(NASA-CR-142725) A RECOMMENDED R EQUALS 1/2, K EQUALS 32, QUICK-LOOK-IN CONVOLUTIONAL CODE FOR NASA USE (Notre Dame Univ.) 14 p HC $3.25 CSCL 09B

N75-23185

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G3/61 19468

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ELECTRICAL ENGINEERING

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#N75-23185

A Recommended R = 1/2, K = 32, Quick-Look-In Convolutional Code for NASA Use*

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Technical Report No. EE-751

April 28, 1975

ABSTRACT

A new R = 1/2 K = 32 Quick-Look-In code is described and compared to the R = 1/2 K = 32 Massey-Costello code now used in some NASA systems. The new code, recently discovered by Johannesson, has the "optimum distance profile" property. This new code is shown, by comparison of Fano sequential decoding performance on a simulated Gaussian noise channel, to be computationally superior to the Massey-Costello code. The new code is also shown to be superior to the Massey-Costello code according to several analytical code criteria.

* This research was supported by the National Aeronautics and Space Administration under NASA Grant NSG 5025 at the University of Notre Dame in liaison with the NASA Goddard Space Flight Center.

I. Introduction

In research under NASA Grant NGL 15-004-026, this author and D. J. Costello [1] developed a new class of rate R = 1/2 non-systematic convolutional codes, called "quick-look-in" (QLI) codes for deep-space applications. The QLI property requires that the two code-generating polynomials, in D-transform notation, satisfy

$$G^{(1)}(D) = G^{(2)}(D) + D. \qquad (1)$$

The QLI property permits the information sequence to be retrieved from the two non-systematic encoded sequences simply by adding these sequences bit-by-bit modulo-two. The QLI property has been found particularly useful in testing the encoding hardware for deep-space missions, and is considered a highly-desirable feature if it can be purchased without significant loss of optimality compared to non-systematic codes without the QLI constraint.

In their paper [1], Massey and Costello gave a K = 48, R = 1/2 QLI code which had the property that, when truncated at every shorter length, the resulting QLI code exhibited good computational performance and low error probability when used with sequential decoding. These shortened QLI codes have been adopted by NASA for several deep-space missions with satisfying results.

In other work under NASA Grant NGL 15-004-026, R. Johannesson [2] formulated the concept of "optimum distance profile" (ODP) convolutional codes and showed the importance of the ODP property for computational performance when the code is used with sequential decoding.

The distance profile $\underline{d}$ of a convolutional code of constraint length K branches is defined as follows: Let D_k denote the minimum separation over the first k branches between encoded sequences resulting from information sequences differing in their first digit. Then the vector of distances

$$\underline{d} = [D_1, D_2, \ldots D_K]$$

is called the distance profile. (Note: In his paper [2], Johannesson uses the "column distance" d_i in place of D_k, the relation between these two distances is $D_k = d_{k+1}$; Johannesson also uses the memory M of the code rather than the constraint length K, the relation between M and K is K = M + 1.) The distance profile $\underline{d}$ is said to be superior to the distance profile $\underline{d}'$ if, for the smallest k such that $d_k \neq d'_k$, it is the case that $d_k > d'_k$.

In his paper [2], Johannesson gives a QLI code that is also ODP among <u>all</u> codes for $K \leq 24$. More recently, using the software, developed at Notre Dame for the UNIVAC 1107 under NASA Grant NGL 15-004-026, with the UNIVAC 1108 computer available to him at the Lund Institute of Technology, Sweden, Johannesson [3] extended his list of QLI ODP codes to $K \leq 51$. Where there was more than one QLI ODP code for a given K, Johannesson chose the one with the smallest number, M_k, of paths at distance d_k for k = K, with any further ties resolved by choosing the code with largest D_{72}. This latter choice ensures a large free distance, $d_{free} = D_\infty$, which is an important determiner of decoding error probability with sequential decoding.

