General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.

- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.

- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.

- This document is paginated as submitted by the original source.

- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

Produced by the NASA Center for Aerospace Information (CASI)
APPLICATION OF LOCAL LINEARIZATION AND THE TRANSONIC EQUIVALENCE RULE TO THE FLOW ABOUT SLENDER ANALYTIC BODIES AT MACH NUMBERS NEAR 1.0

Richard W. Tyson
Ralph J. Muraca

January 30, 1975

This informal documentation medium is used to provide accelerated or special release of technical information to selected users. The contents may not meet NASA formal editing and publication standards, may be revised, or may be incorporated in another publication.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665
Application of Local Linearization and the Transonic Equivalence Rule to the Flow about Bodies at Mach Numbers Near 1.0

Richard W. Tyson and Ralph J. Muraca

The local linearization method for axisymmetric flow is combined with the transonic equivalence rule to calculate pressure distribution on slender bodies at free-stream Mach numbers from .8 to 1.2. This is an approximate solution to the transonic flow problem which yields results applicable during the preliminary design stages of a configuration development. The method can be used to determine the aerodynamic loads on parabolic arc bodies having either circular or elliptical cross sections. The method is particularly useful in predicting pressure distributions and normal force distributions along the body at small angles of attack. The equations discussed in this paper may be extended to include wing-body combinations.
APPLICATION OF LOCAL LINEARIZATION AND
THE TRANSONIC EQUIVALENCE RULE TO THE FLOW ABOUT
SLENDER ANALYTICAL BODIES AT MACH NUMBERS NEAR 1.0

RICHARD W. TYSON
RALPH J. MURACA
SUMMARY

The local linearization method for axisymmetric flow is combined with the transonic equivalence rule to calculate pressure distributions on slender bodies at free-stream Mach numbers from .8 to 1.2. The method can be used to determine the aerodynamic loads on parabolic arc bodies having either circular or elliptical cross sections. The method is particularly useful in predicting pressure distributions and normal force distributions along the body at small angles of attack. The primary application of this method would be to establish trends for the flow variables as a function of geometry, and to evaluate configurations during the preliminary design stage. The equations discussed in this paper may be extended to include wing-body combinations.
The purpose of this paper is to describe an approximate procedure for determining the pressure distributions at free-stream Mach numbers near unity for a number of slender bodies both non-lifting and lifting. The quality of results will be demonstrated by comparison with experimental data. The analysis is based on the small disturbance theory of inviscid transonic flow, and makes use of the approximations of slender body theory, the transonic equivalence rule, and the method of local linearization.

Results are presented for several bodies having identical axial distributions of cross-sectional area and various elliptical cross sections. Angles of attack included in these comparisons range from 0° to 6°. The examples chosen were selected to enable comparison with existing data obtained in wind tunnel tests.

In addition to a description of the method and a presentation of the basic equations used in the derivation of the working equations, the computer program listings and input instructions are included as an appendix.
### SYMBOLS

- **a**: Speed of sound
- **$C_p$**: Pressure coefficient, \( \frac{P - P_\infty}{\frac{\rho_\infty}{2} u_\infty^2} \)
- **$C_N$**: Normal-force coefficient, \( F_N / qS \)
- **d**: Maximum body diameter
- **$F_N$**: Normal force
- **$k$**: \( \frac{M_\infty^2(\gamma + 1)}{U_\infty} \)
- **L**: Length of body
- **M**: Local Mach number
- **p**: Local static pressure
- **$P_\infty$**: Free stream static pressure
- **r**: Radius of body
- **S**: Body cross section area
- **$S \frac{dC_N}{dx}$**: Product of reference area and normal force coefficient derivative
- **t**: Planar body thickness
\( U_\infty \)  Free stream velocity

\( u_1, u_2, u_3 \)  Velocity components

\( u, v, w \)  Perturbation velocity components

\( x, r, \theta \)  Cylindrical coordinates where \( x \) extends in the direction of the free-stream velocity

\( \theta \)  Total cone angle, in radians

\( \alpha \)  Angle of attack

\( \gamma \)  Ratio of specific heats

\( \lambda \)  Ratio of major to minor axis length for an ellipse

\( \rho_\infty \)  Free stream density of air

\( T \)  Thickness ratio \( \ell/d \)

\( \phi \)  Perturbation velocity potential

\( \phi_x \)  Denotes partial derivative with respect to \( x \); \( \frac{\partial \phi}{\partial x} \)

\( \phi_{xx} \)  Denotes second partial derivative with respect to \( x \); \( \frac{\partial^2 \phi}{\partial x^2} \)

Subscripts

\( \infty \)  Free stream conditions

\( i, j, k \)  Dummy subscript used in index notation

\( 0 \)  Stagnation conditions
Superscript

Denotes total derivative with respect to $x$
PRESENTATION OF EQUATIONS

The equations to be presented here assume very small perturbation velocities compared to the free stream velocity \( U_\infty \). The equations of motion are then simplified by neglecting small terms in the perturbation velocities. The Method of Local Linearization described in ref. 1 is used to further simplify the equations and combines with the transonic equivalence rule to give approximate solutions to these equations in the transonic flow regime.

Small Perturbation Equations of Transonic Flow

The equations of motion for steady frictionless flow are given in ref. 2 as:

\[
\frac{du_i}{dt} + u_j \frac{\partial u_i}{\partial x_j} = a^2 \frac{\partial u_k}{\partial x_k}, \quad i, j, k = 1, 2, 3. \tag{1}
\]

The speed of sound is related to the velocity components by

\[
\frac{u_1^2 + u_2^2 + u_3^2}{2} + \frac{a^2}{\gamma - 1} = \frac{a_0^2}{\gamma - 1}, \tag{2}
\]

where \( a_0 \) denotes a reference condition.

Expanding and substituting the velocity field, defined by
\[ u_1 = U_\infty + u \]
\[ u_2 = v \]
\[ u_3 = w, \]

where \( u, v, w \) are small perturbation velocity components, into equations 1 & 2 and neglecting terms containing squares of the perturbation velocities yields the equation:

\[
(1 - M_\infty^2) \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3} = M_\infty^2 (\gamma + 1) \frac{u}{U_\infty} \frac{\partial u}{\partial x_1} + M_\infty^2 (\gamma - 1) \frac{u}{U_\infty} \left( \frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3} \right)
\]

\[
+ M_\infty^2 \frac{v}{U_\infty} \frac{\partial u}{\partial x_2} + M_\infty^2 \frac{w}{U_\infty} \left( \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_3} \right) + M_\infty \left( \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3} \right)
\]

Retaining terms on the right hand side of eq. (4) containing \( \partial u/\partial x_1 \), and dropping all other higher order terms yields the small perturbation equation valid for subsonic, transonic and supersonic flow:

\[
(1 - M_\infty^2) \frac{\partial u}{\partial x_1} + \frac{\partial v}{\partial x_2} + \frac{\partial w}{\partial x_3} = M_\infty^2 (\gamma + 1) \frac{u}{U_\infty} \frac{\partial u}{\partial x_1}
\]

We shall assume the flow to be irrotational and isentropic so that a perturbation velocity potential \( \phi \) exists, where

\[ u = \frac{\partial \phi}{\partial x_1}; v = \frac{\partial \phi}{\partial x_2}; w = \frac{\partial \phi}{\partial x_3} \]

Thus, the equation for frictionless, irrotational flow with small perturba-
Consider a body-fixed Cartesian coordinate system as shown on figure 1, where $x_1, x_2, x_3$ correspond to the body axis $x, y, z$. If the origin is located at the nose with the $x$-axis directed rearward and aligned with the longitudinal axis of the body, the pressure coefficient can be written (ref 1.);

$$C_p = -\frac{U_m}{U_{\infty}} (\phi_x + \omega \phi_z) - \frac{1}{U_{\infty}^2} (\phi_y^2 + \phi_z^2),$$  \hspace{1cm} (7)

where the subscript notation is used to denote derivatives.

The Method of Local Linearization for Axisymmetric Flow

Approximate solutions of good accuracy for axisymmetric flow with $M_\infty = 1$ past a wide class of slender pointed bodies may be obtained by application of the method of local linearization. A discussion of the development of the method and the resulting equations follow.

Equation (6) can be rewritten

$$\lambda \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$  \hspace{1cm} (8)

where

$$\lambda = 1 - M_\infty^2 - k u_\infty.$$
and

\[ k = M_\infty^2 (\gamma + 1)/U_\infty \]

If \( \lambda \) varies sufficiently slowly, it can be considered as a constant in the initial stages of the analysis. The equation is then reduced to the 3-D Laplace equation for which a solution is known. This resulting solution for the velocity component \( u \) is then improved in the following manner. By differentiating the solution for \( u \) with respect to \( x \) and replacing the previously assumed constant \( \lambda \) term appearing in the Laplace solution by its actual value a non-linear ordinary differential equation results. This differential equation may then be integrated numerically.

