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DIRECTIONAL DISCONTINUITIES FROM MARINER 5
AND SOLAR WIND STRUCTURE

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ABSTRACT

Directional discontinuities from Mariner 5 are studied. A substantial majority of the events have properties which are consistent with their being identified as tangential discontinuities. The results presented here agree in several respects with those of Burlaga from Pioneer 6. In addition, new results are presented. Finally, there is a summary of previous Mariner 5 work on this subject with an explanation of apparent inconsistencies.

"DIRECTIONAL DISCONTINUITIES AND SOLAR WIND STRUCTURE"

1. Introduction

In this paper, directional discontinuities from Mariner 5 data will be studied. New findings regarding the nature and structure of these events will be presented. The results will be compared and contrasted with those obtained from Mariner 5 by several other investigators and an erroneous interpretation used by other investigators regarding the results of Turner and Siscoe (1971) will be corrected.

The definition of directional discontinuity to be used here is similar to that used by Burlaga (1969a,b,1971). That is, a change in field direction of 30° or more occurring between consecutive data points and uniform conditions for about 10 minutes on either side. It was found that many of Burlaga's conclusions using data from Pioneer 6 were duplicated in the present case. These include the finding that the population is dominated by tangential discontinuities, that the cross product of the pre- and post-discontinuity magnetic field preferentially laid in the ecliptic plane along the orthospiral direction, that his distribution function for the angle between the fields across the discontinuity fit

this data, that there usually was no change in the field magnitude, and the behavior of discontinuities with large velocity changes also did not agree with the general population.

New results will show that the direction of minimum field fluctuation is nearly the same some distance from the discontinuity as that found near the discontinuity. In addition, it was found that in fast streams, the discontinuities are strongly characterized by little or no change in field magnitude, while in slow streams, there is a definite increase in events with field change.

Previous work by Turner and Siscoe (1971) has been interpreted by others to mean that tangential and rotational discontinuities are comparable in numbers. However, the data show that tangential discontinuities are much more prevalent and this finding is substantiated by the events here.

There will be a summary of previous work on the Mariner 5 data with an explanation of apparent inconsistencies.

1. The Experiment and Data Selection

The data used was from the first forty days of the Mariner 5 mission, during which the data rate was highest.

The plasma samples were averaged over 5.04 minutes. The field was computed every 12.6 seconds and also averaged over 5.04 minutes. The magnetic field experiment (Connor, 1968) and plasma experiment (Lazarus et al., 1967) have been previously described. The coordinate system used is the familiar satellite-centered RTN system.

To be accepted, each event was required to satisfy four criteria. First, the magnetic field must change direction by an angle $\geq 30^\circ$ between consecutive data points. Second, each event was subjectively judged for a relatively steady plasma and field condition on both sides of the discontinuity. The actual points used in the analysis were not those between which the field direction changed, but rather the points preceding and following them. Third, to optimize the reliability of $\underline{B}_1 \times \underline{B}_2$, the standard deviations computed for each component were considered. To be selected as a discontinuity, an event was required to satisfy the relationship $\sin^{-1} 3\sigma/|B| < \cos^{-1} (\underline{B}_1 \cdot \underline{B}_2 / |\underline{B}_1| |\underline{B}_2|)$ where 1 and 2 refer to pre- and post-discontinuity fields and $\frac{3\sigma}{|B|}$ is the larger of $3\sigma_1/|\underline{B}_1|$ and $3\sigma_2/|\underline{B}_2|$. Finally, the points used in the analysis also had to be at least 30° apart in field direction.

It should also be noted that the small subset of tangential and rotational discontinuities reported earlier in Turner and Siscoe (1971) and Turner (1973) were disqualified, since statistics developed from them were used to help identify the character of the directional discontinuities.

A typical event is shown in Figure 1.

2. Results

Using the above criteria, 111 directional discontinuities were found throughout the 40 days under consideration. Only 4 days did not have at least one of the 111 events and these 4 days were scattered randomly among the 40 days.

The events were qualitatively similar in many respects to those mentioned in the studies of Pioneer 6 and Mariner 4 (Burlaga, 1969, a,b, 1971, Siscoe et al., 1968) data. First, the magnitude of the field usually had little or no change across the discontinuity as is shown in Figure 2, where a histogram for the distribution of $|\underline{B}_1|/|\underline{B}_2|$ is plotted. This point will be discussed in greater detail later. From this evidence it may be immediately concluded that fast and slow MHD shocks played little or no role with regard to the

directional discontinuities since for the shocks the field magnitude must change and generally by a significant amount. The distribution of the angle between the pre- and post-discontinuity magnetic field could be fitted very well by the distribution function used by Burlaga, i.e. $A \exp(-w/75^\circ)^2$. The predictions from Burlaga's distribution function and the actual results are compared in Figure 3.

Burlaga also examined separately those discontinuities which had large bulk velocity changes across them, where by large he means ≥ 60 km/s. He found that this subset of the total number of directional discontinuities had a distribution of w which differed from that describing the entire set of events. For these, w tended to cluster around 90° . A similar difference appeared in the discontinuities reported here. Only two events could qualify as large velocity discontinuities in Burlaga's sense, and they had w 's of 74° and 76° . In addition, there were three other events which had bulk velocity changes ≥ 50 km/s. In the five events which had bulk velocity changes ≥ 50 km/s, all five had w 's $\geq 50^\circ$ and four of the five had w 's $\geq 60^\circ$, which is much different than the general distribution.

Some of the rotational and tangential discontinuities observed by Mariner 5 were analyzed by Turner and Siscoe (1971), and Turner (1973). The number of each type of discontinuity was severely limited by stringent criteria for acceptance. These particular events were not included in the directional discontinuities considered above. As mentioned previously, it is unlikely that MHD shocks play significant roles in the directional discontinuities. Since there is no change in the field direction across a contact discontinuity, the directional discontinuities observed must be either rotational or tangential discontinuities. A comparison of the directional, rotational, and tangential discontinuity results should indicate roughly what percentage of the selected directional discontinuities are tangential and what percentage are rotational.

Turner (1973) and Turner and Siscoe (1971) found that tangential and rotational discontinuities had significantly different preferred orientations of $\underline{B}_1 \times \underline{B}_2$. For the tangential discontinuities $\underline{B}_1 \times \underline{B}_2$ was close to the ecliptic plane and perpendicular to the spiral field direction, while for rotational discontinuities $\underline{B}_1 \times \underline{B}_2$ had a significant N component. In

addition, not only was the magnetic field confined to the tangential discontinuity surface at the discontinuity, which it must do, but also was still confined to that plane some distance from the discontinuity. The preferred orientation of $\underline{B}_1 \times \underline{B}_2$ for the set of directional discontinuities was obtained by applying the variance matrix technique (Turner and Siscoe, 1971) to the cross products. The results are shown in Table 1 and the preferred orientation is $\sim 10^\circ$ from that found for tangential discontinuities reported earlier.

