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Feasibility of Hydromagnetic Wave Measurements on Space Shuttle

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Introduction

A. Why should hydromagnetic waves be measured in space?

HM waves are produced by plasma instabilities or currents far out in the magnetosphere and propagate to the earth's surface. For many hm waves there is reason to believe that propagation is field-aligned, or ducted. Thus, the waves seen at the earth's surface provide information about plasma processes going on at known points in space. The wave spectrum, polarization, and direction of propagation can be used to infer the nature of these processes. For example, the existence of Pc 1 waves (left elliptically polarized transverse waves of 5 sec period) has been used to infer the occurrence of the ion cyclotron instability of energetic protons in the magnetosphere. Furthermore the fact that one type of Pc 1 (pearl pulsations) shows disperion in its dynamic spectrum has been used to determine the L shell on which the waves are propagating, the equatorial density of cold plasma on this line, and the parallel energy of the protons responsible for the instability. Another type of Pc 1, IPDP, is beginning to provide information about the point and times of proton injection during substorms as well as details about the drift of these particles. Other possibilities include the high latitude Pc 4,5 seen near the polar cusp, or Pc 3,4 seen near the plasmapause. These waves may eventually be used to provide information about the location of these boundaries as well as processes occurring on them.

In summary, hm waves are diagnostic of plasma processes in the magnetosphere. The location at which these waves occur on the earth's surface reflects locations of important regions or boundaries in space. The properties of the waves reveal information about the plasma properties in these regions. The waves therefore provide the possibility of inexpensive monitoring of these magnetospheric processes with ground observations.

B. Current Feasibility of Monitoring Magnetospheric Processes with hm Waves

The general goal outlined above is not presently attainable for a numer of reasons. Mainly this is a consequence of our lack of understanding of the processes of wave generation and propagation to the earth's surface. This is, in part, the major justification for an hm wave sensor on Space Shuttle. We presently have almost no understanding of the effects of the ionosphere on the transfer of waves. For some waves the ionosphere may act as a resonant cavity, increasing the amplitude of the incident waves much above their initial amplitude. Alternatively, the ionosphere may duct energy away from the point of entry. Almost certainly the polarization of the incident waves is greatly altered by the ionosphere.

These facts make it impossible to use current ground observations to determine the point of entry of hm waves from the magnetosphere, or their polarization as it exists within the magnetosphere. Without this information, no clear inferences can be made about the locations or types of magnetospheric generation mechanisms.

A major goal of an hm wave sensor on Space Shuttle would be to determine the effects of the ionosphere on the transmission of hm waves to the ground.

C. Possible Uses of Space Shuttle In Calibration of Ground Observations of hm Waves

1. Latitude of hm Entry

The most straightforward use of Space Shuttle in hm wave studies would be the determination of the latitude of entry of the waves from space. To accomplish this, the Shuttle would be placed in polar orbit well above the ionosphere. Since the Shuttle orbit would remain relatively fixed in inertial space, the observations would be made at fixed local time. Consequently, it would be necessary to choose this local time on the basis of information about the most probable local time of occurrence of the phenomena to be studied. Continuous monitoring of the output of the sensors would provide a body of data which should include examples of the phenomena to be studied.

Most hm wave phenomena are sporadic in their occurrence as well as localized in space. Consequently analysis of the Shuttle data cannot be carried out without simultaneous ground data monitoring the temporal changes in hm wave amplitude on the ground. If hm waves are present on the ground at a given local time and latitude but not at the Shuttle when it passes over the ground station, one would conclude the waves are propagating in the ionosphere to the ground station. On the other hand, if the waves are also seen at the satellite, then the normalized wave amplitude versus latitude would indicate the latitude of entry of the waves into the ionosphere.

In addition to normalized wave amplitude, the direction of wave propagation and polarization at the Shuttle might reveal the latitude of hm wave entry. For example, it has been reported that Pc 4 waves change their polarization across a demarcation line. Some recent theoretical work suggests there is ample reason for expecting such a result.

2. Transfer Function of the Ionosphere

In some cases hm waves will be observed both on the ground and simultaneously in space. In such cases the incident wave amplitude and polarization can be compared to that observed on the ground. This comparison would define the transfer function of the ionosphere above the station.

3. Hm Wave Propagation in the Ionospheric Cavity

It is well known that some hm waves propagate long distances in the ionosphere. Pc 1 waves have been observed simultaneously at conjugate points of an auroral zone magnetic field line as well as the equator. Dynamic spectra at the conjugate points contain periodic structures such that when one spectrum is displaced by half the period of the structures, the dynamic spectra appear identical. In contrast, the equatorial spectrum appeared to be the sum of the conjugate spectra. Using these kind of data the group delay between stations can be calculated as a function of latitude. The results obtained correspond roughly to the Alfven wave velocity in the F region in the ionosphere.

This ionospheric propagation of hydromagnetic waves has been studied theoretically as well. Among the question of interest are how the waves are injected into the cavity, how large are wave amplitudes in the cavity, how does wave polarization change within the duct, how rapidly are waves attenuated in the duct, and how much wave energy escapes from the duct to the earth's surface?

Space Shuttle could provide useful information on the properties of the ionospheric ducting of hm waves. A single vehicle at 400 km altitude, for example, could determine the wave amplitude and polarization as a function of latitude. Comparison with ground stations below the satellite would show how much energy leaks out of the cavity to the ground.

More important information could be obtained if several or more subsatellites were strung out in orbit behind the Shuttle. Phase delays between successive satellites would allow one to determine the phase velocities of the hm waves in the duct. This information could be used to improve theoretical models of the ducting process.

As we have discussed above, it is not presently possible to use ground observations as diagnostics of magnetospheric processes. A major factor limiting their application is the unknown effects of the ionosphere in the transfer of the hm waves to the ground and the ducting away from the point of entry. Space Shuttle can be extremely valuable in solving some of these problems provided it is possible to make such measurements on the satellite. The following sections are devoted to an examination of some of the requirements the Shuttle must fulfill if such measurements are to be made.

Instrumentation

The construction of a hydromagnetic wave sensor for Space Shuttle presents a number of difficult technical problems. These problems are centered around the necessity of measuring very small fluctuations in the presence of an extremely large, but variable background field. If we accept 10 mγ as the desired resolution and $\pm$50,000 γ as the necessary dynamic range for the earth's field, we require a resolution of $10 \times 10^{-3}/10^5$ or one part in 10^7. If we express this instead in binary form we have roughly:

$$(1/64)/131{,}072. = 1/(2^6)(2^{17}) = 1/2^{23} = 1/8.39 \times 10^6$$

For comparison, binary analog-to-digital converters of 16-bit accuracy have only become commercially available in the last few years. As yet these devices have not been qualified for space applications by NASA. The problem is comparable even when we give up the requirement of absolute field measurement and require only the time derivative of the field. As we show in another section, the maximum rate of change in any component of the earth's file is 50 $\pi\gamma$ per second. For a 10 mγ wave amplitude the time derivative is $2\pi fA = 2\pi \times 10^{-2}$ f. The lower limit for hm waves is roughly 10^{-3} Hz, so we require a resolution of $2\pi \times 10^{-5}/50\pi = 1/2.5 \times 10^6$.

Actually, the foregoing argument exaggerates the problem because we have not considered certain features of the natural geomagnetic spectrum and of electronic noise in instruments. In particular, both electronic noise and amplitude of hm waves decrease as frequency increases. For example, typical wave amplitudes at 10^{-3} Hz are 100 γ. At higher frequencies they decrease roughly in inverse proportion to frequency, being about 100 mγ at 1 Hz. If we require a resolution of one part in 100 of the typical wave amplitude, then we need only 1 γ at 10^{-3} Hz, so that our instrument resolution must be:

$$2\pi(1)(10^{-3})/50\ \pi = 1/(2.5 \times 10^4)$$

Note that $1/2^{14} = 1/(1.6384 \times 10^4)$. Thus, a 14-bit digitizer would be nearly adequate. As we will discuss below, there are similar strategies in the design of absolute instruments that make comparable reductions in the required resolution.

We emphasize that a resolution of one part in 2^{14}-2^{15} is not a simple matter to achieve in a spacecraft environment. A typical instrument using modern integrated circuits will have a maximum voltage output of ±10.24 v. This is $2 \cdot 1024 \times 10^{-2}$ v or $2^{11} \times (10^{-2}$ v). As a rule of thumb, most spacecraft ground lines will have about 5 mv of noise in them. This is $5 \times 10^{-3} = 0.5 \times 10^{-2} = 1/2 \times 10^{-2}$ v. Thus, expressed as a fraction of full scale voltage, the noise is $(1/2 \times 10^{-2})/(2^{11} \times 10^{-2}) = 1/2^{12}$. It would appear pointless to attempt measurement of such a signal to $1/2^{15}$. We note, however, if the noise on ground lines could be reduced within the instrument to 1.25×10^{-3} v $= 0.125 \times 10^{-2} = 1/8 \times 10^{-2}$, we would have $(1/23)/(2^{11}) = 1/2^{14}$, which is roughly adequate as shown above.

In the previous paragraph we have continued to ignore the spectral characteristics of the hm signal, spacecraft noise and the process of digitization. To proceed further in our discussion of the feasibility of an hm wave sensor for Space Shuttle we must consider these. It is a well-known fact that digitization of data introduces white noise. The spectral density of this noise is given by

$$P_D = (D)^2/12B\ (v^2)/Hz$$

where D is the least significant bit of the digitization in volts and B is the bandwidth of the quantized signal in Hz. Since we are considering sampled data, it can be assumed that the bandwidth has been limited to the Nyquist frequency, $f_N = 1/2\Delta t$, where Δt is the time between samples.

Noise spectra of electronic instruments are generally proportional to 1/frequency. Furthermore, quoted noise values are typically determined at low frequencies by recording a few minutes of data on chart paper. Thus, if 5 mv is a correct figure for the bandwidths of interest in ULF wave measurements, it implies that the rms power in a 1/f spectrum between 0.01 and 1 Hz is 5 mv. But

$$(rms(f_L - f_v))^2 = P_N \int_{f_L}^{f_v} (1/f)\, df = P_N \cdot \ln (f_v/f_L)$$

Thus

$$P_N = (rms)^2/\ln (f_v/f_L)$$

For the above assumptions

$$P_N = (5 \times 10^{-3})^2/4.60517 \sim 5 \times 10^{-6}\ (v)^2/Hz$$

Thus, $P_N\ (f) = 5 \times 10^{-6}/f\ (v)^2/Hz$.

Averages of many power spectra for different types of hm waves fall off as $1/(\text{frequency})^2$, i.e., wave amplitude proportional to $1/f$. Thus

$$(\text{rms})^2 = \int_{f_L}^{f_v} (P_s/f^2)\, df = P_S \left(\frac{f^{-2}}{(-2)}\right)_{f_L}^{f_v}$$

or

$$(\text{rms})^2 = P_s\ (1/f_L{}^2 - 1/f_v{}^2)$$

$$P_s = (\text{rms})^2/(1/f_L{}^2 - 1/f_v{}^2)$$

Most quasi-sinusoidal wave events have band widths comparable to center frequency. Thus, if we take 10 γ as a typical rms amplitude at 10^{-3} Hz, we have

$$P_s = (10)^2/(1/5 \times 10^{-4})^2 - 1/(1.5 \times 10^{-3})^2)$$

Thus

$$P_s = 2.81 \times 10^{-5}$$

and

$$P_s\ (f) = 2.8 \times 10^{-5}/f^2$$

Note that this corresponds to a 10 mγ rms amplitude at 1 Hz.

A. Search Coil

If we are considering a search coil magnetometer, we need the spectrum of the derivative. If $B = B_o \sin \omega t$, the $\partial B/\partial t = \omega B_o \cos \omega t = 2\pi f\, B_o \cos \omega t$. Since the 90° phase shift is not important in the power spectrum, we see the signal amplitude differs from that of the original by the factor $(2\pi f)$. Thus, the spectrum of the derivative is given by $(2\pi f)^2$ times the spectrum of the original signal. Actually, we must scale the output of the search coil instrument by a constant (k) so that the largest expected signal remains on scale. Thus, the natural spectrum must be multiplied by the constant $(2\pi fk)^2$. Threfore

$$P_s\ (f) = (4\pi^2 f^2 k^2)(P_s/f^2) = 39.478\ k^2\ P_s$$

or

$$P_s\ (f) \simeq 40\ k^2\ P_s$$

The constant (k) is set by the requirement that the maximum signal $50\pi\ \gamma/\text{sec}$ does not exceed 10.24 v. Thus, 10.24 v = k ($50^-\pi$ γ/sec) or $k = 6.5189 \times 10^{-2}$ v/(γ/sec). Thus

$$V\ (\text{volts}) = k\ (2\pi f\ B_o)$$

or

$$V\ (\text{volts}) = 2\pi(6.5189 \times 10^{-2})\ fB_o$$

$$V\ (\text{volts}) = (0.41)\ fB_o$$

For comparison, a typical ground search coil system for the auroral zone is scaled so that the constant multiplying fB_0 is 1.0. At mid-latitudes it is typically 10.0. Thus, the satellite system must be 2.4 to 24 times less sensitive than ground systems to avoid saturation by motion through the dipole field.

