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THEORETICAL EFFECT OF MODIFICATIONS TO THE UPPER SURFACE OF
TWO NACA AIRFOILS USING SMOOTH POLYNOMIAL ADDITIONAL THICKNESS
DISTRIBUTIONS WHICH EMPHASIZE LEADING EDGE PROFILE AND
WHICH VARY QUADRATICALLY AT THE TRAILING EDGE

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ABSTRACT

This report describes a series of low-speed airfoil designs based on modification to the NACA 64-206 and 64₁-212 airfoils. Designs are based on potential flow theory. The report describes one of a series of airfoil modifications carried out under Contract NAS 2-8599, Application of Multivariable Search Techniques to Optimal Wing Design in Non-Linear Flow Fields. Mr. Raymond Hicks of National Aeronautics and Space Administration's Aeronautical Division, Ames Research Center, served as contract monitor for the study.

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Aerophysics Research Corporation

SUMMARY

An investigation has been conducted on the Lawrence Radiation Center, Berkeley, CDC 7600 digital computer to determine the effects of additional thickness distributions to the upper surface of the NACA 64-206 and 64₁-212 airfoils. The additional thickness distribution had the form of a continuous mathematical function which disappears at both the leading edge and the trailing edge. The function behaves as a polynomial of order ϵ_1 at the leading edge, and a polynomial of order ϵ_2 at the trailing edge. In the present study, ϵ_2 is a constant and ϵ_1 is varied over a range of practical interest. The magnitude of the additional thickness, \bar{y} , is a second input parameter, and the effect of varying ϵ_1 and \bar{y} on the aerodynamic performance of the airfoil was investigated. Results were obtained at a Mach number of 0.2 with an angle-of-attack of 6° on the basic airfoils. All calculations employ the full potential flow equations for two dimensional flow. The relaxation method of Jameson is employed for solution of the potential flow equations.

Earlier studies of these airfoils (References 4-6) used other types of additional thickness distributions. In these studies it was found that increases in the airfoil thickness tended to increase both the lift and the negative (nose-down) pitching moment. In the present investigation, this trend has been partially reversed, apparently because the quadratic curvature at the trailing edge provides a significant downward pressure force at the trailing edge. The result is that large reductions in the pitching moment can occur simultaneously with substantial increases in the

lift coefficient. This trend is most pronounced when the shape parameter ϵ_1 is such as to make the leading edge very blunt, compared to the unmodified airfoil.

For the range of parameters examined, the lift coefficient was nearly insensitive to variations in the shape parameter ϵ_1 (less than 10%), while the moment coefficient showed strong sensitivity to ϵ_1 (from 50% to 200%). Increasing the thickness parameter \bar{y} caused monotonic increases in lift, which were of smaller magnitude than those encountered in earlier studies (References 4-6). The moment coefficient was insensitive to changes in thickness for small values of ϵ_1 , and strongly sensitive to those changes for large values of ϵ_1 .

Additional thickness can be made to produce significant reductions in peak pressure coefficient. This is particularly true for the 64-206 airfoil, which in its unmodified form has a very high pressure peak at the leading edge, due to the small radius of curvature at this point. For both airfoils, it was found that the peak pressure can be reduced to much smaller values while the lift coefficient is increased, by proper choice of the parameters ϵ_1 and \bar{y} . Increasing the thickness first reduced the maximum pressure (which typically occurs at or near the leading edge) and then increased this pressure (which then occurs near the quarter chord).

A large number of parameter combinations were studied and it was found that the minimum pressures obtainable at a given lift are well below the pressures obtained in earlier studies. On the other hand, the maximum lift coefficient obtained with the quadratic trailing edge modification is some what less than the lift obtained in these studies.

It should be noted that viscous effects are neglected in the present analysis. At the higher lift coefficients the effect of viscosity could be significant. Further investigations incorporating a viscous flow model are therefore desirable.

INTRODUCTION

The National Aeronautics and Space Administration and others are currently conducting a series of theoretical and experimental studies to define airfoil sections having improved performance from the aspects of lift, drag, pitching moment, or pressure distribution characteristics, References 1 and 2. Analytic investigations using airfoil surface representations based on high-order polynomials may result in impractical profiles; for example, very thin trailing edge thickness distributions or severe reflexes in the profile. The present study employs a continuous polynomial arc having two free parameters, whose characteristics are selected to avoid such problems. Previous optimization studies using multivariable search techniques, References 1, 4, 5 and 6, generally indicate that shape changes which provide increased lift produce unfavorable changes in moment characteristics. Conversely, profile changes which improve the moment characteristics decrease the lift coefficient.

These characteristic trends were not followed by the results of the present study, however, because of the strong influence of the trailing edge pressure on the pitching moment. On the contrary, it was found to be possible to reduce the magnitude of the moment and the peak pressure while increasing the lift of the airfoil. A systematic variation of the two free parameters was carried out in order to determine the quantitative relationship which holds between lift, moment and peak pressure.

MATHEMATICAL MODELS

Potential Flow Equation

Potential flow analysis is based on solution of the two-dimensional potential flow equation

$$(a^2 - u^2) \phi_{xx} + (a^2 - v^2) \phi_{yy} - 2uv \phi_{xy} = 0$$

where ϕ is the velocity potential, u and v are the velocity components

$$u = \phi_x, \quad v = \phi_y$$

and a is the local speed of sound determined from the energy equation and the stagnation speed of sound

$$a^2 = a_0^2 - \left(\frac{\gamma - 1}{2}\right) (u^2 + v^2)$$

Solutions are obtained by Jameson's finite difference scheme, Reference 6.

AIRFOIL PROFILE REPRESENTATION

Basic Airfoil

Ordinates for the basic NACA 64-206 and 64₁-212 airfoils were approximated by four cubic chain polynomials in the manner of Hicks

$$y_j = a_{0j} F_j + a_{1j} x + a_{2j} x^2 + a_{3j} x^3; j = 1, 2, 3, 4$$

Coefficients in the four polynomial arcs are selected on the following basis:

j = 1 = Arc represents forward portion of upper surface

$$F_1 = \sqrt{x}$$

j = 2 - Arc represents aft portion of upper surface

$$F_2 = 1$$

j = 3 - Arc represents forward portion of lower surface

$$F_3 = \sqrt{x}$$

j = 4 - Arc represents aft portion

$$F_4 = 1$$

The coefficients a_{ij} are determined by introducing four boundary conditions on the airfoil profile in each of the four airfoil arcs. Crout's method for triangularization and back substitution is used to solve the resulting system of linear equations. Note that if four points are specified on the aft portion ($i = 2$ or 4), a discontinuity in slope occurs where the polynomials join. This produces a small ripple in the pressure distribution at the juncture point. However, since the juncture occurs at a region of small slope ($x = .5$) the effect is not significant.

Computer-generated plots of the NACA 64-206 and 64₁-212 airfoils, together with the associated pressure distributions predicted by potential flow theory, are shown in Figure 1.

Additional Thickness

The upper surface of the basic airfoil is modified by addition of the thickness-distribution function,

$$\Delta y(x) = Ax^{\epsilon_1} (1-x)^{\epsilon_2}$$

where $\epsilon_2 = 2$, for the present study. It is shown in Appendix A that the magnitude parameter, A, can be expressed in terms of the maximum thickness, \bar{y} , by the equation,

$$A = \bar{y} \left[\frac{(2 + \epsilon_1)^2 + \epsilon_1}{\epsilon_1} \right]$$

Representative functions $\Delta y(x)$ are shown in Figure 2, for $\epsilon_1 = .25, .75$ and 1.10. The slope of the additional thickness distribution at the leading edge ($x = 0$) is infinite for $\epsilon_1 < 1$, and is zero for $\epsilon_1 > 1$, while the slope at the trailing edge is zero.

The point at which the additional thickness distribution achieves a maximum is given by:

$$\bar{x} = \left[\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \right]$$

measured from the airfoil leading edge. It may be noted that as ϵ_2 increases the point of maximum additional thickness moves forward.

OPTIMIZATION STUDIES

In previous airfoil optimization studies, (references 4 and 5), the modifications to the upper surface took the form of a pair of quadratic arcs, which were cotangential at the point \bar{x} , \bar{y} . These parameters are respectively the chordwise location of the maximum addition thickness and the value of this thickness. Both lift coefficient and moment coefficient were considered as performance indices in this development. The present study is also concerned with a two variable optimization problem using the leading edge thickness distribution exponent, ϵ_1 , and the magnitude of additional thickness, \bar{y} .

Lift Coefficient Maximization

In general, maximization of lift coefficient has the form

$$\phi = \text{Max} [C_L]$$

where

$$C_L = \oint \Delta p(x) dx$$

and the integration is around the airfoil contour. The airfoil contour in the present study and those of references 4 and 5 are completely described in terms of two parameters, α_1 and α_2 . For the present airfoils

$$\phi = \text{Max} [C_L] = \text{Max} [C_L(\epsilon_1, \bar{y})] \equiv \text{Max} [C_L(\alpha_1, \alpha_2)]$$

where

$$\alpha_{1L} \leq \alpha_1 \leq \alpha_{1H}$$

$$\alpha_{2L} \leq \alpha_2 \leq \alpha_{2H}$$

This two variable optimization problem can be solved by use of multi-variable search techniques, for example, a combination of directed random-ray and pattern searches, references 3 and 7.

Examples illustrating this type of search procedure have previously been presented in references 4 and 5. However, the low dimensionality of the present problem (two parameters) permits the solution of optimization problems by inspection of graphical results. This procedure is employed in the present report. Other optimization problems of interest are described below.

Moment Coefficient Minimization

Minimization of the moment coefficient has form

$$\phi = \text{Min} \left[C_M \right] = \text{Min} \left[C_M(\epsilon_1, \bar{y}) \right] = \text{Min} \left[C_M(\alpha_1, \alpha_2) \right]$$

where

$$C_M = \int (x - 1/4) \Delta p(x) dx$$

In previous studies moment minimization resulted in a solution directly opposed to lift maximization. The position of maximum thickness moved forward and the amount of additional thickness was minimized. Thus in those studies the basic airfoil had less adverse moment than any airfoil generated by addition of the specified thickness to the upper surface of the airfoil.

