THE EFFECT OF SUPRATHERMAL PROTONS ON THE PHYSICAL CONDITIONS IN SEYFERT GALAXY NUCLEI

by

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ABSTRACT

The physical conditions in a high density hydrogen gas heated and ionized by suprathermal protons are investigated, with application to the gas in the nuclear region of Seyfert galaxies. The gas is assumed optically thick to Lyman and Balmer line radiation. Mechanisms by which the radiation from the gas can balance the heating by the fast protons are investigated, and minimum values for the mass of gas are estimated. Under certain conditions, the suprathermal atoms themselves can cool the ambient gas by rescattering the line radiation into the optically thin region in the wings of the line. This mechanism, which we call optical reverberation, can enhance the broad component of the hydrogen lines produced by inelastic atomic collisions and yield line widths consistent with those observed in Seyfert and quasar spectra. If this mechanism is important, the Lyman decrement can depend strongly on the temperature and density of the ambient hydrogen. We also discuss the possibility of achieving dynamic equilibrium of the ambient gas by balancing the momentum transfer from the suprathermals with gravitational attraction of a massive central source.
I. Introduction

A radially streaming proton model in which suprathermal protons interact with a stationary ambient gas of hydrogen atoms to give rise to emission lines, has been successful in understanding the broad wings of Balmer lines in Seyfert galaxies (see Ptak and Stoner 1973). In the model, the physical properties of the ambient hydrogen are free parameters that have insignificant effect on the Lyman and Balmer emission line profiles. Variations on the model have produced agreement with the observed emission line profiles of other ions in quasars (Stoner, Ptak and Ellis 1974) and demonstrated that considerably different geometries can produce nearly identical profiles (Stoner and Ptak 1974). Thus it appears that the essential feature of the radially streaming proton model in producing agreement with observed profiles is the role of the inelastic atomic collision processes.

A flux of fast protons large enough to produce emission wings of the observed intensity by the atomic collision mechanism should play a major role in determining the physical state of the ambient gas. The kinetic energy and the momentum of the fast protons will be transferred to the gas, producing heating, ionization and, possibly, large-scale dynamic effects. Here we discuss a simplified model that is designed both to assess the effect of the suprathermal protons on the temperature and state of ionization of the gas and to investigate the ways and forms in which the kinetic energy of the protons can eventually appear in the radiated electromagnetic spectrum of the object. By using the observed spectra of Seyfert
galaxy nuclei and by requiring physical self consistency, we estimate the physical properties of the nuclear gas from which the broad emission lines originate.

II. Description of the Model

The model we consider assumes uniform physical properties for the ambient gas. Detailed self consistent calculations by Kimmer (1975) show that, except for surface effects, this is a reasonable approximation, even when the velocity distribution of the fast protons varies considerably from place to place. The suprathermal protons are assumed to be homogeneously distributed and moving in random directions; they are characterized by an injection energy and an overall rate of injection. While fits to observed profiles require some coherence in the directions of suprathermal ion motions, the observations give few other clues about geometry, and this coherence will affect momentum transfer rates rather than heating rates. Thus, geometrical considerations, necessary for fitting observed profiles, are disregarded, except that the linear dimension and the total volume of the gas are parameters. The gas is assumed to be purely atomic hydrogen, partially ionized.

If the ambient gas were optically thin, the emission from the ambient atoms would contribute strong central cores to the hydrogen lines, with an intensity greater than that contained in the broad component. Since this is not observed, we assume that the density of the ambient gas, its linear dimension, and the \( n = 2 \) population are all sufficiently large to make the gas optically
thick in both Lyman and Balmer lines. We further assume that the gas is optically thick in the Lyman continuum, but transparent to the Balmer continuum, so that only a part of the energy delivered when an atom is ionized can directly escape via recombination. Then the line radiation is trapped until it can be degraded into an optically thin region by one of the mechanisms we describe below.

Most of the energy of the suprathermal protons is first delivered to the electrons of the gas, with the rest going into excitation of the ambient atoms. The electron temperature rises until the electrons are able to transfer energy to the ambient gas atoms at the same rate as they receive it from the suprathermals. Both thermal bremsstrahlung and direct heating of dust by the electron gas are generally negligible. Radiative cooling through forbidden line emission from heavy elements is suppressed by collisional de-excitation at the densities we consider \((10^7 \text{ to } 10^{12}\text{cm}^{-3})\).

