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Final Report

ESTIMATING PROPORTIONS OF OBJECTS FROM MULTISPECTRAL SCANNER DATA

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16. Abstract
Further progress was made in developing and testing methods of estimating, from multispectral scanner data, proportions of target classes in a scene when there are a significiant number of boundary pixels. Procedures were developed to exploit: (1) prior information concerning the number of object classes normally occurring in a pixel, and (2) spectral information extracted from signals of adjoining pixels. Two algorithms, LIMMIX and nine-point mixtures, based on (1) and (2), respectively, are described along with supporting processing techniques. An important by-product of the new procedures, in contrast to the previous method, is that they are often appropriate even when the number of spectral bands is small.

Preliminary tests on LANDSAT data sets -- where target classes were (1) lakes and ponds, and (2) agricultural crops -- were encouraging.

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1

PREFACE

This report describes part of a comprehensive and continuing program of research in multispectral remote sensing of the environment from aircraft and satellites. The research is being carried out for NASA's Lyndon B. Johnson Space Center, Houston, Texas, by the Environmental Research Institute of Michigan (formerly the Willow Run Laboratories, a unit of The University of Michigan's Institute of Science and Technology). The basic objective of this program is to develop remote sensing as a practical tool for obtaining extensive environmental information quickly and economically.

In recent times, many new applications of multispectral sensing have come into being. These include agricultural census-taking, detection of diseased plants, urban land studies, measurement of water depth, studies of air and water pollution. and general assessment of land-use patterns. Yet the techniques employed remain limited by the resolution capability of a multispectral scanner. Techniques described in this report may help to overcome this limitation. They may produce more accurate estimates of target classes in a scene when a significant number of pixels are on boundaries.

To date, our work on estimation of proportions has included: (1) extension of the signature concept to a mixture of ground materials; (2) development of a statistical and geometric model for sets and mixtures of signatures; (3) evaluation of computational methods used to estimate proportions of a mixture by maximum likelihood; (4) creation of a computational technique for assessing the expected accuracy of estimation as a function of the signature set; (5) development of techniques to identify alien objects; (6) testing and evaluating the proportion estimation algorithms on artificial as well as actual multispectral scanner data; (7) extension of the basic proportion estimation techniques to exploit prior and spatial information; and (8) preliminary evaluation of these extensions on space-gathered multispectral scanner data.

The research covered in this report was performed under Contract NAS9-14123, Task IV, and covers the period from 15 May 1974 through 14 March 1975. Dr. Andrew Potter has been Technical Monitor for NASA, and Dr. A.H. Feivison has been Task Monitor. The program was directed by R.R. Legault, Vice-President of the

CONTENTS

FIGURES

TABLES

1
SUMMARY

The potential applications of remote sensing are numerous. However, some of these applications are hampered by the limited spatial resolution of the sensing device. To surmount this difficulty, procedures have been developed to permit more accurate estimates of proportions of target classes in a scene when there are a significant number of boundary pixels.

This report covers a fourth phase in the development of proportion estimation techniques. In the first three phases, a basic solution to the problem was developed and tested, first on artificial data; and later, when it became available, on actual space data. Along with the estimation technique, two ancillary developments were pursued: 1) a statistical test to detect pixels containing alien (unknown) materials, and (2) a geometrical test on the signature set to determine the suitability of the associated data set for proportion estimation processing.

Experience with processing actual space data led to two extensions of the basic proportion estimation technique. These extensions constitute the fourth phase reported herein. One of them (LIMMIX) incorporates prior information in that it is based on the assumption that the number of object classes that can occur simultaneously in a pixel is very limited. The other (nine-point mixtures) is also based on this concept; but, in addition, utilizes spatial information. For a particular pixel, this spatial information is extracted from the signals of the adjoining pixels.

Along with these two extensions, suitable alien object detection procedures were devised. Also, a geometrical test of the signature set was constructed for determining the suitability of the associated data for LIMMIX or nine-point mixtures processing. In addition, it was found necessary to develop a clustering procedure for obtaining signatures when the training fields were narrow. These two procedures have an important advantage over the older procedure (MIXMAP). Whereas, for MIXMAP the size of the signature set can be no larger than the number of spectral channels plus one; for LIMMIX and 9-point mixtures the size of the signature set, in principle, may be unlimited even when the number of spectral channels is as low as two.

Preliminary tests of LIMMIX and nine-point mixtures were made on space data and the results superior to those obtained by conventional recognition processing or the previous proportion estimation procedure. Further investigation is required for solving the problem of setting the parameters of the procedures. Also, it appears that additional experimentation with multiple signatures for single object classes would be fruitful.

9

2

INTRODUCTION

In recent years the staff at ERIM has participated in the development of various techniques for multispectral remote sensing applications, including agricultural land use measurement, geologic classification and water depth measurement.

In conventional multispectral recognition, the total area of each ground material is measured by identifying the material in each ground area (pixel) covered by one resolution element of a multispectral scanner. The total area covered by a ground material is found by adding up the pixels identified with that material. If almost every pixel in the ground scene contains just one of the possible materials, this technique provides adequate estimates of acreages. However, if the pixel contains substantial amounts of more than one material, the pixel cannot be properly classified. For LANDSAT satellite data over agricultural scenes, in which each pixel covers about 1.1 acres, the number of pixels containing significant portions of more than one material may approach 30% of the total.

The purpose of the present effort is to obtain improved area estimates of ground materials in these cases. We attempt to overcome the problem of boundary pixels in two ways. First, we determine which pixels are likely to be on a boundary. Then, for these, we estimate the proportion of materials within.

Since its inception, this effort has consisted of a mix of theoretical model studies and tests with both simulated data and modest amounts of ground-truthed real data. Now that real data sets with adequate associated ground truth are becoming available, we are using these exclusively in testing and developing mixtures procedures. The past history of the effort is summarized below to provide a context for this report.

Our work on estimation of proportions was accomplished in several phases. In the first phase[1,2], a mathematical model was constructed which related the multispectral signatures of a mixture to the signatures of component materials. This model permitted the maximum likelihood estimate of the proportion vector to be formulated in terms of the observed data point. The computational aspects of the problem required this simplification: that all of the covariance matrices of the signatures of the component materials be taken as equal to their average. Theoretical and empirical results supported the validity of this assumption. With this simplification, proportion estimation becomes a quadratic programming problem. Several existing computational methods of quadratic programming were adapted and tested on simulated scanner data. Results indicated that this method for proportion estimation was feasible.

The second phase of the program[3] included investigating the problem of detecting alien objects--i.e., objects in the scene not represented in the signature set. A procedure was devised for rejecting those pixels which probably contained significant amounts of alien materials. In addition, aircraft scanner data were smoothed over LANDSAT sized resolution elements to simulate spaceborne scanner data. When proportion estimation techniques were tested on this data, estimates of crop acreage based on the estimated proportions were found to be better than estimates obtained with conventional recognition techniques.

The third phase of the program was devoted largely to reducing computation time required for the procedures. This was accomplished by improving the basic algorithm. It takes about 20 msec on an IBM 7094 computer

[1]   Horwitz, H. M., R.F. Nalepka, P.D. Hyde, and J.P. Morgenstern, 1971,
      Estimating the Proportions of Objects within a Single Resolution
      on Remote Sensing of Environment, Report No. 10259-1-X, May 1971
      Willow Run Laboratories of the Institute of Science and Technology,
      The University of Michigan, Ann Arbor.
[2]   Nalepka R.F., H. M. Horwitz, and P.D. Hyde, 1972, Estimating
      Proportions of Objects From Multispectral Data, Report No. 31650-73-T
      Willow Run Laboratories of the Institute of Science and Technology,
      The University of Michigan, Ann Arbor.
[3]   Nalepka, R.F., and P.D. Hyde, 1973, Estimating Crop Acreage From
      Space-Simulated Multispectral Scanner Data, Report No. 31650-148-T,
      Environmental Research Institute of Michigan, Ann Arbor.

to process LANDSAT signal assuming there are five signatures. In order to reduce processing time still further, averaging procedures were considered. Averaging improved the speed of estimation by a factor approximately equal to the number of points included in the average; but accuracy of estimation, contrary to theoretical expectations, was unsatisfactory. During this phase, satellite data with associated ground truth information became available. Testing of the procedures on this data, as well as results of other investigators[5,6,7] suggested extensions of the basic proportion estimation procedure.

Investigation of two extensions constitutes the fourth phase of our program and covers the period of this report. One extension is based on the assumption that the number of object classes that can occur simultaneously in a single pixel is very limited. Although our experimental computer program (called LIMMIX) permits taking this limit as large as 4, experience shows that two is an effective value. The other extension (called "nine-point mixtures") incorporates this limiting concept; but, in addition, utilizes spatial information. For a particular pixel, this spatial information is extracted from the signals of adjoining pixels.

These two procedures, LIMMIX and nine-point mixtures, have an important advantage over the original proportion estimation procedure, MIXMAP. A necessary requirement for MIXMAP processing is that the size of the signature set be no larger than the number of spectral channels plus one. However for LIMMIX and nin-point mixtures, the size of the signature set may be , in principle, unlimited even when the number of spectral channels is as low as two.

[5] Malila, W.A., and R.F. Nalepka, 1973, Atmospheric Effects in ERTS-1 Data and Advanced Information Extraction Techniques, Symposium On Significant Results Obtained From the Earth Resources Technology Satellite-1, Vol. 1, Goddard Space Flight Center, Greenbelt, MD.

[6] Thomson, F. J., 1973, Crop Species Recognition and Mensuration in the Sacramento Valley, Symposium on Significant Results Obtained From the Earth Resources Technology Satellite-1, Vol. 1, Goddard Space Flight Center, Greenbelt, Md.

[7] Richardson, W., 1974, A Study of Some Nine-Element Decision Rules, Report No. 190100-32-T, Environmental Research Institute of Michigan, Ann Arbor.

Preliminary tests of these new procedures were·made on ERTS data
sets. One scene contained a number of lakes and ponds and the objective
of the tests was to measure the surface water acreage. The other scenes
were agricultural with selected target crops. Results were encouraging.

The next section reviews our basic approach to proportion estimation.
The LIMMIX procedure is explained in Section 4 and results of tests are
presented. Section 5 contains a description of the nine-point mixtures
algorithm. It also contains comparison tests of this procedure with
selected other procedures. More or less burdensome details of all
sections have been relegated to appendices.

APPROACH TO PROPORTION ESTIMATION

A basic application of remote sensing is the determination of the
proportion of a scene covered by a target class (object class of interest).
For example, what proportion of a 5 x 20 mi. segment of Fayette County,
Illinois was covered by wheat on 12 June 1973? The usual approach to
obtaining an estimate of the proportions of target classes in a scene is
based on the assumption that each pixel contains a single object class.
For multispectral data gathered at space altitudes, we know that pixel
size is relatively large compared to field size for a typical agricultural
scene, and that often 30% of the pixels may be boundary pixels (pixels which
contain more than one object class). Reference[3] contains a discussion
of the mechanism by which errors are introduced into the estimate of the
proportions of target classes by processing procedures which do not
account for boundary pixels.

To the best of our knowledge, ERIM was the first to take into account
boundary pixels by associating signatures with mixtures of object classes[1,2].
Later Detchmendy and Pace [8] published an approach which was quite similar
(see reference [9] for a comparison of the methods. More recently, H. O. Hartley

[1] Horwitz, H. M., R.F. Nalepka, P.D. Hyde, and J.P. Morgenstern, 1971.
     Estimating the Proportions of Objects Within a Single Resolution
     Element of a Multispectral Scanner, Seventh International Symposium
     on Remote Sensing of Environment, Report No. 10259-1-X, May 1971,
     Willow Run Laboratories of the Institute of Science and Technology,
     The University of Michigan, Ann Arbor.

[2] Nalepka, R. F., H. M. Horwitz, and P.D. Hyde, 1972, Estimating
     Proportions of Objects From Multispectral Data, Report No. 31650-73-T
     Willow Run Laboratories of the Institute of Science and Technology,
     The University of Michigan, Ann Arbor.

[3] Nalepka, R. F., andP.D. Hyde, 1973, Estimating Crop Acreage From
     Space-Simulated Multispectral Scanner Data, Report No. 31650-148-T,
     Environmental Research Institute of Michigan, Ann Arbor.

[8] Detchmendy, D.M., and W. H. Pace, 1972, A Model for Spectral
     Signature Variability for Mixtures, Earth Resources Observation and
     Information Analysis Systems Conference, Tullahoma, Tennessee.

[9] Salvato, Jr., P. 1973, Iterative Techniques to Estimate Signature
     Vectors for Mixture Processing of Multispectral Data, Conference
     on Machine Processing of Remotely Sensed Data, Purdue University,
     Lafayette, Indiana.

has suggested a modified moment method approach to account for boundary pixels. Many other current methods for proportion estimation (see, for example, [10]) take as a model what is termed "mixtures of distributions" in the statistical literature. This model does not account for boundary pixels.

This section sketches the basis of ERIM's approach to proportion estimation. Included is a discussion of the correlation assumption implicit in the model for signatures of mixtures of object classes within a single pixel. Evidence supporting the validity of this assumption for LANDSAT-size pixels is presented. The procedure for estimating the proportions of object classes within a pixel is then explained and the rationale for making the simplifying assumption of equal covariance matrices of the signatures is presented. Finally, possible fruitful extensions of the basic proportion estimation procedures are discussed.

3.1 MODEL FOR SIGNATURES OF MIXTURES

When the IFOV (Instantaneous Field of View) of a multispectral scanner is large with respect to the structure of the scene being scanned, a single resolution cell (pixel) may contain more than a single object or material. A mathematical model has been constructed which relates the signature of a mixture of materials to the signatures of the component materials. Suppose the scanner has n spectral channels and that the signature of object class i, where $1 \leq i \leq m$, is represented by the n-dimensional Gaussian distribution with mean A_i and covariance matrix M_i. Let the proportion of object class i be λ^i and let λ be the vector $(\lambda^1, \lambda^2, \dots \lambda^n)^t$, where the superscript t denotes transpose. The signature of the mixture with proportion

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[10] Odell, P.L., J.P. Basu, & W. Coberly, 1974, Concerning Several Methods for Estimating Crop Acreage Using Remote Sensing Data, Progress Report June 1, 1974-August 31, 1974, The University Of Texas at Dallas.

vector is taken to be a Gaussian distribution, with mean A_λ and covariance matrix M_λ given by

$$A_\lambda = \sum \lambda^i A_i = A\lambda$$

$$M = \sum \lambda^i M_i$$

where A is the matrix with i^{th} column A_i. These formulas constitute our model for signatures of mixtures of materials in terms of signatures of the individual materials.

3.2 ERIM CORRELATION ASSUMPTION

Examination of the derivation of the model given in Referende [2], section 2.1, reveals that it is assumed that the correlation is zero between random variables associated with signals from nonoverlapping small areas in a pixel. Critics have pointed to this as being a serious flaw. R. Crane of ERIM suggested an experiment to test the extent of the validity of the ERIM correlation assumption. The general idea is as follows. From Aircraft data, select a number of fields containing the same crop type. Use field center pixels only and assume that the correlation function of the signals from the pixels depends only on the distance between the pixels. Estimate the correlation function for selected channels of data. If we find that the correlation distance is small relative to the size of a LANDSAT size pixel, then the ERIM model would be validated to some extent for LANDSAT size pixels. Although the details of the experiment appear straightforward, there are two complicating factors: between field variations and scan angle effects.

In order to minimize the effect of the first factor, an estimate of the correlation function is made for each field separately and then an average taken over all fields. In order to reduce the effect of the second factor, estimates do not utilize pairs of observations along lines of data, only between lines. Also, a sample mean and variance is used for each angle in a field. Details of the estimation procedure are contained in Appendix A.

[2] Nalepka, R.F., H,M. Horwitz, and P.D. Hyde, 1972, Estimating Proportions of Objects From Multispectral Data, Report No. 31560-73-T Willow Run Laboratories of the Institute of Science and Technology, The University of Michigan, Ann Arbor.

The correlation assumption of the ERIM mixtures model was tested accordingly. The data used was from segment 203 of the Corn Blight Watch Experiment gathered by aircraft at 5000 ft. over Indiana on 13 August, 1973. Seven large fields were chosen at random for the correlation test. For each field and each of four channels,* correlations were computed for distances of up to 47 aircraft pixels or slightly lees than three LANDSAT satellite pixels. The average correlation per channel for all the fields was calculated and plotted.

Figure 1 shows that each of these plots quickly falls to near zero. As separations become large, there are fewer correlation measurements that can be made. Thus, at large distances, this correlation test becomes statistically unreliable. In channel 4 there is clearly some sinusoidal noise superimposed on the signal.**

Figure 2 shows correlation curves of four individual fields in channel 1. They appear to be random when compared to the average curve of channel 1 in Figure 1. The other channels displayed as much or more randomness.

The results of this test, as displayed in Figure 1, support the validity of the correlation assumption in the ERIM model with respect to LANDSAT data. The correlation falls to near zero in a distance that is small with respect to the size of a LANDSAT resolution element. This closely approximates the model's assumption of no correlation between signals from different locations within a LANDSAT pixel. Figure 2 shows that what little correlation there is cannot be used as a correction to the mixtures model because the correlation function seems to be a random variable on a field by field basis.

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*10-channel aircraft data was used for the correlation test. To limit the test to a reasonable amount of computation time, only the first 4 channels were used. It was felt that four was enough to make the correlation test valid, although eventually the longer wavelength bands should be checked.

**The peaks are separated by more than 3 aircraft pixels, which rules out row structure as the reason for the sinusoidal pattern.
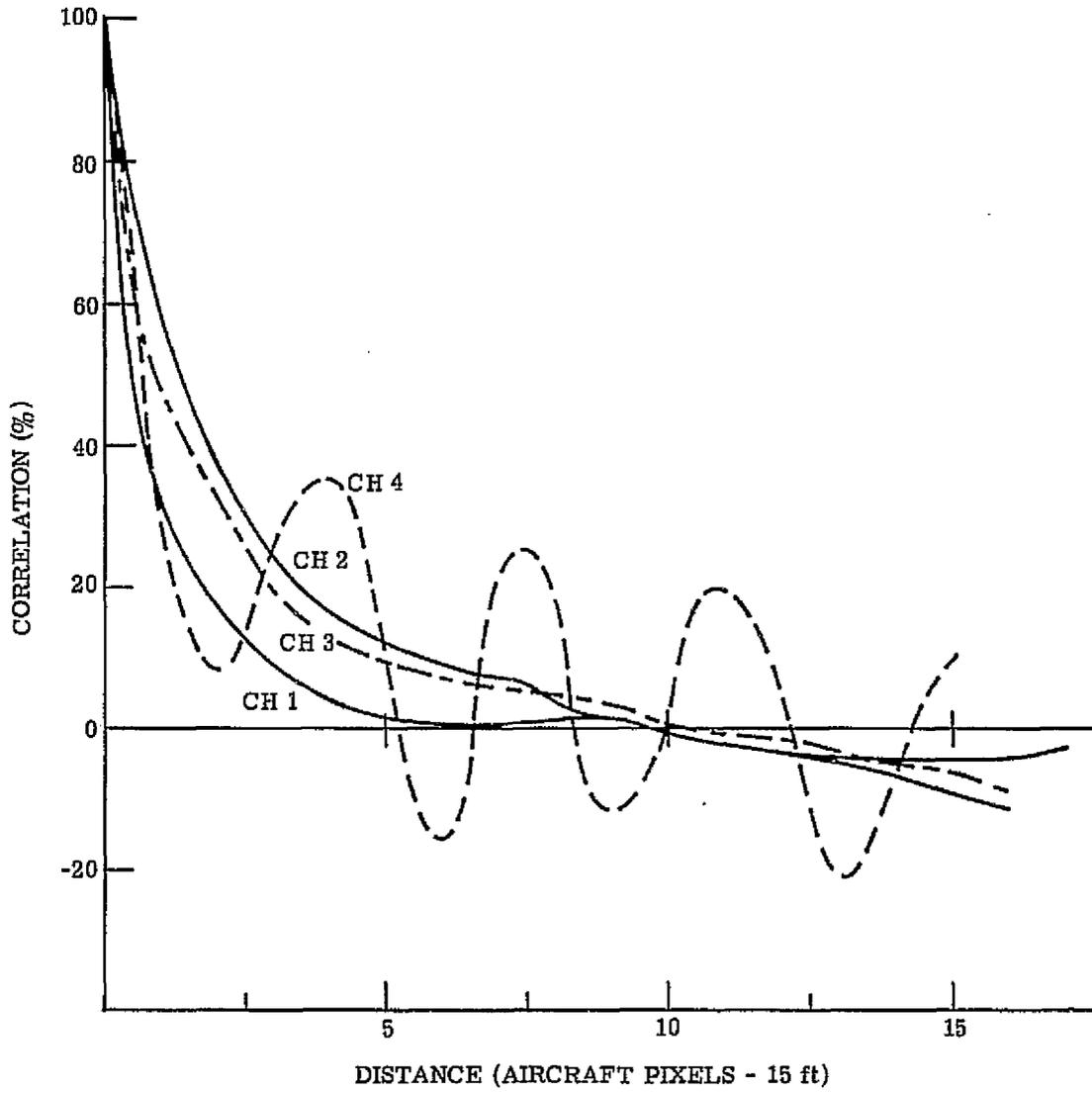
FIGURE 1. CORRELATION (AVERAGE OVER SEVEN FIELDS) VS DISTANCE

18
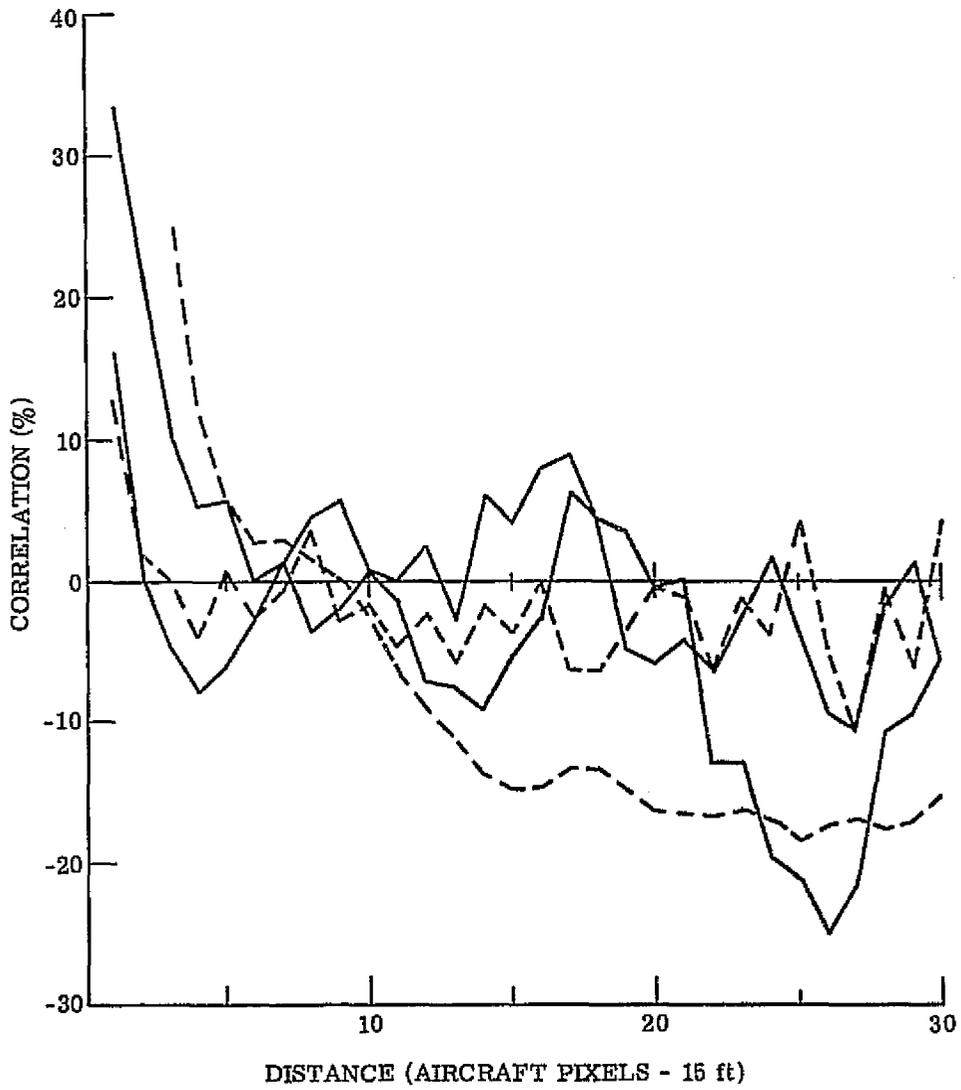
FIGURE 2. CORRELATION IN CHANNEL ONE VS DISTANCE. The plot is of
four individual fields.

