

# Sources of Sound in Fluid Flows

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This paper describes some features of a flow that produce acoustic radiation, particularly when the flow is turbulent and interacting with solid surfaces such as turbine or compressor blades. Early theoretical ideas on the subject are reviewed and are shown to be inadequate at high Mach number. Some recent theoretical developments that form the basis of a description of sound generation by supersonic flows interacting with surfaces are described.

At high frequencies the problem is treated as one of describing the surface-induced diffraction field of adjacent aerodynamic quadrupole sources. This approach has given rise to distinctly new features of the problem that seem to have bearing on the radiating properties of relatively large aerodynamic surfaces.

At low speeds, for fixed geometry, the acoustic power output from turbomachinery increases in proportion to the square of mechanical power. This variation obviously cannot be continued to indefinitely high powers since the acoustic loss would soon exceed the available energy supply. The mechanism of fan noise generation must therefore change its character at high speed, and the critical point seems to be reached when the blade tip speed exceeds the sonic velocity.

Observationally, the change becomes apparent in several distinct ways, one of the most striking being the appearance of the "buzz-saw" phenomenon, a noise characterized by multiple harmonics of the disk rotation frequency. At these high speeds, shock waves form, and the radiation field can hardly remain linear. On the other hand, it seems that the essential features of buzz-saw noise can be explained on the basis of linear theory alone and that nonlinearity is essential only in effecting minor modifications of amplitude. The first part of this paper sets down the central points of the arguments leading to a prediction of the buzz-saw phenomenon from purely linear theory.

The second part of this paper outlines some ideas on how the sound problem changes fundamentally at high speeds because of an inevitable wavelength contraction to a condition where the wavelength cannot be large in comparison with surface dimensions. This noncompactness makes the high-speed problem quite different in nature from that with which we are familiar at low speed. The radiated power is seen to be a decreasing function of jet power, for given geometry. At high speed, the noncompact sources are not so clearly related to the unsteady (or even steady) lift on the surfaces, so there must be some doubt as to the relevance of attempts to eliminate the unsteady forces to the high-speed fan noise problem.

### SOURCES OF SOUND AT DISK ROTATION FREQUENCIES

The sound of an  $N$ -bladed fan is not always confined to harmonics of the blade-passage frequency  $N\Omega$ . There is, inevitably, some residual component at the disk rotation frequency  $\Omega$  that can be attributed to the fact that the blades are not positioned in a completely regular array around the supporting disk and cannot support identical aerodynamic loads, both because of the slight irregularity of support and because of slight geometrical variation from one blade to another. It is known that, when blades travel at supersonic speeds, buzz-saw noise at harmonics of the disk rotation frequency becomes very intense and can become the dominant sound. By precisely what mechanism is this low-frequency sound generated? Is the mechanism essentially nonlinear, and why is the mechanism relatively stronger at supersonic speeds? On the basis of linear theory it is obviously not directly attributable to variations of blade size, incidence, or camber. Each of these effects produces an irregularity in the time history of the sound field that is harmonic on the fundamental rotation frequency of the disk but random on the blade frequency. However, these effects that bring about variations in loading from one blade to another are virtually independent of blade speed; thus the relative level of disk frequency noise to blade frequency noise arising from this cause would be independent of speed, a feature distinctly contrary to observations of the buzz-saw phenomenon. On the other hand, the variation in phase caused by slight circumferential variation of blade support can, on the basis of purely linear theory, induce an effect consistent with observation. This effect becomes more important at supersonic speeds because the frequency spectrum then contains much energy at very short wavelengths; on this short scale, the slight positional irregularities appear very significant. This effect can be quantified as follows.

Consider an  $N$ -bladed disk rotating at an angular frequency  $\Omega$ . The pressure field generated by the motion will be at harmonics of this frequency because, when all effects of turbulence are discarded, the field is

repeated on each rotation of the disk. If the geometry were ideally precise, the sound field would also be repeated at intervals corresponding to the blade-passage frequency. We denote this hypothetical time history of the field at any point by  $g(t)$ , and this signal could be Fourier-analyzed into discrete frequency components:

