DIGITAL PROGRAM FOR SOLVING THE LINEAR STOCHASTIC OPTIMAL CONTROL AND ESTIMATION PROBLEM

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**Abstract**

This report describes a computer program, LSOCE, which solves the linear stochastic optimal control and estimation (LSOCE) problem by using a time-domain formulation. The LSOCE problem is defined as that of designing controls for a linear time-invariant system which is disturbed by white noise in such a way as to minimize a performance index which is quadratic in state and control variables. The LSOCE problem and solution are outlined; brief descriptions are given of the solution algorithms; and complete descriptions of each subroutine, including usage information and digital listings, are provided. A test case is included, as well as information on the IBM 7090-7094 DCS time and storage requirements.
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DIGITAL PROGRAM FOR SOLVING THE LINEAR STOCHASTIC OPTIMAL CONTROL AND ESTIMATION PROBLEM

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SUMMARY

This report describes a computer program, LSOCE, which solves the linear stochastic optimal control and estimation (LSOCE) problem. The LSOCE problem is defined as that of designing controls for a linear time-invariant system which is disturbed by white noise in such a way as to minimize a performance index which is quadratic in state and control variables. In order to do so, the controls must use system measurements which are corrupted by white noise.

The LSOCE problem and solution are outlined; brief descriptions are given of the solution algorithms; and complete descriptions of each subroutine, including usage information and digital listings, are provided. The major subroutine solves the algebraic matrix Riccati equation by using an eigenvector method. Other major subroutines provided are a Lyapunov equation subroutine, an eigenvalue subroutine, an eigenvector subroutine, and a subroutine for solving the matrix Riccati differential equation. Program LSOCE is designed so that the user has options to solve all or part of the LSOCE problem. A test case for LSOCE (for the option which uses all parts of the program) is included, as well as information on the IBM 7090-7094 DCS time and storage requirements.

INTRODUCTION

Over the past few years, there have been great interest and activity in the area of computer-aided design of control systems. This report documents a computer program which has been used at the Lewis Research Center for designing optimal controls for linear systems that are subject to random disturbances and that have measurements corrupted by noise. This program has been applied in designing control systems for supersonic inlets (refs. 1 and 2). It solves the linear stochastic optimal control and
estimation (LSOCE) problem. The LSOCE problem is defined as that of designing controls for a linear time-invariant system which is disturbed by white noise in such a way as to minimize a quadratic performance index. In order to do so, the controls must use system measurements which are corrupted by white noise. The FORTRAN subroutines described in this report either are modifications of or improvements on subroutines available at present, or are implementations of published algorithms which are not now available as computer programs.

The linear stochastic optimal control and estimation problem is presented in Bryson and Ho (ref. 3) and is summarized in this report (see the section THEORETICAL BACKGROUND). The solution to the LSOCE problem is a Kalman filter, which estimates the system states, coupled through a set of optimal regulator gains to produce the desired control signal. The key to solving the LSOCE problem is the solution of the matrix Riccati differential equation. This equation occurs in solving for both the finite-time optimal linear regulator gains and the finite-time optimal linear estimator (Kalman filter) gains. An important special case is the infinite-time LSOCE problem, in which the main equations to be solved use algebraic (steady state) matrix Riccati equations.

Techniques for solving the Riccati differential equation include direct numerical integration, Kalman and Englar's Automatic Synthesis Program (ASP) described in reference 4, the GASP program of Freested, Webber, and Bass (ref. 5), and Vaughan's negative exponential method (ref. 6). Attention has also been given to solving the algebraic Riccati equation, most notably by Kleinman (ref. 7), Greenberg and Bard (ref. 8), and Fath (ref. 9). The approach taken in this report for solving both the differential and algebraic equations is a combination of Fath's and Vaughan's methods, both of which are extensions of the so-called eigenvector method first discussed by MacFarlane (ref. 10), O'Donnell (ref. 11), and Potter (ref. 12). In practice, the solution of the algebraic equation is used in obtaining the solution to the differential equation.

For the infinite-time LSOCE problem, it is of interest to compute system-covariance matrices so as to evaluate system mean-square behavior. This computation requires solution of a Lyapunov matrix equation. Mean-square behavior can be computed for the finite-time LSOCE problem; however, because of computer storage limitations, it was decided to include only computations for the infinite-time case. Solutions to the Lyapunov equation have been developed by such authors as Chen and Shieh (ref. 13) and Smith (ref. 14). The technique adopted in this program is that of Chen and Shieh, which requires the solution of a set of linear equations.

Program LSOCE was written so as to handle systems of any order and is restricted only by computer storage size and accuracy. In particular, maximum dimensions for system state, control, disturbance, and measurement vectors were selected to be 12, 5, 5, and 5, respectively. These dimensions were chosen so that the program would fit on an IBM 7090-7094 computer (32000 storage locations) without overlay. These dimen-
sions can, of course, be changed if the user should want to run larger problems, pro-
vided that the computer being used has sufficient storage. It should be noted that solu-
tion accuracy will be a direct function of system state dimension and also computer word
length.

The report is organized as follows: First, the theoretical background for program
LSOCE is presented. Then a general description of program LSOCE is given, including
a flow chart of the subroutines used; followed by the inputs, outputs, and options; and
then individual descriptions of each subroutine, including FORTRAN listings. Appendix A lists the symbols used. Appendix B gives descriptions and listings for a sample
problem. Appendix C tabulates the time and storage requirements for the sample prob-
lem.

THEORETICAL BACKGROUND

First, the formulation of and solution to the finite-time linear stochastic optimal
control and estimation (LSOCE) problem is presented. The formulation used follows
that of Bryson and Ho (ref. 3, p. 414). The infinite-time LSOCE problem is then de-
efined, the calculation of mean-square values is described, and various options which the
LSOCE program provides to the user are detailed.

FINITE-TIME LSOCE PROBLEM

The finite-time LSOCE problem is defined here as follows: Given the linear time-
invariant system

\[ \dot{x}(t) = Ax(t) + Bu(t) + w(t) \]  
(1)

\[ z(t) = Hx(t) + v(t) \]  
(2)

where

- \( x \) an \( n \)-dimensional state vector
- \( u \) a \( c \)-dimensional control vector
- \( z \) an \( m \)-dimensional measurement vector
- \( w \) an \( n \)-dimensional, white, zero-mean, Gaussian, plant-noise vector
- \( v \) an \( m \)-dimensional, white, zero-mean, Gaussian, measurement-noise vector
- \( A \) an \( n \)-by-\( n \) system matrix
B an n-by-c control input matrix
H an m-by-n measurement matrix
(All symbols are defined in appendix A.) Noises \(w\) and \(v\) are assumed to be uncorrelated with each other and to have covariance matrices given by

\[
E[w(t)w^T(t + \tau)] = Q\delta(\tau)
\]
\[
E[v(t)v^T(t + \tau)] = R\delta(\tau)
\]

where \(Q\) is a symmetric, positive-semidefinite, plant-noise, power spectral density matrix and \(R\) is a symmetric, positive-definite, measurement-noise, power spectral density matrix. Further, it is assumed that all matrices are constant and known, that the pair \((A, B)\) is controllable, and that the pair \((A, H)\) is observable. Also, the initial state is assumed to be a zero-mean random variable with a known covariance,

\[
E[x(t_0)x^T(t_0)] = P_0
\]

Given a set of measurements \(z(\tau)\) for \(t_0 \leq \tau \leq t\), find a control \(u\) as a functional of \(z(\tau)\) which minimizes the quadratic performance index

\[
J = E\left[\frac{1}{2}x^T(t_f)S_f x(t_f) + \frac{1}{2} \int_{t_0}^{t_f} \left(x^T Q_c \dot{x} + 2x^T N u + u^T P_c u\right) dt\right]
\]

Weighting matrix \(P_c\) is positive-definite and matrices \(Q_c\) and \(N\) are such that \((Q_c - NP_c^{-1}N^T)\) is positive-semidefinite.

It has been found that a large number of practical control problems can be formulated in the manner just described. The solution to this problem is well known (see ref. 3 for a derivation of a somewhat more general case). The control which minimizes \(J\) is given by

\[
u(t) = -K_c(t)\hat{x}(t)
\]

where \(\hat{x}(t)\) is the optimal estimate of \(x(t)\). Estimate \(\hat{x}(t)\) is a dynamic function of measurement vector \(z(t)\) and is generated as the output of a Kalman filter, described by
\[ \dot{x}(t) = Ax(t) + Bu(t) + K_e(t)[z(t) - H\hat{x}(t)] \quad (8) \]

The optimal control-gain matrix \( K_c(t) \) is given by

\[ K_c(t) = P_c^{-1}(B^TS(t) + NT) \quad (9) \]

where \( S(t) \) is the (positive-definite, symmetric) solution to the Riccati differential equation

\[ -\dot{S}(t) = S(t)(A - BPc^{-1}NT) + (A - BPc^{-1}NT)^TS(t) - S(t)(BPc^{-1}BT)S(t) + (Q - NPc^{-1}NT) \quad (10) \]

with the boundary condition \( S(t_f) = S_f \). The Kalman-filter-gain matrix \( K_e(t) \) is given by

\[ K_e(t) = P(t)H^TR^{-1} \quad (11) \]

where \( P(t) \) is the covariance of \( x - \hat{x} \), the error in the estimate, and is found as the (positive-definite, symmetric) solution to the Riccati differential equation

\[ \dot{P}(t) = AP(t) + P(t)A^T - P(t)H^TR^{-1}HP(t) + Q \quad (12) \]

with the initial condition \( P(t_0) = P_0 \).

The combination of the plant, Kalman filter, and state-estimate feedback gains is shown in figure 1. The filter can be viewed as a model of the plant which has been modified by insertion of a signal proportional to the difference between the measurement \( z \) and the estimated measurement \( \hat{z} \). This difference is multiplied by Kalman filter gains \( K_e \) and inserted into the plant model at the input to the integrators. The optimal control \( u \) is formed by multiplying the filter output \( \hat{x} \) by control gains \( K_c \).

The finite-time control gains \( K_c(t) \) are computed off-line by first solving equation (10) backwards in time from \( t = t_f \) to \( t = t_0 \), starting with \( S(t_f) = S(f) \) to obtain \( S(t) \). Then \( K_c(t) \) values are computed by using equation (9). Similarly, finite-time filter gains are computed off-line by solving equation (12) forwards in time from \( t = t_0 \) to \( t = t_f \), starting with \( P(t_0) = P_0 \) to obtain \( P(t) \). Then \( K_e(t) \) values are computed by using equation (11).
INFINITE-TIME LSOCE PROBLEM

A case of particular interest is the solution to the infinite-time LSOCE problem. In this case, gains $K_c$ and $K_e$ are constant, resulting in a control which is much simpler to implement than the finite-time solution. If the final time $t_f$ is allowed to be infinite, the solution to the Riccati equation (10) (and hence the control gains eq. (9)) settles out and becomes constant throughout the period $t_0 \leq t \leq t_f = \infty$. This constant value can be computed by solving an algebraic Riccati equation

$$S\left(A - BP_c^{-1}N^T\right) + \left(A - BP_c^{-1}N^T\right)^T S - S\left(BP_c^{-1}B^T\right)S + \left(Q_c - NP_c^{-1}N^T\right) = 0$$

which is obtained from equation (10) by letting $\dot{S} = 0$. Similarly, the constant-Kalman-filter-gain solution results if the final time is infinite, where the behavior of the filter in the region near $t = t_0$ is not of major interest. The solution to the Riccati equation (12) will settle out to a constant value for $t_0 \ll t \leq t_f$. This constant value can be computed by solving an algebraic Riccati equation

$$AP + PA^T - P(H^TR^{-1}H)P + Q = 0$$

which is obtained from equation (12) by setting $\dot{P} = 0$. Thus, the infinite-time (control and filter) solution is the exact solution for the time period of interest, $t_0 \ll t \ll t_f = \infty$.

Performance index $J$ of equation (6) can be examined now where $t_f \to \infty$. By interchanging the integration and expected-value operations, $J$ can be written

$$J = \frac{1}{2} \mathbb{E}\left[x^T(t_f)S_f x(t_f)\right] + \frac{1}{2} \int_{t_0}^{t_f} \mathbb{E}\left[x^TQ_c x + 2x^T Nu + u^TP_c u\right] dt$$

The integrand becomes constant as $t \to t_f$ for large $t_f$. As $t_f \to \infty$, it can be seen that the major contribution to $J$ will come from integrating this constant value. Thus, to minimize $J$, the integrand must be minimized. This leads us to define a new performance index

$$J^* = \mathbb{E}\left[x^TQ_c x + 2x^T Nu + u^TP_c u\right]$$

which will be minimized by a control system with constant $K_c$ and $K_e$ matrices. Thus, in summary, the infinite-time problem can be formulated as follows: Obtain a control $u$...
to minimize $J^*$ of equation (16), given a system described by equations (1) to (4). The solution is obtained by solving equations (9), (11), (13), and (14).

**MEAN-SQUARE BEHAVIOR**

For the infinite-time case, it is of interest to compute various system-covariance matrices and mean-square values by which to judge the control system's effectiveness. Of primary interest is the state-covariance matrix and the covariance of the error in the estimate. The latter is given by the Riccati equation (14). The state-covariance matrix can be obtained by solving the following Lyapunov equation (ref. 3, p. 417):

$$
(A - BK_c)X + X(A - BK_c)^T + BK_cP + PK_c^TB^T + Q = 0
$$

(17)

where $X$ is the covariance of $x$. Also, the covariance of estimate $\hat{x}$ can be shown to be

$$
\hat{X} = X - P
$$

(18)

By using $\hat{X}$, $X$, and $P$, the following covariances can be computed: The control covariance is

$$
U = \hat{K}_c\hat{XK}_c^T
$$

(19)

where $u$ is given by equation (7). The measurement covariance is

$$
Z = HXH^T
$$

(20)

where $z$ is given by equation (2). The estimated measurement covariance is

$$
\hat{Z} = H\hat{XH}^T
$$

(21)

where $\hat{z}$ is the estimated measurement vector given as

$$
\hat{z} = H\hat{x}
$$

(22)

Or, for any general output vector $y$ given as
\( y = Cx \)  

one can compute the output covariance

\[ Y = CXC^T \]  

or the estimated output covariance

\[ \hat{Y} = \hat{C}\hat{X}C^T \]  

where \( \hat{y} \) is the estimated output given as

\[ \hat{y} = C\hat{x} \]  

**LSOCE PROBLEM OPTIONS**

In addition to handling the general finite-time LSOCE solution, the infinite-time solution, and covariance matrix computation, program LSOCE provides the user with a number of options. These options make it possible to solve various subproblems by using portions of the LSOCE program. The subproblem options chosen do not exhaust all possibilities but rather represent the more common variations of the LSOCE problem.

The options available are

1. **Infinite-time Kalman filter**: Here, it is desired to estimate (but not to control) the state of the system of equations (1) to (4) for \( t_0 < t \leq t_f = \infty \). Thus, an infinite-time matrix \( K_e \) and constant-error covariance matrix \( P \) are to be computed, thereby defining a Kalman filter.

2. **Finite-time Kalman filter**: This option is the same as option 1 except that now \( K_e(t) \) and \( P(t) \) are to be computed given \( P(t_0) \) and \( t_f \) for \( t_0 \leq t \leq t_f \).

3. **Finite-time LSOCE problem without mean-square information**: This is the problem defined previously (in the section **FINITE-TIME LSOCE PROBLEM**), where \( K_e \) and \( K_c \) are time varying for \( t_0 \leq t \leq t_f \).

4. **Finite-time LSOCE problem with mean-square information**: This is the problem mentioned in option 3 except that it now includes a calculation of the mean-square behavior (outlined in the section **MEAN-SQUARE BEHAVIOR**). This mean-square calculation is useful in the case where \( t_f \) is large enough so that \( K_e \) and \( K_c \) are constant during some time \( t_0 < t_1 \leq t \leq t_2 < t_f = \infty \). During the interval \( t_1 \leq t \leq t_2 \), the system-covariance matrices will be constant. The covariance matrices computed in this option are those which apply within this interval.
(5) Infinite-time LSOCE problem: This is the problem outlined in the section INFINITE-TIME LSOCE PROBLEM, but including the computation of covariance matrices, as in the section MEAN-SQUARE BEHAVIOR.

(6) Infinite-time, noise-free regulator: Here it is assumed that state $x$ is directly measurable and that the system is described by equation (1), but with no plant noise ($Q = 0$). Thus, $x = \hat{x}$, no Kalman filter is required, and the feedback law (eq. (7)) is $u = -K_c x$. The performance index is that of equation (16), and the equations to be solved are (9) and (13) for the Riccati solution and control gains.

(7) Finite-time, noise-free regulator: It is assumed that state $x$ is directly measurable (no Kalman filter needed) and that the system is described by equation (1), but with no plant noise ($Q = 0$). The performance index of equation (6) is to be minimized, given $t_f$ and $S_f$. The equations to be solved are (9) and (10) for $K_c(t)$ and $S(t)$.

(8) Infinite-time regulator with plant noise: This option is similar to option 6 in that $x$ is assumed to be measurable, but now plant noise is assumed to be present ($Q \neq 0$). The computations required are those of equations (9) and (13). Mean-square information is also provided by solving equations (17), (19), and (24) for covariances $X$, $U$, and $Y$. Note that $X$ will equal $\hat{X}$, since $P$ will be zero (perfect state measurement); thus, $U = K_c X K_c^T$.

(9) Open-loop, plant-covariance matrices: It may be of interest simply to calculate the covariance matrices for an uncontrolled system disturbed by noise $w$ (with $Q \neq 0$), described by equation (1), having an output $y = Cx$ (eq. (23)). This would be done, for example, in comparing open-loop and closed-loop system behavior. In particular, useful information computed in this option is state covariance $X$ and output covariance $Y$.

The use of these options in program LSOCE is explained in more detail in the section DESCRIPTION AND LISTING OF MAIN ROUTINE LSOCE. There, the required inputs and available outputs are listed, and the format for system matrices ($A$, $B$, $H$, etc.) to be input is discussed.

**LSOCE PROGRAM**

This section serves as a user's guide to program LSOCE. It contains first a description of the main routine, followed by sections discussing, in specific terms, the options available in LSOCE, followed by a listing of main routine LSOCE. Then each subroutine is explained and listed, with the exception of subroutines ARRAY, HSBG, MINV, FACTR, and SIMQ, which are IBM Scientific Subroutine Package (SSP) subroutines and consequently are not listed. (The subroutines ARRAY, HSBG, and MINV are defined in the section Description of Subroutine RICSS, subroutine FACTR in the section Description of Subroutine EGVCTR, and subroutine SIMQ in the section Description of...
Subroutine LYPNV.) Background information on the methods used is included when necessary. Finally, there is a discussion of error messages.

All symbols are defined in appendix A. A sample problem (using the option which exercises all parts of the program) is given in appendix B. Time and storage requirements for this sample problem are tabulated in appendix C.

DESCRIPTION AND LISTING OF MAIN ROUTINE LSOCE

The computer program which solves the linear stochastic optimal control and estimation problem consists of a main routine, LSOCE, which calls a set of subroutines. The flow chart of this program (fig. 2) shows the calling sequence and gives a brief description of each subroutine. The main routine LSOCE defines the dimensions of the variables and contains the input data for the LSOCE problem. Main routine LSOCE may call the subroutines ESTMAT, CONTRL, COVM, and TRICCS, depending upon which option the user specifies.

The dimensioned variables are

A(N, N) Lyapunov equation matrix
AA(N, N) system matrix
AAA(N2, N2) Hamiltonian matrix associated with the Riccati equation
ANS(N, N) transient Riccati solution
BB(N, C) control input matrix
CC(R, N) output matrix
CI(N2) imaginary parts of eigenvalues of AAA
CR(N2) real parts of eigenvalues of AAA
HH(M, N) measurement matrix
IOR(N2) block-diagonalizing permutation vector (integer)
KC(C, N) control-gain matrix
KE(N, M) estimator-gain matrix
NBL(N2) size of individual irreducible blocks (integer)
NN(N, C) state-control product weighting matrix
PCINV(C, C) inverse of control weighting matrix
PP(N, N) estimation-error-covariance matrix
PPINIT(N, N) initial condition of transient Riccati solution for estimate
Q(N, N) Lyapunov equation matrix
QC(N, N) state weighting matrix
QQ(N, N) power spectral density matrix of plant disturbance
RRINV(M, M) inverse of power spectral density matrix of measurement noise
SS(N, N) matrix Riccati solution for control problem
SSINIT(N, N) initial condition of transient Riccati solution for control
TS(N2) scaling transformation vector
UU(C, C) control-covariance matrix
X(N2, N2) modified eigenvector matrix of AAA
XX(N, N) state-covariance matrix
YY(R, R) output-covariance matrix
YYEST(R, R) estimated output-covariance matrix
ZZ(M, M) measurement-covariance matrix
ZZEST(M, M) estimated measurement-covariance matrix
AI(N2) temporary storage array
AR(N2) temporary storage array
ASI(KSIM, KSIM) temporary storage array
BSIMQ(KSIM) temporary storage array
EXT(N2, N2) temporary storage array
EX1(N, N) temporary storage array
EX2(N, N) temporary storage array
EX3(N, N) temporary storage array
EX4(N, N) temporary storage array
EX5(N) temporary storage array
EX6(N) temporary storage array
IBL(N2) temporary storage array (integer)
IPER(N2) temporary storage array (integer)
IPERN(N2) temporary storage array (integer)
JMAX(N2) temporary storage array (integer)
TT(N2, N2) temporary storage array
XR(N2, N2) temporary storage array

where \( N \) is the number of state variables, \( M \) is the number of measurements, \( C \) is the number of control inputs, \( R \) is the number of outputs, \( N2 = 2 \times N \), and \( KSIM = (N \times N + N)/2 \).

All these variables are dimensioned at compile time by \( NMAX \), \( MMAX \), \( CMAX \), \( RMAX \), \( N2MAX \), and \( KSIMAX \), where \( NMAX = 12 \), \( MMAX = 5 \), \( CMAX = 5 \), \( RMAX = 5 \), \( N2MAX = 24 \), and \( KSIMAX = 78 \). Integer array dimensions which the user must input in a data statement are \( N \) (less than or equal to 12), \( M \) (less than or equal to 5), \( C \) (less than or equal to 5), and \( R \) (less than or equal to 5). All variables are specifically dimensioned by \( NMAX \), \( MMAX \), \( CMAX \), \( RMAX \), \( N2MAX \), and \( KSIMAX \) so that the user will not be forced to do any dimensioning. The maximum dimensions were chosen such that the problem would fit the 7094 storage capacity without need of overlay.

All data are input in block data format. Integer variables \( N \), \( M \), \( C \), and \( R \) (which are dimensions) and \( ISORT \) (which is the option number) must always be input. Data statements for double-subscripted variables must be in the following column form, for example:

```
DATA (VAR(I,J), I = 1,3, J = 1,2)
/1.0, 2 * 4.5, 85.4, 6.3, 6.0/
```

where matrix \( VAR \) is

\[
\begin{bmatrix}
1.0 & 85.4 \\
4.5 & 6.3 \\
4.5 & 6.0
\end{bmatrix}
\]

The specific data input and printed output are described for each option in the next sections.

The main routine \( LSOCE \) has an input variable, \( ISORT \), which causes various subroutines to be called, depending on the option desired (see the next sections and the section \( LSOCE \) PROBLEM OPTIONS for further description).

The options available are

(1) Infinite-time Kalman filter: calls subroutine \( ESTMAT \) only; calculates infinite-time \( KE \) and \( PP \)

(2) Finite-time Kalman filter: calls subroutines \( ESTMAT \) and \( TRICCS \); calculates infinite-time \( KE \) and \( PP \) and also \( KE \) and \( PP \) at each time point

(3) Finite-time \( LSOCE \) problem without mean-square information: calls subroutines \( ESTMAT \), \( TRICCS \), \( CONTRL \), and \( TRICCS \); calculates infinite-time \( KE \), \( PP \), \( KC \), and \( SS \) and also \( KE \), \( PP \), \( KC \), and \( SS \) at each time point
(4) Finite-time LSOCE problem with mean-square information (the option used in the sample problem, appendix B): calls subroutines ESTMAT, TRICCS, CONTRL, TRICCS, and COVM; calculates all that option 3 does plus the mean-square, system-covariance matrices.

