Technical Memorandum 33-722


Edited by
D. D. Papailiou

JET PROPULSION LABORATORY
CALIFORNIA INSTITUTE OF TECHNOLOGY
PASADENA, CALIFORNIA

March 15, 1975
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PREFACE

The work described in this report was performed by the Propulsion and the Science Divisions of the Jet Propulsion Laboratory, the research in the Propulsion Division being directed by D. D. Papailiou and the research in the Science Division by J. S. Zmuidzinas. The report was compiled and edited by D. D. Papailiou.
LIST OF CONTRIBUTORS TO THIS PUBLICATION

E. J. Roschke: Lasers
A. A. Vetter: Lasers
D. C. Papaiiou: Matter-Antimatter and Energy Exchange
J. S. Zmuidzinas: Excited Helium
T. M. Hsieh: Thermonuclear Fusion
D. F. Dipprey: Appendix
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ABSTRACT

In this report certain novel or advanced concepts are described that presently appear to have the potential for propulsion applications in the post-1990 era of space technology. The studies are still in progress; hence, only the current status of investigation is presented here. The topics for possible propulsion application are lasers, nuclear fusion, matter-antimatter annihilation, electronically excited helium, and energy exchange through the interaction of various fields.
SECTION I
INTRODUCTION
D. D. Papailiou

It is anticipated that the demand for new more sophisticated propulsion schemes with capabilities substantially exceeding those of the conventional propulsion systems will increase as we extend our activities deeper in space. Current chemical propulsion systems being developed for the space program are achieving close to the theoretical maximum attainable performance. The technology for solar electric propulsion is in various stages of development, and the resultant systems are expected to find application well before 1990. These advanced technologies exhibit known limitations and will not be adequate for many of the more ambitious missions.

The general objective of this effort is to generate new propulsion concepts as well as to investigate those already existing, the common characteristic of these concepts being their high potential for application in propulsion in the post-1990 era of space exploration.*

One of the directions to follow in developing such schemes is to investigate the possibility of using energy sources of known capability for releasing large amounts of energy as compared to the chemical propellants used currently. Along this line, work continues in the fields of nuclear fission and fusion, lasers, metastable and radical species, and matter-antimatter annihilation. However, an equally important direction, although perhaps more difficult to pursue, is to exploit the energy resources existing in space and planetary atmospheres. In space and the vicinity of planets, energy is stored in the form of various fields such as gravitational, magnetic, electric, etc., or in elementary particle concentrations. The major prohibitive factor in using this energy is its low-density level.

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*The report studies form part of the activities sponsored by NASA on this subject under the "New Horizons" program for propulsion.
The studies to be discussed are still in progress; hence, the information presented represents the current status of the investigation. The topics included here are lasers, nuclear fusion, matter-antimatter annihilation, metastables and radicals, and energy exchange through the interaction of various fields. The scope adopted in this investigation varies with each subject. The factors considered are: (a) the present state of investigation of each concept, and (b) the objective and direction chosen in pursuing the investigation.

Although a large number of publications on lasers and related subjects appeared in the literature during the past decade, only a few are devoted to laser propulsion. There is a tendency in these publications to oversimplify and to focus attention on a limited number of aspects of the problem. This is perhaps due to the lack of data in many related critical areas. This shortcoming becomes crucial when an attempt is made to evaluate some of the proposed concepts and to conduct comparative studies among them or with other forms of advanced propulsion.

One of the objectives of the work on lasers described in Section II was to form a frame of reference for a subsequent evaluation of various laser propulsion concepts. As a result of this effort a comparative study of solar electric versus laser electric propulsion is currently in progress. Section III is a summary of work on the problems of tracking and pointing. It also discusses some of the limitations inherent to particular laser configurations. In this work, two particular systems have been selected and studied, corresponding to laser beam wavelengths of 0.4 and 10 mm. One of the conclusions obtained in this study is that these configurations cannot compete with solar energy for interplanetary flights. However, laser systems with different characteristics may prove to be capable of competing with solar radiation. The analysis of such systems in terms of optics, energy conversion, wavelength (ultraviolet or shorter wavelengths) and other parameters, should probably be the subject of future studies.

There are two factors which make the use of matter-antimatter annihilation energy attractive for propulsion applications — namely, the extremely high rates of energy released during matter-antimatter annihilation processes and the "clean" products of the reactions involved, from the point of view of radioactive contamination. The released energy is two orders of magnitude higher
than that of a typical fusion, and it is close to the upper limit of energy production as defined by the Einstein's mass-energy relation. However, severe problems associated with antimatter production and storage place the current state of this technology at the conceptual level and project its possible implementation in propulsion beyond the year 2000 A.D. Since the existing work related to the use of matter-antimatter annihilation energy in propulsion is minimal, an attempt was made to first identify and discuss problem areas, based on existing information and then to develop a plan for a systematic investigation of the recognized problems. The results of this effort, which are discussed in Section IV, will form the frame of reference for future research toward evaluating the concept.

Research on electronically excited solid helium in relation to the use of helium metastable species for propulsion application is reported in Section V.

Several concepts have been proposed in the past decades for the utilization of energy available in nature for propulsion applications. Research conducted here consists of a search for energy exchange mechanism which, in principle, would allow transfer of energy from energy sources in space into a working scheme onboard the spacecraft. A concept is discussed along this line in Section VI that pertains to the interaction of a fluctuating magnetic field with an electrically conducting fluid in turbulent motion aboard the spacecraft. Also, the conditions are examined for the complete annihilation of the magnetic field and the transfer of its energy to the fluid in the form of Joule heat. This concept is presently in a conceptual stage. However, if the identified problem of estimated long characteristic decay times of the magnetic field as compared to the interaction time can be solved, this scheme is relatively free from the technical difficulties associated with the development of other concepts. Therefore, its eventual application, at least for certain missions, may not be in the distant future.

In Section VII an analysis is presented of the state of the art and problems involved in the application of nuclear fusion in propulsion. This work is based on information available in the open literature. In this study the results of research for ground applications have been taken as a basis for examining the capabilities of the concept and the anticipated problems for its implementation in propulsion.
The evaluation of the concepts with high potential for propulsion is presently based on two criteria—namely, their state of readiness and the energy-releasing capabilities translated into specific impulse numbers. However, none of the concepts discussed in this work has reached a state of development that allows a complete evaluation of either their potentials or all existing problems that might become evident in subsequent stages of development. It is, therefore, possible that with continuing research in these fields the present evaluation might change in the future. A positive aspect of this research activity is the high payoff potential in propulsion and in other fields of interest, such as energy production.
SECTION II

LASER PROPULSION - CONCEPTS AND PROBLEMS

E. J. Roschke

A. INTRODUCTION

A survey on laser propulsion and related subjects is being conducted as part of the "New Horizons" program. This survey will include reviews of various concepts that have been proposed for laser propulsion, laser devices, power transmission, energy conversion, and applications. As such, there will be no independent analyses performed herein. The purposes of this survey are to (1) identify the most important problems attendant on the various modes of laser propulsion, (2) identify and assess those applications that appear to have the greatest promise, (3) identify areas of further research and study interest, and (4) document pertinent references.

B. REVIEW OF CONCEPTS FOR LASER PROPULSION

This review will provide the historical perspective and overview necessary to achieve the goals set forth. A review of this type can be organized along several lines: (1) the concepts themselves, i.e., the various modes of laser propulsion, (2) mission objective, e.g., launch to orbit, orbit/intra-orbital, escape, translunar, interplanetary, interstellar, etc., or (3) application, e.g., manned/unmanned, scientific, auxiliary function, power transmission, etc. It is judged simpler to begin by reviewing the concepts themselves, because these often dictate the missions or applications anyway.

At the outset it is important to question the justification for any potential laser propulsion systems. If feasibility can be demonstrated, the overriding question becomes one of economics. That is, the laser propulsion system should offer the capability of larger payloads at a lower specific cost, or offer the capability of accomplishing missions that can be achieved in no other way.
There are, however, other considerations such as reliability, or mission duration. For example, if it became imperative in the future that a manned mission to the outer planets (or an interstellar mission) be accomplished in a reasonable time frame, laser propulsion might be acceptable despite enormous financial and resource costs if other forms of propulsion such as photon, matter/antimatter, etc., prove impossible. Nevertheless, the climate of the present times is such that economics probably will remain the deciding factor for a long time to come.

It is convenient to divide laser propulsion systems into two categories: those in which impinging laser energy is utilized directly, and those that utilize the energy as a source for some other form of propulsion. In the first category are the laser-powered sail, various methods for direct heating of propellants, and laser-induced fusion schemes. In the second category are laser-electric propulsion, auxiliary energy sources, etc.

1. Laser-Powered Sail

It is ironic that the concept of the laser-powered sail, which apparently was the first propulsion application suggested for lasers, is probably the most dubious in terms of feasibility. In 1962, Forward\(^1\) mentioned briefly that the concept of the solar sail might be extended by utilizing large lasers in close-solar orbits and impinging their beams on large sails to power vehicles at extreme distances from the sun; he concluded the concept was not feasible. In 1966 Marx\(^2\) suggested that manned interstellar missions might be accomplished with X-ray laser beams impinging on sail-driven vehicles. His claim that propulsion at the velocity of light at an efficiency approaching unity could be achieved in the limit of an infinite time interval was refuted by Redding in 1967.\(^3\)

Moeckel\(^4\) re-examined this problem and concluded that (1) the laser sail was not competitive with other advanced propulsion systems within the solar system and (2) interstellar propulsion, even by X-ray lasers, would require huge beam powers and beam diameters. Thus, the laser-powered sail is probably just an interesting curiosity and is not likely to be a practical mode of
propulsion. A major disadvantage of the laser sail is that only one-way trips or flyby missions would be possible outside of earth-orbit because there would be no deceleration capability in the absence of another onboard propulsion system.\(^2\)

2. Propellant Heating Systems

Kantrowitz\(^5\) called attention to the concept of remote heating of propellant by earth-based lasers but did not elaborate on the technical details. He estimated that approximately 1 GW of beam power would be required to place a 1-ton payload in low earth-orbit. Also considered was orbital insertion of a payload by laser propulsion following earth launch to Mach 15 by other means of propulsion. The major problems of laser propulsion outlined by Kantrowitz are still appropriate and furnish an excellent datum for the discussions to follow: these are (1) scaling up current lasers to the power levels that will be required, (2) improving pointing and tracking accuracies, (3) gaining better understanding of the interaction with, and propagation of high power laser beams in the atmosphere, and (3) better understanding of materials exposed to intense laser beams.

a. Heating of Gaseous Propellants

Rom and Putre\(^6\) have made a simple analysis of launch to low earth-orbit of a vehicle utilizing propellant heating by a ground-based laser beam that is focused into a nozzle to impinge on a propellant injection plate. Hydrogen propellant, seeded to facilitate absorption, is expelled through a supersonic nozzle to provide thrust. Optimum values of specific impulse are in the range 1200 to 2000 seconds. Approximately 250 MW of laser beam power would be required to place 1 ton of gross weight (payload of 200 lb) into earth-orbit. The difference between this value and that of Kantrowitz is due to a more detailed accounting of subsystem efficiencies. The cost analysis of Rom and Putre is confined to the launch vehicle and does not include the ground-based power and laser facility, which is assumed available on a periodic basis.

Minovitch\(^7\) envisions a space tug, operating in spiral earth-orbit, which receives energy by ground-based, high-power laser of the order of 460 MW. In this concept, the laser beam is intercepted by a cylindrical/parabolic reflector
which concentrates the energy on seeded hydrogen propellant located within a transparent core at the reflector focal axis. The heated propellant is directed to a spherical pressure chamber and then expelled through a rocket nozzle. The concept of Minovitch is not well conceived and harbors judgment errors but contains some good mission analysis material and a reasonable discussion of atmospheric transmission, including specific suggestions for laser locations at points of high altitude in the western United States.

b. Heating of Solid Propellants

An alternative to the heating of gaseous propellants is the use of ablative solid propellants; otherwise, the concepts are similar. The most complete discussions of the use of ablative solids in laser propulsion appear to be due to Pirri, et al.\(^8,9\) Avco (AERL), and Harstad\(^10\) (JPL). In general, most concepts involving solid propellants appear more favorable when pulsed, rather than steady, laser energy is utilized. Pirri and Weiss\(^8\) estimate that specific impulse values of $10^3$ to $10^4$ seconds appear feasible in the regime of fully ionized propellant vapor, which is in general agreement with the estimates of Harstad\(^10\). Preliminary experimental results with nonmetallic propellants, however, are not encouraging; Pirri and Monsler\(^9\) obtained values of only 100 to 500 seconds because of heat addition limitations. In principle, additional thrust is provided by expansion of the propellant vapor through a suitable supersonic nozzle.

The physics of steady and pulsed high intensity laser/solid interactions in vacuum and air are being studied by Avco Everett Research Laboratory, Lincoln Laboratory (MIT), and the Naval Research Laboratory\(^9,11-14\). Three regimes are noted: (1) transparent vapor region, (2) partially ionized vapor region, and (3) fully ionized (opaque) vapor region. Beam intensity for ionization threshold has been estimated for metals to be $10^2\;\text{MW/cm}^2$\(^8\) and as much as $10^4\;\text{MW/cm}^2$\(^10\). It is of interest to compare the propellant ionization threshold to the flux required for laser-induced atmospheric breakdown, which for dust-free air has been estimated to be $10^3\;\text{MW/cm}^2$\(^5,8\). The presence of ionized gases is likely to cause laser beam attenuation and also affects the so-called coupling coefficient, which is the thrust developed per unit laser power.
Rob and Turcotte\(^{(15)}\) have shown that vapor/plasma expanding into the atmosphere instead of a vacuum produces a strong detonation wave. Pirri and Monsler\(^{(9)}\) irradiated carbon rods at the focus of a parabolic rocket nozzle with a high-power, long-pulse CO\(_2\) laser and obtained specific impulse values of 100 to 500 seconds. They call this concept "radiation-driven detonation wave" propulsion, and have extended the idea to the so-called "laser pulsejet." The laser pulsejet utilizes no propellant at all other than the air within the nozzle. The concept is, of course, only applicable in a gaseous atmosphere; theoretically, the specific impulse is infinite because no propellant is carried onboard the vehicle. Pirri and Monsler\(^{(9)}\) have conducted experiments with a polished aluminum nozzle that forms a parabolic reflector for the incident laser beam. They used a pulsed CO\(_2\) laser with pulse times between 25 and 100 \(\mu\text{sec}\). Results were not wholly satisfactory because of imperfect focusing and the deposit of oxide on the polished aluminum nozzle.

Detonation propulsion has been proposed for use near planets of high atmospheric pressure, e.g., Jupiter. Theoretical and experimental work on detonation propulsion has been performed at JPL. Varsi and Back\(^{(16)}\) have conducted experiments (single pulse) by electrically detonating a solid propellant and expanding the gas through a conical nozzle with various environmental gases at different ambient pressures. For a 1-bar ambient pressure, typical values of measured specific impulse were about 220 seconds, which increased to about 290 seconds at an ambient pressure of 70 bars. Gases used were N\(_2\) and CO\(_2\).

3. Laser Fusion

The advent of ever higher-energy lasers has promoted consideration of propulsion by nuclear fusion along the lines of the old Orion concept, wherein periodic fission explosions far-removed from a space vehicle were transformed to impulse through a pusher-plate and shock-absorber system at the rear of the vehicle. It is well to note that Project Orion was terminated in 1965 solely on political, and not technical, grounds\(^{(17)}\). Boyer and Balcomb\(^{(18)}\) recount this process and state that the use of fusion rather than fission should remove some of the old objections, namely, atmospheric contamination by fission products, and the prospect of producing much smaller explosions, much lower energy.
release per pulse, and hence much smaller, lighter vehicles. Reaction products are limited to neutrons and gamma rays. Hyde, et al.\(^\text{19}\) envision micro-explosions through fusion of the order of millitons as compared to the kiloton levels associated with nuclear fission.

The bulk of the work on laser-induced fusion, and applications to propulsion, appear to have been accomplished by Lawrence Livermore Radiation Laboratory and Los Alamos Scientific Laboratory.\(^\text{18, 19}\) There are two basic variations of the concept: (1) the external system, i.e., the pusher-plate idea and (2) the internal system, i.e., confinement in a pressure vessel and gaseous plasma discharge through a nozzle or thruster. Thermonuclear burning of a propellant pellet (typically deuterium) is induced by an array of impinging laser beams carried onboard. The laser beams are focused on a very small volume. Confinement is achieved by implosion, which rapidly raises the propellant to extremely high density, initiating fusion and plasma expansion.

Large vehicle sizes and payloads can be accommodated; only the pressure-vessel concept would be used in the earth's atmosphere. Fusion reactions would be initiated at the rate of about one per second at standoff distances of 30 to 100 meters.\(^\text{18}\) Specific impulses of the order of 4000 to 10,000 seconds for the external system, and 800 to 1500 seconds for the internal system, are considered feasible\(^\text{18}\); thrust-to-weight ratios of 3 to 4 appear achievable. Laser fusion propulsion should be most useful for interlunar or extrasolar missions, and possibly for interstellar missions.

The physics of laser-induced propellant implosion are extremely complicated and beyond the scope of this review. Boyer\(^\text{20}\) recently has published a lengthy account of this field. Primary concerns are problems associated with the absorption process and potential instabilities in the compression process. Proper shaping of the laser pulse may be crucial. Burgess\(^\text{21}\) discusses some of the problems that must be faced in the development of fusion lasers. Nonlinear propagation processes lead to pulse distortion, beam self-focusing, and broadening of the pulse spectrum.
4. Laser-Electric System

The laser-electric system is a refinement of the now well-developed solar-electric propulsion system. Spacecraft development is within the state of the art. Other problems associated with laser beam power, optics, pointing and tracking, and atmospheric attenuation (for ground-based lasers) are common to other systems as well, and have been discussed elsewhere. Laser energy could be used to augment solar energy, or to replace solar energy on deep-space missions to the outer planets. A specific problem is energy conversion, and many concepts and devices have been considered. Existing silicon solar cells could operate at increased efficiency using laser irradiation at about 0.8 μm. Unfortunately, the highest power lasers operate at much longer wavelengths.

5. Power Transmission

In 1969, Robinson\(^{(22)}\) examined briefly the use of power transmission by laser from earth-orbit to ground. At that time laser transmission was not yet considered competitive with microwave transmission because of the comparatively low power and efficiency of lasers of that time. It was acknowledged that laser systems would have one decided advantage over microwave systems: much smaller transmitter and receiver antennas. Laser transmission to earth from orbit again raises the problem of atmospheric attenuation and distortion. This problem would not exist for intraorbital applications, e.g., power transmission to other spacecraft, or for direct propulsion utilization in translunar or interplanetary missions. Orbital laser stations would utilize converted solar energy.

Hansen and Lee\(^{(23)}\) re-examined the problem in 1972 and concluded that laser transmission of power by single beams or free-running lasers was feasible for distances several times the earth-moon distance, and distances of up to 2 A.U. for systems using phased-array transmitters. They examined a wide variety of factors including range, power, efficiency, optics, converters, pointing and tracking, radiators, laser systems, etc.

From the standpoint of financial cost, weight and volume, high-power lasers for power transmission will be required to have high efficiency. High
efficiency is especially important for cases where the laser generator is carried aboard the spacecraft, e.g., for power transmission from earth or solar orbit, and for laser fusion propulsion. Hertzberg, et al.\textsuperscript{(24)} have examined theoretically the feasibility of laser power transmission. They envision a closed-cycle gas dynamic laser, called a photon generator, which generates high power with an efficiency approaching that attained in the production of electricity from heat. A similar device, the inverse laser, is suitably designed to receive and convert incoming laser energy so that useful work may be extracted, e.g., conversion device, turbine, etc. The inverse laser system then becomes a photon engine, which could be carried aboard another spacecraft.

Laser power transmission has strong competition from microwave systems. In 1968, Glaser\textsuperscript{(25)} suggested such a system and developed the concept further to what is now known as the Satellite Solar Power Station (SSPS)\textsuperscript{(26)}. Glaser estimates a cost of $500/kW for power transmitted from orbit to ground. More recently, the results of a four-company study team headed by Glaser have been published by Brown\textsuperscript{(27)}. This study team concluded that SSPS was technically feasible and could deliver power to grid at an efficiency of 68% referenced to solar cell output in orbit. However, the financial cost still was considered not competitive with conventional methods of power generation on the ground. From the standpoint of propulsion, microwave systems would appear to be noncompetitive with laser systems because of the excessive size of transmitters and receivers.

6. Previous Reviews and Feasibility Studies

The two best reviews of laser propulsion applications are those due to Arno, et al.\textsuperscript{(28)} and Nakamura, et al.\textsuperscript{(29)} Both appeared in 1972 and treat a wide variety of potential systems and applications. A popularized treatment of advanced chemical propulsion systems was published by Cohen\textsuperscript{(30)}. Moeckel\textsuperscript{(31)} made an analytical, parametric comparison of various propulsion methods that are divided into two types: mission performance limited by maximum specific impulse (Type I) and mission performance limited by minimum specific mass (Type II). For example, propellant heating or pulsed fusion by laser is Type I, whereas the laser-electric system is Type II. A comparison by mission analysis of various nuclear propulsion methods was made by Boyer and Balcomb\textsuperscript{(18)}.
In summary, a wide variety of propulsion and other applications for lasers has been suggested in the literature and is best typified by Table 1, which has been reproduced from Arno, et al.\(^{(28)}\) The present author has suggested the possible feasibility and potential application for various concepts in Fig. 1, as based on this preliminary review. Probable feasibility is dictated partly by comparison/competition with current systems.

C. REVIEW OF LASER DEVICES AND PERFORMANCE

The state of the art of laser devices is changing so rapidly that an overall review in depth is virtually impossible. The literature on lasers appears to be increasing almost exponentially. For example, a 1972 bibliography on just one type of laser, the Nd:YAG solid laser (YAG denotes \(Y_3Al_5O_{12}\)), occupies 15 journal pages.\(^{(32)}\) Current laser development cuts across nearly all the disciplines of science and technology and covers a wide range of scientific and industrial applications. In addition, the current performance of very high power lasers is difficult to determine because of military security associated with thermal weapons. According to one article, the power output of continuous-wave lasers was increased by three orders of magnitude in the 1962-1972 decade.\(^{(33)}\) This performance improvement probably will be equalled or exceeded in the next decade.

An insight into laser devices and development is afforded from past reviews,\(^{(34-37)}\) all of which were limited in scope. An excellent, extensive review of molecular-gas lasers has just been published by Wood.\(^{(38)}\)

1. Laser Classification

There are various ways to classify lasers: (1) according to the lasing material, i.e., solid,\(^{(34)}\) semiconductors,\(^{(39,40)}\) liquid,\(^{(41,42)}\) and gas;\(^{(33,35-38)}\) (2) according to the method of pumping or excitation, i.e., optical, electrical, chemical, or thermal; and (3) according to the mode of operation, i.e., continuous-wave (CW), pulsed, Q-switched, and phase-locked (sometimes called mode-locked). Transient or short-pulse operation often is accomplished by Q-switching (utilizing some form of active or passive shutter system), and is sometimes called the "giant-pulse," "big-bang" technique, etc.
Table 1. Laser system applications from Arno, et al.\(^{(28)}\)

<table>
<thead>
<tr>
<th>ELECTRIC POWER TRANSMISSION</th>
<th>PHOTON TRANSMISSION/ILLUMINATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbit to Earth</td>
<td>Remote area illumination</td>
</tr>
<tr>
<td>Earth to satellite/spacecraft</td>
<td>Atmospheric probe from ground</td>
</tr>
<tr>
<td>Spacecraft to spacecraft</td>
<td>Atmospheric probe from orbit</td>
</tr>
<tr>
<td>Ground to ground</td>
<td>Night scan of clouds, Earth resources</td>
</tr>
<tr>
<td></td>
<td>Planet/comet/asteroid illumination</td>
</tr>
<tr>
<td>ENERGY TRANSFER FOR PROPULSION</td>
<td>Beacon/signal/transit</td>
</tr>
<tr>
<td>Launch vehicle - ablative propellant</td>
<td>Planet atmospheric analysis</td>
</tr>
<tr>
<td>Launch vehicle - (\text{H}_2) propellant</td>
<td>Chemistry/gas and materials properties</td>
</tr>
<tr>
<td>Aircraft takeoff/takeoff assist</td>
<td>Interferometry</td>
</tr>
<tr>
<td>Aircraft flight sustaining</td>
<td>Chemical processing</td>
</tr>
<tr>
<td>Orbit keeping/drag make-up/attitude control</td>
<td>HEAT TRANSFER</td>
</tr>
<tr>
<td>Orbit changing</td>
<td>Cutting, drilling, punching</td>
</tr>
<tr>
<td>Interplanetary electric propulsion</td>
<td>Welding</td>
</tr>
<tr>
<td>Laser-detonated fusion propulsion</td>
<td>Data recording (punching)</td>
</tr>
<tr>
<td>Laser-powered sail spacecraft</td>
<td>Tunneling, mining</td>
</tr>
<tr>
<td>COMMUNICATIONS</td>
<td>Material analysis (ionization, vaporization)</td>
</tr>
<tr>
<td>Interplanetary</td>
<td>Chemical processing</td>
</tr>
<tr>
<td>Terrestrial</td>
<td>Weapons</td>
</tr>
<tr>
<td>Between orbit and ground</td>
<td>Nuclear fusion</td>
</tr>
</tbody>
</table>

\(^{(28)}\) Reference to Arno, et al.
<table>
<thead>
<tr>
<th>CONCEPT</th>
<th>MISSION</th>
<th>LAUNCH TO ORBIT</th>
<th>EARTH TO ORBIT</th>
<th>ORBIT TO EARTH</th>
<th>STATION KEEPING</th>
<th>INTRAOBJECTAL ORBIT CHANGING</th>
<th>TRANS-LUNAR ORBIT</th>
<th>INNER PLANETS</th>
<th>OUTER PLANETS</th>
<th>INTER-STELLAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>LASER-HEATED PROPELLANTS</td>
<td>LAUNCH TO ORBIT</td>
<td></td>
<td></td>
<td></td>
<td>STATION KEEPING</td>
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<td></td>
<td>EARTH TO ORBIT</td>
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<td></td>
<td>ORBIT TO EARTH</td>
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<tr>
<td></td>
<td>STATION KEEPING</td>
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<td></td>
<td>INTRAOBJECTAL ORBIT CHANGING</td>
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<tr>
<td></td>
<td>TRANS-LUNAR ORBIT</td>
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<td>INNER PLANETS</td>
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<td></td>
<td>OUTER PLANETS</td>
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<td></td>
<td>INTER-STELLAR</td>
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</tbody>
</table>

**Fig. 1. Applications for laser propulsion**

- **POTENTIAL APPLICATION**
- **POSSIBLE FEASIBILITY**
A summary of laser classification appears in Table 2; many other sub-
classifications are possible. A disadvantage of current solid-state and liquid
lasers is that they require external optical pumping, e.g., flash-lamps. Semi-
conductor lasers are still relatively low-power devices although they have high
efficiency; homojunction devices are now considered obsolete. Recently, a new
type of laser based on electrochemiluminescence has been suggested by Meas-
sures (43) to take advantage of the best features of liquid and semiconductor
lasers. There are numerous types of gas lasers that utilize neutral (atomic),
ionized, or molecular gases, including metallic vapors, or chemical reactions.
The "purely chemical" laser is so called in reference to reactions obtained
without any external pumping. Thermal pumping in gas lasers may be accom-
ploished in many ways, e.g., combustion, detonation and travelling shock waves,
arc-jet, nuclear reaction, etc. In principle, all current laser types can be
operated in the modes listed in Table 2. Very high power outputs (hundreds of
gigawatts) may be achieved in a single short burst, or in a very short duration
wave-train, by Q-switching or phase-locked operation.

Because gas lasers can achieve high power, although overall efficiencies
are still rather low, and are versatile, they are receiving the most attention
for military applications. It appears that gas lasers are also the most likely
candidates for laser propulsion systems; therefore, they will be discussed in
more detail later.

2. Laser Performance

In general, the important performance factors of lasers include wave-
length and both average and peak power output, e.g., in the pulsed mode. Also
of importance are the duration, the pulse width and shape, and the maximum
repetition rate of the pulses (which often is materials-limited). Lasers have
been operated in a wavelength range that spans the ultraviolet, visible, infrared,
and into the submillimeter region. Gas lasers have the widest range of wave-
lengths to date. Recently, vacuum ultraviolet wavelengths have been achieved
with atomic gas lasers; this wavelength region is of interest for nuclear fusion
lasers. Even shorter wavelengths are desirable for some applications that
require extremely fine focusing. Although an X-ray laser has been reported,
Table 2. Laser classification

<table>
<thead>
<tr>
<th>Lasing Material</th>
<th>Types</th>
<th>Pumping/Excitation</th>
<th>Mode</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid-State</td>
<td>Doped crystals</td>
<td>Optical</td>
<td>Continuous-wave</td>
<td>High-power continuous operation limited by materials</td>
</tr>
<tr>
<td></td>
<td>Crystalline materials</td>
<td></td>
<td>Pulsed</td>
<td>High peak power in pulsed operation</td>
</tr>
<tr>
<td></td>
<td>Glass</td>
<td></td>
<td>Q-switched</td>
<td>Predicted power: $10^4$ W average $10^1$ W peak</td>
</tr>
<tr>
<td>Semiconductor (injection)</td>
<td>Diffused homojunction</td>
<td></td>
<td>Continuous-wave</td>
<td>High efficiency, low power</td>
</tr>
<tr>
<td></td>
<td>Single/double heterojunction</td>
<td></td>
<td>Pulsed</td>
<td>Temperature sensitive</td>
</tr>
<tr>
<td></td>
<td>Large optical cavity (LOC)</td>
<td></td>
<td>Q-switched</td>
<td>May suffer degradation for long-term continuous use</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Poor spectral and coherence properties</td>
</tr>
<tr>
<td>Liquid</td>
<td>Rare earth solutions</td>
<td>Optical</td>
<td>Continuous-wave</td>
<td>Requires optical pumping</td>
</tr>
<tr>
<td></td>
<td>Organic dye solutions</td>
<td></td>
<td>Pulsed</td>
<td>Tunable</td>
</tr>
<tr>
<td></td>
<td>Inorganic dye solutions</td>
<td></td>
<td>Q-switched</td>
<td>Can't be Q-switched at high power</td>
</tr>
<tr>
<td>Gas</td>
<td>Neutral (atomic) gas</td>
<td>Optical</td>
<td>Continuous-wave</td>
<td>Cover the widest range of wavelengths</td>
</tr>
<tr>
<td></td>
<td>Ionized gas</td>
<td></td>
<td>Pulsed</td>
<td>Pressure and temperature dependent</td>
</tr>
<tr>
<td></td>
<td>Molecular gas</td>
<td></td>
<td>Q-switched</td>
<td>Thermally-pumped gas dynamic lasers may operate open or closed cycle</td>
</tr>
<tr>
<td></td>
<td>Chemical</td>
<td></td>
<td>Phase-locked</td>
<td>High power, low overall efficiency</td>
</tr>
<tr>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
some controversy over this claim exists. The Russians are working on a gamma-ray laser, the so-called "graser", a concept that was abandoned by the Americans a decade ago; the essential problem is to develop a nuclear laser that utilizes microfission.

By 1970, average power outputs of gas lasers had been increased significantly, e.g., the Avco CO$_2$ gas dynamic laser had been operated in continuous, single-mode at 30 kW and at 60 kW in multi-mode. It is estimated that this device has now been operated at 200 kW. A sample spectrum of laser performance as reported by Eleccion is shown in Fig. 2, in which atmospheric "windows" have been indicated. The CO gas laser operates at approximately 5 μm, the lower window, as do some of the chemical lasers. Atmospheric windows are of importance for laser systems that must transmit or receive through the earth's atmosphere; a list of atmospheric windows has been given by Minovitch.

Very short pulses of the order of 0.1 to 1 nanosecond at energy levels of $10^5$ to $10^6$ joules will be required for laser-induced nuclear fusion; this is equivalent to power levels of $10^{14}$ to $10^{16}$ watts. Emmett reported in 1971 that 3 picosecond pulses had been achieved by Sandia and 10 to 100 picosecond pulses by Los Alamos, but at much lower energy levels. Recently, Sandia has produced 200 joule pulses of 2 nanosecond duration, and expects to achieve 1000 joule pulses in the near future. Of interest is that conversion to X-ray energy in 1 nanosecond has been predicted for laser-heated plasmas subject to pulses of the order of 25 picoseconds, by Whitney and Davis. Pulse repetition rates in gas lasers are limited by excessive, and nonuniform, gas heating problems. Already in 1971, however, Emmett reported that laser operation at more than 1000 pulses/sec had been achieved in Canada.

Because economics will exert a strong influence on the future development of laser propulsion systems (see Section II-B), the efficiency of energy production by this means will be an important, if not dominant, factor. It is difficult to deal with this subject, however, because there are so many types of laser systems, and because efficiencies are not stated in a consistent way. Indeed, it is difficult to compare efficiencies among different laser systems.
Fig. 2. Average power output of lasers through 1971, adapted from Eleccion[37]
because input or "available" power is not known directly, e.g., purely chemical lasers, because of geometrical differences, and problems of comparing flowing, nonflowing, open, and closed-cycle systems. Quoted efficiencies must be screened carefully with respect to their definitions which, often, are vague.