$$g(t) = \sum_n G_n e^{inN\Omega t} \tag{1}$$

Now suppose that the blades are identical but are positioned in a slightly irregular way around the disk circumference. The time history might then be denoted by  $g(t+\epsilon)$ , where  $\epsilon$  is a small stochastic function of time on the interval corresponding to disk rotation but is exactly repeated every period. The signal  $g(t+\epsilon)$  can be expanded in a Taylor series about  $\epsilon=0$ , and, for sufficiently small  $\epsilon$ , the real signal can be represented by

$$g(t+\epsilon) = g(t) + \epsilon g'(t) \tag{2}$$

$g(t)$  is periodic at the blade frequency, but the second term is not, being periodic at intervals of disk rotation frequency only. Consequently, the real signal is made up of the discrete frequency part of the ideal case, plus a small subharmonic part, smaller by a factor of order

$$\overline{(\epsilon^2)}^{1/2} nN\Omega \tag{3}$$

than the  $n$ th harmonic of the tone at the blade-passage frequency  $N\Omega$ . The frequency  $nN\Omega$  is the ratio of the amplitude of  $g'$  to that of  $g$ . At higher frequencies, the expansion of equation (2) fails and the situation becomes quite different. The spectrum  $\mathfrak{G}(\alpha)$  can then be obtained by the Fourier transformation.

$$\begin{aligned} \mathfrak{G}(\alpha) &= \frac{1}{2\pi} \int g(t+\epsilon) e^{-i\alpha t} dt \\ &= \frac{1}{2\pi} \int \sum_n G_n e^{inN\Omega(t+\epsilon)-i\alpha t} dt \end{aligned} \tag{4}$$

The phase factor,  $nN\Omega(t+\epsilon)$ , is now a rapidly varying function of  $t$ , so that the integral may be evaluated by the method of stationary phase to give

$$\mathfrak{G}(\alpha) = \frac{1}{2\pi} \sum_p \sum_n \sum_m \frac{G_n \exp [i(t_p + 2m\pi/\Omega)(nN\Omega - \alpha) + inN\epsilon\Omega]}{(nN\Omega\epsilon''/2\pi)^{1/2}} \tag{5}$$

where  $\sum_p$  signifies summation over all stationary points occurring in the interval  $(0, 2\pi/\Omega)$ , and  $\epsilon, \epsilon''$  are evaluated at the stationary points  $t_p$ .

The sum over  $m$  forms a delta function so that

$$\mathcal{G}(\alpha) \sim \frac{1}{2} \sum_p \sum_n \frac{G_n e^{-i\alpha t_p}}{(nN\Omega\epsilon''/2\pi)^{1/2}} \delta\left(\sin \frac{\pi\alpha}{\Omega}\right) \quad (6)$$

$$\sim \frac{1}{2\pi} \sum_p \sum_n G_n \left(\frac{2\pi\Omega}{nN\epsilon''}\right)^{1/2} \delta(\alpha - m\Omega) \quad (7)$$

(Note that  $e^{im\Omega t_p} = \text{constant}$ .)

This spectrum is a series of discrete components at harmonics of the disk rotation frequency  $\Omega$  and is utterly independent of the blade-passage frequency. Furthermore, the levels of the discrete tones are independent of the harmonic number, a feature often observed in buzz-saw situations. It seems paradoxical that it is only when the basic waveform contains high enough frequencies that  $nN\Omega\epsilon$  is large that the low-frequency spectrum can become significant. This fact is a reflection of the need that the waveform possess sharp corners, necessitating high frequencies in the spectral decomposition before small errors in phase in  $g(t+\epsilon)$  can materially affect the waveform. Where this is so, the phase variation is random enough for there to be no blade-passage frequency harmonics in the spectrum, and the sound is composed of harmonics of the disk rotation frequency alone. The spectrum is the characteristic of that measured if a definite interval of white noise is repeated harmonically, as it would be in a short loop of taped noise continuously playing on a recorder. That spectrum is an infinite series of equal-strength tones of the loop frequency.

The most significant effect of supersonic motion is that "sharp-cornered" shock waves are formed, making the basic Fourier synthesis of the wave contain very high frequencies. This effect, coupled with slight phase variations due to irregularities of blade positioning, can evidently bring about the array of low-frequency tones that constitute buzz saw.

