ANALYSIS OF EDGE IMPACT STRESSES IN COMPOSITE PLATES

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Abstract

Fiber composite plates, high velocity impact, in-plane edge impact, edge protection, foreign object damage, displacement and stress waves, contact time, fast fourier transform, computer program

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INTRODUCTION

This report is part of a continuing effort by NASA to understand the basic mechanics of foreign object impact of composite materials of particular interest are damage resistant designs of jet engine fan blades under hail or bird impact. In previous reports the central or normal impact response of composite plates was examined [1]. In this report the mechanics of edge impact of composite plates are examined. This is schematically illustrated in Figure 1.

The basic approach to the study of impact of composite plates in this program has been to examine the stress waves generated by the impact forces. For central impact of plates it has been shown that in addition to wave propagation across the plate thickness, bending and extensional waves propagate away from the impact site. The stresses associated with these waves have been studied without considering the effect of boundaries such as the free or clamped edges of a fan blade. This simplification has been made on the premise that for short impact times e.g. less than 10 sec. few edge reflections have taken place, and that the highest stresses occur at the impact site. However, for edge impact, the boundary conditions greatly affect the nature of the wave mechanics.

Edge waves in solids have been studied extensively in seismology. The principal phenomenon is the entrapment of wave energy in a layer near the surface. This surface wave is known as a Rayleigh wave and travels at a velocity below the shear velocity for isotropic solids. For plates, however, two types of edge waves can occur as shown in Figure 2a. For impact transverse to the plate, flexural edge waves can occur. For in-plane, Rayleigh type edge waves are generated. In this report only in-plane edge waves will be discussed.
Wave type solutions to the equations of elastodynamics of an orthotropic plate which exponentially decay away from the edge (X direction) and propagate along the edge (X direction) can be found provided the edge wave velocity satisfies the equation (Reference 9).

\[ \rho v^2 + \left[ \begin{array}{cc} C_{55} & -\rho v^2 \\ C_{13} & C_{33} \end{array} \right] \left[ \begin{array}{cc} C_{11} - \rho v^2 \\ C_{11} \end{array} \right] = 0 \]

where \( \rho \) is the mass density of the plate and \( C_{ij} \) are the effective plate elastic constants (denoted by \( \hat{C} \) in Reference 1). It can be shown that one real root lies in the interval

\[ 0 < \rho v^2 < C_{55} \]

Thus, the edge or Rayleigh wave speed is less than the shear speed in this direction \( \left[ C_{11} / \rho \right]^{1/2} \).

Changing the layup angle will affect the elastic constants \( C_{ij} \) and hence, change the value of the edge wave velocity. The results of this calculation are shown in Figure 2b where the \( C_{ij} \) are obtained from Reference 7. The edge wave speed seems to obtain a maximum between \( \pm 15^0 \) and \( \pm 30^0 \) layup angle which is below the extentional wave speeds labelled "dilational" and "shear" in Figure 2b and which is greater than the bending wave velocity.
In order to prevent damage to composite fan blades under foreign object impact leading edge protection has been used. (See e.g. Ref. (8)). This usually consists of a strip of metal attached to the leading edge of the fan blade. To model the effects of this impact protection strip, the in-plane edge impact of an anisotropic plate, with a beam-strip attached to the impact edge, has been studied (see Figure 1). It will be shown later that the strip will decrease the tensile stress along the edge while producing shear stress between the strip and the plate edge.
I. BASIC EQUATIONS FOR EDGE LOADING ON ANISOTROPIC PLATE

Let the plate be the half-space $x_3 > 0$. The equations of motion are given by [1] as:

$$\begin{align*}
C_{11} u_{1,11} + C_{55} u_{1,33} + (C_{11} + C_{55}) u_{1,3,13} &= \rho u_{1,tt} \\
C_{33} u_{3,33} + C_{55} u_{3,11} + (C_{13} + C_{55}) u_{1,3,13} &= \rho u_{3,tt}
\end{align*}$$

(1.1)

here we have employed $C_{11}, C_{13}, C_{33} \ldots$ to denote $\hat{C}_{11}, \hat{C}_{13}, \hat{C}_{33} \ldots$ of [1]. The in-plane motion is assumed to be independent of the bending deformation. With the loading condition shown in Figure 1, the boundary conditions are (without protection strip):

$$\begin{align*}
t_{13}(x_1,0,t) &= C_{55} (u_{1,3} + u_{3,1}) \bigg|_{x_3 = 0} = 0 \\
t_{33}(x_1,0,t) &= [C_{33} u_{3,3} + C_{13} u_{1,1}] \bigg|_{x_3 = 0} = p f(x_1) g(t)
\end{align*}$$

(1.2)

The following nondimensional parameters are used;
\[
C^* = \frac{C_{11}}{C_{66}}, \quad C_{13}^* = \frac{C_{13}}{C_{66}}, \quad C_{33}^* = \frac{C_{33}}{C_{66}}, \quad C_{55}^* = \frac{C_{55}}{C_{66}} \quad \text{and} \quad P^* = \frac{p}{C_{66}} \quad (1.3)
\]

\[
u^* = \frac{u_1}{\lambda}, \quad u_3^* = \frac{u_3}{\lambda}, \quad x_1^* = \frac{x_1}{\lambda}, \quad x_3^* = \frac{x_3}{\lambda}, \quad t^* = \frac{t \sqrt{C_{66}/\rho}}{\lambda} \quad (1.4)
\]

\(\lambda\) is a length parameter. \(C_{66}\) is a typical elastic constant. In what follows the equations will be assumed to be nondimensionalized using (1.3) and (1.4).

To obtain the solution, we employ transform methods. Define:

\[
F(f) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ikx_1} f(x_1) \, dx_1
\]

as the Fourier transform of \(f(x)\) where we assume that

\[
\left. f(x_1) \right|_{-\infty}^{\infty} = \left. \frac{\partial}{\partial x_1} f(x_1) \right|_{-\infty}^{\infty} = 0
\]

Then

\[
F(\frac{\partial}{\partial x_1} f) = ik \, F(f), \quad F(\frac{\partial^2 f}{\partial x_1^2}) = -k^2 \, F(f)
\]

Also define

\[
\mathcal{L}(f) = \int_{0}^{\infty} e^{-st} f(t) \, dt
\]

as the Laplace transform. The initial conditions are assumed to be,
\[ u_{11}(x_1, x_3, 0) = \frac{\partial}{\partial t} u_{11}(x_1, x_3, 0) = 0 \]
\[ u_{33}(x_1, x_3, 0) = \frac{\partial}{\partial t} u_{33}(x_1, x_3, 0) = 0 \]

(1.5)

Letting \( F(u_1) = \tilde{u}_1(k, x_3, s) \), \( F(u_3) = \tilde{u}_3(k, x_3, s) \), we have the following transformed equation:

\[
\begin{align*}
C_{11} (-k^2) \tilde{u}_1 + C_{55} \tilde{u}_{1,33} + (C_{13} + C_{55}) (ik) \tilde{u}_{1,3,3} &= +s^2 \tilde{u}_1 \\
C_{33} \tilde{u}_{3,3,3} + C_{55} (-k^2) \tilde{u}_3 + (C_{13} + C_{55}) (ik) \tilde{u}_{1,3,3} &= +s^2 \tilde{u}_3
\end{align*}
\]

(1.6)

and at \( x_3 = 0 \), the transformed boundary conditions become

\[
\begin{align*}
\tilde{u}_{1,3} + ik \tilde{u}_1 &= 0 \\
C_{33} \tilde{u}_{3,3} + C_{13} ik \tilde{u}_1 &= -P_0 \tilde{f}(k) \tilde{g}(s)
\end{align*}
\]

(1.7)

Since we are expecting surface wave propagation, we seek the solution of (1.6), (1.7) in the forms:

\[
\begin{bmatrix}
\tilde{u}_1 \\
\tilde{u}_3
\end{bmatrix} = \begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix} e^{-p(k, s) x_3}
\]

(1.8)

(real \( p \geq 0 \))
Therefore, the equations for $\phi_1, \phi_2$ are:

\[
\begin{bmatrix}
-s^2 - C_{11} k_1^2 + p^2 C_{55} & -ik_1 p (C_{13} + C_{55}) \\
-ik_1 p (C_{13} + C_{55}) & -s^2 - k_1^2 C_{55} + p^2 C_{33}
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2
\end{bmatrix} = 0
\]  
(1.9)

or for a non-trivial solution,

\[
\det = C_{33} C_{55} p^n + [C_{33} (-s^2 - C_{11} k_1^2) + C_{55} (-s^2 - C_{55} k_1^2) + k^2 (C_{13} + C_{55})] p^2
\]
\[
+ (s^2 + C_{11} k_1^2) (s^2 + C_{55} k_1^2) = 0
\]  
(1.10)