Next the energy goes by way of the atoms into the trapped radiation field in the vicinity of the emission line centers. Since each Lyman and Balmer photon will be absorbed and reemitted a very large number of times before being degraded or escaping, the atoms are very efficient "heat pipes" connecting the heat baths in the radiation near the line centers. Thus, we assume the radiation in the optically thick region near the line centers rises to the level specified by the Planck formula with a radiation temperature that also describes the distribution over atomic levels of the ambient hydrogen. The free electrons and the Boltzmann distribution over atomic levels are described by different temperatures whose relative
values are to be fixed by requiring that the net rate at which the electrons deliver energy to the radiation field via the atoms is balanced by the rate at which the radiation field is cooled.

We consider three ways by which the energy can finally leave the gas. One possibility is that the two photon decay of the hydrogen 2s state acts as the dominant cooling mechanism, contributing an ultraviolet continuum to the radiated spectrum. Second, if enough dust is present, the line radiation can be absorbed and reemitted in the infrared. Finally, the energy may, under certain conditions, be absorbed and reemitted by the suprathermal atoms themselves: in this case the energy will come out in the broad component of the hydrogen emission lines. The proportion of the energy radiated in each of these three forms will depend on the temperature, the density of ambient gas, the flux of suprathermal particles, and the amount of dust. If the third mechanism is the dominant mechanism in determining the energy balance, then it can also have a large effect on the Lyman and Balmer decrements, and the relative hydrogen line intensities will be determined by the temperature and density of the ambient gas.

III. Ionization and Energy Balance

For a given temperature $T_e$ of the gas of free electrons, the ionization fraction is fixed by a balance between ionization and recombination rates for the ambient hydrogen:

$$N_{La}^{*}c_{ion}(La) + \langle \sigma_{ion}(s)v \rangle (1 - f_i)N_S N_H + \gamma(T_e) f_i (1 - f_i) N_{H}^{2}$$

$$= \alpha(T_e) f_i^2 N_H^2$$  \hspace{1cm} \text{(1)}
where $N_{La}$ is the ambient number density of La photons, $N_H$ is the number density of hydrogen atoms in the $n = 2$ state, $\sigma_{ion}(La)$ is the cross section for ionization of $n = 2$ hydrogen by La photons, $N_H$ is the number density of hydrogen (protons plus atoms), $f_i$ is the ionization fraction of the ambient gas, $\alpha_2(T_e)$ is the $n \geq 2$ recombination coefficient for hydrogen, and $\gamma(T_e)$ is the corresponding rate coefficient for ionization by the free electrons. The effect of the suprathermal protons on the ionization rate is represented in equation (1) by a term containing a number density of suprathermal particles $N_S$, which is related to the overall injection rate $J$ (i.e., suprathermal protons injected per unit time), the time required for an injected proton to thermalize, $\tau$, and the total volume of ambient gas by

$$N_S = J\tau/V \quad (2)$$

The slowing down time will depend on the injection energy, the density of ambient gas, and the ionization fraction. If Coulomb collisions with free electrons is the dominant mechanism for decelerating the protons, then the slowing down time will be approximately

$$\tau = \frac{3.2 \times 10^8}{f_i N_H} \left( \frac{E_0}{25\text{keV}} \right)^{3/2} \quad (\text{sec}) \quad (3)$$

Equation (3) is an approximation based on the formula by Butler and Buckingham (1962) for the deceleration of a suprathermal charged particle in a plasma; it neglects some dependence of the average charge of the suprathermal proton on its velocity and the slight dependence of the DeBye screening factor on density, temperature, and velocity.
For the remaining unspecified quantities in equation (1) we use the following approximations:

\[ \chi(T_e) = 2.07 \times 10^{-11}(5.1 - 0.96 \log_{10}(T_e))/T_e \]  
\[ \gamma(T_e) = 5.3 \times 10^{-11}T_e^{1/2}\exp(-1.58 \times 10^5/T_e) \]  
\[ \langle \sigma_{\text{ion}}(s)\nu \rangle = 2.0 \times 10^{-8}; \quad \sigma_{\text{ion}}(La) = 1.8 \times 10^{-8} \]

each in units sec\(^{-1}\) cm\(^3\). Equation (5) is the ionization rate coefficient of Kalkofen and Strom (1966) and equation (6) represents an approximate average over ionization rates due to suprathermal atoms and protons.