The essential element of the buzz-saw phenomenon seems to be a phase randomization of the elementary blade-passage signals. This effect is inevitable because of slight manufacturing imperfections leading to errors in circumferential positioning. It is also an effect that can arise from nonlinear effects resulting from blades of slightly varying geometry, for the higher pressure regions would tend to travel relatively faster than the low-amplitude signal to induce phase variations away from the rotor disk.

## SOURCES OF BROADBAND SOUND AT HIGH SPEED

Random broadband noise in any turbomachine is caused by some form of turbulence. Steady flow distortions cause broadband noise only indirectly by bringing about turbulence in some part of the machine. The inlet flow into the machine may be turbulent, so that the aerodynamic

loading of the blades is made stochastic, and sound of continuous frequency content is then inevitable. Sound generated in this way can appear relatively discrete only if successive blades experience nearly identical loading histories, as they would if a large sausage-shaped turbulent eddy were being absorbed by a fan over a significant fraction of a disk revolution. Ffowes Williams and Hawkings (ref. 1) show that the bandwidth of the blade-passage tone generated in such a situation is the reciprocal of the number of blades subjected to the distortion field of a single eddy.

At low speeds the random noise is due almost entirely, apart from small (in air) displaced inertia terms, to the unsteady forces on a blade. These forces can be computed by the method of Sears and von Kármán (ref. 2), a method which regards the flow as incompressible. The motions bringing about the unsteady forces are virtually decoupled from the small sound field at low Mach number. At higher Mach number, the coupling is immensely improved, and it is no longer possible to determine the blade loading independently of the sound field; neither is the blade load itself the source of the field. The computation of the field centered on the blades is then properly posed as a diffraction problem, where the incident field is specified with the object of determining the scattered waves. Of course, once conditions are known on the bounding surfaces, the linear radiation field is determined.

Miles (ref. 3) gives many relevant references and describes the field induced by a thin supersonic aerofoil entering a sharp-edged gust. This solution can serve as the Green's function to generate the scattered field for arbitrary two-dimensional incident flow. At first sight, quite a different viewpoint must be taken of the sound generated by turbulence arising on the rapidly moving blading, but in fact recent developments bring the two situations very close together. We consider here blading that moves in a uniform stream.

It is known that in coordinates moving with the stream sound is generated and propagates according to Lighthill's inhomogeneous wave equation

$$\frac{\partial^2 \rho}{\partial t^2} - c^2 \nabla^2 \rho \equiv \square^2 \rho = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j} \quad (8)$$

$T_{ij}$  is only nonzero effectively in a confined region of space where turbulence is present. The boundary conditions under which equation (8) is to be solved usually take the form of a constraint that the normal velocity in the fluid is equal to that of the bounding solid surface, together with a radiation condition. The noise problem is then posed as that of finding the solution to equation (8) with  $T_{ij}$ , a specification of the driving turbulence, known.

It is troublesome to have the boundary conditions specified on a moving surface, so there is some benefit to accrue from a selection of a new coordinate system fixed relative to the surfaces. Unfortunately, the wave operator is not invariant to a Galilean transformation, so the benefit of easily specified boundary conditions is bought at the expense of a more intractable governing equation. This difficulty can be overcome for steadily moving surfaces by the use of a Lorentz transformation that slightly distorts the geometry of the boundaries. However, if the surfaces are thin planes parallel to the flow, the complication in the Lorentz frame is minimal, and the high-speed surface problems can then be reduced to elementary low-speed examples. This scheme can be used, for example, to study sound diffraction from a thin-walled pipe, with flow on both sides of the wall. It is also the scheme by which Berman has been successful in posing the general aerodynamic noise problem for steadily moving surfaces. The results of this approach reproduce the conclusions reached by Ffowes Williams and Hawkings for steady motion (ref. 4). However, their results are applicable to arbitrary motion, so they are more suitable as a basis from which to discuss the issues arising in turbomachinery noise, where rapid rotary accelerations are inevitable.