To find if the field away from the discontinuity was confined to a plane, each day was divided into eight three hour segments. For each segment in which there was a directional discontinuity, the point-by-point change in \underline{B} was computed and the matrix technique applied to the changes in order to find a direction in which \underline{B} was least likely to change. Two conditions were applied to the segments; first, the segment had to contain at least half the maximum possible number of data points in that segment and second, the ϵ_i^2 for the least likely direction of change had to be less than $2/3$ the ϵ_i^2 for the intermediate direction to insure that in that segment there is a clear direction of least likely change. The matrix technique was applied again, this time to

directions of least likely change for segments which met the two conditions. This process determined a least likely direction of change in \underline{B} characteristic of the set of directional discontinuities. The result is given in Table 2 and the direction of least likely change is (very near) the preferred orientation of $\underline{B}_1 \times \underline{B}_2$ given in Table 1. Thus the directional discontinuities are similar in these respects to the tangential discontinuities found.

In Figure 4 a,b, are distributions of w for the tangential and rotational discontinuities, respectively reported earlier by Turner and Siscoe. Forty rotational and 35 tangential discontinuities were found previously. Those for which $\geq 30^\circ$ were selected and normalized to 111 for comparison with the directional discontinuities. More than 70% of these tangential and rotational discontinuities had changes $\geq 30^\circ$. The tangential discontinuity distribution comes closer to matching that for the directional discontinuities. However, it may be noted that the directional discontinuity distribution may be closely approximated by a linear combination of the tangential and rotational discontinuity distributions with the tangential discontinuities weighted much more than the

rotationals. This would mean that the directional discontinuities observed are consistent with a population of tangential and rotational discontinuities with a high percentage (i.e. significantly greater than 50%) of the population being tangential discontinuities.

Two necessary conditions of tangential discontinuities were applied to the 111 directional discontinuities, namely pressure balance and $\Delta \underline{V} \cdot \hat{n} = 0$ (where $\Delta \underline{V}$ is the change in velocity across the discontinuity and $\hat{n} = \underline{B}_1 \times \underline{B}_2 / |\underline{B}_1| |\underline{B}_2|$). Of the 111 events, 98 met both conditions. The 13 events which did not meet the tangential discontinuity conditions were checked for a rotational discontinuity condition, viz $R = \frac{\Delta \underline{V} \cdot \Delta \underline{B}}{|\Delta \underline{V}| |\Delta \underline{B}|} = \pm 1$. For 9 of the cases $|R|$ was greater than 0.70. This was one of the criteria used to identify the rotational discontinuities (Turner and Siscoe, 1971, Turner, 1973). This evidence supports the claim based on the w distribution that a high percentage of the directional discontinuities are tangential discontinuities but that there is also a small, non-negligible percentage of rotational discontinuities present also.

Finally, the distribution in time of the directional discontinuities throughout the 40 days was very similar to

that found for tangential discontinuities. Both were scattered randomly through the 40 days and under all solar wind conditions. On the other hand, the rotational discontinuities were only found during high speed streams. All these results are consistent with the observations by Burlaga and his identification of the majority of the directional discontinuities as being tangential discontinuities.

An interesting feature is found if the $|\underline{B}_1|/|\underline{B}_2|$ distribution shown in Figure 2 is divided on the basis of plasma conditions. In Turner and Siscoe (1971) and Turner (1973) the rotational discontinuities were found primarily in three groups of days (166-168, 178-183, 193-197). These days were all characterized by high speed plasma flow and strong evidence of the presence of Alfvén waves. If the directional discontinuities and the TD's & RD's in the two papers cited above with $w \geq 30^\circ$ are combined and divided between events which occurred within these three groups and those outside a difference is found. The total population of events numbered 168 (111 + 57) with 76 occurring within the groups and 92 outside. Figure 5 a,b show the distribution of $|\underline{B}_1|/|\underline{B}_2|$ based on this division. In both, $|\underline{B}_1|/|\underline{B}_2| \sim 1$

was still preferred although much more strongly within the groups. In addition, events involving field magnitude changes occur far more frequently outside the groups (i.e. generally where V is lowest) than within.

3. Comparison with Other Work

A number of papers on discontinuities in the Mariner 5 data have been published (Turner and Siscoe, 1971, Turner, 1973, Belcher and Davis, 1969, 1971, Smith, 1973 a,b, Martin et al., 1973, Belcher, 1974, Belcher and Solodyna, 1974). Some of the conclusions of these papers are in conflict and they are also at variance with some of the results of this study. This section identifies and attempts to explain these conflicts and points of agreement.

Belcher and Davis (1969,1971) examined the same period as this paper using the same Mariner 5 data. Turner and Siscoe (1971) and Turner (1973) confirmed many of their results. These include the prevalence of high correlation between magnetic field and velocity fluctuations, in fast streams, which suggested the strong presence of Alfvén waves, and the observation that several abrupt changes in the solar wind could be identified as rotational discontinuities.

However, it is not clear from the results presented here that the claim of Belcher and Davis that the abrupt changes are predominantly Alfvénic is necessarily true. If one restricts oneself to abrupt changes which are isolated in the sense that there is a clearly defined solar wind state before and after the abrupt change, then their statement may have to be modified. Significant numbers of directional discontinuities which have been found consistent with the properties of tangential discontinuities have been observed during these fast streams. At best, it is likely that only a qualitative statement may be meaningful due to the number of events, sampling rate, and differences in definition and analysis. Enough tangential discontinuities and structures which appear to be tangential discontinuities have been found relative to rotational discontinuities to assert that both are significant features of fast streams, but the situation is not sufficiently clear to permit a ratio of 50-50 or 60-40 etc, to be quoted reliably. Much of the ambiguity arises in regions where the fluctuations both in field direction and velocity are large, which occurs when \underline{V} is large or increasing, because there it is difficult to isolate discontinuities

due to the absence of uniform conditions.

It has been said that Turner and Siscoe (1971) and Turner (1973) suggested that rotational discontinuities were more abundant than tangential discontinuities (Siscoe, 1974, Mariani et al., 1973, Webb and Quenby, 1973). On the contrary, it was the opinion of the authors that, in fact, the general class of tangential discontinuities of which the reported events were a small subset were more numerous than rotational discontinuities. Because much more rigid restrictions were placed on the tangential discontinuities ($\Delta\rho \geq 20\%$) than on rotational discontinuities, the number of TD's and RD's just happened to be approximately the same.

Two other investigators have been reporting Mariner 5 data also either wholly or partially. Both Smith (1973) and Martin et al. (1973) cite results which would indicate a dominance of rotational discontinuities over tangential discontinuities in general and in high velocity streams respectively. There are many reasons for the apparent conflict between those results and the conclusions reached here. As Martin et al. (1973) point out, differences in

definition and analysis could easily account for many of the apparent conflicts. The criteria chosen by Martin et al., however, need to be examined closely. One criterion is a change in energy density (magnetic plus kinetic) above a threshold. The danger with this type of criterion is that it favors high velocity streams where velocity fluctuations are much stronger and more frequent than in low velocity streams. Figure 6 shows a distribution of abrupt velocity changes ($\Delta V \geq 25$ km/s) during the first 40 days of Mariner 5. No attempt was made at smoothing the velocity fluctuations; they were merely counted. As was pointed out earlier, discontinuities involving field magnitude changes occur more frequently in slow streams, they are not the most common feature and in any case not nearly as strong as the ΔV fluctuations. It has been generally agreed that the population of rotational discontinuities exists in the fast streams. Thus emphasizing the fast streams thereby emphasizes rotational discontinuities preferentially.

