The Electromagnetic Interchange Mode in a Partially Ionized Collisional Plasma

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Abstract

A collisional electromagnetic dispersion relation is derived from two-fluid theory for the interchange mode coupled to the Alfvén, acoustic, drift and entropy modes in a partially ionized plasma. The fundamental electromagnetic nature of the interchange mode is noted; coupling to the intermediate Alfvén mode is strongly stabilizing for finite $k_z$. Both ion viscous and ion-neutral stabilization are included, and it is found that collisions destroy the FLR cutoff at short perpendicular wavelengths.
Introduction

This paper is motivated by a study of the instability of the nighttime equatorial F region called spread F (Hudson 1974; Hudson and Kennel 1974a). We develop a set of plasma equations to suit equatorial F region parameters, which are also applicable to Q machines and other laboratory plasmas. Although Coulomb collisions predominate in the F region, neutral collisions may determine the altitude threshold for spread F onset (Hudson 1974; Hudson and Kennel 1974a), so both have been included in our analysis. Collisonal fluid equations are used (Braginskii 1965), which are appropriate for parallel wavelengths exceeding the total electron mean free path and perpendicular wavelengths greater than the ion Larmor radius. Equatorial localization permits even longer parallel wavelengths, so both finite electron heat conduction along the magnetic field (Tsai et al. 1970) and energy transfer between species must be included. Since this is the first of two papers on the low frequency, density gradient driven modes of a partially ionized collisional plasma, the full dispersion relation for all such modes (interchange, entropy, and drift) will be derived and the modes will be decoupled here. In the second part of this paper we will be concerned with the structure of the interchange mode: its electromagnetic corrections, ion finite Larmor radius (FLR) stabilization (Rosenbluth, et al. 1962) and the relative effect of neutral and coulomb collisional damping.

In order to treat the interchange mode properly, it is necessary to derive an electromagnetic dispersion relation. The interchange mode
is often derived in the electrostatic approximation neglecting perturbations along the magnetic field ($k_z = 0$) (cf. Rosenbluth et al., 1962). While this procedure gives the correct dispersion relation at $k_z = 0$, it conceals the fundamental electromagnetic nature of the interchange mode. The interchange mode appears in lowest order in the ion FLR parameter $b$ in the small $k_z C_A/\omega$ limit of the full electromagnetic dispersion relation; electrostatic modes such as the drift which require finite $k_z$ are obtained in the large $k_z C_A/\omega$ limit, where $C_A$ is the Alfvén speed. It is incorrect to solve for the finite parallel wavelength ($k_z$) corrections to the interchange mode from the electrostatic dispersion relation, since the dominant stabilizing term comes from coupling to the electromagnetic Alfvén mode.

The collisionless interchange or Rayleigh-Taylor mode destabilized by gravity antiparallel to a density gradient and perpendicular to the magnetic field was first suggested by Dungey (1956) as a source of equatorial F region irregularities. Haerendel (Balsley et al., 1972; Haerendel 1974) included neutral collisions but neglected Coulomb collisions in the Rayleigh-Taylor mode, so his analysis is restricted to lower altitudes than typical spread F observations at finite perpendicular wavelengths (Hudson and Kennel 1974a). Both Dungey and Haerendel neglected FLR stabilization, which Rosenbluth et al. (1962) have shown to be important at short perpendicular wavelengths.

The purpose of the second part of this paper which deals exclusively with the interchange mode is to extend the previous work in slab geometry to higher altitudes and answer the following questions. What is the effect of Coulomb collisions on the Rayleigh-Taylor growth rate, and how does it depend on plasma density? This is compared with
the growth rate dependence on neutral collisions and neutral density (Balsley et al. 1972; Haerendel 1974). What is the shortest perpendicular wavelength above the FLR cutoff, and how do collisions affect this FLR cutoff? This is compared with the observation that spread F perpendicular wavelengths can extend below the ion Larmor radius. We will find that collisional particle diffusion drifts oppose the collisionless FLR drift of Rosenbluth et al. (1962), and can extend the unstable perpendicular wavelength range down to the ion Larmor radius where the approximations break down. However, the effect of both neutral and Coulomb collisions at long perpendicular wavelengths is to reduce the maximum growth rate of the interchange mode.

Assumptions and Basic Equations

We assume that a partially ionized slab plasma is immersed in a uniform z-directed magnetic field. There is a constant vertical density gradient in the positive x-direction and gravitational acceleration g in the negative x-direction. We neglect particle sources and sinks, zero order drifts along x and z, static electric fields and zero order temperature gradients.

We will restrict our analysis to perpendicular wavelengths greater than the ion Larmor radius and parallel wavelengths greater than the electron mean free path for momentum transfer to ions and neutrals $\lambda_e = a_e/(v_{ei} + v_{en})$, which depends on the electron thermal speed $a_e$ and the sum of the electron-ion and electron-neutral collision frequencies $v_{ei}$ and $v_{en}$ defined in Table 1.
The following set of fluid equations then applies to the jth species, electrons or ions for \( j = e \) or \( i \)

\[ \partial n_j / \partial t + \nabla \cdot (n_j \mathbf{v}_j) = 0 \]  

(1)

\[ m_j n_j (d \mathbf{v}_j / dt) = - \nabla (n_j T_j) - \nabla \cdot \mathbf{\Pi}_j + n_j e [E + (\mathbf{v}_j/c) \times \mathbf{B}] - C_n n_e \nabla T_e + R_{ei} + R_{jn} \]  

(2)

\[ (3/2) n_j (dT_j / dt) + n_j T_j \nabla \cdot \mathbf{v}_j = - \nabla \mathbf{q}_j + \mathbf{Q}_j \]  

(3)

\[ R_{ei} = -R_{ie} = -C_r m_e n_e \nabla \cdot \mathbf{v}_{ei} (\mathbf{v}_e - \mathbf{v}_i) \]  

(4a)

\[ R_{en} = C_r m_e n_e \nabla \cdot \mathbf{v}_{en} \]  

(4b)

\[ R_{in} = m_i n_i \nabla \cdot \mathbf{v}_i \]  

(4c)

\[ m_e = m, \quad m_i = M \]

Equations (1) - (3) are the momentum, continuity, and heat flow equations written in the neutral rest frame. Neutral dynamics are neglected for oscillation frequencies satisfying \( \omega > > v_{in} n_i / n_n \). However momentum and energy loss to the neutral sink are included.

