ROLL PAPER PILOT

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ABSTRACT

A mathematical model for predicting the pilot rating of an aircraft in a roll task is described. The model includes:
(1) the lateral-directional aircraft equations of motion;
(2) a stochastic gust model; (3) a pilot model with two free parameters; and (4) a pilot rating expression that is a function of rms roll angle and the pilot lead time constant.
The pilot gain and lead time constant are selected to minimize the pilot rating expression. The pilot parameters are then adjusted to provide a 20% stability margin and the adjusted pilot parameters are used to compute a "roll paper pilot" rating of the aircraft/gust configuration. The "roll paper pilot" rating was computed for 25 aircraft/gust configurations. A range of actual ratings from 2 to 9 were encountered and the "roll paper pilot" ratings agree quite well with the actual ratings. In addition there is good correlation between predicted and measured rms roll angle.

1. INTRODUCTION

The paper pilot concept for analytically evaluating the handling qualities of an aircraft is based on the following hypothesis.
1. The numerical pilot rating of an aircraft's handling qualities in a well defined piloted task is a function of the closed loop performance and the pilot work load. This function is called a pilot rating expression.
2. Given the pilot rating expression, the adaptable parameters in the appropriate pilot model are selected to minimize the numerical value of the pilot rating expression for the closed loop system (the lower the rating the better handling qualities). The resultant minimal value for the pilot rating expression provides a "good" analytic indication of the actual numerical pilot rating of the aircraft.
The first part of the hypothesis just suggests that there is some rational basis for a numerical pilot rating. The second part of the hypothesis is based on the assumption that a pilot will adapt so that the closed loop system is the best possible in the sense of what the pilot thinks is best!

The first successful application of the paper pilot concept was the hover task for a VTOL aircraft (Refs 1 and 2). In this particular case only the longitudinal handling qualities were considered.
The pilot rating expression for longitudinal handling qualities in hover developed by Crazy Anderson (Ref 1) is a function of the rms pitch rate, the rms longitudinal hover error, and the pilot work load. The pilot work load is a function of the lead time constants in the pilot model. The pilot work load increases with increasing values of the lead time constants.

The paper pilot rating (the numerical rating derived by minimizing the pilot rating expression) was computed for 79 aircraft/gust intensity combinations (Ref 2). The paper pilot ratings were compared with actual pilot ratings derived from a fixed base simulation as reported in Refs 3 and 4. The correlation is excellent. The paper pilot rating differed from the actual rating by more than 1 rating unit in only 9 of the 79 cases. The difference exceeded 1 1/2 rating units in only 1 case (a difference of 1.69). Actual pilots seldom agree that well.

The paper pilot concept was next applied to a pitch tracking task (Ref 5). The pilot rating expression was based on the limited data of Ref 6. The pilot rating expression for the commanded pitch tracking task was taken to be a function of the rms tracking error and the pilot lead time constant. This time Crazy Anderson didn’t fare so well and some problems were encountered. In particular, for those cases where the short period mode is slightly damped and the short period natural frequency is high, the paper pilot ratings tend to be well below the actual rating (Ref 5) or tend to rate an aircraft as “good” when the open loop characteristics are clearly undesirable (Ref 7).

In this paper, the paper pilot concept is applied to the evaluation of aircraft handling qualities in roll. The piloted task considered is to keep wings level in the presence of side gusts. The aircraft equations, control system equations, gust model, and pilot model are described.

The pilot rating expression was developed using data from Ref 8. In Ref 8 the results of a moving base simulation for 25 aircraft/gust configurations are described. The pilot rating expression was then used to analytically determine a roll paper pilot rating and the closed loop performance for each case. The roll paper pilot ratings are compared with actual ratings and the predicted rms roll angles are compared with measured rms roll angles for these 25 cases in the Conclusion section of this paper.
11. PILOT-VEHICLE MODEL

The pilot-vehicle model used for the roll task was taken from Appendix III of Ref 8. A block diagram of the system is shown in Fig. 1. It can be noted from the block diagram, the pilot is regulating roll angle against a side gust disturbance using only aileron inputs.

\[ \dot{s} = s_{\text{p}} + s_{\text{e}} + s_{\text{d}} + s_{\text{r}} \]

\[ \dot{p} = p_{\text{p}} + p_{\text{e}} + p_{\text{d}} + p_{\text{r}} \]

\[ \dot{r} = r_{\text{p}} + r_{\text{e}} + r_{\text{d}} + r_{\text{r}} \]

\[ \dot{\phi} = \phi_{\text{p}} + \phi_{\text{e}} + \phi_{\text{d}} + \phi_{\text{r}} \]

where \( s \) is sideslip angle, \( p \) is roll rate, \( r \) is yaw rate, \( \phi \) is roll angle, \( \delta_e \) is aileron deflection, \( \delta_r \) is rudder deflection, and \( V_{\text{gust}} \) is the gust input. \( V_{\text{gust}} \) is equal to \( V_{c}/V_0 \) where \( V_{c} \) is the gust intensity along the y axis and \( V_0 \) is the nominal longitudinal velocity.

