

X-621-74-350

PREPRINT

NASA TM X- 70814

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N75-15392

(NASA-TM-X-70814) THEORY OF WAVES  
INCOHERENTLY SCATTERED (NASA) 61 P HC \$4.25  
CSCI 20N

Unclas  
G3/70 07170

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DECEMBER 1974



GODDARD SPACE FLIGHT CENTER  
GREENBELT, MARYLAND

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THEORY OF WAVES INCOHERENTLY SCATTERED

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## THEORY OF WAVES INCOHERENTLY SCATTERED

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### ABSTRACT

Electromagnetic waves impinging upon a plasma at frequencies larger than the plasma frequency, suffer weak scattering. The scattering arises from the existence of electron density fluctuations. The so-called incoherent scattering theory basically deals with fluctuations of random thermal origin; however, for practical purposes, it must also take account of those fluctuations caused by streaming photo-electrons. As is well known, in any scattering experiment, the received signal corresponds to a particular spatial Fourier component of the fluctuations, the wave vector of which is a function of the wavelength of the radio-wave. Wavelengths short with respect to the Debye length of the medium relate to fluctuations due to non-interacting Maxwellian electrons, while larger wavelengths relate to fluctuations due to collective Coulomb interactions. In the latter case, the scattered signal exhibits a spectral distribution which is characteristic of the main properties of the electron and ion gases and, therefore, provides a powerful diagnosis of the state of the plasma, in our case, the ionosphere.

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## INTRODUCTION

The steps leading to the theory of electromagnetic waves incoherently scattered by the ionosphere, or more generally by a plasma, for operating frequencies much larger than the plasma frequency, can be tracked back to the last century. It was not until 1958, however, when Gordon (1958) became aware of the fact that such a scattering was within reach of modern radars, that a large and very successful effort was conducted in order to establish a complete theory.

The first keystone might be thought to be the theory of the scattering of light by the molecules of the atmosphere, developed by Lord Rayleigh (1871). The next and major step is the work of Thomson (1906) who showed that a free electron acting as a dipole is capable of scattering electromagnetic waves at all frequencies. Therefore electromagnetic waves ought to suffer a small scattering from a plasma in a manner similar to the scattering of light by molecules. This last point was further emphasized by Fabry (1928) who indicated that the random motion of the electrons should induce a doppler broadening of the back-scattered signals. The importance of the random motion of the electrons on the scattering has later led to calling the phenomenon "Incoherent scatter" rather than "Thomson scatter". A large effort followed to develop the theory of random motions in plasmas and credit should be given to cite but a few, to Landau (1946), Bohn and Gross (1949a, 1949b), and Pines and Bohn (1952).

Interest in ionospheric research developed rapidly after pioneering experiments of Appleton and Barnett (1925), Breit and Tuve (1925), and Taylor and

Hulburt (1926) which definitely located the ionospheric layer. By 1931 Chapman gave a theory of the ionizing effect of monochromatic radiation in an atmosphere on a rotating earth. The available techniques however put a limit to the study of the ionosphere to essentially the bottom side electron density profiles, leaving out, for example, experimental studies of the energetics of the ionosphere.

The theory of scattering of electromagnetic waves by the earth environment was also in progress. Booker and Gordon (1950) considered radio scattering from the troposphere and Booker (1955) scattering by nonisotropic irregularities in the aurora.

Gordon (1958) finally took the decisive step when he realized that the incoherent scatter from free electrons of the ionosphere would amount, provided powerful radars be used, to a sufficiently large fraction of the sky noise, even taking into account the fact that the doppler broadening due to the thermal motion of the electrons was expected to be quite large. The total cross section was computed to be of the order of  $.25 \text{ cm}^2$  for a  $1^\circ$  antenna beamwidth and a  $67 \mu\text{s}$  pulse length with a mean electron density of  $10^6 \text{ cm}^{-3}$  (Farley 1970). Gordon pointed out that this experiment would provide electron density and temperature data up to several earth radii.

The suggestion made by Gordon met very favorable circumstances. Indeed, it coincided with the advent of the space age, that is, as far as the ionosphere is concerned, with direct probing of the medium and consequently an increasing interest in the understanding of the processes governing it.

The first incoherent scatter experiment was performed the same year by Bowles (1958) using an already existing transmitter together with an array of dipoles. The return power appeared to be approximately of the expected order of magnitude but the bandwidth was observed to be much narrower than predicted by Gordon.

While several groups were in the process of developing incoherent scatter facilities for the purpose of ionospheric studies, several authors undertook the studies of the electron fluctuations including the effect of the ions which rapidly appeared to be responsible for the narrow power spectrum observed by Bowles. The case of independent electrons would appear as a limiting case, and in most instances the small degree of organization of the medium, due in particular to Coulomb interactions, ought to dominate the characteristics of the scattering process. The theory of the scattering had become much more involved than thought in the first place, in turn a much more detailed description of the medium was to be obtained from the incoherent scatter spectrum.

The scope of this paper is to outline the different steps of the building of the theory. The geophysical results as well as the technical description of the existing facilities, which have been extensively reviewed by Evans (1969, 1974) and Farley (1970b), are not covered here.

#### I. Thomson Scattering of an Electromagnetic Wave by a Free Electron

When a field  $\vec{E}_i = \vec{E}_0 e^{i\omega_0 t}$  is applied to a free electron at rest, an oscillatory motion is imparted to such as:

$$\dot{\vec{V}} = \frac{q \vec{E}_0 e^{i\omega_0 t}}{m_e} \quad \text{I.1}$$

where  $\omega_0$  is the angular frequency,  $t$  the time,  $q$  the charge electron,  $m_e$  the electron mass and  $m_e \dot{\vec{V}}$  the time rate of change of momentum of the electron.

In turn the acceleration of the electron gives a rise to a radiation field  $\vec{E}_s$ .

$$\vec{E}_s = \frac{q}{4\pi\epsilon_0 c^2 R^3} \vec{R} \times (\vec{R} \times \dot{\vec{V}}) e^{-i\omega_0 R/c} \quad \text{I.2}$$

where  $\vec{R}$  is the radius vector joining the position of the electron to the field point,  $c$  the speed of light in vacuum,  $\epsilon_0$  the dielectric constant of the vacuum.

Use of I.1 yields:

$$\vec{E}_s = \frac{q^2}{4\pi\epsilon_0 c^2 m_e} \frac{\vec{R} \times (\vec{R} \times \vec{E}_0)}{R^3} e^{-i(\omega_0 R/c)} \quad \text{I.3}$$

therefore the magnitude of the field is

$$E_s = \frac{q^2}{4\pi\epsilon_0 c^2 m_e} \frac{E_0}{R} e^{i\omega_0(t-R/c)} \sin \xi \quad \text{I.4}$$

where  $\xi$  is the angle between  $\vec{E}_0$  and  $\vec{R}$ . The same treatment applies to the ions but because of the mass difference the radiation field is negligible compared to that of the electrons. Equation I.4 can be written as:

$$E_s = \frac{r_e}{R} E_0 e^{i\omega_0(t-R/c)} \sin \xi \quad \text{I.5}$$

where  $r_e = q^2 / (4 \pi \epsilon_0 c^2 m_e)$  is the classical electron

radius:  $r_e = 2.82 \times 10^{-15}$  m.

More generally  $R$  is a function of time, and the frequency of the scattered field

$E_s$  is doppler shifted with respect to the frequency of the incident field  $E_i$ .

The mean radiated power per unit area is

$$P = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{r_e^2}{R^2} E_0^2 \sin^2 \xi \quad \text{I.6}$$

where  $\mu_0$  is the magnetic permeability in vacuum. Equation I.6 can be written

as

$$P = \frac{\sigma_e}{R^2} \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} E_0^2 \quad \text{I.7}$$

where  $\sigma_e = r_e^2 \sin^2 \xi$  is the differential cross section.

## II. Thomson Scattering by Independent Randomly Distributed Free Electrons

The problem is now extended to a set of independent randomly distributed free electrons.

The effect of multiple scattering can be neglected in practical cases because of the smallness of the differential cross section. The working frequency is supposed to be much larger than the frequency of free oscillations (plasma frequency). It follows from the above that the transmitted wave is not significantly altered: The Born approximation is valid. It is further assumed that

the electrons of interest are confined in a small region compared to a sphere of radius  $R$ , where  $R$  is the mean distance between the field point and the scattering electrons.

The total field at the field point can be expressed as

$$\vec{E}_s = \sum_j \vec{E}_{j_s} \quad \text{II.2}$$

$$E_s = \frac{r_e}{R} \sin \xi E_0 e^{i\omega_0 t} \sum_j e^{-i\psi_j} \quad \text{II.2}$$

where  $j$  indicates a summation over all the electrons and  $\psi_j$  is the phase associated with the location of each of the electrons relative to both the transmitter and the receiver sites. However, because of the random nature of the electron distribution in space, the instantaneous field at the field point is of no interest. The average radiated power per unit area is the physically interesting quantity.

$$\langle P \rangle = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} R_e \langle E_s E_s^* \rangle \quad \text{II.3}$$

$$\langle P \rangle = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \left( \frac{r_e}{R} \sin \xi \right)^2 E_0^2 \left[ R_e \left\langle \sum_j e^{-i\psi_j} e^{i\psi_j} \right\rangle + R_e \left\langle \sum_i \sum_{j \neq i} e^{-i\psi_i} e^{i\psi_j} \right\rangle \right] \quad \text{II.4}$$

where  $\Re_e$  designates the real part and the brackets indicate the ensemble average. The first term in the square brackets is just  $N_e$  the number of electrons in the volume of interest. The second term represents the mean phase relationships of the electrons taken two by two; this term has to be zero since the electrons are supposed to be independent.

The radiated power is therefore simply the sum of the radiated power of each electron taken independently

$$\langle P \rangle = \sum_j P_j \quad \text{II.5}$$

and the total differential cross section is

$$\sigma_{\text{tot}} = N_e \sigma_e \quad \text{II.6}$$

This is the result obtained by Gordon (1958). As for the doppler broadening of the scattered field, it simply corresponds to the velocity distribution of the electrons. In the case of backscatter the doppler shift associated with a velocity  $v$  is  $(\omega - \omega_0) = -2\omega_0 v/c$ . Therefore to a Maxwellian distribution of velocities

$$f(v) \propto e^{-\frac{m_e v^2}{2KT_e}}$$

(where  $K$  is the Boltzmann constant and  $T_e$  the electron temperature) there corresponds a power spectrum

$$S(\omega) \propto e^{-\frac{m_e}{2KT_e} (c(\omega - \omega_0)/2\omega_0)^2} \quad \text{II.7}$$

The first experiment of Bowles (1958), however, showed that in general the neglect of the term

$$\left\langle \sum_i \sum_{j \neq i} e^{-i\psi_i} e^{i\psi_j} \right\rangle$$

is not valid. This term represents the degree of organization of the medium which arises from particle interactions.

The object of the rest of this paper is to investigate the degree of organization brought into the medium under various circumstances. It will be shown that in general the total differential cross section obtained for the case of independent electrons is an upper limit.

### III. Formulation of the Scattering Problem

It is first convenient to express the scattered field in the following way

$$E_s = \frac{r_e}{R} \sin \xi E_0 \sum_j e^{i\omega_0 t - i\vec{k}_1 \cdot \vec{r}_j + i\vec{k}_2 \cdot \vec{r}_j} \quad \text{III.1}$$

where  $\vec{k}_1$  is the wave vector of the transmitted electric field,  $\vec{k}_2$  the wave vector of the scattered field in the direction of observation and  $\vec{r}_j$  the radius vector between the  $j^{\text{th}}$  electron and the center of the scattering volume (the incident field at the center of the scattering volume is just  $E_0 e^{i\omega_0 t}$ ).

Defining  $\vec{k}$  as  $\vec{k} = \vec{k}_1 - \vec{k}_2$  leads to

$$E_s = \frac{r_e}{R} \sin \xi E_0 \sum_j e^{i(\omega_0 t - \vec{k} \cdot \vec{r}_j)} \quad \text{III.2}$$

for point charges the electron distribution can be expressed as:

$$N_e(\vec{r}, t) = \sum_j \delta(\vec{r} - \vec{r}_j) \quad \text{III.3}$$

the spatial fourier series of which is

$$N_e(\vec{r}, t) = \sum_k N_e(\vec{k}, t)|_V e^{i\vec{k} \cdot \vec{r}} \quad \text{III.4}$$

such that

$$N_e(\vec{k}, t)|_V = \frac{1}{V} \sum_j e^{-i\vec{k} \cdot \vec{r}_j} \quad \text{III.5}$$

where  $V$  is the scattering volume. Upon substituting Eq. III.5 into III.2 yields

$$E_s(t) = \frac{r_e}{R} \sin \zeta E_0 V e^{i\omega_0 t} N_e(\vec{k}, t)|_V \quad \text{III.6}$$

which means that the scattering only arises from electron density fluctuations and more precisely from the  $\vec{k}$  fourier component of these fluctuations. Stated in other words the scattering only depends upon stratified fluctuations of the density perpendicular to the  $\vec{k}$  direction (Fig. 1).

Equation III.6 also shows that the scattering properties to be investigated depend upon the observing scale  $1/k$ : in fact it will turn out that the degree of organization of the fluctuations of the medium responsible for the scattering depend upon the relative magnitudes of  $1/k$  and  $\lambda_D$  the Debye length of the plasma

$$\lambda_D = \left( \frac{KTe}{N_0 q^2} \right)^{1/2}$$

with  $N_0$  the electron number density. The averaged scattered power is again given by

$$\frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} R_e \langle E_s(t) E_s^*(t) \rangle$$

III.7

$$= \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{r_e^2}{R^2} \sin^2 \xi E_0^2 V^2 R_e \langle N_e(\vec{k}, t) |_{\mathbf{v}} N_e^*(\vec{k}, t) |_{\mathbf{v}} \rangle$$

By virtue of the Wiener-Khinchine theorem the power spectrum is just the Fourier transform of the autocorrelation function:

$$\frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} e \langle E_s(t) E_s^*(t+\tau) \rangle$$

III.8

$$= \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{r_e^2}{R^2} \sin^2 \xi E_0^2 V^2 R_e \langle e^{-i\omega_0 \tau} N_e(\vec{k}, t) |_{\mathbf{v}} N_e^*(\vec{k}, t + \tau) |_{\mathbf{v}} \rangle$$

Thus the power spectrum is then given by

$$P_s(\omega_0 + \omega) = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \langle |E_s(\omega_0 + \omega)|^2 \rangle$$

III.9

$$= \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} \frac{r_e^2}{R^2} \sin^2 \xi V^2 E_0^2 \langle |N_e(\vec{k}, \omega)|_{\mathbf{v}}^2 \rangle$$

In the following the only quantity of interest is consequently

$$\langle |N_e(\vec{k}, \omega)|_v^2 \rangle$$

The associated total differential cross section

$$\sigma_v(\omega_0 + \omega) = r_e^2 \sin^2 \xi V^2 \langle |N_e(\vec{k}, \omega)|_v^2 \rangle \quad \text{III.10}$$

The determination of  $\langle |N_e(\vec{k}, \omega)|_v^2 \rangle$ , the spectrum of the random electron density fluctuations, requires in general the solution of Maxwell equations and of the kinetic equations for the perturbed plasma.