We will choose the p's with positive real parts to ensure the decay in $x_3$ direction of the surface wave. Let the solutions be $p = p_1, p_2$, therefore, we have

\[
p = p_1: \quad \phi_1^{(1)} \equiv C_1 (k_1, s), \quad \phi_2^{(1)} = -\frac{i[-s^2 - C_{11} k_1^2 + C_{55} p^2]}{k_1 p_1 (C_{13} + C_{55})} \phi_1^{(1)} \equiv \psi_{31} C_1
\]
(1.11)

\[
p = p_2: \quad \phi_1^{(2)} \equiv C_2 (k_1, s), \quad \phi_2^{(2)} = -\frac{i[-s^2 - C_{11} k_1^2 + C_{55} p^2]}{k_1 p_2 (C_{13} + C_{55})} \phi_1^{(2)}
\]

Therefore, the displacements have the forms:

\[
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_3
\end{bmatrix} = C_1 (k_1, s) \begin{bmatrix} 1 \\ \psi_{31} \end{bmatrix} e^{-p_1 x_3} + C_2 (k_1, s) \begin{bmatrix} 1 \\ \psi_{32} \end{bmatrix} e^{-p_2 x_3}
\]  
(1.12)
\( C_1, C_2 \) are determined from the boundary conditions (1.7), or

\[
\begin{bmatrix}
-p_1 + i k_1 \psi_{31} & -p_2 + i k_1 \psi_{32} \\
-p_1 C_{33} \psi_{31} + i k_1 C_{13} & -p_2 C_{33} \psi_{32} + i k_1 C_{13}
\end{bmatrix}
\begin{bmatrix}
C_1 \\ C_2
\end{bmatrix} = \begin{bmatrix} 0 \\ -p_0 \bar{f}g \end{bmatrix}
\]

(1.13)

Here, the determinant, \( \Delta(p, s) \), given by,

\[
\Delta = (ik_1 \psi_{31} - p_1) (ik_1 C_{13} - p_2 C_{33} \psi_{32})
\]

\[-(ik_1 \psi_{32} - p_2) (ik_1 C_{13} - p_1 C_{33} \psi_{31})\]

(1.14)

must be non-zero to ensure a solution. (\( \Delta = 0 \) gives the Rayleigh poles of the system, which correspond to a free surface). Therefore,

\[
C_1 = \frac{1}{\Delta} [ik_1 \psi_{32} - p_2] \bar{f}g p_0
\]

(1.15)

\[
C_2 = \frac{1}{\Delta} [p_1 - ik_1 \psi_{31}] \bar{f}g p_0
\]

and the physical displacements \( u_{113}(x, x, t), u_{133}(x, x, t) \) are obtained by inverting their transforms.
From the stress-strain relations

\[ t_{13} = C_{13} (u_{13} + u_{31}), \quad t_{11} = C_{11} u_{11} + C_{13} u_{33}, \]

we obtain the transforms of the stresses

\[ \tilde{t}_{13} = C_{13} \left\{ (-p + ik \psi_1) C_{11} e^{-p_1 x_3} \right\} + \left\{ (-p_2 + ik \psi_2) C_{22} e^{-p_2 x_3} \right\} \]

\[ \tilde{t}_{11} = (C_{11} ik - C_{13} p \psi_1) C_{11} e^{-p_1 x_3} \]

\[ + \left\{ (C_{11} ik - p \psi_2) C_{22} e^{-p_2 x_3} \right\} \]

\[ \tilde{t}_{33} = (-p_1 \psi_1 C_{11} + ik C_{13}) C_{11} e^{-p_1 x_3} \]

\[ + \left\{ (-p_2 \psi_2 C_{22} + ik C_{13}) C_{22} e^{-p_2 x_3} \right\} \]
and the physical stresses can be obtained by inversion.

The particular forcing function employed is

\[ f(x, t) = g(t) = \begin{cases} \sin \left( \frac{\pi t}{\tau_o} \right) & 0 < t < \tau_o \\ 0 & \text{otherwise} \end{cases} \\
| x_1 | \leq a \]

\[ = \left[ 1 - \left( \frac{\partial}{a} \right)^2 x_1^2 \right] \sin \left( \frac{\pi t}{\tau_o^*} \right) \]

where \( \tau_o^* = \frac{a}{\sqrt{C/\rho}} \).

Here \( a \) is a length measuring the impact area, and \( \tau_o \) is the contact time [1]. The transform of this particular forcing function, in non-dimensional form becomes,

\[ \frac{f_g}{k} = \frac{4}{k^2 (a/\lambda)^2} \left[ -\frac{a}{\lambda} \cos \left( \frac{a}{\lambda} \right) + \sin \left( \frac{a}{\lambda} \right) \right] \frac{\pi \tau_o^* (1 + e^{-\pi \tau_o^*/\tau_o^*})}{\pi^2 + \tau_o^* \lambda^2} \]

(1.19)
II. EDGE IMPACT OF PLATE WITH EDGE PROTECTION

To prevent failure of composite fan blades under impact forces, leading edge protective strips have been employed. In practice, these strips of stainless steel are wrapped around the leading edge. To model this device, we consider a beam bonded to the edge of an anisotropic plate (Figure 1). The effect of the beam will be to thwart the force of impact, thereby decreasing the normal stresses in the composite. However, we shall show that with such a reduction in normal stress, sizeable interface shear stresses can be induced.

With the introduction of a beam of thickness b on the edge of the composite plate, the Rayleigh wave behavior will depend on the ratio of the wavelength to thickness ratio of each Fourier component in the $x_1$ direction. Thus one should expect the Rayleigh wave speed to vary with $b/a$, the thickness to impact footprint ratio. In addition the Rayleigh wave will become distorted as it propagates.

To solve the edge strip problem the solution in the composite plate follows the same procedure as the no-strip case except for the boundary conditions on the edge. In place of the zero stress conditions on the edge we relate the edge stresses $t_{33}$, $t_{13}$ to the motion of the beam strip. If one considers a small element of the beam-strip along the $x_1$ direction, the momentum balance equations in the $x_1$, $x_3$ directions become, (for a plate of unit thickness)

$$pb \frac{\partial^2 U}{\partial t^2} = Eb \frac{\partial^2 U}{\partial x_1^2} + t_{13} \quad (2.1)$$
\[
\rho b \frac{\partial^2 W}{\partial t^2} = -E I \frac{\partial^4 W}{\partial x_1^4} + I \rho b \frac{\partial^4 W}{\partial x_1^4} - \frac{b}{2} \frac{\partial^2 t_{33}}{\partial x_1^2} + p f g + p f \phi
\]  
(2.2)

In these equations \( U, W \) are the \( x_1, x_3 \) displacements of the beam element at the half thickness, and \( t_{33}, t_{13} \) are the interface stresses.

We choose the compatibility conditions between the beam and plate displacements

\[
W = u_3, \quad \text{on } x_3 = 0.
\]  
(2.3)

\[
U = u_1 + \frac{b}{2} \frac{\partial u_3}{\partial x_1}, \quad \text{on } x_3 = 0
\]  
(2.4)

In the above equations \( b \) is the depth of the strip, \( E, I, I_0 \) are respectively the Young's modulus, moment of inertia and rotary inertia. Also \( p f(t) g(x_1) \) is the edge loading now applied to the outer protective strip surface.

The equations for the plate remain as in the free edge case and a solution is obtained by taking a Laplace transform on time and a Fourier transform on the space variable \( x_1 \). With nondimensionalization the solution in the plate is assumed in the form of (1.12). The transform of the plate displacements are

\[
\begin{bmatrix}
\tilde{u}_1 \\
\tilde{u}_3
\end{bmatrix} = C_1 \begin{bmatrix} 1 \\
\psi_{31}
\end{bmatrix} e^{-p_1 x_3} + C_2 \begin{bmatrix} 1 \\
\psi_{32}
\end{bmatrix} e^{-p_2 x_3}
\]  
(2.5)

where \( p_1, p_2 \) are defined in (1.10) and \( \psi_{31}, \psi_{32} \) are given in (1.11). \( C_1, C_2 \) are determined from the edge boundary conditions. However, in place of the free edge conditions (1.2) we use the equations of motion for the strip (2.1),
(2.2). \( C_1, C_2 \) are then solutions to the algebraic equations

\[
\begin{bmatrix}
G_1 & H_1 \\
G & H
\end{bmatrix}
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix}
= \begin{bmatrix} 0 \\ -p_0^*f_g \end{bmatrix}
\]

(2.6)

where

\[
G_1 = -k^2Ew + p_1C_55 - \rho ws^2 - \psi_{31} \left( i k^2 w^2 E/2 + i k C_55 + \rho w^2 s^2 i k/2 \right)
\]

\[
H_1 = -k^2Ew + p_2C_55 - \rho ws^2 - \psi_{32} \left( i k^3 w^3 E/2 + i k C_55 + \rho w^2 s^2 i k/2 \right)
\]

\[
G = -ikC_{13} + i k w p_1 C_{55}/2 + \psi_{31} \left( p_1 C_{33} + E w^2 k^4/12 + w k^2 C_{55}/2 \\
+ k^2 s^2 \rho w^3/2 + \rho w s^2 \right)
\]

\[
H = -ikC_{13} + i k w p_2 C_{55}/2 + \psi_{32} \left( p_2 C_{33} + E w^2 k^4/12 + w k^2 C_{55}/2 \\
+ k^2 s^2 \rho w^3/2 + \rho w s^2 \right)
\]