The transport coefficients for a fully ionized plasma including ion dynamics in an arbitrary magnetic field were computed by Braginskii (1965). His resistive, thermo-electric and electron thermal conductivity coefficients along the magnetic field are \( C_r = 0.51 \), \( C_t = 0.71 \) and \( C_x = 3.16 \) respectively. Shkarofsky (1961) has tabulated them as functions of \( v_{en} / v_{ei} \) for a partially ionized plasma neglecting ion dynamics, hence ion drift with respect to neutrals, and Schunk and Walker (1970) have plotted them. These exact numerical coefficients enable us to write
the electron equations (1 - 3) in the identical form to the fully ionized case replacing $v_{ei}$ by the total electron collision frequency $v_e = v_{ei} + v_{en}$.

For example in Equation (2) we have for $V_i = 0$

$$R_{ei} + R_{en} = -C \frac{m_n}{m_e} v_e v_e v_e$$

where $C$ is a function of $v_{en}/v_{ei}$.

Approximations in ion equations

In the limit $|k C_s/\omega_D| << 1$, where $C_s^2 = T_e/M$ is the ion acoustic speed and $|\omega_D| = k T_e C_s^2/\Omega_i$ is the diamagnetic drift frequency, ions primarily move perpendicular to $B$, undergoing a shear rather than a compressional motion (Tsai et al. 1970). As a result, ion temperature fluctuations are scaled down from electron temperature fluctuations by the ion FLR factor $b = k^2 C_s^2/\Omega_i^2 << 1$. We have independently checked that including ion temperature fluctuations does not significantly affect the interchange mode (Hudson 1974). Therefore, we will neglect them here, thus eliminating the ion heat transfer equation from the set (1 - 3).

We retain parallel ion pressure, finite ion inertia, FLR effects and viscosity. We use the fully ionized ion viscosity tensor (Braginskii 1965), since momentum exchange between ions and neutrals is treated separately in (4c). Our separate treatment of the two collision processes makes the reasonable assumption that an ion-neutral collision transfers the total ion momentum to the neutrals, while Coulomb collisions are a diffusion process in a spatial gradient. This is a good approximation for charge exchange collisions, and hard sphere collisions when ion and neutral masses are comparable, since the momentum exchange rate then scales as $M_i/M_n \sim 1$. 
The only contributing terms in the ion stress tensor to order $b = k^2 C_i^2 / \Omega_i^2$ are (Shkarofsky et al. 1963)

\[
\nabla \cdot \tau = -1/(2 \Omega_i) \left\{ \left[ p_i \nabla^2 + \left( \nabla p_i \right)_\perp \cdot \nabla \right] (v \times \hat{z}) + (\hat{\omega} \times \nabla p_i) \cdot \nabla v \right\} \\
- 3/10 \left\{ \left[ (p_i \nu_{ii} / \Omega_i^2 \right) \nabla^2 + \left( \nabla p_i \nu_{ii} / \Omega_i^2 \right) \cdot \nabla \right] v \right\}_\perp \\
- \left[ \hat{\omega} \times \nabla \left( p_i \nu_{ii} / \Omega_i^2 \right) \cdot \nabla (v \times \hat{z}) \right] \\
- 1/3 \nabla \left[ \left( p_i / \nu_{ii} \right) \nabla \cdot v \right] \right\}_\perp
\]

(5)

The collisionless stress term is the ion FLR effect. The first collision term corresponds to shear stress due to ion viscosity, and the second corresponds to compressional stress from the collisional relaxation of $n - n$ differences (Stix 1969). The compressional term is of order $b^3$ in the ion continuity equation (Stix 1969) and will be dropped along with all other terms higher order than $b^2$. Parallel ion viscosity like parallel ion heat conduction, which has been included in the collisional electrostatic drift-acoustic dispersion relation of Coppi and Mazzucato (1971), only affects short parallel wavelength modes $k_z \lambda_e \sim (m/M)^{1/3}$ and will be neglected here. The remaining ion viscosity term then determines the coefficient of perpendicular ion-ion collisional momentum transfer. We will see that this factor is reduced from unity, the coefficient of ion neutral collisional momentum transfer in (4c). In (5) we have neglected collisionless FLR terms of order $b^2 \omega$ (Kennel and Greene 1966) and retained those of order $b^2 \nu_{ii}$ for application to low frequency oscillations $\omega / \nu_{ii} \ll 1$. 
Approximations in electron equations

We neglect electron inertia at frequencies low compared to the electron plasma frequency $\omega < \omega_{pe}$. The perpendicular electron motion can be treated in the guiding center approximation for the modes of interest ($k_z/k_e << 1$). We therefore neglect the perpendicular pressure, diamagnetic drift and off diagonal heat flow terms in the electron equations (1-3). It can be shown that including these does not alter the final result since all additional terms cancel in the perturbed equations.

Electron thermal conductivity along the magnetic field greatly exceeds ion thermal conductivity in general; hence electron temperature fluctuations have been included along with parallel electron pressure.