19

After our experiment had been performed, we learned that Coberly[11] of NASA/JSC had previously conducted a similar investigation. His pixel size was approximately 12 feet. The details of the experiment varied from ours in that he used a single large rye field and a slightly different estimation procedure. Nevertheless, our results and his were very close. Thus we have additional evidency of the validity of the ERIM correlation assumption for LANDSAT data.

3.3 ESTIMATION OF PROPORTIONS (MIXMAP PROCEDURE)

The model for a mixture signature can be used to estimate the proportion vector corresponding to a signal data vector from a multispectral scanner. Let y denote the n-dimensional data vector from the scanner. A maximum likelihood estimate of the proportion vector [2] is a value of which minimizes

$$F(\lambda) = n|M_\lambda| + <y_\lambda - A_\lambda, M_\lambda^{-1}(y - A_\lambda>$$

subject to the constraints that

$$\lambda^i = 1 \text{ and } \lambda^i \geq 0 \quad \text{for} \quad 1 \leq i \leq m$$

Here $|M|$ denotes the determinant of M, M^{-1} is its inverse, and u,v denotes the inner or dot product of the vectors u and v.

In general, minimizing $F(\lambda)$ subject to the given constraints is quite difficult. Investigations[2] showed that a good approximation to the minimal λ could be obtained if a simplifying assumption is made. The assumption is that the average of the covariance matrices of the pure signatures can be substituted for each M_i. By using the simplifying assumption

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[11] Coberly, W.A., 1973, Serial Correlation of Spectral Measurements, NASA Internal Memorandum, JSC, Houston.

[2] Nalepka, R.F., H. M. Horwitz, and P.D. Hyde, 1972, Estimating Proportions of Objects From Multispectral Data, Report No. 31650-73-T, Willow Run Laboratories of the Institute of Science and Technology, The University of Michigan, Ann Arbor.

and applying a linear transformation which reduces the common covariance matrix to the identity, the problem of estimating   becomes one of minimizing a function $G(\lambda)$ of the form

$$G(\lambda) = ||y - A_\lambda||^2$$

subject to the constraints on λ. Now y represents the transformed data point, and A_λ the mean of the signature associated with the proportion vector λ after the pure signature means have also been transformed.

The problem of minimizing $G(\lambda)$ subject to the constraints on λ can b viewed geometrically. The set of points $A_\lambda = A\lambda$, where λ is a proportion vector, is the convex hull of the A_i and is called the signature simplex The problem is to find a proportion vector $\hat{\lambda}$ such that $A\hat{\lambda}$ is the point in the signature simplex closest to the data point y.

The optimal $\hat{\lambda}$ will be unique if the signature simplex is non-degenerate, i.e., has positive m – 1 dimensional volume. This is equivalent to the (n+1)-dimensional vectors A_i^t, 1) being linearly independent. Non-degeneracy of the signature simplex implies that the number of materials m in the pure signature set does not exceed the number of spectral channels n by more than one,

The problem of minimizing $G(\lambda)$ can be identified as a quadratic programming problem. A program adapting the Theil & van de Panne method for solving this type of problem is used to estimate the proportions of object classes within a pixel. Details may be found in References [2,4,12]. The computer program

[2]   Nalepka, R.F., H.M. Horwitz, and P.D. Hyde, 1972, *Estimating Proportions of Objects From Multispectral Data*, Report No. 31560-73-T, Willow Run Laboratories of the Institute of Science and Technology, The University of Michigan, Ann Arbor,

[4]   Horwitz, H.M., P.D. Hyde, and W. Richardson, 1974, *Improvements in Estimating Proportions of Objects From Multispectral Data*, Report No. 190100-25-T, Environmental Research Institute of Michigan, Ann Arbor.

[12]   Kunzi, H.P., W. Krella, and W. Oettli, 1966, *Nonlinear Programming*, Blaisdell Publishing Co., Boston.

is called MIXMAP, and in view of the fact that otherprocedures for estimating proportions are introduced in sections 4 and 5, we shall refer to this basic algorithm as the MIXMAP procedure. It requires about 20 msec to estimate a mixture of 5 materials with 12 channels of data.

3.3.1 DATA AVERAGING

In order to reduce computation time, the MIXMAP program has a data averaging option[4]. This option provides for averaging a number of data points and then estimating proportions of the target classes in the region corresponding to the totality of the data points averaged. This averaging procedure reduces computation time by a factor approximately equal to the number of data points averaged. It also has theoretical advantages in that the estimates of proportions are asympotitically unbiased in an ideal situation. However, up to now, results of limited tests on LANDSAT data using data averaging have not been impressive. More testing is necessary in order to evaluate this procedure more completely.

3.4 EQUAL COVARIANCE ASSUMPTION

The substitution of the average covariance matrix for the individual covariance matrices of the different object classes has been criticized. This assumption was made to facilitate the computation of proportion estimates after making simulation runs using typical agricultural signature sets to test the validity of this substitution. Results indicated that this approximation was reasonable. But the decisive factor in making this substitution was the fact that we know of no reasonable numerical procedure for obtaining the exact maximum likelihood proportion estimate, nor has anyone recommended any appropriate alternative procedure.

3.5 DETECTION OF ALIEN OBJECTS

Estimating proportions of unresolved objects from a signal y is based on the assumption that the signal comes from a pixel which contains a micture of materials. These materials are represented by known signatures that constitute the pure signature set. If the pixel should contain a material not represented in the signature set, significant additional error in the estimate of proportions may result. The amount of this error depends upon

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[4] Horwitz, H.M., P.D. Hyde, and W. Richardson, 1974, Improvements in Estimating Proportions of Objects From Multispectral Data, Report No. 190100-25-T, Environmental Research Institute of Michigan, Ann Arbor.

the proportion of these alien materials and the geometric relationship of their signatures to those in the pure signature set. Those materials occuring in a scene but not represented in the pure signature set are referred to as alien materials or alien objects. Procedures have been designed to reduce the error resulting from the presence of alien objects. These procedures take the form of thresholding tests—hence the designation "alien object threshold."

One might attempt to avoid the alien object problem by obtaining signatures for all materials present in the scene. This approach is usually impractical because of the large number of materials present and the impossibility of obtaining definitive signatures for many of them. An alternative is to use essentiall a chi-square test as in conventional recognition processing.

The new mixtures program contains improved procedures for dealing with alien objects. These procedures can be described most easily in terms of the pure signature set and signals after a linear transformation has been employed. After this transformation, we assume that the i-th material in the pure signature set has mean A_i, and its covariance matrix is the identity. Now given a signal (data point) y from a pixel with unknown proportions of various materials, the estimate $\hat{\lambda}$ of the proportion is obtained as follows. Let Z denote the point in the signature simplex closest to y. Then Z may be represented in the form

$$Z = A\hat{\lambda}$$

where $\hat{\lambda}$ is a proportion vector and is taken as the estimate of proportions in the pixel represented by the signal y. In order to apply an alien object test, we ask, "What is the probability that we would have observed the signal with value exceeding y if the true proportion of the pixel was $\hat{\lambda}$?" Assuming Gaussian signature distributions, this amounts to a chi-square test with n degrees of freedom, where n is the number of spectral channels used. The level of significants is determined by a value χ_o^2, which is the alien object threshold. If

$$||y - z||^2 = ||y - A\hat{\lambda}||^2 > \chi_o^2$$

then the estimate fails the chi-square test; we then say that the pixel contains significant amounts of alien materials and make no estimate of

proportions for the pixel in question. If the estimate passes the test,
we accept it as the estimate of proportions of materials in the pixel
in question.

3.6 SIGNATURE ANALYSIS

The quality of the estimates of proportions one can expect can be
determined to a large extent by examining the pure signature set. In
conventional recognition processing we know that the quality of results
depends upon the distances between pairs of signature means relative to
their spreads (covariances). When these distances are large, good results
can be expected. Not only is this requirement necessary for good proportion
estimates, but a more stringent condition must be satisfied: that no
pure signature be close in a probability sense to any signature of a mixture
of the other materials.

A feature of the MIXMAP program is a simple test called geometric
signature analysis (GEOM). We deal with the transformed signature simplex
with vertices A_i, $1 \leq i \leq m$, and assume that the common covariance matrix
of all the transformed signatures is the identity. Let r_i be the distance
of A_i to the closest point in the hyperplane through the face of the
signature simplex opposite A_i. The face opposite A_i is the convex hull
of all the vertices A_j except for A_i. Then r_i measures this distance, in
standard deviation units of A_i, to the mean of a mixture of the other
materials in the signature set. If some r_i is small, we would expect data
points representing some A_i's to be confused with data points representing
mixtures of the other materials. Figure 3 illustrates a signature simplex
well-conditioned for proportion estimation. The circles at the vertices
indicate the spread of the distributions at the vertices; these circles
were formed by points which are one standard deviation away from the vertex.
Each vertex is several standard units away from the vertex. Each vertex
is several standard units away from the closest point in the opposite
hyperplane. Figure 4, on the otherhand, shows an example of an ill-
conditioned signature simplex. The pure signature mean A_1 is less than a
standard deviation away from the closest point in the opposite hyperplane.
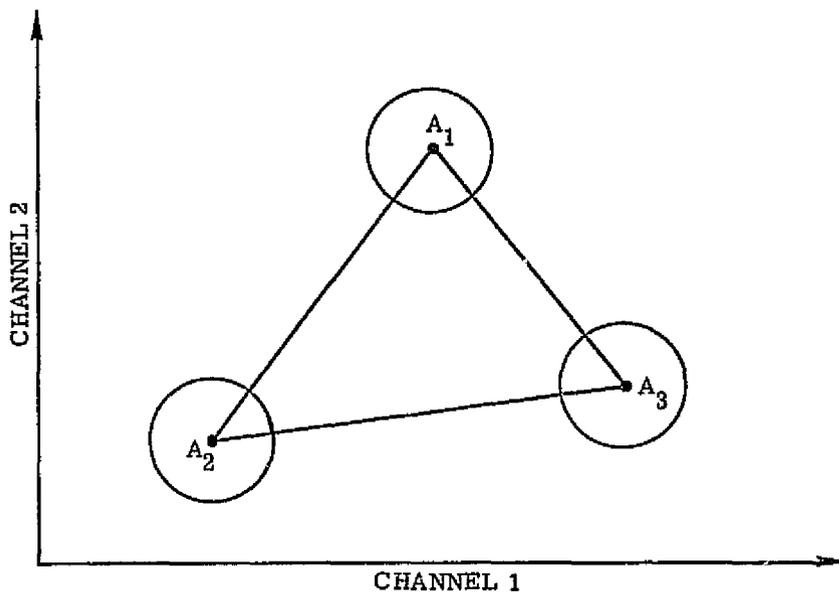
FIGURE 3. WELL-CONDITIONED SIGNATURE SIMPLEX
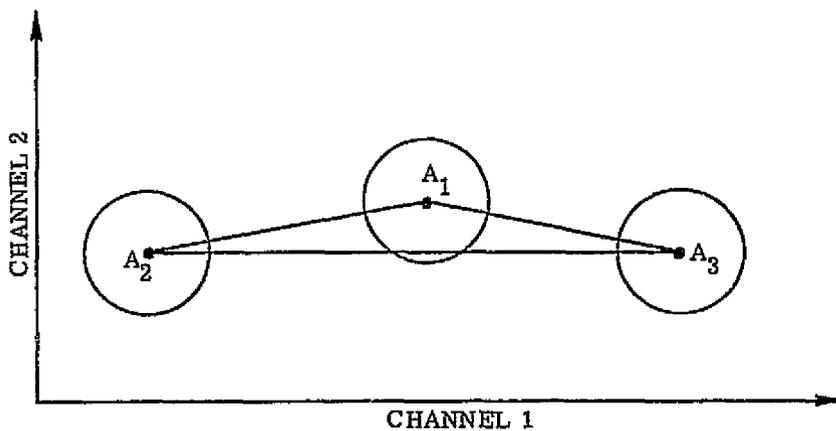
FIGURE 4. ILL-CONDITIONED SIGNATURE SIMPLEX

25

3.7 EXTENSIONS OF BASIC PROPORTION ESTIMATION PROCEDURE

In order to improve performance, the basic ERIM proportion estimation procedure (MIXMAP) has been extended in two directions. One of these extensions results from using prior information about the probable content of pixels. Normally, a majority of pixels are pure (contain a single material). When a mixture pixel occurs, it generally contains a small number of component materials; say 2, 3, or 4. The LIMMIX procedure, described in Section 4, incorporates this kind of prior information.

The other extension results from utilizing spatial information in order to restrict further the combinations of object classes which can occur simultaneously within a single pixel. The spatial information employed consists of the signals from adjoining pixels. The resulting procedure is referred to as nine-point mixtures and is treated in Section 5. It will become clear that nine-point mixtures may be considered an extension of the LIMMIX concept.

Both the LIMMIX and nine-point mixtures procedures have a very important advantage over MIXMAP, especially when the number of spectral channels of information is relatively small as in LANDSAT data. It has been pointed out in Section 3.3 that a necessary requirement for the suitability of MIXMAP processing is that the size m of the signature set and the number n of spectral channels be such that

$$m \leq n + 1$$

Thus, for example, the maximum size of the signature set permissible for MIXMAP processing of LANDSAT data is 5.

The corresponding restriction for LIMMIX and nine-point mixtures processing is much milder although more complicated. Let L denote the maximum number of object classes which are assumed to occur simultaneously in a single pixel. Then a necessary condition for the suitability of LIMMIX or nine-point mixtures processing may be expressed by the following two inequalities:

$$L \leq n + 1 \qquad \text{when } L = m$$

and

$$L \leq n \qquad \text{when } L \neq m$$

Thus for LANDSAT data any size signature set will satisfy this condition as long as L does not exceed 4.

4

UTILIZATION OF PRIOR INFORMATION IN ESTIMATING PROPORTIONS

The experience gained at ERIM with estimating proportions of unresolved objects has led to a number of modifications of the mixtures algorithm. Many of these modifications are similar in that they place limitations on the combinations of object classes which are assumed to occur in a single pixel. Methods for implementing such limitations appear to be of two types. The first type depends on spectral characteristics only, while the second type depends on both spectral and spatial characteristics. The LIMMIX procedure, described in this section is of the first type; while nine-point mixtures, presented in Section 5, is of the second type.

Techniques which support the LIMMIX procedure are also described in this section. In addition, results of preliminary tests are presented.

4.1 LIMMIX PROGRAM

We have found that the number of object classes which occur simultaneously in a single pixel is very limited. LIMMIX exploits this fact. It assumes that no pixel contains more than L, $L \leq 4$ (L is a parameter), object classes simultaneously. In order to facilitate testing and evaluation, the LIMMIX program produces a tape output for further processing. This tape will now be described. Figure 5 is a record of the tape generated for each data point assuming the parameter L was taken to be four. The first four positions give the results for the maximum likelihood single class. Here $\lambda = 1$ because the pixel is all class C_1. Then the likelihood value (a_1) of the data point is stored along with the chi-squared value d_1^2).

The next five entries record the best two at a time choice for the data point. The two λ's are the proportions of the two materials found best and C_2 codes the particular pair chosen. a_2 is the likelihood of the data point with respect to the signature of this best mixture of two objects classes, and d_2^2 is the chi-squared value of the data point with respect to the signature of this pair. Similarly the next six entries on the tape record are the best mixture of a combination of three at a time, and the last seven entries record the best mixture of a combination of 4 object classes. Best is used in the sense of maximum likelihood.

| λ | C_1 | a_1 | d_1^2 | λ | λ | C_2 | a_2 | d_2^2 | λ | λ | λ | C_3 | a_3 | d_3^2 | λ | λ | λ | λ | C_4 | a_4 | d_4^2 |

λ = Proportion
C_i = Combination
a_i = Likelihood

d_1^2 = Chi-Squared Distance

FIGURE 5. ONE RECORD OF THE LIMMIX OUTPUT TAPE

LIMMIX uses the MIXMAP procedure for determining the best mixture of K classes at a time. For example, to find the best three at a time mixture, all subsets of three classes are considered. For each of these subsets, the best mixture of the three classes is obtained via MIXMAP along with the likelihood of the mixture. It is that mixture of three classes yielding maximum likelihood value for the data point that finally appears on the output tape.

In order to obtain results from the LIMMIX tape, further processing must occur. The present processing approach is summarized below. Say the parameter value L is three. Then we choose three threshold values

$$X_1^2, \; X_2^2, \; \text{and} \; X_3^2.$$

If

$$d_1^2 \leqq X_1^2$$

then the pixel is all class C_1. If

$$d_1^2 > X_1^2 \text{ and } d_2^2 \leqq X_2^2$$

then the pixel is taken to contain the mixture associated with the pair C_2 on the LIMMIX tape. If

$$d_1^2 > X_1^2, \; d_2^2 > X_2^2, \text{ and } d_3^2 \leqq X_3^2$$

then the pixel is said to contain the mixture associated with the combination C_3 on the LIMMIX tape. If

$$d_1^2 > X_1^2, \; d_2^2 > X_2^2 \text{ and } d_3^2 > X_3^2$$

then the pixel is taken to contain alien (unknown) materials. Further details of LIMMIX are contained in Appendix B.

4.2 ALIEN2

A computer program, ALIEN2, was developed to operate with LIMMIX to facilitate experimentation. The current version of LIMMIX, as described above, puts all of the calculated results on an output tape, without deciding which k-signatures-at-a-time winner to accept as an overall winner. ALIEN2 then uses this output tape as input, and permits a wide range of decision rules (χ^2 parameters). In effect, LIMMIX is run many times, using only one output tape per scene. ALIEN2 also tabulates the results for each parameter setting, making it relatively easy to evaluate the working parameters of LIMMIX.

In a production set-up (i.e., when it is known how to set the χ_i^2 parameters) the two programs will be combined, with no intermediate tape generated. Since most of the pixels in a scene are pure, it will not always be necessary to calculate the most likely pair triple, etc., of signatures. For instance, if the chi-square distance from the most likely signature to the pixel is within the limit set by the χ_1^2 parameter, the algorithm will call this signature the solution, and go on to the next pixel. If the chi-square distance is greater than χ_1^2, a search will be made to find the most likely signature pair whose distance is less than the χ_2^2 parameter. This process will continue until the pixel is either designated as some combination or is checked as alien. Details of ALIEN2 are in Appendix C.

4.3 GEOMETRICAL SIGNATURE ANALYSIS

A prime factor affecting the performance of LIMMIX is the geometrical configuration formed by the signatures of the object classes occurring in the scene. In the previous ERIM mixtures approach implemented by the program MIXMAP, geometrical signature analysis (program GEOM) is normally performed on the signature sets to determine its adequacy for MIXMAP processing. GEOM supplies measures of how close (in a probability sense) each signature mean is to a point in the hyperplane through the other signature means of the signature simplex.

The larger these distances are, the more non-degenerate is the signature simplex in a probabilistic sense; and the more suitable is the scene for MIXMAP processing. When the number of signatures m in the signature set and the

number n of channels of data are such that

$$m > n + 1$$

it follows that at least one of these distances is zero, which means that
the signature simplex is degenerate, and the associated scene is unsuitable
for MIXMAP processing because the maximum likelihood proportions estimate is
then ambiguous. Thus a necessary requirement for the appropriateness of MIXMAP
processing is that

$$m \leq n + 1$$

This requirement can be a severe limitation, especially when the number of
channels of information is relatively small as in LANDSAT data. The corresponding
conditions for LIMMIX processing which limit the values of parameter L may be
stated as follows:

A necessary condition for the suitability of LIMMIX processing is that
every subset of L + 1 or less signature means form a nondegenerate simplex.
When L = m, the limitation is $L \leq n + 1$. When $L \leq m$, the limitation is

$$L \leq n$$

Thus, in theory, we can use LIMMIX processing with L=4 on LANDSAT four-channel
data with any size signature set. Figure 6 illustrates an example of 6 signature
means and 2 channel data. Any subset of 4 or more of these signature means forms a
degenerate simplex, but any subset of 3 or less forms a nondegenerate simplex;
therefore, the data associated with this signature set might be suitable for LIMMIX
processing with parameter value L=2. To obtain a more quantitative
measure of suitability of a signature set for LIMMIX processing, geometrical
signature analysis is performed on each subset of L+1 signature means. The
requirement for suitability is that each of the L+1 distances obtained for
each of the $\binom{m}{L+1} = \dfrac{m!}{(L+1)!\,(m-L-1)!}$ subsets be adequately large.