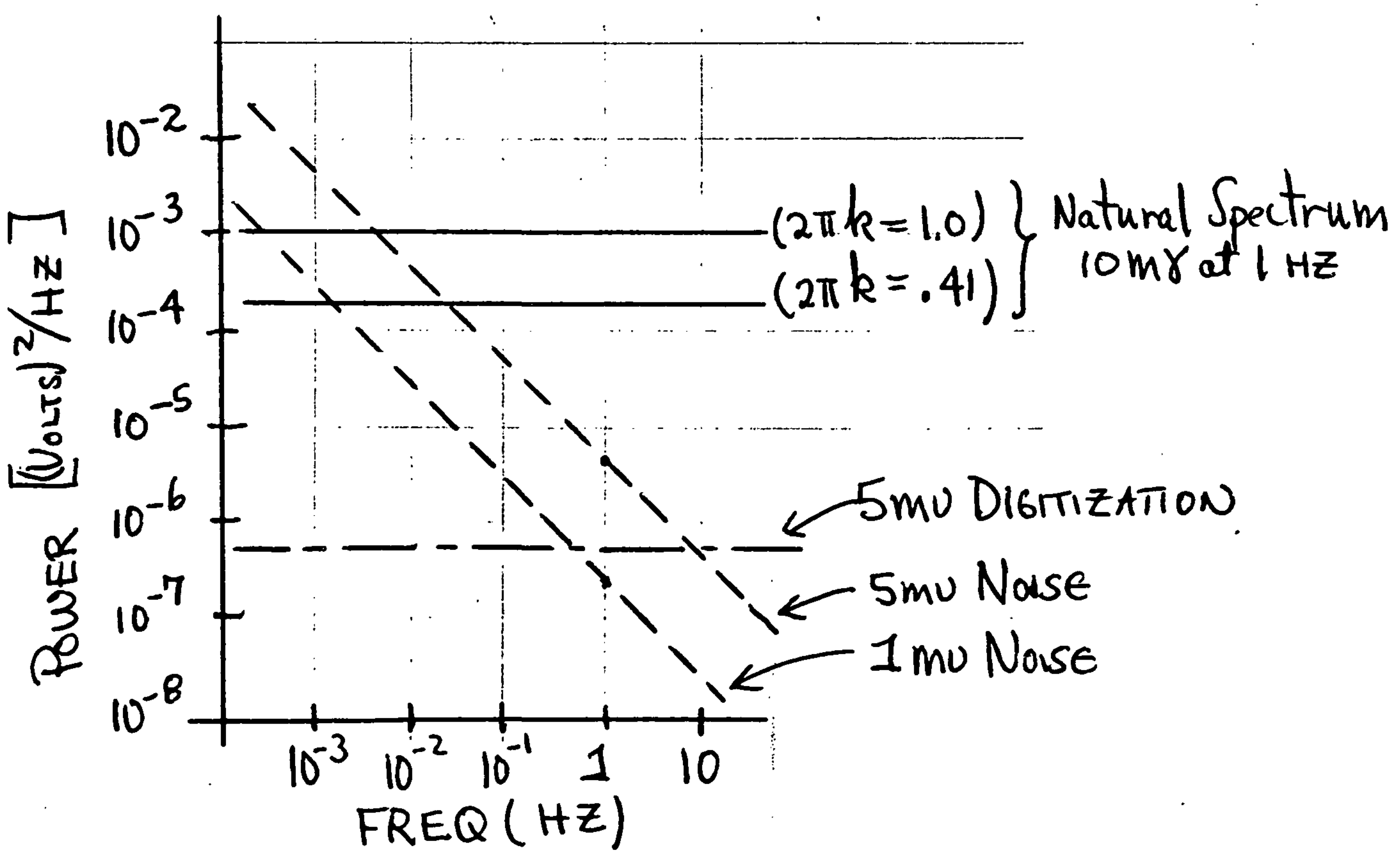
Returning to our "typical" search coil spectrum, we have

$$P_s(f) = 4\pi^2 k^2 P_s = 4\pi^2 \ (4.1 \times 10^{-1})^2 \ (2.81 \times 10^{-5})$$

or

$$P_s(f) = 1.86 \times 10^{-4}$$

If we now plot our typical signal and noise spectra on log-log plots, we have the result shown below.



By our preceding arguments the search coil spectrum for natural signals is flat. In contrast, the instrument noise falls off as 1/f. Using the typical values discussed above, we see that we would not be able to observe natural signals below 0.03 Hz. We note, however, that small changes in the typical noise or the scaling of the search coil change this considerably. For example, if the instrument noise in the band 0.01 to 1 Hz is actually 1 mv rms we would be able to observe all the way to 10^{-3} Hz. Alternatively, we could allow one of the sensors to occasionally saturate and scale the search coil such that $2\pi k = 1.0$. Then, even for the conservative noise estimate of 5 mv, we could observe to 10^{-2} Hz.

As we will show in a later section, there are several reasons why the instrument should not attempt to measure hm waves much below 0.03 Hz. Consequently, we feel that there is no need to allow the instrument to saturate and, furthermore, only small improvements in instrument noise would be required.

Finally, let us decide what the least significant bit (D) should be in our analog-to-digital conversion. This, in turn, depends on desired bandwidth. For discussion purposes, we suppose that we wish to measure up to $f_v = 5$ Hz, i.e., sample 10 times a second. Further, let us require that the digitization noise power density be 3.3 db below the instrument noise at this upper frequency limit. Then

$$P_D = (2/3)\ P_N\ (f_v) = D^2/12\ (f_v)$$

or

$$D = (8P_n(f_v) \cdot f_v)^{1/2} = (8 \cdot P_N)^{1/2} \sim 2.8\ \sqrt{P_N}$$

Substituting, we have

$$D = (8 \cdot (5 \times 10^{-6}/f_v) \cdot f_v)^{1/2} \simeq 6 \text{ mv}$$

Thus, D should be comparable to our assumed noise over two decades of frequency. Thus, our resolution should be about

$$5 \text{ mv}/\ 2 \cdot 10{,}240 \text{ mv} = 1/4046 = 1/2^{12}$$

If instrument noise is actually 1 mv rms, then

$$D = 2.8 \sqrt{2 \times 10^{-7}} \sim 1.25 \text{ mv}$$

and the resolution should be about

$$1.25/20{,}480 = 1/2^{14}$$

As we mentioned earlier, 12-bit digitization is trivial with flight qualified hardware, and 14-bit should be readily obtainable at some increase in cost.

Let us now summarize our discussion of the proposed search coil magnetometer. We assumed that the maximum output voltage of the instrument was ±10.24 volts. Next, we estimated the spectrum of the instrument noise to be a 1/f spectrum and 5 mv rms in the two decades 0.01 to 1 Hz. We then scaled the instrument such that the maximum expected rate of change of field (50π γ/sec) would just saturate the instrument. We chose the least significant bit such that digitization noise power density was 2/3 of the instrument noise power density at 5 Hz. With these assumptions, we found that the allowable resolution was only one part in 2^{12}. We found further that if rms instrument noise was reduced by a factor of 5, the allowable resolution is correspondingly increased to 1 part in 2^{14}.

As a crude estimate of the desired lower limit for the natural hm spectrum, we chose a $1/f^2$ spectrum corresponding to a 10 mγ wave amplitude of 1 Hz bandwidth centered at 1 Hz. These assumptions led to a flat spectrum output from the search coil magnetometer. Because the noise spectrum rises toward lower frequencies, it is impossible to maintain constant ratio of signal to noise as a function of frequency. In fact, scaling the instrument to avoid saturation due to motion in the dipole field and accepting 5 mv rms noise implies instrument noise exceeds our desired lower limit of the hm wave spectrum below 0.03 Hz. By reducing the rms instrument noise to 1 mv, this will not happen until 10^{-3} Hz.

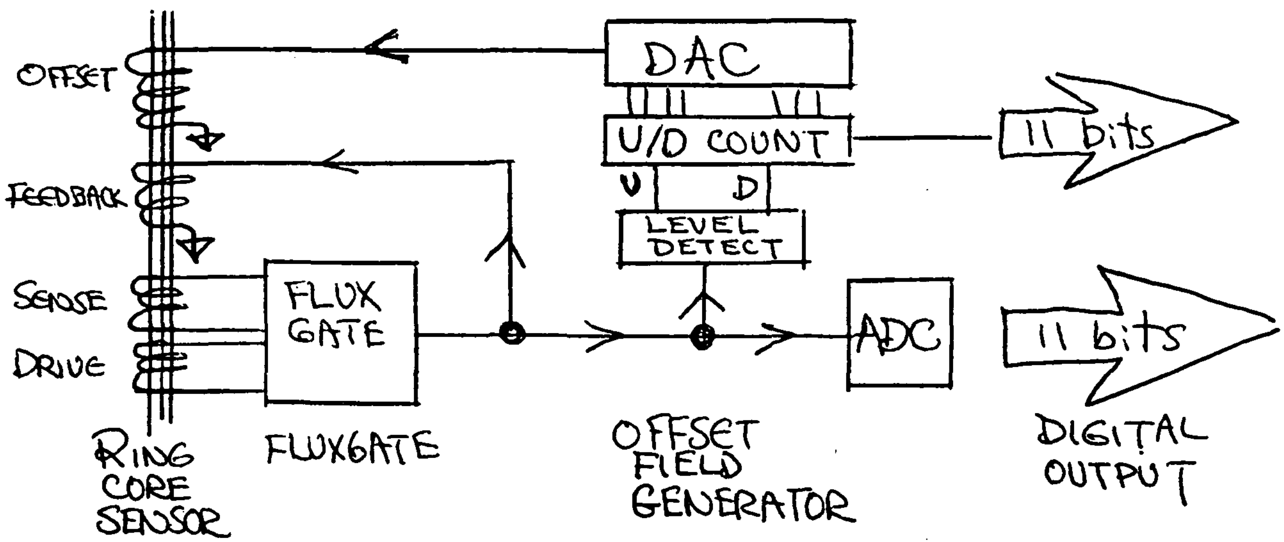
As we show in a later section, there is no point in attempting to observe hm waves below about 0.01 Hz. This could be done according to our previous argument if the instrument noise is about 3 mv in the band 0.01 to 1 Hz. Our allowable resolution would be roughly one part in 2^{13}. We feel this is attainable with good electronic design. Consequently, it appears feasible to measure hm waves in Shuttle orbit with a search coil magnetometer.

B. Fluxgate Magnetometer

If we must measure the absolute field with the same instrument as the hm waves, we have a much more difficult problem. In particular, we must use some device sensitive to the total field component along a given axis. Such devices as fluxgates or alkali vapor with biasing coils can accomplish this. Usually such instruments operate as null detectors, feeding back a current through a biasing coil. The fundamental limitation in such instruments is the fluctuations of the feedback current that are observed by the null detector as field variations. An analysis of instrument noise, and hence feasibility for use in Shuttle orbit, depends on the particular instrument and strategy chosen. As an example, the UCLA fluxgate magnetometer developed for ground use resolves 65,536 γ to one part in 2^{22}, i.e., to 1/16 γ (62 mγ). Instrument noise in the ULF band (10^{-3} to 1 Hz) is less than this resolution. We note, however, that the absolute accuracy is not better than 32 γ. On a long time scale the instrument can drift by amounts comparable to this. Since the time scale of such drifts is days, it is of no consequence in ULF wave measurements.

To demonstrate the feasibility of using such an instrument in Shuttle orbit, we briefly describe the design of the UCLA fluxgate magnetometer. As we will show, there is no reason why this design will not work with the same sensitivity in Shuttle orbit.

A block diagram of a single axis is shown below. The magnetometer includes a ring core sensor, second-harmonic fluxgate detector, analog feedback winding for field nulling, a level detector, an up-down counter, a digital-to-analog converter, and an offset winding for additional field nulling. The fluxgate detector acts as



a highly sensitive null detector of limited dynamic range. Provided the field along the sensor does not exceed this dynamic range, the fluxgate detector generates a feedback current which completely nulls out the field along the sensor axis. If the field exceeds the dynamic range, the fluxgate detector output trips a level detector. This, in turn, enables an up-down counter which adds or subtracts successive counts from its current contents. The contents of this counter are continuously converted to a current proportional to the count. This current passes through an offset coil partially nulling the field along the sensor. Through proper scaling each count corresponds exactly to half the dynamic range of the fluxgate detector. Thus, each time the detector exceeds its dynamic range it is brought back to the center of this range by a change of one count in the offset field generator.