The parallel electron heat flux is

$$q_e = C_t n_e T_e \varepsilon_{ez} - C_x \left( n_e T_e/m_e \nu_e \right) \nu T_e$$

Again $C_t$ and $C_x$ are functions of $\nu_{ei}/\nu_{en}$ (Shkarofsky 1961) defined so as to write the electron equations (1-3) in the fully ionized form, replacing $\nu_{ei}$ by $\nu_e = \nu_{ei} + \nu_{en}$. The collisional energy transfer from electrons to ions and neutrals is given by

$$Q_e = -3(m/M) n_e \nu_e (T_e - T_i) \quad T_n = T_i$$

Here we assume that ion and neutral masses and temperatures are equal, and that all electron energy lost to the ions is subsequently lost by ion neutral collisions to the neutral sink. This assumption is valid for $\nu_{in}/\nu_{ii} > 2 \sqrt{m/M}$ (Hudson 1974). For $\nu_{in}/\nu_{ii} < 2 \sqrt{m/M}$ the ions prefer to give their energy back to the electrons via Coulomb collisions.

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Q_e becomes important when parallel wavelengths are comparable to the energy transfer mean free path from electrons to ions and neutrals, a factor of $\sqrt{M/m}$ longer than the momentum transfer mean free path $\lambda_e$. Such wavelengths are typically long compared to laboratory plasmas, so this term has previously been neglected in the derivation of the following modes (Tsai et al. 1970). However, the possibility of very long parallel wavelengths exists in the ionosphere, so the energy transfer term has been retained.

**Equilibrium**

Perkins (1973) has examined the F region equilibrium, where the Hall conductivity is negligible, and Coulomb collisions make no contribution to the Pederson conductivity. In the frame where neutral winds vanish, the ion momentum equation (2) yields

$$V_{\|i} = \frac{c_E \times \hat{z}}{B} - \frac{c_{T_i}}{eB} \frac{v_n \times \hat{z}}{\Omega} + \frac{v_{in}}{\Omega} \left( \frac{c_E}{B} - \frac{c_{T_i}}{eB} \frac{v_n}{n} + \frac{g}{\Omega} \right) + 0 \left( \frac{v_{in}^2}{\Omega^2} \right)$$

(7a)

Since $v_{in}/\Omega << 1$ in the F region, higher order terms are dropped. The electron momentum equation (2) yields

$$V_{\|e} = \frac{c_E \times \hat{z}}{B}$$

Perkins (1973) finds that only the East-West ($\hat{y}$) component of the current $j_\parallel = ne (V_{\|i} - V_{\|e})$ contributes to the equilibrium. Since the primary electric field in the nighttime F layer is vertical due to polarization (Rishbeth 1970), the only contribution to the equilibrium comes
from
\[
\hat{j}_y = -\frac{cT_1}{B} \nabla n \times \hat{z} + ne \frac{p \times \hat{z}}{\Omega} \tag{7c}
\]
which is independent of electric fields and neutral collisions. This is just the current due to the density gradient drift and gravitational guiding center drift. We will neglect electric fields in the perturbed equations which introduce the \( E \times B \) instability (Simon 1962) to be treated in a separate paper in the finite heat conduction limit.

**Perturbed equations**

We linearize the above set of fluid equations assuming the fluctuations are low frequency oscillations of the form \( e^{i(k \cdot \mathbf{r} - \omega t)} \). For \( k \cdot L ? 1 \) the \( x- \) dependence of the perturbation is weak and a good estimate of the eigenfrequency is obtained by setting \( k_x = 0 \) (Krall 1968).

The parallel ion momentum transfer equation yields
\[
\tilde{v}_{yi} = \frac{i e}{\omega M_i} \left[ \left( 1 + \frac{k_y V_{yi}}{\omega} \right) \tilde{E}_z - \frac{k_z V_{yi}}{\omega} \tilde{E}_y \right] + R_T \frac{k_z C_s^2}{\omega} \tilde{n}_i/n \tag{8a}
\]
We solve the perpendicular ion momentum transfer equation iteratively for low oscillation frequencies and low ion-ion and ion-neutral collision frequencies \( v_{li} \) and \( v_{in} \) defined in Table 1 assuming \( \omega/\Omega_i \), \( v_{li}/\Omega_i \) and \( v_{in}/\Omega_i \sim b < \Omega_i \) appropriate for the F-region where ions are magnetized. We obtain
\[
\tilde{v}_{yi} = -i(\omega/\Omega_i) \tilde{v}_{xi} - g/\Omega_i \tilde{n}_i/n \tag{8b}
\]
\[
\tilde{v}_{xi} = c(T_e/B) - i(k_y C_s^2/\Omega_i) R_T(\tilde{n}_i/n) - \frac{1}{2} \frac{\Omega_i}{\Omega_e} \left( i e/k_y T_e \right) \tilde{E}_y + R_T(\tilde{n}_i/n) \tag{8c}
\]
where \[ \bar{w} = w + i v_L / b \]

\[ v_L / b = v_{in} + 0.3b v_i R_T, \quad R_T = T_i / T_e \]

\( v / b \) is the frequency which scales the perpendicular ion diffusion rate. \( v_{in} \) and \( v_i \) always appear in this set of equations in this combination. Ion-ion collisions only affect transport in the presence of a spatial gradient, where ions in a denser region encounter more collisions with other ions and diffuse toward less dense regions. This is in contrast to a homogeneous plasma, where the center of mass remains stationary in an ion-ion collision and there is effectively no diffusion. It further contrasts with ion neutral collisions where we assume that the total ion momentum is lost. Even if only a fraction of the ion momentum is lost in a single ion-neutral collision, the ions will random-walk and diffuse through the neutrals, which act as a momentum sink.