(2.7)

where \( w = b/\ell \), \( \rho \), \( E \), are nondimensionalized quantities and \( p_1, p_2 \), \( \psi_{31}, \psi_{32} \) are defined in (1.11).
III. NUMERICAL INVERSION

The inversions are accomplished by the Fast Fourier Transform (FFT) techniques [2], which consists of a transformation from Laplace to Fourier transforms, and a two-dimensional numerical inversion using the usual FFT algorithm. Notice the Laplace inversion formula

\[
f(t) = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} f(s) e^{st} ds
\]

Set \( s = C + ia \)

\[
f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(C + ia) e^{ct} e^{iat} da
\]

where \( C \) and \( \alpha \) are both real, and \( C \) is greater than the largest real part of all singularities of \( f(s) \). Numerically, the double Fourier transform (or inversion) has the following form [1]:

\[
f(x,t) = \frac{K_x K_t}{\pi^{2}NM} \sum_{I=1}^{N} \sum_{J=1}^{M} \bar{f}(I,J) e^{2\pi i \left[ \frac{(I-1)x}{N} + \frac{(J-1)t}{M} \right]}
\]

(3.2)
where \( N, M \) are the number of points in \( x \) and \( t \) direction respectively, and \( K_x, K_t \) are the corresponding half-frequency range.

For the present problem, the determination of \( C \) is through the following considerations:

The form of inversion integrals are, in general,

\[
I = \frac{1}{2\pi i} \int_{C-i\infty}^{C+i\infty} \frac{F(k_1, s)}{\Delta(k_1, s)} e^{-px} e^{st} ds 
\]

\[
p = p(k_1, s), \text{Re}(p) \geq 0
\]

(3.3)

and \( \Delta \) is given by (1.14). It is easy to see the singularities of the integral of (2.3) are:

a) Poles at \( \Delta = 0 \),

\[
\Delta = (p_2 - p_1) ik \left( C_{13} - \psi \psi \frac{C_{13}}{C_{33}} \right) + \left( \psi_1 - \psi_3 \right) (k^2 C + p_2 C) = 0
\]

(3.4)

implies

\[
(p_2 - p_1) \left[ C_{13} p_{12} \frac{(C_{13} + C_{33})^2 + C_{33} (C_{13} p_{12}^2 - C_{13} k^2)}{C_{13} k^2} \right] = 0
\]

(3.5)
Interpretation of this condition is best understood for the particular case of an isotropic material, i.e. for

\[ C_{11} = \lambda + 2\mu = C_{33}, \quad C_{13} = \lambda, \quad C_{55} = \mu, \]

choose \( C_{66} = C_{55} = \mu \)

It is easy to see that from (1.10), (1.11)

\[ \det = [\mu p^2 - (\mu k^2 + \rho s^2)] \left\{ (2\mu + \lambda) p^2 - [(\lambda + 2\mu) k^2 + \rho s^2] \right\} \]

\[ p_1^2 = (\mu k^2 + \rho s^2)/\mu, \quad p_2^2 = [(\lambda + 2\mu) k^2 + \rho s^2] / (\lambda + 2\mu) \]

\[ \psi_1 = i k_1 / p_1, \quad \psi_2 = -p_2 / i k_1, \]

Here \( [\mu/\rho]^{1/2} = v_s \), is the shear wave speed and \( [(\lambda+2\mu)/\rho]^{1/2} v_p \), is the longitudinal or pressure wave speed for isotropic materials.
Thus for the isotropic case that, \( \Delta = 0 \) implies

\[
4\mu p_2 + (p_1 + \frac{k^2}{p_1}) [\lambda + \frac{p_2^2}{k^2} (\lambda + 2\mu)] = 0
\]  

(3.6)

where \( p_1, p_2 \) are defined above. This is the equation for the Rayleigh wave speed \( v_R = (\text{is} / k)^{1/2} \) which is found as a real root of (3.6) (see e.g. [3]).

For the case \( \lambda = \mu (\text{Poissons ratio} = 0.25) \), \( v_R = 0.919 v_s \).

For the anisotropic case a computational scheme to calculate the zeroes of (3.5) has been written, (Figure 2).

b) Branch points: The branch points of the integrand in (3.3) are the same as those of the functions \( p_1(k, s), p_2(k, s) \), they are:

i) \( p_1 = 0, \) or \( p_2 = 0 \) which implies \( (C_{11} k^2 + \rho s^2) (C_{55} k^2 + \rho s^2) = 0 \)

i.e., \( k/s = \pm i\sqrt{\rho/C_{55}}, \pm i\sqrt{\rho/C_{11}} \) pure imaginary.

These correspond to longitudinal and shear wave speeds for an isotropic material

\[
p_{1,2} = \pm \sqrt{-B \pm \sqrt{B^2 - 4AC}/2A}
\]  

(3.7)

with \( A = C_{11} C_{55} > 0 \).
\[ B = -ps^2(C_{33} + C_{55}) + k^2[(C_{13} + C_{55})^2 - C_{11}C_{22} - C_{22}^2] \]

\[ C = (C_{11}k^2 + ps^2)(C_{55}k^2 + ps^2) \]

These branch points are those values of \( s/k \) which render \( B - 4AC = 0 \), and are branch points of second order. The distribution of these points is shown in Figure 3. It has been shown [5] that the contribution of these branch points to the value of the integrals (3.3) is important only when one considers the multi-reflected and refracted waves in layered media, or when the position of interest is very close to the impact origin. In this study we were more concerned with how the energy is propagated away from the impact point, which is mainly associated with surface waves, thus we ignored the contribution of these branch points. [5]

The contour of integration for the Laplace inversion is as shown in Figure 3. Notice the branch cuts are extended to negative infinity, in accordance with the requirement that \( C > \max \text{real part of the singularities} \). The requirement that real \( (p) > 0 \) also determines the correct sheet of the Riemann surfaces. Numerically, since the branch points are located at \( s/k = \text{constant} \) as \( k \) gets large \( C \) should be large, and the factor \( e^{ct} \) in the Laplace inversion expression will rise sharply to an unmanageable size. Since, in the last paragraph, we have noticed the contribution of the branch points is unimportant, a path \( \Gamma_2 \) is chosen to replace \( \Gamma_1 \) by the Cauchy's integral theorem. Notice the advantage of integration along \( \Gamma_2 \) is that \( C_0 \) is a positive constant independent of \( k \). The determination of optimal \( C_0 \) is discussed in [2]. Here, in order to minimize the aliasing and round off errors in numerical computations simultaneously, we choose
\[ C_o = \frac{2}{3M\lambda_t} \ln \left( \frac{\hat{g}}{\hat{f}} \right) \quad (3.9) \]

where \( \hat{g} = p_o/C_{\infty} \), \( \hat{f} = 10^{-6} \times \frac{1}{\lambda_t} \). \( \lambda_t \) is a small time interval, less than the impact contact time. Further details are given in Appendix A.
IV. RESULTS

A computer program has been written to calculate the stresses in a plate with an elastic beam on one edge under a transient impact load distribution along the edge. A program description, flow charts, input data formats, and sample printout of the program are contained in the appendices to this report. In this section we will summarize some of the results obtained from this computer program. These results were calculated for an anisotropic plate with effective elastic constants of 55% graphite fiber/epoxy matrix composite obtained from Reference 7 and summarized in the Table.

No-strip case:

The Rayleigh wave can be seen in the stress $t_{11}$ on the edge as shown in Figures 4, 5 for a graphite fiber-epoxy composite for layup angles $0, \pm 15^\circ$. After the initial contact time, the stress is observed to propagate with little change at a speed near the calculated Rayleigh speed (Figure 2). This wave can be observed in the computed output in the space-time ($x_1$, -t) plane Figure 7, as a band of non-zero values along a diagonal from the upper left to the lower right corner of the $x_1$, -t plane. Caution is urged in using this program since spurious waves can enter the calculations due to the periodic nature of the finite numerical Fourier Transform. These spurious waves are data bands which lie along diagonals from upper right to lower left. In other words, only disturbances emanating from the impact source in the upper left corner of the $x_1$ -t plane of Figure 7 should be valid. A computer map of the space time history of the edge impact stress is shown in Figure 6. A contact time of 35 µsec and contact length 2 cm was used in these calculations.
As is characteristic of surface wave effects, the stresses due to impulsive loading on the edge decrease with distance from the edge. This is shown in Figure 8 for two different layup angles. The normal stress $t_{33}$ appears to decrease to about 1/4 of its value on the surface at a depth equal to one half of the loading length $a$. The rate of decay from the edge depends on the layup angle. Another characteristic of edge impact is the development of tension in the normal stress $t_{33}$ under the impact point. This is shown in Figure 9. Thus while the compression part decreases with distance from the edge a tension tail develops in the wave.

The effect of layup angle on the impact stresses can be seen in Figures 10, 11. The stress $t_{11}$ at the edge is larger than the impact pressure and decreases as the layup angle goes from $0^\circ$ to $+45^\circ$ (Figure 10).