2. Gust Model

The Dryden model for the lateral \( V_{\text{gust}} \) is used (Ref 9). The spectral form of \( V_{\text{gust}} \) is

\[ \Phi_{V_{\text{gust}}} (\omega) = \sigma_v^2 \frac{L}{V_0} \left[ 1 + \left( \frac{\omega}{\omega_0} \right)^2 \right] \]

where \( \sigma_v \) is the rms gust level in rad/sec and \( L \) is a scale length of 1000 ft. The corresponding filter that can be used to generate the gust from white noise is

\[ H(s) = \frac{\sigma_v}{s} \frac{s^{1/2}}{\sqrt{V_0^2}} \left[ 1 + \left( \frac{s}{V_0} \right)^2 \right] \]

Fig. 1. Closed Loop System for the Roll Task.

1. Aircraft Equations

The linearized lateral equation of motion for the aircraft in response to control deflections and gust inputs as opted from Ref 8 are (in primed stability axis notation)
where \( N_g \) is white Gaussian noise.

3. **Pilot Model**

   The pilot model is adopted from Ref 5 and is
   \[ Y_p = K_p (T_L s + 1)e^{-\tau s} \]
   where \( K_p \) and \( T_L \) are the pilot's gain and lead, respectively, and \( \tau \) is a pure time delay. A value of \( \tau = 0.3 \) seconds was used to agree with that used in Ref 8 for a pilot with acceleration cues. A 'first order Padé' approximation to the time delay was used. The block diagram for the pilot is shown in Fig. 2.

![Block Diagram of Pilot Model](image)

**Fig. 2. Block Diagram of Pilot Model.**

4. **Control Systems**

   The control system model includes an aileron linkage time lag and a yaw damper.

   (a) **Aileron control**

   An aileron control lag was used to represent the case where a power boosted aileron system results in a time lag between the pilot input and the control surface deflection. The transfer function for the aileron control lag is
   \[ \delta_a = \frac{1/\tau_{con}}{s + 1/\tau_{con}} \]

   (b) **Rudder control**

   The rudder control consists only of a yaw damper. The transfer function for the yaw damper (Appendix III of Ref 8) is
   \[ \delta_r = \frac{-\tau_{aug}}{(\tau_{aug} s + 1)} \]
III. PILOT RATING EXPRESSION

The pilot rating expression was assumed to be of the form

\[ PR = k\sigma_{\phi} + f(T_L) \]  \hspace{1cm} (1)

where \( \sigma_{\phi} \) is the rms roll angle and \( f(T_L) \) in some function of the pilot lead time constant.

The relation between pilot rating and lead was taken to be

\[ f(T_L) = 3.25 \left(1 - e^{-0.77T_L}\right), \quad T_L \geq 0 \]

This hokey function is an exponential approximation to the straight line function of lead used in the longitudinal hover and pitch tracking cases (Refs 2 and 5). The comparison is shown graphically in Fig. 3. It was assumed that \( f(T_L) \) would not change from task to task. For those cases where \( \omega_d/\omega_{\phi} \) is not much more than one, the pilot lead time can be approximated by

\[ T_L = T_T - \frac{1}{\omega_p^2} \]

where \( T_T \) is the roll time constant (Ref 10). This approximation was used as a first cut for values of \( T_L \).

Data for \( PR \) and \( \sigma_{\phi} \) was taken from the results of the moving base simulation described in Ref 8. The pilot rating, \( * \)

\* is the frequency of the second order numerator term in the \( \phi/\dot{\phi} \) transfer function. \( \omega_d \) is the frequency of the dutch roll mode.

Fig. 5. Comparison of Functions Used to Determine Rating Degradation Due to Lead.

PR, is based on the Cooper rating scale (Ref 9) where a rating of 1 is optimal and a rating of 10 is unflyable. The values of \( \sigma_{\phi} \) are in degrees.

With the data for \( PR \) and \( \sigma_{\phi} \) and an approximation for \( T_L \), the value of \( k \) in Eq (1), and hence the performance contribution to the pilot rating, can be estimated by plotting \( PR - f(T_L) \) against the rms roll angle, \( \sigma_{\phi} \). This is done in Fig. 4 and a value of \( k = 1.3 \) was chosen. Having determined \( k \), the pilot rating expression for the roll task is

\[ PR = 1.3\sigma_{\phi} + 5.75 \left(1 - e^{-0.77T_L}\right) \]  \hspace{1cm} (2)
### IV. ROLL PAPER PILOT

Basically the paper pilot rating is determined in the following way. The values of $K_p$ and $T_L$ that minimize the pilot rating expression (Eq. (2)) for the closed loop system are determined. The resulting minimal value of Eq. (2) is taken as the preliminary paper pilot rating.