The various parameters which affect the spectrum are the Coulomb interactions, the magnetic field, the collisions with neutrals, the electron and ion temperatures, the drifts of the ions and of the electrons, the ion composition and the presence of a non maxwellian tail in the electron distribution. The main difficulty is to take account of these by using the proper kinetic equations.

For each situation a different dispersion relation for the longitudinal plasma waves is associated with the scattering problem. The wave modes and the different damping processes involved will be of help for the interpretation of the shape of the spectrums.

The solution of the different sets of equations also requires a statistical treatment because of the random nature of the initial electron distribution in space. The analysis performed by a number of authors differs mostly in the way the statistical treatment is carried out: Fejer (1960, 1961), Salpeter (1960),

Renau (1960), Hagfors (1961), considered the distribution probability of the electrons along lines similar to the one followed Pines and Bohm (1952) while Dougherty and Farley (1960), (1963) Farley et al. (1961) made use of the Nyquist theorem. The identity of the results obtained by the different methods indicates their consistency. For the sake of mathematical simplicity the approach making use of the Nyquist theorem is to be outlined in this paper.

It must be noted (Farley (1970)) that the use of kinetic equations can be avoided for two classes of problems:

Firstly, if just the total scattered power is needed, use can be made, in some cases, of the Debye-Huckel theory (Kahn 1959; Salpeter 1960; Renau 1960). In this connection, see the preceeding section in which the total power scattered by independent electrons was derived without any assumption about the electron velocity distribution.

Secondly, fluid equations can be used instead of kinetic equations in the case of collision-dominated plasmas (Tanenbaum 1968, Seasholt and Tanenbaum 1969).

In summary, the scattering problem involves one particular Fourier component of the electron density fluctuations depending upon the relative locations of the transmitter, the scattering volume and the field point. The determination of the power spectrum of this Fourier component implies the use of Maxwell's and kinetic equations in the  $(\vec{k}, \omega)$  space. The shape of the

spectrum can in general be simply interpreted in terms of heavily damped longitudinal plasma waves.

#### IV. The Nyquist Theorem Approach

The procedure followed by Dougherty and Farley (1960) for obtaining  $\langle |N_e(k, \omega)|_v^2 \rangle$  is outlined in this section.

The Nyquist theorem in its most simple form states that in an isolated resistance  $R$  at a temperature  $T$  there flows a random current  $I$  such that

$$\langle |I(\omega)|^2 \rangle d\omega = \frac{2}{\pi} \frac{KTd\omega}{R} \quad \text{IV.1}$$

The theorem can be extended to a linear system in thermodynamic equilibrium at a temperature  $T$  whose response  $I(\omega)$  to a generalized force  $V(\omega)$  is

$$I(\omega) = Y(\omega) V(\omega) \quad \text{IV.2}$$

where  $Y(\omega)$  is the general admittance function. In the absence of the driving force the response of the system is such that:

$$\langle |I(\omega)|^2 \rangle d\omega = \frac{1}{\pi} G(\omega) KT d\omega \quad \text{IV.3}$$

where

$$G(\omega) = R_e(Y(\omega))$$

and  $\omega$  varies between  $-\infty$  and  $+\infty$ .

When applied to a plasma the theorem is often called the fluctuation-dissipation theorem, whereby random thermal fluctuations in the electron density can be related to a dissipation process such as Landau damping or collisions (Farley, 1970a).

The way of applying the theorem which is relevant to this problem is to calculate the displacement of the electrons in response to a fictitious force  $\vec{F}$  applied to them only. However the motion of the electrons and of the ions are coupled through the space-charge electric field  $\vec{E}$ . Use of Maxwell's equations allows for the determination of the generalized admittance tensor  $\bar{\bar{Y}}'$  in terms of the admittance tensors of the electrons and ions  $\bar{\bar{Y}}_e$  and  $\bar{\bar{Y}}_i$ .

The generalized admittance, in turn, leads to the spectrum of the density fluctuations. The determination of the admittances  $\bar{\bar{Y}}_i$  and  $\bar{\bar{Y}}_e$  calls for the kinetic equations of the electrons and ions. This will be the object of the next sections for various conditions. Assuming charge neutrality and singly charged ions the mean flow vector resulting from  $\vec{F}$  are:

$$\frac{N_{e0} \vec{U}_i}{V} = -q \bar{\bar{Y}}_i \cdot \vec{E} \quad \text{IV.4}$$

$$\frac{N_{e0} U_e}{V} = \bar{\bar{Y}}_e \cdot (\vec{F} + q\vec{E}) \quad \text{IV.5}$$

where  $N_{e0}$  is the unperturbed electron number in the volume  $V$ , and  $\vec{U}_e$  and  $\vec{U}_i$  the electron and ion drift vectors. Introducing the current

$$\vec{J} = \frac{N_{e0}}{V} q(\vec{U}_e - \vec{U}_i) \quad \text{IV.6}$$

and eliminating  $\vec{B}$  in the Maxwell's equations written in the  $(\vec{k}, \omega)$  space yields:

$$\vec{J} = -i\epsilon_0 \vec{E} + \frac{i\vec{k}}{\mu_0 \omega} \times (\vec{k} \times \vec{E}) \quad \text{IV.7}$$

or

$$\vec{J} = \vec{\Gamma} \cdot \vec{E} \quad \text{IV.8}$$

$\vec{\Gamma}$  is a diagonal tensor if a coordinate system is chosen in such a way that one axis is parallel to  $\vec{k}$ . Eliminating  $\vec{E}$ ,  $\vec{J}$  and  $\vec{U}_i$  between equations IV.4, IV.5, IV.6 and IV.8 yields

$$N_{e0} \vec{U}_e = V \vec{Y}' \cdot \vec{F} \quad \text{IV.9}$$

with

$$\vec{Y}' = \left[ \left( \frac{\vec{Y}_i}{q^2} - \vec{\Gamma} \right)^{-1} + \vec{Y}_e^{-1} \right]^{-1} \quad \text{IV.10}$$

Use of the continuity equation of the perturbed electron density  $n_e$  yields

$$\frac{\partial n_e}{\partial t} + \text{div} \left( \frac{N_{e0} \vec{U}_e}{V} \right) = 0 \quad \text{IV.11}$$

which can be linearized in  $(\vec{k}, \omega)$  space:

$$VN_e(\vec{k}, \omega)|_{\mathbf{v}} = -\frac{N_{e0}}{\omega} \vec{k} \cdot \vec{U}_e(\vec{k}, \omega) \quad \text{IV.12}$$

therefore

$$V^2 \langle |N_e(\vec{k}, \omega)|_{\mathbf{v}}^2 \rangle = \frac{k^2}{\omega^2} \langle |N_{e0} U_{ek}^2(\vec{k}, \omega)|^2 \rangle \quad \text{IV.13}$$

and by virtue of the Nyquist theorem

$$V^2 \langle |N_e(\vec{k}, \omega)|_{\mathbf{v}}^2 \rangle = \frac{k^2}{\omega^2} \langle |N_{e0} U_{ek}^2(\vec{k}, \omega)|^2 \rangle = \frac{k^2}{\pi\omega^2} KTV R_e(Y'_{kk}) \quad \text{IV.14}$$

finally

$$\langle |N_e(\vec{k}, \omega)|_{\mathbf{v}}^2 \rangle = \frac{k^2}{\pi\omega^2} \frac{KT}{V} R_e(Y'_{kk}) \quad \text{IV.15}$$

## V. The Case of Collisionless Plasma in Thermal Equilibrium without Magnetic Field

The computation of the admittances  $\bar{Y}_i$  and  $\bar{Y}_e$  for the ions and the electrons, which enter in (IV.15), is performed using the appropriate Boltzmann equation and a perturbation method. The electron velocity distribution can be written as:

$$f(\vec{v}) = f_0(\vec{v}) + f_1(\vec{v}) \quad \text{V.1}$$

where  $f_0$  is a Maxwellian distribution and  $f_1$  is a perturbation.

$$f_0(\vec{v}) = n_0 \left[ \frac{m}{2\pi kT} \right]^{3/2} e^{-m\vec{v}^2/2kT} \quad \text{V.2}$$

where  $m$  is the mass and  $n_0$  the number density of the particle of interest.

The Boltzmann equation, to the first order in perturbations in  $(\vec{k}, \omega)$  space is:

$$i(\omega - \vec{k} \cdot \vec{v}) f_1 + \frac{1}{m} \vec{F} \cdot \frac{\partial f_0}{\partial \vec{v}} = 0 \quad \text{V.3}$$

The next step consists in evaluating  $n_0 \vec{U}$

$$n_0 \cdot \vec{U} = \int \vec{v} f_1 d^3v = \frac{i}{m} \vec{F} \cdot \int \frac{\partial f_0}{\partial \vec{v}} \frac{\vec{v}}{\omega - \vec{k} \cdot \vec{v}} d^3v \quad \text{V.4}$$

$$n_0 \vec{U} = \bar{\bar{Y}} \cdot \vec{F} \quad \text{V.5}$$

Assuming again that  $\vec{k}$  is parallel to one of the coordinate axes it can easily be shown (Dougherty and Farley 1960) that  $\bar{\bar{Y}}$  is diagonal. Therefore all the tensors entering in  $\bar{\bar{Y}}$  are diagonal and only the  $kk$  element of each needs to be considered.

Before giving a final expression for  $\bar{\bar{Y}}$  it is interesting to recall the physical meaning of the singularity in V.4 for  $\omega = \vec{k} \cdot \vec{v}$ .

In terms of waves, the singularity corresponds to particles moving at the phase velocity of the wave. Consequently they tend to see a constant accelerating or decelerating electric field, and interact strongly with the wave. Ultimately

after a net gain or loss of energy the particles oscillate in a reference frame moving at the phase velocity. The linear approach therefore breaks down since it predicts a steady exchange of energy between the particle and the wave. This effect has been considered by Landau (1946) and is known as "Landau damping". It can also be shown (see Bohn and Gross, 1949, for example) that in addition to the solutions  $(\omega, \vec{k})$  of the dispersion relation for a collisionless gas of one type of particles, wave energy can be associated with each value of  $\omega$  for which there exist particles whose velocities satisfy the relationship  $\vec{k} \cdot \vec{v} = \omega$ .

It can also be inferred that the power spectrum of the fluctuations corresponding to these waves decreases when  $k$  decreases, while the opposite is true for the waves predicted from the dispersion relation and for which the contributions of all particles add coherently.

The mathematical difficulty implied by the singularity was overcome by Landau (1946) and the result is (Dougherty and Farley (1960):

$$Y_{kk} = \frac{n_0}{m\omega} 2 \left[ \pi^{1/2} \theta^3 e^{-\theta^2} - i\theta^2 \left( 2\theta e^{-\theta^2} \int_0^\theta e^{p^2} dp - 1 \right) \right] \quad \text{V.6}$$

where

$$\theta = \frac{\omega}{k} \left[ \frac{m}{2KT} \right]^{1/2}$$

or

$$Y_{kk} = \frac{n_0}{m\omega} 2\theta^2 iW(\theta) \quad \text{V.7}$$

where

$$W(\theta) = 1 - \theta e^{-\theta^2} \left( 2 \int_0^\theta e^{p^2} dp + i\pi^{1/2} \right) \quad \text{V.8}$$

Therefore substituting  $Y_{kk}(\theta_e)$  and  $Y_{kk}(\theta_i)$  into eq. IV.10 yields

$$Y'_{kk} = \frac{i n_0 \omega}{k^2 K T} W(\theta_e) \frac{[\alpha^2 W(\theta_i) + 1]}{[\alpha^2 (W(\theta_i) + W(\theta_e)) + 1]} \quad \text{V.9}$$

with

$$\alpha^2 = \frac{n_0 q^2}{k^2 K T \epsilon_0} = \frac{1}{k^2 \lambda_D^2}$$

$\alpha$  relates the scale of the observation, determined by the geometry of the experiment and by the wavelength, to the characteristic scale of the plasma, determined by the plasma number density and the plasma temperature. Therefore the power spectrum of the density fluctuations is:

$$\langle |N_e(\omega, \vec{k})|_{\vec{v}}^2 \rangle = \frac{n_0}{\pi \omega V} \Re_e \left\{ \frac{i W(\theta_e) [\alpha^2 W(\theta_i) + 1]}{[\alpha^2 (W(\theta_i) + W(\theta_e)) + 1]} \right\} \quad \text{V.10}$$

or

$$\langle |N_e(\omega, \vec{k})|_{\vec{v}}^2 \rangle = \frac{n_0}{\pi^{1/2} k V [2KT]^{1/2}} \left\{ \frac{m_e^{1/2} e^{-\theta_e^2} |\alpha^2 W(\theta_i) + 1|^2 + m_i^{1/2} e^{-\theta_i^2} |\alpha^2 W(\theta_e)|^2}{|\alpha^2 [W(\theta_i) + W(\theta_e)] + 1|^2} \right\} \quad \text{V.11}$$

Noting that the  $R_e W(\theta)$  and  $\Im_m W(\theta)$  are of the order of 1 or smaller, two limiting cases of practical interest can be considered, according to the relative magnitudes of the observing and plasma scales.

a)  $\alpha \ll 1$

Equation V.11 becomes

$$\left\langle |N_e(\omega, \vec{k})|_{\vec{v}}^2 \right\rangle = \frac{n_0}{\pi^{1/2} k v} \left[ \frac{m_e}{2KT} \right]^{1/2} e^{-\omega^2 m_e / k^2 2KT} \quad \text{V.12}$$

which is just the result of Gordon (1958): Therefore the spectrum simply corresponds to independent free electrons. The physical interpretation of this situation is that no collective oscillations can be sustained on a scale smaller than the Debye length.

b)  $\alpha \gg 1$

It is first interesting to note that equating to zero the denominator of eqn. V.11 yields the dispersion relation for plasma oscillations (Landau 1946):

$$\alpha^2 (W(\theta_i) + W(\theta_e)) + 1 = 0 \quad \text{V.13}$$

the solution of Eqn. V.13 gives two modes: the first is the pseudo acoustic mode which can be viewed as sound waves of a gaz consisting of particles having the mass of the ions and the sum of the energies of both the electrons and the ions. The second corresponds to electrostatic waves at a frequency close to the plasma frequency.