(5) Infinite-time LSOCE problem: calls subroutines ESTMAT, CONTRL, and COVM; calculates infinite-time KE, PP, KC, and SS and the mean-square, system-covariance matrices.

(6) Infinite-time, noise-free regulator: calls subroutine CONTRL only; calculates infinite-time KC and SS.

(7) Finite-time, noise-free regulator: calls subroutines CONTRL and TRICCS; calculates infinite-time KC and SS and also KC and SS at each time point.

(8) Infinite-time regulator with plant noise: calls subroutines CONTRL and COVM; calculates infinite-time KC and SS and the mean-square, system-covariance matrices.

(9) Open-loop, plant-covariance matrices: calls subroutine COVM only; calculates mean-square, system-covariance matrices.

Option 1 - Infinite-Time Kalman Filter

The block data input for option 1 must include:

<table>
<thead>
<tr>
<th>ISORT</th>
<th>N</th>
<th>M</th>
<th>IOP1</th>
<th>IOP2</th>
<th>AA(N,N)</th>
<th>HH(M,N)</th>
<th>QQ(N,N)</th>
<th>RRINV(M,M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
<td>0</td>
<td>system</td>
<td>measurement</td>
<td>power spectral density</td>
<td>inverse of power spectral density</td>
</tr>
</tbody>
</table>

The output generated by option 1 is:
(1) A top heading:

```
RESULTS OF ESTIMATION PROBLEM
```

13
(2) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART       IMAGINARY PART

where ITER is the number of iterations

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-KE*H)
REAL PART       IMAGINARY PART

(6) The solution to the estimation Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

(7) The estimator-gain matrix with the heading

KE, THE KALMAN FILTER GAIN MATRIX

Option 2 - Finite-Time Kalman Filter

The block data input for option 2 must include

ISORT  2
N    number of state variables, less than or equal to 12
M    number of measurements, less than or equal to 5
IOP1  1 if output from subroutine SCALE is desired
      0 if output from subroutine SCALE is not desired
IOP2  1 if output from subroutine EGVCTR is desired
      0 if output from subroutine EGVCTR is not desired
DT    time step
NTAU  total number of time steps
AA(N, N) system matrix
HH(M,N) measurement matrix
QQ(N,N) power spectral density matrix of plant disturbance
RRINV(M,M) inverse of power spectral density matrix of measurement noise
PPINIT(N,N) initial condition of transient Riccati solution for estimate

The output generated by option 2 is
(1) A top heading

*************************************************************************
RESULTS OF ESTIMATION PROBLEM
*************************************************************************

(2) The scaling vector (if IOP1 ≠ 0) with the heading
TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART   IMAGINARY PART   ITER
where ITER is the number of iterations

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-KE*H)
REAL PART   IMAGINARY PART

(6) The solution to the estimation Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

(7) The estimator-gain matrix with the heading

KE, THE KALMAN FILTER GAIN MATRIX

(8) Followed by a top heading

*************************************************************************
TRANSIENT RICCATI SOLUTION
*************************************************************************

(9) A printout of time (The time initially will be 0.0.)
TIME = 0.

(10) The solution to the Riccati differential equation with the heading

RICCATI SOLUTION MATRIX
(11) The transient estimator-gain matrix with the heading

\textit{KE, THE TRANSIENT ESTIMATE GAIN MATRIX}

(Steps 9 to 11 are repeated for increasing time until NTAU steps have been taken.)

Option 3 - Finite-Time LSOCE Problem without Mean-Square Information

The block data for option 3 must include

\begin{itemize}
  \item \texttt{ISORT} 3
  \item \texttt{N} number of state variables, less than or equal to 12
  \item \texttt{M} number of measurements, less than or equal to 5
  \item \texttt{C} number of control inputs, less than or equal to 5
  \item \texttt{IOP1} 1 if output from subroutine SCALE is desired
  \item \texttt{IOP1} 0 if output from subroutine SCALE is not desired
  \item \texttt{IOP2} 1 if output from subroutine EGVCTR is desired
  \item \texttt{IOP2} 0 if output from subroutine EGVCTR is not desired
  \item \texttt{DT} time step
  \item \texttt{NTAU} total number of time steps
  \item \texttt{AA(N,N)} system matrix
  \item \texttt{HH(M,N)} measurement matrix
  \item \texttt{BB(N,C)} control input matrix
  \item \texttt{QC(N,N)} state weighting matrix
  \item \texttt{NN(N,N)} state-control product weighting matrix
  \item \texttt{QQ(N,N)} power spectral density matrix of plant disturbance
  \item \texttt{RRINV(M,M)} inverse of power spectral density matrix of measurement noise
  \item \texttt{PCINV(C,C)} inverse of control weighting matrix
  \item \texttt{PPINIT(N,N)} initial condition of transient Riccati solution for estimate
  \item \texttt{SSINIT(N,N)} initial condition of transient Riccati solution for control
\end{itemize}

The output generated by option 3 is

(1) A top heading

\begin{verbatim}
******************************************************************************
RESULTS OF ESTIMATION PROBLEM
******************************************************************************
\end{verbatim}
(2) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART    IMAGINARY PART

... (where ITER is the number of iterations)

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX

... (THE ONES WITH NEGATIVE REAL PARTS ARE THE EIGENVALUES OF (A-KE*H))

(6) The solution to the estimation Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

(7) The estimator-gain matrix with the heading

KE, THE KALMAN FILTER GAIN MATRIX

(8) Followed by a top heading

*********************************************************************************************
TRANSIENT RICCATI SOLUTION
*********************************************************************************************

(9) A printout of time (The time initially will be 0, 0.)

TIME = 0.

(10) The solution to the Riccati differential equation with the heading

RICCATI SOLUTION MATRIX

(11) The transient estimator-gain matrix with the heading

KE, THE TRANSIENT ESTIMATE GAIN MATRIX

(Steps 9 to 11 are repeated for increasing time until NTAU time steps have been taken.)

(12) Followed by a top heading

*********************************************************************************************
RESULTS OF CONTROL PROBLEM
*********************************************************************************************

(13) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS
(14) Eigenvalues of the Hamiltonian matrix with the heading

\[
\begin{array}{c|c|c}
\text{REAL PART} & \text{IMAGINARY PART} & \text{ITER} \\
\end{array}
\]

where ITER is the number of iterations.

(15) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(16) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE CNES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF \((A - B*KC)\)
REAL PART IMAGINARY PART

(17) The solution to the control Riccati equation with the heading

SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM

(18) The controller-gain matrix with the heading

KC, THE CONTROL GAIN MATRIX

(19) Followed by a top heading

***********************************************************************************
TRANSIENT RICCATI SOLUTION
***********************************************************************************

(20) A printout of time (The time initially will be set to \(T_f\), which equals NTAU*DT.)

\[
\text{TIME} = T_f
\]

(21) The solution to the Riccati differential equation with the heading

RICCATI SOLUTION MATRIX

(22) The transient controller-gain matrix with the heading

KC, THE TRANSIENT CONTROL GAIN MATRIX

(Steps 20 to 22 are repeated for decreasing time until NTAU time steps have been taken. The last time done will be 0.0 time.)

Option 4 - Finite-Time LSOCE Problem with Mean-Square Information

The block data for option 4 must include

<table>
<thead>
<tr>
<th>ISORT</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>number of state variables, less than or equal to 12</td>
</tr>
<tr>
<td>18</td>
<td></td>
</tr>
</tbody>
</table>
M  number of measurements, less than or equal to 5
C  number of control inputs, less than or equal to 5
R  number of outputs, less than or equal to 5
IOP1 1 if output from subroutine SCALE is desired
      0 if output from subroutine SCALE is not desired
IOP2 1 if output from subroutine EGVCTR is desired
      0 if output from subroutine EGVCTR is not desired
DT  time step
NTAU total number of time steps
AA(N, N) system matrix
HH(M, N) measurement matrix
BB(N, C) control input matrix
QC(N, N) state weighting matrix
NN(N, N) state-control product weighting matrix
CC(R, N) output matrix
QQ(N, N) power spectral density matrix of plant disturbance
RRINV(M, M) inverse of power spectral density matrix of measurement noise
PCINV(C, C) inverse of control weighting matrix
PPINIT(N, N) initial condition of transient Riccati solution for estimate
SSINIT(N, N) initial condition of transient Riccati solution for control

The output generated by option 4 is
(1) A top heading

*****************************************************************************
RESULTS OF ESTIMATION PROBLEM
*****************************************************************************

(2) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART       IMAGINARY PART

   where ITER is the number of iterations
(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF A

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-KE*H)
REAL PART IMAGINARY PART

(6) The solution to the estimation Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

(7) The estimator-gain matrix with the heading

KE, THE KALMAN FILTER GAIN MATRIX

(8) Followed by a top heading

******************************************************************************
TRANSIENT RICCATI SOLUTION
******************************************************************************

(9) A printout of time (The time initially will be 0.0.)

TIME = 0.

(10) The solution to the Riccati differential equation with the heading

RICCATI SOLUTION MATRIX

(11) The transient estimator-gain matrix with the heading

KE, THE TRANSIENT ESTIMATE GAIN MATRIX

(Steps 9 to 11 are repeated for increasing time until NTAU time steps have been taken.)

(12) Followed by a top heading

******************************************************************************
RESULTS OF CONTROL PROBLEM
******************************************************************************

(13) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(14) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART IMAGINARY PART ITER

where ITER is the number of iterations
(15) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 \neq 0) with the heading

**MODIFIED EIGENVECTOR MATRIX OF AAA**

(16) The ordered eigenvalues of the Hamiltonian matrix with the heading

**THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX**
**THE CNES WITH NEGATIVE REAL PARTS ARE THE EIGENVALUES OF (A-B*KC)**
**REAL PART IMAGINARY PART**

(17) The solution to the control Riccati equation with the heading

**SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM**

(18) The controller-gain matrix with the heading

**KC, THE CONTROL GAIN MATRIX**

(19) Followed by a top heading

**************************************************
**TRANSIENT RICCATI SOLUTION**
**************************************************

(20) A printout of time (The time initially will be set to \( T_f \), which equals NTAU×DT.)

**TIME = T_f**

(21) The solution to the Riccati differential equation with the heading

**RICCATI SOLUTION MATRIX**

(22) The transient controller-gain matrix with the heading

**KC, THE TRANSIENT CONTROL GAIN MATRIX**

(Steps 20 to 22 are repeated for decreasing time until NTAU time steps have been taken. The last time done will be 0.0 time.)

(23) Followed by a top heading

**************************************************
**COVARIANCE MATRICES**
**************************************************

(24) The control-covariance matrix with the heading

**UU, CONTROL COVARIANCE MATRIX**

(25) The state-covariance matrix with the heading

**XX, STATE COVARIANCE MATRIX**

(26) The output-covariance matrix with the heading

**YY, OUTPUT COVARIANCE MATRIX**
Option 5 - Infinite-Time LSOCE Problem

The block data input for option 5 must include

| ISORT  | 5
| N      | number of state variables, less than or equal to 12
| M      | number of measurements, less than or equal to 5
| C      | number of control inputs, less than or equal to 5
| R      | number of outputs, less than or equal to 5
| IOP1   | 1 if output from subroutine SCALE is desired
|        | 0 if output from subroutine SCALE is not desired
| IOP2   | 1 if output from subroutine EGVCTR is desired
|        | 0 if output from subroutine EGVCTR is not desired
| AA(N, N) | system matrix
| HH(M, N) | measurement matrix
| BB(N, C) | control input matrix
| QC(N, N) | state weighting matrix
| NN(N, N) | state-control product weighting matrix
| CC(R, N) | output matrix
| QQ(N, N) | power spectral density matrix of plant disturbance
| RRINV(M, M) | inverse of power spectral density matrix of measurement noise
| PCINV(C, C) | inverse of control weighting matrix
The output generated by option 5 is
(1) A top heading

*******************************************************************************
RESULTS OF ESTIMATION PROBLEM
*******************************************************************************

(2) The scaling vector (if IOP1 ≠ 0) with the heading
TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

| REAL PART | IMAGINARY PART | ITER |

where ITER is the number of iterations

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The order eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS ARE THE EIGENVALUES OF (A-K*H)

| REAL PART | IMAGINARY PART |

(6) The solution to the estimation Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

(7) The estimator-gain matrix with the heading

KE, THE KALMAN FILTER GAIN MATRIX

(8) Followed by a top heading

*******************************************************************************
RESULTS OF CONTROL PROBLEM
*******************************************************************************

(9) The scaling vector (if IOP1 ≠ 0) with the heading
TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(10) Eigenvalues of the Hamiltonian matrix with the heading

| REAL PART | IMAGINARY PART | ITER |

where ITER is the number of iterations

(11) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

23
The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-B*KC)
REAL PART IMAGINARY PART

The solution to the control Riccati equation with the heading

SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM

The controller-gain matrix with the heading

KC, THE CONTROL GAIN MATRIX

Followed by a top heading

******************************************************
COVARIANCE MATRICES
******************************************************

The control-covariance matrix with the heading

UU, CONTROL COVARIANCE MATRIX

The state-covariance matrix with the heading

XX, STATE COVARIANCE MATRIX

The output-covariance matrix with the heading

YY, OUTPUT COVARIANCE MATRIX

The estimated output-covariance matrix with the heading

YYEST, ESTIMATED OUTPUT COVARIANCE MATRIX

The measurement-covariance matrix with the heading

ZZ, MEASUREMENT COVARIANCE MATRIX

The estimated measurement-covariance matrix with the heading

ZZEST, ESTIMATED MEASUREMENT COVARIANCE MATRIX

Option 6 - Infinite-Time, Noise-Free Regulator

The block data input for option 6 must include

ISORT 6
N number of state variables, less than or equal to 12
C number of control inputs, less than or equal to 5
IOP1 1 if output from subroutine SCALE is desired  
0 if output from subroutine SCALE is not desired

IOP2 1 if output from subroutine EGVCTR is desired  
0 if output from subroutine EGVCTR is not desired

AA(N, N) system matrix

BB(N, C) control input matrix

QC(N, N) state weighting matrix

NN(N, N) state-control product weighting matrix

PCINV(C, C) inverse of control weighting matrix

The output generated by option 6 is

(1) A top heading

*******************************************************************************
RESULTS OF CONTROL PROBLEM
*******************************************************************************

(2) The scaling vector (if IOP1 ≠ 0) with the heading

TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART IMAGINARY PART ITER

where ITER is the number of iterations

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A - E*KC)
REAL PART IMAGINARY PART

(6) The solution to the control Riccati equation with the heading

SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM

(7) The controller-gain matrix with the heading

KC, THE CONTROL GAIN MATRIX
Option 7 - Finite-Time, Noise-Free Regulator

The block data input for option 7 must include

- **ISORT**: 7
- **N**: number of state variables, less than or equal to 12
- **C**: number of control inputs, less than or equal to 5
- **IOP1**: 1 if output from subroutine SCALE is desired
  0 if output from subroutine SCALE is not desired
- **IOP2**: 1 if output from subroutine EGVCTR is desired
  0 if output from subroutine EGVCTR is not desired
- **DT**: time step
- **NTAU**: total number of time steps
- **AA(N,N)**: system matrix
- **BB(N,C)**: control input matrix
- **QC(N,N)**: state weighting matrix
- **NN(N,N)**: state-control product weighting matrix
- **PCINV(C,C)**: inverse of control weighting matrix
- **SSINIT(N,N)**: initial condition of transient Riccati solution for control

The output generated by option 7 is

1. A top heading

```
********************************************************************************
RESULTS OF CONTROL PROBLEM
********************************************************************************
```

2. The scaling vector (if **IOP1** ≠ 0) with the heading

```
TS, SCALING TRANSFORMATION MATRIX ELEMENTS
```

3. Eigenvalues of the Hamiltonian matrix with the heading

```
REAL PART  IMAGINARY PART  ITER
```
where **ITER** is the number of iterations

4. The modified eigenvector matrix of the Hamiltonian matrix (if **IOP2** ≠ 0) with the heading

```
MODIFIED EIGENVECTOR MATRIX OF AAA
```
(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE CONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-B*KC)
REAL PART IMAGINARY PART

(6) The solution to the control Riccati equation with the heading

SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM

(7) The controller-gain matrix with the heading

KC, THE CONTROL GAIN MATRIX

(8) Followed by a top heading

******************************************************************************
TRANSIENT RICCATI SOLUTION
******************************************************************************

(9) A printout of time (The time initially will be set to $T_f$, which equals NTAU x DT.)

TIME = $T_f$

(10) The solution to the Riccati differential equation with the heading

RICCATI SOLUTION MATRIX

(11) The transient controller-gain matrix with the heading

KC, THE TRANSIENT CONTROL GAIN MATRIX

(Steps 9 to 11 are repeated for decreasing time until NTAU time steps have been taken. The last time done will be 0.0 time.)

Option 8 - Infinite-Time Regulator with Plant Noise

The block data input for option 8 must include

ISORT 8
N number of state variables, less than or equal to 12
M number of measurements, less than or equal to 5
C number of control inputs, less than or equal to 5
R number of outputs, less than or equal to 5
IOP1 1 if output from subroutine SCALE is desired
0 if output from subroutine SCALE is not desired
IOP2 1 if output from subroutine EGVCTR is desired
0 if output from subroutine EGVCTR is not desired

AA(N,N) system matrix
HH(M,N) measurement matrix
BB(N,C) control input matrix
QC(N,N) state weighting matrix
NN(N,N) state-control product weighting matrix
CC(R,N) output matrix
QQ(N,N) power spectral density matrix of plant disturbance
PP(N,N) estimation error-covariance matrix
PCINV(C,C) inverse control weighting matrix

The output generated by option 8 is
(1) A top heading

**************************************************************
RESULTS OF CONTROL PROBLEM
**************************************************************

(2) The scaling vector (if IOP1 ≠ 0) with the heading
TS, SCALING TRANSFORMATION MATRIX ELEMENTS

(3) Eigenvalues of the Hamiltonian matrix with the heading

REAL PART IMAGINARY PART ITER

where ITER is the number of iterations

(4) The modified eigenvector matrix of the Hamiltonian matrix (if IOP2 ≠ 0) with the heading

MODIFIED EIGENVECTOR MATRIX OF AAA

(5) The ordered eigenvalues of the Hamiltonian matrix with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF \((A-KC)\)
REAL PART IMAGINARY PART

(6) The solution to the control Riccati equation with the heading

SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM

(7) The controller gain matrix with the heading

KC, THE CONTROL GAIN MATRIX
Followed by a top heading

COVARIANCE MATRICES

(9) The control-covariance matrix with the heading

UU CONTROL COVARIANCE MATRIX

(10) The state-covariance matrix with the heading

XX, STATE COVARIANCE MATRIX

(11) The output-covariance matrix with the heading

YY, OUTPUT COVARIANCE MATRIX

(12) The estimated output-covariance matrix with the heading

YYEST, ESTIMATED OUTPUT COVARIANCE MATRIX

(13) The measurement-covariance matrix with the heading

ZZ, MEASUREMENT COVARIANCE MATRIX

(14) The estimated measurement-covariance matrix with the heading

ZZEST, ESTIMATED MEASUREMENT COVARIANCE MATRIX

Option 9 - Open-Loop, Plant-Covariance Matrices

The block data input for option 9 must include

ISORT 9
N number of state variables, less than or equal to 12
M number of measurements, less than or equal to 5
C number of control inputs, less than or equal to 5
R number of outputs, less than or equal to 5
AA(N,N) system matrix
HH(M,N) measurement matrix
BB(N,C) control input matrix
CC(R,N) output matrix
QQ(N,N) power spectral density matrix of plant disturbance
PP(N,N) estimation error-covariance matrix
KC(C, N) control-gain matrix

The output generated by option 9 is

1. A top heading

**************************************************************************
COVARIANCE MATRICES
**************************************************************************

2. The control-covariance matrix with the heading

UU, CONTROL COVARIANCE MATRIX

3. The state-covariance matrix with the heading

XX, STATE COVARIANCE MATRIX

4. The output-covariance matrix with the heading

YY, OUTPUT COVARIANCE MATRIX

5. The estimated output-covariance matrix with the heading

YYEST, ESTIMATED OUTPUT COVARIANCE MATRIX

6. The measurement-covariance matrix with the heading

ZZ, MEASUREMENT COVARIANCE MATRIX

7. The estimated measurement-covariance matrix with the heading

ZZEST, ESTIMATED MEASUREMENT COVARIANCE MATRIX
Listing of Main Routine LSOCE

MAIN PROGRAM

*****************************************

PLANT
XDOT = AA * X + BB * U + W

MEASUREMENT
Z = H * X + V

NOISE
COV(W) = QQ, COV(U) = RR

PERFORMANCE INDEX
J = (1/2) * (X**T * QC * X + X**T * NN * U + U**T * PC * U)

DIMENSIONS AND DESCRIPTIONS OF PROGRAM VARIABLES

A(N,N) LYAPUNOV EQUATION MATRIX
AA(N,N) SYSTEM MATRIX
AAA(2N,2N) HAMILTONIAN MATRIX ASSOCIATED WITH THE RICCATI EQUATION
ANS(N,N) RICCATI SOLUTION FROM TRICCS
BB(N,C) CONTROL INPUT MATRIX
CC(N,N) OUTPUT MATRIX
CI(2N) EIGENVALUE IMAGINARY PARTS (OF AAA)
CR(2N) EIGENVALUE REAL PARTS (OF AAA)
HH(N,N) MEASUREMENT MATRIX
IOD(2N) BLOCK DIAGONALIZING PERMUTATION VECTOR
(KC(2N,N) CONTROL GAIN MATRIX
KE(N,M) ESTIMATOR GAIN MATRIX (REAL)
NBL(2N) SIZE OF INDIVIDUAL IRREDUCIBLE BLOCKS
(NN(N,N) STATE / CONTROL WEIGHTING MATRIX
PP(N,N) ESTIMATION ERROR COVARIANCE MATRIX
PPINIT(N,N) INITIAL CONDITION OF RICCATI SOLUTION FOR TRANSIENT ESTIMATE PROBLEM
QQ(N,N) LYAPUNOV EQUATION MATRIX (SYMMETRIC)
QQ(N,N) STATE WEIGHTING MATRIX
QQ(N,N) POWER SPECTRAL DENSITY MATRIX OF PLANT DISTURBANCE
RRINV(M,M) INVERSE OF POWER SPECTRAL DENSITY MATRIX OF MEASUREMENT NOISE
SS(N,N) CONTROL MATRIX RICCATI SOLUTION
SSINIT(N,N) INITIAL CONDITION OF RICCATI SOLUTION FOR TRANSIENT CONTROL PROBLEM
TS(2N) SCALING TRANSFORMATION
UU(N,N) CONTROL COVARIANCE MATRIX
XX(N,N) MODIFIED EIGENVECTOR MATRIX (OF AAA)
XX(N,N) STATE COVARIANCE MATRIX (SYMMETRIC)
YY(R,R) OUTPUT COVARIANCE MATRIX
YYFST(R,R) ESTIMATED OUTPUT COVARIANCE MATRIX
ZZ(M,M) MEASUREMENT COVARIANCE MATRIX
ZZEST(M,M) ESTIMATED MEASUREMENT COVARIANCE MATRIX

DIMENSIONS OF TEMPORARY STORAGE ARRAYS

AI(2N)

ORIGINAL PAGE IS OF POOR QUALITY
C AR(2N)
C ASI(N*N+N)/2,(N*N+N)/2
C BSIMQ((N*N+N)/2)
C EXT(2N,2N)
C EX1(N,N)
C EX2(N,N)
C EX3(N,N)
C EX4(N,N)
C EX5(N)
C EX6(N)
C IBL(2N)
C IP'RR(2N)
C IPERN(2N)
C JMAX(2N)
C TT(2N,2N)
C XR(2N,2N)