Below the surface or edge, the peak normal stress $t_{33}$ at $x_1 = 0$ is a minimum for layup angles near $\pm 30^\circ$, while the shear stress $t_{13}$ increases as the fiber angle goes from $0^\circ$ to $+45^\circ$ (Figure 10).

An unexpected result is the shift of the maximum normal stress $t_{33}$ to points off the impact axis $x_1 = 0$ for layup angles greater than about $\pm 30$. A pronounced peak in $t_{33}$ versus $x_1$ beyond the impact pressure footprint, can be seen in Figure 11 for $\pm 45^\circ$ layup angle.

Impact protection strip case:

The effect of bonding an edge impact protection strip to the half plane is shown in Figures 12-16. In Figures 12, 13, the increase in the thickness of a steel strip produces a decrease in the interface stresses.
but creates an interface shear stress at the strip-composite interface. This shear stress reaches a maximum for strip thickness less than the impact footprint length and decreases for greater strip thicknesses. Thus, if the strip is too thin, debonding can occur under impact due to induced interface shear.

In Figure 14 one can see that increasing the strip thickness decreases the peak normal stress $t_{33}$ and redistributes the load over a longer length under the strip. However, while the peak compression stress is decreased by the strip, tension is created which could also produce debonding of the strip from the composite.

For the no-strip case a Rayleigh wave was seen to propagate along the edge relatively unperturbed (Figures 4, 5). With the strip present, this wave becomes distorted as time increases. In fact, the beam-strip boundary conditions introduce dispersion in the edge waves which make the Rayleigh edge wave velocity dependent in the effective wave length of the disturbance.

Finally in Figure 16 shear stress distributions along the strip-composite interface are shown for a thickness near the shear peak ($b/a = 0.25$) and another for $b/a = 2.0$. In the latter case the shear is distributed over a larger length resulting in a lower peak stress. Also the strip delays the time of maximum shear from $1/2$ to $3/4 T_o$.
Summary of Results

An analytical-numerical method has been developed to solve the response of a composite plate with a bonded edge strip to in-plane impact type forces on the edge. Results of computer simulations reveal the following:

1) Rayleigh edge waves can propagate away from the impact site with tension and compression up to values of the impact pressure, depending on layup angle.
2) Normal to the edge, the initial peak compression pulse decreases as it propagates into the plate but a tension tail develops as it propagates away from the impact site.
3) The edge stress $t_{11}$ under impact is decreased as the fiber layup angle goes from $0^\circ$ to $\pm 45^\circ$.
4) Protection strips of thickness less than half the impact length can develop large interface shear under impact.
5) The normal and edge stresses $t_{33}$, $t_{11}$ at the edge can be decreased significantly by protection strips of thickness greater than the half impact length.
REFERENCES


**TABLE 1.** - STRESS-STRAIN COEFFICIENTS FOR 55 PERCENT GRAPHITE FIBER-EPOXY MATRIX COMPOSITE

[All constants to be multiplied by $10^6$ psi; data obtained from ref. 7.]

<table>
<thead>
<tr>
<th>Layup</th>
<th>0° Layup</th>
<th>±15° Layup</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>27.95 0.3957 0.3957 0 0 0</td>
<td>24.56 0.4000 1.986 0 0 0</td>
</tr>
<tr>
<td></td>
<td>1.170 0.4601 0 0 0</td>
<td>1.170 0.4558 0 0 0</td>
</tr>
<tr>
<td></td>
<td>1.170 0 0 0</td>
<td>1.374 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0.3552 0 0</td>
<td>0.3552 0 0</td>
</tr>
<tr>
<td></td>
<td>0.7197 0</td>
<td>0 0</td>
</tr>
<tr>
<td>±30° Layup</td>
<td>0.3552</td>
<td>2.310 0</td>
</tr>
<tr>
<td></td>
<td>±45° Layup</td>
<td>0.3552</td>
</tr>
<tr>
<td>±30° Layup</td>
<td>16.48 0.4118 5.167 0 0 0</td>
<td>8.197 0.4279 6.758 0 0 0</td>
</tr>
<tr>
<td></td>
<td>1.170 0.4400 0 0 0</td>
<td>1.170 0.4279 0 0 0</td>
</tr>
<tr>
<td></td>
<td>3.093 0 0 0</td>
<td>8.179 0 0 0</td>
</tr>
<tr>
<td></td>
<td>0.3552 0 0</td>
<td>0.3552 0 0</td>
</tr>
<tr>
<td></td>
<td>5.491 0</td>
<td>0 0</td>
</tr>
<tr>
<td>±45° Layup</td>
<td>0.3552</td>
<td>7.082 0</td>
</tr>
</tbody>
</table>
APPENDIX A

The Determination of Parameter $C_0$

Consider the Laplace inversion of a function $g(t)$ as

$$g(t) = \frac{1}{2\pi i} \int_{C_0-i\infty}^{C_0+i\infty} \tilde{g}(s) e^{st} \, ds$$  \hspace{1cm} (A.1)

and let $s = C_0 + i\omega$, $C_0$ and $\omega$ are real. We can change (A.1) into the Fourier inversion formula

$$g(t) = \frac{1}{2\pi} e^{C_0 t} \int_{-\infty}^{\infty} \tilde{g}(C_0 + i\omega) e^{i\omega t} \, d\omega = e^{C_0 t} x(t)$$  \hspace{1cm} (A.2)

which is then inverted by the Fast Fourier Transform technique. In the FFT scheme, a continuous function $\tilde{g}(C_0 + i\omega)$ is discretized and the infinite interval of integration is truncated. The error due to truncation depends on each problem but doesn't depend on $C_0$, thus in the determination of $C_0$, we will assume the truncation error is negligible.

It is shown [2] that the discretizing of the transform in one domain will cause aliasing error in the other domain, e.g. sampling $\tilde{g}$ at $N$ points in a frequency interval, $0 < \omega < \Omega$, will produce a transformed function $x_p(t)$ which is periodic and which differs substantially from $x(t)$ for large enough $t$. For even $x(t)$ this difference can be shown [2] to be given by,

$$x_p(t) = x(t) + x(t-T)$$

for $0 < t < T/2$ where $T = N/\Omega$. 


It has been shown [2] also that the aliasing error is approximated by $E_a(t) = e^{-C_o(T-2t)}g(T-t)$ for the Laplace inversion (A.2).

Notice the aliasing error is a decreasing function of $C_o$.

The other source of error is of course the round off error in computation. Since we multiply the resulting $x(t)$ by $e^{C_0 t}$ to get $g(t)$, the rounding error is of the form

$$C_0 T$$
$$E_r(t) = e^{C_0 t} r(t).$$

The error bounds are then

$$\epsilon_1 = \max_{0 \leq t \leq T} |E_a(t)| = e^{-C_0(T-2t)} \max_{0 \leq t \leq T} |g(T-t)|$$

$$\epsilon_2 = \max_{0 \leq t \leq T} |E_r(t)| = e^{C_0 t} \max_{0 \leq t \leq T} |r(t)|$$

Equating $\epsilon_1$ and $\epsilon_2$, the optimal $C_o$ is then

$$C_o = \frac{\ln \left( \frac{\max g(T-t)}{\max r(t)} \right)}{T-T}$$

Choosing $T = T/4$, therefore

$$C_o = \frac{4}{3T} \ln(\hat{g}/\hat{r}), \quad \hat{g} \equiv \max g(T-t), \quad \hat{r} \equiv \max r(t)$$

Notice, empirically, $\hat{r} \approx 2 \frac{N}{T} 10^{-6}$ on single precision IBM 360 systems.
A FLOW CHART OF THE PROGRAM

1. Main Program

START

Read in material data, strip data

Calculate wave speeds

Calculate $X_{\text{max}}$ and $T_{\text{max}}$

Non-dimensionalization

Calculate Laplace Inversion Parameter

Calculate singularities including Rayleigh speed

Calculate the transformed expressions

A Test Case?

Yes

Inversion

CALL Fourt

WRITE OUT

No

CALL Calcul

Obtain transformed expressions for disp. and stresses

CALL Fourt for inversion

Plot the Relative Values

Repeat for different $x_3$

END
3. Subroutine calcul

START

CALCULATE $p_1$, $p_2$, $\psi_1$, $\psi_2$

CALCULATE forcing

CALCULATE $\Delta$

Write $p_1$, $p_2$, $C_1$, $C_2$, $\psi_1$, $\psi_2$, $k_1$, $k_2$
on Disc

RETURN TO MAIN
APPENDIX C

NOTES ON COMPUTER PROGRAM

The choice of scales is very essential to the success of the present computational method. It is noticed that the accuracy depends on the number of points employed and the range of frequency spectra covered. Considering limitations of both computer storage and time, a time-space grid of 32 x 64 points was chosen for \( t > 0, \ x > 0 \). Thus, the non-dimensionalization of all equations and quantities are both necessary and important to the obtaining of meaningful data from the limited grid size.

