STeady-state and transient analysis of a squeeze film damper bearing for rotor stability

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16. Abstract

This report presents a study of the steady-state and transient response of the squeeze film damper bearing. Both the steady-state and transient equations for the hydrodynamic bearing forces are derived. The steady-state equations are used to determine the bearing equivalent stiffness and damping coefficients. These coefficients are used to find the bearing configuration which will provide the optimum support characteristics based on a stability analysis of the rotor-bearing system. The effects of end seals and cavitated fluid film are included. The transient analysis of rotor-bearing systems is performed by coupling the bearing and journal equations and integrating forward in time. The effects of unbalance, cavitation, and retaining springs are included in the analysis. Methods of determining the stability of a rotor-bearing system under the influence of aerodynamic forces and internal shaft friction are discussed. Particular emphasis is placed on solving the system characteristic frequency equation, and stability maps produced by using this method are presented. The study shows that for optimum stability and low force transmissability the squeeze bearing should operate at an eccentricity ratio $\epsilon < 0.4$. 

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CHAPTER 1
INTRODUCTION

Modern turbomachines are highly complex systems. Current design trends are producing machines that consist of several process stages joined together. The rotors in these machines are highly flexible shafts, often mounted in more than two bearings, that rotate at very high speeds. It is not uncommon to see machines that operate above the second critical speed. As a result the system dynamics are very complicated.

One of the major problems encountered in these machines is instability produced by aerodynamic forces on impeller wheels, friction in the stressed rotor and hydrodynamic forces in the bearings. The instability is characterized by large amplitude whirl orbits and often results in bearing or total machine failure. It is often aggravated by unbalance and other external forces transmitted to the machine. Production losses from failed machines are very high and it may take many months to repair or replace the failed unit. In addition operator safety is jeopardized when machines fail and occasional loss of life occurs.

From the earliest investigations of rotor instability, it has been known that the use of flexible, damped supports has an effect on instability and can eliminate it or alter the speed at which it occurs. Recent research has produced a large body of knowledge on the use of these supports and their effect on instability.

The squeeze film damper bearing is one type of flexible
support that is currently being investigated. This study examines the squeeze bearing and through computer simulation shows its effects on several rotor-bearing systems. The equations for the hydrodynamic bearing forces are developed in both fixed and rotating coordinate systems. The use of two coordinate systems allows for both steady-state and transient analysis of bearing performance. This results in more efficient bearing analysis and a savings in time and money when experimental testing of the bearings is conducted.

The steady-state behavior of the bearing results in the formulation of bearing stiffness and damping coefficients which can be used to set the bearing configuration. This is accomplished by comparing the coefficients with required values obtained from a stability analysis of the rotor-bearing system. Several methods of determining the system stability are discussed. The effects of end seals and cavitation of the fluid film are also included in the steady-state coefficients.

The transient analysis is very useful in determining the bearing response to particular forms of external and internal forces as noted previously. Also the effect of bearing retainer springs and fluid film cavitation can be found. The transient response is found by tracking the journal motion forward in time by integrating the equations of motion under the influence of the system forces.

The limitations of and assumptions used in deriving the steady-state and transient equations are discussed in order to
obtain meaningful interpretation of the results and to es-
tablish useful design criteria.

Dr. R. Gordon Kirk developed the computer programs used
to perform the transient analysis in this report.
CHAPTER 2
THEORETICAL ANALYSIS

2.1 REYNOLDS EQUATION

The configuration of the squeeze film damper bearing is shown in Figure (2-1) where the clearance has been exaggerated. Both fixed and rotating coordinate systems are shown, and the bearing equations are derived for both systems. The definitions of the various parameters are listed in the nomenclature section of this report.

The basic bearing equation is the Reynolds equation which is derived from the Navier-Stokes equations for incompressible flow. With the proper bearing parameters the equation for the fluid film forces are derived. [1]

The Reynolds equation for the short, plain journal bearing is given in both fixed and rotating coordinates by:

Fixed coordinates:

$$\frac{\partial}{\partial Z} \left[ \frac{h^3}{6\mu} \frac{\partial P}{\partial Z} \right] = (\omega_b + \omega_j) \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \quad (2-1)$$

Rotating coordinates:

$$\frac{\partial}{\partial Z} \left[ \frac{h^3}{6\mu} \frac{\partial P}{\partial Z} \right] = (\omega_b + \omega_j - 2\phi) \frac{\partial h}{\partial \theta'} + 2 \frac{\partial h}{\partial t} \quad (2-2)$$

As shown in Figure (2-1), the angle $\theta$ in the fixed coordinate expression is measured from the positive $x$-axis in the direction of rotation whereas the angle $\theta'$ in the rotating coordinate expression is measured from the line of centers in the
Figure 2-1 Squeeze Film Damper Bearing Configuration in Fixed and Rotating Coordinate Systems
direction of rotation. The assumptions used in the derivation of equations (1) and (2) include:

1. The fluid inertia terms in the Navier-Stokes equations have been neglected due to their small magnitude.
2. Body forces in the fluid film have been neglected.
3. The fluid viscosity is constant.
4. The flow in the radial direction has been neglected, that is, the short bearing approximation has been used.

Figure (2.2) shows a comparison of the short bearing solution and the general solution of the Reynolds equation solved by a finite difference technique for the plain journal bearing under steady state conditions. It can be seen that the short bearing solution is highly accurate for a wide range of eccentricities for $L/D < 1/4$ and is acceptable for $L/D$ values up to 1 if the eccentricity ratio is low. The normal design range of the squeeze film bearings will be $L/D < 1/2$ and eccentricity ratios $< 0.4$.

By assuming the bearing is perfectly aligned ($h$ not a function of $Z$) equations (1) and (2) are integrated directly to yield expressions for the fluid film forces.

2.2 BEARING FORCES IN FIXED COORDINATES

For the plain bearing with full end leakage the appropriate boundary conditions are:

$$P(\theta, 0) = P(\theta, L) = 0$$ (2-3)
Figure 2-2 Comparison of Finite Length and Short Bearing Solutions
In the fixed coordinate system the film thickness, $h$, is given by:

$$h = c - x \cos \theta - y \sin \theta \tag{2-4}$$

Substituting into equation (2-1) and integrating yields:

$$p(\theta,z) = \frac{3\mu}{h^3} \left[ Z^2 - LZ \right] \left[ (\omega_b + \omega_j) \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \right] \tag{2-5}$$

Differentiation of equation (2-4) yields:

$$\frac{\partial h}{\partial \theta} = x \sin \theta - y \cos \theta \tag{2-6}$$

$$\frac{\partial h}{\partial t} = -\dot{x} \cos \theta - \dot{y} \sin \theta \tag{2-7}$$

The incremental force acting on the journal is:

$$\Delta F = -p(\theta,z)Rd\theta dz \left. \right|_{n_R} \tag{2-8}$$

The relationship between the unit vectors in the fixed and rotating reference frames is:

$$\hat{i} = \cos \theta \left| n_R \right| - \sin \theta \left| n_\theta \right| \tag{2-9}$$

$$\hat{j} = \sin \theta \left| n_R \right| + \cos \theta \left| n_\theta \right| \tag{2-10}$$

$$\left| n_R \right| = \cos \theta \hat{i} + \sin \theta \hat{j} \tag{2-11}$$

$$\left| n_\theta \right| = -\sin \theta \hat{i} + \cos \theta \hat{j} \tag{2-12}$$
Substituting equation (2-11) into equation (2-8) yields:

$$
\Delta F = - P(\theta, Z) R d\theta dZ (\cos \theta \hat{i} - \sin \theta \hat{j})
$$

(2-13)

The elemental x and y force components may be found by taking the dot product of equation (2-13) with unit vectors in the x and y directions.

$$
\Delta F_x = (\Delta F \cdot \hat{i}) \hat{i} = -(P(\theta, Z) R d\theta dZ \cos \theta) \hat{i}
$$

(2-14)

$$
\Delta F_y = (\Delta F \cdot \hat{j}) \hat{j} = -(P(\theta, Z) R d\theta dZ \sin \theta) \hat{j}
$$

(2-15)

The total force components in the x and y directions are found by integrating equations (2-14) and (2-15) over the entire journal surface.

$$
F_x = - \int_0^{2\pi} \int_0^L p(\theta, Z) R \cos \theta \, dZ \, d\theta
$$

(2-16)

$$
F_y = - \int_0^{2\pi} \int_0^L p(\theta, Z) R \sin \theta \, dZ \, d\theta
$$

(2-17)

Substituting the expressions for \( \frac{\partial h}{\partial \theta} \) and \( \frac{\partial h}{\partial t} \) into the pressure equation and integrating around the bearing circumference gives:

$$
\begin{align*}
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} &= -\frac{u RL^3}{2} \int_0^{2\pi} \left( \omega_B + \omega_J \right) (x \sin \theta - y \cos \theta - 2(x \cos \theta + y \sin \theta) \left( \frac{\cos \theta}{c - x \cos \theta - y \sin \theta} \right)^{\frac{3}{2}}) \, d\theta \\
\end{align*}
$$

(2-18)
The above equation is applicable to the evaluation of the forces developed in the plain journal bearing as well as the squeeze film damper bearing for arbitrary values of journal displacement, velocity, and shaft and bearing housing angular velocities. Hence the analysis can also be used for the general floating bush bearing with rotation.

For the case of the squeeze film damper where the journal and housing are constrained from rotating, \((\omega_b = \omega_j = 0)\), the force expressions become

\[
\begin{bmatrix}
F_x \\
F_y
\end{bmatrix} = \frac{-\mu RL^3}{2} \int_0^{2\pi} \frac{-2(\dot{x} \cos \theta + \dot{y} \sin \theta)}{(c - x \cos \theta - y \sin \theta)^3} \begin{bmatrix}
\cos \theta \\
\sin \theta
\end{bmatrix} d\theta \quad (2-19)
\]

These non-linear fluid film forces are easily combined with the rotor-bearing system dynamical equations providing a complete non-linear dynamical analysis of the system. Because the bearing force equations are written in fixed Cartesian coordinates a transformation from one coordinate system to another is not required. This is very important for conservation of computation time since the bearing pressure profile must be integrated at each time step of the system motion.

2.3 BEARING CAVITATION

If the complete pressure profile is calculated without regard to cavitation or rupture of the film, then the bearing pressure will be similar to Figure (2.3). This figure represents the three dimensional pressure generated in the bearing.

The exact mechanism causing cavitation in fluids is not
Figure 2.3 Uncavitating Pressure Profile Showing Region of Negative Hydrodynamic Pressure
fully known. It is known that film rupture is influenced by gas and solid content of the fluid. Recent investigations have shown that a fluid may stand large tensile stresses [2], and its ability to withstand rupture is dependent on its past history. In this investigation it is assumed that cavitation occurs when the pressure in the film drops below ambient pressure. The cavitated film then extends over only a section of the bearing circumference as shown in Figure (2-4). Recent experimental research has shown that cavitation in the squeeze bearing occurs in streamers of bubbles which extend around the entire bearing [3]. These streamers initially appear at the center of the bearing and extend outward as the rotor speed increases. It is beyond the scope of this present research to analyze this type of cavitation effect. Therefore the conventional cavitated film is assumed to occur when $P < P_c$ where $P_c$ is the assumed cavitation pressure.