Because of the mass difference between ions and electrons the numerator of eqn. V.II. is dominated by the second term when  $\theta_i \lesssim 1$  and by the first term when  $\theta_e \gtrsim 1$ . The corresponding simplified expressions are termed respectively ionic and electronic spectrum.

b.1) The ionic spectrum ( $\theta_i \lesssim 1$ ,  $\theta_e \ll 1$ )

Eqn. V.II. reduces to:

$$\langle |N_e(\omega, \vec{k})|_V^2 \rangle = \frac{n_0}{k\pi^{1/2}V} \left[ \frac{m_i}{2KT} \right]^{1/2} \frac{e^{-\theta_i^2}}{|[W(\theta_i) + 1]|^2} \quad \text{V.14}$$

This shows that the spectrum corresponds to an ion velocity distribution modified by the term in the denominator. Figure 2 (after Farley 1970) is a diagram of the normalized differential cross section for backscatter per unit volume  $\sigma(\theta_i)\pi^{1/2}/(n_0 r_e^2)$  where  $\sigma(\theta_i)$  is defined by the relation

$$P_s(\omega_0 + \omega) d\omega = \frac{1}{2} \sqrt{\frac{\epsilon_0}{\mu_0}} V E_0^2 \sigma(\theta_i) d\theta_i \quad \text{V.15}$$

Also shown on the diagram is the normalized ion velocity distribution  $e^{-\theta_i^2}$ .

Such a spectrum was first observed by Bowles (1958). From the width of the spectrum be concluded that the ions are playing a major role. The influence of the denominator affects however significantly the spectrum which is depressed in the center and enhanced on the sides because of the presence of heavily damped pseudo acoustic waves.

The damping of the waves reflects the fact that the wave phase velocity lies in a range of the ion velocity distribution such that many particles interact with the wave and therefore cause damping.

It might be worth noticing that at the time the first incoherent scatter measurements were made, the ions were generally considered as a uniform charged background.

The measurements therefore provided an immediate experimental support to new theories of plasma oscillations taking into account the discrete nature of the ions such as the one of Kahn (1959). Incoherent scatter observations therefore played a significant part towards the understanding of plasma oscillations.

Recalling that the electrons are responsible for the scatter it is interesting to give an interpretation of the reason why the observed characteristics match closely these of the ions.

If the ions were by themselves thermal oscillations on a scale much larger than the Debye length  $\lambda_D$  (that is  $a \gg 1$ ), would be highly restricted because of the opposing effect of the space charge electric field which would arise from those fluctuations. The conditions are however quite different because of the presence of the electrons. The electrons which have a large mobility compared to that of ions respond very effectively, to the space charge which tends to develop and this has the following effects; (1) the electron fluctuations nearly match the ion fluctuations, and therefore the spectrum of the electron fluctuations pictures

the ion fluctuations. (ii) the space charge electric field is considerably reduced with respect to the one obtained from the same fluctuations in the absence of the electrons. The residual electric field is comparable to the ambipolar (polarization) electric field and leads to pseudo neutral having the mass of the ions and the sum of the energies of the electrons and the ions. (iii) the residual electric field, however small, is sufficient to distort significantly the spectrum of the ion fluctuations with respect to the one obtained for non interacting ions.

b.2) The Electron Spectrum  $\theta_e \gtrsim 1$ ,  $\theta_i \gg 1$

Equation V.1 reduces to

$$\left\langle |N_e(\omega, \vec{k})|_v^2 \right\rangle \simeq \frac{n_0}{k\pi^{1/2}V} \left[ \frac{m_e}{2KT} \right]^{1/2} \frac{e^{-\theta_e^2} |\alpha^2 W(\theta_i) + 1|^2}{|\alpha^2 W(\theta_e) + 1|^2} \quad \text{V.16}$$

This is generally a small quantity except for values of  $\theta_e$  for which the denominator is a minimum, that is for the solutions of the dispersion relation corresponding to electrostatic waves at about the plasma frequency.

In the corresponding ranges of

$$\theta_e \left( |\theta_e| \sim \frac{\alpha}{\sqrt{2}} \right)$$

equation V.II further reduces to

$$\left\langle |N_e(\omega, \vec{k})|_v^2 \right\rangle \simeq \frac{n_0}{k\pi^{1/2}V} \left[ \frac{m_e}{2KT} \right]^{1/2} \frac{e^{-\theta_e^2}}{\left[ 1 - \frac{\alpha^2}{2\theta_e^2} \right]^2 + [\alpha^2 \pi^{1/2} e^{-\theta_e^2} \theta_e]^2} \quad \text{V.17}$$

The resonant pulsation is given by (Bohm and Gross 1949 for example):

$$\omega_r^2 = \omega_p^2 + \left( \frac{3KT}{m_e} \right) k^2 \quad \text{V.18}$$

where  $\omega_p$  corresponds to the plasma frequency, or in normalized units:

$$\theta_{er}^2 = \frac{a^2 + 3}{2} \quad \text{V.19}$$

the corresponding spectrum consists in two very narrow lines called the plasma lines; their width is, according to the initial assumptions, determined only by the Landau damping:

$$\delta\theta \simeq \frac{\pi^{1/2} a^4 e^{-a^2/2}}{2} \quad \text{V.20}$$

The Landau damping being small, because of the small number of electrons moving at the phase velocity of the waves, other processes such as collisions, not taken into account here might play a significant role. This will be investigated in other sections.

c) Figure 3 from Hagfors (1961) shows complete normalized spectra for various values of  $\alpha$ .

For  $\alpha/\sqrt{2} = 300$  the spectrum has a width corresponding to the ion velocity distribution. The plasma line, missing in this diagram, would be found for  $\theta \simeq 300$ ; however the corresponding power would be negligible. Therefore the spectrum is essentially an ion spectrum, for  $\alpha/\sqrt{2} = .1$  the spectrum corresponds

to independent free electrons. In particular the plasma line which should be found for  $\theta \approx \sqrt{3/2}$  is completely damped because the corresponding phase velocity is close to the thermal velocity.

Intermediate cases such as  $\alpha/\sqrt{2} = 3$  exhibit, with different amplitudes, both the electronic and ionic velocity distributions characteristics; the plasma line is also present but contributes to a negligibly small part of the total scattered power.

d) Total differential cross sections.

The cross section  $\sigma(\theta_i)$  per unit volume can be integrated analytically or even computed directly without considering the frequency domain (e.g. Hagfors 1961, Salpeter 1960, Kahn 1959, Renan 1960). The result for backscatter is:

$$\sigma = \sigma_e n_0 \frac{1 + \alpha^2}{1 + 2\alpha^2} \quad \text{V.21}$$

This expression gives the result obtained by Gordon (1958) for  $\alpha \ll 1$  and reduces to

$$\sigma = \sigma_e \frac{n_0}{2} \quad \text{V.22}$$

for  $\alpha \gg 1$ .

It must be noticed that the power contained in the plasma line being proportional to  $1/(2\alpha^2)$ , the expression V.21 gives in fact the power contained in the ion spectrum for  $\alpha \gg 1$ .

e) Conclusion:

It is worth noting that, fortunately, the problem of incoherent scatter was approached on the basis of random electron thermal fluctuations. Indeed the current theories of collective oscillations in the plasma at that time (e.g. Pines and Bohm 1952) would have led to the predictions of the plasma lines only and to a correspondingly negligible small scattered power, since the ions were then considered as a uniform background.

The next sections concern the inclusion of magnetic field, unequal ion and electron temperatures, collisions, drifts of the electrons or ions and the effects of photoelectrons.

## VI. Case of Unequal Ion and Electron Temperatures

The problem of nonthermal equilibrium is of considerable interest for the ionospheric plasma. Indeed cascading of energy from the photo-electrons to the thermal electrons, to the ions and finally to the neutrals implies that generally electrons, neutrals and ions have different temperatures. The extension of the computations of the previous section to the case of nonthermal equilibrium between the ions and the electrons was undertaken by Salpeter (1960, 1963), Fejer 1961, Renau, Camnitz and Flood (1962), Rosenbluth and Rostoker (1962), Farley (1966).

The results obtained by Renau et al. (1962) differed from those of the other authors. The disagreement was due to an improper extension of the Nyquist

theorem made by Renau, et al. (1962). Farley (1966) later indicated the valid extension of the Nyquist theorem to the case of nonthermal equilibrium between the ions and the electrons. The approach of Farley is again outlined here.

For a system in quasi equilibrium (Farley 1970), in which the electron and ion temperatures are not equal the equivalent electrical circuit is one containing impedances at different temperatures; the noise output is such that:

$$\langle |E(\omega)|^2 \rangle d\omega = \frac{K}{\pi} \sum_n T_n R_e [Z_n(\omega)] d\omega \quad \text{VI.1}$$

where  $Z_n$  is the  $n^{\text{th}}$  impedance, and  $E$  the open circuit voltage. The corresponding expression applying to the incoherent scatter problem is (Farley 1966):

$$\langle |N_{e0} U_{ek}(\vec{k}, \omega)|^2 \rangle = \frac{KV}{\pi} \frac{R_e [T_i Z_i + T_e Z_e]}{|Z_i + Z_e|^2} \quad \text{VI.2}$$

where

$$Z_i = [Y_{ikk} + i\omega\epsilon_0/q^2]^{-1}$$

and

$$Z_e = Y_{ekk}^{-1}$$

As a consequence the generalized expression for the power spectrum of the electron density fluctuations is:

$$\langle |N_e(\vec{k}, \omega)|^2 \rangle = \frac{n_0}{k\pi^{1/2}V}$$

VI.3

$$\frac{\frac{m_e^{1/2} e^{-\theta_e^2}}{[2KT_e]^{1/2}} [|\alpha_i^2 W(\theta_i) + 1|^2] + \frac{m_i^{1/2} e^{-\theta_i^2}}{[2KT_i]^{1/2}} |\alpha_e^2 W(\theta_e)|^2}{|1 + \alpha_i^2 N(\theta_i) + \alpha_e^2 W(\theta_e)|^2}$$

where

$$\alpha_{e,i}^2 = \frac{n_0 q^2}{KT_{e,i} \epsilon_0} \frac{1}{k^2}$$

in the following  $\alpha$  is defined as  $\alpha = \alpha_e$ . While the electron spectrum is unchanged, the shape of the ion spectrum undergoes large changes when Te/Ti varies as illustrated in Figure 4 for  $\alpha \gg 1$ . For negligibly small values of the ratio Te/Ti, the ion spectrum tends to the gaussian velocity distribution of the ions. Indeed for the range of velocities of interest the momentum equation for the electrons reduces to a balance between the electron pressure gradient and the space charge electric field. The pressure becoming negligible the space charge electric field tends to zero and the ion fluctuations are those of non-interacting particles.

Increasing the ratio of Te/Ti has two effects: the first is to increase the space charge electric field thus reducing the ion fluctuations, this corresponds to the general decrease of the amplitude of the power spectrum for increasing Te/Ti.

The second effect is to enhance the pseudo acoustic waves, indeed the phase velocity of the waves increases as  $(T_e + T_i)^{1/2}$  and therefore less and less ions interact strongly with the waves when  $T_e$  increases, thus reducing the damping of such waves. However further increases in the electron temperature lead to a damping of the pseudo sound waves dominated by the electrons rather than the ions (Farley 1966) with the net effect of increasing the total scattered power.

For moderate values of  $T_e/T_i$  the total differential cross section has been computed to be (Buneman (1962), Farley (1966)):

$$\sigma = \sigma_e n_0 \left[ \frac{1}{1 + \alpha^2} + \frac{\alpha^4}{(\alpha^2 + (T_e/T_i) \alpha^2 + 1) (1 + \alpha^2)} \right] \quad \text{VI.4}$$

which for large values of  $\alpha$  yields:

$$\sigma = \frac{\sigma_e n_0}{1 + T_e/T_i} \quad \text{VI.5}$$

This expression would be valid if the ratio of the ion to the electron masses was infinite.

The actual variation of  $\sigma$  with  $T_e/T_i$  was computed by Moorecroft (1963).<sup>†</sup> Figure 5 shows such a variation for  $O^+$  and  $H^+$ , for  $n_0$  equal to unity and for  $\alpha \gg 1$ . Also shown on the figure is the variation given by the approximate expression VI.5 which is a limit for an infinite ratio of ion to electron masses.

The transition between a spectrum dominated by random ion thermal fluctuations ( $T_e/T_i \ll 1$ ) to a spectrum of weakly damped pseudo sound waves corresponds to a minimum of the total scattered power.

The variation of the cross section with respect to  $1/\alpha$  for various values of  $T_e/T_i$  and for  $n_0$  equal to unity is given on Figure 6 (After Moorcroft, 1963).

The variation given by the approximate formula is also shown for  $T_e/T_i \rightarrow \infty$ . A monotonic increase of the cross section as a function of  $1/\alpha$  is observed for  $T_e/T_i = 1$ , corresponding to a transition from an ionic to an electronic spectrum.

A large departure from the approximate formula is observed for  $T_e/T_i \geq 15$  and for small values of  $1/\alpha$ ; Indeed the approximate formula is equivalent to neglecting the damping of the pseudo sound waves by the electrons: for large values of  $T_e/T_i$  the waves are in thermal equilibrium with the electrons rather than with the ions because the former participate most in the damping (Farley 1966).