This method results in the following first order nonlinear ordinary differential equations in \( u/U_\infty \). For \( M_\infty < 1 \) the following equation results:

\[
\frac{d(u/U_\infty)}{dx} = \frac{S'''(x)}{4\pi} \ln \left(1 - M_\infty^2 - k u\right) + E'(x),
\]

for \( 0 < x < L \)

where

\[
E(x) = \frac{S''(x)}{4\pi} \ln \frac{S}{4\pi(x-L-x)} + \frac{1}{4\pi} \int_0^L \frac{S''(x)S''(\xi)}{|x-\xi|} d\xi.
\]

For \( M_\infty > 1 \) the following equation results:

\[
\frac{d(u/U_\infty)}{dx} = \frac{S''''(x)}{4\pi} \ln \left(M_\infty^2 - 1 + k u\right) + H'(x),
\]

where

\[
H(x) = \frac{\gamma''(x)}{4\pi} \ln \left[\frac{S}{4\pi x^2}\right] + \frac{1}{2\pi} \int_0^x \frac{S''(x)S''(\xi)}{x-\xi} d\xi.
\]
for $0 < x < \ell$.

For $M_\infty = 1$ the following equations result:

If $u < 0$ we have

$$\frac{d}{dx} \left( \frac{u}{U_\infty} \right) = \frac{S''(x)}{4 \pi} \ln (-k u) + P_E'(x), \quad (13)$$

$$P_E(x) = \frac{S''(x)}{4 \pi} \ln \frac{S(x)}{4 \pi K \ell^2} + \frac{1}{4 \pi} \int_0^x \frac{S''(x) - S''(\xi)}{x - \xi} \, d\xi. \quad (14)$$

If $u > 0$ we have

$$\frac{d}{dx} \left( \frac{u}{U_\infty} \right) = \frac{S''''(x)}{4 \pi} \ln (k u) + P_H'(x), \quad (15)$$

and

$$P_H(x) = \frac{S''(x)}{4 \pi} \ln \frac{S(x)}{4 \pi K \ell^2} + \frac{1}{2 \pi} \int_0^x \frac{S''(x) - S''(\xi)}{x - \xi} \, d\xi. \quad (16)$$

To solve these equations it is necessary to define a value for $u$ at some point along the body in order to evaluate the constant of integration. The initial value of $u$ may be defined in several different ways. If there is no experimental data available for the particular body under consideration then there are two classes of cases to be distinguished. One class includes the foreparts of bodies with a convex corner, such as a cone-cylinder, for which the necessary condition is supplied by the fact that the velocity must be sonic at the corner, that is;

$$\frac{u}{U_\infty} = \frac{1 - M_\infty^2}{M_\infty^2(\gamma + 1)} \quad (17)$$
The other includes smooth bodies for which a value for $u$ cannot be
specified a priori. The procedure to be followed in the latter case is to
find that point $(x)$ along the body which satisfies the condition $S''(x) = 0$.
The solution to Laplace's equation at this point will be input as the initial
$u$. Numerical integration must progress in both directions from this point.
The local linearization program which was developed performs this integra-
tion and provides initial conditions as required.

The local linearization program solves equations 9 through 16 for
$u/U_\infty$ using trapezoidal and the Runge-Kutta integration techniques. The
program then computes pressure coefficients using equation 7. A listing
of this program and sample input/output are included in the appendix.

The Transonic Equivalence Rule

The transonic equivalence rule first stated by Oswatitsch
(references 3 and 4) relates the flow around a slender body of arbitrary
cross section to that around an "equivalent" nonlifting body of revolution
having the same longitudinal distribution of cross-sectional area.

Following the development of reference 1, the general solution
for the flow over a body can be written

$$\Phi(x, y, z) = U_\infty (x + az) + \phi(x, y, z)$$

(18)

and the pressure coefficient is defined by equation (7).

From slender body theory:

$$\phi = \phi_2 + g(x)$$

(19)

where $\phi_2$ is the solution of the 2-D Laplace equation for the actual body
shape, and $g(x)$ is a function of the cross sectional area distribution.
Using the equivalence rule the potential can be written

\[ \phi = \phi_{2,a} + \phi_{2,t} - \phi_{2,B} + \phi_B \]  

(20)

where \( \phi_{2,a} \) is the angle of attack solution from Laplace's equation, \( \phi_{2,t} \) is the thickness solution from Laplace's equation, \( \phi_{2,B} \) is the solution to the 2-D Laplace's equation for the equivalent body, \( \phi_B \) is the equivalent body contribution.

Consider the application of this method to the case of a body having an elliptical cross section. The equivalent body will be a body of revolution (circular cross section) having an identical \( S(x) \) distribution.

The equivalent body potential solution is:

\[ \phi_{2,B} = \frac{U_\infty}{2\pi} S' \ln r + 2U_\infty \alpha r \sin \theta, \]  

(21)

and

\[ g'(x) = -\frac{U_\infty}{2} \left[ (c_p)_B \frac{S''}{2\pi} \ln \left( \frac{S}{\pi} \right) + \frac{S'^2}{4\pi S} \right] \]  

(22)

The thickness solution is obtained from the potential solution for the case of an expanding ellipse. Physically it can be interpreted as the effect of passing a body through a plane of fluid normal to the path of motion.

From reference 5 the solution for a uniformly expanding circle in terms of a complex potential, \( W \), is:

\[ W = r V_r \log \zeta, \]  

(23)

Where \( V_r \) represents the velocity at which the radius of the circle is growing, and \( \zeta \) is the complex variable in the transformed plane. The
relationship between $\xi$ and $\sigma$, the complex variable in the physical plane is

$$\xi = \frac{1}{2} \left[ \sigma + (\sigma^2 - a^2 + b^2)^{1/2} \right]$$  \hspace{1cm} (24)$$

where $\xi = \gamma + i\zeta$ and $a$ and $b$ are the length of the major and minor axis of the ellipse. The quantity $r_o \nu_r$ can be written

$$r_o \frac{dr}{dt} = r \frac{dr}{dx} \frac{dx}{dt} = r \nu_r U_\infty$$ \hspace{1cm} (25)$$

but, $S' = 2\pi r \nu_r$

therefore

$$r \nu_r = \frac{S'U_\infty}{2\pi}.$$ \hspace{1cm} (26)$$

The relationship between $r$, and $a$ and $b$, is $r = \sqrt{ab}$. Now $W(\xi)$ is the complex potential defined by

$$W(\xi) = \phi + i\psi$$

where $\phi$ is the perturbation potential in the equation for $C_p$.

Substituting equations 24 and 26 and into equation 23 yields the thickness solution for an ellipse

$$\phi_{2,t} = R.P. \left[ \frac{U_\infty S'}{2\pi} \ln \frac{\sigma + (\sigma^2 - a^2 + b^2)^{1/2}}{2} \right]$$ \hspace{1cm} (28)$$

where R.P. denotes the real part of the potential function.

Similar analysis of flow over a cylinder (ref. 6) yields the expression for angle of attack effects:
\[ \phi_{2,t} = \text{R.P.} \left[ \frac{iU_0 a}{2 \pi} \left[ -\sigma - \left( \sigma^2 - a^2 + b^2 \right)^{1/2} \right] + \frac{\left( a^2 + b^2 \right)^{1/2}}{\sigma = \left( \sigma^2 - a^2 + b^2 \right)^{1/2}} \right] \] (29)

The constants \( a \) and \( b \) in eq. 28 may be related to the cross sectional area of the ellipse, \( S = \pi ab \) and \( \lambda = \frac{a}{b} \).

