor of 2 arises from the symmetry of the quadrupole term with respect to interchange of \( i, j \).

**DETERMINATION OF FLUCTUATING VELOCITIES DUE TO THE ROTOR POTENTIAL FLOW FIELD**

For the determination of the fluctuating velocity field of the rotor, the most convenient quantity is the (nondimensional) lift coefficient of the steady lift exerted by the rotor blades on the fluid. However the quantities most conveniently available in practical terms are the pressure ratio across the rotor, its wheel tip and axial flow Mach numbers, etc. So we first evaluate \( C_L \) in terms of these quantities.

The work per lbm of the flow is

\[
C_L = \frac{\rho W_r^2 \cos(\alpha_r)V}{2p U d} = \frac{\rho C_L}{2} \frac{U^2 \sin(\alpha_r)}{\cos^2(\alpha_r)}
\]

\[
= C_p T_01 \left( \frac{T_{02}}{T_{01}} - 1 \right)
\]
\[
\frac{\gamma - 1}{(Y - 1)} = \frac{p_{02}}{p_{01}} \gamma - 1
\]

\[
\frac{\sigma C_L \sin(\alpha_r)}{\cos^2(\alpha_r)\left(\frac{2}{(Y - 1)M_r^2} + 1\right)} = \frac{p_{02}}{p_{01}} \gamma - 1
\]

This gives the desired expression for \(C_L\) as:

\[
C_L = \frac{\cos^2(\alpha_r)}{\sigma \sin(\alpha_r)} \left(1 + \frac{2}{(Y - 1)M_r^2}\right) \frac{p_{02}}{p_{01}} \gamma - 1
\]

To calculate the \(u', v'\) (x and y components of velocity) requires a linearized, compressible analysis of the flow field of the isolated rotor.

Let \(W_r = a M_r\). We first find the solution for velocity components \(u', v'\), parallel to the \(x' - y'\) coordinate system of Figure 1, due to equally spaced concentrated unit forces at the origin and its corresponding points as shown in Figure 1. (Note that the blade exerts a force on the fluid equal and opposite to the force by the fluid on the blade.) We use a frame of reference fixed w.r.t. the translating blade row so that we have a steady state problem. We have to consider the effect of a sum of forces:

\[
1 \sum_{n=-\infty}^{\infty} \delta(x') \delta[y' - 2n \frac{d}{2}]
\]

where \(1\) denotes a unit force vector, and \(\delta\) stands for the Dirac delta function. By using a result on page 68 of (1) concerning the sum of an infinite row of equally spaced delta functions, clearly the above is equal to:

\[
\frac{1}{d} \delta(x') \sum_{n=-\infty}^{\infty} \exp[j \frac{2\pi n y'}{d}]
\]

The linearized equations of motion and continuity are:

\[
\rho \left[ \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} \right] + W_r \left[ \cos(\alpha_r) \frac{\partial p'}{\partial x'} + \sin(\alpha_r) \frac{\partial p'}{\partial y'} \right] = 0
\]

\[
\cos(\alpha_r) \frac{\partial u'}{\partial x'} + \sin(\alpha_r) \frac{\partial u'}{\partial y'} = -\frac{1}{\rho W_r} \frac{\partial p'}{\partial x'} + \frac{\sin(\alpha_r) \delta(x')}{\rho d W_r}
\]

\[
\sum_{n=-\infty}^{\infty} \exp[j \frac{2\pi n y'}{d}] = 0
\]

and
\[
\cos(\alpha_r) \frac{\partial v'}{\partial x'} + \sin(\alpha_r) \frac{\partial v'}{\partial y'} = -\frac{1}{\rho W_r} \frac{\partial p'}{\partial y'}
\]

\[
-\cos(\alpha_r) \frac{\delta(x')}{\rho W_r} \int_0^\infty \exp(j \frac{2\pi ny'}{d}) \, dy'
\]

Eliminating \( p' \) from (1b, c) we derive that:

\[
[\cos(\alpha_r) \frac{\partial}{\partial x'} + \sin(\alpha_r) \frac{\partial}{\partial y'}] \left[ \frac{\partial u'}{\partial y'} - \frac{\partial v'}{\partial x'} \right] = -\frac{\delta(x')}{\rho W_r} \int_0^\infty \exp(j \frac{2\pi ny'}{d}) \, dy'
\]

and since \( u', v' \) and the delta function term vanish far from the blade row

\[
\frac{\partial u'}{\partial y'} - \frac{\partial v'}{\partial x'} = -\frac{\delta(x')}{\rho W_r} \int_0^\infty \exp(j \frac{2\pi ny'}{d}) \, dy'
\]

[Kutta Joukowski Law]

Next we eliminate the force terms in (1b, c) and assuming an isentropic relation between \( p' \) and \( \rho' \) one obtains:

\[
\frac{\partial u'}{\partial x'} [1 - M_r^2 \cos^2(\alpha_r)] + \frac{\partial v'}{\partial y'} [1 - M_r^2 \sin^2(\alpha_r)]
\]

\[
= M_r^2 \sin(\alpha_r) \cos(\alpha_r) \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'}
\]

(modified continuity equation)

Using (2), (3) single equations for \( u', v' \) may be obtained which may be solved by requiring that \( u', v' \) vanish as \( x' \to \pm \infty \). We omit the details and give the result.

\( u' \) for \( x' \geq 0 = \)

\[
\frac{1}{2\rho d W_r} \left[ \operatorname{sgn}(n) \sqrt{1 - M_r^2} \pm M_r^2 \sin(\alpha_r) \cos(\alpha_r) \right]
\]

\[
\frac{\sqrt{1 - M_r^2} \cos^2(\alpha_r)}{d(1 - M_r^2 \cos^2(\alpha_r))}
\]

\[
\sum_{n=-\infty}^{\infty} \exp[j \frac{2\pi nx'}{d}] \exp[-\frac{2\pi n x'}{d}] \left[ \frac{2\pi n x'}{d} \right]
\]

(4a)

Similarly:

\( v' \) for \( x' \geq 0 = \)

\[
\frac{1}{2\rho d W_r} \sum_{n=-\infty}^{\infty} \exp[j \frac{2\pi nx'}{d}] \exp[-\frac{2\pi n x'}{d}] \left[ \frac{2\pi n x'}{d} \right]
\]

*\( \operatorname{sgn}(n) = 1 \) if \( n > 0 \), \( = -1 \) if \( n < 0 \) and \( = 0 \) if \( n = 0 \).
$$[j M_r^2 \cos(\alpha_r) \sin(\alpha_r) + \text{sgn}(n) \sqrt{1 - M_r^2}]$$

When viewed in a frame of reference fixed with respect to the casing of the machine, these \( u' \), \( v' \) velocities given by (4a), (4b) appear as:

\[
\begin{align*}
\text{for } x > 0 & = -C_l \frac{a \omega}{d} \exp\left(j \frac{2\pi n y}{d} \exp\left[\frac{2\pi n x \left( \text{sgn}(n) \sqrt{1 - M_r^2} + j M_x M_t \right)}{d(1 - M_x^2)}\right]\right) + \text{sgn}(n) / l - M_x^2) \right)\right]
\end{align*}
\]

\[
\text{for } x < 0 =
\end{align*}
\]

\[
\exp(j \frac{2\pi n V_t}{d}) \ldots (5a)
\]

\[
\left(\text{5b}\right)
\]

THE AXIAL VELOCITY OF THE INLET DISTORTION

Far upstream from the rotor, the inlet distortion appears as regions of axial velocity defect superposed on a uniform inlet velocity.

Some assumption regarding the form of the inlet distortion is necessary in order to analyze its effect. The form assumed is sketched in Figure 2.

Let the axial velocity, in Mach number, associated with the uniform total pressure region be \( M_a \). The axial velocity defect \( \Delta M_a \) associated with the inlet distortion may be deduced as follows from the condition that far upstream there is no variation of static pressure.

\[
\frac{P_t - \Delta p_t}{P} = \left(1 + \frac{\gamma - 1}{2} (M_a^2 - \Delta M_a^2)\right) \frac{\gamma}{\gamma - 1} \quad (6)
\]

(\( \gamma \) is the specific heat ratio of the gas)

Now

\[
\frac{P_t}{P} = \left(1 + \frac{\gamma - 1}{2} M_a^2\right) \frac{\gamma}{\gamma - 1} \quad (7)
\]

Thus one may solve for \( \Delta M_a \) in (6) as:
\[ \Delta M_a = M_a - \left( \frac{2}{\gamma - 1} \left( 1 - \frac{\Delta p_t}{p_t} \right)^{\gamma - 1} \left( 1 + \frac{\gamma - 1}{2} M_a^2 \right) - 1 \right)^{1/2} \] (8)

The form of Eq. (8) indicates that \( \Delta p_t/p_t \) and \( M_a \) are subject to a restriction that

\[ \frac{\gamma - 1}{(1 - \frac{\Delta p_t}{p_t}) (1 + \frac{\gamma - 1}{2} M_a^2) > 1} \] (9)

(Other than the above restriction, \( M_a \) and \( \Delta p_t/p_t \) may be specified arbitrarily.)

