(NASA-CR-140857) A PROPOSAL TO STUDY POLARIZATIONS FOR HYDROMAGNETIC WAVES OBSERVED AT ATS-1 Final Report (Grambling State Univ., La.) 17 p HC $3.25 CSCL 03B G3/46
N75-12464
Unclas
02883

FINAL TECHNICAL REPORT

SUBMITTED TO:	NASA Scientific and Technical Information Facility P.O. Box 33 College Park, Maryland 20740
REFERENCE NUMBER	NGR 19-011-013
INSTITUTION	Grambling State University Grambling, Louisiana 71245
TITLE:	A Proposal to Study Polarizations for Hydromagnetic Waves Observed at ATS-1
PRINCIPAL INVESTIGATOR	N. Gajendar Associate Professor Department of Mathmatics
DURATION:	One (1) Year
SUM GRANTED BY NASA	$9,267
DATE:	November 7, 1974
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A STUDY OF POLARIZATIONS FOR HYDROMAGNETIC WAVES OBSERVED AT ATS-1

Final Report for NGR 19-011-013
by N. Gajendar

Introduction

Low frequency oscillations of the magnetic field have been observed in the magnetosphere since 1960; several of these fluctuations have been interpreted in terms of hydromagnetic waves (Coleman et al., 1960; Sonett et al., 1962; Judge and Coleman, 1962; Nishida and Cahill, 1964; Patel and Cahill, 1964; Patel, 1965, 1966). In all of these cases, the satellite was apparently in motion at rather high velocities relative to the plasma in which the waves were propagating at the time of their observation. As a result, the recorded oscillations in most of the observations were of short duration and seldom included more than a few cycles. Recently, however, satellites at synchronous and near synchronous orbit have provided an opportunity to observe these low frequencey oscillations over long durations (Cummings et al., 1969; Dwarkin et al., 1970).

The data collected using the UCLA magnetometer that was placed on the ATS-1 satellite, indicate that at least three types of oscillations occur:

1. Oscillations with the magnetic component perpendicular to the background field (Cummings et al., 1969). These so called "transverse oscillations" occur predominantly during geomagnetically quiet conditions. The ATS-1 satellite is presumably sometimes within and sometimes without the plasma sphere when these oscillations occur. These oscillations aremost frequently observed during

the afternoon with a peak in the occurrence rate between 1400 and 1500 L.T. The most often observed period is in the range 72 - 84 seconds (Cummings et al., 1971). Even under geomagnetically quiet conditions, the oscillations apparently occur in a high β plasm (β is the ratio of the particle energy density to the field energy density). Frank (1967, 1971) reports particle energy densities in the range $3^{-9} \times 10^{-9}$ joules/m^3 at the synchronous orbit during quiet conditions. This corresponds to a β of $\simeq$ 1-2 for a magnetic field of 100γ.

2. Oscillations during geomagnetically quiet conditions that have the magnetic component parallel to the background field (Barfield et al., 1971). These oscillations occur much less frequently than the transverse oscillations.
3. Oscillations during magnetic storms that have the magnetic component in the magnetic meridional plane (Barfield and Coleman, 1970; Barfield et al., 1971). These oscillations are composed of harmonically related components. They typically occur during the main phase and are confined to the afternoon sector of the magnetosphere.

Cummings et al (1969) have used the simplest possible set of hydromagnetic equations and solved for Standing wave eigen frequencies as functions of the background plasma distribution in the magnetosphere. From the results of the analysis it was argued that the observed transverse oscillations are the second harmonic of a Standing Alfven wave. Under this interpretation the data are consistent with the hypothesis that the plasma pressure beyond $6.6R_E$ only during very quiet periods.

Cummings and Gajendar (1971) have extended the theoretical work done by Cummings et al., (1969), to include the effects of a gradient in background plasma pressure. They used a two fluid approach, following Spitzer (1962). The following basic set of linearized equations were used in this study (Cummings and Gajendar, Final Report for NAS-9-10244, 1971):

a) $\vec{\nabla}\times\vec{B} = 4\pi\vec{J}$ Ampere's Law

b) $\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$ Faraday's Law

c) $\vec{E}+\vec{v}\times\vec{B}_o+\vec{V}_o+\vec{B}_o+\frac{c}{en_o}\vec{\nabla}p_e+\frac{n}{n_o}\vec{V}_o\times\vec{B}_o=0$ Ohm's Law

d) $\rho_o\, d\vec{v}/dt = \vec{J}\times\vec{B}_o - 2\vec{\nabla}p_e$ Momentum Eqn

e) $\vec{\nabla}p_e = KT\vec{\nabla}n$ Iso-thermal Condition

f) $\partial n/\partial t = -[\vec{V}_o\cdot\vec{\nabla}n + n_o\vec{\nabla}\cdot\vec{v} + \vec{v}\cdot\vec{\nabla}n_o]$ Continuity Eqn