Smith (1973a) cites as a requirement for tangential discontinuities that $\Delta |B| \neq 0$. Obviously this is not

necessary (Colburn and Sonnet, 1966). In addition, the selection criterion required that at least one component of the field change by a minimum of 3γ . This is discriminatory on two counts. First, strong tangential discontinuities may be found with changes substantially less than this. Second, Belcher and Davis (1970, 1971) report that the power spectrum of fluctuations in fast streams showed that most of the power went into fluctuations near the N direction. This kind of criterion assumes that the RTN system is physically significant at most times for solar wind fields. However, as Belcher and Davis point out, this is not necessarily true, particularly over the short term. Finally, Turner and Siscoe (1971) and Turner (1973) report a strong preference for ΔB across rotational discontinuities to be in the N direction. On the other hand, the only constraint on ΔB in tangential discontinuities is that it remain in the plane of the discontinuity. Thus a tangential discontinuity may undergo a significant change in field direction without that change being manifested in only one component. So in this light Martin et al.'s results are not surprising.

In Belcher and Solodyna (1974) discontinuities were selected based on the criteria of Burlaga (1969a). These criteria are probably inadequate for selecting discontinuities in disturbed intervals, because the important condition of uniformity before and after the discontinuities is left to subjective judgement. There are serious questions relating to both selection and analysis. A steady state requirement was said to have been imposed on the pre- and post-discontinuity states. This is a particularly important criterion at this stage of solar wind research since basic questions are being asked about the various structures. Thus it is necessary to try to remove ambiguity by selecting fairly isolated events with clearly definable states on both sides of the discontinuity. A few of the examples shown in figures 2-4 of Belcher and Solodyna have obvious steady states, but there are others in which a steady state is highly questionable. For the times chosen in that study, there is general agreement that there is strong evidence of Alfvén waves in the plasma and a consequent absence of steady states that are crucial in differentiating between a large amplitude wave and a discontinuity.

Next the primary conclusions of their paper rest heavily on two tables relating the angles θ and α , where

$$\tan \alpha = \left(\frac{4\pi}{\rho_1 + \rho_2} \right)^{1/2} \left(\frac{|\Delta \underline{V}|}{\frac{|\underline{B}_1 - \underline{B}_2|}{\rho_1 \rho_2}} \right) \text{ and } \theta = \lambda \Delta \underline{V}, \frac{B_1}{\rho_1} - \frac{B_2}{\rho_2}$$

They conclude from these tables that most of their events are rotational discontinuities. (The tables are shown in Figure 7 a,b here.). Their conclusions are doubtful for three reasons. First, an α , θ distribution derived from tangential discontinuities reported in Turner and Siscoe is shown in parentheses in Figure 7a. The distribution is obviously equivalent to that of Belcher and Solodyna. So applying their criterion one would erroneously conclude that the TD's are RD's. The tangential discontinuities all had $|\Delta \rho| \geq 20\%$ and were found throughout the first 40 days of Mariner 5. Second, to require that α lie between $30^\circ - 40^\circ$ is not meaningful. For an isotropic plasma $\tan \alpha$ becomes

$$\frac{1}{21.8} \frac{|\Delta \underline{V}| \text{ (km/s)}}{\left[\frac{\Delta \underline{B}}{\sqrt{n}} \text{ (\gamma/cc)} \right]}$$

A number density of 5/cc, a $|\Delta \underline{V}| = 20$ km/s, and a $|\Delta \underline{B}| = 3\gamma$ give

$30^\circ < \alpha < 40^\circ$, but those values are typical of random fluctuations in fast streams of the type examined by Belcher and Solodyna. Finally, Belcher and Solodyna project the events shown in Figure 7a as being the more reliable due to smaller variances relative to field changes. But Figure 7b is essentially the same as Figure 7a so that the reliability of the measurements does not seem to affect the results.

Belcher and Solodyna chose to consider the mathematical possibility for θ to lie anywhere for a tangential discontinuity as a physical probability. The tangential discontinuity is a structure which is restricted neither to the solar wind nor to plasmas in general. Thus the mathematics of tangential discontinuities do not incorporate the inherent structure and properties of the solar wind. Thus to duplicate the θ distribution only a small angle change of \underline{V} , which is weakly correlated with $\Delta\underline{B}$, is needed. This situation is not implausible for a tangential discontinuity since $\Delta\underline{V}$ and $\Delta\underline{B}$ are confined to the same plane for physical reasons.

In Belcher (1974) tables similar to those of Belcher and Solodyna are presented with similar conclusions. Here

too ambiguity is introduced in categorizing abrupt changes as discontinuities, and it was assumed without justification that events with a certain position in the α, θ Table are rotational discontinuities.

4. Summary

Directional discontinuities were found in Mariner 5 and the vast majority appear to be tangential discontinuities based on comparisons with other published data. These discontinuities occurred in both fast and slow streams and exhibited many characteristics reported by Burlaga (1969a,b,1971). In addition, to the field being confined to a plane at the discontinuity, there was evidence that this confinement continued on into the flow on both sides of the discontinuity. The events which occurred in fast streams had a far greater percentage of little or no change in field magnitude whereas the slow stream events showed that field magnitude changes were important, though little or no change was somewhat more frequent. Finally an explanation was given for the apparent conflict between this work and other reports.

ACKNOWLEDGEMENT

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Table 1 - Eigenvectors for the Directional
Discontinuity Normals

	<u>R</u>	<u>I</u>	<u>N</u>	$\frac{\epsilon_i^2}{i}$	<u>θ</u>	<u>φ</u>
1.	.770	.464	-.438	55.2	40°	317°
2.	.340	.282	.897	33.7	70°	73°
3.	-.539	.840	-.059	22.1	147°	356°

Table 2 - Direction of Minimum Fluctuation in B for
Segments Containing Directional Discontinuities

	<u>R</u>	<u>I</u>	<u>N</u>	$\frac{\epsilon_i^2}{i}$	<u>θ</u>	<u>φ</u>
1.	.895	.300	-.329	40.6	26°	312°
2.	-.290	.954	.080	24.7	107°	5°
3.	.338	.023	.941	19.7	70°	89°