C DIMENSION AA(12,12), QQ(12,12), QC(12,12), PP(12,12), SS(12,12),
C IXX(12,12), A(12,12), Q(12,12), ANS(12,12), EX1(12,12), EX2(12,12),
C 2EX3(12,12), EX4(12,12), PPINIT(12,12), SSINIT(12,12), AAA(24,24),
C 3TT(24,24), EXT(24,24), X(24,24), XR(24,24), ASI(78,78), YY(5,5),
C 4ZZ(5,5), UU(5,5), YYEST(5,5), ZZEST(5,5), RRINV(5,5), PCINV(5,5),
C 5BB(12,5), NN(12,5), KE(12,5), KC(5,12), CC(5,12), HH(15,12),
C 6EX5(12), EX6(12), TS(24), CR(24), CI(24), AR(24), AI(24),
C 7IPER(24), IPERN(24), BSIMQ(78), IDR(24), NBL(24), IBL(24),
C 8JMAX(24)

C REAL KE, NN, KC
C REAL INIT
C INTEGER C, R, CMAX, RMAX

C INSERT BLOCK DATA HERE

C NMAX = 12
C MMAX = 5
C CMAX = 5
C RMAX = 5
C N2MAX = NMAX + NMAX
C KMAX = NMAX * NMAX
C KSIMAX = (KMAX + NMAX) / 2
C N2=N*N
C K=N*N
C KSIM=(K+N)/2
C
C IF (ISORT GT 5) GO TO 10
C
C CALL ESTMAT (AA,HH,QQ,RRINV,KE,PP,CR,CI,X,TS,XR,TT,AAA,EXT,AR,AT,
C 1IPER,IPERN,IOR,NBL,IBL,JMAX,N,M,N2,IOPI,IOP2,NMAX,MMAX,N2MAX)
C
C IF (ISORT EQ 1) STOP
C IF (ISORT EQ 5) GO TO 10
C
C CALL TRICCS (X,PPINIT,CR,CI,ANS,DT,NTAU,N,C,M,N2,EX1,EX2,EX3,EX4,
C 1EX5,EX6,IPER,IPERN,HH,RRINV,NN,PCINV,BB,1,NMAX,CMAX,MMAX,N2MAX)
C
C 10 IF (ISORT EQ 2) STOP
C IF (ISORT EQ 9) GO TO 20
C
C 32
CALL CONTRL (AA, BB, QC, NN, PCINV, KC, SS, CR, CI, X, TS, XR, TT, AAA, EXT, AR, RI, IPER, IPERN, IOR, NBL, IBL, JMAX, N, C, N2, IOP1, IOP2, NMAX, CMAX, N2MAX)

C
IF (ISORT.EQ.6) STOP
IF (ISORT.EQ.5 .OR. ISORT.EQ.8) GO TO 20
C
CALL TRICCS (X, SSINIT, CR, CI, ANS, DT, NTAU, N, C, M, N2, EX1, EX2, EX3, EX4, EX5, EX6, IPER, IPERN, HH, RRINV, NN, PCINV, BB, 2, NMAX, CMAX, MMAX, N2MAX)
C
20 IF (ISORT.EQ.3 OR ISORT.EQ.7) STOP
C
CALL COVM (AA, BB, HH, CC, QQ, PP, KC, N, M, CR, XX, YY, ZZ, UU, YVEST, ZVEST, KSIM, A, Q, BSIMQ, ASI, NMAX, MMAX, CMAX, RMAX, KSIMAX)
C
RETURN
END

DESCRIPTIONS AND LISTINGS OF SUBROUTINES

Description of Subroutine ESTMAT

Subroutine ESTMAT solves the optimal linear state estimation problem. Using AA, HH, QQ, and RRINV, it sets up an N2xN2 matrix called AAA, which is the Hamiltonian matrix associated with the matrix Riccati equation. Subroutine ESTMAT then calls subroutine RICCSS to solve the Riccati equation and to obtain the estimation error-covariance matrix PP, which subroutine ESTMAT then uses to compute the estimator gains KE.

The subroutine statement for ESTMAT is as follows:

SUBROUTINE ESTMAT (AA, HH, QQ, RRINV, KE, PP, CR, CI, X, TS, XR, TT, AAA, EXT, AR, RI, IPER, IPERN, IOR, NBL, IBL, JMAX, N, M, N2, IOP1, IOP2, NMAX, MMAX, N2MAX)

The input variables are

AA(N, N) system matrix
HH(M, N) measurement matrix
QQ(N, N) power spectral density matrix of plant disturbance
RRINV(M, M) inverse of power spectral density matrix of measurement noise
IOP1 scaling printing option
IOP2 eigenvector printing option

The input integer dimensions are

N number of state variables
M number of measurements
N2 \quad 2 \times N, \text{ dimension of Hamiltonian matrix}

NMAX \quad \text{dimension, greater than or equal to } N

MMAX \quad \text{dimension, greater than or equal to } M

N2MAX \quad \text{dimension, greater than or equal to } N2

The output variables are

PP(N, N) \quad \text{estimation error-covariance matrix}

KE(N, M) \quad \text{estimator-gain matrix}

The intermediate variables are

AAA(N2, N2) \quad \text{Hamiltonian matrix associated with the Riccati equation}

CR(N2) \quad \text{real parts of eigenvalues of } AAA

CI(N2) \quad \text{imaginary parts of eigenvalues of } AAA

X(N2, N2) \quad \text{modified eigenvector matrix of } AAA

NBL(N2) \quad \text{size of each irreducible block (integer)}

IOR(N2) \quad \text{block-diagonalizing permutation vector (integer)}

TS(N2) \quad \text{scaling transformation}

The temporary storage arrays are

XR(N2, N2)

TT(N2, N2)

EXT(N2, N2)

AR(N2)

AI(N2)

IPER(N2) \quad \text{integer}

IPERN(N2) \quad \text{integer}

IBL(N2) \quad \text{integer}

JMAX(N2) \quad \text{integer}

Vectors CR and CI contain the eigenvalues of the Kalman filter; that is, the eigenvalues of \((A - K_e H)\). If IOP1 is not equal to zero, the elements of the scaling transformation matrix are printed. If IOP2 is not equal to zero, the modified eigenvectors of AAA are printed.

There are no error messages output from this subroutine. It prints the following message:
After the output from subroutines EGVCTR and RICSS, it prints the ordered eigenvalues with the heading

THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE CNES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF (A-KE*H)
REAL PART IMAGINARY PART

It then prints the solution to the Riccati equation with the heading

PP, THE ESTIMATION ERROR COVARIANCE MATRIX

Finally, it prints the K_e's with the heading

K_e, THE KALMAN FILTER GAIN MATRIX

Using the method of reference 9, subroutine ESTMAT forms a 2n×2n Hamiltonian matrix \( M \)

\[
M = \begin{bmatrix}
-A^T & H^T R^{-1} H \\
Q & A
\end{bmatrix}
\] (27)

where matrices \( A, H, Q, \) and \( R \) are defined in the section FINITE-TIME LSOC Optimization PROBLEM. Matrix \( M \) is required as input to the Riccati-equation-solving subroutine RICSS. The particular Riccati equation solved is equation (14). After forming \( M \), subroutine ESTMAT calls subroutine RICSS to obtain the Riccati solution matrix \( P \), which is the covariance of the estimation error. Kalman gains \( K_e \) are then computed as

\[
K_e = PH^T R^{-1}
\] (28)
DOUBLE PRECISION SUM
REAL KE

DIMENSION A(NMAX,1), HH(MMAX,1), QQ(NMAX,1), RRINV(MMAX,1),
IKE(NMAX,1), PP(NMAX,1), CR(1), CI(1), X(N2MAX,1), TS(IN,
2XR(N2MAX,1), TT(N2MAX,1), AAA(N2MAX,1), LXX(N2MAX,1), AR(1),
3AI(1), IPER(1), IPERN(1), IOR(1), NBL(1), IBL(1), JMAX(1)

WRITE (6,130)

FORM AAA MATRIX

GO 50 J=1,N2
JJ=J-N
DO 50 I=1,N2
II=I-N
IF (I.JT.N) GO TO 20
IF (I.GT.N) GO TO 10
AAA(I,J)--AA(J,I)
GO TO 50
AAA(I,J)=QQ(II,J)
GO TO 50
IF (I.GT.N) GO TO 40
SUM=0.000
DO 30 L=1,M
DO 30 K=1,M
30 SUM=SUM+HH(K,I)*RRINV(K,L)*HH(L,J)
AAA(I,J)=SUM
GO TO 50
AAA(I,J)=AA(I,J)
CONTINUE

OBTAIN SOLUTION TO ESTIMATION RICCATI EQ. AND STORE IN PP

CALL RICSS (AAA,X,PP,CR,CI,TS,XR,EXT,TT,IPER,IPERN,IOR,NBL,IBL,
1JMAX,AR,Al,IOP1,IOP2,N,N2,NMAX,N2MAX)
WRITE (6,140)
WRITE (6,150) (CR(I),CI(1),I=1,N2)
WRITE (6,160)
LL=1
LLL=10
60 IF (N.LT.LLL) LLL=N
DO 70 I=1,N
WRITE (6,170) (PP(I,L),L=LLL,LLL)
70 CONTINUE
WRITE (6,180)
IF (N.EQ.LLL) GO TO 80
LL=LL+10
LLL=LLL+10
GO TO 60

COMPUTE ESTIMATOR GAIN MATRIX KE

GO 80 J=1,M
GO 80 I=1,N
SUM=0.000
DO 90 L=1,M
DO 90 K=1,N
90 SUM=SUM+PP(I,K)*HH(L,K)*RRINV(L,J)

36
```
100  KE(I,J)=SUM
    WRITE (6,190)
    LL=1
    LLL=10
110  IF (M.LT.LLL) LLL=M
    DO 120 I=1,N
    WRITE (6,170) (KE(I,L),L=LL,LLL)
120  CONTINUE
    WRITE (6,180)
    IF (M.EQ.LLL) RETURN
    LL=LL+10
    LLL=LLL+10
    GO TO 110

C
130  FORMAT (1H1/1X,5OH*******************************************************************************
1**/15X,29HRESULTS OF ESTIMATION PROBLEM/1X,50H*******************************************************************************
2*******************************************************************************//)
140  FORMAT (1X,49HTHE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX/11X,32HTHE ONES WITH NEGATIVE REAL PARTS/1X,31HARE THE EIGENVALUES
20F (A-KE*H1/1X,9HREAL PART,14X,14H1MAGINARY PART)
150  FORMAT (1X,E15.8,8XE15.8)
160  FORMAT (1X,1P10E12.4)
170  FORMAT (1X,33HKE, THE KALMAN FILTER GAIN MATRIX)
180  FORMAT (1X,33HKE, THE KALMAN FILTER GAIN MATRIX)
190  FORMAT (1X,33HKE, THE KALMAN FILTER GAIN MATRIX)
END

Description of Subroutine CONTRL

Subroutine CONTRL solves the optimal linear regulator problem. Using AA, BB, QC, NN, and PCINV, it sets up an N2xN2 matrix called AAA, which is the Hamiltonian matrix associated with the matrix Riccati equation. Subroutine CONTRL then calls subroutine RICSS to solve the Riccati equation and to obtain the control matrix Riccati solution SS, which subroutine CONTRL then uses to compute the control gains KC.

The subroutine statement for CONTRL is as follows:

SUBROUTINE CONTRL (AA, BB, QC, NN, PCINV, KC, SS, CR, CI, X, TS, XR, TT, AAA, EXT, AR, AI, IPER, IPERN, IOR, NBL, IBL, JMAX, N, C, N2, IOP1, IOP2, NMAX, CMAX, N2MAX)

The input variables are

AA(N,N)  system matrix
BB(N,C)  control input matrix
QC(N,N)  state weighting matrix
NN(N,C)  state-control product weighting matrix
PCINV(C,C) inverse of control weighting matrix
```
IOP 1  scaling printing option
IOP 2  eigenvector printing option

The input integer dimensions are

N  number of state variables
C  number of control inputs
N2  $2 \times N$, dimension of Hamiltonian matrix
NMAX  dimension, greater than or equal to N
CMAX  dimension, greater than or equal to C
N2MAX  dimension, greater than or equal to N2

The output variables are

SS(N, N)  matrix Riccati solution for control problem
KC(C, N)  control-gain matrix

The intermediate variables are

AAA(N2, N2)  Hamiltonian matrix associated with the Riccati equation
CR(N2)  real parts of eigenvalues of AAA
CI(N2)  imaginary parts of eigenvalues of AAA
X(N2, N2)  modified eigenvector matrix of AAA
NBL(N2)  size of each irreducible block (integer)
IOR(N2)  block-diagonalizing permutation vector (integer)
TS(N2)  scaling transformation

The temporary storage arrays are

XR(N2, N2)
TT(N2, N2)
EXT(N2, N2)
AR(N2)
AI(N2)
IPER(N2)  integer
IPERN(N2)  integer
IBL(N2)  integer
JMAX(N2)  integer
Vectors CR and CI contain the eigenvalues of the optimal linear regulator; that is, the eigenvalues of \((A - BK_c)\). If IOP1 is not equal to zero, the elements of the scaling transformation matrix are printed. If IOP2 is not equal to zero, the modified eigenvectors of AAA are printed.

There are no error messages output from this subroutine. It prints the following message:

```
*******************************
RESULTS OF CONTROL PROBLEM
*******************************
```

After the output from subroutines EGVCTR and RICSS, it prints the ordered eigenvalues with the heading

```
THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX
THE ONES WITH NEGATIVE REAL PARTS
ARE THE EIGENVALUES OF \((A - B*K_C)\)
REAL PART IMAGINARY PART
```

It then prints the solution to the Riccati equation with the heading

```
SS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM
```

Finally, it prints the \(K_c\)'s with the heading

```
KC, CONTROL GAIN MATRIX
```

The method used by subroutine CONTRL, given in reference 9, is as follows:

Subroutine CONTRL forms a \(2n \times 2n\) Hamiltonian matrix \(M\) given as

\[
M = \begin{bmatrix}
-\left(A - B P^{-1}_c N^T\right) & \left(B P^{-1}_c B^T\right) \\
\left(Q_c - N P^{-1}_c N^T\right) & \left(A - B P^{-1}_c N^T\right)^T
\end{bmatrix}
\]

(29)

where the matrices \(A, B, N, P_c,\) and \(Q_c\) are defined in the section FINITE-TIME LSOCE PROBLEM. Matrix \(M\) is required as input to the Riccati-equation-solving subroutine RICSS. The particular Riccati equation solved is equation (13). After forming \(M\), subroutine CONTRL calls subroutine RICSS to obtain the Riccati solution matrix \(S\). It then computes the control gains \(K_c\) as

\[
K_c = P^{-1}_c \left(B^T S + N^T\right)
\]

(30)
SUBROUTINE CONTRL (AA, BB, QC, NN, PCINV, KC, SS, CR, CI, X, TS, XR, TT, AAA, 1EXT, AR, AI, IPER, IPERN, IOR, NBL, IBL, JMAX, N, C, N2, IOP1, IOP2, NMAX, CMAX, 2N2MAX)

C
C CONTRL SOLVES OPTIMAL LINEAR REGULATOR PROBLEM. IT SETS UP N2 BY
C N2 MATRIX AAA, USING AA, BB, QC, NN, AND PCINV.
C CONTRL CALLS RICSS TO OBTAIN SOLUTION TO RICCATI EQUATION, SS.
C CONTROL GAINS ARE THEN COMPUTED, KC.

C DOUBLE PRECISION SUM1, SUM2, SUM3
REAL NN, KC
INTEGER C, CMAX

DIMENSION AA(NMAX,1), BB(NMAX,1), QC(NMAX,1), NN(NMAX,1),
1PCINV(CMAX,1), KC(CMAX,1), SS(NMAX,1), CR(1), CI(1),
2TS(1), XR(N2MAX,1), TT(N2MAX,1), AAA(N2MAX,1), EXT(N2MAX,1),
3AR(1), AI(1), IPER(1), IPERN(1), IOR(1), NBL(1), IBL(1), JMAX(1)

WRITE (6,100)

C
C FORM AAA MATRIX

C 20 J=1,N
JJ=J+N
C 20 I=1,N
IJ=I+N
SUM1=AA(I,J)
SUM2=QC(I,J)
SUM3=0.0
DO 10 L=1,C
DO 10 K=1,C
SUM1=SUM1+BB(I,K)*PCINV(K,L)*NN(J,L)
SUM2=SUM2-NN(I,K)*PCINV(K,L)*NN(J,L)
SUM3=SUM3+BB(I,K)*PCINV(K,L)*BB(J,L)
AAA(I,J)=SUM1
AAA(JJ,I)=SUM1
AAA(IJ,I)=SUM2
AAA(I,JJ)=SUM3

C
C OBTAIN SOLUTION TO CONTROL RICCATI EQ. AND STORE IN SS
C
CALL RICSS (AAA, X, SS, CR, CI, TS, XR, EXT, TT, IPER, IPERN, IOR, NBL, IBL,
1JMAX, AR, AI, IOP1, IOP2, N2, NMAX, N2MAX)
WRITE (6,110)
WRITE (6,120) (CR(I), CI(I), I=1,N2)
WRITE (6,130)
LL=1
LLL=10
30 IF (N.LT.LLL) LLL=N
DO 40 I=1,N
WRITE (6,140) (SS(I,L), L=LLL,L)
40 CONTINUE
WRITE (6,150)
IF (N.EQ.LLL) GO TO 50

40
LL=LL+10
LLL=LLL+10
GO TO 30
C
C COMPUTE CONTROL GAIN MATRIX KC
C
DC 70 J=1,N
DO 70 I=1,C
SUM1=0.000
DO 60 K=1,C
SUM1=SUM1+PCINV(I,K)*NN(J,K)
DO 60 L=1,N
SUM1=SUM1+PCINV(I,K)*BB(L,K)*SS(L,J)
70 KC(I,J)=SUM1
WRITE (6,160)
LL=1
LLL=10
80 IF (N.LT.LLL) LLL=N
DO 90 I=1,C
WRITE (6,140) (KC(I,L),L=LL,LLL)
90 CONTINUE
WRITE (6,150)
IF (N.EQ.LLL) RETURN
LL=LL+10
LLL=LLL+10
GC TO 80
C
100 FORMAT (1H1/1X,50H*******************************************************************************/
1**/15X,26HRESULTS OF CONTROL PROBLEM/1X,50H*******************************************************************************/
2*******************************************************************************/
110 FORMAT ('/1X,49HTHE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX/
11X,33HTHE ONES WITH NEGATIVE REAL PARTS/1X,31HARE THE EIGENVALUES
20F (A-B*KC)/1X,9HREAL PART/14X,14HIMAGINARY PART)
120 FORMAT ('/1X,E15.8,8X,E15.8)
130 FORMAT ('/1X,47HSS, MATRIX RICCATI SOLUTION FOR CONTROL PROBLEM)
140 FORMAT ('/1X,1P10E12.4)
150 FORMAT ('/1H0)
160 FORMAT ('/1X,27HKC, THE CONTROL GAIN MATRIX)
END

Description of Subroutine RICSS

Subroutine RICSS computes the solution (called OUTPUT) to the algebraic Riccati equation. The input is an N2xN2 Hamiltonian matrix, AAA, formed by either subroutine ESTMAT or subroutine CONTRL. Subroutine RICSS calls subroutines CONDI, EIGQR, ORDER, EGVCTR, and the IBM SSP subroutines HSBG (upper Hessenberg transformation), ARRAY (conversion from vector storage to double-dimensioned storage or the reverse), and MINV (matrix inversion). Subroutine RICSS first conditions AAA (CONDI), then transforms it to upper Hessenberg form (HSBG) and computes its eigenvalues (EIGQR). The symmetric set of eigenvalues are then ordered (ORDER) (those with positive real parts occupying the first N positions), and the modified eigenvector
matrix is computed (EGVCTR) using the original AAA matrix. The matrix OUTPUT is then calculated by inverting and multiplying submatrices of the modified eigenvector matrix. For a detailed explanation of the method, see the following and reference 9.

The subroutine statement for RICSS is as follows:

```
SUBROUTINE RICSS (AAA, X, OUTPUT, CR, CI, TS, XR, EXT, TT, IPER,
                  IPERN, IOR, NBL, IBL, JMAX, AR, AI, IOP1,
                  IOP2, N, N2, NMAX, N2MAX)
```

The input variables are

- **AAA(N2, N2)** Hamiltonian matrix associated with the Riccati equation
- **IOP1** scaling printing option (used in subroutine SCALE)
- **IOP2** eigenvector printing option (used in subroutine EGVCTR)

The input integer dimensions are

- **N** number of state variables
- **N2** 2xN
- **NMAX** dimension, greater than or equal to N
- **N2MAX** dimension, greater than or equal to N2

The output variable is

- **OUTPUT(N, N)** matrix Riccati solution

The intermediate variables are

- **X(N2, N2)** modified eigenvector matrix of AAA
- **CR(N2)** real parts of eigenvalues of AAA
- **CI(N2)** imaginary parts of eigenvalues of AAA
- **NBL(N2)** size of each irreducible block (integer)
- **IOR(N2)** block-diagonalizing permutation vector (integer)
- **TS(N2)** scaling transformation vector

The temporary storage arrays are

- **XR(N2, N2)**
- **EXT(N2, N2)**
- **TT(N2, N2)**
- **IPER(N2)** integer
- **IPERN(N2)** integer

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There is one error message from this subroutine. During calculation of OUTPUT, the upper left quadrant of the X matrix is inverted prior to multiplication by the lower left quadrant. The following message may then be printed:

\textbf{X11 IS SINGULAR}

There is no other printed output from this subroutine.

The method used by subroutine RICSS is as follows: subroutine RICSS solves the algebraic Riccati equation

\[
VM_{11} + M_{11}^T V - VM_{12} V + M_{21} = 0 \tag{31}
\]

for an \( n \times n \) positive-definite, symmetric matrix \( V \). A \( 2n \times 2n \) Hamiltonian matrix \( M \) given by

\[
M = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\tag{32}
\]

where \( M_{22} = -M_{11}^T \), is required as input. The method consists of solving the equation

\[
MT = TA \tag{33}
\]

for the modified eigenvector matrix \( T \), where \( A \) is defined as

\[
A = \begin{bmatrix}
C & 0 \\
0 & C
\end{bmatrix}
\tag{34}
\]

\( C \) is the \( n \times n \) block-diagonal matrix of the \( n \) eigenvalues of \( M \) that have positive real parts. The block-diagonal form of \( C \) is such that an eigenvalue pair \( \lambda = \alpha \pm j\beta \) appears as a \( 2 \times 2 \) block

\[
C = \begin{bmatrix}
\alpha & -\beta \\
+\beta & \alpha
\end{bmatrix}
\tag{35}
\]
on the diagonal. Except that the diagonal elements of $\bar{C}$ are all negative, $C$ is the same as $\bar{C}$. Thus, with $A$ so defined, the modified eigenvector matrix $T$ is a block-diagonalizing transformation matrix of $M$.

After reducing and scaling $M$, transforming it to upper Hessenberg form, and obtaining its eigenvalues, subroutine RICSS calls subroutine ORDER, which sorts the eigenvalues into the required order. The modified eigenvector matrix of the original form of $M$ is then computed by using subroutine EGVCTR to obtain $T$. The solution $V$ is then obtained from the $n \times n$ submatrices of $T$ as

$$V = T_{21}T_{11}^{-1}$$  (36)

Listing of Subroutine RICSS

SLBROUTINE RICSS (AAA,X,OUTPUT,CR,CI,TS,XR,EXT,TT,IPER,IPERN,ICR,
INBL, IBL, MAX, AR, AI, IOP1, IOP2, N, N2, NMAX, NMAX)

RICSS COMPUTES SOLUTION TO STEADY STATE MATRIX RICCATI EQUATION,
OUTPUT. INPUT IS N2 BY N2 MATRIX AAA, WHICH IS THE HAMILTONIAN
MATRIX ASSOCIATED WITH THE MATRIX RICCATI EQUATION.
RICSS CALLS CONDI, EIGQR, ORDER, EGVCTR AND IBM SSP SUBROUTINES
FSBG, MINV, AND ARRAY.