The above set of equations contains the following simplifying assumptions:

a') the displacement current in Ampere's Law is ignored.

c') only the electron pressure gradient in Ohm's Law (Spitzer, 1962) is included.

e') it is assumed that the fluctuations in the pressure gradient are caused by fluctuations in the density.

Further, it is assumed that $n_i = n_e$, $T_i = T_e = T$, and finite Larmor radius effects are ignored. The analysis is further simplified by considering the case of off-axis propagation, where the propagation vector $\vec{k}$ is assumed to lie in the plane formed by n_0 and B_0; $E_z = V_z = 0$ and , $(\vec{\nabla} n_0)_x = 0$ and $(\vec{\nabla} n_0)_y \neq 0$.

We established the following conclusions during the above study:

1. Two modes of polarizations exist: one corresponds to the propagation along the background magnetic field which is referred to as the "parallel mode". The second mode of polarization referred to as "perpendicular mode", is actually composed of two submodes; both submodes correspond to propagation perpendicular to the plane defined by the background magnetic field vector and the gradient in the background plasma density.
2. The polarization of the magnetic vector for the "parallel mode" is perpendicular to both the background magnetic field and the gradient in the background plasma density. Assuming the background plasma density has an approximately radial gradient at ATS-1, the polarization for this mode would be in the azimuthal direction (the D-direction in ATS-1 coordinates). The oscillations at ATS-1 that occur on geomagnetically quiet days are rarely polarized in the aximuthal direction (<u>Cummings et al.,</u> 1969). The polarization is infact more often observed near the radial direction.
3. The polarization of the magnetic vector for the first submode of the perpendicular mode is essentially parallel to the background magnetic field vector, for conditions of both high and low β. This submode might correspond to the compressional oscillations that is infrequently observed at ATS-1 on geomagnetically quiet days (<u>Barfield et al.,</u> 1971).
4. The polarizations of the magnetic vector for the second submode of the perpendicular mode is in the plane defined by the magnetic field vector and the gradient in the

background plasma density. For low β, the magnetic vector is approximately parallel or antiparallel to the gradient in the background plasma density. This submode could therefore correspond to the "transverse oscillations" that are often observed at ATS-1 during geomagnetically quiet days (Cummings et al., 1969).

5. For $\beta \simeq 1$, the magnetic vector for the second submode has a significant component in the direction of the background magnetic field. This second submode of the perpendicular mode could therefore also correspond to the storm-related oscillations observed at ATS-1 (Barfield and Coleman, 1970; Barfield et al., 1971).

Analysis

A more general study of the theoretical model presented by Cummings and Gajendar (1971) for the low frequency oscillations observed at ATS-1 during magnetically quiet conditions, is made by removing the apparent inconsistencies of the model given by Cummings and Gajendar (1971).

It might be noted that in the Momentum equation (Eqn. d), the term involving the background current density is ignored. When this is included, the Momentum equation would read,

d') $$\rho_o \frac{\partial \vec{v}}{\partial t} = \vec{J} \times \vec{B}_o + \vec{J}_o \times \vec{B} - 2\vec{\nabla} p_e$$

It is the purpose of this study to include the background current density term which may have interesting effects on the polarizations of the low frequencies oscillations during geomagnetically quiet conditions.

The new set of basic equations would be the same as those already considered by Cummings and Gajendar (1971), except for the momentum equation which takes the form as given in eqn.(d'); the basic equations using the simplified assumptions a',c' and e', are:

(1) $$\vec{\nabla} \times \vec{B} = 4\pi \vec{J}$$

(2) $$\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$$

(3) $$\vec{E} + \vec{V} \times \vec{B}_o + \vec{V}_o \times \vec{B} + \frac{c}{e n_o} \vec{\nabla} p_e + \frac{n}{n_e} \vec{V}_o \times \vec{B}_o = 0$$