Differentiating eq (28) with respect to \( x \) yields:

\[ \frac{\partial \phi_{2,t}}{\partial x} = \text{R.P.} \frac{U_0}{2\pi} \frac{2}{\partial x} \left[ S' \ln \left( \frac{\left( \sigma^2 - a^2 + b^2 \right)^{1/2}}{2} \right) \right] \] (30)

Substituting for \( \sigma \), where \( \sigma = re^{i\theta} \) yields,

\[ \frac{\partial \phi_{2,t}}{\partial x} = \text{R.P.} \frac{U_0}{2\pi} \frac{2}{\partial x} \left\{ S'' \ln \left[ \frac{r}{r} \left( e^{i\theta} + (e^{i2\theta} - \lambda + \frac{1}{\lambda})^{1/2} \right) \right] \right. \\
+ \left. S' \left[ \frac{r'}{r} + \ln \left( \frac{e^{i\theta} + e^{i2\theta} - \lambda + \frac{1}{\lambda}}{2} \right) \right] \right\} \] (31)

Differentiating \( \phi_{2,t} \) with respect to \( y \) & \( z \) yields,

\[ \frac{\partial \phi_{2,t}}{\partial y} = \text{R.P.} \frac{U_0}{2\pi} S' \left[ 1 + \frac{re^{i\theta}}{(r^2e^{i2\theta} - a^2 + b^2)^{1/2}} \right] \frac{1}{re^{i\theta} + (r^2e^{i2\theta} - a^2 + b^2)^{1/2}} \] (32)

and

\[ \frac{\partial \phi_{2,t}}{\partial z} = \text{R.P.} \frac{iU_0}{2\pi} S' \left[ 1 - \frac{re^{i\theta}}{(r^2e^{i2\theta} - a^2 + b^2)^{1/2}} \right] \frac{1}{re^{i\theta} + (r^2e^{i2\theta} - a^2 + b^2)^{1/2}} \] (33)
In the angle of attack solution for an ellipse, \( r = \frac{a + b}{2} \), and \( \lambda = \frac{a}{b} \), form the two relationships between \( a \), \( b \) and \( r \).

Differentiating eq 29 with respect to \( x \) yields:

\[
\frac{\partial \phi_2 a}{\partial x} = R.P. \left\{ \frac{iU_\infty a}{2} r' \left[ -e^{i\theta} - \left( e^{i\theta} + \frac{4(1-\lambda)}{1 + \lambda} \right)^{1/2} \right] + \frac{4r}{\sqrt{e^{i\theta} + \left( e^{i\theta} + \frac{4(1-\lambda)}{1 + \lambda} \right)^{1/2}}} \right\} 
\]

Differentiating eq 29 with respect to \( y \) and \( z \) yields:

\[
\frac{\partial \phi_2 a}{\partial y} = R.P. \left\{ \frac{iU_\infty a}{2} \left\{ -1 - \frac{re^{i\theta}}{(r^2 e^{i\theta} a^2 + b^2)^{1/2}} \right] \right\} 
\]

and

\[
\frac{\partial \phi_2 a}{\partial z} = R.P. \left\{ \frac{iU_\infty a}{2} \left\{ -1 - \frac{ire^{i\theta}}{(r^2 e^{i\theta} a^2 + b^2)^{1/2}} \right] \right\} 
\]
With the derivatives of \( \phi \) defined all of the terms on the right hand side of equation 7 can be evaluated except the pressure coefficient distribution on the equivalent body \( C_{p_B} \). This data must be input from either experiment or from the local linearization program previously discussed. Evaluation of equation 7 is performed in the Transonic Aerodynamics Program. These results may then be integrated to obtain values for lift, drag, and pitching moments on slender bodies. Since the aerodynamic loading, lift and all lateral forces and moments may be expressed in terms of differences in pressure between points at the same longitudinal station, these quantities depend solely on \( \phi_2 \). A listing of the Transonic Aerodynamics Program with sample input/output appears in the Appendix.

APPLICATIONS

The bodies chosen for analysis fall into three categories, bodies of revolution, elliptical bodies, and planar bodies. All were previously tested in wind tunnels and experimental data for comparison were readily available. Data for four models tested in the Ames 14-Ft Transonic Wind Tunnel at Mach numbers from .8 to 1.2 are presented in reference 7. Configurations were analyzed and comparisons with experimental data are shown in figure 2 through figure 6.
The body chosen initially is a body of revolution with axial location of maximum cross-sectional area at $x/l = .5$, and a fineness ratio of 12. The radius of the body is given by

$$r(x, \theta) = \frac{\ln r_{\text{MAX}} (x - x^2)}{\sqrt{\cos^2 \theta + \lambda^2 \sin^2 \theta}},$$

(37)

where $r_{\text{MAX}} = 3.0$.

Figures 2a and 2b show the pressure coefficients computed by the local linearization program, and experimental data from reference 7. The data shown on Figure 2a at $M_\infty = 1.0$ are in good agreement up to the location $x/l = 0.7$, where a shock occurs. The results shown on Figure 2b at $M_\infty = 1.2$ are in good agreement over the entire body. Both cases were run with initial conditions obtained from experimental data. Figure 3 shows the differences which result when the initial condition is derived from experimental data and when it is calculated using the solution at the point where $S''(x) = 0$. The major difference between the results obtained using a computed $u_o$ versus an experimentally determined value is a shifting of the curve along the $C_p$ axis.

The second body chosen for analysis was a parabolic arc body of elliptical cross-section. The body radius is given by equation 37 above.
The Transonic Aerodynamics Program is used to calculate angle of attack variation over this body with $\lambda = 1$. Experimental data was used for the equivalent body at zero angle of attack. Comparisons with wind tunnel results are shown in figures 4a and 4b with Mach number equal to 1.0 and 1.2. The pressure coefficient, $C_p$, is plotted versus $x/l$ at an angle of attack of 6°.

The agreement between computed and measured results is not good for the $M_\infty = 1.0$ case. The general trend of the data is correct, however, the analysis predicts lower value of $C_p$ over most of the body. At $M_\infty = 1.2$ the agreement is better although the analysis continues to underpredict $C_p$.

Figure 5a shows results for a configuration with $\lambda = 3$ at Mach number equal to 1.0. The equivalent body pressure coefficient is again obtained from experimental data. The same configuration is shown in figure 5b at 6° angle of attack. This analysis was done using the local linearization program to predict the equivalent body pressure coefficient used in the Transonic Aerodynamic Program.

For the case at $\alpha = 0$ the agreement between computed and measured data is good. The analysis predicts slightly lower values of $C_p$ over the forward portion of the body. Since measured data was used to obtain $C_{pE}$ the sudden increase in $C_p$ on the rear portion of the body caused by a shock is reflected in the computed results.

For the case at $\alpha = 6^\circ$ the agreement between computed and measured data is not as good. The data at $\theta = -90^\circ$ shows better agreement than that at $\theta = +90^\circ$. The general trend in $C_p$ is predicted but the computed values
are consistently lower.

The final body chosen for comparison is the planar body shown in figure 6a. The elliptic cone has a major to minor axis ratio of 16.5 and is compared to the model tested in reference 1. Shown on figure 6a are the computed data for a circular and an elliptical cone with identical cross-sectional area distribution. Also shown is experimental data for the elliptic cone. As can be seen the agreement is quite good over the forward portion of the body; however, as the base of the cone is approached, the calculated pressures are higher than measured. This is probably due to wall interference effects as cited in reference 1.

Results for the elliptic cone at $\alpha = 4\degree$ are presented on figure 6b. The pressure coefficient distribution along two meridonal locations on the body are shown. The data at $\theta = +90\degree$ represents the windward meridian and the $\theta = -90\degree$ the leeward meridian. In both instances the theory predicts higher pressures than measured. The reason for this could be the same as cited for the $\alpha = 0\degree$ results.

CONCLUSION

The approximate method presented herein represents a useful tool for establishing the variations of pressure and velocity along the surface of analytic bodies. The method will appropriately reflect the effects of geometry changes on the local pressure coefficients. The computer solutions require minimal input and short computation times consequently the method is well suited to the analysis of configurations during the preliminary design stages.
In this context the quality of agreement between computed and measured results is acceptable. The procedures by which these solutions have been obtained are not restricted to the particular examples selected for display in this paper, but possess much greater generality.
Figure 2(a). - Comparison of pressure coefficients on a parabolic arc body at $M_\infty = 1.0$. $S_{max}$ at $x/l = .50$
Figure 2(b).- Comparison of pressure coefficients on a parabolic arc body at $M_\infty = 1.2$, $S_{ax}$ at $x'/\xi = 0.50$. 
Figure 3.- Effect of initial conditions on the pressure coefficients on a parabolic arc body.
Figure 4(b). - Comparison of pressure coefficients on a parabolic arc body at $\alpha = 60^\circ$ and $M_\infty = 1.2$. 
Figure 5(a).- Comparison of pressure coefficients on a parabolic arc body with elliptical cross section at $\alpha = 0^\circ$ and $M_\infty = 1.0$. 

\[ \lambda = 3.0, \; \alpha = \pm 90^\circ \]
Figure 5(b).- Comparison of pressure coefficients on a parabolic arc body with elliptical cross section at \( \alpha = 6^\circ \) and \( M_\infty = 1.0 \).
Figure 6(a). - Comparison of pressure coefficients on a planar body at $\alpha = 0^\circ$ and $M_\infty = 1.0$.
Figure 6(b).- Comparison of pressure coefficients on a planar body at $\alpha = 4^\circ$ and $M_\infty = 1.0$. 