We assume the dominant mechanism by which the free electron gas exchanges energy with the radiation field is the collisional excitation and de-excitation of the \( n = 2 \) state of the hydrogen atom. If there were nearly complete thermal equilibrium between the electron gas and the radiation field, the excitation and de-excitation rates would be in nearly perfect balance, and the two temperatures, \( T_e \) and \( T_a \), would be equal; however, the two-photon decay of the hydrogen 2s state provides a minimum rate of cooling that reduces the effective temperature of the radiation relative to that of the electron gas. If we equate the rate of collisional excitation of the \( n = 2 \) state with the sum of the rates for collisional de-excitation and two-photon production, we obtain an approximate relationship between the two temperatures:

\[ T_a = 11.84 \times 10^4 \sqrt{\frac{11.84 \times 10^4}{T_e} + 2n \left(1 + \frac{10^6 T_e^{1/2}}{f_1 N_H} \right)} \]
where, as a simplifying assumption, the cross section for excitation of \( n = 2 \) by electron impact has been taken to vary inversely with the impact energy when the electron kinetic energy is above threshold.

If other cooling mechanisms are significant, then the depopulation of the \( n = 2 \) state by emission of Lyman alpha photons will not be entirely balanced by the reverse absorption process, and the radiation temperature will be correspondingly smaller than that given by equation (7). The remaining requirement for a steady state of ionization and temperature is that the total rate of heating of the gas by the suprathermals equals the rate at which radiated energy escapes the gas. The overall rate at which energy is delivered to a unit volume of the gas is clearly \( J \frac{E_o}{V} \), or, using equations (2) and (3), the heating rate is:

\[
\frac{N_s E_o}{\tau} = \frac{N_s f_i N_i H_o}{3.2 \times 10^8} \left( \frac{25 \text{keV}}{E_o} \right)^{3/2}
\]  

(8)

If we assume that the Ly\( \alpha \) photons produced by excitation of the ambient gas are immediately degraded (through absorption by dust, for example), then the above heating rate must be in balance with the rate at which energy is transformed into Ly\( \alpha \) photons by electron excitation, which is

\[
f_i N_i^2 \sigma_{\text{exc}(e)} <v> h\nu_o
\]

(9)

where \( \sigma_{\text{exc}(e)} \) is the cross section for collisional excitation of Ly\( \alpha \) by electrons, \( h\nu_o \) is the energy of a Ly\( \alpha \) photon, and the average is over the Boltzmann distribution of the electron velocities.
Equating expressions (8) and (9) gives us the energy balance, and this, together with equation (1) determines the ionization and temperature of the gas for a given $N_H$ and $N_s$. For this situation we use the total recombination coefficient in equation (1), and we take the Ly$\alpha$ number density to be negligible. In this case only the ratio of these two densities is important. Some representative values are given in Table 1, in the column headed by "Extreme Dust".

If the two-photon decay of the 2s state of hydrogen is the dominant cooling mechanism, then the rate at which energy is radiated by the gas is given by

$$\frac{(1 - f_i)N_H A' \exp \left[-1.18 \times 10^5/T_a\right]}{h \nu_O}$$

where $A'$ is the transition probability for the two-photon decay. In this case the ionization and the temperature are determined by solving equations (1) and (7) simultaneously with the equation of expressions (8) and (10). Results for this situation are also presented in Table 1.

Once the ionization and temperature are determined, we can estimate the amount of ambient gas that is required for a particular source; this estimate also depends on the cooling mechanism that dominates. Assuming that Ly$\alpha$ is immediately degraded, the energy balance takes the form

$$\frac{J E_O}{\nu} = (1 - f_i) f_i N_H^2 \sigma_{\text{exc}} (v) h \nu_O$$

Thus the required amount of gas is

$$N_H V = \frac{J E_O}{(1 - f_i) f_i N_H \sigma_{\text{exc}} (v) h \nu_O}$$
We can easily see what this number is for a typical case. For the average observed luminosity in broad Balmer lines, we require $J$ to be a few solar masses of suprathermals per year. So, if we choose $E_o = 10^6$ eV, $f_i = 0.1$, and $N_H = 10^{10}$ cm$^{-3}$, about 100 solar masses of gas is required in a volume of about $10^{49}$ cm$^3$ or $10^{-5}$ cubic light years. We should note that if the optical reverberation process discussed in the next section is important, the value for $J$ could be considerably reduced since the number of broad-component photons produced by each suprathermal proton could be greatly increased.

If instead the two-photon decay is doing the cooling, the energy balance becomes

$$\frac{JE_o}{V} = (1 - f_i)N_H A' \exp \left[ -1.18 \times 10^4 / T_a \right] h\nu_o$$

Again we can solve for the amount of gas needed:

$$N_H V = \frac{JE_o}{(1 - f_i)A' h\nu_o} \exp \left[ \frac{11.8 \times 10^4}{T_a} \right]$$

Using the same values as before and taking $T_a \approx 11,000^\circ$K, we find that a minimum of about 100 solar masses of ambient hydrogen is required in this case as well.