The restriction of (9) follows from the assumption that the distortion consists of regions of constant static pressure and varying axial velocity. For such a distortion, (9) simply expresses the requirement that the static pressure not exceed the total pressure anywhere. For small \( \Delta p_t/p_t \) and small \( M_a^2 \), (9) may be expressed approximately as

\[ \frac{\Delta p_t}{p_t} < \frac{\gamma}{2} M_a^2 \] (9)

The interaction of the inlet distortion with the isolated rotor may be analyzed (as sketched in Figure 2) as the interaction of a shear wave, convecting along the fan inlet duct at an axial Mach number \( M_a \), with an isolated rotor.

Let the inlet distortion be Fourier analyzed in an x-y coordinate system at the mean radius of analysis "a" as follows. The basic periodicity of the distortion in the y-direction is \( 2\pi a/S \) (for an S lobed distortion). Let the distorted velocity profile be expressed as:

\[ \sum_{i=-\infty}^{\infty} \exp(-j i S \frac{\gamma}{a}) C_i \] (10)

Here \( j = \sqrt{-1}, i = \) dummy index of summation.

\[ C_i = \frac{2\pi a}{S} \int_{0}^{2\pi a} \Delta M_a(y) e^{j i S \frac{\gamma}{a}} dy \] (11)

**THEORY OF QUADRUPOLE NOISE GENERATION BY INLET DISTORTION**

Let \( A = \frac{C_L \sigma W M_a M_t}{4(1 - M_a^2)} \), \( A' = 2A \)
\[ B = \frac{C_L \sigma W \sqrt{1 - M_a^2}}{4(1 - M_a^2)} \cdot B' = 2B \] (Sometimes \( B \) is also used to denote the number of rotor blades and the context of its use makes this fairly clear.)

\[ C = \frac{+C_L \sigma W}{4} \cdot C' = 2C \]

\[ \alpha_n = \frac{2\pi |n| \sqrt{1 - M_a^2}}{d(1 - M_a^2)} \quad (-\infty < n < \infty) \]

\[ \beta_n = \frac{2\pi n}{d} \quad (-\infty < n < \infty) \]

\[ \gamma_n = \beta_n M_t \quad (-\infty < n < \infty) \]

\[ \delta_n = \frac{2\pi n M_a M_t}{d(1 - M_a^2)} \quad (-\infty < n < \infty) \]

\[ \varepsilon_n = \beta_n S_n B \quad (-\infty < n < \infty) \]

Only an axial quadrupole of type \( p \), \( u \), \( u \) and a transverse one of type \( p \), \( v \), \( u \) where \( u \), \( v \) are the axial and tangential velocity distortions associated with the rotor potential flow field and \( u \), the axial velocity distortion associated with the inlet distortion contributes to the noise (at blade passing frequency and its multiples).

Exploiting the three facts that (a) there is no interest in the zeroth harmonic of blade passing frequency (which is a dc field) (b) that the rotor is subsonic (\( M_r < 1 \)) and (c) we are only interested in propagating noise fields, we may show that the noise generation for a particular harmonic \( n \) \((\geq 1)\) of blade passing frequency noise is governed by:

\[
\begin{align*}
\left\{ \frac{\partial}{\partial t} + M_a \frac{\partial}{\partial x} \right\}^2 p - \nabla^2 p &= -\frac{\partial^2}{\partial x^2} \rho_o \left\{ (\pm A' + j B') \sum_{i=1}^{\infty} C_i^1 \right. \\
\left. \exp(-\alpha_n |x|) \exp(j \delta_n x) \exp(j \beta_n (1 - \frac{is}{nB})y) \right. \\
\left. \sum_{i=1}^{\infty} C_i^1 \exp(-\alpha_n |x|) \exp(j \delta_n x) \exp(j \beta_n (1 - \frac{is}{nB})y) \right. \\
\end{align*}
\]
\[
\exp[j \beta_n M_t a_0 t] \ldots \tag{1}
\]

(where \( C'_i = a_0 C_i \))

Clearly \( p \) depends on \( y, t \) (given \( i \)) as

\[
\exp[j \beta_n (1 - \frac{is}{nB}) y] \exp[j \beta_n M_t a_0 t].
\]

Thus we deduce that the axial part of \( p \) depends on \( x \) as:

\[
\frac{d^2 p}{dx^2} (1 - M_a^2) - 2j M_a M_t \beta_n \frac{dp}{dx} + \beta_n^2 (M_t^2 - (1 - \frac{is}{nB})^2) p = \rho_o \frac{\partial^2}{\partial x^2} (\pm A' + j B') C'_i \exp(-\alpha_n |x|) \exp(j \delta_n x)
\]

\[
\pm \frac{\partial}{\partial x} \rho_o j C'_i \beta_n (1 - \frac{is}{nB}) \exp(-\alpha_n |x|) \exp(j \delta_n x) \tag{2}
\]

To solve (2) introduce the Fourier exponential transform \( P(z) \) of \( p \) by:

\[
P(z) = \int_{-\infty}^{\infty} p e^{-jzx} \, dx
\]

so that

\[
p = \frac{1}{2\pi} \int_{-\infty}^{\infty} P(z) e^{jzx} \, dz
\]

Then:

\[
(z - z_+)(z - z_-)P(z) = \frac{-2j z [A'(z - \delta_n)z - \alpha_n z B' + C'(z - \delta_n) \beta_n (1 - \frac{is}{nB}) C'_i \rho_o]}{[\alpha_n^2 + (z - \delta_n)^2](1 - M_a^2)} \tag{3}
\]

where

\[
z_{\pm} = \frac{\beta_n}{(1 - M_a^2)} \left\{ M_a M_t \pm \sqrt{M_t^2 - (1 - M_a^2)(1 - \frac{is}{nB})^2} \right\}
\]

Let

\[
G(z) = \frac{-2j z C'_i [A'(z - \delta_n)z - \alpha_n z B' + C'(z - \delta_n) \beta_n (1 - \frac{is}{nB}) \rho_o]}{[\alpha_n^2 + (z - \delta_n)^2](1 - M_a^2)} \tag{4}
\]

By the method of residues, the downstream/upstream waves have amplitudes given by:
Independent waves (above cut-off) for each \( n \) will be produced over a range of \( i \) given by:

\[
\frac{\frac{1}{S} \left( 1 - \frac{M_t}{\sqrt{1 - M_a^2}} \right)}{\frac{1}{S} \left( 1 + \frac{M_t}{\sqrt{1 - M_a^2}} \right)} < i < \frac{\frac{1}{S} \left( 1 - \frac{M_t}{\sqrt{1 - M_a^2}} \right)}{\frac{1}{S} \left( 1 + \frac{M_t}{\sqrt{1 - M_a^2}} \right)}
\]

For each of these \( i \), amplitudes of upstream and downstream waves are computed from (5). It is demonstrable that the rms values of the axial components of the intensity are (power/cross sectional area of annulus):

\[
\frac{|A|^2 |\Delta M_t (1 - M_a^2)|^2}{2 \rho_o \alpha O (M_t + \Delta M_a)^2}
\]

where

\[
\Delta = [M_t^2 - (1 - M_a^2)(1 - \frac{iS}{nB})]^{1/2}
\]

This completes the theory of inlet distortion quadrupole noise generation.

**NOISE GENERATION BY INLET TURBULENCE**

We represent the turbulent velocity components by:

\[
(u_t, v_t, w_t) = \int dZ_{u,v,w} (\vec{k}) e^{i(k \cdot \vec{r})} \rho \partial^2 \partial^2
\]

adopting the usual Fourier Stieltjes form where

\[
\partial Z_{u} (k^1) dZ_{v} (k^1') = \delta (k^1 - k^1') \phi_{uv} (k^1) d^3 \vec{k}^1 d^3 \vec{k}^1'
\]

and \( \phi_{uv} (k) \) is the usual spectral density tensor.