In the UCLA ground magnetometer the offset field generator is scaled so that 11 bits encompass the complete range of possible earth fields, i.e.

$$2^{11} \text{ bits} = 65{,}536\ \gamma$$

or

$$2048 \text{ bits} = 131{,}072\ \gamma$$

Thus, the least significant bit (LSB) corresponds to

$$\text{LSB} = 64\ \gamma$$

The fluxgate detector is, in turn, scaled so that it has a dynamic range of $\pm 64\ \gamma$ which is also encompassed by 11 bits. Thus

$$2^{11} \text{ bits} = 64\ \gamma$$

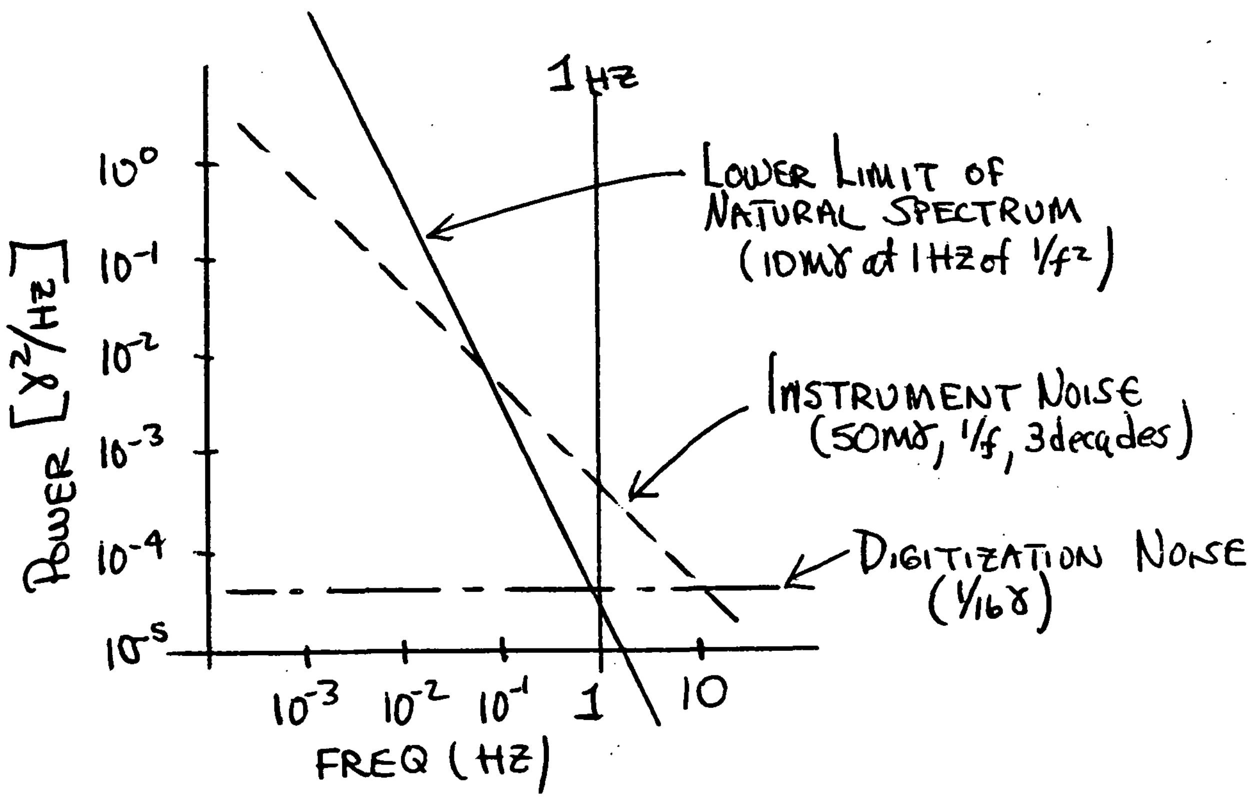
or

$$\text{LSB} = 1/16\ \gamma$$

The digital output of the instrument is obtained by appending 11 zeroes to the binary count in the up-down counter and algebraically adding the 11-bit output from the analog-to-digital converter.

Instrument noise in this system is very difficult to evaluate theoretically. Past experience has shown that it depends heavily on the geometry of the circuits, the degree of isolation of the magnetometers' power supplies from other instruments (particularly digital), the degree of integration (used in circuit construction and improvements in solid state technology (particularly ADC's and DAC's).

To demonstrate the feasibility of this instrument we adopt the measured noise spectrum at the output of the fluxgate detector in the UCLA magnetometer as typical. This is found to be a 1/f spectrum with spectral density of $4 \times 10^{-4}\ \gamma^2$/Hz at 1 Hz. According to our discussion of the natural spectrum given previously, an acceptable lower limit to the natural spectrum is a $1/f^2$ spectrum of spectral density $2.8 \times 10^{-5}\ \gamma^2$/Hz at 1 Hz. These spectra are plotted below as a function of frequency.



Using the above estimates it is apparent that this fluxgate is usually adequate for frequencies below 0.1 Hz. However, above this frequency instrument noise exceeds the lower limit of the natural spectrum. However, in our earlier discussion of the search coil magnetometer we concluded the search coil would probably be inadequate below about 0.03 Hz. Thus, both search coil and fluxgate would be required to observe the entire ULF band. In a later section we will demonstrate that the lower the frequency the more likely it will be that spatial variations will be confused with temporal variations in Shuttle orbit. In particular, 0.01 Hz appears to be a lower limit of observable variations. Consequently, it appears that a fluxgate is not an appropriate sensor for hm wave measurements in Shuttle orbit.

The origin of the noise in the fluxgate is not clear. If we assume it is due to electronics associated with the fluxgate detector, we might be able to improve the overall noise by rescaling the instrument. For example, since this detector is scaled so that 64 γ = 10.24 v, we have a sensitivity of 16×10^{-2} v/γ. Using this to convert our observed noise spectrum in γ^2/Hz to $(v)^2$/Hz, we obtain a noise spectral density at 1 Hz of 7×10^{-7} $(v)^2$/Hz. This gives a two decade rms noise power of 1.8 mv. This is somewhat better than we assumed possible for the search coil in a spacecraft environment.

If we reduce the dynamic range of the fluxgate detector by 2 and assume that the electronic noise remains unchanged, we reduce the noise spectral density in γ units by a factor of four. We note, however, that one additional bit must be added to the offset field generator. Noise power from it will now be four times as important. In the present magnetometer design it appears that noise from the sensor, offset field generator and fluxgate detector are comparable. Consequently, compromises such as those suggested above cannot significantly improve the performance.

Thus, we conclude that simple modifications of the current fluxgate design are not likely to significantly change the instrument noise. Consequently, the search coil remains the best choice for an hm wave sensor in Shuttle orbit.

Difficulties Associated with hm Wave Measurements at Shuttle Altitude

In a preceding section we discussed various instruments with sufficient sensitivity to measure hm waves in space. If one of these instruments is placed on a spacecraft at Shuttle altitude, there will be a number of factors which limit its ability to detect such waves. In this section we consider only those factors which are important even if we had an ideal measurement platform. These are nearly all consequences of motion of the sensor around the earth. They include separation of the spatial and temporal features of the hm waves, effects of spatial gradients in the main field, crustal anomalies, ionospheric currents (Sq, equatorial electrojet, auroral electrojet), and field aligned currents.

A. Separation of Spatial and Temporal Aspects of hm Waves

Let us suppose that we are able to make perfect hm wave measurements on a platform in low altitude polar orbit. With a single moving platform it is impossible to distinguish between temporal changes of the wave and spatial changes. For example, suppose that a 100 sec period wave is localized in latitude to about 1000 km. At the velocity of a spacecraft in low altitude polar orbit ($\sim$10 km/sec) it will take 100 sec to travel across the region of localization. In this time the wave amplitude at a fixed spatial point will have undergone one cycle. For the moving magnetometer it will be impossible to decide whether the observed change in $\underset{\sim}{B}$ was due to temporal changes or spatial variations.

To demonstrate the above, suppose the spatial variation in a given field component is Fourier analyzed. Then if the field is localized in latitude with scale x, the most important wave numbers will be those for which $kx_0 \sim 2\pi$, or $\lambda \sim x_0$, i.e., wavelengths comparable to the scale. If the field component is quasi-sinusoidal in time, we have

$$B_i\ (x,\ t) \propto \exp\ [i(2\pi/\lambda)x]\ \exp\ [i\omega t]$$

or

$$B_i\ (x,\ t) \propto e\ [i2\pi(x/\lambda + t/T)]$$

where $\omega = 2\pi/T$. But for a moving spacecraft $x = vt$, so

$$B_i(t) \propto \exp\ [i2\pi(v/\lambda + 1/T]\ t$$

Clearly spatial variations in the field are seen by the moving magnetometer as temporal changes. In particular the problem is most serious for wave periods $T \sim \lambda/v$.

For latitudinal localization of order 1000 km and satellite velocities $\sim$10 km/sec, we have $T \sim 100$ sec. This is within the period of interest to us in studies of hm waves.

In a recent study of a stormtime Pc 5 wave event, Lanzerotti et al. (1974) examined the latitude dependence of a hm wave of 400 sec period. Their analysis suggests that the wave had a spatial scale of $\sim$1000 km (10° of latitude) centered about 62° magnetic latitude. From our preceding analysis it is clear that time and space variations would have been badly mixed in a Shuttle observation of this wave event.

B. Effects of Motion Through the Earth's Dipole Field

As a polar orbiting satellite passes around the earth, the direction and magnitude of the field at the satellite are continually changing. Because the earth's field is so large, the rate of change of field due to satellite motion dominates any changes due to hm waves. To demonstrate this, consider the following.

In spherical coordinates the dipole field is given by

$$B_r = -(2B_o/L^3) \cos \theta$$

$$B_\theta = -(B_o/L^3) \sin \theta$$

$$B_\phi = 0$$

The output of a search coil sensor aligned in the radial direction will be proportional to dB_r/dt.

$$[(d/dt)B_r] = [(\partial/\partial t)B_r + (\underset{\sim}{v} \cdot \underset{\sim}{\nabla})B_r]$$

We assume there is no time variation at a fixed point, so $(\partial/\partial t)B_r = 0$. Expanding $(\underline{v} \cdot \underline{\nabla})B_r$ in spherical coordinates gives

$$(d/dt)B_r = [(\underset{\sim}{v} \cdot \underset{\sim}{\nabla})\underset{\sim}{B}]_r = v_r/(1)(\partial/\partial r)B_r + (v_\theta/r)(\partial/\partial\theta)B_r$$

$$v_\phi/r \sin\theta\ (\partial/\partial\phi)B_r + [B_\theta/(1)(r)]\ [v_r\ (\partial/\partial\theta)(1) - v_\theta$$

$$(\partial/\partial r)(r)] + [B_\phi/(1)(r \sin\theta)]\ [v_r(\partial(1)/\partial\phi) - v_\phi$$

$$(\partial(r \sin\theta)/\partial r)]$$

But $B_\phi = 0$ and we assume for simplicity a perfect circular, polar orbit with $v_r = v_\phi = 0$. Thus

$$(d/dt)B_r = (v_\theta/r)(\partial/\partial\theta)B_r + B_\theta/r\,[-v_\theta] = (v_\theta/r)(2B_0 \sin\theta/L^3)$$

$$-(v_\theta B_\theta/r) \quad (d/dt)B_r = (v_\theta/r)(-2B_\theta) - (v_\theta B_\theta/r) = -3v_\theta B_\theta/r$$

Finally,

$$(d/dt)B_r = (3v_\theta/r)(B_0/L^3)\sin\theta = (3v_\theta/Re)(B_0/(L)^4)\sin\theta$$

For a near-earth circular orbit $L \sim 1$ and $\theta = \Omega t$, where $\Omega_S = v_\theta/Re$. Thus

$$[(d/dt)B_r]_{L=1} = (3\Omega_S B_0)\sin\Omega_S t$$

The output of the sensor is a sine wave with amplitude

$$3\,\Omega_S B_0 = 50\,\pi\ \gamma/\text{sec}$$

This is approximately 160 γ/sec.

For comparison, consider a hm wave of 100 sec period and 2.5 γ amplitude. $B = B_0 \sin\omega t$ so, $\partial B/\partial t = \omega B_W \cos\omega t$.

The output of the sensor is again a sine wave with amplitude ωB_W.

$$\omega B_W = (2\pi/T)B_W = 2\pi/100 \cdot (2.5) \sim 0.157\ \gamma/\text{sec}$$

The ratio of these signals is

1/(signal wave/signal dipole) = $1(2\pi B_W/T)/50\pi = 25\ T/B_W \sim 1000.$

Clearly even a large amplitude hm wave has an effect a thousand times smaller than the effects of motion through the earth's field.

Attempts to remove the earth's field using field models and information on the sattelite orbit can introduce serious problems in the data from a satellite in shuttle orbit. An example shown in Figure 1 of one such attempt made by Cain et al, 1967, illustrates the problem. Residuals left after subtracting the total field of a model from the observations show large amplitude oscillations in the field magnitude. The correlation of these with satellite perigee makes the results very suspicious. Possible sources of error include higher order terms in the field model and small errors in the determination of the satellite orbit. For a vector field instrument the problem would be more serious since errors in atttitude would enter as well. Clearly it would not be easy to determine whether these oscillations were waves of natural orgin or not.

C. Effects of Crustal Anomalies

The earth's field has higher order terms than a dipole, which will also affect the measurements made by an hm wave sensor at Shuttle altitude. Furthermore, close to the earth's surface anomalies caused by variations in the composition of the crust become important. Since the magnetic field due to these anomalies satisfies Laplace's equation, it is possible to calculate the field at Shuttle altitude if it is known at the earth's surface. Unfortunately, very little is currently known about the spatial distribution of anomalies of scale important at Shuttle altitude.

The reason for this lack of knowledge is that the process of upward continuation of the surface field acts like a low pass filter. Roughly, one can say that only features of scale comparable to the distance from the source to the observation point can be observed. At Shuttle altitude this is 500-1000 km. This distance is so large that, to date, there are no surface magnetic maps showing features of this scale. The reason is simply that it takes so long to survey a region of this size that temporal variations in the surface field dominate the observations. With the current sparse distribution of fixed observatories it is impossible to remove these temporal effects from the survey observations.

There has been one attempt to generate a map of crustal anomalies using satellite data, however. Zietz et al. (1970) used data from the USSR satellite Cosmos 49 to contour anomalies observed between 261 and 488 km altitude over the United States. Their results, shown in Figure 2 suggest that a significant problem exists for an hm wave sensor at Shuttle altitude. It is evident from this figure that a satellite pass over the U.S. would observe quasiperiodic variations of order 50 γ in distances of order 1000 km. At 10 km/sec satellite velocity these would appear to be hm waves of 50 γ amplitude and 100 sec period.