Many values of efficiency quoted in the literature refer to just the laser itself, and sometimes are called "electrical", "thermal", or "chemical" efficiencies. But more important, the efficiency of the power source and, in some cases, the output optics, should be included in an "overall" efficiency. For propulsion, power transmission and other applications, the only really meaningful efficiency is the "system" efficiency, which includes the efficiency of the optics/transmitting system, attenuation/dispersion, etc., of the media (atmosphere) where applicable, and the efficiency of the receiver/converter. The receiver/converter efficiency may be highly variable depending on the type of system, e.g., directly heated propellant, silicon-cell receivers for transmitted power, and so on. System efficiency will be discussed in a later section. For the present purposes this discussion will be confined to laser efficiency and/or overall efficiency.

The highest efficiencies have been obtained for semiconductor lasers. As much as 70% of the input electrical power has been converted to laser radiation in these devices;\(^3\) they are, however, low power devices. Solid-state devices yielded laser efficiencies of 30% in 1968,\(^3\) and probably have been improved considerably since then. The overall efficiencies of solid lasers are low, however, because of the generally low efficiency of the optical pumping system.

A 15% laser efficiency for converting heated gas to laser radiation was achieved by Gerry\(^3\) utilizing an open-cycle, combustion-driven CO\(_2\) gas dynamic laser (GDL). Pulsed gas-lasers have developed laser efficiencies ranging from 5 to 25% at one atmosphere pressure.\(^3\) In principle, closed-cycle operation should improve GDL efficiencies but the supersonic diffusers required are not highly efficient so that predicted results are not as good as might be expected. Tulip and Seguin\(^3\) have predicted theoretical laser efficiencies of 25% for thermally-driven, closed-cycle, CO\(_2\) gas lasers.
In contrast, the maximum (quantum) efficiency to be expected of CO₂ lasers is 40%. This upper limit is due to inability to extract energy from the rotational molecular states of CO₂. When the efficiency of the power source is considered, the overall efficiency of the GDL is rather low; values of 0.5 to 1% are quoted by Klass for open-cycle operation.

An improvement has been possible in electric discharge lasers (EDL); 24% laser efficiency has been achieved by the Air Force for continuous wave EDL gas lasers, and 30% is expected in the near future. Overall efficiencies are of the order of 10%, much better than for the thermally-driven GDL. Plummer and Glowacki have predicted laser efficiencies exceeding 50% for the CO supersonic EDL, where efficiency is defined as the ratio of output optical power to input electrical power.

The "chemical" efficiency of purely chemical lasers is the order of 5%, but values as large as 20% are anticipated. An overall chemical efficiency of 10% has been achieved in a chemical GDL. By itself the chemical laser is fundamentally simple, light, and compact, but the associated support equipment is bulky and heavy.

To close this discussion on laser performance it is of interest to list the "best" performance results achieved in various laser fields in 1971, as given by Emmett. Although these values have probably been exceeded since then, the list will serve as a milestone summary.

**Laser Performance: Best Reported Results (1971)**

- Average Power: 60 kW at 10.6 microns (multi-mode)
- Peak Power: 2.5 x 10¹³ watts at 1.06 microns
- Conversion of Chemical Energy: 4 to 5% in purely chemical laser
- Wavelength Range: Submillimeter to 1523 angstroms
- Tunability: Many contiguous ranges from 3410 angstroms to 3.5 microns; some tunable ranges at longer wavelengths
3. Gas Lasers

The waste power in lasers, i.e., that portion not converted to useful laser radiation, must be removed by cooling, which depends on the thermal conduction or diffusion properties of the laser material. In solids there are obvious limitations. In fluids, great advantage may be gained by forced convection or high-speed flow. In general, gas lasers are high-power devices that are more suitable than solid lasers for continuous wave operation and are amenable to a greater variety of pumping or excitation methods. Versatility in performance is achieved by varying parameters such as pressure and temperature and, in flowing systems, the mass rate of flow. A wide range of wavelengths already has been achieved (Fig. 2), and there is expectation that this range will be extended further. As stated earlier, gas lasers are the prime candidates for military applications, particularly thermal weapons. (33)

Gas lasers may be operated without flow, e.g., in sealed tubes, or with flow. Examples of the former are the electric discharge laser (EDL) or the electron beam laser (E-beam laser), which are particularly well suited to high-energy short bursts as typified by the Avco "big-bang" concept. (See references 33, 38, 52.) The term "gas dynamic laser" (GDL), which originally was applied to a thermally-driven, flowing-gas laser, has become a general term because GDL systems now exist that utilize electrical, chemical, or thermal pumping. It is becoming common to preface "GDL" by a term describing the pumping or excitation, e.g., thermally-driven GDL. Gerry (35) points out that the application of flow to various gas lasers varies with the pumping method. All three pumping methods mentioned above rely on flow for removal of waste energy. But, in addition, chemically-pumped GDL utilize flow for temperature control, mixing, and reactant replenishment, and thermally-pumped GDL utilize flow for the production of gas inversion from equilibrated hot gas.

Current GDL are open-cycle devices, and most of them utilize gases expanded through nozzles at supersonic velocities. If operated for extended time intervals, these devices require a significant fuel storage. Additional information on GDL devices may be obtained from references 33, 36, 37, 50.
Conceptually, higher efficiencies could be obtained by closed-cycle operation. In this case, however, diffusers are required for pressure recovery. Aerodynamic factors affecting GDL design have been discussed by Director. At low environmental pressures, such as high-altitude or space, diffusers would no longer be required. Theoretical analyses pertaining to closed-cycle GDL systems are available.

The first reported work on high-pressure, pulsed gas lasers appeared in 1969-1970, according to Wood. Transversely excited atmospheric-pressure lasers are now called TEA lasers, and form a fast-growing field of research. High gas pressure is beneficial because laser power output is roughly proportional to the square of the pressure, at least up to 4 atmospheres. Also, high pressure would tend to alleviate the diffuser problem. Pressures up to 10 or even a 100 atmospheres are envisioned. A problem is to generate uniform discharge; one approach to this problem is the so-called electro-ionization (ELION) laser. High pressure also tends to enhance the prospect of tunability in some gas lasers. An excellent and current review of high-pressure pulsed lasers has been published by Wood.

To quote Wood, "Today, the most successful lasers built based upon the high-pressure concept exhibit the following properties: large bandwidths (leading to the possibility of continuous wavelength tunability and ultrashort pulse generation), high peak-power outputs, large pulse energies, high efficiencies, low construction and operating costs, and outputs spanning wavelengths from the vacuum ultraviolet to the far infrared." High-pressure operation has been achieved in nearly 30 different gases. The peak power achieved in pulsed molecular lasers has been plotted in Fig. 3; data was adapted from Wood. Wood lists many applications for these lasers, including space propulsion and nuclear fusion, but does not mention thermal weapons or other military uses. These latter areas are of interest because the problems closely parallel those anticipated for propulsion.

According to Klass, the leading contenders for high-power laser weapons are (1) the thermally-pumped gas dynamic laser (GDL) - highest power levels (200 kW). (2) the electric discharge laser (EDL) - second highest...
Fig. 3. Observed performance of pulsed molecular lasers during years 1971-1973 adapted from Wood (38)
power levels (100 kW), and (3) chemical lasers - power levels now approaching 10 kW. In a more recent article, Klass\(^{57}\) reports that the U. S. Navy has now shifted its research interests to chemical lasers. The main reason for this is that chemical lasers can operate in the 2.6 to 5.0 micron range, where relatively little absorption by the atmosphere or moisture over the ocean surface occurs (shipboard operation). Additional information on chemical lasers may be found in references 58-61.

D. PROBLEM AREAS

Considerations for laser propulsion begin with the power source and the pumping/excitation energy for the laser(s) and end with the laser target and/or receiver/converter located on the appropriate vehicle. In some instances, e.g., proposed laser-fusion propulsion concepts, the entire system is contained on or within a spacecraft launched by other means. Although such concepts eliminate some problems, e.g., atmospheric interactions, they introduce others, e.g., a self-contained energy/power source is required. Because laser propulsion systems are concepts of the future, little work has been done to clarify subsystem problems or systems integration problems relative to the design of ground installations and space vehicles, or to mission analysis. Aside from this, many problems, obvious or anticipated, arise from consideration of the various subsystems. In some instances work relative to other areas of interest may be utilized in this regard. As an example, satellite ranging work\(^{62}\) is useful as adjunct information on atmospheric problems.

It is difficult to discuss problems associated with laser propulsion except by means of a generalized approach. Although many problems are common to all concepts, there must be specialized treatment according to mission, or when considering such diverse schemes as direct propellant heating, laser-fusion, or laser-electric systems. Vital to all systems is the development of adequate energy flux levels. Arno, et al.,\(^{28}\) lists the following factors that influence the flux level: laser wavelength and power level, optics quality and size, pointing (and tracking) accuracy, beam attenuation and dispersion, and distance dilution (flux levels vary inversely with the square of the distance from the source). Most problems of laser propulsion systems arise directly or in-
Table 3. General problems associated with high-power laser systems
for propulsion

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Present Limitations</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy/Power Source</td>
<td>Future availability for projected needs is difficult to anticipate or plan. Power source development for laser propulsion alone not economical.</td>
<td>Availability of existing power systems</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Operational efficiency and cost (including auxiliary facilities)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Site/location</td>
</tr>
</tbody>
</table>
| Lasers and Laser Systems   | 1. Low conversion efficiency  
2. Power levels still too low  
3. Practical wavelength range: 0.03 μm < λ < 20 μm  
4. Nonuniform excitation in high-power lasers causes optical scattering  
5. Trace impurities in lasing materials may limit power level due to self-focusing or local breakdown  
6. High gain hard to achieve at high power levels  
7. Ultra-short pulses at high power have not yet been achieved  
8. Multiple laser focusing difficult  
9. Present research diagnostics not sufficient (time/space resolution)  
10. Present auxiliary equipment is heavy and bulky, lack of development  
11. Fuel toxicity/storage/handling (some chemical lasers toxic)                                                                                     | 1. Greatly improved efficiencies needed  
2. Power scale-up (feasibility probable)  
3. Lower wavelengths desirable for improved range/focusing, optics problems  
4. Uniform excitation (large lasers) is hard to achieve, particularly gas lasers  
5. Improved lasing materials requires research/development, types: solid, liquid, gas  
6. Superradiance causes amplification of stray signals  
7. Research required to develop ultra-short, shape-controlled pulses (fusion)  
8. High accuracy required for fusion  
9. Diagnostics for ultra-short pulses for laser research needed  
10. Development of small lightweight systems needed for onboard propulsion  
11. Continuous operation of lasers in space requires efficient storage/handling of fuels/propellants                                                                               |
| Optics and Optics Systems  | 1. Size currently limited to about 1 meter but 10 meters considered possible - large optics tend to be bulky/heavy, causing pointing/tracking problems  
2. Fraction of wavelength accuracy is possible at 1 meter diameter. Fabrication difficulties limit large optics  
3. Current upper bound of 100 MW with 10 meter optics limited by materials heating, which increases with decreasing λ (current lower bound is λ = 0.3 μm)  
4. Materials performance limited by current reflectivity, transmissivity, and absorption for windows, lenses, and mirrors  
5. Systems integration studies lacking. In space, cooling systems and/or radiators may be required for optics systems in some applications                                                                 | 1. Large optics (10 to 100 meters) may be needed. Weight reductions needed for large, high-quality optics. Problems include bending stresses and thermal distortion. |
<p>|                            |                                                                                                                                                                                                                      | 2. High accuracy needed, becomes more difficult with decreasing wavelength (far UV, X-ray)   |
|                            |                                                                                                                                                                                                                      | 3. Higher power densities desirable will require improved materials and design. Optical non-linearities occur in many materials at high power densities |
|                            |                                                                                                                                                                                                                      | 4. Research needed for materials of greater purity and improved performance, particularly at low wavelengths |
|                            |                                                                                                                                                                                                                      | 5. Long-term dimensional stability thermal degradation of surfaces, uneven solar heating, accurate movement of large optics systems |</p>
<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Present Limitations</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pointing and Tracking</td>
<td>Divergence of $10^{-5}$ rad demonstrated in lunar ranging. Pointing accuracy of $2 \times 10^{-8}$ rad achieved in Orbiting Astronomical Observatory. NASA 1985 goal for pointing is $5 \times 10^{-8}$ rad (MOT program).</td>
<td>Total angular uncertainty (including beam divergence and atmospheric disturbances) of $10^{-8}$ to $10^{-6}$ rad will be required in earth orbit. Advanced phased arrays of lasers will require $\sim 3 \times 10^{-11}$ rad accuracy.</td>
</tr>
<tr>
<td>Transmitting Medium (Atmosphere)</td>
<td>Current state of art not yet well developed. Treatment difficult because effects may be localized, but must be integrated over path length in space and, possibly, time. Knowledge limited by difficulty of modeling large beam, high power experiments in the atmosphere. There are beam size and energy flux level effects.</td>
<td>Problems include: 1) Effects of winds, humidity, lapse rate (temperature), density changes, particulate matter (liquid, solid), and turbulence. 2) Dispersion, refraction, scintillation. 3) Slant paths/oblique angles, line of sight, horizon approach. 4) Scattering, e.g., Rayleigh, stimulated-Raman, stimulated-Brillouin. 5) Laser interactions such as beam steering, thermal blooming, atmospheric heating/breakdown, thermal defocusing.</td>
</tr>
<tr>
<td>Target Interactions</td>
<td>Studies of laser/target interactions are in beginning stages. Range of phenomena difficult to explain/classify, there are numerous variables. Modeling difficulties are apparent.</td>
<td>Surface and bulk damage to optics elements, especially for CW operation. Plasma formation/absorption (beam blocking). Optimization pulsing for heating/cooling and vaporization of propellants. Intense/precise focusing for fusion.</td>
</tr>
<tr>
<td>Receiver/Converter</td>
<td>No developments to date for large-scale laser systems (paper studies only). May require breakthroughs beyond current state of art.</td>
<td>Matching with laser systems, low efficiency, large size, thermal and space-related degradation, mass, radiator design for waste heat removal, simulation/testing.</td>
</tr>
</tbody>
</table>

ORIGINAL PAGE IS OF POOR QUALITY.
directly from these factors. Information on general and specific problems has been gleaned from references 5, 6, 21, 23, 28, 29, 33, 36, 33, 58, and 63. A reasonable classification may then be made according to the various subsystems involved, as shown in Table 3. The remainder of this section will be devoted mainly to lasers and optics; problems related to transmission, atmospheric disturbances, and reception will be discussed briefly in subsequent sections.

1. Power Sources

If it is assumed that the use of high-power lasers for propulsion purposes would be too infrequent to justify special power sources for their activation, then existing terrestrial power sources are the alternative. These may be commercial electric or nuclear power generation facilities. Thus, a ground-based laser facility would require location in close proximity to a large power station or power grid. Kantrowitz (5) estimated that approximately 1 GW of beam power on target would be required to place a 1-ton payload in low earth-orbit. For comparison, Arno, et al., (28) rate the output of the Grand Coulee Dam as 1.9 GW. In this case, an overall efficiency (of laser-delivered power) of approximately 50% would be needed, a value far too optimistic. Rom and Putre (6) assume that overall efficiencies of converting ground-based electrical energy to heat onboard a vehicle may be in the range of 10 to 50%; these values are not consistent with the current state of the art and are much too high. The useful energy output of the receiver/converter is more likely to be much less than 1% of the laser input energy, even ignoring the efficiency of the power source. This is due to the rather low efficiencies of the laser/optics/transmission system, as well as receiver/converter efficiency.

2. Lasers

Because lasers have been the subject of extensive research and development, many problems related to propulsion already may be anticipated. Increasing laser output is not simply a matter of increasing their size, because lasers are difficult to scale. This is due to the fact that the combination of physical phenomena involved does not usually scale linearly, and also because
spatial inhomogeneities may become more critical with increasing size. In addition, preparation of large lasing elements, e.g., solid rods, without imperfections is difficult. Emmett\(^{36}\) states that two of the most significant obstacles to achieving high average power are low conversion efficiency and nonuniformity of excitation. It becomes more difficult to achieve uniform excitation as the size is increased, particularly in the case of large gas lasers; nonuniformities cause optical scattering and power loss. In molecular gas lasers, optical breakdown of gases at high power density may cause uncontrolled oscillations, according to Wood\(^{(38)}\). Other problems are listed in Table 3.

Lasers for induced fusion have special problems of their own because they require ultra-short, high-energy pulses.\(^{(20, 21, 47, 48)}\) If multiple laser beams are used, the beams must be accurately synchronized, and accurately focused on a very small volume; the width and shapes of the pulses must be carefully controlled to avoid destructive influences.\(^{(21)}\) It has just been reported\(^{(64)}\) that kilojoule pulses for fusion research have been achieved in France, and by Sandia and Los Alamos in the United States. The French achieved this level in 100 picoseconds, yielding a brief power of about 10 terawatts. Soon it may be possible to test the prediction of X-ray energy from short pulses as suggested by Whitney and Davis.\(^{(49)}\) As laser pulses become increasingly shorter, there will be a corresponding need for diagnostics with sufficient time resolution and increased capabilities in the X-ray, ultraviolet, and infrared wavelength regions.\(^{(47)}\) To requote reference 47, "The major long-range difficulty in developing a laser-fusion power reactor will be an array of engineering problems that will make the physics problem look trivial."

Although not directly relevant to the development of lasers and laser propulsion systems, the effects of laser radiation on the human eye are pertinent to safety at ground facilities as well as to manned missions. A comprehensive treatment of this subject was given by Makous and Gould.\(^{(65)}\) They list five possible effects on the human eye: (1) thermal (heating), (2) electric field, (3) electrostrictive stress, (4) radiation pressure, and (5) photochemical effects. Of course, high-energy lasers also may cause damage to other tissues besides the eye. The extent of damage due to the five effects depends on wavelength, energy level, and duration of exposure. A large number of cases are possible. Theoretical damage thresholds for a 25 \(\mu\)sec laser pulse may occur.
in the range 0.2 to 8.4 ergs, depending on various conditions. It is surprising that a kilojoule pulse directed into the eye has sufficient momentum to impart a speed of 4000 km/sec to absorbing tissues having the density of water. (65)

3. Optics

Further developments in high-power lasers are being hampered by problems related to optics and optics systems for transmitting and receiving. Optics elements include windows (for transmission), lenses (for focusing and beam adjustment), and mirrors. Considerations for laser propulsion introduce additional problems. A brief discussion of optics problems related to orbital laser power-stations was given by Hansen and Lee; (23) some optics problems are listed here in Table 3. Aside from cost, the main optics problems for laser space applications fall into one of the following categories:

1) Design and performance.
2) Fabrication and quality control.
3) Materials development.
4) Laser/optics interactions.
5) Testing methods.
6) Long-duration performance.

Many problems are related to size, because optics of the order of 10 meters or larger are anticipated for space applications. The present state of the art is certainly 1 meter, with 10 meters considered feasible; optics of the order of 100 meters would require several technological breakthroughs. Only a brief discussion of problems can be presented here. The scope of problems is large indeed, but the reader may acquire an insight into the field from a collection of 83 papers published by NASA in 1970. (66)

The design and fabrication of large optics with surface accuracy to a fraction of a wavelength, and acceptable surface roughness, is a challenging problem. Large optics tend to be bulky and heavy, which magnifies problems associated with bending and thermal stresses, mounting, and mobility required by pointing and tracking, and tracking response. Quality control is important with respect to surface accuracy but also to materials uniformity, i.e., highly
homogeneous materials without minute voids, inclusions, or gradients in physical properties are desirable. New and improved materials are needed for wavelength applications outside the visible, e.g., X-ray, ultraviolet, and infrared. Fabrication of large metallic mirrors has been improved through the use of electroforming, (67) which permits high accuracy, decreased weight, and reduced time and cost of fabrication. General Electric has produced 3 meter, and larger, mirrors using the process. Glass technology has been improved significantly through the impetus of laser development. (68)

The interaction of high-power laser beams with optical components produces local heating and may lead to nonlinear optics (NOL) phenomena, e.g. (see reference 69), various scattering effects, and damage, degradation, or destruction of surfaces or bulk materials. Thermal or mechanical stress concentrations may lead to optical distortion, fracture, or local melting in materials subjected to pulsed lasers. Bulk and surface damage has been discussed by several investigators. (70, 71, 72) Damage mechanisms include those due to particulate inclusions (high local stresses), thermal stress concentrations, self-focusing (nonlinear dependence between refractive index and local electric field), and electron avalanche (caused by self-focusing). Experiments have been performed on a variety of materials and typical power fluxes that lead to breakdown in optical materials have been published. (70, 72) If the duration of ultra-short pulses is of the order of the acoustic wave propagation, thermally-induced plane-wave stresses may occur in window materials. (73) All of the problems discussed have some dependence on wavelength, power flux level, and duration of exposure to laser radiation.

Other areas of concern for large optics include means for assessing materials' homogeneity, polishing and testing, and dimensional stability. Dimensional stability of optics on the ground is important and is influenced by atmospheric interactions; in space, long-duration dimensional stability may be crucial. Nonthermal factors that affect dimensional stability include (1) relaxation of residual stresses, (2) phase changes, (3) anisotropic properties, (4) chemical gradients, (5) physical property changes, and (6) in-orbit or space changes in physical properties. (74, 75) Radiation forces produce bending moments as well. (23)
E. PROBLEMS ASSOCIATED WITH POWER TRANSMISSION

In this section, some problems associated with beam divergence, pointing and tracking, atmospheric influences, and laser-target interactions will be discussed briefly.

1. Laser Beam Divergence

Even if optics, and pointing and tracking are perfect, laser beams diverge slightly because the optics are diffraction-limited.\(^{(23)}\) The diffraction of a parallel beam of radiation of wavelength \(\lambda\) by a circular aperture of diameter \(d\) scatters the radiation through a half-angle \(\theta = 1.22 \lambda / d\), where the constant 1.22 arises from the mathematics of Fraunhofer diffraction. Because real (especially large) optics cannot be fabricated to approach the diffraction limit, many authors\(^{(5, 23)}\) use a larger value, such as 2. If \(D\) is the diameter of the receiver, then the range \(R\) at which focusing is accomplished is approximately \(R = dD / 2\lambda\). The range may be increased by increasing the receiver and transmitter diameter, or decreasing the wavelength. At distances larger than \(R\), the beam power density decreases inversely as \(R^2\). It is to be noted that the diffraction angle of one beam in a phased array of \(N\) lasers is reduced by a factor of \(N\).\(^{(23)}\)

Laser beam divergence in previous lunar ranging experiments has been estimated variously as \(2 \times 10^{-6}\) to \(10^{-5}\) radian.\(^{(6, 23)}\) Rom and Putre\(^{(6)}\) estimate that a beam divergence of about \(5 \times 10^{-6}\) radian would be acceptable for orbital systems at 1000 km with 10 meter optics. To maximize the power reception of a vehicle with fixed receiver size moving away from earth, it may be necessary to control beam divergence as a function of time. Techniques of the type suggested by Massey\(^{(76)}\) may be useful in this regard.

2. Pointing and Tracking

Pointing and tracking precision of \(5 \times 10^{-6}\) radian has been demonstrated in earth-based astronomical telescopes\(^{(23)}\) despite the fact that expected dispersion due to atmospheric disturbance is of the order of \(10^{-5}\) radian.\(^{(5, 23)}\)
In various balloon experiments pointing accuracies in the range $4 \times 10^{-8}$ to $2 \times 10^{-5}$ radian have been achieved.\(^{(23, 77)}\) The Orbiting Astronomical Observatory (OAO) has achieved a pointing accuracy of $2 \times 10^{-7}$ to $2 \times 10^{-6}$ radian.\(^{(23)}\) NASA programs will require tracking accuracies of the order of $10^{-8}$ radian by 1985.

Pointing and tracking requirements for laser propulsion are difficult to anticipate and, of course, are dependent on the system and mission. Hansen and Lee\(^{(23)}\) suggest that pointing accuracy should be half the diffraction limit. Transmission through the atmosphere will have great impact on pointing and tracking, and tracking response accuracies for any laser propulsion system where such transmission is required. Minovitch\(^{(7)}\) has suggested Western mountain-top sites for laser transmitters. It appears that many laser applications will require overall accuracies of the order of $10^{-8}$ radian, or better, which is the NASA 1985 goal.

An insight into potential requirements is afforded with reference to some simple calculations by Arno, et al.,\(^{(28)}\) who have calculated required laser power fluxes as a function of mission distance if the combined diffraction and pointing accuracy is $10^{-7}$ radian. Their result, based on the assumption that beam energy is uniformly distributed over the target spot, is reproduced in Fig. 4.

Figure 4 indicates that present optics will limit laser system use to roughly earth synchronous orbit.

Lee and Hansen\(^{(23)}\) point out that limits are imposed on pointing and tracking by the uncertainty principle, i.e., if $h$ is Planck's constant, $h < I \omega \delta$, where $I$ is polar moment of inertia of a tracking element with residual rotational speed $\omega$, and $\delta$ is the allowable angular uncertainty. It appears that requirements for precision phased arrays in the future may approach this limit.
Fig. 4. Power flux and application regimes (reproduced from Arno, et al.)

3. Atmospheric Disturbances

Various conditions in the atmosphere may lead to the attenuation, refraction, dispersion, etc., of laser beams. These conditions may be classified roughly into two groups: (1) disturbances that occur naturally and are present irrespective of any laser beams and (2) disturbances that occur because of atmospheric interaction with a laser beam. Because this is a complex, relatively new, and poorly understood field at present, only token discussion is possible.
a. Natural Disturbances

Because the atmosphere contains varying temperature and density gradients, winds, turbulence, humidity, and particulate matter, it is capable of producing a wide variety of optical effects. A typical example is scintillation (a twinkling effect), which is caused by the presence of variable density regions that result from turbulence or moving stratified layers crossing an optical line-of-sight. Recent satellite experiments\(^{(76)}\) confirm that scintillation at orbital distances is within the limits measured for stellar scintillation. Atmospheric absorption may cause a loss of beam power of the order of \(15\%\).\(^{(6)}\) Charts for atmospheric transmissivity as a function of wavelength are given in reference 79, and a table of atmospheric "windows" has been given by Minovitch.\(^{(7)}\) If signal velocity is an important factor, then two regions of the atmosphere must be considered, according to Hopfield:\(^{(62)}\) (1) the un-ionized part (troposphere and stratosphere) and (2) the ionized part (ionosphere). Signal velocity will vary differently in these two regions.

Atmospheric turbulence is difficult to treat because of its random character and variations in its scale and intensity; it may cause attenuation and dispersion. Several analyses,\(^{(80, 81, 82)}\) and some experimental data\(^{(82, 83)}\) are available. From the standpoint of focusing, Bakker and Vriend\(^{(81)}\) find that favorable weather conditions for laser transmission include (1) cloudy or overcast weather, (2) light or no winds, and (3) during, or just after, light rainfall. Presumably these conditions tend to favor reduced turbulence; however, water vapor introduces absorption problems. Dowling\(^{(83)}\) suggests a first-order correction for beam divergence due to turbulence over a path length \(L\) such that \(\theta^2 = \theta_0^2 + (K C_n)^2 L\), where \(\theta\) is total divergence, \(\theta_0\) is divergence in a vacuum, \(K\) is a proportionality constant, and \(C_n\) is a structure constant. Fine-scale turbulence may cause scattering, which also may result from water vapor and/or particulate matter.

Few studies have been made of high-power laser beams over long paths, and most of them were performed at ground level. Recently, Mason and Lindberg\(^{(84)}\) studied the propagation of a small 15 mW He-Ne laser over an 80 km path high above the desert floor in New Mexico; experiments were conducted over a period of hours, including day and night. Beam deflections of as
much as 20 meters were observed which, in this case, amounts to deflection angles of as much as $2.5 \times 10^{-3}$ radian. This is one type of beam steering. In general, most laser applications will require beams directed along slant paths through the atmosphere. Not only does this introduce greater atmospheric absorption because of a longer path length, but the probability of increased refraction and nonhomogeneities is also increased.

b. Laser-Atmosphere Interactions

As yet, there is little experience with high-power laser beam propagation through the atmosphere. On a laboratory scale, it has been shown by Chodako and Lin\textsuperscript{85} that modest laser power may induce turbulence in an otherwise laminar atmosphere, and this is accomplished at surprisingly low Reynolds numbers. This is but one effect of gas/atmospheric heating. Instabilities associated with optical-acoustic coupling of a laser beam in an absorbing gas have been discussed by Brueckner and Jorna.\textsuperscript{86} Localized heating may cause thermal defocusing, which is divergence resulting from changes in refractive index transverse to the beam center line.

Various theoretical studies of nonlinear thermal processes in the atmosphere resulting from laser beam propagation are available, e.g., see references 87 and 88. Such thermal processes depend on optics, acoustics, thermodynamics, fluid dynamics, and electromagnetic theory; they are very complex. Various regimes are described according to whether cooling processes are dominated by conduction, free convection, or forced convection. The theoretical results of Bissonnette\textsuperscript{88} have been replotted here in Fig. 5. According to Bissonnette the threshold below which thermal distortion is practically negligible corresponds to a Fresnel number $F = 1$ (indicated in Fig. 5).

So-called thermal blooming occurs when the atmosphere behaves as a lens, which may defocus and disperse the beam. Thermal blooming is not a direct attenuation effect but rather an energy dispersion effect; it is considered a serious obstacle to thermal weapons development.\textsuperscript{83} According to Bradley and Herrmann\textsuperscript{89} thermal blooming is different according to whether the laser
Fig. 5. Various regimes of horizontal propagation in a quiescent atmosphere at sea level of a CW CO₂ laser beam. Redrawn from Bissonnette. 

\( t_d, t_g = \text{time constants for conduction and free convection respectively} \)

\( F = \text{Fresnel number} \)
is operating in a continuous wave or a pulse mode. For CW operation the heated atmosphere behaves like a thick lens with aberrations, but in pulsed operation the lens is thin and located near the focus. A means of compensation has been suggested. Pirri and Weiss state that thermal blooming is not a problem when laser pulses are less than $10^{-4}$ sec. According to Klass, thermal distortion is influenced by many factors including atmospheric optical properties, the range and power of the beam, beam diameter, strength of focusing, and winds.

If laser beam intensity becomes sufficiently large, gas breakdown and the production of plasma occurs. This leads to beam blocking; if regions of totally ionized gas are formed, up to 100% of the beam energy may be absorbed. Plasma, and radiation driven shock waves may propagate up the laser beam toward its source. Obviously, these phenomena would be unacceptable for both thermal weapons applications and earth-based laser systems for propulsion. In clean, dust-free air, a threshold power flux for breakdown at atmospheric pressure is of the order of 1 GW/cm$^2$, or 1 to 10 GW/cm$^2$ for pulsed lasers, depending on pulse duration. Although these flux levels would appear to be safe for most laser propulsion applications, it is noted that much lower values occur when particulate matter is present in the air. Presumably, lower breakdown values would occur at high altitude where the air density is very low. In addition, Klass reports that threshold breakdown is reduced as the beam size increases, by as much as two orders of magnitude, but it is also true that plasmas can be cleared out by substantial cross winds.

The presence of particulate matter in air can cause optical scattering, e.g., Rayleigh scattering. Outside the earth's atmosphere, in solar space, some laser beam attenuation from Rayleigh scattering may be expected due to residual matter, but this has been shown to be a negligible effect. Other forms of scattering include stimulated-Raman scattering and stimulated-Brillouin scattering.
4. Laser-Target Interactions

Laser-target interactions are to be expected whenever the laser beam is intercepted by an object or a substance. Excluding the atmosphere, which already has been discussed and which is not usually an intentional target except in instances when energy extraction from plasma is sought, there are numerous targets to consider. These may vary from optics and receiver/collector surfaces, to propellants situated to produce heating and/or vaporization. A summary of possibilities is given in Table 4. Current research appears to be concentrated in three areas: (1) surface damage to optics, (70-73) (2) momentum transfer and thrust from solid propellants, (8-11) and (3) basic studies on various solid targets in vacuum and in different gaseous atmospheres. (12-14, 63, 91) Presumably, a great deal of military research is being conducted on the behavior of candidate targets exposed to high-power laser beams; much of that work is classified. Beam blocking and thermal blooming are important aspects of categories (2) and (3).