In the near future, an R = 1/2, K = 32 convolutional code must be chosen for the International Ultraviolet Explorer (IUE) spacecraft. At the request of the Goddard Space Center, we have undertaken a thorough comparison of the K = 32 code obtained by shortening the Massey-Costello code and the K = 32 ODP QLI code of Johannesson to see if the possible advantages of the latter code would justify its choice over the former in the IUE mission. This report gives the results of this comparison, as well as our recommendation that, in light of its demonstrated superiority, the Johannesson K = 32 ODP QLI code should be selected for the IUE spacecraft.

II. Analytical Comparison of the Codes

Hereafter, we refer to the K = 32 truncated Massey and Costello QLI code as the M-C QLI code, and we refer to the Johannesson K = 32 ODP QLI code as the ODP QLI code. The actual generators which define these two codes are listed in Table I.

We first consider analytical measures of comparison for these two codes. Using assembly-language software developed for the IBM 370/158 computer in the University of Notre Dame Computing Center, we calculated the minimum distance D_k and number of erroneous paths, M_k, at distance D_k from the correct path for all $k \leq 74$ in the case of the M-C QLI code, and for all $k \leq 72$ in the case of the ODP QLI code. Each of these calculations required well over an hour of central-processor time. The results of this calculation are given in Table II and, with the M_k data deleted, in Figure 1.

Recall that the distance profile $\underline{d}$ involves only D_k for $1 \leq k \leq K = 32$. Thus, the ODP QLI code is guaranteed to be superior, in the D_k sense, to the M-C QLI code only for $k \leq 32$. Fig. 1 shows, however, that the ODP QLI code is in fact superior for all $k \leq 72$ which is as far as time permitted the calculation to extend. Moreover, the superiority of the ODP QLI is evident for k as small as 6. Even at those points (k = 7, k = 16, k = 63, k = 65, k = 66) where D_k of the M-C QLI code "catches up" with that of the ODP QLI, Table II shows the latter code is clearly superior in the smaller number M_k of erroneous paths at distance D_k from the correct path. In many places over the range of k, we see from Figure 1 that the ODP QLI code has D_k a full two units greater than the M-C QLI code.

The importance of the distances D_k is twofold. First, a rapid increase of these distances as k increases (particularly for k small) is important for good computational performance with sequential decoding. This follows from the fact that a larger separation D_k between the correct path and all incorrect paths of k branches will allow the sequential decoder to reject the incorrect paths with less searching. Second, the D_k's for k large are important determiners of decoding error probability since they measure the "long-term" ability of an incorrect path to appear as an attractive path to the sequential decoder. This is generally acknowledged by the interest in $d_{free} = D_\infty$ of convolutional codes.

For two codes with the same d_{free}, the depth k at which first one obtains $d_k = d_{free}$ is a measure of their relative error probability; the larger this depth, then the poorer the error probability. For both the M-C QLI code and the ODP QLI code, we see from Fig. 1 that $d_{free} \geq 20$. Hence, both codes will be good from an error probability viewpoint. But the earlier depths at which the ODP QLI code attains each distance, particularly distance 20, is strong evidence that this code will be somewhat superior in terms of decoding error probability.

In Fig. 1, we have also included the profile of D_k for an ODP code with $K = \infty$ as far as this function has been determined to date, namely $k \leq 51$. [3] No code can have a distance profile superior to this one. We see that the $K = 32$ ODP QLI code matches this "ultimate" profile for $k \leq 34$, and is no more than one unit inferior for $k \leq 50$.

We conclude that, as far as analytical measures are concerned, the $K = 32$ ODP QLI code is a very good code indeed and clearly superior to the earlier $K = 32$ M-C QLI code.

III. Experimental Comparison of the Codes

To put the conclusions drawn from the analytical comparisons of the previous section to the acid test, software was written for the IBM 370/158 computer to simulate the additive white Gaussian noise channel and to perform Fano sequential decoding with both codes. The channel simulated was the

Gaussian channel with a symbol energy, E, to one-side noise spectral density, N_0, ratio of 0 db or, equivalently since $R = 1/2$ coding was used, an energy per information bit, E_b, to N_0 ratio of +3 db. The channel was quantized to eight levels (3 bit quantization.)