The noise generation problem is given by:

\[
\begin{aligned}
\left[ \partial_0^2 + \rho_a \frac{\partial}{\partial x} \right] &+ \left( \frac{\partial^2}{\partial x^2} \right)^2 u_p = \rho_o \left( \frac{\partial^2}{\partial x^2} (u_p u_t) \right) \\
+ \frac{\partial^2}{\partial x \partial y} (u_p v_t + v_p u_t) + \frac{\partial^2}{\partial y^2} (v_p v_t)
\end{aligned}
\]

A typical term of the RHS of (7) may be written as:
\[- \rho_o \frac{\partial^2}{\partial x^2} (\pm A + \text{sgn}(n)j B) dZ_u(\ldots) \]
\[- \rho_o \frac{\partial^2}{\partial x \partial y} (\pm A + j \text{sgn}(n)B) dZ_v(\ldots) \]
\[\pm C dZ_u(\ldots) + \rho_o \frac{\partial^2}{\partial y^2} C dZ_v(\ldots). \tag{8}\]

where (\ldots) stands for:
\[\exp\{-\alpha_n |x|\} \exp[j(\delta_n + k_x) x] \]
\[\exp[j(\beta_n + k_y)y] \exp[j(\beta_n M_t - k_x a_o t)] \tag{9}\]

A word of explanation is in order here concerning the model of
turbulence employed in the analysis. The model is the same as was
employed in (3). A three dimensional model of turbulence is employed
but the spectrum functions are integrated over the third coordinate
(z) so that nowhere does the z dependence appear explicitly.

Substituting for the y dependence from (9), the source term (8) is
effectively:
\[- \rho_o \frac{\partial^2}{\partial x^2} (\pm A + j \text{sgn}(n)B) dZ_u(\ldots) \]
\[- \rho_o \frac{\partial^2}{\partial x} j(\beta_n + k_y)((\pm A + j \text{sgn}(n)B) dZ_v \pm C dZ_u)(\ldots) \]
\[\pm (\beta_n + k_y)^2 \rho_o C dZ_v(\ldots). \tag{10}\]

As before the Fourier exponential transform of the term in (10) may be
written as:
\[-2j \rho_o \frac{\partial^2}{\partial x^2} \{z[z(\delta_n + k_x)A - \text{sgn}(n)B \alpha_n] \]
\[+ C(\delta_n - k_x)(\beta_n + k_y)]dZ_u \]
\[+ (\beta_n + k_y)(z[\delta_n - k_x]A - \alpha_n \text{sgn}(n)B] \]
\[+ (\beta_n + k_y)(\delta_n - k_x)C) dZ_v \} \tag{11}\]
\ [= G_u(z)dZ_u + G_v(z)dZ_v \] \tag{12}\]

where the definitions of \(G_u(z)\) and \(G_v(z)\) are obvious.

As before the downstream and upstream waves will have form:

(i) if \((\beta_n M_t - k_x M_a) > 0\), then with
The waves are given by

\[ \frac{j[G_u(z_+dZ_u) + G_v(z_+dZ_v)]}{(1 - M_a^2)|z_+ - z_-|} \]

\[ \exp[jz_+x]\exp[j(\beta_n + k_y)y] \]

\[ \exp[j(\beta_n M - k_x M_a)\Delta t] \quad (13) \]

(ii) If \((\beta_n M - k_x M_a) < 0\), then

\[ z_+ = \frac{1}{(1 - M_a^2)} \{(\beta_n M - k_x M_a)M_a \]

\[ \pm \{(\beta_n M - k_x M_a)^2 - \]

\[ (1 - M_a^2)(\beta_n + k_y)^2\}^{1/2} \quad (14) \]

and the rest is as in (13).

Let \(\Delta = [(\beta_n M - k_x M_a)^2 - (1 - M_a^2)(\beta_n + k_y)^2]^{1/2} \quad (15a) \]

and

\[ f = |\beta_n M - k_x M_a| \quad (15b) \]

Then the acoustic energy produced downstream and upstream by such waves is given by taking the mean square of (13) and multiplying the resulting quantity by:

\[ \frac{1}{2\rho_0 a_0} \frac{\Delta f(1 - M_a^2)^2}{(f + M_a \Delta)^2} \quad (16) \]

The remaining procedure is more or less mechanical. Let us say that the interest is in the acoustic energy between wave numbers lying between \(\chi\) and \(\chi + \Delta \chi\). (Wave number = frequency in radians/sec. \(\div\) speed of sound). This determines that for each \(n\) from \(-\infty\) to \(\infty\), a range of \(k_x\) lying between:
\[ \beta_n \tan(\alpha_r) - \frac{X}{M_a} \] to \( \beta_n \tan \alpha_r - \frac{X}{M_a} - \frac{\Delta X}{M_a} \) \quad (17)

is of interest. This determines (for each \( n \)) a specific value of \( \chi \). A range of \( k_y \) from

\[ \frac{X}{\sqrt{1 - M^2_a}} - \beta_n \leq k_y \leq \frac{X}{\sqrt{1 - M^2_a}} - \beta_n \] \quad (18)

is to be considered (depending on whether \( \chi \) is positive or negative). The quantity

\[ G^2(z_\pm)\phi_{uu} + 2G^2(z_\pm)G(z_\pm)\phi_{uv} + G^2(z_\pm)\phi_{vv} \]

times the factor in (16) is to be integrated over the range of \( k_y \) indicated in (18), given \( \chi \), \( \beta_n \), and hence \( k_y \) by (17) for each \( n \) in range \(-\infty \) to \( \infty \). For given physical wave number of interest both energy corresponding to \( +\chi \) and \(-\chi \) need to be considered. By adding up the infinite series of contributions from \( n = -\infty \) to \( n = \infty \) for \(+\chi \) and \(-\chi \), taking note of the fact that the width in \( k_y \) is \( (\Delta X/M_a) \), etc., the required spectral density \( dI^2/d\chi \) may be deduced. \( \phi_{uu} \), \( \phi_{uv} \), and \( \phi_{vv} \) should first be integrated over \( k_y \) from \(-\infty \) to \( \infty \), in keeping with two dimensional or plane nature of present analysis. Actually it turns out to be more pertinent to think in terms of \( \chi dI^2/d\chi \) as this quantity has the units of power and is related directly to measurements obtained by constant percent bandwidth or constant octave filters. If we assume a specific form of the longitudinal velocity correlation function for the turbulence of type \( \exp(-r/L) \), \( \phi_{uu} \), \( \phi_{uv} \), and \( \phi_{vv} \) can be easily written down for given turbulence intensity.

DISCUSSION OF RESULTS

Previously developed analyses (3, 4) of dipole noise were employed to estimate the dipole contribution to inlet distortion and inlet turbulence noise.

We show in Figures 3(a), (b), (c) first the nondimensional constant percent filter spectra due to inlet turbulence rotor interaction noise. We study a fixed operating line characterized by a steady lift coefficient of unity associated with a rotor whose solidity is unity. In turbulence noise calculations, one parameter that enters the calculation is the ratio of the integral length scale of turbulence to the blade spacing designated herein as \( (L/D) \). We have independently verified that the specific value of the ratio \( (L/D) \) does not materially affect the ratio of dipole/quadrupole noise contribution. For rotors with axial Mach numbers of order two-thirds the wheel tip Mach number, at wheel tip Mach numbers of order 0.8, the quadrupole noise contribution does indeed begin to exceed the dipole contribution especially at the higher frequencies. The low frequency end of the spectrum (frequencies less than half the blade passing frequency) is still dominated by dipole noise. Similarly calculations were carried out for a rotor with \( (M_a/M_t) = 0.5 \) and \( C_L = 0.5 \) and are shown in Figures 4(a), (b) and (c). Similar trends are evident insofar as the fact that the ratio of quadrupole/dipole contribution increases
with $M_t$ and also, at a given $M_t$, increases with frequency. However one finds that the ratio of quadrupole/dipole noise, at a given $M_t$, increases with the ratio $(M_a/M_t)$. To bring this out in somewhat sharper focus, a calculation is shown in Figure 4(d) of a rotor with $(M_a/M_t)$ being unity, at the same relative Mach number and lift coefficient, etc., as the rotor in Figure 4(c). The ratio of quadrupole/dipole noise is much greater in Figure 4(d) than in Figure 4(c). In Figures 5(a), (b), (c) and (d) are shown calculations of noise spectra at a fixed axial Mach number and total pressure ratio but with varying tip speed. Again as might be expected, higher tip speeds (and associated higher frequencies) do increase the relative ratio of quadrupole/dipole noise. However no evidence is available in these results to support the notion that fan noise could be minimized for constant work by choice of high tip speed and low loading. For other specific designs over some limited tip speed ranges, some such result may be true but it is certainly not a generally valid principle. Indeed the results of Figure 5 support the notion that for subsonic fans, to minimize noise for a given flow and total pressure ratio one should employ high blade loading and low tip speed. This conclusion applies of course only to the physical generation process and not to aspects such as effectiveness of treatment or perceived noise considerations.