The distances obtained for the geometrical signature analysis for
LIMMIX processing (GEOM2) will now be defined more precisely. To avoid
notational complexity we will assume that a specific subset of L+1 signatures
has been chosen and relabeled, if necessary, so that their means are denoted
by A_1, $A_2, \ldots, A_{L+1}$ and covariance matrices by M_1, $M_2 \ldots, M_{L+1}$. Let H_1
denote the hyperplane of dimension L–1 though the means $A_2, \ldots A_{L+1}$ and let
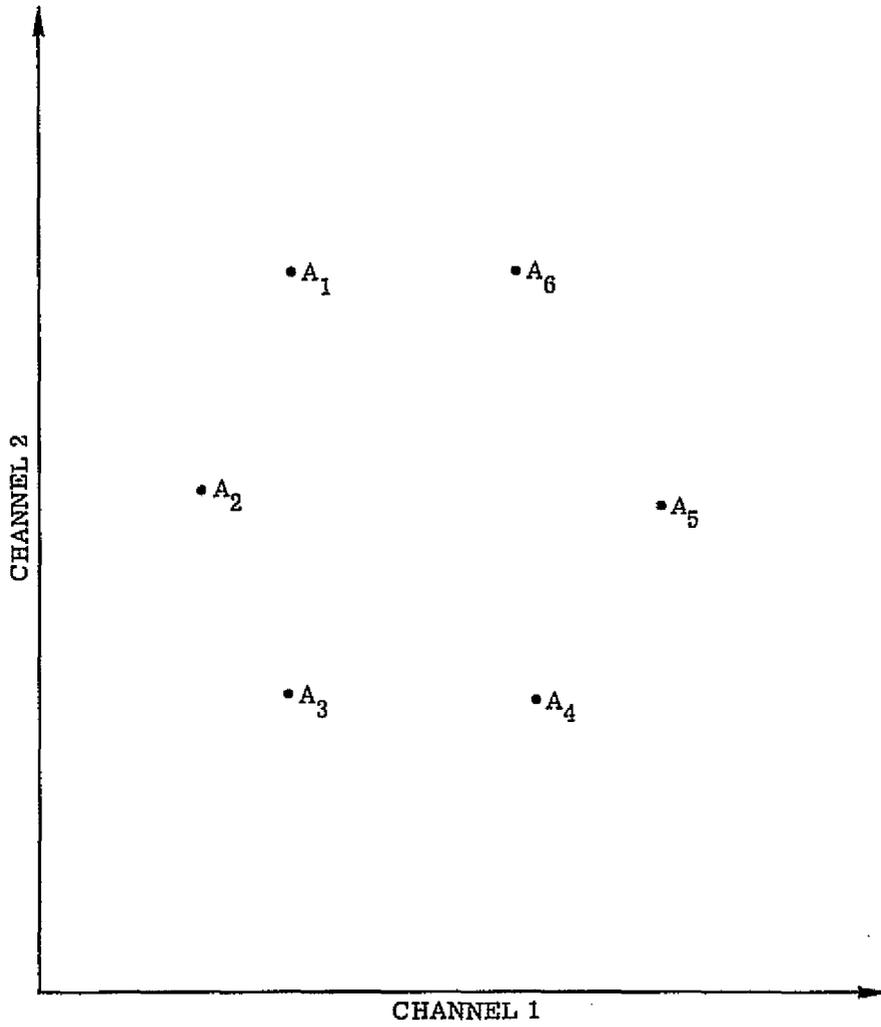
FIGURE 6. SIX SIGNATURE MEANS TWO CHANNEL DATA

Z be the point in H_1 which maximizes the Gaussian density with parameters A_1, M_1. Then d_1 is defined by

$$d_1^2 = <Z-A_1, \; M_1^{-1} \; (Z-A_1)>$$

Figure 7 is an illustration for the case L=2 and n=2. In this example, d_1 is approximately 3.

There is an interpretation of the distances d_i associated with a simplex that may be helpful. It is understood most easily when the covariance matrices are all equal and the usual transformation to the identity is utilized. Then the radius r of the largest inscribed sphere is given by

$$\frac{1}{r} = \sum_{i=1}^{L+1} \frac{1}{d_i}$$

Then r may be taken as a summary measure of the suitability of the simplex. When covariance matrices are not equal, then r as given by this formula, although lacking a simple geometric interpretation, appears to have merit.

Table 1 displays the output from GEOM2 with respect to a CITARS[*][13] data set. This data was gathered 21 August 1973 over Fayette County, Illinois. The target crops were corn and soybeans. The data was used in tests reported in Section 5.2. The signature set contained six classes and the limit L was taken to be two. Thus all possible combinations of three materials required examination. Since there are 20 of these combinations, there are 20 rows in the table. In the first row, first column, for example, 1.7 is the closest distance (measured in standard deviation units of the corn signature) of the mean of the corn signature to the line through the means of soy and trees. In the second column of the first row the entry 2.3 is the closest distance (measured in standard deviation units of the soy signature) of the mean of the soy signature to the line through

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*CITARS was a joint research task for Crop Identification Technology Assessment for Remote Sensing.

[13] Malila, W.A., D.P. Rice, and R.C. Cicone, 1975, Final Report on the Citars Effort by the Environmental Research Institute of Michigan, Report 109600-12-F, ERIM, Ann Arbor, Michigan.
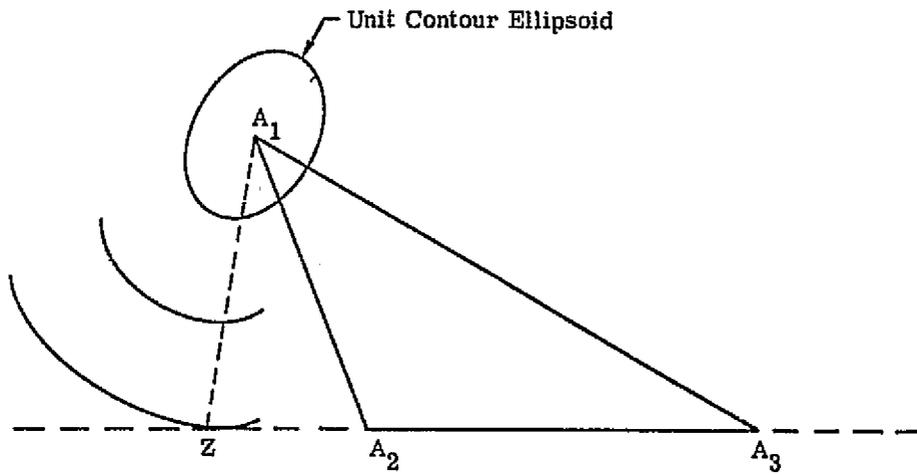
FIGURE 7. GEOMETRICAL SIGNATURE ANALYSIS (GEOM2)

TABLE 1 – DISTANCES CALCULATED BY GEOM2, Fayette Co., 12 August 1973 (Units of Standard Deviation)

CORN	SOY	TREE	BARE	CLOVER	WEED
1.7	2.3	9.3			
2.7	2.2		3.6		
1.6	2.4			4.4	
1.9	2.2				9.9
1.1		2.6	5.0		
2.5		9.2		2.9	
1.6		3.5			10.8
2.2			1.5	5.2	
3.4			1.0		1.8
2.5				5.1	2.4
	2.7	9.1	3.8		
	3.7	7.7		2.8	
	2.9	9.4			9.8
	3.3		1.5	4.1	
	5.9		0.7		3.5
	3.4			4.1	14.9
		3.4	1.5	2.7	
		18.9	1.1		1.0
		3.0		2.7	12.1
			1.5	11.5	1.5

the means of corn and trees. Overall, the distances are fairly large, although the 0.7 for bare soil versus soy and weeds may indicate possible difficulties.

4.3 CLUSTERING PROGRAM

We have found that poor multispectral data processing results are often due to signatures which are not representative of ground class distributions. This in turn may stem from two sources: (1) an insufficient number of data points to obtain a good estimate, and (2) the incorrect determination of the number of modes of the distribution.

The large error that may be introduced in this manner often makes it difficult to evaluate the efficacy of a classification procedure. Clustering algorithms offer hope of a solution. These algorithms may be loosely defined as algorithms which identify data points which are 'alike'. Because this project has been hampered by the errors arising from this problem, suitable algorithms were developed.

4.4.1. Description

To provide versatility, three different algorithms were incorporated into the program.

Algorithm one uses small, normal distributions to approximate the cumulative distribution function of the ground classes in a scene. Then it combines these elements, on the basis of high probability of misclassification, to form signatures. A description of this follows.

(1) Suppose we have m cells $\Gamma_1 \ldots \Gamma_m$, with mean A_i, variances $(\sigma_{i\,1}^2 \ldots \sigma_{i\,n}^2)$ $1 \leq i \leq m$, where n is number of channels. Let K_i denote the number of samples within the ith cell. Given a new sample X, calculate the distance of X from each cell center by

$$d(X, A_i) = \sum_{j=1}^{n} (X_j - A_{ij})/\sigma_{ij}^2 \quad (i=1, \ldots m)$$

Find r such that $d(X, A_r) = MIN_i\ d(X, A_i), \ 1 \leq i \leq m$

Then X is classified as one of the following.

If $\quad\quad d(X,A_r) < \quad \tau$ then X assigned to Γ_r

If $\quad\quad d(X,A_r) > \quad \theta$ then X creates a new cell Γ_{m+1} otherwise X is stored.

(2) When a new sample is classified to the ith cell, this cell's

parameters are adjusted as follows:

(a) increase the number of samples (K_i) by one

(b) calculate a new mean vector (A_i)

$$A_i = \frac{1}{K_i} \sum_{\text{all } x_i \text{ in cell } \Gamma_i} X_i$$

(c) determine new variances by

$$\sigma^2_{i,j} = MAX(\sigma^2_{i,j}(0), S^2_{i,j})$$

where

$$S^2_{i,j} = \frac{1}{K_i} \sum_{\ell=1}^{K_i} (X_{\ell j} - A_{ij})^2$$

where the X_ℓ are classified to the ith cell and $\sigma^2_{ij}(0)$ is an initial

assignment of σ^2_{ij}. Only when S^2_{ij} exceeds $\sigma^2_{ij}(0)$ do we replace $\sigma^2_{ij}(0)$

with S^2_{ij}.

37

(3) The first sample always creates a new cell. The second sample is tested and classified by (1) and so on. When all samples have been classified, the stored samples are forced into the nearest cells according to (1). Each cluster is then tested against every other cluster for a high probability of misclassification. Whenever two clusters are found to have a high probability of misclassification, they are combined with a weighting based on the number of points in the clusters. This process is iterated until one cluster has more than a certain percentage of the points, or the largest several clusters have more than some other percentage of the points. The measure of probability of misclassification used is:

$$p=(P(W_1) \ P(W_2))^{1/2} \ \exp(-(A_2-A_1)^T((M_1+M_2)/2)^{-1}(A_2-A_1))$$

W_1 and W_2 are the two classes involved. The A and M symbols stand for the mean vectors and covariance matrices of the two clusters.

Algorithm two is almost identical to algorithm one, except that it is a supervised algorithm, i.e., each data point is labeled (by crop class) and algorithm one is carried out separately for each class.

Algorithm three is an unsupervised, iterative algorithm which estimates the means and variances of ground class distributions. It is, in part, similar to NSPACE, developed by Eigen and Northouse at the University of Wisconsin[14]. Algorithm three proceeds as follows. First, the user inputs his initial guess of starting means and variances, or allows the program to spread starting means evenly throughout the data space, with a common starting variance. Data points are then classified to these means using either the standard L_1 metric or the linear Bayes decision rule. The estimates of each mean and variance may be updated every time a data point is classified to that mean, or after each scan line or region. The new means and variances are used for further classification. This process is repeated until the estimates of the means and variances change very little from iteration to iteration. Further details are contained in Appendix E.

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[14]   Eigen, D.J., and R.A. Northouse, 1972, N Space—An Unsupervised Clustering Algorithm Based on Discretized Marginal Distributions, Report No. TR-AI-72-3, The University of Wisconsin, Milwaukee.

4.4.2 EFFECTIVENESS

It was found that these algorithms, especially one and two, produce highly accurate signatures. They have been useful in analyzing variations in the data, multi-modality, and identifying troublesome 'other' classes. The use of these algorithms has reduced the error stemming from poor correspondence between signatures and ground class distributions. This has resulted in better evaluation of classification schemes.

4.5 PROBLEM OF ESTABLISHING LIMMIX PARAMETERS

The effectiveness of the LIMMIX procedure is dependent on setting the parameters properly. As an algorithm becomes more sophisticated, it is usually more difficult to set the parameters, because there are more of them. Such is the case with LIMMIX. Even when pixels are limited to mixtures of two signatures, there are three parameters to set. They are χ_1^2, χ_2^2, and τ, the proportion threshold. There is also the option of renormalizing the remaining proportions after thresholding. In MIXMAP there were only the τ and one χ^2 parameter (the alien object threshold) to set. The only known method for establishing parameters is to run the algorithm on training data. A wide variety of parameter combinations are used. The parameter set giving the closest estimate of the training area ground truth is then used on the test area. It is also difficult to set the parameters in the nine-point algorithm as explained in Section 5. In Section 4.6, tests are made on LANDSAT data in order to devise techniques for establishing parameters.

4.6 PRELIMINARY TESTS

Two data sets were chosen for preliminary testing of the LIMMIX algorithm: (1) A water data set consisting of 20 generally small lakes and ponds in an eight square mile area near Lansing, Michigan, and (2) a fourteen section agricultural data set from Hill County, Montana.

The first data set was chosen for an initial test because water is a relatively high contrast target. Also, other algorithms had already been tried on the water data. This provided a basis for comparing the results that LIMMIX generated.

The Hill County Data was selected as the agricultural test of LIMMIX. for two reasons. The area's main crop is wheat, the target crop of the soon to be implemented LACIE*[15] project, and the area contains many narrow fields. The latter insures that there will be numerous mixture pixels to exercise the LIMMIX algorithm.

4.6.1 WATER DETECTION

A water detection project [3] previously done with MIXMAP was redone using LIMMIX. As before, the data set was divided into water and non-water regions. The detection rate is defined as the area of water found in the water region as compared to the area known from ground truth. The false alarm rate is the area of water detected in the non-water region divided by the area of that region. When the detection rate is plotted against the false alarm rate, we obtained the so-called operating curves of the algorithm.

Figures 8, 9, and 10 shows the operating curves for MIXMAP and LIMMIX. These curves represent the best performance of each algorithm, and will be compared as such.

The MIXMAP graph (Figure 8) is for various rejection probabilities and thresholds (water only). The thresholding, needed to cut down the numerous false alarms, gives the best operating curves.

LIMMIX, on the other hand, thresholds all materials. Thresholding all of the signatures will reduce the detections and false alarms. False alarms are not as large a problem with LIMMIX due to the recognition portion of the algorithm. The renormalization process, which increases the detections, is therefore the preferred operating mode. The operating curves of LIMMIX for various combinations of χ_1^2 and χ_2^2 values are presented in Figure 9.

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*LACIE is a joint project for a Large Area Crop Inventory Experiment. LACIE
    results will contribute to a future operational system for worldwide crop
    inventory using remote sensing and computer technology.

[15] Large Area Crop Inventory Project Plan, November 18, 1974, NASA-NOAA-USDA
    Report No. LAP01, NASA/JSC, Houston, Texas

[3] Nalepka, R.F., and P.D. Hyde, 1973, Estimating Crop Acreage From Space-
    Simulated Multispectral Scanner Data, Report No. 31650-148-T, ERIM,
   Ann Arbor, Michigan.
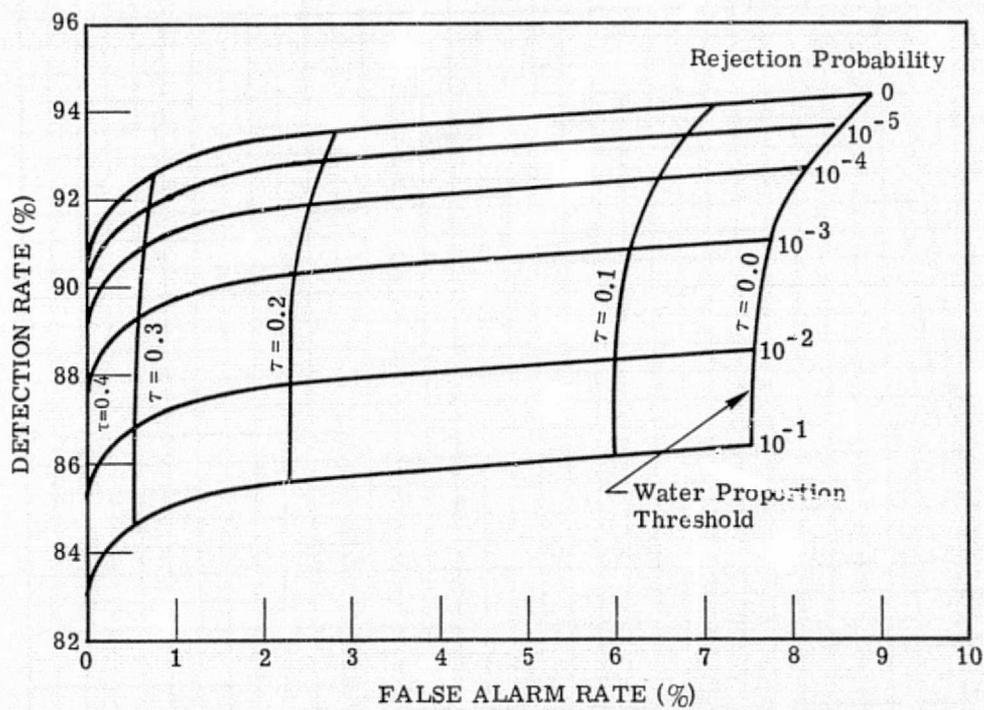
FIGURE 8. OPERATING CHARACTERISTICS OF PROPORTION ESTIMATION (3 CHANNEL)
(MIXMAP)
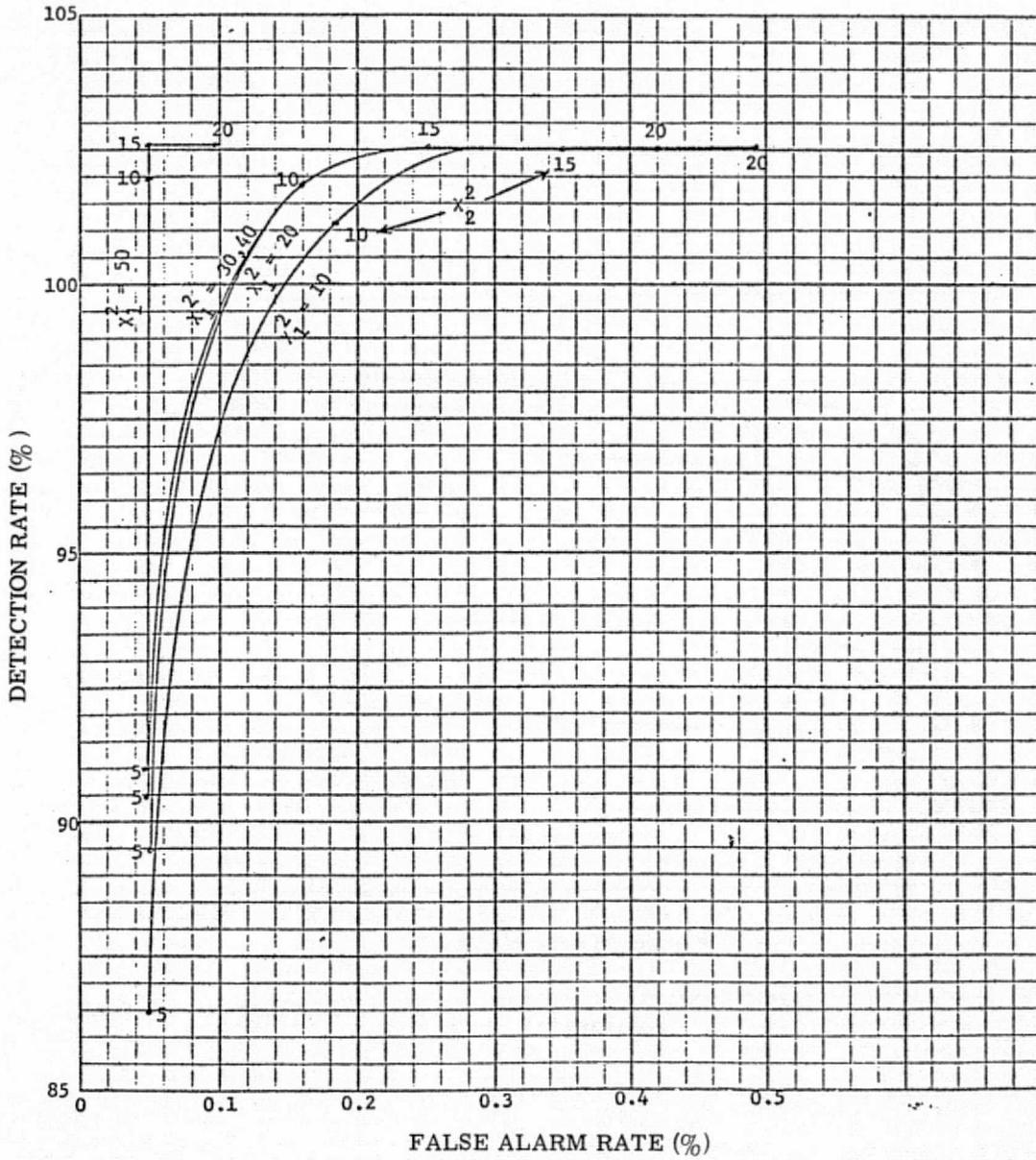
FIGURE 9.   LIMMIX OPERATING CURVES FOR WATER DETECTION.
Threshold = 0.4 with renormalization.

A much higher detection rate with a smaller false alarm rate is
evident in the LIMMIX operating curves. The LIMMIX curves show that it
is possible to detect 100% of the water while having only about 0.5% false
alarms. MIXMAP is only able to detect about 93% of the water for the same
rate of false alarms. Figure 10 was included to show that even the
operating curves for LIMMIX without thresholding are the equal of those
for MIXMAP at its best.

4.6.2 ESTIMATING THE PROPORTION OF WHEAT

LIMMIX was tried with wheat as the target crop. The data set selected
was from Hill County, Montana. Its long, narrow fields create many mixture
pixels making recognition difficult (Figure 11). The purpose of the experiment
was not to train parameters to be used on test data, but rather to see if LIMMIX
had the capability of achieving good and consistent results.

The data consisted of several different LANDSAT passes over Hill County.
On the basis of previous unpublished results generated by NASA/JSC/Earth
Observation Division personnel, the July 16 pass was selected for
processing. Unfortunately, the data tapes were unlabeled, and thus a
considerable amount of effort was required to discover which data set corresponded
most closely to known characteristics of the July 16 data (these characteristics
were mean signal levels of various crops in two channels).

When the July 16 data set was identified the conventional process
for identifying field location was carried out, i.e., various features
were identified on a line-printer map of one channel, and then a regression
fit was performed to determine the coordinate transformation from an aerial
photograph to the data set. Signatures for the data set were then obtained.