(4) $\rho_0\, \partial\vec{v}/\partial t = \vec{J}\times B_0 + \vec{J}_0\times\vec{B} - 2\vec{\nabla} p_E$

(5) $\vec{\nabla} p_e = KT\vec{\nabla} n$

(6) $\partial n/\partial t = -\left[\vec{v}_0\cdot\vec{\nabla} n + n_0\vec{\nabla}\cdot\vec{v} + \vec{v}\cdot\vec{\nabla} n_0\right]$

Using a complex manipulation, the basic set of equations 1 through 6, are reduced to give the wave equation,

(7) $$\vec{A}\times[\vec{A}\times \mathrm{curl}\,\mathrm{curl}\,\vec{E}] = \frac{\partial^2\vec{E}}{\partial t^2} - \vec{v}_0\times\frac{\partial}{\partial t}(\mathrm{curl}\,\vec{E}) + \frac{i c K T \omega^2 \vec{k} n}{e n_0} - \omega^2 n\frac{\vec{v}_0\times\vec{B}_0}{n_0} + \frac{2KT\omega n}{\rho_0}(\vec{B}_0\times\vec{k}) + \frac{1}{\rho_0}\vec{B}_0\times\vec{J}_0\times(\mathrm{curl}\,\vec{E})$$

and,

(8) $$n = \frac{A^2 n_0 k_x k^2 E_y}{B_0\omega^3\Delta},$$

where $|\vec{A}| = B_0/\sqrt{4\pi m_i n_0} \equiv$ the Alfven Speed

$$\Delta = \left[1 - \left(\frac{c_s}{\omega}\right)^2 k^2 + \left(\frac{\Omega_i}{\omega}\right) R_i^2 k_x \frac{(\nabla n_0)_y}{n_0}\right]$$

$\Omega_i = eB_0/m_i c \equiv$ the ion Cyclotron frequency

$R_i = \sqrt{kT/m_i}\,/\,\Omega_i \equiv$ the ion Cyclotron radius

$c_s = \sqrt{2kT/m_i} \equiv$ the Sound Speed

Using equations (7) and (8) and assuming all wave quantities vary as

$$\exp\{i(\omega t - k_x x - k_z z)\}, \quad \text{we have}$$

(9) $$\frac{\partial^2 \vec{E}}{\partial t^2} = \vec{A} \times \left[\vec{A} \times \text{curl}\, \text{curl}\, \vec{E}\right] + \Omega_i R_i^2 \omega k_z E_x \frac{(\nabla n_o)_y}{n_o} \hat{e}_z$$
$$- \Omega_i R_i^2 \omega k_x E_y \frac{(\nabla n_o)_y}{n_o} \hat{e}_y$$
$$- A^2 \left(\frac{\Omega_i}{\omega}\right) \frac{R_i^2}{\Delta} k_x k^2 E_y \Big[i(k_x \hat{e}_x + k_z \hat{e}_z)$$
$$+ \frac{(\nabla n_o)_y}{n_o} \hat{e}_y \Big]$$
$$- \frac{C_s^2}{\omega^2} \frac{A^2}{\Delta} k_x^2 k^2 E_y \hat{e}_y$$
$$+ \frac{B_o}{\rho_o} \left(\frac{\Omega_i}{\omega}\right) \frac{R_i^2}{\Delta} j_o k_z^2 E_x (k_x \hat{e}_x + k_z \hat{e}_z)$$
$$- i \frac{B_o}{\rho_o} \left(\frac{\Omega_i}{\omega}\right) \frac{R_i^2}{\Delta} j_o k_z^2 E_x \frac{(\nabla n_o)_y}{n_o} \hat{e}_y$$
$$- i \frac{C_s^2}{\rho_o \omega^2} \frac{j_o}{\Delta} k_z^2 k_x B_o E_x \hat{e}_y$$
$$- \frac{i}{\rho_o} B_o j_o k_x E_y \hat{e}_x$$

Thus, we have the following dispersion relations:

$$\left[\omega^2 - A^2 k_z^2 + \frac{B_0}{\rho_0}\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2}{\Delta}\vec{J}_0 k_z^2 k_x\right] E_x$$

(10)
$$-\left[i A^2 \left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2}{\Delta} k_x^2 k^2 + \frac{B_0}{\rho_0}\vec{J}_0 k_x\right] E_y = 0$$

$$\left[\omega^2 - A^2 k^2 - \Omega_i R_i^2 \omega k_x \frac{(\nabla n_0)_y}{n_0} - A^2\left(\frac{\Omega_i}{\omega}\right)\cdot\right.$$