F. ENERGY CONVERSION

All laser propulsion concepts that do not directly utilize the energy of an impinging laser beam for impulse (laser-powered sail), or propellant heating to produce thrust, will require some form of energy conversion. In the case of laser-electric propulsion, (28, 29) laser radiation could be used to augment or replace solar radiation for lunar or near-earth missions; this energy would then be converted to electricity for an electric propulsion system. If the transmitter is earth-based, then atmospheric transmission problems in pointing and tracking must be compensated. If, on the other hand, the power station is located in earth-orbit as suggested by Hansen and Lee, (23) then atmospheric problems are avoided but the laser power-system design becomes more complex due to size and weight limitations. Energy for an orbital laser transmitter could be derived from the sun, e.g., sun-pumped solid laser or a self-contained nuclear source.
Table 4. Laser/target interactions

<table>
<thead>
<tr>
<th>Item</th>
<th>Target</th>
<th>Results of Interactions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optics</td>
<td>Windows, lenses, mirrors</td>
<td>Surface/bulk degradation and damage nonlinear optics</td>
</tr>
<tr>
<td>Propellants</td>
<td>1. Gas (seeded)</td>
<td>1. Heating</td>
</tr>
<tr>
<td></td>
<td>2. Solid</td>
<td>2. Heating, vaporization and/or detonation</td>
</tr>
<tr>
<td></td>
<td>3. Pellets (fusion)</td>
<td>3. Rapid heating/implosion</td>
</tr>
<tr>
<td>Collectors</td>
<td>Spaceborne mirrors</td>
<td>Surface damage/degradation</td>
</tr>
<tr>
<td>Receivers/Converters</td>
<td></td>
<td>Surface damage/degradation</td>
</tr>
<tr>
<td>Weapons Systems</td>
<td>Military vehicles, aircraft</td>
<td>Damage/destruction</td>
</tr>
<tr>
<td></td>
<td>and/or missile surfaces and</td>
<td></td>
</tr>
<tr>
<td></td>
<td>subsystems</td>
<td></td>
</tr>
</tbody>
</table>

Recently Shimada of JPL\(^{(92)}\) summarized potential laser converters, reproduced here in Table 5. Of the various concepts, the state of the art is most advanced for photovoltaic and thermionic/thermoelectric devices. The quantum efficiency of most photodiode materials has a strong wavelength dependence that peaks sharply in some cases.\(^{(28)}\) Silicon cells, for example, have a maximum quantum efficiency approaching 70% at 0.8 to 0.85 \(\mu\)m.\(^{(23, 28)}\) Even higher efficiencies may be obtained at lower wavelengths (about 0.6 \(\mu\)m) by means of antireflective coatings.\(^{(28)}\) Nakamura\(^{(29)}\) has pointed out that silicon cells begin to exhibit nonlinear properties when the incident power flux approaches 100 MW/cm\(^2\), which exceeds solar flux outside the earth's atmosphere by a factor of more than seven. Unfortunately, there does not seem to be a current laser that operates near the wavelength of peak efficiency for silicon cells. Thermionic and thermoelectric systems may operate at 10.6 \(\mu\)m (Table 5), but not at very high conversion efficiency. However, CdHgTe photovoltaic material may operate at 10.6 \(\mu\)m with efficiencies as high as 70%.\(^{(23)}\) The CO\(_2\) gas laser operates at 10.6 \(\mu\)m.
Table 5. Laser converters, after Shimada, JPL, 1974 (92)

<table>
<thead>
<tr>
<th>Device</th>
<th>Principle</th>
<th>Optimum Wavelength (μm)</th>
<th>Max. Input Power (W/cm²)</th>
<th>Max. Efficiency</th>
<th>Problem Areas</th>
<th>Technology Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Si solar cell</td>
<td>Photovoltaic</td>
<td>0.7 - 0.9</td>
<td>&lt;100</td>
<td>30</td>
<td>Surface recombination should be minimized</td>
<td>Now</td>
</tr>
<tr>
<td>2. GaAs junction cell</td>
<td>Photovoltaic</td>
<td>0.5 - 0.7</td>
<td>&lt;100</td>
<td>40</td>
<td></td>
<td>1977</td>
</tr>
<tr>
<td>3. Schottky barrier cell</td>
<td>Photovoltaic</td>
<td>0.3 - 0.6</td>
<td>&lt;100</td>
<td>30</td>
<td>Fabrication of increased active areas</td>
<td>1975</td>
</tr>
<tr>
<td>4. Photocell</td>
<td>Electron emission</td>
<td>~0.7</td>
<td>10²</td>
<td>30</td>
<td>Need electron collection with extremely low work function (&lt;1.0 eV)</td>
<td>1980</td>
</tr>
<tr>
<td>5. Laser plasma-dynamic</td>
<td>Ion emission</td>
<td>~0.3</td>
<td>10² - 10³</td>
<td>30</td>
<td>Ion yield is not known</td>
<td>1980</td>
</tr>
<tr>
<td>6. Thermionic</td>
<td>Electron emission</td>
<td>10.6</td>
<td>10² - 10³</td>
<td>15</td>
<td>Good heat absorber is required</td>
<td>Now</td>
</tr>
<tr>
<td>7. Thermoelectric</td>
<td>Seebeck Effect</td>
<td>10.6</td>
<td>10²</td>
<td>10</td>
<td>Good heat absorber is required</td>
<td>Now</td>
</tr>
<tr>
<td>8. Laser rectifier</td>
<td>Nonlinearity</td>
<td>&gt;1,000</td>
<td>10⁻¹²</td>
<td>80</td>
<td>Electronic circuit technology above 100 GHz is nonexistent</td>
<td>1985</td>
</tr>
<tr>
<td>9. Inverse laser</td>
<td>Quantum</td>
<td>?</td>
<td>10³</td>
<td>30</td>
<td>Means for efficient coupling of laser with working fluid is required</td>
<td>?</td>
</tr>
<tr>
<td>10. Turbine</td>
<td>Thermal</td>
<td>10.6</td>
<td>10³</td>
<td></td>
<td></td>
<td>1980</td>
</tr>
<tr>
<td>11. Controlled fusion</td>
<td>Nuclear</td>
<td>1.06</td>
<td>10³ W. peak</td>
<td>?</td>
<td></td>
<td>?</td>
</tr>
</tbody>
</table>
It is clear that much research and development will be required to produce laser conversion systems that can be optimized with respect to efficiency, wavelength, lifetime, size, and weight for laser propulsion purposes.

G. OPTIMIZATION STUDIES FOR APPLICATIONS

An important consideration in the development of future laser propulsion systems is the amount of laser radiation that can be delivered to a given target in a given interval of time. To maximize this delivery, two objectives must first be accomplished: (1) large lasers and power generation facilities must be developed that are both technically and economically feasible and (2) means for transmission of large high-power, well-collimated laser beams must be developed. To this end, many of the problems listed in Table 3 must be surmounted. It is anticipated by most experts that ever more wavelengths will become available, that the wavelength spectrum will be broadened (hopefully into the X-ray region), and that much higher power levels will become possible. It is likely that pulsed lasers will begin to show more and more advantages over continuous wave types, but much work is needed to develop pulses of optimal shape, duration, and repetition rate.

Pirri and Weiss\(^{(b)}\) raise the following technical questions regarding laser propulsion:

1) How is efficiency maximized with respect to pulse energy, duration and wavelength, ambient pressure, and working fluid?
2) How efficient are the energy conversion processes (absorption, heating, and expansion) basic to laser propulsion?
3) How efficiently may laser-propelled structures (vehicles/spacecraft) be designed?

The question of efficiency is important because it will, in part, determine cost. Most treatments of laser system efficiency are either vague or incomplete (Section II-C, Laser Performance) and do not include the efficiency of subsystem components, e.g., the power source or the optics. For example, Pirri
and Weiss\(^8\) define laser propulsion efficiency as the ratio of vehicle kinetic energy to utilized laser energy. Even with perfect energy conversion, they obtain a typical value of 60\% for propulsion efficiency.

A more realistic and sober examination of the facts, e.g., Nakamura\(^{29}\), does not give great cause for optimism. More recently, Shimada\(^{92}\) compared current technology with future goals for laser propulsion applications, reproduced here in Fig. 6. Overall (system) efficiencies are currently as much less than 1\% if the laser, and its power source, optics, and receiver/converter are considered. An increase of several orders of magnitude would be required to achieve Shimada's system efficiency goal of 10\% or greater, i.e., 10\% of the input power to a laser would be available for propulsion as the output of the receiver/converter.

It appears that definitive optimization studies for laser propulsion must await further developments and substantial advancements in the state of the art. Even so, it appears that more realistic analyses than previous treatments (for examples, see references 6, 7, 8, 18, 28) are possible at this time.

An important question has been raised by Pirri and Weiss\(^8\) in that further theoretical analyses on laser propulsion systems will require supportive experimental data in a number of areas. To accomplish this, experiments utilizing prototype models will be required under properly simulated conditions. Typical examples are the propagation of high-power laser beams through the atmosphere, and laser/target interactions. This is a difficult field because many physical phenomena associated with these processes do not scale readily, and even may be shifted to entirely different regimes. Thus, it will be difficult to simulate the interactions of large, pulsed laser beams with large targets on a laboratory scale. Because full-scale experiments will be costly and difficult to justify a priori, means must be found to accomplish meaningful model experiments.
CURRENT TECHNOLOGY – GOAL

- **Power Source**: 
  - Efficiency: $\eta \sim 30\%$
  - Life: 2-3 years

- **Laser**: 
  - Efficiency: $\eta \sim 10\%$
  - Cycle: Open

- **Optics**: 
  - Mass: $\sim 10M$
  - Efficiency: $\eta \sim 50\%$

- **R/C**: 
  - Efficiency: $\eta \sim 10\%$

**CURRENT SYSTEM EFFICIENCY**

- **Receiver/Converter**: 
  - Efficiency: $\sim \eta \ll 1\%$

**SYSTEM EFFICIENCY GOAL > 10\%**

- **Power Source**: 
  - Efficiency: $\sim 100\%$
  - Life: 5-10 years

- **Laser**: 
  - Efficiency: $\sim 40-60\%$

- **Optics**: 
  - Efficiency: $\sim 50M$
  - Cycle: Closed

**Fig. 6.** Subsystem and system efficiencies for laser propulsion, after Shimada, JPL, 1974(92).
H. SUMMARY AND CONCLUSIONS

The various concepts of laser propulsion that have been proposed in recent years have been described and discussed briefly; various applications are listed in Table 1 and Fig. 1. Although there has been little research and development directed specifically towards laser propulsion, an immense amount of work has been done on lasers themselves, and laser systems. Laser devices and performance have been reviewed here and a laser classification appears in Table 2. A true perspective of the potential for laser propulsion, and competition with other forms of advanced propulsion, can be achieved only by considering the entire system: the power source, the laser, optics, receiver/converter and, in some applications, atmospheric transmission and energy conversion. Unfortunately, such a perspective is impossible to achieve at this time because information is lacking in many areas. For example, much work has been devoted to energy conversion systems, particularly in the photovoltaic and thermionic areas, but none of this work has been directed specifically toward laser applications. The literature review given here is believed to contain all of the significant work in laser propulsion and, in other areas, is thought to represent current thought and progress.

Before laser propulsion becomes technically and economically practicable, it will be necessary to develop (1) large power sources (electric or nuclear), (2) large, high-power lasers that can be pulsed rapidly, and (3) means for transmission of large, well-collimated beams with highly accurate pointing and tracking. It is probably safe to assume that adequate optics can be developed in the future. Many general problems have been identified in Table 3. The development of large beam power levels appears feasible; if this proves to be marginal, the use of multiple lasers in phased arrays is a likely alternative.

A formidable obstacle to laser propulsion systems is that projected system efficiencies are undoubtedly much higher than could be achieved currently, e.g., see Fig. 6. Extremely accurate pointing and tracking systems will be required for at least two reasons: (1) failure to intercept a significant portion of the laser beam at the receiver will cause power loss and greatly reduce system efficiency even though the other subsystems are adequate and (2) a partial
miss could cause catastrophic damage to other vehicle parts if a beam of high-power flux were to impinge on areas other than the intended receiver. For long-range transmission, this may require a substantial physical separation between vehicle and receiver. In this regard, the difficulties inherent in direct propellant heating concepts are obvious.

Laser transmitters may be ground-based, located in earth-orbit, or carried aboard a vehicle. Because laser beam interactions with the atmosphere introduce a special set of problems (discussed in Section II-E), remarks concerning the feasibility of various propulsion concepts will be divided according to transmitter location.

1. Ground-Based Laser Transmitters

Launch to orbit by direct heating of suitable propellants to produce thrust has been discussed. This concept is not attractive and may be feasible only for quite small payloads. (28, 29) This concept appears better adapted to orbital insertion, station keeping, intraorbital maneuvers and, possibly, acceleration to escape velocity. At present the concept appears to be confined to modest power requirements. In addition to adequate pointing and tracking capability, adequate tracking response is a problem.

The concept of an orbiting relay station at first appears attractive. Reflector optics would be used to direct an earth-based laser beam toward other vehicles/spacecraft in orbit or near earth. The addition of yet another subsystem, however, may be questioned on the grounds of efficiency.

Atmospheric interaction problems may limit the usefulness of such concepts but may be overcome in part by using pulsed rather than continuous wave lasers. High response systems would be needed to compensate for changing beam paths and local, dynamic changes in atmospheric conditions.
2. Transmitters in Orbit or Space

An obvious solution to atmospheric propagation problems and uncertainties is to locate the laser transmitter outside the earth's atmosphere, e.g., see reference 23. The problem then will be to develop compact, high-power self-contained power sources for the laser(s). Conventional nuclear reactors might be used for this purpose. Another possibility is to utilize large solar collectors/converters, either for sun-pumped solid lasers, or for conversion to electrical power for the laser input. Laser output could then be beamed to other vehicles to heat propellants for thrust, to provide auxiliary electric power, or for laser-electric systems. As pointed out by Arno, et al., systems that require multiple energy conversion steps always suffer the consequence of reduced efficiency. Nakamura, et al., conclude that laser-electric systems, even for near-earth missions, would require pointing and tracking accuracies beyond the present technology.

Of interest, but not directly related to laser propulsion, is the orbital power station utilized for earth power consumption. Solar power unattenuated by the earth's atmosphere is collected and converted, then transmitted to earth either by microwave or laser beam. The microwave concept is well-developed and is considered feasible in a very recent analysis by Patha and Woodcock. The laser concept merits further attention because it has not been restudied recently.

Progress toward laser fusion is advancing rapidly. If successful, application to laser propulsion is an attractive alternative. Micropellets subjected to multiple laser irradiation are compressed rapidly and sequentially; implosion triggers the fusion reaction. A recent article by Emmett, Nuckolls, and Wood describes the theory, problems, and applications of laser fusion. It appears, however, that definitive and meaningful studies of propulsion by laser-induced fusion are not yet possible.
In summary, meaningful conclusions concerning laser propulsion would be premature. Laser propulsion literature is not extensive and suffers from some general deficiencies that arise partly from lack of data: (1) treatments are incomplete and not sufficiently comprehensive, (2) cost analyses are incomplete and often ignored, especially power source costs, (3) systems analyses and subsystem integration studies are incomplete, (4) incomplete mission analyses, and (5) lack of conceptual design with engineering detail. Thus, comparisons with competing propulsion systems are very difficult.

I. RECOMMENDATIONS

The following areas should be investigated: (1) mission and systems analysis, (2) thermal control (spacecraft), (3) receivers/converters, (4) pointing and tracking, and (5) lunar ranging. To initiate further studies, we need to develop a deeper and more complete understanding of current progress in high-power lasers and associated optics systems.

More comprehensive system/subsystem studies of the more promising laser propulsion concepts should be conducted. These studies include: (1) orbiting power stations, (2) laser-electric systems, and (3) laser-fusion systems. Modeling and simulation studies of laser-target interactions and/or laser-atmosphere interactions should also be investigated.
SECTION III

CHARACTERIZATION OF LASER PROPAGATION
AND COUPLED POINTING

A. A. Vetter

Laser energy has recently been seriously considered for use in propulsion systems that represent the entire spectrum of performance level requirements, from attitude control to launch booster. In many of the proposed schemes for the conversion of laser energy to propulsive thrust, the laser is an external power supply from which the energy must be transmitted to the user vehicle. The problems concerning the capabilities of transmission and pointing of the laser beams have been given only cursory analyses, with insufficient detail to actually determine where laser propulsion could be applicable to a specific mission. The analyses contained in this report are attempts to determine the important functional relationships of laser transmission system parameters to the average power impinging on an extraterrestrial spacecraft. The objective is to improve the accuracy of the laser transmission system model with regard to propagation and pointing effects, without relying on empirical formulations or complex numerical computations.

This report does not contain all of the answers for all cases of laser propulsion. More precisely, it is a summary of the work done by the author in attempting to determine how and where laser propulsion would be feasible with state-of-the-art capabilities. The determinations and comparisons are totally analytical; no experimental work was performed. Many important aspects of laser propulsion are given only passing mention, and the reader is asked to consider the work that is described herein, not the work which, due to lack of time, could not be considered. It should also be emphasized that it is the methods that are of importance, not the particular results obtained from particular choices of exemplars. The reader is invited to choose his own state-of-the-art capabilities and substitute them freely, provided that the restraints on the approximations are not violated.
A. INTRODUCTION

The concept of a laser beam should include the notion that it is coupled electromagnetic energy, with the interaction accounting for the resultant beam characteristics. Because its temporal coherence is vastly superior to that of other light sources, diffraction effects dominate the description of laser beam propagation. In attempts to transmit energy by laser beam, these diffraction effects will set a limit to the efficiency of the transmission. The efficiency of transmission also is dependent upon the alignment of the beam with the energy receiver.

The two most important approximations to electromagnetic wave propagation, the Fraunhofer and Fresnel approximations, will be presented and utilized to illustrate the characteristics of diffraction-limited laser beams. The examples chosen will be those cases which are often used to approximate real laser beams. The coupling of diffraction patterns with the accuracy of pointing the laser beams will be presented for two cases; one considers a single laser and the other considers an array of phase-coupled lasers.

B. ELECTROMAGNETIC WAVE PROPAGATION APPROXIMATIONS

For the cases to be considered, laser light will be represented as an electric vector propagating along a directional vector which will define one axis in three-dimensional space. The temporal fluctuation will be taken as sinusoidal and perfectly coherent, allowing a representation in which temporal dependence need not be explicitly shown and pure phase dependencies can be ignored. Let the light amplitude in a plane perpendicular to the propagation vector be given by \( U(x, y) \), where \( x, y \) are rectilinear coordinates of this plane and \( z \) is the third rectilinear coordinate whose axis coincides with the propagation vector.\(^\star\) Let the coordinates \( \xi, \eta \) correspond to \( x, y \) for \( z = 0 \), which will be taken as the laser aperture. Then the light intensity (energy flux) in the laser aperture, \( I_0 \), is given by the complex square of the light amplitude:

\[
I(\xi, \eta) = U(\xi, \eta) U^*(\xi, \eta)
\]

\(^\star\)See paragraph III-J below for nomenclature list.
The far-field or Fraunhofer approximation can be applied for ranges which are large compared to the aperture,

\[ z \gg \frac{\pi (\xi^2 + \eta^2)_{\text{max}}}{\lambda} \]  \hspace{1cm} (2)

where \((\xi^2 + \eta^2)_{\text{max}}\) indicates the square of the largest radius of the aperture and \(\lambda\) is the laser wavelength. The resulting approximation for the light amplitude in the \(x, y\) plane at range \(z\), \(U\), is given by \(95\)

\[ U(x, y) = \frac{1}{\lambda z} \int_{-\infty}^{+\infty} U_0(\xi, \eta) \exp \left\{ i \frac{2\pi}{\lambda z} (x\xi + y\eta) \right\} d\xi d\eta \] \hspace{1cm} (3)

and resembles the Fourier transformation of the aperture amplitude function.

The near-field or Fresnel approximation can be applied for closer ranges than the Fraunhofer approximation, but is considerably more complex. This approximation, which is valid for ranges such that

\[ z \gg \left[ \frac{\pi}{4\lambda} (\xi^2 + \eta^2)_{\text{max}}^2 \right]^{1/3} \] \hspace{1cm} (4)

can be obtained by replacing the spherical Huygens wavelets with a quadratic surface and is given by

\[ U(x, y) = \frac{1}{\lambda z} \int_{-\infty}^{+\infty} U_0(\xi, \eta) \exp \left\{ i \frac{2\pi}{\lambda z} \left[ (x - \xi)^2 + (y - \eta)^2 \right] \right\} d\xi d\eta \] \hspace{1cm} (5)

The Fresnel approximation is often valid for ranges much less than given in Eq. (4), depending on the amplitude distribution in the aperture. The Fresnel approximation can be valid for essentially all ranges \(z > 0\), as will be shown for the case of a gaussian distribution in a large aperture.

For both of the above relations, given in Eqs. (3) and (5), and for all of the analysis to follow, phase factors which contribute nothing to the intensity distribution have been deleted.
C. DIFFRACTION PATTERNS FOR SOME SIMPLE APERTURE AMPLITUDE FUNCTIONS

As a first approximation, assume that the laser output is uniform in intensity and phase across a circular aperture of diameter \( d \). Then the aperture amplitude function is given by

\[
U(\xi, \eta) = I_0^{1/2} \quad 0 \leq \sqrt{\xi^2 + \eta^2} \leq \frac{d}{2}
\]

\[
0 \quad \frac{d}{2} < \sqrt{\xi^2 + \eta^2}
\]

The far-field intensity distribution pattern is then given by

\[
I(x, y) = I_0 \left[ \frac{\pi d^2}{4\lambda z} \right]^2 \left[ 2 \frac{J_1 \left( \frac{\pi d r}{\lambda z} \right)}{\frac{\pi d r}{\lambda z}} \right]^2
\]

where \( r \) is the radial coordinate of polar coordinates in the \( x, y \) plane and \( J_1 \) is the first order Bessel function. This distribution is known as the Airy pattern.

The diffraction angle of this pattern is taken as the angle to the first zero, which occurs for \( rd/\lambda z = 1.22 \). If the beam diameter, \( D \), is defined as the distance in the far-field plane between the first zeroes of the diffraction pattern, then for the Airy pattern,

\[
D = 2.44 \frac{\lambda}{d} z
\]

Let the coefficient on the right-hand side be called the diffraction coefficient \( \sigma \) so that this diffraction equation can be written as

\[
D = \sigma \frac{\lambda}{d} z
\]

For the circular aperture the fraction of energy intercepted within the beam diameter is 0.838.
The dependence of the relative intensity of the diffraction pattern on the range is most readily displayed by considering the maximum intensity, which occurs at the center of the pattern.

\[ I(0, 0) = I_0 \left( \frac{\pi d^2}{4\lambda z} \right)^2 \]  

(10)

The characteristic fall-off distance, \( L \), can be seen from this relation to be the aperture area divided by the wavelength

\[ L = \frac{\pi d^2}{4\lambda} \]  

(11)

Conservation of energy requires this inverse-distance-squared decrease in energy for the far-field region.

The far-field intensity distribution for a laser with an output which is uniform in intensity and phase across a rectangular aperture, so that

\[ U(\xi, \eta) = I_0^{1/2} \quad \frac{-a}{2} \leq \xi \leq \frac{a}{2}, \quad \frac{-b}{2} \leq \eta \leq \frac{b}{2} \]  

(12)

\[ = 0 \quad \text{otherwise} \]

is given by

\[ I(x, y) = I_0 \left( \frac{ab}{\lambda z} \right)^2 \left[ \text{sinc} \left( \frac{\pi a x}{\lambda z} \right) \text{sinc} \left( \frac{\pi b y}{\lambda z} \right) \right]^2 \]  

(13)

where the sinc function is defined by \(^{(97)}\)

\[ \text{sinc} \ x = \frac{\sin x}{x} \]  

(14)

The maximum intensity of the pattern is found at the center,

\[ I(0, 0) = I_0 \left( \frac{ab}{\lambda z} \right)^2 \]  

(15)
From which the energy flux characteristic fall-off distance is equal to the aperture area divided by the wavelength.

In order to find the fraction, $\epsilon$, of total power, $P$, in the diffraction pattern as a function of the distance from the center, the normalization of $I_0 = P/ab$ is required. The fraction $\epsilon$ is defined by

$$\epsilon(x_1, y_1) = \frac{1}{P} \int_{-x_1}^{+x_1} \int_{-y_1}^{+y_1} I(x, y) \, dx \, dy$$

Since the integrals are separable, due to the symmetry of the aperture, and are of the same form, integration over $y$ leads to clarification. Let

$$\epsilon_1(q) = \int_{-q}^{+q} \left[ \text{sinc} \left( \frac{\pi w}{2} \right) \right]^2 \, dw$$

where $w$ and $q$ are dummy parameters; then

$$\epsilon(x, y) = \epsilon_1 \left( \frac{ax}{\lambda z} \right) \epsilon_1 \left( \frac{by}{\lambda z} \right)$$

The fraction $\epsilon_1$ is plotted as a function of the distance parameter $ax/\lambda z$ for $0 \leq ax/\lambda z \leq 2$ in Fig. 7, which is the normalized integration of the single dimension diffraction pattern. The fraction $\epsilon_1$ is tabulated in Table 6 as a function of the distance parameter $ax/\lambda z$ for larger values of the parameter. A square aperture, with sides of length $a$, will have the first zero of its far-field pattern at $x = \lambda z/a$. The fraction of the power intercepted in the far-field plane in a square of sides $2\lambda z/a$ will be, from Table 6 and Fig. 7, $[\epsilon_1(1)]^2 \approx 0.815$. The diffraction coefficient is 2 where the beam and aperture sides are employed in Eq. (9).

The laser output is most often approximated with a circularly symmetrical aperture, a constant phase amplitude, and a gaussian distribution of light intensity. The light intensity distribution in the aperture plane for a gaussian beam is given by
Fig. 7. Fraction of power within the nondimensional distance and the related sinc function.
Table 6. Integration of the intensity distribution of a single aperture

<table>
<thead>
<tr>
<th>Distance Parameter</th>
<th>Fraction of Power Intercepted</th>
<th>Distance Parameter</th>
<th>Fraction of Power Intercepted</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{ax}{\lambda z}$</td>
<td>$\xi_1$</td>
<td>$\frac{ax}{\lambda z}$</td>
<td>$\xi_1$</td>
</tr>
<tr>
<td>0</td>
<td>0.0</td>
<td>5.5</td>
<td>0.982</td>
</tr>
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<td>0.5</td>
<td>0.774</td>
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<td>0.932</td>
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<td>0.986</td>
</tr>
<tr>
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<td>7.5</td>
<td>0.987</td>
</tr>
<tr>
<td>2.5</td>
<td>0.960</td>
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</tr>
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</tr>
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</tr>
<tr>
<td>4.5</td>
<td>0.978</td>
<td>10.0</td>
<td>0.991</td>
</tr>
<tr>
<td>5.0</td>
<td>0.981</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[
I(\xi, \eta) = I_0 \exp \left[ -\pi \frac{\xi^2 + \eta^2}{\ell^2} \right]
\]  \hspace{1cm} (19)

where $\ell$ is the characteristic length of the distribution, corresponding to $\sqrt{2\pi}$ standard deviations, and the power of the laser, $P$, is given by $P = \ell^2 I_0$. The Fresnel approximation yields an intensity pattern which is gaussian,

\[
I(x, y) = \frac{I_0}{\left(\frac{\lambda z}{2\ell^2} \right)^2 + 1} \exp \left\{ -\pi \frac{(x^2 + y^2)}{\ell^2 + \left(\frac{\lambda z}{2\ell} \right)^2} \right\}
\]  \hspace{1cm} (20)

Although the Fresnel approximation is strictly not applicable close to the laser aperture, the above result is correct for all ranges, including the aperture plane where $z = 0$. For the far-field, where $z \gg \frac{2\ell^2}{\lambda}$, the Fresnel approximation reduces to the Fraunhofer approximation.
The maximum intensity of the pattern is found on the center:

\[ I(x, y) = I_0 \left( \frac{2l^2}{\lambda z} \right)^2 \exp \left| -\pi \left( \frac{2l^2}{\lambda z} \right) (x^2 + y^2) \right| \]  \hspace{1cm} (21)

From which the fall-off distance, \( L \), can be obtained in the limit of the far-field as

\[ L = \frac{2l^2}{\lambda} \]  \hspace{1cm} (23)

The beam diameter and diffraction coefficient cannot be chosen based on the distance to the first zero of the intensity pattern since the gaussian distribution has no nodes. Choosing the beam diameter by twice the distance corresponding to \( \sqrt{2\pi} \) standard deviations, the beam diameter is given by

\[ D = 2l \sqrt{1 + \left( \frac{\lambda z}{2l^2} \right)^2} \]  \hspace{1cm} (24)

so that the diffraction coefficient can be found as a function of range

\[ \sigma = 2 \sqrt{1 + \left( \frac{2l^2}{\lambda z} \right)^2} \]  \hspace{1cm} (25)

where the aperture diameter has been taken as \( 2l \). For the far-field, the diffraction coefficient reduces to \( \sigma = 2 \). The fraction of energy flux contained within this defined beam diameter is 0.957.
D. COUPLING POINTING ERROR WITH DIFFRACTION PATTERNS

The small divergence of the laser makes it suitable for power transmission over distances where other wireless methods fail. Small divergence is advantageous, however, only if the laser can be pointed at a target with sufficient accuracy to effect energy transfer.

A measure of transmission efficiency is the instantaneous ratio of energy intercepted by the target to the maximum intercepted energy at the given range. Consider the pointing problem as that of a moving target in a fixed diffraction pattern; this approximation will be quite adequate for the case of ranges large enough so that

\[
\sin \left( \frac{r}{z} \right) \approx \frac{r}{z}
\]  

(26)

Since the diffraction patterns to be considered have their maximum intensities at the center of the patterns, the maximum intercepted power is found when the target is centered at the origin. If the target function, \( T \), is given by

\[
T(x, y) = \begin{cases} 
1 & \text{if } x, y \text{ are within the geometric target boundary} \\
0 & \text{if } x, y \text{ are outside}
\end{cases}
\]  

(27)

then the maximum intercepted power, \( P_0 \), is given by

\[
P_0 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T(u, v) I(u, v) \, du \, dv
\]  

(28)

where \( u, v \) are dummy Cartesian coordinates corresponding to \( x, y \). The ratio of intercepted power at a given point in the far-field plane, \( \beta \), is then given by
\[ \beta(x, y) = \frac{1}{P_0} \iint_{-\infty}^{+\infty} T(u - x, v - y) I(u, v) \, du \, dv \]  

(29)

for which the range dependence is contained implicitly in the intensity function.

The time-averaged value of the intercepted power ratio, \( \beta \), is a measure of the expected value of power received at the target. This value must take into consideration the time-dependent pointing error. If the pointing error is considered in a probabilistic sense in order to eliminate explicit temporal considerations, then the averaged value is most appropriately termed the expectation value. Let the pointing function in the target plane, i.e., the probability that the target will be at a particular place in the diffraction pattern, be given by the function \( G \); then the expectation value, \( E \), is given by

\[ E = \frac{1}{P_0} \iint_{-\infty}^{+\infty} G(x, y) \left[ \iint_{-\infty}^{+\infty} T(u - x, v - y) I(u, v) \, du \, dv \right] \, dx \, dy \]  

(30)

if the function \( G \) is normalized so that its integral over the two-dimensional \( x, y \) plane is 1. The average power intercepted by the target, \( P_A \), is the product of the maximum power and the expectation value.

\[ P_A = E P_0 \]  

(31)

1. Perfect Pointing

Consider the case of perfect pointing so that the pointing function is represented by a two-dimensional delta function at the center of the diffraction pattern. Then the expectation value is equal to unity and the average power is equal to the maximum power. The maximum power is a function of the target size and intensity pattern through which range dependency appears.
2. Square Aperture Laser

For a simple approximation to the problem of coupling laser propagation with pointing, consider a uniformly illuminated square laser aperture which transmits power to a square target. Further assume, in order to allow for separation of the integrals, that the aperture and target are aligned in the same orientation. Then the maximum power for a target of side $2s$ can be found by substitution of Eq. (13) for $b = a$ and the target function

$$T(x, y) = \begin{cases} 
1 & |x| \leq s \text{ and } |y| \leq s \\
0 & \text{otherwise}
\end{cases} \quad (32)$$

into Eq. (28), with the result in the far-field as

$$P_0 = I_0 \ a^2 \left[ \frac{\pi s}{\lambda z} \right]^2 \ \left( \frac{\pi w}{\lambda z} \right)^2 dw \quad (33)$$

where $w$ is a dummy parameter which is defined by $w = ax/\lambda z$. By comparison with the previously defined function $\epsilon$, and since $I_0 = P/a^2$,

$$P_0 = P \left[ \epsilon \left( \frac{ax}{\lambda z} \right) \right]^2 \quad (34)$$

Assume, as a first approximation, that the pointing function can be represented by a two-dimensional step function whose orientation coincides with that of the aperture. Then the pointing function as seen in the target plane is

$$G(x, y) = \frac{1}{4t^2} \ \text{for} \ |x| \leq t \ \text{and} \ |y| \leq t$$

$$= 0 \ \text{otherwise} \quad (35)$$

The meaning of this function is that a bound exists on the ability to point and that the target is equally likely to be found at any point inside that limit. Since each of the functions $I$, $G$, and $T$ are separable into $x$ and $y$ components, consider the one-dimensional expectation value $E_1$ whose square yields the previously defined expectation value $E$. For this case,
\[ E_1 = \left( \frac{P}{P_0} \right)^{1/2} \int_{-t}^{+t} \int_{x-s}^{x+s} \left[ \frac{a}{\lambda z} \left( \text{sinc} \left\{ \frac{au}{\lambda z} \right\} \right)^2 \right] du \ dx \]  

(36)

Consider a slightly more general form of the single dimension expectation value:

\[ E_1 = \frac{1}{2t} \int_{-t}^{+t} \int_{x-s}^{x+s} F(u) \ du \ dx \]  

(37)

where \( F \) is an appropriate function which depends on the diffraction pattern; for the case of the square aperture, the definition of \( F \) is obtained from Eq. (36).