In Figures 6 and 7, calculations corresponding to Figure 5 and Figure 3 are shown for 1st and 2nd harmonic pure tone (blade passing frequency) noise generated by an inlet distortion - rotor interaction. In both cases a maximum total pressure defect of 2% of the inlet total pressure was assumed. A four lobed distortion is assumed to impinge on a 36 bladed rotor. Other details are given in Figures 6, 7. A fairly smooth profile of the distortion as shown in Figures 6, 7 was assumed. The results of Figures 6, 7 are similar to those of Figures 5, 3 in that the ratio of quadrupole/dipole noise is greater for second harmonic rather than first harmonic noise and also increases with tip Mach number when a fixed operating line study is considered (ratio of $(M_a/M_t)$ fixed, fixed $C_L$).

Finally an attempt was made to compare the results of the current analysis with experimental data from (5). The data given in (5) pertains to 50 Hz bandwidth forward radiated power at the blade passing frequency measured at several speeds and several pressure ratios at each speed. The data for two rotors (designated rotor 1 and rotor 2 in (5)) were employed.

Figure 8 indicates the results for rotor 1. This case has also been analyzed by the authors of (6) and, as assumed by them, a four lobed distortion with maximum velocity defect equal to 1% of the axial velocity is assumed to impinge on the rotor. The shape of the distortion was assumed to be a triangular pulse of width 10% of the extent of the lobe. This width corresponds roughly to the width of each of four struts placed upstream of the rotor in the experiments reported in (6).

Shown in Figure 8 are the actual measured data at 50, 60, 70 and 80% speeds and predictions by the current analysis using the results of the current study and those of (4). The dipole and quadrupole contributions are shown separately (as predicted by the theory). The dipole noise levels predicted by the theory are roughly in the ballpark of the results observed in (5) by Gelder and Soltis but in one major respect the theory fails completely. The present calculations employ the two dimensional Sears gust formula for estimation of unsteady forces needed to obtain the dipole noise. According to these results, the dipole noise greatly
exceeds the quadrupole noise (by at least 10 dB) over the operating range of rotor 1 of (5). Since dipole noise is independent of blade loading and is proportional to the rotor relative velocity (which decreases as the loading is raised at constant wheel tip speed), the inlet noise is always predicted to decrease with increasing pressure ratio (and associated lower weight flow and rotor relative velocities). The data of (5) of course fail to show this trend at all.

In case of rotor 2, an attempt was made to predict the noise (in a 50 Hz bandwidth) with the aid of the turbulence noise prediction procedures. Measurements of scale and intensity of turbulence are pretty scarce but a good set of measurements for one flow condition and one inlet is available in (9) and these were crudely scaled to the conditions of the inlet in (5). A 4% intensity of turbulence effective over the outer 10% of the rotor blade span and an (L/D) of about 0.5 was assumed for these calculations. Again one notices a fair ability to account for the gross noise levels by the inlet turbulence dipole noise theoretical estimates but a total failure to predict the variation with pressure ratio and flow at fixed speed largely due to the extreme predominance of dipole noise according to the theoretical estimates.

To put these results in perspective, we start by remarking that the quadrupole noise mechanism pointed out in (2) is a second order effect while dipole noise is a first order effect. Thus, with a dipole mechanism, the acoustic pressures scale roughly as \((p_0 W_r u')\) where \(p_0 = \text{mean fluid density}, W_r = \text{steady relative velocity through the rotor and } u'\) the fluctuating solenoidal velocity (whether due to distortion or turbulence). With a quadrupole noise mechanism, they scale as \((p_0 u_p u')\) where \(u_p\) is the potential flow field induced by the rotor and generally \(u_p < W_r\).

Indeed the very procedure adopted to estimate \(u_p\) is a small perturbation calculation assuming that \(u_p \ll W_r\).

Again the full quadrupole source term \(\rho_0 \frac{\partial^2}{\partial x_i \partial x_j} (p_0 u_i u_j)\) may be expanded as \((\theta^2 + 2u_i \frac{\partial \theta}{\partial x_i} + (\frac{\partial u_j}{\partial x_i})(\frac{\partial u_i}{\partial x_j}))\) where \(\theta\) is the divergence of the velocity. So far as the inflow distortion or inflow turbulence is concerned, these velocity fields have zero divergence. Even the potential flow field of the rotor does not have a significant divergence at modest axial and wheel tip Mach numbers. For \(M_a = M_t\), for example, the divergence of the rotor velocity field is \(M_a^2(\theta + 2(\partial u/\partial y))\) and thus of second order in the axial Mach number. Hence only the term \(\rho_0 \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j}\) may be expected to contribute significantly.

Finally the quadrupole source is an extended source subject to extensive phase cancellation while the dipole source is generally assumed to be compact (as is the case in the present model). In this regard it is interesting to note that the authors of (6) considered only the potential flow field upstream of the rotor plane as contributing to the upstream noise and it is possible that such an approach will involve less phase cancellation.

Rao and Chu (8) concur with the results of the current calculations in obtaining estimates of the ratio of dipole/quadrupole noise of the order of 20 dB. Morfey (7) has calculated noise from the axial quadrupole \((p_0 u_p u_s)\) but it is difficult to see why he did not include the contribution of the x-y quadrupole \((p_0 v_p u_s)\). Thus the term that Morfey calculates is...
\[ \rho_o \frac{\partial^2}{\partial x^2}(u_p u_s) \]  
Actually if one includes the term \[ \rho_o \frac{\partial^2}{\partial x \partial y}(\rho_o v_p v_s) \] the quadrupole noise source term can be written as

\[ \rho_o \frac{\partial u_s}{\partial y} \frac{\partial v_p}{\partial x} + \rho_o u_s \frac{\partial \rho}{\partial x} \]

where \( \rho_o \) = divergence of rotor potential flow field = \( \frac{\partial u_p}{\partial x} + \frac{\partial v_p}{\partial y} \). For the low Mach number applications considered by Morfey (he has \( 0 < M_t < M_a \) and \( M_a < 0.7 \)) and especially for the Mach numbers \( M \sim 0.25 \) where he claims the quadrupole mechanism overtakes the dipole mechanism, \( \theta_p \) was probably negligible and thus the term he should have dealt with was really

\[ \rho_o \frac{\partial u_s}{\partial x} \frac{\partial v_p}{\partial x} \]. The term that he really employed viz. \( \rho_o \frac{\partial^2}{\partial x^2}(u_p u_s) \) is actually more or less cancelled by a part of \( \rho_o \frac{\partial^2}{\partial x \partial y}(\rho_o v_p v_s) \). Thus a serious question arises with regard to the correctness of his relative estimates of the quadrupole/dipole mechanism.