It would be difficult to get representative signatures from such
narrow fields by conventional methods since many of them are less than 1
pixel wide. For this reason it was decided to use a clustering algorithm
to obtain the signatures. The equivalent of 5.5 sections was clustered
(farms N-1, 2, 3, 5, 6, 7, 8, 14, 15, 16, 17) and 13 signatures were
obtained. To show that they were indeed different, program EPLOT was run.
The program plotted the mean and covariance matrix for each signature for
3 pairs of channels (2 vs 1, 3 vs 2, 1 vs 4). The plots were examined

43
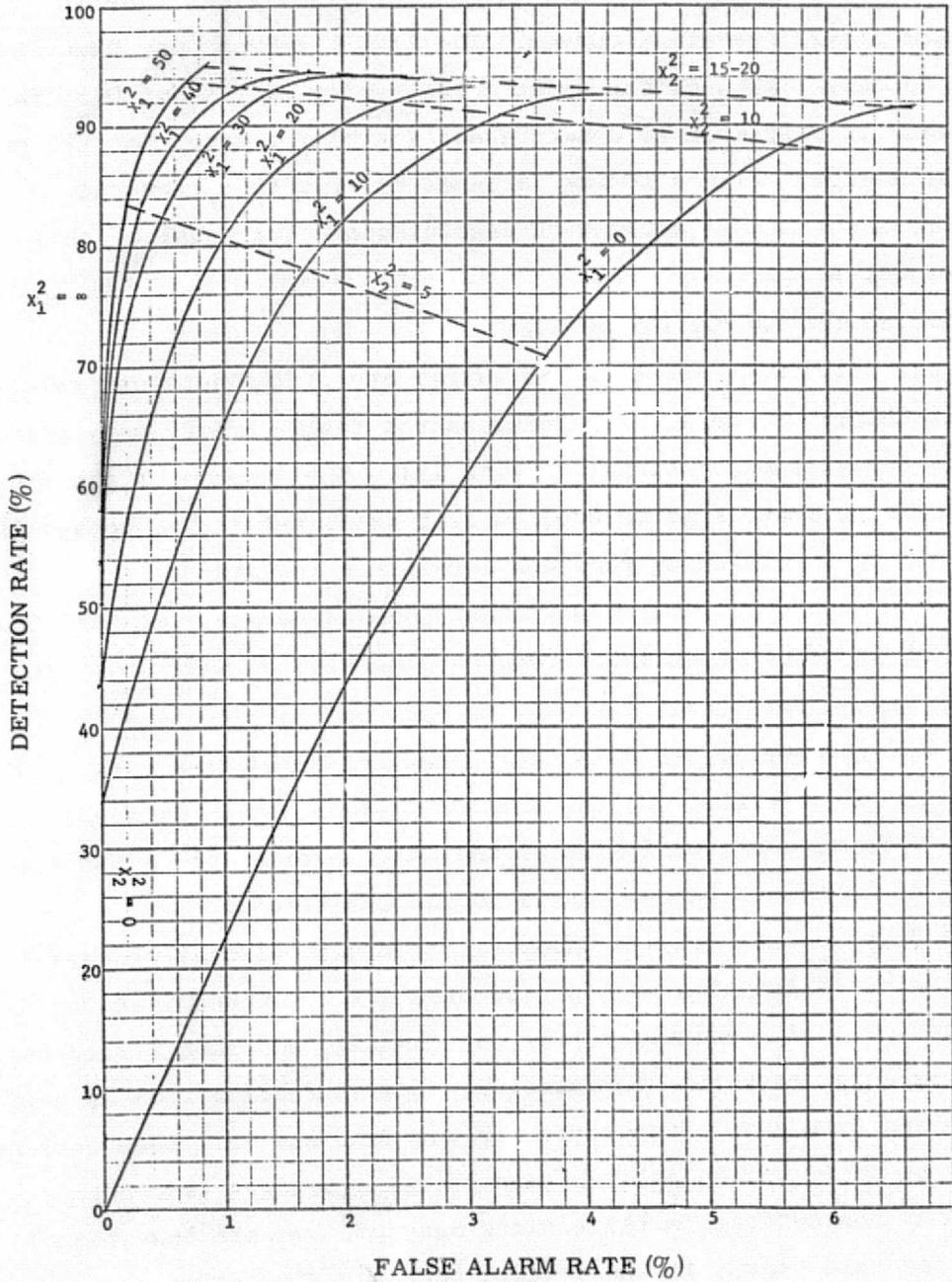
FIGURE 10. LIMMIX OPERATING CURVES FOR WATER DETECTION, NO THRESHOLDING

44

HILL COUNTY, MONTANA (NORTH)
MAY 5, 1973 GROUND TRUTH

Small Area #1   Small Area #2

LEGEND

▨ WATER		▨ OATS		☐ STUBBLE	
▨ ALFALFA		▨ SPRING WHEAT		▨ GRASS	
☐ NO CROPS		▨ WINTER WHEAT		☐ BARLEY	
▨ SUMMER FALLOW		☐ CRESTED WHEAT GRASS			
▨ RUSSIAN WILD RYE		☐ BUILDING SITES, UNKNOWN, ROADS			

PREPARED BY
MAPPING SCIENCES BRANCH, EOD.
NASA, JOHNSON SPACE CENTER
HOUSTON, TEXAS
FEBRUARY, 1974

N-1	N-3	N-4	N-5	N-6	N-7	N-9
N-2					N-8	
N-21	N-19 N-20	N-18	N-17	N-16 N-15 N-14 N-13 N-12	N-11	N-10

	R-13			R-14		
T34N	13	18	17	16	15	14
	24	19	20	21	22	23

FIGURE 11. GROUND TRUTH PAPER PRINT OF 6 SECTION TEST AREA AND SMALL
AREAS ONE AND TWO

45
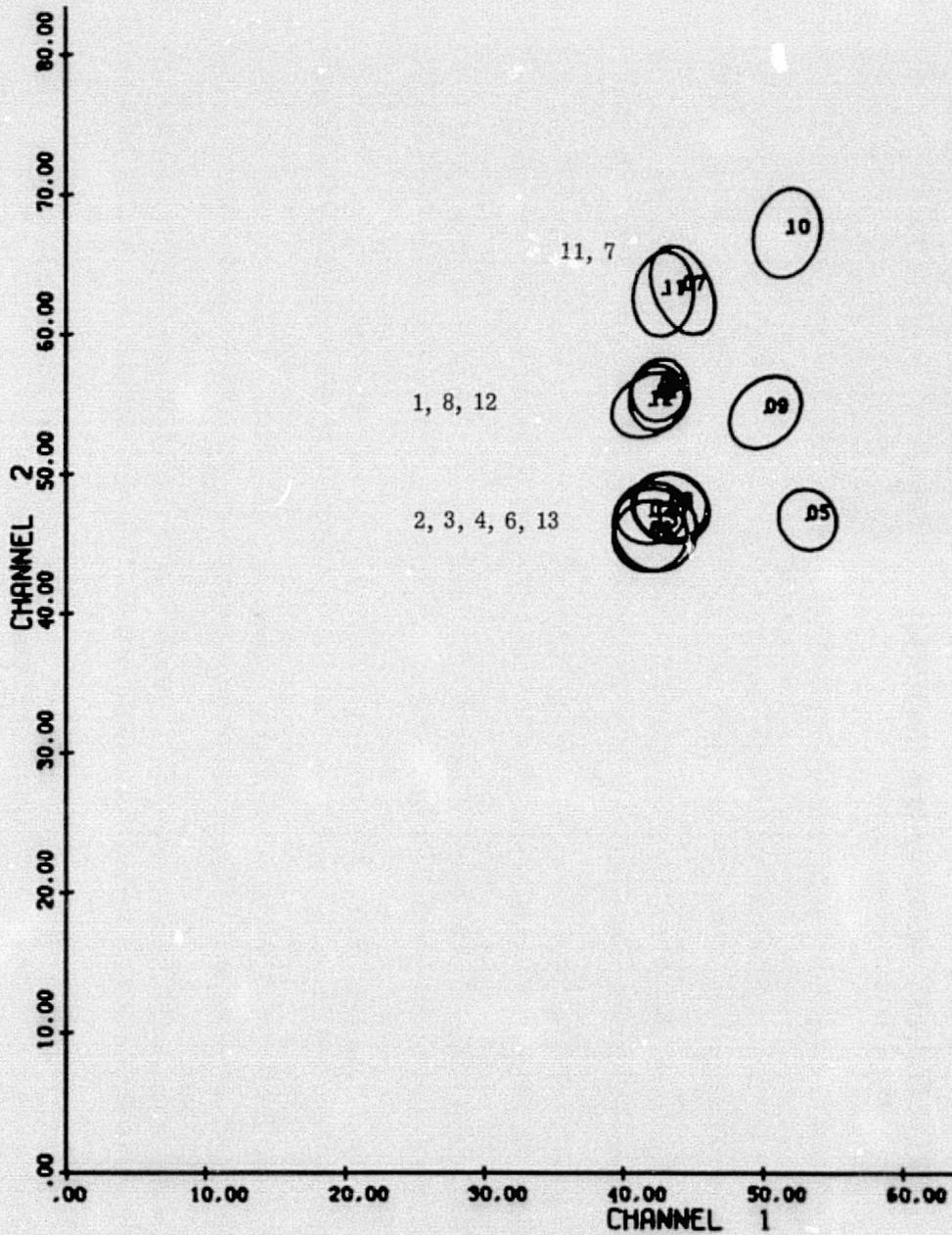
FIGURE 12. HILL COUNTY SIGNATURES (CHANNEL 1 VS CHANNEL 2)

46
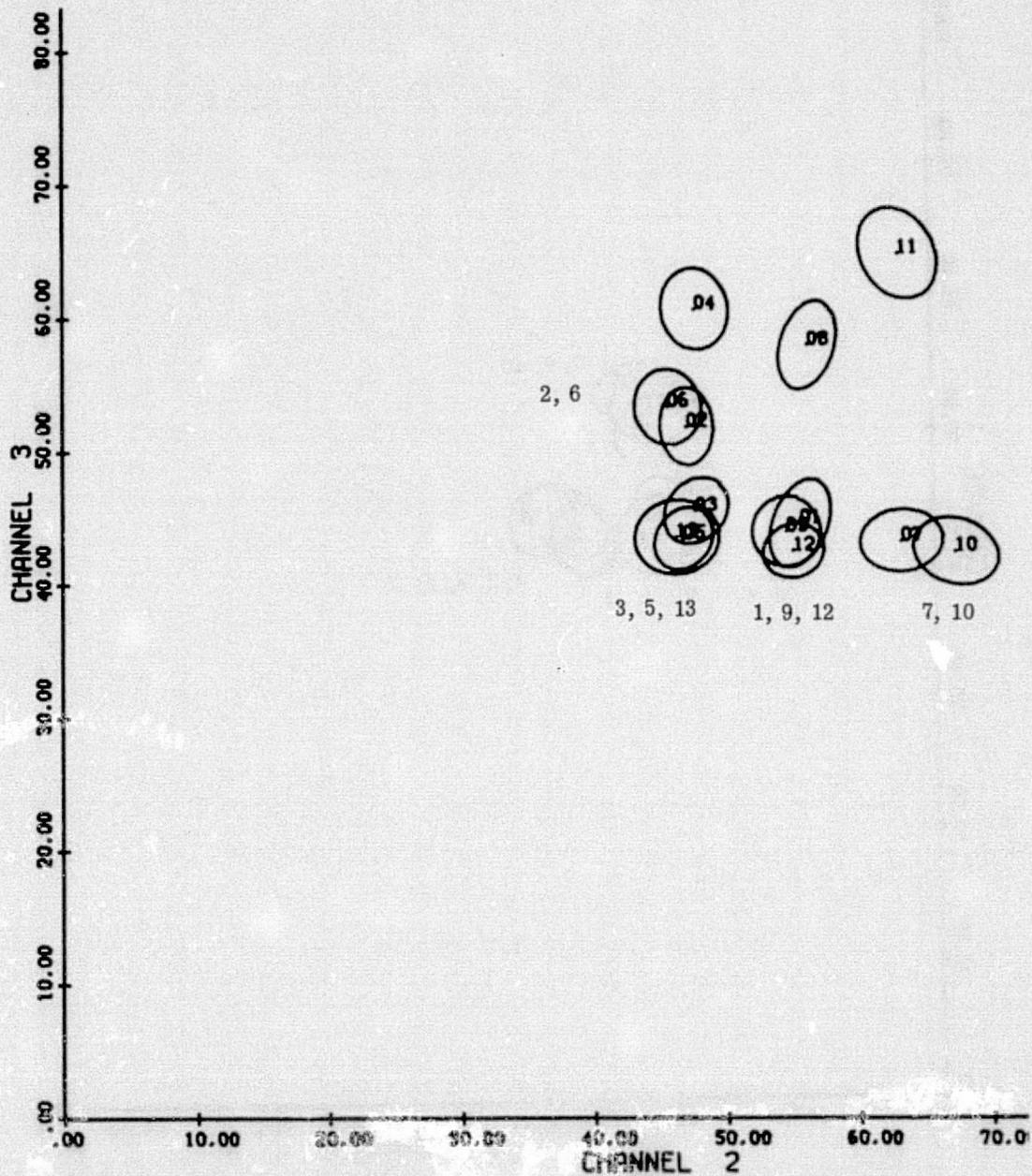
PLOT NUMBER 002   B00911 5685 28 AUG 1974

FIGURE 13. HILL COUNTY SIGNATURES (CHANNEL 2 VS CHANNEL 3)

47
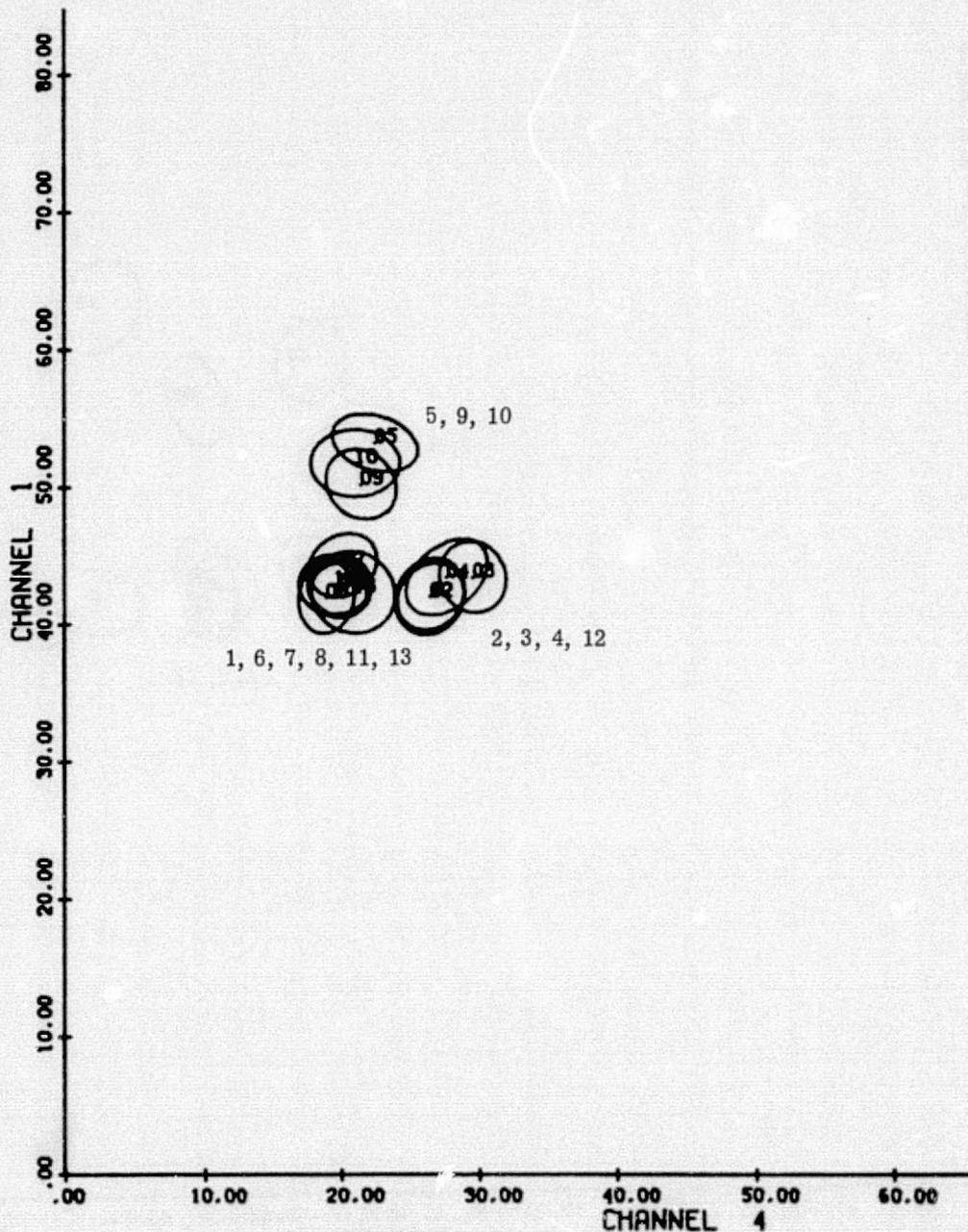
PLOT NUMBER 004   800911 5685 28 AUG 1974

FIGURE 14.   HILL COUNTY SIGNATURES (CHANNEL 1 VS CHANNEL 4)
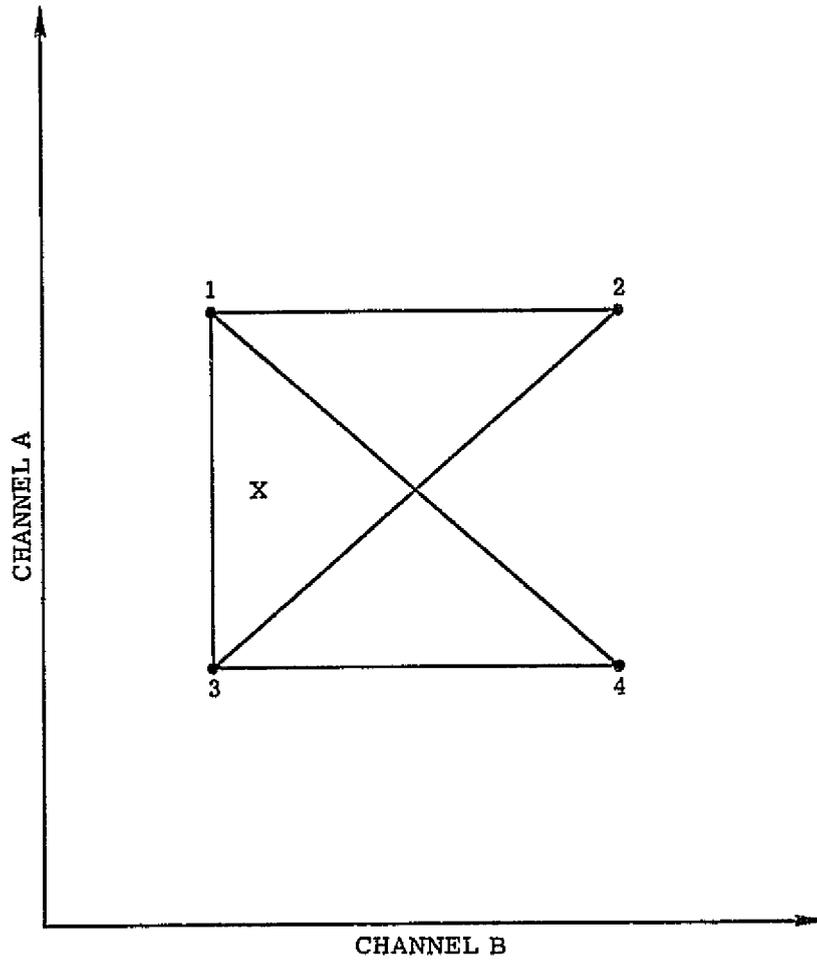
FIGURE 15. EXAMPLE OF A LIMMIX SOLUTION AMBIGUITY

and the signatures were found to be distinct. For the 78 combinations
of signature pairs. none of the covariance plots overlapped on all three
graphs, and only 9 pairs overlapped on two graphs. (Figures 12, 13, 14).

The next task was to correlate the signatures to crop types known to
be in the scene. Recognition processing was run on Hill County for the
13 signatures. The ground truth map and a recognition map were used to
identify the clustered signatures. Three of the signatures were found
to be wheat. (Numbers 2, 6, 11). LIMMIX, was then used to classify the Hill
County area.

Mixtures of no more than two materials were used in processing Hill
County. That two signatures is the maximum which can be used can be clearly
seen in Section 3.7. The reason for this is perhaps less clear, considering
that MIXMAP is capable of using one more signature than the number of channels
of data. Here is an explanation by example: For 2 channel data and a signature
set of three members, LIMMIX and MIXMAP can both consider mixtures of at most 3
object classes in a single pixel. When the set has four members, MIXMAP
breaks down completely, since it must consider mixtures of four, and there can be
many ambiguities. LIMMIX, of course, cannot calculate the best four at a time
either, again because of the ambiguities; but it can find the best one and two
at a time. The 3 at a time is a special case where there is usually just one
ambiguity. Figure 15 shows four signatures in two channels. The data point
(x) could represent a combination of signatures 1, 3, and 4 or 1, 2, and 3, since
the likelihood for either is the same. It is for this ambiguity that three
at a time must be discarded for LIMMIX.

The criterion chosen for determining classification accuracy was the
percentage of each material found in a relatively large area as compared to the
true percentage of each material in that area. This was because the normal
method of determining classification accuracy (testing field center pixels) is
inappropriate for the LIMMIX. algorithm, since much of its value lies in its
potential to deal with mixture pixels.

LIMMIX was run on Hill County data using the 13 signatures for combinations
up to two at a time. To save processing time, only 6 sections (N-1-8, 12, 13)
were chosen for further analysis.