$\lambda = 16.5$  $\alpha = 4^\circ$
REFERENCES


APPENDIX

Local Linearization Program

Computer Program Input/Output - All input data are input according to the NAMELIST type format. In columns 2 through 7 the name $ DATA is punched. Column 8 is left blank. Beginning in column 9 the variable names which are shown below and their values are punched. Each variable must be separated by a comma and the value of the final variable must be followed by a $ sign. If optional variables are not known, they need not be punched. The variables may be punched out to column 80 in any order.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>XO</td>
<td>Initial value for X along body</td>
</tr>
<tr>
<td>CP</td>
<td>Pressure coefficient at XO</td>
</tr>
<tr>
<td>A, B, C, D</td>
<td>Geometry Variables</td>
</tr>
<tr>
<td></td>
<td>Appearing in body defining</td>
</tr>
<tr>
<td>C</td>
<td>expression $ R = A(BX + CX^D)$</td>
</tr>
<tr>
<td>G</td>
<td>Ratio of specific heats</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
</tbody>
</table>

EXAMPLE:

$ DATA M = 1.0, XO = .48, CP = 0., A = .071, B = 1, 
C = -1, D = 6.03, G = 1.4, $ 

If more than one case is to be run, then the input data for the second case may be placed immediately after the data for the first.
The output consists of a listing of the input parameters, Mach number, Initial station and corresponding velocity, and ratio of specific heats for the medium. In addition, the equation defining the body shape is shown. The variation of velocity \( v \), pressure coefficient \( C_P \), and body radius \( R \), are also shown versus the longitudinal coordinate \( X \).
LOCAL LINEARIZATION PROGRAM

PROGRAM TAP 1 (INPUT,OUTPUT,TAPES=INPUT,PUNCH)
THIS PROGRAM DETERMINES TRANSONIC AERODYNAMIC CHARACTERISTICS ON
BODIES OF REVOLUTION USING THE
METHOD OF LOCAL LINEARIZATION.
NAMELIST /DATA/M, X0, U0, A, B, C, D, G, CP
REAL M
DOUBLE VAR(2), CVAR(2)
DIMENSION U(52), X(52), CVAR(2)
COMMON /DLK/CVAR, VAR, UER, M, X0, U0
COMMON /FC/ A, B, C, D, G, K, P
PI = 3.14159265
1 X0 = 100.
READ(S, DATA)
IF (EOF(S), 2, 3)
2 STOP
3 IF (X0 NE 100.) PRINT 4
IF (X0 EQ 100.) PRINT 5
4 FORMAT(1H19, X0 IS INPUT TO THIS PROGRAM, U0 IS CALCULATED FROM C
1P INPUT*)
5 FORMAT(1H19, X0 AND U0 ARE CALCULATED BY THIS PROGRAM*)
IF (X0 EQ 100.) GO TO 10
U0 = (-CP - R1(X0)**2)/2.
GO TO 11
CALCULATE X0 AND U0 IF NECESSARY
10 CALL XUO(X0, U0, M)
11 PRINT 6, M, X0, U0, G, A, B, C
6 FORMAT(1H MACH NUMBER = *E16.8, X0 = *E16.8/
1, 1H GAMMA = **E16.8//
2, 1H *R(X) = *F5.2.4H * (F5.2.1H X *(F5.2.1H 1-X) * (F5.2.1H (1-
3-X)**(F5.2.1H))
CALCULATE X AND U VALUES ALONG THE BODY
CALL XU(U, X, N)
PRINT OUT X, R, R1, S, S1, S2, U, CP
PRINT 7
8 PRINT 8, I = 1, N
U = X(I) S RD = R(E) S R1D = R1(E) CPD = CP(U(I), E)
SD = PI*K(E)**2
S1D = 2.*PI*K(E)*R1(E)
SD = 2.*PI*K(R(E)*R2(E) + R1(E)**2)
8 PRINT 9, E = U(I), CPD, RD, R1D, SD, S1D, S2D
9 FORMAT(1H, 8E16.8)
PLT S1, S2 VS. X AND U VS. X AND CP VS. X
CALL PLOT(X, U, N)

ORIGINAL PAGE IS
OF POOR QUALITY
SUBROUTINE XU0(X0,U0,M)
REAL M,K
DIMENSION XI(52), YI(52)
COMMON /FC/ A,B,C,D,G,K,PI
EPS=1.E-15

CALCULATE X0
 X0=X SUCH THAT S2(X)=0
 X0 LIES BETWEEN .02 AND .98

BISECTION

X=0.
S2X=0.
1 S2P=S2X
   X=X+.01
   S2X =2.*PI*(R(X)*R1(X)**2)
   1F(ABS(S2X).LT.EPS)GO TO 6
   IF(S2X*S2P.GE.0.) GO TO 1
   X1=X-.010
   X2=X-.005
   X3=X
   2 S2X1 =2.*PI*(R(X1)*R2(X1) + R1(X1)**2)
   S2X2 =2.*PI*(R(X2)*R2(X2) + R1(X2)**2)
   1F(ABS(S2X1).LT.EPS)GO TO 6
   IF(S2X2.SLT.0.) GO TO 4
   IF(S2X1 .LT.O.) GO TO 5
   X1=X2
   X2=(X3+X2)/2.
   GO TO 2
   4 IF(S2 X1 .LT.Oo)GO TO 3
   5 X3=X2
   X2=(X1+X2)/2.
   GO TO 2
   6 XU=X2
   IF(X0.6E.02.AND.X06LE.9.98)GO TO 8
   PRINT 7,X0
   7 FOKMAT(1H 923H** • XO OUT OF RANGE **/1.1H 9*X0 = +E10.
   STOP

CALCULATE U0

8 IF(M.GE.1.0)GO TO 13
   DO 9 I=1,51
   XI(I)=(I-1.)*.01
   IF(X0.LE.XI(I)-.01.0R.X0.GT.XI(I)+.01) GO TO 10
   XI(I)=X0
   9 XI(I)=XI(I)•[2.]*PI*(R(X0)*R3(X0) + 3.*R1(X0)*R2(X0))
   GO TO 9
   10 XI(I)=2.*PI*(R(X1(I))*R2(X1(I)) + R1(X1(I)**2)/ABS(X0-X1(I))
   Y CONTINUE
   CALL TRAP(YI,XI,N,U0)
   GO TO 16
13 XI(I)=0.
   DO 14 I=2,51
14 XI(I)=XI(I-1)+X0/50.
   DO 15 I=1,50
15 XI(51)=2.*PI*(R(X1(I))*R2(X1(I)) + R1(X1(I)**2)/(X0-X1(I))
    YI(51)=-2.*PI*(R(X0)*R3(X0) + 3.*R1(X0)*R2(X0))
   N=51
   CALL TRAP(YI,XI,N,U0)
16 U0=U0/(4.*3.14159265)
SUBROUTINE XU(U, X, N)
EXTERNAL DERSUB, CHSUB
REAL M, K
DOUBLE PRECISION CUVAR(2), VAR(2)
DIMENSION UER(2), ELE1(2), ELE2(2), X(52), U(52)
COMMON /DER/ CUVAR, VAR, DER, M, X(0:0)
COMMON /FC/ A, B, C, D, G, K, P
ELE1(1) = 0.01
ELE2(1) = 0.01
K = M * 2 * (1.0 * G)

INTEGRATE FORWARD

I = 0
II = 0
VAR(1) = DBLE(X0)
VAR(2) = DBLE(U0)
C1 = 0.015625
1 I = I + 1
CALL INT1(I, 1.0, C1, 0.02, 0.0, IERR, VAR, CUVAR, DER, ELE1, ELE2, ELT, ERRVAL)
1, DERSUB, CHSUB, 0)
X(1) = SNGL(VAR(1))
U(1) = SNGL(VAR(2))
IF(UABS(VAR(1) - 0.98).GT.0.0001) GO TO 1
DO 2 J = 1, I
JJ = J - 1
X(50 - JJ) = X(I - JJ)
2 U(50 - JJ) = U(I - JJ)