We are still left however with the question of the total failure of the present results in Figures 8, 9 to agree with the data of (5). Two remarks are in order here. First of all, absolute prediction is really much less of a goal than the prediction of relative trends. This is because absolute prediction requires a whole host of good inputs other than the acoustic data such as distortion, turbulence data which have only been estimated in this case. Paradoxically of course the present predictions have done somewhat better on absolute prediction than on the relative trends. We notice consistently from Figures 8, 9 that if the dipole-quadrupole levels were more closely matched, the relative trends in the data would be much better explained. In other words, the actual data in Figures 8, 9 often exhibit trends intermediate between those of the dipole and quadrupole noise trends considered separately. Needless to say, since the dipole noise is predicted to be so much higher than the quadrupole source, any attempt to simply add them algebraically would of course fail to explain the data trends. This brings us to the second remark. It appears that the present calculations may be considerably overestimating the ratio of dipole/quadrupole energy. The quadrupole noise calculation is, in many respects, the more exact calculation. The dipole noise estimate employs the Sears gust formula and there are indeed several reasons for believing that it leads to overestimation of the unsteady force. Note that the Sears gust formula applies only to plane, incompressible flows involving isolated, flat plate airfoils at zero angle of attack. Firstly accounting for longitudinal or chordwise gusts generally leads to reduced values of gust loads (7). For inlet distortion noise, the reduction may be estimated as \( (\tan(\alpha_f) - C_L/\pi)/\tan(\alpha_f) \). Secondly suggestions have been made that the Sears function could be revised to accommodate real fluid effects by multiplying it by the steady value of \( (dC_l/da) \) at finite loading to the steady value of \( (dC_l/da) \) at zero loading. This correction would have to be determined experimentally but \( (dC_l/da) \) generally decreases with increasing loading (or \( C_L \) becoming zero before the airfoil stalls. Similarly inclusion of finite aspect ratio effects or of gust obliquity would also lead to
lowered estimates of unsteady loads (see (6) for suggestions on including
the aspect ratio effect - the suggested procedure is very similar to the
one concerning real fluid effects). Finally compressibility and cascade
effects also generally tend to lower estimates of the gust loads. The
work of Timman (10) on compressibility effects at high Mach numbers for
isolated airfoils does predict a reduced gust load (at fixed reduced
frequency). The source compactness assumed in the present calculations
for dipole noise may again be a source of overestimation in view of the
tendency of phase cancellation to occur with a distributed source.

One final remark with regard to Morfey's study (7) is that by choice
of a distortion with period equal to the blade spacing (one that is not
likely to occur in practice), he starts, a priori, with a situation of
low gust loads due to the very high reduced frequencies involved. Also he
considered a rotor of unusually low tip solidity = 0.5, a model which
again inherently favors the quadrupole mechanism.

CONCLUSIONS

1. A systematic theory of quadrupole noise generation in fans/compressors
has been worked out both for inlet distortion and inlet turbulence -
rotor noise. The calculations along with previously developed analyses
of dipole noise due to these agencies have been coded into working pro-
grams included in the present report.

2. Parametric calculations with hypothetical constraints indicate that:

   a) At any fixed operating point, the ratio quadrupole/dipole noise
      increases with frequency whether one considers inlet turbulence
      or inlet distortion to be the source of noise.
   
   b) Along a fixed operating line, i.e. for fixed \( M_a/M_t \) and for
given \( C_L \), the ratio of quadrupole to dipole noise increases with
tip Mach number.
   
   c) At fixed \( M_a \) and given total pressure ratio, the variation of
      ratio of quadrupole to dipole noise with tip Mach number is not
      very simple. This is because with increasing tip speed and
      fixed pressure ratio, the loading (or \( C_L \)) decreases with tip
      speed but the frequencies increase. These features are illustrated
      in Figure 5 where the ratio of quadrupole to dipole noise is
      minimum in Figure 5(c) at \( M_t = .6 \).
   
   d) The ratio of quadrupole/dipole noise is largely a function of
      \( M_a, M_t, \) rotor solidity and pressure ratio (or \( C_L \)) and largely
      independent of the specific form of the distortion or turbulence
      (within broad limits).
   
   e) The analysis shows the quadrupole noise to be independent of
      rotor solidity (at fixed \( M_a, M_t \) and total pressure ratio) but
      dipole noise increases with rotor solidity and hence the ratio
      of quadrupole to dipole noise decreases with increasing rotor
      solidity.

3. No general support is obtained for the idea that for given total
pressure ratio, fan noise can be lowered by operating at high tip speed
and low loading based on present calculations (see Figure 5).
4. Comparisons with data from two rotors as reported in (5) reveal that the analysis does fairly well in predicting the gross PWL levels at given speeds but fails substantially in predicting the trends of the variation of PWL at fixed speed with flow and pressure ratio. When this deficiency is examined in detail, it is found that if one looks at the separate theoretical predictions of the trends of variation of PWL due to dipole and quadrupole mechanisms, the data of (5) often exhibit trends intermediate to those of the dipole and quadrupole predictions. However, the theory consistently places the dipole noise levels at values much higher than the quadrupole levels for the design parameters of the two rotors studied in (5) and hence a composite of the two theoretical predictions does not exhibit the trend of the real data of (5) at all. A possible inference from this is that the ratio of dipole/quadrupole noise is overestimated by the theory and it is argued that this may be chiefly due to overestimate of the gust loads by relying on the Sears gust formula to predict them. Modifications to the gust formula to allow for real fluid, chordwise gust, aspect ratio, spanwise variation, compressibility and cascade effects are all suggested refinements likely to bring the predicted levels of dipole and quadrupole noise much closer together. If this happens, it is possible that a composite of the two will result in predicted trends in much better agreement with the data of (5).
APPENDIX 1

COMPUTER PROGRAMS FOR CALCULATION OF DIPOLE AND QUADRUPOLE NOISE DUE TO INLET DISTORTION OR TURBULENCE

INLET DISTORTION NOISE

Inputs needed are:

- **TPD**: total pressure defect, maximum value of \((\Delta p_t) / p_t\).
- **IS**: number of lobes in distortion.
- **EMA**: \(M_a\)
- **EMT**: \(M_t\)
- **NDP**: number of points used to specify the distortion shape (see Figure A1), should be less than 51.
- **GAM**: \(\gamma\)
- **RH0**: gas density in lbm/cft.
- **C**: speed of sound in fps.
- **A**: area of annulus being studied in square feet.
- **N**: harmonic of blade passing noise of interest.
- **IB**: number of rotor blades.
- **SIGR**: \(\sigma\)
- **TPR**: \((p_{02} / p_{01})\)
- **TPDS(I)**: NDP values specifying relative shape of total pressure distortion (max. value would be unity: \(0 \leq TPDS(I) \leq 1\)). See Figure A1.

Outputs given are:

1) All inputs except NDP, TPDS(I).

2) Sound power in dB re: \(10^{-13}\) watts in each lobe number \(M\) where \(M = N \cdot IB - I \cdot (IS)\). \(M, I\) are also printed. The dipole and quadrupole noise upstream and downstream are given as PUDDB, PDDDB, PUQDB, PDQDB. The sum over all propagating \(M\) is given as

- **SPUDDB**: (dipole noise upstream),
- **SPDDDB**: (dipole noise downstream),
- **SPUQDB**: (quadrupole noise upstream) and
- **SPDQDB**: (quadrupole noise downstream).
APPENDIX 1

INLET DISTORTION NOISE

Inputs needed are:

TPD  total pressure defect, maximum value of \( \Delta p_t / p_t \).
IS   number of lobes in distortion.
EMA  \( M_a \)
EMT  \( M_t \)
NDP  number of points used to specify the distortion shape (see Figure A1), should be less than 51.
GAM  \( \gamma \)
RHØ  gas density in lbm/cft.
C    speed of sound in fps.
A    area of annulus being studied in square feet.
N    harmonic of blade passing noise of interest.
IB   number of rotor blades.
SIGR \( \sigma \)
TPR  \( (p_{02}/p_{01}) \)
TPDS(I) NDP values specifying relative shape of total pressure distortion (max. value would be unity: \( 0 \leq \text{TPDS}(I) \leq 1 \)). See Figure A1.

Outputs given are:

1) All inputs except NDP, TPDS(I).
2) Sound power in dB re: \( 10^{-13} \) watts in each lobe number \( M \) where \( M = N \cdot IB - I \cdot (IS) \). \( M, I \) are also printed. The dipole and quadrupole noise upstream and downstream are given as PUDDB, PDDB, PUQDB, PDQDB. The sum over all propagating \( M \) is given as

SPUDDB (dipole noise upstream),
SPDDDB (dipole noise downstream),
SPUQDB (quadrupole noise upstream) and
SPDQDB (quadrupole noise downstream).
INLET TURBULENCE NOISE

Inputs needed are:

- EMA \( M_a \)
- EMT \( M_t \)
- TPR \( \frac{P_{02}}{P_{01}} \)
- NELMAX number of \((L/D)\) needed to be investigated.
- \( M \) number of frequencies at which \( \frac{dI_f}{df} \) is desired.
- TI turbulence intensity \( <u'^2>_U \)
- GAM \( \gamma \)
- RHØ \( \rho \) in lbm/cft.
- CS speed of sound in fps.
- AI cross sectional area of annulus of interest, sq. ft.
- SIGR \( \sigma \)

- \( F(I)(I = 1, M) \): M values of frequency expressed as \( \frac{f}{f_b} \) where \( f_b \) = blade passing frequency.
- \( ELIN(I)(I = 1, NELMAX) \): NELMAX values of \((L/D)\) of interest.