Other Optimization Criteria

Other airfoil performance criteria can be considered, which typically involve compromises between lift, moment and peak pressure coefficients. Such modified criteria can take any of the following forms:

1. Maximize a linear combination of lift and moment:

$$\phi = \text{Max} \left[C_L - aC_M \right]$$

2. Minimize the moment at a specific value of lift:

$$\phi = \text{Min} \left[C_M \right]_{\bar{C}_L}$$

3. Minimize the peak pressure at a specific value of lift:

$$\phi = \text{Min} \left[C_{P_{\max}} \right]_{\bar{C}_L}$$

In the studies reported in references 4 and 5, the parameters available for airfoil modification (\bar{x} and \bar{y}), both were varied over a large range. This permitted a straightforward interpretation of results, such that optimal parameter pairs for a given performance criterion could be determined by inspection. As noted above, this procedure is also followed in the present study. Free variables for the present study are the parameters ϵ_1 and \bar{y} .

SYSTEMATIC AIRFOIL SHAPING

The present study is primarily concerned with a limited but systematic investigation on the effects of varying the leading edge thickness magnitude and distribution for the NACA 64-206 and 64₁-212 airfoils. The parameters \bar{y} and ϵ_1 are varied over ranges which are sufficient to permit qualitative conclusions as to their effects on lift, moment, and peak pressure coefficients. For each pair of such parameters, a plot of the airfoil and its associated calculated pressure distribution are given in Figures 3(a) to 3(o) and 4(a) to 4(o).

The first 15 of these plots relate to modifications of the 64-206 airfoil, and the last 15 are related to modifications of the 64₁-212 airfoil. These pressure signatures differ from those of the basic airfoils (Figure 1) chiefly in the magnitude of the peak pressure at the leading edge. Increases in \bar{y} and in ϵ_1 tend to soften the pressure variations over the upper surface, by reducing the leading edge peak and by increasing pressures over the central and trailing edge regions. In the case of the modified 64-206 airfoil high values of ϵ_1 ultimately reverse this trend and the strong overpressure peak reappears.

Lift and moment coefficient variation for the two airfoils are shown in Figure 5, and the effects of varying the parameter ϵ_1 and \bar{y} are apparent. For both airfoils, the lift increases only slightly with the exponent ϵ_1 , while the thickness \bar{y} has a more pronounced influence on the lift increment. The adverse moment coefficient also rises sharply with additional thickness, while the exponent ϵ_1 has a more significant influence on C_M . Combined lift vs. moment results are given in Figure 6, which shows that the least adverse moment is obtained at a given \bar{y} , or at a given C_L , with the smallest value of ϵ_1 . This corresponds to a relatively blunt leading edge on the airfoil.

Variations of the peak pressure with the parameters ϵ_1 and \bar{y} are shown in Figure 7 for the two airfoils being studied. The pressure variation for both airfoils is minimized at a particular \bar{y} by a specific choice of ϵ_1 . For the 64₁-212 airfoil peak overpressure at a given lift coefficient

is minimized by using the largest leading edge exponent value, ϵ_1 . The C_L - $C_{P_{max}}$ variation is more complex for the 64-206 airfoil, however, in that the constant ϵ_1 loci cross each other. The envelope of these curves is given in Figure 8, and it defines the minimum peak pressure for a given lift. The small number of data points available in this study does not permit accurate cross-plotting, so the estimated minimum pressure peak values are shown in the cross-hatched area. Despite the uncertainties in the cross-plotting procedure it is evident that the present family of airfoil designs produce lower peak overpressures for a given C_L than the airfoils previously obtained through biquadratic modifications.

Qualitative results of this parametric study are summarized in Table I. These conclusions follow directly from the results shown in Figures 4 to 7. Figure 7 also presents the minimum peak overpressures obtained with biquadratic airfoil modifications in References 4 and 5.

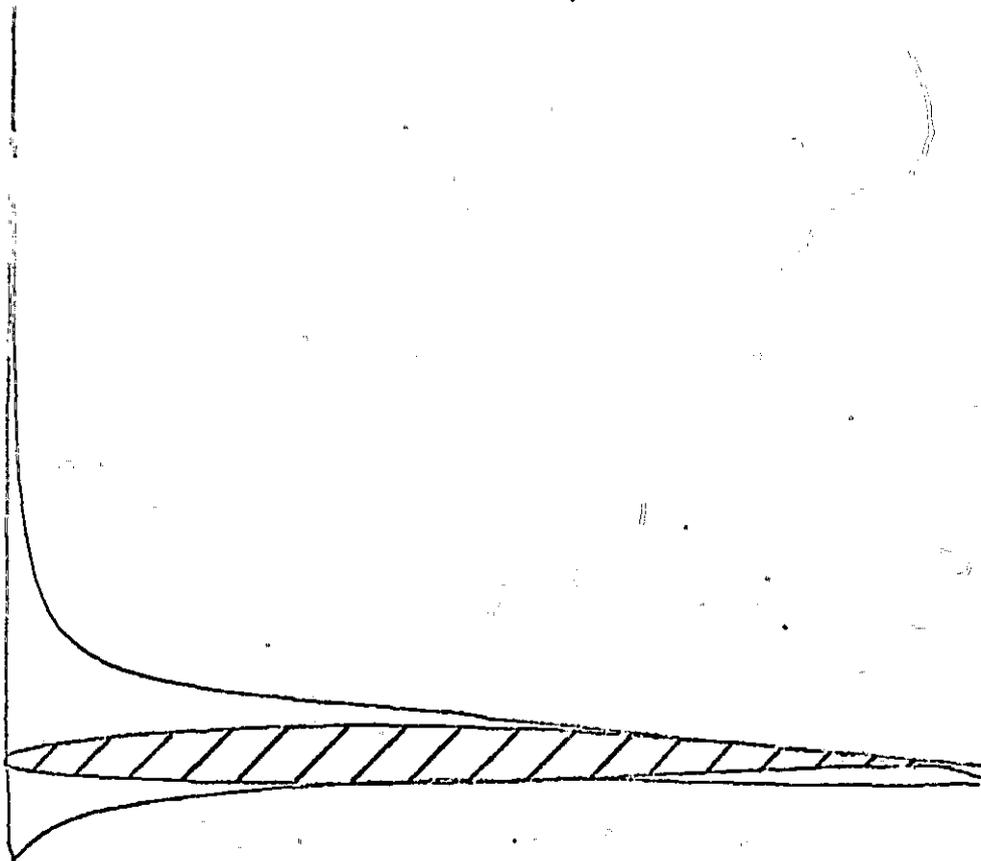
TABLE I. PARAMETRIC VARIATIONS FOR OPTIMIZING VARIOUS CRITERIA

	Criterion	Exponent ϵ_1	Thickness \bar{y}	Comment
64-206 Airfoil	Max $\left[C_L \right]$	Max	Max	
	Min $\left[C_m \right]$	Min	Max	
	Min $\left[C_m \right]$ \bar{C}_L	Min	$\bar{y} = \bar{y} (C_L)$	
	Min $\left[C_{P_{max}} \right]$ \bar{C}_L	$\epsilon_1 = \epsilon_1 (C_L)$	$\bar{y} = \bar{y} (C_L)$	
64 ₁ -212 Airfoil	Max $\left[C_L \right]$	Max	Max	Insensitive to ϵ_1
	Min $\left[C_m \right]$	Min	Min	
	Min $\left[C_m \right]$ \bar{C}_L	Min	$\bar{y} = \bar{y} (C_L)$	
	Min $\left[C_{P_{max}} \right]$ \bar{C}_L	Max	$\bar{y} = \bar{y} (C_L)$	

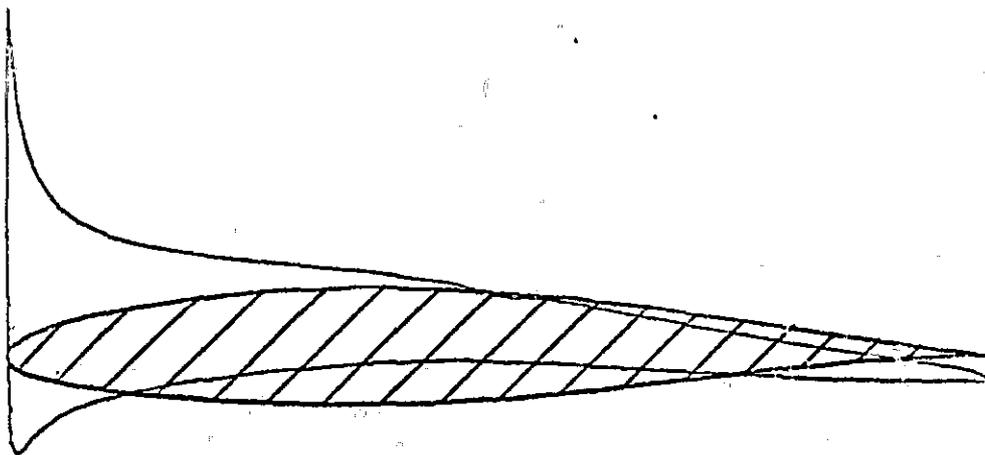
CONCLUSION

A numerical study has been completed of a class of modifications to the NACA 64-206 and 64₁-212 airfoils. Systematic changes in the upper surfaces of these airfoils were studied by independent variations in the thickness and leading edge thickness distribution exponent. The Mach number and angle-of-attack were constant during the study, and the results are summarized as follows:

1. Pressure distribution is moderately sensitive to leading edge profile and to additional thickness.
2. Lift coefficient is nearly independent of the leading edge profile, but increases with additional thickness.
3. Adverse pitching moment increases with additional thickness and with increases in the leading edge profile exponent.
4. Peak pressure for a given lift coefficient can be considerably reduced by careful selection of the leading edge additional thickness distribution exponent. Somewhat lower peak pressures at a given lift are possible using the present airfoil modifications, as compared with the "biquadratic" modifications of References 4 and 5.
5. For a given lift coefficient, adverse pitching moment is minimized by reducing the leading edge profile exponent.