## VII. Plasma containing several kinds of ions

The case of a mixture of ions was studied by Buneman (1961) and Moorcroft (1964). It is easily shown that  $W(\theta_i)$  in eqn. VI.2 must be replaced by

$$\sum_n P_n W(\theta_{in})$$

where  $P_n$  is the relative concentration of the  $n^{\text{th}}$  ion

$$\left( \sum_n P_n = 1 \right).$$

The effect of a transition from  $O^+$  to  $He^+$  is illustrated on Figure 7 (after Moorcroft 1964).

More generally for several types of ions with multiple charges  $W(\theta_i)$  must be replaced by

$$\sum_n P_n Z_n^2 W(\theta_{in})$$

where  $Z_n$  is the charge number such that

$$\sum_n P_n Z_n = 1$$

### VIII. Effect of the magnetic field

The effect of a uniform magnetic field on the incoherent scatter spectrum was studied by Laaspere (1960), Fejer (1961), Hagfors (1961), Farley, Dougherty and Barron (1961), Salpeter (1961a), Renau et al. (1961) Farley (1966).

The calculation, once again, can be conducted through the use of the Nyquist theorem approach (Farley et al., 1961). The problem reveals significant complications because of off diagonal terms in  $\bar{Y}_i$  and  $\bar{Y}_e$ . However, it can be shown that except for values of  $\phi$  (the angle between the magnetic field and the wave vector  $\vec{k}$ ) extremely close to  $\pi/2$  only the diagonal elements are important. This is equivalent to letting  $c \rightarrow \infty$  and is called the "longitudinal approximation" (Bernstein 1958, Farley et al. (1961).

Therefore equation VI.2 is still valid if  $W(\theta)$  is replaced by

$$W(\theta, \phi, \Phi) = 1 - i\theta \int_0^\infty e^{(-i\theta t - \Phi^{-2} \sin^2 \phi \sin^2 1/2 \Phi t - 1/4 t^2 \cos^2 \phi)} dt \quad \text{VIII.1}$$

where  $\Phi$  is the normalized gyro frequency:

$$\Phi = \frac{q B}{m k} \left( \frac{m}{2KT} \right)^{1/2} \quad \text{VIII.2}$$

The integral is called the Gordeyev integral.

It can be shown (Farley et al. 1961) that for the ionospheric case no change in the spectrum is to be expected from the presence of the magnetic field for  $\phi \lesssim 85^\circ$ . For  $\phi > 85^\circ$  the dispersion relation has new roots corresponding approximately to integral multiples of the ion or electron gyro frequencies ( $\theta_i \approx n\Phi_i$ ,  $\theta_e \approx n\Phi_e$ ) and called Bernstein modes (Bernstein, 1958). They correspond to particles which are periodically in the same phase relationship with the plasma waves.

However due to the effect of the mass factor upon the gyrofrequency, the ion spectrum for practical purposes ( $\alpha \gg 1$ ) will only be affected by ion gyroresonances as illustrated on Figure 8 for  $\phi = 88^\circ$ . The general shape of the spectrum is similar to the one obtained for  $\phi = 0^\circ$  (or no magnetic field) with in addition a modulation at approximately the ion gyro frequency as originally suggested by Bowles (1959). When  $\phi \rightarrow 90^\circ$  the gyroresonances peaks are more and more pronounced and eventually the spectrum becomes a spectrum of  $\delta$  functions; however simultaneously all the power tends to be concentrated at the central frequency.

It will be shown later that when ion collisions are included the modulation tends to be damped.

The total power scattered can be shown to be independent of the magnetic field for  $T_e = T_i$ . The effect of the magnetic field for  $T_e \neq T_i$  can be shown to be equivalent to a change in the ratio of the mass of the electrons to the ions.

The only quantity of interest is

$$\gamma = \left( \frac{m_e}{m_i} \right)^{1/2} \sec \phi.$$

This is due to the fact that the electrons are strongly bound to the magnetic field.  $\alpha \gg 1$  corresponds to wavelengths which are large both with respect to the Debye length and to the electron Larmor radius; as a result only the field aligned component of the electron velocity plays a part in the collective plasma oscillations:  $\vec{k} \cdot \vec{v}_e$  must be replaced by  $\vec{k} \cdot \vec{v}_{e\parallel}$  where  $\vec{v}_{e\parallel}$  is the field aligned electron velocity. This is equivalent to changing  $m_e^{1/2}$  into  $m_e^{1/2} \sec \phi$ . (This change of mass obviously only applies to the determination of the plasma oscillations not to the scattering of electromagnetic waves).

Figure 9 shows the variation of the normalized cross section as a function of  $T_e/T_i$  for various values of  $\gamma$  (after Farley (1966)),

The interpretation given by Farley is that sound waves get into thermal equilibrium with the particle population which contributes most to their Landau damping. The amplitude of the wave is then proportional to  $T_m/(T_e + T_i)$  in which  $T_m$  is the temperature of the population which contributes most to the damping. For small values of  $T_e/T_i$ ,  $T_m$  is equal to  $T_i$  and for large values

of  $T_e/T_i$ ,  $T_m$  is equal to  $T_e$ . This explains why for both large or small values of  $T_e/T_i$  the ratio  $T_m/T_e + T_i$  tends to 1. The transition corresponds to a minimum and depends upon the ratio of the effective masses: the larger the effective electron mass, the smaller the values of  $T_e/T_i$  for which the Landau damping of the sound waves is dominated by the electrons.

The location of the plasma line is also modified by a uniform magnetic field (Salpeter, 1961b; Perkins and Salpeter 1965):

$$\omega_r^2 = \omega_p^2 + \left( \frac{3KT_e}{m_e} \right) k^2 + \Omega_e^2 \sin^2 \phi \quad \text{VIII.3}$$

provided  $\Omega_e^2 \ll \omega_p^2$  where  $\Omega_e$  is the electron gyrofrequency. In normalized units eqn. VIII.3 can be written as:

$$\theta_{er}^2 = \frac{\alpha^2 + 3}{2} + \Phi_e^2 \sin^2 \phi \quad \text{VIII.4}$$

Salpeter (1961b) has shown that an additional root of the dispersion relation is associated with the presence of the magnetic field (see also Perkins (1967):

$$\omega_h^2 = [m_e/m_i + \cos^2 \phi] \Omega_e^2 \omega_p^2 / (\omega_p^2 + \Omega_e^2) \quad \text{VIII.5}$$

in the ionosphere  $\omega_p$  is generally larger than  $\Omega_e$  so that for  $\phi \rightarrow \pi/2$

$$\omega_h \simeq [\Omega_e \Omega_i]^{1/2}$$

which is the geometric mean gyrofrequency. An additional line therefore appears in the incoherent scatter spectrum.

The effect of the magnetic field has been considered in this section for the case of practical interest for the ionosphere. However a number of other situations might be considered according to the relative amplitudes of  $k^{-1}$  and the ion and electron Larmor radii. (See Farley et al. 1961) for example).

#### IX. Effect of bulk motions of the plasma or of one type of particles.

##### a. Motion of the whole plasma $\vec{v}_e = \vec{v}_i \neq 0$

It is straightforward to show that the whole ion spectrum is not changed in shape but is simply doppler shifted by  $\delta\omega = -\vec{k} \cdot \vec{v}_e$

##### b. relative motion of the electrons with respect to the ions.

This case has been studied by Rosenbluth and Rostoker (1962) and Lamb (1962).

The spectrum becomes asymmetrical: the peak corresponding to the pseudo sound waves propagating in the direction of the electron drift is enhanced, while the other peak is damped. Indeed the electrons tend to feed the pseudo sound waves propagating in their direction and to damp the ones propagating in the opposite direction. This effect, however, only takes place when the electron drift ( $v_d$ ) is a significant fraction of the electron thermal velocity ( $v_{Te}$ ) as shown in Figure 10 for  $H^+$  ions,  $T_e/T_i = 2$ , and backscatter (after Lamb 1962).

The peak grows rapidly when  $v_d$  approaches  $v_{Te}$  corresponding to the two streams instability.

#### X. Effect of Collisions

The collisionless case corresponds to many situations encountered in the ionosphere. It ceased to be valid, however, everytime a collision frequency

becomes of the same order of magnitude as the width of some line in the spectrum: this is the case in the E region when the ion neutral collision frequency is not negligible as compared to the width of the ion spectrum. This is also the case when gyroresonances tend to develop with widths of an order of magnitude close to the ion-ion collision frequency. And also, it is the case when the Landau damping of the plasma line becomes very small, electron-ion collisions must then be taken into account.

The effect of collisions has been studied by Dougherty (1963), Cohen (1963), Dougherty and Farley (1963), Taylor and Comisar (1963), Waldteufel (1963, 1965), Tanenbaum (1968), Seasholtz and Tanenbaum (1969), Hagfors (1970), Hagfors and Brockelman (1971).

a. Ion-neutral collisions.

The complexity of the problem is increased considerably when collisions are included. Indeed the simple relaxation term generally used for the Boltzmann equation is not convenient here since the scattering properties depend sensitively upon the detailed kinetic behavior of the plasma (Hagfors 1970).

Efforts have been made to include phenomenologically collision terms which would conserve some of the physical invariants such as particle number density, momentum, energy, thus following the procedure of Bhatnagar-Gross-Krook (1954) who proposed a term preserving charged particles through collisions.

Figure 11 (after Dougherty and Farley 1963) shows the effect of collisions on the ion spectrum using a term preserving charged particles through collisions.

$\psi$  is a normalized collision frequency:

$$\psi = \frac{\nu}{k} \left( \frac{m}{2KT} \right)^{1/2}$$

For increasing ion neutral collision frequencies the ion spectrum tends to become a Lorentz curve of width proportional to  $\psi^{-1}$ .

Hagfors (1970) has described a new approach to the problem, in which random forces exerted by the collisions with the neutrals are included among the forces which determine the unperturbed motion of the charged particles. The problem is then "to determine a transition probability which describes the diffusion of a charged particle through position velocity space" (Hagfors 1970). Hagfors and Brockelman (1971) applied it to hard spheres elastic collisions. The gross features are identical to the one of Figure 10, however the central depression of the spectrum does not disappear as rapidly when  $\psi_i$  increases.

b. Ion-Ion collisions.

In the F region of the ionosphere the frequency of ion-ion 90° collisions is generally smaller than the Ion gyro frequency. However small angle deviations through collisions are sufficient to affect significantly the ion trajectories over one gyroperiod. As a consequence gyroresonances are completely destroyed over most of the F region.

c. Electron-Ion collisions

It was mentioned earlier that Landau damping tends to bring plasma waves in thermal equilibrium with the plasma. This holds for the plasma lines,

however when the Landau damping becomes small because of the small number of electrons participating in it, damping through electron-ion collisions becomes dominant. In these circumstances this last mechanism is responsible for bringing the plasma waves in thermal equilibrium with the plasma (Perkins and Salpeter (1965)).

The total intensity of the plasma lines is consequently unchanged but their width is now determined by collisional damping.

Interesting situations develop when the excitation of the plasma line by photoelectrons is taken into account. This is the subject of the next section.

#### XI. Plasma lines enhanced by the photoelectrons

Plasma lines enhancement by photoelectrons have been studied by Perkins and Salpeter (1965), Perkins et al. (1965), Yngvesson and Perkins (1968).

When a tail of photoelectrons is added to the thermal electron Maxwellian distribution, the plasma lines reach thermal equilibrium with either the thermal plasma or the photoelectron population; in the latter case the equivalent temperature is much higher and the intensity of the line is correspondingly much higher. The particle population which determines the intensity of the wave depends upon the relative strengths of the Landau dampings by the thermal electrons or the photoelectrons and of the electron ion collisions damping: if either the thermal electron Landau damping or the electron-ion collision damping dominates, the plasma line is in thermal equilibrium with the thermal plasma;

while, if the Landau damping by photoelectrons dominates, the plasma line is in thermal equilibrium with the photoelectron population.

Perkins and Salpeter (1965) derived the following formula for the enhancement of the plasma line intensity in a plasma without magnetic field with respect to its level when photoelectrons are absent:

$$I = \frac{f_{th}(v_\psi) + f_p(v_\psi) + \chi}{f_{th}(v_\psi) - \frac{KT_e}{m_e} v_\psi f'_p(v_\psi) + \chi} \quad \text{XI.1}$$

where  $f_{th}$  is the one dimensional velocity distribution of the thermal electrons in the  $\vec{k}$  direction,  $f_p$  is the photoelectron velocity distribution in the same direction,  $\chi$  describes the electron ion collision damping and  $v_\psi$  is the wave phase velocity.

$$\chi = \frac{k^3}{6\pi^2} \left( \frac{m}{2\pi KT} \right)^{1/2} \text{Log}_e(4\pi n_0 \lambda_D^3) \quad \text{XI.2}$$

The dominant term in eqn. XI.1 is the thermal electron damping for low values of  $v_\psi$ , the collisional damping for large values of  $v_\psi$  and the photoelectron damping for intermediate values of  $v_\psi$ .

Therefore the plasma line is enhanced only over a limited range of values of  $v_\psi$  or in other words over a limited range of resonant frequencies.

The effect of the magnetic field is to considerably increase the Landau damping by the thermal electrons and therefore to generally decrease the range of frequencies over which the plasma line is enhanced.

Salpeter (1961b) has shown that the effect of the magnetic field can be taken into account by modifying the velocity distribution of the thermal electrons in the following way:

$$f_{th} = n_0 \left( \frac{m_e}{2\pi K T_e} \right)^{1/2} \sum_{n=-\infty}^{+\infty} \frac{e^{-1/2 \Phi_e^{-1} \sin^2 \phi}}{\cos \phi}$$

XI.3

$$I_m \left( \frac{1}{2} \Phi_e^{-1} \sin^2 \phi \right) e^{-(\omega_r / \Omega_e - n)^2 / \Phi_e \cos^2 \phi}$$

where  $I_m$  is the Bessel function of imaginary argument. When  $\phi$  varies from  $0^\circ$  towards  $90^\circ$ , the frequency range dominated by thermal electron Landau damping increases steadily. However, when  $90^\circ$  is approached the Landau damping by thermal electrons becomes strongly modulated with maxima at frequencies equal to multiples of the electron gyrofrequency. Eventually the Landau damping by thermal electrons disappears when  $\phi$  reaches  $90^\circ$ .

