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PROJECT SPACE SHUTTLE

ERROR PROTECTION CAPABILITY
OF
SPACE SHUTTLE DATA BUS DESIGNS

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SYMBOLS

A	peak signal amplitude
B_n	noise bandwidth
dB	decibel
MHz	Megahertz
P_c	probability of a correct bit decision
P_{cw}	probability of a correct word
P_{HBC}	probability of a correct half-bit decision
P_{HBe}	probability of a half-bit decision error
P_{HBi}	probability of an invalid half-bit
P_i	probability of an invalidated bit
P_{iw}	probability of an invalidated word
P_{ow}	probability of an outputted word
P_{ue}	probability of an undetected bit error
P_{uwe}	probability of an undetected word error
T	bit period
V	detector threshold level
V_n	rms noise voltage
$\Phi()$	Gaussian distribution function
(n,k)	an n-bit codeword consisting of k information bits and n-k check bits
p_c	probability of a signal sample being correct
p_e	probability of a signal sample being in error

p_i probability of a signal sample being improper
 $p(r)$ probability density function of data bus signal
 $p(s)$ probability density function of signal prior to
sampling
 r correlation coefficient
 rms root mean square

1.0 INTRODUCTION

Error protection provides assurance in the reliability of digital data communications. The need for error protection on the Space Shuttle data bus system has been recognized and specified as a hardware requirement.

The error protection techniques of particular concern are those designed into the Shuttle Main Engine Interface (MEI) and the Orbiter Multiplex Interface Adapter (MIA). The techniques and circuit design details proposed for these hardware are analyzed in this report to determine their error protection capability. The capability is calculated in terms of the probability of an undetected word error. Calculated results are reported for a noise environment that ranges from the nominal noise level stated in the hardware specifications to burst levels which may occur in extreme or anomalous conditions. The results provide a direct comparison of the capability of each proposed technique and information concerning expected performance limits at burst noise levels.

The scope of the analysis is limited to the protection provided by prefiltering and bit detection circuits in the receiver in conjunction with coding of the data word. Although error protection is also provided by the sync preamble, the address structure, bit counts per word, and coded commands, these factors are not considered in the analysis. The analysis assumes that data bus noise is the only cause of data errors and ignores causes such as power supply and ground noise, component tolerances and variations,

and environmental effects. The significance of these causes will increase as bus noise decreases. In addition, the analysis assumes the bus noise is a stationary random process (i.e. a process whose statistics are constant with time). Although impulsive noise is a nonstationary process, the analysis is also indicative of performance in presence of impulsive noise that spans an entire word. Noise impulses that span less than one word length cause fewer bit errors which are more likely to be detected.

The report presents a functional description of the receiver and validity checks for each design, a statement of the decision criteria implemented, and a statistical formulation of the quantities to be calculated. Calculated results are tabulated and graphically illustrated for comparison purposes.

2.0 DATA BUS DESIGNS

Bilevel digital data are serially communicated on the data bus as Manchester coded signal waveforms. The Manchester code provides a simple but effective means of detection and reconstruction of each data bit at the receiver. Noise signals which occur on the data bus introduce an additive component that causes random variations of the received signal waveform.

Bit errors occur when the random variations satisfy the decision criteria designed into the receiver. Such errors may be detected by performing validity checks on unique characteristics of the signal waveform and/or by employing an error detection code to check the entire sequence of bits in a data word transmission. The resultant error detection capability is dependent upon several specified and design characteristics:

- Signal waveform
- Noise level and spectrum
- Receiver design
- Waveform validity checks
- Error detection code

Specified characteristics are summarized in table I for three data bus designs:

- I. Main Engine Interface (Honeywell design)
- II. Orbiter MIA (Singer design)
- III. Alternate MIA (Receiver per Rockwell specifications with NASA requested code)

TABLE I. — SPECIFIED DATA BUS CHARACTERISTICS

	Main Engine Interface	Orbiter MIA	Alternate MIA
<u>Source Document</u>	I.C.D. No. 13M1500F (per IRN No. 13415, 4/16/74)	Specification MC615-0010 (3/29/74)	NASA RECP 115 (11/3/73)
<u>Signal</u>			
Rate	1MBPS	1MBPS	1MBPS
Type	Manchester II code	Manchester II code	Manchester II code
Amplitude (peak)	±2.5V to ±5.3V	±1.5V to ±4.0V	±1.5V to ±4.0V
Rise/Fall Time	<100n seconds	<250n seconds	<250n seconds
<u>Noise</u>	(None specified)	0.3Vrms of white, Gaussian noise in 4MHz bandwidth	0.3Vrms of white, Gaussian noise in 4MHz bandwidth
<u>Receiver</u>			
Line coupling	Transformer isolation	Transformer isolation	Transformer isolation
Input filter	(None specified)	-23dB at 1kHz and 4MHz	-23dB at 1kHz and 4MHz
Theshold level	(None specified)	±0.5 volt	±0.5 volt
<u>Waveform Checks</u>	(None specified)	Manchester code transition	(None specified)
<u>Error Detection Code</u>	Cyclic (31,16) BCH	Odd Parity (25,24)	Cyclic (47,24)
<u>Other Error Checks</u>	Valid word sync Bits/word	Valid word sync Bits/word	Valid word sync Bits/word

The principal differences between specified characteristics are the error protection techniques: the MEI implements a cyclic (31, 16) BCH code; the Orbiter MIA employs a Manchester code check along with single bit parity; the Alternate MIA uses a cyclic (47, 24) code. Other differences of significance are the increased signal amplitude specified for the MEI, and the input filter and null zone detector specified for the MIA's.

The receiver design characteristics summarized in table II were obtained from contractor drawings and descriptions. Figures 1, 2 and 3 illustrate the receiver functions which affect the error protection capability.

The MEI receiver converts data bus signals directly to standard logic levels with an unbiased differential amplifier, takes one level sample per bit period for bit decisions, and performs a cyclic code check on the detected sequence of bits. A word is outputted when the following conditions are satisfied:

1. No errors are detected by the (31, 16) cyclic code.
2. A total count of 31 mid-bit transitions occur between word syncs.
3. A total count of 33 bits occur between word syncs.
4. An all zero-bit sequence (idle bus state) has not occurred.

Thus, a word error that is undetected by the cyclic code is outputted the same as a word without an error. The undetected word error statistics are determined from the bit error statistics corresponding to the data bus signal to noise ratio

TABLE II. - RECEIVER DESIGN CHARACTERISTICS

	Main Engine Interface	Orbiter MIA	Alternate MIA
Line coupling	Transformer	Transformer	Transformer
Input filter	None	6-pole Gaussian filter -3dB at 1.5MHz	6-pole Gaussian filter -3dB at 1.5MHz
Input circuit	Unbiased differential amplifier	+0.5 volt and -0.5 volt threshold detectors	+0.5 volt and -0.5 volt threshold detectors
Bit decision	1 sample/bit	2 consecutive in 4 samples/ half-bit	1 sample/bit



Figure 1. - Main Engine Interface receiver functions.

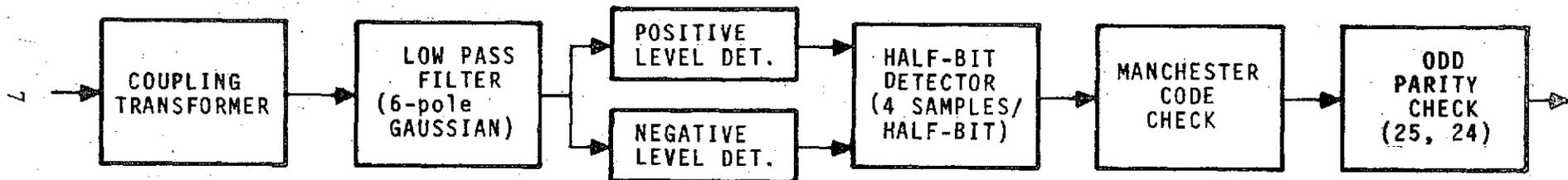


Figure 2. - Orbiter MIA receiver functions.

