THE COMPLEX MAGNETIC FIELD OF JUPITER

(NASA-TM-X-70833) THE COMPLEX MAGNETIC FIELD OF JUPITER (NASA) 26 p HC $3.75

CSCL 03B

Unclas

G3/90 10734

FEBRUARY 1975

M. H. ACUNA
N. F. NESS

GSFC
GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
For information concerning availability of this document contact:

Technical Information Division, Code 250
Goddard Space Flight Center
Greenbelt, Maryland 20771

(Telephone 301–982–4488)

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THE COMPLEX MAGNETIC FIELD OF JUPITER

by

Mario H. Acuna
Norman F. Ness

Submitted to Science
14 February 1975
ABSTRACT

The main magnetic field of Jupiter has been measured by the GSFC Fluxgate Magnetometer on Pioneer 11 and analysis reveals it to be considerably more complex than expected. In a spherical harmonic representation, the dipole term (with opposite polarity to Earth's) has a magnitude of $4.02 \text{ Gauss-R}_j^3$, at a tilt angle of $9.0^\circ$ and a system III longitude of $221^\circ$. However, the quadrupole and octupole moments are proportionately very large, 50% and 90% of the dipole moment, and this leads to significant and complex deviation of the planetary magnetic field at distances $<4 \text{ R}_j$ from a simple dipole topology. The north polar field strength is 22 Gauss and in the northern hemisphere the "footprint" of the Io associated flux tube is localized to system III longitudes of $75^\circ-215^\circ$. Associated L shell splitting in the radiation belts, warping of the equatorial planes, and enhanced absorption effects due to the satellites Amalthea and Io are expected as a result of the field complexity.
INTRODUCTION

Results from a preliminary analysis of the data for the NASA/GSFC magnetic field experiment on Pioneer 11 are summarized in this report. The high field triaxial fluxgate magnetometer (1) was provided by NASA/GSFC to extend the field range coverage up to 17 gauss, values believed to be representative of high latitude, low altitude field intensities from radio astronomy observations (2). The data have been analyzed in terms of a traditional Schmitt normalized spherical harmonic expansion fitted to the observations in a least squares sense. We show that the measurements can be interpreted in such a way as to add considerably to our knowledge of the main magnetic field of the planet. Our results have significant implications regarding theories of trapped radiation and radio emissions by Jupiter, absorption effects by the natural satellites and on general theories of planetary dynamos.

Previous Studies

The first in-situ studies of the magnetosphere of Jupiter were conducted in late 1973 by the Pioneer 10 spacecraft. An analysis of the Helium Vector Magnetometer data (3) indicated that the planetary field was well represented by an offset tilted dipole at distances from 2.8 to 10 R\textsubscript{j}. Nevertheless, the moment (4.0 gauss-R\textsubscript{j}\textsuperscript{3}) and tilt (10.6° at λ\textsubscript{III}=222°) of this model yielded a field configuration and intensity which were inconsistent with a number of independently and previously derived estimates obtained from ground based observations of radio emissions (2). In contrast to the limited latitude and longitude coverage provided by Pioneer 10, the Pioneer 11 trajectory, which passed within 0.6 R\textsubscript{j} of the planetary surface, covered 660° in longitude, from λ\textsubscript{III}=30° to λ\textsubscript{III}=330°, and 80° in latitude, from -30° to +50° during the period of closest approach on December 3, 1974 from 0000 to 1100 GMT.
The measurements obtained over such a wide range of latitude and longitude provide a more complete sampling of the planetary magnetic field, which can then be analyzed with increased confidence to predict field values at other positions not covered by the spacecraft trajectory. The results of a preliminary analysis of the quick-look real time data obtained during Pioneer 11 Jupiter encounter by the GSFC-FGM instrument (4) indicated that within 3 $R_J$, the planetary field was much too complex to be represented by a simple offset tilted dipole and that higher harmonic multipoles were required. In this paper, we report the results obtained from an analysis of the preliminary experimenter's data tape covering the time interval 0120 to 0926 GMT (S/C time).