DOUBLE PRECISION SUM

DIMENSION AAA(N2MAX+1), X(N2MAX+1), OUTPUT(NMAX+1), CR(1), CI(1),
TS(1), XR(N2MAX+1), EXT(N2MAX+1), TT(N2MAX+1), IPER(1), IPERN(1),
2AR(1), AI(1), IOR(1), NBL(1), IBL(1), JMAX(1)

EPS IS THE ERROR CRITERION USED IN DETERMINING WHETHER TWO
EIGENVALUES ARE SYMMETRIC ABOUT THE IMAGINARY AXIS

EPS=.001

PUT AAA IN BLOCK DIAGONAL FORM (IF REDUCIBLE) AND SCALE
CALL CONDI (AAA,X,XR,IPER,IPERN,IOR,NBL, IBL, MAX, TS, IOP1, N2, N2MAX)

TRANSFORM CONDITIONED AND SCALED AAA TO UPPER HESSENBERG
CALL ARRAY (2,N2,N2,N2MAX,N2MAX,XR,XR)
CALL HSBG (N2,XR,N2)
CALL ARRAY (1,N2,N2,N2MAX,N2MAX,XR,XR)

OBTAIN EIGENVALUES OF HESSENBERG MATRIX
CALL EIGQR (XR,N2,CR,CI,1,N2MAX)

ORDER EIGENVALUES
CALL ORDER (CR, CI, N2, EPS)

CALL EGVCTR (AAA, CR, CI, X, N2, TT, EXT, AR, A1, IPERN, IPER, IOP2, N2MAX)

DO 40 J=1,N
    DO 40 I=1,N

40 OUTPUT(I, J) = X(I, J)

CALL INVERT X11

CALL ARRAY (2, N, N, NMAX, NMAX, OUTPUT, OUTPUT)
CALL MINV (OUTPUT, N, DET, IPERN, IPER)
CALL ARRAY (1, N, N, NMAX, OUTPUT, OUTPUT)
IF (DET.EQ.0.0) WRITE (6, 110)

IF (CET.EQ.0.0) WRITE (6, 110)

 DC 70 J = 1,N
 DO 60 I = 1,N
    IF = I + N
    SUM = 0.000
     DO 50 K = 1,N
    SUM = SUM + X(II, K) * OUTPUT(K, J)
     50

60 AR(I) = SUM
    DC 70 M = 1,N
70 OUTPUT(M, J) = AR(M)
RETURN

110 FORMAT (1X, 15HX11 IS SINGULAR): END

Description of Subroutine CONDI

Subroutine CONDI conditions an N x N matrix, VARO, before the eigenvalues of VARO are obtained. This conditioning is done in order to improve the accuracy of the eigenvalues. The matrix VARO is first transformed into block-diagonal form (if reducible) and then each individual block is scaled. In order to accomplish this, subroutine CONDI calls subroutines REDU and SCALE. If VARO is not reducible, it is simply scaled.

The subroutine statement for CONDI is as follows:

SUBROUTINE CONDI (VARO, SS, S, IN, JBL, IOR, NBL, IBL, JMAX, D, IOP1, N, NMAX)

The input variables are

VARO(N, N)    matrix to be conditioned
IOP1           scaling printing option (used in subroutine SCALE)
The input integer dimensions are

N size of matrix to be reduced and scaled
NMAX dimension, greater than or equal to N

The output variable is

S(N,N) matrix in block-diagonal and scaled form

The intermediate variables are

D(N) vector containing the diagonal elements of the diagonal scaling matrix
IOR(N) block-diagonalizing permutation vector (integer)
NBL(N) size of each irreducible block (integer)

The temporary storage arrays are

SS(N,N)
IN(N) integer
JBL(N) integer
IBL(N) integer
JMAX(N) integer

There is no printed output from this subroutine.

Listing of Subroutine CONDI

SUBROUTINE CONDI (VARO, SS, S, IN, JBL, IOR, NBL, IBL, JMAX, D, IDP1, N, NMAX)
C
C ************ CHANGE CONDITION OF MATRIX BY PUTTING IT IN BLOCK DIAGONAL FORM
C (IF REDUCIBLE) AND THEN SCALING.
C CONDI CALLS SUBROUTINES REDU AND SCALE.
C
DIMENSION VARO(NMAX,1), SS(NMAX,1), S(NMAX,1), IN(1), JBL(1),
1 IOR(1), NBL(1), IBL(1), D(1), JMAX(1)
C
CALL REDU (VARO, SS, S, IN, JBL, IOR, NBL, IBL, JMAX, N, NMAX)
DO 10 I = 1, N
10 C(I) = 1.0
NSUM = 0
DO 30 K = 1, INBL
NSMOLD = NSUM + 1
NSUM = NBL(K) + NSUM
IF (NBL(K) .LE. 1) GO TO 30
DO 20 J = NSMOLD, NSUM
JJ = J - NSMOLD + 1
DO 20 I = NSMOLD, NSUM
46
**Description of Subroutine REDU**

Subroutine REDU reduces an \( N \times N \) matrix \( \text{VARO} \), if \( \text{VARO} \) is reducible. A simple permutation transformation is used on \( \text{VARO} \). Subroutine REDU calls no subroutines.

The subroutine statement for REDU is as follows:

```
SUBROUTINE REDU (VARO, SS, S, IN, JBL, INBL, IOR, NEL, IBL, JMAX, N, NMAX)
```

The input variable is

\( \text{VARO}(N,N) \) matrix to be reduced to block-diagonal form

The input integer dimensions are

\( N \) size of matrix to be reduced

\( NMAX \) dimension, greater than or equal to \( N \)

The output variables are

\( S(N,N) \) matrix in block-diagonal form

\( \text{NBL}(N) \) size of each irreducible block (integer)

\( \text{INBL} \) number of irreducible blocks (integer)

The intermediate variable is

\( \text{IOR}(N) \) block-diagonalizing permutation vector (integer)

The temporary storage arrays are

\( \text{SS}(N,N) \)

\( \text{IN}(N) \) integer

\( \text{JBL}(N) \) integer

\( \text{IBL}(N) \) integer

\( \text{JMAX}(N) \) integer

There is no printed output from this subroutine.
The following is a simplified description of the subroutine REDU algorithm, which is based on Harary's method for reducing a matrix to block-diagonal form (ref. 15). The aim is to permute the rows and columns of a matrix \( A \) so that (if possible) the following block-diagonal form results:

\[
\begin{bmatrix}
A_{11} & A_{12} \\
0 & A_{22}
\end{bmatrix}
\]

A Boolean matrix \( B \) corresponding to the input matrix \( A \) is formed by inserting ones in the positions of \( B \) where nonzero entries occur in \( A \) and zeroes elsewhere. For example,

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & C2 & C8 & 0 & C9 & 0 \\
2 & 0 & 0 & 0 & C3 & 0 \\
3 & C10 & C11 & C1 & 0 & 0 \\
4 & 0 & C4 & 0 & C5 & C6 \\
5 & 0 & 0 & 0 & C7 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 1 & 1 & 0 & 1 & 0 \\
2 & 0 & 0 & 0 & 1 & 0 \\
3 & 1 & 1 & 1 & 0 & 0 \\
4 & 0 & 1 & 0 & 1 & 1 \\
5 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

Matrix \( B \) is raised to the \( n - 1 \) power by using the rules of Boolean multiplication and addition. Then, the resulting matrix is checked for coincident ones, that is, where ones appear in both the \((i,j)\) and the \((j,i)\) positions when \( i \neq j \). This checking is done to find which variables are associated with irreducible submatrices of \( A \). In this example, \( B^{n-1} \) is

\[
\begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 1 & 1 & 0 & 1 & 1 \\
2 & 0 & 1 & 0 & 1 & 1 \\
B^4 = 3 & 1 & 1 & 1 & 1 & 1 \\
4 & 0 & 1 & 0 & 1 & 1 \\
5 & 0 & 1 & 0 & 1 & 1
\end{bmatrix}
\]

The coincident ones occur in locations \((2, 5\) and \(5, 2)\), \((2, 4\) and \(4, 2)\), and \((4, 5\) and \(5, 4)\). Thus, variables 2 and 5, 2 and 4, and 4 and 5 are associated with irreducible submatrices. However, 2 is common to the first two submatrices and 4 is common to the
last two submatrices. Thus, 2, 4, and 5 are all associated with one 3×3 irreducible submatrix. Therefore, the grouping of variables is (2, 4, 5), (1), and (3).

Now that the variables associated with the irreducible submatrices are identified, the original matrix $A$ is rearranged into block-diagonal form. The rows and columns of $A$ are permuted so that the variables associated with each irreducible submatrix are adjacent. An $INBL \times INBL$ Boolean matrix $B$ is formed, whose off-diagonal elements are ones if any of the elements in the corresponding block of the permuted $A$ matrix are nonzero; otherwise, they are zero. All diagonal elements of $B$ must be set to 1. For example,

$$
A = \begin{bmatrix}
2 & 4 & 5 & 1 & 3 \\
2 & 0 & C3 & 0 & 0 \\
4 & C4 & C5 & C6 & 0 \\
5 & 0 & C7 & 0 & 0 \\
1 & C8 & C9 & 0 & C2 \\
3 & C11 & 0 & 0 & C10 & C1
\end{bmatrix}
$$

The block-diagonal form

$$
A = \begin{bmatrix}
2 & 4 & 5 & 1 & 3 \\
0 & C3 & 0 & 0 & 0 \\
C4 & C5 & C6 & 0 & 0 \\
0 & C7 & 0 & 0 & 0 \\
C8 & C9 & 0 & C2 & 0 \\
C11 & 0 & 0 & C10 & C1
\end{bmatrix}
$$

The $B$ matrix is checked to see which column has the most zeroes. Then the block of $A$ corresponding to that column is placed in the first position, and that row and column of $B$ are eliminated. Thus, in this example, the third block of $A$ is placed in the first position, and $B$ is changed by eliminating the third row and third column, resulting in

$$
B = \begin{bmatrix}
1 & 1 & 0 \\
3 & 1 & 1 & 1
\end{bmatrix}
$$

The process is repeated; for example, the second block of $A$ is placed in the second position and the second row and column of $B$ are eliminated, resulting in

$$
B = \begin{bmatrix}
1 & 1 & 0 \\
1 & 1
\end{bmatrix}
$$

Finally, the first block of $A$ is placed in the third position. The final $A$ is now in the correct block-diagonal form with the irreducible blocks on the diagonal.
Listing of Subroutine REDU

```
SUBROUTINE REDU (VARO, SS, S, IN, JBL, INBL, IOR, NBL, IBL, JMAX, N, NMAX)

************************************************************************************
HARARYS METHOD FOR REDUCTION OF A REDUCIBLE MATRIX TO BLOCK
DIAGONAL FORM. REDU CALLS NO SUBROUTINES.
************************************************************************************

DIMENSION VARO(NMAX,1), SS(NMAX,1), S(NMAX,1), IN(1), JBL(1),
1 IOR(1), NBL(1), IBL(1), JMAX(1)

INBL = 0
DO 5 I = 1,N
  IN(I) = 0
  JBL(I) = 0
  IOR(I) = 0
  NBL(I) = 0
  JMAX(I) = 0
5  IBL(I) = 0

TAKE S MATRIX WHICH IS VARO MATRIX TO THE N-1 POWER IN BOOLEAN

DC 10 I = 1,N
IN(I) = I
DC 10 J = 1,N
10 SS(I,J) = VARO(I,J)
  IF (N .GT. 2) GO TO 30
  DO 20 J = 1,N
    DC 20 I = 1,N
    S(I,J) = 0.0
    IF (VARO(I,J) .NE. 0.0 .AND. SS(J,I) .NE. 0.0) GO TO 50
    CONTINUE
    GO TO 60
  50 S(I,J) = 1.0
20 CONTINUE
  GO TO 80
30 N2 = N - 2
  DO 70 L = 1,N2
    DC 70 I = 1,N
    DO 60 K = 1,N
      S(K,I) = 0.0
      DO 40 J = 1,N
        IF (VARO(K,J) .NE. 0.0 .AND. SS(J,I) .NE. 0.0) GO TO 50
        CONTINUE
        GO TO 60
50 S(K,I) = 1.0
```

A (final form) =

```
3 1 2 4 5
3 [ C1  C10  C11  0  0 ]
1 0  C2  C8  C9  0
4 0  0  C4  C5  C6
5 0  0  0  C7  0
```

ORIGINAL PAGE IS OF POOR QUALITY
CONTINUE
DO 70 J = 1,N
DO 70 I = 1,N
SS(I,J) = S(I,J)

FIND EQUIVALENCE BLOCKS AND IDENTIFY

INBL = 1
KSUM = 1
KSUMO = KSUM
DO 130 I = 1,N
DO 130 J = 1,N
IF (I .EQ. J) GO TO 130
IF (S(I,J) .NE. S(J,I)) GO TO 130
IF (IN(I) .EQ. 0 .AND. IN(J) .EQ. 0) GO TO 130
IF (S(I,J) .EQ. 0.0 .AND. S(J,I) .EQ. 0.0) GO TO 130
IF (KSUMO .NE. KSUM) GO TO 100
IN(I) = 0
IN(J) = 0
ICR(KSUM) = J
KSUM = KSUM + 2
GO TO 130

K1 = KSUM - 1
DO 110 L = KSUMO,K1
IF (IOR(L) .NE. I) GO TO 110
IF (IN(J) .EQ. 0) GO TO 130
IN(J) = 0
ICR(KSUM) = J
KSUM = KSUM + 1

CONTINUE
DO 120 L = KSUMO,K1
IF (IOR(L) .NE. J) GO TO 120
IF (IN(I) .EQ. 0) GO TO 130
IN(I) = 0
ICR(KSUM) = I
KSUM = KSUM + 1

CONTINUE
IF (KSUMO .NE. KSUM) GO TO 150
DO 140 I = 1,N
IF (IN(I) .EQ. 0) GO TO 140
NBL(INBL) = 1
INBL = INBL + 1
ICR(KSUM) = IN(I)
IN(I) = 0
KSUM = KSUM + 1

CONTINUE
INBL = INBL - 1
KSUM = KSUM - 1
GO TO 180

NBL(INBL) = KSUM - KSUMO
NSUM = 0
DO 160 I = 1,INBL
NSUM = NBL(I) + NSUM

CONTINUE
IF (NSUM .NE. N) GO TO 170
KSUM = KSUM - 1
GO TO 180
170  INBL = INBL + 1
GO TO 90
C
C REARRANGE MATRIX TO FORM EQUIVALENCE BLOCKS
C
180  GO 190  I = 1,N
K = IOR(I)
GO 190  J = 1,N
190  SS(I,J) = VARO(K,J)
GO 200  J = 1,N
K = IOR(J)
GO 200  I = 1,N
200  S(I,J) = SS(I,K)
C
C MAKE BOOLEAN MATRIX FROM EQUIVALENCE BLOCKS
C
NSUM = 0
GO 210  L = 1,INBL
NSMOLD = NSUM + 1
NSUM = NSUM + NBL(L)
JSUM = 0
GO 210  K = 1,INBL
JSMOLD = JSUM + 1
JSUM = JSUM + NBL(K)
SSL(K,L) = 0.0
GO 210  J = JSMOLD,JSUM
GO 210  I = NSMOLD,NSUM
IF (SSI(J,I) .NE. 0.0) SS(L,K) = 1.0
210 CONTINUE
GO 220  I = 1,INBL
IBL(I) = NBL(I)
IF (SSI(I,I) .NE. 1.0) SSI(I,I) = 1.0
220 CONTINUE
C
C REARRANGE EQUIVALENCE BLOCKS IN ORDER OF BOOLEAN COLUMNS WITH
C DECREASING NUMBERS OF ZEROS
C S = MATRIX IN EQUIVALENCE BLOCK ORDER
C INBL = NUMBER OF EQUIVALENCE BLOCKS
C NBL(I) = SIZE OF ITH BLOCK, (I = 1,...,INBL)
C IBL(I) = NBL(I)
C ICR(J) = NEW ORDER OF ROWS AND COLUMNS OF S IN COMPARISON
C WITH VARO, (J = 1,...,N)
C SS = BOOLEAN MATRIX OF EQUIVALENCE BLOCKS IN S
C
INBL1 = INBL - 1
INUM = 0
IVAR = 1
230 IMAX = 0
IZERMX = 0
DO 250  J = 1,INBL
IZER = 0
DO 240  I = 1,INBL
IF (INUM .EQ. 0) GO TO 245
DO 235  JJJ = 1,INUM
IF (J .EQ. JMAX(JJJ)) GO TO 250
IF (I .EQ. JMAX(JJJ)) GO TO 240
235 CONTINUE
240 IF (SSI(I,J) .EQ. 0.0) IZER = IZER + 1
240 CONTINUE
IF (IZER LE IZERMX) GO TO 250
IZERMX = IZER
IMAX = J

250 CONTINUE
INUM = INUM + 1
JMAX(INUM) = IMAX
KEL = INBL - INEW + 1
IF (IMAX EQ 0) GO TO 260
JBL(KBL) = IBL(IMAX)
IBL(IMAX) = 0
GO TO 290

260 DO 280 I = KBL, INBL
DO 270 J = 1, INBL
IF (IBL(J) EQ 0) GO TO 270
JBL(I) = IBL(J)
IBL(J) = 0
GO TO 280

270 CONTINUE
280 CONTINUE
IF (IZERMX EQ 0) GO TO 340
NSUM = 0
DO 300 I = 1, IMAX
300 NSUM = NSUM + NBL(I)
NSMOLD = NSUM - NBL(IMAX) + 1
GO 310 I = NSMOLD, NSUM
J = IVAR + I - NSMOLD
IN(J) = IOR(I)

310 IOR(I) = 0
IVAR = IVAR + NSUM - NSMOLD + 1
INEW = INEW - 1
IF (INEW LE 0) GO TO 370
GO TO 230

340 DO 360 I = 1, N
IF (IN(I) NE 0) GO TO 360
DO 350 J = 1, N
IF (IOR(J) EQ 0) GO TO 350
IN(I) = IOR(J)
IOR(J) = 0
GO TO 360

350 CONTINUE
360 CONTINUE

C REARRANGE ROWS AND COLUMNS OF ORIGINAL MATRIX VARO TO FORM FINAL
C MATRIX S IN BLOCK DIAGONAL FORM

C

370 CONTINUE
DO 380 I = 1, INBL
380 NBL(I) = JBL(I)
DO 390 I = 1, N
IOR(I) = IN(I)
K = IOR(I)
DO 390 J = 1, N
390 SS(I,J) = VARO(K, J)
DO 400 J = 1, N
K = IOR(J)
DO 400 I = 1, N
400 S(I,J) = SS(I,K)
RETURN
END
Description of Subroutine SCALE

Subroutine SCALE transforms an irreducible \( N_2 \times N_2 \) matrix \( A \), by using a diagonal matrix \( T_S \), so that the norm of \( A \) is minimized. The purpose of using subroutine SCALE is to condition \( A \) so that accuracy is improved in taking its eigenvalues. The resulting scaled matrix is stored in \( A \). Subroutine SCALE calls no subroutines. See reference 9 for further description.

The subroutine statement for SCALE is as follows:

\[
\text{SUBROUTINE SCALE } (A, T_S, N_2, IOP_1, N_2\text{MAX})
\]

The input variables are

- \( A(N_2, N_2) \) matrix to be scaled
- \( IOP_1 \) controls printing option

The input integer dimensions are

- \( N_2 \) size of matrix to be scaled
- \( N_2\text{MAX} \) dimension, greater than or equal to \( N_2 \)

The output variables are

- \( A(N_2, N_2) \) input matrix in scaled form
- \( T_S(N_2) \) vector containing the diagonal elements of the diagonal scaling matrix

There are no error messages output from this subroutine. If \( IOP_1 \) is not equal to zero, the \( T_S \) vector is printed with the heading

\[
T_S, \text{SCALING TRANSFORMATION MATRIX ELEMENTS}
\]

Listing of Subroutine SCALE

```
SUBROUTINE SCALE (A, T_S, N_2, IOP_1, N_2MAX)

C *********** ************ ****
C SCALE TRANSFORMS N_2 BY N_2 MATRIX A USING DIAGONAL MATRIX T_S SO
C THAT THE NORM OF A IS MINIMIZED. THE RESULTING SCALED MATRIX IS
C STORED IN A. SCALE CALLS NO SUBROUTINES.
C
DOUBLE PRECISION YNUM, XDEN
C
DIMENSION A(N_2MAX,1), T_S(1)

CC 10 I=1,N_2
10 T_S(I)=1.0
20 K=0
FLAG=1.0

54
```
Subroutine EIGQR computes the eigenvalues of a real matrix XR, by using the QR algorithm of Francis (refs. 16 and 17). The input matrix must be in upper Hessenberg form.
form. The maximum number of iterations of the QR algorithm used in finding an
eigenvalue is 50. Subroutine EIGQR calls no subroutines.