(a) 64-206 Airfoil



(b) 64₁-212 Airfoil

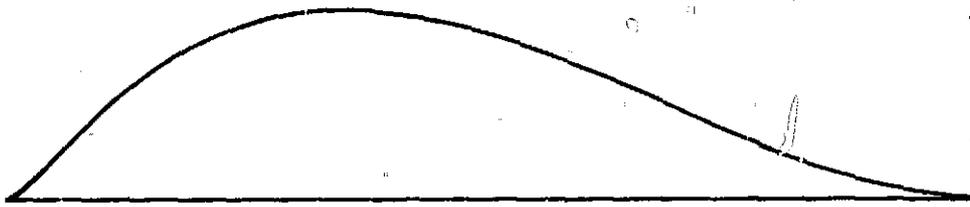
FIGURE 1. UNMODIFIED AIRFOILS AND PRESSURE DISTRIBUTIONS



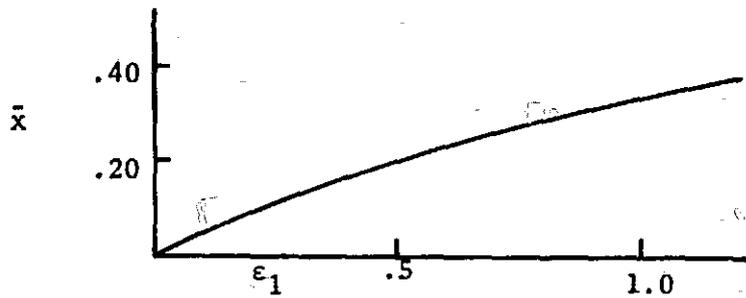
(a) $\epsilon_1 = .25$



(b) $\epsilon_1 = .75$



(c) $\epsilon_1 = 1.10$



(d) Chordwise Location of Maximum Thickness

FIGURE 2. UPPER SURFACE MODIFICATIONS ($\epsilon_2 = 2$)

FIGURE 3(a). MODIFIED 64-206 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

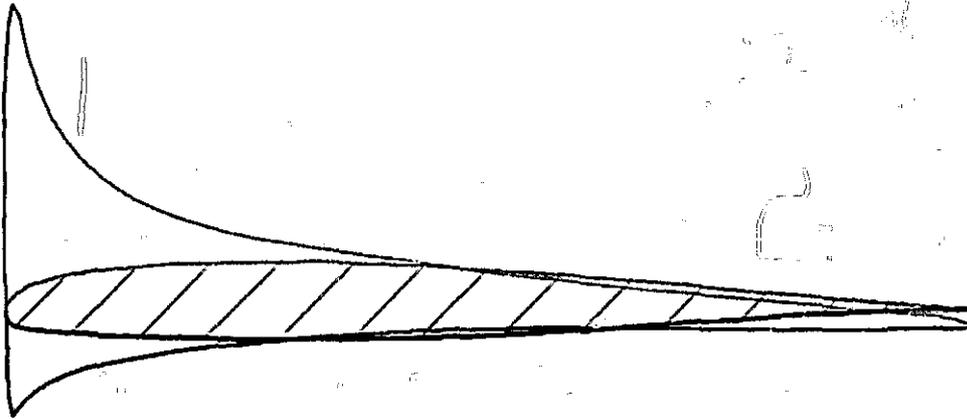


FIGURE 3(b). MODIFIED 64-206 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .50$, $\epsilon_2 = 2$

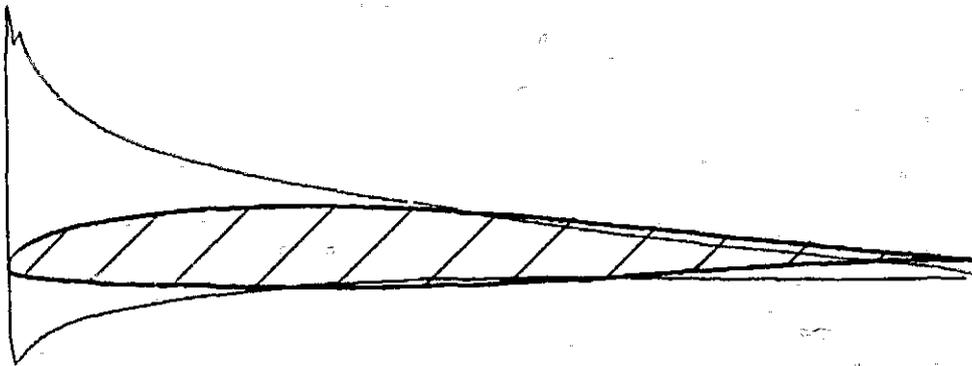


FIGURE 3(c). MODIFIED 64-206 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .75$, $\epsilon_2 = 2$

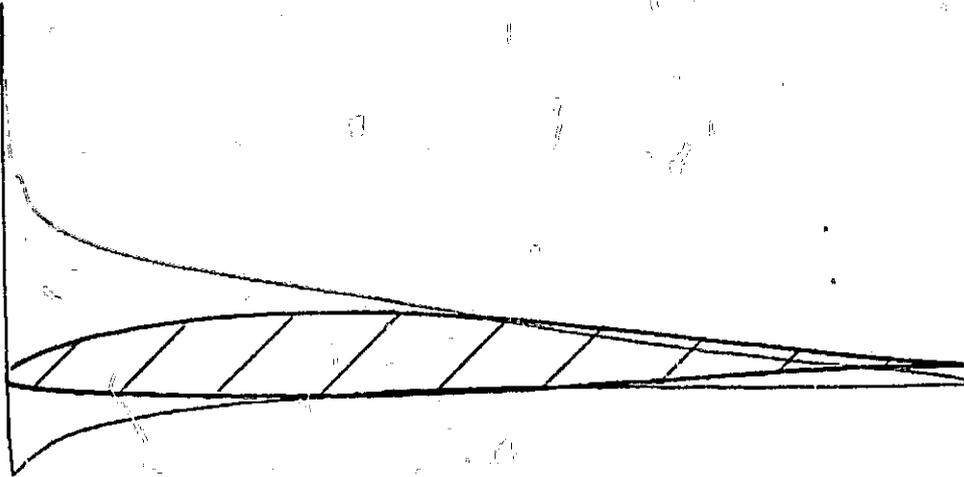


FIGURE 3(d). MODIFIED 64-206 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .9$, $\epsilon_2 = 2$

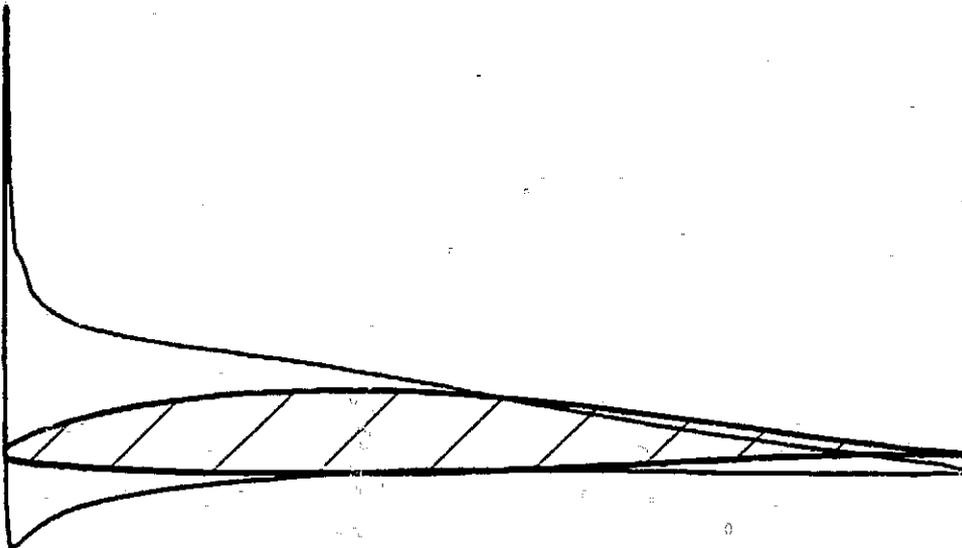


FIGURE 3(e). MODIFIED 64-206 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$

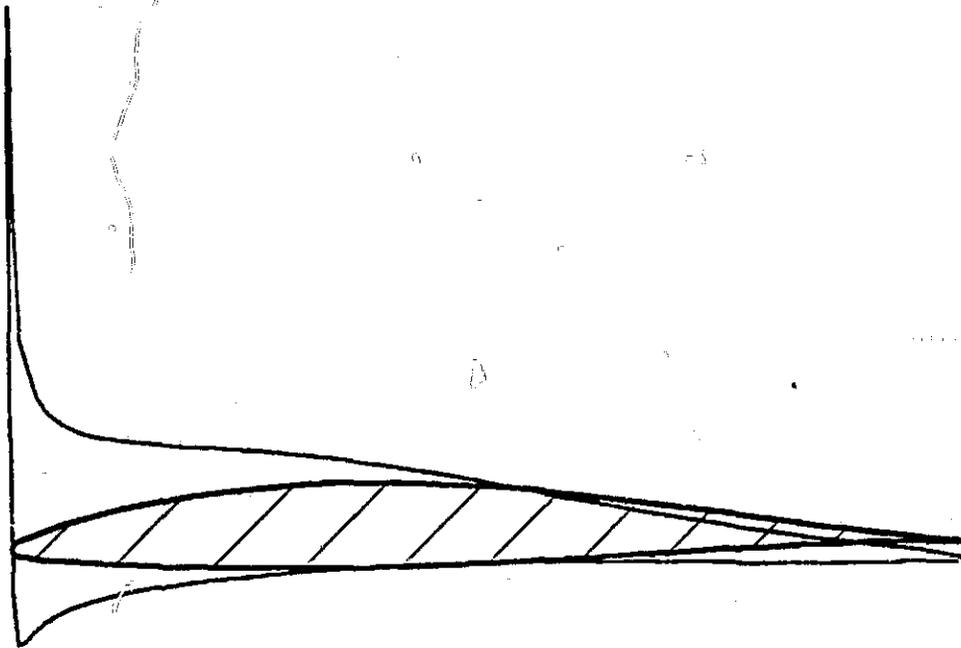


FIGURE 3(f). MODIFIED 64-206 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