INTEGRATE BACKWARD

I = 50 - I + 2
II = 0
VAR(1) = DBLE(X0)
VAR(2) = DBLE(U0)
C1 = -0.015625
3 I = I - 1
CALL INT1(I, 1.0, C1, 0.02, 0.0, IERR, VAR, CUVAR, DER, ELE1, ELE2, ELT, ERRVAL)
3, DERSUB, CHSUB, 0)
X(1) = SNGL(VAR(1))
U(1) = SNGL(VAR(2))
IF(UABS(VAR(1) - 0.02).GT.0.0001) GO TO 3
N = 50
IF(N .EQ. 1) GO TO 7
KK = 0
DO 6 J = 1, 50
KK = KK + 1
X(KK) = X(J)
6 U(KK) = U(J)
N = KK
7 CONTINUE
RETURN
END
SUBROUTINE DERSUB
REAL M, K
DOUBLE PRECISION CUVAR(2), VAR(2)
DIMENSION DER(2), X(52), Y1(52)
COMMON /DER/ CUVAR, VAR, DER, M, X(0:0)
COMMON /FC/ A, B, C, D, G, K, P
N = 51
X = SNGL(CUVAR(1))
U=SNGL(CUVR(2))
S = PI*RS2(X)**2
S1 = 2.*PI*RS2(X)*R1(X)
S2 = 2.*PI*(R(X)**2)*R2(X) + R1(X)**2)
S3 = 2.*PI*(R(X)**2)*R3(X) + 3.*R1(X)*R2(X))
S4=2.*PI*(R(X)**2)*R4(X) + 4.*R1(X)*R3(X) + 3.*R2(X)**2)
IF(M-1.) 21,7
1 IF(U.GT.0.) GO TO 7

C M EQUAL TO 1 AND U LESS THAN 1

C CALCULATE INTEGRAL FOR USE IN DERIVATIVE EXPRESSION
C DELTA=X/50.
C EXTEND LIMITS OF INTEGRATION FOR U .LT.0
IF(A*GT.XO) GO TO 11
DELTA=X/50.
GO TO 13
11 DELTA=X/50.
13 CONTINUE
DO 4 I=1,51
XI(I)=(I-1.)*DELTA
IF(X.GT.XI(I)-.01.*AND.X.LT.XI(I)+.01) GO TO 5
S2I=2.*PI*(R(XI(I))**2)*R2(XI(I)) + R1(XI(I))**2)
YI(I)=(X-XI(I))**2*S3 = S2 + S2I/(X-XI(I))**2)
GO TO 4
5 YI(I)=S4
XI(I)=X
4 CONTINUE
CALL THAP(YI,XI,N,SUM)
C CALCULATE DERIVATIVE
DEK(2)=(S3*ALOG(-K*U) + S3 + S3*ALOG((S/(4.*PI*X*(1.-X)))) + S2*S1/S
1 - S2*(1.-2.*X)/(X*(1.-X)) + SUM)/(4.*PI)
RETURN

C M LESS THAN 1

C CALCULATE INTEGRAL FOR USE IN DERIVATIVE EXPRESSION
2 DO 3 I=1,51
XI(I)=(I-1.)*DELTA
IF(X .LE.XI(I)-.01.0R.X .GE.XI(I)+.01) GO TO 6
X1(I)=X
Y1(I)=S4
GO TO 3
6 S2I= 2.*PI*(R(XI(I))**2)*R2(XI(I)) + R1(XI(I))**2)
Y1(I)=(ABS(X-XI(I))**2*S3 = S2 + S2I/(X-XI(I))**2)
Y1(I)=(X-XI(I))**2*S3 = S2 + S2I/(X-XI(I))**2)
3 CONTINUE
CALL THAP(YI,XI,N,SUM)
C CALCULATE DERIVATIVE
AIA=-M**2 - K*U
IF(AIA.LT.0.) PRINT 20+AIA
20 FORMAT(1H *ABA = *E16.8)
IF(AIA.LT.0.) ABA=-ABA
DEK(2)=(S3*ALOG(ABA) + S3*ALOG((S/(4.*PI*X*(1.-X)))) + S2*S1/S
1 - S2*(1.-2.*X)/(X*(1.-X)) + SUM)/(4.*PI)
1 - S2*(1.-2.*X)/(X*(1.-X)) + SUM)/(4.*PI)
Q1=S3*ALOG(ABA)
Q2=S3*ALOG((S/(4.*PI*X*(1.-X))))
Q3=S2*S1/S
Q4=S2*(1.-2.*X)/(X*(1.-X))
PRINT 129+X,Q1,Q2,Q3,Q4,SUM
129 FORMAT(1H *E16.8)
RETURN
M GREATER THAN 1
OR M EQUAL TO 1 AND U GREATER THAN OR EQUAL TO 0

CALCULATE INTEGRAL FOR USR IN DERIVATIVE EXPRESSION

7 XI(1)=0.
DELTA=X/50.
DO 8 I=2,N
8 XI(I)=XI(I-1) + DELTA
DO 9 I=1,N
9 S2I= 2.*PI*(R(XI(I))*R2(XI(I)) + R1(XI(I))**2)
Y1(I)) = ((X-XI(I))*S3 - S2 * S2I)/(X-XI(I))**2
Y1(S1)=S4
CALL TRAP(Y1*X1,N,SUM)

CALCULATE DERIVATIVE
ABA=M**2 - 1. + K*U
IF(ABA.LT.0.) PRINT 20,ABA
IF(ABA.LT.0.) ABA=-ABA
DER(2)=S3 *ALOG( ABA )/(4.*PI) +
1 S3 *ALOG(S /((4.*PI*X**2)))/(4.*PI) +
2 S2 *(X*S1 -2.*S1 )/(4.*PI*X**2) +
3 S3 /(2.*PI) +
4 SUM/(2.*PI)
RETURN
END

SUBROUTINE TRAP(Y,X,N,SUM)
DIMENSION X(N),Y(N)
SUM=0.
DO 1 I=1,N
1 SUM=SUM*(Y(I)+Y(I-1))*(X(I)-X(I-1))/2.
RETURN
END

FUNCTION H(X)
COMMON /FC/ A,B,C,D,G,K,P1
EXPONENT CHECK
IF(U.NE.0.) GO TO 1
R =A*(B*(1.-X) + C)
GO TO 2
1 R=A*(B*X+C*X*X*D)
2 CONTINUE
RETURN
END

FUNCTION R1(X)
COMMON /FC/ A,B,C,D,G,K,P1
EXPONENT CHECK
IF(U.NE.0. AND D.NE.0.) GO TO 1
R1=-A*B - A*C*D
GO TO 2
1 R1=-A*B-A*C*D*(1.-X)**(D-1.)
2 CONTINUE
RETURN
END

FUNCTION R2(X)
COMMON /FC/ A,B,C,D,G,K,P1
EXPONENT CHECK
IF(U.NE.0. AND D.NE.0. AND D.NE.2.) GO TO 1
R2=A*C*D*(U-1.)
GO TO 2
1 R2=A*C*D*(U-1.)*A**(D-2.)
2 CONTINUE

ORIGINAL PAGE IS OF POOR QUALITY
FUNCTION R3(X)
COMMON /FC/ A*B*C*D+G*K+P
C EXPONENT CHECK
IF (D*NE.0..AND.*D*NE.1..AND.*D*NE.2..AND.*D*NE.3..) GO TO 1
R3=A*C*0*(D-1.)*(U-2.)
GO TO 2
1 R3=A*C*0*(D-1.)*(D-2.)*(1.-X)*(D-3.)
2 CONTINUE
RETURN
END
FUNCTION R4(X)
COMMON /FC/ A*B*C*D+G*K+P
C EXPONENT CHECK
IF (D*NE.0..AND.*D*NE.1..AND.*D*NE.2..AND.*D*NE.3..AND.*D*NE.4..) GO TO 11
R4=A*C*0*(D-1.)*(D-2.)*(D-3.)
GO TO 2
11 R4=A*C*0*(D-1.)*(D-2.)*(D-3.)*(1.-X)*(D-4.)
11 CONTINUE
RETURN
END
FUNCTION CP(U,X)
COMMON /FC/ A*B*C*D+G*K+P
CP=2.*U-R1(X)**2
RETURN
END
SUBROUTINE CHSUB
RETURN
END
SUBROUTINE PLOT(X+U+N)
DIMENSION X(N),U(N)
RETURN
END
DATA M=1.0,X0=.489,CP=0.5,A=.07125,B=1.0,U=6.039,G=1.49
DATA M=1.2,X0=.569,CP=0.5,A=.07125,B=1.0,U=6.039,G=1.49
**X0 IS INPUT TO THIS PROGRAM. L0 IS CALCULATED FROM CP INPUT**