Outputs are:

1) All inputs except NELMAX.

2) For each \((L/D)\) (written as EL) and \( F \), \( \text{10 log}_{10} \left[ \frac{dI_f}{df} + \frac{5}{2} \rho <u'^2> \right] \)
for both dipole and quadrupole sources as SDVDBR, SDDDBR, SQVDBR, SQDDBR. Also sum of dipole and quadrupole contributions to \( [f \frac{dI_f}{df}] \), upstream and downstream re: \( 10^{-13} \) watts as PVDB and PDDB.
INLET TURBULENCE NOISE

09999 FILENAME OUTPUT
10000 DIMENSION AEV(46), AED(46), ZMM(46), ZPP(46), DCV1(91), DCI2(91),
10001 & F(20), DCV2(91), DCI2(91), STH0SR(91), AEVE(2, 91),
10002 & AEDET(2, 91), ELIN(25)
10010 OUTPUT="ITOUT"
10020 BEGIN FILE OUTPUT
10030 END FILE OUTPUT
10100 100 FORMAT(V)
10200 READ ("ITINPUT", 100) LN, EMA, EMT, TPR
10205 READ ("ITINPUT", 100) LN, NELMAX, M, TI
10210 READ ("ITINPUT", 100) LN, GAM, RH0, CS, AI, SI GR
10220 IBEG=1
10230 IEND=10
10240 240 IF (I END .GT. M) IEND=M
10250 READ ("ITINPUT", 100) LN, (F(I), I=IBEG, I END)
10260 IF (I END .EQ. M) G0 T0 300
10270 IBEG=IBEG+10
10280 IEND=IEND+10
10290 G0 T0 240
10300 300 IBEG=1
10310 IEND=10
10320 320 IF (I END .GT. NELMAX) IEND=NELMAX
10330 READ ("ITINPUT", 100) LN, (ELN(I), I=IBEG, I END)
10340 IF (I END .EQ. NELMAX) G0 T0 500
10350 IBEG=IBEG+10
10360 IEND=IEND+10
10370 G0 T0 320
10500 500 WRITE (OUTPUT, 510)
10510 510 FORMAT(//, 32H EMA EMT M TI)
10520 WRITE (OUTPUT, 530) EMA, EMT, M, TI
10530 530 FORMAT(2F9.3, I6, F9.2)
10535 C=CS
10536 A=AI
10540 WRITE (OUTPUT, 550)
10550 550 FORMAT(//, 53H GAM RH0 C A SI GR)
10551 & TPR)
10560 WRITE (OUTPUT, 570) GAM, RH0, C, A, SI GR, TPR
10600 DBL=130.*4.*342945.*AL0G(1.*105*RH0*(C*EMA)**3*TI**2*A)
10610 PI=3.1415926
10615 TPI=2.*PI
10620 G10V2=(GAM-1.)/2.
10630 G10VG=(GAM-1.)/GAM
10640 EMR=SQRT(EMA**2*EMT**2)
10650 T11=TPR**G10VG-1.
10660 T112=1.+1./(G10V2*EMA**2)
10670 SRIMM2=SQRT(1.-EMA**2)
10680 SRIMR2=SQRT(1.-EMR**2)
10690 CR=EMA/EMR
10695 CR2=CR**CR
10700 SP=EMT/EMR
10705 SR2=SR**SR
10710 SRCR=SR**CR

21
10715  TR=EMT/EMA
10720  EMRC=EMA/(S*CR)
10730  CL=CR2*T12*T11/(SIGR*SR)
10740  AA=1.-EMA**2
10750  A=CL*SIGR*EMA*EMT/(4.*AA)
10760  B=CL*SIGR*SR1MP2/(4.*AA)
10770  C=CL*SIGR/4.
10780  ALC=TPI*SR1MR2
10790  BETC=TPI
10800  CHIC=BETC*EMT
10810  DELC=TPI*EMA*EMT
10820  CDP=PI*SIGR/2.
10830  CDP=CDP*CDP
10840  CAE=1./(EMA*SR1MM2)
10850  NINC02=10
10860  NINC=2*NINC02
10870  FNINC=NINC
10880  D0 1230  I = 1/IMAX
10890  CTH=COS(THETA)
10900  EMAMC=EMA-CTH
10910  EMAPC=EMA+CTH
10920  IF (I .GT. (NINC02+1)) G0 T0 1090
10930  ZMMU)»EMAMC
10940  ZPP(I)=EMAPC
10950  IF (I .GT. (NINC02+1)) INEIX=*=IMAX+1-I
10960  AEV(I)=CAE/(1.+EMA*CTH)**2
10970  AEDC(I)=CAE/(1.-EMA*CTH)**2
10980  STH=SIN(THETA)
10990  TERM=SR1MM2*STH*CR
11000  DCV1(I)=(EMAPC*SR-TERM)**2
11010  DCV2(I)=(EMAPC*SR+TERM)**2
11020  DCD1(I)=(EMAPC*SR-TERM)**2
11030  DCD2(I)=(EMAPC*SR+TERM)**2
11040  STH0SR(I)=STH/SR1MM2
11050  INDEX=I
11060  IF (I .GT. (NINC02+1)) INDEX=IMAX+1-I
11070  AEVETC(1,I)=AEV(INDEX)*CDP2*DCV1(I)
11080  AEVETC(2,I)=AEV(INDEX)*CDP2*DCV2(I)
11090  AEDETC(1,I)=AED(INDEX)*CDP2*DCD1(I)
11100  AEDETC(2,I)=AED(INDEX)*CDP2*DCD2(I)
11110  THETA=THETA+DELTH
11120  CONTINUE
11130  D0 1230  J=UM
11140  WRITE (OUTPUT,1261) EL
11150  1261  FORMAT(/11H ***** EL=1P1E12.3,6H *****)
11160  CPHI=1./(4.*PI*EL)
11170  ELC=1./(EL*EL)
11180  D0 3000  J=1,M
11190  WRITE (OUTPUT,1320) F(J)
11200  1320  FORMAT(/7H F(J)=E12.3)
11330  CHI=F(J)*CHIC
11335  CHI2=CHI*CHI
11340  SDV=0.
11350  SDD=0.
11360  SQV=0.
11370  SQD=0.
11380  NVAL=8
11390  NVALP1=NVAL+1
11500  DO 2800 NN=1,NVALP1
11510  N=NN-1
11520  IF (N) 1530,1550,1570
11530  1530  SGN=-1.
11540  G0 T0 1580
11550  1550  SGN=0.
11560  G0 T0 1580
11570  1570  SGN=1.
11580  1580  EN=N
11590  AL=ABS(EN)*ALC
11595  AL2=AL*AL
11600  ALSGNB=AL*SGN*B
11605  BET=EN*BETC
11610  DEL=DELC*EN
11630  SUMDV=0.
11640  SUMDD=0.
11650  SUMQV=0.
11660  SUMQD=0.
11670  THETA=-PI/2.
11800  DO 2600 I=1,IMAX
11805  AKX=BET*TP-CHI/EMA
11810  AKY=-BET+CHI*STH0 SR(I)
11820  AKY2=AKY*AKY
11830  INDEX=I
11840  IF (I .GT. (NINC02+1)) INDEX=IMAX+I-1
11850  ZM=ZMM(INDEX)*CHI
11860  ZP=ZPP(INDEX)*CHI
11900  DO 2310 Ll =1,2
11910  AKX2=AKX*AKX
11920  ELAKX2=ELC+AKX2
11930  CPH1=CPHI/(ELAKX2+AKY2)**2.5
11940  PHIXX=(4.*AKY2+ELAKX2)*CPHI1
11950  PHIYX=-3.*AKY*AKX*CPHI1
11960  PHIYY=(4.*AKX2+ELC+AKY2)*CPHI1
11965  TPH1XY=2.*PH1XY
11970  ØMR=SI GR*(AKX*CR+AKY*SR)/2.
11980  SRF=1./(1.+TPI*ABS(ØMR))
11990  PHIT=PHIXX*SR2-TPH1XY*SRCR+PHIYY*CR2
11995  PHITS=PHIT*SRF
12000  IF (L1 .EQ. 2) G0 T0 2040
12010  FNDVP=AEVETC(1,I)*PHITS
12020  FNDDP=AEDETC(1,I)*PHITS
12030  G0 T0 2060
12040  2040  FNDVM=AEVETC(2,I)*PHITS
12050  2050  FNDDM=AEDETC(2,I)*PHITS
ZTERM=ZF
AKX=AKX*AA
D0 2200 L2=1,2
ZDELAK=ZTERM-DEL-AKXA
DEN=(AL2+ZDELAK**2)**2
PART=ZTERM*(ZDELAK*A-ALSGB)+C*ZDELAK*CHI*STH0SR(I)*AA
GX=ZTERM*PART
GY=CHI*STH0SR(I)*PART/AA
TQ=(GX**2*PHIXX+GY**2*PHIYY+GX*GY*TPHIXY)/DEN
IF (L2.EQ.2) G0 T0 2180
FNQV=TQ*AEV(INDEX)
FNQD=TQ*AED(INDEX)
ZTERM=ZM
CONTINUE
IF (L1.EQ.2) G0 T0 2250
FNQVP=FNQV
FNQDP=FNQD
AKX=BET*TR+CHI/EMA
ZP=-ZP
ZM=-ZM
G0 T0 2310
FNQVM=FNQV
FNQDM=FNQD
CONTINUE
FDV=FDVP+FNDV
FDD=FNDDP+FNDDM
FQV=FNQVP+FNQVM
FQD=FNQDP+FNQDM
IF ((I .NE. 1) .AND. (I .NE. IMAX)) G0 T0 2490
FDV=FDV/2.
FDD=FDD/2.
FQV=FQV/2.
FQD=FQD/2.
SUMDV=SUMDV+FDV
SUMDD=SUMDD+FDD
SUMQV=SUMQV+FQV
SUMQD=SUMQD+FQD
THETA=THETA+DELTH
CONTINUE
SNDV=CHI2*SUMDV*DELTH
SNDD=CHI2*SUMDD*DELTH
SNQV=SUMQV*DELTH
SNQD=SUMQD*DELTH
IF (N.GT.0) G0 T0 2750
SDV=SDV+SNDV
SDD=SDD+SNDD
SQV=SQV+SNQV
SQD=SQD+SNQD
G0 T0 2800
SDV=SDV+2.*SNDV
SDD=SDD+2.*SNDD
SQV=SQV+2.*SNQV
12780 SQD = SQD + 2 * SNQD
12800 CONTINUE
12802 SDV = SDV * EMRC
12803 SDD = SDD * EMRC
12804 SQV = SQV * EMRC
12805 SQD = SQD * EMRC
12810 SDVDBR = 4.342945 * ALOG(SDV)
12820 SDDDBR = 4.342945 * ALOG(SDD)
12830 SQVDBR = 4.342945 * ALOG(SQV)
12840 SQDDBR = 4.342945 * ALOG(SQD)
12850 PV = SDV + SQV
12860 PD = SDD + SQD
12870 PVDB = 4.342945 * ALOG(PV) + DBL
12880 PDDB = 4.342945 * ALOG(PD) + DBL
12900 WRITE (OUTPUT, 2910)
12910 2910 FORMAT(/40H SDVDBR, SDDDBR, SQVDBR, SQDDBR)
12920 WRITE (OUTPUT, 2930)
12930 2930 FORMAT(4F10.1)
12940 WRITE (OUTPUT, 2950)
12950 2950 FORMAT(/20H PVDB, PDDB)
12960 WRITE (OUTPUT, 2970)
12970 2970 FORMAT(2F10.1)
13000 CONTINUE
13010 CONTINUE
13100 STOP
13110 END
INLET DISTORTION NOISE