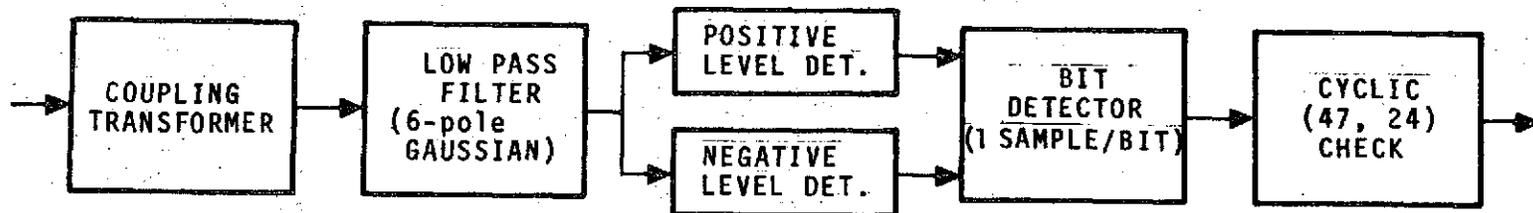


Figure 3. - Alternate MIA receiver functions.

and the error detection capability of the (31, 16) code (see table III). The MEI design requires a larger data bus signal to noise ratio to provide the same bit error statistics as the MIA's.

The Orbiter MIA design is substantially different from the MEI. Data bus signals are bandlimited by a 6-pole Gaussian filter and then converted into two discrete signal channels by a null zone detector which consists of a positive and a negative level detector. The level detector outputs are asynchronously sampled at an 8-MHz rate to obtain up to four level samples per half-bit. Half-bit detections require two consecutive samples to be identical. Failure to satisfy this criterion is decided to be an improper condition that invalidates the word. Detected half-bits, in a bit period, are checked to be complements of each other in order to be a valid Manchester coded bit. If half-bits are not complements, a bit error is detected and the word is invalidated. Detected bits are then subjected to an odd parity check. A word is outputted when the following conditions are satisfied:

1. Two consecutive level samples occur each half-bit.
2. Detected half-bits are a valid Manchester code.
3. No errors are detected by the odd parity check.
4. A total count of 25 bits for the data bus (or 17 bits for a serial I/O channel) occur in a word.

The Orbiter MIA performs error checks in three stages; level samples, half-bits, and bits per word. A word error that satisfies all of these checks passes undetected and is

TABLE III. - ERROR DETECTION CAPABILITY OF SELECTED CODES

<p><u>Main Engine Interface</u></p> <p>(31,16) cyclic code detects:</p> <ul style="list-style-type: none">• All combinations of 6 or less random bit errors• All bursts with length 15 or less• 99.993 897% of bursts with length 16• 99.996 949% of bursts with length greater than 16
<p><u>Orbiter MIA</u></p> <p>(25,24) odd parity code detects:</p> <ul style="list-style-type: none">• All combinations of bit errors which are odd in number
<p><u>Alternate MIA</u></p> <p>(47,24) cyclic code detects:</p> <ul style="list-style-type: none">• All combinations of 10 or less random bit errors• All bursts with length 23 or less• 99.999 976% of bursts with length 24• 99.999 988% of bursts with length greater than 24

outputted. The corresponding statistics for such an occurrence are derived from the sample statistics, the decision criteria, and the error detection capability of the odd parity check. The sample statistics are improved by the frequency and amplitude characteristics of the receiver which increase the predetection signal to noise ratio relative to that on the data bus.

The Alternate MIA design also utilizes the Orbiter MIA input filter and null zone detector to improve the sample statistics. One sample per bit is used for bit decisions as in the MEI. However, if the signal is within the null zone formed by the positive and negative level detectors, an improper condition exists and the bit is invalidated. Detected bits are serially checked for errors by a (47, 24) cyclic code. A word is outputted when the following conditions are satisfied:

1. Signal amplitude exceeds the level detector thresholds at each bit sample time.
2. No errors are detected by the (47, 24) cyclic code.
3. A total count of 47 bits occur in a word.

A word error that satisfies these criteria is an undetected error that is outputted. The corresponding statistics are determined from the sample statistics and the error detection capability of the code listed in table III.

The three designs provide the basic functions of a receiver augmented with different checks and techniques for purposes of detecting errors. The resultant error detection capabilities of these designs are analyzed in the next section.

3.0 ANALYSIS OF DESIGNS

The data bus designs are analyzed to determine their performance as a function of data bus noise. The noise is assumed to be additive with the signal waveform, and characterized as white with zero mean Gaussian amplitude statistics. The resultant signal and noise input to the receiver is a random process that requires analysis on a probabilistic basis. The objective is to formulate expressions for the probabilities of all possible outcomes of a received word transmission.

The receiver either outputs or invalidates a word on the basis of built-in checks. Outputted words are either correct replicas of the transmitted word, or contain bit errors which are undetected by the receiver checks. Invalidated words are those in which an improper or erroneous condition has been detected. Thus, each transmitted word has three possible outcomes with probabilities as follows:

P_{cw} - probability of a correct word

P_{uwe} - probability of an undetected word error

P_{iw} - probability of an invalidated word

where $P_{cw} + P_{uwe} + P_{iw} = 1$.

The probability of an undetected word error (P_{uwe}) is of particular interest since it is a measure of the error protection capability. It is noted that the probability

of a word being outputted is

$$P_{ow} = P_{cw} + P_{uwe}.$$

The word throughput rate for continuous transmission of an (n, k) code at $1/T$ bits per second is,

$$\text{Word throughput rate} = \left(\frac{1}{nT}\right) \cdot (P_{cw} + P_{uwe}).$$

The word probabilities are related to the probabilities associated with the possible outcomes for each bit decision which are as follows:

P_i - probability of an invalidated bit

P_{ue} - probability of an undetected bit error

P_c - probability of a correct bit decision.

Since bit decisions are statistically independent, the probability that an n -bit word contains exactly i invalidated bits and j undetected bit errors with the remaining $n-i-j$ bits correct is described by the trinomial distribution,

$$P(i, j: n) = \frac{n!}{i! j! (n-i-j)!} p_i^i p_{ue}^j p_c^{n-i-j}.$$

Since an invalidated bit invalidates the word, an outputted word occurs only if $i = 0$ for which,

$$\begin{aligned} P(0, j: n) &= \frac{n!}{j! (n-j)!} p_{ue}^j p_c^{n-j} \\ &= \binom{n}{j} p_{ue}^j p_c^{n-j} \end{aligned}$$

where $\binom{n}{j}$ is the number of combinations of j undetected bit errors in n -bits.

A correct word occurs only if $i = j = 0$, so that

$$P_{cw} = \frac{n!}{n!} p_i^0 p_{ue}^0 p_c^n = p_c^n.$$

A code provides the means for detecting certain combinations of bit error patterns which are termed random errors or burst errors. Random bit errors are independent of one another whereas burst errors may be dependent. A burst error of length b is any pattern of errors over b consecutive bits in which the first and last bits are in error. Although an (n, k) cyclic code detects all bursts of length $(n-k)$ and a large percentage of longer bursts, the random error detection capability alone provides an adequate performance bound for purposes of this report. For the code capabilities listed in table III, the probability of an undetected word error for each design is expressed as:

$$\text{Main Engine Interface} - P_{uwe} = \sum_{j=7}^{31} \binom{31}{j} p_{ue}^j p_c^{31-j}$$

$$\text{Orbiter MIA} - P_{uwe} = \sum_{j=2,4,6,\dots}^{24} \binom{25}{j} p_{ue}^j p_c^{25-j}$$

$$\text{Alternate MIA} - P_{uwe} = \sum_{j=11}^{47} \binom{47}{j} p_{ue}^j p_c^{47-j}$$

These equations relate the effectiveness of the code in terms of the bit decision probabilities. The bit decision probabilities are a function of the input signal and noise and dependent upon receiver design characteristics.