50
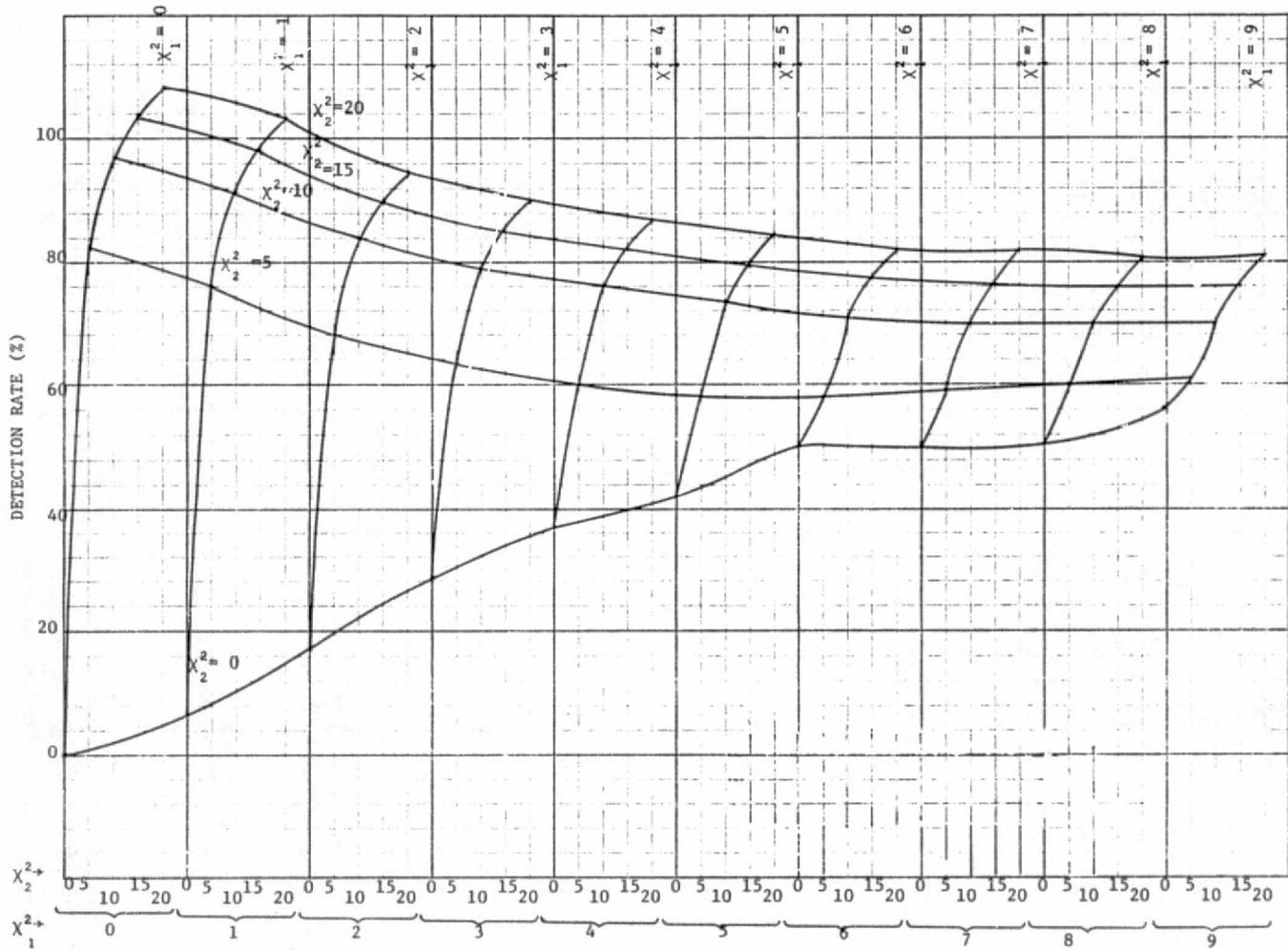
FIGURE 16. DETECTION RATE VS CHI-SQUARES FOR 6 SECTIONS OF HILL COUNTY, MONTANA
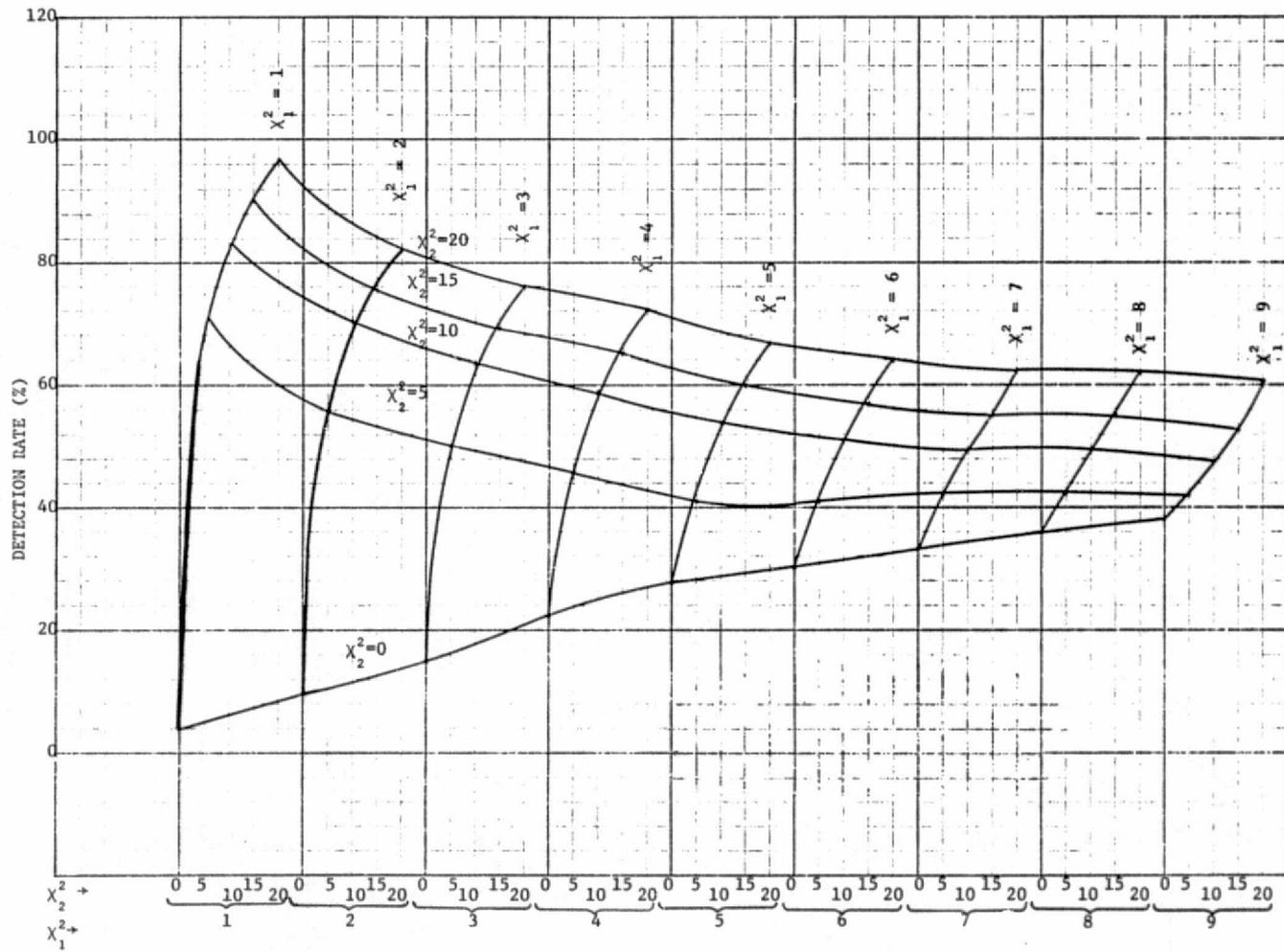
FIGURE 17. DETECTION RATE VS CHI-SQUARES FOR SMALL AREA #1, HILL COUNTY, MONTANA
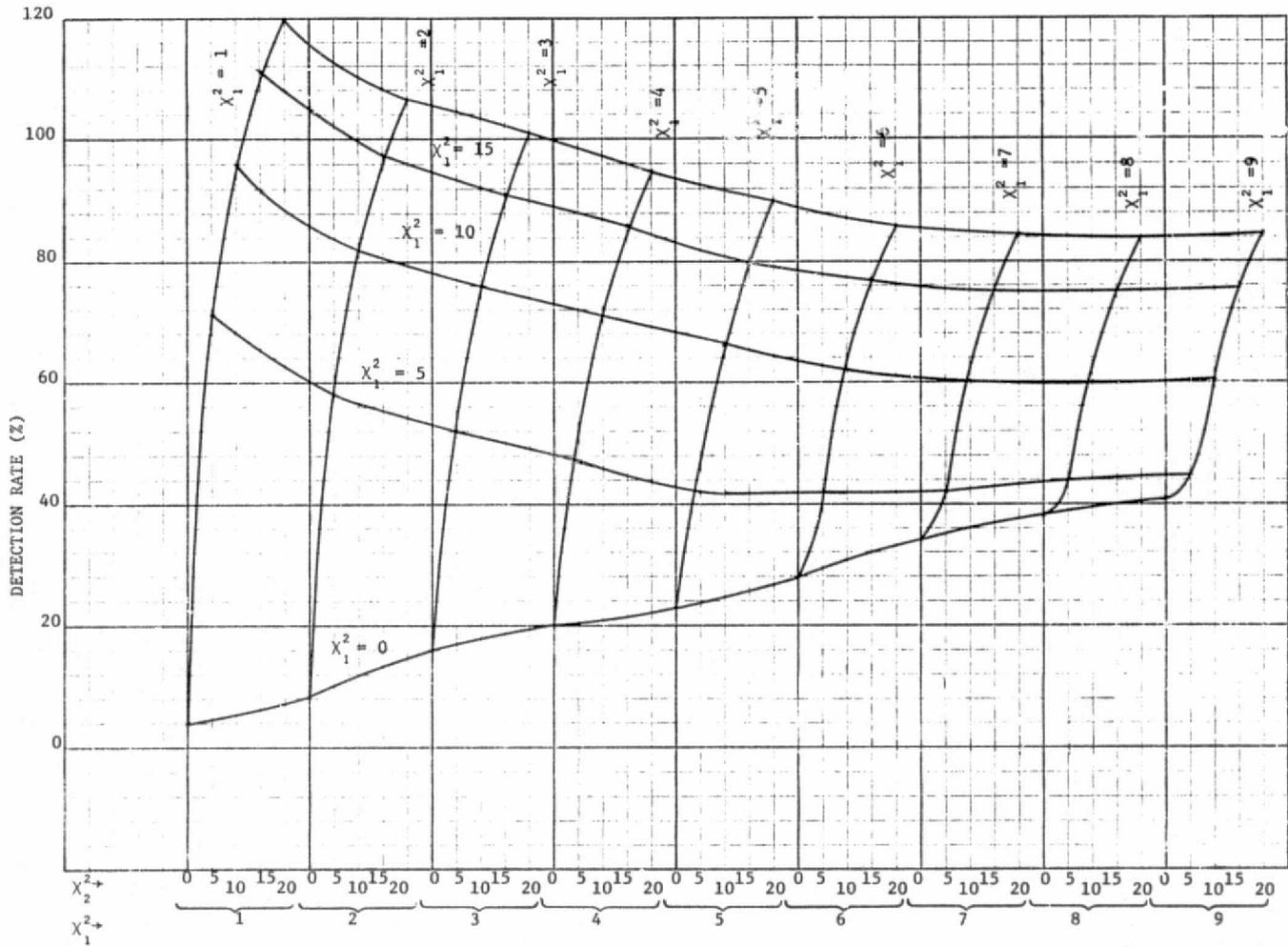
FIGURE 18. DETECTION RATE VS CHI-SQUARES FOR SMALL AREA #2, HILL COUNTY, MONTANA

Program ALIEN2 was run on the LIMMIX tape for a variety of χ_1^2 and χ_2^2 values. The output, in number of pixels detected for each signature and each pair of Chi-Squares, was compared with the ground truth to obtain the detection rate for wheat over the 6 sections.

Due to the small field sizes, it was not possible to define non-wheat areas and therefore to record false alarms by use of the ALIEN2 program, and consequently the usual operating curves (i.e., detection rate vs false alarm rate) are replaced by a graph of detection rate vs. the chi-square values. Results are presented in Figure 16. Since the graph does cross 100% detection (including false alarms), it was decided to use these parameter values in two subset areas to test their universality. Small areas 1 and 2 are defined in Figure 11. The results for the 2 smaller areas are presented in Figure 17 and 18. These figures are the same general shape as Figure 16 but are shifted along the detection rate axis. Parameter settings of $\chi_1^2=1$ and $\chi_2^2=17$ where the detection rate is 100% for the 6 section area would give detection rates of 92% and 114% for small areas 1 and 2. Even though we did not use separate test and training regions, this preliminary experiment indicates that there may be parameters settings which are approximately correct over subregions.
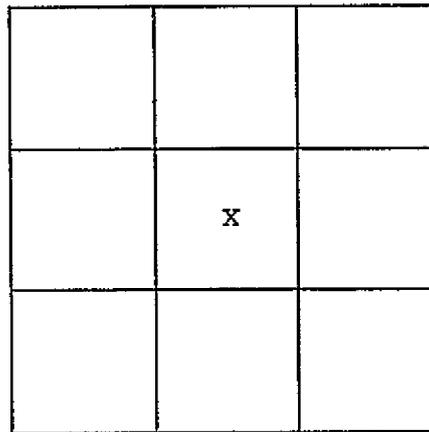
FIGURE 19. SCHEMATIC FOR NINE
ELEMENT RULE

5

UTILIZATION OF SPATIAL INFORMATION IN ESTIMATING PROPORTIONS

Many current multispectral data processing schemes classify pixels on
the basis of their associated signals; the signals from neighboring pixels do not
influence the outcome. But for many applications, schemes which take neighboring
data into account would be expected to perform better than these single element
rules. In addition, such schemes should make the distinction between pure and
mixture pixels better than a single element scheme.

Nine element rules are designed to gain these advantages while preserving
simplicity and speed. Such rules are applied in turn to each pixel of the scene
in the context of its eight immediate neighbors arranged in a 3 x 3 grid as
diagrammed in Figure 19. These rules assume that when most of these nine pixels
are assigned the same classification on a preliminary recognition pass, then the
center pixel is unequivocally this material. When there is no clear consensus
amont these nine pixels, the center pixel may then be a mixture. Modest storage
requirements and the small number of pixels playing a role in each decision make
these rules practical.

After a study of investigations of nine-point rules by Richardson[7], the
voting rule was selected as the one most likely to detect boundary pixels.
The voting rule is applied after a preliminary recognition pass has been
made on the nine pixels. The center pixel is assigned the material recognized
most frequently among the nine if N_1 or more pixels of the nine have been
recognized as that material (N_1 is a parameter of the procedure). If no material
gets at least N_1 votes, than the center pixel may be either a pure pixel or a
mixture pixel.

The advantage of the voting rule in proportion estimation is that a large
number of pixels contain a single material, and this rule detects most of them.
For these pixels, the procedure terminates after the vote. For the remainder
of the pixels, the procedure terminates after the vote. For the remainder of

[7] Richardson, W., 1974, A Study of Some Nine-Element Decision Rules,
     Report No. 190100-32-T, Environmental Research Institute of Michigan,
     Ann Arbor, Michigan.

the pixels, the voting rule provides contextual information which may be used
to determine which materials are present in a mixture.

5.1 NINE-POINT MIXTURES PROCEDURE

The voting rule was combined with the LIMMIX processing scheme. Three
algorithms were developed for testing. These are described below. Additional
details are contained in Appendix F.

5.1.1 ALGORITHM 1

A. Make a preliminary pass through the data, classifying each pixel
according to the quadratic Bayes decision rule.

B. For each pixel, look at it and the adjoining eight pixels, and take a
'vote' as to their identity (pixels may participate in the vote only if their
associated Chi-Squared level is less than η_1^2). If at least N_1 of the pixels
agree as to identity, the center pixel is classified as this material.

C. If less than N_1 of the pixels agree as to identity, examine the Chi-
Squared level of the center pixel's classification. If this Chi-Squared level
is less than η_2^2, accept the recognition.

D. If the Chi-Squared level of the center pixel is greater than η_2^2, find
the two largest vote winners in the vote of (B). Call the pixel a mixture of
these two materials, i.e., if 4 pixels 'voted' for corn, 3 pixels 'voted' for
wheat, and 2 pixels 'voted' for soy, call the center pixel 4/7 corn and 3/7 wheat.

5.1.2. ALGORITHM 2.

This is the same as Algorithm 1 except for step D, which becomes:

D. If the Chi-Squared level of the center pixel is greater than η_2^2, find
the best two-at-a-time mixture via the LIMMIX procedure.

5.1.3 ALGORITHM 3.

This is the same as Algorithm 1 except for D, which becomes:

D. If the Chi-Squared level of the center pixel is greater than η_2^2, and
if the totals of the two largest vote winners in the vote of (B) are greater
han or equal to N_2, the pixel is assumed to be a mixture of these two materials.
Find their proportions via the LIMMIX procedure (The signature set contains only
these two materials). If the totals of at least one of the two largest vote
winners is less than N_2, find the best two-at-a-time mixture via the LIMMIX procedure
(all signatures are included in the signature set).

5.2 TEST RESULTS

In order to determine which of the three algorithms performed the best, and to determine proper parameter settings for each, these algorithms were tested on three types of data sets: (1) a water data set from an eight square mile area near Lansing, Michigan, consisting of 20 small lakes and ponds, which ranged in size from seventy acres to one-third of an acre, averaging about 10 acres (Section 5.2.1). (2) An agricultural data set, gathered 21 August 1973, with target crops of corn and soybeans (one of the CITARS data sets), with training and test data taken from a 5 x 20 mile area in Fayette County, Illinois (Section 5.2.3). (3) Two agricultural data sets with wheat as the target crop. The first was a 14-section data set from Hill County, Montana with 6 of the sections taken to be test data (Section 5.2.2). The second was a CITARS data set, gathered 10 June 1973, with training and test data taken from a 5 x 20 mile area in Fayette County, Illinois (Section 5.2.4).

Preliminary testing was done on the water data set and on the Hill County data set. These preliminary test results showed that the performance of algorithm one was markedly inferior to that of conventional recognition, and it was discarded. Algorithms two and three were found to perform approximately the same in all cases, although algorithm three is preferable because of shorter processing time. Consequently, only algorithm three was tested further, and it will be referred to as 'the nine-point mixtures algorithm'.

Examination of the four parameters, N_1, N_2, η_1^2, and η_2^2, in algorithms two and three showed that the best values of both N_1 and N_2 were invariant over the data sets studied. N_1 was found to be optimum at eight, and severely degraded performance resulted from any other setting. The optimal value of N_2 was found to be four.

The best settings of η_1^2 and η_2^2 vary from data set to data set, much as the parameters χ_1^2 and χ_2^2 do in LIMMIX. And as in LIMMIX, training is the only method we now have for selecting parameter settings.

5.2.1 WATER DETECTION

The water data was the first set used for testing of nine-point mixtures. Because conventional recognition and LIMMIX processing results were already
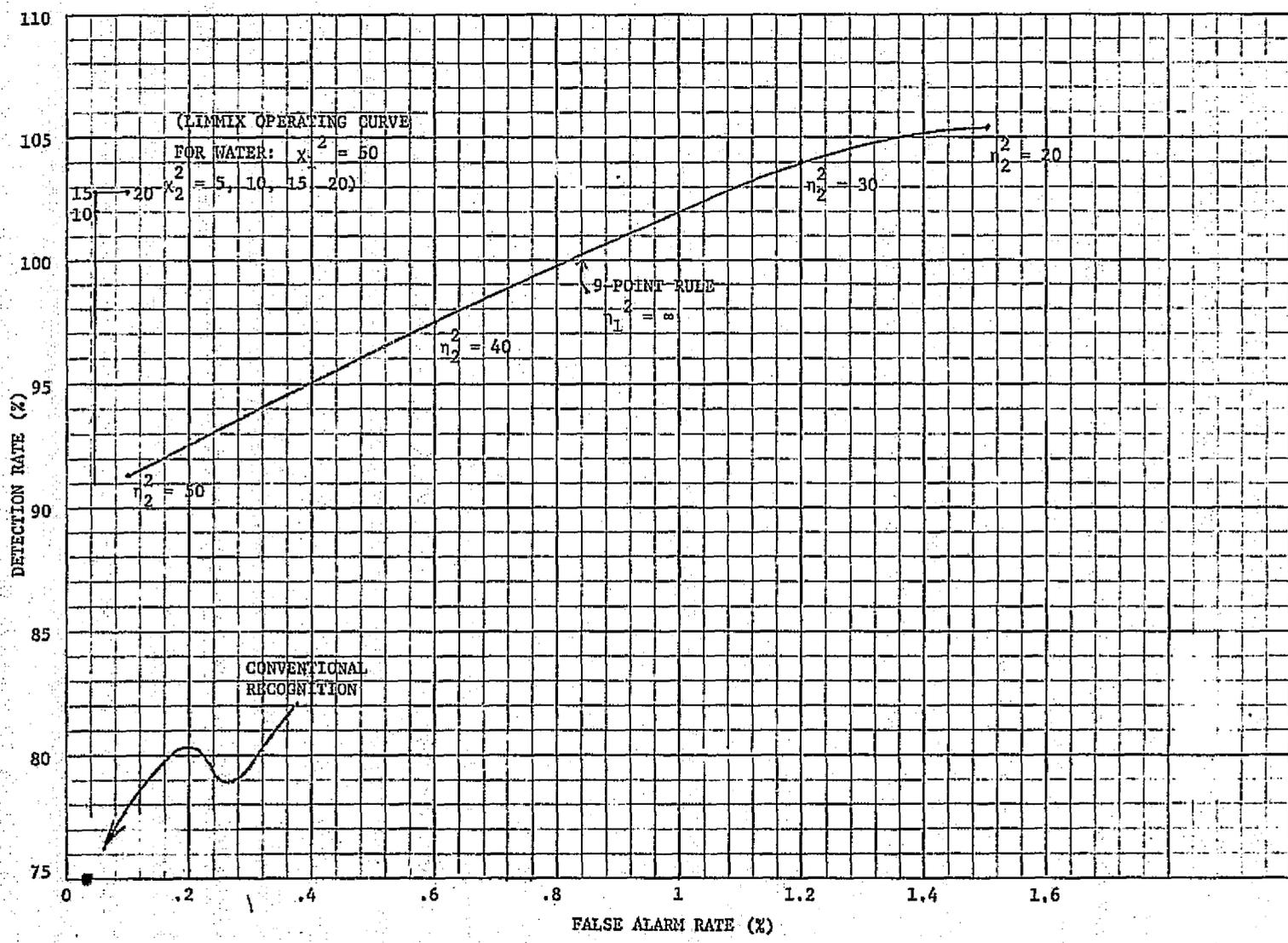
FIGURE 20. WATER OPERATING CURVE FOR THE 9-POINT RULE, LIMMIX, AND RECOGNITION

available for this data set, a direct comparison was possible. Figure 20 shows
a comparison of results for a range parameter settings for LIMMIX and nine-point
mixtures processing, and the best parameter setting for conventional recognition
processing.

In the nine-point mixtures processing on the water data, it was found that
the results were quite sensitive to changes in η_2^2, while the results were almost
invariant for any η_1^2 greater than 25.

It can be seen from the figure that both LIMMIX and nine-point mixtures
performed better than conventional recognition. It is noteworthy that for
nine-point mixtures when the detection rate was 100%, the false alarm rate was
only about 0.8%. In addition, nine-point mixtures was quite accurate even on
a lake by lake basis.