The results of decoding 10,000 frames of 256 information bits (plus a "tail" of $K - 1 = 31$ dummy zeroes) by the Fano algorithm for each code are given in Table III and in Figure 2. In Figure 2, the computation has been put on a per bit basis (so that this curve may be used for other frame lengths) by dividing the computation to decode the frame by $256 + 31 = 287$.

From Figure 2, we see the clear computational superiority of the ODP QLI code over the M-C QLI code. The practical import of this superiority is that there will be a significant savings in computer time to perform Fano sequential decoding when the ODP QLI code is used rather than the M-C QLI code. This is, of course, in agreement with the conclusions of our analytical comparison of these two codes.

With both codes, all 10,000 frames were decoded with 100,000 or fewer computations per frame. No decoding errors were made with either code in the 2.56×10^6 decoding decisions. Thus, one can assert with certainty, from the simulations, only that the decoding error probability is very small for both codes. As the simulation required an hour of central-processor time for each code, it was not feasible to enlarge the number of decoded frames to the point where the decoding error probability difference between the two codes would be evident. However, there is a significant difference in the nature of the computational curves of Figure 2 that supports the analytical conclusion that the ODP QLI code will also prove to be superior to the M-C QLI code from an error probability viewpoint. Note that the ODP QLI curve in Figure 2 has the expected "Pareto form" over the entire range out to 200 computations per bit, whereas the M-C QLI curve in Fig. 2 begins to plunge below the Pareto commencing at about 100 computations per bit. From extensive experience with past simulations, we know that such a "plunge" represents cases where the decoder has, because of weakness in the code, rushed through to the end of the tree and accepted a path without all the computation that should have been done to guarantee that this was the best path through the tree. Such a plunge is a tell-tale sign that the decoder is entering a region of less reliable decoding. From these signs, we would estimate that the error probability of the M-C QLI code would be on the order of 10^{-8} while that of the ODP QLI code, which has not given evidence of similar unreliability in its computational behavior, would be several times smaller.

IV. Recommendation and Acknowledgment

In view of the clear evidence of its marked superiority over the K = 32 M-C QLI code, we recommend that the K = 32 ODP QLI code found by Johannesson be adopted for the IUE spacecraft. It is unlikely that any K = 32 code, whether QLI or not, will ever be found which is significantly better than the recommended code.

We owe an obvious debt of gratitude to Mr. Rolf Johannesson of the Lund Institute of Technology, Sweden, for furnishing us with the K = 32 ODP QLI code discussed herein and the other codes he has found in continuation of the work which he began under NASA Grant NGL 15-004-026. We are also extremely grateful to Mr. Teofilo C. Ancheta, Jr., Research Assistant under NASA Grant NSG 5025 for developing all of the software described in this report -- a formidable task since the IBM 370/158 computer is not well matched to the task of convolutional coding.

References

[1] J. L. Massey and D. J. Costello, "Nonsystematic Convolutional Codes for Sequential Decoding in Space Applications," IEEE Trans. on Comm. Tech., COM-19, October, 1971, pp. 806-813.

[2] R. Johannesson, "Robustly-Optimal Rate One-Half Binary Convolutional Codes." Tech. Rpt. No. EE-7403, Dept. of Electrical Engineering, University of Notre Dame, July 3, 1974. (To appear in IEEE Trans. Info. Th., July 1975.)

[3] R. Johannesson, Private Communication, April, 1975.

$G^{(1)}$	(binary)	111,100,000,100,010,100,000,010,000,111,01
$G^{(1)}$	(octal)	74042402072
$G^{(1)}$	(hexadecimal)	F045021D
$G^{(2)}$	(binary)	101,100,000,100,010,100,000,010,000,111,01
$G^{(2)}$	(octal)	54042402072
$G^{(2)}$	(hexadecimal)	B045021D

A. The R = 1/2 ODP QLI K = 32 Code

$G^{(1)}$	(binary)	111,011,011,101,011,011,110,111,110,111,01
$G^{(1)}$	(octal)	73353367672
$G^{(1)}$	(hexadecimal)	EDD6F7DD
$G^{(2)}$	(binary)	101,011,011,101,011,011,110,111,110,111,01
$G^{(2)}$	(octal)	53353367672
$G^{(2)}$	(hexadecimal)	ADD6F7DD