When evaluating the integral of equation (2-19), negative pressures are equated to zero if the film is assumed to cavitate. If the oil supply pressure is sufficiently high and suitable operating conditions exist the film does not cavitate.

2.4 BEARING FORCES IN ROTATING COORDINATES

The Reynolds equation in rotating coordinates was given by equation (2-2). Assuming steady-state circular synchronous precession of the journal about the bearing center and no axial misalignment, equation (2-2) can be integrated in closed form. The resulting equations for the bearing forces give the equivalent stiffness and damping of the bearing.
Figure 2-4: Cavitated Pressure Profile With Negative Hydrodynamic Pressure Equated to Zero
Applying the boundary conditions:

\[ P(\theta, 0) = P(\theta, L) = 0 \] (2-20)

and integrating equation (2-2) yields:

\[ P(\theta, Z) = \frac{3\mu}{h^3} \left[ -2\phi \frac{\partial h}{\partial \theta} + 2 \frac{\partial h}{\partial t} \right] \left[ Z^2 - LZ \right] \] (2-21)

where \( \theta' \) is measured from the line of centers in the direction of rotation as shown in Figure (2-1).

In rotating coordinates the film thickness, \( h \), is given by:

\[ h = c(1 + \varepsilon \cos \theta') \] (2-22)

where the eccentricity ratio, \( \varepsilon \), is defined as:

\[ \varepsilon = \frac{e}{c} \] (2-23)

Differentiation of equation (2-22) gives:

\[ \frac{\partial h}{\partial \theta'} = -c \varepsilon \sin \theta' \] (2-24)

and

\[ \frac{\partial h}{\partial t} = c \varepsilon \cos \theta' \] (2-25)

Substituting the expressions for \( h \) and its derivatives into equation (2-21) and integrating gives the fluid film force:

\[ \tilde{F} = \frac{\mu RL^3}{c^2} \int_{\theta_1}^{\theta_2} \left( \frac{\phi \varepsilon \sin \theta' + \varepsilon \cos \theta'}{(1 + \varepsilon \cos \theta')^3} \right) d\theta' \mid_{\theta_1}^{\theta_2} \] (2-26)
The transformation into the $\hat{n}_r$, $\hat{n}_\theta$ coordinates is

$$|\hat{n}_r = -\cos \theta' \hat{n}_r - \sin \theta' \hat{n}_\theta$$  \hspace{1cm} (2-27)$$

The components of the fluid film force in the radial and tangential directions, $\hat{n}_r$ and $\hat{n}_\theta$ are found by taking the dot product of the force with unit vectors in the radial and tangential directions. Thus:

$$\hat{F}_r = (\hat{F} \cdot \hat{n}_r) \hat{n}_r$$  \hspace{1cm} (2-28)$$

and

$$\hat{F}_\theta = (\hat{F} \cdot \hat{n}_\theta) \hat{n}_\theta$$  \hspace{1cm} (2-29)$$

and the force components are:

$$\begin{align*}
\begin{bmatrix}
F_r \\
F_\theta
\end{bmatrix} &= \frac{-\mu RL^3}{c^2} \int_{\theta_1}^{\theta_2} \left( \dot{\epsilon}_r \sin \theta' + \dot{\epsilon}_\theta \cos \theta' \right) \left( \frac{\cos \theta'}{(1 + \epsilon \cos \theta')^3} \right) d\theta' \hspace{1cm} \text{(2-30)}
\end{align*}$$

The limits of integration, $\theta_1$ and $\theta_2$, define the area over which a positive pressure profile exists and are dependent on the type of journal motion and whether or not cavitation occurs.

It is assumed that the journal is precessing in steady-state circular motion about the origin. Therefore $\dot{\epsilon} = 0$ and the pressure expression, equation (2-21) becomes:

$$P(\theta', Z) = \frac{6 \mu \omega e \sin \theta'}{c^2 (1 + \epsilon \cos \theta')^3} \left[ Z^2 - ZL \right]$$  \hspace{1cm} (2-31)$$
The maximum pressure in the axial direction occurs at $Z = L/2$ and equation (2-31) is rewritten as:

$$P(\theta', L/2) = \frac{-3\mu L^2 \omega \varepsilon \sin \theta'}{2c^2(1 + \varepsilon \cos \theta')^3}$$

(2-32)

By differentiating equation (2-32) with respect to $\theta'$ and equating to zero, the tangential location of the maximum pressure may be found. The angle, $\theta'_\text{max}$, where the pressure is maximum is given by:

$$(1 + \varepsilon \cos \theta'_\text{max}) \cos \theta'_\text{max} + 3 \varepsilon \sin^2 \theta'_\text{max} = 0$$

(2-33)

The angle $\theta'_\text{max}$ varies with $\varepsilon$ and shifts from

$$\theta'_\text{max} = \frac{3\pi}{2} \text{ when } \varepsilon = 0$$

to

$$\theta'_\text{max} = \pi \text{ when } \varepsilon = 1$$

and the maximum pressure is given by:

$$P_{\text{max}} = \frac{-3\mu L^2 \omega \varepsilon \sin \theta'_\text{max}}{2c^2(1 + \varepsilon \cos \theta'_\text{max})^3}$$

(2-34)

The pressure expressed by equation (2-32) is positive over the region $\theta' = \pi$ to $\theta' = 2\pi$ and for the cavitated fluid film the limits of integration in equation (2-30) are:

$$\theta'_1 = \pi, \quad \theta'_2 = 2\pi$$

(2-35)
The radial and tangential components of the fluid film force are given by:

\[
\begin{align*}
\begin{pmatrix}
F_r \\
F_\theta
\end{pmatrix} &= \frac{-\mu RL^3 \varepsilon \omega}{c^2} \int_\pi^{2\pi} \sin \theta' \left(1 + \varepsilon \cos \theta' \right)^3 \begin{pmatrix}
\cos \theta' \\
\sin \theta'
\end{pmatrix} \ d\theta'
\end{align*}
\]  \hspace{1cm} (2-36)

The integrals in equation (2-36) were integrated using Booker's method [4]. The resulting force components are:

\[
F_r = -\frac{2\mu RL^3 \varepsilon \omega c^3}{(1 - \varepsilon^2)^2}
\]  \hspace{1cm} (2-37)

and

\[
F_\theta = \frac{-\mu RL^3 \pi \varepsilon \omega}{2c^3 (1 - \varepsilon^2)^{3/2}}
\]  \hspace{1cm} (2-38)

The force in equation (2-37) appears as a stiffness coefficient times a displacement acting in line of the displacement towards the bearing center. The equivalent bearing stiffness is:

\[
K_0 = \frac{2\mu RL^3 \varepsilon \omega}{c^3 (1 - \varepsilon^2)^2}
\]  \hspace{1cm} (2-39)

Since the journal is precessing and not rotating, every point in the journal has a velocity equal to \(\varepsilon \omega\). The force in equation (2-38) therefore appears as a damping coefficient times a velocity acting in the direction opposite the journal motion.

The equivalent bearing damping is:

\[
C_0 = \frac{\mu RL^3 \pi}{2c^3 (1 - \varepsilon^2)^{3/2}}
\]  \hspace{1cm} (2-40)
For the uncavitated film the limits of integration in equation (2-36) become:

\[ \theta_1' = 0, \theta_2' = 2\pi \]  
\[ (2-41) \]

Integrating and evaluating at those limits yields force components given by:

\[ F_r = 0 \]  
\[ (2-42) \]

\[ F_\theta = \frac{-\mu R L^3 \pi \omega}{c^3 (1 - \varepsilon^2)^{3/2}} \]  
\[ (2-43) \]

It is therefore evident that a complete fluid film does not produce an equivalent bearing stiffness but doubles the damping of the cavitated film.

Although the equations for the bearing characteristics were derived for a plain bearing with no circumferential oil groove they are applicable to other bearing configurations. For example Figure (2-5a) represents a plain bearing with circumferential oil groove and full end leakage.

The total length of the bearing, \( L \), corresponds to the length of the plain bearing with no oil groove. The bearing in Figure (2.5a) consists of two plain bearings without an oil groove whose length is \( L/2 \). Thus the bearing parameter \( L \) in the equations may be replaced by \( L/2 \) and the equations multiplied by 2 to obtain the total effect of the two half bearings. The net effect is to decrease the maximum pressure by a factor of 2:
Similarly the damping and stiffness values are decreased by a factor of 4:

\[
\frac{2 \left( \frac{L}{2} \right)^2}{L^2} = \frac{1}{2}
\]  

(2-44)

\[
\frac{2 \left( \frac{L}{2} \right)^3}{L^3} = \frac{1}{4}
\]  

(2-45)

The bearing represented in Figure (2.5b) is a plain bearing with circumferential oil groove and end seals to prevent end leakage. If there is no end leakage the boundary conditions are:

\[
\frac{\partial P}{\partial Z} \bigg|_{Z = 0, L} = 0
\]  

(2-46)

and the net effect leaves the pressure and bearing characteristic equations unchanged.

Figure 2-5  
a. Axial Pressure Distribution of Bearing With Circumferential Oil Groove  
b. Axial Pressure Distribution of Bearing With End Seals and Circumferential Oil Groove
The bearing equations derived in this section are summarized in Table (2-1). Also included in the table are the equations for pure radial squeeze motion. For this type of operation $\dot{\phi} = 0$ and results from a purely unidirectional load on the journal. The radial and tangential force components are derived from equation (2-30) where only the term containing $\dot{\varepsilon}$ in the integral is retained. The pressure equation is also modified to include only the $\dot{\varepsilon}$ term. The maximum pressure occurs at $\theta = \pi$ for all values of journal eccentricity. Examination of the pressure equation reveals that the hydrodynamic pressure is positive only in the region $\theta' = \frac{\pi}{2}$ to $\frac{3\pi}{2}$. These values of $\theta'$ are the limits of integration in equation (2-30) for the cavitated film.

The table also shows that for purely radial motion no bearing stiffness is obtained in either the cavitated or uncavitated bearing. Thus if this type of motion exists retainer springs must be included to provide support flexibility.