D. Effects of Ionospheric and Field Aligned Currents

For a polar orbiting satellite at a few hundred km altitude, ionospheric currents will cause significant variations in the field. Distinct currents which must be considered include the solar quiet day variation, Sq; the polar cap currents due to magnetospheric convection, Sqp, the equatorial electrojet and the auroral electrojets. All of these current systems have considerable spatial structure and are time varying as well. This is particularly true of the auroral electrojet with a latitudinal extent of a few hundred km and time variations on the minute time scale.

Effects of the Sq current system at the earth's surface 100 km below the source are of order 50 γ. On occasions this current system has spatial structure, with scale tens of degrees (several thousand kilometers) (Schieldge, 1974). Effects of the equatorial electrojet are even larger at the earth's surface and have been studied by satellite magnetometers at Shuttle altitude (Cain and Sweeney, 1973). Figures 3 and 4 taken from this paper illustrate the nature of the observed variations in total field. Changes as large as 20 γ are seen in less than 5° of latitude as the satellite passes over the magnetic equator. This corresponds to 500 km or 50 sec, i.e., a rate of change of order 0.4 γ/sec. Clearly, the variations appear to be quasiperiodic and would be indistinguishable from hm waves of long period. Comparison of different passes over the equator show that this phenomenon is highly variable and probably impossible to predict and remove from the data.

Perturbations due to the auroral electrojet are much larger than those of the equatorial electrojet at Shuttle altitude. Figure 5 taken from a paper by Langel and Cain (1968) shows the effects in the total field. For the pass shown there was a change of 500 γ within 10° of latitude.

Passage of the satellite through sheets of field-aligned current flowing into the auroral electrojet will also cause perturbations of large amplitude. In a recent paper by Armstrong et al. (1974) vector field observations made by a fluxgate magnetometer on the TRIAD satellite as it flew through an auroral arc were reported. Figure 6 taken from this paper shows that in approximately 20 sec the east component of the field decreased by almost 400 γ. Superimposed on this decrease were small-scale perturbations of many tens of γ amplitude, presumably due to fine scale structure in the field-aligned currents.

E. Summary

In the foregoing discussion we have outlined a number of sources of magnetic field variation which would be recorded by a satellite in a low altitude polar orbit. All of these sources would cause perturbations that would have to be removed from the data in order to make an unambiguous identification of an hm wave in the data. As shown above, most of these sources are unpredictable, have relatively small spatial scale, significant time variations and magnitude much larger than the waves of interest. Assuming that a strategy is devised for removing these effects, one is still faced with the problem of separating the spatial aspects of the perturbations from the temporal.

Difficulties Associated with Measurements on Space Shuttle

A. AC Interference Created by Spacecraft

The difficulty most likely to be encountered on the Space Shuttle is low-frequency ac noise generated by the spacecraft itself. The Shuttle will be an extremely complex device constructed with magnetic materials, many of which will be moved as sensors are pointed or booms are deployed and retracted. There will be a large variety of electric motors used to actuate devices as well as numerous complex electrical circuits. It seems unlikely that much attention can be paid during spacecraft construction to minimizing the effects of these different sources of ac interference.

An example of such circumstances of high levels of magnetic noise is that of the UCLA fluxgate magnetometer on ATS 1. This spacecraft was not designed for a low noise environment and the magnetometer was attached to the spacecraft itself. Since the spacecraft electronics were located only a few feet from the magnetometer, changes in the state of various subassemblies were detected as changes in ambient field. Dc offsets of 50 γ were regularly recorded in a sensor parallel to the satellite spin axis as one or another system was turned on and off. More serious from the point of view of wave measurements was an 82-second period, sawtooth variation in the dc field. Although the amplitude of this variation was only about 3 γ pp, spectral analysis of waveform produced a harmonic spectrum spanning most of the band of hydromagnetic waves. Finite resolution of the spectral analysis, smoothing of the spectral estimates, etc., spread the power in these harmonics producing a background spectrum above which a natural signal must rise in order to be observed.

A problem similar to the above was observed at higher frequencies in the transverse field sensor on ATS 1. In this case a ramp of period equal to the basic telemetry sequence (5.12 sec) was present with amplitude of order 5 γ. Much of the time this signal dominated the waveform of the magnetometer making it impossible to identify visually the occurrence of hm waves in the Pc 1 band.

Examples of these problems are shown in Figures 7 to 9. In Figure 7 a sawtooth of several gamma amplitude in the Y component is phased with the vertical lines corresponding to the start of telemetry sequences. In the spectra of Figure 8 at least five harmonics of this 5.12-sec periodic waveform are evident. Figure 7 also shows the longer period ramp in the Z component (see step in Z at second vertical line). At least 18 harmonics are present in the spectrum, shown in Figure 9. Finally, Figure 7 shows the transient effects of a dc offset in the field at the eleventh vertical line from the right edge.

These examples clearly demonstrate that a magnetometer located close to an improperly designed spacecraft will be influenced by a variety of noise sources. We believe it is extremely unlikely that Shuttle can be constructed to avoid such problems. Consequently, we conclude that an hm wave sensor cannot be mounted on the Shuttle itself. In the following section we consider the possibility of using a long boom.

B. Problems Associated with Long Booms

From the foregoing discussion, it is clear that the hm wave sensor should be located at some distance from the Shuttle itself. Long booms are one means of accomplishing this, provided it is physically possible to mount the sensor on the boom. Search coil sensitivity is roughly proportional to weight. Typical coils used for ground measurements weigh 100 lbs. apiece. A 300 pound sensor array mounted near the end of a 50 meter boom would generate a considerable moment of inertia when any attempt is made to change the spacecraft attitude.

If we assume that sensor weight is not a consideration, we must sill consider the importance of boom vibrations. For example, let us examine the situation of a sensor oriented at an angle $\theta = \theta_o + \Delta\theta$ to the earth's field, B_o. The component of field along the sensor is then

$$B_S = B_o \cos (\theta_o + \Delta\theta)$$

Expanding the cosine function gives

$$B_S = B_o (\cos \theta_o \cos \Delta\theta - \sin \theta_o \sin \Delta\theta)$$

Since $\Delta\theta$ is a small angle, we have

$$B_S \simeq B_o \cos \theta_o (1) - B_o \sin \theta_o \cdot \Delta\theta$$

The magnitude of the perturbation due to boom oscillations $\Delta\theta$ is maximum for a sensor perpendicular to the field, $\theta_o = 90°$.

$$\Delta B_S (90°) \simeq [2\pi/360 \cdot 50{,}000\ \gamma] \cdot \Delta\theta\ (\mathrm{deg})$$

or

$$\Delta B_S\ (90°) \simeq 870\ \gamma \cdot \Delta\theta\ (\text{deg})$$

This is roughly 14 γ per minute of arc.

The foregoing is a serious problem which has limited the ability of previous magnetometers to sense hm waves close to the earth. For example, the UCLA fluxgate magnetometer on OGO 5 was mounted on a 20-foot boom. As the satellite approached perigee, frequent changes in attitude were made to keep the antennas pointed earthward and the solar cell arrays perpendicular to the sun. Impulses caused by firing of attitude control jets caused damped sinusoids of 0.3 and 3 Hz to be generated. As many as 20 cycles of 5-γ amplitude oscillations were recorded in a 1000-γ field. These imply boom oscillations of order 0.25°. From our calculations above, this would give boom oscillations of 200 γ at Shuttle altitude. Clearly, the booms used on Shuttle would have to be extremely rigid if attitude control maneuvers are continuously carried out.

It should be pointed out that boom vibrations generate highly coherent, monochromatic signals. In addition, since the vibration cannot alter the ambient field, the field magnitude must remain constant. Thus, the oscillations will have no compressional component. As a consequence of these facts, their magnetic signature is easily recognized with spectral analysis. Their importance arises from the possibility of saturating search coil amplitudes or obscuring weaker signals in waveform plots.

C. Problems Associated with the Determination and Control of Spacecraft Attitude

Difficulties may arise in hm wave measurements because of uncertainties in Shuttle attitude. Present plans are to determine this to about 0.5° accuracy. Using the calculations of the preceding section, this indicates a possible error in some component of the field as large as 400 γ. In addition, we note that the booms to be used on Shuttle will be more than 100 feet in length. Maintaining sensor alignment at the end of such a boom seems likely to be quite difficult. It may be necessary to have a separate attitude determination system at the end of the boom to determine sensor alignment with sufficient accuracy.

Errors in sensor attitude are most serious for a device measuring the vector field rather than its derivative. The problem arises when we try to use two or more satellites in the same orbit as a gradiometer. Subtraction of the vector field observations made at the two different satellites at a given time can produce a large difference vector. This difference will include the random errors due to attitude determination which would add to a difference vector as large as 1000 γ.

Changes in spacecraft attitude may be serious for a search coil if attitude control maneuvers are carried out in discrete steps rather than continuously. For example, suppose the rate of rotation is 1° per second (6 min rotation period). From above, the rate of change of field could be as high as 870 γ/sec, which would saturate any search coil system.

D. Summary

Measurement of hm waves on the space Shuttle itself will be quite difficult for several reasons. First, the Shuttle is likely to be a source of low-frequency noise that will contaminate the observations. This noise could originate from a wide variety of electrical circuits, motors, and slowly moving magnetic materials. In order to reduce the effects of this noise, the hm wave sensor could be placed on a long boom. However, if the sensor consists of a highly sensitive search coil, it will be very heavy. This could adversely affect the moment of inertia of the spacecraft. Even if this is not a problem, vibrations of the boom as the attitude of the spacecraft changes could produce extremely large field oscillations that would either saturate the measuring system or swamp any natural signal in the data. Assuming that proper boom construction can damp the oscillations, attitude control maneuvers can produce rates of change of field that would saturate the system. Errors in attitude are another problem, particularly when more than one satellite in the same orbit is being compared.

Difficulties Associated with Measurements on Tethered and Free Subsatellites

In the preceding section we showed that it is difficult to measure hm waves on the Shuttle itself. Two ways in which this problem might be solved are to use either a tethered subsatellite or a free subsatellite. As we will show, there are difficulties associated with both of these.

A. Tethered Subsatellite

A tethered subsatellite appears to have a number of advantages for hm wave measurements. First, the subsatellite can be located far enough from the Shuttle to minimize low frequency noise. Second, power to the subsatellite and data communications can be through the tether to the Shuttle. Third, the attitude of the subsatellite can be maintained fixed in inertial space.

In order to realize the foregoing advantages, there are several problems which must be considered. First, the tether would have to be attached to the Shuttle along an axis which remains fixed in inertail space. If this is not the case, any attitude control maneuver would set the combined system of Shuttle and subsatellite into rotation. This could be done, for example, by orienting the longitudinal axis of the Shuttle transverse to the Shuttle velocity and tethering the subsatellite to the nose of the Shuttle. As the Shuttle traveled around the earth, it could roll so that the top (or bottom) of the Shuttle was always along the local vertical.

The foregoing configuration could cause another problem to arise, however. Because of the size and shape of the Shuttle, drag from the atmosphere will be greater on it than on the subsatellite. The unbalanced forces on the two ends of the tethered system constitute a torque that would set the combined system into rotation about the center of mass. How significant this would be depends on the altitude of the orbit and the shapes of the tethered Shuttle and subsatellite.

Another problem which must be considered is how to keep the subsatellite fixed in inertial space. Clearly, spin stabilization is not possible because of the large field changes this would cause in the hm wave sensor. If the subsatellite carries a heavy array of three orthogonal search coils the subsatellite might have a tendency to tumble as a consequence of forces due to its own attitude control system or impulses transmitted to it via the tether. It seems likely that gyros may be required to stabilize the subsatellite. Finally, we note that there must be a very accurate system for determining the attitude on this subsatellite to allow reduction of observations from multiple satellites to a single coordinate system.

B. Free Subsatellites

Free subsatellites are another alternative which might make it possible to observe hm waves in Shuttle orbit. Clearly, the problems of determining and maintaining fixed inertial attitude of a free subsatellite is the same as for a tethered subsatellite. In addition, however, there are new problems of power and communications. For a short-lived mission the power problem does not appear to be too serious, but it should be kept in mind that the subsatellite will be in darkness half the time. Communications are more difficult since it cannot be assumed that ground stations will always be under the subsatellite. Possible solutions include on-board data storage, communications via synchronous satellite and communications directly to or through other subsatellites with the Shuttle.

Another problem with a free subsatellite is maintenance of its position in orbit relative to the Shuttle and other free subsatellites. For short missions this would probably not be serious, but if a number of subsatellites are left in orbit after Shuttle returns to earth, the problem would become important.

C. Summary

Both tethered and free subsatellites provide a better opportunity for hm wave measurements than Shuttle itself. Both types of subsatellites require a method of precisely determining and maintaining the attitude of the subsatellite in inertial space. The interaction of the Shuttle with the subsatellite via the tether is a problem of some concern. The tether does, however, provide a simple means of providing power and communications to the subsatellite. A free subsatellite requires a source of power, means for controlling its orbital position, and communications with the ground, considerably increasing the cost and complexity of the free subsatellite as compared with a tethered subsatellite.