The solution to equation (8) can be written in a useful form as long as all source distributions are compact. That is, all surface and eddy sizes must be very much smaller than any wavelength in the radiation field. That solution is given in equation (7.4) of Ffowes Williams and Hawkings (ref. 4):

$$4\pi c^2(\rho - \rho_0)(\mathbf{x}, t) = \frac{\partial^2}{\partial x_i \partial x_j} \left[ \frac{Q_{ij} + \rho_0 v_i v_j V_0}{r |1 - M_r|} \right] - \frac{\partial}{\partial x_i} \left[ \frac{P_i + \rho_0 \dot{v}_i V_0}{r |1 - M_r|} \right] \quad (9)$$

$Q_{ij}$  is the integrated stress tensor,  $v_i$  the surface velocity,  $V_0$  the volume displaced by the surfaces, and  $P_i$  the force exerted by the surface on the fluid. The field is driven by a dipole system of strength equal to the body force excess over that required to change the displaced momentum and by a quadrupole made up in part by the integrated turbulence stress tensor and in part by the surface Reynolds stresses.  $|1 - M_r|$  is the Doppler factor by which the length scale in the sound field is reduced because of source motion at Mach number  $M_r$  toward the field point  $\mathbf{x}$ .

At low speeds, the dipole term is dominant, as can be seen from the usual estimating procedure of setting the force proportional to  $\rho U^2 L^2$  ( $L$  being a characteristic source length) and the integrated Reynolds stress to  $\rho U^2 L^3$ . The source radiates a field of wavelength  $\lambda$ , and the operator  $\partial/\partial x_i$  changes the magnitude of the differentiated quantity by a factor of order  $\lambda^{-1}$ . According to equation (9), the magnitude of the quadrupole field is, therefore,

$$c^2 \rho \sim \frac{\rho U^2 L^3}{r |1 - M_r|} \cdot \frac{1}{\lambda_0^2 |1 - M_r|^2} = \rho U^2 \left(\frac{L}{r}\right) \left(\frac{L}{\lambda}\right)^2 |1 - M_r|^{-1} \quad (10)$$

while that of the dipole-induced field is

$$c^2 \rho \sim \frac{\rho U^2 L^2}{r |1 - M_r|} \cdot \frac{1}{\lambda_0 |1 - M_r|} = \rho U^2 \left(\frac{L}{r}\right) \left(\frac{L}{\lambda}\right) |1 - M_r|^{-1} \quad (11)$$

$\lambda_0$  is the wavelength emitted by the source at rest, and  $\lambda = \lambda_0 |1 - M_r|$  is the wavelength at convection Mach number  $M_r$ .

The quadrupole is negligible evidently only as long as  $L/\lambda \ll 1$ , as it always is in the regime where equation (9) is relevant. Then, the dominant source term is the force exerted by the fluid on the surface, and the job of quieting a low-speed fan is essentially one of reducing the level of the unsteady forces on the fan surfaces. Not so when  $L/\lambda \ll 1$ , as is inevitably the case because of the Doppler contraction and the tendency for the wavelength to vary inversely with fan speed. The general solution to equation (8) was rewritten by Ffowes Williams and Hawkings in a form suitable for the noncompact distribution, and for the special case of steady high-speed motion this equation becomes

$$\begin{aligned} 4\pi c^2 (\rho - \rho_0) (\mathbf{x}, t) = & \int_{\bar{\Omega}, \tau} \frac{\partial^2 T_{rr}}{\partial r^2} \frac{c \, d\bar{\Omega} \, d\tau}{r} \\ & + \int_{\Gamma, \tau} \left\{ \operatorname{div} [\mathbf{m} \rho \cot \theta (u_r^2 - c^2)] + \cos \theta \frac{\partial T_{rr}}{\partial r} \right\} \frac{c \, d\Gamma \, d\tau}{r \sin \theta} \end{aligned} \quad (12)$$

where the suffix  $r$  is the tensor component in the direction of radiation (in a frame moving with the fluid),  $\bar{\Omega}$  is the surface  $r = \text{constant}$ ,  $\Gamma$  is the curve of intersection of  $\bar{\Omega}$  with the boundary surface  $S$ ,  $\mathbf{m}$  is a unit vector normal to  $\Gamma$  in  $S$ , and  $\theta$  is the angle between the surface normal and the radiation direction.  $\tau$  is a time variable.

The usual dimensional analysis of this equation shows that both the noncompact quadrupoles and surface dipoles radiate fields of comparable magnitude, of order

$$c^2 \rho \sim \rho U^2 \frac{c\tau}{r} \quad (13)$$

where  $\tau$  is the coherent radiating lifetime of a source. For steady fields, this lifetime is infinite and leads to the shock fields of steady supersonic flow. For the broadband noise, however, the time is the inverse frequency  $L/U$ , so that at high speeds the broadband noise amplitude is

$$c^2 \rho \sim \rho U^2 \left( \frac{L}{r} \right) M^{-1} \quad (14)$$

This is initially the same magnitude as that of the steady pressure waves radiated from a blade in rapid supersonic motion, but the steady field is not subjected to the same geometrical spreading law.