Finally, it is possible to set a lower limit on the amount of gas needed irrespective of how the energy is ultimately radiated. This is because the self-consistent calculation indicates that $N_s / N_H \lesssim 10^{-6}$. If the relative density is larger than this, the
ambient gas becomes completely ionized and the atomic processes are not efficient enough to be of interest. If we combine this inequality with equations (2) and (3) we obtain:

$$N_{H}V > 5 \times 10^{14} \frac{J}{f_{1}N_{H}} \left( \frac{E_{0}}{25 \text{keV}} \right)^{3/2}$$

(15)

with the numerical values previously used, this relation yields a lower limit of about $10M_{\odot}$ of ambient gas.

If the physical conditions of the ambient gas are specified, we can begin to make estimates concerning dynamic stability. Here we examine the possibility that the force exerted on the gas by the suprathermal particles is balanced by the gravitational attraction of a massive central body. We will consider the extreme case where all the suprathermals move in the same direction and have the same initial energy.

Let us consider a cloud of ambient gas at a distance $r$ from the source of the fast particles, which has an area $A$ perpendicular to the motion of the suprathermals. Then the number of suprathermals stopping in the cloud is $\frac{J}{4\pi r^{2}} A$. If the initial particle energy is $E_{0}$, and if all the particles stop in the cloud, then the rate of momentum transfer to the cloud is $\frac{J}{4\pi r^{2}} A (2m_{p}E_{0})^{1/2}$. The most extreme case occurs when the cloud is just thick enough to stop the particles; this means that the volume of the cloud is $A \cdot R$, where $R$ is the stopping distance for the fast particles. For this case the rate of momentum transfer to unit volume of the ambient gas is

$$\frac{J}{4\pi r^{2}} \frac{(2m_{p}E_{0})^{1/2}}{R}.$$ 

(16)
The gravitational force on unit volume of the gas due to the central source is

\[ \frac{G N_H m_H M_s}{2 r^2} \]

where \( M_s \) is the mass of the central object. Equating this force to the momentum transfer rate gives us

\[ \frac{J}{4\pi R} (2m_p E_o)^{1/4} = G N_H m_H M_s \]  \hspace{1cm} (17)

So equilibrium obtains when

\[ M_s = \frac{J}{4\pi} \left( \frac{2m_p E_o}{G m_H} \right)^{1/4} \left( \frac{1}{R N_H} \right) \]  \hspace{1cm} (18)

The factor \( R N_H \) has previously been calculated for suprathermal protons in hydrogen gas (Ptak and Stoner 1973, figure 6), and it depends on \( E_o \) and the ionized fraction of the gas \( f_i \).

A few solar masses of protons per year are required to account for the observed H\( \alpha \) luminosity (the optical reverberation mechanism could reduce this); this makes \( J \lesssim 10^{50} \text{ sec}^{-1} \). If we assume that the initial energy of the protons is 1MeV, and take the gas to be 10% ionized, then \( R N_H \sim 10^{21} \text{ cm}^{-2} \). Inserting these values in equation (18) we obtain

\[ M_s \lesssim 10^{44} \text{ gm} \sim 5 \times 10^{10} M_{\odot} \]
This is quite a large mass, but it is of the order of magnitude of a spinar (Morrison and Cavaliere 1970). We should note that this quantity depends more or less linearly on $f_I$, and that it goes like $E_o^{-3/2}$. Furthermore, we have considered the most extreme case where the cloud of gas is just thick enough to stop all the particles. For the large ambient gas densities we have discussed, the value of $R$ is quite small, and we should expect a typical gas cloud to be one or more orders of magnitude thicker than $R$. The value of $M_s$ required by the equilibrium condition would then be reduced by one or more orders of magnitude.

IV. Radiation From a Suprathermal Atom

A photon with frequency near the center of a Lyman or Balmer line will be repeatedly scattered by the ambient hydrogen atoms with small frequency changes due to the thermal motions of the atoms (which we assume are described by the temperature $T_e$). This scattering will both produce a distribution in frequency near the line centers (see Avery and House, 1968 and Auer 1968) and establish a quasithermal equilibrium between the distributions over the lower levels of the ambient atoms and the radiation field. As an approximation, we assume that this equilibrium is established over the frequency range in which the gas is optically thick ($\tau > 1$) and that a photon scattered outside this frequency range always escapes. The frequency range in which the radiation field rises to the level of black body radiation then lies $\Delta \nu$ on either side of the line center, where
\[ \frac{\Delta \nu}{U} = \left[ 2 \ln \left\{ \frac{(1 - f)}{8 \pi^3/2 U} \right\} - \frac{\varepsilon}{k_B T_a} \right]^{1/2} \]  