For the case of circular journal precession, the table shows the stiffness and damping of the cavitated film and damping of the uncavitated film remain essentially constant for low eccentricity ratios. As the eccentricity ratio increases above 0.4 there is a rapid increase in these properties and they approach infinity as $\varepsilon$ approaches 1. This variation of stiffness in the cavitated film is very important. As the eccentricity becomes large the support becomes more rigid with a corresponding increase in the rotor critical speed. If the rotor critical speed is increased above the operating speed, the phase angle between the
<table>
<thead>
<tr>
<th>TYPE OF MOTION</th>
<th>MAXIMUM PRESSURE</th>
<th>EQUIVALENT DAMPING Ko (lb/in)</th>
<th>EQUIVALENT DAMPING Co (lb-sec/in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CIRCULAR SYNCHRONOUS PRECESSION</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \dot{\phi} = \omega, \dot{\epsilon} = 0 )</td>
<td>( \frac{-3\mu L^2 \omega \epsilon \sin \theta_m}{2c^2(1 + \epsilon \cos \theta_m)^3} )</td>
<td>( \frac{2\mu R L^3 \epsilon \omega}{c^3(1 - \epsilon^2)^2} )</td>
<td>( \frac{\mu R L^3 \pi}{2c^3(1 - \epsilon^2)^{3/2}} )</td>
</tr>
<tr>
<td>CAVITATED FILM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNCAVITATED FILM</td>
<td>( (1 + \epsilon \cos \theta_m) \cos \theta_m + 3\epsilon \sin^2 \theta_m = 0 )</td>
<td>( 0 )</td>
<td>( \frac{\mu R L^3 \pi}{c^3(1 - \epsilon^2)^{3/2}} )</td>
</tr>
<tr>
<td>PURE RADIAL SQUEEZE MOTION</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \dot{\phi} = 0, \dot{\epsilon} \neq 0 )</td>
<td>( \frac{-3\mu L^2 \dot{\epsilon} \cos \theta_m}{2c^2(1 + \epsilon \cos \theta_m)^3} )</td>
<td>( 0 )</td>
<td>( \frac{\mu R L^3 (\pi - \cos^{-1}(\epsilon))(2\epsilon^2 + 1)}{c^3(1 - \epsilon^2)^{5/2}} )</td>
</tr>
<tr>
<td>CAVITATED FILM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UNCAVITATED FILM</td>
<td>( \theta_m = \pi )</td>
<td>( 0 )</td>
<td>( \frac{\mu R L^3 \pi (2\epsilon^2 + 1)}{c^3(1 - \epsilon^2)^{5/2}} )</td>
</tr>
</tbody>
</table>

Table 2-1. Summary of Equivalent Stiffness and Damping Coefficients for Squeeze Film Damper Bearings.
rotor unbalance vector and amplitude vector becomes less than 90°. When this condition occurs the force transmitted through the support structure will always be greater than the unbalance load. With an uncavitated film this problem does not occur because no bearing stiffness is generated. To obtain the stiffness required to stabilize a rotor (see Chapter 3) it is necessary to use retainer springs in the support bearings.

One of the most significant parameters affecting damper performance is the length to clearance ratio. The stiffness and damping coefficients vary as \((L/C)^3\) and therefore either doubling the bearing length or decreasing the clearance by 1/2 will increase the coefficients by a factor of 8.
3.1 ROTOR-BEARING STABILITY

After Jeffcott's [5] analysis in 1919 of the single mass flexible rotor on rigid bearings, manufacturers began producing light, flexible rotors operating above the first critical speed. However, some manufacturers encountered severe operating difficulties with some of their designs. These machines underwent violent whirling while running above the critical speed and often failed.

Experimental and analytical investigations by Newkirk and Kimball [6] [7] revealed that the whirl instability was not caused by unbalance in the rotor, but by internal shaft effects such as internal friction. Kimball theorized that forces normal to the plane of the deflected rotor could be produced by alternating stresses in the metal fibers of the shaft. In light of this theory, Newkirk concluded also that the same normal forces could be produced by shrink fits on the rotor shaft. By incorporating these forces in Jeffcott's model Newkirk showed that the rotor was unstable above twice the rotor critical speed.

Further investigation by Newkirk showed cases of rotor instability which were not produced by shaft effects but by effects in the journal bearings. [8] One cause of journal bearing instability was later shown to be due to lack of radial stiffness in the bearing and the instability occurred at twice the rotor critical speed.
critical speed. These instabilities were especially common in lightly loaded rotors and larger bearing loads tended to promote stability. The effect of the larger loads is to cause cavitation of the fluid film which results in a radial stiffness component of the bearing forces being produced. [9], [10], [11]

In 1965 Alford reported on the effects of aerodynamic forces on rotors [12]. He showed that these forces couple the rotor equations of motion and can produce instability. He also noted that labyrinth seals and balance pistons also produce forces that can promote instability.

Recent investigators including Gunter, Kirk and Choudhury [13] [14][15] have analyzed the effects of support flexibility and damping on reducing rotor instability produced by the forces just described. As a result they have derived stability criteria for determining the necessary support characteristics.

One of the most general methods for determining rotor stability is to derive the characteristic frequency equation of the system. The stability is given by the roots of this equation. The real part of the root corresponds to an exponentially increasing or decreasing function of time. Thus a positive real part indicates instability whereas a negative real part indicates a stable system. This type of stability analysis of a rotor-bearing system therefore requires that the characteristic equation be known. This equation is not always easy to obtain.

The characteristic equation is derived from the homogeneous second order differential equations of motion of the system [15].
By assuming solutions of the form

\[ x_i = A_i e^{\lambda t} \quad i = 1, 2, \ldots, n \]

and differentiating, the equations are substituted back into the equations of motion. This produces a matrix known as the characteristic matrix. The determinant of this matrix gives the characteristic equation, a polynomial of degree 2n in \( \lambda \), where \( n \) is the number of degrees of freedom of the system.

The computer program SDSTB [16] was used to produce the stability maps shown in this chapter. The program calculates the characteristic equation for a three-mass symmetric flexible rotor mounted in journal bearings and supported in squeeze film damper bearings. The rotor-bearing model is shown in Figure (3-1). The rotor is assumed to remain stationary in the axial direction so the rotor has six degrees of freedom and the characteristic equation is therefore of degree twelve. The characteristic matrix is shown in Figure (3-2). The determinant of this matrix gives the characteristic equation. The unknown variable in this equation is \( \lambda \), the natural frequency of the system. An examination of the characteristic matrix shows that the coefficients of \( \lambda \) are functions of the rotor and bearing properties as well as internal shaft friction, absolute rotor damping and aerodynamic cross coupling. The natural frequencies and stability of the system are found by finding the roots of this equation.

The journal and support bearing characteristics can either be inserted directly as linear coefficients or they may be calculated
Figure 3-1. Three-Mass Flexible Rotor Mounted in Flexible, Damped Supports
Figure 3-2  Characteristic Matrix For Three-Mass Model
Including Aerodynamic Cross-Coupling Internal Shaft Friction and Absolute Shaft Damping
in the program from the bearing parameters by solving for the equilibrium positions of the journal and support. These characteristics are non-linear functions of the journal eccentricity. The stability maps in this chapter were produced with the linearized journal and support bearing characteristics given as input data to the program. The assumption of linear bearing characteristics is useful because for low eccentricity the characteristics do not vary greatly with changes in eccentricity. This assumption allows a large savings in computer time. If the non-linear characteristics are calculated, the amount of computer time increases because an iterative procedure is used to find the equilibrium position.

As an example of how a stability map is produced, consider the following system:

**ROTOR CHARACTERISTICS**

- **ROTOR WEIGHT**: 675 lbs
- **JOURNAL WEIGHT**: 312 lbs (each)
- **SUPPORT WEIGHT**: 15 lbs (each)
- **SHAFT STIFFNESS**: 280000 lb/in
- **SHAFT DAMPING**: .10 lb-sec/in
- **INTERNAL DAMPING**: 0.0 lb-sec/in
- **ROTOR SPEED**: 10000 RPM

**BEARING CHARACTERISTICS**

- \( k_{xx} \) \( = 1.287 \times 10^6 \) lb/in
- \( k_{yy} \) \( = 1.428 \times 10^6 \) lb/in
- \( C_{xx} \) \( = 1200 \) lb-sec/in
Two values of aerodynamic cross coupling were selected, \( Q = 20000 \text{ lb/in} \) and \( Q = 100,000 \text{ lb/in} \). For each value of \( Q \), several values of support stiffness were selected ranging from 50,000 \( \text{lb/in} \) to 500,000 \( \text{lb/in} \). For each value of support stiffness a range of support damping values from 0 to 10000 \( \text{lb-sec/in} \) was used. Using this method a stability contour was found for a given value of aerodynamic cross coupling and support stiffness. The rotor and bearing characteristics remained unchanged.

Figures (3-3) and (3-4) show the stability maps for the above system for the two values of aerodynamic cross coupling. There is an intermediate range of support damping values for which the system is stable for a given value of the support stiffness. As the stiffness is increased the system becomes less stable. With \( Q = 20000 \text{ lb/in} \) the optimum amount of damping ranges from 500 to 2500 \( \text{lb-sec/in} \) as the stiffness increases from 50000 to 500000 \( \text{lb/in} \). For damping less than 100 \( \text{lb-sec/in} \) the system is unstable for all values of stiffness. The same is true if the damping exceeds 10000 \( \text{lb-sec/in} \).

For \( Q = 100000 \text{ lb/in} \) the optimum damping is 1000 \( \text{lb-sec/in} \) and does not shift over the stiffness range selected. When the stiffness reaches 250000 \( \text{lb/in} \) the system is unstable for all values of damping.
Figure 3-3  Stability of a Flexible Rotor With Aerodynamic Cross Coupling (Q = 20,000 lb/in., N = 10,000 RPM)
The stability calculation depends upon the accuracy of the root solving technique applied to the characteristic equation. For symmetric bearing and support characteristics repeated roots of the equation occur and many root solving routines break down under this condition. Although no cases have been encountered where the root solving routine has failed, it is possible that some combinations of rotor-bearing properties might cause this to happen. However because root solving routines are generally easy to obtain it would be easy to replace the one currently in SDSTB if such a situation arose.

The amount of computer time required to solve the characteristic equation depends upon the root solving technique and the order of the characteristic equation. Many studies do not require extensive stability maps and it is only necessary to determine whether the system is stable and not how stable. In these cases application of the Routh stability criteria [15] gives the required information without solving for the roots of the characteristic equation. This results in a savings of computer time. The option of using only the Routh stability criteria is available in SDSTB.

The method of determining the stability of the system from the characteristic equation becomes less practical when the order of the system is large. The elements of the characteristic matrix must be found from the equations of motion and unless the determinant is found by a computer routine, the characteristic equation must be expanded by hand. Therefore complicated systems may re-
quire a prohibitive amount of formulation time.

Recent research in transfer matrix and finite element techniques for determining the stability and natural frequencies of rotor bearing systems has been directed toward overcoming these difficulties [17] [18]. However, because iterative procedures are required in the solution, higher order modes may be expensive to obtain from the standpoint of computer time. The type of method used will depend on the amount of computer funds available and the availability of programs using the various techniques.

3.2 STEADY-STATE ANALYSIS

The steady-state stability maps just discussed provide information on the support characteristics needed to promote stability in a given rotor-bearing system. There remains the problem of relating these characteristics to the actual support bearing. The squeeze bearing equations derived in Chapter 2 in rotating coordinates are used to determine the preliminary bearing design. As noted in Chapter 2, these equations were derived assuming steady-state circular, synchronous precession of the journal.

The bearing characteristics, stiffness, damping and pressure are functions of the amplitude of the journal orbit, fluid viscosity and bearing geometry. The addition of oil supply grooves, end seals and cavitation affect the bearing characteristics.

The steady-state equations have been programmed on a digital
computer. This program, SQFDAMP, analyzes three basic bearing configurations:

1. Plain bearing, no oil supply groove or end seals.
2. Bearing with oil supply groove but without end seals.
3. Bearing with both oil supply groove and end seals.

Both cavitated and uncavitated fluid films can be analyzed. A listing of the program with a description of the input data requirements and sample output are contained in Appendix A.

The program calculates the bearing characteristics and plots them as functions of the journal eccentricity ratio, ε. By varying the bearing parameters the designer is able to determine the bearing characteristics and select a bearing configuration that will provide the stability requirements of the system under consideration.