**Instrumentation**

The instrument consists of a single range triaxial fluxgate magnetometer sensor and associated electronics capable of measuring fields up to 10 Gauss along each orthogonal axis. Instantaneous vector measurements of the three components of the field, using a ten-bit precision A-D converter, yields a quantization step size of $\pm 600$ gamma for fields less than 2 Gauss. These are made once every three revolutions of the spacecraft (36 seconds), in synchronization with a reference axis crossing through the ecliptic. The digitized data are sent directly to ground without further processing on board the spacecraft. The complete instrument weighs 272 grams and uses 300 milliwatts of power from the GSFC-Cosmic Ray Telescope Experiment. A more complete description has been given in (1).

The raw magnetic field data are translated to a Jupiter centered spherical coordinate system and combined with spacecraft trajectory positional information to yield a triad measurement set. A total of 683 vector measurements
were obtained during the close encounter period, corresponding to radial distances in the range $1.7 \, R_J < R < 6 \, R_J$. Due to the low spacecraft bit rate during occultation when the S/C was placed into a memory storage mode, no measurements were obtained in this period by this experiment.

**Observations and Analyses**

The predicted magnetic field components for the $D_2$ model of Smith et al. (3), and the GSFC-FGM observations have been plotted in a Jupiter centered spherical coordinate system as a function of radial distance as shown in Figure 1(a). For radial distances greater than $4 \, R_J$ there is reasonable agreement with the predicted values, both in magnitude and polarity, although the quantization step size of the FGM instrument precludes a closer comparison when the measured field is less than 1000 $\gamma$. The largest deviation is observed for $R > 4 \, R_J$ in the radial component of the field. For distances less than $4 \, R_J$ the observed field increases rapidly, much faster than the inverse cube law for a dipole model such as $D_2$. This implies that the contributions from higher order multipoles are significant and must be included in the analysis to obtain an adequate representation of the main planetary field.

The magnetic field $\vec{B}$ in a region containing no sources ($\nabla \times \vec{B} = 0$) can be expressed as the gradient of a scalar potential $V$ which represents the contribution of sources internal and external to the region of interest. Thus we have

$$\vec{B} = -\nabla V = -\nabla (V^e + V^i)$$  \hspace{1cm} (1)$$

and it is customary to express the potential in terms of spherical harmonics as

$$V = V^e + V^i = a \sum_{n=1}^{\infty} \left\{ \left( \frac{\ell}{a} \right)^n T_n^e + \left( \frac{\ell}{r} \right)^{n+1} T_n^i \right\}$$  \hspace{1cm} (2)$$
where in our case $r$ denotes the distance from Jupiter's center, $a$ is Jupiter's radius (71372 km) and the $T_n^{e,i}$ are given by

$$T_n^{e,i} = \sum_{m=0}^{n} P_n^m (\cos \theta) [g_n^m \cos m\phi + g_n^m \sin m\phi]$$

$$T_n^e = \sum_{m=0}^{n} P_n^m (\cos \theta) [G_n^m \cos m\phi + H_n^m \sin m\phi]$$

(3)

The angles $\phi$ and $\theta$ denote zeugographic east longitude and colatitude respectively, $P_n^m(\theta)$ are the associated Legendre functions with Schmitt normalization and $g_n^m$, $h_n^m$, $G_n^m$ and $H_n^m$ are the Schmitt coefficients.

From a well known theorem of potential theory, a unique representation of the magnetic field in the source free region is derivable from vector measurements over a simple surface which completely encloses the internal sources. In the case at hand, we do not have such a "complete" set of observational data and thus it is impossible to find a unique solution. Said another way, the data do not form an orthogonal set which span the same space as the harmonic representation. In practice, this means that cross coupling exists between the various harmonic coefficients $g_n^m$, $h_n^m$, $G_n^m$ and $H_n^m$ in that as the order of the representation increases from $n=1$, the values of the coefficients so derived vary, dependent upon the highest order employed. We shall see these effects in the hierarchy of solutions presented later.

With respect to the validity of the source free region approximation, we have estimated the diamagnetic effects of the trapped charged particles (5) and the possible ring current effects due to their drift in the region $R < 10 R_j$. We have also considered the fields due to the highly distorted distant magnetosphere or magnetodisc (3) which occurs at
distances $>25 \, R_j$. The combined effects appear to be significantly less than the quantization step size, $\pm 600\gamma$. Thus a representation of a scalar potential is justified and all external fields have been assumed negligible in our data set (i.e. $C_n^m = H_n^m = 0$ for $n \geq 1$, $m > 0$).