(19)

where \( U \) is the thermal doppler width in frequency =

\[ \frac{\nu_0 \sqrt{2kT_e}}{c \sqrt{m_H}}, \]

\( N_H L \) is the hydrogen column density, \( A \) is the appropriate rate constant for absorption of the line, \( \varepsilon \) is the energy of the lowest level for the transition relative to the ground state, and \( \lambda \) is the wavelength of the photon.

The suprathermal proton travelling through the ambient gas will spend a fraction of its time as a suprathermal atom, which can then interact by resonant absorption with the radiation field described above. In the rest frame of the suprathermal atom, the radiation field will not appear isotropic; only those photons travelling in directions approximately perpendicular to the suprathermal atom's velocity will be available for resonant scattering. Thus, while an ambient hydrogen atom "sees" isotropic line radiation at the black body level, only a fraction \( \Delta \Omega/4\pi \) of the full solid angle of this radiation is available for resonant scattering with the suprathermal atom, and, at speed \( v \), this is

\[ \frac{\Delta \Omega}{4\pi} = \frac{c}{v} \frac{\Delta \nu}{\nu_0} \]  

or unity when \( v \) is less than \( c\Delta \nu/\nu_0 \). At a speed of \( \alpha c = 1 \) atomic unit (2188 km/sec), temperatures of about \( 10^{40} \) K, an ambient density
of $10^8 \text{ cm}^{-3}$, and for an ambient gas cloud $10^{15} \text{ cm}$ thick, $\Delta \Omega/4\pi$ is about 0.02 for the first few lines of the Lyman and Balmer series.

At reasonably high temperatures and reasonably low densities, the fast atom will absorb and re-emit many line photons between atomic collisions; in this low density regime, the distribution over atomic states of the suprathermal atom will be determined by the size of $\Delta \Omega/4\pi$ and by the radiation temperature. At higher ambient gas densities certain inelastic collisions with the electrons, protons and atoms of the ambient gas begin to play a significant role.

The rate at which the suprathermal atoms emit photons in a given line is the product of the probability the atom is in the appropriate upper state and an emission rate coefficient; thus, a calculation of the distribution over atomic states will yield predictions for the relative intensities of the broad components of the various hydrogen emission lines. We have made such a calculation using a truncated set of eight hydrogen atomic states: $1s$, $2s$, $2p$, $3s$, $3p$, $3d$, all $n = 4$ states combined, and the ionized state. The relative probability the suprathermal hydrogen exists in each of these states is determined by equating the rates for populating and depopulating each one by the several radiative and collisional interactions at work.

The radiative interactions include resonant absorption of Ly$\alpha$, Ly$\beta$, Ly$\gamma$, H$\alpha$ and H$\beta$ from the radiation field as well as spontaneous emission of the same quanta. Collisional processes included are the excitations from the ground state, collisional mixing of the
2s and 2p states, population of all bound states by charge transfer from ambient atoms to suprathermal protons, and ionization from the ground state. Rates of other processes that could lead to transitions among these states, such as stimulated emission, were estimated and found negligible. Free parameters in the calculation were the radiation temperature, the ambient hydrogen number density, the ambient ionization fraction, and the value of $\Delta \Omega/4\pi$ at $v = \alpha c$. The velocity-dependent cross sections used are generally those used for the previous computation of emission profiles (Ptak and Stoner 1973) and the collisional 2s-to-2p cross sections of Seaton (1955).

Table 2 gives some typical results of the calculation described above. The table presents quantities analogous to the "b's" commonly used to describe the distribution over states of hydrogen atoms in planetary nebulae. The quantity $b(nl)$ is the ratio of the probability a suprathermal atom is in the state $(nl)$ to the probability an ambient gas atom (in thermal equilibrium with a radiation field of temperature $T_a$) is in the same state. If the suprathermal atom could interact resonantly with all the line photons rather than just those with momenta in the fractional solid angle $\Delta \Omega/4\pi$, and if inelastic collisional processes were unimportant, then all of the $b(nl)$ would be unity.