As illustrated in figure 4, the received signal, $r(t)$ is filtered, quantized and then sampled. The samples which form the basis for bit decisions determine the decision probabilities. The sample statistics are a probabilistic description of the instantaneous waveform amplitude at the sample times. These statistics are derived below for a positive half-bit level which are identical to those for a negative half-bit level because of the symmetry in peak signal amplitudes ($\pm A$), threshold levels ($\pm V$), and zero mean noise. During the steady-state portion of the positive half-bit waveform in figure 5, the received signal amplitude is a Gaussian random variable whose mean value is equal to the peak value of the Manchester waveform ($+A$) and whose variance is the mean square noise voltage on the bus (V_n^2). Hence, the probability density function of the received signal amplitude is

$$p(r) = \frac{1}{\sqrt{2\pi}V_n} e^{-\frac{1}{2}\left(\frac{r-A}{V_n}\right)^2}$$

The MEI design provides no filtering; it simply quantizes the received signal about a zero level and samples once near the middle of the first half-bit period (at $T/4$). The resulting MEI sample statistics are derived from the probability density function as below:

p_c = probability the sample is the correct level

$$= P[r(t) > 0] = \int_0^{\infty} p(r) dr = \int_{-\frac{A}{V_n}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \Phi\left(\frac{A}{V_n}\right)$$

p_e = probability the sample is incorrect

$$= P[r(t) < 0] = \int_{-\infty}^0 p(r) dr$$

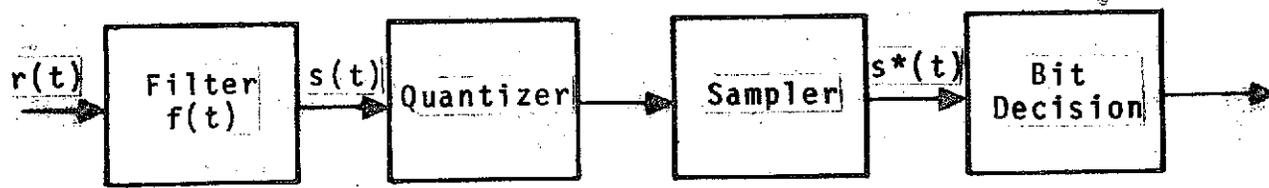


Figure 4. - Functional model of receiver.

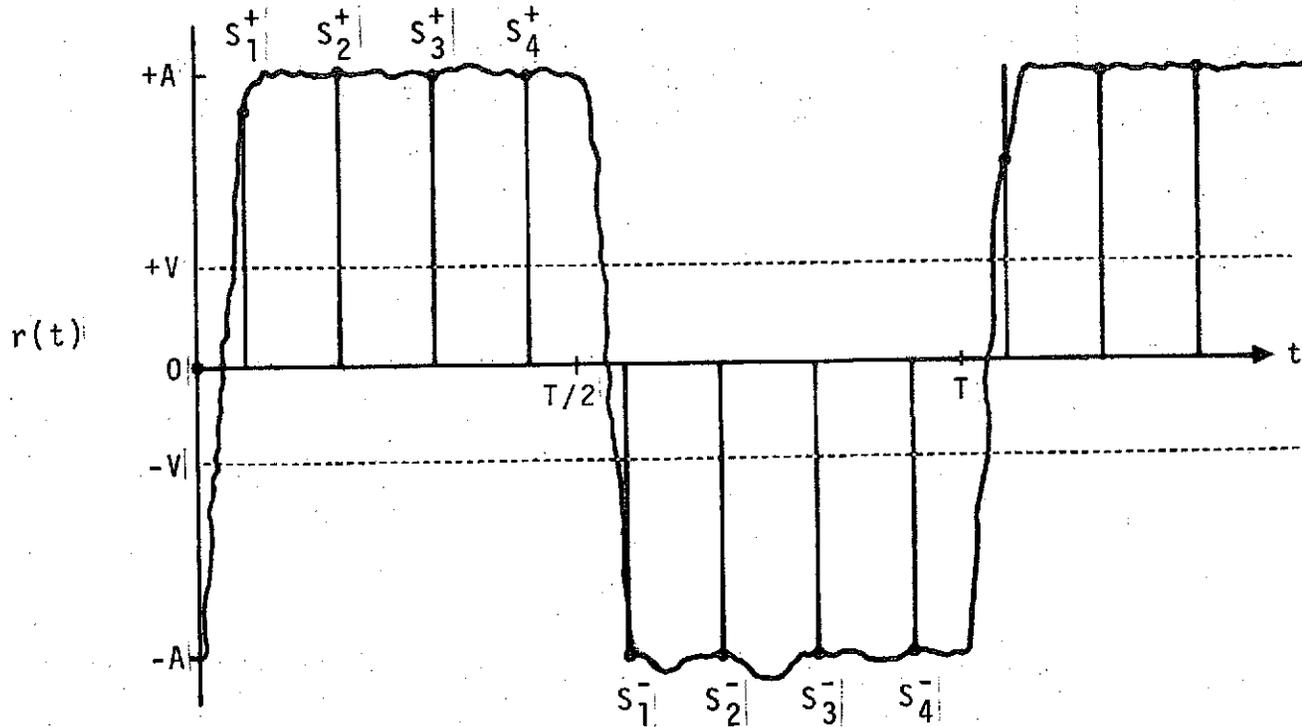


Figure 5. - Typical received waveform and samples.

$$= \int_{-\infty}^{-A/V_n} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 - \Phi\left(\frac{A}{V_n}\right)$$

$$\text{where } \Phi(\alpha) \triangleq \int_{-\infty}^{\alpha} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = 1 - \Phi(\alpha)$$

is the widely tabulated Gaussian distribution function. Since a MEI sample is accepted as a bit without further checks, the bit decision probabilities are identical to the corresponding sample statistics;

$$P_c = p_c = \Phi\left(\frac{A}{V_n}\right)$$

$$P_{\mu e} = p_e = 1 - \Phi\left(\frac{A}{V_n}\right)$$

$$P_i = 0.$$

The alternate MIA sample statistics are different from the MEI because of the filter and null zone detector. The filter affects the transient response and bandlimits the noise. The step response for a 6-pole low pass Gaussian filter with -3db cutoff at 1.5MHz is shown in figure 6. The filter response has less than one percent overshoot, a rise time of 90 nanoseconds, and an equivalent noise bandwidth of 2.4MHz. The Alternate MIA samples the filtered signal once each bit period at $(T/4)$ which is sufficient for the signal to reach its peak steady-state value ($\pm A$). The filtered noise voltage is related to the data bus noise by