B. The R = 1/2 M-C QLI K = 32 Code

Table I: Generators for the R = 1/2 ODP QLI K = 32 Code and the R = 1/2 M-C QLI K = 32 Code in Binary, Octal, and Hexadecimal

k	ODP QLI Code D_k	ODP QLI Code M_k	M-C QLI Code D_k	M-C QLI Code M_k
1	2	1	2	1
2	3	2	3	2
3	3	1	3	1
4	4	3	4	3
5	4	2	4	1
6	5	6	4	1
7	5	3	5	4
8	6	11	5	2
9	6	6	5	1
10	6	1	5	1
11	7	12	6	3
12	7	5	6	2
13	8	29	7	9
14	8	12	7	4
15	8	6	7	3
16	8	3	8	12
17	9	18	8	6
18	9	7	8	2
19	9	3	8	1
20	10	31	8	1
21	10	13	9	7
22	10	4	9	4
23	10	1	9	2
24	11	28	9	1
25	11	13	9	1
26	11	7	9	1
27	11	2	10	4
28	12	21	10	3
29	12	9	10	2
30	12	3	10	1
31	13	43	11	3
32	13	15	11	1
33	13	11	11	1
34	13	5	12	9
35	13	3	12	7
36	13	1	12	7
37	13	1	12	2
38	14	9	12	2
39	14	5	12	1
40	14	2	12	1
41	15	25	12	1
42	15	11	13	2
43	15	3	13	1
44	15	1	14	6
45	15	1	14	2
46	16	10	14	1
47	16	9	15	8
48	16	5	15	3
49	16	5	15	3
50	16	4	15	2
51	16	1	15	2
52	16	1	15	2
53	17	3	15	1
54	17	2	15	1
55	17	1	16	4
56	18	12	16	2
57	18	6	16	1
58	18	5	17	6
59	18	2	17	3
60	18	1	17	3
61	18	1	17	2
62	18	1	17	2
63	18	1	18	4
64	19	2	18	2
65	19	1	19	11
66	19	1	19	5
67	20	4	19	5
68	20	2	19	2
69	20	1	19	1
70	20	1	19	1
71	20	1	19	1
72	20	1	19	1
73			20	3
74			20	2

Table II: Minimum Distance D_k between Correct Path and All Incorrect Paths over k Branches, and the number, M_k, of Incorrect Paths at Distance D_k for the R = 1/2 ODP QLI Code and the R = 1/2 M-C QLI Code.

Computations N Required to Decode Frame	Number of Frames Decoded	
	R = 1/2 ODP QLI K = 32 Code	R = 1/2 M-C QLI K = 32 Code
$N \leq 400$	952	875
$400 < N \leq 550$	3,182	2,918
$550 < N \leq 600$	737	705
$600 < N \leq 850$	2,039	1,999
$850 < N \leq 1{,}000$	641	611
$1{,}000 < N \leq 1{,}500$	944	1,147
$1{,}500 < N \leq 4{,}000$	968	1,085
$4{,}000 < N \leq 5{,}000$	113	120
$5{,}000 < N \leq 10{,}000$	179	244
$10{,}000 < N \leq 20{,}000$	107	128
$20{,}000 < N \leq 50{,}000$	69	99
$50{,}000 < N \leq 100{,}000$	69	69
$100{,}000 < N$	0	0

Table III. Results of Decoding 10,000 Frames of 256 Information Bits Each by Fano Sequential Decoding on the Simulated 8-Level Quantized Gaussian Noise Channel with $E_b/N_o = 3$ db.