The subroutine statement for EIGQR is as follows:

```
SUBROUTINE EIGQR (XR,N2,CR,CI,IOP,N2MAX)
```

The input variables are

- `XR(N2,N2)`: input matrix in upper Hessenberg form
- `IOP`: controls printing option

The input integer dimensions are

- `N2`: size of input matrix
- `N2MAX`: dimension, greater than or equal to `N2`

The output variables are

- `CR(N2)`: real parts of eigenvalues
- `CI(N2)`: imaginary parts of eigenvalues

There is one error message from subroutine EIGQR. If it cannot find the eigenvalue
or eigenvalues in 50 iterations, it prints

```
THE FOLLOWING ANSWER(S) DID NOT CONVERGE
```

If `IOP` is not zero, the output from subroutine EIGQR prints the list of real parts and
imaginary parts of the eigenvalues with the number of QR iterations associated in ob-
taining each one. The heading is

```
REAL PART    IMAGINARY PART    ITER
```

Listing of Subroutine EIGQR

```
SUBROUTINE EIGQR (XR,N2,CR,CI,IOP,N2MAX)

C**************************************************************
C EIGQR COMPUTES THE EIGENVALUES USING THE QR ALGORITHM. THE INPUT
C MATRIX MUST BE IN UPPER HESSEBERG FORM. THE MAXIMUM NUMBER OF
C ITERATIONS IS 50, EIGQR CALLS NO SUBROUTINES.
C**************************************************************
CDIMENSION XR(N2MAX,1), CR(1), CI(1), PSI(2), GR(3)
FPZ=1.0E-15
N=N2
DO 10 I=1,N
   CR(I)=0.0
   CI(I)=0.0
10 IF (IOP) 20,30,20
   IF (IOP) 20,30,20
56
```
WRITE (6,800)
ZERO=0.0
JJ=1
XNN=0.0
XN2=0.0
AA=0.0
B=0.0
C=0.0
DD=0.0
R=0.0
SIG=0.0
ITER=0
IF (N-2) 50,100,110
50 IF (IOP) 60,70,60
60 WRITE (6,810) XR(1,1)
70 CR(1)=XR(1,1)
CI(1)=0.0
80 CONTINUE
NE=N2/2
CC 90 I=1,NE
NN=N2-I+1
B=CR(I)
C=CI(I)
CR(I)=CR(NN)
CI(I)=CI(NN)
CR(NN)=B
90 CI(NN)=C
RETURN
100 JJ=-1
110 Y=(XR(N-1,N-1)-XR(N,N))**2
S=4.0*XR(N,N-1)*XR(N-1,N)
ITER=ITER+1
IF (ABS(S)-Y*1.0E-8) 120,120,160
120 IF (ABS(XR(N-1,N-1))-ABS(XR(N,N))) 140,130,130
130 E=XR(N-1,N-1)
G=XR(N,N)
GO TO 150
140 G=XR(N-1,N-1)
E=XR(N,N)
150 F=0.0
H=0.0
GO TO 230
160 T=Y+S
IF (ABS(T)-4MAX1(Y,ABS(S))*1.0E-6) 170,170,180
170 T=0.0
180 Y=XR(N-1,N-1)+XR(N,N)
SQ=SQRT(ABS(T))
IF (T) 220,190,190
190 F=0.0
H=0.0
IF (Y) 210,200,200
260 E=(Y-SQ)/2.0
G=(Y+SQ)/2.0
GO TO 230
210 G=(Y-SQ)/2.0
E=(Y+SQ)/2.0
GO TO 230
220 F=SQ/2.0
E=Y/2.0
G=E
F=F
230 IF (JJ) 250, 240, 240
240 C=1.0E-10*(ABS(G)+F)
IF (ABS(XR(N-1,N-2)).GT.D) GO TO 280
250 IF (IOP) 260, 270, 260
260 WRITE (6,910) E,F,ITER
WRITE (6,910) G,H
270 CR(N)=E
CI(N)=F
CR(N-1)=G
CI(N-1)=H
N=N-2
IF (JJ) 80, 40, 40
280 IF (ABS(XR(N,N-1)).GT.1.0E-10*ABS(XR(N,N))) GO TO 320
290 IF (IOP) 300, 310, 300
300 WRITE (6,810) XR(N,N),ZERO,ITER
310 CR(N)=XR(N,N)
CI(N)=0.0
N=N-1
GO TO 40
320 VQ=ABS(XR(N,N-1))-ABS(XR(N-1,N-2))
IF (ABS(AABS(XN2,XR(N-1,N-2))-1.0E-6) GO TO 340, 340, 330
330 IF (ABS(AABS(XNN,XR(N,N-1))-1.0)-1.0E-6) 340, 340, 350
340 IF (VQ) 290, 250, 250
350 IF (ITER.LT.50) GO TO 360
WRITE (6,8201
IF (VQ) 290, 250, 250
360 IF (ITER.GT.50) GO TO 380
Z1=ABS(E-AA)+ABS(F-B)-.5*(ABS(E)+ABS(F))
Z2=ABS(G-C)+ABS(H-DD)-.5*(ABS(G)+ABS(H))
IF (Z1) 370, 400, 400
370 IF (Z2) 380, 390, 390
380 R=E*G-F*H
SIG=E+G
GO TO 430
390 R=E*E
SIG=E+E
GO TO 430
400 IF (Z2) 410, 420, 420
410 R=G*G
SIG=G+G
GO TO 430
420 R=0.0
SIG=0.0
430 XNN=XR(N,N-1)
XN2=XR(N-1,N-2)
N1=N-1
IA=N-2
IP=IA
IF (N-3) 480, 480, 440
440 DO 470 J=3,N1
450 IF (ABS(XR(J1+1,J1))-D) 480, 480, 450
460 DO 470 J=3,N1
470 IP=J1
58
48C IQ=IP
490 IP1=IP-1
IQ=IP1
IF (IP1-1) 520,520,500
500 GO 510 J=2,IP1
J1=IP-J+1
IF (ABS(XR(J1+1,J1))=0) 520,520,510
510 IQ=IQ-1
520 GO 790 I=IP,N1
IF (I-IP) 54C,530,540
530 GR(1)=XR(IP,IP)*(XR(IP,IP)-SIG)+XR(IP+1,IP)*XR(IP+1,IP)+R
GR(2)=XR(IP+1,IP)*(XR(IP,IP)+XR(IP+1,IP)-SIG)
GR(3)=XR(IP+1,IP)*XR(IP+2,IP+1)
XR(IP+2,IP)=0.0
GO TO 570
540 GR(1)=XR(I+1,IP)
GR(2)=XR(I+1,IP-1)
IF (I+IA) 550,550,560
550 GR(3)=XR(I+2,IP-1)
GO TO 570
560 GR(3)=0.0
570 CONTINUE
IF (ABS(GR(1)).LT.FPZ) GR(1)=0.0
IF (ABS(GR(2)).LT.FPZ) GR(2)=0.0
IF (ABS(GR(3)).LT.FPZ) GR(3)=0.0
XR=XK=SIGN(SQRT(GR(1)**2+GR(2)**2+GR(3)**2),GR(1))
XR=XR(I,J)+PSI(1)*XR(I+1,J)+PSI(2)*XR(I+2,J)
GO TO 670
60C IF (I-IQ) 610,640,610
610 IF (I-IP) 630,620,630
620 XR(I,I-1)=XR(I,I-1)
GO TO 640
630 XR(I,I-1)=XR(I,J)
640 DO 690 J=1,N
IF (I-N1) 65C,660,660
650 DR=PSI(2)*XR(I+2,J)
GO TO 670
66C ER=AL*XR(I,J)+PSI(1)*XR(I+1,J)+DR
IF (ABS(ER).LT.FPZ) ER=0.0
XR(I,J)=XR(I,J)-ER
XR(I+1,J)=XR(I+1,J)-PSI(1)*ER
IF (I-N1) 680,690,690
680 XR(I+2,J)=XR(I+2,J)-PSI(2)*ER
69C CONTINUE
IF (I-N1) 700,710,71C
70C L=I+2
GO TO 720
710 L=N
720 DO 770 J=IQ,L
Description of Subroutine ORDER

Given a set of NE eigenvalues stored in vectors CR and CI and symmetrically located with respect to the imaginary axis, subroutine ORDER places ones with positive real parts in the first NE/2 locations of CR and CI. Corresponding symmetric eigenvalues with negative real parts are placed in locations (NE/2) + 1 through NE of CR and CI. The convergence criterion EPS is used in determining if a pair of eigenvalues is symmetric. Subroutine ORDER calls no subroutines.

The subroutine statement for ORDER is as follows:

```
SUBROUTINE ORDER (CR, CI, NE, EPS)
```

The input variables are:

- CR(NE) real parts of eigenvalues (not ordered)
- CI(NE) imaginary parts of eigenvalues (not ordered)
- EPS criterion for symmetry

The input integer dimension is:

- NE number of eigenvalues
The output variables are

\begin{align*}
\text{CR(NE)} & \quad \text{real parts of eigenvalues (ordered)} \\
\text{CI(NE)} & \quad \text{imaginary parts of eigenvalues (ordered)}
\end{align*}

There is one error message from this subroutine. When an eigenvalue pair \((\lambda_i, \lambda_j)\), where \(\lambda_i = \alpha_i + j\beta_i\), is not symmetric within the \(\epsilon\) criterion, that is,

\[
\left(\frac{|\alpha_i| - |\alpha_j|}{|\alpha_i|}\right) \leq \epsilon,
\]

the program will stop and the following message will be printed:

\textit{EIGENVALUES ARE NOT SYMMETRIC}

There is no other printed output from this subroutine.

Listing of Subroutine ORDER

SUBROUTINE ORDER (CR, CI, NE, EPS)

GIVEN A SET OF NE EIGENVALUES, SYMMETRICALLY LOCATED WITH RESPECT TO THE IMAGINARY AXIS. ORDER PLACES ONES WITH POSITIVE REAL PARTS IN FIRST NE/2 LOCATIONS. CORRESPONDING SYMMETRIC EIGENVALUES ARE PUT IN LOCATIONS NE/2 + 1 THRU NE. EPS IS CONVERGENCE CRITERION USED IN DETERMINING IF PAIR OF EIGENVALUES ARE SYMMETRIC. ORDER CALLS NO SUBROUTINES.

DIMENSION CR(1), CI(1)

NE2=NE/2
KMAX=NE2+1
IMAX=NE+1
K=0

PUT EIGENVALUES WITH POSITIVE REAL PARTS IN TOP HALVES OF CR AND CI AND REST IN BOTTOM HALVES

10  K=K+1
20  IF (K.EQ.KMAX) GO TO 40
   IF (CR(K).GT.0.0) GO TO 13
   TCR=CR(K)
   TCI=CI(K)
   I=K+1
30  L=I-1
   CR(L)=CR(I)
   CI(L)=CI(I)
   I=I+1
   IF (I.NE.IMAX) GO TO 30
   CR(NE)=TCR
   CI(NE)=TCI
   GO TO 20
40  CONTINUE
Description of Subroutine EGVCTR

Subroutine EGVCTR obtains the \( N2 \times N2 \) modified eigenvector matrix \( X \), given the real input matrix \( AAA \) and its eigenvalues \( CPR \) and \( CPI \). The modified eigenvector matrix differs from the eigenvector matrix only in that where a complex pair of eigenvectors (columns) would occur in the eigenvector matrix, the corresponding two columns in the modified eigenvector matrix contain the real plus the imaginary parts and the real minus the imaginary parts, respectively. The internal variable IEND specifies the number of passes which will be made through the inverse iteration algorithm in obtaining each eigenvector. It is set nominally at 4. Subroutine EGVCTR calls subroutine PRMUTE and the IBM SSP subroutines FACTR and ARRAY.

The subroutine statement for EGVCTR is as follows:

\[
\text{SUBROUTINE EGVCTR} (AAA, CPR, CPI, X, N2, TT, EXT, AR, AI, IPEBN, IPER, IOP2, N2MAX)
\]

The input variables are

- \( AAA(N2, N2) \) matrix for which eigenvectors are to be obtained
- \( CPR(N2) \) real parts of eigenvalues of \( AAA \)
- \( CPI(N2) \) imaginary parts of eigenvalues of \( AAA \)
\textbf{IOP2} controls printing option

The input integer dimensions are

\textbf{N2} size of matrix for which eigenvectors are to be found

\textbf{N2MAX} dimension, greater than or equal to \textbf{N2}

The output variable is

\textbf{X(N2, N2)} modified eigenvector matrix of AAA

The temporary storage arrays are

\textbf{TT(N2, N2)}

\textbf{EXT(N2, N2)}

\textbf{AR(N2)}

\textbf{AI(N2)}

\textbf{IPERN(N2)} integer

\textbf{IPER(N2)} integer

If IBM SSP subroutine FACTR finds that the matrix cannot be factored, it returns with an error signal. Subroutine EGVCTR reads this signal and, if there is an error, it prints a message. If it is a real eigenvector, it prints

\textbf{FACTR IS WRONG}

If it is a complex eigenvector, it prints

\textbf{FACTR 2 IS WRONG}

If \textbf{IOP2} is not 0, the modified eigenvector matrix is written with the heading

\textbf{MODIFIED EIGENVECTOR MATRIX OF AAA}

Subroutine EGVCTR uses the inverse iteration method of Wilkinson (ref. 18) to solve the eigenvector problem \( Aq = \lambda q \), given an \( n \times n \) matrix \( A \) and an eigenvalue \( \lambda \).

The inverse iteration algorithm for finding \( q \) is given by Wilkinson as

\[
(A - \lambda I)z^{i+1} = q^i
\]  

(37)

\[
q^{i+1} = \frac{z^{i+1}}{(z^{i+1})_{\text{max}}}
\]  

(38)

where \((z^{i+1})_{\text{max}}\) is the element of \( z^{i+1} \) having the largest magnitude and \( q^i \) is the \( i^{\text{th}} \) approximation to the eigenvector \( q \). Initially, \( q^0 \) is chosen to be all ones. Vector \( q^i \)
usually converges to the true eigenvector after only a few iterations. Since \((A - \lambda I)\) is almost singular (except for computational errors), it is not inverted in solving for \(z^{i+1}\) but (as suggested by Wilkinson) is factored into lower and upper triangular factors, \(L\) and \(U\), by using IBM SSP subroutine FACTR. Subroutine FACTR obtains \(L\) and \(U\) such that

\[ P(A - \lambda I) = LU \]  

(39)

where \(P\) is a permutation matrix accounting for row interchanges which occurred in performing the factoring. Once the factors are obtained, it is easy to solve equation (37) for \(z^{i+1}\). To do this, we can write

\[ P(A - \lambda I)z^{i+1} = Pq^i \]  

(40)

If we let

\[ y^{i+1} = Uz^{i+1} \]  

(41)

then

\[ Ly^{i+1} = Pq^i \]  

(42)

which can be solved for \(y^{i+1}\) by using forward substitution (solving first for the first element of \(y^{i+1}\)). Having obtained \(y^{i+1}\), we obtain \(z^{i+1}\) by back substitution (solving first for the last element of \(z^{i+1}\)). The iteration is continued until \(q^i \approx q^{i+1}\) (three or four iterations have been found to suffice). Note that IBM SSP subroutine FACTR need only be called once for each value of \(\lambda\).

The preceding algorithm can be used either for \(\lambda\)-real or \(\lambda\)-complex. However, to avoid using complex arithmetic for the complex case, equations (37) and (38) can be expanded as follows: let \(\lambda = \alpha + j\beta\), where \(\beta \geq 0\), and its corresponding eigenvector \(q = q_R + jq_I\). Then we can write

\[
[(A - \alpha I) - j\beta I](z_R^{i+1} + jz_I^{i+1}) = q_R^i + jq_I^i
\]  

(43)

and

\[
q_R^{i+1} + jq_I^{i+1} = \frac{z_R^{i+1} + jq_I^{i+1}}{(z_R^{i+1})_{\max} + j(z_I^{i+1})_{\max}}
\]  

(44)
Following Wilkinson, equation (43) can be expanded and separated into two equations, resulting in

\[ Cz_{i+1}^R = Dq_i^R - \beta q_i^I \]  (45)

and

\[ Dz_{i+1}^R + \beta z_{i+1}^I = q_i^R \]  (46)

where

\[ C \triangleq \beta^2 I + (A - \alpha I)^2 \]  (47)

and

\[ D \triangleq A - \alpha I \]  (48)

Matrix C can be factored (as was done previously for A in the real eigenvector case) as \( PC = LU \). Then we can write

\[ y_{i+1}^R = U z_{i+1}^R \]  (49)

so that equation (45) becomes

\[ Ly_{i+1}^R = P Dq_i^R - P \beta q_i^I \]  (50)

Now equation (50) is solved for \( y_{i+1}^R \), and then \( q_{i+1}^R \) is solved by means of back substitution while using equation (49). Next, equation (46) can be rewritten as

\[ z_{i+1}^I = \frac{q_i^R - Dz_{i+1}^R}{\beta} \]  (51)

which can easily be solved for \( z_{i+1}^I \).

Then, equation (44) can be expanded into the two equations

\[ q_i^R = \frac{z_{i+1}^R (z_{i+1}^R)_{\max} + z_{i+1}^I (z_{i+1}^I)_{\max}}{|z_{i+1}^I|_{\max}^2} \]  (52)
and

\[ q^i_I = \frac{z^{i+1}_I (z^{i+1}_R)^\text{max} - z^{i+1}_R (z^{i+1}_I)^\text{max}}{|z^{i+1}_\text{max}|^2} \]  

(53)

which are solved for \( q^i_R \) and \( q^i_I \).

Subroutine EGVCTR stores the resulting eigenvectors in matrix \( T \) in column form. However, complex conjugate pairs of eigenvectors (for instance, \( q^R_R \pm jq^I_I \)) are stored with \( q^R_R + q^I_I \) in the first column and \( q^R_R - q^I_I \) in the second. The matrix \( T \) is then actually the modified eigenvector matrix, that is, it satisfies

\[ AT = TA \]

(54)

where \( \Lambda \) is the block-diagonal form of \( A \), described as

\[
\Lambda = \begin{bmatrix}
\alpha_1 & & \\
\alpha_2 & \alpha_3 & |\beta_3| \\
|\beta_3| & \alpha_3 & \\
0 & & 0
\end{bmatrix}
\]

(55)

Here, one typical complex eigenvalue pair, \( (\alpha_3 \pm j|\beta_3|) \), is shown along with two real eigenvalues.

Listing of Subroutine EGVCTR

```
SUBROUTINE EGVCTR (AAA,CPR,CPI,X,N2,T,XT,AR,AL,IPERM,IPER,IOIP2,
                     1N2MAX)

C**********************************************************************
C EGVCTR OBTAINS THE N2 BY N2 MODIFIED EIGENVECTOR MATRIX X, GIVEN
C INPUT MATRIX AAA AND ITS EIGENVALUES, STORED IN CPR AND CPI, IEND
C SPECIFIES THE NUMBER OF PASSES THRU THE INVERSE ITERATION ALGO-
C RITHM. EGVCTR CALLS SUBROUTINE PRMUTE AND IBM SSP SUBROUTINES
C FACTR AND ARRAY.
C**********************************************************************
```

66
DIMENSION AAA(N2MAX,1), CPR(1), CPI(1), X(N2MAX,1), TT(N2MAX,1), 
1EXT(N2MAX,1), AR(1), AI(1), IPER(1), IPERN(1)

DO 10 J=1,N2
DO 10 I=1,N2
10 X(I,J)=1.0
IEND=4

L=1
20 IF (L.GT.N2) GO TO 450
DO 30 J=1,N2
DO 30 I=1,N2
30 EXT(I,J)=AAA(I,J)
30 CONTINUE
30 IF (CPI(L).NE.0) GO TO 180

CALL REAL EIGENVECTOR
CALL FACTR (EXT,AR,N2,N2,IER)
CALL ARRAY (1,N2,N2,N2MAX9N2MAXEXT)

40 IPERN(I)=AR(I)
40 IF (IER.NE.3) GO TO 50
WRITE (6,490)
50 CONTINUE

80 I=1,N2
80 IF (EXT(I,I).NE.0) GO TO 80
II=I+1
AMAX=ABS(EXT(I,II))
80 CONTINUE
IF (AMAX.EQ.0) AMAX=1.0
EXT(I,I)=.5*AMAX/(2.0**35)
80 CONTINUE
ITER=0
GO TO 130

GO TO BACK SUBSTITUTION PHASE
90 ITER=ITER+1
90 IF (ITER.NE.IEND) GO TO 100
L=L+1
GO TO 20
100 CONTINUE
CALL PRMUTE (X,IPERN,IPER,N2,L,N2MAX)
BEGIN FORWARD SUBSTITUTION

DO 120 I = 2, N2
   M = I - 1
   SLM = X(I, L)
   DO 110 J = 1, M
      SLM = SLM - EXT(I, J) * X(J, L)
   END DO
   X(I, L) = SLM
   X(N2, L) = X(N2, L) / EXT(N2, N2)

BEGIN BACKWARD SUBSTITUTION

CC 130 I = 2, N2
   J = N2 - I + 1
   M = J + 1
   SLM = X(J, L)
   DO 140 K = M, N2
      SLM = SLM - X(K, L) * EXT(J, K)
   END DO
   X(J, L) = SLM / EXT(J, J)
   ZMAX = ABS(X(1, L))
   DO 160 I = 2, N2
      ZINT = ABS(X(I, L))
      IF (ZINT .LE. ZMAX) GO TO 160
   END DO
   ZMAX = ZINT
   IZ = I
   CONTINUE
   ZMAX = X(IZ, L)
   DO 170 I = 1, N2
      X(I, L) = X(I, L) / ZMAX
   END DO
   IF (ABS(X(1, L)) .LT. 1.0E-10) X(1, L) = 0.0
   GC TO 90

CALCULATE COMPLEX EIGENVECTOR

FORM LHS MATRIX (BETA**2 + (A - ALPHA*I)**2)

DO 190 J = 1, N2
   DO 190 I = 1, N2
      T(I, J) = EXT(I, J)
      DO 220 J = 1, N2
         DO 210 I = 1, N2
            SUM = 0.0
         END DO
         SLM = SUM + T(I, K) * EXT(K, J)
      END DO
      AR(I) = SLM
      DO 220 M = 1, N2
         EXT(M, J) = AR(M)
      END DO
      IF (M .EQ. J) EXT(M, J) = AR(M) + CPI(L) * CPI(L)
      CONTINUE

FACTOR LHS MATRIX INTO LOWER AND UPPER TRIANGULAR FACTORS

CALL ARRAY (2, N2, N2, N2MAX, EXT, EXT)
CALL FACTR (EXT, AR, N2, N2, IER)
CALL ARRAY (1, N2, N2, N2MAX, EXT, EXT)
DO 230 I = 1, N2
   IPEFN(I) = AR(I)
   IF (IER .NE. 3) GO TO 240
WRITE (6,500)

C
C CHECK FOR ZEROS ON DIAGONAL OF UPPER TRIANGULAR MATRIX
C
CC 270 I=1,N2
AR(I)=1.0
IF (EXT(I,I).NE.0.0) GO TO 270
II=II+1
AMAX=ABS(EXT(I,II))
IF (II.EQ.N2) AMAX=1.0
IF (II.GE.N2) GO TO 250
II=I+2
250 DO 260 J=II,N2
IF (ABS(EXT(I,J)).GT.AMAX) AMAX=ABS(EXT(I,J))
260 CONTINUE
IF (AMAX.EQ.0.0) AMAX=1.0
EXT(I,I)=.5*AMAX/(2.0**35)
270 CONTINUE

ITER=0
LL=LL+1
C
C GC TO BACK SUBSTITUTION PHASE
C
CC 280 ITER=ITER+1
IF (ITER.NE.IEND) GO TO 290
GO TO 430
C
C FORM RHS VECTOR FOR FINDING REAL PART OF EIGENVECTOR PAIR
C
290 CONTINUE
DO 300 I=1,N2
AR(I)=XT(I,L)
300 AI(I)=XT(I,LL)
DO 320 I=1,N2
SUM=C.0 DO
DO 310 J=1,N2
SUM=SUM+XT(I,J)*AR(J)
310 X(I,L)=SUM
CALL PRMUTE (XT,IPERN,IPER,N2,L,N2MAX)
CALL PRMUTE (XT,IPERN,IPER,N2,LL,N2MAX)
330 X(I,L)=-X(I,LL)*ABS(CPI(L))+X(I,L)
C
C BEGIN FORWARD SUBSTITUTION
C
DO 350 I=2,N2
M=I-1
SUM=X(I,L)
DO 340 J=1,M
SLM=SUM-EXT(I,J)*X(J,L)
340 X(I,L)=SUM
X(N2,L)=X(N2,L)/EXT(N2,N2)
350 CONTINUE
C
C BEGIN BACKWARD SUBSTITUTION
C
CC 360 I=2,N2
J=N2-I+1
M=J+1
SUM=X(J,L)
DO 370 K=M,N2
370 SLM=SUM-X(K,L)*EXT(J,K)
380 X(J,L)=SUM/EXT(J,J)
DO 400 I=1,N2
SUM=SUM-X(K,L)*EXT(J,K)
DO 390 K=1,N2
390 X(I,LL)=SUM/ABS(CPL(I))

C C
BEGIN Normalization of Eigenvectors
C
IZ=1
ZMAX=X(IZ,L)*X(IZ,L)*X(IZ,LL)*X(IZ,LL)
DO 410 I=2,N2
ZINT=X(I,L)*X(I,L)*X(I,LL)*X(I,LL)
IF (ZINT.LE.ZMAX) GO TO 410
ZMAX=ZINT
IZ=I
410 CONTINUE
XRMAX=X(IZ,L)
XIMAX=X(IZ,LL)
DO 420 I=1,N2
XR=(X(I,L)*XRMAX+X(I,LL)*XIMAX)/ZMAX
XI=(X(I,LL)*XRMAX-X(I,L)*XIMAX)/ZMAX
X(I,L)=XR
X(I,LL)=XI
IF (ABS(X(I,L)).LT.1.0.E-10) X(I,L)=0.0
IF (ABS(X(I,LL)).LT.1.0.E-10) X(I,LL)=0.0
420 CONTINUE
GO TO 280

C
AC AND subtract columns of X to form modified eigenvector matrix
C
DO 440 I=1,N2
SUM=X(I,L)
440 X(I,LL)=SUM-X(I,LL)
L=L+2
GO TO 20

C C
END OF Complex eigenvector calculation
C
C PRINT OUT eigenvector matrix
C
450 IF (IOP2.EQ.0) GO TO 480
WRITE (6,510)
510 LL=1
LLL=10
460 IF (N2.LT.LLL) LLL=N2
DO 470 I=1,N2
WRITE (6,520) (X(I,L),L=LL,LLL)
470 CONTINUE
WRITE (6,530)
480 CONTINUE
RETURN
Description of Subroutine PRMUTE

Subroutine PRMUTE permutes elements in column LA of an $N \times NE$ matrix $X$, as dictated by the transposition vector $ITRANS$, which is produced by IBM SSP subroutine FACTR. Subroutine PRMUTE calls no subroutines.

The subroutine statement for PRMUTE is as follows:

```plaintext
SUBROUTINE PRMUTE (X, ITRANS, IA, NE, LA, NEMAX)
```

The input variables are

- $X(NE, NE)$: input matrix
- $ITRANS(NE)$: transposition vector
- $LA$: column of $X$ to be permuted

The input integer dimensions are

- $NE$: size of $X$ matrix
- $NEMAX$: dimension, greater than or equal to $NE$

The output variable is

- $X(NE, NE)$: input matrix in permuted form

The temporary storage array is

- $IA(NE)$: integer

There is no printed output from this subroutine.