Figures (3-5) - (3-7) show the characteristics for a bearing being considered for the 675 lb rotor system described earlier. The bearing has an oil supply groove and end seals, and the fluid film is assumed to be cavitated. The bearing parameters are, length, 1.0 inches, radius, 1.2 inches and fluid viscosity 10 microreyns. For the case where Q = 20000 lb/in it was determined that the optimum support damping is about 500 lb-sec/in and the support stiffness should be less than 100000 lb/in. Because it is desirable to keep the eccentricity ratio of the journal low, Figures (3-5) and (3-6) reveal that this bearing will provide the necessary stiffness and damping characteristics with a clearance of about 4 mils at an eccentricity ratio of ε = .10 to .20.
Figure 3-5. Damping Coefficient for Squeeze Film Bearing With Cavitated Film - End Seals and Oil Supply Groove Included.
Figure 3-6. Stiffness Coefficient for Squeeze Film Bearing With Cavitated Film - End Seals and Oil Supply Groove Included.
Figure 3-7. Maximum Pressure for Squeeze Film Bearing with Cavitated Film - End Seals and Oil Supply Groove Included.
This corresponds to a journal orbit of 0.4 to 0.8 mils amplitude. The maximum hydrodynamic pressure in the bearing is about 100 psi for this clearance. If the fluid film cavitates, the resulting characteristics are shown in Figures (3-8) and (3-9). A slightly larger clearance, 5.0 mils, will produce the optimum damping. However, because the uncavitated film does not produce an equivalent stiffness, retainer springs must be incorporated in the bearing. If the end seals are flexible the required spring rate may be obtained from them.

One advantage of the uncavitated film is that if the journal eccentricity ratio should become very large, there is no rise in stiffness that could cause the system to become unstable or raise the critical speed above the operating speed. Figures (3-5) and (3-8) indicate that even at eccentricity ratios of 0.9 the damping value still remains acceptable. For the cavitated film at $\varepsilon = 0.9$, the stiffness exceeds 2,000,000 lb/in and the system would be bordering on instability. For both films the maximum pressure exceeds 70000 psi. at $\varepsilon = 0.9$ and this large a rotating pressure field could result in bearing failure.

If $Q = 100000$ lb/in, the bearing characteristic graphs reveal that for a cavitated film the clearance must be as small as possible because of the limitations on stiffness shown in Figure (3-4). With a 3.0 mil clearance, the stiffness is 250000 lb/in at $\varepsilon = 0.23$ and this stiffness will produce system instability. For $Q = 100000$ lb/in it is desirable to study other bearing lengths and radii to obtain a cavitated bearing which pro-
Figure 3-8. Damping Coefficient for Squeeze Film Bearing with Uncavitated Film - End Seals and Oil Supply Groove Included
Figure 3-9. Maximum Pressure for Squeeze Film Bearing with Uncavitated Film - End Seals and Oil Supply Groove Included.
duces the required damping at low eccentricity ratios and also produces a stiffness of 100000 lb/in or less. For an uncavitated film a 3.0 mil clearance provides adequate damping.

The steady-state bearing characteristic equations used in conjunction with the stability analysis based on steady-state motion provide an excellent means of determining bearing configurations. Using the methods just described, good preliminary designs can be obtained which can be more thoroughly analyzed. The steady-state analysis described here and the transient analysis described in the next chapter provide bearing design criteria which will eliminate the experimental testing of unsuitable designs. The costs of the analysis more than offset the experimental losses when designs are tested that result in bearing or machine failure.
CHAPTER 4
TRANSIENT ANALYSIS

4.1 INTRODUCTION

There are a number of operating conditions in which the squeeze bearing journal does not orbit the bearing center in circular synchronous precession. These conditions can occur when there is a unidirectional load on the rotor or when there is a suddenly applied load such as the application of unbalance when blade loss occurs. Intermittent or cyclic forces transmitted to the machine from nearby equipment can also result in non-linear orbiting. Under these conditions the bearing stiffness and damping coefficients developed in Chapter 2 are no longer applicable and a time-transient analysis of the bearing is necessary to determine the squeeze bearing support's ability to restabilize the system.

The equation for the squeeze film damper fluid film forces, in fixed coordinates, equation (2-19) provides a useful means of determining the time dependent transient behavior of a rotor bearing system. The force equations have been programmed on a digital computer and combined with the journal equations of motion. The resulting program, BRGTRAN [19], tracks the journal motion forward in time under the influence of the bearing forces, journal weight and unbalance. The program is capable of including the effects of retainer springs and cavitation.

Because both the bearing pressure equation and the journal
equations of motion must be integrated, the accuracy of the simulation depends upon the numerical integration method used. Two integration methods, Adam-Bashforth-Moulton Predictor-Corrector, and 4th Order Runge-Kutta, are provided as options in BRGTRAN. Although the 4th Order Runge-Kutta method is highly accurate, four functional evaluations of the pressure equation are required for each step in time. Even though the required time step size may be larger using the Runge-Kutta method, the total computer time required is still greater than other methods because of the many functional evaluations.

The Adams-Bashforth-Moulton method has been found to be sufficiently accurate for most cases run if the time step has been made sufficiently small, or about 0.01 cycles. At low eccentricities no problems are encountered in the integration process and the solutions are accurate for both methods. The changes in the bearing forces are relatively small from one time step to the next and even a very simple integration method such as the Modified Euler Method provides reasonable accuracy. However, at high eccentricities the bearing forces change drastically with even a very small change in eccentricity. Even the more sophisticated integration schemes lose accuracy when the eccentricity is high unless the time step is made very small. The amount of computer time and core storage required for a small time step becomes prohibitive. This is especially true in light of the fact that at high eccentricities the validity of the short bearing approximation used in reducing the Reynolds equation is
doubtful. In using the short bearing approximation it was assumed that the pressure gradient in the tangential direction is small, and this assumption may be violated at high eccentricities.

It was also shown in Chapters 2 and 3 that at high eccentricities the bearing stiffness becomes very large. This can result in system instability as indicated in the stability maps, Figures (3-3) and (3-4), or in raising the system critical speed above the operating speed causing large forces to be transmitted to the machine structure. For these reasons the design criteria of $\varepsilon<0.4$ was established and it is unnecessary to use excessive computer time to obtain greater accuracy at higher eccentricities. The information provided at these eccentricities is very useful in showing trends in the ability of a bearing to perform adequately and should be used with this restriction kept in mind. Also recent transient analysis has been performed using hybrid computer simulation thereby avoiding the difficulties inherent in numerical integration. [20]

4.2 ANALYSIS

Using a time transient bearing program, a design engineer can make an analysis of the bearing effects without resorting to either a complete time-transient analysis of the entire rotor-bearing system or a costly experimental program during the preliminary design stage. During later design stages when the
bearing configuration has been tentatively fixed, a more complete theoretical and experimental analysis of the entire system may be performed. The bearing force calculations have also been incorporated as a subroutine in a computer program which analyzes the transient behavior of certain rotor-bearing models [21]. By using these programs to determine the bearing parameters experimental verification of the bearing effects can be performed with more assurance that the design is feasible, and costly and time consuming machine prototype failures can be reduced.

The computer program BRGTRAN was recently used as part of an analysis of an existing turbomachine which had suffered frequent bearing failures. The manufacturer had decided to use a squeeze film damper bearing to reduce the vibration amplitudes at the failing bearings. Without performing a complete analysis of the bearing effects, an experimental program was initiated where various bearing configurations were installed on a test machine. The damper bearings used did not dampen out the vibrations and much time and money was lost during the project.

The following discussion of the computer simulation of this system shows the effect of varying the rotor-bearing parameters including bearing length, unbalance, cavitation and retainer springs. In this study of the problem, the analysis made using BRGTRAN showed why the bearings used were unable to reduce vibration amplitudes. Figure (4-1) shows the journal orbit in a cavitated
Figure 4-1. Unbalanced Rotor in Cavitated Squeeze Film Bearing - \( L = 0.45 \) IN. - Unbalance Eccentricity = 0.002 IN. - No Retainer Spring.
film during the first 10 cycles of transient motion. The bearing parameters are:

- LENGTH - 0.45 inches
- RADIUS - 2.55 inches
- CLEARANCE - 0.004 inches
- FLUID VISCOSITY - 0.38 x 10^{-6}
- JOURNAL WEIGHT - 74 lbs.
- UNBALANCE ECCENTRICITY - 0.002 inches
- RETAINER SPRING RATE
  - Kxx - 0 lb/in
  - Kyy - 0 lb/in
- N - 16800 RPM

In the transient orbit figures a standard right-hand coordinate system has been adopted with positive journal rotation in the counterclockwise direction. The asterisk on the orbit represents the point where the maximum force is generated. The small dots represent timing marks denoting one revolution of shaft motion. These marks can be used to determine the relative location of the unbalance with respect to the amplitude in the x-direction. The timing mark sensor is assumed to be located on the positive x-axis, and the phase angle is measured from the x-axis to the timing mark in the clockwise direction.

Returning to Figure (4-1), it is seen that the journal very quickly spirals out to an eccentricity ratio \( \epsilon = 0.95 \), where a limit cycle is formed due to the non-linearity of the bearing forces. The maximum force transmitted through the support
structure is 7405 lbs, which is over 6 times the rotating unbalance load of 1181 lbs. as shown by the parameter TRD Figure (4-1). This large rotating force will eventually lead to bearing failure and is therefore undesirable.

The phase angle between the maximum amplitude in the x direction and the timing mark is approximately 30°, and this indicates that the precession rate is less than the natural frequency of the bearing. This has been caused by the large stiffness developed in the bearing. Figure (4-2) shows that the stiffness is approximately 487,000 lb/in. The damping is given in Figure (4-3) as 71.5 lb-sec/in.

One possible design change considered was to increase the bearing length. Figure (4-4) shows the effect of increasing the bearing length of 0.90 inches, all other parameters remaining the same. The journal still spirals outward, however the limit cycle is produced at an eccentricity ratio, ε = 0.88. This results in a reduction in the force transmitted to the support from 7405 to 3371 lbs., but it is still greater than the rotating unbalance load, and is undesirable since bearing failure will result. The phase angle has shifted from 30° to 60° and this would have been accompanied by an increase in the force transmitted except that the stiffness and damping values have also changed with the net effect being a reduction in the transmissability. The stiffness and damping coefficients are shown in Figures (4-5) and (4-6) to be 880000 lb/in and 22 lb-
Figure 4-2. Stiffness Coefficient for Squeeze Film Bearing of Figure (4-1).
Figure 4-3. Damping Coefficient for Squeeze Film Bearing of Figure (4-1).
### Squeeze Film Bearing

**Cavitated Film**

**Horizontal**

<table>
<thead>
<tr>
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<tr>
<td>( W )</td>
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</tr>
<tr>
<td>( L )</td>
<td>0.900 IN</td>
</tr>
<tr>
<td>( C )</td>
<td>4.00 MILS</td>
</tr>
<tr>
<td>( PS )</td>
<td>0.00 PSI</td>
</tr>
<tr>
<td>( WX )</td>
<td>0.00 LBS</td>
</tr>
<tr>
<td>( FU )</td>
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<tr>
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<tr>
<td>( R )</td>
<td>2.550 IN</td>
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<tr>
<td>( MU )</td>
<td>0.302 MICREYN5</td>
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<tr>
<td>( FMAX )</td>
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<tr>
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<td>( EMU )</td>
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<td>( KRY )</td>
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<tr>
<td>( PMAX )</td>
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---

**Figure 4-4.** Unbalanced Rotor in Cavitated Squeeze Film Bearing \( L = 0.90 \) IN. - Unbalance Eccentricity = \( 0.002 \) IN. - No Retainer Spring.
Figure 4-5. Stiffness Coefficient for Squeeze Film Bearing of Figure (4-4).
Figure 4-6. Damping Coefficient for Squeeze Film Bearing of Figure (4-4).
Figure (4-7) shows the first 10 cycles of motion for the bearing configuration of Figure (4-1) with the addition of retainer springs with a stiffness of 123000 lb/in. Only a very slight improvement in force transmission results, and the bearing is still unacceptable.