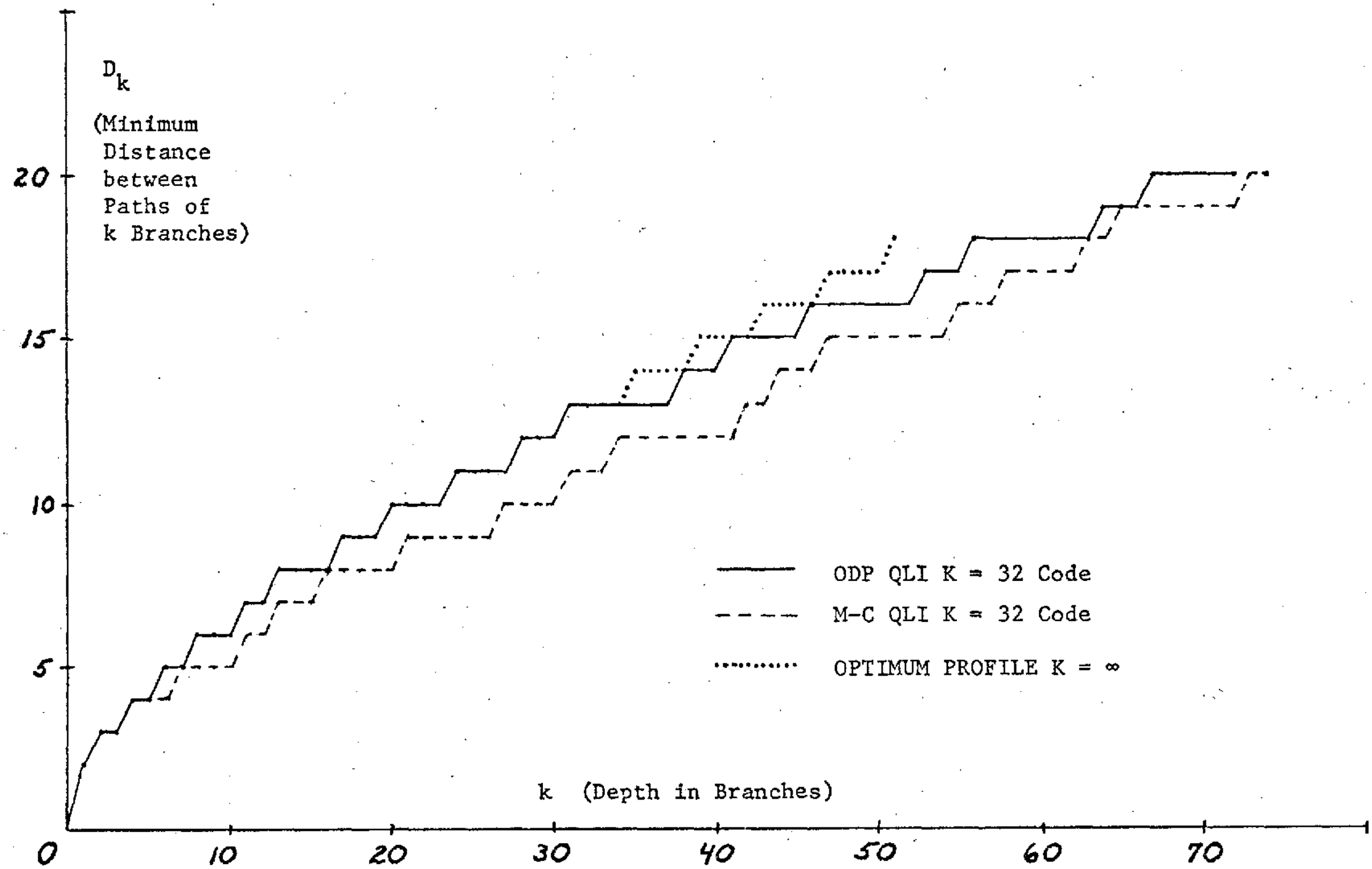

Figure 1: Comparison of the Distance Profiles for the Rate R = 1/2 ODP QLI K = 32 Code and the Rate R = 1/2 M-C QLI K = 32 Code.