**MACH NUMBER = 1.30**

**X0 = -0.37073353E-01**

**L0 = -1.8324112**

**GAMMA = 1.480338E+00**

**R(X) = 0.77 * ( 1.00 * ( 1 - X ) + 1.75 * ( 1 - X )** 2 **6.03)**

**OUTPUT**

<table>
<thead>
<tr>
<th>X</th>
<th>U</th>
<th>CP</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0000000E-01</td>
<td>-3.02076530E-02</td>
<td>5.53387436E-02</td>
<td>1.42500000E-03</td>
</tr>
<tr>
<td>4.0000000E-01</td>
<td>-2.84295944E-02</td>
<td>5.17836933E-02</td>
<td>2.84999974E-03</td>
</tr>
<tr>
<td>6.0000000E-01</td>
<td>-2.73621334E-02</td>
<td>4.96877430E-02</td>
<td>4.27499794E-03</td>
</tr>
<tr>
<td>8.0000000E-01</td>
<td>-2.66379709E-02</td>
<td>4.81568597E-02</td>
<td>5.65952854E-03</td>
</tr>
<tr>
<td>1.0000000E-01</td>
<td>-2.60582201E-02</td>
<td>4.70406688E-02</td>
<td>7.12493346E-03</td>
</tr>
<tr>
<td>1.2000000E-01</td>
<td>-2.55515251E-02</td>
<td>4.67869704E-02</td>
<td>8.54970368E-03</td>
</tr>
<tr>
<td>1.4000000E-01</td>
<td>-2.51724832E-02</td>
<td>4.65270376E-02</td>
<td>9.97494255E-03</td>
</tr>
<tr>
<td>1.6000000E-01</td>
<td>-2.48337476E-02</td>
<td>4.62939876E-02</td>
<td>1.15988684E-02</td>
</tr>
<tr>
<td>1.8000000E-01</td>
<td>-2.45132474E-02</td>
<td>4.60566169E-02</td>
<td>1.22938913E-02</td>
</tr>
<tr>
<td>2.0000000E-01</td>
<td>-2.42010811E-02</td>
<td>4.58342506E-02</td>
<td>1.31346546E-02</td>
</tr>
<tr>
<td>2.2000000E-01</td>
<td>-2.37236353E-02</td>
<td>4.56181133E-02</td>
<td>1.40478746E-02</td>
</tr>
<tr>
<td>2.4000000E-01</td>
<td>-2.32361180E-02</td>
<td>4.54047200E-02</td>
<td>1.49025183E-02</td>
</tr>
<tr>
<td>2.6000000E-01</td>
<td>-2.28622846E-02</td>
<td>4.52013469E-02</td>
<td>1.57345956E-02</td>
</tr>
<tr>
<td>2.8000000E-01</td>
<td>-2.24987192E-02</td>
<td>4.49455951E-02</td>
<td>1.65393570E-02</td>
</tr>
<tr>
<td>3.0000000E-01</td>
<td>-2.21562725E-02</td>
<td>4.46905150E-02</td>
<td>1.72530304E-02</td>
</tr>
<tr>
<td>3.2000000E-01</td>
<td>-2.18347902E-02</td>
<td>4.44332062E-02</td>
<td>1.79344865E-02</td>
</tr>
<tr>
<td>3.4000000E-01</td>
<td>-2.15340332E-02</td>
<td>4.41731275E-02</td>
<td>1.85666086E-02</td>
</tr>
<tr>
<td>3.6000000E-01</td>
<td>-2.12551908E-02</td>
<td>4.39078975E-02</td>
<td>1.91116518E-02</td>
</tr>
<tr>
<td>3.8000000E-01</td>
<td>-2.09875592E-02</td>
<td>4.36365351E-02</td>
<td>1.95866135E-02</td>
</tr>
<tr>
<td>4.0000000E-01</td>
<td>-2.07290250E-02</td>
<td>4.33670710E-02</td>
<td>2.00045024E-02</td>
</tr>
<tr>
<td>4.2000000E-01</td>
<td>-2.04776213E-02</td>
<td>4.30992537E-02</td>
<td>2.03717623E-02</td>
</tr>
<tr>
<td>4.4000000E-01</td>
<td>-2.02252551E-02</td>
<td>4.28307376E-02</td>
<td>2.07008776E-02</td>
</tr>
<tr>
<td>4.6000000E-01</td>
<td>-2.19607287E-02</td>
<td>4.25594005E-02</td>
<td>2.09951485E-02</td>
</tr>
<tr>
<td>4.8000000E-01</td>
<td>-2.17952707E-02</td>
<td>4.22932772E-02</td>
<td>2.12568125E-02</td>
</tr>
<tr>
<td>5.0000000E-01</td>
<td>-2.16293260E-02</td>
<td>4.20283197E-02</td>
<td>2.14866687E-02</td>
</tr>
<tr>
<td>5.2000000E-01</td>
<td>-2.14626953E-02</td>
<td>4.17621783E-02</td>
<td>2.17040908E-02</td>
</tr>
<tr>
<td>5.4000000E-01</td>
<td>-2.12943820E-02</td>
<td>4.14933872E-02</td>
<td>2.19102698E-02</td>
</tr>
<tr>
<td>5.6000000E-01</td>
<td>-2.11244273E-02</td>
<td>4.12203028E-02</td>
<td>2.21062839E-02</td>
</tr>
<tr>
<td>5.8000000E-01</td>
<td>-2.09518346E-02</td>
<td>4.09394550E-02</td>
<td>2.22926930E-02</td>
</tr>
<tr>
<td>6.0000000E-01</td>
<td>-2.07764781E-02</td>
<td>4.06587190E-02</td>
<td>2.24732133E-02</td>
</tr>
<tr>
<td>6.2000000E-01</td>
<td>-2.05974202E-02</td>
<td>4.03764992E-02</td>
<td>2.26482768E-02</td>
</tr>
<tr>
<td>6.4000000E-01</td>
<td>-2.04124316E-02</td>
<td>4.00925004E-02</td>
<td>2.28166332E-02</td>
</tr>
<tr>
<td>6.6000000E-01</td>
<td>-2.02224704E-02</td>
<td>4.07664953E-02</td>
<td>2.29772569E-02</td>
</tr>
<tr>
<td>6.8000000E-01</td>
<td>-2.00249103E-02</td>
<td>4.04379371E-02</td>
<td>2.31309417E-02</td>
</tr>
</tbody>
</table>

**ORIGINAL PAGE IS OF POOR QUALITY**
Computer Program Input/Output - All input data are input according to
NAMELIST type format. In columns 2 through 7 the name and INPUT is punched.
Column 8 is left blank. Beginning in column 9 the variable names which
are shown below and their values are punched. Each variable must be
separated by a comma and the value of the final variable must be followed
by a $ sign. The variables may be punched out to column 80 in any order.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>MACH</td>
<td>Mach number</td>
</tr>
<tr>
<td>CPB(1)</td>
<td>The array of pressure coefficients for the equivalent body of revolution. These values are required from $X = 0.05$ to $X = 0.95$ in increments of .05. Dimension of the array should be 17.</td>
</tr>
<tr>
<td>ITRIG = 0</td>
<td>If $\lambda \neq 1$, elliptical cross section.</td>
</tr>
<tr>
<td>= 1</td>
<td>If $\lambda = 1$ and body of revolution to be run at $\alpha \neq 0$.</td>
</tr>
<tr>
<td>LAM</td>
<td>Ratio of major to minor axis of ellipse $\lambda = a/b$.</td>
</tr>
<tr>
<td>ALPHA</td>
<td>Angle of attack in radians.</td>
</tr>
<tr>
<td>THET(1)</td>
<td>Values of $\theta$ around body at which pressure coefficients are desired in degrees.</td>
</tr>
<tr>
<td>NTHETA</td>
<td>Dimension of THET(1) array.</td>
</tr>
</tbody>
</table>

EXAMPLE:

```
$INPUT MACH = 1., CPB(1) = .15, .1, .05, .02, -.01, 
-.04, -.06, -.085, -.095, -.1, -.1, -.1, -.1, -.01, 
.05, .01, ITRIG = 0, LAM = 3, ALPH' = 1.047198, 
THET(1) = -90., -40., -0., 40., 90., NTHETA = 5, $
```

If more than one case is to be run, then the input data for the second case may be placed immediately after the data for the first.
All parameters used in this program are complex and in the form $A + iB$. The real or imaginary parts or both may be zero. This reserves space for the values when they are not zero. The built-in functions for handling floating point are integer computations which have a complex counterpart.
Output

The output consists of a listing of the input parameters, \( X \), \( \theta \), and the real and imaginary parts of \( C_p \).
PROGRAM CCP (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

REAL LAM, MACH, LAM1, LAM2, LAM3, LAM4, LAM+/LAM弱点


DIMENSION CPB(25), THET(25), SI(J(25), COST(25), .1, .1)

EQUALITY (W(1) + CP)