Equation (12) makes it clear that the fluctuating force on the blade does not appear explicitly as a source term. In place of the surface stress we have the Reynolds stress gradient, which is the same term as that responsible for volume sound. Also, the lifetime of the source appears directly in the field strength, so that the random field would be quieter if the turbulent eddies were made to lose their coherence more quickly.

A point that is perhaps worth making also is that at the high supersonic speeds the noise seems to increase in direct proportion to thrust and that the noise per unit power is a decreasing function of speed.

The differences that feature in the high-speed noise are due entirely to the inevitable contraction in wavelength to a point where sound is no longer easily distinguishable from the driving unsteady flow. Then the process of attempting a specification of surface conditions in the absence of sound, followed by a prediction of the sound field from these estimated boundary values, fails. The problem has become inevitably one of diffraction, where both surface loads and the radiated sound must be estimated together. This procedure is only in its early stages in the study of aerodynamic noise, but already effects are being predicted quite contrary to the qualitative ideas one gathers from extrapolating known results for compact source distributions. The most readily measured property of noise is the variation in energy level with increasing speed. Compact quadrupoles and dipoles have a  $U^8$  and  $U^6$  variation, respectively, while both have a  $U^2$  variation in the noncompact condition. Away from these limits, one can observe a very wide range of velocity indices resulting from the scattering of the quadrupole field by noncompact surfaces. No doubt this work is relevant to the broadband noise inside fan systems wherever the fan chord exceeds the radiated wavelength. However, the ideas discussed here have yet to be applied to that situation.

## REFERENCES

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## DISCUSSION

J. VRANA (McGill University): It is interesting to note the comment by Dr. Ffowes Williams that nonlinearity need not be invoked to explain the occurrence of combination noise. Let us face it, the nonlinearity is there; that is the way shock waves propagate.

In thinking about the phenomenon, I was intrigued by the small effects of attempts to make the blading as uniform as possible. Could the bunching of shock waves be an inherent instability of certain flow fields, where, for instance, a shock displaced slightly forward would "lose" more expansion Mach waves behind it than it gained in front (speaking in the 2-D blade-to-blade terminology of the compressor designer), thereby increasing in strength and moving further forward to a new stable position? Blading imperfections would then merely act as triggers for this departure from periodicity.

T. G. SOFRIN AND G. F. PICKETT (Pratt & Whitney Aircraft): Professor Ffowes Williams has presented a linear theory explaining the generation of tones at harmonics of engine rotation frequency. Although he shows that these tones may be produced by small circumferential variations in shock spacing, the lack of a mechanism to account for evolution of greater proportions of extra tones with increasing distance is a significant limitation. The probe data outlined in our paper clearly show that small circumferential variations in shock spacing are amplified because of the nonlinear nature of the propagating shock waves, and we contend that this is the dominant factor in the distribution of the power in the direct rotor field throughout the harmonics of engine rotation frequency.

FFOWCS WILLIAMS (author): Mr. Vrana suggests an interesting possibility, and I know that some people have been working on the stability of the shock train although I haven't myself. I think it's quite likely that it has an important bearing on this problem, but if this is just simply an instability issue, I can't really see why the experiments show that the signature is exactly the same every time the disk goes around. Your comment about the nonlinearity being there anyway: Of course it's there. But it's certainly not necessary to "buzz saw." You could get buzz saw for vanishingly low amplitude at supersonic speeds. That was the point I was making.

The information given by Mr. Sofrin and Mr. Pickett is a very convincing account of the observation that, in their experiment, nonlinear effects are very important. They are likely important in all real engine situations where buzz-saw noise is observed. The point I make, however, is that buzz saw is also inevitable for arbitrarily small amplitude. The linear theory I describe shows how blade spacing errors will account for a buzz-saw phenomenon, but I have no doubt that, in the experiment reported, the main phasing errors were induced by nonlinear effects. However, nonlinearity is not *essential* to the occurrence of buzz saw.