In this test only three signatures were used, and we found that the speed
of processing with nine-point mixtures was approximately that of conventional
recognition. As the number of signatures increases, processing time of nine-point
mixtures increases more rapidly than that of conventional recognition. In a
production setup, the processing time of nine-point mixtures would be approximately

$$\frac{2}{3} + \frac{m!}{6(m=2)!}$$

times that of conventional recognition, where m is the number of signatures.

5.2.1.1 Comparison of Surface Water Detection Procedures

For purposes of comparison, the water data set was processed with two
other procedures. These procedures were developed at NASA [16] and NASA personnel
suggested that this comparison be made. One of them employs a two-channel
discriminant with a universal decision algorithm. The other uses a tailored two-
channel discriminant established with training data via procedures obtained from
the reference. It should be emphasized that these discriminant techniques were

[16] Anderson, A.C., 1973, Development of a Two-Channel Linear Discriminant
Function for Detecting and Identifying Surface Water Using ERTS-1 Data,
Report No. JSC-08450, NASA, Houston.

developed for detecting all water bodies of 10 acres or more*, and they did just that. Results are given for these discriminant methods as well as for nine-point mixtures in Table 2.

It is clear that nine-point mixtures is best for this scene insofar as the number of lakes detected and % total water detected is concerned. However, processing time for this procedure is much slower than for the other two, by about two orders of magnitude.

Signatures for water and non-water were obtained from a training area which comprised approximately 5% of the scene. These signatures are shown in Figure 21. In this figure, the universal discriminant obtained from the reference is shown in line 1. The tailored discriminant, shown as line 2, was drawn by eye.

The universal discriminant requires no signature extraction or experimentation, and is extremely rapid. This procedure was found to detect lakes of greater than ten acres, however it functioned erratically on lakes of significantly smaller size. Overall accuracy was the lowest of the three in area determination. It should also be mentioned that it found two lakes where there was actually one narrow lake.

The tailored discriminant requires signature extraction and some experimentation to determine the linear discriminant function. The speed of classification is equal to that of the universal discriminant. Performance, however, was better in as much as lakes of ten acres or more were again reliably found, but the determination of lake size was more accurate. This procedure correctly identified a narrow lake as just one lake instead of two.

Nine-point mixtures requires both signature extraction and experimentation to establish operating parameters. This requires more effort than the tailored discriminant. Nine-point mixtures detected all but one lake with an area of one-half acre or more while detecting a lake whose area was less than one-half acre. This procedure can be expected to reliably detect lakes of one acre or more. Area determination accuracy is also very high -- the average error on each lake was less than one acre, with almost zero total error. The main disadvantage is processing time.

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*Report JSC 08449, table 8, page 7-4, documents the fact that the procedure was developed according to the criteria required for the National Program for the Inspection of Dams. These requirements were that the procedure must accurately detect the existance of lakes of 10 acres or more. Further, the procedure was not required to estimate sizes of water b :ies.

TABLE 2

COMPARISON OF WATER DETECTION PROCEDURES

PROCEDURE	NO. OF LAKES DETECTED (OUT OF TWENTY)	EQUIVALENT NO. OF WATER PIXELS FOUND	% DETECTION
Universal Discriminant	13	162	67.1%
Tailored Discriminant	12	193	79.9%
Nine-Point Mixtures	19	245	101.4%
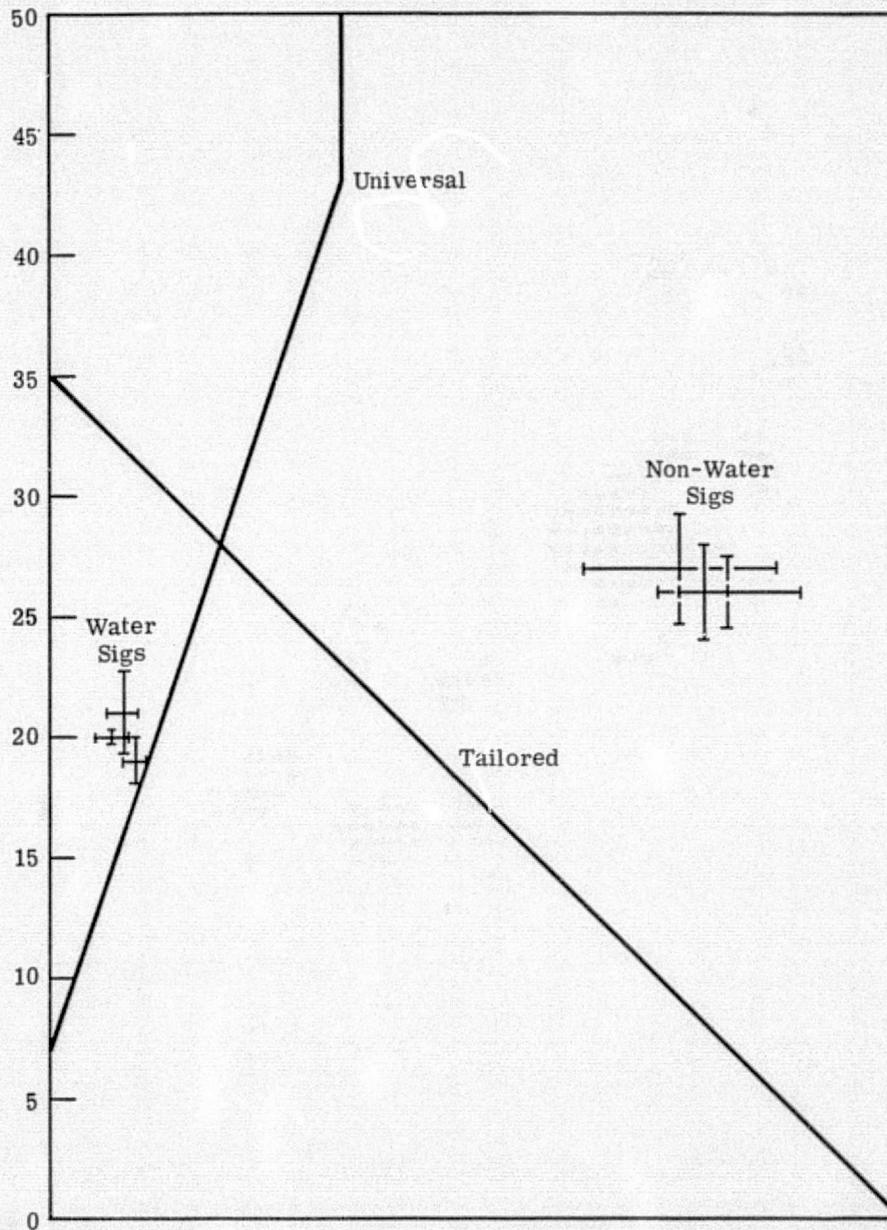
FIGURE 21. 2 CHANNEL LINEAR DISCRIMINANTS
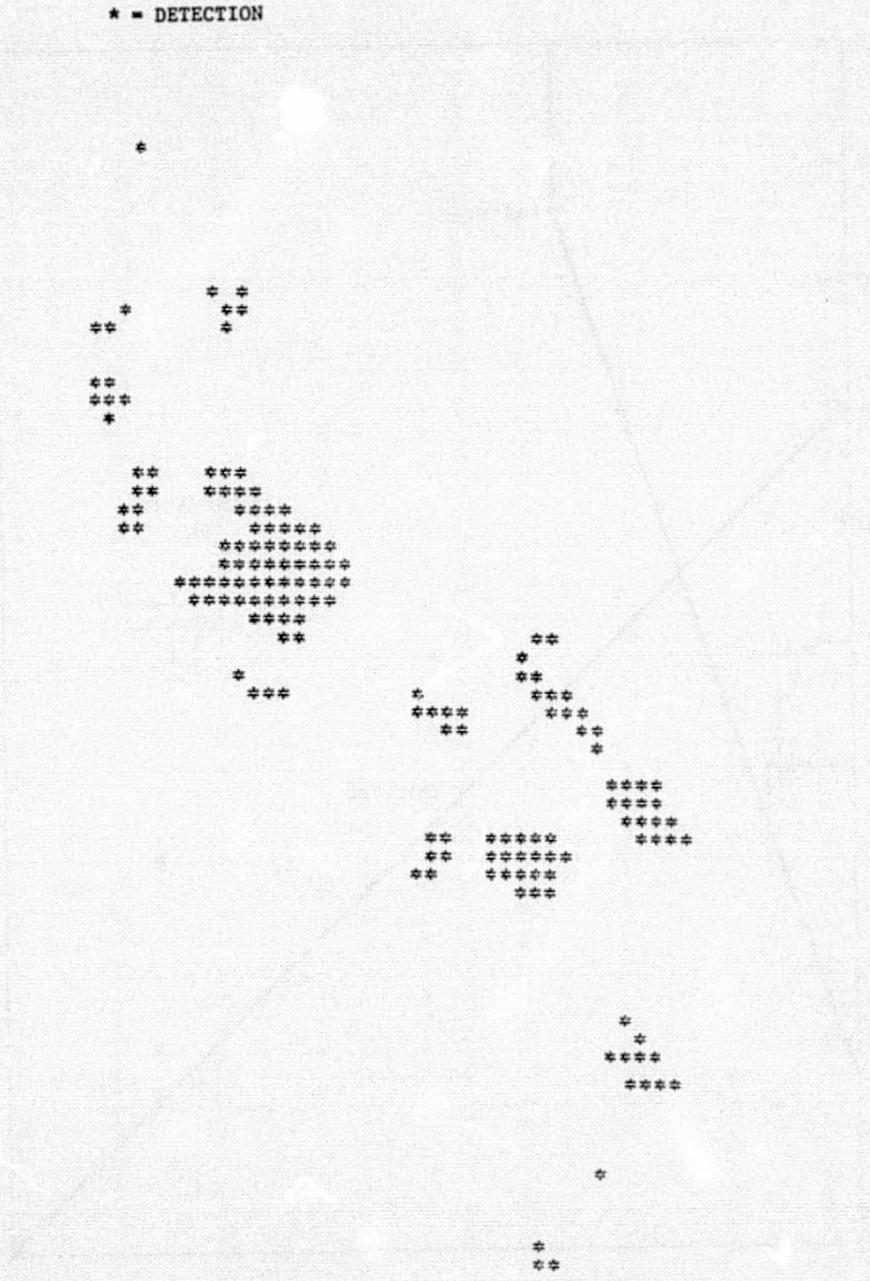
* = DETECTION

FIGURE 22. CLASSIFICATION MAP FOR THE UNIVERSAL 2-CHANNEL
DISCRIMINANT ON WATER

* = DETECTION
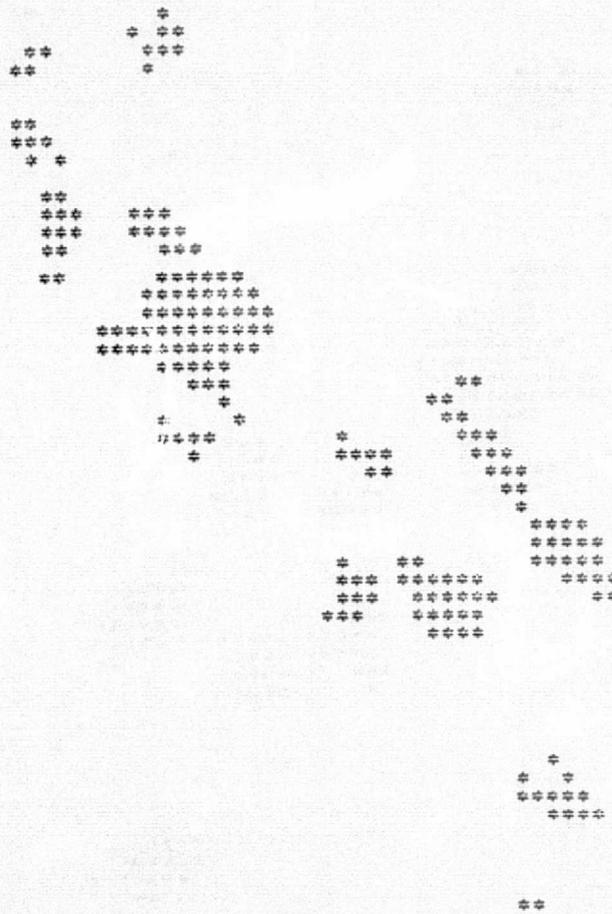
FIGURE 23. CLASSIFICATION MAP FOR THE TAILORED 2-CHANNEL
DISCRIMINANT ON WATER

$$x_1^2 = 50$$

$$x_2^2 = 7$$

$$\tau = .4$$

✿ = RECOGNITION
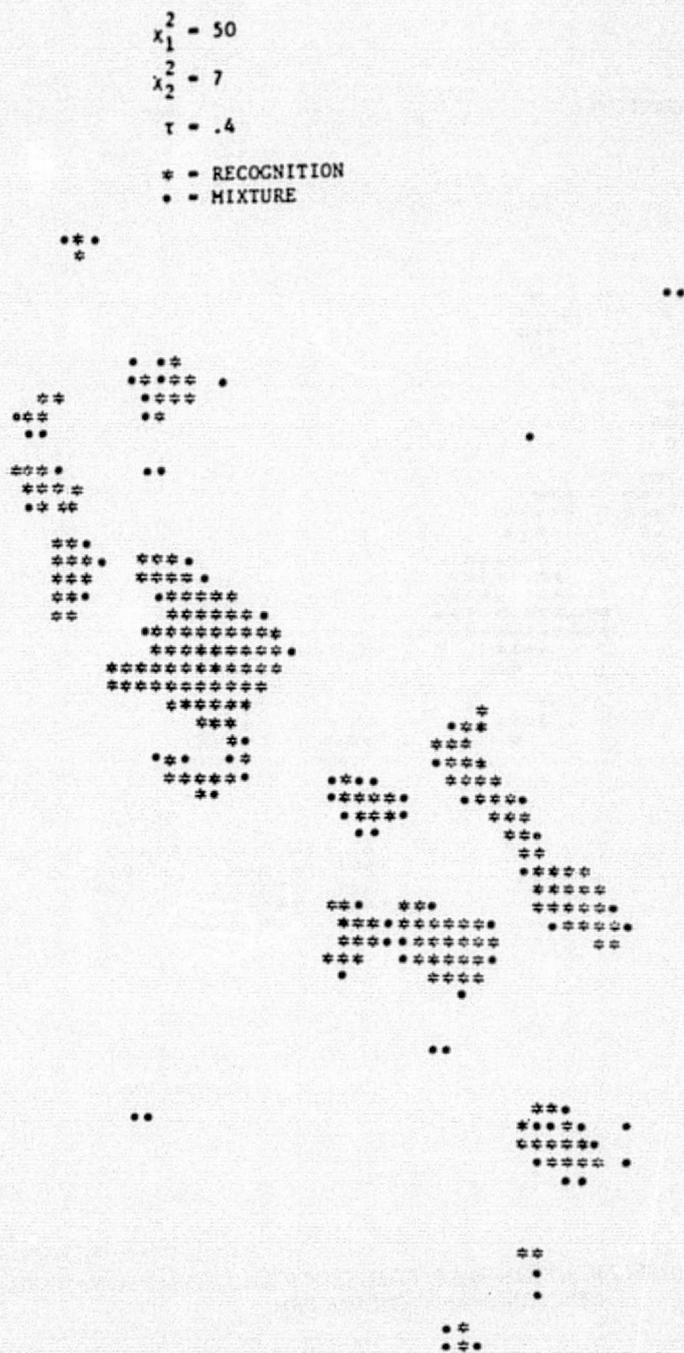
• = MIXTURE

FIGURE 24. CLASSIFICATION MAP FOR NINE-POINT MIXTURE ON WATER

66

For each of the above procedures there were an insignificant number of false alarms.

Figures 22, 23, and 24 are classification maps for the three procedures. The tailored discriminant (Figure 23) fills out the lake areas more completely than the universal discriminant (Figure 22) even though the latter detected one more lake. The classification map for nine-point mixtures (Figure 24) shows how this procedure not only detects interior water pixels (denoted by the symbol M), but also delineates boundary, or mixture, pixels (denoted by the symbol *).

5.2.2 PRELIMINARY WHEAT DETECTION TEST

The nine-point mixtures algorithm was applied to the Hill county data set, where results of conventional recognition and LIMMIX processing were available for comparison. Table 3 shows results of the three processing procedures on this data. The conventional recognition shown is the quadratic rule with a rejection threshold of ∞. LIMMIX is shown employing parameter values of $\chi_1^2=5$, $\chi_2^2=\infty$ and proportion threshold $\tau= .4$ (proportions less than τ are set to zero, and the remaining proportions renormalized so that they sum to one). The nine-point mixtures rule is shown using parameter values of $N_1=8$, $N_2=4$, $\eta_1^2=30$, and $\eta_2^2=5$.

Table 3 compares the detection of wheat for the three procedures. Thirteen signatures were employed. Three of these represented wheat. Detection rates were obtained as follows. All the pixels in the test area were designated, using ground truth information, as wheat or non-wheat. The detection rate is defined to be the equivalent percentage of wheat pixels found by the procedure to be wheat. The false alarm rate is defined to be the equivalent percentage of non-wheat pixels found by the procedure to be wheat.

The results in Table 3 were obtained by the use of classification maps and overlays; as such they should be treated as close estimates, rather than exact figures. For purposes of comparison, we note that field center recognition is about 80% for wheat on this data. However, on all wheat pixels recognition was only 63.4%. It can be seen from this table that the nine-point mixtures algorithm shows itself superior to both conventional recognition and LIMMIX. LIMMIX, on the other hand, is significantly better than conventional recognition as a wheat detector.

TABLE 3

WHEAT DETECTION (HILL COUNTY)

Procedure	Detection Rate	False Alarm Rate
9-Point Mixtures	78.7%	4.4%
Recognition	63.4%	12.8%
LIMMIX	71.4%	6.7%

5.2.3 ESTIMATING PROPORTIONS OF CORN AND SOYBEANS

Tests were conducted on Fayette county data employing a set of signatures derived during the post-project analysis of the CITARS project results. These signatures were obtained after breaking the data set into three parts: training (20 quarter sections), pilot (10 sections), and test (10 sections). These three parts contain 2880 pixels, 5630 pixels, and 5529 pixels, respectively. The 'other' signatures were those previously obtained from the training quarter sections, while the signatures for corn and soybeans were obtained from the pilot sections.

This was done because the corn and soybean fields in the training area had been found to be unrepresentative of the corn and soybean fields in the test data.

The parameters of the nine-point mixtures rule were then established on the training quarter-sections, using accuracy of area determination as the criterion for selecting the best parameters. The results of the effort to establish parameters are shown in Table 4.

Nine-point mixtures was then used on the test sections with parameter values $\eta_1^2=20$, $\eta_2^2=5$. Results were poor. Examination of field center pixels showed a problem with misclassified 'other' pixels. Investigation showed that the poor results were due principally to the fact that there was no rejection threshold used for mixtures. To correct this, a third chi-squared parameter, η_3^2, was added to the algorithm which sets a rejection threshold on mixtures, as η_2^2 does in LIMMIX. Mixtures which are rejected are called 'other'. With this addition, the parameters were again established on the training quarter-sections. The results obtained are shown in Table 5.

The best settings of the parameters ($\eta_1^2=20$, $\eta_2^2=5$, $\eta_3^2=5$) were employed on the test sections but again the results were poor.

The parameters were then established on the larger set consisting of both the pilot sections and training quarter-sections. Results are shown in Table 6.

A selection of four parameter settings including the best settings of the parameters ($\eta_1^2=20$, $\eta_2^2=2.5$, $\eta_3^2=2.5$) were then used on the test sections.

TABLE 4

RESULTS OF ESTABLISHING NINE-POINT MIXTURE PARAMETERS

PARAMETER SETTINGS		PROPORTION ESTIMATES (%)	
η_1^2	η_1^2	Corn	Soybeans
20	2.5	26.4	45.56
20	5	22.52	47.02
20	7.5	18.05	50.66
20	10	15.76	53.43
Ground Truth		23.53	45.31

TABLE 5

RESULTS OF ESTABLISHING NINE-POINT MIXTURES PARAMETERS

(Training Quarter-Sections Only)

Parameter Settings			Proportion Estimation (%)	
η_1^2	η_2^2	η_3^2	Corn	Soybeans
20	5	2.5	18.12	43.83
20	5	5	22.96	45.47
20	2.5	2.5	18.76	40.47
20	2.5	5	26.46	43.35
Ground Truth			23.53	45.31

TABLE 6

RESULTS OF ESTABLISHING NINE-POINT MIXTURE PARAMETERS

(Training Quarter-Sections Plus Pilot Sections)

Parameter Settings			Detection (%)	
η_1^2	η_2^2	η_3^2	Corn	Soybeans
20	5	2.5	21.75	38.13
20	5	5	18.30	28.75
20	2.5	2.5	21.50	34.88
20	2.5	5	28.99	36.85
Ground Truth			24.54	33.63

The "best" parameters ($\eta_1^2=20$, $\eta_2^2=2.5$, $\eta_3^2=2.5$) yielded excellent results when used on the test sections as shown in Table 7.

Why then were results so poor when parameters were established only on the training quarter-sections? We know based on detailed examinations, that the corn and soybeans fields in the training region are not adequately representative of the corn and soybean fields in the test region. This is why the corn and soybean signatures were obtained from the training quarter-sections plus pilot sections, rather than from the training sections alone. We believe that this is the explanation for the poor results obtained when the parameters were established on the training quarter-sections alone.

An analysis of the error was made in order to establish the consistency of nine-point mixtures as an estimator of crop proportions. To do this the RMS error between nine-point mixtures crop proportion estimate and ground truth proportions was computed for each of the 10 test sections (averaging 553 pixels each). The RMS error between the true percentage corn and the estimated percentage corn over the ten test sections was 3.53(%). For soybeans the corresponding figure was 4.33(%).

5.2.4 ESTIMATING PROPORTIONS OF WHEAT

Another test of nine-point mixtures on a data set with a target crop of wheat was made using the CITARS data set of 10 June 1973 on a 5 x 20 mile area of Fayette County, Illinois. There were twenty training quarter sections containing a total of 2880 pixels and nineteen test sections containing a total of 10,223 pixels.

Because of time limitations, it was decided that we should at first limit ourselves to the two best values of η_1^2, η_2^2, and η_3^2, as indicated by the corn and soybean test. When using these two parameter settings, an attempt to establish parameter settings on the training data was made. The results of this test are given in Table 8. The parameter setting of $\eta_1^2=20$, $\eta_2^2=2.5$, $\eta_3^2=2.5$ gives the closer estimate.