Finally, Table 3 gives some typical results for the production of line photons by the suprathermal protons, obtained by integrating the production rate over the deceleration of the suprathermal proton from 6500 km/sec to about 500 km/sec.
An important result is that both the Lyman decrement and the ratio of the emission in Balmer lines to the emission in Lyman lines depend strongly on the physical conditions in the ambient gas. This happens primarily because of the changing relative importance of atomic collisions and optical reverberation in this range of temperature and density. The importance of optical reverberation is increased by either increasing the temperature (and thus the photon density in the radiation field) or decreasing the ambient gas density (decreasing the rate of atomic collisions).

When optical reverberation dominates, the line emission rate per suprathermal is inversely proportional to density. At densities of a few times $10^8 \text{ cm}^{-3}$ and at $T_a = 10^4 \text{ K}$, a single suprathermal is capable of scattering 1 MeV of line radiation into the wings, so that this mechanism can easily remove all of the heat energy delivered to the gas by the suprathermals. This suggests the possibility that the temperature and density of the gas may adjust itself at $\sim 10^4 \text{ K}$ and $\sim 10^9 \text{ cm}^{-3}$ to balance the heating and cooling effects of the suprathermals.

In the limit of high density and low temperature, the emission line profiles will be the same as previously reported (Ptak and Stoner 1973) since the atomic collisions will the dominate the emission processes. As optical reverberation becomes important, the full width at zero intensity will remain the same since the characteristic speed at which the suprathermal proton spends a significant fraction of its time as a hydrogen atom will not change; the details of the emission profiles may change, however.
V. Discussion

The picture that emerges is one in which a small volume of relatively high density gas is involved in the region where the broad emission lines originate. On the other hand, the ratio of the density of suprathermal particles to the density of gas particles is small; the upper limit is about $10^{-6}$. We find that the minimum amount of ambient gas which is required is moderate, about $100 M_\odot$ for an ambient gas density of $10^{10} \text{cm}^{-3}$.

Under these conditions, we can construct a self-consistent picture in which the temperature and ionization of the ambient gas are determined by the flux of suprathermal protons, and where the hydrogen emission excited in the gas is trapped and degraded before it can escape. By finding reasonable physical conditions in a nuclear gas, consistent with the presence of suprathermal particles and with the removal of the "narrow component" of Balmer radiation, we have removed two major objections to the suprathermal ion hypothesis for the origin of the broad line emission. The physical conditions of the gas will depend somewhat on the mechanism which converts the kinetic energy of the suprathermals into radiation. Detailed self-consistent calculations have been performed by Kimmer (1974) in two limiting cases: 1) the Lyα photons excited in the gas are immediately destroyed, 2) the energy escapes from the gas only by way of the two-photon decay of the hydrogen 2s state. His results are qualitatively the same as those presented in Table 1, for which we have assumed that the gas and the suprathermal protons are everywhere the same.
The first case provides us with a lower limit for the temperature and ionization of the gas, and the second case gives us an upper limit for the electron temperature and ionization fraction. Intermediate cases could be provided by varying amounts of dust in the gas absorbing Lyα photons and reradiating the energy in the infrared.

Another possible cooling mechanism which lies between the two extreme cases is the optical reverberation from suprathermal hydrogen atoms. We have not yet calculated the physical conditions of the gas in a self-consistent way with this process dominating the cooling of the gas. This does not seem crucial since the two limiting cases do not produce radically different results. What we have calculated are the population of states for the suprathermal atoms as a function of the temperature and density of the ambient gas.

With these population ratios available, it is a simple matter to calculate the intensity ratios for the broad components of the hydrogen emission lines. The calculations we have done so far determine only the Lyman decrement reliably (since the angular momentum sublevels of $n = 4$ are not treated separately), and these results appear in Tables 2 and 3. This intensity ratio has not yet been measured for any Seyfert galaxy, of course. However, we can see from the table that the calculated ratio is very sensitive to the density and temperature of the ambient gas, and we can expect that the Balmer decrement calculated in this way will also be a sensitive indicator of the physical conditions in the gas. We are
presently working to extend the calculation in this direction.

The dynamic picture is still rather unclear. We have seen that a kind of force balance is possible if a very massive central object binds the ambient gas gravitationally. How massive this object must be depends on the geometry and on the sorts of restrictions one wishes to place on the size of the emitting region. Geometrical factors have been suppressed in the model presented in this paper. However, it is possible to make some statements of this type based on our calculations.

We found that for an ambient gas density of about $10^{10} \text{cm}^{-3}$ and an ionized fraction of 0.1, the required volume of gas is about $10^{49} \text{cm}^3$. So if the emitting region is to be confined to within a sphere of radius one light year, the minimum value of the filling factor is about $2 \times 10^{-6}$. This small value implies that even with a small size for the emission region, a good deal more ambient gas may be present than is needed, or, in other words, the gas clouds may be considerably thicker than the stopping distance of the suprathermal particles. The necessary mass for a central object to produce some quasi-static equilibrium is then correspondingly reduced.