(11)
$$\left.\cdot \frac{R_i^2}{\Delta} k_x k^2 \frac{(\nabla n_0)_y}{n_0} - \left(\frac{C_s}{\omega}\right)^2 \frac{A^2}{\Delta} k_x^2 k^2\right] E_y$$

$$-\left[i \frac{B_0}{\rho_0}\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2}{\Delta}\vec{J}_0 k_z^2 \frac{(\nabla n_0)_y}{n_0}\right.$$
$$\left.+ i \frac{C_s^2}{\rho_0 \omega^2}\frac{\vec{J}_0}{\Delta} k_z^2 k_x B_0\right] E_x = 0$$

$$\left[\Omega_i R_i^2 \omega \frac{(\nabla n_0)_y}{n_0} + \frac{B_0}{\rho_0}\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2}{\Delta}\vec{J}_0 k_z^2\right] E_x$$

(12)
$$- i A^2 \left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2}{\Delta} k_x k^2 E_y = 0$$

The following notation is used in the dispersion relations given by eqns. 10, 11 and 12:

$$\Delta = \left[1-\left(\frac{C_s}{\omega}\right)^2 k^2 + \left(\frac{\Omega_i}{\omega}\right) R_i^2 k_x \frac{(\nabla n_o)_y}{n_o}\right]$$

$A = B_o/\sqrt{4\pi m_i n_i} \equiv$ the Alfven speed

$\Omega_i = eB_o/m_i c \equiv$ the ion cyclotran frequency

$R_i = \left(\sqrt{kT/m_i}\right)/\Omega_i \equiv$ the ion cyclotran radius

$C_s = \sqrt{2kT/m_i} \equiv$ the sound speed

Eliminating E_x/iE_y from equations (11) and (12) we have the following expression for k_z^2:

$$k_z^2 = \frac{\omega^2}{A^2}\cdot\frac{\left[1-k_x^2\left\{\frac{A^2}{\omega^2}+2\left(\frac{\Omega_i}{\omega}\right)^2 R_i^2\left[1+\frac{1}{2}R_i^2\left(\frac{\nabla n_o}{n_o}\right)^2\right]\right\} - k_x^3\left\{\frac{2A^2}{\omega^2}\left(\frac{\Omega_i}{\omega}\right)R_i^2-2\left(\frac{\Omega_i}{\omega}\right)^3 R_i^4\right\}\left(\frac{\nabla n_o}{n_o}\right)\right]}{\left\{1+2\left(\frac{\Omega_i}{\omega}\right)R_i^2 k_x\left(\frac{\nabla n_o}{n_o}\right)\right\}} \tag{13}$$

Using the above expression for k_z^2 in the equations (10) and (12), and eliminating E_x/iE_y we get a sixth degree polynomial equation for k_x. When $R_i^2 \quad \Omega_i^2 - A^2 = 0$, the sixth degree polynomial equation is k_x, reduces to a fifth degree polynomial equation in k_x. These polynomial eauations for k_x are shown in equations (14) and (15) respectively.