Examination of these results from the training area showed that most of the errors came from wheat recognitions in hay and summer fallow fields. As the η_1^2 level for accepting a pixel into the vote was rather large, it appeared that decreasing η_1^2 should help the results. Parameter settings with $\eta_1^2=14$ and $\eta_1^2=7$ were then tried on training data, and the results of this test are given in Table 9.

TABLE 7

PROPORTION ESTIMATION ON TEST DATA

(Fayette County, August 21, 1973)

Parameter Settings			Proportion Estimation (%)	
n_1^2	n_2^2	n_3^2	Corn	Soybeans
20	5	2.5	13.56	33.58
20	5	5	20.12	37.30
20	2.5	2.5	15.85	31.06
20	2.5	5	24.56	33.63
Ground Truth			14.16	31.41

*Note: The parameter set established by training
        gives the best results on the test sections.

TABLE 8

PROPORTION ESTIMATION ON TRAINING DATA

Parameter Settings			Proportion Estimate (%)
η_1^2	η_2^2	η_3^2	WHEAT
20	2.5	5.0	23.3
20	2.5	2.5	18.1
Ground Truth			13.1

TABLE 9

PROPORTION ESTIMATION ON TRAINING DATA (CONTINUED)

Parameter Settings			Proportion Estimate (%)
η_1^2	η_2^2	η_3^2	WHEAT
14	2.5	5.0	22.5
14	2.5	2.5	16.5
7	2.5	5.0	21.5
7	2.5	2.5	13.5
Ground Truth			13.1

These parameter settings and results were then graphed. Extrapolation from this graph indicated that the correct parameter settings would be $\eta_1^2=6$, $\eta_2^2=2.5$, $\eta_3^2=2.5$. The results for this test is shown in Table 10. The results on the test data were then computed for each of the parameter settings previously tried. These results are given in Table 11. A graph of the training and test results plotted against the parameters is shown in Figure 25.

5.3 DISCUSSION

Analysis of the results obtained by nine-point mixtures reveals that:
(1) Nine-point mixtures has performed significantly better than conventional recognition as a crop proportion estimator for each data set examined. (2) Nine-point mixtures has shown itself capable of extremely accurate crop proportion estimation on one agricultural data set (Section 5.2.3). (3) On the other agricultural data set (Section 5.2.4), while nine-point mixtures performed better than conventional recognition, it is clear that better methods of setting the parameters should be investigated. (4) Nine-point mixtures has shown itself capable of extremely accurate proportion estimation of water, even with very small (.3 acre) lakes. (5) Nine-point mixtures appear to be consistent in this respect: it retains much of its accuracy even over small areas, as indicated by both the corn and soybeam test, and the water test. (6) Nine-point mixtures is comparable to conventional recognition in processing time for a reasonable number of signatures.

TABLE 10
PROPORTION ESTIMATING ON TRAINING DATA
(CONTINUED)

Parameter Settings			Proportion Estimate(%)
η_1^2	η_2^2	η_3^2	WHEAT
6	2.5	2.5	12.6
Ground Truth			13.1

TABLE 11

PROPORTION ESTIMATION ON TEST DATA

Parameter Settings			Proportion Estimate (%)
η_1^2	η_2^2	η_3^2	WHEAT
20	2.5	5.0	32.7
20	2.5	2.5	26.6
14	2.5	5.0	29.3
14	2.5	2.5	22.1
7	2.5	5.0	23.1
7	2.5	2.5	12.4
6	2.5	2.5	11.4
Ground Truth			9.0
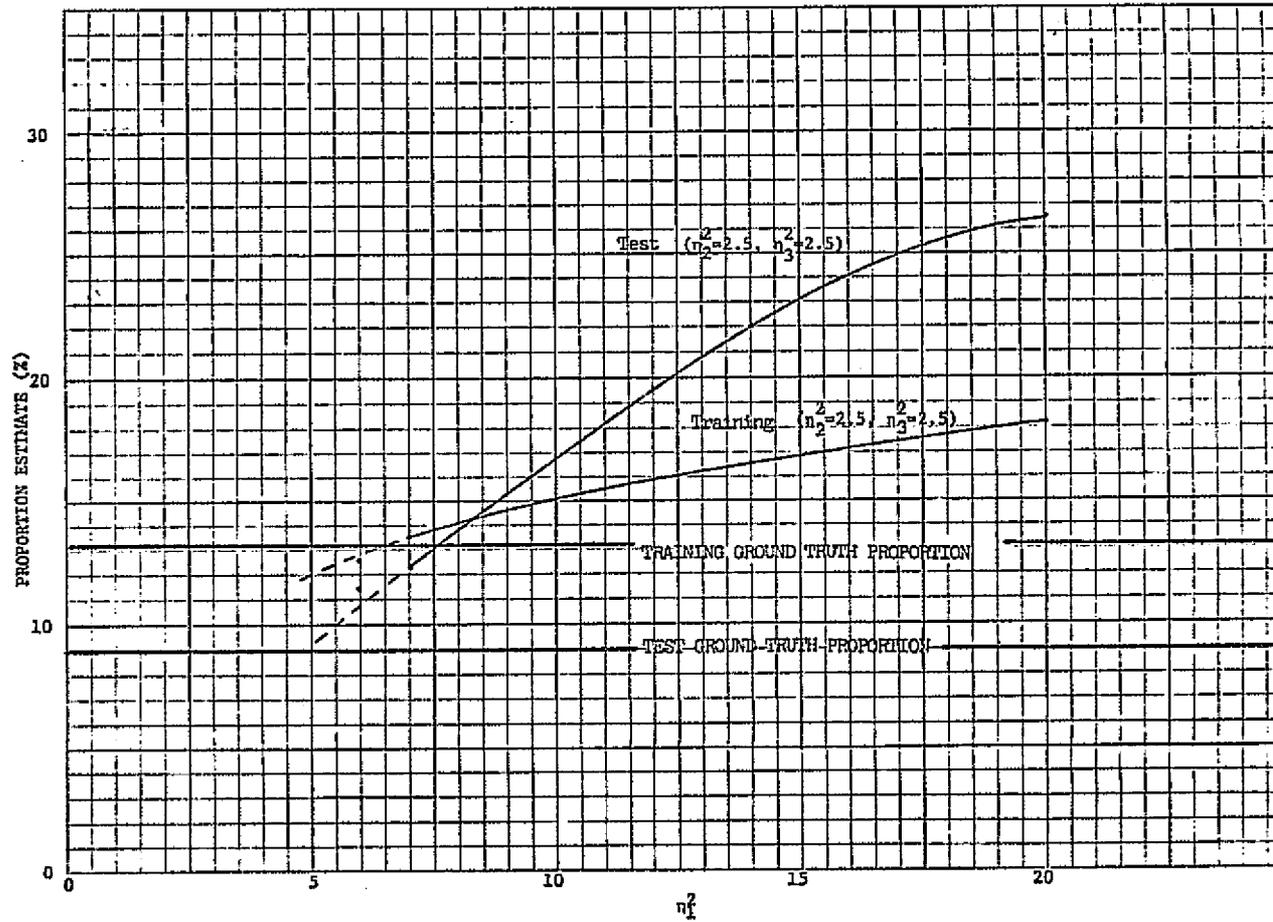
FIGURE 25. VARIATION OF THE PROPORTION ESTIMATE WITH η_1^2 IN THE 9-POINT
MIXTURE ALGORITHM (FAYETTE COUNTY DATA, 10 JUNE 73)

6

CONCLUSIONS AND RECOMMENDATIONS

Results of tests performed on LANDSAT data sets show that the LIMMIX and nine-point mixtures processing schemes offer significant improvement over both conventional recognition and MIXMAP processing. The reason for this superior performance seem to stem from the incorporation of prior information about mixture pixels and their spatial arrangement. The reduced number of spectral channels required for these procedures offers a further advantage over MIXMAP. For these reasons we believe that further testing of these newer concepts is warranted. In addition, reevaluation of data averaging should be considered.

The attainment of superior performance via LIMMIX and nine-point mixtures is possible only when the parameters of these procedures are correctly set; thus the problem of setting these parameters warrants further study. This is especially true for nine-point mixtures because of it's greater number of parameters.

Analysis of signatures shows they are often clearly multimodal, and the employment of unimodal signatures may degrade performance severely. This indicates that the effect of the utilization of several unimodal signature to represent a single object class be investigated in conjunction with these newer proportion estimation procedures. The possibility of doing this with MIXMAP is limited because of the restriction on the size of the signature set permitted relative to the number of spectral channels.

APPENDIX A

ESTIMATION OF CORRELATION FUNCTION

The computation of the estimate of the correlation function for a single field and single element channels is as follows. For simplicity, we assume that the field center pixels form a rectangular grid with line numbers $L, L_1 \leq L \leq L_2$, $L_2 - L_1 + 1 = N_L$; point numbers $P, P_1 \leq P \leq P_2$, $N_p = P_2 - P_1 + 1$. The signal at coordinates (L,P) is denoted by $X(L,P)$. The sample mean along point P is denoted by $\overline{X}_P$ where

$$\overline{X}_P = \frac{1}{N_L} \sum_{L=L_1}^{L_2} X(L,P)$$

The sample variance along point P is denoted by s_P^2 and is computed by

$$s_P^2 = \frac{1}{N_L} \sum_{L=L_1}^{L_2} [X(L,P) - \overline{X}_P]^2 = \frac{1}{N_L} \sum_{L=L_1}^{L_2} X^2(L,P) - (\overline{X}_P)^2$$

Then an estimate $\hat{R}(j)$, $0 \leq j \leq N_L - 1$, of the correlation function $R(j)$ for the field, crop type, and channel is taken as:

$$\hat{R}(0) = R(0) = 1$$

$$R(j) = \frac{1}{N_p(N_L - j)} \sum_{P=P_1}^{P_2} \sum_{L=L_1}^{L_2-j} \frac{[X(L,P) - \overline{X}_P][X(L+j,P) - \overline{X}_P]}{s_P^2}$$

$$1 \leq j \leq N_L - 1$$

If we transform the data by

$$y(L,P) = \frac{X(L,P) - \overline{X}_P}{s_P}$$

then we have

$$\hat{R}(j) = \frac{1}{N_P(N_L-j)} \sum_{P=P_1}^{P_2} \sum_{L=L_1}^{L_2-j} y(L,P) \cdot y(L+j,P) \qquad 1 \leq j \leq N_L - 1$$

The transformation from X to Y may be thought of as correcting for a simultaneous multiplicative and additive scan angle effect.

Now let us assume that there are K fields and that the estimated correlation function for the kth field is denoted by:

$$\hat{R}_k(j) \qquad 1 \leq k \leq K, \qquad 0 \leq j \leq N_{L,k} - 1$$

Let the average of the correlation functions over the K fields be denoted by

$$\tilde{R}(j)$$

where j ranges over $\qquad 0 \leq j \leq N_{min} - 1$

and $\qquad N_{min} = \min_k N_{L,k}$

Then

$$\tilde{R}(0) = 1$$

$$\tilde{R}(j) = \frac{1}{K} \sum_{j=1}^{K} \hat{R}_k(j) \qquad 1 \leq j \leq N_{min} - 1$$

APPENDIX B

DESCRIPTION OF LIMMIX

The following is a step by step explanation of the LIMMIX algorithm. The step numbers correspond to the accompanying flowchart. (Figure 26). Also included is a list of program variables.

EXPLANATION OF ALGORITHM

1. The mean (A) and covariance matrix (M) for each signature is entered and stored. A is an $n \times 1$ matrix and M is $n \times n$ (n = number of channels).

2. To save time, four frequently called terms are precomputed and stored.

   A. $\overline{M}_c^{-1}$

   The inverse of the average of the covariance matrix for all combinations of up to L at a time are calculated. The subscript c designates the combination number. For combinations of one at a time, c goes from 1 to m (m = number of signatures.) The combination numbers for 2 at a time begin at m+1 and are in this order: signatures 1 and 2, 1 and 3,..., 1 + m, 2 and 3, 2 and 4,...,etc.

   B. $\overline{M}_c^{-1} A_i$

   Each of the previously stored matrices are multiplied by each of the A (mean) vectors which correspond to the component matrices used to compute each $\overline{M}_c^{-1}$ matrix.

   Example: $\overline{M}_{2,5}^{-1}$ is multiplied by A_2 and the result stored

   Then $\overline{M}_{2,5}^{-1}$ is multiplied by A_5 and the result stored

   Each of these products result in an $n \times 1$ matrix.

   C. Γ^{*-1}

   First, the Γ (gamma) matrix is calculated. Here is an example where the matrix combination containing covariances of signatures one,

two, and four are used. M will stand for $\overline{M}_{1,2,4}^{-1}$.

$$\Gamma = \begin{pmatrix} A_1^t MA_1 & A_1^t MA_2 & A_1^t MA_4 \\ A_2^t MA_1 & A_2^t MA_2 & A_2^t MA_4 \\ A_4^t MA_1 & A_4^t MA_2 & A_4^t MA_4 \end{pmatrix}$$

Γ is an m x m matrix, independent of the number of channels. (Each $A^t MP$ is 1x1 matrix, or just a number).

Γ^*, the augmented matrix, is the Γ matrix with an extra row and column of ones, and a zero in the m+1, m+1 position.

Example:

$$\Gamma^* = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & 1 \\ \Gamma_{21} & \Gamma_{22} & \Gamma_{23} & 1 \\ \Gamma_{31} & \Gamma_{32} & \Gamma_{33} & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

The inverse of the above matrix is stored for all the combinations.

D. $\ln|\overline{M}_c|$

As a biproduct of taking the inverse of the average covariance matrix, the determinant is computed. The natural log of these determinants are stored for use later in the likelihood and chi-squared calculations.

3-6. The likelihood, chi-squared, and proportion vector storage bins are given initial values. The data vector (n x 1) from the first pixel is multiplied by the transpose of the first MB vector (1 x n) to yield g. For the first m calculations, there is one A per M and g is just a number. When MA's are called from storage in sequences where two or more A's are multiplied by a common M, g is a vector whose length is the number of components of M.

7. The g vector is augmented and then multiplied by the Γ^{*-1} matrix that used the same $\overline{M}_c^{-1}$ matrix. The product gives the proportion vector of the component signatures and Δ (which serves the role of a Lagrange multiplier).

8-10. If any one of the proportions is negative, that signature set is rejected as a solution and the next $M\overline{A}$ and Γ^{*-1} matrices are used to find a new λ vector. When an all positive proportion vector is found, the likelihood and chi-squared values are calculated.

11-14. If this calculated likelihood is greater than (i.e., less likely) the stored value (a_o), the signature set is rejected as a solution. However, if $a_o > a$, the new values for the likelihood, chi-square, and proportion vector are substituted into temporary storage. When the level is complete (i.e., when all the one at a time or two at a time etc. combinations have been looked at) a_o, χ_o^2 and the λ_o's are stored to be output on tape later. If the level has not been completed, the matrices for the next combination of signatures are brought from storage and the calculations starting from step 6, are done again.

15-17. When a level has been completed, the values that will be calculated in the next level are initialized with those calculated from the previous level. This is not really a step, since the winning values from the previous level are already in temporary storage (step 13).

   Step 16 is included to clarify the fact that the values are not initialized to the same numbers in the succeeding levels as they were in the first (step 5). This method of initialization prevents solutions at the i + 1 level being less likely than the solutions at the i level. e.g., the most likely two at a time may be a recognition with the second proportion equal to zero.

   When all four (or less) levels are complete, the likelihood, chi-square, and proportion vector for each level are put on tape, and the algorithm proceeds to the next pixel.

FIGURE 26. LIMMIX FLOW CHART
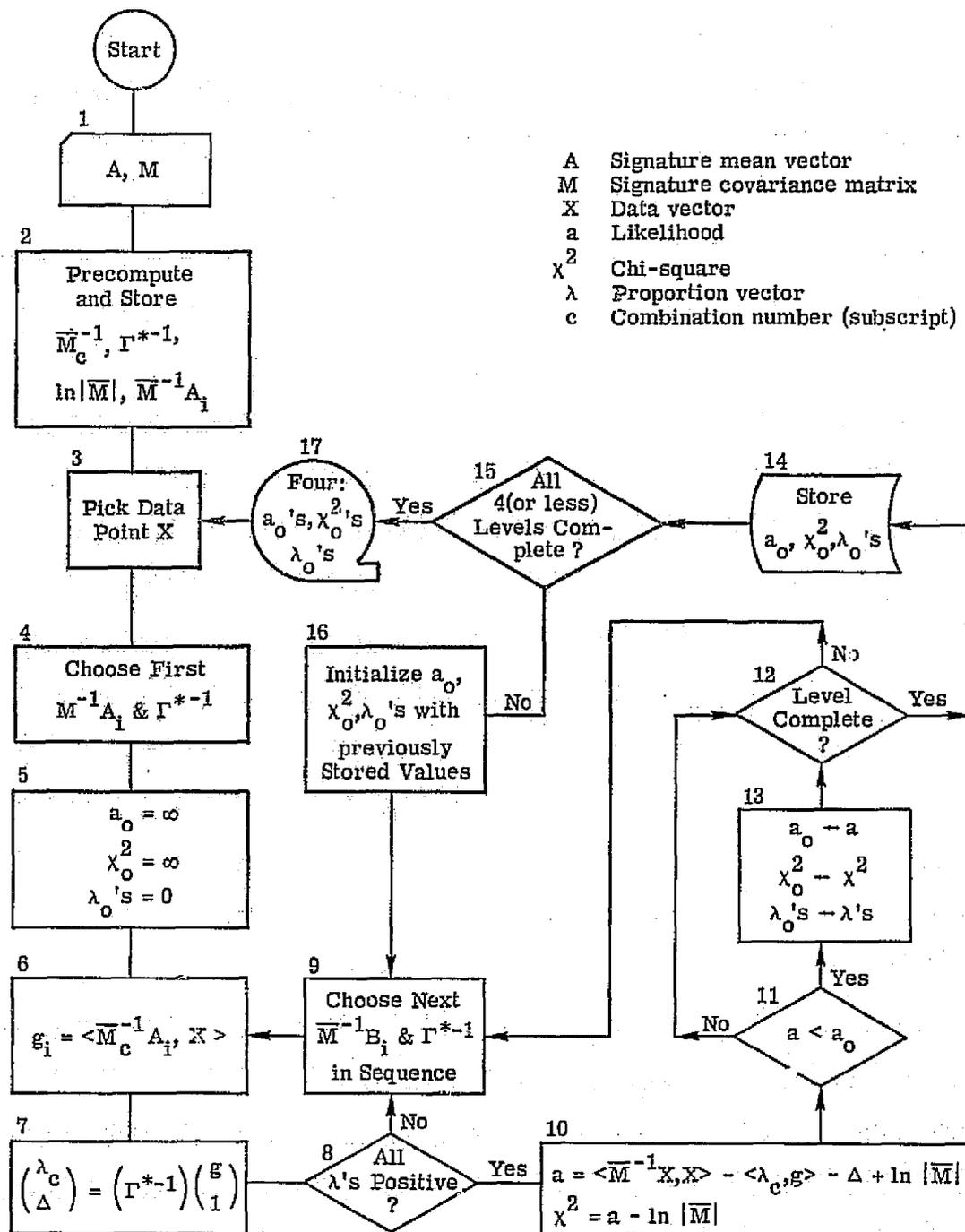
A Signature mean vector
M Signature covariance matrix
X Data vector
a Likelihood
χ^2 Chi-square
λ Proportion vector
c Combination number (subscript)

LIMMIX. is a module of POINT. A program whose main function is to transfer data points to its modules in accordance with control data. This control data specifies the ground area to be processed. The format of POINT. is such that modules may be called at several different stages of the data processing procedure. At step one of POINT., before any data has been read in, LIMMIX. control variables are read, initialization is completed, and pre-computations are done. In step two, the number of output channels is set to 22. This is after the POINT. control variables have been read but before any processing is done. Lastly, POINT. calls the processing part of LIMMIX. (Flowchart Step 3 to the end) for each data point until the area is complete. The next page contains a list of the LIMMIX control variables.

```
CALLING SEQUENCE:        $COMPILE MAD, EXECUTE
                                POINT. (LIMMIX.)
                                E'M

                         $ BINARY
                             LIMMIX,BINARY DECK

                         $ DATA
                             READ AND PRINT DATA CARD(S) FOR LIMMIX.*
                             SIGNATURES
                             INBIN,OUTBIN,FILE,OFILE,...
                             NSA'S (OR POLYGON COORDINATES[1])*
```

[1] POLYGON coordinates have been run with LIMMIX. on the ERIM ERTS project with no apparent errors.

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LIMMIX VARIABLES

VARIABLE	DEFAULT	IT IS RESET TO DEFAULT AFTER MODE1=$START$*	EXPLANATION
NSIG(2)	None	No	Number of signatures (each with NV channels – subsets of channels not allowed for signatures)
ND	None	No	Maximum number of channels on the data tape.
NV(2)	None	No	Number of channels on the data tape that will be used (NV ≡ NCHAN).
ICODE	1,2,3,4,...	No	Which channels on the data tape Example: ICODE(1) = 1,2,4
RANK(2)	4	Yes	Maximum number of signatures considered in the identification of each pixel. RANK must be ≤4 (RANK ≡ NCOMP (ALIEN 2))
SCALE	1	Yes	Scale factor for χ^2 (chi-squared)
MODE	0	Yes	= 0 means read the first NSIG signatures = 1 means search tape for NSIG signatures whose names are read, one to a card, by 2C6* at the time of the search. = 2 means search tape but use the previous name list. =-1 means return without reading new signatures. Do not use this option in LINMAP or CLASFY.
CC	0	Yes	=-1 print nothing = 0 print the i.d. card only = a character. Print the i.d. card, mean vector and covariance matrix with CC as the carriage control character for the i.d.

(2)The values that are set for NSIG, NV, and RANK are interrelated. For a given NSIG and NV, there is a maximum setting for RANK. The following table shows the relationship.

RANK ≤ NV+1   when RANK = NSIG

and

RANK ≤ NV   when RANK ≠ NSIG

88

APPENDIX C
DESCRIPTION OF ALIEN 2

This program, written in the programming language MAD as implemented on an IBM 7094 computer, performs the analysis algorithm for LIMMIX.

This program accepts the output of LIMMIX., and produces the following output:

1. How many pixels were classified by recognition, mixtures with 2 signatures, mixtures with 3 signatures, mixtures with 4 signatures, and the number unclassified.

2. The amount of each material classified by each of the above methods.

3. The amount of each material classified by each of the above methods, but as a fraction of the total number of pixels.

4. The amount of each material classified by each of the above methods, but as a fraction of the total number of classified pixels.

5. The amount of each material from recognition, recognition plus 2 signature mixtures, recognition plus 2 and 3 signature mixtures, and the total amount of each material.

6. The mean square error (in pixels) of the subject material, both for each area and the sum of all of the areas.

7. The percent mean square error of the subject material, both for each area, and the sum of all the areas.

Portions of this output can be suppressed.