The effect of changing the rotating unbalance load is shown in Figure (4-8). The bearing configuration is the same as Figure (4-4) except the unbalance has been reduced by one half. The journal orbit has been greatly reduced and the force transmitted to the support structure is reduced to only 80% of the unbalance load. The journal is now precessing about a point offset from the bearing center.

By adding retainer springs to the bearing of Figure (4-8) the motion in Figure (4-9) results. The retainer spring rate is 123,000 lb/in. The journal is orbiting about the center of the bearing at an eccentricity ratio of \( \epsilon = 0.35 \). The journal motion is stabilizing more quickly than without the retainer springs and the journal is precessing synchronously as indicated by the small dots on the orbit which represents one cycle of motion. The transmitted force has been further reduced to only 65% of the unbalance load. From the standpoint of producing a small amplitude orbit and attenuating the unbalance load such a bearing configuration is desirable.

If the unbalance eccentricity is again increased to 0.002 inches, the bearing configuration of Figure (4-9) is no longer
Figure 4-7. Unbalanced Rotor in Cavitated Squeeze Film Bearing - \( L = 0.45 \) IN. - Unbalance Eccentricity = 0.002 IN. - Retainer Spring Stiffness, \( KR = 123,000 \) LB/IN.
**SQUEEZE FILM BEARING**
**CAVITATED FILM**
**HORIZONTAL**

<table>
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</tr>
<tr>
<td>C</td>
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<tr>
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Figure 4-8. Unbalanced Rotor in Cavitated Squeeze Film Bearing - L = 0.90 IN. - Unbalance Eccentricity = 0.001 IN. - No Retainer Spring
Figure 4-9. Unbalanced Rotor in Cavitated Squeeze Film Bearing - $L = 0.90$ IN - Unbalance Eccentricity = 0.001 IN. - Retainer Spring Stiffness, $KR = 123,000$ LB/IN.
acceptable. The resulting motion is shown in Figure (4-10) and a large amplitude limit cycle with large force transmission again occurs. However if the fluid film does not cavitate the bearing performance improves and is marginally acceptable as shown in Figure (4-11). The journal is orbiting at \( \epsilon = 0.65 \) and the transmitted force is only 5% less than the unbalance load.

Although this particular analysis includes only the journal weight and the unbalance loading, it shows the usefulness of this program in determining the bearing effects with different bearing parameters. The ability to perform the analysis without extensive preliminary experimental work provides a great savings in time and money. By systematically varying the bearing parameters design guidelines are established.

For instance Figures (4-12 - (4-15) shows the effect of varying the unbalance on a 675 lb. journal operating in a squeeze bearing with a 7 mil clearance. The unbalance eccentricity is increased from 1.75 to 3.5 mils with an accompanying increase in the journal amplitude and the force transmitted to the support structure. Although the exact unbalance may not be known precisely for a given rotor, a design estimate can be made based on the effect of a suddenly applied known unbalance due to blade loss or loss of chemical deposits from the blade surfaces. Prior to the sudden unbalance the rotor is assumed to be perfectly balanced. The ability of the bearing to reduce the amplitude to tolerable
Figure 4-10. Unbalanced Rotor in Cavitated Squeeze Film Bearing – L = 0.90 IN. – Unbalance Eccentricity = 0.002 IN. – Retainer Spring Stiffness, KR = 123,000 LB/IN.
SQUEEZE FILM BEARING
UNCAVITATED FILM
HORIZONTAL

<table>
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</table>

Figure 4-11. Unbalanced Rotor in Uncavitated Squeeze Film Bearing - L = 0.90 IN. - Unbalance Eccentricity = 0.002 IN. - Retainer Spring Stiffness, KR = 123,000 LB/IN.
Figure 4-12. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 1.75 Mils
Figure 4-13. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 2.10 Mils.
Figure 4-14. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 2.45 Mils.
Figure 4-15. Vertical Unbalanced Rotor in Squeeze Film Bearing - Effect of Unbalance Magnitude - Unbalance Eccentricity = 3.50 Mils.
levels until the machine is shut down or its operating conditions changed can be determined. Note the shift in phase angle from 180° to 90° as the eccentricity increases.

It has been shown that retainer springs help center the journal and reduce the vibration amplitude. They may also be used to prevent oil leakage from the end of the bearing if they are of the O ring type. As noted in Chapter 2, the uncavitated film provides no equivalent stiffness when the journal is operating in synchronous precession about the bearing center. In this case the use of a retainer spring to provide a restoring force in the bearing is necessary. Although an increase in stiffness results in centering the journal in the bearing, (see Figures (4-16) (4-19) the magnitude of the transmitted force is minimized for some intermediate value of stiffness which depends on the bearing parameters and loading. Also the ability of the journal to quickly return to a stable, steady state operating condition is impaired with increasing stiffness. Both of these operating conditions were indicated by the stability maps in Chapter 3.

After the preliminary bearing design, a more thorough analytic study may be made by incorporating the damper bearing effects into a program which includes the dynamic effects of the entire system. Although such programs are not readily available for many complex systems, one has been developed for a three mass rotor in journal bearings on a squeeze film bearing support as noted earlier. The rotor bearing model is shown in Figure (3-1). This program may be used to calculate the ability of squeeze
Figure 4-16. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, KR = 0 LB/IN.
Figure 4-17. Horizontal Unbalanced Rotor in Squeeze Film Bearing — Effect of Retainer Springs — Retainer Spring Stiffness, \( K_R = 50,000 \, \text{LB/IN} \).
Figure 4-18. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, $K_R = 100,000$ LB/IN.
Figure 4-19. Horizontal Unbalanced Rotor in Squeeze Film Bearing - Effect of Retainer Springs - Retainer Spring Stiffness, \( K_R = 200,000 \) LB/IN.
film bearings to stabilize a multi-mass rotor. In addition, information may be obtained to verify the stability maps obtained by other means (see Chapter 3). Because the bearing is initially designed using the criteria derived from a stability analysis such verification is useful in judging the overall worth and limitations of the analytical design process. Using these analytical methods leads to a more efficient testing program because unacceptable bearing designs are eliminated before testing begins.
CHAPTER 5
CONCLUSIONS AND RECOMMENDATIONS

The problem of rotor stability is of current importance because of the high speeds and complex dynamics of modern rotor bearing systems. Damped flexible supports have a great effect on the ability of a system to suppress unstable whirl. Therefore there is a need for methods of predicting rotor instability and obtaining bearing designs that provide the necessary support characteristics.

5.1 PREDICTING ROTOR INSTABILITY

Methods for determining the stability of rotor bearing systems include:

1. Using the rotor-bearing system equations of motion including the effects of aerodynamic forces, shaft damping and internal friction to obtain the system characteristic equation. The roots of this equation show the stability and natural frequencies of the system.

2. Using finite element and transfer matrix methods to obtain the system stability and natural frequencies.

3. Applying Routh stability criterion to the system characteristic equation.

The third method does not yield information on relative stability but only determines whether or not the system is absolutely
stable. The first two methods provide relative stability information that can be used to plot stability contours as functions of the support characteristics. The stability maps presented show that for a given value of support stiffness there is a range of damping values which will stabilize the system. If the support stiffness becomes too large, the system will be unstable for all values of support damping. The optimum stiffness and damping values for a particular system depend upon the rotor-bearing properties and the nature and magnitude of the forces acting on the system that produce instability.

5.2 DETERMINING THE STIFFNESS AND DAMPING COEFFICIENTS OF THE SQUEEZE FILM DAMPER BEARING

The assumption of steady state circular synchronous precession of the journal and the use of a rotating coordinate system allow the bearing forces to be equated to equivalent stiffness and damping forces. This establishes stiffness and damping coefficients for the bearing. These coefficients are functions of the bearing geometry, the use of end seals and cavitation of the fluid film. The coefficients obtained from the steady state bearing analysis are compared with the values from the stability maps to determine a bearing configuration that promotes system stability.

The stiffness and damping coefficients are non-linear functions of the journal eccentricity. However for \( \varepsilon < 0.4 \) the coefficients do not change appreciably with changes in eccentricity.
For values of $e > 0.4$ a rapid increase in the coefficients occurs. The high stiffness developed can cause system instability or raise the system critical speed above the operating speed resulting in force transmissibilities greater than 1. For these reasons a design criteria of $e < 0.4$ has been established. If the fluid film does not cavitate a radial stiffness is not developed and must be supplied by retainer springs.

5.3 TRANSIENT ANALYSIS

The bearings designed using steady state analysis are further analyzed using transient response programs. The motion of a system under the influence of unbalance and other external forces is monitored. Effect of retainer springs to preload the bearing can be determined and the bearing design further refined.

The accuracy of transient response programs are dependent on the accuracy of the numerical integration methods employed. At high eccentricities very small integration step sizes are required to retain high accuracy in the solution because of the rapid variation in the bearing forces. The cost of obtaining high accuracy at high eccentricity does not justify using small time steps or complicated integration methods requiring many functional evaluations because optimum bearing design requires operation at low eccentricities. The information provided by simpler integration methods is sufficient to indicate trends in bearing operation at high eccentricities.
5.4 ADVANTAGES OF BEARING SIMULATION

The analytic simulation of the squeeze film damper bearing eliminates many bearing designs that would result in bearing or machine failure in a test installation. Because the cost of constructing and instrumenting a test rig is very high, preventing the failure of these machines is important. The time involved in manufacturing and testing bearings is also great and the elimination of unsuitable designs by analytic procedures results in a substantial savings in time and money.

A good test program is essential, however, to determine the actual rotor-bearing response under various conditions. The analytic simulation provides a means of interpreting the test data. Often the nature of the actual system excitation is unknown and the actual system response must be used to infer the nature of these excitations. Where it is possible to systematically vary the excitation the experimental results provide a check on the accuracy and limitations of the analytic simulation.

5.5 LIMITATIONS OF ANALYTICAL INVESTIGATIONS

In any analytical investigation it is very important to know the assumptions made in analyzing the problem. In deriving the bearing equations for this study several assumptions were made. To obtain the Reynolds equation from the Navier-Stokes equations it was assumed that:

1. The viscosity is constant
2. The flow is steady state
3. The fluid inertia terms are negligibly small
4. The density is constant
5. There is no flow in the radial direction
6. There is no pressure gradient in the radial direction

The Reynolds equation obtained was modified by using the short bearing approximation. The assumption made was that the pressure gradient in the tangential direction is small and when multiplied by $h^3$ it is very small in comparison to other terms containing only $h$. This assumption is valid only if the tangential pressure gradient is small, and at high eccentricity ratios this assumption may not be valid. For this reason the design criterion is that the eccentricity ratio be less than 0.4. In addition the short bearing approximation is valid for $\frac{L}{D} < 0.5$. For ratios greater than this the axial pressure gradient is not large compared to the tangential gradient. For $\frac{L}{D} > 0.5$ either the long bearing approximation or finite bearing techniques should be used.