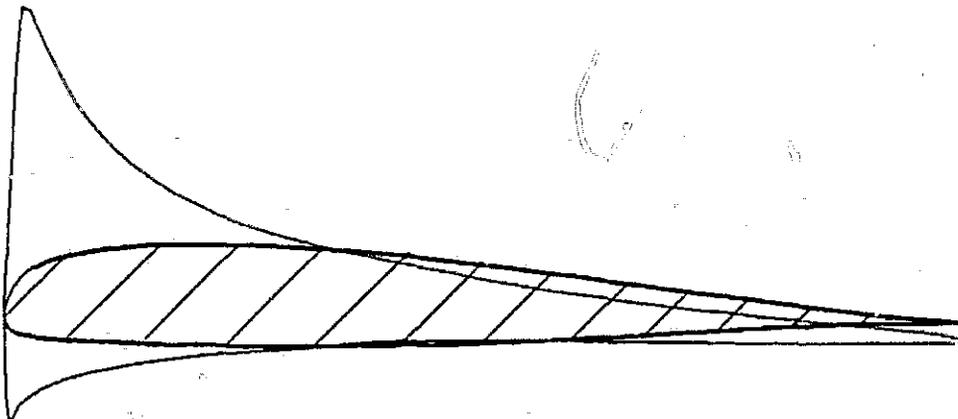


FIGURE 3(g). MODIFIED 64-206 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .50$, $\epsilon_2 = 2$

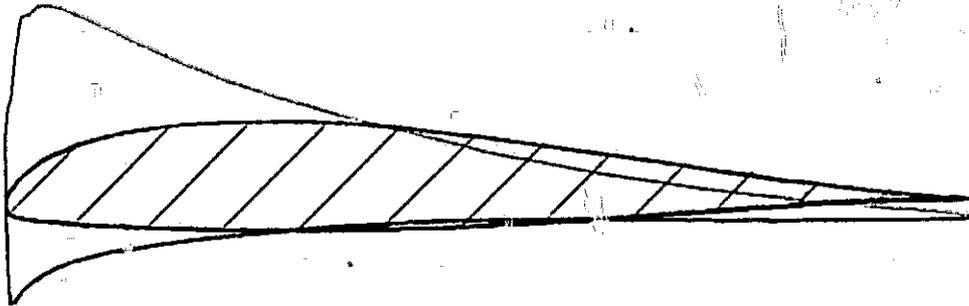


FIGURE 3(h). MODIFIED 64-206 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .75$, $\epsilon_2 = 2$.

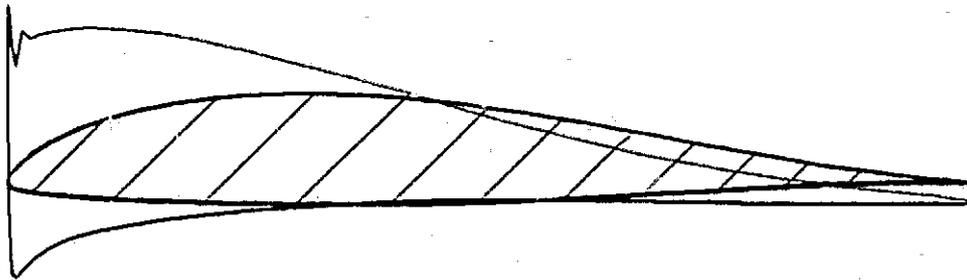


FIGURE 3(i). MODIFIED 64-206 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .9$, $\epsilon_2 = 2$

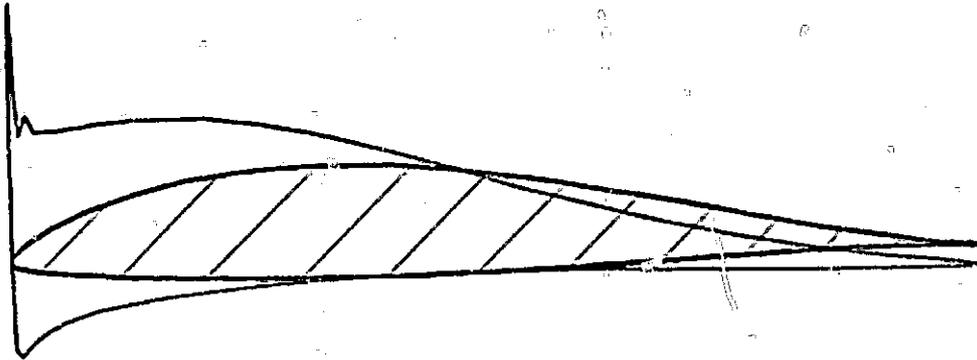


FIGURE 3(j). MODIFIED 64-206 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$

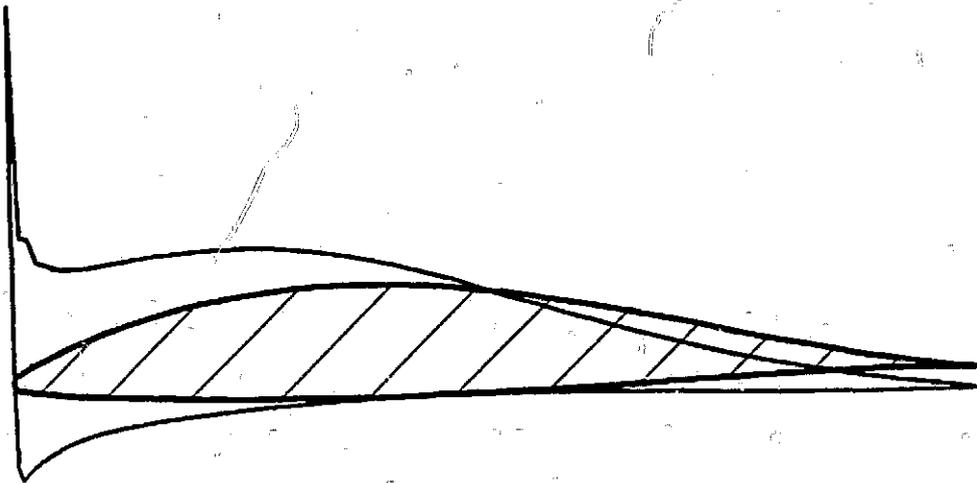


FIGURE 3(k). MODIFIED 64-206 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

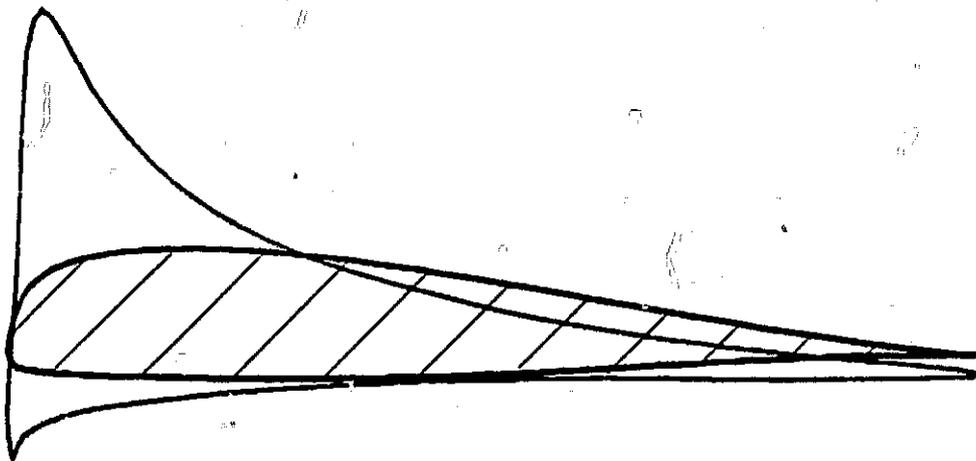
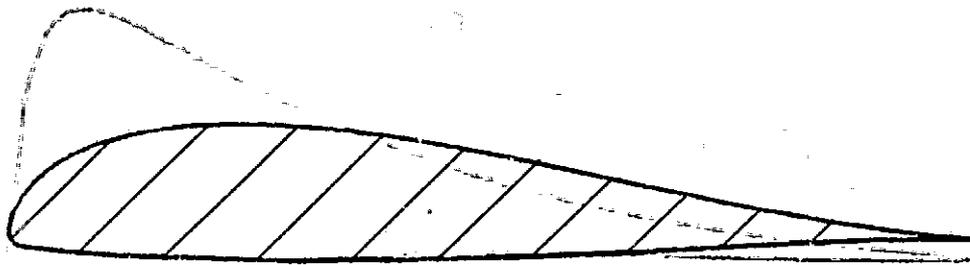


FIGURE 3(l). MODIFIED 64-206 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .50$, $\epsilon_2 = 2$



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FIGURE 3(m). MODIFIED 64-206 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .75$, $\epsilon_2 = 2$

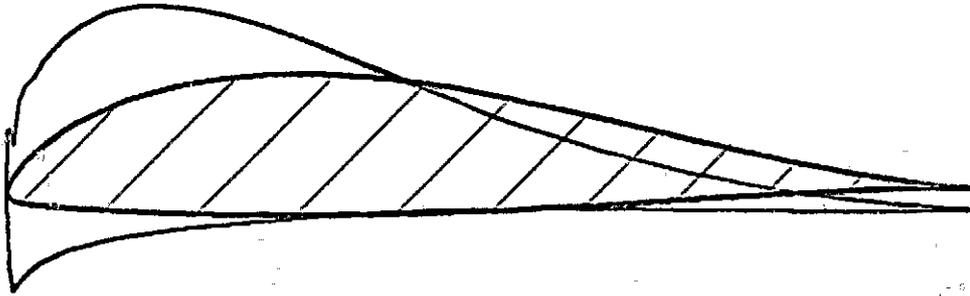


FIGURE 3(n). MODIFIED 64-206 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .09$, $\epsilon_2 = 2$