$$V_n' = \sqrt{\frac{B_n'}{B_n}} V_n$$

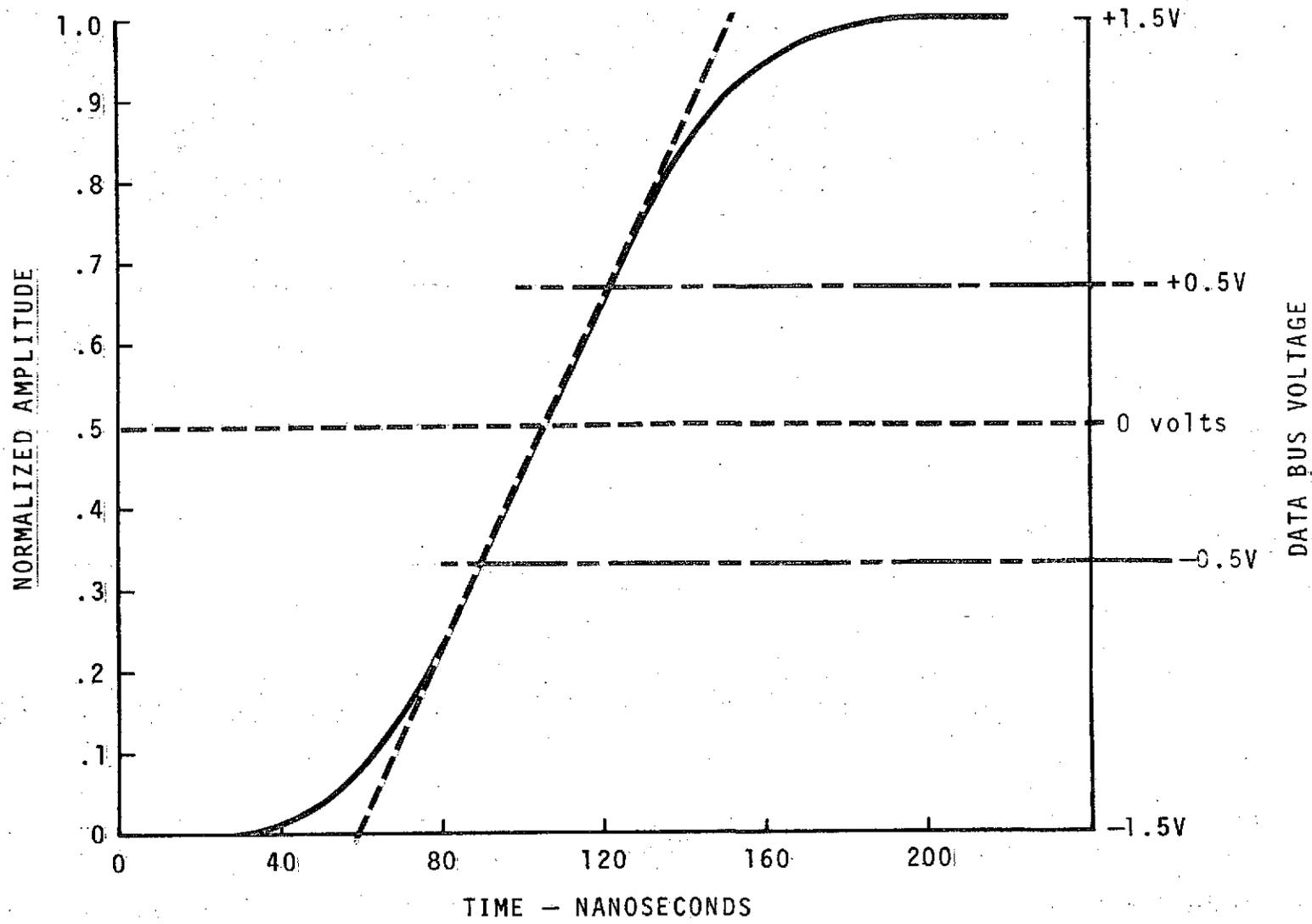


Figure 6. - Step response: 6-pole low pass Gaussian filter with $f_c = 1.5$ MHz.

where B'_n is the filter noise bandwidth

V_n is the bus noise voltage whose nominal value is specified as 0.3 Vrms in a noise bandwidth (B_n) of 4MHz.

Thus, the filtered noise voltage is,

$$V'_n = \sqrt{\frac{2.4\text{MHz}}{4.0\text{MHz}}} V_n = \sqrt{0.6} V_n$$

and the probability density function of the filtered signal is,

$$p(s) = \frac{1}{\sqrt{2\pi} \sqrt{0.6} V_n} e^{-\frac{1}{2} \left(\frac{s - A}{\sqrt{0.6} V_n} \right)^2}$$

The sample statistics for quantizing threshold levels at $\pm V$ are derived from the probability density function as below:

$$p_c = P[s(t) > +V] = P[V'_n > +V - A] = \Phi\left(\frac{A - V}{\sqrt{0.6} V_n}\right)$$

$$p_i = P[-V < s(t) < +V] = P[-V - A < V'_n < +V - A] \\ = \Phi\left(\frac{A + V}{\sqrt{0.6} V_n}\right) - \Phi\left(\frac{A - V}{\sqrt{0.6} V_n}\right)$$

$$p_e = P[s(t) < -V] = P[V'_n < -V - A] = 1 - \Phi\left(\frac{A + V}{\sqrt{0.6} V_n}\right)$$

The Alternate MIA accepts a sample that exceeds either threshold as a bit; otherwise, the bit is invalidated.

For this criteria, the bit decision probabilities are:

$$P_c = p_c = \Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right)$$

$$P_e = p_e = 1 - \Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right)$$

$$P_i = p_i = \Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right) - \Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right).$$

The statistics for a single Orbiter MIA sample are the same as the Alternate MIA. Since the Orbiter MIA makes half-bit decisions on the basis of consecutive samples, it is necessary to consider all possible sample sequences which result in a particular decision and calculate the sequence probabilities. Let C_k , E_k , and I_k denote the outcome of the k^{th} sample as correct, incorrect, and improper, respectively. Since these sample outcomes are exhaustive and mutually exclusive,

$$P(C_k + E_k + I_k) = 1 = p_c + p_e + p_i.$$

A correct half-bit decision occurs when two consecutive samples are correct in a maximum of four samples. The sample sequences in which this can occur are:

2 samples - (C_1, C_2)

3 samples - $(E_1, C_2, C_3), (I_1, C_2, C_3)$

4 samples - $(C_1, E_2, C_3, C_4), (C_1, I_2, C_3, C_4),$

$(E_1, I_2, C_3, C_4), (I_1, E_2, C_3, C_4),$

$(I_1, I_2, C_3, C_4).$

Since these sequences are mutually exclusive, the probability of a correct half-bit decision is the sum of the joint probabilities,

$$\begin{aligned}
P_{\text{HBC}} &= P(C_1, C_2) + P(E_1 + I_1, C_2, C_3) \\
&+ P(C_1, E_2 + I_2, C_3, C_4) + P(E_1, I_2, C_3, C_4) \\
&+ P(I_1, E_2 + I_2, C_3, C_4).
\end{aligned}$$

The probability of an undetectable half-bit decision error is similarly derived;

$$\begin{aligned}
P_{\text{HBE}} &= P(E_1, E_2) + P(C_1 + I_1, E_2, E_3) \\
&+ P(E_1, C_2 + I_2, E_3, E_4) + P(C_1, I_2, E_3, E_4) \\
&+ P(I_1, C_2 + I_2, E_3, E_4).
\end{aligned}$$

Since all other sequences, not included in the above expressions, result in an invalid half-bit,

$$P_{\text{HBI}} = 1 - P_{\text{HBC}} - P_{\text{HBE}}.$$

These joint probability expressions cannot be evaluated except to determine their maximum and minimum values. These bounding values are obtained by expanding the joint probabilities in terms of conditional probabilities which are readily determined for the extremes of correlation between samples. The equation for P_{HBC} in terms of conditional probabilities is;

$$\begin{aligned}
P_{\text{HBC}} &= P(C_1) \cdot P(C_2/C_1) + P(E_1 + I_1) \cdot P(C_2/E_1 + I_1) \\
&\cdot P(C_3/C_2, E_1 + I_1) + P(C_1) \cdot P(E_2 + I_2/C_1) \\
&\cdot P(C_3/E_2 + I_2, C_1) \cdot P(C_4/C_3, E_2 + I_2, C_1) + P(E_1) \\
&\cdot P(I_2/E_1) \cdot P(C_3/I_2, E_1) \cdot P(C_4/C_3, I_2, E_1) + P(I_1) \\
&\cdot P(E_2 + I_2/I_1) \cdot P(C_3/E_2 + I_2, I_1) \\
&\cdot P(C_4/C_3, E_2 + I_2, I_1).
\end{aligned}$$

Completely dependent samples correspond to a unity correlation coefficient ($r = 1$) in which case the conditional probabilities of mutually exclusive events A and B are:

$$P(A/A) = P(B/B) = 1$$

$$P(A/B) = P(B/A) = 0.$$

Thus, for completely dependent samples ($r = 1$),

$$P_{HBc} = P(C) \cdot 1 = p_c$$

$$P_{HBe} = P(E) \cdot 1 = p_e$$

$$P_{HBi} = 1 - p_c - p_e.$$

The other extreme is complete independence between samples corresponding to a correlation coefficient of zero ($r = 0$) for which the conditional probabilities of mutually exclusive events A and B are:

$$P(A/A) = P(A/B) = P(A)$$

$$P(B/B) = P(B/A) = P(B).$$

The probabilities for completely independent samples ($r = 0$) are evaluated to be:

$$P_{HBc} = p_c^2 (3 - 2p_c - p_e^2)$$

$$P_{HBe} = p_e^2 (3 - 2p_e - p_c^2)$$

$$P_{HBi} = 1 - P_{HBc} - P_{HBe}.$$

The correlation between samples is caused by the input filter which bandlimits the noise. An estimate of the correlation between samples can be obtained from the correlation coefficient of an ideal low pass filter with cutoff frequency equal to the equivalent noise bandwidth