Magnetometer Array for Separating Spatial and Temporal Effects in Hm Wave Measurements at Shuttle Orbit

We have previously dsicussed a number of sources of magnetic field which would be observed by an hm wave sensor in Shuttle orbit. It is apparent that satellite motion through any spatial changes in the magnetic field would be recorded as a time variation. Furthermore, the absence of time variations at a particular ground station beneath the satellite could not be taken as evidence that the satellite signal is due to purely spatial effects because of the possibility that wave phenomena are spatially localized. Because of these facts, we must devise a strategy for separating spatial and temporal effects. As we show below, this requires an array of magnetometers if the results are to be unambiguous.

A. Observations with a Single Observatory

To begin, let us consider the simpler problem of separating temporal and spatial effects in ground observations. If we have a single observatory, we can only measure the vector field as a function of time. If observations are made over a sufficiently long time, the mean value of the observations will be due to spatial effects, i.e., local sources of magnetic field. The fluctuations about this mean will have various causes including propagating waves, standing waves, and time varying noise. We may Fourier transform the observations obtaining the frequency spectrum of the fluctuations. If the Fourier spectrum were entirely due to waves and we knew their dispersion relation, we could calculate the spatial distribution of the waves as well. This follows from the fact that the waves satisfy the wave equation which enables one to write for any field component

$$f(x,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A(\omega) e^{-ik(\omega)x + i\omega t} d\omega$$

In the above $k(\omega)$ is the dispersion relation for the particular wave phenomenon and $A(\omega)$ is the Fourier transform of the observations at a particular location.

Since our major experimental goal is to define the dispersion relation, we must invert the above procedure. In other words, we wish to use observations of $f(x, t)$ to define $k(\omega)$. Thus, we must make observations at more than one point as a function of time.

B. Observations with Two Observatories

Suppose the signal consists of a single frequency component, ω_0. Then

$$f(x,t) \propto \mathrm{Re}\left[e^{-ik(\omega)x + i\omega t}\right] = \cos(kx - \omega t)$$

Note that this implies that only a single wavelength is involved. From the observations at a location X, we can determine the frequency ω_0. Then from observations at a second location we can determine the difference in phase, i.e.,

$$f(x_1, t) = \cos(kx_1 - \omega t)$$

$$f(x_2, t) = \cos(kx_2 - \omega t)$$

$$\Delta\phi = k(x_2 - x_1) = k\Delta x$$

Then $k = \Delta\phi/\Delta x$ and $V_{phase} = \omega/k$. For this simple case, two observatories are sufficient to determine the wave number and phase velocity. If nature were sufficiently kind as to provide repeated monochromatic examples of this wave phenomenon over a wide range of frequencies, we could map out the dispersion relation.

In many natural situations we may have $\Delta\phi = 0$, i.e., standing waves. If this happens, we learn nothing about the dispersion relation. More frequently the signal consists of a broad spectrum of waves and no constant phase difference exists. Again, we are unable to determine the dispersion relation.

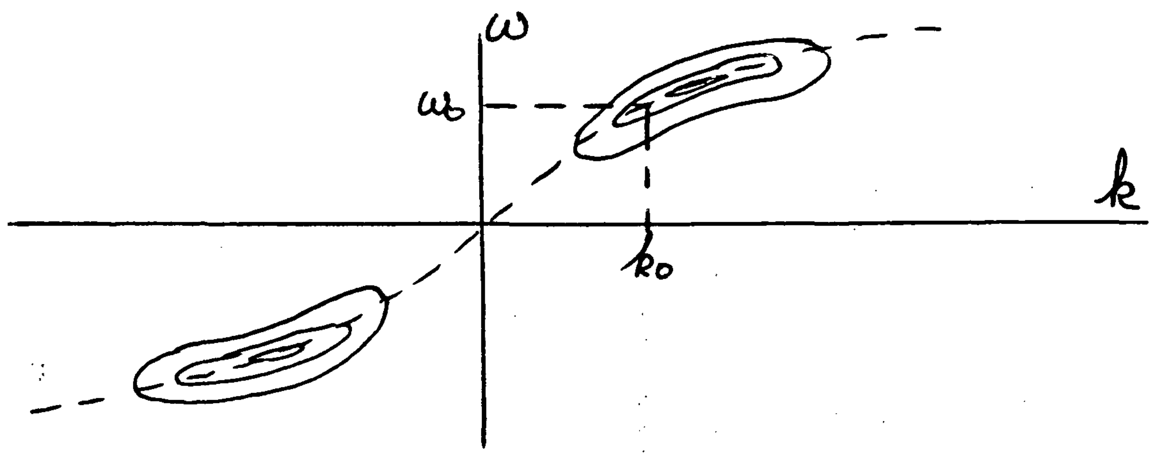
C. Observations with a "Dense" Array of Observatories

In the preceding discussion, our ability to determine the dispersion relation depended on an assumption about the shape of the signal waveform. In order to solve the general problem we must make independent measurements of both the time variations and the spatial variations of the signal. To do this, we must have an array of observatories.

Let us assume for the present that we can continuously measure both the time and space variations of our signal with infinite resolution. In this case, we can evaluate the double Fourier transform for the observations

$$F(k,\omega) = \frac{1}{2\pi} \iint_{-\infty}^{\infty} f(x,t)\, e^{ikx - i\omega t}\, dk\, d\omega$$

This transform may be displayed as contour maps of the real and imaginary parts in the ω, k plane.



In such a display any fluctuations which satisfy the wave equation will have f(k, ω) organized along trajectories corresponding to the dispersion relation. This follows from the fact that for a given ω_0, only those values of k which satisfy the dispersion relation are allowed. Note it is this fact which enabled us to express $f(x, t)$ as a single integral when we discussed the spatial distribution earlier.

The plot of $f(k, \omega)$ in the ω, k plane is an extremely useful tool because it allows us to distinguish between propagating waves, standing waves, noise and purely spatial variations. First, since purely spatial variations are time independent by definition, they appear at zero frequency. Because they are unrelated to the wave phenomena present in the signal, they will appear as a singularity along the k axis. Second, noise will not satisfy the wave equation and should not be organized along trajectories. Third, monochromatic waves appear as a pair of impulses in opposite quadrants. The phase velocity of this wave is given by the slope of a line from the origin to the locations of these two impulses. Finally, since a standing wave is the superposition of two waves propagating in opposite directions, it will be represented by two pairs of impulses symmetrically located in all four quadrants.

D. Limitations Resulting from Finite Extent and Separation of Observatories in Array

In the discussion above, we assumed that we could measure the spatial distribution of the waves with infinite resolution over all space and all time. In a typical situation both the length of the array and the time duration of the records are of limited extent. Furthermore, the data are usually sampled at discrete intervals. These effects of truncation and discrete sampling can be quite significant, particularly in the case of the spatial variable where each additional observatory represents a considerable increase in cost.

The effects of truncating a time or space series in the transform domain are easily calculated. Truncation is accomplished by multiplying the series by a "rectangular" function. Using the convolution theorem, multiplication in one domain is equivalent to convolution in the other. Thus, we convolve the sin X/X transform of the rectangle function with the transform of the original data. For a rectangle of length L the transform will have a width of order 1/L. As a result of the convolution, detail in the spectrum of width less than 1/L is lost. In particular, we cannot determine any information about frequencies less than 1/L, i.e., wavelengths longer than the length of the array.

Effects of sampling are calculated in a similar way. Sampling is accomplished by multiplying the series by a Dirac comb of delta functions spaced ΔL apart. The transform of the Dirac comb is also a Dirac comb with reciprocal spacing, $1/\Delta L$. Convolving this transform with that of the original series replicates the spectrum an infinite number of times. Problems arise if the spectrum of the original data extends more than half way to the first delta function of the reciprocal Dirac comb, i.e., beyond $1/2\Delta L$. When this is the case, convolution folds the spectrum on top of itself and it is impossible to determine whether the original signal is above or below the folding frequency. In the original domain this process of folding corresponds to sampling the original waveform at intervals greater than half its shortest wavelength.

If observations are made with a fixed ground array, the main limitation is the number of observatories which is available. Usually we can sample with as much time resolution and for as long a time as necessary. Consequently, the frequency resolution and folding frequency that result from truncation and sampling in the time domain are not important. In the wave number domain, however, serious problems arise. For example, suppose 10 observatories are used, then only five wavelengths can be measured. In many cases this would be adequate to determine considerable information about the dispersion relation. Considerable improvement could be achieved if one adopts the strategy of removing every other station and using them to double the lengths of the array. Over a long interval of time this procedure would define the phase velocities of a large number of wavelengths.

The foregoing procedure provides a possible means for separating temporal and spatial effects in Shuttle orbit. Suppose that the Shuttle places a number of subsatellites into the same orbit each separated by a variable distance. Data from this Shuttle array could be double Fourier transformed and reduced to a stationary coordinate system using the shift theorem of Fourier analysis. This theorem states that the Fourier transform of a function shifted by an amount a, i.e., f(x - a) is given by

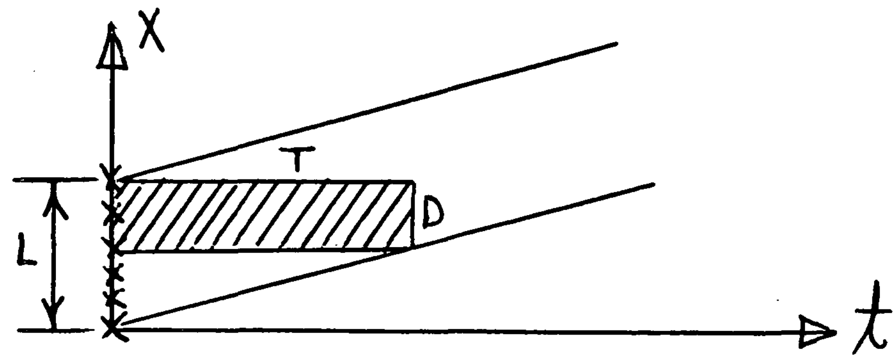
$$F'(k) = e^{ika} F(k)$$

where

$$F(k) \underset{F.T.}{\Leftrightarrow} f(x)$$

For the Shuttle array, $a = V_S t$, where V_S = satellite velocity.

The need to define the time variation by a moving array imposes a new restriction not discussed above. Since the array is of finite extent the time variation in a given spatial interval can only be measured while the array is passing by. This fact is illustrated below where we show the location of the array as a function of time by diagonal lines in the x-t plane



From the diagram it is clear that no time variations can be measured at a given spatial location after the last observatory has passed overhead. Note that the longer time, T, one attempts to measure, the shorter is the spatial interval D over which this can be done. If we optimize both D and T by maximizing the area of the rectangle, then D =L/2 and T = L/2 V_S. As an example, suppose the length of the array is 10^4 km and 10 observatories are used. Then from above the effective length is 5000 km and T = 500 seconds. Note that at any one time only half the observatories are contributing to the definition of the function of x and t. Because of this, only half as many wavelengths may be defined, as was the case for the fixed array. Thus, 10 moving observatories will define only two wavelengths.

Provided it is possible to progressively increase the separation of the observatories, it would still be possible to define the dispersion relation of a frequently occurring wave phenomenon.

Summary of Feasibility Study

The fundamental question we have examined in this report is whether it is feasible to measure hm waves on space Shuttle. The answer appears to be that it is just possible technically to make such a measurement. However, for a number of reasons, the results would be very difficult to interpret.

The basic technical problem is that hm waves are extremely weak in comparison with the earth's field at Shuttle orbit. Any instrument designed to measure these waves must simultaneously have a wide dynamic range and high resolution. For example, a fluxgate magnetometer would have to have a resolution of order 0.01 γ and dynamic range of $\pm 50{,}000\ \gamma$, or 1 part in 10^7.

For a search coil on a non-spinning spacecraft, the requirements are less severe, but still significant. The primary requirement is that the induced emf due to satellite motion through the earth's field does not saturate the system amplifier. This requires that we be able to measure a signal of amplitude $\Omega_\omega B_\omega$ due to a wave superimposed on a signal as large as 50 $\pi\gamma$/sec, the maximum dB/dt due to satellite motion in the dipole field. Thus, we must resolve to better than $2\pi B_\omega / 50\pi\ T_\omega = 4 \times 10^{-4}$ for a 0.01 γ wave at 1 Hz.

If the spacecraft is subject to frequent changes in attitude, the measurement becomes more difficult for either type of instrument. For a search coil we must still be able to resolve $\Omega_\omega B_\omega$ compared with $\Omega_{rot} B_o$, i.e., to better than $(2\pi B_\omega / T_\omega) / 2\pi B_o / T_{rot}$ = $(T_{rot}/T_{wave})\ (B\omega/B_o)$.

If we assume that a typical attitude control maneuver is carried out at a rate corresponding to a full roll in 500 sec, then resolution $\approx (500/1)\,(10^{-2}/5 \times 10^{4}) = 1 \times 10^{-4}$ for the 0.01 γ wave at 1 Hz.

While these requirements are severe, existing instruments can probably be modified to meet them. Assuming this is done, we still believe it is extremely unlikely that hm wave measurements can be made on Shuttle itself. This is because the Shuttle observatory will be so large and complex that there will be abundant sources of DC and AC field to contaminate the observations. Since the guiding philosophy of Shuttle is to be a low cost, general purpose facility there is little possibility that it will be constructed to minimize these sources of noise.