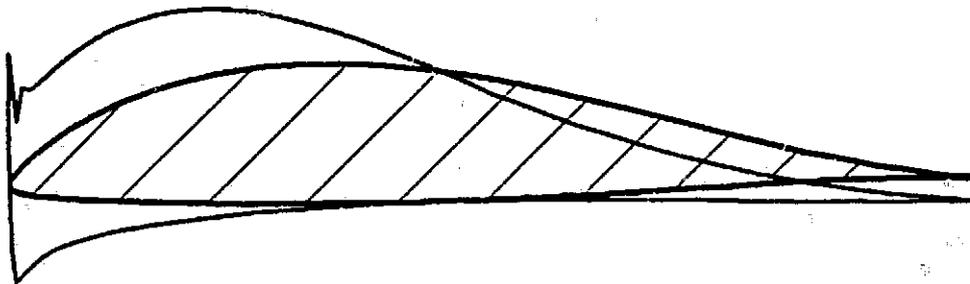
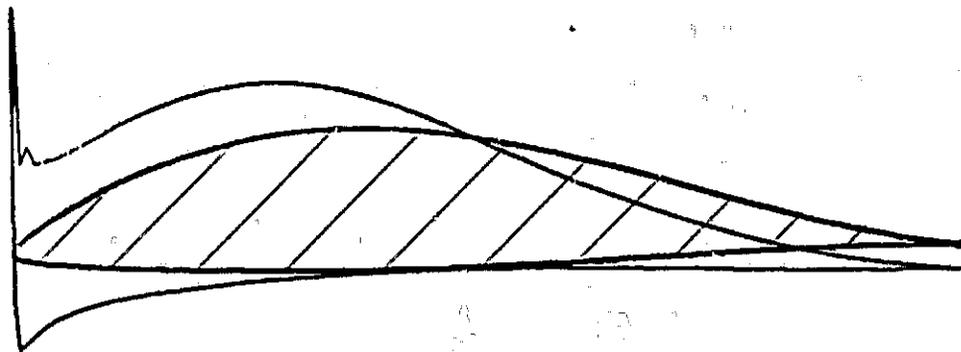


FIGURE 3(o). MODIFIED 64-206 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$



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FIGURE 4(a). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

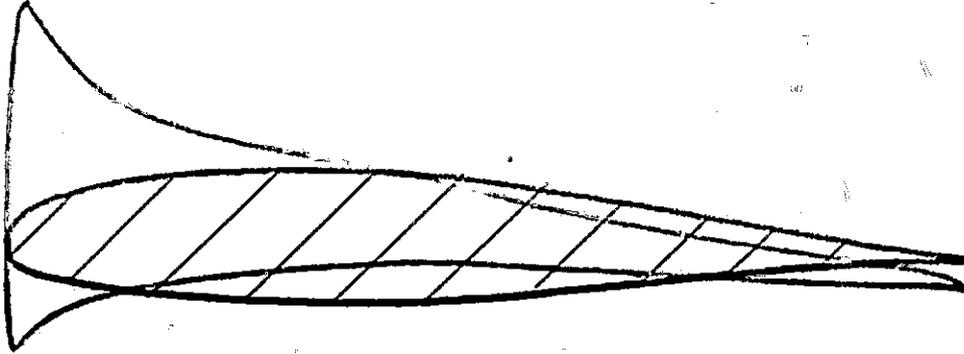
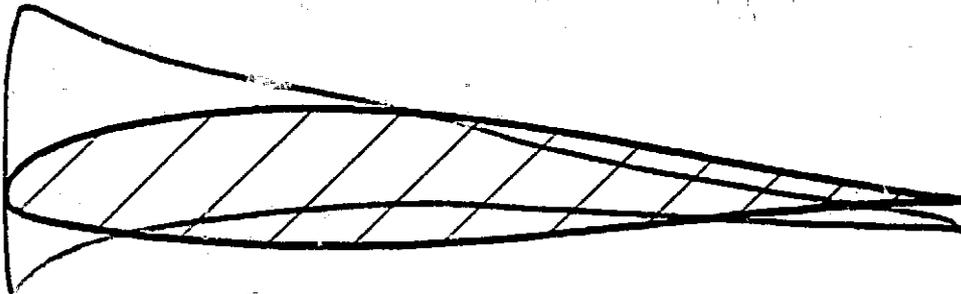


FIGURE 4(b). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .5$, $\epsilon_2 = 2$



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FIGURE 4(c). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .75$, $\epsilon_2 = 2$

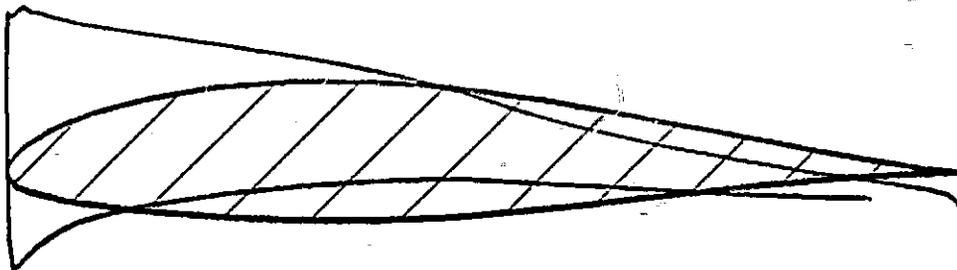


FIGURE 4(d). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = .9$, $\epsilon_2 = 2$

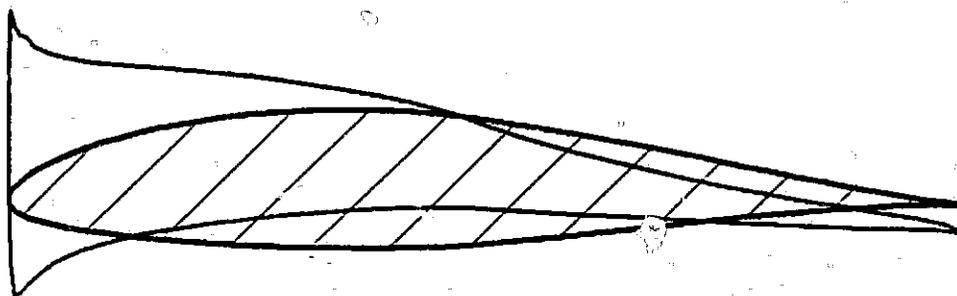


FIGURE 4(e). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .03$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$

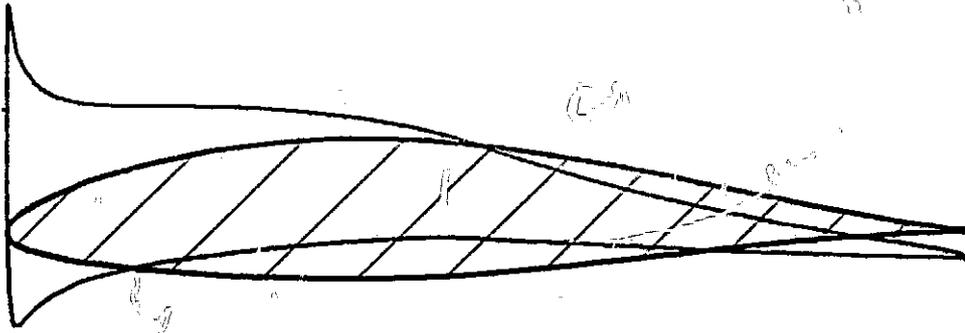


FIGURE 4(f). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

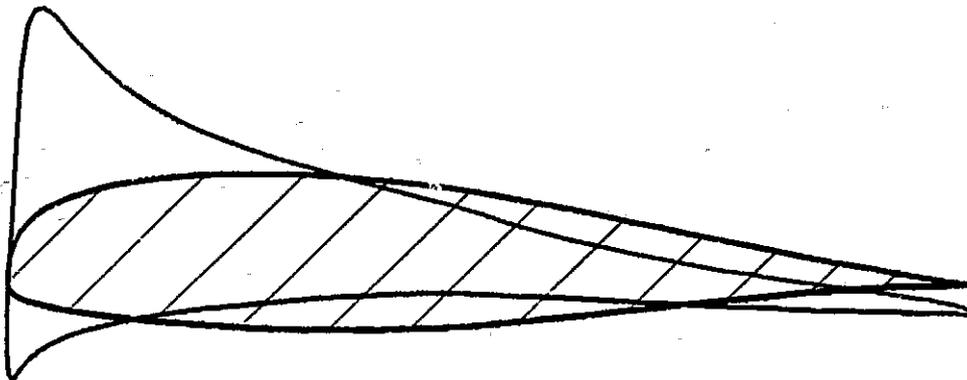


FIGURE 4(g). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .5$, $\epsilon_2 = 2$

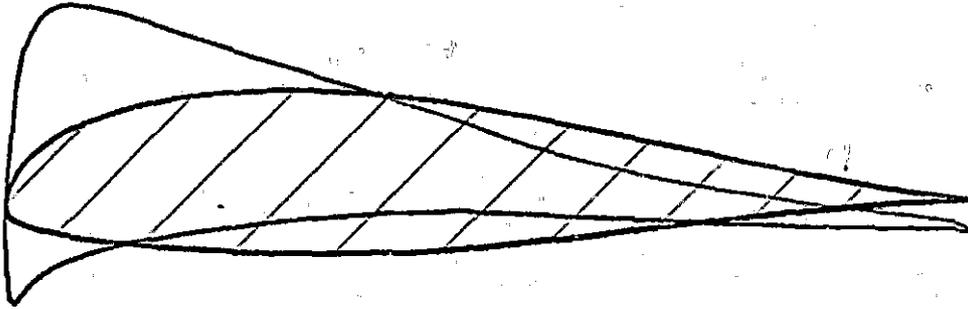


FIGURE 4(h). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .75$, $\epsilon_2 = 2$