REFERENCES

- Belcher, J.W., MIT Center for Space Research Report CSR-P-74-113, 1974.
- Belcher, J.W. and L. Davis, EOS, 51, 413, 1970.
- Belcher, J.W. and L. Davis, J. Geophys. Res., 76, 3534, 1971.
- Belcher, J.W., L. Davis, and E.S. Smith, J. Geophys. Res., 74, 2303.
- Belcher, J.W. and C.V. Solodyna, MIT Center for Space Research Report, CSR-P-74-125, 1974.
- Burlaga, L.F., Solar Phys. I, 54, 1969a.
- Burlaga, L.F., Solar Phys. 7, 72, 1969b.
- Burlaga, L.F., J. Geophys. Res., 76, 4360, 1971.
- Connor, B.V., IEEE Trans. Magnetics, MAG-4, 391, 1968.
- Lazarus, A.J., H.S. Bridge, J.M. Davis, and C.W. Snyder, Space Res., 7, 1296, 1967.
- Mariani, F. Bavassano, B., Villante, U., and N.F. Ness, J. Geophys. Res., 78, 8011, 1973.
- Martin, R.N., J.W. Belcher, and A.J. Lazarus, J. Geophys. Res., 78, 3653, 1973.
- Siscoe, G.L., Discontinuities in the Solar Wind, presented to Asilomar Solar Wind Conference, 1974.
- Siscoe, G.L., L. Davis, Jr., P.J. Coleman, Jr., E.J. Smith, and D.E. Jones, J. Geophys. Res., 73, 61, 1968.
- Smith, E.J., J. Geophys. Res., 78, 2054, 1973.
- Smith, E.J., J. Geophys. Res., 78, 2088, 1973b.
- Turner, J.M., J. Geophys. Res., 78, 59, 1973.
- Turner, J.M. and G.L. Siscoe, J. Geophys. Res., 76, 1816, 1971.
- Webb, S. and Quenby, J.J., Imperial College Report, 1973.

Figure Captions

- Figure 1 - Solar wind parameters for a typical event. The angles shown are the polar angle θ_B (R taken as the polar axis), and azimuthal angle φ_B ($\varphi_B = 0$, $\theta = 90^\circ$ corresponds to T direction, $\varphi_B = 90^\circ$, $\theta = 90^\circ$ corresponds to N direction)
- Figure 2 - Histogram of $|\underline{B}_1|/|\underline{B}_2|$ for all events.
- Figure 3 - Distributions of the angle ψ . Events reported here are given by the solid lines, Burlaga's distribution function shown by dotted lines.
- Figure 4 - a) Distribution of ψ for tangential discontinuities reported by Turner and Siscoe scaled to 111 events.
b) Distribution of ψ for rotational discontinuities reported by Turner and Siscoe scaled to 111 events.
- Figure 5 - a) Histogram of $|\underline{B}_1|/|\underline{B}_2|$ for total population of events occurring within the three groups
b) Histogram of $|\underline{B}_1|/|\underline{B}_2|$ for total population of events occurring outside the three groups.
- Figure 6 - Distribution of number times per day $\Delta V \geq 25$ km/s occurs between consecutive data points.
- Figure 7 - a) Table 1a from Belcher and Solodyna (1974). Events for which the field variance was small relative to ΔB .
b) Table 1b from Belcher and Solodyna (1974). Events for which the field variance was comparable to or greater than ΔB .

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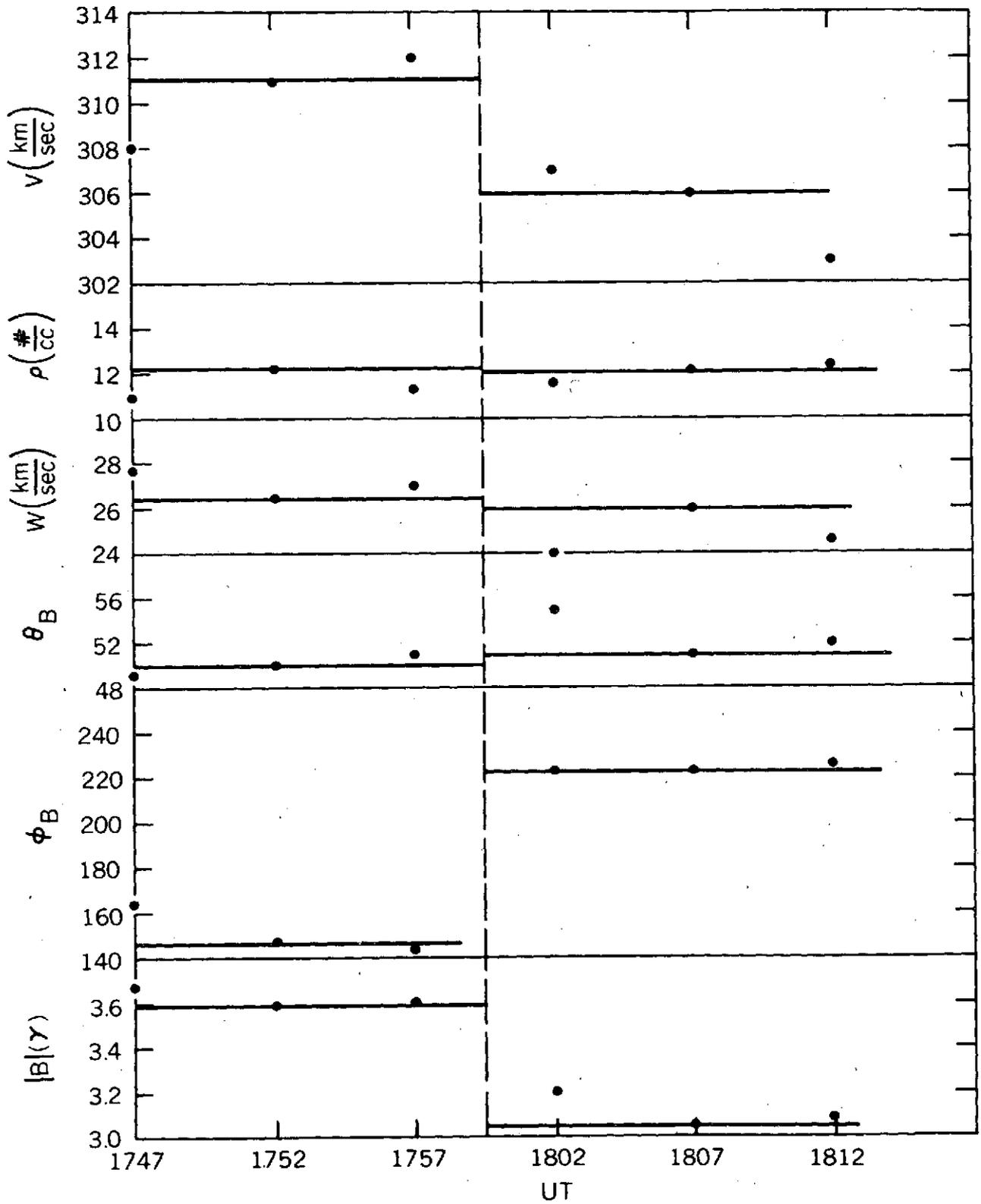


FIGURE 1.