The method used by subroutine PRMUTE is as follows: given a set of linear equations to be solved,

$$Ax = b$$

(56)

IBM SSP subroutine FACTR can be used to solve the equation

$$PA = LU$$

(57)

for $P$, $L$, and $U$, given $A$, where $P$ is a permutation matrix, $L$ is lower triangular, and $U$ is upper triangular. Thus, if factoring is used, the original problem becomes
\[ PAx = Pb \quad \text{(58)} \]

or

\[ LUx = Pb \quad \text{(59)} \]

This equation can be easily solved for \( x \) by forward and backward substitution (see the section Description of Subroutine EGVCTR). Subroutine PRMUTE is used to take permutation information represented by matrix \( P \), which is output from IBM SSP subroutine FACTR and stored in vector ITRANS, and to permute the elements of \( b \) to form \( Pb \). However, subroutine PRMUTE considers vector \( b \) to be the LA column of an \( NE \times NE \) matrix \( X \).

Listing of Subroutine PRMUTE

```
SUBROUTINE PRMUTE (X,ITRANS,IA,NE,LA,NMAX)

C *********************************************************
C PRMUTE PERMUTES ELEMENTS IN LA COLUMN OF NE BY NE MATRIX X AS
C DICTATED BY TRANSPOSITION VECTOR ITRANS. ITRANS IS PRODUCED,
C IN OUR CASE, BY SUBROUTINE FACTR. IT CALLS NO SUBROUTINES.
C *********************************************************

DIMENSION X(NMAX,1), ITRANS(1), IA(1)

DO 10 I=1,NE
  IA(I)=I
DO 6C JJ=1,NE
  J=NE+1-JJ
  ITRN=ITRANS(J)
  I=C
20   I=I+1
  IF (IA(I).EQ.ITRN) GO TO 30
  GO TO 20
30   L=0
40   L=L+1
  IF (IA(L).EQ.J) GO TO 50
  GO TO 40
50   XTEMP=X(L,LA)
   X(L,LA)=X(I,LA)
   X(I,LA)=XTEMP
   ITRN=ITRANS(J)
   IA(I)=J
63   CONTINUE
RETURN
END
```
Description of Subroutine COVM

Subroutine COVM computes various covariance matrices that describe the system which represents the solution to the LSOCE problem. It sets up matrices for subroutine LYPNV (Lyapunov equation), which is then called to obtain the state-covariance matrix XX. Then XX, the estimation error covariance PP, and the control gains KC are used to obtain various other system-covariance matrices. These are the output covariance YY, the measurement covariance ZZ, the control covariance UU, the estimated output covariance YYEST, and the estimated measurement covariance ZZEST.

The subroutine statement for COVM is as follows:

\[
\text{SUBROUTINE COVM (AA, BB, HH, CC, QQ, PP, KC, N, M, C, R, XX, YY, ZZ,}
\text{ \hspace{1cm} UU, YYEST, ZZEST, KSIM, A, Q, BSIMQ, ASI, NMAX,}
\text{ \hspace{1cm} MMAX, CMAX, RMAX, KSIMAX)}
\]

The input variables are

- AA(N, N) system matrix
- BB(N, C) control input matrix
- HH(M, N) measurement matrix
- CC(R, N) output matrix
- QQ(N, N) power spectral density matrix of plant disturbance
- PP(N, N) estimation error-covariance matrix
- KC(C, N) control-gain matrix

The input integer dimensions are

- N number of state variables
- M number of measurements
- C number of control inputs
- R number of outputs
- KSIM \((N \times N + N)/2\)
- NMAX dimension, greater than or equal to N
- MMAX dimension, greater than or equal to M
- CMAX dimension, greater than or equal to C
- RMAX dimension, greater than or equal to R
- KSIMAX dimension, greater than or equal to KSIM

The output variables are
The intermediate variables are
\[ A(N, N) \] Lyapunov equation matrix
\[ Q(N, N) \] Lyapunov equation matrix

The temporary storage arrays are
\[ BSIMQ(KSIM) \]
\[ ASI(KSIM, KSIM) \]

There are no error messages output from this subroutine. It prints the message

```
**********************************************
COVARIANCE MATRICES
**********************************************
```

and then prints these arrays with the following headings:

\[ UU, \] CONTROL COVARIANCE MATRIX
\[ XX, \] STATE COVARIANCE MATRIX
\[ YY, \] OUTPUT COVARIANCE MATRIX
\[ YYEST, \] ESTIMATED OUTPUT COVARIANCE MATRIX
\[ ZZ, \] MEASUREMENT COVARIANCE MATRIX
\[ ZZEST, \] ESTIMATED MEASUREMENT COVARIANCE MATRIX

The mean-square state equation for the system obtained as the solution to the infinite-time LSOCE problem is given as equation (17). Subroutine COVM sets up the matrices \( A - BK_c \) and \( BK_c P + PK_TBT + Q \) and calls subroutine LYPNV with these matrices, obtaining state-covariance matrix \( X \). Then the various covariances are calculated.

Listing of Subroutine COVM

```
SUBROUTINE COVM (AA, BB, HH, CC, QQ, PP, KK, N, M, C, R, XX, YY, ZZ, UU, YYEST,
                  ZZEST, KSIM, AQ, BSIMQ, ASI, NMAX, MMAX, CMAX, RMAX, KSIMAX)
```

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**C**

`C` **COVM SETS UP MATRICES FOR SUBROUTINE LYPNV (LYAPUNOV EQ.)**

`C` WHICH IS THEN CALLED TO OBTAIN STATE COVARIANCE MATRIX XX.

`C` XX, ESTIMATION ERROR COVARIANCE PP, AND CONTROL GAINS KC ARE

`C` USED TO OBTAIN VARIOUS OTHER SYSTEM COVARIANCE MATRICES

****************************************************************

`DOUBLE PRECISION SUM1, SUM2`

`INTEGER C, R, CMAX, RMAX`

`REAL KC`

`DIMENSION AA(NMAX,1), BB(NMAX,1), HH(MMAX,1), CC(RMAX,1),`

`1QQ(NMAX,1), PP(NMAX,1), KC(CMAX,1), XX(NMAX,1), YY(RMAX,1),`

`2ZZ(MMAX,1), UU(CMAX,1), YYEST(RMAX,1), ZZEST(MMAX,1), A(NMAX,1),`

`3Q(NMAX,1), BSIMQ(1), ASI(KSIMAX,1)`

`WRITE (6,260)`

`FORM COEFFICIENT MATRICES FOR LYAPUNOV EQUATION WHOSE SOLUTION`

`IS STATE COVARIANCE MATRIX`

`DC 20 J=1,N`

`DC 20 I=1,N`

`SUM1=AA(I,J)`

`SUM2=QQ(I,J)`

`DO 10 K=1,C`

`SUM1=SUM1-BB(I,K)*KC(K,J)`

`DO 10 L=1,N`

`SUM2=SUM2+BB(I,K)*KC(K,L)*PP(L,J)+PP(I,L)*KC(K,L)*BB(J,K)`

`A(I,J)=SUM1`  

`Q(I,J)=SUM2`

`CALL LYPNV (A,Q,XX,ASIBSIMQ,N,KSIMAX)`

`CALL LYPNV (A,Q,XX,ASIBSIMQ,N,KSIMAX)`

`OBTAIN STATE COVARIANCE MATRICES FOR VARIABLES U(CONTROL), Z(MEASUREMENT), Y(OUTPUT), PLUS Z ESTIMATED AND Y ESTIMATED`

`DC 40 J=1,R`

`DC 40 I=1,R`

`SUM1=0.0D0`

`SUM2=0.0D0`

`DC 30 L=1,N`

`CC 30 K=1,N`

`SUM1=SUM1+CC(I,K)*XX(K,L)*CC(J,L)`

`SUM2=SUM2+CC(I,K)*PP(K,L)*CC(J,L)`

`YY(I,J)=SUM1`  

`YYEST(I,J)=SUM2`

`DC 60 J=1,M`

`DC 60 I=1,M`

`SUM1=0.0D0`

`SUM2=0.0D0`

`CC 50 L=1,N`

`CC 50 K=1,N`

`SUM1=SUM1+HH(I,K)*XX(K,L)*HH(J,L)`

`SUM2=SUM2+HH(I,K)*PP(K,L)*HH(J,L)`

`ZZ(I,J)=SUM1`  

`ZZEST(I,J)=SUM2`

`DC 80 J=1,C`
DD 80 I=1,C
DD 70 L=1,N
DO 70 K=1,N
SUM1=SUM1+KC(IK)*(XX(KL)-PP(K,L)*KC(J,L)
UU(I,J)=SUM1
WRITE (6,270)
LL=1
LLL=10
90 IF (C.LT.LLL) LLL=C
CO 100 I=1,C
WRITE (6,280) (UU(I,L),L=LLL)
CONTINUE
WRITE (6,290)
IF (C.EQ.LLL) GO TO 110
LL=LL+10
LLL=LLL+10
GO TO 90
110 WRITE (6,300)
LL=1
LLL=10
120 IF (N.LT.LLL) LLL=N
DO 130 I=1,N
WRITE (6,280) (XX(I,L),L=LL,LLL)
CONTINUE
WRITE (6,290)
IF (N.EQ.LLL) GO TO 140
LL=LL+10
LLL=LLL+10
GO TO 120
140 WRITE (6,310)
LL=1
LLL=10
150 IF (R.LT.LLL) LLL=R
CO 160 I=1,R
WRITE (6,280) (YY(I,L),L=LLL)
CONTINUE
WRITE (6,290)
IF (R.EQ.LLL) GO TO 170
LL=LL+10
LLL=LLL+10
GO TO 150
170 WRITE (6,320)
LL=1
LLL=10
180 IF (M.LT.LLL) LLL=M
CO 190 I=1,M
WRITE (6,280) (ZZ(I,L),L=LL,LLL)
CONTINUE
WRITE (6,290)
IF (M.EQ.LLL) GO TO 200
LL=LL+10
LLL=LLL+10
GO TO 180
200 WRITE (6,330)
LL=1
LLL=10
210 IF (M.LT.LLL) LLL=M
DO 220 I=1,M
WRITE (6,280) (ZZ(I,L),L=LL,LLL)
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SUBROUTINE LYPNV (A,Q,XX,ASI,BSIMQ,N,KSIM,WMAX,KSIMA)

The input variables are

A(N,N)       Lyapunov equation matrix
Q(N,N)       Lyapunov equation matrix

The input integer dimensions are

N       dimension of A, Q, and XX
KSIM  (N×N + N)/2

Description of Subroutine LYPNV

Subroutine LYPNV solves the Lyapunov equation. It first converts this equation,

\[ A_L X_L + X_L A_L^T + Q_L = 0 \]

into a set of linear equations. Subroutine LYPNV then calls IBM SSP subroutine SIMQ (solves a set of linear, algebraic equations) to solve this equation set for the matrix \( X_L \), called XX in the subroutine (ref. 13). Subroutine LYPNV also calls IBM SSP subroutine ARRAY.

The subroutine statement for LYPNV is as follows:

SUBROUTINE LYPNV (A,Q,XX,ASI,BSIMQ,N,KSIM,WMAX,KSIMA)
NMAX dimension, greater than or equal to N
KSIMAX dimension, greater than or equal to KSIM

The output variable is
XX(N,N) output matrix

The temporary storage arrays are
ASI(KSIM, KSIM)
BSIMQ(KSIM)

There is one error message output from this subroutine. If IBM SSP subroutine SIMQ finds that the matrix of the equation set being solved is singular and cannot complete its calculations, subroutine LYPNV prints the following message:

MATRIX IS SINGULAR

There is no other printed output from this subroutine.

Listing of Subroutine LYPNV

SUBROUTINE LYPNV (A, Q, XX, ASI, BSIMQ, N, KSIM, NMAX, KSIMAX)
C
C**************************************************************************
C LYPUNOV EQUATION
C A*X + X*A**T + Q = 0
C
C LYPNV CONVERTS LYAPUNOV EQ. TO SET OF LINEAR EQUATIONS. LYPNV
calls IBM SSP SUBROUTINES SIMQ (SOLVES EQUATION SET) AND ARRAY.
C KSIM =(N*N + N)/ 2
C FORM KSIM BY KSIM LHS MATRIX AND KSIM ORDER RHS VECTOR USING
C INPUT MATRICES A AND Q.
C**************************************************************************
C DIMENSION A(NMAX,1), Q(NMAX,1), XX(NMAX,1), ASI(KSIMAX,1),
C BSIMQ(1)
C
DO 10 J=1,KSIM
BSIMQ(J)=0.0
CG 10 I=1,KSIM
ASI(J,I)=0.0
10 CONTINUE
JS=0
CG 90 JJ=1,N
CG 9C II=JJ,N
JS=JS+1
IF (II.NE.JJ) GO TO 20
BSIMQ(JS)=-Q(II, JJ)
CG TO 30
20 BSIMQ(JS)=-Q(II, JJ)+Q(JJ, II))
30 CONTINUE

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Description of Subroutine TRICCS

Subroutine TRICCS is used to compute the solution to the constant-coefficient matrix Riccati differential equation at NTAU time points, DT apart. A call to subroutine RICSS (providing the modified eigenvector matrix TT of the Hamiltonian matrix and the real and imaginary parts CR and CI of the eigenvalues of the Hamiltonian matrix)
must precede the call to subroutine TRICCS. That is, subroutine TRICCS requires that the results of the algebraic Riccati equation be available for use in solving for the transient (see the section Description of Subroutine RICSS). The initial (or terminal) condition matrix is specified as INIT, and the Riccati solution matrix appears as ANS. The gains are also calculated at each time point, either the KE's or the KC's depending on whether the transient is being calculated for the estimate or control problem. Subroutine TRICCS calls subroutine BDSTM and the IBM SSP subroutines ARRAY and MINV.

The subroutine statement for TRICCS is as follows:

```fortran
SUBROUTINE TRICCS
  (TT,INIT,CR,CI,ANS,DT,NTAU,N,C,M,NE,
   EX1,EX2,EX3,EX4,EX5,EX6,IPER,IPERN,
   HH,RRINV,NN,PCINV,BB,IEC,NMAX,CMAX,
   MMAX,NEMAX)
```

The input variables are

- **TT(NE,NE)**: modified eigenvector matrix
  (called X in subroutine RICSS)
- **CR(NE)**: real parts of ordered eigenvalues
  (called CR in subroutine RICSS)
- **CI(NE)**: imaginary parts of ordered eigenvalues
  (called CI in subroutine RICSS)
- **INIT(N,N)**: initial condition of Riccati solution
- **HH(M,N)**: measurement matrix
- **RRINV(M,M)**: inverse of power spectral density matrix of measurement noise
- **NN(N,C)**: state-control product weighting matrix
- **PCINV(C,C)**: inverse of control weighting matrix
- **BB(N,C)**: control input matrix
- **IEC**: switch (1, if call is for estimate; 2, if call is for control)
- **NTAU**: number of time points
- **DT**: time step

The input integer dimensions are

- **N**: order of Riccati equation
- **C**: number of control inputs
- **M**: number of measurements
- **NE**: 2xN
NMAX dimension, greater than or equal to N
CMAX dimension, greater than or equal to C
MMAX dimension, greater than or equal to M
NEMAX dimension, greater than or equal to NE

The output variable is
ANS(N, N) solution to Riccati equation

The temporary storage arrays are
EX1(N, N)
EX2(N, N)
EX3(N, N)
EX4(N, N)
EX5(N)
EX6(N)
IPER(N) integer
IPERN(N) integer

There are two error messages from subroutine TRICCS. During the inversion of
ANS the following message may be printed:

    ANS IS SINGULAR

And during the inversion of EX1 the following message may be printed:

    EX1 IS SINGULAR

The normal output from subroutine TRICCS is the message

  *****************************************
  TRANSIENT RICCATI SOLUTION
  *****************************************

It then prints the time with the heading

    TIME =

followed by the ANS array with the heading

    RICCATI SOLUTION MATRIX

followed by either the estimate gains or the control gains (depending on the call) with
one of these headings:

    KE, THE TRANSIENT ESTIMATE GAIN MATRIX

or
KC, THE TRANSIENT CONTROL GAIN MATRIX

There will be a series of these outputs, one set for each time point calculated.

The method used by subroutine TRICCS, given in reference 6, is as follows:

Subroutine TRICCS computes the \( n \times n \), positive-definite, symmetric solution matrix for the matrix Riccati differential equation

\[
\dot{V} = \text{VM}_{11} \quad -\text{MT}_{11} V = \text{VM}_{12} V + \text{M}_{21}
\]  

(60)

with the initial condition, \( V(0) = V_0 \), and where the \( 2n \times 2n \) Hamiltonian matrix \( M \) is defined as

\[
M = \begin{bmatrix}
\text{M}_{11} & \text{M}_{12} \\
\text{M}_{21} & \text{M}_{22}
\end{bmatrix}
\]  

(61)

and \( M_{22} = \text{MT}_{11} \). Subroutine TRICCS uses the results of subroutine RICCS, namely, eigenvalues and modified eigenvectors of \( M \), plus the matrix exponential subroutine BDSTM, in computing its results. The method used follows that of Vaughan (ref. 6), except that where Vaughan used complex eigenvalues and eigenvectors involving complex arithmetic, this algorithm requires only real arithmetic. Since the modified-eigenvector-method matrix \( T \) and the block-diagonal-system matrix \( \text{C} \) (see the section Description of Subroutine RICSS) are used, the basic equation solved by subroutine TRICCS is

\[
V(k \Delta t) = \left[ T_{21} - T_{22}W(k \Delta t) \right] \left[ T_{11} - T_{12}W(k \Delta t) \right]^{-1}
\]  

(62)

where

\[
W(k \Delta t) = \exp (Ck \Delta t) \left[ V_0 T_{12} - T_{22} \right]^{-1} \left[ V_0 T_{11} - T_{21} \right] \left[ \exp (Ck \Delta t) \right]^{T}
\]  

(63)

Here, \( \Delta t \) is the time step, \( k \) is an integer, \( k \Delta t \) is the time at which \( V \) is to be calculated. Matrices \( \text{C} \) and \( T_{ij} \) are defined in the section Description of Subroutine RICSS. The state transition matrix \( \exp (C \Delta t) \) is calculated by using subroutine BDSTM, and then \( \exp (Ck \Delta t) \) is calculated as

\[
\exp (Ck \Delta t) = \exp (C(k - 1) \Delta t) \times \exp (C \Delta t)
\]  

(64)

Depending on whether IEC is 1 or 2, subroutine TRICCS computes \( K_e \) or \( K_c \), using equation (9) or (11).
Listing of Subroutine TRICCS

SUBROUTINE TRICCS (TT, INIT, CR, CI, ANS, DT, NTAU, N, C, M, NE, EX1, EX2, EX3, EX4, EX5, EX6, IPER, IPERN, HH, RRINV, NN, PCINV, BB, IEC, NMAX, CMAX, MMAX, 2NEMAX)

C
C TRICCS COMPLETES THE SOLUTION TO THE RICCATI DIFFERENTIAL EQUATION AT NTAU TIME POINTS, DT APART, AND A TRANSIENT GAIN AT EACH POINT. TRICCS CALLS SUBROUTINE BDSTM AND IBM SSP SUBROUTINES MINV AND ARRAY.
C
C REAL INIT, NN
INTEGER C, CMAX
DOUBLE PRECISION SUM, SUM1

DIMENSION TT(NEMAX,1), INIT(NMAX,1), CR(I), CI(1), ANS(NMAX,1), 
1 EX1(NMAX,1), EX2(NMAX,1), EX3(NMAX,1), EX4(NMAX,1), EX5(I), 
2 EX6(I), IPER(1), IPERN(1), HH(MMAX,1), RRINV(MMAX,1), 
3 PCINV(CMAX,1), BB(NMAX,1)

C
C ANTAU=NTAU
XTIME=XTIME*DT
WRITE (6, 360)
IF (IEC.EQ.1) TIME=0.0
IF (IEC.EQ.2) TIME=XTIME
WRITE (6, 370) TIME
WRITE (6, 380)
LL=1
LLL=10
10 IF (N.LT.LLL) LLL=N
DO 20 I=1,N
WRITE (6, 390) (INIT(I,L),L=LL,LLL)
20 CONTINUE
WRITE (6, 400)
IF (N.EQ.LLL) GO TO 30
LL=LL+10
LLL=LLL+10
GO TO 10
30 DC 50 J=1,N
JJ=J+N
DO 50 I=1,N
SUM=0.000
SUM1=0.000
CO 40 K=1,N
SUM=SUM+INIT(I,K)*TT(K,JJ)
40 SUM1=SUM1+INIT(I,K)*TT(K,J)
ANS(I,J)=SUM
50 EX1(I,J)=SUM1
CO 60 J=1,N
JJ=J+N
DO 60 I=1,N
II=I+N
SUM=0.000
SUM1=0.000
SUM=ANS(IJ)-TT(II,JJ)

83
SUM1 = EX1(I, J) - TT(I, J)
ANS(I, J) = SUM

CALL ARRAY (2, N, N, NMAX, NMAX, ANS, ANS)
CALL MINV (ANS, N, DET, IPER, IPERN)
CALL ARRAY (1, N, NMAX, NMAX, ANS, ANS)
IF (DET.EQ.0.0) WRITE (6, 410)
CC 80 J = 1, N
CC 80 I = 1, N
SUM = 0.000
CC 70 K = 1, N

SUM = SUM + ANS(I, K) * EX1(K, J)
INIT(I, J) = SUM
CC 90 I = 1, N
EX6(I) = - CR(I)
CC 100 I = 1, N
EX5(I) = CI(I)
CC 100 I = 1, N

CALL BOSTM (DT, EX6, EX5, EX4, N, NMAX)
CC 120 K = 1, N
CC 120 K = 1, N
SUM = 0.000
CC 160 K = 1, N
CC 160 K = 1, N
SUM = 0.000
CC 180 K = 1, N

EX1(I, J) = SUM

IF (STM VALUES .LT. 1.0E-15, SET TO 0.0)
DO 140 J = 1, N
CC 140 I = 1, N
IF (ABS(EX2(I, J)) .LE. 1.0E-15) EX2(I, J) = 0.0

CONTINUE
C EX3 NOW CONTAINS THE BLOCK DIAGONAL STATE TRANSITION MATRIX
CC 170 J = 1, N
CC 170 I = 1, N
SUM = 0.000
CC 180 K = 1, N

SUM = SUM + EX3(I, K) * INIT(K, L) * EX3(J, L)
ANS(I, J) = SUM
CC 190 J = 1, N
JJ = J + N
DO 190 I = 1, N
II = I + N
SUM = 0.000
SUM1 = 0.000
CC 180 K = 1, N
KK = K + N
SLM = SUM + TT(I, KK) * ANS(K, J)
SLM1 = SUM1 + TT(I, KK) * ANS(K, J)
EX1(I, J) = SUM

84
\begin{verbatim}
190   EX2(I,J)=SUM1
   DO 200 J=1,N
   DO 200 I=1,N
   I=I+N
   SUM=-EX1(I,J)+TT(I,J)
   SUM1=-EX2(I,J)+TT(I,J)
   EX1(I,J)=SUM
   200   EX2(I,J)=SUM1
   CALL ARRAY (2,N,N,NMAX,NMAX,EX1,EX1)
   CALL MINV (EX1,N,DET,IPER,IPERN)
   CALL ARRAY (1,N,N,NMAX,NMAX,EX1,EX1)
   IF (DET.EQ.0.0) WRITE (6,420)
   DO 220 J=1,N
   DO 220 I=1,N
   SUM=0.0D0
   DO 210 K=1,N
   SUM=SUM+EX2(I,K)*EX1(K,J)
   210   ANS(I,J)=SUM
   LL=1
   LLL=10
   IF (N.LT.LLL) LLL=N
   DO 240 I=1,N
   WRITE (6,390) (ANS(I,L),L=LL,LLL)
   240   CONTINUE
   WRITE (6,400)
   IF (N.EQ.LLL) GO TO 300
   LL=LL+10
   LLL=LLL+10
   GO TO 230
250   IF (IEC.NE.1) GO TO 300
   DO 270 J=1,M
   DO 270 I=1,N
   SUM=0.0D0
   DO 260 L=1,M
   SUM=SUM+ANS(I,K)*HH(L,K)*RRINV(L,J)
   260   EX2(I,J)=SUM
   WRITE (6,430)
   LL=1
   LLL=10
   IF (M.LT.LLL) LLL=M
   DO 290 I=1,N
   WRITE (6,390) (EX2(I,L),L=LL,LLL)
   290   CONTINUE
   WRITE (6,400)
   IF (M.EQ.LLL) GO TO 300
   LL=LL+10
   LLL=LLL+10
   GO TO 280
300   IF (IEC.NE.2) GO TO 350
   DO 320 J=1,N
   DO 320 I=1,C
   SUM=0.0D0
   CC 310 K=1,C
   SUM=SUM+PCINV(I,K)*NN(J,K)
   310   EX2(I,J)=SUM
   WRITE (6,440)
   \end{verbatim}
Description of Subroutine BDSTM

Subroutine BDSTM computes the state transition matrix for a matrix in block-diagonal form. The matrix's eigenvalues are the subroutine's inputs.