The observations obtained between 1.7 and 6.0 $R_j$ were fitted in a least squares sense to first, second and third degree spherical harmonic expansions of the form \[2\], $(n=1,2,3)$, that is, terms corresponding to a centered dipole, quadrupole and octupole moments of internal origin only. Thus, given a set of measurements we compute a set of coefficients \(\{g_n^m, h_n^m\}\) such that the sum over the $N$ measurements of the vector residuals squared

\[
\sum_{k=1}^{N} |\vec{e}_k|^2 = \sum_{k=1}^{N} \left( |\vec{B}_{\text{model}} - \vec{B}_{\text{obs}}|^2 \right)
\]

is minimized. We find that the inclusion of quadrupole and octupole terms $(n=2,3)$ leads to a significant reduction in the RMS of the residuals.

From the quadrupole expansion $(n=2)$, an offset tilted dipole representation can be obtained using 6 of the 8 dipole plus quadrupole coefficients to model the field. In Table I, we show the results obtained for the dipole terms in different representations as well as the vector RMS of the residuals for each model. As previously noted, the dipole term is observed to vary, depending upon the highest order $n$ of the representation used.
The dipole, OTD and quadrupole models yield vector RMS deviations which are considerably larger than the quantization step size of the instrument. Also, the dipole axis tilt is smaller than that inferred from radio astronomy observations although the longitude is in reasonable agreement with prior work (2). The octupole model on the other hand provides a much better agreement with the observations with a significantly reduced residual RMS. The dipole term in the octupole expansion also agrees much better with Pioneer 10 results derived from energetic particle measurements (5) and magnetic field observations (3). These facts are a clear indication that the main magnetic field of Jupiter is extremely complex and cannot be represented by any simple dipole models, especially for distances less than 4 R\textsubscript{J}.

To illustrate the goodness-of-fit obtained with the octupole model, which we shall denote as O\textsubscript{3}, a plot similar to the one shown in Figure 1a for the D\textsubscript{2} model of Smith et al. is shown in Figure 1b. The overall agreement with the observations is excellent in all three components of the field. There are suggestions however, that the inclusion of higher order terms of internal origin and external terms to at least first order, could further improve the fit to the observations and provide additional insight into the mechanisms responsible for the complexity of the observed field.

**Interpretation**

The magnitudes of the quadrupole and octupole moments calculated from the O\textsubscript{3} model are 49% and 89% of the dipole moment respectively. These large values lead to a very complex field topology which extends to significant distances from the planet. Figure 2 shows a plot of isointensity contours obtained from the model at the surface of the planet and at 2 R\textsubscript{J} (a 1/15.4 flattening has been assumed for the surface plot). It is clearly evident that even at 2 R\textsubscript{J} the magnetic equator, as defined by the minimum field
intensity, is highly distorted and bears little resemblance to that expected from any simple dipole model. Thus, in any attempts to reconcile the observed decimetric radiation from Jupiter with models of the radiation belts (6), a study which uses a specific field model, the real deviations from a dipole will be incorrectly treated by any interpretation which admits only a dipole in the modeling.

In general, the maps in Figure 2 show considerable hemispherical and azimuthal asymmetries. On the surface of the planet, the maximum field strength at the poles is highly asymmetrical, 22.8 and 15.8 Gauss respectively in the north and south polar regions, and much higher than that inferred from the $D_2$ dipole model. The map also shows that subsidiary maxima exist. These departures from symmetry lead to a very complex distortion of the motion of trapped particles forming the radiation belts (7). These observed strong azimuthal asymmetries are no doubt in part responsible for the periodic escape of relativistic electrons, as discussed in (8). The magnitude of the higher order moments is such that considerable L shell splitting is expected to occur even at large distances from the planet (7). We have calculated the corresponding L values derived from the $O_3$ model along the spacecraft trajectory for the particular case of $90^\circ$ pitch angle particles, as shown in Figure 3. We have also shown the corresponding values for the $D_2$ model and a centered dipole model with $9.5^\circ$ tilt at $233^\circ$ longitude (5), as well as the range of L values covered by Io and Amalthea, based on $O_3$. Figure 4 provides more detailed information about Io and Amalthea L shell parameters. In particular, it can be observed that a broader range of L values than that predicted from dipole models is covered and the curves are asymmetrical
with respect to the zenographic phase of the subsatellite points.