Long booms are one possible way to reduce the effects of Shuttle noise sources to a tolerable level. Such booms introduce a number of problems, however. The most serious problem is boom vibrations. A long boom is a resonant structure which will oscillate in response to each change in attitude of the Shuttle vehicle. Effects of these vibrations could be much more serious than those due to both satellite motion through the dipole field and attitude rotation, depending on their amplitude and period. For example, if the boom vibrations were only 10 minutes of arc in 10 sec, the amplitude of the corresponding field oscillations would be about 150 γ. This is enormous compared with any naturally occurring signal of this period. Spectral analysis would be required in order to extract any natural signals from the resulting waveform.

A more attractive solution to the problems imposed by noise and attitude control on the Shuttle is to place the hm wave sensor on a subsatellite. If the subsatellite were tethered to the space Shuttle, power, ground communications, and orbital control could be supplied by the Shuttle, simplifying construction of the subsatellite. For a free subsatellite these would have to be built into the subsatellite in addition to other requirements. The most important of these requirements for magnetic field measurements would be inertial stabilization and a very accurate means of determining sensor attitude. If the instrument measures the absolute field rather than its derivative, very negligible errors in attitude (i.e., of order 1/10°) would cause large errors in the field components (as large as 87 γ). If hm measurements on two satellites were compared with attitude determined no more accurately than the 1/2° planned for the Shuttle itself, differences in measured field as large as 1000 γ could result. Again spectral analysis would be required to eliminate these differences in the measured signals. We note that the requirement that the subsatellites be inertially stabilized may be hard to realize, particularly for a tethered subsatellite because of forces transmitted to it from the Shuttle.

Let us now assume that a suitable designed magnetometer is placed in an inertially stabilized, noise-free subsatellite. There are numerous natural sources of magnetic field which will be observed by the instrument as time variations with amplitude greater than thse of hm waves. These include motion of the satellite over crustal anomalies in the earth's field, motion over the ionospheric Sq current system, motion over the equatorial and auroral electrojets, and passage through sheets of field aligned currents. External to the satellite are the magnetopause, ring and tail currents. Effects due to the latter are not likely to be localized in latitude but they do change on a time scale comparable to the satellite's orbital period. The electrojets can be highly structured and passage over them would be recorded as large amplitude waves of short duration (several hundred gamma in tens of seconds).

None of the foregoing effects can easily be removed from the data of a single satellite. Most of them are due to phenomena which are highly variable on a day-to-day basis. Even the crustal anomalies, which are essentially constant in time, would require very long intervals of recording, perfectly circular orbits and synchronized orbital periods to remove from the data. If we limit our observations to the $\simeq 45°$ latitude between electrojets, we can only observe waves of 500 sec period or less since this is the time the satellite takes to traverse this region. For a typical Shuttle orbit of 500 km altitude, any observed time variation with period longer than 500 km/10 km/sec = 50 sec may be due to crustal anomalies. For periods shorter than this, it is more likely they are real time variations since far from a magnetic source only wavelengths comparable to or larger than the distance of observation can be observed.

Let us now assume that we have successfully measured something between 10 and 500 cycles of what are probably hm waves of period less than 50 sec. These measurements by themselves are of little value, but if they are combined with ground data, then it becomes possible to make inferences about wave propagation, source regions, etc. As an example, suppose nearly the same frequency is seen at both ground and satellite. A cycle-by-cycle comparison of wave polarization between satellite and ground starting when the satellite passed poleward over the station might reveal a progressive change in phase due to wave propagation, or a systematic change in amplitude due to a standing wave.

Since one observation is made above the ionosphere and one below, it is quite possible that some of the observed differences are a result of propagation through the local ionosphere. In order to remove possible ionospheric effects from the observations, one needs at least two satellites in the same orbit. Intercomparisons between these and a ground station would reveal additional information.

In general, most wave phenomena consist of a broad band of frequencies and wavelengths. In such cases a comparison of the waveforms at two locations will not be particularly meaningful. This is especially true if the medium is dispersive. In this case the only recourse is to establish an array of observatories. The largest wavelength which can be studied corresponds to the length of the array; the shortest wavelength to twice the array spacing. Similarly, the lowest and highest frequencies are limited by the duration and sampling of the time record at each station in the array.

If we assume that data are available from such an array, we can best study wave phenomena by Fourier transforming the data to frequency-wavenumber space, i.e., the w-k plane. A wave packet propagating through a dispersive medium will have a Fourier transform organized along a curved trajectory in the w-k plane. A standing wave would consists of a symmetric pair of such trajectories corresponding to pairs of waves at each frequency propagating in opposite directions. Truly spatial variations will appear as a singularity function along the k axis.

If we wish to make an unambiguous determination of the phase velocity of hm waves, we must carry out such a procedure. If this were our only goal, it would be sufficient to use a fixed ground array. But if we wish to remove effects of satellite motion over spatial variations from temporal effects, the array must be in Shuttle orbit. As we mentioned above, this is a matter of necessity for wave periods longer than about 50 sec. Unfortunately, a moving array of finite extent is handicapped by the fact that it is passing over a given spatial interval for a limited time. Since the double Fourier transform effectively requires equal length time records at each spatial point, we are only able to use data from half the spatial array. Thus, definition of only four wavelengths would require 4 x 4 = 16 observatories in the Shuttle array.

It seems extremely unlikely that inertially stabilized subsatellites capable of obtaining and maintaining their positions in a spaced array can be made and launched cheaply. Consequently, we feel a Shuttle array is unfeasible. The only alternative appears to be a detailed mapping of the vector magnetic field with a single satellite in Shuttle orbit. Such a map would require an extensive interval of data acquisition and reduction to produce. Given such a map, knowledge of satellite position and attitude would allow one to remove purely spatial variations from the observations of a given orbit.

Conclusions

1. An hm wave sensor is technically feasible for Shuttle orbit.

2. Existing sensors are inadequate in terms of resolution, dynamic range, and frequency response, but can probably be modified to make the necessary measurements.

3. It would be impossible to mount the sensor on the Shuttle itself because of high levels of magnetic noise.

4. A long boom attached to the Shuttle is unlikely to be very helpful unless attitude control maneuvers are made continuously rather than in discrete steps. High rotation rates or boom vibrations are likely to mask any natural signals.

5. A tethered subsatellite appears to be an inexpensive way of removing the hm sensor from the influence of the Shuttle provided it can be inertially stabilized and will not be influenced through the tether by attitude control maneuvers of the Shuttle.

6. A free subsatellite that can be positioned as well as stabilized is a better location for an hm wave sensor.

7. Hm wave measurements can probably only be made in the ≈45° of latitude between the highly structured and unpredictable equatorial and auroral electrojets.

8. Because of magnetic effects due to crustal anomalies, wave periods longer than ≈50 sec would be difficult to study.

9. Studies of long period waves would require either an array of sensors in Shuttle orbit or a long-term mapping of the crustal anomalies.

10. Effective wave studies would require at least two variably spaced sensors in Shuttle orbit and one ground station.

11. An extensive linear array on the ground would contribute greatly to the study.

Acknowledgments

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Zietz, I., G.E. Andreasen and J.C. Cain, Magnetic anomalies from satellite magnetometer, J. Geophys. Res., 75, (20), 4007-4015, 1970.

Figure Captions

1. Fluctuating values of total magnetic field obtained by a polar orbiting satellite after subtraction of a model field. (Cain and Hendricks, 1967).
2. Magnetic effects of crustal anomalies observed at satellite altitudes over the United States (Zietz, et al, 1970).
3. Magnetic variations observed by a polar orbiting satellite as it passes over the equatorial electrojet (Cain and Sweeney, 1973).
4. Anomalies in the equatorial electrojet signature illustrating day to day variability (Cain and Sweeney, 1973).
5. Total field changes caused by satellite passage over the auroral electrojet (Langel and Cain, 1968).
6. Transverse field variations observed by a vector field magnetometer passing through a sheet of field aligned current flowing into the auroral oval (Armstrong, et al, 1974).
7. High resolution ATS-1 magnetic field data showing effects of spacecraft field as discussed in text. Vertical scale 5 gamma per division, horizontal scale 5.12 seconds per division.
8. Power spectrum of the derivative of ATS-1 magnetic field data showing harmonics interference in transverse components at multiples of (1/5.12) HZ.
9. Power spectrum of quiet ATS-1 field data showing harmonics of (1/81.92) HZ due to ramp in Z component.

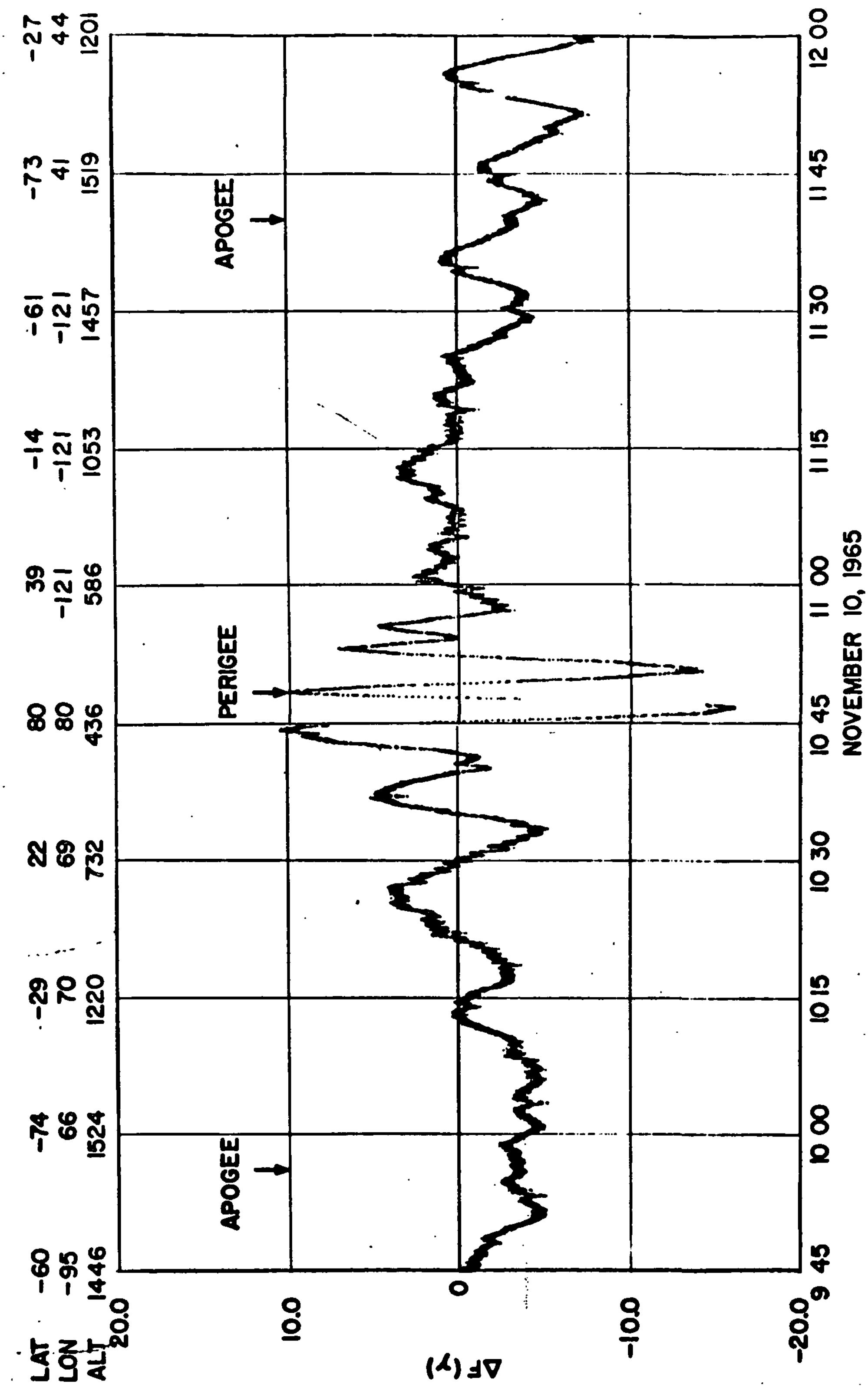
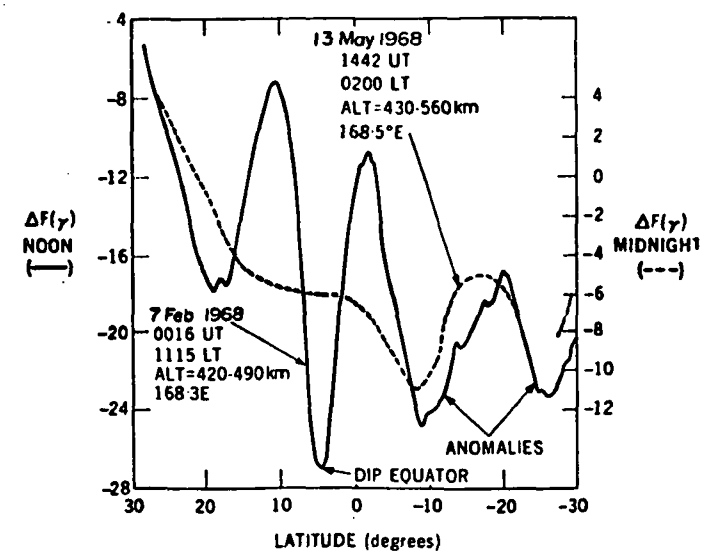

LAT
LON
ALT
-60 -95 1446
-74 66 1524
-29 70 1220
22 69 732
80 80 436
39 -121 586
-14 -121 1053
-61 -121 1457
-73 41 1519
-27 44 1201
APOGEE
PERIGEE
APOGEE
20.0
10.0
0
-10.0
-20.0
ΔF(γ)
9 45
10 00
10 15
10 30
10 45
11 00
11 15
11 30
11 45
12 00
NOVEMBER 10, 1965

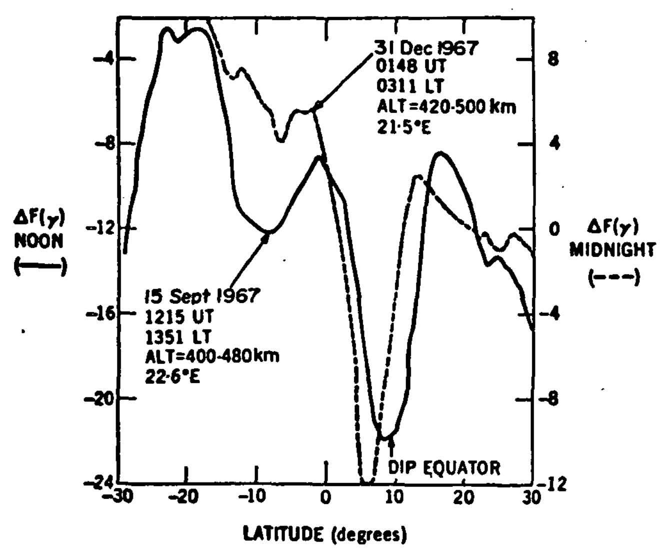
Figure 1

Residual magnetic map contoured from data taken by USSR satellite Cosmos 49. Contour interval is 5 γ. The altitude ranges from 261 to 488 km.