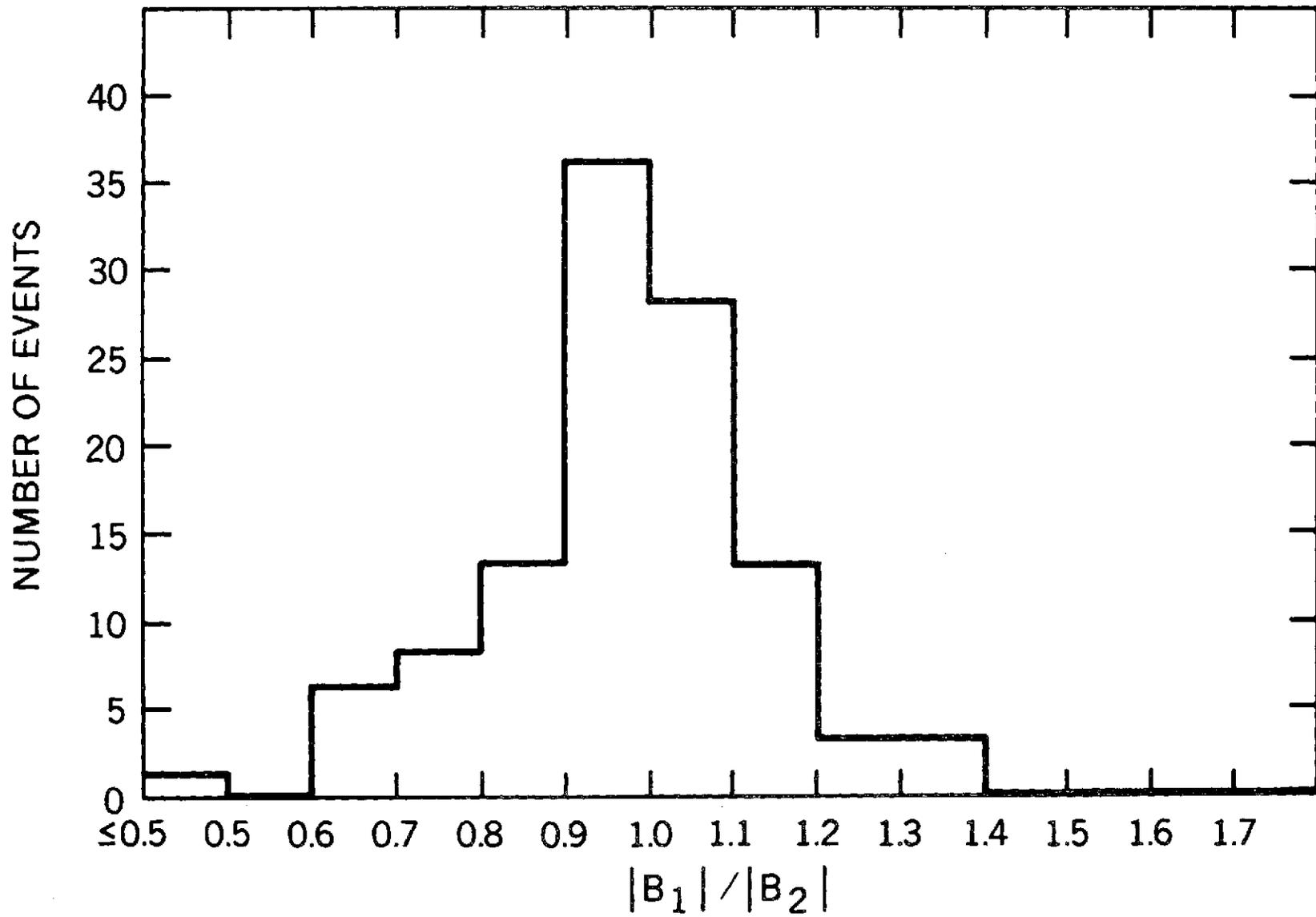


FIGURE 2.

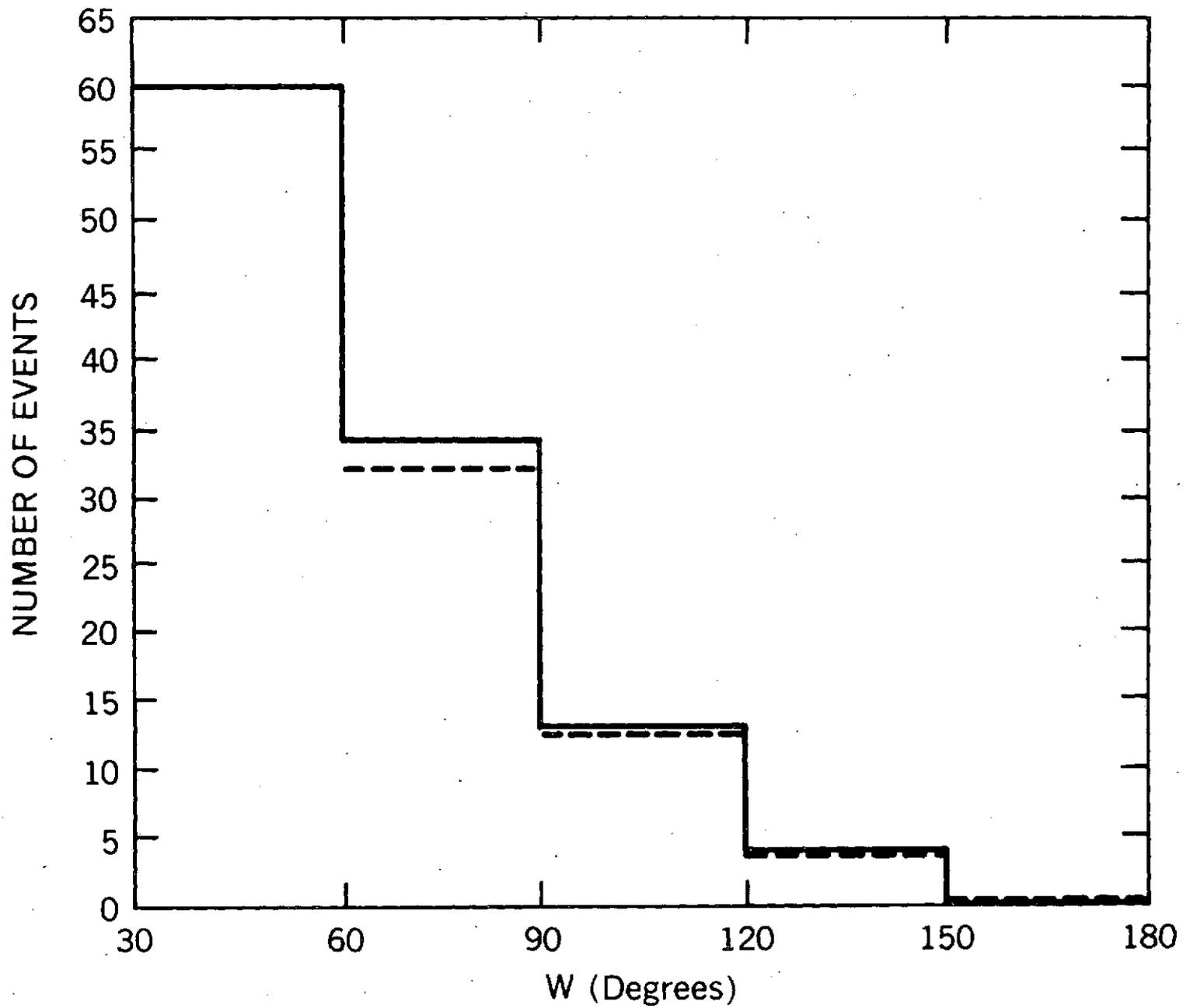


FIGURE 3.