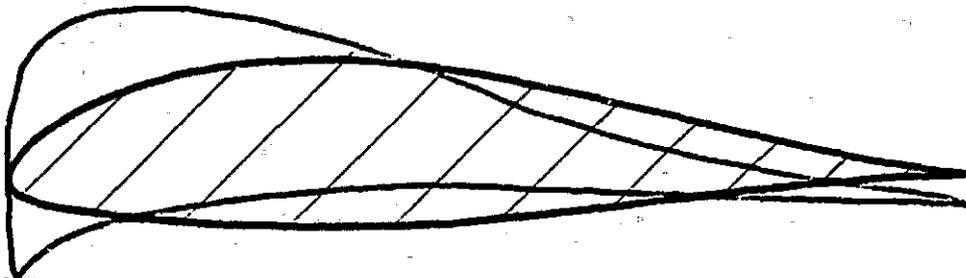


FIGURE 4(i). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = .9$, $\epsilon_2 = 2$

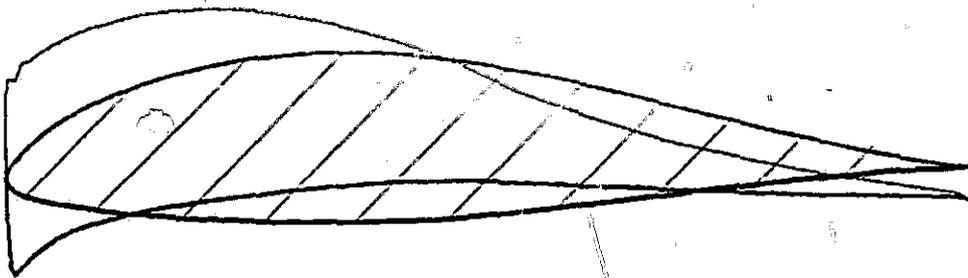


FIGURE 4(j). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .06$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$

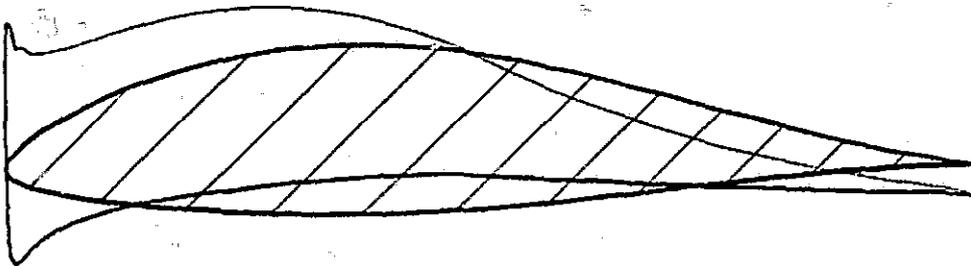


FIGURE 4(k). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .25$, $\epsilon_2 = 2$

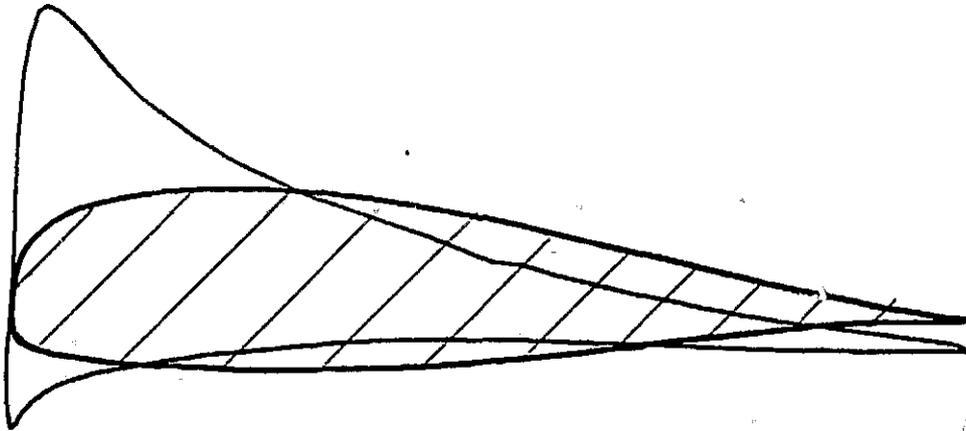


FIGURE 4(l). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .50$, $\epsilon_2 = 2$

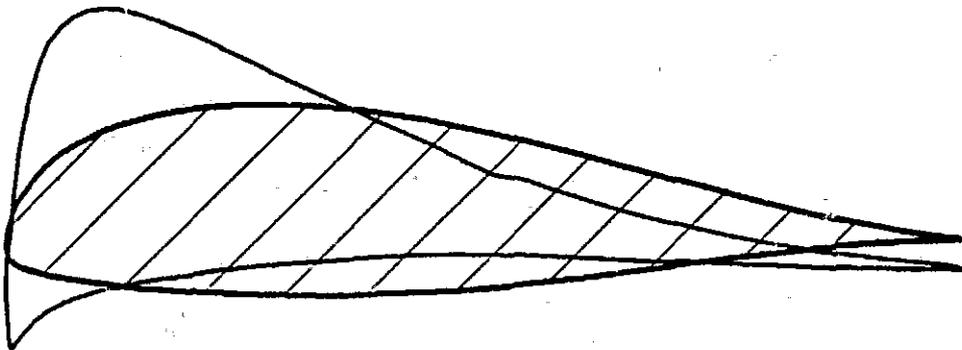


FIGURE 4(m). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .75$, $\epsilon_2 = 2$

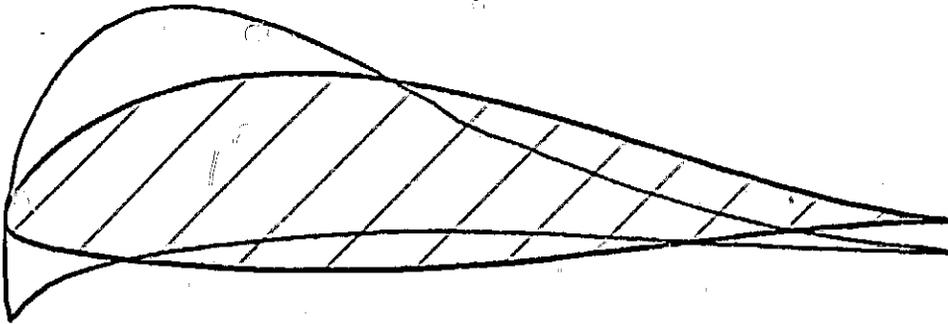


FIGURE 4(n). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = .9$, $\epsilon_2 = 2$

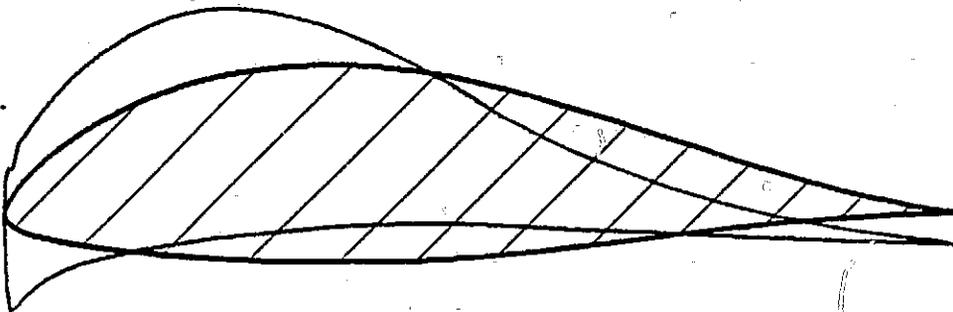
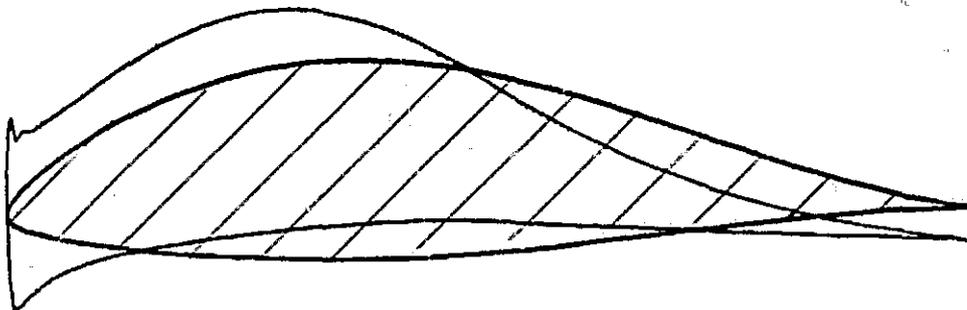
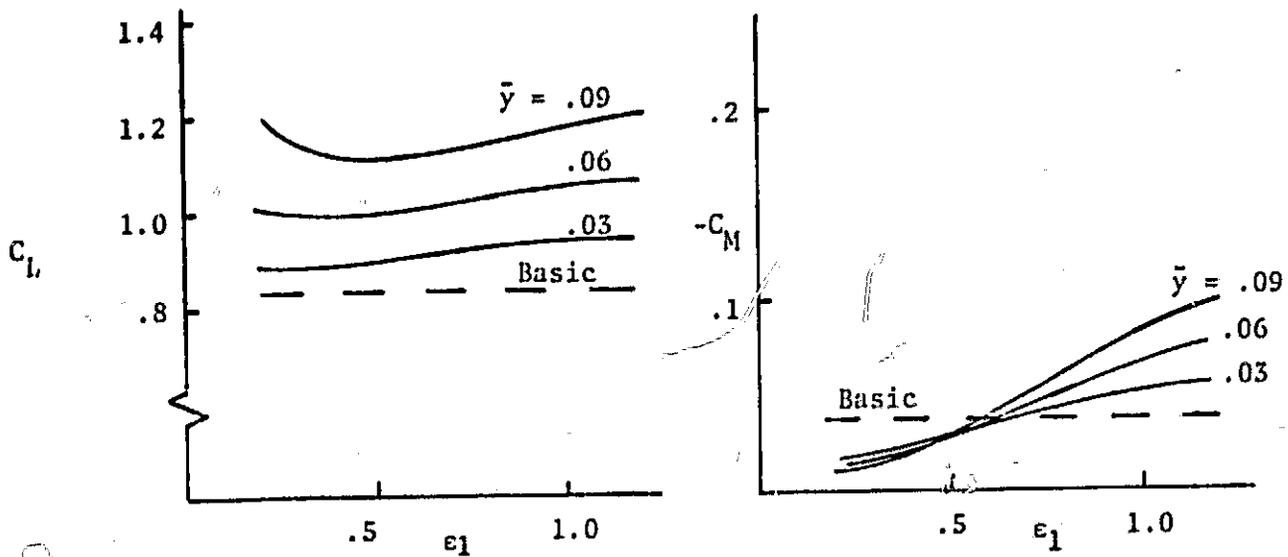
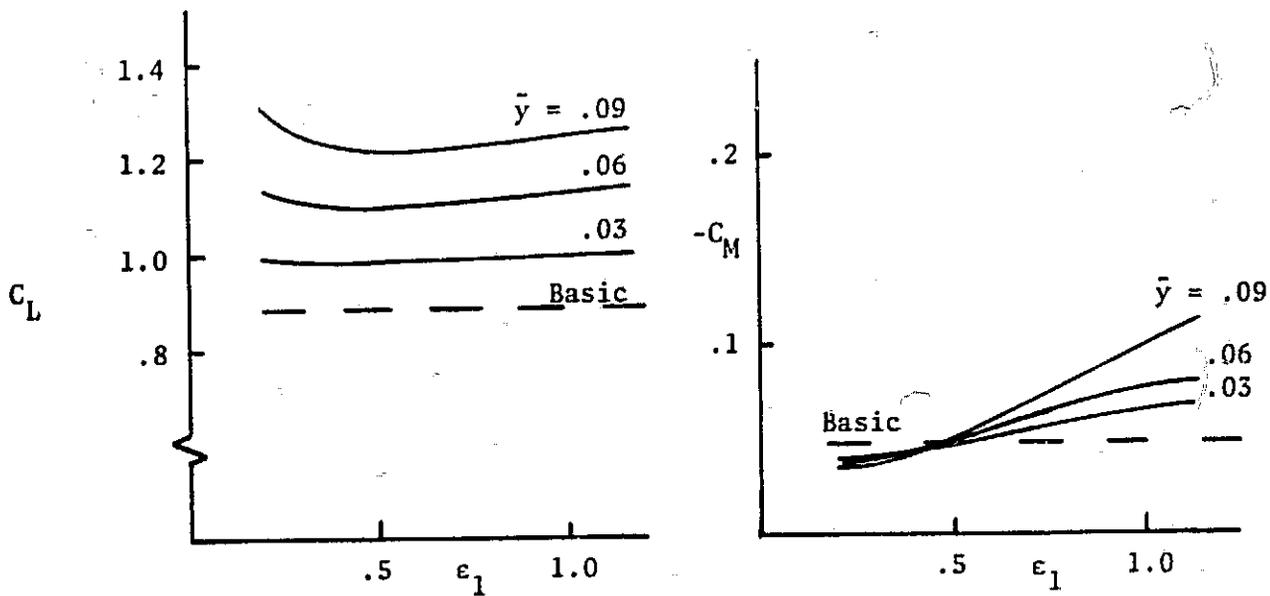