For \( k_L >> 1 \) the important scale length is the perpendicular wavelength. The different \( b \) dependences of \( v_{ii} \) and \( v_{in} \) in \( v \) suggest that the two collision processes will be significant over different perpendicular wavelength ranges. We will subsequently find that ion-neutral collisions damp the drift mode at long perpendicular wavelengths. Ion viscosity will also be found to increase the collisional damping of the interchange mode at short perpendicular wavelengths.

Substituting \( \bar{v}_{xi} \), \( \bar{v}_{yi} \), and \( \bar{v}_{zi} \) into the perturbed ion continuity equation, assuming \( k_L L >> 1 \) and dropping terms of higher order than \( b^2 \) eliminates the collisional terms in (8c) and yields the following relation
between ion density and electric field fluctuations

\[
\left(\frac{n_i}{n}\right)\left[\frac{\omega + R \bar{b} \bar{w} - k_y g/\Omega - R k_z c_s^2/\omega}{\Omega - R^2 k_z c_s^2/\omega}\right]
\]

\[
= + \left[\frac{e/k Y e}{\omega D - b w - \left(k z c_s^2/\omega^2\right) k_y V_y^\perp} \right] E_y
\]

\[
+ \left(\frac{k_z e}{\omega}\right) \left(1 + k_y V_y^\perp/\omega\right) E_z
\]

(9)

The electron momentum transfer equation yields, assuming \(\omega/\Omega_e\),

\[
\nu_{ei}/\Omega_e \text{ and } \nu_{en}/\Omega_e \ll 1
\]

\[
\tilde{\nu}_{xe} = c E_y / B
\]

(10a)

\[
\tilde{\nu}_{ye} = 0
\]

(10b)

\[
\tilde{\nu}_{ze} = -\frac{1}{k_z} \left[\frac{n_e}{n} + \left(1 + C_t^e \right) \frac{T_e}{T_e^e}\right]

- \frac{e}{T_e} \frac{\nu_{ze}}{k_z} \left(1 + \frac{k_y V_y}{\omega} \right) E_z - \frac{k_z V_y E_y}{\omega}
\]

(10c)

where \(\nu_{||} = k_z a_e^2/2 C_r \nu_e\) and \(\nu_e = \nu_{ei} + \nu_{en}\). \(\nu\) scales inversely with the sum of the collision frequencies. It must be small compared to the electron transit frequency over a parallel wavelength, \(k a_e\), since \(k a_e/\nu_e = k \lambda_e \ll 1\).

By assumption many collisions must occur over a parallel wavelength for this collisional fluid analysis to apply.
We substitute $\bar{v}_{xe}$ and $\bar{v}_{ze}$ into the electron heat flow equation and retain energy transfer from electrons to ions and neutrals. At very low oscillation frequencies $\omega \ll (v_{ei} + v_{en})$ the collision frequencies adjust to the electron density and temperature fluctuations; therefore we must perturb the electron density and temperature dependences of the collision frequencies. From Table 1 $v_{ei} \propto n_e T_e^{-3/2}$, while $v_{en} \propto n_e T_e^{1/2}$ is independent of electron density. The perturbed electron heat flow equation yields

$$\frac{T_e}{T_e^*} = -\left\{ \frac{2}{3} i v_{i \parallel} \left( 1 + C_t \right) \left[ n_e - \frac{n_e}{n} \left( i e/k_z T_e \right) E_{z \parallel} \right] \right\}$$

$$+ i 2 \left( 1 - R_T \right) (m/M) \left( 2 v_{ei} + v_{en} \right) \frac{n_e}{n}$$

$$/ (\omega + i v_{i \parallel} \bar{\chi})$$

(11a)

where

$$\bar{\chi} = \frac{2}{3} \left[ C_r C_x + \left( 1 + C_t \right)^2 \right] + i 2 (m'/M) v_e / v_{i \parallel}$$

$$+ i \left( 1 - R_T \right) (m/M) (v_{en} - 3 v_{ei}) / v_{i \parallel}$$

(11b)

Substituting (10a), (10c) and (11) into the electron continuity equation yields the following relation between electron density and electric field fluctuations
\[
\frac{\alpha}{n} \left[ (w + i\nu_{\parallel}) (w + i\nu_{e}) + \gamma^2 \nu_{\parallel}^2 \right] \\
= i \left( \frac{E_y}{k_y T_e} \right) \left[ w_0 - i (\nu_{\parallel}/w) k_y V_y \right] (w + i\nu_{e}) \\
+ i \left( \frac{E_z}{k_z T_e} \right) \left[ i\nu_{\parallel} (1 + k_y V_y/w) \\
+ (w + i\nu_{e}) + \gamma^2 \nu_{\parallel}^2 \right]
\]

(12a)

where

\[
w_0 = k_y V_{e_y} = -k_y C_s^2/\Omega \eta
\]

(12b)

\[
\gamma = 2/3 \left( 1 + C_t \right)^2 + 2 \left( 1 + C_t \right) \left( 1 - R_t \right) \\
\cdot \frac{(m/M)(2\nu_{e} + \nu_{en})}{\nu_{\parallel}}
\]

(12c)

\[\chi \text{ and } \gamma \text{ are combinations of terms which arise from finite parallel} \]

electron heat conduction which permits electron temperature fluctuations, 
and energy transfer between species. They are discussed further and 
plotted as functions of \( \nu_{en}/\nu_{e} \) by Hudson and Kennel (1974b).

Electromagnetic Dispersion Relation

To the set of five fluid equations we add Faraday's law

\[
\nabla \times E = \frac{1}{c} \frac{\partial B}{\partial t} = 0
\]

(13a)

and Ampere's law, neglecting the displacement current for \( w < w_{pi} \)

\[
\nabla \times B = \frac{4\pi}{c} J
\]

(13b)
Neglecting the displacement current is equivalent to assuming quasi-neutrality in place of Poisson's equation.