of the Gaussian filter,

$$r(\tau) = \frac{(\text{Sin } 2\pi B'_n \tau)}{(2\pi B'_n \tau)}$$

For $B'_n = 2.4\text{MHz}$ and an 8MHz sampling rate, the correlation between the first and k^{th} sample is

$$r_k = \frac{\text{Sin } 1.89k}{1.89k}$$

TABLE IV. - CORRELATION BETWEEN SAMPLES

Sample	1	2	3	4	5	6	7	8
Correlation	1.0	.50	-.16	-.10	+.13	-.00	-.08	+.05

Calculated values for the eight samples in a bit period are listed in table IV. For engineering purposes, only adjacent samples have a significant correlation. Samples taken two or more sampling periods later are uncorrelated and therefore independent Gaussian random variables. The sample correlations also reveal that half-bit decisions and bit decisions are both independent. Since the correlation between samples must be in the range ($0 \leq r \leq 1$), the half-bit decision probabilities are bounded by:

$$p_c^2 (3 - 2p_c - p_e^2) < P_{HBc} < p_c$$

$$p_e^2 (3 - 2p_e - p_c^2) < P_{HBue} < p_e$$

where p_c, p_e are single sample statistics. The bit decision probabilities resulting from the independent half-bit decisions are:

$$P_c = P_{HBc}^2$$

$$P_{ue} = P_{HBue}^2$$

$$P_i = 1 - P_c - P_{ue}$$

The principal expressions derived in this section are summarized in table V for reference. The next section presents calculated values.

TABLE V. - SUMMARY OF RECEIVER STATISTICS

	Main Engine Interface	Orbiter MIA	Alternate MIA
<u>Sample statistics</u>			
P_c	$\Phi\left(\frac{A}{V_n}\right)$	$\Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right)$	$\Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right)$
P_e	$1 - \Phi\left(\frac{A}{V_n}\right)$	$1 - \Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right)$	$1 - \Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right)$
P_i	0	$\Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right) - \Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right)$	$\Phi\left(\frac{A + V}{\sqrt{.6} V_n}\right) - \Phi\left(\frac{A - V}{\sqrt{.6} V_n}\right)$
<u>Bit decision probabilities</u>			
		(r = 0) (r = 1)	
P_c	P_c	$\left[p_c^2 (3 - 2p_c - p_e^2)\right]^2 < \bar{P}_c < p_c^2$	P_c
P_{ue}	P_e	$\left[p_e^2 (3 - 2p_e - p_c^2)\right]^2 < P_{ue} < p_e^2$	P_e
P_i	0	$1 - P_c - P_{ue}$	$1 - P_c - P_e$
<u>Word probabilities</u>			
P_{cw}	P_c^{31}	P_c^{25}	P_c^{47}
P_{uwe}	$\sum_{j=7}^{31} \binom{31}{j} p_{ue}^j P_c^{31-j}$	$\sum_{j=2,4,\dots}^{24} \binom{25}{j} p_{ue}^j P_c^{25-j}$	$\sum_{j=11}^{47} \binom{47}{j} p_{ue}^j P_c^{47-j}$
P_{iw}	$1 - P_{cw} - P_{uwe}$	$1 - P_{cw} - P_{uwe}$	$1 - P_{cw} - P_{uwe}$

4.0 CALCULATED PERFORMANCE DATA

The statistical expressions for receiver performance are numerically evaluated to determine predicted performance of each design and to compare their overall error detection effectiveness. The data are calculated as a function of data bus noise voltage in the range from 0.25 to 2.00 volts rms.

The performance data are calculated from the expressions in table V using appropriate values specified in table I for the peak input signal amplitude and the threshold level. MEI data are calculated for a ± 2.5 volt minimum input signal amplitude to obtain specified performance and for a ± 1.5 volt signal amplitude to allow a direct comparison with the two MIA's. The MIA data are calculated for the specified ± 1.5 volt minimum signal amplitude and ± 0.5 volt threshold values. Calculated results are presented in table VI for the MEI, tables VII and VIII for the Orbiter MIA with independent and dependent samples, and table IX for the Alternate MIA.

The overall error detection performance for specified parameter values of each design is graphically illustrated in figure 7. The performance of each design typically decreases exponentially with increasing noise voltage; however, the Orbiter MIA performance approaches a limiting value of about 10^{-8} at very high noise levels. This particular characteristic of the Orbiter MIA is desirable compared to the MEI and Alternate MIA which both decrease to a limiting value of about 1.

These performance limits can be determined from direct evaluation of the receiver statistics as the noise voltage increases without limit. The limiting value of

TABLE VI. - MAIN ENGINE INTERFACE PERFORMANCE

V_n (volts rms)		$P_c = P_c$	$P_e = P_{ue}$	P_{cw}	P_{uwe}
A=1.5V	A=2.5V				
0.25	0.42	.99999 99990	9.87×10^{-10}	.99999 9969	2.40×10^{-57}
0.30	0.50	.99999 97133	2.87×10^{-7}	.99999 1112	4.22×10^{-40}
0.35	0.58	.99999 11	8.90×10^{-6}	.99972 41	1.16×10^{-29}
0.40	0.67	.99991 16	8.84×10^{-5}	.99726 32	1.11×10^{-22}
0.50	0.83	.99865	.00135	.95899	2.08×10^{-14}
0.60	1.00	.99379	.00621	.82439	8.22×10^{-10}
0.70	1.17	.98394	.01606	.60538	5.16×10^{-7}
0.75	1.25	.97724	.02276	.48982	5.14×10^{-6}
0.80	1.33	.96960	.03040	.38404	3.31×10^{-5}
0.90	1.50	.95224	.04776	.21935	5.40×10^{-4}
1.00	1.67	.93319	.06681	.11724	3.75×10^{-3}
1.10	1.83	.91374	.08626	.06102	1.46×10^{-2}
1.20	2.00	.89440	.10560	.03144	3.95×10^{-2}
1.25	2.08	.88493	.11507	.02260	5.86×10^{-2}
1.30	2.17	.87574	.12426	.01635	8.19×10^{-2}
1.40	2.33	.85792	.14208	.00865	1.41×10^{-1}
1.50	2.50	.84134	.15866	.00472	2.12×10^{-1}
1.75	2.92	.80427	.19573	.00117	4.05×10^{-1}
2.00	3.33	.77337	.22663	.00035	5.75×10^{-1}

Parameter values: Input signal amplitude, A = 1.5 volt, 2.5 volt
 Threshold level, V = 0
 Noise bandwidth, $B_n = 4.0$ MHz

TABLE VII. - ORBITER MIA PERFORMANCE FOR INDEPENDENT SAMPLES ($r = 0$)

V_n (volts rms)	P_c	P_{ue}	P_i	P_{cw}	P_{uwe}
0.25	.99999 9999	1.35×10^{-92}	.00000 0000	.99999 9999	$\sim 10^{-182}$
0.30	.99999 9999	6.52×10^{-76}	.00000 0005	.99999 9995	$\sim 10^{-148}$
0.35	.99999 9925	1.07×10^{-47}	.00000 1885	.99999 8115	3.43×10^{-92}
0.40	.99999 7620	3.80×10^{-36}	.00000 2380	.99994 0492	4.33×10^{-69}
0.50	.99986	9.55×10^{-28}	.00014	.99640	2.73×10^{-52}
0.60	.99854	2.26×10^{-20}	.00146	.96406	1.48×10^{-37}
0.70	.99378	6.24×10^{-16}	.00622	.85548	1.01×10^{-28}
0.75	.98943	3.07×10^{-14}	.01057	.76678	2.21×10^{-25}
0.80	.98365	5.49×10^{-13}	.01635	.66227	6.19×10^{-23}
0.90	.96754	8.26×10^{-11}	.03246	.43822	9.58×10^{-19}
1.00	.94647	2.76×10^{-9}	.05353	.25271	6.45×10^{-16}
1.10	.92172	3.90×10^{-8}	.07828	.13029	7.00×10^{-14}
1.20	.89456	3.02×10^{-7}	.10544	.06169	2.11×10^{-12}
1.25	.88053	7.12×10^{-7}	.11947	.04155	8.15×10^{-12}
1.30	.86624	1.54×10^{-6}	.13376	.02760	2.62×10^{-11}
1.40	.83774	5.76×10^{-6}	.16226	.01196	1.70×10^{-10}
1.50	.80988	1.68×10^{-5}	.19010	.00513	6.63×10^{-10}
1.75	.74381	1.24×10^{-4}	.25607	.00061	5.10×10^{-9}
2.00	.68502	4.76×10^{-4}	.31450	.00008	1.13×10^{-8}