09990 FILENAME OUTPUT
10000 DIMENSION SIGN(2), ETA(2), TPDS(50), DELM(500), PD(2), PQ(2)
10010 OUTPUT="IDOUT"
10020 BEGIN FILE OUTPUT
10030 END FILE OUTPUT
10100 FORMAT(V)
10200 READ("IDINPUT",100) LN, TPD, IS, EMA, EMT, NDP
10210 READ("IDINPUT",100) LN, GAM, RH0, C, A, N, IB, SIGR, TPR
10220 IBEG=2
10230 IEND=11
10240 240 IF (IEND .GT. (NDP+1)) IEND=NDP+1
10250 READ("IDINPUT",100) LN, (TPDS(I), I=1 BEG, IEND)
10260 IF (IEND .EQ. (NDP+1)) G0 TO 500
10270 IBEG=IBEG+10
10280 IEND=IEND+10
10290 G0 TO 240
10500 500 S=IS
10510 EN=N
10520 B=IB
10600 WRITE (OUTPUT,610)
10610 610 FORMAT(/36H TPD IS EMA EMT)
10620 WRITE (OUTPUT,630) TPD, IS, EMA, EMT
10630 630 FORMAT(F10.3,I7/2F10.3)
10640 WRITE (OUTPUT,650)
10650 650 FORMAT(/65H GAM RH0 C A N IB
10651 & SIGR TPR)
10660 WRITE (OUTPUT,665) GAM, RH0, C, A, N, IB, SIGR, TPR
10670 PI=3.1415926
10675 IMAX=500
10676 IMAX1=IMAX-1
10677 FIMAX1=IMAX1
10680 JMAX=NDP+2
10685 JMAX1=JMAX-1
10686 FJMAX1=JMAX1
10687 TPDS(1)=0.
10688 TPDS(JMAX)=0.
10690 DELM(1)=0.
10700 DELM(IMAX)=0.
10710 GI0V2=(GAM-1.)/2.
10720 GI0VG=(GAM-1.)/GAM
10730 T1=1.*GI0V2*EMA**2
10740 T11=TPR**GI0VG-1.
10750 T12=1.*1./(GI0V2*EMA**2)
10760 D0 800 1=2, IMAX1
10762 FI=1
10764 TEMI1=(FI+1.)/FIMAX1
10766 J1=TERMI*FJMAX1+1.
10768 J2=J1+1
10770 FJ1=J1
10772 FJ2=J2
10774 TEMJ1=(FJ1-1.)/FJMAX1
10776 TEMJ2=(FJ2-1.)/FJMAX1