There is however, an additional detail that must be considered and two adjustments are made to arrive at the final value of the paper pilot rating. To be specific:

1. The minimizing value of $T_L$ is constrained to the range
   \[ 0 \leq T_L \leq 5 \text{ sec} \]

2. Once the minimizing pilot parameters and the resulting closed loop performance is determined, the paper pilot rating is determined by the following formula.

   \[
   PR = g(\sigma_\phi) + f(T_L) + h(\omega_\phi/\omega_d)
   \]

   where

   \[
   g(\sigma_\phi) = \begin{cases} 
   1 & , 1.3 \sigma_\phi < 1 \\
   1.3 \sigma_\phi & , 1.3 \sigma_\phi \leq 6.75 \\
   6.75 & , 1.3 \sigma_\phi > 6.75
   \end{cases}
   \]

   and

   \[
   h(\omega_\phi/\omega_d) = 6.66[1 - \omega_\phi/\omega_d]
   \]

   The constraint on $T_L$ is imposed to account for the physical limitations of the pilot.

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**Table:**

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<tr>
<th>Case</th>
<th>$\omega_\phi/\omega_d$</th>
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</table>

**Fig. 4:** Rating Degradation Due to RMS Roll Angle.
The modification on the rating contribution due to performance \( g(\sigma) \) can be justified in three ways: (1) the results are better, (2) it insures that paper pilot rating is in the same range as that of the actual pilot, between 1 and 10, and (3) it is reasonable to expect there is an upper and lower limit on the performance influence on the pilot's rating of the handling qualities of the aircraft.

The final modification to the paper pilot rating is a function of \( \omega_\theta/\omega_d \). A detailed explanation of the effect of \( \omega_\theta/\omega_d \) on the pilot rating can be found in Ref 9 or Ref 10.

The basic idea is that an aircraft with \( \omega_\theta/\omega_d = 1 \) is considered ideal by a pilot; and if there is a departure from \( \omega_\theta/\omega_d = 1 \), the resulting dutch roll oscillation is considered by the pilot to be a nuisance. The correlation of pilot rating with \( \omega_\theta/\omega_d \) as reported in Ref 10 is shown in Fig. 5. The function \( h(\omega_\theta/\omega_d) \) that is added on to the paper pilot rating is a straight line approximation to the data shown in Fig. 5.

The main reason for adding this function of \( \omega_\theta/\omega_d \) to the paper pilot rating is that it improves the correlation of paper pilot ratings with the actual ratings. Whether or not this function of \( \omega_\theta/\omega_d \) detracts from the paper pilot concept depends upon the point of view. The party of the second part believes that this term represents a failure of the pilot rating expression to adequately express the pilot rating as a function of the characteristics of the closed loop system. The party of the first part, on the other hand, considers this modified rating as the proper blend of the closed loop and open loop characteristics of the aircraft in the evaluation of aircraft handling qualities.

Fig. 5. Correlation of Pilot Rating with \( \omega_\theta/\omega_d \).
(From Ref 10).
V. RESULTS

The roll paper pilot results are computed for the 25 aircraft gust intensity configurations of Ref. 8. The digital computer program used to compute the roll paper pilot results is described in Ref. 11. The aircraft simulated was the T-33 and the detail data is given in both Refs 8 and 11.

The actual pilot's rating, roll paper pilot's rating, actual RMS roll angle, and predicted RMS roll angle are tabulated in Table I by configuration. A comparison of the results is shown in the scatter diagrams of Fig. 5, 6, and 7.

The correlation in pilot rating appears to be good. In fact, the paper pilot results may be better than the actual ratings, especially in the ratings below 4.0 or in the acceptable range. In this range there are two data points that lie outside of the one rating unit boundary. Both of these points are for the AB2.6 aircraft configuration with n = 6.0 and the same gust intensity inputs. The simulation that resulted in an actual rating of 2 had a gust input of 13.03 ft/sec and that with an actual rating of 5 had a gust input of 16.50 ft/sec. Also note that two other simulations of the AB2.6 configuration resulted in a rating of 2.5 for a gust input of 7.46 ft/sec and a rating of 3 for a gust input of 34.27 ft/sec. Perhaps these two ratings should have been somewhat closer to the paper pilot rating.

<table>
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<tr>
<th>Configuration</th>
<th>Gust Intensity</th>
<th>Actual $\bar{g}_{\theta}$ deg</th>
<th>Paper Pilot $\bar{g}_{\theta}$ deg</th>
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Fig 6. Comparison of Paper Pilot Rating and Actual Pilot Rating.

Fig 7. Comparison of RMS Bank Angle Between Pilot and Actual Pilot.
VI. CONCLUSIONS

The paper pilot predicted ratings agree well with actual ratings for those cases considered. However, a better evaluation of the agreement could be made if there was some indication of the actual pilot rating spread for each aircraft-gust intensity configuration. Also, since there was only 25 data points available for comparison, it is premature to assert that the roll paper pilot is an unqualified success at predicting ratings.

The apparent success of the roll paper pilot to predict roll angle performance in the roll tracking task is also encouraging. As with the rating correlation, a better evaluation of the agreement could be made if some information in the actual spread were known and the results were compared to a larger data base.

Press on paper pilot—you done good.

REFERENCES