Changing the limits of integration and integrating:

\[ E_1 = 2 \frac{s}{t} \int_0^t F(u) \ du + \frac{1}{t} \int_{-s}^t (t - s - u) F(u) \ du + \frac{1}{t} \int_t^{t+s} (t + s - u) F(u) \ du \]  

(38)

Let \( \Delta = \xi - t \), then \( |\Delta| \leq s \), and consider the limiting, but useful, case of \( s << t \).

Expand \( F(t+\Delta) \) in a Taylor series about \( \Delta = 0 \) for the last two terms of Eq. (38). Integrate to obtain

\[ E_1 = 2 \frac{s}{t} \int_0^t F(u) \ du + \frac{1}{t} \int_0^t (t - s - u) F(u) \ du + \frac{1}{t} \sum_{n=2}^{\infty} \frac{s^{2n+1}}{n(2n+1)} \int_0^t \frac{2n-1}{(2n-1)!} F^{(2n-1)}(u) \ du \]  

(39)

where the even derivative terms drop out. Since \( F^{(1)}(t) \sim t^{-2} \), and similarly \( F^{(2n-1)}(t) \sim t^{-2n} \), the first term in Eq. (39) represents a good estimate for \( E_1 \) if \( (s/t)^{2} << 1 \).

Again considering the special case of the square aperture laser, the single dimension expectation value is

\[ E_1 \approx \left( \frac{P}{P_0} \right)^{1/2} \left[ \frac{s}{t} \right] \left( \frac{at}{\lambda z} \right) + \frac{1}{3\pi} \left( \frac{s}{t} \right)^3 \left( \sin \left\{ \frac{2\pi at}{\lambda z} \right\} - \frac{2\lambda z}{2\pi at} \sin^2 \left\{ \frac{\pi at}{\lambda z} \right\} \right) \]  

(40)
where the previous normalization of $I_0 = \frac{P}{a^2}$ has been substituted. Since symmetry was assumed throughout, the expectation value is then the square of the partial value, which will then be approximated, for $(s/t)^2 \ll 1$, by

$$E = \frac{P}{P_0} \left(\frac{s}{t}\right)^2 \left[\epsilon_1(\frac{at}{\lambda z})\right]^2$$

(41)

The pointing accuracy can be represented by the half-angle $\vartheta$ of the furthest limit of the pointing function:

$$\vartheta = \frac{t}{z}$$

Substituting this relation and the definition of Eq. (31),

$$P_A \approx P \left(\frac{s}{\vartheta z}\right)^2 \left[\epsilon_1(\frac{s}{\lambda \vartheta})\right]^2$$

(42)

Therefore, for the first approximation, the time-averaged intercepted power is directly proportional to the area of the target and inversely proportional to the square of the range.

3. Circular Aperture Laser

Since the result for the rectangular aperture laser has been shown to be relatively simple, it is natural to extend the result to the case of a single circular laser aperture with a gaussian intensity distribution. The maximum power intercepted by a circular target of radius $s$ is found by substitution of Eq. (20) and the circular target function

$$T(r) = \begin{cases} 1 & r \leq s \\ 0 & r > s \end{cases}$$

(43)

into Eq. (28), with the result of
where the laser power is given by \( P = I_0 l^2 \).

Assume a circular step function pointing function which is described in the target plane by

\[
G(r) = \frac{1}{\pi t^2} \quad \text{for} \quad r \leq t \quad \text{(45)}
\]

\[
0 \quad \text{for} \quad r > t
\]

Since the pointing function and the intensity distribution pattern are dependent only on the radial coordinate, the expectation value can be obtained for \((s/t)^2 \ll 1\) by calculation of the mean of the ratio of intercepted power, \( \beta \), within the area defined by the pointing function. This corresponds to calculation of the first term of the series in \( \beta \). This term is integrable analytically, yielding the expression for the approximation to the expectation value,

\[
E = \left( \frac{P}{P_0} \right)^2 \left( 1 - \exp \left( -\frac{\pi s^2}{\left( \frac{\lambda z}{2} \right)^2 + l^2} \right) \right) \quad \text{(46)}
\]

Consider the simplification of the far-field (Fraunhofer) approximation and convert to the pointing angle \( \nu \) to find the average power intercepted as

\[
P_A = P \left( \frac{s}{\nu} \right)^2 \left[ 1 - \exp \left( -\frac{2 \pi l u}{\lambda} \right)^2 \right] \quad \text{(47)}
\]

From which the dependence can be seen to be similar to the previous result.
4. Gaussian Pointing Function

Consider the case of a laser with a gaussian distribution of intensity in the aperture plane so that the diffraction pattern is given by Eq. (20). Define the diffraction pattern parameter $A$ by

$$A = \frac{\lambda z}{2\ell}$$

for Fraunhofer diffraction

$$= \frac{\lambda z}{2\ell} \left[ 1 + \left( \frac{2\ell^2}{\lambda z} \right) \right]^{1/2}$$

for Fresnel diffraction

(48)

to provide a simplified form for the intensity

$$I(r) = \frac{P}{A^2} \exp \left[ -\pi \left( \frac{r}{A} \right)^2 \right]$$

(49)

where $P$ is the power of the laser.

Let the pointing function, $G(r)$, be described by a normal probability function with the square of the parameter $B$ as twice the variance of the distribution.

$$G(r) = \frac{1}{B^2} \exp \left[ -\pi \left( \frac{r}{B} \right)^2 \right]$$

(50)

Define the pointing accuracy by the angle $\nu$ where

$$\nu = \frac{B}{z}$$

(51)

so that this angle corresponds to $\sqrt{2\pi}$ standard deviations.

Assume, for the first approximation, that the mean incident power is given by that at the center of the target. This approximation is equivalent to taking only the first term in Eq. (39), which has been shown to be valid for cases of small target or large pointing error. Since the shape of the target is not important for this approximation, as only the surface area perpendicular to the propagation vector is considered, choose any target with area $S^2$. The target function can be approximated by a delta function with area $S^2$. 

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The ratio of intercepted power, \( \beta \), is given by

\[
\beta(r) = \frac{S^2 I(r)}{P_0}
\]  

(53)

where the maximum power is approximated by

\[
P_0 = P \left( \frac{S}{A} \right)^2
\]

(54)

The expectation value can be integrated analytically to find the simple function

\[
E = \frac{1}{1 + \left( \frac{B}{A} \right)^2}
\]

(55)

The average power intercepted by the target is given by

\[
P_A = P \frac{S^2}{A^2 + B^2}
\]

(56)

Substituting for the grouped parameters \( A \) and \( B \),

\[
P_A = P \frac{S^2}{(\nu z)^2 + \left( \frac{\lambda z}{2f} \right)^2 + \ell^2}
\]

(57)

The dependence of average intercepted power on the laser and pointing parameters can be found from Eq. (57). For the case of poor pointing ability, the first term in the denominator, \( \nu z \), will dominate, yielding the average intercepted power as inversely proportional to the square of the pointing parameter \( \nu \).

For cases in which the beam spread is the most important factor, the middle term, \( \lambda z/2f \), will dominate, yielding the same dependence as would be calculated from modified ray optics as the power within a beam of diameter \( 2\ell \). For cases where the target is close enough for the beam spread to be small and the pointing is good, then the initial beam characteristics dominate. The third term in the denominator, which is the important one for this case, is independent of range, wavelength, and pointing.
E. ANALYSIS OF A COUPLED SYSTEM

The gaussian pointing function and gaussian laser beam are the best approximations to reality that will be discussed. Since this approximation tends to be a relatively simple form for the expectation value and average intercepted power and since it is applicable in the near-field as well as the far-field, this system is chosen for more detailed analysis.

The approximation is not valid if the target size is of the same order of magnitude as either the pointing parameter B or the beam spread parameter A. In fact there is a size for which this approximation indicates that all of the laser power is intercepted. This size, \( S_m \), is found from Eq. (56) with the condition that \( P_A = P \),

\[
S_m = \sqrt{A^2 + B^2} \tag{58}
\]

For targets larger than \( S_m \), the average power should be taken as the maximum, i.e., the laser power, \( P \). The accuracy of this approximation is shown in Fig. 8, where the approximation is compared to exact numerical solutions for a circular target with \( B = A \). The exact solution is at most about 40% below the approximation, as is quite accurate for target sizes almost equal to the parameters A and B.

The sensitivity of the average intercepted power to changes in the laser parameters, \( \lambda \) and \( L \), pointing parameter, \( \nu \), and range, \( z \), is obtained by calculation of the partial derivatives in the form of

\[
\frac{\delta P_A}{P_A} = \alpha \frac{\delta q}{q} \tag{59}
\]

Thus a fractional change of \( q \) will result in approximately an \( \alpha \)-fold fractional change in \( P \). The influence coefficient is a means to present the instantaneous relation in a power form:

\[
P_A = q^\alpha \tag{60}
\]
Fig. 8. Relative accuracy of the approximation as a function of target size
Thus, for example, an influence coefficient of unity implies a linear relationship and zero implies independence. The coefficients are

\[
\alpha_\lambda = \frac{-2}{\left(\frac{2L}{\lambda} \right)^2 + 1 + \left(\frac{2L^2}{\lambda z} \right)^2}
\]

\[
\alpha_\ell = \frac{+2}{\left(\frac{2L}{\lambda} \right)^2 + 1 + \left(\frac{2L^2}{\lambda z} \right)^2}
\]

\[
\alpha_\nu = \frac{-2}{1 + \left(\frac{\lambda}{2Lv} \right)^2 + \left(\frac{L}{\nu z} \right)^2}
\]

\[
\alpha_z = \frac{-2}{1 + \left(\frac{\nu}{L} \right)^2 + \left(\frac{\lambda z}{L} \right)^2}
\]

(61)

where the order of the terms in the denominator has been maintained so that the influence of the parameters in the three special cases of poor pointing, large beam spread, and small range can be obtained by inspection.

Given a laser with parameters \( \lambda \) and \( \ell \), the minimum pointing accuracy, represented by \( \nu_m \), corresponding to these parameters is to be obtained. Pointing systems with pointing errors greater than \( \nu_m \) will be pointing-limited; systems with pointing errors less than \( \nu_m \) will be beam-spread-limited. Let the minimum acceptable ratio of centroid energy flux to the maximum energy flux of the diffraction pattern be given by \( \gamma \). Then the radial distance \( r_o \), at which the energy flux will drop by a factor of \( \gamma \), will be given from Eq. (50) by

\[
r_o = A \sqrt{\frac{\ln \frac{1}{\gamma}}{\pi}}
\]

(62)

Let \( \tau \) be the fraction of time for which the pointing system has a distance error in the target plane less than or equal to \( r_o \). Then the required pointing parameter B is given by integration of Eq. (50).
The choice of $\gamma$ and $\tau$ will define the system sufficiently well to yield the pointing parameter in terms of the laser parameters.

\[ B = r_o \sqrt{\frac{\pi}{\ln \left( \frac{1}{1 - \tau} \right)}} \]  

(63)

Employing the forms for $A$ and $B$ defined in Eqs. (48) and (51), respectively,

\[ B = A \left[ \frac{\ln (\gamma)}{\ln (1 - \tau)} \right]^{1/2} \]  

(64)

Now consider some numerical examples of the pointing accuracy required for a pointing system to maintain a sufficiently small target within the half-power points of a gaussian laser beam at a 3 sigma performance level. Then the minimum acceptable ratio of centroid energy flux to maximum energy flux, $\gamma$, is taken as $1/2$. For the pointing system performance level to be 3 sigma (three standard deviations), the pointing system remains within the pointing error $\nu_m$ for the fraction of time, $\tau$, of 0.9973. This means that the intercepted power ratio $B$ to $A$ is $B/A = 0.3423$. The aperture diameter must be larger than the characteristic fall-off distance, $\ell$, in order to allow the beam to assume a representation of a gaussian profile. Assume that the aperture is twice the characteristic distance, then

\[ \nu_m = 0.3423 \left( \frac{\lambda}{2\ell} \right) \left[ 1 + \left( \frac{(2\ell)^2}{2\lambda z} \right)^2 \right]^{1/2} \]  

(65)

This pointing accuracy requirement has two limits of interest. First, consider the far-field, where the gaussian laser beam has its divergence determined by the laser wavelength and aperture and the required pointing accuracy is tied directly to that divergence. Placing the far-field limit of $2\lambda z \gg (2\ell)^2$ into Eq. (66) yields the expected behavior, which is independent of $\tau$.
\[ \nu_m = k \frac{\lambda}{(2\ell)} \]  

(67)

where \( k \) is the constant coefficient of 0.3423. Now consider the near-field, where the gaussian beam can be represented by a cylindrical projection of the aperture as represented in Fig. 9. The required pointing accuracy is determined by the aperture and range and is independent of the laser wavelength. This behavior is obtained by assuming \((2\ell)^2 \gg 2\lambda z\) for which Eq. (66) degenerates to

\[ \nu_m = k \frac{\ell}{z} \]  

(68)

Fig. 9. Near-field region where the range is sufficiently close to the aperture so that diffraction effects are negligible. The maximum pointing error can then be approximated by the ratio of beam radius to range.

These limits as well as the intermediate behavior of \( \nu_m \) are illustrated in Figs. 10-13, where lines of constant \( \nu_m \) are given as functions of wavelength and effective aperture, \( 2\ell \). The far-field results are illustrated in Fig. 10 for the range of 1 A.U. The transition to the near-field limit is shown in the successive figures through Fig. 13. In these figures, as well as Tables 7 through 10, the FORTRAN E-format representation of numbers has been employed (e.g., 2.15E-06 is \( 2.15 \times 10^{-6} \)).
Fig. 10. Lines of constant pointing accuracy requirement (in radians) for the range of 1 A.U.
Fig. 11. Lines of constant $\nu_m$ (radians) for the earth-moon range.
Fig. 12. Lines of constant $\nu_m$ (radians) for synchronous orbit range.
Fig. 13. Lines of constant $\nu_m$ (radians) for low earth-orbit range
The maximum target size for which the approximation is valid is tabulated for a few choice values of wavelength and effective aperture in Tables 7 through 10. The maximum target size has been taken as the square root of the target area, $S^2$, and is determined from Eq. (58). The validity of the approximation is illustrated in Fig. 14 where the exact numerical results for a circular target are given. For target sizes smaller than half of the maximum size, the approximation becomes quite good.

F. COMPARISON WITH SOLAR ILLUMINATION

It is interesting, if not requisite, to determine the conditions for a laser system to provide more power to a target than can be received from the solar flux. Detailed analyses will be omitted in favor of a simple comparison. Consideration is not given to analyses of vehicle construction or trajectory, nor to laser pointing or propagation. In order to minimize the clutter, the phase angle will be taken as zero so that the range will be taken from the earth to the vehicle along the line of syzygy.

The solar flux as a function of distance from earth, $z$, is given by

$$\Phi_K \frac{1}{(1 + \frac{z}{K})^2}$$

(69)

where $\Phi_K$ is the solar constant and $K$ is the sun-laser distance. Thus the power per unit area that can be obtained from the solar flux, $P_S$, is approximated by

$$P_S = \eta_S \Phi_K \frac{1}{(1 + \frac{z}{K})^2}$$

(70)

where $\eta_S$ is the conversion efficiency for solar radiation. The maximum power per unit area, $P_L$, for a gaussian laser beam is found at the center of the beam and is obtained from Eq. (22),
Table 7. Maximum target size for the approximations to be valid at a range of 1 A.U.

Range is 4.00E 05 meters

<table>
<thead>
<tr>
<th>Wavelength (m)</th>
<th>Aperture (m)</th>
<th>Pointing Accuracy (rad)</th>
<th>Max. Size (m)</th>
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N.B.: 1. Aperture is twice the characteristic distance.

2. Pointing accuracy is for half power with 3 sigma probability.

3. Approximations valid only for smaller targets.

4. Atmosphere invalidates results for apertures larger than 0.2 M.
Table 8. Maximum target size for the approximations to be valid at earth-moon distance

Range is 4.00E07 meters

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4. Atmosphere invalidates results for apertures larger than 0.2 M.

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Table 9. Maximum target size for the approximations to be valid at synchronous orbit distance

Range is $4.00 \times 10^8$ meters

<table>
<thead>
<tr>
<th>Wavelength (m)</th>
<th>Aperture (m)</th>
<th>Pointing Accuracy (rad)</th>
<th>Max. Size (m)</th>
</tr>
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<td>4.23E 03</td>
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</tbody>
</table>

N.B.: 1. Aperture is twice the characteristic distance.
   2. Pointing accuracy is for half power with 3 sigma probability.
   3. Approximations valid only for smaller targets.
   4. Atmosphere invalidates results for apertures larger than 0.2 M.
### Table 10. Maximum target size for the approximations to be valid at low earth-orbit range

**Range is 1.50E 11 meters**

<table>
<thead>
<tr>
<th>Wavelength (m)</th>
<th>Aperture (m)</th>
<th>Pointing Accuracy (rad)</th>
<th>Max. Size (m)</th>
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</tbody>
</table>

N.B.: 1. Aperture is twice the characteristic distance.
2. Pointing accuracy is for half power with 3 sigma probability.
3. Approximations valid only for smaller targets.
4. Atmosphere invalidates results for apertures larger than 0.2 M.
Fig. 14. Comparison of the approximation with the exact numerical solution for a circular target.
\[ P_L = \eta_L \frac{P}{l^2 \left[ 1 + \left( \frac{\lambda z}{2 \lambda^2} \right)^2 \right]} \] (71)

where \( P \) is the total power of the laser and \( \eta_L \) is the efficiency of conversion of laser radiation. Since the conditions that were chosen illustrate the best case for laser energy transfer, the results should be treated cautiously. However, it is useful to know when a laser system cannot, even with perfect pointing, collect more power than can be obtained from the constant solar source.

One condition sought is that for which \( P_L > P_S \) for all distances from the laser. This criterion is satisfied when the parameters are such that

\[ \frac{\eta_L}{\eta_S} \left( \frac{\xi}{\lambda} \right)^2 P > \frac{1}{4} \phi_K K^2 \] (72)

From conservation of energy considerations \( \phi_K K^2 \) is found to be constant at \( 8 \times 10^{24} \text{W} \) when evaluated at earth, where \( \phi_K \) is \( 1.4 \times 10^3 \text{W} \) and \( K \) is \( 1.5 \times 10^{11} \text{m} \). However, even if this condition were satisfied, the power per area at large distances could be too small to be useful. For interplanetary distances, the condition can be relaxed slightly. For \( z \gg \frac{l^2}{\lambda} \), the parameters must satisfy the approximate relationship

\[ \frac{\eta_L}{\eta_S} \left( \frac{\xi}{\lambda} \right)^2 P > \frac{1}{4} \phi_K K^2 \left( \frac{1 + \frac{2K}{Z} + \frac{K^2}{z^2}}{1 + \frac{2K}{Z} + \frac{K^2}{z^2}} \right) \] (73)

With a range from earth or earth orbit to the closest approach of Jupiter, where the range is about four times the earth-sun distance, the requirement, as imposed by Eq. (73), is lowered by about a third. The corresponding range to Mars is about half the earth-sun distance, consequently the requirement is an order of magnitude lower than for the stellar range.

If the laser is situated at Jupiter, the denominator in Eq. (73) becomes larger as compared to an earth location, and the requirement lower for fixed and finite ranges. At Jupiter's distance \( K \) is \( 7.8 \times 10^{11} \text{m} \), and the minimum distance...
to Saturn is about the same, so that for Jupiter to Saturn transmission the requirement would be reduced by 4 from the stellar condition.

G. MULTIPLE LASERS AND PHASE CONTROLLED LASER ARRAYS

The comparative performance of phase and free-running (non-phase controlled) laser arrays are evaluated for extraterrestrial energy transfer. Mathematical models of the arrays, diffraction effects, and pointing capability are developed to obtain parametric descriptions of the time-averaged power intercepted by a spacecraft. The analysis provides an improvement over ray optics techniques for the transmission approximations while maintaining simplicity of application. The boundaries for advantage of linear and square arrays of phased lasers are determined for transmission distances for which the Fraunhofer approximation of light propagation is valid. Projections from contemporary capabilities are made to illustrate the limits of advantage of the phased array concept for laser energy transfer; earth-based phased arrays present no advantage, while spaceborne phased arrays are advantageous for systems with very accurate pointing capabilities.

1. The Phased Array

The phased array concept has been suggested as a method for increasing the energy flux at the spacecraft. The coherence of the laser allows the phased array concept to be applied to manipulate the diffraction pattern of the array in order to decrease its effective divergence. However, smaller effective divergence is advantageous only if the laser system can be pointed at a target, i.e., a spacecraft, with sufficient accuracy. Some of the characteristics of the phased arrays will be detailed and compared with corresponding arrays of uncoupled lasers to determine the bounds of advantageous application of the phased array concept.

This section presents mathematical models of laser transmission systems with regular arrays. Rectangular apertures are chosen because the functional dependencies can be obtained without the obfuscation of numerical analysis. The difference in the result caused by the choice of rectangular over circular apertures is not significant for this analysis. Also, for clarification, the function which describes the response of the pointing system has been idealized to a step function from the more realistic normal probability function. The details of laser generation, phase control, energy conversion, and ultimate energy usage have been excluded, as have any discussion on the feasibility, justification, or cost of building a phased array.
2. Analysis

The phased array is a group of regularly spaced lasers with parallel optical axes and with all lasers in the array controlled so that each emits the same wavelength light at the same phase. If the frequency and phase controls are removed so that the phase of each of the lasers is uncorrelated, then the array will be termed a free-running array. Conceptual diagrams for both types of arrays are shown in Fig. 15. For both arrays the 'beam' is assumed to be composed of light which is constant in amplitude and phase across the exit aperture. Phase control, when applied, is assumed to be perfect in order to present the best case.

A row of N identical phased lasers with square apertures and spatial period c will yield the light intensity pattern given by

\[ I_p(x, y) = P(a/\lambda z)^2 \left[ \frac{\sin (\pi N c y / \lambda z)}{\sin (\pi c y / \lambda z)} \right]^2 \left[ \text{sinc} \left( \pi a y / \lambda z \right) \right]^2 \left[ \text{sinc} \left( \pi a x / \lambda z \right) \right]^2 \]

(74)

The first and last two terms are identical with the single aperture which is given in Eq. (13) with \( b = a \); the second term, called the array factor, is determined by the characteristics of the array.

Diffraction effects for the linear array of phased lasers depend upon the direction in the far-field plane. In the direction perpendicular to the array, the diffraction coefficient is the same as for the single aperture, while in the direction of the array, the diffraction angle is a factor of \( a/Nc \) smaller. However, the fraction of power intercepted in this angle is a factor of approximately \( a/c \) smaller; this approximation is accurate to within 10% for all \( N \) and 1% for \( N \geq 6 \).

If each of the lasers in the row of \( N \) is free-running, then the time-averaged light intensity pattern in the far-field is given, within the Fraunhofer approximation, by \( N \) times that of a single aperture. This is valid for \( N \) laser apertures, even if they are not in a row, whenever the criterion of Eq. (2) is satisfied. The normalized diffraction pattern is the same as that of a single aperture; hence, the diffraction coefficients are identical.
Fig. 15. Conceptual diagrams for phased and free-running arrays
Since the diffraction patterns for the square aperture lasers uncouple with respect to the two cartesian coordinates, the square array of lasers has a diffraction pattern which can be obtained directly from the form of the linear array. If the previous nomenclature is retained, with \( N \) a perfect square, then the light intensity pattern in the far-field for a square array of \( N \) phased lasers is

\[
I(x, y) = P (a/\lambda z)^2 \left[ \frac{\sin (\pi N^{1/2} cx/\lambda z) \sin (\pi N^{1/2} cy/\lambda z)}{\sin (\pi cx/\lambda z) \sin (\pi cy/\lambda z)} \right]^2 
\times \left[ \text{sinc} (\pi ax/\lambda z) \text{sinc} (\pi ay/\lambda z) \right]^2
\]

The first and last terms are the same as for the single aperture; the second term is the array factor. The square phased array has a diffraction coefficient identical to that of a linear array of \( N^{1/2} \) lasers.

In order for energy to be transferred by laser beam it is necessary for the laser or lasers to be pointed so that part of the beam is incident upon the target. Of interest herein is the time-averaged power as defined previously. Let the target be square with side \( 2s \) and assume the step function pointing function as given by Eq. (35). Then the expectation value is separable with respect to the \( x, y \) axes, so that only the single-dimension expectation value, \( E_1 \), need be considered in detail.

An array of free-running lasers has the same normalized expectation value as the single laser, provided that the Fraunhofer approximation remains valid. The array need not be regular, but the configuration is constrained by the requirement of parallel optical axes.

Consider the case of a linear array of phased lasers. The normalized far-field pattern yields a component expectation value which is given as

\[
E_1 = \left( \frac{NP}{P_0} \right)^{1/2} (s/\lambda z) \epsilon_2 (a\nu/\lambda, c/a, N) + \text{higher order terms}
\]

where the fraction \( \epsilon_2 \) is given by

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Then for the phased linear array, the first order approximation is

$$\epsilon_2 \left( ay/\lambda z, c/a, N \right) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[ 1 \right] P \left( \xi, \eta \right) / NP \ d\xi \ d\eta \quad (77)$$

The first order approximation for the case of the phased square array can be obtained from the case of a linear array of $N^{1/2}$ lasers by squaring the single dimension expectation value

$$E_{p, s} = \frac{P}{P_0} \left( s/uz \right)^2 \left[ \epsilon_1 \left( ay/\lambda \right) \epsilon_2 \left( ay/\lambda, c/a, N \right) \right]$$

The first order approximation for the case of the phased square array can be obtained from the case of a linear array of $N^{1/2}$ lasers by squaring the single dimension expectation value

$$E_{p, s} = \frac{NP}{P_0} \left( s/uz \right)^2 \left[ \epsilon_2 \left( ay/\lambda, c/a, N^{1/2} \right) \right]^2 \quad (79)$$

The time-averaged power transmitted to a vehicle can be found by multiplying the normalized expectation value by the number of lasers and by the power per laser, i.e., a factor of NP. Although the fractions $\epsilon_1$ and $\epsilon_2$ must be evaluated, their functional forms are easily adapted to approximation since they are monotonic as shown in Fig. 16 and approach a limit of 1 for large values of the distance parameter. Since these factors are independent of the range, $z$, they need be evaluated only once to find the average power intercepted as a function of the range.

3. Comparison of Concepts

A linear array of $N$ phase-controlled lasers yields a far-field pattern in which the center intensity is proportional to $N^2$ and the diffraction coefficient in the direction of the row is proportional to $1/N$ as compared to the single laser. A free-running linear array of $N$ lasers has its center intensity proportional to $N$ and diffraction coefficient independent of $N$. In order to compare the two systems, consider the case with $N = 5$ and with spacing equal to the aperture so that $c = 2a$. The relative light intensity in the direction along the array is shown for these cases in Fig. 17.
Fig. 16. Fractions $\epsilon_1$ and $\epsilon_2$ as a function of the distance parameter

Fig. 17. Intensity distribution for a linear array of five lasers
Let \( R \) be the ratio of the expectation values of the phased to free-running linear arrays, so that when \( R > 1 \), the phased array would provide a larger expectation value:

\[
R \approx \frac{\epsilon_2 (a\nu/\lambda, c/a, N)}{\epsilon_1 (a\nu/\lambda)}
\]

(80)

For the limiting case of small values of \( a\nu/\lambda \), the phased array has an expectation value which is \( N \) times that of the free-running array; the maximum value of the ratio occurs at this limit. For large values of \( a\nu/\lambda \), the two expectation values are identical, \( R = 1 \). The case of \( N = 5 \) and \( c/a = 2 \), given in Fig. 18, illustrates typical behavior of this ratio. For \( a\nu/\lambda \leq 0.27 \) the phased array has theoretical advantage, but for greater values it is either disadvantageous to or not significantly different from the free-running array.

A square array of \( N \) phase-controlled lasers yields a far-field pattern in which the center intensity is proportional to \( N^2 \) and the diffraction coefficient is proportional to \( N^{1/2} \) as compared to a single laser. A corresponding free-running array has its center intensity proportional to \( N \) and diffraction coefficient independent of \( N \). The diffraction pattern \( I \) of the two array directions is given by that of the linear array, as shown in Fig. 17. Let \( R_s \) be the ratio of the expectation values for the square arrays, in an analogous manner to \( R \). The maximum value of \( R_s \) is \( N \), which is approached in the limit of small \( a\nu/\lambda \). The behavior of this ratio, shown in Fig. 18, is similar to that for a linear array. Again the distinction between the two systems disappears for large values of \( a\nu/\lambda \).

The previous comparisons contained the implicit assumption that the addition of phase and frequency control did not change the power output of the lasers. It is more reasonable to assume that when constraints are placed on a laser, the total power output would decrease. Let \( \zeta \) be the ratio of the power output of the individual phase-controlled laser to that of the free-running laser. Then the previous discussion concerned cases with \( \zeta = 1 \). If \( \zeta \leq 1/N \), then the expectation value of the phased array is always less than that of the corresponding free-running array; hence, the range of interest is \( 1/N \leq \zeta \leq 1 \). The ratios \( R \) and \( R_s \) are altered to include this effect by a multiplication by \( \zeta \).
Fig. 18. Ratio of the relative power of the phased to free-running systems for a linear array of five lasers and a square array of 25 lasers.
\[ R \ell \approx \zeta \left( \epsilon_2 / \epsilon_1 \right) \]
\[ R_s \approx \zeta \left( \epsilon_2 / \epsilon_1 \right)^2 \]  \hspace{1cm} (81)

Choose the smallest value of \( au / \lambda \) for which \( R \) equals 1 as the limit for advantage of the phased array. The phased array will have an advantage only for values of \( au / \lambda \) smaller than these limits, which are shown in Figs. 19 and 20 for the previously discussed cases.

H. CONTEMPORARY CAPABILITIES

State-of-the-art capabilities for laser systems will be projected to provide a baseline upon which the utility of these systems may be evaluated. The analysis will be applied to earth-based and earth-orbiting laser systems which simultaneously maintain all of the state-of-the-art values. Some of the parameters involved in the analysis are not independent of the others; however, only first-order dependency will be considered.

Although the highest power CW lasers have wavelengths in the middle infrared, equal power levels are expected to be developed in the shorter wavelength regions. The CO\(_2\) laser at 10.6 \( \mu \)m is presently considered as leading the power race, with the CO lasers at 5.3 \( \mu \)m and the HF lasers at 2.3 \( \mu \)m as its leading competitors.\(^{(98)}\) The visible lasers have power which is growing at a rate similar to the infrared lasers and the UV lasers are gaining in numbers. For this exercise, two typical wavelengths of 10 \( \mu \)m and 0.4 \( \mu \)m (blue) will be chosen to illustrate the range of probable laser systems.

The optical equipment for high power lasers must be sufficiently large to avoid high power densities. A limit of roughly \( 10^6 \) W m\(^{-2}\) for distortion and \( 10^8 \) W m\(^{-2}\) for fracture may be applied to transmitting optical materials.\(^{(99)}\) Reflective optics have a significantly higher limit of at least \( 10^8 \) W m\(^{-2}\) for distortion. The state of the art for the size of diffraction-limited optics is 6 meters for ground-based systems\(^{(100)}\) and 3 meters for the earth-orbiting Large Space
Fig. 19. Limits of advantage of phased linear arrays
Fig. 20. Limits of advantage of phased square arrays
Telescope. Since diffraction-limited lasers with significant power presently exist, all lasers will be assumed to be diffraction limited. The current prediction of 10 meter optics will be used. Consequently, the upper limit on the total power per laser, \( P \), is 100 MW. If windowless high-powered lasers are developed, then this total power can be two orders of magnitude larger.

Static high energy laser systems can be aligned to within \( 2 \times 10^{-6} \) radian. Atmospheric inhomogeneities limit earth-based telescopes to a seeing accuracy of about \( 5 \times 10^{-6} \) radian, thus their pointing accuracy has been designed to perform at that level. Spaceborne telescopes have done much better; the Stratoscope system can point a 1 meter telescope with an accuracy of \( 8 \times 10^{-8} \) radian and the 3 meter Large Space Telescope is expected to point within \( 2.4 \times 10^{-8} \) radian. A projected pointing accuracy of \( 2 \times 10^{-8} \) radian will be assumed for an earth orbiting system.

Distortion and attenuation of light by the earth's atmosphere makes a high power density earth-based system unlikely. Under good conditions, the atmospheric diffraction for a lower power visible laser is about \( 10^{-5} \) radian. Attenuation by absorption limits the choice of laser wavelength to those for which the atmosphere is relatively transparent. However, the atmosphere is not completely transparent to any wavelength, and adverse weather conditions will add to the absorption. At the power densities which are considered, only a very small fraction of the beam need be absorbed for the index of refraction of the air to change significantly. The results are thermal blooming and beam steering; the former causes the beam to spread and the latter causes it to miss the target. Atmospheric turbulence will also cause a loss in beam strength, and a smoothing of diffraction structure. Phase-compensated optical systems can be employed to compensate for some of the phase disturbances caused by the atmosphere. These systems apply coherent optical adaptive techniques to introduce a phase variation in the output beam which compensates for the phase distortions produced by turbulence and thermal blooming. The target provides a feedback which enables the adaptive system to seek the optimum phase alignment. Although only small, low power density lasers have been shown to be
adaptable, the upper limit on power density is determined by atmospheric breakdown or optical material breakdown. For the earth-based laser array, a pointing accuracy of $1 \times 10^{-6}$ radian will be assumed, and the beam spread will be optimistically taken as that of the diffraction limit.