Figure 2

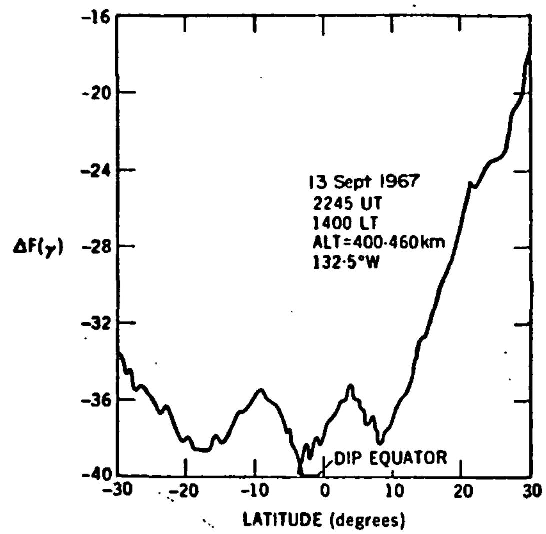


Examples of day and night traversals over western Pacific. Both day (solid line) and night (dashed) show structure from 10° to 30°S probably due to magnetic anomalies. High 'shoulders' on each side of electrojet minimum in day trace is atypical.

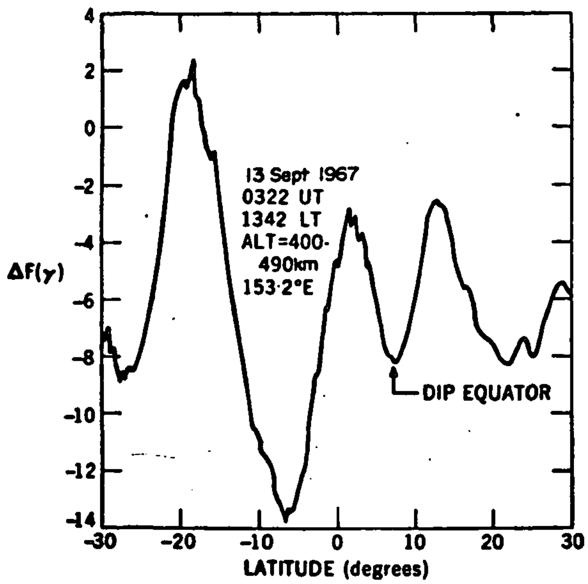


Magnetic anomaly around Bangui (Central African Republic) near position of electrojet on both day and night passes.

Figure 3

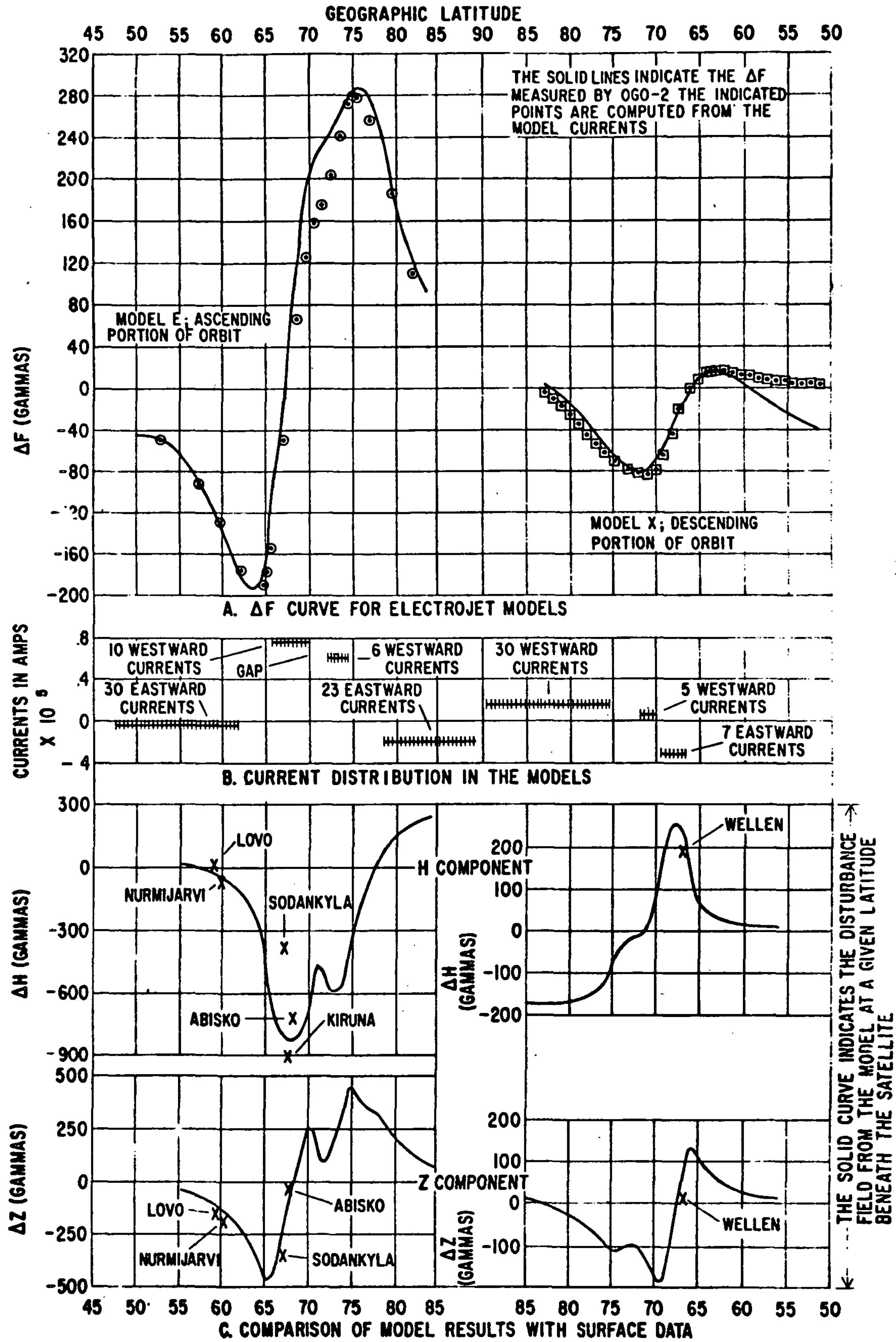


Complex signature in field residuals during moderate magnetic disturbance.



. Complex variation with depression on one side of jet exceeding that of the jet.

Figure 4



Disturbance fields from A polar current model compared with disturbances observed at OGO-2 and at surface observatories.

Figure 5

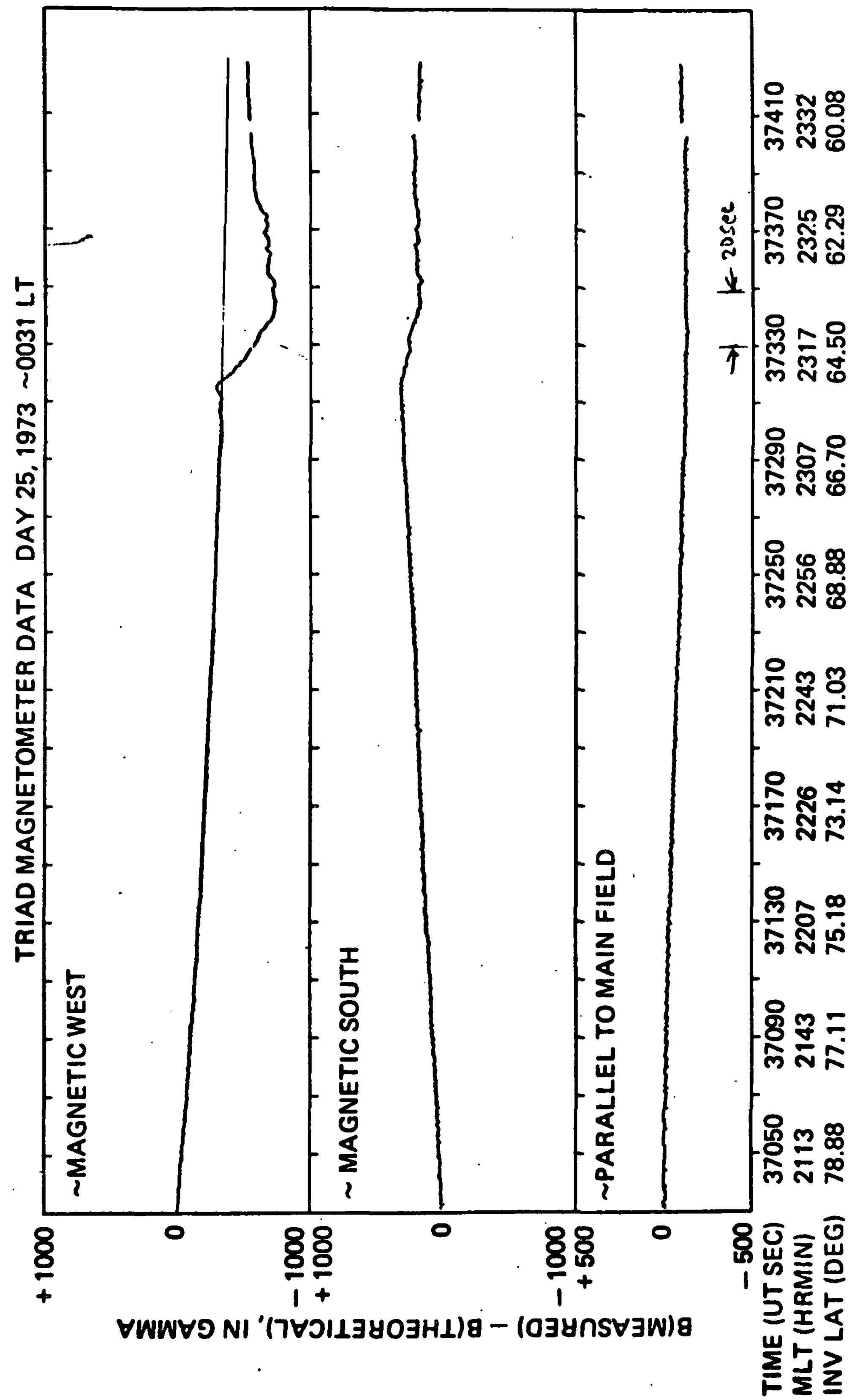

TRIAD MAGNETOMETER DATA DAY 25, 1973 ~0031 LT
~MAGNETIC WEST
~MAGNETIC SOUTH
~PARALLEL TO MAIN FIELD
B(MEASURED) – B(THEORETICAL), IN GAMMA
+1000
0
−1000
+1000
0
−1000
+500
0
−500
TIME (UT SEC) 37050 37090 37130 37170 37210 37250 37290 37330 37370 37410
MLT (HRMIN) 2113 2143 2207 2226 2243 2256 2307 2317 2325 2332
INV LAT (DEG) 78.88 77.11 75.18 73.14 71.03 68.88 66.70 64.50 62.29 60.08
20sec