The numerical inaccuracies introduced have several origins:
1. Theoretically, error has been introduced by the neglecting of the outer branch points contributions. It has been shown, for isotropic cases, the contributions of these branch points behaved like \( r^{-2} \) at large distance from a delta function loading at the origin \( r = 0 \). Compared with the \( r^{-1} \) decreases of the contribution of the residue. Thus, for small \( r \), or near the origin, the errors might be significant. An asymptotic form of the behavior of small \( r \) has been deduced for a simple delta loading at origin on an isotropic half-space [ ]. It is shown the error thus introduced is of the order of 5% maximum response in stress.
2. The aliasing error introduced through the periodizing of the functions.
3. The round-off error in Laplace inversion along with aliasing error.

have been discussed in the determination of \( C_0 \) (Appendix A). It is
found that error 3 is more serious of the two. Hence in calculations the maximum non-dimensionalized time should be restricted to below 6 or 8 for reasonably good results.

4. Errors due to reflections at the boundary. Since the space grid is finite, it has been observed that whenever a wave hit the boundary of chosen space, a sizable numerical error will start propagating in, as if the wave were reflected from the boundary. The basic reason is due to the periodization of the space \((x_1)\) domain. In computation, this error should be avoided. To correctly determine the extent of the space \((x_1)\) domain, a priori recognition of significant wave speed (at which most of the energy travels) is important. Usually an estimation will be sufficient. Then the proper nondimensionalizing constants can be chosen.
APPENDIX D
PROGRAM INPUT AND OUTPUT

A. Method:

FFT algorithm for numerical inversion

B. Input Data Cards:

Card 1. NTEST, NSTRIP, NP

NTEST = 1 A test program for FFT (2-D) (beam).

NTEST = 0 The present program

NSTRIP = 1 With strip

NSTRIP = 0 No strip

NP = 1 Calculate: \( u_1 \) only

NP = 2 \( u_1, u_3 \)

NP = 3 \( u_1, u_3, t_{33} \)

NP = 4 \( u_1, u_3, t_{33}, t_{11} \)

NP = 5 \( u_1, u_3, t_{33}, t_{11}, t_{13} \)

Card 2. CC(I) I = 1, 9, RHO, ANGLE (8E10,4)

CC(I=1,9) Material constants of composite, in the order

\( C_{11}, C_{22}, C_{33}, C_{44}, C_{55}, C_{66}, C_{12}, C_{13}, C_{23} \) (psi)

RHO Density of composite (\( \text{g/cm}^3 \))

ANGLE (degrees) lay-up angle

Card 3. VEL, DM, E1, ANU, DEN (8E10,4)

VEL - Velocity of incoming particle m/sec

DM - Diameter of impacting object cm

E1 - Young's Modulus of impacting object (psi)

ANU - Poisson's ratio of impacting object

DEN - Density of impacting object (gm/cc)
Card 4. NSTRES, NK3, DX3  (215, F10,3)

NSTRES = 1

NK3: Number of steps in $x_3$ direction

DX3: $\Delta x_3/\lambda$, step size in $x_3$ direction

Photo elastic fringe order computer map in the x-t plane.

Card 5. RO, W, ES  (3E10,4)

RO: density of strip beam (g/cm$^3$)

W: depth of strip beam (cm)

ES: Young's modulus of strip beam (psi)

C. Output:

1. Test problem: Appendix III

2. Values of displacements

   1 : $u_1$
   2 : $u_3$

   (stress) 3 : $t_{33}$
   4 : $t_{13}$
   5 : $t_{11}$

3. Relative magnitude computer maps of displacements and stresses.

\[ \rightarrow x_1/\lambda \]
\[ \downarrow \]
\[ t/t^* \]
This program calculates the elastic response of an anisotropic plate to an in-plane edge impact force on \( x_3 = 0.0 \). When \( n_{\text{strip}} = 1 \), the program places an impact protection strip, or elastic beam on the edge. The impact force is a half sine function in time and is non-zero for \( 0 < t < t_0 \), (microsec). The force is distributed along the edge as \( P_0 (1 - x_1^2) \), where \( x_1 \) is normalized by the half width of the impact contact length.

The method employs a Fourier transform in the edge direction \( x_1 \) and a Laplace transform in the time dimension. The transform of the forcing function is given in the subroutine \( \text{CALCUL} \) and the solution is obtained using a 2-dimensional Fast Fourier inversion routine called 'FOURT'.

The output for a given depth \( x_3 \) consists of displacements \( u_1, u_3 \) and stresses \( t_{33}, t_{11}, t_{13} \) in the \( x_1 \)-time plane. The stresses are normalized by the elastic constant \( c(6,6) \).

**Impact Force**

\[ \]
W, THICKNESS TO WIDTH RATIO OF BEAM
C IS, YOUNG'S MODULUS FOR BEAM
C WL--FOURIER WAVELENGTH (CM) OF THE ORDER OF TO OR LESS
C WT--FOURIER WAVE PERIOD (SEC) OF THE ORDER OF TO OR LESS
C CHOICE OF WL,WT DETERMINES DX,DT--DX=WL/2,DT=WT/2
C FG, TRANSFORM OF NORMALIZED FORCING FUNCTION F(X1)*G(T)
C THIS IS PROVIDED IN PROGRAM BUT CAN BE CHANGED BY THE USER
C
C OUTPUT DATA
C TO,TC CONTACT TIME (SEC,1.E-6 SEC ) (FROM HERTZ THEORY)
C AC,A, HALF THE IMPACT CONTACT LENGTH,CM
C PO, MAX IMPACT FORCE FROM HERTZ THEORY, NEWTONS
C E, WAVE SPEED IN PLATE SORT(C66/RHO)
C OR WAVE SPEED IN BEAM STRIP,SORT(E1/DEN) UNITS CM/SEC
C CL=SORT(C11/RHO) LONGITUDINAL WAVE SPEED ALONG EDGE IN PLATE
C CS=SORT(C55/RHO), SHEAR SPEED ALONG EDGE OF PLATE,CM/SEC
C CR, RAYLEIGH WAVE SPEED ALONG FREE EDGE OF PLATE
C
C DX,DT SPACE TIME INCREMENTS IN X1-T SPACE UNITS--CM AND SEC
C DATA(I,J) ,NORMALIZED TRANSFORM OF ONE OF DISPL. OR STRESSES
C --BEFORE CALL FOURT,AFTER CALL FOURT DATA IS A 2 DIM MATRIX OF
C DISPL. OR STRESSES IN X1-T SPACE, DEPENDING ON VALUE OF K
C IN THE LOOP 'DO 4 K=1,NP'
C U1,U3, NORM. DISPL. IN PLANE OF PLATE (E.G. U1/A0)
C T33,T11,T13, NORM. STRESSES (E.G. T33/C66)
C --NOTE-- T33 ON X3=0 SHOULD REPRODUCE THE FORCING FUNCTION
C WHEN THERE IS NO STRIP
C --NOTE-- AS A CHECK T13=0 ON X3=0 WHEN THERE IS NO STRIP
C THE FRINGE ORDER MAP PLOTS THE DIFF IN PRINCIPAL STRESSES AND
C MAY BE USED TO COMPARE WITH PHOTOELASTIC EXPERIMENTS OR TO
C LOOK FOR POINTS OF MAX IN PLANE SHEAR STRESSES
C
C********************
C
C THIS PROGRAM HAS BEEN WRITTEN BY F.MOON AND C-K KANG UNDER
C A GRANT TO PRINCETON UNIVERSITY FROM THE NASA LEWIS RESEARCH
C LAB.
C
C********************
C
C --NOTE TO THE USER-- IN THE OUTPUT MAPS OF STRESSES OR DISPL.,
C YOU WILL NOTICE BANDS OF SIMILAR NUMBERS RUNNING FROM THE
C UPPER LEFT CORNER TO THE LOWER RIGHTCORNER -- THESE ARE WAVES
C WHICH EMINATE FROM THE IMPACT POINT--HOWEVER- WAVES RUNNING
C FROM RIGHT UPPER TO LEFT LOWER CORNER ARE SPURIOUS DUE TO THE
C DESCRIPTENESS OF THE NUMERICAL FOURIER INVERSION PROGRAM
C
C --ALSO DATA FOR TIMES NEAR MAX AT THE BOTTOM OF THE MAPS ARE
C USUALLY SPURIOUS AND SHOULD NOT BE USED
C
0001 COMMON /MMC/ D,DK1
0002 COMMON /MC/DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRTP,AO
0003 DIMENSION DATA(128,32),MM(40),CC(9)
0004 DIMENSION NN(2)
0005 DIMENSION DATA(40)
0006 DIMENSION CIDATA(40)
0007 DIMENSION FRNGE(64,32)
0008 DIMENSION T11(64,32),T33(64,32),T13(64,32)
0009 CCMPLEX DATA,S
0010 CCMPLEX P1,P2,S1,S2,C1,C2,B,C,D,SI
0011 CCMPLEX DK2,FG,SLAP,DK1
C*** READ IN AND WRITE OUT DATA AND PARAMETERS
C*** RHO,A0,TO MUST BE IN C.G.S. UNITS, PO MUST BE CONSISTENT WITH CC
CALL INDUMP
READ (5,102) NTEST,NSTRIP,NP
READ (5,100) CC,RHO,ANGLE
WRITE(6,490) CC
WRITE(6,491) RHO,ANGLE
READ(5,100) VEL,DM,E1,ANU,DEN
PO=CC(6)
C11= CC(1)-CC(7)**2/CC(2)
C33= CC(3)-CC(9)**2/CC(2)
C13= CC(8)-CC(7)*CC(9)/CC(2)
C55= CC(5)
PI= 3.14159265
RHO= RHO/6.895*1.E-4
READ (5,101) NSTRES,NK3,DX3
WRITE (6,504) NSTRES,NK3,DX3
SI= CMPLX(0.,1.)
C******** CALCULATE THE IMPACT CONTACT TIME, RADIUS, AND PRESSURE
C******** BASED ON HERTZ CONTACT THEORY
R=DM/2.0
R=R*1.E-2
E1=E1*6895.0
E2=CC(2)*6895.0
DEN=DEN*1.0E3
AMASS=4./3.*PI*R**3*DEN
AK2=4./3.*SQRT(R)*E1/((1.0-ANU*ANU)+E1/E2)
ALF=5./4.*AMASS*VEL*VEL/ALF
ALF= (ALF)**0.4
TC=2.943*ALF/VEL
F0=1.14*AMASS*VEL*VEL/ALF
A=SQRT(ALF*R)
A=1.0E2*A
TC=TC*1.0E6
WRITE(6,710) VEL,TC,A,F0
C**************,*******************
C*** CALCULATE THE NON-DIMENSIONAL PARAMETERS
EE= CC(6)
E= SQRT(EE/RHO)
RE= RHO
C** DEFINE TRANSFORM SPACE AND DISTANCE-TIME SPACE
C** UNIT DISTANCE -- CM.
A0=A
T0=TC*1.0E-6
TEST CASE TO== 35E-6, A0=1 CM
C
A0=1.0
T0=35.0E-6
AL=A0
C** UNIT TIME--SEC.
T0=AL/2
C** SMALLEST WAVELENGTHS
WL=A0/1.5
WT=TC/10.0
C*** DIMENSIONS OF TRANSFORM SPACE
XK=2.0*PI*AL/WL
TK=2.0*PI*TE/WT
C*** NCNDIMENSIONALIZE CONSTANTS
AO=AO/AL
T0=T0/TE
DT=WT/2.0
DX=WL/2.0
WRITE(6,621) WT,WL,TK,XK
C CALCULATE RAYLEIGH WAVE SPEED
X1=0.0
X2=C55
N=0
DO 90 I=1,100
N=N+1
X=(X1+X2)/2.0
D1=(C55-X)/(C11-X)
D1=SQRT(D1)
D2=(C13)**2-C33*(C11-X)
D3=C55*C33
D3=SQRT(D3)
F=D1*D2/D3
F=X+F
IF(F) 81,82,83
81 X1=X
GO TO 92
82 GO TO 91
83 X2=X
92 CONTINUE
F1=F/C55
DIFF=ABS(F1)-1.0E-4
IF(DIFF) 91,91,90
90 CONTINUE
91 CONTINUE
WRITE(6,701) C11,C33,C55,C13
701 FORMAT(/,10X,6H C11 =,E12.4,6H C33 =,E12.4,6H C55 =,E12.4,
16H C13 =,E12.4,/) CL=C11/RHO
CL=SQRT(CL)
CS=C55/RHO
CS=SQRT(CS)
CR=X/RHO
CR=SQRT(CR)
RDS=CR/CS
WRITE(6,702) CL,CS,CR,RDS,N
702 FORMAT(/,10X,12HLONG SPEED =,E12.4,13HSHEAR SPEED =,E12.4,1
1,10X,16HRAYLEIGH SPEED =,E12.4,8H CR/CS =,E10.5,5X,15)
C*** CONSTANTS FOR STRIP CASE
READ (5,100) RO,W,ES
WRITE(6,521) RO,W,ES
RO= RO/6.895*1.0E-4
IF (RO) 51,51,50
50 CONTINUE
E= SQRT(ES/RO)
WRITE (6,510) E
51 CONTINUE
RO = RO/RE
ES = ES/EE
W= W/AO/AL