The method of conversion of the laser energy to electrical energy is not of concern here. Since the efficiency of conversion is involved, a little will be said about photovoltaic devices. The solar cell presently has an efficiency of about 0.1; however, improvements, led by a decrease in impurities and defects, could raise this to nearly 0.22.\(^{(112)}\) The monochromatic laser has a large theoretical advantage over the polychromatic sun as much of the solar flux is wasted in both the long wavelength and short wavelength regions, owing to insufficient quantum energy and to energy in excess of that required for a photoelectric event, respectively. A cell which is tailored to the laser wavelength can be expected to have an efficiency of about 0.4 and possibly higher.\(^{(10)}\) A conversion efficiency of 0.5 will be projected, with the reservation that it could be low.

For large aperture systems, the ranges for validity of the Fraunhofer approximation include all interplanetary space beyond the moon. It is possible for the results for these exemplars to be extrapolated to ranges on the order of kilometers, if focusing optics are applied. The light intensity pattern of the far-field is valid for closer ranges within the Fresnel approximation if the beam is focused with non-vignetting diffraction-limited optics.

I. CONCLUSIONS

The ability to estimate the propagation and pointing characteristics of laser systems has been developed for energy transfer over large distances. These analyses will be combined with the estimates of contemporary capabilities to determine the utility or futility of laser energy transfer to an extraterrestrial target.
The single laser analysis with the assumption of gaussian pointing and light distribution will be employed as an exemplar of individual lasers. The assumption of no less than half the possible power with a three sigma probability yields pointing criteria shown in Fig. 21. Variable range focusing can be applied if the laser is composed of a large number of elements which are controlled by one of the coherent optical adaptive techniques. This focusing is accomplished by phasing the elements to form a representation of a converging spherical wave whose radius of curvature is variable. In the far-field (Fraunhofer diffraction region) diffraction phenomena dominate so that focusing has a minimal effect.

For ranges greater than near earth-orbit, the earth-based laser system is pointing-limited, while the ranges around near earth-orbit require less stringent pointing than is projected. For the 10 meter optics, diffraction effects are not important to the beam shape at the low earth-orbit range, as is illustrated in Fig. 21 where the required pointing accuracy is independent of laser wavelength. The situation is different for the spaceborne laser system where the excellent pointing capability makes the 10 \( \mu \)m laser beam spread-limited for all ranges. For ranges greater than synchronous orbit, the pointing accuracy and pointing criteria are well matched. Thus an earth-based laser system will be pointing-limited if it attempts to transmit farther than low earth-orbit, but a spaceborne laser system could point as well as required for a 0.4 \( \mu \)m laser but much better than required for a 10 \( \mu \)m laser.

None of the contemporary systems can provide more power than the sun for all interplanetary distances. The beam spread of the 10 \( \mu \)m laser allows the sun to provide more power at distances farther than the earth-moon range. The 0.4 \( \mu \)m laser can supply more than enough flux to better the sun at the earth-moon range, but it cannot beat the sun at interplanetary range. The power constants of these two lasers, which correspond to the left-hand side of Eq. (73), are 1.3 \( \times 10^{20} \) W for 10 \( \mu \)m and 7.8 \( \times 10^{22} \) W for 0.4 \( \mu \)m.
Fig. 21. Pointing criteria $\nu_m$ for half power with three sigma probability as a function of range for a 10 meter aperture.
The phased array of lasers has a theoretical advantage over a corresponding array of free-running lasers for laser energy transfer systems which have diffuse beams and excellent pointing capabilities. When the previously discussed values for the earth-based system are substituted, the parametric group $a \nu/\lambda$ has values of 1 for the 10 $\mu$m lasers and 25 for the 0.4 $\mu$m lasers. Assuming the best case of $\zeta = 1$, a factor of 4 decrease in $a \nu/\lambda$ is required before any advantage is obtained by the phased arrays. Lower values for $\zeta$ would require correspondingly larger improvement factors. Increasing the wavelength or decreasing the aperture diameter to improve the relative position of the phased array would be counterproductive for far-field ranges as this would cause the average power received to decrease (Fig. 16). There is no distinct advantage for phase-controlled over free-running arrays for earth-based operations. When the values for the space-borne system are substituted, the parametric group $a \nu/\lambda$ has values of 0.02 for the 10 $\mu$m lasers and 0.5 for the 0.4 $\mu$m lasers. Even for the best case of $\zeta = 1$, the visible phased array does not have a significant advantage over the corresponding free-running array (Fig. 18). However, the phased array of middle infrared lasers at 10 $\mu$m does have a significant advantage over the free-running array if there are enough lasers in the array. As can be seen from Fig. 19 and Fig. 20, the phased array will have a theoretical advantage for reasonable values of $\zeta$ and all values of $N$.

Since the phased and free-running arrays have the same characteristic distance for energy flux dispersion, the range is not a factor in the comparison; thus the relative worth of these two alternatives depends only on the diffraction and pointing parameters. The object has been to determine if the addition of phase control to an array of large lasers would significantly improve the power transfer to a target. The conclusion of this work is that phase control of laser arrays can yield a significant improvement in power transfer, but it should not be applied indiscriminately as there are cases where it will harm the performance of a laser energy transmission system.
### J. NOMENCLATURE

<table>
<thead>
<tr>
<th>Letter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Diffraction pattern parameter for the gaussian distribution</td>
</tr>
<tr>
<td>B</td>
<td>Pointing function parameter for a gaussian distribution</td>
</tr>
<tr>
<td>D</td>
<td>Nominal beam diameter</td>
</tr>
<tr>
<td>E</td>
<td>Expectation value of normalized intercepted power</td>
</tr>
<tr>
<td>F</td>
<td>Diffraction pattern function</td>
</tr>
<tr>
<td>G</td>
<td>Pointing function</td>
</tr>
<tr>
<td>I</td>
<td>Light intensity</td>
</tr>
<tr>
<td>J</td>
<td>Bessel function of first order</td>
</tr>
<tr>
<td>K</td>
<td>Sun-earth distance, $1.5 \times 10^{11}$ meters</td>
</tr>
<tr>
<td>L</td>
<td>Laser energy fall-off distance</td>
</tr>
<tr>
<td>P</td>
<td>Total power of one laser</td>
</tr>
<tr>
<td>PA</td>
<td>Average intercepted power</td>
</tr>
<tr>
<td>PL</td>
<td>Power per unit area from the laser source</td>
</tr>
<tr>
<td>PS</td>
<td>Power per unit area from the sun</td>
</tr>
<tr>
<td>P0</td>
<td>Maximum intercepted power</td>
</tr>
<tr>
<td>R</td>
<td>Ratio of expectation values of phased to free-running arrays</td>
</tr>
<tr>
<td>S</td>
<td>Square root of target area</td>
</tr>
<tr>
<td>T</td>
<td>Target function in target plane</td>
</tr>
<tr>
<td>U</td>
<td>Light amplitude function</td>
</tr>
<tr>
<td>a, b</td>
<td>Half of the sides of a rectangular aperture</td>
</tr>
<tr>
<td>c</td>
<td>Spatial period of an array of lasers</td>
</tr>
<tr>
<td>d</td>
<td>Diameter of circular aperture</td>
</tr>
<tr>
<td>k</td>
<td>Constant</td>
</tr>
<tr>
<td>l</td>
<td>Characteristic length of gaussian distribution of light intensity</td>
</tr>
<tr>
<td>q</td>
<td>Dummy parameter</td>
</tr>
<tr>
<td>r</td>
<td>Radial distance in target plane</td>
</tr>
<tr>
<td>s</td>
<td>Half of the side of a square target</td>
</tr>
<tr>
<td>t</td>
<td>Maximum pointing error in the target plane</td>
</tr>
<tr>
<td>u, v</td>
<td>Dummy cartesian coordinates corresponding to $x, y$</td>
</tr>
<tr>
<td>w</td>
<td>Dummy parameter</td>
</tr>
<tr>
<td>x, y</td>
<td>Cartesian coordinates in the target plane</td>
</tr>
<tr>
<td>z</td>
<td>Range from laser to target</td>
</tr>
</tbody>
</table>
\( \alpha \) Influence coefficient
\( \beta \) Ratio of intercepted power at a point in the target plane to maximum intercepted power in that plane
\( \gamma \) Minimum acceptable ratio of centroid energy flux to maximum energy flux
\( \Delta \) Difference in target plane
\( \epsilon \) Fraction of laser power within a distance from the center of the beam
\( \zeta \) Ratio of laser power without to with phase control
\( \xi, \eta \) Cartesian coordinates of the aperture plane
\( \eta_L \) Conversion efficiency of laser energy
\( \eta_S \) Conversion efficiency of solar energy
\( \lambda \) Laser wavelength
\( \sigma \) Diffraction coefficient
\( \nu \) Half-angle pointing angle for gaussian pointing function
\( \tau \) Fraction of time for which the pointing system has less than a specified error
\( \varphi \) Half-angle representation of furthest extent of pointing function
\( \Phi_K \) Solar constant, \( 1.4 \times 10^3 \) W m\(^{-2}\)
SECTION IV

THE USE OF MATTER-ANTIMATTER
ANNIHILATION ENERGY IN PROPULSION

D. D. Papailiou

A. INTRODUCTION

The discovery of the positron in 1932 justified the predictions of Dirac and verified the existence of a particle-antiparticle symmetry in nature. Since then, advances in linear accelerator technology have led to the production of heavier antiparticles such as antiprotons, antideuterons, and antialphas.

When a particle and an antiparticle are brought together, annihilation occurs simultaneously with the release of large amounts of energy in the form of various particles, created during the annihilation process. A typical annihilation is that of a proton-antiproton, which develops as follows: Initially, positive, negative, and neutral $\pi$-mesons are produced according to the reaction:

$$p^+ + p^- \rightarrow \pi^+ + \pi^- + \pi^0 \quad \text{(average number of mesons produced } = 5)$$

Since $\pi$-mesons are unstable, they decay to muons, neutrinos, and $\gamma$-rays as follows:

$$\pi^+ \rightarrow \mu^+ + \nu_{\mu}, \quad \pi^- \rightarrow \mu^- + \bar{\nu}_{\mu}, \quad \pi^0 \rightarrow 2\gamma \quad (h\nu \approx 200 \text{ MeV})$$

Finally, the muons decay, producing positrons, electrons, and neutrinos:

$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu, \quad \mu^- \rightarrow e^- + \nu_e + \bar{\nu}_\mu$$
Therefore, the final products of annihilation are electrons, positrons, and \( \gamma \)-ray radiation. The energy associated with these particles represents approximately half of the total annihilation energy, the remainder being lost in the form of neutrinos.

The energy per unit mass released from the above annihilation reaction is approximately \( 4.5 \times 10^{13} \) joules per gram. This figure is about two orders of magnitude higher than the corresponding energy released from a typical nuclear fusion reaction. For instance, the fusion of four protons to form an alpha particle is followed by an energy release of \( 0.75 \times 10^{12} \) joules per gram. As indicated in the preliminary analysis presented in the Appendix, the amount of antimatter needed for propulsion is very small, of the order of \( 10^{-1} \) gm. According to these values it appears that, at least in principle, the use of matter-antimatter annihilation energy for propulsion is very attractive. However, several difficult problems must be solved before it would become feasible.

The purpose of the present section is to summarize existing information on the subject and to discuss problems related to the use of the matter-antimatter annihilation energy for propulsion. The material presented is as follows: The question of antimatter production is examined (paragraph IV-B); existing information on atom-antiatom (ion-antiion) interatomic potential energy is presented (IV-C)—this information is necessary for calculations of annihilation cross sections and annihilation energy rates; questions of matter-antimatter separation and the problem of controlling matter-antimatter annihilation processes are considered (IV-D); possible schemes for antimatter storage are presented (IV-E); and a number of studies are suggested (IV-F) on problems related to the application of antimatter to propulsion.

B. ANTIMATTER PRODUCTION

There are two possible ways of obtaining antimatter: It can be produced in the laboratory, or it can be obtained from existing sources in our galaxy. The possibility of utilizing antimatter available in nature is attractive; however, its existence in large quantities, even within the metagalaxy\(^*\), is debatable.

\(^*\) Total number of galactic systems.
Direct detection of antimatter in the universe is presently not possible, because of the symmetric structure of matter and antimatter (particles and antiparticles have equal mass and opposite quantum numbers and electric charges). The only possible way of detecting antimatter in the universe is by studying matter-antimatter annihilation reactions. The detectable final products of these reactions are γ-ray radiation (average energy $\hbar \nu \approx 200 \text{ MeV}$) and radio waves.

The question concerning the existence of antimatter in the universe is of interest to cosmologists in relation to the development of theories explaining the origin and evolution of the universe (113-116) and also theories explaining the enormous amounts of energy needed for quasars, Seyfert galaxies, and radio stars. (117)

Arguments for the existence of large amounts of antimatter in the universe are presented by the proponents of the theory of a "symmetric universe" containing equal amounts of matter and antimatter (Klein, (113) Alfven and Klein (118)). However, according to Steigman and Hoyle, (119) existing measurements (120, 121) of energetic γ-rays ($\hbar \nu \approx 200 \text{ MeV}$) set limits on the amount of antimatter in the universe. These measurements are interpreted by these investigators as indicating that interstellar gas is overwhelmingly of one kind ($\overline{M} \ll M_\odot$). It should be noted that the above-mentioned measurements indicate the existence of a line source of γ-ray radiation of strength $\frac{dF_\gamma}{d\Omega} \approx 2 \times 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1} \text{ rad}^{-1}$ in the galactic plane, and also of a source of strength $F_\gamma \approx 10^{-4} \text{ cm}^{-2} \text{ sec}^{-1}$ in the direction of the galactic center. Therefore, it appears that the amount of antimatter in the universe rather than its existence is questioned by cosmologists.

At present antimatter is produced in the laboratory either in the form of a positron beam which, together with an electron beam, circulates in "storing rings" (122) or in the form of antinuclei (antiprotons, antideuterons, etc.) produced by means of large linear accelerators. (123) The electrons and positrons produced during $p - p^-$ annihilation will also interact; however, the released annihilation energy during $e - e^+$ interaction is very low (0.5 MeV) compared to that produced during $p - p^-$ annihilation. Therefore, $e - e^+$ annihilation will be excluded from the present discussion.
Large linear accelerators (such as, for instance, the one in use at Los Alamos) can produce a proton beam of \( \sim 10^{15} \) protons per second. When such a beam collides with a target, antiprotons are produced as part of the debris. The antiproton yield is independent of beam energy and is of the following order of magnitude:

\[
yield \equiv \frac{n^-_P}{n_{\text{total}}} \lesssim 10^{-2}
\]

Factors such as limitation in the number of antiprotons which can be captured by an applied magnetic field and optimum size of the target in order to avoid collisions of antiprotons with target nuclei reduce further the production of antiprotons by two orders of magnitude. Hence, the rate of antiproton production is of the order

\[
\frac{dn^-_P}{dt} \approx 10^{11} \text{ antiprotons/second}
\]

Based on this number the estimated time to produce one kilogram of antiprotons is

\[
t \approx 1.9 \times 10^8 \text{ years}
\]

This number led Steigman \(^{(123)}\) to the conclusion that the use of antimatter for propulsion purposes is not feasible. However, given that the energy released by annihilating one gram of antimatter is approximately equal to \( 4.5 \times 10^{13} \) joules, it appears that even a small antimatter mass of the order of a mgr might be sufficient for propulsion purposes (see Appendix A). According to Kantrowitz \(^{(124)}\) the amount of energy required to place 1 lb in a lower orbit is approximately \( 1.62 \times 10^7 \) joules. The time required to produce 1 mgr of antimatter is

\[
t \approx 1.9 \times 10^2 \text{ years/mgr}
\]
Furthermore, advancement in the areas of producing proton beams of increased intensities and improving the remainder of the factors affecting antimatter production which most likely will occur in the future are expected to reduce the production time. It should also be pointed out that the presently used methods are not designed for antimatter production but rather for studies in the field of the physics of elementary particles.

The question of producing heavier antiparticles such as antideuterons, antialphas, and even heavier ones, has been examined by Hagedorn. The results of this study are rather pessimistic since the indication is that the production of antinuclei with $A$ antinucleons relative to antiprotons drops according to the relation

$$\frac{n(A)}{n(p)} \sim 10^{-5}(A-1)$$

The following conclusions pertaining to production can be stated:

1) The question of antimatter occurrence in nature cannot be answered conclusively at the present time. Furthermore, even if antimatter were to be found in the future at distances within our reach, its practical utilization might present serious problems. To demonstrate the point, consider the simple question of how a matter-spacecraft can survive in an antimatter environment.

2) The laboratory production of antimatter in sufficient quantities will possibly be feasible in the not-too-distant future. A study to investigate the possibility of increasing production rates at the level required for propulsion applications is presently needed. The problems of antimatter storage, annihilation rate control, and development of practical propulsion schemes are those on which future studies should also be focused.
C. MATTER-ANTIMATTER ANNIHILATION PROCESSES - ANNIHILATION CROSS SECTION

Knowledge of the important annihilation reactions occurring in a matter-antimatter mixture is essential to the derivation of annihilation cross-section expressions corresponding to these processes. Factors defining the dominant annihilation mechanisms in a matter-antimatter mixture are particle-antiparticle relative velocities (kinetic energies), their degree of ionization, and their number densities. The information concerning annihilation cross section is needed for estimating the rates of annihilation and energy release during the annihilation processes. The knowledge of cross sections is needed for the studies of the application of the matter-antimatter annihilation concept to propulsion, for use in solving problems related to particle-antiparticle separation, the storage of antimatter, and the control of annihilation rates.

Matter-antimatter interactions take place on three different scales: (1) large scale interactions, (2) atomic scale interactions, and (3) nuclear scale interactions. Interactions (1) and (2) are discussed below. The proton-antiproton nuclear reactions discussed in the Introduction are sufficient for the purpose of this study.

1. Large Scale Interactions

Large scale interactions involve the presence of electric, magnetic, and gravitational fields. Typical of studies on large scale interactions are those made by Alfven and Klein(118) and by Alfven(126). Although the emphasis in these studies is placed on the cosmological aspects of matter-antimatter interaction, several elements of these studies, referring to the behavior of an "ambiplasma" in the presence of various fields, are conductive to the study of the problems related to the application of the concept in propulsion.

2. Atomic Scale Interactions

Several interaction mechanisms leading to matter-antimatter annihilation have been described, mainly by Morgan and Hughes(127, 128) and by Morgan(129). In general, they can be classified as follows:
1) Direct annihilation.
2) Radiative capture leading to the formation of a positronium (Ps), in the case of e\(^+\) - e\(^-\) interactions, a protonium (Pt) in the case of p - p\(^-\) interaction, or a nucleonium* in the case of heavier particles, with the simultaneous emission of a photon. (127)
3) Rearrangement collisions in which atoms, ions, or molecules form bound states before actual annihilation occurs.

The conditions for which a particular annihilation process assumes the predominant role in a matter-antimatter mixture are as follows:

a. Direct Annihilation

For collision energies above about 100 eV or for a completely ionized particle-antiparticle mixture, the direct annihilation cross sections apply. (127) Morgan and Hughes have investigated the cases of e\(^+\) - e\(^-\) and p - p\(^-\) direct-annihilation cross sections.

For kinetic energies above 10 keV for e\(^-\) - e\(^+\) interactions and 10 MeV for p - p\(^-\) interactions, measured in the center of mass system, the Coulomb field effect is negligible and therefore the plane wave approximation(130) can be applied to estimate annihilation cross sections. Morgan and Hughes(127) applied the plane wave approximation to calculate e\(^-\) - e\(^+\) annihilation cross sections above 10 keV. They also used available experimental data of p - p\(^-\) annihilation cross sections(131) in the range 20 MeV to 6 GeV to suggest an estimate for p - p\(^-\) cross sections in the range 10 MeV to 1 GeV. In this range they noticed that the p - p\(^-\) cross section curve is approximately fitted by the expression

\[ \sigma_{ap}(v) = 0.19 \sigma_{ae}(v) \] (where v is velocity)

*The definitions of Ps and Pt are given in paragraph IV-C-2-b. The formation of a nucleonium is the result of the interaction of a nucleus and an antinucleus with the simultaneous emission of a photon.
where, for nonrelativistic velocities, the $e^- - e^+$ cross section $\sigma_{ae}$ is given by the expression

$$\sigma_{ae} = \left(\frac{c}{v}\right) \pi r_o^2$$

For kinetic energies below 10 keV and 10 MeV for $e^- - e^+$ and $p - p^-$ interactions, respectively, Coulomb field interactions cannot be neglected. Morgan and Hughes (127) considered this effect in estimating direct annihilation cross sections by multiplying $\sigma_{ap}$ and $\sigma_{ae}$ respectively, by the ratio of the square of the amplitude of the Coulomb wave functions for zero $e^- - e^+$ and $p - p^-$ separation, and by the corresponding Coulomb wave functions in the plane wave approximation.

b. Radiative Capture

As already mentioned, radiative capture is another mechanism which can contribute to particle-antiparticle annihilation. The following radiative capture reactions can occur in a mixture of ionized $\text{H} - \overline{\text{H}}$.

$$e^- + e^+ \rightarrow \text{Ps} + h\nu$$

$$p + \overline{p} \rightarrow \text{Pn} + h\nu$$

For particle densities less than $10^{11}$ cm$^{-3}$ the so-formed Ps and Pn remain intact until annihilation occurs and therefore, for these densities, radiative capture should be considered in estimating total annihilation cross sections. Morgan and Hughes (127) derived expressions for radiative capture annihilation cross sections for both $e^- - e^+$ and $p - p^-$ reactions ($\sigma_{re}$, $\sigma_{rp}$). However, they also found that although, for $e^- - e^+$ reactions, $\sigma_{ae}$ and $\sigma_{re}$ have comparable values, for $p - p^-$ reactions, values of $\sigma_{ap}$ far exceed corresponding values of $\sigma_{rp}$ for all energies of interest. Given that the annihilation energy released during $e^- - e^+$ reactions is negligible in comparison with that released from $p - p^-$ reactions, we can conclude that, in general, the contribution of radiative capture reactions should be neglected in calculating annihilation energy rates. However, they might influence other factors such as, for
instance, the consistency of an "ambiplasma". The consequences of changes in electron-positron concentrations will be discussed below in paragraphs IV-D and IV-E.

c. Rearrangement Collisions

Rearrangement collisions become important at energy levels below those corresponding to the total atomic binding of the electrons in an atom and positrons in an antiatom (about 100 eV for an H - \( \bar{H} \) mixture). Within this low energy range, rearrangement cross sections are far greater than direct annihilation cross sections and, therefore, particle-antiparticle annihilation cross sections are equal to rearrangement cross sections. The above statements are correct, provided that particle-antiparticle densities are not sufficiently high to lead to breakup and provided, also, that atoms and antiatoms are not wholly ionized (completely stripped of their electrons or positrons). Several forms of rearrangement collisions can occur in a matter-antimatter mixture \(^{(128, 129)}\); however, they all fall into three main categories: (1) atom-antiatom, (2) atom-antiion (ion-antiatom), and (3) ion-antiion collisions. To estimate annihilation cross sections for a given atom-antiatom pair, the interatomic potential energy of this pair should be known. This is not easy to determine and in almost all cases it involves extensive calculations. For instance, in the work by Junker and Bardsley, \(^{(132)}\) which represents the best existing calculations on H - \( \bar{H} \) interaction, a wave function containing up to 75 terms was used to estimate interatomic potential energies. Considerable work on atom-antiatom interatomic potential energies has been carried out by Morgan and Hughes in relation to the study of cross sections for rearrangement collisions. The method is based on a perturbation expansion for the estimation of the interatomic potential \( V \), in which the perturbation potential energy is the sum of the Coulomb energies between the particles of the atom and the particles of the antiatom. Morgan \(^{(129)}\) estimated upper and lower limits of \( V \) for spherically symmetric atoms and antiatoms, from which he derived corresponding lower and upper bounds of rearrangement annihilation cross sections. In modeling the perturbation potential, the following common features of atom-antiatom interaction were considered. First, for an atom-antiatom
pair, there is no interatomic exchange energy as in the case of an atom-atom pair because of the opposite signs of electrons and positrons, a state which prohibits their interchange in an atom-antiatom pair. Second, for close enough atom-antiatom distances \((R \rightarrow 0)\), the potential \(V\) is given by the relation

\[
V = -\frac{Z\bar{Z}}{R}
\]

which is the Coulomb potential energy between the positive nucleus with \(Z\) protons and the negative antinucleus with \(\bar{Z}\) antiprotons. For further description of the potential \(V\), the particular character of each of the three cases mentioned above should be considered. For instance, for atom-antiatom pairs (case (I)), the lowest order term of the expansion, \(E_1\), can be used as the upper bound for \(V\). \(E_1\) is the electrostatic potential energy between the nucleus and the undistorted electron charge distribution of the atom and the antiatom. For the lower bound the long range van der Waals potential of the form \(V = C/R^6\) can be employed. It should be emphasized that, obtained this way, the upper and lower bounds of annihilation cross section can be different by several orders of magnitude and therefore can only be used to obtain a very rough estimate of the range within which annihilation cross sections and annihilation rates lie. The accuracy of these calculations will be demonstrated in paragraph IV-E, where some estimates will be given in relation to the problem of antimatter storage. The various particle-antiparticle collision mechanisms and the conditions under which they are significant in defining annihilation rates are summarized in Table 11.

In conclusion, it should be mentioned that more accurate annihilation cross-section calculations are needed for the study of rearrangement collisions which, as will be discussed in paragraph IV-E, are necessary for the study of annihilation processes in liquids and solids.
Table 11. Particle-antiparticle collision mechanisms

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Conditions for Which the Mechanism Contributes Significantly to the Annihilation Cross Section</th>
</tr>
</thead>
</table>
| Direct annihilation: Coulomb field not important | a) Kinetic energy levels above those corresponding to the total atomic binding of the electrons of an atom or positrons of an antiatom (for a H - H mixture above about 100 eV).  
   b) Completely ionized atoms.                                                                 |
| Direct annihilation: Coulomb field important    | Kinetic energies above 10 keV and 10 MeV for $e^+ - e^-$ and $p - p^-$ interactions, respectively. |
| Radiative capture                               | a) In general, important at particle densities less than $10^{11}$ cm$^{-3}$.  
   b) Negligible for $p - p^-$ interactions for all energies of interest.  
   c) $e^- - e^+$ interaction values of radiative-capture annihilation cross sections are comparable to those due to direct annihilation. |
| Rearrangements collisions                      | Kinetic energies below those corresponding to the total atomic binding of the electrons of an atom or positron of an antiatom, provided that particle densities are low (below $10^{11}$ cm$^{-3}$) and the atoms and antiatoms are not completely ionized. |
Adequate information exists for accurate studies of matter-antimatter annihilation processes in an "ambiplasma" consisting of protons, antiprotons, electrons, and positrons. In this case, annihilation occurs through direct interactions since the antiplasma is completely ionized.

D. MATTER-ANTIMATTER SEPARATION, MATTER-ANTIMATTER ANNIHILATION REACTIONS CONTROL

Alfven\(^{118}\) has considered the problem of matter-antimatter separation. He proposed a separation mechanism involving the presence of electric, magnetic, and gravitational fields. In what follows, the concepts introduced by Alfven will be discussed in relation to problems associated with the possible development of a matter-antimatter propulsion scheme.

In his first approach to the problem, Alfven investigated an "electrolysis process" for matter-antimatter separation. Two adjacent clouds of "ambiplasma," one rich in electrons-positrons and the other in protons-antiprotons, were assumed. This particular configuration is essential for matter-antimatter separation since it was found that a "symmetric ambiplasma" cannot separate under the influence of a combined electric and magnetic field. The means by which such a configuration can be achieved will be discussed later in this section. If this configuration (Fig. 22) starts expanding, or contracting, in the presence of a magnetic field, a current loop will be formed resulting in an electron-proton motion toward the area A and a positron-antiproton motion toward the area B, thus separating matter from antimatter. This process will also result in the formation of a thin layer ("Leidenfrost") between the separated matter and antimatter, within which annihilation will occur at a lower rate. The amount of separating mass \(M\) is given by a relation similar to that describing an electrolysis process, that is

\[ M = m_H I T/e \]

where \(m_H\) = H atom mass, \(I\) = current, and \(T\) = time. The electric current appearing in this expression is related to the applied magnetic field. The properties of the "Leidenfrost" are of particular importance for separating
matter-antimatter and also for controlling annihilation rates. In particular, the qualitatively described increase of pressure in this layer caused by the high energy particles produced is of importance for separation. This pressure is transferred to the matter-antimatter clouds by means of the applied magnetic field, thus separating the two clouds further. On the other hand, the characteristics of the "Leidenfrost," including temperature and particle densities, presumably can be controlled by the applied fields and initial clouds condition, thus achieving control of the rate of annihilation in the layer.

Alfvén\textsuperscript{126} studied also the behavior of a "symmetric ambiplasma" in the presence of a gravitational field and found that diffusion of particles of different mass produced a final state of separation into regions of high $e^- - e^+$ and $p^+ - p^-$ concentrations. As already mentioned, this condition is necessary for further separation of matter-antimatter in cosmological problems; however, in propulsion applications, particle concentration might be controlled without the presence of a gravitational field.
Another important aspect of the problem of using antimatter in propulsion is that of the radiation emitted by the electrons and positrons spiraling in the magnetic field. It has been mentioned that a total of two to four electrons and positrons per annihilated couple \( p^+ - p^- \) are produced from the decay of muons with an average kinetic energy of 100 MeV. Although \( e^+ - e^- \) annihilation energy is negligible (0.5 MeV), the presence of electrons and positrons is very important in the separation processes and the release of their kinetic energy in the form of radiation. The fact that the frequency of this radiation can be controlled by the magnitude of the applied magnetic field is especially significant. In a magnetic field \( B \) the electrons and positrons emit synchrotron radiation. The decay time, \( T_d \), is

\[
T_d = \frac{5 \times 10^8}{B^2} \frac{1}{1 + \left(\frac{W}{W_o}\right)^2} \text{ sec}
\]

where \( W \) is their kinetic energy and \( W_o = m_e c^2 \) (see, for instance, Alfven and Falthammar, reference 133). If \( T_d << T_o \), where \( T_o \) is the average electron (positron) lifetime before annihilation occurs, the electrons (positrons) radiate most of their energy in the form of synchrotron radiation. If, however, \( T_d >> T_o \), very little synchrotron radiation is emitted. The energy maximum of the synchrotron radiation \( (h\nu_{\text{max}}) \) depends on the kinetic energy of the electrons and the applied magnetic field \( B \) in the following way:

\[
\nu_{\text{max}} = \frac{(eB/2\pi mc)}{(W/W_o)^2} = 3 \times 10^6 \left(\frac{W}{W_o}\right)^2 B
\]

Depending on the applied propulsion scheme, control of the wavelength might be desirable in using the synchrotron radiation energy, for instance, for the heating of a given mass of propellant. (The average kinetic energy of electrons and positrons represent the one-third of the useful annihilation energy.)

The preceding discussion leads to the following conclusions:

1) A mechanism is available which, in principle, can provide separation of a matter-antimatter mixture and control of
the annihilation reaction rates in this mixture. This mechanism involves a four-component \((e^-, e^+, p^-, p^+)\) "ambiplasma" in the presence of magnetic and electric fields.

2) Of particular importance is the study of the physical processes occurring in the "Leidenfrost" layer.

E. ANTIMATTER STORAGE

It has been discussed that the attractive feature of matter-antimatter annihilation, favoring the application of the concept in propulsion, is the large amount of energy released during annihilation. However, the same feature is the source of a number of difficult problems, including that of antimatter storage. In the latter case annihilation should be avoided.

The separation mechanism proposed by Alfvén could provide the basis for the development of an antimatter storage scheme; however, other schemes involving antimatter in liquid or solid states should also be considered. The existence of an interatomic potential barrier at a certain distance separating an atom-antiatom pair could, in principle, allow the storage of antimatter in the form of matter-antimatter mixture. Such a barrier was found by Janker and Bardsley\(^{(132)}\) in their calculation of the interatomic potential of a hydrogen-antihydrogen pair.*

The interaction of matter-antimatter in a solid or liquid state has been examined recently by Morgan.\(^{(129)}\) The annihilation processes occurring in a solid mixture of matter-antimatter are different from those occurring in a gas mixture, owing to the fixed positions of atoms and antiatoms in a solid, which prohibits atom-antiatom collisions. Annihilation in this case would first occur

* According to information released to the author by Dr. D. Morgan recently, more accurate calculations carried out by Kolos and Schraeder of Marquette University showed no bump in the potential V.
among the electrons of the atoms and positrons of antiatoms through direct annihilation or positronium formation. This initial process would leave the atom-antiatom pair with opposite charges resulting in their attraction and the formation of nucleonium and probably more positronium, which would subsequently undergo annihilation.

Morgan applied the method of upper and lower estimates of annihilation cross section to investigate the stability of the following matter-antimatter schemes.

1) Solid or liquid matter-antimatter mixture.
2) Matter-antimatter in surface contact.
3) Matter-antimatter contact involving the gaseous state.
4) Antimatter in a vacuum.