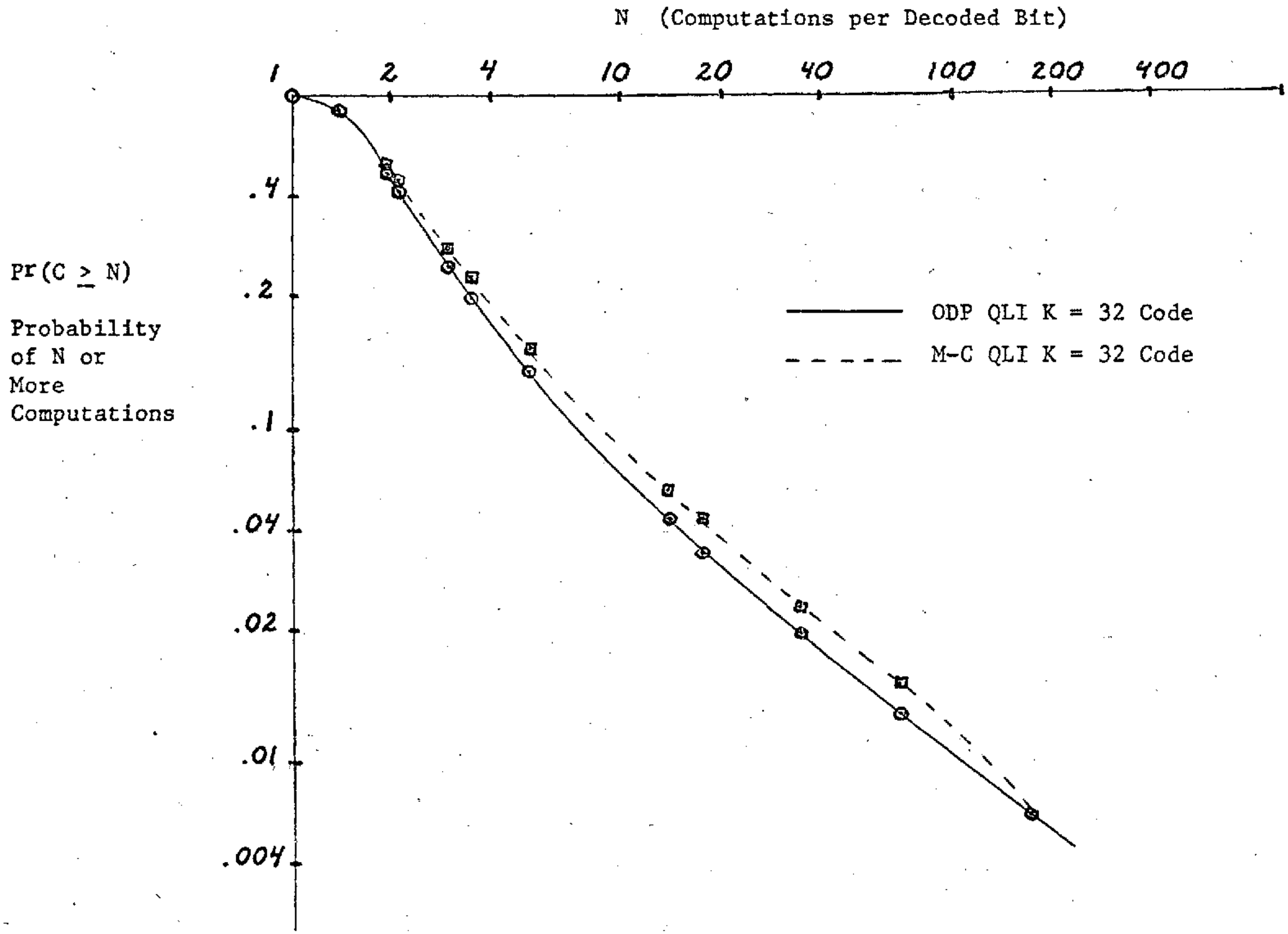

Figure 2: Comparison of the Computational Performance of the Rate R = 1/2 ODP QLI K = 32 Code and the Rate R = 1/2 M-C QLI K = 32 Code with Fano Sequential Decoding on the Simulated 8-level Quantized Gaussian Channel with E_b/N_o = 3 db, as Determined by Decoding of 10,000 Frames with 256 Information Bits per Frame.