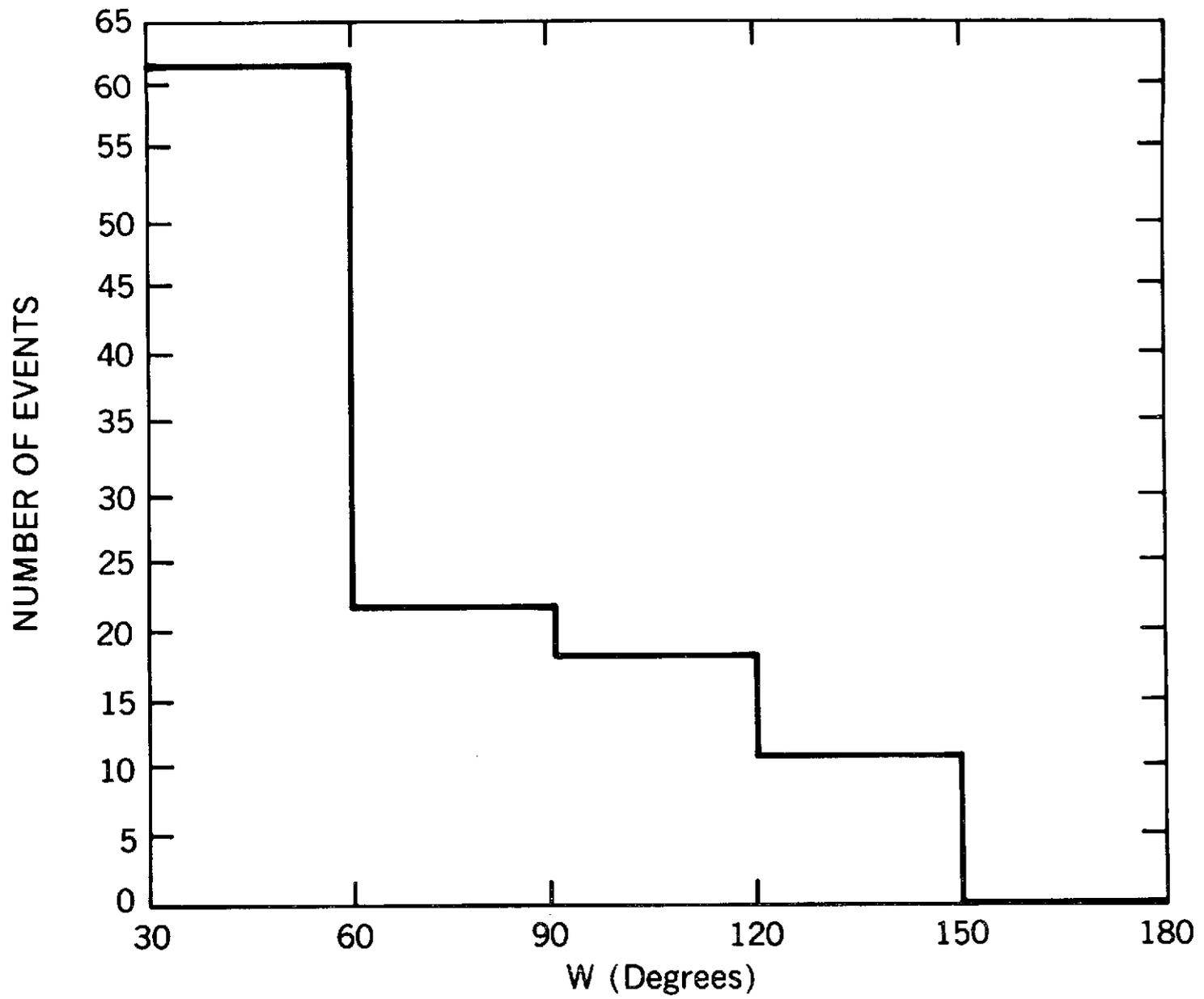


FIGURE 4a.

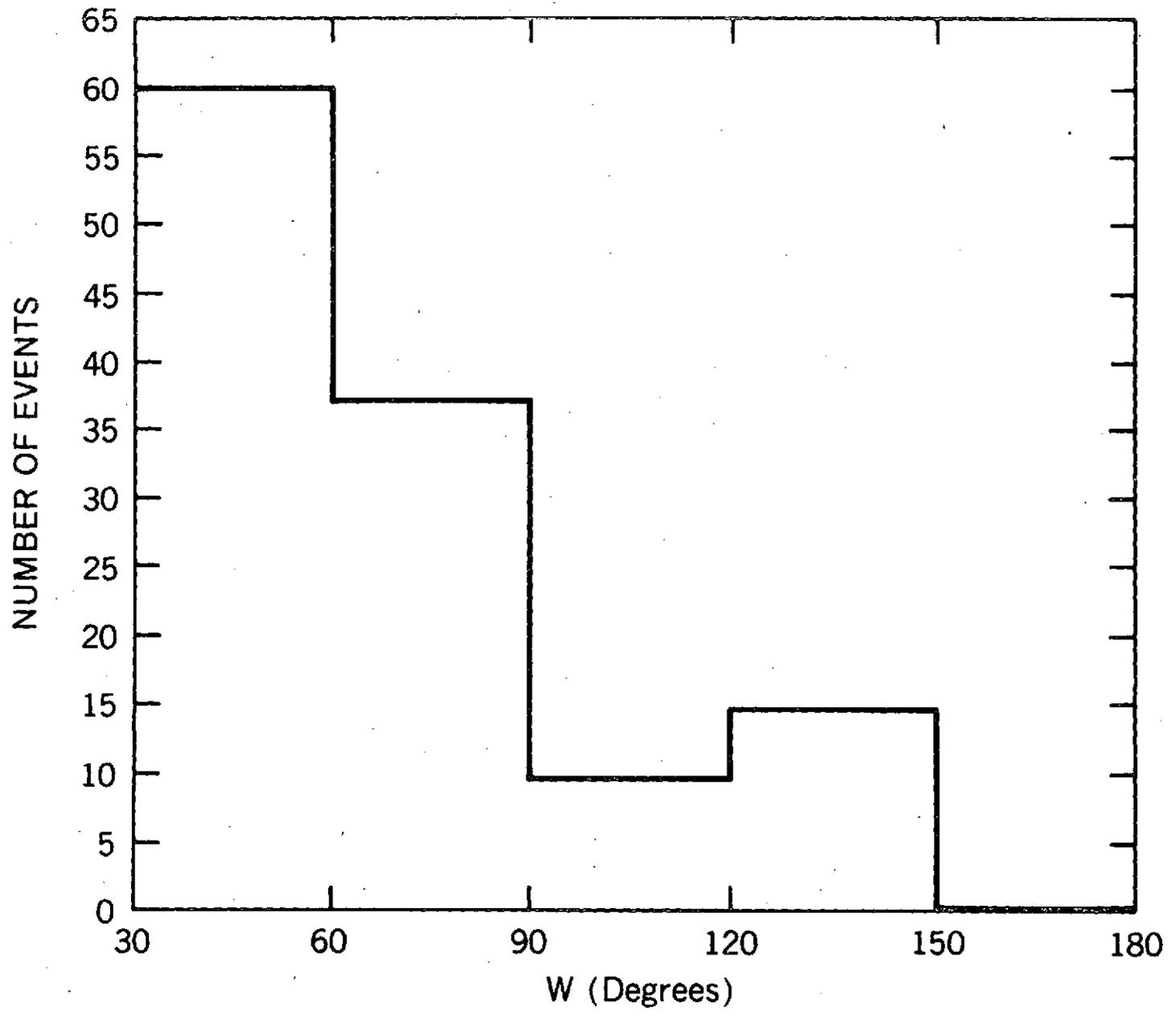


FIGURE 4b.