In the case of a solid matter-antimatter mixture, Morgan assumed the form \( V = E_1 + E_2 \) for the potential energy, where \( E_1 \) was calculated by using the atomic model and \( E_2 \) was taken to be proportional to \( R^{-6} \) (van der Waals potential energy for large interatomic distances). Based on this model, expressions for annihilation rates and corresponding energy production were obtained as functions of the interatomic distance \( R \). Subsequently, a tolerable amount of heat production was assumed to be equal to 1 cal mol\(^{-1}\) sec\(^{-1}\) and the interatomic distance and lifetime corresponding to this amount were estimated for H - H and He - He mixtures. These values were compared to calculated values of the interatomic distance \( R_{\text{min}} \) at which a minimum in \( V \) appears, and to the lifetimes and energy production rates corresponding to it. These values for an H - H mixtures are shown in Table 12.
Table 12. Minimum and maximum lifetimes of an $H - \overline{H}$ mixture

<table>
<thead>
<tr>
<th>$R$ (atomic units)</th>
<th>$W$ (cal/mol sec)</th>
<th>Lifetime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 28.5$</td>
<td>1</td>
<td>14,203 years</td>
</tr>
<tr>
<td>$R_{\text{min}} = 9.7$</td>
<td>$2.932 \times 10^{15}$</td>
<td>$1.5 \times 10^{-4}$ sec</td>
</tr>
</tbody>
</table>

It is evident from these results that this storage scheme is not feasible.

The same model was used to estimate the degradation rates of matter-antimatter at surface contact. The calculations showed that the velocity with which the two surfaces are approached in order to maintain contact ranges from approximately $10^{-1}$ to $10$ cm/hr and the energy released due to annihilation is of the order of $10^{14}$ cal cm$^{-2}$ sec$^{-1}$. The large rates of released energy rule out the possibility of using this scheme for antimatter storage. It should be mentioned, however, that besides the uncertainty in these results due to the inadequacies of the model of $V$ employed, the approach that was followed is oversimplified, since it does not take into account important physical processes, such as the formation of a "Leidenfrost" layer between the contact surfaces. Consideration of such phenomena might completely change the results contained.

Because of the estimated high rate of release of annihilation energy, Morgan rejected the possibility of storing a gaseous mixture of matter-antimatter in a magnetic bottle or in a solid-matter container, and also the possibility of storing gaseous antimatter in a solid-matter container.
In the case of electromagnetic suspension of solid antimatter in a vacuum, annihilation would occur only if vaporized antimatter atoms collide with the solid-matter walls of the enclosure. An upper limit to the annihilation energy rates can be estimated by assuming that the annihilation cross section of vaporized antiatoms colliding with the matter wall atoms is the same as that of an antiatom interacting with an individual atom of the wall. It is also assumed that all the antiatoms striking the wall surface undergo annihilation. Under those conditions the energy production rate \( \omega \) is related to the lifetime in years by the simple expression

\[
\omega = 6.83 \times 10^5 \frac{\overline{A}}{F} \text{ cal mol}^{-1} \text{ sec}^{-1}
\]

where \( \overline{A} \) is the atomic number of antimatter. This scheme for storing antiatoms was suggested by Morgan\(^{129}\) as feasible.

Based on the preceding discussion the following conclusions concerning antimatter storage can be drawn:

1) Presently, it appears that the most promising scheme for antimatter storage is that of solid antimatter magnetically suspended in vacuum.

2) Besides the mechanism of matter-antimatter separation discussed in paragraph IV-D, which was proposed by Alfven, the possibility of storing antimatter in the form of a matter-antimatter mixture and also the possibility of storing solid matter in contact with solid antimatter, should be explored. Specific problems in these areas are discussed in paragraph IV-F.

F. PLANS FOR FUTURE STUDIES ON PROBLEMS RELATED TO THE USE OF MATTER-ANTIMATTER ANNIHILATION ENERGY IN PROPULSION

The preceding sections summarize the existing information on antimatter and the annihilation processes associated with particle-antiparticle reactions. In addition certain problem areas such as antimatter production and storage, matter-antimatter separation, and control of annihilation
rates are discussed in relation to the possibility of developing a matter-
antimatter propulsion scheme. In this section problems are suggested for
future studies in the above areas.

1. Antiparticle Production Methods

The antiparticles produced in large linear accelerators are currently
used for the study of elementary particle reactions. No specific efforts have
been directed toward increasing their production rates. Therefore, the
initiation of a program should be considered that would study methods specifi-
cally designed to increase antiparticle production to the required level for
propulsion application. This program should be developed in collaboration with
workers in the field of elementary particle physics. Some of the problems
which should be considered are

1) Increase of the production rate of protons in linear
   accelerators (presently $10^{15}$ protons/sec).
2) Increase of antiproton yield, which as already discussed
   is of the order $10^{-2}$.
3) Maximize the number of antiprotons collected by an applied
   magnetic field in the area of antiproton production.
4) Optimize the size of the target on which the proton beam
   impinges in order to obtain minimum annihilation between
   the produced antimatter and the matter-target.

2. Matter-Antimatter Separation Mechanism

The matter-antimatter separation mechanism proposed by Alfven
(paragraph IV-D) should be studied. The configuration for study would be
two clouds - one of electrons-positrons and the other of protons-antiprotons -
moving in a magnetic field. The assumption that the particles are in thermo-
dynamic equilibrium corresponding to a certain temperature can be made and
a velocity distribution function can be defined. The physical processes occur-
ring in this configuration would be studied, including annihilation processes and
synchrotron radiation. Of particular importance is the study of the character of
and the phenomena occurring in the "Leidenfrost" layer in relation to the possible use of the layer for annihilation-reaction control. The formulation of the equations describing these phenomena and an initial dimensional analysis of these equations might reveal some essential characteristics of the problem. Also, the question of "criticality" in matter-antimatter reactions should be studied; that is, the conditions under which the rates of annihilation energy released are too high, leading to "explosion."

3. Antimatter Storage

In the area of antimatter storage the following problems should be studied:

1) The possibility of reducing annihilation rates in the case of matter-antimatter in contact, due to "Leidenfrost" formation. The influence of a magnetic field on the character of this layer is of importance.

2) The possibility of changing the interatomic potential of an atom-antiatom pair by applying a strong magnetic field. This possibility, which would allow storage in the form of a matter-antimatter mixture, has been suggested by Morgan.

3) Finally, Morgan's proposed storage scheme of solid antimatter magnetically suspended in vacuum should be further investigated.

4. Annihilation Cross Sections

In the field of particle-antiparticle interactions, more effort should be devoted to developing methods for accurate estimation of annihilation cross sections.
SECTION V

ELECTRONICALLY EXCITED SOLID HELIUM

J. S. Zmuidzinas

In this section the problems of energy storage in electronically excited solid helium are discussed, and some of the work done in this area at JPL and at the University of Wisconsin is reported. Earlier considerations of the problems are contained in references 134 and 135.

For reasons of economy, only the abundant isotope of helium, $^4\text{He}$, is of interest, although most of our considerations would be applicable to $^3\text{He}$, perhaps with minor modifications to reflect the isotopic mass difference. The effects of nuclear spin and statistics will not be important in what follows.

A. ENERGY-STORAGE CAPABILITIES OF SOLID HELIUM

From among an infinite number of possible excited electronic states of the helium atom, only the metastable $2^3\text{S}$ state is of practical interest for energy-storage applications. The reason for this is that only the $2^3\text{S}$ state has a sufficiently long lifetime (~2.3 hours); all of the other states are much shorter-lived. The excitation energy of the $2^3\text{S}$ state is 19.8 eV. For the case of a 100% electronically excited solid helium, this translates to a specific energy of approximately 500 megajoules/kg or 0.5 megajoules/cm$^3$, the density of solid helium being close to that of water at pressures $\geq 25$ atm. This specific energy is almost two orders of magnitude higher than that of the most energetic chemical fuel presently available, the hydrogen-fluorine system.

B. LIFETIME CONSIDERATIONS

The theoretically predicted lifetime of the $2^3\text{S}$ state of the helium atom is quite long;\,(136,137) however, this lifetime may, in general, be considerably shortened when the atom finds itself in a solid-helium lattice, because in that environment various perturbing influences are present. Thus the problems
arise: (1) how to estimate the influence of the lattice on the lifetime and (2) how to stabilize the excited atoms and thereby prolong their lifetime. A number of physical processes tending to modify the He(2\(^3\)S) lifetime in solid helium have been identified and described in references 134 and 135. Later in this section some recent work on the problem will be discussed, specifically the influence of phonons and excitons on the He(2\(^3\)S) lifetime.

C. EXPERIMENTS IN PROGRESS

Experiments have recently been started at the University of Wisconsin to produce electronically excited solid helium and investigate its physical properties. The principal investigator is Professor W. A. Fitzsimmons, who has had a great deal of experience in the investigation of electronically excited superfluid helium.

In the Wisconsin experiments, about 1 cm\(^3\) of solid helium is produced in a pressure cell at a temperature of about 1.45\(^\circ\)K, with pressures varying from 27 to 29 atm. At this temperature, liquid helium II solidifies to the bcc phase at about 27 atm, as verified in these experiments. When the pressure is raised, a bcc \(\rightarrow\) hcp phase transition, also observed, occurs at about 28 atm. Solid helium is excited by an incident pulsed, 200-keV electron beam of approximately 0.1 \(\mu\)A. The presence of excited a\(^3\Sigma\) helium molecules (bound states of He(2\(^3\)S) and ground-state atoms) in the target is inferred by observing the 2.1 micron a\(^3\Sigma \rightarrow b\(^3\)\Pi\) molecular absorption band in the sample. At present it is not clear whether the observed transition occurs in solid or liquid helium; the latter could be produced locally as a result of heating the solid by an excessive electron beam current. Experiments are now being performed to resolve this question; after these experiments, the lifetimes, mobilities, concentrations, etc. of the various electronic excitations in solid helium will be studied.

D. THEORETICAL WORK AT JPL

So far, theoretical work at JPL has been mainly concerned with the case of relatively low (\(\lesssim 10\%\)) concentrations of excited atoms/molecules in solid helium, because such concentrations are expected to be typical of the initial
experiments on solid helium. Later, both experimental and theoretical studies will be extended to the high-concentration regime.

To simplify the theoretical analysis, it is convenient to study the behavior of a single excited atom (henceforth denoted by He*) inside a host lattice of normal helium atoms. Such treatment neglects He*-He* interactions and is expected to be valid for the condition of low He* concentrations being investigated. In analogy to the situation in superfluid helium, it can be expected that excimers (excited atoms and molecules) will be trapped inside cavities or "bubbles" in solid helium. The bubble effect should help to isolate the excimer from the perturbing influence of the surrounding lattice and thus should help to moderate any lifetime-shortening effects attributable to the lattice. To understand the physics of excimer trapping, calculations are being made at JPL to determine bubble parameters (principally the lattice distortion around an excimer and the energy of the distorted lattice).

To this end, a computationally very economical cell model of quantum solids has been developed and tested to compute the ground-state properties of hcp solid helium. The predictions of this simple model agree remarkably well with experimental data and with the much more sophisticated and expensive Monte Carlo and dynamical field calculations. The model is now being used to compute the lattice properties of an He* atom in hcp solid helium.

In parallel with the above calculations, studies have been made of the effects of phonons and excitons on the radiative lifetime of an He* atom in solid helium. The basic physics of the situation is briefly this: As the excited atom radiatively decays, the bubble around it collapses and in so doing overshoots, thereby exciting lattice vibrations or phonons. The excitation of the phonons has two effects. The first is that the emitted line is shifted from its normal free-atom position. The second effect is that the emission of phonons, along with the photon, contributes to an effective shortening of the radiative lifetime of the 2\(^3\)S state. Fortunately, this shortening effect is rather small, of the order of

\[
\frac{\text{lattice distortion energy}}{\text{electronic excitation energy}} \lesssim 0.01
\]
A much more dramatic effect is expected as a result of the coupling of the emitted photon to excitonic modes of solid helium. Preliminary calculations show that, under favorable circumstances, the phenomenon of photon trapping may occur. This happens if the emitted photon has a frequency which falls into the opacity band of the solid helium lattice, i.e., into a region of frequencies for which wave propagation with real wave vectors is not allowed by the dispersion relation of the medium. Since there is a finite, even though generally small, probability that an arbitrary configuration of phonons can be emitted along with the photon, the frequency of the emitted photon can vary considerably and hence the conditions for perfect photon trapping can never be realized in practice. Our studies indicate, however, that partial photon trapping is feasible and could lead to a substantial lengthening of the radiative lifetime of the $\text{He}^*$ state in solid helium. To provide unambiguous answers about the feasibility of photon trapping, much more refined calculations are needed, and these are now being undertaken. One of the basic ingredients in those calculations is the dielectric tensor of solid helium at vacuum ultraviolet wavelengths. In general, calculations of dielectric properties of materials from "first principles" are very difficult. In the case of solid helium, these difficulties are mitigated by the fact that atoms in solid helium interact very weakly (by van der Waals forces). Because of this, it can be expected that the electronic structure of solid helium will be characterized by narrow conduction bands, and the simple tight-binding approach should be adequate. Work along these lines is now in progress.
SECTION VI

ENERGY EXCHANGE MECHANISM FOR PROPULSION APPLICATION

D. D. Papailiou

A. INTRODUCTION

It is known that enormous energy resources exist in space associated with electric, magnetic, gravitational, and other fields. It is also known that the major difficulty in tapping this energy arises from the extremely low density level at which this energy is stored in space. Several concepts have been proposed during the last 20 years for utilization of this energy. Examples of such efforts are Alfvén's "sailing in the solar wind" and Forward's "zero thrust vector control" concepts.

The main objective of this study is to identify and investigate schemes for efficient utilization of these energy resources for application to propulsion.

To tap the energy stored in the various fields in space, one can in principle seek a physical mechanism which, for example, can be utilized to produce an exchange of energy between the magnetic field and an appropriate second field coupled to it, located aboard the spacecraft. Once such a mechanism is identified, the following questions concerning its feasibility must be examined.

1) Under what conditions would the exchange process result in a net increase of energy associated with the field on the spacecraft?

2) Under what conditions would the energy exchange rates be high enough for practical utilization of the concept in propulsion?

Some of the pertinent problems for an additional investigation are those concerning energy storage and the design of an efficient scheme for utilization of the "trapped" energy. In the following sections a novel, conceptually promising energy exchange mechanism is presented and the questions regarding
its feasibility are examined. The concept incorporates a mechanism for the utilization of the energy associated with a fluctuating magnetic field. These fields, which according to observations are particularly strong in the vicinity of planets such as the earth and Jupiter, (145) can be coupled with velocity fields of electrically conducting fluids in turbulent motion located on the spacecraft. Under certain conditions, to be discussed in the following section, the total energy of the turbulent flow field can be increased at the expense of the energy associated with the fluctuating magnetic field.

B. INTERACTION OF FLUCTUATING MAGNETIC AND FLOW FIELDS

The energy transfer processes which occur between a fluctuating magnetic field and the velocity field of a conducting fluid in turbulent motion have been studied by astrophysicists and fluid mechanics investigators in relation to questions concerning the origin and amplification of weak magnetic fields observed in stars. Actually, these studies are part of a more general effort to develop plausible theories (146, 147, 148) (dynamo theories) to explain the mechanism by means of which the magnetic fields existing in the stars sustain themselves.

Batchelor (149) first investigated the conditions under which the turbulent energy of a conducting fluid could be transferred to a weak magnetic field and result in its amplification.

The equation describing the rate of change of a magnetic field \( \overrightarrow{\mathbf{B}} \) coupled with a velocity field \( \overrightarrow{\mathbf{V}} \) of a conducting medium is of the form

\[
\frac{\partial \overrightarrow{\mathbf{B}}}{\partial t} = \nabla \times (\overrightarrow{\mathbf{V}} \times \overrightarrow{\mathbf{B}}) + \nu_m \nabla^2 \overrightarrow{\mathbf{B}} ; \nu_m = \frac{1}{\mu \sigma} \tag{1}
\]

where \( \nu_m \) is the magnetic diffusivity, \( \mu \) is the magnetic permeability and \( \sigma \) is the electric conductivity. In deriving Eq. (1) it was assumed that \( \nu/\sigma \ll 1 \) (c = speed of light). The equation for the rate of change of the energy in the fluctuating magnetic field per unit volume can be derived from the above equation. This equation has the following form: (148)
The above equation can also take the form: (149)

\[
\frac{1}{2} \frac{d|B|^2}{dt} = \vec{B} \cdot \nabla x (\nabla \times \vec{B}) + \nu_m \frac{\nabla^2}{B^2} \nabla \times \vec{B}
\]

In Eq. (3) the subscript \( B \) means a component in the direction of magnetic force and subscript \( i \) indicates a summation over three orthogonal components. The magnetic and velocity fields are coupled through the first term in the righthand part of Eqs. (1), (2), and (3). This term in Eq. (3) can be interpreted physically as representing the flow of energy to or from the turbulent velocity field, the direction depending on the effect that the velocity gradient \( \nabla \times \vec{B} \) has on the magnetic lines. A stretching of the magnetic lines corresponds to an increase in the magnetic energy \( |B|^2 \) at the expense of the velocity field, while a contraction of the magnetic lines indicates a flow of energy in the opposite direction. The second term in Eqs. (2) and (3) represents, as can be easily seen, losses in the magnetic energy in the form of Joulean dissipation, \( \frac{J^2}{\sigma} \) (\( J = \) current density).

An efficient dynamo-mechanism capable of transferring energy from the turbulent fluid to the magnetic field requires a Joulean dissipation term smaller than the coupling term (Eqs. (2) and (3)). As will be discussed in the following section, this is a very restricted, if at all possible, case, the common procedure being that of a rapid decay of the magnetic field energy into Joule heat. Since the successful use of the concept in propulsion requires a transfer of energy from the magnetic to the turbulent flow field rather than a flow of energy in the opposite direction (which is required for an effective dynamo mechanism), the above results are favorable for this application.
C. APPLICATION OF THE CONCEPT IN PROPELLION

The basis for the application of the concept of fluctuating magnetic field-turbulent conducting flow interaction in propulsion is shown in Fig. 23. In this figure a conducting fluid (e.g., mercury, liquid sodium, or ionized gas) which is in turbulent motion, is shown aboard a spacecraft moving with a velocity $U_0$ through an external fluctuating magnetic field $B$. The conducting turbulent fluid occupies the volume $S_0L_0$, as shown in Fig. 23, which might be, in the case of mercury or sodium, a channel of length $L_0$ and cross section $S_0$.

![Diagram showing fluctuating magnetic field and turbulent flow interaction](image)

**Fig. 23.** Fluctuating magnetic field/turbulent conducting flow interaction

The energy density of the fluctuating magnetic field $B$ is $\frac{1}{8\pi\mu}B^2$. The conducting fluid, as it moves with the velocity $U_0$, sweeps a volume per unit time equal to $S_0U_0$. If the magnetic energy contained in this volume could be annihilated and transferred to the conducting turbulent fluid in another form, through the energy transfer mechanism explained above, then the power $P$ gained by the conducting fluid is

$$P = U_0S_0 \frac{|B^2|}{8\pi\mu}$$
For a value \( B^2 \approx 1 \text{(gauss)}^2 \), which is a characteristic order of magnitude value of a fluctuating magnetic field (for instance, near Jupiter\(^*\)) and for values \( U_0 = 3 \times 10^4 \text{ m/sec} \) and \( S_0 = 1 \text{ m}^2 \), the power gain is

\[
P = 120 \text{ watts}
\]

In the following section an attempt will be made to answer the questions posed in the Introduction concerning the conditions under which the application of the concept in propulsion could be feasible.

D. FEASIBILITY CONSIDERATIONS

The question concerning the direction of energy flow between the coupled fluctuating magnetic and velocity fields was first considered by Batchelor.\(^{(149)}\)

His analysis was based on the similarity between Eq. (3) and the vorticity equation in fluid mechanics,

\[
\frac{1}{2} \frac{d}{dt} \left| \omega \right|^2 = \left| \omega \right|^2 \left| \frac{\Delta V}{\Delta x} \right| \omega - \nu \left| \nabla \omega \right|^2
\]

where \( \omega \) represents vorticity fluctuations and \( \nu \) is the kinematic viscosity of the fluid. He concluded that a hydromagnetic dynamo, in which energy flows from the velocity field to the magnetic field, can operate only under the condition \( \nu_m < \nu \). In the case in which \( \nu_m > \nu \), the Joulean dissipation term in Eqs. (2) and (3) is much larger than the coupling term in the same equations. In this case the magnetic field decays very rapidly into heat, imparted to the electrically conducting medium. Table 13 shows values of the magnetic diffusivity \( \nu_m \) for mercury, liquid sodium, and ionized hydrogen. It can be seen in Table 13 that, in all three cases, \( \nu_m > \nu \).

\(^*\)Precise estimates of the energy associated with the fluctuating magnetic field can be obtained only from related power spectra. This information is currently not available. However, available results show that the fluctuating magnetic field near Jupiter is much stronger than that of the earth and that the characteristic value of \( 1 \text{(gauss)}^2 \) adopted here is reasonable.
Table 13. Values of physical quantities for various conducting media

<table>
<thead>
<tr>
<th>Media</th>
<th>$\sigma$ (mho/m)</th>
<th>$\nu$ $(m^2/sec)$</th>
<th>$\nu$ $(m^2/sec)$</th>
<th>$T$ $(^\circ K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Hg</td>
<td>$1.07 \times 10^6$</td>
<td>$7.74 \times 10^{-1}$</td>
<td>$1.14 \times 10^{-7}$</td>
<td>293</td>
</tr>
<tr>
<td>Liquid Na</td>
<td>$10.4 \times 10^6$</td>
<td>$7.65 \times 10^{-2}$</td>
<td>$7.6 \times 10^{-7}$</td>
<td>373</td>
</tr>
<tr>
<td>Ionized H$_2$</td>
<td>$5.3 \times 10^4$</td>
<td>15</td>
<td>10</td>
<td>$10^5$</td>
</tr>
</tbody>
</table>

The above analysis indicates that any of the conducting fluids considered is suitable as a working fluid in the energy exchange scheme shown in Fig. 22. It also answers question 1) posed in the Introduction concerning the direction of energy flow between coupled turbulent flow fields and fluctuating magnetic fields.

To answer the second question posed in the Introduction, we will examine the conditions under which complete annihilation of the magnetic field can be achieved by transferring this energy to the conducting fluid in the form of Joule heat.

It has been found that for the working fluids considered $\nu_m > \nu$ and, therefore, the coupling term in Eq. (3) is negligible as compared to the diffusion term $\nu_m |\nabla B|^2$. In this case Eq. (3) takes the form

$$\frac{1}{2} \frac{d |B|^2}{dt} = -\nu_m |\nabla B|^2$$

This is a diffusion equation indicating that the magnetic field decays into the conducting fluid, of conductivity $\sigma$, with decay time $\tau_D$ of the order of

$$\tau_D \approx \frac{L_0^2}{\nu_m}$$
The characteristic time indicating the period during which a given fluctuation of the magnetic field interacts with the turbulent velocity field of the conducting fluid is of the order

$$\tau_I = \frac{L_0}{U_0} \quad (8)$$

In order to achieve complete annihilation of the fluctuating magnetic field, the decay time $\tau_D$ should equal or be less than $\tau_I$; that is,

$$\tau_D \leq \tau_I \quad (9)$$

Table 14 gives values of the decay time $\tau_D$ for the conducting fluids considered in this work and also the interaction time $\tau_I$ for the following characteristic values of velocity and length.

$$U_0 = 3 \times 10^4 \text{ m/sec} \text{ and } L_0 = 1 \text{ m}$$

Table 14. Representative values of decay and interaction times for various conducting media

<table>
<thead>
<tr>
<th>Conducting Fluid</th>
<th>$\tau_D$ (sec)</th>
<th>$\tau_I$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid Hg</td>
<td>0.645</td>
<td></td>
</tr>
<tr>
<td>Liquid Na</td>
<td>6.55</td>
<td>3.33 x 10^{-5}</td>
</tr>
<tr>
<td>Ionized H$_2$</td>
<td>3.325 x 10^{-2}</td>
<td>3.33 x 10^{-5}</td>
</tr>
</tbody>
</table>

The values in Table 14 show that even in the most favorable case (ionized hydrogen) the decay time of the magnetic field is three orders of magnitude higher than the interaction time and therefore no complete annihilation of the magnetic field can be achieved. Furthermore, the magnitude of the spacecraft's velocity has an opposing effect on the power gain and the interaction time. As can be seen from Eq. (1), high values of $U_0$ are desirable.
since the power gain $P$ is directly proportional to the velocity $U_0$. However, according to its definition, the interaction time, $\tau_I$, decreases with increasing values of $U_0$, resulting in an increasingly unfavorable effect on the efficiency of the proposed scheme.

The preceding discussion leads to the conclusion that a key factor in the efficiency of the system is the characteristic decay time $\tau_D$. Therefore, at the present stage of development of the concept, effort should be devoted to investigating ways of minimizing the values of the decay time $\tau_D$ of the fluctuating magnetic field.

According to Eqs. (1) and (7), the decay time $\tau_D$ can be reduced by using conducting media with low electrical conductivity, $\sigma$; however, there is another important aspect of the problem which is expected to have favorable results in decreasing the time, $\tau_D$. It has been argued by several investigators (146, 149, 150) that, in the case of the interaction of fluctuating magnetic fields with turbulent fields of conducting fluids, the decay time $\tau_D$ is much shorter than that calculated from corresponding values of the electrical conductivity, $\sigma$. Very little is known concerning the exact mechanism by which turbulence accelerates the decay of the magnetic field into Joule heat. In general, it appears that the interaction of the turbulent motion of the conducting fluid and the fluctuating magnetic field reduces the size of the magnetic fluctuations, therefore decreasing their decay time.

Several models of turbulent "eddy magnetic diffusivity" have been suggested, mainly in relation to the problem of the hydromagnetic dynamo. Elsesser (147) introduced an eddy magnetic diffusivity $\nu_m^\prime$ equal to

$$\nu_m^\prime = \text{Re}_m \nu_m$$

where $\text{Re}_m$ is the magnetic Reynolds number equal to $U_0 L_0 / \nu_m$. Based on the above relations, the turbulent "eddy magnetic diffusivity" is

$$\nu_m^\prime = L_0 U_L$$
Similar models have been discussed by Parker, Chandrachoochar, Piddington, and others.

Although no physical insight exists at present concerning the processes involved in the decay of the magnetic fluctuations in a turbulent conducting fluid, an indication of their effect on the characteristic decay time $\tau_D$ can be obtained by using Elsseser's model. Introducing the representative values $L_0 = 1 \text{ m}$ and $U_0 = 3 \times 10^4 \text{ m/sec}$ in Eq. (11), the obtained value of the eddy magnetic diffusivity, $\nu_m^1$, is $3 \times 10^4 \text{ m}^2/\text{sec}$ corresponding to a decay time $\tau_D = 1.5 \times 10^{-5} \text{ sec}$. This value is comparable to the value of the interaction time $\tau_I = 3.3 \times 10^{-5} \text{ given in Table 13.}$

E. CONCLUSIONS AND SUGGESTIONS

It has been demonstrated in the preceding sections that the interaction of a fluctuating magnetic field with an electrically conducting fluid in turbulent motion can produce an energy flow from the magnetic field to the conducting fluid, in the form of Joule heat. It was also recognized that a major difficulty, at the present stage of development of the concept, is the long decay time of the magnetic field as compared to its interaction time with the turbulent conducting fluid. This difference between the two characteristic times $\tau_D$ and $\tau_I$, which amounts to several orders of magnitude, prohibits the complete annihilation of the magnetic field, and therefore causes a reduction in the amount of energy transferred to the conducting fluid.

The preliminary analysis conducted in the preceding sections leads to the two general conclusions:

1) The application of the concept in propulsion appears promising for flights in areas of relatively strong magnetic fields such as, for instance, those existing in the vicinity of Jupiter and other planets.

2) The rate at which energy is gained by the conducting fluid in the form of heat during interaction with the fluctuating magnetic field is low, typically of the order of $10^2$ watts. However, the concept becomes attractive in the case in which this energy is continuously
collected and stored during prolonged flights in the areas of strong magnetic fields. For instance, the energy collected over a period of 24 hours at the rate of 120 watts is equal to approximately $6 \times 10^5$ joules.

Based on the preceding discussion, the following topics are suggested for further investigation.

1) A theoretical-experimental effort should be conducted to examine the mechanism of decay of a fluctuating magnetic field in an electrically conducting fluid in turbulent motion.

2) An evaluation study of the application of the concept (i.e., the interaction of magnetic fields with conducting fluids in turbulent motion) to propulsion should be conducted, including the kind of missions for which it appears suitable.

3) An appropriate scheme should be developed for the utilization of the collected energy in propulsion.

Finally, it should be mentioned that although the analysis presented refers to the case of a fluctuating magnetic field interacting with a turbulent conducting fluid, the concept can also serve, in principle, as an exchange mechanism for collecting energy from other fields existing in space - for example, fluctuating electric fields and Alfvén waves.
SECTION VII

THERMONUCLEAR FUSION TECHNOLOGY AND ITS APPLICATION
IN SPACE PROPULSION

Teh-Ming Hsieh

A. INTRODUCTION

In space exploration, chemical and solar electric propulsion systems have inherent limitations. Nuclear propulsion systems outperform them\(^{(153-156)}\) and, for effective deep space exploration, are indispensable. Three types of nuclear reactions are possible sources of the required energy:

1) Nuclear Fission Reaction: Heavy nuclei, such as uranium, can release energy by fission following the absorption of a neutron. For \(U^{235}\), about 200 MeV\(^*\) of energy is released per fission.\(^{(157)}\) For fission energy, both the technology of instantaneous release (as in the A-bomb) and of controlled release (as in a light water reactor) have been achieved.

2) Thermonuclear Fusion Reaction: Light nuclei, such as deuterons, can fuse into heavier nuclei under certain conditions. When the fusion reactions take place, energy is released at the rate of about 7 to 9 MeV per deuteron.\(^{(158)}\) Since a deuteron weighs about 1/120th as much as a uranium nucleus, this type of reaction releases about 4.5 times the amount of energy of the fission reaction on the same weight basis. Although the technology of instantaneous fusion energy release has been achieved (as in the H-bomb), the technology of controlled fusion energy release remains to be developed.

3) Thermonuclear Fission Reaction: Some light nuclei, such as boron-11, can interact with a proton and split into several helium nuclei\(^{(159)}\) with the release of energy. The only fission products are charged

\[^{\text{*1 MeV} = 1.602 \times 10^{-13} \text{ joule} = 4.449 \times 10^{-20} \text{ kW-hr.}}\]
particles (alpha particles) - this means that superclean (particularly with respect to radioactivity) power and direct conversion to electrical energy are possible. This concept is in the earliest stages of exploration.

These nuclear reactions are summarized in Table 15.

<table>
<thead>
<tr>
<th>Type of Reaction</th>
<th>Nuclear Fission</th>
<th>Thermonuclear Fusion</th>
<th>Thermonuclear Fission</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy released per reaction (MeV)</td>
<td>~200</td>
<td>~8</td>
<td>~8</td>
</tr>
<tr>
<td>Relative energy released per unit mass (based on fission)</td>
<td>1</td>
<td>~4.5</td>
<td>~1</td>
</tr>
<tr>
<td>Relative radioactivity present in system (based on fission)</td>
<td>1</td>
<td>~10^-4</td>
<td>~0</td>
</tr>
<tr>
<td>Reaction temperature</td>
<td>Room temperature and up</td>
<td>~10^8 °K</td>
<td>~10^9 °K</td>
</tr>
<tr>
<td>Direct energy conversion without intermediate medium</td>
<td>No</td>
<td>Possible</td>
<td>Most suitable</td>
</tr>
<tr>
<td>Status of technology development</td>
<td>Well developed</td>
<td>Being developed</td>
<td>Earliest stages of exploration</td>
</tr>
</tbody>
</table>

Nuclear fission propulsion concepts have been extensively studied and some programs are well developed. (160, 161) Space propulsion systems utilizing nuclear fission energy can produce major gains in payload and trip time over chemical rockets for manned trips to Mars and Venus. (154) For unmanned missions, the mass ratio of payload to spacecraft and the length of journey are also much improved. On the other hand, thermonuclear fusion propulsion offers
even longer mission life, shorter trip time, higher ratio of jet power to propulsion system weight, and higher specific impulse. (154, 156, 162, 163)

Most literature on space fusion propulsion starts with the assumption that the fusion technology will be available some day, despite the many problems yet to be solved and the many new technologies to be advanced. It is hoped that, by briefly reviewing here the status of ground power fusion technology development, a better understanding of the potential of fusion power for space propulsion will be created. Thus the purpose of this report is to describe the state of the art of thermonuclear fusion power technology and to discuss its potential application to space propulsion. Many thermonuclear fusion problems have been repeatedly discussed in the literature; these will be mentioned only briefly here. Those problems that are important to space applications, and are either neglected elsewhere or not particularly emphasized, will be discussed in more detail.