Following Mikhailovskii and Rudakov (1963) and Coroniti and Kennel (1970) we will neglect the parallel component of the oscillatory magnetic field. This eliminates the fast, or magnetosonic mode of hydromagnetics, which does not couple significantly to the modes of interest here, with perpendicular phase velocities the order of the diamagnetic drift. However, the intermediate and slow modes are well known to couple to drift and flute modes (Kennel and Greene 1966) so the Alfvén and acoustic terms will be included in our initial set of equations.

For \( B_z = 0 \) and \( k_x = 0 \) the linearized Faraday's law expresses the only nonvanishing component of the oscillatory magnetic field in terms of the nonzero electric field components

\[
B_x = \frac{c}{\omega} \left( k_z E_y - k_y E_z \right) \tag{14}
\]

Substituting (14) into the linearized Ampere's law yields the oscillatory current components

\[
J_y = ne\tilde{\nu}_y i + \tilde{n}_i e\nu_i = \frac{1}{\omega} \left( \frac{c^2}{\mu_0 m} \right) \left( k_z \tilde{E}_y - k_y \tilde{E}_z \right) \tag{15a}
\]

\[
J_y = ne \left( \tilde{\nu}_y i - \tilde{\nu}_{ze} \right) = -\frac{1}{\omega} \left( \frac{c^2}{\mu_0 m} \right) \left( k_z \tilde{E}_y - k_y \tilde{E}_z \right) \tag{15b}
\]
We substitute for $\tilde{v}_z$, $\tilde{v}_y$, $\tilde{n}$ and $\tilde{v}_x$ from (8ab), (9) and (10c), making use of (11a) to eliminate $T_e/T_i$. Equations (15a, b) reduce to a 2x2 matrix (16) in $\tilde{E}_y$ and $\tilde{E}_z$. $C_A^2 = B^2/4\pi n M$ is the Alfvén speed. The determinant of (16) must vanish for a nontrivial solution; the cross term cancels much of the main diagonal, yielding the collisional electromagnetic dispersion relation. This reduces to the fluid limit of the collisionless electromagnetic dispersion relation obtained by Coroniti and Kennel (1970) from kinetic theory. We will now examine the structure of (16) in the large and small $k_z C_A/\omega$ limits.
Electrostatic Limit $k_z^2 C_s^2 / \omega^2 \rightarrow \infty$

In the very low $\beta < m/M$ limit appropriate for ionospheric application where $C_s < a_e < C_A$, we let $k_z^2 C_s^2 / \omega^2 \rightarrow \infty$ in (16), and the Alfvén terms are eliminated from the electromagnetic dispersion relation, which reduces to

$$b \left\{ \left( \omega + R_T \omega_0 \right) \left( \omega + i \nu_\perp / b \right) + g / L_\perp \right\}$$

$$- \left( k_z^2 C_s^2 / \omega \right) \left( \omega + R_T \omega_0 \right)$$

$$= - i \nu_\parallel \left[ \frac{\omega + i \nu_\parallel (\nu - \nu_e)}{\omega + i \nu_\parallel \nu_e} \right]$$

$$\times \left\{ \omega \left[ 1 + \rho \beta \right] - \omega_0 + i \nu_\perp \rho \right\}$$

$$- \left( k_z^2 C_s^2 / \omega \right) \rho_j$$

$$\rho = 1 + R_T$$

(17a)

The second term on the left hand side of (17a) (an electromagnetic correction) is negligible for $\nu \gg k_z^2 C_s^2 / \omega$ or $\omega / \nu_e \gg m/2M$. The rest of (17a) corresponds to the electrostatic limit which can be obtained directly by assuming quasi-neutrality, $\tilde{n}_i = \tilde{n}_e$, and substituting $\tilde{E}_y = - i k \tilde{\omega}$ and $\tilde{E}_z = - i k z \tilde{\varphi}$ in (9) and (12a). As required by the continuity equation (1) this electrostatic result can also be obtained by making the same substitutions in the conductivity matrix $J = \sigma \cdot E$ (16) and setting $\nu \cdot J = 0$, recalling that $k_x = 0$. Neglecting the energy transfer terms in $\tilde{\nu}_y$ and $\tilde{\nu}_z$, neutral collisions, acoustic terms and gravity, this electrostatic dispersion relation (17a) reduces to that obtained by Tsai et al. (1970).

We have written (17a) in a form which separates the interchange mode on the left hand side from the product of entropy and drift mode terms respectively on the right hand side. Note that the interchange mode appears
only as FLR and acoustic corrections in the large $k_z C_A$ limit. Also note that in the large or small $v$ limit where $(\bar{\chi} - \bar{\xi}) - \bar{\chi}$, the entropy mode (Hudson and Kennel, 1974c) is eliminated from (17a) which becomes

$$b\left[\left(\omega + R_T w_D\right)\left(\omega + iv_\perp/b\right) + g/L_\perp\right]$$

$$-\left(k_z^2 C_s^2/\omega\right)\left(\omega + R_T w_D\right)$$

$$= -iv_\parallel\left\{\omega [1 + \rho b] - \omega_D + iv_\perp - \left(k_z^2 C_s^2/\omega\right)\rho\right\} \quad (17b)$$

Neglecting neutral collisions, acoustic terms, and gravity this quadratic isothermal dispersion relation reduces to that obtained by Chu et al. (1969).

Electromagnetic Flute Mode Limit $k_z \to 0$

The limit $k_z \to 0$ requires that $v_\parallel \to 0$ and (16) becomes

$$\left\{\left(\omega + R_T w_D\right)\left(\omega + iv/b\right) + g/L_\perp\right\}$$

$$\left\{(\omega) \left(\omega + iv \bar{\chi}\right) \left(\omega - k_y V_D\right)\right\} = 0 \quad (18)$$

where $v_\parallel \bar{\chi} - 2 (m/M) v_e$ as $v \to 0$.