INSIDE 3 GROUPS

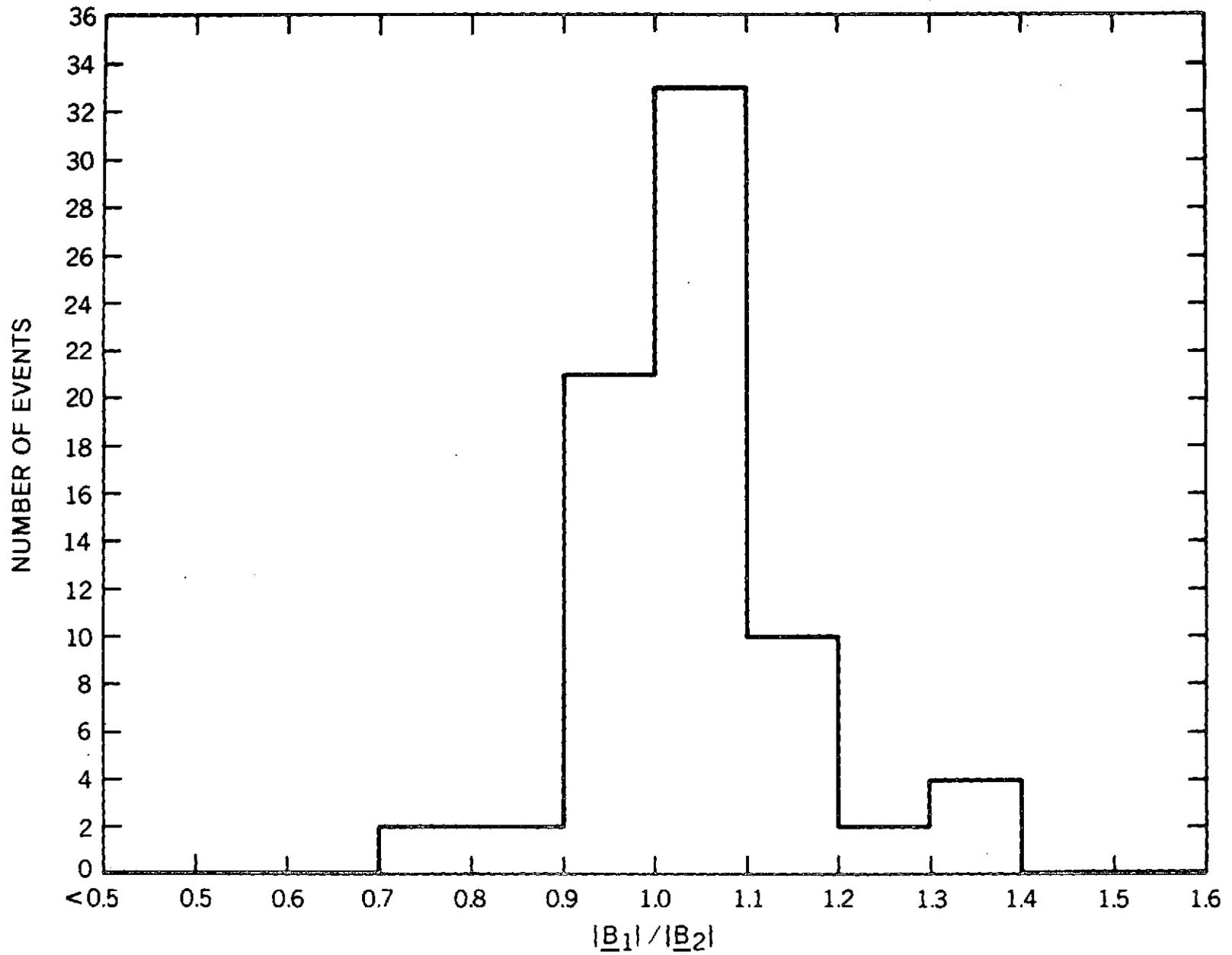


FIGURE 5a.