These features are illustrated on Figure 12 which shows the frequency ranges of plasma line enhancements for various angles  $\phi$  and wavelengths after Yngvesson and Perkins (1968) and for a given  $f_p$  distribution. The enhancement is expressed in terms of plasma temperatures, which implies that when fully enhanced the line is in thermal equilibrium with the local photoelectron velocity distribution characterized by an apparent temperature  $T_p$ . Changing the wavelength has the effect of changing the phase velocity associated with a given plasma frequency; as a consequence the range of frequencies over which the plasma line is enhanced depends upon the wavelength. In turn, since for practical

purposes the plasma frequency in the ionosphere covers a limited range the overlap of the two frequency ranges will be more or less favorable according to the wavelength used.

For a wavelength  $\lambda$  of .70 m there is a good overlap and one can see the three regions: the region dominated by thermal electron Landau damping for low frequencies and its expansion for increasing values of  $\phi$ , the region where the line is enhanced because of the predominance of the Landau damping by the photoelectrons for intermediate values of the plasma frequency, and finally the region dominated by electron-ion collisional damping. For  $\lambda = .23$  m, the region dominated by thermal electron damping has expanded, while for  $\lambda = 200$  m most of the range corresponds to collisional damping. The modulation at the electron gyrofrequency is quite pronounced in all cases for  $\phi = 80^\circ$ .

## XII. Conclusion

The simple concept of independent free electrons initiated the theory of incoherent scatter. The problem became much more complex when it was realized that collective interactions had to be taken into account for various situations.

The nature of the complexity being associated with some of the essential properties of the plasma, the elaboration of the theory became more and more rewarding for both the theorist and the experimenter.

In particular in practically all instances, every effort made by the theorists to include a new parameter in the theory was accompanied by an immediate

experimental verification, the experimental work thus helping considerably the theoretical work.

In turn the addition of new parameters was to permit the experimenters a more and more complete diagnosis of the state of the ionosphere. The simultaneous determination of 4 to 5 ionospheric parameters has certainly made incoherent scatter the most powerful ground based mean for studying the dynamics and thermodynamics of the ionosphere and the thermosphere: a considerable contribution to these fields has already been made by incoherent scatter sounding (e.g. reviews by Evans (1969, 1964) and by Farley (1970b)). Incoherent scatter sounding using laser beams is also more and more used for laboratory plasma diagnosis.

The future of incoherent scatter sounding of the ionosphere involves, in addition to observations from the existing facilities, important projects in North America and Northern Europe and even a possible implementation of an incoherent scatter radar on board the Space Shuttle Space Laboratory. In all instances the future of incoherent scatter for the study of the ionosphere is to result from a joint effort of scientists from many countries.

#### Acknowledgements:

Helpful comments by Drs. S. J. Bauer, R. F. Benson, K. D. Cole, and P. Waldteufel are gratefully acknowledged. The author is indebted to the National Research Council of USA for the award of a Senior Postdoctoral Research Associateship and to the Centre National d'Etudes des Telecommunications and the Centre National de la Recherche Scientifique of France.

## REFERENCES

- Appleton, E. V. and Barnett M. A. F., (1925) Proc. Roy. Soc., 109, 621-641.
- Bernstein, I. B. (1958), Phys. Res., 109, 10-21.
- Bahtnagar, P. L., Gross E. P. and Krook M., (1954) Phys. Rev., 94, 511-525.
- Bohm, D. and Gross, E. P. (1949a) Phys. Res., 75, 1851-1864.
- Bohm, D. and Gross, E. P. (1949b) Phys. Res., 75, 1864-1876.
- Booker, H. G. and Gordon, W. E. (1950) Proc. I.R.E., 38, 401-412.
- Booker, H. G. (1956), J. Atmos. Terr. Phys., 8, 204-221.
- Bowles, K. L. (1958) Phys. Rev. Lett., 1, 454-455.
- Bowles, K. L. (1959), N.B.S., Boulder, Colo. Rept. 6070.
- Breit, G. and Tuve, M. (1925) Nature, 116, p. 357.
- Buneman, O. (1961) J. Geophys. Res., 66, 1978-1979.
- Buneman, O. (1962), J. Geophys. Res., 67, 2050-2053.
- Chapman, S. (1931) Proc. Roy. Soc. (London), 43, 26-45.
- Cohen, M. H. (1963), J. Geophys. Res., 68, 5675-5679.
- Dougherty, J. P. and Farley, D. T. (1960) Proc. Roy. Soc. (London) A, 259, 79-99.
- Dougherty, J. P. (1963) J. Fluid. Mech. 16, 126-137.
- Dougherty, J. P. and Farley, D. T. (1963) J. Geophys. Res., 68, 5473-5486.
- Evans, J. V. (1969), Proc. IEEE, 57, 496-530.
- Evans, J. V. (1974), J. Atmos. Terr. Phys., 36, 2183-2234.
- Fabry, C. (1928) C. R. Acad. Sci. (Paris), 187, 777-781.
- Farley, D. T., Dougherty, J. P. and Barron, D. W. (1961) Proc. Roy. Soc. (London), A, 263-258.

- Farley, D. T. (1966), *J. Geophys. Res.*, 71, 4091-4098.
- Farley, D. T. (1970a), *Methods of Experimental Physics*, Volume 9B, Chapter 4,  
Griem, H. R. and Lorbert, R. H., Eds. Academic Press, New York.
- Farley, D. T. (1970b) *J. Atmos. Terr. Phys.*, 32, 693-704.
- Fejer, J. A. (1960) *Can. J. Phys.*, 38, 1114-1133.
- Fejer, J. A. (1961) *Can. J. Phys.*, 39, 716-740.
- Gordon, W. E. (1958) *Proc. I.R.E.*, 46, 1824-1829.
- Hagfors, T. (1961) *J. Geophys. Res.*, 66, 1699-1712.
- Hagfors, T. (1970) *Phys. Fluids*, 13, 2790-2794.
- Hagfors, T. and Brockelman, R. A. (1971), *Phys. Fluids*, 14, 1143-1151.
- Kahn, F. D. (1959) *Astrophys. J.*, 129, 205-216.
- Laaspere, T. (1960) *J. Geophys. Res.*, 65, 3955-3959.
- Lamb, G. L. (1962) Los Alamos Scientific Laboratory, Report LA 2715.
- Landau, L. (1946) *J. Phys.*, USSR, 10, 25-34.
- Moorcroft, D. R. (1963) *J. Geophys. Res.*, 68, 4870-4872.
- Moorcroft, D. R., (1964) *J. Geophys. Res.*, 69, 955-970.
- Perkins, F. W. and Salpeter, E. E. (1965) *Phys. Res.*, 139, A, 55-62.
- Perkins, F. W., Salpeter, E. E. and Yngvesson, K. O. (1965) *Phys. Rev. Lett.*  
14, 579-581.
- Perkins, F. W. (1967) MATT-555, Plasma Physica Laboratory, Princeton  
University.
- Pines, D. and Bohm, D. (1952) *Phys. Res.*, 85, 338-353.

- Lord Raleigh (1871) *Phil. Mag.* XLI, 4, 107-120.
- Renau, J. (1960) *J. Geophys. Res.*, 65, 3631-3640.
- Renau, J., Camnitz, H. and Flood, W. (1961) *J. Geophys. Res.*, 66, 2703-2732.
- Renau, J., Camnitz, H. and Flood, W. (1962) Cornell Aeronautical Lab.  
Rep. No. 121.
- Rosenbluth, M. N. and Rostoker, N. (1962) *Phys. Fluids*, 5, 776-788.
- Salpeter, E. E. (1960) *Phys. Rev.*, 120, 1528-1535.
- Salpeter, E. E. (1961a) *J. Geophys. Res.*, 66, 982-984.
- Salpeter, E. E. (1961b) *Phys. Rev.*, 122, 1663-1674.
- Salpeter, E. E. (1963) *J. Geophys. Res.*, 68, 1321-1333.
- Seasholtz, R. G. and Tanenbaum, B. S. (1969) *J. Geophys. Res.*, 74, 2271-2275.
- Tanenbaum, B. S. (1968) *Phys. Rev.*, 171, 215-221.
- Taylor, A. H. and Hulburt, E. O. (1926) *Phys. Rev.*, 27, 189-215.
- Taylor, E. C. and Comisar, G. C. (1963) *Phys. Rev.*, 132, 2379-2384.
- Thomson, J. J. (1906) *Conduction of Electricity through gases*, p. 321,  
Cambridge University Press, London.
- Waldteufel, P. (1963) *C. R. Acad. Sci. (Paris)* 257, 3835-3837.
- Waldteufel, P. (1965) *Ann. Geophys.*, 21, 106-120.
- Yngvesson, K. O. and Perkins, F. W. (1968) *J. Geophys. Res.*, 73, 97-110.

# GEOMETRY OF INCOHERENT SCATTERING

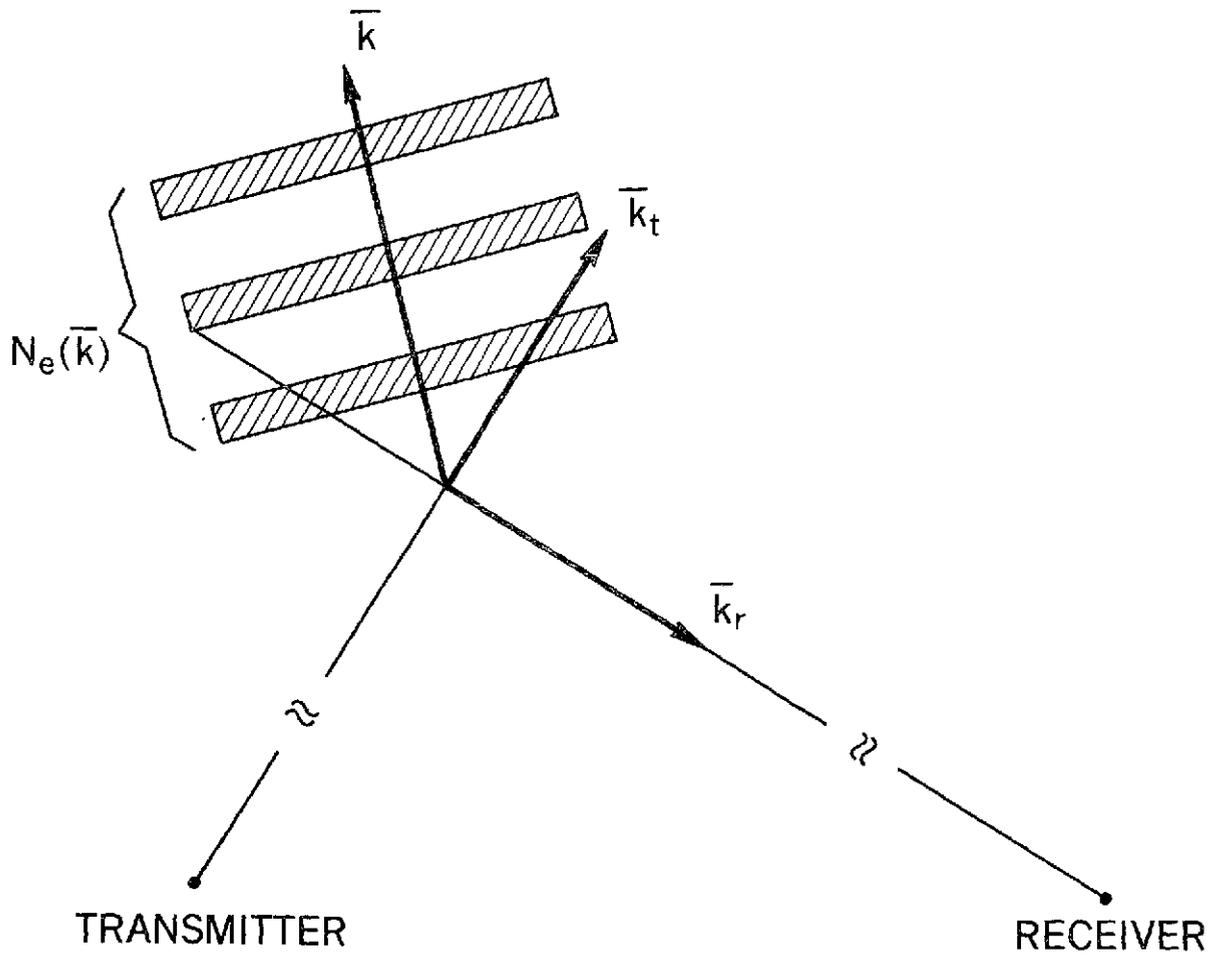


Figure 1. Geometry of incoherent scattering.

