January 2, 1975

FINAL REPORT
CONVOLUTIONAL CODING TECHNIQUES
FOR DATA PROTECTION

NASA Grant NGL 15-004-026

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RESEARCH PERIOD REPORTED: September 15, 1967 to July 31, 1974
(Grant was inactive during the period
Sept. 16, 1971 to Oct. 15, 1972)

PRICES SUBJECT TO CHANGE
1. Introduction

Under this grant, NASA has supported for six years the work of the Principal Investigator and his associates centering on the use of convolutional codes in data communications. The breadth of this work can be seen from a perusal of the list in Section IV of the publications issuing under this grant.

Because the research in the first five years of this grant has previously been described in reports to NASA, we give a description in Section II only of the research performed in the final year of the grant. In Section III, we list the personnel who have been involved in the research under this grant since its inception and, as mentioned previously, we give a complete listing of the grant publications in the final Section IV.

II. Summary of Research in Final Year of Grant (October 16, 1973 to July 31, 1974)

It has been the policy of the Principal Investigator to communicate research results under this grant to NASA as timely as possible through appropriate technical reports and/or preprints of submitted journal articles. Most of the research performed during the final year of this grant has already been communicated in this manner to NASA and hence this section will be devoted primarily to citing of the pertinent reports together with a description of certain fragmentary results that have not been previously reported. The following four sub-sections contain this description of the research as divided into topical areas.

A. Convolutional Coding Fundamentals

The nearly six years of extensive research into convolutional coding techniques conducted under this grant has given the Principal Investigator an unparalleled opportunity to explore the advantages of "non-block" coding for
communications purposes. In an effort to share the fruits of this research with the communications engineering community, the Principal Investigator has recently prepared the following self-contained and extensive treatment of non-block coding:


The above paper, which runs to 78 typewritten pages, while written in a tutorial style is also repository for a number of new results including:

1. A useful definition of the general classes of tree and trellis codes with proofs of their error-correcting power,

2. A quantitative demonstration of the advisability of choosing the "memory" greater than the "tail length" for convolutional codes used with sequential decoding, and

3. A description of the smallest class of convolutional codes for which the random coding error bounds apply.

Motivated by (1) and (2) above, Mr. Rolf Johannesson, research assistant under this grant, conducted extensive simulations to verify the required excess of memory over tail length for minimal error probability and also generalized the bounds for tree and trellis codes. These results are reported in:


Motivated further by the differing importance of the parameters "row distance," "column distance," and "free distance" of convolutional codes [all of which parameters were first identified as useful by previous research under this grant] dependent on the type of decoder used [sequential decoder, Viterbi decoder, or algebraic decoder], Johannesson sought to find codes which would
be simultaneously optimal or near-optimal for all three criteria. This work
was successfully concluded and is described in:

R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional
Codes," Tech. Rpt. No. EE-7403, Department of Electrical Engineering,
University of Notre Dame, July 3, 1974 (to appear in IEEE Trans. on Information
Theory.)

and was in part presented orally as

R. Johannesson, "Some Rate 1/2 Binary Convolutional Codes Which Are
Optimal for Various Criteria," Presented at IEEE International Symposium on

These codes are expected to be prime candidates for adoption by NASA in
future coding standards for deep-space missions.

B. Modulation and Coding Interaction

The concern about the effect of the bit tracking loop memory on sequential
decoding computation that arose in the HELIOS program motivated the principal
investigator to pay increased attention to the interaction of the coding scheme
with the modulation scheme in a communication system. Such considerations
led to a "new" communications theory adequate to treat simultaneously coding
and modulation and first reported in:

International Zurich Seminar on Digital Communications, pp. E2(1)-E2(4),
Zurich, Switzerland, March 12-15, 1974.

This paper gives (1) the first analytical approach to optimal "soft-
decision" demodulation for binary modulation and (2) a simple method to evalu-
ate the quality of a modulation signal set. The approach taken in this paper
is that the function of the modulation system is not to minimize bit error
probability, as has been traditional, but to create the best channel for coding.
"Best" here means having the largest cut-off rate R_0 (the "two-codeword random
coding exponent" or, equivalently, the rate above which the average computation
for sequential decoding becomes infinite.) As an application of the approach,
it is shown that the simplex signal set is optimal for the $R_0$ criterion for the additive white Gaussian noise channel.

Johannesson has continued the investigation of the $R_0$ approach to channels with memory and some fragmentary results were obtained before the expiration of the grant period. This work was continued under a new grant (NASA Grant NSG 5025 "Coordinated Design of Coding and Modulation Systems") and a technical report now under preparation under the new grant will contain these earlier fragmentary results as well as more extensive further results. Johannesson also began a study for the analytical calculation of the computational distribution for sequential decoding and obtained some results under this grant which will be reported in another technical report, under the new grant, now in preparation.

To illustrate the importance of soft-decision modulation, particularly for channels with memory, if coding is to be profitably employed, the Principal Investigator applied the experience gained under this grant to the problem of communication against ionospheric scintillations as has recently been encountered in communication satellite systems. This research was reported in:


C. Concatenated Coding Systems

Mr. Lin-nan Lee, research assistant under this grant, has been investigating the use of convolutional codes as the inner codes in concatenated coding systems. An early realization from this study was of the need for extending Massey's optimal soft-decision theory for binary modulation to the non-binary case because the digits in the outer code of a concatenated coding system are non-binary. The results of this work were reported in:

Lee has continued this research under the new grant mentioned above (NASA Grant NSG 5025) and is preparing a technical report thereunder which will include the detailed derivation of the results in the above presentation. The object of his research is to produce simple, yet powerful, concatenated coding systems in which the inner decoder will be a "soft-decision Viterbi decoder."