The sixth degree polynomial eqn. in k_x is

(14)
$$
\begin{aligned}
& k_x^6\left[4\left(\frac{\Omega_i}{\omega}\right)^2 R_i^8\,\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^2-4A^2\left(\frac{\Omega_i}{\omega}\right)^2 R_i^6\,\Omega_i^2\left(\frac{\nabla n_o}{n_o}\right)^2\right] \\
& + k_x^5\left[2A^2\left(\frac{\Omega_i}{\omega}\right)^2\Omega_i^2 R_i^4\left(\frac{\nabla n_o}{n_o}\right)-8\left(\frac{\Omega_i}{\omega}\right)\Omega_i^4 R_i^6\left(\frac{\nabla n_o}{n_o}\right)\right. \\
& \qquad \left. + 2A^2\Omega_i^2\left(\frac{\Omega_i}{\omega}\right)R_i^6\left(\frac{\nabla n_o}{n_o}\right)^3\right] \\
& + k_x^4\left[4\Omega_i^4 R_i^4+2A^2\Omega_i^2 R_i^2-2R_i^6\,\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^2\right. \\
& \qquad + A^2\Omega_i^2 R_i^4\left(\frac{\nabla n_o}{n_o}\right)^2-R_i^8\,\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^4 \\
& \qquad \left. -16R_i^8\left(\frac{\Omega_i}{\omega}\right)^2\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^4\right] \\
& + k_x^3\left[-2A^2\omega\Omega_i R_i^2\left(\frac{\nabla n_o}{n_o}\right)+4\omega\Omega_i^3 R_i^4\left(\frac{\nabla n_o}{n_o}\right)\right. \\
& \qquad + \omega^2\Omega_i^2 R_i^6\left(\frac{\nabla n_o}{n_o}\right)^3-16\left(\frac{\Omega_i}{\omega}\right)R_i^6\,\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^3 \\
& \qquad \left. + 8R_i^8\left(\frac{\Omega_i}{\omega}\right)^2\Omega_i^3\,\omega\left(\frac{\nabla n_o}{n_o}\right)^5\right] \\
& + k_x^2\left[-A^2\omega^2-2\Omega_i^2\,\omega^2 R_i^2+R_i^4\,\Omega_i^2\,\omega^2\left(\frac{\nabla n_o}{n_o}\right)^2\right. \\
& \qquad \left. -4R_i^4\,\Omega_i^4\left(\frac{\nabla n_o}{n_o}\right)^2+16\Omega_i^4 R_i^6\left(\frac{\nabla n_o}{n_o}\right)^4\right] \\
& + k_x\left[10\Omega_i^3\,\omega R_i^4\left(\frac{\nabla n_o}{n_o}\right)^3-\omega^3\Omega_i R_i^2\left(\frac{\nabla n_o}{n_o}\right)\right] \\
& + 2R_i^2\,\omega^2\Omega_i^2\left(\frac{\nabla n_o}{n_o}\right)^2=0
\end{aligned}
$$

When $R_{i\circ}^{2}\Omega_{i\circ}^{2} = A^{2}$, the polynomial equation under consideration becomes:

$$
\begin{aligned}
& k_x^5\Big[2A^2\Big(\frac{\Omega_{i\circ}}{\omega}\Big)^2\Omega_{i\circ}^2 R_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big) - 8\Big(\frac{\Omega_{i\circ}}{\omega}\Big)\cdot \\
& \qquad \cdot\,\Omega_{i\circ}^4 R_{i\circ}^6\Big(\frac{\nabla n_0}{n_0}\Big) + 2A^2\Omega_{i\circ}^2\Big(\frac{\Omega_{i\circ}}{\omega}\Big)R_{i\circ}^6\Big(\frac{\nabla n_0}{n_0}\Big)^3\Big] \\
& + k_x^4\Big[4\Omega_{i\circ}^4 R_{i\circ}^4 + 2A^2\Omega_{i\circ}^2 R_{i\circ}^2 - 2R_{i\circ}^6\Omega_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^2 \\
& \qquad + A^2\Omega_{i\circ}^2 R_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^2 - R_{i\circ}^8\Omega_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^4 \\
& \qquad - 16R_{i\circ}^8\Big(\frac{\Omega_{i\circ}}{\omega}\Big)^2\Omega_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^4\Big] \\
& + k_x^3\Big[-2A^2\omega\,\Omega_{i\circ} R_{i\circ}^2\Big(\frac{\nabla n_0}{n_0}\Big) + 4\omega\,\Omega_{i\circ}^3 R_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big) \\
& \qquad + \omega^2\Omega_{i\circ}^2 R_{i\circ}^6\Big(\frac{\nabla n_0}{n_0}\Big)^3 - 16\Big(\frac{\Omega_{i\circ}}{\omega}\Big)R_{i\circ}^6\Omega_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^3 \\
& \qquad + 8R_{i\circ}^8\Big(\frac{\Omega_{i\circ}}{\omega}\Big)^2\Omega_{i\circ}^3\,\omega\Big(\frac{\nabla n_0}{n_0}\Big)^5\Big] \\
& + k_x^2\Big[-A^2\omega^2 - 2\Omega_{i\circ}^2\omega^2 R_{i\circ}^2 + R_{i\circ}^4\Omega_{i\circ}^4\omega^2\Big(\frac{\nabla n_0}{n_0}\Big)^2 \\
& \qquad - 4R_{i\circ}^4\Omega_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^2 + 16\Omega_{i\circ}^4 R_{i\circ}^6\Big(\frac{\nabla n_0}{n_0}\Big)^4\Big] \\
& + k_x\Big[10\Omega_{i\circ}^3\omega R_{i\circ}^4\Big(\frac{\nabla n_0}{n_0}\Big)^3 - \omega^3\Omega_{i\circ} R_{i\circ}^2\Big(\frac{\nabla n_0}{n_0}\Big)\Big] \\
& + 2R_{i\circ}^2\omega^2\Omega_{i\circ}^2\Big(\frac{\nabla n_0}{n_0}\Big)^2 = 0
\end{aligned}
\tag{15}
$$