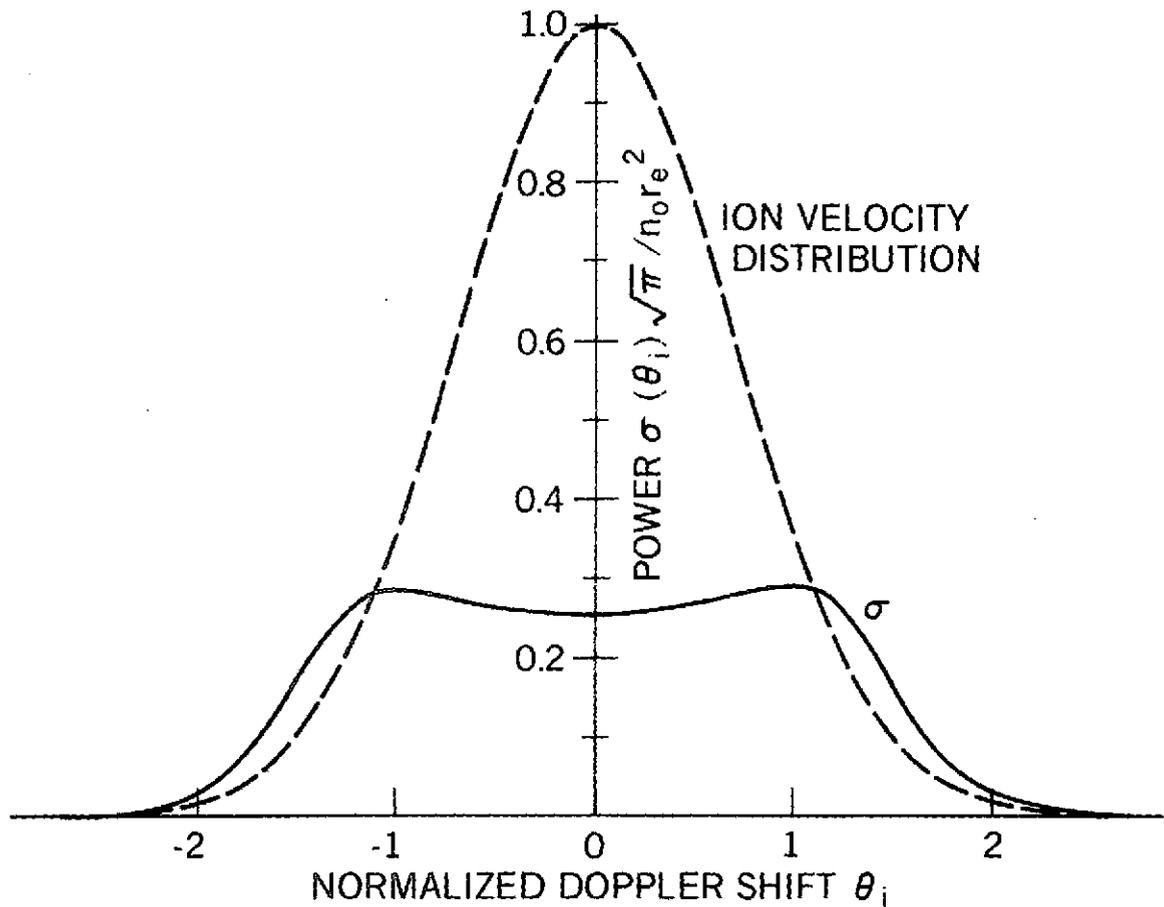


Figure 2. Normalized spectrum of the ionic component for  $\alpha \gg 1$  and  $T_e = T_i$  (full line). Ion velocity distribution (dashed line). The normalized doppler shift is defined by

$$\theta_i = \frac{\omega}{k} \left[ \frac{m_i}{2KT_i} \right]^{1/2} = v \left[ \frac{m_i}{2KT_i} \right]^{1/2}$$

(after Farley, 1970a).

## COMPLETE SPECTRA FOR VARIOUS VALUES OF $\alpha$

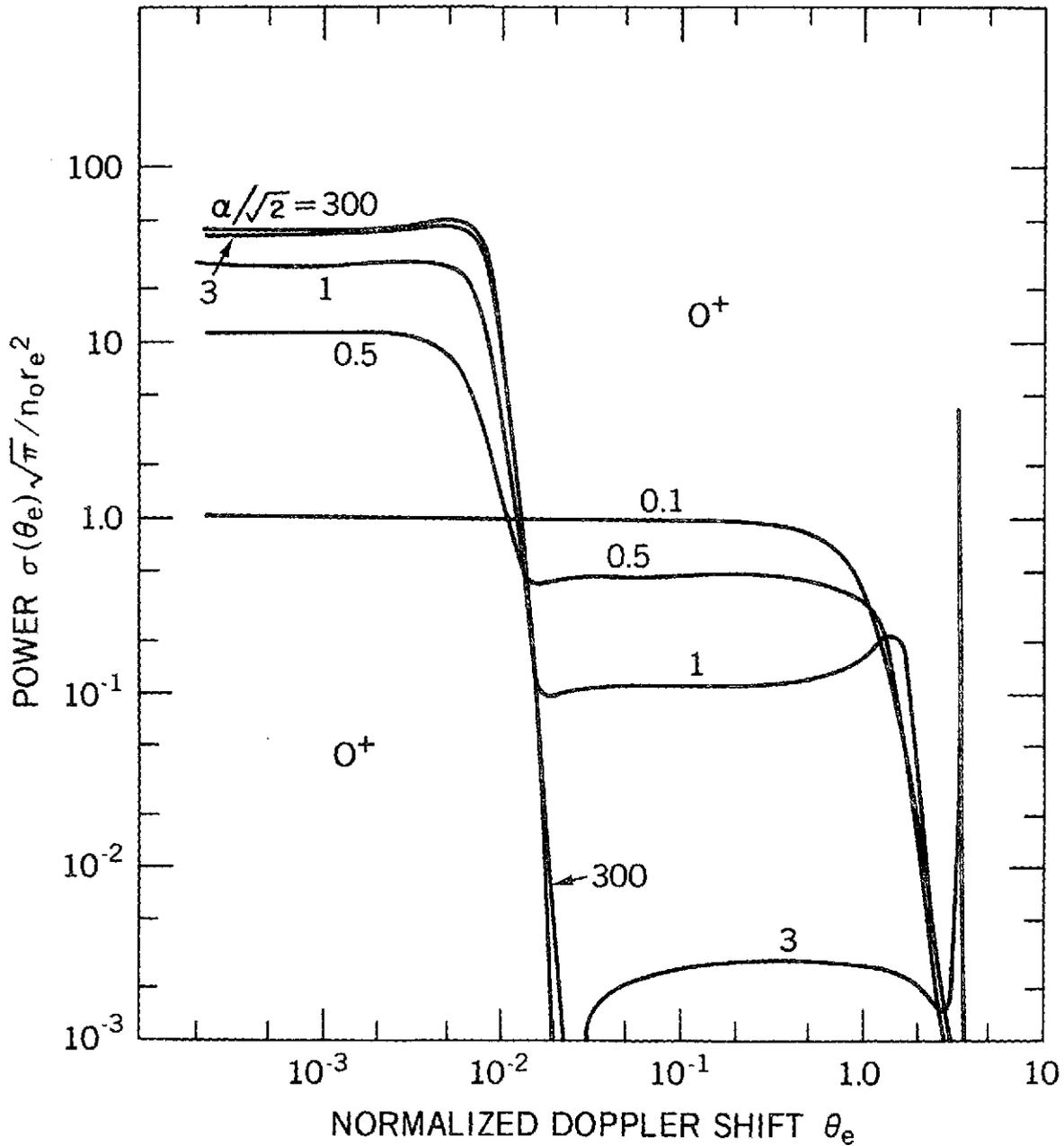


Figure 3. Normalized complete spectra for different values of  $\alpha = 1/k\lambda_D$ , ( $T_e = T_i$ ). The normalized doppler shift is

$$\theta_e = \frac{\omega}{k} \left[ \frac{m_e}{2KT_e} \right]^{1/2}$$

(after Hagfors, 1961).

# IONIC COMPONENT VS. $T_e/T_i$

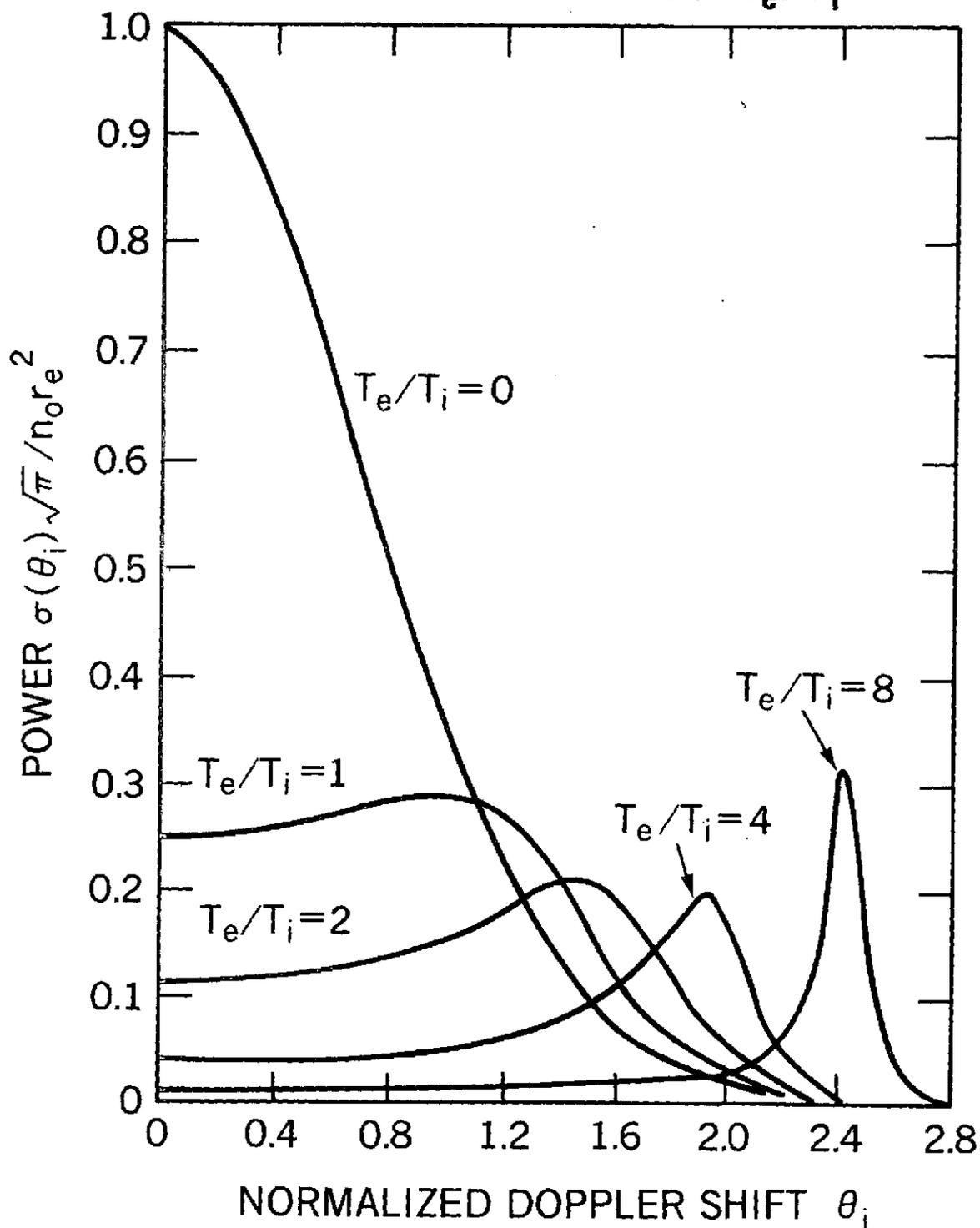


Figure 4. Normalized half spectra of the ionic component for  $\alpha \gg 1$  and for different values of  $T_e/T_i$ . (After Fejer, 1961).

### CROSS SECTION VS. $T_e/T_i$

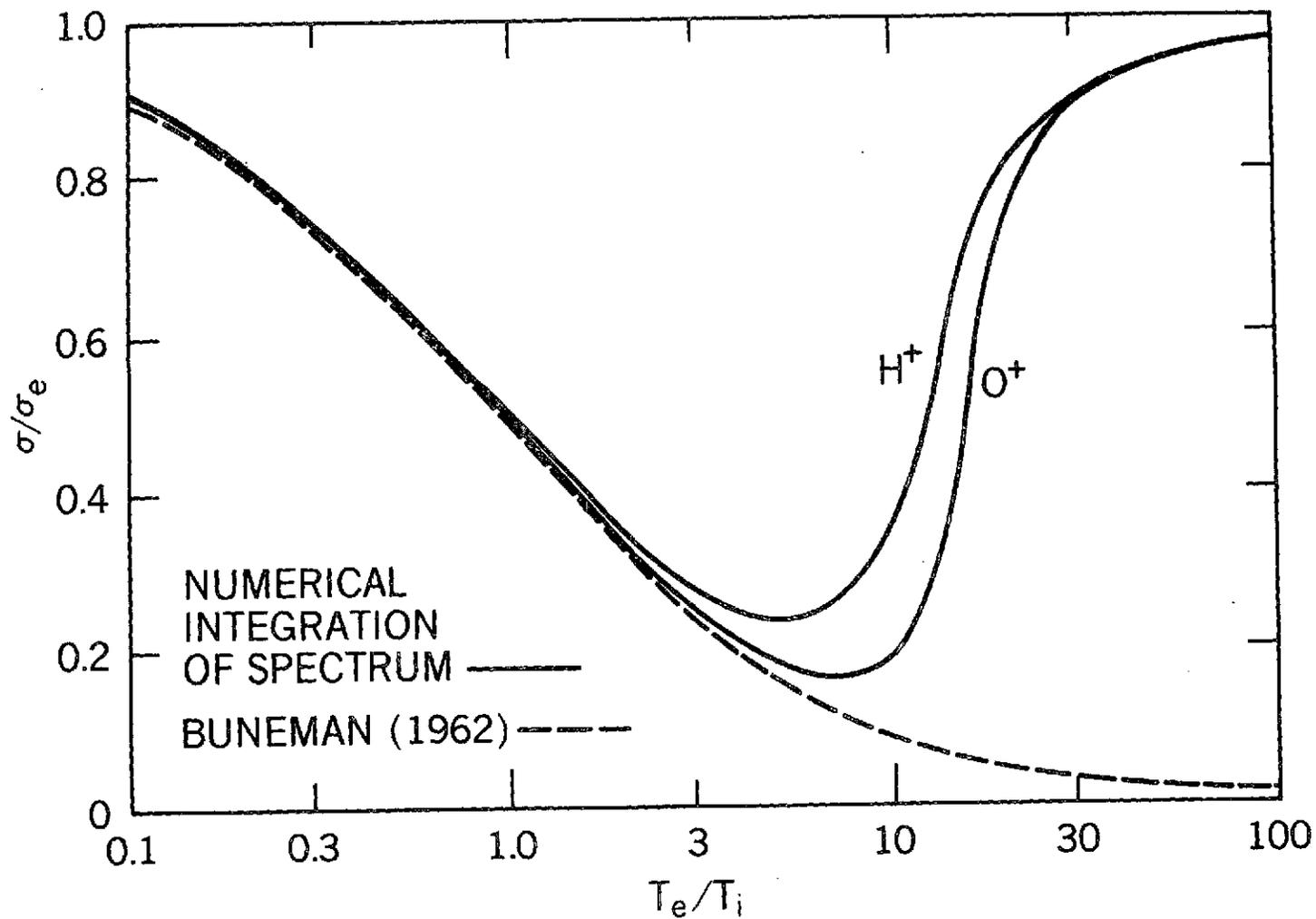


Figure 5. Variation of the ratio of the total differential cross section for  $\alpha \gg 1$  to the total differential cross section for independent electrons as a function to  $T_e/T_i$  (after Moorcroft, 1963). The dashed curve corresponds to the approximate expression VI.5 given by Buneman (1962).