The conditions under which cavitation occurs must be modified in light of the current experimental results. In this study cavitation conditions were assumed to be the same in squeeze bearings as in journal bearings.

5.6 RECOMMENDATIONS FOR FUTURE RESEARCH
1. Construct an experimental rotor with squeeze damper supports to provide data to verify the analytic squeeze bearing model.
2. Conduct analytic and experimental research into the conditions under which the squeeze bearing fluid film cavitates
and to determine how cavitation propagates through the film.

3. Investigate the heat transfer characteristics of the bearings and provide modifications to the bearing programs to account for variable viscosity of the lubricant.


APPENDIX A

DESCRIPTION OF PROGRAM SQFDAMP

This program analyzes the stiffness, damping and pressure characteristics of the squeeze film damper bearing. Three bearing configurations may be analyzed:

1. Plain bearing without end seals or circumferential oil supply groove.
2. Bearing without end seals but with circumferential oil supply groove.
3. Bearing with both end seals and circumferential oil supply groove.

In addition the fluid film may be assumed to be cavitated or uncavitated. If cavitated the film extends from $\theta = \frac{\pi}{2}$ to $\theta = \frac{3\pi}{2}$. If uncavitated it extends from $\theta = 0$ to $\theta = 2\pi$. The $\theta$ is measured from the line of centers of the bearing in the direction of journal precession. The evaluation of the bearing characteristics assumes that the journal precesses synchronously about the bearing center in a circular orbit.

The following is a description of the program input data:

CARD 1 -80 column free-field comment card.
CARD 2 -80 column free-field comment card.
CARD 3 -Namelist/BRGTYPE/ TYP, CAV, PS

TYP - 0 for bearing type 1 (see above)
    - 1 for bearing type 2 (see above)
    - 2 for bearing type 3 (see above)

CAV - 0 for uncavitated film
    - 1 for cavitated film

79
PS - oil supply pressure, psi.

CARD 4  Namelist/BEARING/ L, R, MU, N
L - Bearing length, in.
R - Bearing radius, in.
MU - Lubricant viscosity, microreyns
N - Rotor speed (journal precession rate), RPM

CARD 5  Namelist/ECRATIO/ES, EF
ES - initial journal eccentricity ratio  ES>0
EF - final journal eccentricity ratio  EF<1

CARD 6  Namelist/CLEARNC/ C(I), NC
C(I) - clearance
NC - Number of clearance values

CARD 7  Namelist/PLOTSEM/ CS, PC, PK, PP
CS - Plot control, .T. if plot desired, otherwise .F.
PC - .T. if damping plot desired, otherwise .F.
PK - .T. if stiffness plot desired, otherwise .F.
   (if CAV=0, PK = .F.)
PP - .T. if pressure plot desired, otherwise .F.

Sample input data.

CARD DATA
1 SAMPLE DATA FOR SQFDAMP
2  1 MARCH 1973
3 $BRGTYP TYP = 0 , CAV = 1, PS = 0.0$
4 $BEARING L = 0.90, R = 2.55, MU = 0.382, N = 16800.0$
5 $ECRATIO ES = 0.1, EF = 0.9$
6 $\text{$\text{\$CLEARANCE}\ C(1) = .003, C(2) = .004, C(3) = .005}$
$\text{C(4) = .006, NC = 4$}

7 $\text{$\text{\$PLOTSEM}\ CS = .T., PC = .T., PK = .T., PP = .T.$}$

The following is a listing of the program and a sample output.
SET UP HE أعلن LOCK FLD6020.7

PhC

bOVE GRIGIN SCFD0430

-WRXJE(6v5_)

SQFDO5G

0

FORMAT (1x9 SQFD51)

--lSBHTHIS_258HCHARACTERISTICS CF THE CQUEEZE FILM CAMPER BEARING. THREE /lX, SCFD0530

458X 0

.. PLAIN EEACING hITkC LEAKAGE SEALS CR CIRCUH-

SCFD0550

_358HeEARIhG_CCNFIGUCATIoN

YAY BE AhALYZED- __ /lX* -SQFDC540.

/1Xg

SGFD0600

___ 258HTATEC OR UNCAVITATED. IF CAVITATED THE FILX

IS ASSUMED -/lX*

SQFO0620

358HTO EXTEhO FROI! THETA=FI/2 -TO THETA=3FI/Z, HHERE THETA

/1X~

SQFO0630

458KIS

NEPSUREO FROt' THE LIhE CF- CEEtTERS IN THE DIRECTION OF

-./IX~

SQFO0640

558HJOURhPL PRECESSICN. THE EVALUATION CF THE BEARING CHARAC- /lX, SOFDU65J

++58HTERISIICS ASSUHEI THAT. THE .JCURhAL FRECESSES SYNCHRONOUS-- /lX,. SQFO0660-

%8H. ___ __ ENTIAL OIL SUPFLY GROOVE

156HIK ADGITION, THE

FILI.:

hAY BE ASSUHEO TO BE-EITHER-CAVI- 11x9 SQFDGblO
THE FOLLOWING IS A DESCRIPTION OF THE INPUT PARAMETERS...

WRITE(6,6) SOD0660

6 FORMAT(1X, 16H4 NAMELIST/BRGTYPE/ TYP,CAV,PS)
   156HCARD 1, -60 COLUMN FREE-FIELD COMMENT CARD /1X, SOD0700
   259HCARD 2, -60 COLUMN FREE-FIELD COMMENT CARD /1X, SOD0700
   358HCARD 3, -NAMELIST/BRGTYPE/ TYP,CAV,PS /1X, SOD0730
   458H TYP - 0 FOR BEARING TYPE 0 (SEE ABOVE) /1X, SOD0740
   558H 1 FOR BEARING TYPE 1 (SEE ABOVE) /1X, SOD0750
   658H 2 FOR BEARING TYPE 2 (SEE ABOVE) /1X, SOD0760
WRITE(6,8) SOD0770

8 FORMAT(1X, 156HCAV - 0 FOR UNCAVITATED FILM /1X, SOD0790
   258H 1 FOR CAVITATED FILM /1X, SOD0800
   358H IF CAV=0 PK=1.F, (SEE CARD 7) /1X, SOD0810
   458H PS - OIL SUPPLY PRESSURE, PSI /1X, SOD0820
   558H L - BEARING LENGTH, IN. /1X, SOD0840
   658H R - BEARING RADIUS, IN. /1X, SOD0850
   758H MU - LUBRICANT VISCOSITY, MICROREYN. /1X, SOD0860
   858HCARD 4, -NAMELIST/BEARING/ L,M,NU,N /1X, SOD0880
   958HCARD 5, -NAMELIST/ECCRATIO/ ES,EF /1X, SOD0890
   158H ES - INITIAL JOURNAL ECCENTRICITY RATIO, ES>0 /1X, SOD0910
   158H EF - FINAL JOURNAL ECCENTRICITY RATIO, EF<1.0 /1X, SOD0930
   458HCARD 6, -NAMELIST/CLEARANCE/ C(i),NC /1X, SOD0940
   558H C(i) - CLEARANCE VALUES, IN. 055/1X, SOD0950
   658H NC - NUMBER OF CLEARANCE VALUES /1X, SOD0960
WRITE(6,7) SOD0970

7 FORMAT(1X, 158HCARD 7, -NAMELIST/FLCISEM/ CS,PC,PK,FP /1X, SOD0980
   258H CS - PLOT CONTROL, T, IF PLOT DESIRED, /1X, SOD0990
   358H OTHERWISE -F. /1X, SOD1010
   458H PC - T, IF DAMP PLOT DESIRED, OTHERWISE -F. /1X, SOD1030
   558H PK - T, IF STIFF PLT DESIRED, OTHERWISE -F. /1X, SOD1050
   658H PP - T, IF PRESS PLOT DESIRED, OTHERWISE -F. /1X, SOD1070
   758HSAMPLE DATA /1X, SOD1080
   858HCARD 1 /1X, SOD1090
   958HCARD 2 /1X, SOD1100
   158H SBRGTYPE TYP=0,CAV=1,FS=0.0$ /1X, SOD1120
   258H BEARING L=99.0,R=2.55,NU=0.382,N=16800$ /1X, SOD1140
   358H SECRATIO ES=0.1,EF=0.9% /1X, SOD1160
   458H CLEARANCE C(1)=.003, C(2)=.004, C(3)=.005, C(4)=.006, NC=4$ /1X, SOD1180
   558H PLOTSEM CS,.T.,PC,.T.,PK,.T.,PP,.T.$ /1X, SOD1200
WRITE(6,25) COMMENT1,COMMENT2

C WRITE OUT INPUT DATA
C
WRITE(6,25) COMMENT1,COMMENT2

FOMAT(1H1,1X,2(8A10/1X)) SOD1130
IF(TYP.EQ.0) WRITE(6,27) SOD1140
IF(TYP.EQ.1) WRITE(6,28) SOD1150
IF(TYP.EQ.2) WRITE(6,29) SOD1160
IF(CAV.EQ.0) WRITE(6,31) SOD1170
IF(CAV.EQ.1) WRITE(6,32) SOD1180

FOMAT(27) SOD1190
C BEARING WITH NO ENC SEALS OR OIL SUPPLY GROOVE
C
FOMAT(28) SOD1200
C BEARING WITH NO ENC SEALS BUT WITH OIL SUPPLY GROOVE
C
FOMAT(29) SOD1210
C
C
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</tr>
<tr>
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<td>41X,15HPS=NU,F8.1,12H_INCHES</td>
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<tr>
<td>52</td>
<td>520HP=ES,F8.3</td>
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<tr>
<td>61</td>
<td>61X,15HREF=ES,F8.3,12H_INCHES</td>
</tr>
<tr>
<td>72</td>
<td>WRITE(6,11) CO(J),J=1,KC.</td>
</tr>
<tr>
<td>11</td>
<td>FORMAT(6X,19HCLEARANCES - INCHES//5(64X,F7.5)//)</td>
</tr>
<tr>
<td>C</td>
<td>CinisLI bağlantıConstants</td>
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<tr>
<td>C</td>
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</tbody>
</table>