The first bracket in (18) contains interchange mode terms. The second bracket in (18) contains a zero frequency ion acoustic mode $(\omega)$, a damped entropy mode $(\omega + iv \bar{\chi})$, and a purely oscillatory drift mode $(\omega - k_y V_D)$. These three modes are basically electrostatic, and require finite $k_z$ for growth. In low $\beta << 1$ plasmas the Alfvén speed $C_A >> C_s$, the ion acoustic speed, which is the characteristic electrostatic phase velocity along the magnetic field. Hence these finite $k_z$ electrostatic modes are properly treated in the large $k_z C_A/\omega$ limit of the electromagnetic dispersion relation (16).
Decoupled interchange mode: \( k_z^2 C_A^2 / \omega^2 < 1 \)

In the preceding section we saw that the interchange mode was obtained in the small \( k_z^2 C_A^2 / \omega^2 \) limit of the collisional electromagnetic dispersion relation (16). Retaining terms of order \( k_z^2 C_A^2 / \omega^2 \) and dropping terms of order \( b k_z^2 C_A^2 / \omega^2 \), the interchange mode dispersion relation is

\[
(w + R_T w_0)(w + i \nu_L / b) + g/L_L - k_z^2 C_A^2 = 0 \quad (19a)
\]

The solutions are

\[
w = 1/2 \left\{ - \left( R_T w_0 + i \nu_L / b \right) \pm \sqrt{\left( R_T w_0 - i \nu_L / b \right)^2 + 4 \left( k_z^2 C_A^2 - g/L_L \right)} \right\} \quad (19b)
\]

The \( k_z^2 C_A^2 \) term in (19b) is strongly stabilizing in most applications.

Solving the electrostatic dispersion relation (17a) for the finite \( k_z \) correction to the interchange mode gives a less restrictive and incorrect result. The \( k_z^2 C_A^2 \) term restricts the interchange instability to \( k_z^2 = 0 \) in slab geometry, and to the lowest order flute perturbations of an entire flux tube in general geometry. Haerendel (1974) has performed a dipole geometry calculation treating magnetic field lines as equipotentials. Assuming that the plasma density is field-aligned, he has averaged all density-dependent quantities in the local dispersion relation (20d) over an entire flux tube. Although the plasma density is altitude - rather than field - aligned below the F maximum, the step to averaging over altitude-aligned plasma density in dipole geometry is not straightforward. We will not pursue the general geometry flute mode further; instead we will focus on new results in slab geometry.
Flute Mode; \( k_z = 0 \) in Slab Geometry

Setting \( k_z = 0 \) and neglecting gravity in (19b), one root is damped and the other is marginally stable

\[
\begin{align*}
    w_+ &= -i \frac{\nu_1}{b} \\
    w_- &= -R_TW_D
\end{align*}
\]

Including gravity, the roots are

\[
\begin{align*}
    w &= \frac{1}{2} \left\{ -\left( R_TW_D + i \nu_1/b \right) \pm \sqrt{\left( R_TW_D - i \nu_1/b \right)^2 - 4g/L_1} \right\} \\
\end{align*}
\]

\( w_+ \) becomes unstable and can be written in the form

\[
\begin{align*}
    w_+ &= \frac{1}{2} \left\{ \left( R_TW_D + i \nu_1/b \right) + \left( x^2 + y^2 \right)^{1/2} \right\} \exp\left[ -i/2 \tan^{-1}(y/x) + i\pi/2 \right] \\
\end{align*}
\]

where

\[
\begin{align*}
    x &= \left( R_TW_D \right)^2 - (\nu_1/b)^2 - 4g/L_1 \\
    y &= 2(R_TW_D\nu_1/b), w_D < w \\
    n = 0 \quad x > 0 \quad n = 1 \quad x < 0
\end{align*}
\]

At \( b = 0 \) the real frequency vanishes and the growth rate is maximum

\[
\begin{align*}
    w_+ &= -i \frac{\nu_1}{2} \pm \frac{i}{2} \sqrt{\frac{2}{\nu_1} + \frac{4g}{L_1}}
\end{align*}
\]
In this limit $v_\perp/b = v_{in}$ and $R_\| w_D = 0$, hence the growth rate is independent of Coulomb collisions and FLR effects. This limit corresponds to that obtained by Haerendel (Balsley et al., 1972; Haerendel 1974) for slab geometry. We have plotted this maximum growth rate as $\nu / \sqrt{g/L_\perp}$ vs $v_{in} / \sqrt{g/L_\perp}$ from (20d) in Figure 1. We see that the collisionless growth rate is maximum and that the growth rate decreases with increasing neutral collision frequency.

In the collisionless limit $v_\perp \to 0$, (20c) yields precisely the result obtained by Rosenbluth et al. (1962) from kinetic theory

$$w_+ = \frac{1}{2} \left\{ - R_\perp w_D + \sqrt{(R_\perp w_D)^2 - 4g/L_\perp} \right\}$$  \hspace{1cm} (21a)

where FLR effects are stabilizing for

$$\left( R_\perp w_D \right)^2 < \left( R_\perp C_\perp / L_\perp \right)^2 \Rightarrow R_\perp > 1 / L_\perp$$  \hspace{1cm} (21b)

This sets an upper limit on unstable $b$ or lower limit on unstable perpendicular wavelengths.