The range and predicted crossing times of the L shells affected by these satellites are given in Table 2. The range of L swept out by the satellites Io and Amalthea is considerably larger than that expected for a dipole model and this predicts enhanced absorption effects of these satellites. The predicted L shell crossing times agree very well with the times derived from charged particle measurements (9) which show satellite sweeping effects. Thus, this provides a necessary independent test of the validity of the O₃ model. By comparison with the inferior predictions of the D₂ model, it is an indication of the improvement achieved with O₃.

The charged particle data obtained during the close encounter period (9) show 4 minima in count rate and 3 interior peaks. The two exterior minima can be explained in terms of Amalthea sweeping effects by the O₃ model. There exist three possible explanations for the one peak and remaining two minima which occur near closest approach. The most plausible is that the O₃ model does not include sufficiently high order terms (i.e. hexadecapole and beyond) and as such is limited in latitude and longitude resolution to determine fine L shell structure variations on this scale. The second explanation adds the existence of a significant longitudinal variation of the loss cone on an L shell and across L shells. Finally, although we consider it remote, the possibility exists that the two minima are due to charged particle sweeping effects by an unknown satellite or ring of particles not yet visually observed.

In Figure 4 is shown a set of parameters useful in studying the relationship of the trapped particle motions with the absorption effects
of the satellites Amalthea and Io. These data indicate in more
detail than Figure 3 permitted the rather large extent of the L region
affected by the satellites and the asymmetries which exist in the
parameters due to the complex planetary field.

The magnetic equatorial distortion may also have significant
implications regarding the efficiency with which satellites can sweep
up trapped particles (10). We expect a large fraction of near equatorial
pitch angle particles to interact with any satellite orbiting Jupiter
in or near its equatorial plane. Further, if such is the case, the
pitch angle distributions will not peak as markedly around $90^\circ$ as
predicted for a dipole model.

The concentration of field lines around the strong north pole,
i.e., the localization in longitude and latitude, and the relationship
to the field lines passing through Io may play an important role in
explaining the modulation of the decametric emissions by this satellite.
We show the "footprint" of the field line through the satellite Io in
Figure 2 and is seen to pass through the north polar region where it
is restricted to system III longitudes lying between $75^\circ$ and $215^\circ$. In
the southern hemisphere, the footprint passes near the south pole and
covers a greater longitude range between $130^\circ$ and $360^\circ$ with a minor
trace from $0^\circ$ to $30^\circ$. These features indicate that particles mirroring
on Io's L shell also mirror in the Zenovian auroral regions. With the
larger fields in the north, then the primary source of decametric radio
emissions is probably sporadic precipitation of particles into the
northern hemisphere. If we assume that the observed frequencies
correspond to electron gyrofrequencies near the surface of the planet,
we predict 25 to 64 MHz for the north polar region, and 22 to 33 MHz for the south polar region. This implies, of course, that the precipitating particles radiate their energies and interact with the ionosphere at higher altitudes since the maximum observed frequency of emission is 40 MHz (2).

In addition to testing the validity of the $O_3$ model by comparison of predicted and observed satellite sweeping effects, we have conducted another special test. This consists of assuming that the centered dipole representation derived by the Iowa group (5) is a better approximation to the dipole term than that of $D_2$ (3) or $O_3$. In order that our octupole representation approach that of a simple centered dipole at large distances, the charged particle data require that we constrain the ratios $g_1^0/g_1$ and $h_1^0/g_1$ so as to achieve a dipole tilt of 9.5° at a system III longitude of 233° (extrapolated by 3° from Pioneer 10 epoch).

The least squares fitting to our data with such a constraint yields a harmonic coefficient set nearly identical to the unconstrained $O_3$ model. The vector residual RMS is exactly the same as before, 0.015 Gauss and the dipole moment increases by 0.8% to 4.05 Gauss-R$_j^3$. The 15 individual harmonic coefficients change by very small amounts, ranging from 0.3% to 16% with an RMS value of 2.6%. Lastly, the isointensity contour maps show negligible deviations from those of $O_3$, as expected with such a coefficient set. Thus we conclude that $O_3$ represents a superior model of the main magnetic field of Jupiter for distances $<6 R_j$ and is probably valid at distances up to $12 R_j$, the distance at which the effects of external fields of the magnetodisc will become important and must be included.
The large quadrupole and octupole moments have significant implications regarding the origin of the planetary magnetic field. It is widely accepted that the most plausible explanation of the Jupiter field is associated with an internal dynamo (11). Whether or not the mechanism is identical to Earth is of considerable importance. The large higher multipole harmonics suggest a much larger source region proportionately, than the Earth, i.e. a much larger "core" on Jupiter, proportionately.