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**Notes:**

- Lines 31, 32, and 41 are part of a FORTRAN program for calculating various engineering parameters.
- The program includes calculations for bearing length, bearing radius, and clearance values.
- The program also includes definitions for various constants such as MU, PS, ES, and EF.
- The program calculates values for clearance in inches and micoreyns per inch.
- The final section of the program continues with similar calculations for eccentricity range.
300  NN=NEO*NC
     DD..30.III=II,NN,NC
I=III
     WRITE(6,13).EO(I),CG(I),KO(I),PMAX(I),THETAM(I)
13     FORMAT(5(5X,F12.3,5X))
     CONTINUE
100   CONTINUE
     IF(.NOT..CS) GO TO 1000
     NP1=NC*NEO+1
     NP2=NC*NEO+NC
400   DO 500 I=NP1,NSP2
     EO(I)=0.5
     NP1=NP1+NC
     NP2=NP2+NC
500   DO 500 I=NP1,NSP2
     EO(I)=.16667
     NP=NC*NEO
     CALL Platzier(NP,L,R,MU,K,PS,ORING,COMENT1,COMENT2,NC,CS
     EO,GO,KO,PMAX,PC,PK,PP,TYP,CAV)
     K=0
1000  GO TO 900
9995  STOP
10000  END
SUBROUTINE PLOTTER(NP,L,R,MU,NC,PS,ORING,COMENT1,COMENT2,NC,G,IE0,CG,KO,PMAX,PC,PK,PP,TYP,CAV)
    C THIS SUBROUTINE PLOTS CO,KO,AND PMAX AS FUNCTIONS OF EO
    C EACH SET OF CLEARANCE VALUES REQUIRES ONE BLOCK
    C DIMENICL: C(5),COMENI(8),COMENT2(8),CLABEL(2)
    INTEGER COMENT1,COMENT2,CLABEL,Typ
    INTEGER CAV
    REAL L,M,N,KO,MU1
    LOGICAL ORING,PC,PP
    DIMENSION EO(135),CO(135),KO(135),PMAX(135)
    COMMON NEO
    DATA CLABEL (1),CLABEL(2)/
    110HC= M,10HILS /
    C NNF=NP/NC
    C DELE=(EO(NP)-EO(1))/NNP
    C NC=NNP
    C M1=M1*1.066
    IF (.NOT. PC) GOTO 101
    C PLOT CO
    C JAXIS=1
    CALL GRID(L,R,MU1,PS,COMEN1,COMEN2,JAXIS,EO,CG,KO,PMAX)
    NPI=NC*NEO+1
    NP2=NC*NEO+NC
    DO 400 I=NPI,NP2
      C(I)=0.0
    DO 400 C(I)=EO(I)
    NNP1=NP1+NC
    NNP2=NP2+NC
    DO 500 I=NP1,NP2
      C(I)=1.0
    DO 500 C(I)=1.0
    CALL LINE(NNP1),C(NP1),NNF,NC,0,0)
    C DECIDE WHETHER LABELING SHOULD BE OVER CURVE OR
    C AT END OF CURVE, IF MAXIMUM ECCENTRICITY I 0.7
    C PLACE LABELING OVER CURVE, OTHERWISE AT END
    C IF ECC(NE),GE, 0.7) GOTO 60
    C DETERMINE THE X AND Y COORDINATES OF THE LOWER
    C LEFT HAND CORNER OF THE LABELING AND THE ANGLE
    C IT MAKES WITH THE X-AXIS
    XX=(EO(NE))/0.16667 + 0.1
    YY=CO(III) + 0.1
    THETA=ATAN((CO(III)-CG(III-10*NC))/(EO(III)-EO(III-10*NC))*6.0)
    CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
    C DETERMINE THE X AND Y COORDINATES OF THE LOWER
    C LEFT_HAN D CORNER OF THE CLEARANCE VALUE TO BE
C PLOTTED SQFD2630

C

XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*C(I)
CALL NUMBER(XX,YY,0.1,CV,THETA,4,HE5,2)
GOTO 30

C DETERMINE THE X AND Y COORDINATES OF THE LOWER
LEFT_HAND_CORNER_OF_THE_LABELING_AND_THE_ANGLE
IT MAKES WITH THE X-AXIS

C

60 XX=ECINC*{(NEO-1)/2}+1)*6.0
III={(NEC-1)/2}+NC+1
YY=COIII+0.1
IP=IPX*(1,3,129/DELE))%NC+III
THETA=ATAN(GO(IP)-CO(III))/((EO(IP)-EO(III))*6.0))
THETA1=THETA*57.2958
CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)

C DETERMINE THE X AND Y COORDINATES OF THE LOWER
LEFT_HAND_CORNER_OF_THE_CLEARANCE_VALUE_TO_BE
PLOOTED

C

XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*C(I)
CALL NUMBER(XX,YY,0.1,CV,THETA,4,HE5,2)

C CONTINUE

C

CALL PLOT(15.0,0.0,0.0)

C

IF (NOT, PK) GOTO 201

JAXIS=2
CALL GRID(L,R,N,NU1,PS,COMEN1,COMEN2,JAXIS,EO,CO,KO,PMAX,
1NF,1NP,CAY)
NP1=NC%NEO+1
NP2=NC%NEO+NC
DO 401 I=NP1,NP2

K(I)=0.0
NP1=NP1+NC
NP2=NP2+NC
DO 501 I=NP1,NP2

K(I)=1.0

DO 40 I=1,NC

CALL LINE(EO(I),KO(I),XH,F,NC,0,0)
IF (EO(NF)+GE.0.7) GOTO 70
XX=EC(NF)*6.0+0.1
III=NP%NC+1
YY=KO(III)+0.1
THETA=ATAN((KO(III)-KO(III-10*NC))/((EO(III)-EO(III-10*NC))*6.0))
THETA1=THETA*57.2958
CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*C(I)
CALL NUMBER(XX,YY,0.1,CV,THETA1,4,HE5,2)
GOTO 40

70 XX=ECINC*{(NEO-1)/2}+1)*6.0
III={(NEC-1)/2}+NC+1
IP=(IFIX(0,13129/DELE))**NC + III
YY=K0(III)+0.1
THETA=ATAN((K0(IP)-K0(III))/((EO(IP)-EO(III))*6.0))
THETA1=THETA57.2958
CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*G(1)
CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
40 CONTINUE
CALL PLECT(15.0,0.0,-6)
JAXIS=3
CALL GRID(1,R,N,MU1,PS1,GCMM1,GCMEN1,JAXIS,EO,CO,KO,PMAX)
1NP,TYP,CAV)
NP1=NC+NEO+1
NP2=NC*NEO+NC
DO 402,1=NP1,2
402 PMAX(I)=0.0
NP1=NP1+NC
NP2=NP2+NC
DO 502,1=NP1,2
502 PMAX(I)=1.0
DO. 502,1=1,NC
CALL LINE(EG(I),PMA(I),HF,NG,0,0)
IF(NGNE0)GE. 0.7)GOTO 60
XX=EC(NP)*6.0+0.1
III=NP-NC+1
YY=PMA(I)+0.1
THETA=ATAN((PMA(I)-PMA(III)-10*NC))/
11((EO(III)-EO(III-10*NC))*6.0))
THETA1=THETA57.2958
CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*G(1)
CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
GOTO 70
80 XX=EO(NC*((NEO-1)/2)+1)*6.0
III=LINE(1)/2.0*NC+1
IP=(IFIX(0.13129/DELE))**NC + III
THETA=ATAN((PMA(IP)-PMA(III))/((EO(IP)-EO(III))*6.0))
THETA1=THETA57.2958
CALL SYMBOL(XX,YY,0.1,CLABEL,THETA1,13)
XX=XX+0.26*COS(THETA)
YY=YY+0.26*SIN(THETA)
CV=1000.0*G(1)
CALL NUMBER(XX,YY,0.1,CV,THETA1,4HF5.2)
50 CONTINUE
CALL PLECT(15.0,0.0,-6)
RETURN
END
| CALL SYMBOL (1.29,0.96,0.21,HEADER1,0.0,19) | SQFD4310 |
| CALL SYMBOL (0.30,0.57,0.21,HEADER2,0.0,30) | SQFD4320 |
| IF(TYP,EQ.0) CALL SYMBOL (0.30,0.57,0.21,HEADER3,0.0,30) | SQFD4330 |
| IF(TYP,EQ.1) CALL SYMBOL (0.30,0.57,0.21,HEADER4,0.0,30) | SQFD4340 |
| IF(CAV,EQ.0) CALL SYMBOL (1.20,0.18,0.21,HEADER5,0.0,20) | SQFD4350 |
| IF(CAV,EQ.1) CALL SYMBOL (1.20,0.18,0.21,HEADER6,0.0,20) | SQFD4360 |
| CALL SYMBOL (4.6,1.6,0.10,LEGEND1,0.0,16) | SQFD4370 |
| CALL SYMBOL (4.6,0.6,0.10,LEGEND2,0.0,12) | SQFD4380 |
| CALL SYMBOL (4.6,0.6,0.10,LEGEND3,0.0,12) | SQFD4390 |
| CALL SYMBOL (4.6,0.6,0.10,LEGEND4,0.0,16) | SQFD4400 |
| CALL SYMBOL (4.6,0.2,0.10,LEGEND5,0.0,21) | SQFD4410 |
| CALL_NUMBER (4.26,1.6,0.10,H,0.0,4HF6.1) | SQFD4420 |
| CALL_NUMBER (4.26,0.8,0.10,R,0.0,4HF6.2) | SQFD4430 |
| CALL_NUMBER (4.26,0.8,0.10,L,0.0,4HF5.2) | SQFD4440 |
| CALL_NUMBER (4.26,0.4,0.10,F5,0.0,4HF7.4) | SQFD4450 |
| CALL_NUMBER (4.26,0.2,0.10,M1,0.0,4HF6.3) | SQFD4460 |
| CALL SYMBOL (-0.43,-0.75,0.10,CONTENT1,0.0,80) | SQFD4470 |
| CALL SYMBOL (-0.43,-0.90,0.10,CONTENT2,0.0,80) | SQFD4480 |
| RETURN | SQFD4490 |
| END | SQFD4500 |
SUBROUTINE LOGSCAL(Y, K, S, LO, CYCLES, KH)

THIS SUBROUTINE TAKES AN ARRAY OF DATA *Y* ANDSCALES ITS VALUESفقFGOF0450
A LOGARITHMIC PLOT. THERE ARE TWO ENTRIES——

LOGSCAL IS USED IF THE RANGE OF DATA VALUES ARE UNKNOWN.  .SGFQDF0450
LOGCYC IS USED IF THE RANGE OF DATA ARE KNOWN AND CAN BE PROVIDED."GFGQDF0450
INPUT PARAMETERS FOR BCTH_ROUTINES***************FGQDF0450
Y= THE ARRAY IN WHICH THE DATA TO BE SCALLED IS STORED  .GFGQDF0450
K= THE NUMBER OF THE POINTS IN THE ARRAY  .GFGQDF0450
S= THE LENGTH OF THE AXIS IN INCHES  .GFGQDF0450
K= A REPLICATION FACTOR FOR ACCESSING DATA IN Y  .GFGQDF0450
INPUT PARAMETERS FOR LOGCYC BUT OUTPUT PARAMETERS OF LOGSCAL  .SGFQDF0450
CYCLES = THE NUMBER OF CYCLES OF REPETITIONS OF THE GRAPH  .SGFQDF0450
LO= THE LOWEST EXPONENT OF THE DATA  .SGFQDF0450
IT IS AN OUTPUT PARAMETER OF LOGSCAL *** INPUT FOR LOGCYC  .SGFQDF0450
IE THE DIFFERENCE BETWEEN HIGHEST AND LOWEST EXPONENTS + ONE  .SGFQDF0450
IE, IF Y(I) IS GREATER THAN 1.0E10**IE THEN LO = IE  .SGFQDF0450
INTERNAL VARIABLES  .SGFQDF0450
HI= THE MAXIMUM VALUE OF THE DATA  .SGFQDF0450
AMI AND ALO ARE REAL VALUES FOR INTEGER HI AND LO  .SGFQDF0450
CL= A SCALE_FACTOR  .SGFQDF0450
***********************************************************************FGQDF0470
INTEGER HI, CYCLES  .SGFQDF0470
DIPENسا Y(NN)  .SGFQDF0470
AMI=1.0E**321  .SGFQDF0470
ALO=10.**321  .SGFQDF0470
************************************************************FGQDF0470
C FIND THE MAXIMUM OF THE DATA  .SGFQDF0470
DO 10 I=1, N, K  .SGFQDF0470
   AMI=A MAX(AMI, Y(I))  .SGFQDF0470
   ALO=A MIN(ALO, Y(I))  .SGFQDF0470
10 CONTINUE  .SGFQDF0470
C COMPUTE HI, CYCLES
C ALO=ALOG10(ALO)  .SGFQDF0470
AMI=ALOG10(AMI)  .SGFQDF0470
LO=IFIX(ALO)  .SGFQDF0470
IF(ALO.LT.0.0) LO=LO-1  .SGFQDF0470
HI=IFIX(AMI)+1  .SGFQDF0470
CYCLES=HI-LO  .SGFQDF0470
GOTO 11  .SGFQDF0470
C THIS ENTRY POINT IS USED WHEN LO AND CYCLES ARE KNOWN  .SGFQDF0470
C ENTRY LOGCYC
C CL=S/CYCLES  .SGFQDF0470
11 DO 20 I=1, N, K  .SGFQDF0470
C GET LOG_10 OF THE DATA AND ADJUST TO SCALE  .SGFQDF0470
C IF(Y(I),LE.0.) PRINT 30  .SGFQDF0470
20 FORMAT(*,NEGATIVE OR 0 VALUES HAVE BEEN SCALLED BY THE SCALING SGFQDF0500)
1SUBROUTINE *********** THE ABSOLUTE VALUE OF THE VALUE HAS BEEN USED "SGFQDF0500"
1ED FOR PLOTTING PURPOSES