C********************************************************
205 CONTINUE
C11 = C11/EE
C13 = C13/EE
C33 = C33/EE
C55 = C55/EE
PO = PO/EE
RHO = RHO/RE
FG1 = PO*TO

C********************************************************
206 CONTINUE
N = NN(1)
M = NN(2)

C*** CALCULATE THE LAPLACE INVERSION PARAMETER
WT = WT/TE
RH = P0*WT*1.E6
CH = 2./3./M/WT*ALOG(RH)
CLAP = CH

C*** CALCULATE THE SECOND ORDER BRANCH POINTS
A = RHO*(C33-C55)**2*RHO
E = -2.*RHO*((C33+C55)*(C13*C13+2.*C13*C55-C11*C33)+2.*C33*C55
1*(C11+C55))
P = (C13*C13+2.*C13*C55-C11*C33)**2-4.*C11*C33*C55*C55
BS = 1.DO*E*E-1.DO*4.*A*P
D = CSQRT(BS)
P1 = .5/A*(-E+D)
P2 = (P1/A)
P1 = CSQRT(P1)
P2 = CSQRT(P2)

WRITE (6,509)
WRITE (6,506) P1, P2

C*** GENERATE THE TRANSFORMED EXPRESSIONS
REWIND 2

DO 1 I = 1,N
DK1 = 2.*XK/N*(I-.5)-XK
DO 1 J = 1,M
DK2F = 2.*TK/M*(J-.5)-TK
SLAP = CMPLX(CLAP,DK2F)
DK2 = -SI*SLAP

IF (NTEST.EQ.1) GO TO 211

IF (NSTRIP.EQ.1) WRITE (6,506) P1, P2

C*** TEST FOR A STRING
DATA(I,J)=-FG/(SLAP*SLAP+DK1*DK1)
GO TO 1

CALL CALCUL(0)

CONTINUE
IF (MTEST,NE,1) GO TO 202
WRITE (6,507)
CALL FOUTT(DATA,NN,2,1,1,0)
DO 15 KI= 1,N2
B= -SI*PI/N*(N-1)*(KI-1)
DO 15 KJ= 1,M
C= -SI*PI/M*(M-1)*(KJ-1)
T= PI/TK*(KJ-1)
DATA(KI,KJ) = DATA(KI,KJ) *XK*TK/PI/PI/N/M*CEXP(B)*CEXP(C)
15 DATA(KI,KJ) = DATA(KI,KJ)*EXP(CLAP*T)
WRITE (6,506) ((DATA(I,J), J= 1,M), I= 1,N2)
GO TO 203

C**** MAIN LOOP FOR DIFFERENT DEPTHS X3
DO 6 KJ= 1,NK3
X3= (K3-1)*DX3

C**** LOOP FOR CALCULATING DISPLACEMENTS AND STRESSES AT A GIVEN DEPTH
DO 4 K= 1,NP
WRITE(6,680)
WRITE (6,501) X3
REWIND 2
DO 7 I= 1,N
DO 7 J= 1,M
READ (2) P1,P2,C1,C2,S1,S2,DK1,DK2
B= CEXP(-X3*P1)
C= CEXP(-X3*P2)

C**** DISPLACEMENTS
IF (K.EQ.1) DATA(I,J) = C1*H+C2*C
IF (K.EQ.2) DATA(I,J) = C1*S1*B+C2*S2*C

C**** STRESSES
IF (K.EQ.3)
1DATA(I,J) = (C13*SI*DK1-C33*P1*S1)*C1*B+(C13*SI*DK1-C33*P2*S2)*C2*C
IF (K.EQ.4)
1DATA(I,J) = (C11*SI*DK1-C13*P1*S1)*C1*B+(C11*SI*DK1-C13*P2*S2)*C2*C
IF (K.EQ.5)
1DATA(I,J) = C55*((-P1*SI*DK1*S1)*C1*B+(-P2*SI*DK1*S2)*C2*C)
IF (K,LT,6) GO TO 7

C**** DISPL. FOR A BEAM ON AN ELASTIC FOUNDATION
EP=0.1

C**** FORCING FUNCTION
FG= F1*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI*DK2*DK2*T0*T0)*
14./(DK1*A0)**2*CSIN(DK1*A0)/DK1-A0*CCOS(DK1*A0)

D=W*DK1/12.*((ES*W*W*DK1*DK1-BO*W*W*DK2*DK2)-BO+W*DK2*DK2)
1.*EP

IF (K.EQ.6)
1DATA(I,J)=FG/D

7. CONTINUE

CALL FOUTT(DATA,NN,2,1,1,0)
DO 3 KI= 1,N2
B= -SI*PI/N*(N-1)*(KI-1)
DO 3 KJ= 1,M
C= -SI*PI/M*(M-1)*(KJ-1)
T= PI/TK*(KJ-1)
DATA(KI,KJ) = DATA(KI,KJ) *XK*TK/PI/PI/N/M*CEXP(B)*CEXP(C)
E= CLAP*T
3 DATA(KI,KJ) = DATA(KI,KJ)*EXP(E)
**ORTTRAN IV G LEVEL 21**