D. Data Compression with Convolutional Codes

Mr. Teofilo C. Ancheta, Jr., research assistant under this grant, has been continuing his investigation of the "syndrome-source-coding" technique by which error-correcting codes are used for data compression in a novel manner. His work under this grant is reported in:


To elucidate the difference and the potential advantage of "syndrome-source-coding" over the conventional or "codeword method" of using error-correcting codes for data compression, the Principal Investigator prepared the following tutorial paper:


In general, it can be stated that the "codeword method" is more suited to data sources that exhibit much symmetry and little memory whereas the "syndrome method" matches better to sources with little symmetry and much memory. Since the latter type of source seems to approximate real sources better than the former, we expect syndrome-source-coding to find practical
applications in data compression systems. In order to study source models with memory, it was found necessary to enlarge the present understanding of Markov source models which seem to be the most realistic and convenient models for sources with memory. Our results in this area were reported in:

III. Summary of Personnel Supported by Grant

One of the valuable aspects of a research grant to a University is the resultant educational value to the graduate students who perform research to meet grant objectives. Through Grant NGL 15-004-026, NASA has provided such support to ten graduate students, six of whom have already received the Ph.D. degree from the University of Notre Dame for dissertations based on their grant research and two more of whom are expected to do so in the near future.

A ninth graduate student, Rolf Johannesson, spent the period September 1, 1973 to October 31, 1974 on leave from the University of Lund, Sweden, to perform research under the Principal Investigator which he will now submit as his doctoral research at Lund. In this respect, NASA has through this grant played a remarkably extensive and vital role in support of graduate education.

Several of these past graduate students have already won considerable recognition for their research under this grant. The following is a table of all graduate students who performed research under this grant under the direction of the Principal Investigator.

<table>
<thead>
<tr>
<th>Name</th>
<th>Period of Research Support under Grant</th>
<th>Remarks</th>
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<tbody>
<tr>
<td>Teofilo C. Ancheta, Jr.</td>
<td>10/16/72 to 7/31/74</td>
<td>Ph.D. expected August 1975.</td>
</tr>
<tr>
<td>Daniel J. Costello, Jr.</td>
<td>9/16/68 to 9/16/69</td>
<td>Ph.D. received, Sept. 1969.</td>
</tr>
<tr>
<td>John M. Geist</td>
<td>9/16/69 to 9/15/70</td>
<td>Ph.D. received, Sept. 1970.</td>
</tr>
<tr>
<td>Rolf Johannesson</td>
<td>(unsupported)</td>
<td>Ph.D. expected, Univ. of Lund, Sweden, June 1975.</td>
</tr>
<tr>
<td>Lin-nan Lee</td>
<td>10/16/72 to 7/31/74</td>
<td>Ph.D. expected, June 1975.</td>
</tr>
</tbody>
</table>
Raymond R. Olson  9/16/67 to 8/15/68  Ph.D. received, Sept. 1970.
Gerald E. Seguin  9/16/69 to 8/15/71  Ph.D. received, Jan. 1972.

Besides the above graduate students, research support under this grant was furnished during the period 6/16/68 to 8/15/68 to Dr. Michael K. Sain and to Dr. Kasivanathan Vairavan, the former of whom acted as a co-investigator during this period and the latter of whom served as a postdoctoral research assistant. Dr. James L. Massey has been the Principal Investigator throughout the grant period.
IV. Summary of Grant Publications

Research is generally of little value, even to its sponsor, unless it is communicated rapidly and clearly to the general scientific world and contributes thereby to the advance of its technical field. The research from this grant has resulted in 20 papers published in archival journals, 21 conference presentations and/or conference publications and 21 technical reports. These publications are listed in sub-sections A, B and C below. (Two further papers are presently in process of review for possible journal publication as mentioned in Section II.) Preprints and/or reprints of all these publications were distributed previously to NASA in accordance with the established grant procedures and contained acknowledgment of NASA support.

A. Journal Articles


B. Conference Presentations and Articles
Published in Conference Proceedings


C. Technical Reports


J. L. Massey and M. K. Sain, "Trunk and Tree Searching Properties of the Fano
Sequential Decoding Algorithm," Elec. Engr. Memo. No. EE-6817, Univ. of Notre
Dame, Notre Dame, Ind., October 1, 1968.

T. N. Morrissey, Jr., "A Unified Markovian Analysis of Decoders for Convolutional
Dame, Ind., October 24, 1968.

D. J. Costello, Jr., "A Construction Technique for Random Error Correcting
Convolutional Codes," Memo EE-6822, Dept. of Elec. Engr., U. of Notre Dame,
Notre Dame, Ind., November 14, 1968.

D. J. Costello, Jr., "Construction of Convolutional Codes for Sequential

J. J. Uhran, Jr. and J. L. Massey, "Analysis of Satellite Communications in
a Multipath Environment," Elec. Engr. Memo EE-701, Univ. of Notre Dame, Notre
Dame, Indiana, January 1970.

J. M. Geist, "A Comparison of the Fano and Jelinek Sequential Decoding Algorithms,"
Ind., January 31, 1970.

No. EE-703, Dept. of Elec. Engr., Univ. of Notre Dame, Notre Dame, Indiana,


W. F. Hartman, "Note on Arithmetic Codes and Arithmetic Distance," Tech. Report

Univ. of Notre Dame, Notre Dame, Ind., July 26, 1971. (Submitted to IEEE
Transactions on Information Theory.)


L. Lee, "Real-Time Minimal-Bit-Error-Probability Decoding of Convolutional Codes,"

No. EE-7313, Dept. of Elec. Engr., U. of Notre Dame, Notre Dame, Ind., July 25,
1973 (Revised July 1974.)

R. Johannesson, "On the Error Probability of General Tree and Trellis Codes with
Univ. of Notre Dame, December 1973.

R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional Codes,"