In the collisional limit of (20c)

$$\left( R_\perp w_D - i v_\perp / b \right)^2 >> 4g/L_\perp$$  \hspace{1cm} (22a)

we can expand the radical to obtain one growing root

$$w_+ = - R_\perp w_D + g/L_\perp \frac{\left( R_\perp w_D + i v_\perp / b \right)}{\left( R_\perp w_D \right)^2 + \left( v_\perp / b \right)^2}$$  \hspace{1cm} (22b)
In the limit $v_\perp/b \gg R_\perp \omega_D$, the growth rate is

$$\gamma = \frac{1}{L_\perp v_\perp} \cdot v_\perp/b = v_{in} + 0.3b v_{ii} R_\perp$$

(22c)

which agrees with the collisional Rayleigh-Taylor mode growth rate of Haerendel (1974; Balsley et al. 1972), here modified to include ion viscous damping, which we have seen vanishes for $b = 0$.

The full dispersion relation (20c) is more general than either the result of Haerendel (20d) or Rosenbluth et al. (21a). In addition to including Coulomb collisions, (20c) demonstrates the effect of neutral collisions as well on the collisionless FLR cutoff. In Figure 2 we have plotted the complete Rayleigh-Taylor growth rate (20c) at $k_z = 0$ as a function of $b$ at fixed $v_{ii}$, $L_\perp$ and two different values of $v_{in}$.

The top curve is essentially collisionless ($v_\perp < 2/\sqrt{g/L_\perp}$); the bottom curve is collision-dominated ($v_\perp > 2/\sqrt{g/L_\perp}$). The growth rate is maximum at $b = 0$ in both cases, and smaller in the collision-dominated case, as indicated in Figure 1. Collision dominated growth extends to shorter perpendicular wavelengths than permitted by the collisionless FLR cutoff (Rosenbluth et al. 1962) of the top curve. The $y$ term in (20c), which vanishes in the collisionless limit and becomes significant as $b - 1$ in the collisional case, is responsible for the extension of the instability region down to the ion Larmor radius, where the present fluid analysis breaks down. Physically the collisional particle diffusion drift opposes and cancels the collisionless FLR drift. This diffusion drift depends on $v_\perp/b = v_{in} + 0.3 b v_{ii}$, hence both neutral and Coulomb collisions at finite perpendicular wavelengths.
Ion viscosity only affects transport when there exists a spatial gradient. For $k_y L_y \gg 1$ the important spatial gradient is the perpendicular wavelength or $b$. It is clear from (22c) that in a plasma where $\nu_{ii} \gg \nu_{in}$, such as the nighttime equatorial F region, ion viscosity previously neglected will dominate ion-neutral collisions in determining the perpendicular ion diffusion rate $\nu_i/b$ and Rayleigh-Taylor growth rate at short perpendicular wavelengths.

**Conclusion**

From two-fluid theory we have derived a collisional electromagnetic dispersion relation for the interchange mode coupled to the Alfvén, acoustic, drift and entropy modes. We have demonstrated the fundamental electromagnetic nature of the interchange mode which appears in the opposite limit $(\nu_z^2 C_A^2 / w^2 < 1)$ of the electromagnetic dispersion relation (16) from the electrostatic modes $(k_z^2 C_A^2 / w^2 \gg 1)$. The main stabilizing affect for finite $k_z$ comes from coupling to the electromagnetic Alfvén mode rather than electrostatic modes, even though interchange mode terms appear in the electrostatic dispersion relation. Coupling to the Alfvén mode restricts the interchange mode to $k_z = 0$ in slab geometry.

In all limits the interchange mode destabilized by gravity has a positive growth rate only when gravity is directed antiparallel to the density gradient. For example, this precludes the growth of the Rayleigh-Taylor instability on the topside of the F-layer. The growth rate maximizes at $k_z = 0$ and $b = 0$. Only neutral collisions affect the growth rate at $b = 0$, and the growth rate decreases with $\nu_{in}$. For finite $b$, ion-ion collisions contribute to the perpendicular ion diffusion rate $\nu_i/b = \nu_{in} + 0.3 b \nu_{ii}$. There is competition between the collisional
diffusion drift and the collisionless FLR drift at short perpendicular wavelengths. When \( \nu_\perp/b < 2 \sqrt{g/L_\perp} \), the collisionless FLR drift mode (Rosenbluth et al. 1962) stabilizes short perpendicular wavelengths. When \( \nu_\perp/b > 2 \sqrt{g/L_\perp} \) collisions eliminate the FLR cutoff and extend the unstable spectrum down to the ion Larmor radius where the small \( b \) expansion breaks down.

Since \( \nu_{i\parallel} >> \nu_{i\perp} \) at the altitudes where spread F is generally observed, including ion viscosity is an important extension of the collisional Rayleigh-Taylor instability theory (Balsley et al. 1972; Haerendel 1974). Our consideration of finite perpendicular wavelengths has shown that it is possible for the Rayleigh-Taylor instability to cover the entire range of perpendicular wavelengths above the ion Larmor radius in the limit \( \nu_\perp/b > 2 \sqrt{g/L_\perp} \) where collisions eliminate the FLR cutoff. However, typically \( \nu_\perp/b < 2 \sqrt{g/L_\perp} \) at the altitudes where spread F is observed (Hudson and Kennel 1974b). Hence, the Rayleigh-Taylor instability is primarily collisionless and Haerendel's limit does not apply. The FLR cutoff then restricts the Rayleigh-Taylor instability to perpendicular wavelengths the order of a hundred meters or greater, and the drift mode which is investigated in the companion paper (Hudson and Kennel 1974b) has a larger growth rate at shorter perpendicular wavelengths.
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References