### CROSS SECTION VS. $1/\alpha$

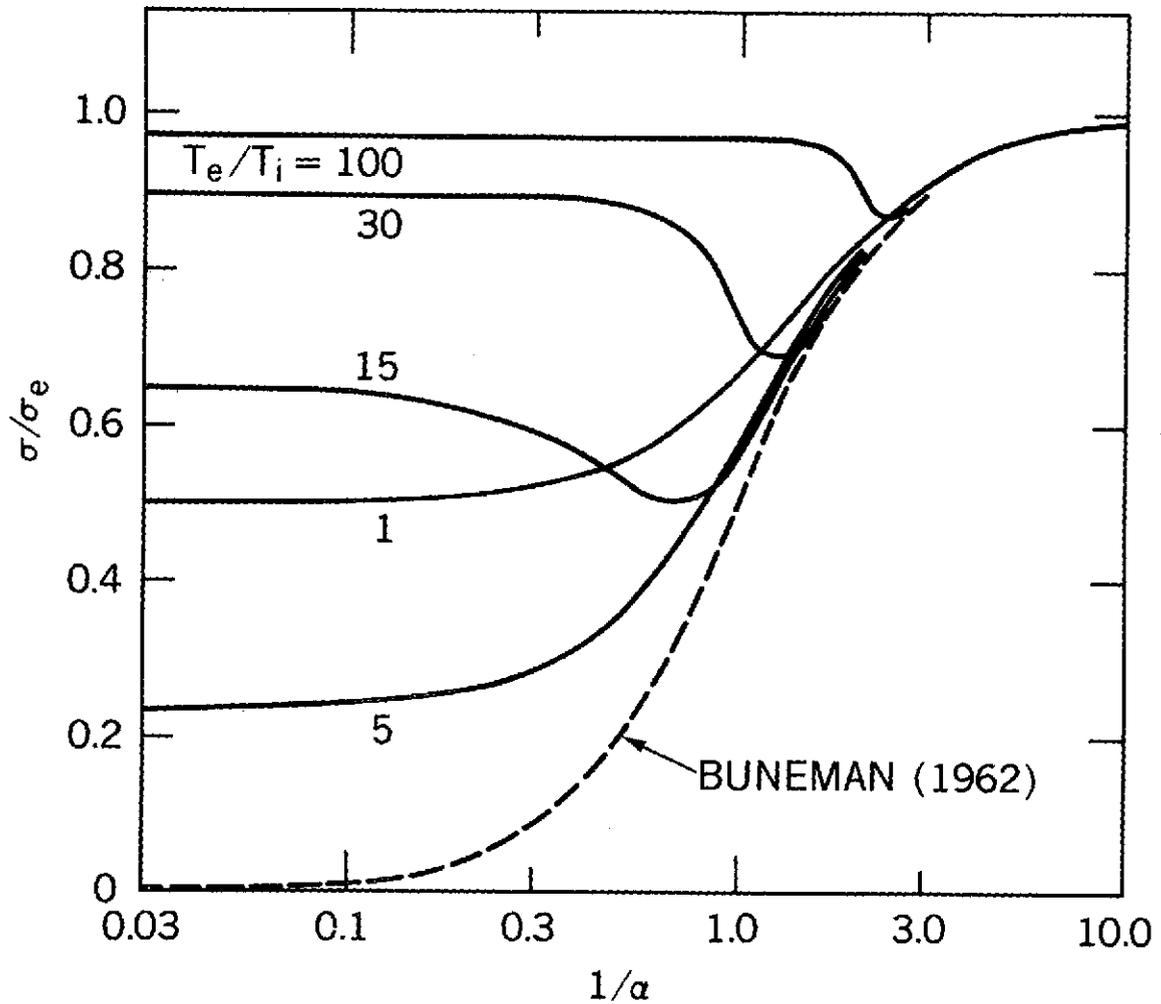


Figure 6. Variation of the ratio of the total differential cross section  $\sigma$  to the total differential cross section for independent electrons as a function of  $\alpha$ , for  $H^+$  and for different values of  $T_e/T_i$  (after Moorcroft, 1963). The dashed line corresponds to the approximate expression given by Buneman (1962).

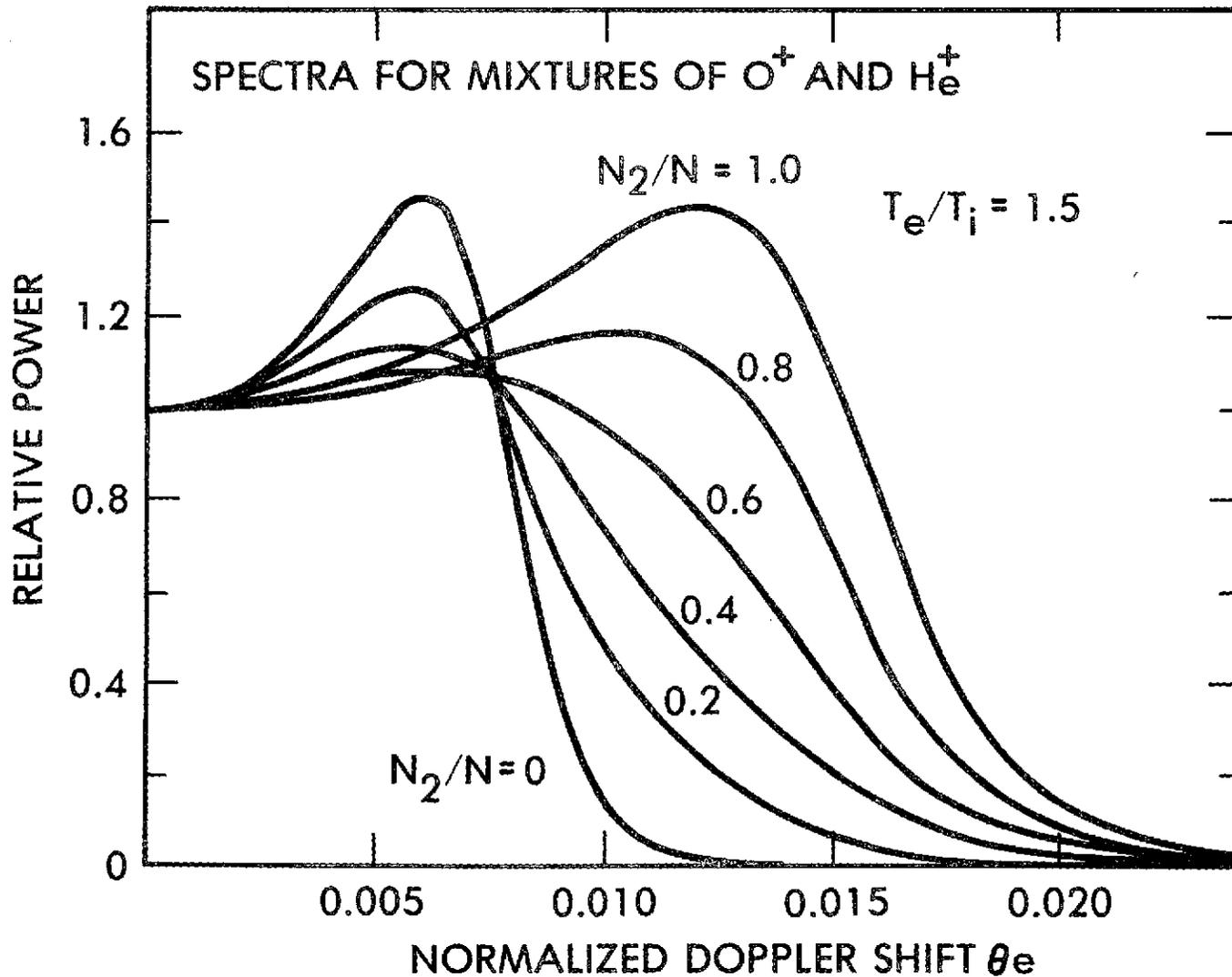


Figure 7. Half spectra of the ionic component (normalized to a value of 1 for zero doppler shift) for  $T_e/T_i = 1.5$ ,  $\alpha \gg 1$  and for different mixtures of  $O^+$  and  $He^+$ ,  $N_2/N$  is the relative concentration  $He^+$ . (After Moorcroft, 1964).

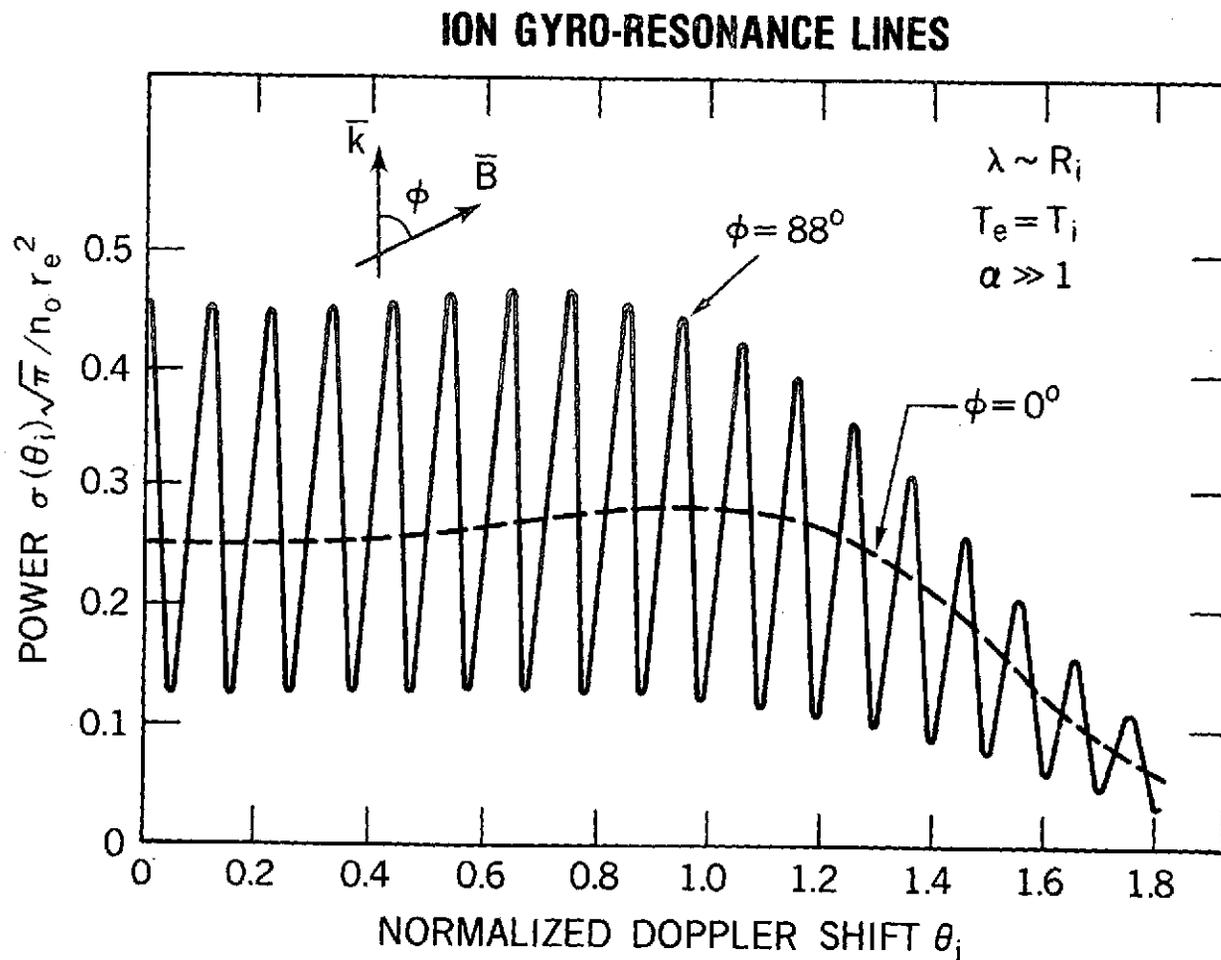


Figure 8. Normalized half spectra of the ionic component for  $T_e = T_i$ ,  $\alpha \gg 1$ ,  $0+$  ions,  $\lambda \sim R_i$  (the Lamor radius of the ions), and for  $\vec{k}$  either parallel to the magnetic field ( $\phi = 0^\circ$ , or no magnetic field) or nearly perpendicular to it ( $\phi = 88^\circ$ ). Ion-Ion collisions have been assumed to be negligible. (After Farley et al., 1961).