Parameter values: Input signal amplitude, $A = 1.5$ volt
 Threshold level, $V = 0.5$ volt
 Noise bandwidth, $B_n = 2.4$ MHz

TABLE VIII. -- ORBITER MIA PERFORMANCE FOR DEPENDENT SAMPLES (r=1)

V_n (volts rms)	P_c	P_{ue}	P_i	P_{cw}	P_{uwe}
0.25	.99999 9754	5.81×10^{-47}	.00000 0246	.99999 3850	3.38×10^{-93}
0.30	.99998 30	1.28×10^{-38}	.00001 70	.99957 51	4.91×10^{-74}
0.35	.99977 58	1.64×10^{-24}	.00022 42	.99441 04	8.03×10^{-46}
0.40	.99875	9.74×10^{-19}	.00125	.96914	2.77×10^{-34}
0.50	.99020	1.54×10^{-14}	.00980	.78184	5.67×10^{-26}
0.60	.96885	7.40×10^{-11}	.03115	.45329	7.93×10^{-19}
0.70	.93588	1.21×10^{-8}	.06412	.19078	9.57×10^{-15}
0.75	.91656	8.41×10^{-8}	.08344	.11324	2.86×10^{-13}
0.80	.89632	2.81×10^{-7}	.10368	.06479	1.91×10^{-12}
0.90	.85416	4.24×10^{-6}	.14584	.01943	1.44×10^{-10}
1.00	.81297	2.41×10^{-5}	.18701	.00565	1.49×10^{-9}
1.10	.77405	8.95×10^{-5}	.22586	.00166	6.64×10^{-9}
1.20	.73795	2.46×10^{-4}	.26180	.00050	1.67×10^{-8}
1.25	.72114	3.77×10^{-4}	.27848	.00028	2.31×10^{-8}
1.30	.70501	5.53×10^{-4}	.29444	.00016	2.96×10^{-8}
1.40	.67526	1.06×10^{-3}	.32368	.00005 5	4.03×10^{-8}
1.50	.64864	1.82×10^{-3}	.34954	.00002 0	4.72×10^{-8}
1.75	.59250	4.92×10^{-3}	.40258	.00000 21	4.31×10^{-8}
2.00	.54839	9.67×10^{-3}	.44194	.00000 03	2.84×10^{-8}

Parameter values: Input signal amplitude, $A = 1.5$ volt
 Threshold level, $V' = 0.5$ volt
 Noise bandwidth, $B'_n = 2.4$ MHz

TABLE IX. - ALTERNATE MIA PERFORMANCE

V_n (volts rms)	$p_c = P_c$	$p_e = P_{ue}$	$p_i = P_i$	P_{cw}	P_{uwe}
0.25	.99999 9877	7.62×10^{-24}	.00000 0123	.99999 4219	$\sim 10^{-254}$
0.30	.99999 15	1.13×10^{-19}	.00000 85	.99960 06	$\sim 10^{-199}$
0.35	.99988 79	1.28×10^{-12}	.00011 21	.99474	$\sim 10^{-132}$
0.40	.99937	9.87×10^{-10}	.00063	.97097	$\sim 10^{-89}$
0.50	.99509	1.24×10^{-7}	.00491	.79347	1.55×10^{-66}
0.60	.98430	8.60×10^{-6}	.01569	.47533	1.88×10^{-46}
0.70	.96741	.00011	.03248	.21072	1.51×10^{-34}
0.75	.95737	.00029	.04234	.12905	4.43×10^{-30}
0.80	.94674	.00053	.05273	.07636	2.25×10^{-27}
0.90	.92421	.00206	.07373	.02462	2.89×10^{-21}
1.00	.90165	.00491	.09344	.00771	1.68×10^{-17}
1.10	.87980	.00946	.11074	.00243	9.41×10^{-15}
1.20	.85904	.01570	.12526	.00079	1.05×10^{-12}
1.25	.84920	.01941	.13139	.00046	7.14×10^{-12}
1.30	.83965	.02351	.13684	.00027	3.91×10^{-11}
1.40	.82174	.03259	.14567	.00010	6.54×10^{-10}
1.50	.80538	.04262	.15200	.00003 82	6.54×10^{-9}
1.75	.76974	.07011	.16015	.00000 46	3.85×10^{-7}
2.00	.74053	.09835	.16112	.00000 07	4.68×10^{-6}

Parameter values: Input signal amplitude, $A = 1.5$ volt
 Threshold level, $V = 0.5$ volt
 Noise bandwidth, $B_n = 2.4$ MHz

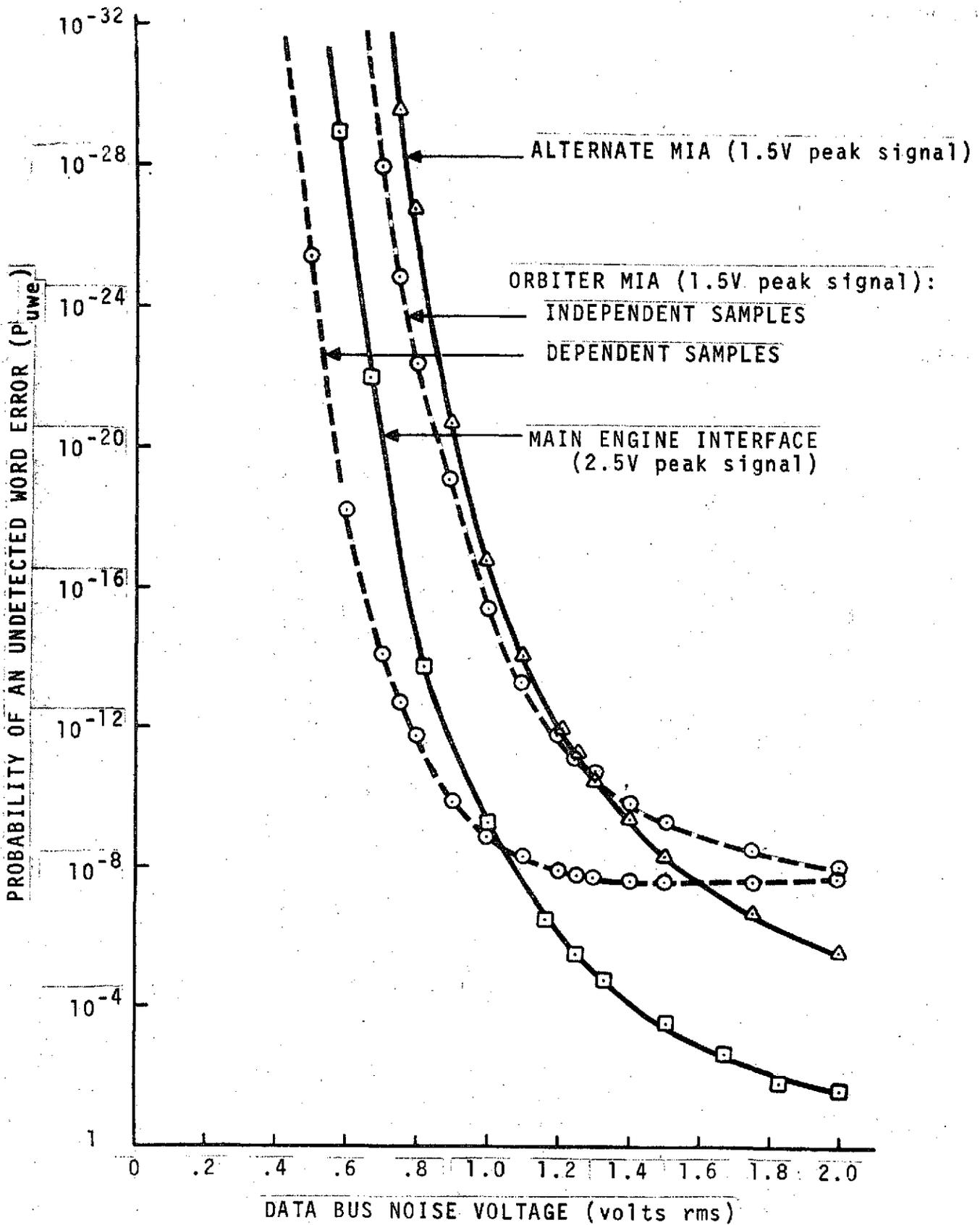


Figure 7. - Error detection performance.

the receiver statistics calculated in this manner are listed in table X. The error detection performance of all designs is better than 10^{-40} at the nominal noise level specified as 0.3 Vrms. However, there are several orders of magnitude difference between designs and the performance of each is extremely dependent upon small changes in the rms noise at low levels.