Figure 6

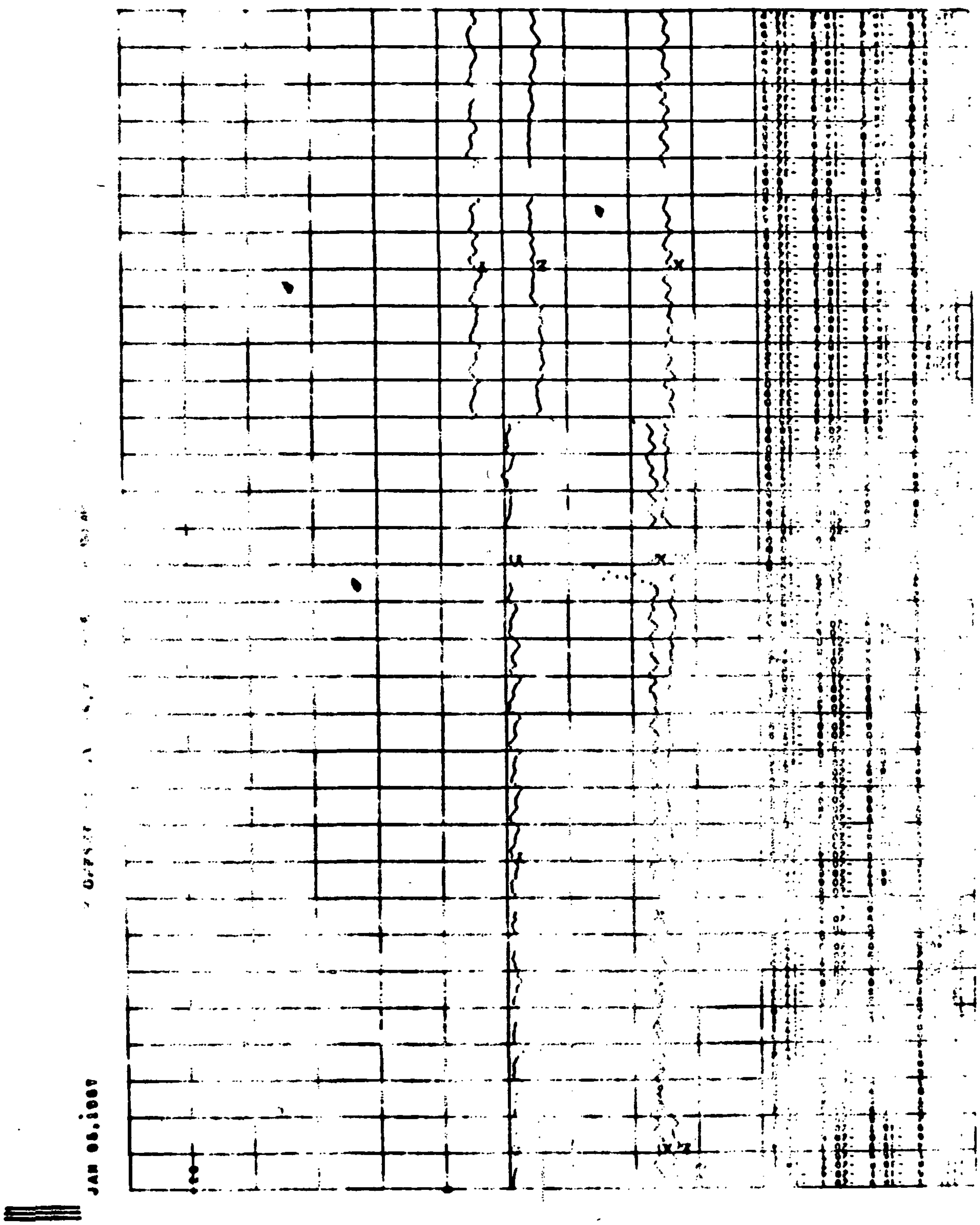


Figure 7

QUIET DATA
0200-0211 UT OCT 9, 1969
UCLA ATS-1 MAGNETOMETER

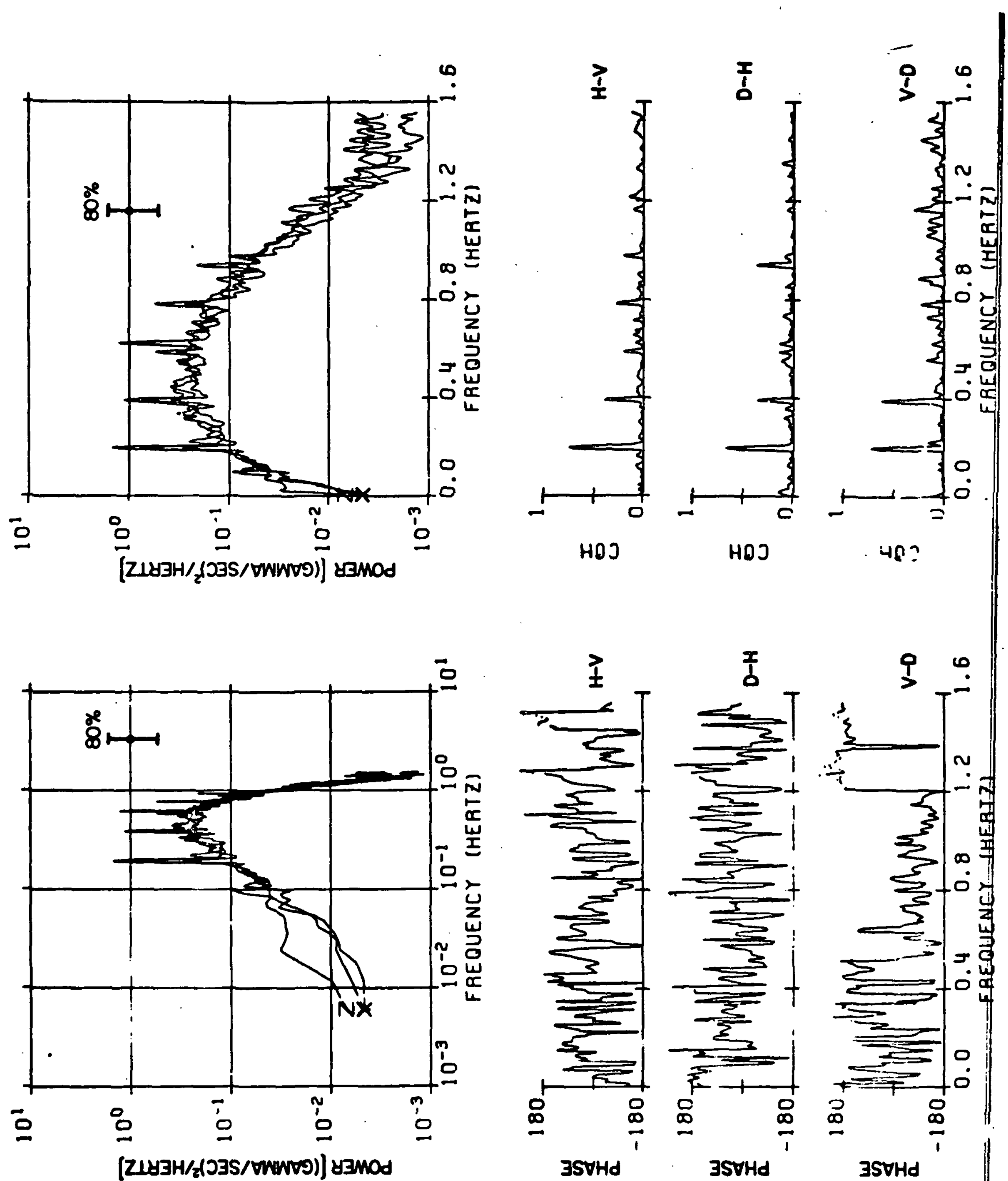


Figure 8

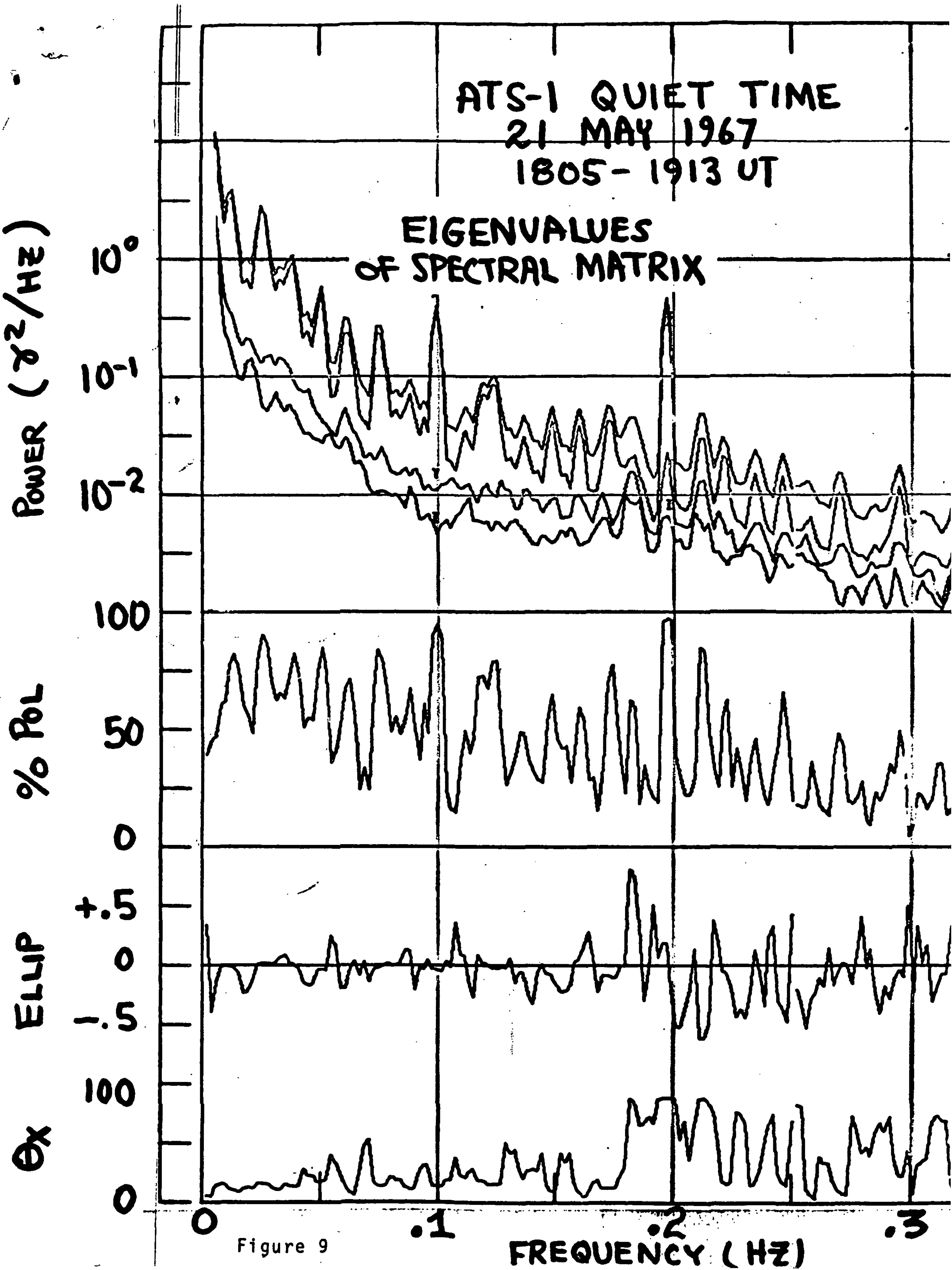

ATS-1 QUIET TIME
21 MAY 1967
1805 - 1913 UT
EIGENVALUES
OF SPECTRAL MATRIX
POWER (γ^2/HZ)
10^0
10^{-1}
10^{-2}
% POL
100
50
0
ELLIP
+.5
0
-.5
θ_x
100
0
0
.1
.2
.3
FREQUENCY (HZ)

Figure 9

Introduction

A. Why should hydromagnetic waves be measured in space?

HM waves are produced by plasma instabilities or currents far out in the magnetosphere and propagate to the earth's surface. For many hm waves there is reason to believe that propagation is field-aligned, or ducted. Thus, the waves seen at the earth's surface provide information about plasma processes going on at known points in space. The wave spectrum, polarization, and direction of propagation can be used to infer the nature of these processes. For example, the existence of Pc 1 waves (left elliptically polarized transverse waves of 5 sec period) has been used to infer the occurrence of the ion cyclotron instability of energetic protons in the magnetosphere. Furthermore the fact that one type of Pc 1 (pearl pulsations) shows disperion in its dynamic spectrum has been used to determine the L shell on which the waves are propagating, the equatorial density of cold plasma on this line, and the parallel energy of the protons responsible for the instability. Another type of Pc 1, IPDP, is beginning to provide information about the point and times of proton injection during substorms as well as details about the drift of these particles. Other possibilities include the high latitude Pc 4,5 seen near the polar cusp, or Pc 3,4 seen near the plasmapause. These waves may eventually be used to provide information about the location of these boundaries as well as processes occurring on them.

In summary, hm waves are diagnostic of plasma processes in the magnetosphere. The location at which these waves occur on the earth's surface reflects locations of important regions or boundaries in space. The properties of the waves reveal information about the plasma properties in these regions. The waves therefore provide the possibility of inexpensive monitoring of these magnetospheric processes with ground observations.

B. Current Feasibility of Monitoring Magnetospheric Processes with hm Waves

The general goal outlined above is not presently attainable for a numer of reasons. Mainly this is a consequence of our lack of understanding of the processes of wave generation and propagation to the earth's surface. This is, in part, the major justification for an hm wave sensor on Space Shuttle. We presently have almost no understanding of the effects of the ionosphere on the transfer of waves. For some waves the ionosphere may act as a resonant cavity, increasing the amplitude of the incident waves much above their initial amplitude. Alternatively, the ionosphere may duct energy away from the point of entry. Almost certainly the polarization of the incident waves is greatly altered by the ionosphere.

These facts make it impossible to use current ground observations to determine the point of entry of hm waves from the magnetosphere, or their polarization as it exists within the magnetosphere. Without this information, no clear inferences can be made about the locations or types of magnetospheric generation mechanisms.

A major goal of an hm wave sensor on Space Shuttle would be to determine the effects of the ionosphere on the transmission of hm waves to the ground.