**MAIN**

**DATE = 74186**

11/28/2

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**C*** CALCULATE THE PHOTOELASTIC FRINGE ORDER

0197 DO 290 I=1,40
0198 DO 290 J=1,32
0199 IF (K.EQ.3) T33(I,J)=REAL(DATA(I,J))
0200 IF (K.EQ.4) T11(I,J)=REAL(DATA(I,J))
0201 IF (K.EQ.5) T13(I,J)=REAL(DATA(I,J))
0202 IF (K.EQ.5) GO TO 280
0203 GO TO 290
0204 280 CONTINUE
0205 FNGE=(T11(I,J)-T33(I,J))**2+4.0*T13(I,J)*T13(I,J)
0206 FRNGE(I,J)=SQRT(FNGE)
0207 290 CONTINUE
0208 214 WRITE (6,511) K

**C*** FIND THE MAXIMUM VALUE

0209 RS=1.E-3
0210 DO 14 I=1,40
0211 DO 14 J=1,32
0212 S=DATA(I,J)
0213 TP=REAL(S)/RS
0214 IF (ABS(TP).LT.1.) GO TO 14
0215 RS=REAL(S)
0216 14 CONTINUE
0217 WRITE (6,516) RS

**C*** PRINT REAL PART OF DISPL. AND STRESSES

0218 NIJ=NIJ/4
0219 NIJ=10
0220 DO 310 I=1,NIJ
0221 I1=I-1
0222 WRITE(6,600) I1
0223 DO 300 J=1,32
0224 SR=REAL(DATA(I,J))/RS
0225 CIDATA(J)=SR
0226 300 CONTINUE
0227 WRITE(6,620) (PDATA(L),L=1,M)
0228 WRITE(6,650)
0229 WRITE(6,620) (CIDATA(L),L=1,11)
0230 310 CONTINUE
0231 WRITE(6,680)
0232 IF (K.EQ.1) WRITE(6,630)
0233 IF (K.EQ.2) WRITE(6,631)
0234 IF (K.EQ.3) WRITE(6,632)
0235 IF (K.EQ.4) WRITE(6,633)
0236 IF (K.EQ.5) WRITE(6,634)
0237 IF (K.EQ.6) WRITE(6,635)
0238 WRITE(6,640) DX,DT

**C*** PLOT THE RELATIVE VALUES

0241 DO 12 J=1,M
0242 DO 12 I=1,40
0243 S=DATA(I,J)
0244 MM(I)=REAL(S)/RS*100
0245 WRITE (6,515)
0246 12 CONTINUE

**C*** PLOT A MAP OF PHOTOELASTIC FRINGE ORDER

0247 IF (K.LT.5) GO TO 4
RS = 1.8-3
DO 312 I = 1, 20
DO 312 J = 1, 20
SS = FRNGE(I, J)
TP = SS / RS
IF (ABS(TP) .LT. 1.) GO TO 312
RS = SS
312 CONTINUE
WRITE (6, 680)
WRITE (6, 516) RS
WRITE (6, 690)
DO 320 J = 1, 32
DO 315 I = 1, 40
SS = FRNGE(I, J)
315 MM(I) = SS / RS * 100
WRITE (6, 515) MM
320 CONTINUE
4 CONTINUE
6 CONTINUE
100 FORMAT (8E10.4)
101 FORMAT (215, 3F10.3)
102 FORMAT (315)
490 FORMAT (5X, 55H ELASTIC CONSTANTS C11, C22, C33, C44, C55, C66, C12, C13, C32, 12, /, 9E12.4, /)
491 FORMAT (5X, 25H DENSITY OF PLATE GM/CC-- , P10.4, 5X, 20H FIBER LAYUP ANGLE-- , P10.4)
500 FORMAT (2F12.4, 4E15.4)
501 FORMAT (5H X3 = P10.4)
502 FORMAT (10H STRESSES I3/(8E15.7))
503 FORMAT (5X5H DATA, 15X10H PARAMETERS /8E15.7)
504 FORMAT (5X8H STRESS = I3, 5X4HNX3 = I4, 5X4HDX3 = P10.4, /)
505 FORMAT (14H SIMPLE POLES)
506 FORMAT (8E15.7)
507 FORMAT (15H THIS IS A TEST )
508 FORMAT (30H LAPLACE INVERSION PARAMETER = P12.4)
509 FORMAT (28H SECOND ORDER BRANCH POINTS)
510 FORMAT (5X, 27H LONG. WAVE SPEED IN BEAM = P12.3, 7H CM/SEC)
511 FORMAT (14H DISPLACEMENTS I4)
512 FORMAT (24H RAYLEIGH SPEED CS/CR*1 )
513 FORMAT (8P15.8)
514 FORMAT (/50X 10H WITH STRIP)
515 FORMAT (2X, 40I3)
516 FORMAT (16H MAXIMUM VALUE = E15.7)
520 FORMAT (/5X, 23H IMPACT PRESSURE (PSI) = P12.4, 5X, 25H 1/2 CONTACT LENGTH (CM) = P12.4, 5X, 23H IMPACT DURATION SEC = P12.4, /)
521 FORMAT (/5X, 18H DENSITY OF BEAM = P12.4, 5X, 17H NORMAL THICKNESS = , .P12.4, 5X, 15H BEAM MODULUS = , P12.4, /)
522 FORMAT (/5X, 18H DENSITY OF BEAM = P12.4, 5X, 17H NORMAL THICKNESS = , .P12.4, 5X, 15H BEAM MODULUS = , P12.4, /)
524 FORMAT (/5X, 16H DISPLACEMENT U1, /)
525 FORMAT (/5X, 16H DISPLACEMENT U3, /)
526 FORMAT (/5X, 11H STRESS T33, /)
527 FORMAT (/5X, 11H STRESS T11, /)
0298 634 FORMAT(30X,11H STRESS T13,/) 0299 635 FORMAT(30X,41H TEST-DISPL. OF BEAM ON ELASTIC FOUNDATION,/) 0300 640 FORMAT(20X,14H-->> X1, DX=, F12.4,/, 20X, 2H 1,/, 20X, 2H 1,/, 20X, 12H 1,/, 20X, 2H V,/, 20X, 2H V,/, 20X, 9H TIME, DT=, E12.4,/) 0301 650 FORMAT(10H IMAG PART) 0302 680 FORMAT(1H1) 0303 690 FORMAT(/, 20X, 17H FRINGE ORDER MAP,/, 20X, 31H SQRT((T11-T33)**2+4.*1T13*T13)) 0304 710 FORMAT(5X, 6H VEL= , F12.4, 5X, 5H TO= , F12.4, 5X, 4H A= , F12.4, 5X, 15H PO= , F12.4,/) 0305 203 STOP 0306 END
SUBROUTINE CALCUL(NPOLE)

COMMCN /MC/ D,DK1
COMMON /MC/ DK2,FG1,T0,C11,C13,C33,C55,RHO,RO,W,ES,NSTRIP,A0

COMMON /MC/ D,DK1
COMMON G0,G01,H0,H01,G10,G11,H10,H11

COMPLEX G,H,G1,G10,G11,H10,H11

COMPLEX D,DK1,DK2,P1,P2,S1,S2,C1,C2,SI,B,C,D,FG
COMPLEX*16 BS

PI = 3.14159265
SI = CMPLX(O.,1.)

A = C33*C55
B = C33*(RHO*DK2*DK2-C11*DK1*DK1)+C55*(RHO*DK2*DK2-C55*DK1*DK1)+
1DK1*DK1*(C13+C55)**2
C = (RHO*DK2*DK2-C11*DK1*DK1)*(RHO*DK2*DK2-C55*DK1*DK1)

BS = 1.0*P1*A-1.0*4.*A*C

D = CLSQBT(BS)
P1 = 5/A*(-B+D)
P2 = C/P1/A

IF (REAL(P1).LT.0.0.AND.NPOLE.NE.2) P1 = -P1

P2 = CSQRT(P2)

IF (REAL(P2).LT.0.0.AND.NPOLE.NE.2) P2 = -P2

S1 = -SI/DK1 /
            P1/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P1*P1)
S2 = -SI/ DK1 /
            P2/(C13+C55)*(RHO*DK2*DK2-C11*DK1*DK1+C55*P2*P2)

IF (NPOLE.EQ.2) GO TO 201

C*** FORCING FUNCTION

FG = PI*FG1*(1.+CEXP(-SI*DK2*T0))/(PI*PI-DK2*DK2*T0*T0)*
14./(DK1*AO)**2*(CSIN(DK1*AO)/DK1-AO*CCOS(DK1*AO))

201 IF (NSTRIP.EQ.1) GO TO 202

D = (SI*DK1*S1-P1)*(SI*DK1*C13-P2*C33*S2)-
(SI*DK1*S2-P2)*(SI*DK1*C13-P1*C33*S1)