Y(I)=ALOG10(ABS(Y(I)))

20  Y(I)=(Y(I)-LC)*CL

Y(N+1)=C*D
Y(N+2)=1.0
RETURN
END
SUBROUTINE LOGAXIS(X,Y,IBCD,NCH,SZ,THTA,LOEXP,ICYCLES)
C THIS ROUTINE CREATES A LOGARITHMIC AXIS WHICH MAY BE HORIZONTAL
C ALONG THE LENGTH OF THE PAPER, OR THTA=0.0) OR VERTICAL (ACROSS
C THE WIDTH OF THE PAPER OR THTA =90.0) OR AT ANY OTHER ANGLE. FURTHER.
C THE AXIS CAN BE DRAWN WITH DESIRED LABELS.
C PRINTED
C VARIABLES
C X,Y-THE COORDINATES OF THE BEGINNING POINT OF THE AXIS
C IBCD=AN ARRAY OF HOLLRITH CHARACTERS PRINTED AS LABELING
C NCHAR=NUMBER OF CHARACTERS IN IBCD TO _FLOAT
C SIZE = THE LENGTH OF THE AXIS IN INCHES
C THTA =THE ANGLE AT WHICH THE AXIS IS DRAWN
C LOEXP-MINIMUM EXPONENT (FACHER OF TEN) OF ANY VALUE TO BE PLOTTED
C IYCLES-MINIMUM NUMBER OF SECTIONS OF THE AXIS TO BE PLOTTED
C INTERNAL VARIABLES
C FINE DETERMINES FINE OR COARSE TICK MARKS
C HGT-CONTAINS THE HEIGHTS OF THE TICK MARKS
C COORD-CONTAINS COORDS. USED TO MOVE THE PEN THE CORRECT LENGTH
C WHILE THE GRAPH IS BEING DRAWN
C DIMENSION COORD(65), HGT(65), IBCD(8)
C LOGICAL FINE
C NOTE WE NEED THE ABSOLUTE VALUES OF SIZE, THTA, NCHAR
C
           SIZE=SZ
C       NCHAR=NCH
C       THTA=THTA
C       Z=COORD(1)
C       J=ALOG(FLOAT(11)/10.0)
C       IF(Z.EQ.J) GO TO 1
C THIS PROCEDURE FILLS THE ARRAYS WITH NEEDED INFORMATION
C
           ITEN=6 M 1X10
C           K=10
C           INCR=1
D0 5 I=1,65
C             K=K+INCR
C             A=FLOAT(K)/10.
C             COORD(I)=ALOG10(A)
C             HGT(I)=.04
C             J=K-(K/5)*5
C             IF(J.EQ.0) HGT(I)=.06
C             J=K-(K/10)*10
C             IF(J.EQ.0) HGT(I)=.10
C             IF(K.EQ.50) INCR=2
C
C       5 CONTINUE
C THIS SECTION SETS THE VALUES FOR THE DIFFERENT ICTIONS IN TICK MARKS LABELING ETC.
C
       DIRECT=1
       IF(INCHAR.LT.0) DIRECT=(-1)
       TICK =DIRECT
       IF(THTA.LT.0.) TICK =(-TICK)
C
93
FINE=.FALSE.
IF(SIZE.LT.0), FINE = .TRUE.
THETA=ABS(THETA)
NCHAR =ABS(NCHAR)
SIZE =ABS(SIZE)
CSTHETA=COS(THETA*.017455)
SNTHETA=SIN(THETA*.017455)

C THIS DRIVES THE AXIS AT THE PROPER ANGLE AND LENGTH
C IT USES_COORD.ARRAY FOR GIVING THE RIGHT LENGTH FOR TH AXIS AND TICS
C MARKS TICK, DIRECT AND FINE SET THE DIRECTION OF TICK MARKS,
C DIRECTION OF THE LABELING AND OF THE NUMBER OF TICK MARKS

CYCLES2=SIZE/FLOAT(IYCLES)
W=.10*(-TICK)*SNTHETA+X
K=5
DO 20 I=1,ICYCLES
    J=0
    CALL WHERE(C, E, IDUMMY)
    CALL PLOT(V, W, 3)
    CALL PLOT(K, X, 2)
    CALL PLOT(Y, H, 1)
    CALL PLOT(A, B, 1)
    IF(J.EQ.0) GO TO 20
    J=J+K
    A=COORD(J)*CSTHETA*CYCLES2+C
    B=COORD(J)*SNTHETA*CYCLES2+C
    CALL PLOT(A, B, 2)
    CALL PLOT(Y, H, 1)
    CALL PLOT(A, B, 1)

20 CONTINUE
IF(NCHAR.EQ.0) GO TO IC_020

C THIS ROUTINE PROVIDES LABELING IN THE PROPER DIRECTION,
C FOR THE IDENTIFICATION OF THE TICK MARKS

IQ=5
IUL=40
BB=*0.07*TICK +.21*DIRECT -.07 -(TICK+DIRECT)*.03
DX=CSTHETA*-.28) +EE*SNTHETA
DY=SNTHETA*(-.28) +BB*CSTHETA
XA=SIZE*CSTHETA*X+DX
YA=SIZE*SNTHETA*Y+DY
CALL SY60A(XA, YA, .18, .14, ITEN, _THEMA, 6)
INDEX=ICYCLES+LOEXP
XA=CSTHETA-.72*SNTHETA*+.05+XA
YA=SNTHETA-.72+CSTHETA*.05+YA
POWER=2*H10
CALL NUMER(XA, YA, .10, 11, THETA, POWER)
INDEX=ICYCLES

C THIS MOVES THE PEN DOWN THE AXIS.Putting the tick mark labeling
C AT THE CORRECT POSITION

320 K=IQ
II=II-1.
INCR=-5
I=IUL

330 QA= (COORD(I)+ FLOAT(I))*CYCLESZ +.25
XA=QA*CSTHETA+X+DX
YA=QA*SINTHETA+Y+DY
K=K+1
IF (I.EQ.40) INCR=-10
I=I+INCR
IF (I.GT.0) GO TO 330

GA=FLOAT(I)*CYCLESZ
XA=GA*CSTHETA+X+DX
YA=GA*SINTHETA+Y+DY
CALL SYMBOL (XA, YA, .14, ITEK, THETA, 6)
II=II+LOEXP
XA=CSTHETA+.72-SINTHETA*.05+XA
YA=SINTHETA*.72+CSTHETA*.05+YA
POWER=2HI3
CALL NUMBER (XA, YA, .10, II, THETA, POWER)

820 RETURN
**Squeeze Film Capper**

This program analyzes the stiffness, damping, and pressure characteristics of the squeeze film capper bearing. Three bearing configurations may be analyzed:

0 - Plain bearing without end leakage seals or circumferential oil supply groove
1 - Bearing without end leakage seals but with circumferential oil supply groove
2 - Bearing with both end leakage seals and circumferential oil supply groove

In addition, the film may be assumed to be either cavitated or un-cavitated. If cavitated, the film is assumed to extend from $\Theta=\pi/2$ to $\Theta=3\pi/2$, where $\Theta$ is measured from the line of centers in the directick of journal precession. The evaluation of the bearing characteristics assumes that the journal precesses synchronously about the bearing center.

The following is a description of the input parameters:

**Card 1. -80 Column Free-Field Coment Card**

**Card 2. -80 Column Free-Field Coment Card**

**Card 3. -NAMELIST/PBRTYPE / TYP,CA,P**

**TYP - 0 FOR BEARING TYPE O (SEE ABOVE)**

1 FOR BEARING TYPE 1 (SEE ABOVE)

2 FOR BEARING TYPE 2 (SEE ABOVE)

**CAV - 0 FOR UCNAVITATED FILM**

IF CAV=0 PK=+.F. (SEE CARD 7)

**PS - OIL SUPPLY PRESSURE, PSI**

**Card 4. -NAMELIST/EARING/ L,R,Pu,N**

**L - BEARING LENGTH, IN.**

**R - BEARING RADIUS, IN.**

**Pu - LUBRICANT VISCOSITY, PICCROPS**

**N - ROTOR SPEED (J645E13 7IE65895 I1CE, RPM)**

**Card 5. -NAMELIST/ECCRATIO/ ES,EF**

ES - INITIAL JOURNAL ECCENTRICITY RATIO, ES=J.

**EF - FINAL JOURNAL ECCENTRICITY RATIO, EF<1.J**

**Card 6. -NAMELIST/CLEARANCE/ C(1),NC**

**C(1) - CLEARANCE VALUES, IN. 0<155**

**NC - NUMBER OF CLEARANCE VALUES**

**Card 7. -NAMELIST/PLOT/ ES,PC,P,K,C**

**CS - PLOT CONTROL, .T. IF PLOT DESIRED, OTHERWISE .F.**

**PC - .T. IF DAMP PLOT DESIRED, OTHERWISE .F.**

**PK - .T. IF STIFF. PLOT DESIRED, OTHERWISE .F.**

**PF - .T. IF PRESS PLOT DESIRED, OTHERWISE .F.**

**Sample Data**

**Coment Card 1**

**BRINTYPE TYP=0,CAV=1,FE=0.0**

**BEARING L=0.50,R=2.50,PU=0.382,TP=IEEE88**

**ECCRATI ES=0.1,EF=0.5**

**CLEARANCE C(1)=.003,C(2)=.004,C(3)=.005,C(4)=.006,NC=41**

**PLOT/ ES=.T.,PC=.T.,PK=.T.,PF=.T.**

96
\[ c = 0.030 \text{ in.} \]

<table>
<thead>
<tr>
<th>EC (O/F)</th>
<th>CO LE-SEC/IN</th>
<th>KU LE/IN</th>
<th>PMAX LR/IN**2</th>
<th>THETA DEGREES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.130</td>
<td>1.802</td>
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—National Aeronautics and Space Act of 1958

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