Future Work

The results reported herein represent only our initial efforts of a more complete and detailed study of the Jovian magnetic field, its origin and effects on charged particles. In particular, we intend to improve on the $O_3$ model given here, by incorporating additional internal and external terms in the analysis, as well as global field characteristics deduced from charged particle measurements. From these studies an improved model should emerge which can be used with confidence to calculate L-shell structures, bounce and drifts periods, trapping boundaries and other relevant parameters to a more complete description of the Jovian magnetosphere.

Equally significant, the study of the model in the context of radio astronomy observations should provide us with an increased understanding of the mechanisms responsible for the emissions, and their modulation by Io as in the case of the decametric bursts. And lastly, a valid representation of the main field and its possible time variations, by comparison with similar analyses of Pioneer 10 data, should provide insight on any temporal variation of the magnetic field and the properties of the planetary interior. The differences between system III and system II may be due to a secular variation as observed in the Earth's field.
References


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12. We thank C. F. Hall, J. Lepetich and D. Sinott of the Pioneer Project
    Office at NASA Ames for their support during integration of this
    experiment. We especially thank Professor J. G. Roederer of the
    University of Denver for making certain of his computer programs
    relating to trapped particle motion available to us and G. Burgess,
    F. W. Ottens and R. F. Thompson of NASA/GSFC for their outstanding
    support in the prompt analysis of these data. We appreciate
    opportunities to discuss these results with J. K. Alexander,
S. Gulkis, J. Warwick and colleagues on Pioneer II. Finally, the support provided by Mr. D. Stillwell of NASA/GSFC and the Space and Tactical Systems Corporation, Bedford, Mass. during the development of the instrument is deeply appreciated.
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<th>OTD (n=2)</th>
<th>Quadrupole</th>
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List of Figures

1. Comparison plots of spherical coordinate components of the GSFC-FGM magnetic field measured on Pioneer 11 with the simple offset tilted dipole model, \( D_2 \), (la, left panel) and the GSFC octupole model, \( O_3 \) (lb, right panel).

2. Isointensity contour maps of the Zenovian magnetic field at the "surface" (upper panel) and at the assumed centroid of the decimetric radio emission region, \( R = 2 R_j \) (lower panel). The significant distortion of the minimum B surface from a plane is evident, as is the localization of the northern footprint of the Io associated flux tube. The trace of the Pioneer 11 trajectory near perijove is indicated in the upper panel.

3. Comparison of three different magnetic field models in terms of the derived equivalent L shell parameter. Note the convoluted behavior of the GSFC \( O_3 \) model result near closest approach when compared to either Pioneer 10 models, \( D_2 \) or "Iowa", both extrapolated to the epoch of Pioneer 11.

4. Illustration of the effect of the complex main magnetic field of Jupiter on the L shells affected by absorption effects of the satellites Amalthea and Io. The equatorial points of the field lines threading the satellites, field magnitude and distance and presented in the top two panels, and the actual L value at the satellite in the lower panels.
FIGURE 1a
FIGURE 1b
GSFC MAGNETIC FIELD MODEL O₃ - PIONEER II

FIGURE 2
PIONEER II TRAJECTORY AT JUPITER

+ DIPOLE (9.5°, 233°)

O OTD (D₂)

- GSFC O₃

FIGURE 3
AMALTHEA L SHELL PARAMETERS

\[ B_{\text{EQUAT.}} \]

\[ D \]

\[ L(R_J) \]

\[ \lambda_{\text{III}} \]

IO L SHELL PARAMETERS

\[ B_{\text{EQUAT.}} \]

\[ D \]

\[ L(R_J) \]

\[ \lambda_{\text{III}} \]

\[ \alpha = 90^\circ \]

\[ 045315 \quad 061645 \]

\[ 045915 \quad 060245 \]

\[ 033015 \quad 070345 \]

\[ 034815 \quad 065745 \]

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PIONEER II TRAJECTORY
(S/C TIME 3 DEC 1974)

DISTANCE TO EQUATORIAL POINT (R_J)

ZENOGRAPHIC PHASE OF SUBSATELLITE POINT

\[ \lambda_{\text{III}} \]