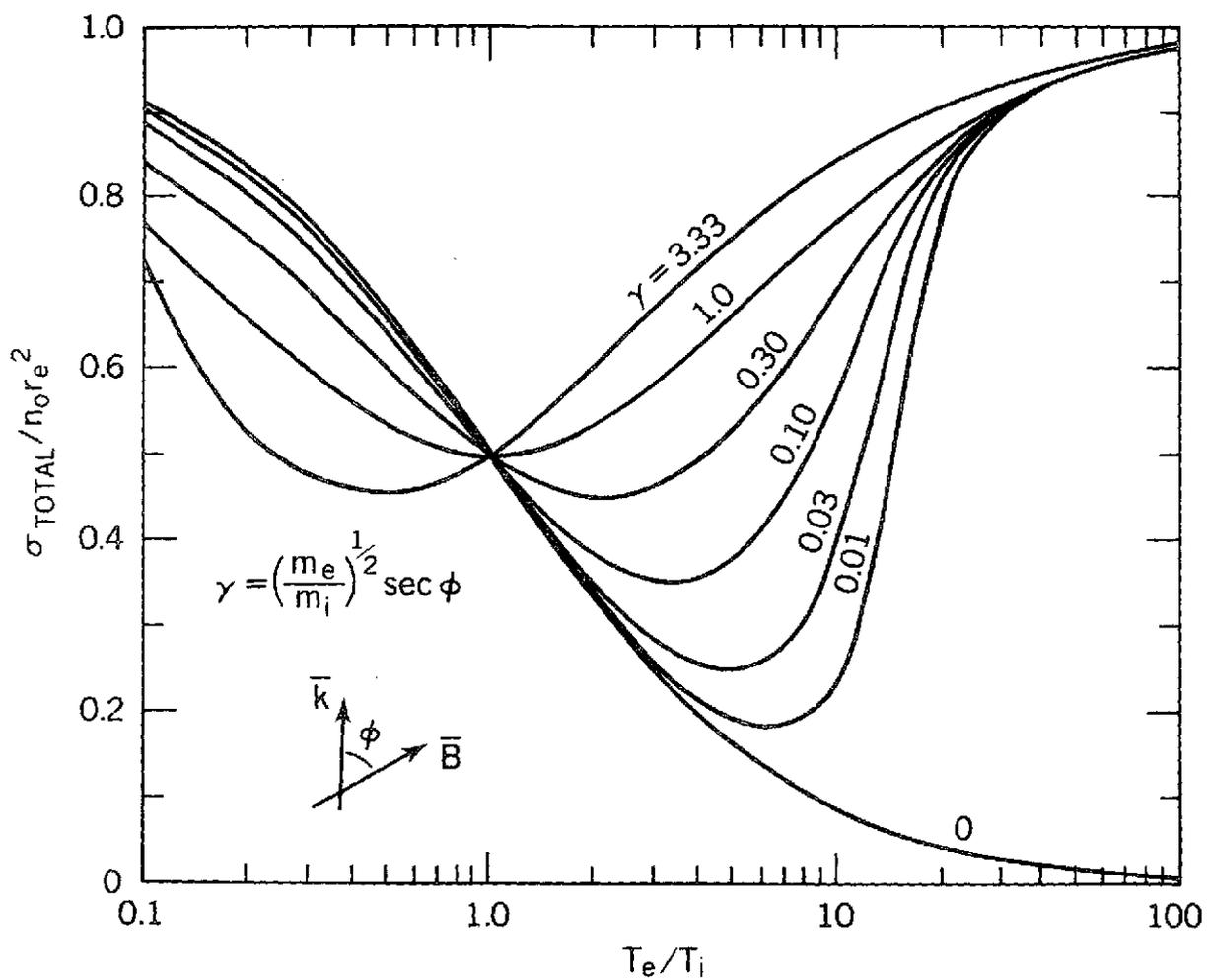


Figure 9. Variation of the normalized total differential cross section as a function of  $T_e/T_i$  for different values of

$$\gamma = \left(\frac{m_e}{m_i}\right)^{1/2} \sec \phi$$

(After Farley, 1966).

# EFFECT OF ELECTRON DRIFT ON IONIC COMPONENT

$$T_e/T_i = 2.0 \quad \alpha = 10^2$$

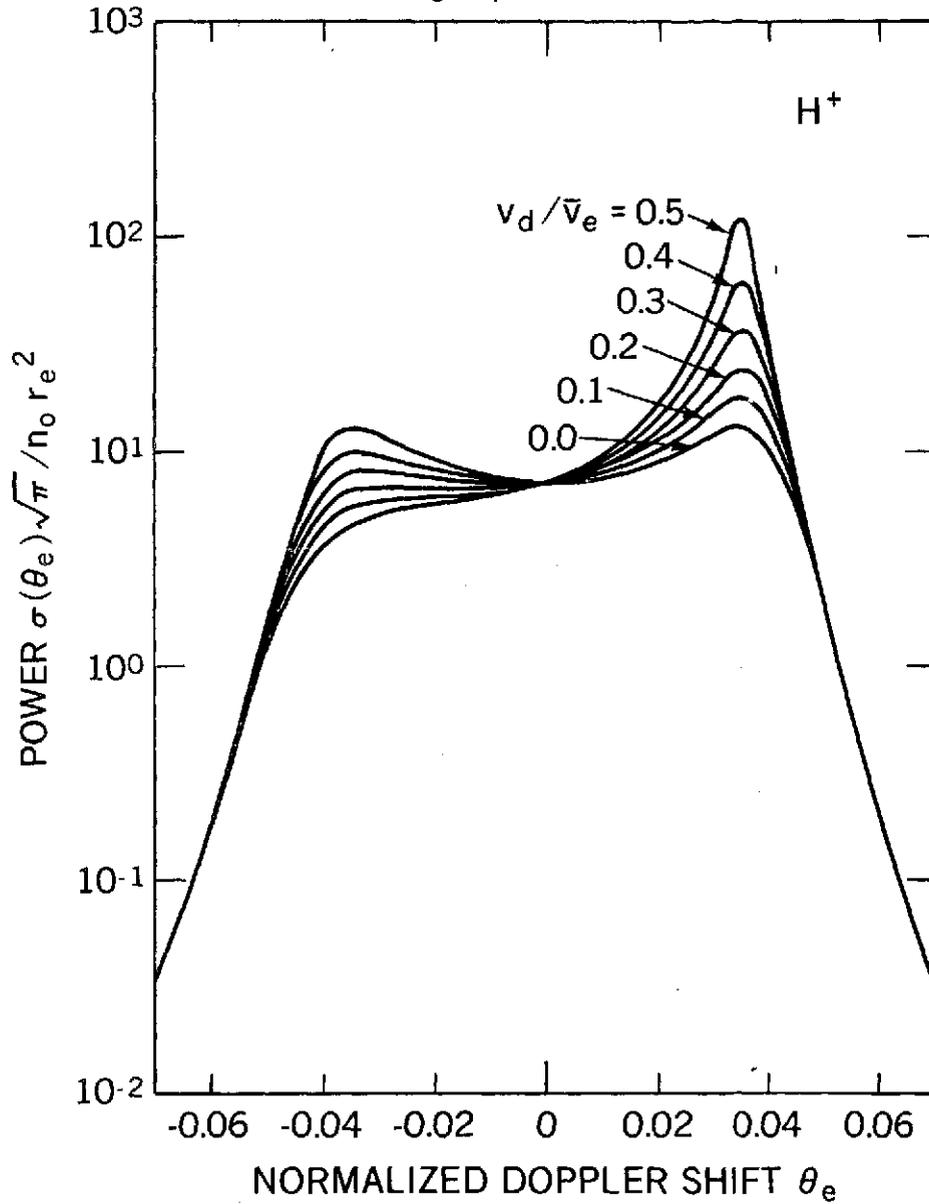


Figure 10. Normalized spectra of the ionic component for  $T_e/T_i = 2.0$ ,  $\alpha = 10^2$ ,  $H^+$  ions and different electron drifts antiparallel to  $\vec{k}$ .  $v_d/\bar{v}_e$  is the ratio of the drift velocity to the electron thermal velocity. (After Lamb, 1962).

## EFFECT OF COLLISIONS ON IONIC COMPONENT

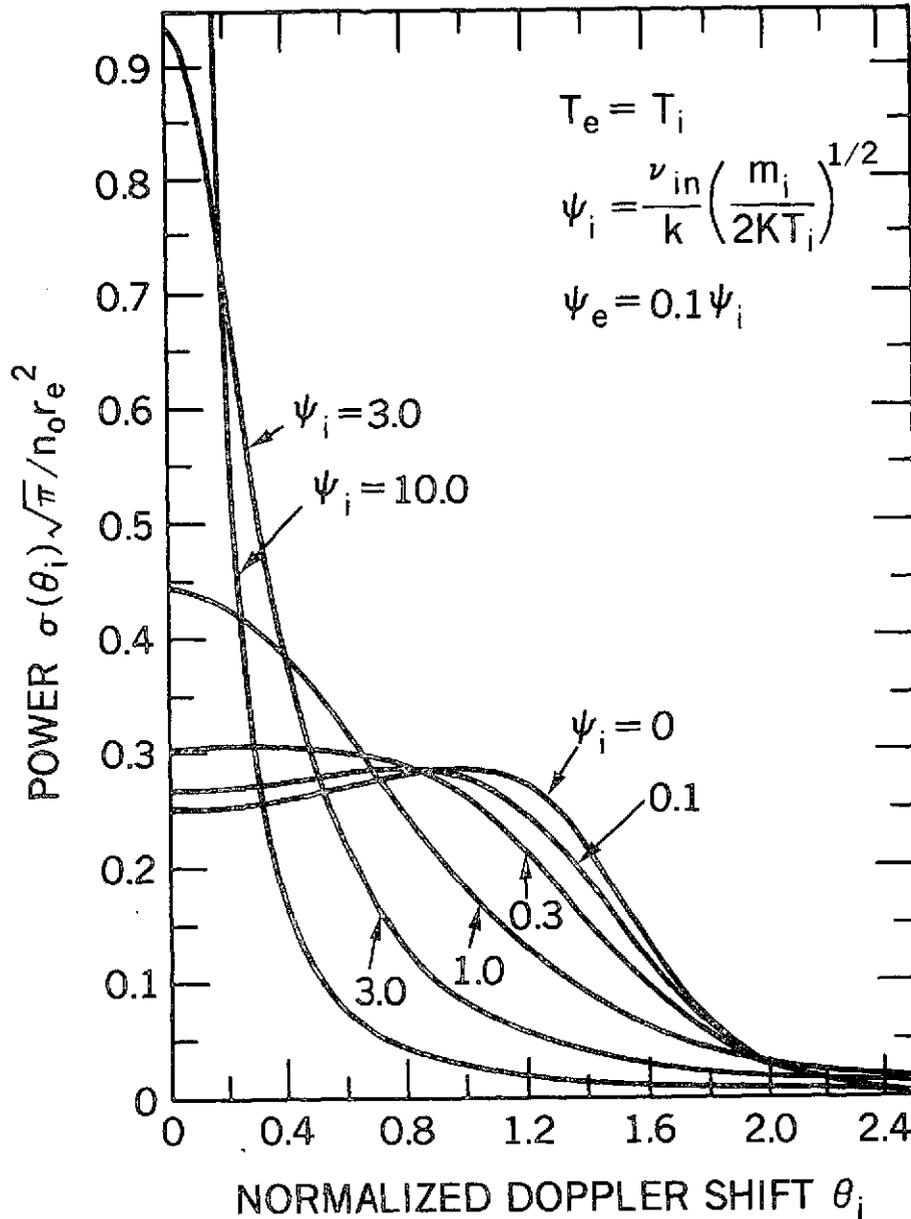


Figure 11. Normalized half spectra of the ionic component for  $T_e = T_i$ ,  $\alpha \gg 1$  and for different normalized ion neutral collisions frequencies

$$\psi_i \left( = \frac{v_{in}}{k} \left( \frac{m_i}{2KT_i} \right)^{1/2} \right)$$

(After Dougherty and Farley, 1963).

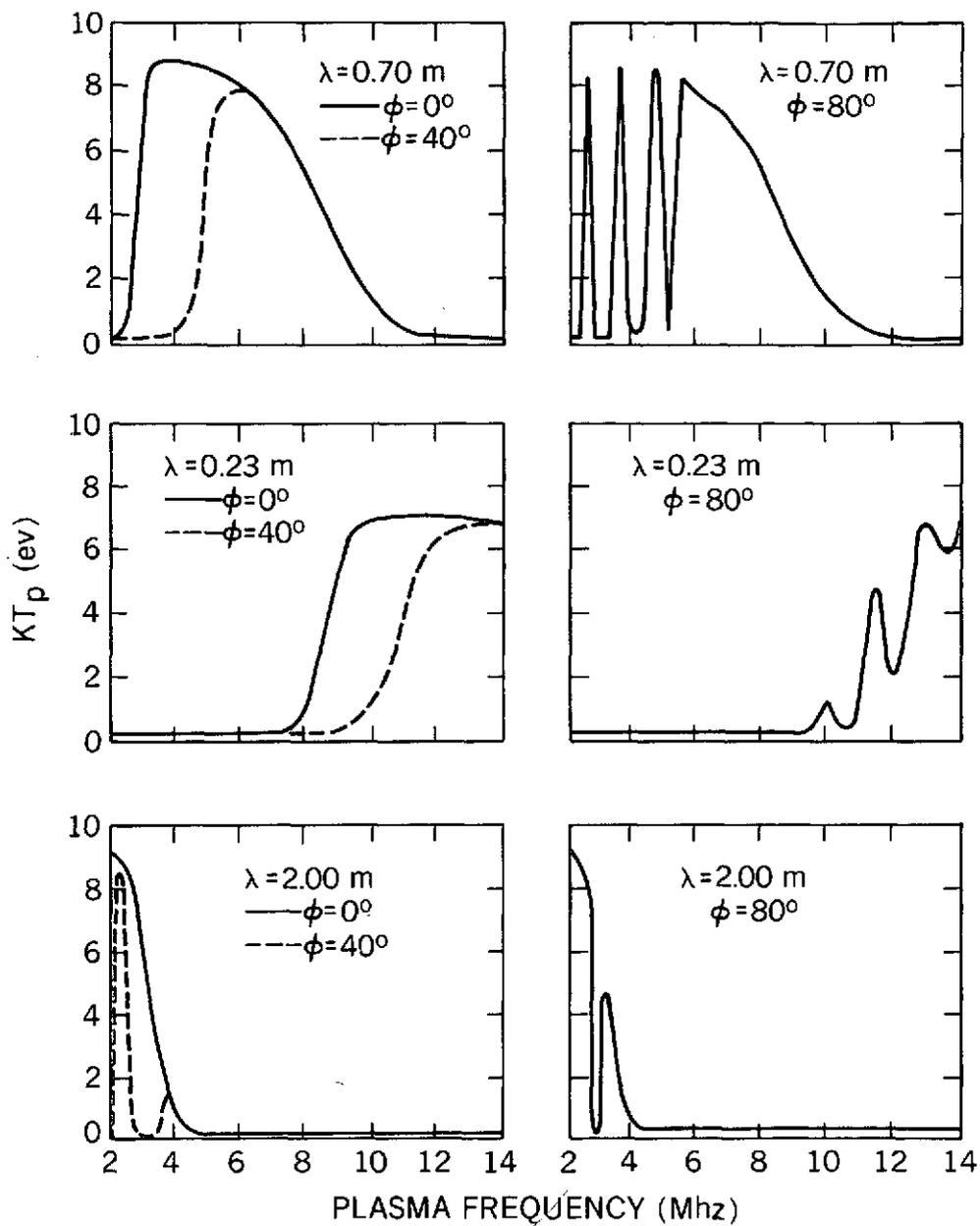


Figure 12. Plasma line enhancements for various wave-lengths and angles  $\phi$  between the  $\vec{k}$  vector and the magnetic field as functions of the plasma frequency. The photoelectron velocity distribution  $f_p$  was deduced from measurements and  $T_e$  was chosen equal to 2000 K (after Yngvesson and Perkins, 1968).