B. BASIC CONCEPTS OF THERMONUCLEAR FUSION POWER

1. Fusion Fuels

Although ordinary hydrogen - the most abundant and lightest isotope - can be used as a fusion fuel, its cross sections for fusion reactions are, unfortunately, too low. The principal thermonuclear fusion fuels which have been considered are deuterium, tritium, and helium-3. The corresponding fusion reactions are (158, 164)

\[ \begin{align*}
1D^2 + 1T^3 & \rightarrow 2He^4 (3.5 \text{ MeV}) + n (14.1 \text{ MeV}) \\
1D^2 + 1D^2 & \rightarrow 2He^3 (0.82 \text{ MeV}) + n (2.45 \text{ MeV}) \\
& \quad \rightarrow 1T^3 (1.0 \text{ MeV}) + 1H^1 (3.02 \text{ MeV})
\end{align*} \]

* The cross section is a measure of probability of interaction. It has the dimension of area. The commonly used unit is the barn. One barn is equal to \( 10^{-24} \text{ cm}^2 \). (157)
\[ D^2 + 2\text{He}^3 \rightarrow \text{He}^4 (3.6 \text{ MeV}) + \text{H}^1 (14.7 \text{ MeV}) \]

The cross sections for these reactions are shown in Fig. 24.

For ground power applications, the D-T fuel is the most promising: it has a higher cross section, lower ignition temperature, larger Q value - the ratio of energy produced to energy needed to sustain the fusion reaction, etc. (165) For space propulsion applications, however, D-He\(^3\) is the most suitable, because only charged particles are produced; (166) such particles can be captured by an external magnetic field. (167, 168) Compared to a system using D-T, which produces neutrons, the shielding problems would be greatly reduced and the weight penalty from shielding would be much less severe.

Deuterium is an abundant natural element. * The total amount of deuterium in the sea is estimated at \(10^{17}\) pounds, a supply which, if used for fusion power, should last much longer than the sun is expected to last. (169) Tritium is very scarce in nature. It may be produced by letting the neutrons from the fusion reactions react with lithium. (170, 171) Since lithium is also a good coolant, many conceptual designs of fusion powerplants involve the utilization of lithium surrounding the reactor for cooling and breeding. (172, 173) Tritium is volatile and radioactive (with a half-life of about 12 years). The large tritium inventory in the blanket of the reactor (~1 to 10 kg, or \(\sim 10^7\) to \(10^8\) curies) (169) and the induced radioactivity of the reactor structure, produced by neutrons from the tritium, would be a public hazard; this hazard, however, would be much smaller than that from fission products. (174)

\(\text{He}^3\) is also very scarce in nature. If D-He\(^3\) fuel is used for space propulsion, the simple thermonuclear reaction of D-He\(^3\) will not produce neutrons. An additional advantage of using the D-He\(^3\) fuel is that the tritium problems of recycling and radiation can be avoided. The presence of the deuterons, however, will permit side reactions of the D-D type. Furthermore, for every neutron generated from a D-D reaction, one atom of tritium will be produced. This tritium then results in a certain number of D-T reactions which produce neutrons.

---

*Deuterium is present as one part in every 6800 atoms of hydrogen in ordinary water. One gallon of sea water contains \(1/6\) teaspoon of heavy water, which can be easily extracted at very low cost. (175)
Fig. 24. Cross sections for D-T, D-D (total), and D-He$^3$ reactions \(^{(158)}\)
In order to minimize the production of neutrons, it is beneficial to have an excess of $^3\text{He}$. This reduces the net power density to some extent, but this reduction is smaller than the reduction in the rate of neutron production.\(^{(163)}\)

2. Conditions for Controlled Thermonuclear Fusion

Because of the strong Coulomb repulsion forces between two approaching nuclei, fusion reactions can take place only at very high energy.\(^{(164)}\) For example, D and T nuclei must have kinetic energies of at least $\sim 10$ keV or, correspondingly, $\sim 10^8^\circ \text{K}$.\(^{(170)}\) At such high temperatures, D-T fuel can exist only in the ionized state, i.e., plasma. The high temperature of the plasma gives rise to the following problems: (1) no known solid material can contain the plasma in steady-state condition\(^{(164)}\) and (2) once the plasma contacts the container wall, the fusion reactions will be quenched, because the plasma is very tenuous and the energy loss rate is too high.\(^{(176)}\)

The charged particles must be confined long enough for fusion reactions to take place. Lawson\(^{(177)}\) set a criterion for determining the length of time a plasma must be confined at a given density and temperature to reach a break-even point in the input vs. output power balance.\(^{(166,170,178)}\)

\[
\begin{align*}
n\tau &> 10^{14} \text{ sec-particles/cm}^3 \quad \text{D-T at } > 10 \text{ keV} \\
n\tau &> 10^{15} \text{ sec-particles/cm}^3 \quad \text{D-He}^3 \text{ at } > 100 \text{ keV} \\
n\tau &> 10^{16} \text{ sec-particles/cm}^3 \quad \text{D-D at } > 50 \text{ keV}
\end{align*}
\]

where $n$ is the ion density in particles/cm\(^3\) and $\tau$ is the plasma confinement time in seconds. In order not to produce an explosion in the D-T fusion reactor, the density $n$ must be limited to approximately 1,000-10,000 times less than that of air (i.e., $n \leq \sim 10^{15}$ to $\sim 10^{16}$ particles/cm\(^3\)), while the confinement time must reach at least a few fractions of a second.\(^{(179)}\) For highly compressed laser-induced fusion, in which the energy is released in an explosive mode, densities could be of the order of $10^{26}$ particles/cm\(^3\) or more, so that confinement times of picoseconds would be adequate.\(^{(180)}\)
At present, the minimum conditions for plasma confinement time, plasma density, and plasma temperature at the break-even point in power balance have been achieved separately. Figure 25 shows quantitatively how the plasma conditions \((n, \tau, T)\) of some devices stood at the beginning of this decade. It is optimistically believed that, by the end of this decade or early in the next decade, the break-even requirements \(n, \tau, T\) can be met simultaneously in a single device.

3. Plasma Confinement and Instabilities

As mentioned previously, the plasma must be confined for a sufficient time and cannot be in contact with any material wall. The fact that a plasma consists of charged particles makes it possible to confine them by applying a strong external magnetic field. In strong magnetic fields, individual charged particles are confined to movement along field lines in tight helical trajectories. Despite the magnetic confinement, however, some of these charged particles escape without undergoing fusion reactions. Collisions between particles result in a slow leak of particles from the magnetic container, referred to as the classical loss. This kind of loss cannot be eliminated completely and it sets an upper limit on confinement time, which has been termed the classical confinement time. Until recently, all plasma experiments yielded loss rates considerably in excess of classical values. These anomalous losses seemed to be associated with plasma turbulence. For a fully turbulent plasma, the confinement time is termed the Bohm time, which has been used as a basis for comparing the quality of plasma confinement. In a number of types of research devices, classical confinement times are now several hundred times the Bohm time, and this is adequate for a fusion reactor.

There are other kinds of plasma instability which cause the hot plasma to be lost before it has reached the required temperature:

1) MHD instability - plasma is a diamagnetic material and will always move to the weaker magnetic field.
2) Stream instability - a condition arising from the presence of a directed beam of energetic particles in a plasma.
Fig. 25. Development of various fusion reactor devices, compared with the requirements for controlled thermonuclear fusion \cite{181,182}
3) Hydromagnetic instability arising from imposed currents within the plasma itself producing $\mathbf{J} \times \mathbf{B}$ forces.

4) Other instabilities arising from density gradients, velocity gradients, etc. (184,185)

During the past decade, plasma stability has been achieved (e.g., by the use of magnetic wells and sheared fields). More advanced stabilization techniques such as dynamic stabilization, feedback stabilization, and a new technique used in the topolotron are being developed. (185-187) Today, it can be said that most plasma instabilities, which have been one of major stumbling blocks in fusion technology, have been identified and understood, and methods of dealing with them in specific machines have been developed.

4. Fusion Energy Release and Energy Losses from Plasma

After a fusion reaction, the total mass of the particles - which generally consist of a heavier fused nuclei plus a nucleon - is less than the total mass of the two fuel nuclei before the reaction. It is this "mass defect" which is converted into energy and released. A rough estimate of the power per unit volume available from fusion reactions is given as (158,163,188)

$$P_{\text{DT}} = 5.6 \times 10^{-13} n_D n_T \overline{\sigma v} \text{watts/cm}^3$$

$$P_{\text{He}^3D} = 2.93 \times 10^{-12} n_{\text{He}^3} n_D \overline{\sigma v} \text{watts/cm}^3$$

$$P_{\text{DD}} = 3.3 \times 10^{-13} n_D^2 \overline{\sigma v} \text{watts/cm}^3$$

where $\overline{\sigma v}$ is the average fusion reaction rate, $\sigma$ is the cross section, $v$ is the relative velocity between colliding ions, and $n$ is the ion density in particles/cm$^3$. Although $\overline{\sigma v}$ depends on the precise ion distributions, it is estimated that the differences are no more than 10% if one adopts $\overline{\sigma v}$ calculated for a Maxwellian distribution. (189) The values of $\overline{\sigma v}$ based on Maxwellian distribution for D-T, D-D, and D-He$^3$ reactions are given in Fig. 26. (158)
Fig. 26. Values of $\overline{\sigma v}$ based on Maxwellian distribution for D-T, D-D (total), and D-He reactions (158).
In addition to energy losses due to runaway particles and charge exchange collisions, as discussed in the section on plasma confinement and instabilities, there are energy losses associated with radiation. These are (1) Bremsstrahlung, (2) gyromagnetic radiation, and (3) Cherenkov radiation.

a. Bremsstrahlung Loss

The Bremsstrahlung loss is an unavoidable loss, primarily in the form of X-rays, that occurs when electrons collide with nuclei. It is well known that Bremsstrahlung losses increase dramatically with plasma impurity content. Experiments have shown that the presence of less than 1% impurity causes a great dissipation of the kinetic energy of the plasma particles as Bremsstrahlung radiation. The radiation losses by Bremsstrahlung from the accelerated electrons are mainly in the ultraviolet and X-ray region. The losses are partially recoverable, but such recovery would make it more difficult to heat the plasma to the required temperature, if impurities are present. A rough estimate of this kind of loss can be obtained from the following formula:

\[ P_{br} \approx 5.3 \times 10^{-31} n^2 T_e^{1/2} \text{ watts/cm}^3 \]

where \( T_e \) is the kinetic temperature of the electrons in KeV (assuming a Maxwellian distribution).

b. Gyromagnetic Radiation

Gyromagnetic radiation is also called cyclotron or synchrotron radiation and is principally in the microwave harmonic radiation range. The ions in a plasma move moderately slowly and the associated emission of cyclotron radiation is insignificant. For electrons, the rate of energy loss may exceed the Bremsstrahlung loss. A rough estimate of gyromagnetic radiation loss is as follows:

\[ P_{gr} \approx 5.0 \times 10^{-32} n^2 T_e^2 \text{ watts/cm}^3 \]
It can be seen that while the Brønnsstrahlung loss increases as $T_e^{1/2}$, the gyromagnetic radiation varies as $T_e^2$. At temperatures below 5 keV, the gyromagnetic radiation loss is less than that due to Brønnsstrahlung. At higher temperatures, the gyromagnetic radiation rate increases very rapidly and may greatly exceed that of Brønnsstrahlung. However, the energy losses due to gyromagnetic radiation can largely be reflected into the plasma by the wall of the vacuum vessel.

c. Cherenkov Radiation

When the electrons in a plasma attain relativistic velocities, they emit Cherenkov radiation. This radiation appears in the electromagnetic wave spectrum at microwave, infrared, and visible frequencies, but not beyond these frequencies. At $T_e < 50$ keV the Cherenkov radiation is insignificant. It increases rapidly with the fifth power of temperature. It also depends on the dielectric and magnetic characteristics of the plasma. The expression for the estimate of this radiation loss is too complicated to be presented here.

5. Plasma Heating

The plasma must be heated to a certain temperature before a thermonuclear fusion reaction can proceed to a significant extent. A number of methods of heating the plasma have been developed. A few examples of these methods are as follows:

1) Ohmic heating by a large electric current flowing through the plasma. A current induced in the plasma will heat the plasma because of its resistance. Since the classical resistivity of a hot plasma drops rapidly with electron temperature, the ohmic heating rate falls too low for the ignition temperature to be reached. Hence, this heating method is restricted to the first stage of plasma heating. To reach fusion reaction temperatures, additional heating must be applied.

2) Adiabatic compression. The magnetic coil system and the fusion reaction chamber are specially designed to allow the major radius
and minor radius of the plasma (while it is undergoing the initial ohmic heating) to be reduced by means of a pulsed vertical magnetic field. \(^{(193)}\) (Adiabatic heating results from compression of the plasma at a relatively slow rate; shock heating, which will be discussed below, results from the rapid compression \(^{(158)}\) created by a shock wave.)

3) Ion cyclotron. Adjusting the alternating frequency of the local magnetic field to be slightly lower than the frequency at which the ions spiral along the magnetic field, causes a wave motion to develop in the plasma. The damping of these waves results in the conversion of their energy into heat. \(^{(158)}\)

4) Neutral beam injection. Injection of a current of high energy atoms (> 100 keV) into the established plasma causes part of the beam to become ionized through a combination of Lorentz and collisional ionization. Deuterium gas is first ionized and then accelerated by an electrical field. The ion beam is neutralized by picking up electrons from a gas or metal vapor in a neutralizer cell. Once neutralized, the energetic particles can enter the plasma through the strong magnetic field and become ionized and trapped. This method of plasma heating is also useful for reactor control, since the particle energy and composition can be controlled from the outside. \(^{(176)}\)

5) Magnetic pumping. The magnetic field strength is continuously and rapidly increased and decreased. If the frequency of the alternations is chosen correctly, the plasma heating that occurs during the increase in field strength exceeds the plasma cooling that occurs during the decrease in field strength, and the plasma is heated. \(^{(194, 195)}\)

6) Shock heating. If the strength of the confining magnetic field is increased suddenly, the plasma is compressed and heated. Both collisional and collisionless shocks have been studied and are possible heating mechanisms. \(^{(195)}\)

C. THERMONUCLEAR FUSION SCHEMES FOR GROUND POWER

Most efforts in controlled thermonuclear fusion research have been concentrated on the concept of employing strong magnetic fields to keep the hot plasma from coming in contact with the vacuum container walls, which otherwise
would quench the fusion reaction. Recently, in contrast to this magnetic confinement concept, much interest has been directed toward using laser or relativistic electron beams to compress the plasma and heat it to the reaction temperature in a very short period of time, before the plasma can start to expand and diffuse. Thus the plasma is said to be confined inertially. In addition, a few researchers are exploring other possible methods. The various thermonuclear fusion schemes will be discussed below under the three basic categories.

1. Magnetically-Confined Thermonuclear Fusion Systems

The magnetically-confined thermonuclear fusion systems are the steady-state toroidal systems (principally, the tokamak), the magnetic mirror systems, and the pulsed high-beta pinch systems (principally, the theta pinch). The most notable features of these systems are (1) the magnetic confinement of plasmas (which takes the advantage of the fact that charged particles in strong magnetic fields are forced to move along the field lines) and (2) plasma instabilities (which are the main problem in such systems). Present main efforts are to scale up the plasma conditions (n, \(T\), \(T\)) for achieving a positive power balance.

a. Steady-State Toroidal Systems

A schematic arrangement of a tokamak is shown in Fig. 27. A strong current pulse on the primary winding ionizes the gas and generates a secondary plasma current, \(I_\varphi\), in the torus. This current produces a new poloidal magnetic field, \(B_\varphi\). The induced azimuthal current has an upper limit, the so-called Kruskal-Shafranov limit, beyond which a helical kink hydromagnetic instability can occur. This K-S limit also places a strong limit on \(\beta\), the ratio of plasma pressure to magnetic pressure. For a tokamak, \(\beta\) must be limited to 1-4%. The poloidal magnetic field, \(B_\theta\), plus the azimuthal field, \(B_\varphi\), which is induced by the external current, \(I_\varphi\), in the winding wires make up the confining magnetic structure. The hot plasma is confined on magnetic surfaces composed of helical
field lines, resulting from the superposition of the toroidal field and the poloidal field.\(^{(164, 176)}\) The plasma current, \(I_q\), is also used for heating. It compresses and resistively heats the plasma to a moderate temperature (\(\sim 600\;\text{eV}\)). It disappears with a time constant of \(L/R\) (\(L\): inductance; \(R\): resistance of the plasma, which could be rather high, \(\sim 10^3\) seconds).\(^{(196)}\) A heating enhancement technique such as neutral beam injection must be applied to heat up the plasma to fusion temperature.\(^{(197)}\)

Fig. 27. The tokamak plasma confinement scheme\(^{(174)}\)
Other toroidal devices are also being studied. The stellarator, which uses a rotational transform of the axial magnetic field, can operate in true steady state, because the confining magnetic fields are all external. Internal ring or multipole systems in which the current-carrying conductors have been suspended or levitated within the plasma can provide a longer confinement time than the tokamak and a higher β. \(184,193\)

b. Magnetic Mirror Systems

Magnetic mirror systems are open at both ends. In such systems, the particles must be reflected or "plugged" from the ends, where the magnetic field is much stronger than in the center. Figure 28 shows schematically how the particles are partially plugged in the regions of high field strength. The simplified theory of plugging is as follows: The velocity component of a particle along the z-axis decreases as it moves into a region of high field strength. It can fall to zero and be reflected if

\[
\frac{B_2}{B_1} - 1 > \frac{u_{\parallel}^2}{u_\perp^2}
\]

where \(B_2/B_1\) is the ratio of field strength at the end to that at the center; \(u_{\parallel}\) is the particle velocity component along the z-axis at the center; \(u_\perp\) is the particle velocity component perpendicular to the z-axis at the center. \(178\) Any plasma ions with sufficient momentum along the field line can escape through the end mirrors, which lie in a so-called mirror loss cone, and thus significantly decrease the containment efficiency, probably to an intolerable extent. \(176\)

A mirror machine itself cannot provide a confinement time that is long enough to meet the equilibrium condition. Energy injection is necessary to maintain the reactor in steady system operation. \(198\)

In dense plasmas, it has been found that increasing the field at the ends (i.e., the mirrors or plugs) is not sufficient to achieve confinement. Instead, the magnetic field must be increased in every direction, surrounding the plasma and forming a magnetic well. \(176\)
Fig. 28. Magnetic mirror particle (and plasma) confinement configuration\(^{(163)}\)
High efficiency direct conversion of fusion energy to electrical current is a popular concept with the mirror systems. It was first proposed by Post in 1970.\(^1\) Particles escaping through the mirrors (open ends) can be guided and expanded radially to the periphery of a large disk at each end, where the plasma density is so low that electrons can be separated from the ions.

A possible way to improve the power balance is provided by the concept of a chain of mirrors. Kelley (1967) proposed a chain of three mirror machines, with a longer machine at the center and the two shorter machines at the ends. The potential is maintained constant along field lines between the outer machines so that mirror loss in the center machine is reduced.\(^2\)

It can be seen that plasma plugging is a unique problem for the open-ended systems. High-energy neutral beam injection is essential, and the large amounts of electric power needed for the injector could make the mirror system infeasible.

c. Theta Pinch Systems

The Culham theta pinch device was the first device to demonstrate how the anomalous Bohm diffusion can be largely suppressed.\(^3\) The pinch effect on the plasma is produced by a magnetic field which is generated by a current flowing through the plasma itself.\(^4\) In a theta pinch reactor, a one-turn coil encloses a long cylindrical tube. A capacitor discharge through the single-turn coil generates a rapid-rising, strong magnetic field (15 tesla in less than \(10^{-6}\) sec).\(^5\) This rapid-rising, strong magnetic field ionizes, shock-heats, and subsequently compresses and further heats the plasma. The induced currents flow in the plasma in the theta direction by transformer action.\(^6\) The ratio \(\beta\) is inherently very high (near unity), which means very efficient use of the magnetic field.

The length requirement for a linear open-ended theta pinch reactor depends on the field strength. Lengths are typically on the order of several hundred meters to about one kilometer.
A reference theta pinch reactor considered by Thomassen, et al. (1973) is designed to operate in a pulsed mode and use a combination of fast implosion heating and slow adiabatic compression, via magnetic fields, to ignite, burn, and confine a D-T plasma. Shock and compression heating are followed by a burn phase, purging, and the introduction of new fuel (Post, 1973).

For a theta pinch reactor, a very large capacity for inductive energy storage is required, for use in generating a rapidly-increasing, strong axial magnetic field. The $n \tau$ values achievable in linear theta pinches depend only on the length. To minimize the length of the power plant, a theta pinch reactor should operate with the strongest possible magnetic field. The advantages of this concept are:

1) The plasma can be stably confined in the radial direction, escaping only by diffusing out the tube ends.
2) Access from the ends is easy.
3) The magnetic field is used more efficiently than in toroidal devices.

To reduce the magnetic energy storage requirement and reduce the plant size, a hybrid system involving the use of a laser to heat the plasma from the ends has been proposed. The laser heating replaces the shock-heating stage and is followed by the conventional adiabatic magnetic compression. A laser with an output energy of 5 megajoules or more is required.

In addition, a toroidal theta pinch reactor concept is being advanced by the Los Alamos Scientific Laboratory. The aspect ratio of a closed-end theta pinch reactor is much larger than that of a steady-state toroidal device, which means that the shape of the reactor looks more or less like the wheel tube of a bicycle rather than a torus. As the first step, physics questions such as the toroidal equilibrium, hydromagnetic stability, and shock heating must be answered.
2. Inertially-Confined Thermonuclear Fusion Systems

For a long time most scientists have believed that controlled thermonuclear fusion can only be achieved through a magnetically-confined, stable, and hot plasma. (185) Even today, most research programs are concentrated on the magnetic confinement approach, which is reflected in the $10 billion, 5-year energy R&D program budget. (202, 203) However, advancement in laser and relativistic electron beam technology has promoted the inertially-confined thermonuclear fusion approach to a very strong candidate position.

The principal idea of these systems is to heat a D-T plasma of density $n$, radius $r$ to a temperature, $T$, of $\sim 10^4$ eV instantaneously. Assuming the plasma expands at the ion thermal velocity $V_{th} = \sqrt{2KT/M}$, the effective containment time $\tau = r/V_{th}$ is kept so short that equilibrium, MHD, and microstability problems are avoided. (188, 204)

The minimum input energy is about $10^7$ to $10^9$ joules, which must be delivered, in a period of a few nanoseconds, to a fuel pellet up to a few millimeters in diameter, (204-206) corresponding to a power density roughly equal to $10^{16}$ to $10^{18}$ W/cm$^3$. To produce laboratory break-even power, however, an energy of at least $10^3$ joules with a 1-nanosecond or shorter pulse is required. (206, 207) One popular concept of supercompression is to use a hollow, high-density shell to compress and heat a D-T gas mixture within that shell. (208) Precompression, which reduces the required trigger energy, can perhaps be achieved by the use of a laser-generated shock. A high-atomic-number material tamp, if feasible, could enhance the inertial confinement time further. (204)

a. Laser-Induced Fusion

For power generation by laser-induced fusion to be economically competitive, it is generally considered that the fuel pellet, when it fuses, must release $\sim 75$ times the energy it absorbs from the laser pulse (i.e., a gain factor of $\sim 75$). (206) This concept assumes (1) that 10% of the input energy is actually absorbed$^*$ by

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$^*$10% is a frequently-projected - but not yet achieved - efficiency; current technology is <1%.
the pellet and (2) that 30\% of the power produced is recirculated to provide power to operate the plant.

The current output energy capabilities of the most completely developed lasers are typically a few hundred joules of output energy, with a pulse duration of a few nanoseconds. Thus, present laser technology falls short of what is required for fusion power generation. Recently, Battle claimed to have developed the world's most powerful laser: a 12-beam, 7-stage neodymium-doped glass laser producing 900 to 1500 joules of energy, in pulses ranging from 1.5 to 5.0 nanoseconds. It is reported that the University of Rochester Laboratory's laser is capable of delivering more than 1 kilojoule of energy in 10^-10 sec. Designs are in progress for a 10^4 joule module which could provide the basis for a 10^5 joule CO_2 laser system. It should be possible soon to demonstrate the feasibility of laser fusion, since the break-even point (i.e., fusion energy = laser energy) is about 1 x 10^3 joules, which is readily achievable with present glass laser technology. (Recently, KMS Fusion, Inc., claimed to have produced thermonuclear neutrons from laser implosion.)

Because of the high plasma density, 10^8 to 10^10 joules or more of fusion energy output is expected. The amount of energy released per pellet will be equivalent to that from somewhere between a fraction of a ton of TNT up to a few tons.

Since the energy is released explosively, a rather innovative design of the first wall of the combustion chamber is required. At Oak Ridge National Laboratory a "Blascon" reactor vessel concept was studied. In the Blascon, a free-standing vortex is formed by rotating a lithium pool about a vertical axis. Because of the rotating thick lithium inner layer, radiation damage and buildup of radioactive impurities in the structure are reduced. However, the access to the fuel pellet is very restricted and achievement of the desired spherical symmetry seems to be impossible. Owing to the time required to establish the lithium vortex, the frequency of operation is limited to 0.1 Hz. A wetted wall concept has been pursued at Los Alamos Scientific Laboratory, in which a thin ablative layer of lithium protects the inner porous wall of the reactor vessel. It is estimated that 1 second is required to achieve an adequate vacuum
between micro-explosions, which limits the frequency of operation to 1 Hz. The dry wall concept developed at the Lawrence Livermore Laboratory is essentially a simple cavity structure 3 to 4 meters in radius. It might be economically feasible for energy releases of less than $10^7$ joules.\(^{(215)}\) The characteristics of these concepts are shown in Table 16.

**Table 16. Some parameters for three proposed laser-induced fusion chambers\(^{(204)}\)**

<table>
<thead>
<tr>
<th>System</th>
<th>Blascon (Oak Ridge)</th>
<th>Wetted wall (Los Alamos)</th>
<th>Dry Wall (Livermore)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermonuclear energy per pulse</td>
<td>1500 MJ</td>
<td>200 MJ</td>
<td>10 - 80 MJ</td>
</tr>
<tr>
<td>Repetition rate of laser</td>
<td>0.1 Hz</td>
<td>1 Hz</td>
<td>~100 Hz</td>
</tr>
<tr>
<td>Mean power</td>
<td>150 MW (th)</td>
<td>200 MW (th)</td>
<td>~1000 - 8000 MW (th)</td>
</tr>
<tr>
<td>Inner wall</td>
<td>Lithium vortex (with bubbles)</td>
<td>Porous wall wetted by 2 mm Li</td>
<td>10 chambers</td>
</tr>
<tr>
<td>Inner wall diameter</td>
<td>4.5 m</td>
<td>2 m</td>
<td></td>
</tr>
</tbody>
</table>

**b. Fusion Induced by Relativistic Electron Beams**

Tightly focused relativistic electron beams have only recently been seriously considered as an alternative to laser beams as a means of heating and compressing fuel pellets for achieving fusion.\(^{(206,208,216,217)}\) It is expected that megajoule electron beam accelerators with a 10 nanosecond pulse duration can be developed that will offer a more efficient conversion of stored energy to beam energy than do sub-nanosecond short-wavelength lasers. At least one multimegajoule relativistic electron beam accelerator is already operating at a 50% efficiency level. The required gain factor can be reduced to 15.\(^{(208)}\)
Intense relativistic electron beam experiments have been carried out for nearly a decade, and much progress has been made toward understanding the electron beam formulation, equilibrium, stability, and propagation. Large pulsed electron accelerators were developed primarily for application to the study of radiation effects in materials. The accelerator technology has proven to be comparatively simple, inexpensive, and scalable to higher power. (206)

Only one beam is theoretically necessary for implosion because the thermal conductivity of the target electrons is so high that the fuel pellets quickly become spherical. The confinement time can be extended by surrounding the pellet with a high atomic number material and by the self-induced magnetic field of the focused electron beam. (216)

The Aurora accelerator (Harry Diamond Operation laboratory) has achieved a beam pulse energy of 2.5-3.0 megajoules for 125 nanoseconds (~10^{13} watts) with an efficiency of ~50%, but the area of the beam is fairly large. Another device, at North Carolina State University, has achieved a current density as high as 25 megamps/cm^2. It is estimated that to deliver a few megajoules, in ~10 nanoseconds, onto a sphere several millimeters in diameter would require a current density of several times 10^8 amps/cm^2 for a beam in the MeV range. (216)

Much of the power of present electron beams is wasted because their pulses last longer than nuclear reaction times by one to two orders of magnitude. Foremost among the problems is how to focus the electron beam to achieve energy densities suitable for triggering thermonuclear fusion. For pulsed relativistic electron beam power technology, developments in synchronized switching are particularly needed, in order to achieve a 10 nanosecond pulse. (208) To consider the application of electron beams to the supercompression approach, the additional problem of spherical loading must be solved.

The realization of the rather impressive decreases in the break-even energy requirement that could be obtained by compressing the D-T fuel many orders of magnitude (~10^4) beyond "solid" density is perhaps the factor most responsible for the increased level of interest in the inertial confinement concept. (208) One major advantage of inertial confinement with implosion over magnetic confinement for power production is that the system can be relatively small. (207)
The problems to be overcome are those involved in the supercompression of the fuel pellet by the laser or relativistic electron beams; the development of an efficient, high power beam machine; and finding a way to couple the beam energy efficiently into the D-T fuel. The reactor vessel must be capable of containing the explosive energy without damage.

3. Unconventional Systems

Although much experimental progress has been made in improving both magnetic and inertial confinement, there is not as yet conclusive evidence that either approach will be successful in providing the ultimate power of thermonuclear fusion for all mankind, at least not in the near future. Growing interest in concepts other than these two is natural; several of these are discussed below,

a. Fusion-Fission Hybrid

The concept of fusion-fission hybrid systems was initiated as early as the beginning of the USAEC program in controlled thermonuclear fusion research for the purpose of providing an intense source of neutrons to produce fissionable material. The hybrid reactor would operate with a sub-Lawson plasma and a subcritical fission blanket. The 14 MeV neutrons from a controlled D-T fusion plasma are used to produce neutron source multiplication (by a factor of about 5) and energy multiplication (by a factor of about 50) through fission in a fissionable material blanket surrounding the fusion plasma. Through utilization of fission energy, the fusion energy yield can be increased by an order of magnitude. The values of the plasma characteristics $n$, $T$, $\tau$ necessary to achieve fusion-fission reactions appear to be significantly lower than those required for pure fusion systems. However, the potential contribution of hybrid systems to fission power economy seems to be the primary objective of this research.

b. Migmatron (Rutgers University)

The main idea of the migmatron is to trigger the nuclear fusion reaction in colliding beams of ions instead of in a plasma. The ions, in a magnetic field, are looping in a figure-8 course and colliding with themselves. No heating process
is involved. Electric power is obtained by direct conversion. A small-scale experiment was successful in proving the theory, but the power output is too small ($\sim 10^{-6}$ watts). Some people are skeptical about the basic theory of the device, contending that, when the device is scaled up, there will be particle scattering which will create a plasma and invalidate the theory. (219)

c. Collision of Ions with a Relativistic Beam (Brookhaven National Laboratory)

This concept is basically the same as that of the migmatron in that the fusion reaction takes place in colliding beams instead of a plasma. A system of beams of deuterons and tritons traveling in the same direction and being focused by a relativistic electron beam is found to be capable of yielding kilowatts of fusion power per meter of beam path. (220) The concept is in the early stages of exploration. Many basic problems such as beam instabilities, the mechanism of beam collapse, and the nonlinear forces developed in the composite beams need further study.

d. Magnetically-Confined Fusion Induced by Laser of Relativistic Electron Beam (Massachusetts Institute of Technology)

In previous discussions on theta pinch systems, the application of a laser to heat the plasma was mentioned. The main idea behind this relatively new and very promising concept is to use a laser or relativistic electron beam to induce fusion reactions in a plasma which is magnetically confined. By this hybrid technique it is hoped to eliminate the most immediate problems faced separately by the magnetically confined and inertially confined approaches. The required magnetic field strength is greatly reduced as compared to that for the magnetic confinement alone. At the same time, the beam pulse length and beam power requirements are relaxed to a great extent as compared with those for the inertial confinement approach alone. Electron beam generators presently exist in the terawatt power range, and their application to a magnetically-confined toroidal device is very promising. At MIT's Francis Bitter National Magnet Laboratory, a hybrid between the tokamak and laser method is being studied. (221) Much more research is needed on problems such as beam/plasma interaction, beam trapping and coupling with the plasma before the beam
can escape, effects of the beam on the gross stability of the magnetically
confined plasma, and the method for injecting the laser or electron beam into
a magnetic confinement system.