NAMELIST INPUT/MACH, CPB, LAM1, LAM+, THETA, ALPHA, THE1, THETA

CP DATA WITH ALPHA=0 FOR ACT BODY

I_T=0=1 YES
I_T=0=0 NO

CI=CMPLX(W*1.0)
1 READ(S, INPUT)
1 IF (EOF) STOP

3 WRITE(S, INPUT)
1 ALPHA=ALPHA* 0.0174533
1 ALPHI=CMPLX(A, B, ALPHA)

PRINT 7# MACH, LAM, ALPHA

7 FORMAT(1H1# 15X, X, MACH, LAM, THETA, THETA, CP, REAL, 9X, CP, IMAG, CP, MACH
1 IF (I_TI=0) WRITE(10)
1 IF (I_TI=0) WRITE(10)

10 FORMAT(1H1# 1X, CP DATA WITH ALPHA=0 FOR ACT BODY)

LAM1=-LAM+1. /LAM
LAM2=LAM/LAM+1.0
LAM3=1.0/(LAM+1.0)
LAM4=4.0*(1.0-LAM)/(1.0+LAM)
FLAM=CMPLX(LAM4, 0.0)
SLAM=CMPLX(LAM1, 0.0)

EVALUATE EXPITHE1, AND EV12THE1

DO 20 J=1, NTHETA
1 THET(J)=THET(J)*0.0174533
1 THET=THET(J)
1 THTHE1=2.0*THET1(J)
1 E1J=CMPLX(COS(THT1, J), SIN(THT1, J))
1 E1J=CMPLX(COS(THT1, J), SIN(THT1, J))
1 SINT(J)=SIN(THT1, J)
1 COST(J)=COS(THT1, J)
20 CONTINUE

A VARIES FROM .35 TO .05 IN STEPS OF .05
I = COUNTI FOR X
1 = 0

40 X=X+.05
1=I+1
1 IF (X.GT(.04)) GO TO 10
CALCULATE $k \cdot k_1 \cdot k_2$

$k_1 = k(\text{Prime})$

$k_2 = k(\text{Double Prime})$

$k = 122 \cdot k$

$k_1 = 122$

$k_2 = 0$

$\text{ALK}_{SU} = \text{ALUG}(k \cdot k)$

$\text{HSU}_{1} = k_1 \cdot k_1$

$\text{TKSU}_{1} = \gamma \cdot k \cdot \text{HSU}$_1

$\text{CMSU}_{1} = \text{CMPLA}(\text{HSU}_{1}, 0)$

$\text{CTKSU}_{1} = \text{CMPLA}(\text{TKSU}_{1}, 0)$

$LAM7 = (2 \cdot k \cdot \text{LAM}^2) \ast \ast 2 \cdot (2 \cdot k \cdot \text{LAM}^3) \ast \ast 2$

$\text{ELAM} = \text{CMPLA}(LAM7, 0)$

$LAMY = -k \cdot \text{LAM} + k \cdot \text{LAM}^4 / \text{LAM}$

$\text{TLAM} = \text{CMPLA}(LAM4, 0)$

DO 50 J = 1, NTHTA

$A = E1T(J) \cdot \text{CSQRT}(E1T(J) \cdot \text{FLAM})$

$B = 5^9 (E1T(J) \cdot \text{CSQRT}(E1T(J) \cdot \text{SLAM}))$

$C1 = k \cdot \text{SINT}(J)$

$C2 = k \cdot \text{COST}(J)$

$C = \text{CMPLX}(C1 \cdot C2)$

$D = \text{CSQRT}(R \cdot k \cdot E1T2(J) \cdot \text{LLAM})$

$E1 = (2 \cdot k \cdot \text{LAM}^2 + \text{LAM}) \ast \ast 2$

$\text{LAM} = \text{CMPLA}(E1T, 0)$

$F = \text{CSQRT}(R \cdot k \cdot E1T2(J) \cdot \text{TLAM})$

$G1 = k \cdot \text{COST}(J)$

$G2 = k \cdot \text{SINT}(J)$

$G = \text{CMPLX}(G1 \cdot G2)$

$H1 = k \cdot R2 \cdot k \cdot k_1$

$H = \text{CMPLX}(H1, 0)$

$AA = 5 \cdot \text{ALPH}1 \cdot (-C1 \cdot C) - E1 \cdot (R \cdot E1T(J) + 1) - E \cdot C / (D \cdot (R \cdot E1T(J) + 1))$

$BB = k \cdot R1 \cdot (1 / (R \cdot E1T(J) + 1)) \cdot (C1 \cdot C / F)$

$CP = \text{CMPLA}(\text{CPH}(1) \cdot u_* )$

$CP = \text{ALPH1} \cdot k_1 \cdot (4 \cdot k \cdot A - 2 \cdot k \cdot \text{ALPH1} \cdot (R \cdot A)) - 2 \cdot k \cdot \text{ALPH1} \cdot (R \cdot A) - \text{CTKSU}_{1} \cdot 2 \cdot k \cdot k_1 \cdot \text{CLUG}(R) + H \cdot \text{ALPH}^2$

$1 \cdot \text{CTKSU}_{1} \cdot 2 \cdot \text{ALPHA} \cdot (AA + 3) - (-5 \cdot \text{ALPH1} \cdot 3 \cdot \text{ALPH}1 \cdot (R / A) - \text{E1T}(J) - \text{E1T}(J) + 1) / (D \cdot (R \cdot E1T(J) + 1))$

$\text{THETA} = \text{THEF}(J)$

$\text{CP} = CP \cdot CP^Q$

PRINT 47*$\text{THEF}(J) \ast \ast (1 \ast \ast 2)$

47 FORMAT(1H \*4E10.0)

50 CONTINUE

IF (MOD(J, 7)) 40 60 40

60 PRINT 7*$\text{MACH} \cdot \text{LAM} \cdot \text{ALPHA}$

GO TO 40
INPUT
C:CH = 0.1E+01
C:EB = 0.1E+00, 0.1E+04, 6.5E-01, 6.2E-01, -0.1E-01, -0.4E-01,
-0.4E-01, -0.85E-01, 0.9E-01, -0.1E+00, -0.1E+00, 0.1E+00,
-0.1E+00, -0.1E+00, -0.1E-01, 0.1E-01, 0.1E+00, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0.
LAM = 0.3E+01
L:HG = 0
ALPHA = 0.10E+00
L:ET = -0.137779E+01, -2.69816E+00, 0.0, 0.69816E+00, 0.127079E+01, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0.
N:HE = 5
N:END