The subroutine statement for BDSTM is as follows:

```
SUBROUINI BDSTM (DT, CR, CI, EAT, N, NMAX)
```

The input variables are

- **DT**: time step
- **CR(N)**: real parts of eigenvalues
- **CI(N)**: imaginary parts of eigenvalues

The input integer dimensions are

- **N**: matrix size
- **NMAX**: dimension, greater than or equal to **N**

The output variable is

- **EAT(N, N)**: block-diagonal state transition matrix

Subroutine BDSTM prints no error messages and no output.
The method used by subroutine BDSTM is as follows: given an \( n \times n \) matrix \( A \) having eigenvalues \( \lambda = \alpha_i + j\beta_i (i = 1, 2, \ldots, n) \), \( \tilde{A} \), the block-diagonal form of \( A \), can be displayed as

\[
\tilde{A} = \begin{bmatrix}
\alpha_{k-1} & 0 \\
0 & \tilde{B}
\end{bmatrix}
\]

where

\[
\tilde{B} = \begin{bmatrix}
\alpha_k & \beta_k \\
\beta_{k+1} & \alpha_{k+1}
\end{bmatrix}
\]  

(65)

Shown here are how two real \((\alpha_{k-1} \text{ and } \alpha_{k+2})\) eigenvalue coefficients and the coefficients of a typical complex conjugate pair \((\alpha_k + j\beta_k \text{ and } \alpha_{k+1} + j\beta_{k+1})\) are placed on the diagonal of \( \tilde{A} \), either singly, or as a 2x2 block, \( \tilde{B} \). Note that for the complex conjugate pair, \( \alpha_k = \alpha_{k+1} \) and \( \beta_{k+1} = -\beta_k \). Subroutine BDSTM assumes that the vectors (CR and CI) of the eigenvalues are such that a conjugate pair is stored in successive locations. This will be the case if subroutine EIGQR is used to find the eigenvalues.

We desire to find the state transition matrix \( \exp(\tilde{A} \Delta t) \), where \( \Delta t \) is the desired time interval. It can be displayed as

\[
\exp(\tilde{A} \Delta t) = \begin{bmatrix}
\exp(\alpha_{k-1} \Delta t) & 0 \\
0 & \exp(\tilde{B} \Delta t)
\end{bmatrix}
\]

(67)
where \( \exp(\mathbf{B} \Delta t) \) is a 2\( \times \)2 matrix,

\[
\exp(\mathbf{B} \Delta t) = \begin{bmatrix}
\exp(\alpha_k \Delta t) \times \cos(\beta_k \Delta t) & -\exp(\alpha_k \Delta t) \times \sin(\beta_k \Delta t) \\
\exp(\alpha_k \Delta t) \times \sin(\beta_k \Delta t) & \exp(\alpha_k \Delta t) \times \cos(\beta_k \Delta t)
\end{bmatrix}
\]  \hspace{1cm} (68)

In order to see that \( \exp(\mathbf{B} \Delta t) \) is in fact of this form, consider the 2\( \times \)2 diagonal matrix,

\[
\mathbf{B} = \begin{bmatrix}
\alpha + j \beta & 0 \\
0 & \alpha - j \beta
\end{bmatrix}
\]  \hspace{1cm} (69)

where the diagonal elements are conjugates. Then for a transformation matrix \( \mathbf{U} \) given as

\[
\mathbf{U} = \begin{bmatrix}
1 - j & 1 + j \\
1 + j & 1 - j
\end{bmatrix}
\]  \hspace{1cm} (70)

it can be shown that \( \mathbf{U}^{-1} \mathbf{B} \mathbf{U} = \mathbf{\tilde{B}} \). Thus,

\[
\exp(\mathbf{\tilde{B}} \Delta t) = \exp(\mathbf{U}^{-1} \mathbf{B} \mathbf{U}) \]  \hspace{1cm} (71)

or

\[
\exp(\mathbf{\tilde{B}} \Delta t) = \mathbf{U}^{-1} \exp(\mathbf{B} \Delta t) \mathbf{U}
\]

\[
= \mathbf{U}^{-1} \begin{bmatrix}
\exp(\alpha + j \beta \Delta t) & 0 \\
0 & \exp(\alpha - j \beta \Delta t)
\end{bmatrix} \mathbf{U}
\]

\[
= \mathbf{U}^{-1} \begin{bmatrix}
\exp(\alpha \Delta t) \times \left[ \cos(\beta \Delta t) + j \sin(\beta \Delta t) \right] & 0 \\
0 & \exp(\alpha \Delta t) \times \left[ \cos(\beta \Delta t) - j \sin(\beta \Delta t) \right]
\end{bmatrix} \mathbf{U}
\]

\[
= \begin{bmatrix}
\exp(\alpha \Delta t) \cos(\beta \Delta t) & -\exp(\alpha \Delta t) \sin(\beta \Delta t) \\
\exp(\alpha \Delta t) \sin(\beta \Delta t) & \exp(\alpha \Delta t) \cos(\beta \Delta t)
\end{bmatrix}
\]  \hspace{1cm} (72)
Listing of Subroutine BDSTM

SUBROUTINE BDSTM (DT,CR,CI,EAT,N,NMAX)

C
C ****************************************************
C ** BCSTM Computes the state transition matrix for a matrix in block diagonal form. **
C ** BDSTM calls no subroutines. **
C ****************************************************
C
DIMENSION CR(1), CI(1), EAT(NMAX,1)

C 10 J=1,N
DO 10 I=1,N
10 EAT(I,J)=0.0
I=1
20 CONTINUE
IF (I.GT.N) RETURN
IF (CI(I).EQ.0.0) GO TO 30
Z=EXP(CR(I)*DT)
X=Z*COS(ABS(CI(I))*DT)
IF (ABS(X).LE.1.0E-15) X=0.0
Y=Z*SIN(ABS(CI(I))*DT)
IF (ABS(Y).LE.1.0E-15) Y=0.0
L=I+1
EAT(I,I)=X
EAT(I,L)=-Y
EAT(L,I)=Y
EAT(L,L)=X
I=I+1
GO TO 20
30 EAT(I,I)=EXP(CR(I)*DT)
IF (ABS(EAT(I,I)).LE.1.0E-15) EAT(I,I)=0.0
I=I+1
GO TO 20
END

LSOCE PROGRAM ERROR MESSAGES

The error messages that are printed by program LSOCE and the subroutines in which they occur are as follows:

LYPNV prints MATRIX IS SINGULAR
if IBM SSP subroutine SIMQ finds the matrix being solved is singular. The program then continues.

RICSS prints X11 IS SINGULAR
if the upper left quadrant of X is found to be singular during inversion. The program then continues.
EIGQR prints THE FOLLOWING ANSWER(S) DID NOT CONVERGE if an eigenvalue or eigenvalue pair cannot be found in 50 iterations. The program then continues to the next eigenvalue iteration.

ORDER prints EIGENVALUES ARE NOT SYMMETRIC if an eigenvalue pair is not symmetric within the EPS criterion. The program then stops.

EGVCTR prints FACTR IS WRONG if the matrix for a real eigenvector cannot be factored into upper and lower triangular segments by IBM SSP subroutine FACTR. The program then continues.

EGVCTR prints FACTR 2 IS WRONG if the matrix for a complex eigenvector cannot be factored into upper and lower triangular segments by IBM SSP subroutine FACTR. The program then continues.

**CONCLUDING REMARKS**

This report has presented a program for solving the LSOCE problem. Options that allow solution of all or part of the problem have been included. The LSOCE program has been used for systems as large as 10th order on an IBM 7090-7094 computer with a DCS system. For these cases, the program used reasonable amounts of time and storage. A similar version of the LSOCE program has been run on an IBM 360-67 computer for problems as large as 41st order, with good results. Certain of the subroutines used in LSOCE, such as those for reducing and scaling a matrix, for obtaining eigenvectors and eigenvalues, and for solving the Lyapunov equation, have been made general enough so as to be useful by themselves in the analysis of dynamic systems.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, September 3, 1974,
### APPENDIX A

### SYMBOLS

<table>
<thead>
<tr>
<th>Mathematical symbol</th>
<th>FORTRAN symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AA</td>
<td>system matrix, ( n \times n )</td>
</tr>
<tr>
<td>A</td>
<td></td>
<td>general matrix, ( n \times n )</td>
</tr>
<tr>
<td>( \tilde{A} )</td>
<td></td>
<td>block-diagonal form of ( A ), ( n \times n )</td>
</tr>
<tr>
<td>( A_L )</td>
<td>A</td>
<td>Lyapunov equation matrix, ( n \times n )</td>
</tr>
<tr>
<td>B</td>
<td>BB</td>
<td>control input matrix, ( n \times c )</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>diagonal matrix, ( 2 \times 2 )</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>Boolean matrix, ( n \times n )</td>
</tr>
<tr>
<td>( \tilde{B} )</td>
<td></td>
<td>diagonal block matrix, ( 2 \times 2 )</td>
</tr>
<tr>
<td>b</td>
<td></td>
<td>right-hand-side vector, ( n \times 1 )</td>
</tr>
<tr>
<td>C</td>
<td>CC</td>
<td>output matrix, ( r \times n )</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>intermediate matrix in complex eigenvector computation, ( n \times n )</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>upper diagonal submatrix of ( A ), ( n \times n )</td>
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<tr>
<td>( \overline{C} )</td>
<td></td>
<td>lower diagonal submatrix of ( A ), ( n \times n )</td>
</tr>
<tr>
<td>c</td>
<td>C</td>
<td>integer, number of control inputs</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>intermediate matrix in eigenvector computation, ( n \times n )</td>
</tr>
<tr>
<td>H</td>
<td>HH</td>
<td>measurement matrix, ( m \times n )</td>
</tr>
<tr>
<td>I</td>
<td></td>
<td>identity matrix, ( n \times n )</td>
</tr>
<tr>
<td>J</td>
<td></td>
<td>performance index for LSOCE problem</td>
</tr>
<tr>
<td>( J^* )</td>
<td></td>
<td>performance index for infinite-time LSOCE problem</td>
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<tr>
<td>j</td>
<td></td>
<td>( \sqrt{-1} )</td>
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<td>( K_c )</td>
<td>KC</td>
<td>control-gain matrix, ( c \times n )</td>
</tr>
<tr>
<td>( K_e )</td>
<td>KE</td>
<td>estimator-gain matrix, ( n \times m )</td>
</tr>
<tr>
<td>k</td>
<td></td>
<td>integer index</td>
</tr>
<tr>
<td>L</td>
<td></td>
<td>lower triangular matrix, ( n \times n )</td>
</tr>
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<td>Mathematical symbol</td>
<td>FORTRAN symbol</td>
<td>Description</td>
</tr>
<tr>
<td>---------------------</td>
<td>----------------</td>
<td>-------------</td>
</tr>
<tr>
<td>M</td>
<td>AAA</td>
<td>Hamiltonian matrix, $2n \times 2n$</td>
</tr>
<tr>
<td>m</td>
<td>M</td>
<td>integer, number of measurements</td>
</tr>
<tr>
<td>N</td>
<td>NN</td>
<td>state-control product weighting matrix, $n \times c$</td>
</tr>
<tr>
<td>n</td>
<td>N</td>
<td>integer, number of states</td>
</tr>
<tr>
<td>n</td>
<td></td>
<td>integer, dimension of a general matrix</td>
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<td>P</td>
<td>PP</td>
<td>estimation-error covariance matrix, $n \times n$</td>
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<tr>
<td>P</td>
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<td>permutation matrix, $n \times n$</td>
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<tr>
<td>$P^{-1}$</td>
<td>PCINV</td>
<td>inverse of control weighting matrix, $c \times c$</td>
</tr>
<tr>
<td>Q</td>
<td>QQ</td>
<td>power spectral density matrix of plant disturbance, $n \times n$</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>QC</td>
<td>state weighting matrix, $n \times n$</td>
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<tr>
<td>$Q_L$</td>
<td>Q</td>
<td>Lyapunov equation matrix, $n \times n$</td>
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<tr>
<td>q</td>
<td></td>
<td>eigenvector, $n \times 1$</td>
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<tr>
<td>$R^{-1}$</td>
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<td>inverse of power spectral density matrix of measurement noise, $m \times m$</td>
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<tr>
<td>r</td>
<td>R</td>
<td>integer, number of outputs</td>
</tr>
<tr>
<td>S</td>
<td>SS</td>
<td>control matrix Riccati solution, $n \times n$</td>
</tr>
<tr>
<td>T</td>
<td>X</td>
<td>modified eigenvector matrix of a Hamiltonian matrix, $2n \times 2n$</td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>modified eigenvector matrix of a general matrix, $n \times n$</td>
</tr>
<tr>
<td>t</td>
<td></td>
<td>time</td>
</tr>
<tr>
<td>$\Delta t$</td>
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</tr>
<tr>
<td>U</td>
<td>UU</td>
<td>control-covariance matrix, $c \times c$</td>
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<tr>
<td>U</td>
<td></td>
<td>transformation matrix, $2 \times 2$</td>
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<tr>
<td>U</td>
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<td>upper triangular matrix, $n \times n$</td>
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<tr>
<td>u</td>
<td></td>
<td>control vector, $c \times 1$</td>
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<tr>
<td>V</td>
<td>ANS</td>
<td>Riccati solution matrix, $n \times n$</td>
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<tr>
<td>v</td>
<td></td>
<td>white, zero-mean, Gaussian, measurement-noise vector, $m \times 1$</td>
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<tr>
<td>W</td>
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<td>intermediate matrix for transient Riccati solution, $n \times n$</td>
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<tr>
<td>Mathematical symbol</td>
<td>FORTRAN symbol</td>
<td>Description</td>
</tr>
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<td>---------------------</td>
<td>----------------</td>
<td>-------------</td>
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<tr>
<td>$w$</td>
<td></td>
<td>white, zero-mean, Gaussian, plant-noise vector, $n \times 1$</td>
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<tr>
<td>$x$</td>
<td>$XX$</td>
<td>state-covariance matrix (symmetric), $n \times n$</td>
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<tr>
<td>$\hat{x}$</td>
<td>$\mathbf{X}_{L}$</td>
<td>covariance of $\hat{x}$, $n \times n$</td>
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<td>$X_L$</td>
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<td>Lyapunov output matrix, $n \times n$</td>
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<td>state vector, $n \times 1$</td>
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<td>estimate of $x$, $n \times 1$</td>
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<tr>
<td>$Y$</td>
<td>$YY$</td>
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<tr>
<td>$\hat{Y}$</td>
<td>$YY_{EST}$</td>
<td>estimated output-covariance matrix, $r \times r$</td>
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<td>output vector, $r \times 1$</td>
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<tr>
<td>$\hat{y}$</td>
<td></td>
<td>estimated output vector, $r \times 1$</td>
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<tr>
<td>$\mathbf{Z}$</td>
<td>$ZZ$</td>
<td>measurement-covariance matrix, $m \times m$</td>
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<tr>
<td>$\hat{\mathbf{Z}}$</td>
<td>$ZZ_{EST}$</td>
<td>estimated measurement-covariance matrix, $m \times m$</td>
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<td>measurement vector, $m \times 1$</td>
</tr>
<tr>
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<td></td>
<td>estimated measurement vector, $m \times 1$</td>
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<td>$\alpha$</td>
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</tr>
<tr>
<td>$\beta$</td>
<td></td>
<td>imaginary part of eigenvalue</td>
</tr>
<tr>
<td>$\delta$</td>
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<td>Dirac delta function</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>$EPS$</td>
<td>symmetry criterion</td>
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<tr>
<td>$\Lambda$</td>
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<td>block-diagonal form of Hamiltonian matrix, $2n \times 2n$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td></td>
<td>eigenvalue</td>
</tr>
<tr>
<td>$\tau$</td>
<td></td>
<td>time</td>
</tr>
</tbody>
</table>

**Subscripts:**

- $f$: final value
- max: element of complex vector having largest magnitude
- $R$: real part of
<table>
<thead>
<tr>
<th>Mathematical symbol</th>
<th>FORTRAN symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td></td>
<td>imaginary part of</td>
</tr>
<tr>
<td>i, j, k,</td>
<td></td>
<td>integers, elements of</td>
</tr>
<tr>
<td>k-1, k+1,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td>initial value</td>
</tr>
<tr>
<td>Superscripts:</td>
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<td></td>
</tr>
<tr>
<td>T</td>
<td></td>
<td>transpose</td>
</tr>
<tr>
<td>-1</td>
<td></td>
<td>matrix inverse</td>
</tr>
<tr>
<td>()</td>
<td></td>
<td>derivative with respect to time</td>
</tr>
<tr>
<td>i; i+1</td>
<td></td>
<td>value after i or (i + 1)th iteration</td>
</tr>
<tr>
<td>Operators:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E[     ]</td>
<td></td>
<td>expected value of</td>
</tr>
<tr>
<td></td>
<td></td>
<td>absolute value of</td>
</tr>
<tr>
<td>-</td>
<td></td>
<td>approaches, in the limit</td>
</tr>
</tbody>
</table>
APPENDIX B

SAMPLE PROBLEM

The main routine LSOCE and its related subroutines were used to run a sample problem for a 10th-order system ($N = 10$) having two inputs ($C = 2$), three outputs ($R = 3$), and four measurements ($M = 4$). The option used was $ISORT = 4$, which exercised all subroutines. The state variable description in terms of $A$, $B$, $C$, and $H$ matrices is shown in the following table:

$$\begin{array}{cccccccccccc}
0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
-.025 & -.29 & -.5 & 0. & 0. & 0. & 0. & -17360. & -826.7 & -8.267 \\
.09 & .3 & 1. & -1. & 0. & 0. & 0. & 0. & 0. & 0. \\
.09 & 1. & 0. & -.5. & 0. & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. & 0. \\
0. & 0. & 0. & 1.667 & 1.667 & -100. & -12. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 0. & 1. \\
0. & 0. & 0. & 0. & 0. & 500. & 25. & -2.5E5 & -1.05E4 & -180.
\end{array}$$

$$\begin{array}{cccccccccccc}
0. & 0. & 0. & 1.667 & 1.667 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 62500. & 2976. & 29.76 \\
0. & 0. & 0. & 0. & 0. & 500. & 25. & 0. & 0. & 0. \\
\end{array}$$

$$\begin{array}{cccccccccccc}
0. & 0. & 0. & 1.667 & 1.667 & 0. & 0. & 0. & 0. & 0. \\
0. & 0. & 0. & 0. & 0. & 0. & 0. & 62500. & 2976. & 29.76 \\
0. & 0. & 0. & 0. & 0. & 500. & 25. & 0. & 0. & 0. \\
.09 & .3 & 1. & 0. & 0. & 0. & 0. & 0. & 0. & 0. \\
\end{array}$$
For the LSOCE problem, the following noise and weighting matrices were chosen:

\[ Q = \text{diag} (.1,.1,.1,.1,.1,.1,.1,.001,.001,.001) \]
\[ R^{-1} = \text{diag} (.01,.01,.01,.01) \]
\[ Q_c = \text{diag} (1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0,1.0) \]
\[ N = 0 \]
\[ P_c^{-1} = \text{diag} (1.0,1.0) \]

For computing the finite-time Kalman filter gains, the initial value of \( P(t) \) was

\[ P_0 = \text{diag} (1.1,1.1,1.1,1.1,1.1,1.1,1.1) \]

For computing the finite-time control gains, the terminal weighting \( S(t) \) was

\[ S_f = \text{diag} (1.1,1.1,1.1,1.1,1.1,1.1,1.1) \]

The time interval over which the performance index was minimized was chosen to be 12.0 seconds (that is, 60 time steps with \( \Delta t = 0.2 \)). Transient results were calculated and printed each 0.2 second.