26
10778  TPDI=TPDS(J1)+(TPDS(J2)-TPDS(J1))*(TERM1-TERMJ1)/(TERMJ2-TERMJ1)
10780  TPDD=TPD*TPDI
10785  T2=(1.-TPDD)**GI0VG
10790  DELM(I)=EMA-SQRT((T1*T2-1.)/GI0V2)
10800  800 CONTINUE
10810  EMR=SQRT(EMA**2+EMT**2)
10820  SR1MM2=SQRT(1.-EMA**2)
10830  SR1MR2=SQRT(1.-EMR**2)
10840  IMIN=(1.-EMT/SR1MM2)*EN*B/S+.999
10850  IMAX=(1.+EMT/SR1MM2)*EN*B/S
10860  ALSRR=ATAN(EMT/EMA)
10870  VR=C*EMA/COS(ALSRR)
10880  CONST=(VR*VR*SIGN/2.*)**2*A*PHI*746./(C*32.2*550.)
10890  SR=SIN(ALSRR)
10900  CX1=SR
10910  TXI=-COS(ALSRR)/SR
10920  CLT=COS(ALSRR)**2*T12*T11/(SIGN*SR)
10930  AA=1.-EMA**2
10940  AP=CLT*EMA*EMT/AA
10950  BP=CLT*SR1MR2/AA
10955  CP=CLT
10960  BB=EN*B*EMT
10970  BB2=BB*BB
10980  CC=EMA*BB
10990  DD=EMA/BB
11000  ALPH=EN*B*SR1MR2/AA
11010  SPUD=0.
11020  SPDD=0.
11030  SPUG=0.
11040  SPDO=0.
11050  SIGN(1)=1.
11060  SIGN(2)=-1.
11070  COM=PI*SR*SINRE/S
11080  CCLC=PI*SIN(2.*ALSRR)/EMA
11090  WRITE (0UTPUT, 1800)
11100  D0 1900 I=IMIN,IMAX
11110  FI=I
11120  M*N*I+B-I*IS
11130  EM=M
11140  DELTH=2.*PI/(S*FINX1)
11150  THETA=DELTH
11160  SUM1=0.
11170  SUM2=0.
11180  D0 400 K=2,IMAX1
11190  SUM1=SUM1+F1
11200  SUM2=SUM2+F2
11210  THETA=THETA+DELTH
11220  400 CONTINUE
11230  CIR=TERM1*SUM1
```fortran
CII = TERM1 * SUM2
CI = SQRT(OUT**2 + CII**2)
DEL = SQRT(BB2 - AA * EM**2)
T0PD = (CXI / DEL) * CCL * CI
EMTXI = EM * TXI
T0PQ = CI / ((ALPH**2 + (DEL / AA)**2) * EM)
M = FI * C0M
APP = 18.11
SEARS = SQRT((APP + 0M) / (APP + (P1 * APP + 1) * 0M + 2 + P1 * 0M * 0M))
D0 1700 K = 1, 2
CAY = (SIGN(K) * CC + DEL) / AA
A0VFQ = T0PD * CAY * ((1 + EMTXI * SIGN(K) / CAY) * SEARS
F1 = CAY / SQRT(CAY**2 + EM**2)
F2 = 1 + SIGN(K) * CAY * DD
TACK) = -0.5 * (F1 - SIGN(K) * EMA) / F2
A0VFD = T0PD * CAY * (1 - T - EMTXI * SIGN(K) / CAY) * SEARS
F1 = CAY / SQRT(CAY**2 + EM**2)
F2 = 1 + SIGN(K) * CAY * DD
TACK) = -0.5 * (F1 - SIGN(K) * EMA) / F2
A0VFQ = CAY * (AP + DEL * CAY / AA - ALPH * BP * CAY * SIGN(K) + CP * SIGN(K) * DEL*
& EM / AA) * T0PD / DEL
PD(K) = ETA(K) * A0VFQ**2
PQ(K) = ETA(K) * A0VFQ**2
1700 CONTINUE
1710 PUD = PD(1) * CONST
1720 PDD = PD(2) * CONST
1730 PUQ = PQ(1) * CONST
1740 PDQ = PQ(2) * CONST
1750 PUDDB = 130 + 4.342945 * ALOG(PUD)
1760 PDDDB = 130 + 4.342945 * ALOG(PDD)
1770 PUDQDB = 130 + 4.342945 * ALOG(PUD)
1780 PDDQDB = 130 + 4.342945 * ALOG(PDD)
1800 F0ORMAT (//50H I M PUDDB PDDDB PUQDB PDQDB
1801 & PDQDB)
1810 WRITE (OUTPUT, 1820) I, M, PUDDB, PDDDB, PUQDB, PDQDB
1820 1820 F0ORMAT (2I5/4F10.1)
1830 SPUD = SPUD + PUD
1840 SPDD = SPDD + PDD
1850 SPUQ = SPUQ + PUQ
1860 SPDQ = SPDQ + PDQ
1900 1900 CONTINUE
1910 SPUDDB = 130 + 4.342945 * ALOG(SPUD)
1920 SPDDDB = 130 + 4.342945 * ALOG(SPDD)
1930 SPUDQDB = 130 + 4.342945 * ALOG(SPUD)
1940 SPDDQDB = 130 + 4.342945 * ALOG(SPDD)
1950 WRITE (OUTPUT, 1960)
1960 1960 F0ORMAT (//41H SPUDDB SPDDDB SPUDQDB SPDDQDB)
1970 WRITE (OUTPUT, 1980) SPUDDB, SPDDDB, SPUDQDB, SPDDQDB
1980 1980 F0ORMAT (4F10.1)
2000 STOP
20010 END
```
APPENDIX 2

LIST OF SYMBOLS

\( a_0, a \) speed of sound.

\( a \) sometimes used to denote radius of fan annulus being studied.

\( B \) number of blades in the rotor.

\( C_L \) lift coefficient of the rotor.

\( C \) rotor chord.

\( C_p \) specific heat at constant pressure.

\( C_i \) coefficients of inlet distortion.

\( d \) pitch of rotor.

\( j \) \( \sqrt{-1} \)

\( \frac{L}{D} \) integral length scale of turbulence/rotor pitch.

\( M_a, M_r \) and \( M_t \) axial, relative and wheel tip Mach numbers.

\( p \) pressure.

\( p_{02}/p_{01} \) pressure ratio across rotor

\( p_t \) local total pressure

\( R \) gas constant

\( S \) number of lobes in distortion.

\( T_{01}, T_{02}, T_1 \) total temperature upstream and downstream of rotor, temperature ahead of rotor.

\( U \) axial velocity entering rotor.

\( V \) wheel velocity.

\( W_r \) relative velocity through rotor.

\( \alpha_r \) stagger angle of rotor.

\( \delta_{ij} \) Kronecker delta.

\( \delta \) delta function.

\( \rho \) density.

\( \sigma \) rotor solidity.
REFERENCES


FIGURE 1

CALCULATION OF ROTOR POTENTIAL FLOW FIELD
$p_t(y)$

$a = \text{mean radius of annulus}$

**FIGURE 2**

VARIATION OF TOTAL PRESSURE AROUND THE ANNULUS
FIGURE 3. INLET TURBULENCE - ROTOR NOISE

- DIPOLE NOISE
- QUADRUPOLE NOISE

\( f_b \): blade passing frequency
\( C_L = 1.00, \sigma = 1.00 \)
\( L/D = 1.8 \)

\[
\frac{10 \log_{10} \left( \frac{d}{\rho} \right)}{2 \nu \langle u' \rangle^2}
\]

\( M_a = 0.2, M_t = 0.3 \)
\( M_a = 0.4, M_t = 0.6 \), Rest as in Fig. 3(a)
$M_a = 0.55, M_t = 0.825$, Rest as in Figure 3(a)
FIGURE 4. INLET TURBULENCE - ROTOR NOISE (Contd.)

\[ \sigma = 1.00, \text{CL} = 0.5, \text{L/D} = 0.5 \]

-20

-30

-40

-50

\[ 10 \log_{10}\left(\frac{d^2}{df_c} \cdot \frac{\frac{1}{2}}{p_0 U \left< u'^2 \right>} \right) \]

(a)

M_a = 0.2, M_t = 0.4

(b)

M_a = 0.35, M_t = 0.7, Rest as in 4(a)

DIPOLE

QUADRUPOLE
FIGURE 4 (Concluded)

\[ M_a = 0.425, \ M_t = 0.85, \ \text{Rest as in 4(a)} \]

\[ M_a = 0.7, \ M_t = 0.7, \ \text{Rest as in 4(a)} \]
FIGURE 5. EFFECT OF TIP SPEED AT CONSTANT PRESSURE RATIO

- DIPOLE
- QUADRUPOLE

\[ M_a = 0.4, \sigma = 1.00, \text{pressure ratio} = 1.4, \]
\[ L/D = 1.8 \]

\[ 10 \log_{10} \left[ \frac{dI}{df} \cdot \frac{5}{2} \rho_o \langle U^2 \rangle \right] \]

(a) \[ M_t = 0.4 \]

(b) \[ M_t = 0.5, \text{Rest as in 5(a)} \]
**FIGURE 6. INLET DISTORTION NOISE**

\[ \frac{\Delta p_t}{p_t} = 0.02, \; M_a = 0.4, \; B = 38, \; S = 4, \; \sigma = 1.00, \]
\[ C_L = 1.00 \]

**FIGURE 7. INLET DISTORTION NOISE**

\[ M_a = \frac{2}{3} M_t, \; \frac{\Delta p_t}{p_t} = 0.02, \; C_L = 1.00, \; S = 4, \]
\[ B = 38, \; \sigma = 1.00 \]

Diagram showing inlet distortion noise with one lobe, 1st harmonic, 2nd harmonic, dipole, and quadrupole.

10 dB

1.2.4.6.8

1.2.4.6.8

1.2.4.6.8

1.2.4.6.8
Figure 8. Comparison with (5): Rotor 1.
FIGURE 9. COMPARISON WITH (5): ROTOR 2

Data from NASA TMX-2191

- 80% Speed
- 70% Speed
- 60% Speed
- 50% Speed

DIPOLE NOISE DUE TO INLET TURBULENCE (THEORETICAL)

QUADRUPOLE NOISE DUE TO INLET TURBULENCE (THEORETICAL)
One lobe

1 2 3 4 (NDP - 3) NDP

$\Delta p_t / (\Delta p_t)_{max} = TPDS$

**FIGURE A1**

DEFINITION SKETCH FOR INPUT FUNCTION TPDS
The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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