Another way of viewing the problem is to ignore gravity and to consider only inertia. With the conditions described above, the minimum value of ambient gas in the emission region is about $100 M_\odot$. If this is all the gas present, and if about $1 M_\odot$ of fast protons with parallel initial velocities stop in the gas each year, then the system is dynamically unstable with a time scale on
the order of 100 years. However, if there is much more gas present than the minimum required, then the time scale is correspondingly increased.

An important feature of the self-consistent calculation is that when the relative density is not so large as to make the gas completely ionized, then for a wide range of conditions, the ambient gas is quite neutral; the ionized fraction is like 0.1 or less. This means that when the atomic collision mechanism is able to operate, it does so at close to the best efficiency possible. It also means that a moderate amount of gas can have a large optical thickness in Lyman and Balmer lines, and thus provide the necessary trapping of the line photons excited in the ambient gas.

We are in the process of calculating detailed line profiles and the Balmer decrement for the case where optical reverberation dominates the emission from the suprathermal particles. The results of this calculation will be presented in a subsequent paper.
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Table 1

Temperature and Ionization of the Ambient Gas

<table>
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<th>$N_H$ (cm$^{-3}$)</th>
<th>$N_S$ (cm$^{-3}$)</th>
<th>$T_e$ ($10^3$ K)</th>
<th>$f_i$</th>
<th>$T_e$ ($10^3$ K)</th>
<th>$T_a$ ($10^3$ K)</th>
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<td>$10^3$</td>
<td>10</td>
<td></td>
<td>NO SOLUTION</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^9$</td>
<td>10</td>
<td>8.4</td>
<td>7.6</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{10}$</td>
<td>$10^2$</td>
<td>8.4</td>
<td>9.1</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{11}$</td>
<td>$10^3$</td>
<td>8.4</td>
<td>9.1</td>
<td>0.03</td>
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<td></td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>$10^4$</td>
<td>12.3</td>
<td>12.3</td>
<td>0.14</td>
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<td></td>
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<tr>
<td>$10^{10}$</td>
<td>10</td>
<td>7.8</td>
<td>7.5</td>
<td>0.008</td>
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<td></td>
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<tr>
<td>$10^{11}$</td>
<td>$10^2$</td>
<td>7.7</td>
<td>0.006</td>
<td>8.8</td>
<td>8.8</td>
<td>0.01</td>
</tr>
<tr>
<td>$10^{12}$</td>
<td>$10^3$</td>
<td>10.3</td>
<td>10.3</td>
<td>0.02</td>
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</table>
The distribution over the lowest few excited states of a suprathermal hydrogen atom with velocity $v = \omega c$ in an optically thick hydrogen gas. In all cases the gas is 5 percent ionized and the quantity $A\Omega/4\pi = 0.02$. See text for the definition of the $b(nl)$.

<table>
<thead>
<tr>
<th>$T$ Ambient Temperature (K)</th>
<th>$N_H$ Ambient Gas Density ($cm^{-3}$)</th>
<th>$b(2s)$</th>
<th>$b(2p)$</th>
<th>$b(3s)$</th>
<th>$b(3p)$</th>
<th>$b(3d)$</th>
<th>$b(4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^4$</td>
<td>$10^8$</td>
<td>1.06</td>
<td>0.020</td>
<td>0.014</td>
<td>0.021</td>
<td>0.0019</td>
<td>0.010</td>
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<tr>
<td>$10^4$</td>
<td>$10^9$</td>
<td>2.40</td>
<td>0.020</td>
<td>0.081</td>
<td>0.025</td>
<td>0.0029</td>
<td>0.014</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$10^{10}$</td>
<td>15.0</td>
<td>0.023</td>
<td>0.745</td>
<td>0.067</td>
<td>0.013</td>
<td>0.052</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$10^{11}$</td>
<td>122.0</td>
<td>0.0492</td>
<td>7.37</td>
<td>0.441</td>
<td>0.111</td>
<td>0.406</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$10^{12}$</td>
<td>501.0</td>
<td>0.365</td>
<td>73.0</td>
<td>2.55</td>
<td>0.980</td>
<td>3.14</td>
</tr>
<tr>
<td>$0.8 \times 10^4$</td>
<td>$10^{10}$</td>
<td>480.0</td>
<td>0.073</td>
<td>24.6</td>
<td>1.59</td>
<td>0.364</td>
<td>1.59</td>
</tr>
<tr>
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<td>$10^{10}$</td>
<td>2.24</td>
<td>0.020</td>
<td>0.079</td>
<td>0.025</td>
<td>0.003</td>
<td>0.013</td>
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<tr>
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<td>$10^4$</td>
<td>.921</td>
<td>0.020</td>
<td>0.0068</td>
<td>0.020</td>
<td>0.0017</td>
<td>0.0097</td>
</tr>
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</table>
Table 3