The probability of outputting a correct word is plotted in figure 8 as a function of bus noise voltage. All three designs have a probability greater than 0.999 at the specified 0.3 Vrms noise which rapidly decreases for noise greater than 0.4 to 0.5 Vrms. The MEI characteristic is better than that for the MIA's because it requires a 2.5 volt peak signal instead of 1.5 volts. Comparison of tabulated data for 1.5 volt signals reveals a MEI characteristic between the lower and upper bounds of the Orbiter MIA.

The correct word throughput rate for each design is presented in table XI and figure 9. The higher output rate of the Orbiter MIA at low noise results from use of a single code check bit instead of a highly redundant code. However, the Orbiter MIA requires a more complex bit detector to obtain overall error detection performance equivalent to that for the MEI and Alternate MIA which use simpler bit detectors.

The relative attributes of the bit detection techniques are revealed by comparing the undetected bit error probabilities for identical input signal amplitude (1.5 V peak) and noise as plotted in figure 10. The MEI design has the greatest probability for bit errors and is the least

TABLE X. - LIMITING VALUE OF RECEIVER STATISTICS ($V_n \rightarrow \infty$)

	Main Engine Interface	Orbiter MIA	Alternate MIA
<u>Sample statistics</u>			
P_c	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
P_e	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
P_i	0	0	0
<u>Bit decision probabilities</u>			
P_c	$\frac{1}{2}$	$\frac{1}{4} \left(\frac{7}{8}\right)^2 < P_c < \frac{1}{4}$	$\frac{1}{2}$
P_{ue}	$\frac{1}{2}$	$\frac{1}{4} \left(\frac{7}{8}\right)^2 < P_{ue} < \frac{1}{4}$	$\frac{1}{2}$
P_i	0	$\frac{1}{2}$	0
<u>Word probabilities</u>			
P_{cw}	2^{-31}	2^{-50}	2^{-47}
P_{uwe}	$1 - 2^{-21} \approx 1$	$2^{-26} \approx 10^{-8}$	$1 - 2^{-18} \approx 1$
P_{iw}	2^{-21}	$1 - 2^{-26} \approx 1$	2^{-18}

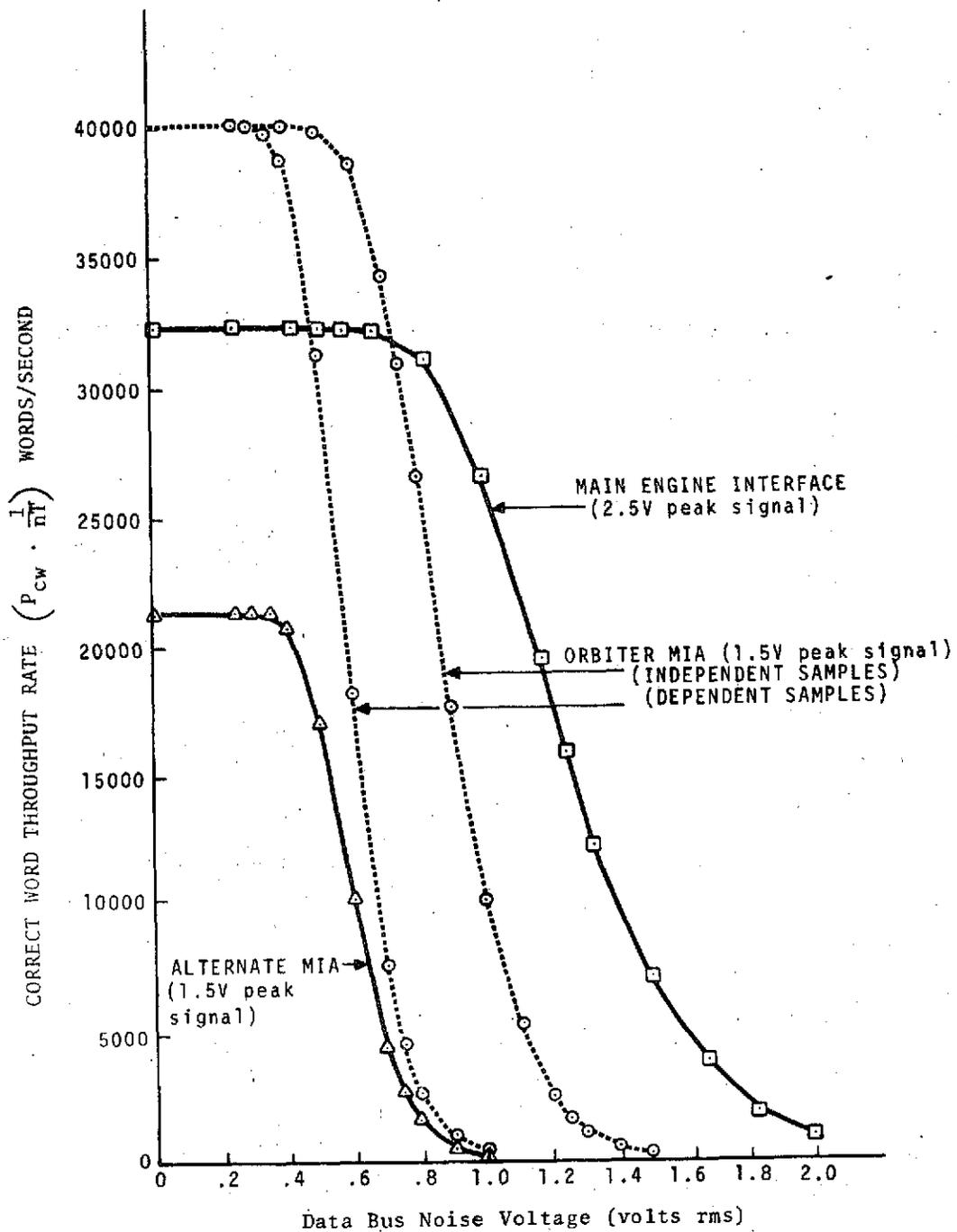


Figure 8. - Probability of a correct word.

TABLE XI. - CORRECT WORD THROUGHPUT RATE

V_n (volts rms)	Main Engine Interface (2.5V signal)	Orbiter MIA (1.5V signal)		Alternate MIA (1.5V signal)
		r = 0	r = 1	
0.25	32,258	40,000	40,000	21,277
0.30	32,258	40,000	39,983	21,268
0.35	32,258	40,000	39,776	21,165
0.40	32,258	39,998	38,766	20,659
0.50	32,258	39,856	31,274	16,882
0.60	32,249	38,562	18,132	10,113
0.70	32,170	34,219	7,631	4,483
0.75		30,671	4,530	2,746
0.80	30,935	26,491	2,592	1,625
0.90		17,529	777	524
1.00	26,593	10,108	226	164
1.10		5,212	66	52
1.20	19,528	2,468	20	17
1.25	15,801	1,662	11	10
1.30	12,388	1,104	6	6
1.40		478	2	2
1.50	7,076	205	1	1
1.75	2,875	24	0	0
2.00	1,014	3	0	0

Note: Data tabulated in words per second.