OUTSIDE 3 GROUPS

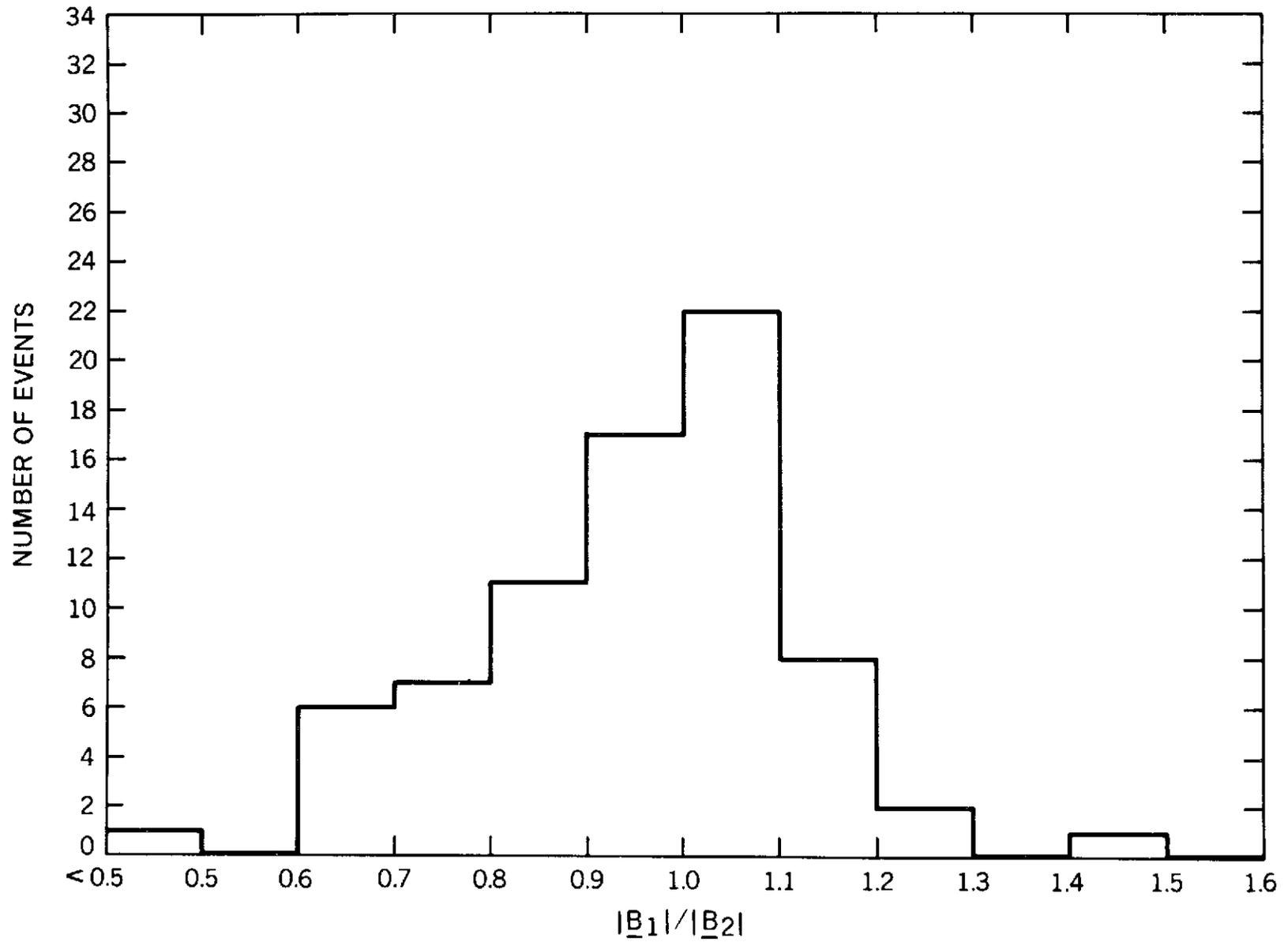


FIGURE 5b.

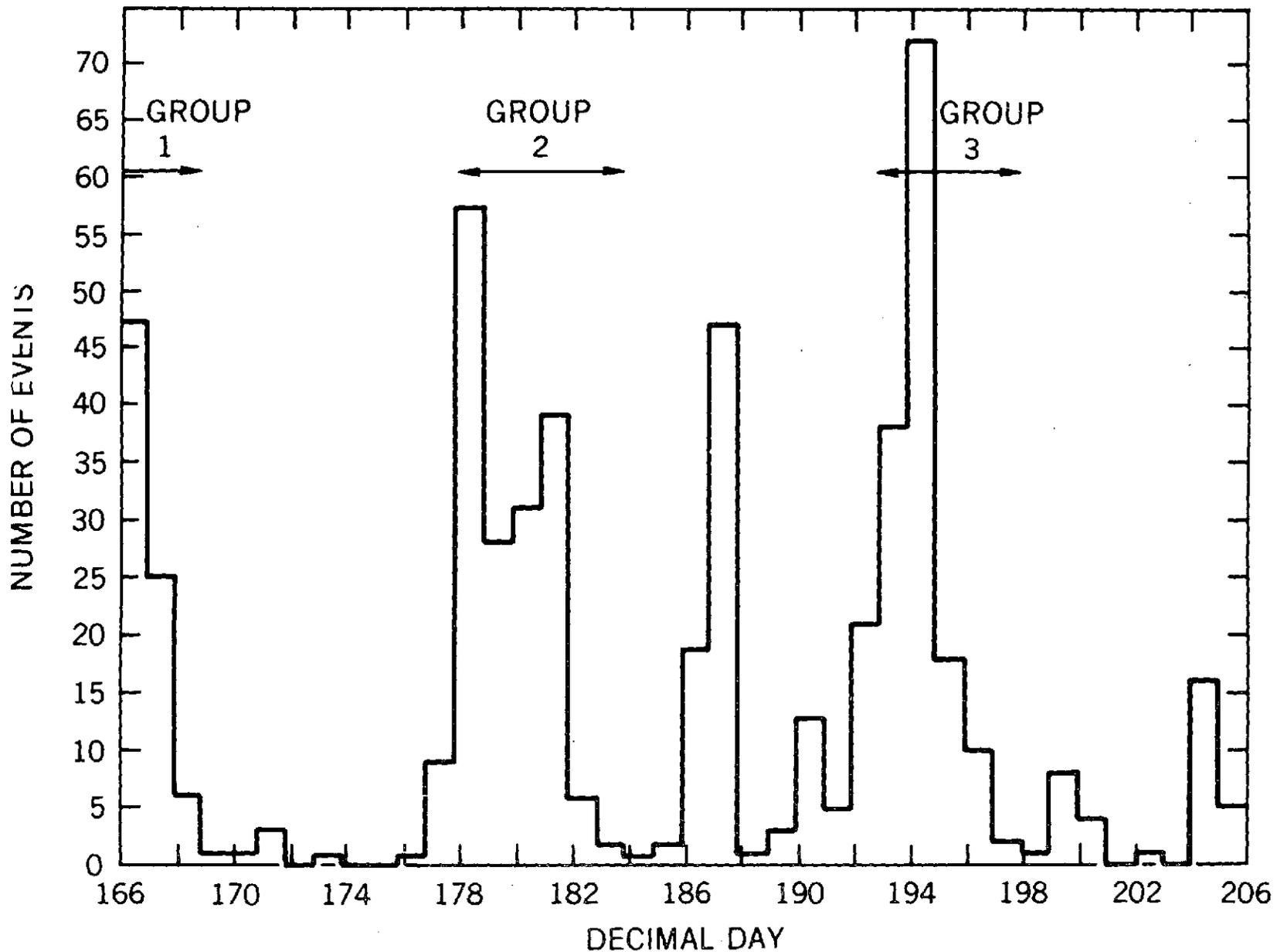


FIGURE 6.

TABLE 1a. DISTRIBUTIONS OF α VERSUS θ FOR $\mu \leq 0.5$
 FROM BELCHER AND SOLODYNA (1974)

α , DEG.										
θ , DEG.	0/10	10/20	20/30	30/40	40/50	50/60	60/70	70/80	80/90	0/90
0/39	0	0(1)	0	0(2)	0(4)	0(6)	0(5)	0(1)	0	0
39/56	0	0	0	0	0(2)	0(2)	0	0	0	0
56/71	0	1	0	0	0	0(1)	0(1)	0	0	1
71/84	0	0	0(1)	0(1)	0	0	0(1)	0	0	0
84/96	0(1)	0	0	0	0	0	0	0	0	0
96/109	0	0	0	0	0(1)	0(1)	0(1)	0	0	0
109/124	0	0	0	1	0	0	0	0	0	1
124/141	0	0	2	4	1	0	0(1)	0	0	7
141/180	0	2	13	24	4	2	0(2)	0	0	45
0/180	0	3	15	29	5	2	0	0	0	

FIGURE 7a.

TABLE 1b. DISTRIBUTIONS OF α VERSUS θ FOR $\mu > 0.5$
 FROM BELCHER AND SOLODYNA (1974)

α , DEG.										
θ , DEG.	0/10	10/20	20/30	30/40	40/50	50/60	60/70	70/80	80/90	0/90
0/39	0	1	0	0	0	0	0	0	0	1
39/56	0	0	0	0	0	1	0	0	0	1
56/71	0	0	0	0	0	1	0	0	0	1
71/84	0	0	0	0	0	0	1	0	0	1
84/96	0	0	0	0	1	0	0	0	0	1
96/109	0	1	0	2	1	1	0	0	0	5
109/124	0	1	0	0	1	0	1	1	0	4
124/141	0	0	0	1	2	1	0	0	0	4
141/180	0	1	1	6	5	2	0	0	0	15
0/180	0	4	1	9	10	6	2	1	0	

FIGURE 7b.