ORIGINAL PAGE IS OF POOR QUALITY
<table>
<thead>
<tr>
<th>A</th>
<th>Theta</th>
<th>CR REAL</th>
<th>CR IMAG</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.00000000E-02</td>
<td>-1.57079632E+00</td>
<td>4.52852429E-02</td>
<td>6.60435199E-02</td>
</tr>
<tr>
<td>5.00000000E-02</td>
<td>-0.98130000E-01</td>
<td>6.46910252E-02</td>
<td>3.48324109E-02</td>
</tr>
<tr>
<td>5.00000000E-02</td>
<td>0.00000000E+00</td>
<td>1.49812699E-01</td>
<td>1.06920110E-02</td>
</tr>
<tr>
<td>5.00000000E-02</td>
<td>6.98130000E-01</td>
<td>1.58292005E+00</td>
<td>5.00335719E-02</td>
</tr>
<tr>
<td>5.00000000E-02</td>
<td>5.7079632E+00</td>
<td>1.57079632E+00</td>
<td>6.58375824E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>-1.57079632E+00</td>
<td>1.00000000E+00</td>
<td>4.62951749E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>-6.98130000E-01</td>
<td>2.61276300E-02</td>
<td>2.15020683E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>0.00000000E+00</td>
<td>9.79777777E-02</td>
<td>2.13649636E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>6.98130000E-01</td>
<td>9.19737744E-02</td>
<td>3.46894196E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>5.7079632E+00</td>
<td>1.44056333E-01</td>
<td>6.58375824E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>-1.57079632E+00</td>
<td>-2.51163554E-02</td>
<td>2.13649636E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>-6.98130000E-01</td>
<td>-1.31071104E-02</td>
<td>4.62951749E-02</td>
</tr>
<tr>
<td>1.00000000E-01</td>
<td>0.00000000E+00</td>
<td>4.94057424E-02</td>
<td>9.46318299E-03</td>
</tr>
<tr>
<td>1.50000000E-01</td>
<td>6.98130000E-01</td>
<td>4.47727750E-02</td>
<td>2.19774060E-02</td>
</tr>
<tr>
<td>1.50000000E-01</td>
<td>5.7079632E+00</td>
<td>5.15741795E-02</td>
<td>2.71082514E-02</td>
</tr>
<tr>
<td>2.00000000E-01</td>
<td>-1.57079632E+00</td>
<td>-4.32499415E-02</td>
<td>1.13584433E-02</td>
</tr>
<tr>
<td>2.00000000E-01</td>
<td>-6.98130000E-01</td>
<td>-3.32310506E-02</td>
<td>1.75676811E-03</td>
</tr>
<tr>
<td>2.00000000E-01</td>
<td>0.00000000E+00</td>
<td>1.95179334E-02</td>
<td>1.77491101E-02</td>
</tr>
<tr>
<td>2.00000000E-01</td>
<td>6.98130000E-01</td>
<td>1.64163891E-02</td>
<td>7.14550316E-03</td>
</tr>
<tr>
<td>2.00000000E-01</td>
<td>5.7079632E+00</td>
<td>2.38233525E-02</td>
<td>1.39998440E-02</td>
</tr>
<tr>
<td>2.50000000E-01</td>
<td>-1.57079632E+00</td>
<td>-6.01326660E-02</td>
<td>3.41435456E-03</td>
</tr>
<tr>
<td>2.50000000E-01</td>
<td>-6.98130000E-01</td>
<td>-5.41476738E-02</td>
<td>1.14853209E-02</td>
</tr>
<tr>
<td>2.50000000E-01</td>
<td>0.00000000E+00</td>
<td>-1.36146060E-02</td>
<td>2.43679384E-03</td>
</tr>
<tr>
<td>2.50000000E-01</td>
<td>6.98130000E-01</td>
<td>1.19845353E-02</td>
<td>4.46666136E-03</td>
</tr>
<tr>
<td>2.50000000E-01</td>
<td>5.7079632E+00</td>
<td>-4.95218537E-02</td>
<td>4.22837045E-03</td>
</tr>
<tr>
<td>3.00000000E-01</td>
<td>-1.57079632E+00</td>
<td>-6.55591660E-02</td>
<td>1.06242004E-02</td>
</tr>
<tr>
<td>3.00000000E-01</td>
<td>-6.98130000E-01</td>
<td>-7.59996813E-02</td>
<td>1.94812304E-02</td>
</tr>
<tr>
<td>3.00000000E-01</td>
<td>0.00000000E+00</td>
<td>-4.69931794E-02</td>
<td>3.30332136E-02</td>
</tr>
<tr>
<td>3.00000000E-01</td>
<td>6.98130000E-01</td>
<td>-4.78788802E-02</td>
<td>1.45596797E-02</td>
</tr>
<tr>
<td>3.00000000E-01</td>
<td>5.7079632E+00</td>
<td>-1.44568434E-02</td>
<td>1.70123090E-02</td>
</tr>
<tr>
<td>3.50000000E-01</td>
<td>-1.57079632E+00</td>
<td>-9.05890449E-02</td>
<td>2.69831402E-02</td>
</tr>
<tr>
<td>3.50000000E-01</td>
<td>-6.98130000E-01</td>
<td>-8.09826351E-02</td>
<td>2.66376115E-02</td>
</tr>
<tr>
<td>3.50000000E-01</td>
<td>0.00000000E+00</td>
<td>-6.44631111E-02</td>
<td>3.46050667E-02</td>
</tr>
<tr>
<td>3.50000000E-01</td>
<td>6.98130000E-01</td>
<td>-7.21914027E-02</td>
<td>2.30491004E-02</td>
</tr>
<tr>
<td>3.50000000E-01</td>
<td>5.7079632E+00</td>
<td>-5.37655310E-02</td>
<td>2.71784484E-02</td>
</tr>
<tr>
<td>Value 1</td>
<td>Value 2</td>
<td>Value 3</td>
<td>Value 4</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-1.570790000E+00</td>
<td>-3.94328174E-02</td>
<td>3.14365789E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-6.181300000E+01</td>
<td>-3.05658927E-01</td>
<td>3.18357892E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.91624726E-02</td>
<td>3.45472916E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-6.981300000E+01</td>
<td>-2.86831647E-02</td>
<td>2.47530294E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>1.570790000E+00</td>
<td>-8.19830614E-02</td>
<td>3.58552695E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-1.570790000E+00</td>
<td>-1.04314061E-01</td>
<td>-4.15451321E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.06797201E-01</td>
<td>-3.38719735E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>0.000000000E+00</td>
<td>-9.91532808E-02</td>
<td>3.07347145E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>-6.981300000E+01</td>
<td>-9.59137671E-02</td>
<td>3.49423130E-02</td>
</tr>
<tr>
<td>4.500000000E+01</td>
<td>1.570790000E+00</td>
<td>-4.10523733E-02</td>
<td>4.15743903E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-1.570790000E+00</td>
<td>-1.07804444E-01</td>
<td>-4.51470417E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.11870347E-01</td>
<td>-3.70911387E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.20159444E-01</td>
<td>3.66908414E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.92437067E-01</td>
<td>3.51686438E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>1.570790000E+00</td>
<td>-1.03490183E-01</td>
<td>4.03237717E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-1.570790000E+00</td>
<td>-1.02482508E-01</td>
<td>-4.01226078E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.08481413E-01</td>
<td>-3.98949885E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.15133345E-01</td>
<td>3.95832483E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.43275949E-01</td>
<td>3.93485953E-02</td>
</tr>
<tr>
<td>5.000000000E+01</td>
<td>1.570790000E+00</td>
<td>-1.11116347E-01</td>
<td>4.04484324E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-1.570790000E+00</td>
<td>-9.40321936E-02</td>
<td>-4.08263789E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.05145237E-01</td>
<td>-3.97029545E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.16653449E-01</td>
<td>3.86666871E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.18749676E-01</td>
<td>3.65556625E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>1.570790000E+00</td>
<td>-1.17847841E-01</td>
<td>4.37889838E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-1.570790000E+00</td>
<td>-1.56535615E-02</td>
<td>-4.50597803E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.63141399E-01</td>
<td>-3.03920529E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.72311602E-01</td>
<td>2.66754789E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.74644871E-01</td>
<td>3.58416758E-02</td>
</tr>
<tr>
<td>6.000000000E+01</td>
<td>1.570790000E+00</td>
<td>-1.50916714E-01</td>
<td>3.68407478E-02</td>
</tr>
<tr>
<td>7.000000000E+01</td>
<td>-1.570790000E+00</td>
<td>-9.43690575E-02</td>
<td>-3.19894328E-02</td>
</tr>
<tr>
<td>7.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.62123036E-01</td>
<td>-2.30674948E-02</td>
</tr>
<tr>
<td>7.000000000E+01</td>
<td>0.000000000E+00</td>
<td>-1.26213007E-01</td>
<td>2.18601132E-02</td>
</tr>
<tr>
<td>7.000000000E+01</td>
<td>-6.981300000E+01</td>
<td>-1.24101217E-01</td>
<td>3.98850712E-02</td>
</tr>
<tr>
<td>7.000000000E+01</td>
<td>1.570790000E+00</td>
<td>-1.35374673E-01</td>
<td>3.18735560E-02</td>
</tr>
<tr>
<td>( \theta \times 10^3 )</td>
<td>( CP_{\text{REAL}} )</td>
<td>( CP_{\text{IMAG}} )</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>-------------------</td>
<td>-----------------</td>
<td></td>
</tr>
<tr>
<td>7.50000000E-01</td>
<td>-1.57079000E+00</td>
<td>-3.2229224E-03</td>
<td></td>
</tr>
<tr>
<td>7.50000000E-01</td>
<td>-6.19130000E-01</td>
<td>-1.1205708E-02</td>
<td></td>
</tr>
<tr>
<td>7.50000000E-01</td>
<td>0.0</td>
<td>-4.0292630E-02</td>
<td></td>
</tr>
<tr>
<td>7.50000000E-01</td>
<td>6.19130000E-01</td>
<td>-5.0397375E-02</td>
<td></td>
</tr>
<tr>
<td>7.50000000E-01</td>
<td>1.57079000E+00</td>
<td>-5.0414340E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>-1.57079000E+00</td>
<td>5.5909404E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>-6.19130000E-01</td>
<td>-5.0397375E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>0.0</td>
<td>1.5454102E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>6.19130000E-01</td>
<td>3.1394107E-04</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>1.57079000E+00</td>
<td>-8.1360082E-03</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>-1.57079000E+00</td>
<td>1.0371426E-01</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>-6.19130000E-01</td>
<td>9.5224100E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>0.0</td>
<td>-6.1041346E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>6.19130000E-01</td>
<td>3.9977424E-02</td>
<td></td>
</tr>
<tr>
<td>8.00000000E-01</td>
<td>1.57079000E+00</td>
<td>2.5434313E-02</td>
<td></td>
</tr>
</tbody>
</table>