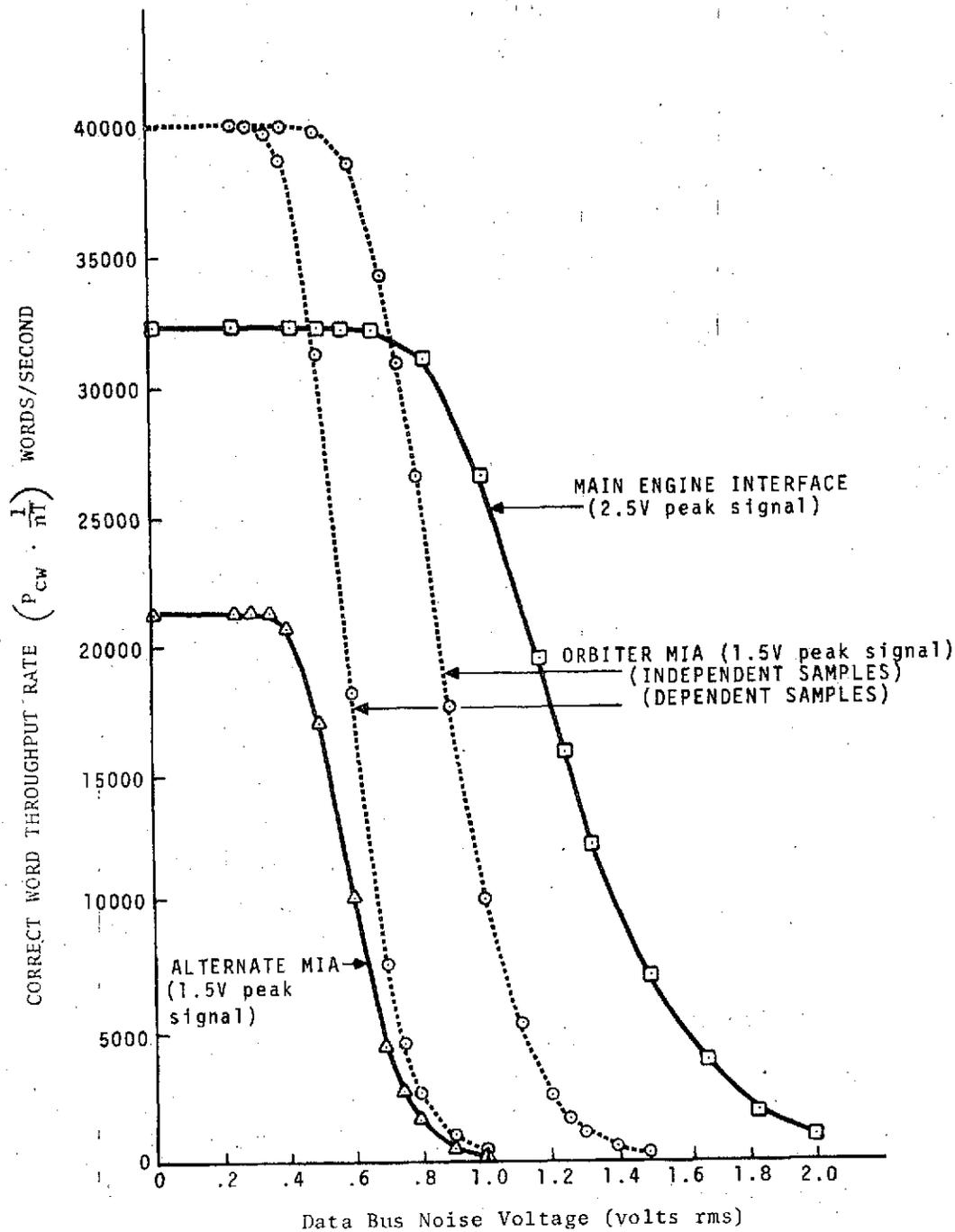


Figure 9. - Correct word throughput rate.

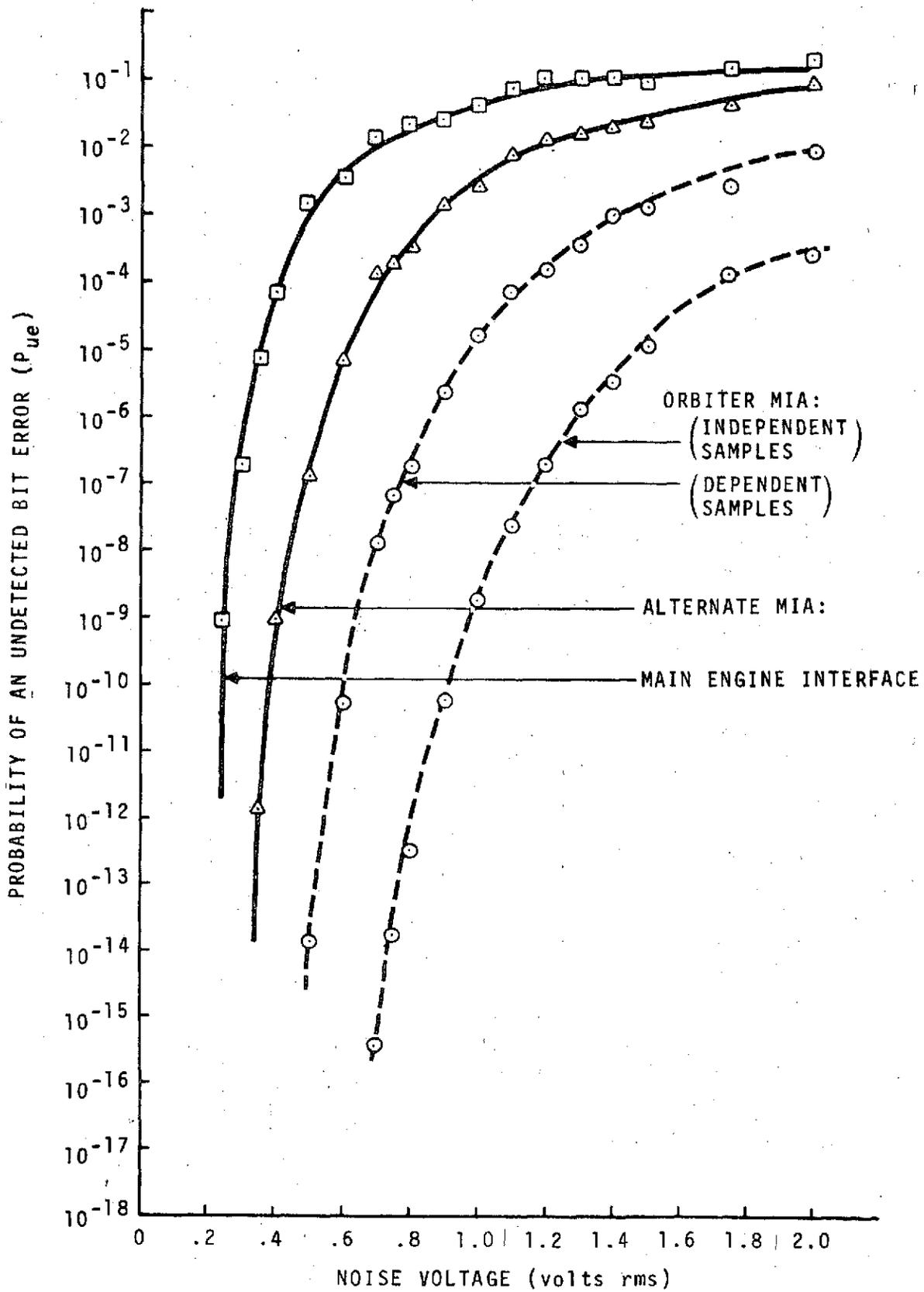


Figure 10. - Undetected bit error performance.

effective design in this regard. The effectiveness of the input filter and null zone detector accounts for the difference between the MEI and the Alternate MIA performance. They provide several orders of magnitude improvement at high signal to noise, and about one order of magnitude improvement at unity signal to noise ratio. The improvement between the Alternate MIA and the Orbiter MIA with dependent samples represents the effectiveness of implementing one sample per half-bit and a valid Manchester code check instead of simply one sample per bit. It is interesting to note that the dependent sample case corresponds to sampling at a 2MHz rate and performing a validity check on the Manchester coded bits. The maximum improvement realized by sampling at 8MHz and the half-bit decision criteria implemented in the Orbiter MIA design is the difference between the Alternate MIA and the independent sample case.