IF (NPOLE.EQ.2) RETURN

C*** MATEIX FOR STRIP PROBLEM

B1 = W*W*(ES*DK1*DK1-RO*DK2*DK2)*DK1*DK1/12.0-RO*DK2*DK2
B2 = -W*(ES*DK1*DK1-RO*DK2*DK2)

G10 = (SI*DK1*S1-P1)
H10 = (SI*DK1*S2-P2)

G11 = (1.0+SI*DK1*S1*W/2)
H11 = (1.0+SI*DK1*S2*W/2)

G0 = -(SI*DK1*C13-P1*C33)
H0 = -(SI*DK1*C13-P2*C33)

G01 = -SI*DK1*W*G10/2.0 + B1*S1
H01 = -SI*DK1*W*H10/2.0 + B1*S2

H1 = H10+H11
G1 = G10+G11
G0 = G0+G01
H0 = H0+H01

D = G10*H0-G0*H10 + (H0*G11+H01*G10-G0*H11-G01*H10) + (H01*G11-H11*G01)

IF (NPOLE.EQ.2) RETURN
C1 = -F/G*D*H1
C2 = F/G*D*G1
203 WRITE (2) P1, P2, C1, C2, S1, S2, DR1, DR2.
RETURN
END
ELASTIC CONSTANTS C11, C22, C33, C44, C55, C66, C12, C13, C32:
0.2456E 08 0.1171E 07 0.1374E 07 0.3525E 06 0.2310E 07 0.3525E 06 0.4000E 06 0.1986E 07 0.4558E 06

DENSITY OF PLATE GM/CC-- 1.4400  FIBER LAYUP ANGLE-- 15.0000
NSTRESS= 1  NX3= 1  DX3= 1.0000

VEL= 300.0000  TO= 87.3169  A= 1.3342  PO= 1042938.7500

WAVELEN(Time-SEC)= 0.3500E-05  WAVELEN(Dist-CM)= 0.6667
MAX FREQ NO(NON DIM)= 13.7654  MAX WAVE NO(NON DIM)= 9.4248

C11 = 0.2442E 08  C33 = 0.1196E 07  C55 = 0.2310E 07  C13 = 0.1830E 07

LONG. SPEED = 0.1081E 07  SHEAR SPEED = 0.3326E 06
RAYLEIGH SPEED = 0.3020E 06  CR/CS = 0.90800  12

DENSITY OF BEAM= 2.7000  NORM. THICKNESS= 0.5000  BEAM MODULUS= 0.1124E 08
LONG. WAVE SPEED IN BEAM = 535757.750 CM/SEC

SECOND ORDER ERRACH POINTS
0.1997043E 02 0.0 0.0 0.2266004E 01
LAPLACE INVERSION PARAMETER= 0.5948

WITH STRIP

* This program has test cards (#51, 52) to override the Hertz calculation TO=3510^-6 s A=1 cm.
\[
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X_3 &=& 0.0 \\
\text{DISPLACEMENTS} &=& 3 \\
\text{MAXIMUM VALUE} &=& -0.7104896 \times 10^0
\end{array}
\]

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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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\[
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\]

\[
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**SIRESS T11**

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DISPLACEMENTS

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Stress values are given in units of ksi.

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**DISPLACEMENTS 5**  
**MAXIMUM VALUE = -0.462303500**

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**NORMALIZED DIST. = 2**

| REAL PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| IMAG PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

<table>
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<tr>
<th>NORMALIZED DIST.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00004</td>
<td>0.02103</td>
<td>0.10073</td>
<td>0.24333</td>
<td>0.41035</td>
<td>0.56703</td>
<td>0.70144</td>
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<tr>
<td>0.95511</td>
<td>0.92661</td>
<td>0.87394</td>
<td>0.80010</td>
<td>0.70587</td>
<td>0.59526</td>
<td>0.46931</td>
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<tr>
<td>-0.15673</td>
<td>-0.15637</td>
<td>-0.11853</td>
<td>-0.08225</td>
<td>-0.04707</td>
<td>-0.02788</td>
<td>-0.00446</td>
</tr>
</tbody>
</table>

**NORMALIZED DIST. = 3**

| REAL PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| IMAG PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

<table>
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<tr>
<th>NORMALIZED DIST.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00004</td>
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<td>0.00997</td>
<td>0.07754</td>
<td>0.19345</td>
<td>0.31982</td>
<td>0.43364</td>
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<tr>
<td>0.67025</td>
<td>0.65479</td>
<td>0.62217</td>
<td>0.57325</td>
<td>0.51049</td>
<td>0.43492</td>
<td>0.36924</td>
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<tr>
<td>-0.14683</td>
<td>-0.17773</td>
<td>-0.15636</td>
<td>-0.11319</td>
<td>-0.07501</td>
<td>-0.04060</td>
<td>-0.02147</td>
</tr>
</tbody>
</table>

**NORMALIZED DIST. = 4**

| REAL PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| IMAG PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |

<table>
<thead>
<tr>
<th>NORMALIZED DIST.</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
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<td>0.00030</td>
<td>0.00548</td>
<td>0.02595</td>
<td>0.02357</td>
<td>0.03348</td>
<td>0.12663</td>
<td>0.22238</td>
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<tr>
<td>0.45759</td>
<td>0.45334</td>
<td>0.43645</td>
<td>0.40685</td>
<td>0.36774</td>
<td>0.31856</td>
<td>0.26281</td>
</tr>
<tr>
<td>-0.10517</td>
<td>-0.16989</td>
<td>-0.18071</td>
<td>-0.14489</td>
<td>-0.10571</td>
<td>-0.06167</td>
<td>-0.03831</td>
</tr>
</tbody>
</table>

**NORMALIZED DIST. = 5**

| REAL PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
| IMAG PART        | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 | 0.00000 |
JOB 0350  7/5/74  IBM 360-91
EXECUTION TIME  34 SEC.
25 PAGES
CORE  150 K
FIGURE 1

GEOMETRY OF IN-PLANE EDGE IMPACT OF A COMPOSITE PLATE WITH PROJECTION STRIP
Rayleigh Wave

In Plane Plate Edge Wave

Flexural Edge Wave

FIGURE 2a
Comparison of in-plane plate edge wave speed with body wave speeds versus fiber layup angle for 55% graphite fiber/epoxy matrix composite.
FIGURE 3.

Poles, Branch Points and Integration Contours in the complex Plane for the Numerical Solution.
FIGURE 4

Edge Stress $t_{11}$ Surface Wave for 0° Fiber layup Angle: 55% graphite fiber/epoxy matrix composite
$t_{11}$ vs $X_1$

ON EDGE, $X_3=0$

±15° FIBER LAYUP ANGLE

NORMALIZED BY 4.62 $P_0$

$a = 1$ cm., $T_0 = 35 \mu$ sec.

$V_R = 3$ mm/µ sec.

FIGURE 5

Edge Stress $t_{11}$ Surface Wave for ± 15° Fiber Layup Angle: 55% Graphite fiber/epoxy matrix composite.
**FIGURE 7**

Computer Map of Edge Stress $t_{11}$ Surface Wave in the Space-Time $(x_1, t)$ Domain
FIGURE 8
Decrease of Stress $t_{33}$ with Distance from the Impact Edge.
FIGURE 9
Stress $t_{33}$ versus Time at Different Distances from the Edge under the Contact Point.
Maximum Stresses versus Layup angle for 55% Graphite Fiber/epoxy Matrix Composite.
FIGURE 11
Distribution of Stress $t_{33}$ Along the Edge for Fiber Layup angles $0^\circ$, $\pm 30^\circ$, $\pm 45^\circ$. 

NORMAL STRESS $t_{33}/P_0$
AT DEPTH $X_3 = 0$

FIBER ANGLE

0° LAYUP ANGLE

$\pm 45^\circ$

$\pm 30^\circ$
Effect of Edge Strip Thickness on Interface Stresses.

DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX
±15° LAYUP - STEEL STRIP

FIGURE 12a
Effect of Edge Strip Thickness on Interface Stresses.

DATA FOR 55% GRAPHITE FIBER/EPOXY MATRIX
±45° LAYUP ANGLE - STEEL STRIP

FIGURE 12b
Effect of Edge Strip Thickness on Interface Stresses.

0° LAYUP ANGLE ALUMINUM STRIP
\( X_3 = 0, \ a = 1 \text{ cm}, \ T_0 = 35 \mu \text{ sec} \)

FIGURE 13
FIGURE 14
Effect of Edge Strip Thickness on Stress $t_{33}$ Distribution Along the Edge.
FIGURE 15
Effect of Edge Strip Thickness on the Rayleigh Edge Wave Shape.

ALUM STRIP \( b/a = 0.1 \)

RAYLEIGH EDGE WAVE
0° LAYUP ANGLE \( t_{11}/P_0 \), TIME \( t = 1.3 T_0 \)
\( X_3 = 0 \) \( a = 1 \text{cm} \) \( T_0 = 35 \mu \text{sec} \)
Effect of Edge Strip Thickness of Interface Shear Stress $t_{13}$ Distribution Along the Edge.