FIGURE 4(o). MODIFIED 64₁-212 AIRFOIL, $\bar{y} = .09$, $\epsilon_1 = 1.1$, $\epsilon_2 = 2$



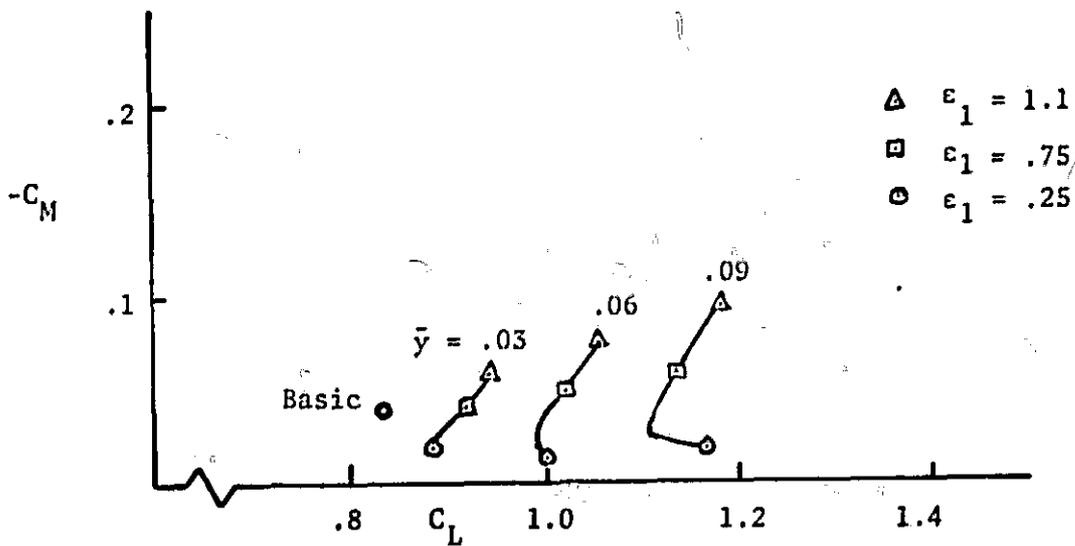


(a) 64-206 Airfoil

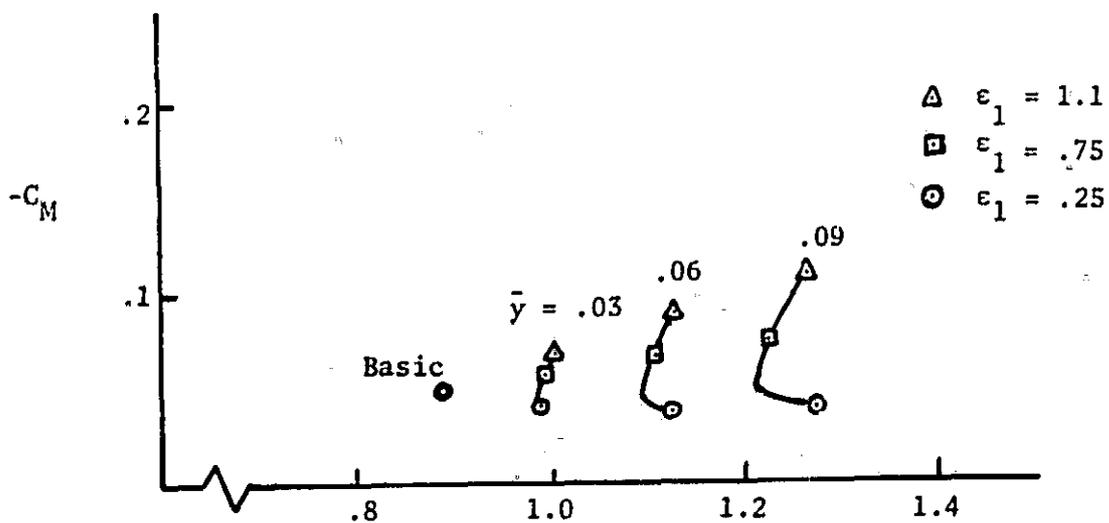


(b) 64₁-212 Airfoil

FIGURE 5. AERODYNAMIC COEFFICIENT VARIATIONS WITH EXPONENT ϵ_2 AND THICKNESS \bar{y}

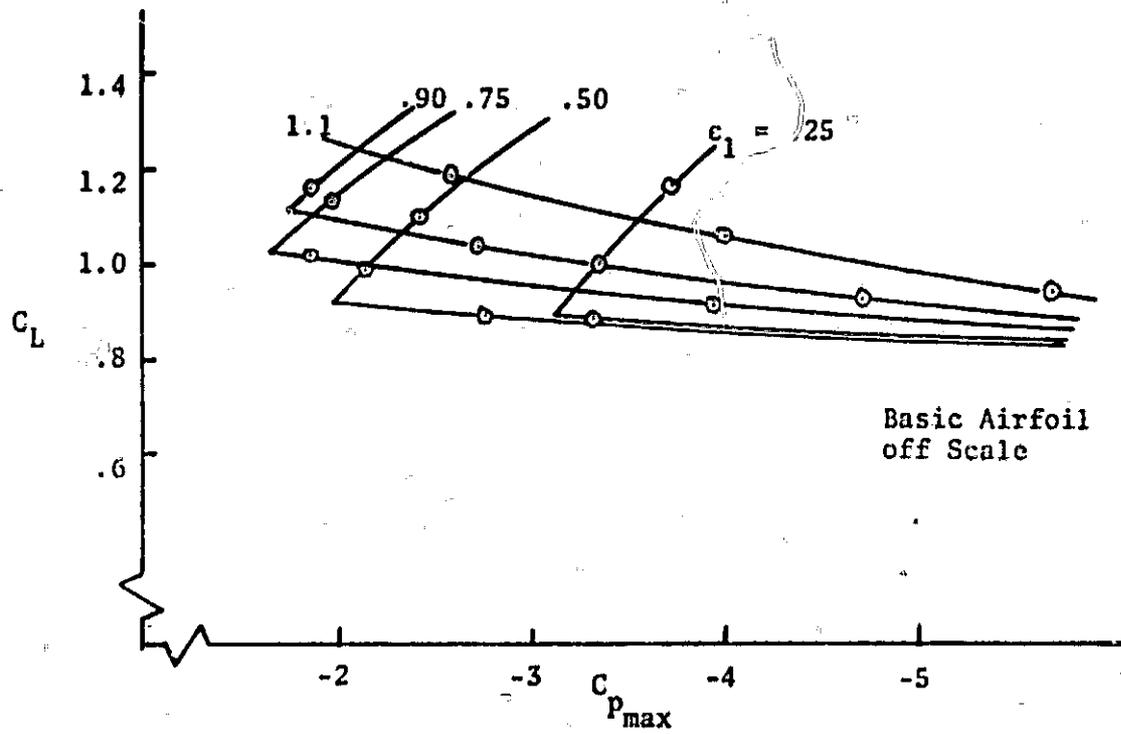


(a) 64-206 Airfoil

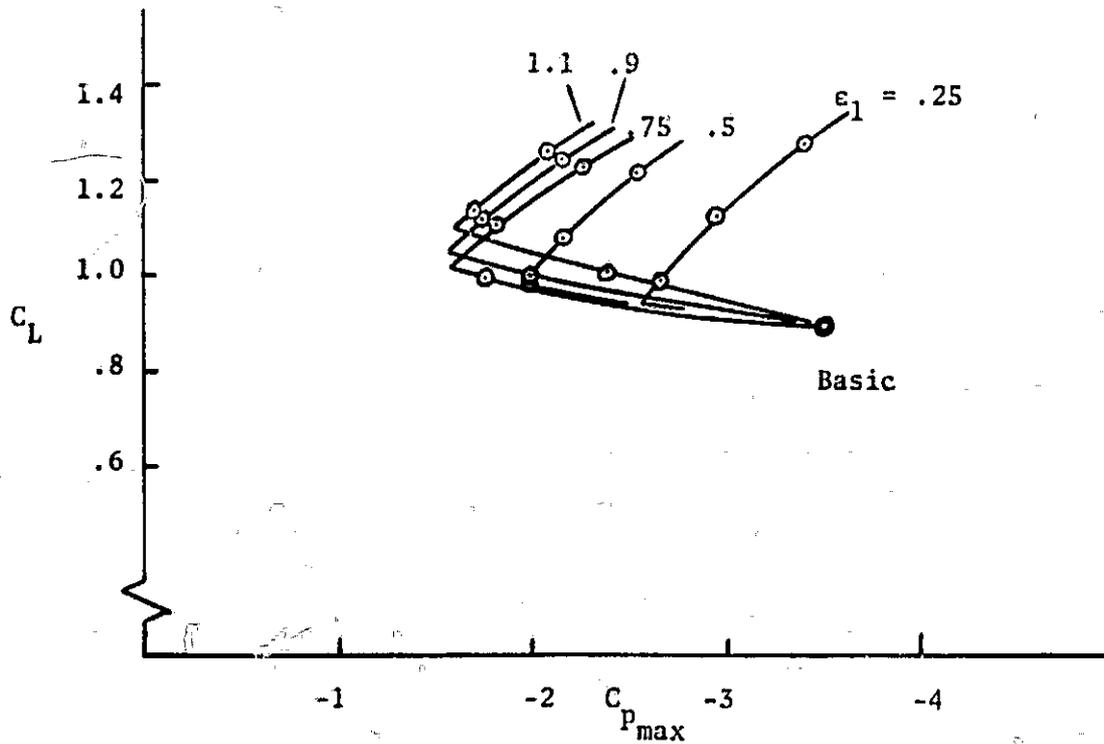


(b) 64₁-212 Airfoil

FIGURE 6. LIFT AND MOMENT VARIATIONS

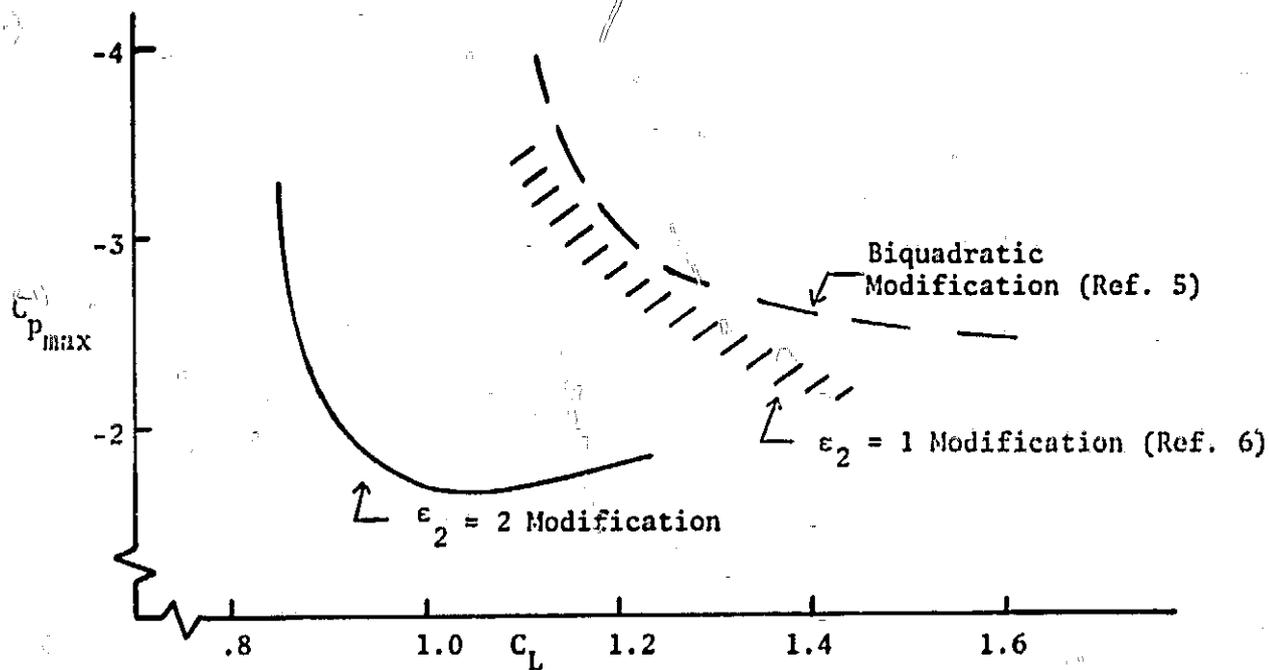


(a) 64-206 Airfoil

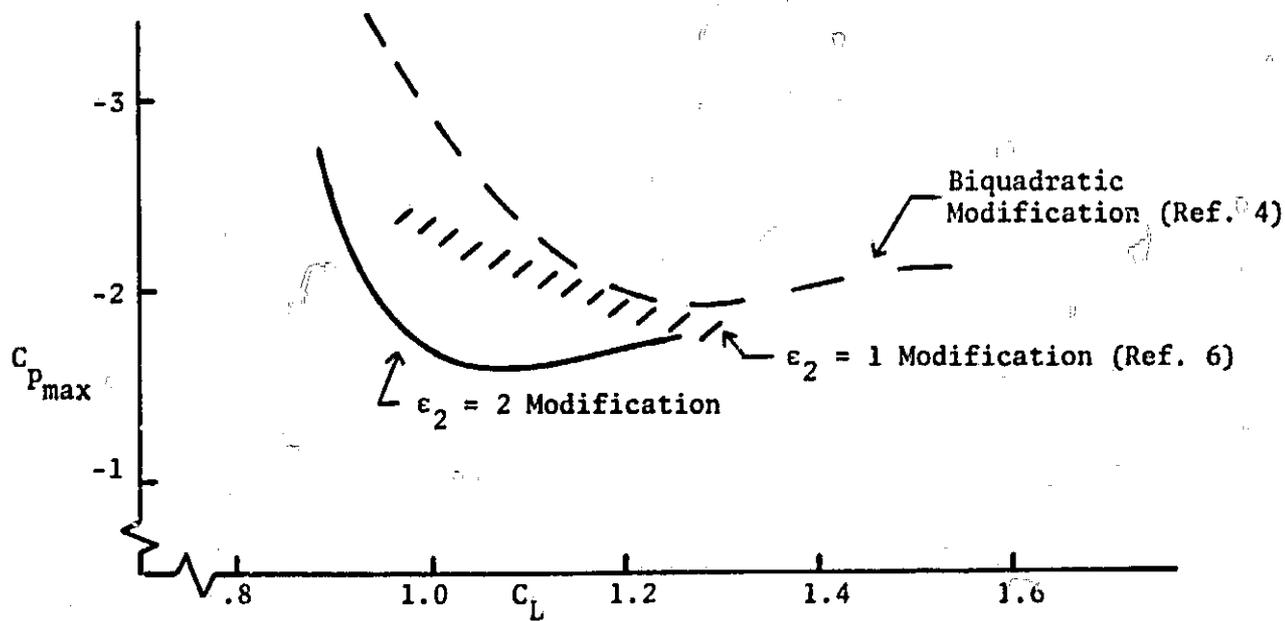


(b) 64₁-212 Airfoil

FIGURE 7. LIFT AND PEAK PRESSURE VARIATIONS



(a) 64-206 Airfoil



(b) 64₁-212 Airfoil

FIGURE 8. MINIMUM PEAK PRESSURE OBTAINABLE FOR GIVEN LIFT

APPENDIX A

CHORDWISE LOCATION OF MAXIMUM THICKNESS

The distribution function used in this study is

$$\Delta y(x) = Ax^{\epsilon_1} (1-x)^{\epsilon_2} \quad (\text{A-1})$$

and the variation of this function is smooth for $0 \leq x \leq 1$. The point of maximum additional thickness occurs when

$$\Delta y^{(1)}(x) = \Delta y(x) \left[\epsilon_1 x^{-1} - \epsilon_2 (1-x)^{-1} \right] = 0 \quad (\text{A-2})$$

This shows that the chordwise location of the point of maximum thickness is

$$\bar{x} = \left[\frac{\epsilon_1}{\epsilon_1 + \epsilon_2} \right] \quad (\text{A-3})$$

and the value of the maximum thickness is then found in terms of the parameters as

$$\Delta y_{\max} = \bar{y} = A \left[\frac{\epsilon_1^{\epsilon_1} \epsilon_2^{\epsilon_2}}{(\epsilon_1 + \epsilon_2)^{\epsilon_1 + \epsilon_2}} \right] \quad (\text{A-4})$$

For the case studied in this report, $\epsilon_2 = 1$, and the parameter A is

$$A = \bar{y} \left[\frac{1 + \epsilon_1}{\epsilon_1^{\epsilon_1}} \right] \quad (\text{A-5})$$

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