D. PROBLEM AND RESEARCH AREAS FOR GROUND POWER
THERMONUCLEAR FUSION

Much work remains to be done to demonstrate the scientific feasibility of
thermonuclear fusion. Part of this work has been mentioned above. Although
much progress has been made in plasma physics, some important physics
problems such as microscopic stability of plasma, laser beam and plasma inter-
action, intermediate $\beta$ plasma, etc. are not yet well understood. Enginee-
ring studies on thermonuclear fusion reactor systems have been underway for only
a few years. The engineering problems may turn out to be much more difficult
than the physics problems. (222)

1. Problems Peculiar to Magnetically-Confined Systems

In a magnetically-confined system, the magnet must be maintained at about
liquid helium temperature ($\sim 4^\circ$K). (223) The use of superconductors is required
in order to limit electrical power consumption to an acceptable level. The
superconducting magnets produced thus far are much smaller than those that
would be required for a reactor. Insulation of a superconductor seems to be a
difficult task because of irradiation effects, and cooling the magnet will take a
long time because of the enormous stored energy. The production of a strong
magnetic field of the order of $10^5$ gauss in a volume of several thousands of m$^3$
is required. (197) The magnet coils must be prevented from quenching (i.e.,
going from superconducting to normal) which, if it happens, will cause asymme-
tric fields and produce powerful tangential forces on the coils of the order of
20,000 tons. (169) A sturdy supporting structure is needed and must be kept
cooled to the same temperature as the superconductor. A good insulation material
which has adequate thermal expansion properties and will not deteriorate under
heavy load is yet to be found. (197, 224, 225)

The mechanical problem faced in designing the vacuum wall is obvi-
ous: the wall surfaces face, on one side, the fusing plasma at zero pressure and, on
the other side, a coolant at higher pressure. The wall has to withstand erosion from the plasma inside and chemical erosion from the coolant outside. In addition, fast neutrons (14 MeV, $\sim 10^{15}$ n/cm$^2$/sec), a smaller number of slow neutrons, primary $\gamma$ rays, secondary $\gamma$ rays, $\alpha$ rays, and $\beta$ rays (from tritium) will irradiate the wall. There will also be thermal cycling and other stresses. Nuclear transmutation of the wall material, and tritium and helium diffusion will induce wall damage such as severe surface blistering, volume swelling, and void and defect formation. Photon-induced desorption may be an important effect at wavelengths less than 2000 Å, a region which has not yet been investigated. Physical sputtering due to particle bombardment may be the most important among all the processes that affect the lifetime of the first wall. Helium bubbles due to $(n,\alpha)$ reaction can occur in most structural materials (0.1 - $\sim$1% of helium may be formed) by the 14 MeV neutrons, and this will lead to the loss of wall ductility. Direct experiments with the 14 MeV neutrons are not yet possible. Little work has been done on radiation damage specifically for thermonuclear fusion. There are significant uncertainties in the energy and particle fluxes impinging on the wall. Experience from fission reactors does not offer much help either, because the nuclear data for particles with energies of 5 MeV or more are either lacking or inaccurate. The lacking of sputtering yield data makes the engineering design study of fusion reactors of limited value. Chemical sputtering is even less understood. Impurities in the wall material may diffuse to the wall surface and be thermally desorbed. The mechanism of impurity interaction with the wall is another unknown area. Impurities will increase the radiation loss and may stop the fusion reaction. In a toroidal device the impurities will be accumulated and must be purged constantly. Technology for diverting impurities needs to be developed.

The feasible operating temperature of the vacuum wall has been assumed to be about 1000 to 1300 K, whereas the plasma temperature is well above $10^8$ K. The early values assumed for the heat flux of the first wall have been reduced to the same order of magnitude as that for a liquid metal fast breeder reactor ($\sim 0.1$ kW/cm$^2$). Efficient heat removal methods, involving a minimum in pumping power requirements, and suitable for use in a very strong magnetic field and a high radiation environment, should be developed.
For a D-T fuel fusion power plant, the tritium must be bred in the lithium blanket surrounding the reactor. With a thermal efficiency of \(\sim 30\%\), the reactor would burn about 0.5 kg of tritium per day per gigawatt of electricity. The concentration of tritium in the lithium must be kept very low, perhaps on the order of 1 gram of tritium per cubic meter of lithium (or \(\sim 1\) ppm), \(^{(171)}\) so that the structure will not be embrittled. Tritium is volatile. The recovery system must minimize the tritium inventory and the amount percolating into the atmosphere or the steam system. \(^{(171,226)}\) No fast and efficient ways of recovering such a low concentration of tritium are known.

Introducing new fuel into an area where a plasma is confined by a strong magnetic field is certainly not a trivial problem. The refueling process should not disturb the self-sustaining fusion reactions. Ingenious techniques for refueling and for the extraction of spent fuel will be needed before fusion power can be economically harnessed.

2. Problems Peculiar to Inertially-Confined Systems

The most pressing problem of the inertially-confined concept is the development of a laser or electron beam sufficiently powerful and efficient for fusion power generation.

The combustion chamber corresponding to the first wall of a magnetic confinement system is also a challenging engineering problem. Because the fuel pellet will be a solid, and highly precompressed, the fusion energy will be released in an explosive mode. The chamber must be protected from being damaged by the blast waves. In a pulsed reactor, the flash of Bremsstrahlung radiation can raise the walls to a temperature above their melting point. Proper cooling of the inner wall will be necessary. \(^{(188)}\)

The physics of the absorption of beam energy by the plasma is not well understood. \(^{(228)}\) The role of plasma instabilities in coupling the laser energy into the pellet, and the possible effects of other instabilities in limiting the densification of the pellet under the influence of laser heating are unanswered questions. \(^{(155,205)}\)
Other problems may occur, such as the possibility of premature heating of the compressed pellet core at longer laser wavelengths and the possibility, at the highest laser intensities, of strong stimulated backscatter of the incident radiation. The optical surfaces transporting the laser beam must be protected to some extent from pellet debris and from X-rays. Target design, fabrication, and delivery are also important engineering problems.

E. THERMONUCLEAR FUSION FOR SPACE PROPULSION

In a propulsion system for space exploration, the primary use of thermonuclear fusion energy would be to produce thrust, whereas terrestrial uses would involve the production of electricity. It would seem that the extremely high temperature plasma present in a fusion reactor could be used directly to produce thrust, without a conventional heat cycle; for electricity production, however, a heat cycle is considered necessary. Thus far our discussion has been largely concerned with ground power fusion technology. The availability of almost inexhaustible and cheap fusion fuel (for instance, deuterium from water), the inherent freedom from reactor power excursion, and the relatively small radioactive release hazard ($\sim 1/10,000$ of that for a nuclear fission power plant)$^{(229,230)}$ should be strong incentives to apply all possible, worldwide efforts to research and development on fusion technology. However, commercial fusion power may not be realized until it is economically competitive with other power systems such as light water reactor systems. This may tend to slow down the pace of development of fusion power technology, particularly for the ground-based application. As will be discussed below, it seems that the fusion power for space propulsion is an attractive alternative for interplanetary exploration in the solar system. Hence, the development of the fusion propulsion technology for space propulsion need not wait until the ground-based fusion power technology has been well developed. In addition to being free of economic restraint, the exoatmospheric environment is particularly suitable for magnetically-confined fusion systems, since no special vacuum requirement is imposed on the inner wall of the fusion reactor.$^{(231)}$ Some problems (such as tritium recycling) that must be solved for ground power fusion systems can be circumvented or alleviated.
Fusion propulsion has many attractive features. A few of them are as follows:

1) The amount of energy released per unit mass of fuel by fusion reactions is larger than that from chemical or nuclear fission reactions. This means that the maximum obtainable specific impulse \( I_{sp} \) is the highest (2.6 x 10^6 sec for fusion; 1.3 x 10^6 sec for fission; and 5 x 10^2 sec for chemical).\(^{(232)}\) Conceptual fusion rocket studies indicate that \( I_{sp} \) can be as high as 10,000 sec (high thrust system)\(^{(232)}\) to 200,000 sec (low thrust system),\(^{(233)}\) in contrast to an \( I_{sp} \) of about 1000 sec for solid core nuclear rockets; or 2500 sec for gas core nuclear rockets;\(^{(153)}\) or about 5000 sec for nuclear electric propulsion systems. For the low thrust systems, the specific weight \( \alpha \) can be as low as 0.5 kg/kW,\(^{(233)}\) as compared to \( \sim 30 \) kg/kW for nuclear electric propulsion systems.\(^{(234)}\)

2) Direct conversion of fusion power to electricity is possible without applying a conventional thermal cycle.\(^{(199)}\) The waste heat can be rejected at a higher temperature (i.e., propellant mixing temperature). The radiator mass is thus minimized.\(^{(235)}\)

3) The radioactive product problem is much less severe than in nuclear fission reactions. Lower shielding mass for the spacecraft, and lower development and testing costs on the ground are possible. No critical mass of the fuel is required for fusion propulsion. Thus one can choose the smallest feasible operating unit mass, so as to minimize the total vehicle mass.\(^{(232)}\)

4) The fusion propulsion system may have a high and variable specific impulse (2500 to 200,000 sec), a low specific weight (as low as 0.5 kg/kW), and a low propellant-to-payload weight ratio.\(^{(233)}\) The overall performance is estimated to be an order of magnitude better than nuclear (fission) electric propulsion system.

5) For manned missions beyond Mars, in a practical journey time, all high performance systems are inferior to fusion propulsion systems with respect to payload ratio and trip time.\(^{(154)}\)

*The specific weight \( \alpha \) is is usually defined as follows:

\[
\alpha = \frac{\text{total weight of the propulsion system (kg)}}{\text{total thrust power (kW)}}
\]
6) The prospect for fusion propulsion would be even brighter if the protons from the D-He\textsuperscript{3}, etc. fusion reactions could be utilized for the energetic thermonuclear fission reactions discussed above in paragraph VII-A.

F. PROPOSED FUSION PROPULSION SYSTEMS

Current thermonuclear fusion propulsion concepts can be grouped into two categories, namely, the steady-state low-thrust systems and the pulsed high-thrust systems. In a steady-state fusion rocket, part of the fusion reaction products would be directed out of the reaction chamber to the magnetic nozzle\textsuperscript{*} to produce thrust, or to an energy conversion device to produce electricity for ion engines. It is a magnetically-confined system. By contrast, the pulsed fusion rocket would be an inertially-confined system. The pulses would be derived from a "pulse unit" such as a D-T pellet, which would be induced to explode by an intense energy beam. The cloud of explosion debris would be used to push the spacecraft or to produce thrust through a nozzle. The thrust that can be obtained from the steady-state systems is limited by the allowable temperature of the structure. Pulsed propulsion systems, however, would operate in very short bursts and the thermal waves produced would have little time to penetrate the structure; hence the system can be exposed to a much higher temperature, and produce higher thrust, without structural damage.\textsuperscript{(233)} The thrust/weight ratios of steady-state systems are much smaller than unity\textsuperscript{(155)} (of the order of 0.01 and smaller), whereas for pulsed systems these ratios can be as high as 3.6.\textsuperscript{(232)} A propulsion system with a thrust/weight ratio of less than unity would be limited to use for ferry missions between orbits.

Comparisons between thermonuclear fusion propulsion systems and nuclear fission rockets for interplanetary exploration are shown in Figs. 29 and 30.\textsuperscript{(155,234)} It can be seen that fusion propulsion systems have superior capabilities. Also, as indicated in these two figures, for a high-thrust fusion rocket, the specific impulse ($I_{sp}$) should be as high as possible (Fig. 29) and, for a low-thrust fusion rocket, the specific weight, $\alpha$, should be as low as possible (Fig. 30). For the

\textsuperscript{*}A magnetic nozzle guides the escaping plasma in the desired direction by a magnetic field. The basic principle is the same as for the magnetic confinement of the plasma.
Fig. 29. Comparison of pulsed high-thrust fusion rocket with nuclear rockets (after Moeckel)
Fig. 30. Comparison of steady-state low-thrust fusion rocket with nuclear electric propulsion system (after Moeckel)
specific weight, \( \alpha \), of a 1 kg/kW of a steady-state low-thrust fusion rocket and specific impulse \( I_{sp} \) of 10,000 sec of a high-thrust pulsed fusion rocket, the crosspoint is about at Neptune for the same trip time. For missions beyond the crosspoint (in this case, near Neptune), the low-thrust fusion rocket would provide a shorter trip time than the pulsed high-thrust rocket. It should be noted, however, that this crosspoint will shift for different values of \( \alpha \) and \( I_{sp} \).

As discussed previously, D-He\(^3\) is the most favored fusion fuel, primarily because of the relatively desirable fusion products (mainly charged particles). The severity of the problems of radiation shielding and thermal insulation of the superconducting subsystem would be greatly reduced. However, the operating temperature would necessarily be higher, resulting in more rapid energy loss and more difficult triggering requirements.\(^{156,236}\) One possible way of dealing with the problem of a higher triggering temperature requirement would be to use D-T fuel for starting. When a self-sustaining fusion reaction had been achieved, the reactor would be switched to the D-He\(^3\) fuel.\(^{190}\)

1. Steady-State Low-Thrust Propulsion Systems

The steady-state thermonuclear fusion propulsion systems are characterized by relatively low specific weight and very high specific impulse, as compared to other low-thrust systems (e.g., nuclear electric system, solar electric system).\(^{155}\) Two possible approaches to a fusion propulsion system are shown in Fig. 31.\(^{156}\) In the first approach the kinetic energy of the charged particles from the plasma is used indirectly, by converting it to electric power for use in ion engines to produce thrust. In the second approach the energetic plasma is used directly, by being expelled from the reactor together with a suitable amount of propellant added to bring the exhaust velocity down to the optimum value. Various schemes for converting the energy of the plasma particles into electricity are possible. Direct conversion is preferred because it requires no turbine and electric generator, which are very heavy and involve failure-prone moving parts.\(^{237}\) Direct conversion schemes, such as letting the charged particles from fusion reactions expand against a magnetic field, have been proposed for ground-based fusion power.
Fig. 31. Steady-state fusion propulsion systems (156)
a. Magnetic Mirror Engine

One of the earlier and simpler concepts for steady-state propulsion systems was the magnetic mirror engine, shown schematically in Fig. 32. The upper drawing shows a configuration involving the application of an asymmetrical magnetic field at the open ends of the reactor chamber. The field strength at one end is made to be much stronger than at the other end. The charged particles flow preferentially out through the weak mirror, at which point the propellant is introduced and mixed with the escaping plasma to obtain the optimum exhaust velocity. The plasma escaping through the stronger mirror could be used to generate electricity by direct conversion for onboard use. The second configuration involves the application of a symmetrical magnetic field at both ends of the reactor chamber, which are bent 90°. The plasma leaking out the two ends is guided by a magnetic field (magnetic nozzle) into unidirectional flow and expanded, at which point propellant mixing takes place.

Magnetic mirror engines are not self-sustaining; the steady-state injection of an energetic neutral beam may be required. This, in turn, would require a very heavy energy storage subsystem and would result in a specific weight that, for space applications, would be unacceptably high. To reduce end-losses to an acceptable level, a minimum length of the order of 1 km for the reactor may be required, which in turn would require a shuttle craft of extremely large capacity to service it.

b. Toroidal Fusion Reactor Engine

In the toroidal fusion reactor engine, there would be no need for the steady-state injection of fusion fuel, a process which might take up a great portion of the power output of the fusion power plant. The direct fusion rocket concept is shown schematically in Fig. 33. The plasma, which is confined in the toroidal magnetic field, is gradually lost by diffusing toward the walls. A diverter, similar to the one suggested for the ground-based toroidal reactor, is a special section of the torus which can prevent the particles from diffusing to the torus wall. It collects the particles out of the reaction chamber and guides them into the magnetic nozzle, where the propellant is introduced. In an indirect fusion propulsion system, the kinetic energy of the charged particles from the fusion reactor...
Fig. 32. Magnetic mirror fusion propulsion engines\(^{276}\)
Fig. 33. A direct fusion rocket based on a toroidal fusion reactor\textsuperscript{(156)}
is converted to electrical power by the van de Graff generator.\(^{(156)}\) Ion engines are needed to produce thrust. The toroidal fusion reactor engine is considered to be the more promising concept.

At present, the toroidal fusion reactor utilizes a heavy iron core transformer through which a current is induced in the plasma to provide the first stage plasma (ohmic) heating. Since the specific weight should be as small as possible for space propulsion, this plasma heating technology may not be appropriate. Also, \(\beta\) has been limited to a few percent, to ensure stable operation. Much higher \(\beta\) values are necessary in order to reduce cyclotron radiation losses to a minimum, which in turn would reduce the weight of the radiator to a minimum. More advanced plasma heating and stabilization techniques are to be developed for the purpose.

Another major weight penalty of a steady-state fusion propulsion system is the heavy cryoplant required for cooling the superconductivity magnet.\(^{(162)}\) The allowable structure temperature of the magnetic nozzle limits the performance to some extent.

2. Pulsed Propulsion System

The advantage of the pulsed thermonuclear fusion propulsion concept is that it offers greatly improved performance in thrust and specific impulse. D-T fuel is allowable for pulsed propulsion systems. The fusion reactions take place in an explosive mode. The resulting particles, moving at extremely high velocities, are available to produce high thrust. The smallness of the interaction time per pulse - of the order of 100 \(\mu\)sec or less - provides a means of circumventing the overheating barrier inherent in steady-state systems.\(^{(232)}\) The total operating time of the "propellant" - i.e., fusion plasma plus added propellant - interaction for even a high mission velocity might be under 1 second.\(^{(238)}\) Pulsed fusion systems can be grouped into two categories - external and internal, as shown in Fig. 34.
a. External Systems

In an external system, the pulse unit, which consists of a charge of propellant plus a small charge of fusion fuel, is ignited outside the vehicle by a laser beam,\textsuperscript{(239)} a relativistic electron beam,\textsuperscript{(240)} or a nuclear fission bomb.\textsuperscript{(240)} A "fissionless" triggering device is preferable because its energy output can be made much smaller; the minimum energy output of a thermonuclear bomb detonated by a fission trigger is always above the kiloton TNT equivalent energy range.\textsuperscript{(240)} It is estimated that $10^{7}$ joules of input energy is sufficient for triggering a fusion reaction in a fuel pellet a fraction of a cubic centimeter in volume. If the fuel pellet is optimally compressed and ignited, the amount of triggering energy required would be lower, as is discussed under the ground power application.\textsuperscript{(228)} When the fusion reaction occurs, the cloud of propellant plus fusion products immediately impacts a portion of the vehicle, producing accelerations of the order of $10^{5}$ g. It is desirable that maximum momentum be imparted to the vehicle with as little increase in internal energy as possible. The portion of the vehicle that receives the impact energy may be a pusher plate which is connected to a shock absorber, a rotating cable pusher (in which the cables act as a parasol spinning about its shaft and closing under the impact of
an impulsive gust), \(^{(241)}\) or a concave magnetic mirror (in which the plasma-induced electromagnetic force serves to propel the spacecraft, which is rigidly connected to the mirror). \(^{(240)}\) The shock is thus reduced to a manageable acceleration that the instruments in the payload and the structure of the vehicle can withstand. If the flow of particles of the cloud is collimated, the maximum impulsive efficiency is apparently 50\%. \(^{(232)}\) However, the flow within the explosive cloud is generally isotropic. As a result, the impulse imparted to the vehicle depends on the shape and size of the pusher plate (or its variations), and the maximum impulsive efficiency is hence much lower than it would be for collimated flow, since the projected area of the pusher plate subtends only a small solid angle. It would be desirable for the flow of the explosive cloud to be collimated; however, technology in this area has not been explored.

b. Internal Systems

In an internal system, a pulse unit is exploded within a pressure vessel which is attached to the vehicle through a relatively small shock absorber. The wall of the pressure vessel is cooled by a flow of propellant. The gas mixture, which is at high temperature and pressure, expands through the nozzle. Because the pressure vessel and the nozzle are designed to direct the "propellant" in a well-collimated stream, the internal system has a higher impulse efficiency \(^{(232)}\) than the external system. The specific impulse is limited, however, to about 2500 sec because of limits to the temperature, pressure, and neutron or gamma radiation that can be tolerated in the chamber. The various schemes of fusion propulsion are summarized in Table 17.

In either the external or the internal system, the size of the pulse unit determines the size of shock absorber required. If a smaller pulse unit can be used, the shock absorber, and hence the total vehicle mass, can be smaller. \(^{(232)}\) Thus a propulsion system with high thrust and low specific mass is possible through use of the pulsed fusion device. Further studies are needed to solve the ablation problem of the pusher plate or its variations, and to develop a practical triggering device.
Table 17. Summary of most significant characteristics (projected) of thermonuclear fusion propulsion schemes

<table>
<thead>
<tr>
<th>Pulsed High-Thrust Systems</th>
<th>Steady-State Low-Thrust Systems</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{sp} = \sim 10,000$ sec</td>
<td>$\alpha \leq 1$ kg/kW</td>
</tr>
<tr>
<td><strong>External System</strong></td>
<td></td>
</tr>
<tr>
<td>Triggering device</td>
<td>Indirect use of plasma energy</td>
</tr>
<tr>
<td>- Laser beam</td>
<td>- Toroidal reactor plus ion engines</td>
</tr>
<tr>
<td>- Relativistic electron beam</td>
<td>- Van de Graaff generator for electricity</td>
</tr>
<tr>
<td>- Atomic bomb</td>
<td></td>
</tr>
<tr>
<td>Momentum conditioner</td>
<td>Direct use of plasma energy</td>
</tr>
<tr>
<td>- Pneumatic or other mechanical shock absorber plus pusher plate</td>
<td>Mirror Engine</td>
</tr>
<tr>
<td>- Rotating cable</td>
<td>- Symmetrical</td>
</tr>
<tr>
<td>- Magnetic mirror</td>
<td>- Asymmetrical</td>
</tr>
<tr>
<td><strong>Internal System</strong></td>
<td>Toroidal engine - diverter plus magnetic nozzle</td>
</tr>
<tr>
<td>- Pressure vessel</td>
<td></td>
</tr>
<tr>
<td>- Relatively small shock absorber</td>
<td></td>
</tr>
<tr>
<td>- High efficiency energy utilization</td>
<td></td>
</tr>
<tr>
<td>- Lower $I_{sp}$</td>
<td></td>
</tr>
</tbody>
</table>

G. RESEARCH AREAS OF SPACE FUSION PROPULSION APPLICATION

Many problems remain to be solved before fusion power can be realized. In the propulsion application, the basic physics and engineering problems are similar to those for the ground based application, which have been discussed previously. To conclude this study, the problems peculiar to the space propulsion application, some of which have been mentioned, are summarized below:
1) In the application of fusion energy to space propulsion, the most important factor to be considered is minimization of the masses of the subsystems. The waste heat radiator and cryogenic plant are usually very massive. It may be worthwhile to conduct a detailed and extensive study on efficient heat removal methods in a near-zero gravity environment, to find the lightest heat removal system. Heat removal subsystems using helium, potassium, and lithium have been studied for ground-based fusion power systems; however, for these systems, the weight of the subsystem is not very important. The nucleate boiling mechanism, in which numerous tiny bubbles form and leave the heat transfer surface, greatly increases the heat transfer rate. In zero gravity, however, the buoyancy force no longer exists to remove the gas bubbles, although in a strong magnetic field environment, other kinds of mechanisms such as ponderomotive, electrostrictive, and magnetostrictive forces may retard or help the movement of the bubbles; these mechanisms need further study. Heat pipe radiators, magnetic caloric pumps, conventional heat removal methods with different coolants, or other heat removal methods may have significantly different characteristics, with consequent important system parameter changes.

2) Examination of the exceptionally low specific weight (< 1 kg/kW) projected for a low-thrust fusion rocket may reveal some areas of difficulty to be overcome:

a) The aspect ratio (major diameter/Minor diameter of the torus) is assumed to be 2, which is much smaller than the average value of 3 to 5 used for the ground power application. A smaller aspect ratio means a higher heat flux. At the same time, less space is available for cooling. Thus, in space propulsion, the heat transfer problem is even more challenging.

*The basic principle involved is that the attractive force between a magnetofluid and the regions of high magnetic field will decrease with increasing temperature and a pressure head will result when the fluid is heated to high temperature.
b) The specific weight of the radiator is assumed to be 0.015 kg/kW at 2000°K; for a nuclear electric propulsion system, the corresponding values are ~1 kg/kW at 1000°K. Of course, for a higher radiator temperature, the weight can be greatly reduced; however, the vacuum wall must operate at an even higher temperature than the ~1000°K used for the ground power application. At such a high temperature, few materials can satisfy some of the basic requirements of a vacuum wall, such as strength and radiation reflectivity.

c) The specific weight of the shadow shield\(^\text{a}\) for a nuclear electric propulsion system ranges from a few kg/kW to about 10 kg/kW, whereas for a fusion rocket, it is assumed to be only a small fraction of 1 kg/kW.

d) The specific weight of the helium cryoplant\(^\text{b}\) is assumed to be 0.005 kg/kW, which is said to be a factor of five below that of existing systems. Also, the cryoplant is assumed to operate with a current density in the magnet windings as high as 10\(^9\) amp/m\(^2\), in order that the magnet heat absorption is at a minimum. This current density is about a factor of 10 higher than what is practical in today's superconductivity magnets.

e) The specific weights of some major subsystems, the concepts for which are still in a nebulous stage, such as energy storage, triggering mechanism, plasma heating, fuel delivery, etc. are assumed to be negligible.

3) Even if D-He\(^3\) fuel is used for fusion, side reactions can produce significant neutron flux.\(^{(166,233,245)}\) In an application to unmanned spacecraft, these neutrons need not be shielded completely. They can be reflected from the magnetic coils and channeled to space. The neutron energy deposited in the coils can be decreased by making the materials thinner and by choosing materials with smaller cross sections. Owing to a lack of neutron and gamma ray spectra for a

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\*A contradictory estimate of the specific weight of the shielding and refrigeration plant of 3.5 kg/kW has been reported.\(^{(154)}\)
fusion system, rigorous shielding analysis has been impossible in the past. A much improved study on shielding problems is needed.

4) Although some studies of the ablation of pusher plates have been made, the problems are not yet solved. The design of the pusher plate and the shock conditioner determines the useful life and efficiency of the system. Exposure distance between the pulse unit and the pusher plate, fatigue of the structures, and the ablation rate of the pusher plate are all important factors.

5) The effects on the plasma equilibrium due to the diverter or magnetic nozzle need to be demonstrated. The diverter, the jet mixing chamber, and the magnetic nozzle technologies need to be developed.

6) Efficient methods for removing high energy particles from the reactor and guiding them into a unidirectional beam should be developed.

7) Very light, highly reliable, repeatable, and efficient startup or triggering systems await breakthrough technology.

8) More detailed systems studies are needed to guide the direction of development among the various concepts. Some concepts, such as high beta theta pinch systems and inertially-induced magnetically-confined hybrid concepts, have not been explored for space propulsion application.

9) A more careful estimate of energy balance of the D-He\(^3\) fueled reactor would be valuable.
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APPENDIX

MATTER-ANTIMATTER ANNIHILATION AS AN ENERGY SOURCE IN PROPULSION

D. F. Dipprey

In this appendix a first-order analysis is presented to establish the amount of antimatter which must be stored on board a rocket vehicle using matter-antimatter annihilation as an energy source. The following simplifying assumptions are made:

1) The mass of antimatter used as the energy source is negligible as compared with the mass of propellant.
2) The mass of the rocket system used to contain the propellant and the antimatter and to convert the annihilation energy into kinetic energy of the propellant is neglected as compared with the payload mass. Conversely, this system mass could be defined as part of payload mass.
3) The energy conversion to propellant kinetic energy is 100% efficient.
4) The rocket velocity and the propellant exhaust velocity are both so much less than the speed of light that relativistic effects can be neglected.
5) The rocket operates in free space, far from accelerating fields.

Nomenclature

\( m_a \) = mass of antimatter and matter converted to energy
\( m_p \) = mass of propellant expelled
\( \Delta V \) = rocket velocity increase
\( V_e \) = exhaust velocity of propellant with respect to the rocket
\( m_{PL} \) = payload mass. If assumption 2) is not used this also includes the rocket system mass
\( c \) = the speed of light

Subscript \( m \) refers to the value that corresponds to minimum \( m_a \)
The Problem

1) Given \( m_{PL} \) and \( \Delta V \), find the minimum amount of annihilation mass (energy) that must be stored on board the rocket.

2) Compare this result with that obtained when propellant mass is zero and the photons produced during matter-antimatter annihilation are released as the propellant (photon rocket).

3) As an example of the mass of antimatter needed for an energetic mission within the solar system, find a value for \( m_a \) when \( \Delta V \) is about twice the speed of the earth around the sun (i.e., \( \Delta V = 7 \times 10^4 \text{ m/s} \)) and \( m_{PL} = 10^4 \text{ kg} \).

Formulation

Under the assumptions, rocket momentum conservation gives for \( V_e \) constant in time:

\[
\frac{\Delta V}{V_e} = \ln \frac{m_{PL} + m_p}{m_{PL}}
\]  

(A-1)

With assumptions 3) and 4), the energy conservation equation is

\[
m_a c^2 = m_p \frac{V_e^2}{2}
\]  

(A-2)

Eliminating \( m_p \) between Eqs. (1) and (2) yields

\[
\frac{m_a}{m_{PL}} = \frac{V_e^2}{2c^2 \left( \frac{\Delta V}{V_e} \right)^2 - 1}
\]  

(A-3)

Let \( \gamma = \frac{\Delta V}{V_e} \)

and \( \mu = \frac{2c^2 \frac{m_a}{m_{PL}}}{(\Delta V)^2} \)
Then Eq. (3) can be written

\[ \mu(y) = \frac{1}{y^2} \left[ \exp(y) - 1 \right] \quad (A-4) \]

The first part of the problem is then to find if a minimum exists in this function. An extremum exists if

\[ 0 = \frac{d\mu(y)}{dy} - \frac{2}{y^3} \left[ \exp(y) - 1 \right] \]

or if

\[ \exp(y) = \frac{1}{1 - y^2} \]

By iteration of the transcendental equation, the solution point for the extremum is found to be

\[ y(m) \approx 1.6 \]

Consideration of the second derivative of the function in Eq. (A-4) indicates that the extremum at this point is indeed a minimum. Thus \( m_a \) is a minimum when

\[ V_e(m) = \frac{\Delta V}{1.6} \quad (A-5) \]

For this condition

\[ \frac{2 \Delta V^2}{(\Delta V)^2} \frac{m_a(m)}{m_{PL}} = \frac{(\exp 1.6 - 1)}{(1.6)^2} = 1.55 \]

or

\[ m_a(m) = 0.775 m_{PL} \left( \frac{\Delta V}{c} \right)^2 \quad (A-6) \]
The above relation gives the minimum mass, \( m_a \), required for a given set of values of \( m_{PL} \) and \( \Delta V \) and therefore constitutes the answer to the first part of the problem. The propellant mass corresponding to minimum \( m_a \) can be estimated by substituting the value \( Y(m) = 1.6 \) into Eq. (1)

\[
\frac{m_p(m)}{m_{PL}} = \exp(1.6) - 1
\]

or

\[
\frac{m_p(m)}{m_{PL}} \approx 4 \tag{A-7}
\]

To deal with the second part of the problem we must negate assumption 1), since the "propellant" mass as distinct from the antimatter mass goes to zero, and also assumption 4), since the exhaust velocity becomes the speed of light. The mass consumed, \( m_a \), will be negligible with respect to the \( m_{PL} \), and the momentum equation can be written

\[
m_{PL} \Delta V = p \tag{A-8}
\]

where \( p \) is the momentum of the departing photons. The average momentum associated with the photons produced during the matter-antimatter annihilation of a total mass \( m_a \) is

\[
p = \frac{E}{c} = \frac{m_a c^2}{c} = m_a c \tag{A-9}
\]

\[
\frac{m_p \{V_e = c\}}{m_{PL}} = \frac{\Delta V}{c} \tag{A-10}
\]
Comparing Eqs. (6) and (10), we obtain

\[
\frac{m_a \left( V_e = c \right)}{m_a(m)} = \frac{1}{0.775} \frac{c}{\Delta V} \tag{A-11}
\]

which is the answer to the second part of the problem. For rockets designed to travel within the solar system, \(c >> \Delta V\). One can then see from Eq. (A-10) that a very large penalty is incurred if photons are used for the propellant instead of an optimum separate propellant mass.

For \(\Delta V = 7 \times 10^4 \text{ m/s}\) and \(m_{PL} = 10^4 \text{ kg}\), Eq. (A-6) gives

\[
m_a(m) \approx 0.46 \text{ gm} \tag{A-12}
\]

which is the answer to the third part of the problem.

We have assumed that \(m_a\) is the total mass of the interacting matter and antimatter; therefore, the mass of antimatter needed would be half of this amount. However, as already discussed in the introduction, in a H–H interaction, about half of the annihilation products appear as neutrinos which cannot be utilized. Thus the net effect is that about one-half gram of antiprotons would be needed for the mission in the above example.

Two additional observations can be made with regard to this example. First, the specific impulse (exhaust velocity) corresponding to minimum annihilation mass is very high. Using Eq. (A-5) we obtain

\[
I_s(m) = V_e(m) = \frac{\Delta V}{1.6} = \frac{7 \times 10^4}{1.6}
\]

\[
\approx 4.4 \times 10^4 \text{ m/s}
\]

\[
\approx 4400 \text{ lbf \cdot s/lbm}
\]
This shows that a very high energy density \( (V_e^2/2) \) must be imparted to the propellant mass. Difficulties in containing the energetic propellant and converting its energy content to kinetic energy may force consideration of using a larger, nonoptimum mass of propellant and hence a larger mass of antimatter. The second observation is that if one goes to the other extreme of the photon rocket allowing the photons produced to act as propellant, a very large amount of antimatter is required:

From Eq. (A-11)

\[
\frac{m_a (V_e - c)}{m_a (m)} = \frac{3 \times 10^8}{0.775 \times 7 \times 10^4} = 5500
\]

and, using the result of Eq. (12)

\[
m_a (V_e = c) \approx 2 \text{ kg}
\]