Both \( \text{IOP1} \) and \( \text{IOP2} \) were set to 1 to obtain full printout. The following is a listing of main routine LSOCE plus the program outputs:

```plaintext
C MAIN PROGRAM
C
C ********~****************** *************************************
C
C PLANT
C XDOT = AA * X + BB * U + W
C MEASUREMENT
C Z = H * X + V
C NOISE
C COV(W) = QQ, COV(V) = RR
C PERFORMANCE INDEX
C J = E (1/2 * ( X**T * QC * X + X**T * NN * U + U**T * PC * U ) )
C
C DIMENSIONS AND DESCRIPTIONS OF PROGRAM VARIABLES
C
C A(N,N) LYAPUNOV EQUATION MATRIX
C AA(N,N) SYSTEM MATRIX
C AAA(2N,2N) HAMILTONIAN MATRIX ASSOCIATED WITH THE
C RICCATI EQUATION
C ANS(N,N) RICCATI SOLUTION FROM TRICCS
C BB(N,C) CONTROL INPUT MATRIX
C CC(N,N) OUTPUT MATRIX
C CI(2N) EIGENVALUE IMAGINARY PARTS (OF AAA)
C CR(2N) EIGENVALUE REAL PARTS (OF AAA)
C HHtMN) MEASUREMENT MATRIX
C IOR(2N) BLOCK DIAGONALIZING PERMUTATION VECTOR
C (INTEGER)
C KC(C,N) CONTROL GAIN MATRIX
C KE(N,M) ESTIMATOR GAIN MATRIX (REAL)
C NBL(2N) SIZE OF INDIVIDUAL IRREDUCIBLE BLOCKS
C (INTEGER)
C NN(N,C) STATE / CONTROL WEIGHTING MATRIX
C PCINV(C,C) INVERSE OF CONTROL WEIGHTING MATRIX
C PP(N,N) ESTIMATION ERROR COVARIANCE MATRIX
```
INITIAL CONDITION OF RICCATI SOLUTION FOR TRANSIENT ESTIMATE PROBLEM
LYAPUNOV EQUATION MATRIX (SYMMETRIC)
STATE WEIGHTING MATRIX
POWER SPECTRAL DENSITY MATRIX OF PLANT DISTURBANCE
INVERSE OF POWER SPECTRAL DENSITY MATRIX OF MEASUREMENT NOISE
CONTROL MATRIX RICCATI SOLUTION
INITIAL CONDITION OF RICCATI SOLUTION FOR TRANSIENT CONTROL PROBLEM
SCALING TRANSFORMATION
CONTROL COVARIANCE MATRIX
MODIFIED EIGENVECTOR MATRIX (OF AAA)
STATE COVARIANCE MATRIX (SYMMETRIC)
OUTPUT COVARIANCE MATRIX
ESTIMATED OUTPUT COVARIANCE MATRIX
MEASUREMENT COVARIANCE MATRIX
ESTIMATED MEASUREMENT COVARIANCE MATRIX

DIMENSIONS OF TEMPORARY STORAGE ARRAYS

INTEGER A(12,12), QQ(12,12), QC(12,12), PP(12,12), SS(12,12),
EX1(12,12), EX2(12,12), EX3(12,12), EX4(12,12), PPINIT(12,12), SSINIT(12,12),
AAA(24,24), 3TT(24,24), EXT(24,24), XR(24,24), ASI(78,78), YY(5,5),
ZZ(5,5), UU(5,5), YYEST(5,5), ZZEST(5,5), RRINV(5,5), FCINV(5,5),
BB(12,5), NN(12,5), KE(12,5), KC(5,12), CC(5,12), HH(5,12),
6EX5(12), EX6(12), TS(24), CH(24), CI(24), AR(24), AI(24),
7IPER(24), IPERN(24), BSIMQ(78), 10R(24), NBL(24), IBL(24),
8JMAX(24)
REAL KE, NN, KC
REAL INIT
INTEGER C, R, CMAX, RMAX

DATA ((AA(I,J), 1 = 1,10), J = 1,10)
1 / 2 * 0.0, -0.075, 2 * 0.09, 5 * 0.0, 1.0, 0.0, -0.29, 2 * 0.3,
2 * 0.0, 1.0, -0.5, 2 * 1.0, 8 * 0.0, -1.0, 2 * 0.0, 1.667,
3 * 0.0, -5.0, 0.0, 1.667, 9 * 0.0, -100.0, 2 * 0.0, 50.0.
4 5 * 0.0, 1.0, -12.0, 2 * 0.0, 25.0, 2 * 0.0, 2 * 0.0, 17360.0, 6 * 0.0, 5 * 2.5E5, 2 * 0.0, -826.7, 4 * 0.0, 1.0, 0.0, -1.05E4, 2 * 0.0, 6 * 8.267, 5 * 0.0, 1.0, -180.0 /
DATA ((BB(I,J), I = 1, 10), J = 1, 2)
1 / 2 * 0.0, -277.0, 13 * 0.0, 1.0, 3 * 0.0 /
DATA ((HH(I,J), I = 1, 4), J = 1, 10)
1 / 3 * 0.0, -0.9, 3 * 0.0, -3, 3 * 0.0, 1.0, 1.667, 3 * 0.0, 1.667, 2 * 0.0, 500.0, 3 * 0.0, 25.0, 2 * 0.0, 62500.0, 3 * 0.0, 2976.0, 3 * 0.0, 29.76, 2 * 0.0 /
DATA ((III(I,J), I = 1, 10), J = 1, 10)
1 / 0.10, 10 * 0.0, 0.10, 10 * 0.0, 0.10, 10 * 0.0, 0.10, 10 * 0.0, 0.10, 10 * 0.0, 0.10, 0.0, 10 * 0.0, 0.10, 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 3 * 0.0, 1.0 /
DATA ((QQ(I,J), I = 1, 4), J = 1, 10)
1 / 1.0E-2, 4 * 0.0, 1.0E-2, 4 * 0.0, 1.0E-2, 4 * 0.0 /
DATA ((II(I,J), I = 1, 10), J = 1, 10)
1 / 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 10 * 0.0, 1.0, 3 * 10 * 0.0, 1.0 /
DATA ((NN(I,J), I = 1, 10), J = 1, 2)
1 / 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0 /
DATA ((PP(I,J), I = 1, 10), J = 1, 10)
1 / 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0 /
DATA ((QQQ(I,J), I = 1, 4), J = 1, 10)
1 / 1.0, 1.0, 1.0, 1.0, 1.0 /
DATA ((NNN(I,J), I = 1, 10), J = 1, 10)
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1 / 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0 /
DATA ((QQQQ(I,J), I = 1, 4), J = 1, 10)
1 / 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0, 1.0 /
DATA (N / 10, /, M / 4, /, C / 2, /, R / 3, /, IOP1 / 1, /, IOP2 / 1, /, DATA DT / .200 /, NTAU / 60, /, ISORT / 4, /)
C
C NMAX = 12
C MMAX = 5
C CMAX = 5
C N2MAX = NMAX + NMAX
C KMAX = NMAX * NMAX
C KSIMAX = (KMAX + NMAX) / 2
C N2=N*N
C K=N*N
C KSIM=(K+N)/2

C IF (ISORT.GT.5) GO TO 10
C
C CALL ESTMAT (AA, HH, QQ, RRINV, KE, PP, CR, CI, X, TS, XR, TT, AAA, EXT, AR, AI, LPER, IPERN, IOR, NBL, IBL, JMAX, N, M, N2, IOP1, IOP2, NMAX, MMAX, N2MAX)
C
C IF (ISORT.EQ.1) STOP
C IF (ISORT.EQ.5) GO TO 10
C
C CALL TRICCS (X, PPINIT, CR, CI, ANS, DT, NTAU, N, M, N2, EX1, EX2, EX3, EX4, 1EX5, EX6, IPER, IPERN, HH, RRINV, NN, PCINV, RB, 1, NMAX, CMAX, MMAX, N2MAX)
C
C 98
IF (ISORT.EQ.2) STOP
IF (ISORT.EQ.9) GO TO 20

CALL CONTRL (AA, BB, QC, NN, PCINV, KC, SS, CR, CI, X, TS, XR, TT, AAA, EXT, AR, LAI, IPER, IPERN, IOR, N8L, IBL, JMAX, N, C, N2, IOP1, IOP2, NMAX, CMAX, N2MAX)

IF (ISORT.EQ.6) STOP
IF (ISORT.EQ.5 OR ISORT.EQ.8) GO TO 20

CALL TRIGCS (X, SSINIT, CR, CI, ANS, DT, NTAU, N, C, M, N2, EX1, EX2, EX3, EX4, EX5, EX6, IPER, IPERN, HH, RRINV, NN, PCINV, BB, Z, NMAX, CMAX, MMAX, N2MAX)

IF (ISORT.EQ.3 OR ISORT.EQ.7) STOP

CALL COVM (AA, BB, HH, CC, QQ, PP, KC, N, M, C, R, XX, YY, ZZ, UU, YYEST, ZZEST, KSIM, A, Q, BSIMQ, ASI, NMAX, MMAX, CMAX, RMAX, KSI MAX)

RETURN
END
### Results of Estimation Problem

**TS: Scaling Transformation Matrix Elements**

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**Modified Eigenvector Matrix of AAA**

| 4.227E-02 | 6.811E-02 | 4.780E+00 | 5.249E-01 | 1.800E+02 | 1.629E+01 | 1.237E+01 | 2.691E+00 | 1.722E+00 | 2.458E+00 | 5.816E-01 | 2.438E-02 | 2.336E+01 | 6.875E-04 | 7.599E-03 | 1.000E+00 |
|------------|------------|-----------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 0.17554749E+00 | 0.25538571E+00 | 10  |
| 0.17554749E+00 | -0.25538571E+00 | 4   |
| -0.17554749E+00 | 0.25538571E+00 | 5   |
| -0.17554749E+00 | -0.25538571E+00 | 1    |

**Real Part**

| 0.17554749E+00 | 0.25538571E+00 | 10  |
| 0.17554749E+00 | -0.25538571E+00 | 4   |
| -0.17554749E+00 | 0.25538571E+00 | 5   |
| -0.17554749E+00 | -0.25538571E+00 | 1    |

** Imaginary Part**

| 0.17554749E+00 | 0.25538571E+00 | 10  |
| 0.17554749E+00 | -0.25538571E+00 | 4   |
| -0.17554749E+00 | 0.25538571E+00 | 5   |
| -0.17554749E+00 | -0.25538571E+00 | 1    |

**Iter**

| 0.17554749E+00 | 0.25538571E+00 | 10  |
| 0.17554749E+00 | -0.25538571E+00 | 4   |
| -0.17554749E+00 | 0.25538571E+00 | 5   |
| -0.17554749E+00 | -0.25538571E+00 | 1    |
THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX

THE ONES WITH NEGATIVE REAL PARTS
APF THE EIGENVALUES OF (A-KE*H)

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## Transient Riccati Solution

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**KE, The Transient Estimate Gain Matrix**

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**Note:** The table continues with similar entries and structures. The data appears to be related to transient gain matrices with various coefficients. The values are presented in scientific notation, indicating they are likely numerical data sets used in a mathematical or engineering context.
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**TIME = 0.620000E+01**

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The document contains matrices and transient estimates with various coefficients and values. It appears to be a technical or scientific document, possibly related to control systems or engineering, given the context of the matrices and constants presented. The matrices are labeled with different descriptions, such as "KE, THE TRANSIENT ESTIMATE GAIN MATRIX" and "RICCATI SOLUTION MATRIX," indicating their purpose in the analysis or simulation of a system. The values within the matrices range widely, suggesting a complex system analysis with a focus on transient behavior. The time stamps at the end of each section indicate the simulation time points, which are rounded to high precision, possibly for accuracy in the simulation process.
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**Riccati** = the transient Riccati matrix

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\[
\text{KE, THE TRANSIENT ESTIMATE GAIN MATRIX}
\]

**TIME = 0.10000E+02**
The transient estimate gain matrix

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Riccati solution matrix

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The transient estimate gain matrix

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THE TRANSIENT ESTIMATE GAIN MATRIX

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RICCATI SOLUTION MATRIX

TIME = 0.12000E+02

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### RESULTS OF CONTROL PROBLEM

**TS: SCALING TRANSFORMATION MATRIX ELEMENTS**

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### MODIFIED EIGENVECTOR MATRIX OF AAA

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**SS**, MATRIZ RICATI SOLUTION FOR CONTROL PROBLEM

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**THE ORDERED EIGENVALUES OF THE HAMILTONIAN MATRIX**

**THE ONES WITH NEGATIVE REAL PARTS**

**ARE THE EIGENVALUES OF (A-BK)C**

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**TIME = 0.86000E+01**
I want to learn more about the transient response of a system. Can you explain how to analyze it using the Riccati equation and the control gain matrix?

The transient response of a system can be analyzed using the Riccati equation, which is a nonlinear differential equation that describes the behavior of the system over time. The solution to the Riccati equation provides the system's gain matrix, which is used to calculate the system's response to an input.

In the context of control systems, the Riccati equation is used to find the optimal control law that minimizes a certain performance criterion. The control gain matrix is a matrix of gains that are applied to the system's inputs to achieve the desired output.

To analyze the transient response, we first need to solve the Riccati equation. This can be done using numerical methods or analytical techniques, depending on the complexity of the system.

Once we have the gain matrix, we can use it to calculate the system's response to an input signal. The response can be visualized using a time domain or frequency domain analysis.

In summary, the Riccati equation and the control gain matrix are powerful tools for analyzing the transient response of a system. By solving the Riccati equation and calculating the gain matrix, we can gain insight into how the system behaves under different conditions.

If you have any specific questions or need further clarification, please let me know!
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### Riccati Solution Matrix

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### RC Gain

- \(5.6785E+00\)
- \(1.3259E+00\)
- \(1.1615E+00\)
- \(-3.1278E-01\)
- \(-1.1034E+00\)
- \(-2.8905E+00\)
- \(-3.8982E+00\)
- \(1.0826E+00\)
- \(-5.0020E+00\)
- \(-8.0901E+00\)

### RC Solution

- \(5.6785E+00\)
- \(1.3259E+00\)
- \(1.1615E+00\)
- \(-3.1278E-01\)
- \(-1.1034E+00\)
- \(-2.8905E+00\)
- \(-3.8982E+00\)
- \(1.0826E+00\)
- \(-5.0020E+00\)
- \(-8.0901E+00\)

### Riccati Solution

- \(5.6785E+00\)
- \(1.3259E+00\)
- \(1.1615E+00\)
- \(-3.1278E-01\)
- \(-1.1034E+00\)
- \(-2.8905E+00\)
- \(-3.8982E+00\)
- \(1.0826E+00\)
- \(-5.0020E+00\)
- \(-8.0901E+00\)

### Control Gain

- \(5.6785E+00\)
- \(1.3259E+00\)
- \(1.1615E+00\)
- \(-3.1278E-01\)
- \(-1.1034E+00\)
- \(-2.8905E+00\)
- \(-3.8982E+00\)
- \(1.0826E+00\)
- \(-5.0020E+00\)
- \(-8.0901E+00\)
### KC, THE TRANSIENT CONTROL GAIN MATRIX

\[
\begin{array}{cccccc}
-5.0962E+00 & -1.1878E+01 & -6.6745E-01 & -5.3489E-01 & -1.4447E+00 & -1.1259E+00 \\
3.5795E+00 & 7.6633E+00 & 4.7731E+00 & -4.6992E+00 & -1.1878E+01 & -1.5796E+00 \\
-2.0675E+00 & -4.1812E+00 & -1.1259E+00 & 1.0368E+00 & 1.3492E+00 & 2.6975E+00 \\
-6.6746E-01 & -1.1878E+01 & 2.6795E+00 & 9.3518E+00 & 1.4091E+00 & 1.6783E+00 \\
-8.9068E+00 & -1.8418E-01 & -1.8418E-01 & -1.1259E+00 & -1.1216E+00 & 4.7260E+00 \\
-1.4033E+00 & -3.2377E-01 & -3.2377E-01 & -1.1259E+00 & -1.1216E+00 & -1.3575E+00 \\
\end{array}
\]

**Riccati Solution Matrix**

\[
\begin{array}{cccccc}
-1.1034E+01 & -1.0218E+00 & -3.8871E+00 & -1.0218E+00 & -1.3575E+00 & 2.6975E+00 \\
-1.0368E+00 & -1.0406E+01 & -1.4033E+00 & 2.6975E+00 & 9.3518E+00 & 1.4091E+00 \\
-1.3492E+00 & -1.3492E+00 & -1.1259E+00 & 1.0368E+00 & 1.3492E+00 & 2.6975E+00 \\
-9.3518E+00 & -1.4091E+00 & -1.1259E+00 & -1.0368E+00 & -1.0368E+00 & 9.3518E+00 \\
-1.4091E+00 & -1.4091E+00 & -1.1259E+00 & -1.3492E+00 & -1.3492E+00 & -1.1259E+00 \\
-3.8871E+00 & -3.8871E+00 & -1.1259E+00 & -2.6975E+00 & -2.6975E+00 & -1.1259E+00 \\
\end{array}
\]

**Time = 0.28000E+01**

### KC, THE TRANSIENT CONTROL GAIN MATRIX

\[
\begin{array}{cccccc}
5.6791E-01 & 1.3259E+00 & 1.615E+00 & -3.1278E-01 & -1.1034E-01 & -2.809E+00 \\
-6.6743E-01 & -1.5796E00 & -1.4033E+00 & 3.9913E-01 & 1.4091E+00 & 3.5298E+00 \\
\end{array}
\]

**Riccati Solution Matrix**

\[
\begin{array}{cccccc}
-2.9062E+00 & -2.0444E+00 & -5.3493E-01 & -1.8418E-01 & -1.1259E+00 & 2.0444E+00 \\
-2.0444E+00 & -2.0444E+00 & -5.3493E-01 & -1.8418E-01 & -1.1259E+00 & 2.0444E+00 \\
-5.3493E-01 & -1.8418E-01 & -1.1259E+00 & 2.0444E+00 & 2.0444E+00 & 2.0444E+00 \\
\end{array}
\]

**Time = 0.26000E+01**

### KC, THE TRANSIENT CONTROL GAIN MATRIX

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5.6795E-01 & 1.3260E+00 & 1.615E+00 & -3.1278E-01 & -1.1034E-01 & -2.809E+00 \\
-6.6745E-01 & -1.5796E00 & -1.4033E+00 & 3.9913E-01 & 1.4091E+00 & 3.5298E+00 \\
\end{array}
\]

**Riccati Solution Matrix**

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-5.3493E-01 & -1.8418E-01 & -1.1259E+00 & 2.0444E+00 & 2.0444E+00 & 2.0444E+00 \\
\end{array}
\]

**Time = 0.24000E+01**

---

**NOTE:** The numbers are in scientific notation. For example, $5.6791E-01$ means $5.6791 	imes 10^{-1}$.
### KC, THE TRANSIENT CONTROL GAIN MATRIX

\[
\begin{array}{cccccccc}
-0.970E+00 & -2.05E+00 & -1.900E+01 & 4.726E+00 & 1.678E+00 & 4.320E+00 & 5.909E+00 & -8.55E+00 & -1.293E+02 & 1.235E+02 & 1.333E+00 \\
1.427E-01 & -3.327E+00 & -2.912E+00 & 2.759E-02 & 7.258E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
\end{array}
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### TIME = 0.22000E+01

### RICCATI SOLUTION MATRIX

\[
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3.579E+00 & 7.663E+00 & 4.773E+00 & -1.126E+00 & -4.994E-01 & -1.879E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
2.044E+00 & 4.773E+00 & 4.181E+00 & -3.972E+00 & -1.879E+00 & 1.349E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
\end{array}
\]

### KC, THE TRANSIENT CONTROL GAIN MATRIX

\[
\begin{array}{cccccccc}
5.679E+00 & 1.326E+00 & 1.161E+00 & -3.127E+00 & -1.103E+00 & -2.890E+00 & -3.898E+00 & 1.062E+00 & -5.002E+00 & -8.090E-02 \\
-6.717E+00 & -1.579E+00 & -1.403E+00 & 3.991E+00 & 1.409E+01 & 3.530E+00 & 5.215E+00 & -1.357E+02 & 5.909E+00 & 9.761E+00 \\
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### TIME = 0.20000E+01

### RICCATI SOLUTION MATRIX

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3.579E+00 & 7.663E+00 & 4.773E+00 & -1.126E+00 & -4.994E-01 & -1.879E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
2.044E+00 & 4.773E+00 & 4.181E+00 & -3.972E+00 & -1.879E+00 & 1.349E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
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### KC, THE TRANSIENT CONTROL GAIN MATRIX

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5.679E+00 & 1.326E+00 & 1.161E+00 & -3.127E+00 & -1.103E+00 & -2.890E+00 & -3.898E+00 & 1.062E+00 & -5.002E+00 & -8.090E-02 \\
-6.717E+00 & -1.579E+00 & -1.403E+00 & 3.991E+00 & 1.409E+01 & 3.530E+00 & 5.215E+00 & -1.357E+02 & 5.909E+00 & 9.761E+00 \\
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### TIME = 0.18000E+01

### RICCATI SOLUTION MATRIX

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3.579E+00 & 7.663E+00 & 4.773E+00 & -1.126E+00 & -4.994E-01 & -1.879E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
2.044E+00 & 4.773E+00 & 4.181E+00 & -3.972E+00 & -1.879E+00 & 1.349E+00 & 2.697E+00 & 9.532E+01 & 3.530E+00 & 5.215E+00 & 9.761E+00 & -2.712E+00 & 1.333E+00 & 2.351E+00 \\
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The transient control gain matrix:

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The Riccati solution matrix:

| 5.601E-01 | 1.326E+00 | 1.161E+00 | -3.128E+00 | -1.035E-01 | -2.890E+00 | -3.098E+00 | 1.092E-01 | 3.501E+00 | -1.576E+00 | 5.910E+00 | 9.762E+00 |
| 5.601E-01 | -1.579E+00 | -1.403E+00 | 3.991E+00 | 1.409E-01 | -2.890E+00 | -3.098E+00 | 1.092E-01 | 3.501E+00 | -1.576E+00 | 5.910E+00 | 9.762E+00 |

Time = 0.60000E+00

The Riccati solution matrix:

<p>| 5.601E-01 | 1.326E+00 | 1.161E+00 | -3.128E+00 | -1.035E-01 | -2.890E+00 | -3.098E+00 | 1.092E-01 | 3.501E+00 | -1.576E+00 | 5.910E+00 | 9.762E+00 |
| 5.601E-01 | -1.579E+00 | -1.403E+00 | 3.991E+00 | 1.409E-01 | -2.890E+00 | -3.098E+00 | 1.092E-01 | 3.501E+00 | -1.576E+00 | 5.910E+00 | 9.762E+00 |</p>
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**KC, The Transient Control Gain Matrix**

5.6802E-01 1.3261E+00 1.6161E+00 -3.128E-01 -1.1035E-01 -2.8909E+00 -3.897E-01 1.0828E+00 -5.0027E+00 -8.0912E-02
-6.6756E-01 -1.5798E+00 -1.4034E+00 3.919E+00 1.4092E-01 5.3502E+00 5.2156E-01 -1.3576E+00 5.9101E+03 9.7623E-02

**Time = 0.400000E+00**

5.6802E-01 1.3261E+00 1.6161E+00 -3.128E-01 -1.1035E-01 -2.8909E+00 -3.897E-01 1.0828E+00 -5.0027E+00 -8.0912E-02
-6.6756E-01 -1.5798E+00 -1.4034E+00 3.919E+00 1.4092E-01 5.3502E+00 5.2156E-01 -1.3576E+00 5.9101E+03 9.7623E-02

**KC, The Transient Control Gain Matrix**

5.6802E-01 1.3261E+00 1.6161E+00 -3.128E-01 -1.1035E-01 -2.8909E+00 -3.897E-01 1.0828E+00 -5.0027E+00 -8.0912E-02
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**Time = 0.200000E+00**

5.6802E-01 1.3261E+00 1.6161E+00 -3.128E-01 -1.1035E-01 -2.8909E+00 -3.897E-01 1.0828E+00 -5.0027E+00 -8.0912E-02
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**KC, The Transient Control Gain Matrix**

5.6802E-01 1.3261E+00 1.6161E+00 -3.128E-01 -1.1035E-01 -2.8909E+00 -3.897E-01 1.0828E+00 -5.0027E+00 -8.0913E-02
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<tr>
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<td>3.9913E-01 1.4091E+00 3.5298E+00 \ 5.2151E-01 -1.3574E+02 5.9094E+00 9.7611E-02</td>
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<td>3.9913E-01 1.4091E+00 3.5298E+00 \ 5.2151E-01 -1.3574E+02 5.9094E+00 9.7611E-02</td>
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<td>3.9913E-01 1.4091E+00 3.5298E+00 \ 5.2151E-01 -1.3574E+02 5.9094E+00 9.7611E-02</td>
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<td>3.9913E-01 1.4091E+00 3.5298E+00 \ 5.2151E-01 -1.3574E+02 5.9094E+00 9.7611E-02</td>
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### UU, CONTROL COVARIANCE MATRIX

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### XX, STATE COVARIANCE MATRIX

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### YY, OUTPUT COVARIANCE MATRIX

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### ZZ, MEASUREMENT COVARIANCE MATRIX

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### ZVEST, ESTIMATED MEASUREMENT COVARIANCE MATRIX

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APPENDIX C

TIME AND STORAGE REQUIREMENTS

The computer used for the sample LSOCE problem was the IBM 7090-7094 with DCS (Direct Couple System). This computer has $77,000_8 (32000_{10})$ core storage locations with capabilities of overlay. For the sample problem, no overlay was used. The 10th-order sample problem described in appendix B used $67,000_8$ storage and took 1.75 minutes to run.

The following table shows various subroutines and approximate time (in sec) per call for the sample problem:

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Time, sec/call</th>
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<td>ESTMAT</td>
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<tr>
<td>CONTRL</td>
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<td>RICSS</td>
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<tr>
<td>REDU</td>
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In addition, problems as large as 41st-order have been run on an IBM 360-67 using TSS (Time Sharing System) with good results. The only change was the use of double precision instead of single precision throughout the program.
REFERENCES


Figure 1. - Plant, Kalman filter, state-estimate feedback combination.
Solves linear stochastic optimal control and estimation problem

Solves estimation problem
Solves quadratic control problem
Obtains state-covariance matrix
Solves time-varying matrix Riccati equation for estimate or control

ESTMAT
Solves steady-state matrix Riccati equation

RICSS uses ARRAY
Performs reduction

Conditions matrix
CONDI
Hessenberg transformation (SSP)

HSBG
Obtains eigenvalues

EIGOR
Orders eigenvalues

ORDER
Obtains eigenvectors

EIGVCTR uses ARRAY
Factors into upper and lower triangular factors (SSP)

FACTR

Performs permutation transformation

PRMUTE

MINV

LYPMV uses ARRAY
Solves set of linear equations (SSP)

SIMQ

Performs matrix inversion (SSP)

MINV

CONVL
Solves Lyapunov equation

CDVM

TRICCS uses ARRAY
(Performs matrix inversion (SSP))

MINV

Obtains state transition matrix (block-diagonal form)

BDSTM

MINV

Figure 2. - Flow chart for linear stochastic optimal control and estimation (LSOCE) problem solution.

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