The total line radiation from a suprathermal hydrogen stopping in an optically thick hydrogen gas under different physical conditions. In all cases the quantity in equation (20) is taken to be 0.02 for \( v = v_c \).

<table>
<thead>
<tr>
<th>Temperature ( T_a ^{(\text{oK})} )</th>
<th>Ambient Gas Density ( N_H \text{(cm}^{-3}\text{)} )</th>
<th>Fractional Ionization of the Ambient Gas</th>
<th>( L_\alpha \text{ Photons Proton} )</th>
<th>( L_\beta \text{ Photons Proton} )</th>
<th>( H_\alpha \text{ Photons Proton} )</th>
<th>Luminosity Ratio ( \frac{L_\beta}{L_\alpha} )</th>
<th>Luminosity Ratio ( \frac{H_\alpha}{L_\alpha} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>( 10^8 )</td>
<td>0.1</td>
<td>( 16.3 \times 10^4 )</td>
<td>4950.0</td>
<td>1040.0</td>
<td>0.036</td>
<td>0.0012</td>
</tr>
<tr>
<td>10,000</td>
<td>( 10^9 )</td>
<td>0.1</td>
<td>( 16.6 \times 10^3 )</td>
<td>639.0</td>
<td>169.0</td>
<td>0.046</td>
<td>0.0019</td>
</tr>
<tr>
<td>10,000</td>
<td>( 10^9 )</td>
<td>0.5</td>
<td>586.0</td>
<td>35.0</td>
<td>11.8</td>
<td>0.071</td>
<td>0.0037</td>
</tr>
<tr>
<td>10,000</td>
<td>( 10^{11} )</td>
<td>0.1</td>
<td>525.0</td>
<td>134.0</td>
<td>68.4</td>
<td>0.303</td>
<td>0.0241</td>
</tr>
<tr>
<td>10,000</td>
<td>( 10^{12} )</td>
<td>0.1</td>
<td>440.0</td>
<td>72.5</td>
<td>57.0</td>
<td>0.195</td>
<td>0.0240</td>
</tr>
<tr>
<td>9,000</td>
<td>( 10^9 )</td>
<td>0.1</td>
<td>4,700.0</td>
<td>262.0</td>
<td>92.7</td>
<td>0.066</td>
<td>0.0036</td>
</tr>
<tr>
<td>9,000</td>
<td>( 10^9 )</td>
<td>0.5</td>
<td>192.0</td>
<td>22.6</td>
<td>9.25</td>
<td>0.140</td>
<td>0.0089</td>
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<tr>
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<td>( 10^9 )</td>
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<td>1,170.0</td>
<td>177.0</td>
<td>75.4</td>
<td>0.179</td>
<td>0.0119</td>
</tr>
<tr>
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<td>( 10^9 )</td>
<td>0.5</td>
<td>75.8</td>
<td>19.9</td>
<td>8.66</td>
<td>0.311</td>
<td>0.0211</td>
</tr>
<tr>
<td>7,000</td>
<td>( 10^9 )</td>
<td>0.1</td>
<td>429.0</td>
<td>165.0</td>
<td>72.5</td>
<td>0.456</td>
<td>0.0313</td>
</tr>
<tr>
<td>7,000</td>
<td>( 10^9 )</td>
<td>0.5</td>
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<td>19.4</td>
<td>8.51</td>
<td>0.446</td>
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</tr>
<tr>
<td>10,000</td>
<td>( 10^9 )</td>
<td>0.05</td>
<td>53,200.0</td>
<td>1880.0</td>
<td>465.0</td>
<td>0.042</td>
<td>0.0016</td>
</tr>
<tr>
<td>10,000</td>
<td>( 10^{10} )</td>
<td>0.05</td>
<td>5,890.0</td>
<td>481.0</td>
<td>182.0</td>
<td>0.097</td>
<td>0.0057</td>
</tr>
<tr>
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<td>307.0</td>
<td>148.0</td>
<td>0.305</td>
<td>0.0230</td>
</tr>
</tbody>
</table>
REFERENCES