The polynomial equation k_x is solved for the following constants and parameter values as a test case:

Constants

$m_i = 1.61 \times 10^{-24}$

$e = 4.803 \times 10^{-10}$

$c = 3 \times 10^{10}$ cm/sec

$\pi = 3.1416$

Parameters

$B_o = 1.2 \times 10^{-3}$ gauss

$n_o = 5$ particles/cm^3

$\beta = 0.25\ (0.25)\ 1.5$

$\lambda = 3.2 \times 10^{8}$ cm

$w = 0.082$

The roots of the polynomial equation is shown for different values of β in the following table:

k_x / β	$k_x(1)$	$k_x(2)$	$k_x(3)$	$k_x(4)$	$k_x(5)$	$k_x(6)$
0.25	0.131202E-01	0.710023E04	-0.345787E.03 -i0.430916E-03	-0.345787E-03 +i0.430916E-03	-0.131099E-01	0.632671E-03
0.50	0.234223E-01	0.583645E04	-0.367176E-03 -i0.434761E-03	-0.367176E-03 +i0.434761E-03	-0.234258E-01	0.652622E-03
0.75	0.344022E01	0.569801E04	-0.357304E-03 -i0.399383E-03	-0.357304E-03 +i0.399383E-03	-0.344143E-01	0.639591E-03
1.00	0.456009E-01	0.614606E04	-0.274727E-03 -i0.631589E-04	-0274727E-03 +i0.631589E-04	-0.456219E-01	0.631389E-03
1.25	0.569710E-01	0.730311E04	-0.426458E-05 -i0.264374E.02	-0.426458E-05 +i0.264374E-02	-0569445E-01	0.177775E-04
1.50	0.683382E-01	0.996392E04	0387256E-05 -i0.193962E-02	-0.387256E-05 +i0.193962E-02	-0.683456E-01	0.266411E-04

In the work by Cummings and Gajendar, a gradient in the background plasma pressure was considered in Ohm's Law. The gradient of the magnetic field, however, was not considered. It is infact actually necessary to include the gradient in the magnetic field which certainly affects the drift velocity of plasma.

When the gradient in the magnetic field considered by assuming that the background magnetic field B_o is dependent on y, we have the following additional terms in the expression for n:

$$+ \frac{i n_o \frac{\partial B_o}{\partial y} k_x k_z E_x}{4\pi \rho_o \omega^3 \Delta}$$

The additional terms in the wave equation are:

$$\frac{\Omega_i R_i^2 B_o}{4\pi \rho_o \omega \Delta} \frac{\partial B_o}{\partial y} E_x (k_x \hat{e}_x + k_z \hat{e}_z) k_x k_z$$

$$- \frac{\Omega_i R_i^2 B_o}{4\pi \rho_o \omega \Delta} \frac{\partial B_o}{\partial y} E_x k_x k_z \left(\frac{\vec{\nabla} n_o}{n_o}\right)$$

$$- i \frac{C_s^2}{4\pi \rho_o \omega^2 \Delta} B_o \frac{\partial B_o}{\partial y} E_x k_x^2 k_z \hat{e}_y$$

The additional terms in the three dispersion relations are:

for equation (10):

$$+\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2 B_0 \frac{\partial B_0}{\partial y}}{4\pi \rho_0 \Delta} E_x k_x^2 k_z$$

for equation (11):

$$-\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2 B_0 \frac{\partial B_0}{\partial y}}{4\pi \rho_0 \Delta} k_x k_z \left(\frac{\nabla n_0}{n_0}\right)_y E_x$$

$$- i\left(\frac{c_s}{\omega}\right)^2 \frac{B_0 \frac{\partial B_0}{\partial y}}{4\pi \rho_0 \Delta} k_x^2 k_z E_x$$

for equation (12):

$$+\left(\frac{\Omega_i}{\omega}\right)\frac{R_i^2 B_0 \frac{\partial B_0}{\partial y}}{4\pi \rho_0 \Delta} k_x k_z^2 E_x$$

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