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Table 1 Notation

\begin{align*}
\text{parameter perpendicular to magnetic field } B \\
\text{parameter parallel to magnetic field } B \\
\text{perturbed quantities which vary as } e^i (k \cdot r - \omega t) \\
\text{ion electron temperature ratio} \\
\text{electron-ion mass ratio} \\
\text{electron-ion collision frequency, single charged ions} \\
\text{ion-ion collision frequency} \\
\text{electron-neutral collision frequency (Nicolet 1963)} \\
\text{ion-neutral collision frequency} \\
\text{total electron collision frequency} \\
\text{Coulomb log (Spitzer 1967)} \\
\text{electron thermal speed} \\
\text{ion thermal speed} \\
\text{ion acoustic speed}
\end{align*}
\[ \Omega = \frac{eB}{Mc} \]
ion cyclotron frequency

\[ \Omega_e = \frac{eB}{mc} \]
electron cyclotron frequency

\[ \rho_i = \frac{A_i}{\Omega} \]
ion Larmor radius

\[ \sigma_e = \frac{a_e}{\Omega_e} \]
electron Larmor radius

\[ \omega = \frac{4\pi ne^2}{M} \]
ion plasma frequency

\[ \lambda_e = \frac{a_e}{\nu_e} \]
total electron mean free path

\[ \lambda_{ii} = \frac{A_i}{\nu_{ii}} \]
ion-ion mean free path

\[ L_\perp = (\nabla n/n)^{-1} \]
perpendicular density gradient scale length

\[ b = k_\perp C_s^2 / \Omega^2 \]
ion finite Larmor radius parameter

\[ \omega_D = -\left( \frac{cT_e}{eB^2} \right) k_{\perp} \cdot (\nabla n \times B)/n \]
Diamagnetic drift frequency, defined to be positive for waves propagating in the electron drift direction

\[ \nu_{\perp} = b \nu_{in} + 0.3b^2 \nu_{ii} \]
perpendicular ion diffusion rate

\[ \nu_\parallel = \frac{k_\perp a_e^2}{2C_r \nu_e} \]
parallel electron streaming rate

\[ C_r = \sigma_\perp / \sigma_\parallel \]
ratio of perpendicular to parallel electrical conductivities (Braginskii 1966)

\[ C_t \]
dimensionless thermoelectric coefficient

\[ C_x \]
dimensionless electron thermal conductivity coefficient
\( \bar{X} = (2/3) \left[ C_x C_x + (1 + C_t)^2 \right] \\
+ i 2 (m/M) v_e / v_{||} \\
+ i \left( 1 - R_T \right) (m/M) \left( v_{en} - 3 v_{ei} \right) \\
\bar{\varepsilon} = (2/3) \left( 1 + C_t \right)^2 \\
+ 2 \left( 1 - C_t \right) \left( 1 - R_T \right) (m/M) \\
\cdot \left( 2 v_{ei} + v_{en} \right) / v_{||} \\
\) heat conduction and energy transfer coefficients

All other notation standard
Figure 1

Plot of maximum Rayleigh-Taylor growth rate in slab geometry (20d) which occurs for \( b = 0 \) and \( k_z = 0 \), normalized as \( \gamma / \sqrt{g/L_\perp} \) vs. \( \nu_{in} / \sqrt{g/L_\perp} \). Note that the growth rate is maximum in the collisionless limit and decreases with increasing neutral collision frequency; ion viscosity and FLR stabilization do not affect the \( b = 0 \) mode.

We have scaled the \( x = \nu_{in} / \sqrt{g/L_\perp} \) axis with altitude for an average model neutral atmosphere (Johnson 1965) and the \( y = \gamma / \sqrt{g/L_\perp} \) axis with growth rate for two reasonable choices of \( L_\perp \) for the bottom of the nighttime \( F \) layer, \( L_\perp = 9.42 \) and \( 25.6 \) km. The altitude dependence is affected more than the magnitude of the growth rate by this change in \( L_\perp \).

Figure 2

Rayleigh-Taylor growth rate (2.0c) at \( k_z = 0 \) as a function of \( h \) at fixed \( L_\perp = 9.42 \) km and \( \nu_{ii} = 0.55 \) sec\(^{-1}\) for two different values of \( \nu_{in} = 0.014 \) and \( 0.08 \) sec\(^{-1}\). The top curve is essentially collisionless \( (\nu_{in} < 2 \sqrt{g/L_\perp}) \); the bottom curve is collision-dominated \( (\nu_{in} > 2 \sqrt{g/L_\perp}) \). The growth rate is maximum at \( b = 0 \) in both cases and smaller in the collision-dominated limit. Collision dominated growth extends to shorter perpendicular wavelengths than permitted by the collisionless FLR cutoff (Rosenbluth et al. 1962) of the top curve, but the theory breaks down as \( b \rightarrow 1 \) (dashed line).

The two different neutral collision frequencies \( \nu_{in} = 0.014 \) and \( 0.08 \) sec\(^{-1}\) correspond to two different altitudes \( h = 400 \) and \( 280 \) km for an average model neutral atmosphere (Johnson 1965).
Rayleigh-Taylor Flute Mode

Maximum Growth Rate

$b = 0$

Figure 1
$\gamma_{\text{RT}}$ $h = 400 \text{ km}$
$\nu_{\parallel} = 0.014 < 2\sqrt{\frac{g}{L_\perp}} \text{ sec}^{-1}$
$\nu_{\parallel} = 0$

$2\sqrt{\frac{g}{L_\perp}} = 0.0645 \text{ sec}^{-1}$

$\nu_{\perp} = 0.001$ $0.01$ $0.1$ $1.0$

$\gamma \times 10^{-3} \text{ sec}^{-1}$

$\gamma_{\text{RT}}$ $h = 280 \text{ km}$
$\nu_{\perp} = 0.08 > 2\sqrt{\frac{g}{L_\perp}} \text{ sec}^{-1}$

Figure 2
PPG-125 "Electromagnetic Wave Functions for Parabolic Plasma Density Profiles," Alfredo Baños, Jr. and Daniel L. Kelly (September 1972). Accepted by Physics of Fluids.