The program is a module of POINT, a program which provides data points to it's modules in accordance with control data (NSA cards) and in a rigid format which includes calls to all of the modules before any control data is read, after control data is read, before each line of data is read, and after each area has been processed. For each data point, a call to the internal function of each module is made, for processing of data.

The program ALIEN2 is organized as follows:

STEP(1) - (This step is called before POINT's control data is read)
setup and initialization, obtain control variables.

STEP(2) - (POINT. calls this after initial control data is read),
zero sums of number of pixels of each class if starting
new region,

STEP(5) - (POINT. calls this after each area is processed) if this
wasn't the last area to be combined into one region,
return to POINT. Otherwise compute and print out statistics.

Internal Function PSUM - (POINT. calls this for each data point). It
is here that the decision as to whether a point is one material,
two materials or more is made, and here the thresholding
and/or renormalization is done, and finally the pixel is
added to the running sum of the number of pixels of each
material.

The variables J(1) through J(4), (in the THROUGH loops, lines 106-109)
correspond to the variables X_1^2, X_2^2, X_3^2, X_4^2, of LIMMIX, which are used as
thresholds (in lines 114 and 115) to decide whether the pixel is one
material, two or alien. If the pixel is alien, the alien COUNT is
increased (in line 117), otherwise, the correct N-materials at a time count
is incremented in line 119.

Then, in lines 121 to 129, the combination of materials is decoded,
and the proportions of these materials are stored in the OUT array.

At this point, either thresholding, or thresholding with renormalization
is done to the proportions (in lines 136 to 138 or 139 to 150, respectively),
and finally these proportions are added to the SUM array, which holds the
accumulated totals for each material. Optionally, a likelihood weighting
can be used as a decision rule, and this is done in lines 151-162. See
Figure 27 for ALIEN2 flowchart.

The arrays COUNT and SUM are indexed by the variable Z, which is
incremented each time the data point has been processed by a set of
parameters, and thus the proportion of each material (in SUM) and the count
of how many pixels were pure, or of two materials, etc., (in COUNT) is kept
separate for each parameter setting. Further, the array SUM is indexed

ΣERIM

by N, the number of materials in that pixel, so that the amount of pure material, two material mixtures, etc., is kept separate for each parameter setting and each material.

Control variables for this program are as follows:

VARIABLE	POSSIBLE VALUES	DEFAULT	FUNCTION
THRESH	ON	off	When THRESH=ON, any proportion less than TAU* is set to zero
NORMAL	ON	off	When NORMAL=ON, any proportion less than TAU* is set to zero. Then all remaining proportions are re-normalized to sum to one.
LIKELY	ON	off	When LIKELY=ON, then the likelihood decision rule is used, i.e., ℓ_1, ℓ_2, ℓ_3, ℓ_4, are 'weighted', and the minimum is decided upon (see description of LIMMIX output tape). When LIKELY=ON, P* must be specified. When LIKELY is not ON, the Chi squared decision rule is used.
HAFOUT	ON	off	When HAFOUT=ON, output items (1)-(4) are not calculated.
ERROUT	ON	off	When ERROR=ON, output items (5) & (6) are calculated. When ERROR=ON, TRUTH*,CHAN* must be specified.
NCOMP	1-4	1	A maximum of NCOMP-signatures per mixtures was used.
NSIG	1-9	1	Number of signatures.
TAU	any	0	Thresholding value, see NORMAL*

*See Specification of Variable

VARIABLE	POSSIBLE VALUES	DEFAULT	FUNCTION
START(1-4)	INTEGER	1	Starting value of index. See Figure 28.
STP(1-4)	INTEGER	1	Increment of index. See Figure 28.
JEND(1-4)	INTEGER	1	Final value of index. See Figure 28.
CHAN	1-9	1	Material or signature under consideration in calculating mean square error.
P(1-4,1-21)	any	0	Weights used in likelihood decision rule, indexed by START*, STP*, JEND*, see Figure 28.
TRUTH	any	Garbage	The amount of material under consideration for each area.
LCUT(1-4)	any	1000	Alien thresholds for likelihood decision rule.
SUMOUT	ON	off	When SUMOUT=ON only output Item (4) is calculated.

*See Specification of Variable
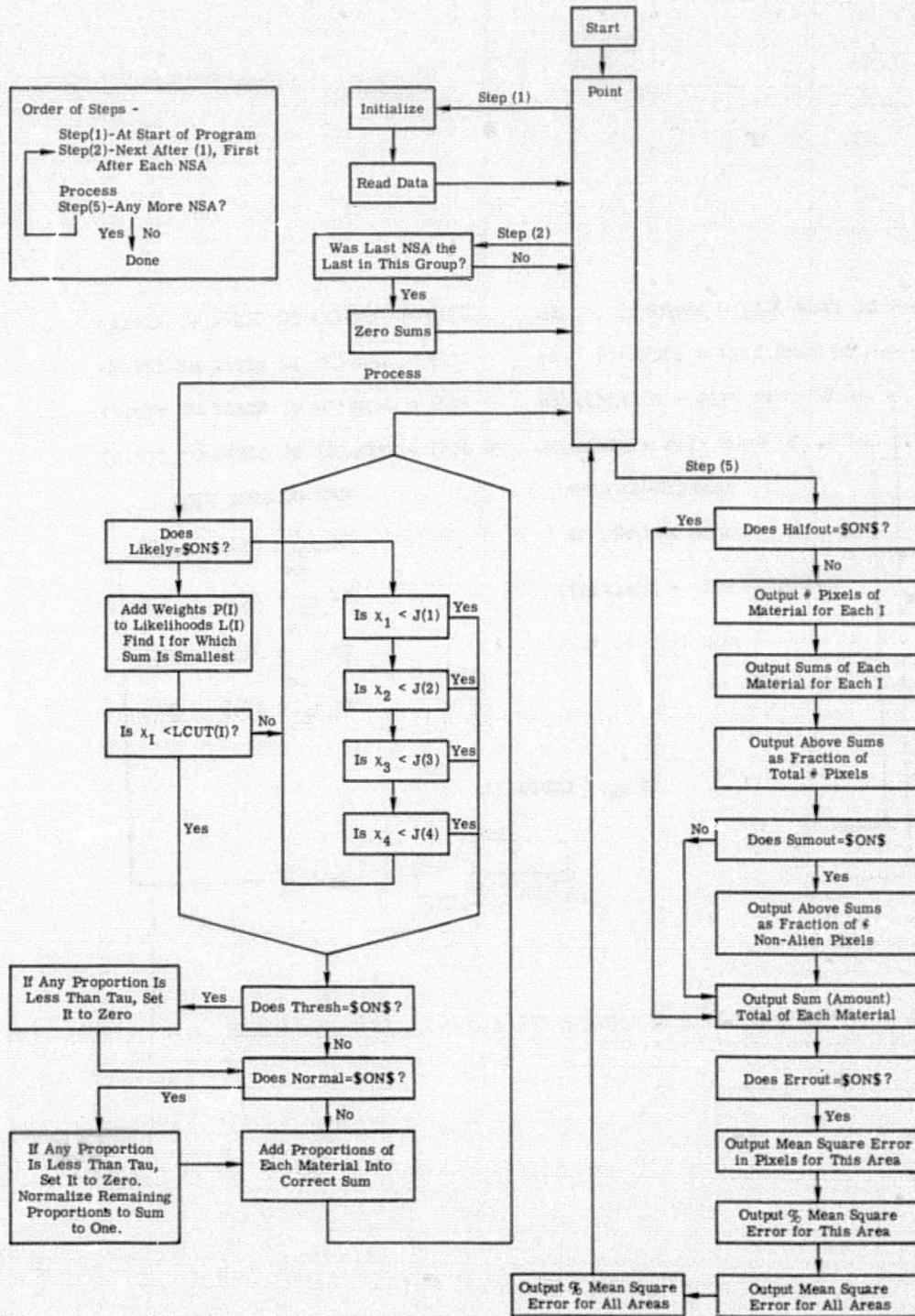
FIGURE 27. ALIENZ FLOW CHART

93

```
┌──── DO FROM J(1) = START(1)    to     J(1) = JEND(1) BY STEPS OF STP(1)
│ ┌──── DO FROM J(2) = START(2)    to     J(2) = JEND(2) BY STEPS OF STP(2)
│ │ ┌──── DO FROM J(3) = START(3) to     J(3) = JEND(3) BY STEPS OF STP(3)
│ │ │ ┌──── DO FROM J(4) = START(3)   TO J(4) = JEND(4) BY STEPS OF STP(4)
```

LIKELIHOOD RULE	CHI-SQUARED RULE

TAKE MINIMUM OF

$L(I) + P(I,J(1))$

FOR I=1, 2, 3, 4

IS $\chi_1 < J(1)$ YES
no
IS $\chi_2 < J(2)$ YES
no
IS $\chi_3 < J(3)$ YES
no
IS $\chi_4 < J(4)$ YES

IS $\chi_I < LCUT(I)$?

YES

CLASSIFIED
AS I^{th} DATA SET
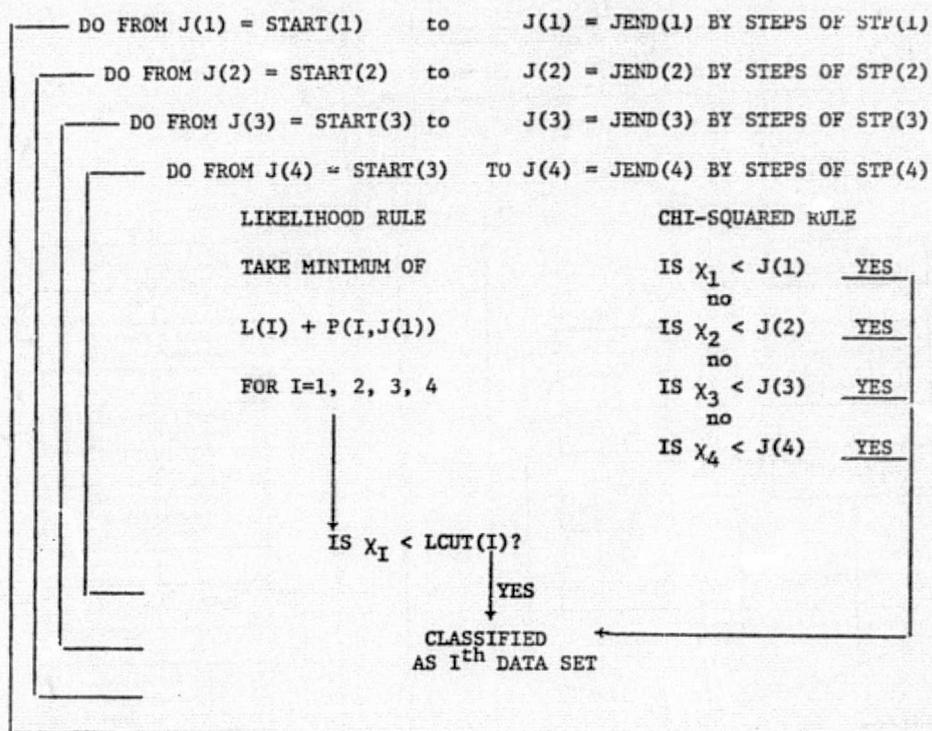
FIGURE 28. EXPANSION OF ALIENZ DECISION RULE

APPENDIX D

DESCRIPTION OF GEOM2

The distances obtained for the geometrical signature analysis for IMMIX processing (GEOM2) are defined as follows. To avoid notational complexity we will assume that a specific subset of $L+1$ signatures has been chosen and relabeled, if necessary, so that their means are denoted by A_1, A_2,...,A_{L+1} and covariance matrices by M_1, M_2,...,M_{L+1}. Let H_1 denote the hyperplane of dimension $k=1$ though the means A_2,...A_{L+1} and let Z be the point in H_1 which maximizes the Gaussian density with parameters A_1, M_1. Then d_1 is defined by

$$d_1^2 = <Z-A_1, M_1^{-1} (Z-A_1)>$$

It has been shown by W. Richardson in Reference [2] that d_1 may be computed in the following fashion. Let Γ denote the $(L+1) \times (L+1)$ matrix with entries $<A_i, M_1^{-1}A_j>$; $1 \le i$, $j \le L+1$.

Let e_k denote the column vector of length L with all components equal to 1, and let Γ_1^* be the $(L+2) \times (L+2)$ matrix defined by

$$\Gamma_1^* = \begin{bmatrix} \Gamma_1 & e_k \\ e_k^t & 0 \end{bmatrix}$$

Then $\dfrac{1}{d_1^2}$ is the $(1,1)$ element of the inverse of Γ_1^*. More generally

$\dfrac{1}{d_i^2}$ is the $(i.i)$ element of the inverse of Γ_i^*, $1 \le i \le k+1$.

By some manipulation one can obtain a more convenient form of the Richardson result for the computation of the d_i. Let $B_{i1} = A_i-A_1$ and let $\bar{\Gamma}_1$ be the $L \times L$ matrix defined by

$$\overline{\Gamma}_1 = \begin{bmatrix} <B_{2,1}, \ M_1^{-1} B_{2,1}> & \cdots & <B_{2,1}, \ M_1^{-1} B_{L+1,1}> \\ <B_{L+1,1}, \ M_1^{-1} B_{2,1}> & \cdots & <B_{L+1,1} \ M_1^{-1} B_{L+1,1}> \end{bmatrix}$$

Set

$$\overline{\Gamma}_1^* = \begin{bmatrix} \overline{\Gamma}_1 & e_{L-1} \\ e_{k-1}^t & 0 \end{bmatrix}$$

Then $-d_1^2$ is the (L,L) element of the inverse of $\overline{\Gamma}_1^*$. In general, $-d_i^2$ is the (L,L) element of the inverse of $\overline{\Gamma}_i^*$.

D.1 OUTLINE OF THE PROGRAM

A. Input control data, signatures. Store covariance terms in the GAMMA matrix, and means in the MATRIX matrix.

B. For each signature (say, the mth signature) repeat through B(3).

   B(1) Assign MATRX2 (I,J) ← MATRIX(J,I) − MATRIX(M,I)

   This moves the mth corner of the signature complex to the origin.

   B(2) Assign MATRIX(I,J) ← MATRX2(J,I) Invert this translated signature complex

   B(3) Assign GAMMA (M,I,J) ← MATRIX(I,J) (GAMMA(M,I,J)$^{-1}$)MATRX2(I,J)

C. For each signature (say, the mth signature) repeat through C(5).

   C(1). For each combaination of (NO−1) signatures, $(X_1 \ X_2 \ X_{NO-1})$ which does not include the mth signature, select the following elements of GAMMA(M,I,J)

$$\begin{matrix} (X_1, \ X_1) & \cdots & (X_1, \ X_{NO-1}) \\ \vdots & & \vdots \\ (X_{NO-1}, \ X_1) & \cdots & (X_{NO-1}, \ X_{NO-1}) \end{matrix}$$

   and arrange these elements of GAMMA(M,I,J) in a (NO−1)X(NO−1) matrix, in the above order, assign these elements to MATRIX(I,J).

96

C(2). Augment MATRIX(I,J) by placing 1.0 in the $(NO-1)^{th}$ row and column, and 0.0 in the ((NO-1), (NO-1)) position.

C(3). Invert MATRIX(I,J) and place the inverse back into MATRIX(I,J)

C(4). Assign MATRIX(NO, NO) $(-MATRIX(NO,NO))^{1/2}$

C(5). The element MATRIX(NO,NO) then contains the desired answer, and is printed out.

D. End of Program

D.2 HOW TO RUN THE PROGRAM

GEOM2 needs the following input:

NO - the size of simplex used measuring the distances, i.e., in Figures   and   , NO=3.

NSIG   - the total number of signatures to be input (less than twelve)

NCHAN   - he number of channels in the data (less than twelve)

A typical deck might look like

```
$COMPILE MAD, EXECUTE
        GEOM2.
        E'M
$BINARY
        {GEOM2 BINARY DECK}
$DATA
        NO=5, NSIG=6, NCHAN=4*
        {SIGNATURE DECKS IN STANDARD ERIM FORMAT}
```

C-2

APPENDIX E

DESCRIPTION OF CLUSTR

This program written in the programming language MAD as implemented
on an IBM 7094 computer, implements the clustering algorithms described
in Section 4.3.

E.1 ALGORITHM ONE

This is the default algorithm. The following variables must be set:

> NCHAN - number of channels, if different from
> 4 (must be $\leq$ 8)
>
> Any POINT control variables (i.e., NC, NV, ICODE(1))
>
> LASTID- ID FIELD of last NSA to be clustered. Default
> is $TTTTTT$

Rare:

If only a very few clusters are produced, it may be necessary to
set RHOSRT to less than 8. With more than 5 channels, increase
RHOSRT as follows:

RHOSRT - this should be set to

$$\left(\frac{1}{2} \cdot \left[\frac{\prod\limits_{i=1}^{i=NCHAN} (\text{Range of 2 standard deviations in channel i})}{NCHAN \cdot (K_{NCHAN})^{\frac{1}{NCHAN}}} \right] \right)^2$$

Where K_{NCHAN} = 150 for NCHAN = 1 to 6, 100 for NCHAN - 7, 70 for NCHAN = 8
Also, set REPLAC = $\sqrt{RHOSRT}$

If some clusters contain too many data points, or if there are too
many clusters produced, it may be necessary to reset PERCT1 and PERCT2.

PERCT1 – whenever a cluster holds more than PERCT of the points, clustering is stopped. Default is .166.

PERCT2 – whenever the largest NUM clusters hold more than PERCT2 of the points, clustering is stopped, default is .666.

E.2 ALGORITHM TWO

All variables are the same as in algorithm one, except

INIT – if INIT=ON, points from NSA's with differing FIRST SIX CHARACTERS in the ID field will be separated, and these first six characters will be used as the name of the cluster; this is useful to identify multimodality.

E.3 ALGORITHM THREE

POINT CONTROL VARIABLES (i.e., NC, NV, ICODE(1))

NSPACE – must be set to ON

SEQ – if SEQ=ON, updating of means and variances will occur after each point, otherwise after each NSA. Recommended ON for $\leq$ 2000 points only.

NNIEB – if NNIEB=OFF, the linear classification rule is used for point assignment, otherwise, euclidian distance is used.

NUM – only the largest NUM clusters are displayed.
NUM $\leq$ 30, default is 15.

LASTID – as above.

NCHAN – as above but $\leq$ 5.

E.4 HISTOGRAMMING

Histogramming: After the completion of any of the above algorithms, a histogram of all the major clusters can be obtained at no additional cost. This type of histogram has the advantage that it represents the data set, sans noise and mixtures, and it requires no tape mounts.

HIST=ON-default is ON

MIN – the smallest data value displayed, default is 1.

MAX – the largest data value displayed, default is 100.

After clusters have been produced, it may be necessary to further combine them.

This program has the capability of combining signatures together on the basis of probability of misclassification, and this combining is stopped whenever more than PERCT1 of the points in all of signatures have been combined, or whenever the most populous NUM clusters have more than PERCT2 of the points in all signatures.

To use this capability, simply specify COMCOV=ON, and NSIG=Number of signatures; this is to be followed by the signatures. If it is desired that signatures with unlike ID fields (i.e., the first six characters of the ID) not be combined, specify INIT=OFF also. If it is desired to combine the signatures with weighting by the number of points in the signature, specify WEIGHT=ON.

SAMPLE SET UPS:

> CLUSTERING ALGORITHM ONE, HISTOGRAMMING,
>
> CHANNELS 4, 5, 6, 7, 8 used out of 24
>
> $COMPILE MAD
>
> > POINT.(CLUSTR.)
> >
> > E'M

CLUSTR.
OBJECT
DECK
{
> $BINARY
>
> $DATA
>
> > NC=24, NV=5, NCHAN=5, ICODE(1) = 4,5,6,7,8
> >
> > HIST=ON*

NSA'S TO BE
CLUSTERED
{
> NSA=
>
> NSA= $TTTTTT$*

CLUSTERING OF SIGNATURES, 4 CHANNELS, 28 SIGNATURES, WEIGHTED COMBINING

> $COMPILE MAD, EXECUTE
>
> > POINT.(CLUSTR.)
> >
> > E'M

CLUSTR
OBJECT
DECK
{
> $BINARY
>
> $DATA

```
                COMCOV=$ON$, NSIG=28, WEIGHT=$ON$*
SIGNATURE
DECKS

        CLUSTERING ALGORITHM TWO, NO HISTOGRAMMING, LAST NSA ID IS $QQ100$

        FOUR CHANNELS

        $COMPILE MAD
                POINT.(CLUSTR.)
                E'M
        $BINARY
CLUSTR.
OBJECT
 DECK

        $DATA
        INIT=$ON$, LASTID=$QQ100$*
                    NSA=            $SOY$
                    NSA=            $SOY$
NSA'S               NSA=            $CORN$
TO BE                                etc.
CLUSTERED           NSA=            $QQ100$*


        CLUSTERING ALGORITHM THREE, UPDATE AFTER EACH POINT, LINEAR RULE FOUR CHANNELS

        $COMPILE MAD
                POINT.(CLUSTR.)
                E'M
        $BINARY
CLUSTR.
OBJECT
 DECK

        $DATA
        NSPACE=$ON$, SEQ=$ON$, NNIEB=$OFF$*
                    NSA=
NSA'S
TO BE
CLUSTERED           NSA=            $TTTTTT$*
```

E.5 LIST OF THE CONTROL VARIABLES

VARIABLE NAME	DEFAULT VALUE	EXPLANATION
NCHAN	4	Number of channels to be used (<8) (This must always be specified if different from 4)
INIT	OFF	An option to 'name' clusters, see algorithm 2.
NSPACE	OFF	To effect use of algorithm 3, set NSPACE=ON
NUM	10	The most populous NUM clusters are used for display (NUM $\leq$ 30)
LASTID	$TTTTTT$	This is the ID field of the last NSA used for each operation. See examples
NNIEB	ON	This specifies that the distance measure to be used with algorithm 3 is the L_1 or Euclidian metric
SEQ	OFF	When SEQ=ON, the means and variances in algorithm 3 are updated after each point; when off, after each pass
CUT(3)	10	Any cluster with > CUT(3) points in it will have a signature deck punched up for it, unless CARDS=OFF
CARDS	ON	When CARDS=OFF, no signature decks are punched.
PERCT1	.166	In algorithms 1 and 2 whenever a cluster contains more than PERCT1 of the points, the combining is halted.
PERCT2	.666	In algorithms 1 and 2 whenever the NUM largest clusters contain more than PERCT2 of the points, combining is halted.
HIST	ON	Whenever HIST=ON, histograms of the clusters will be made for each channel.
CMS		Number of approximating cells. Program runs faster with fewer cells, but less accurately.

VARIABLE NAME	DEFAULT VALUE	EXPLANATION
		CMX Defaults to: 150, for NCHAN=1-6 100, for NCHAN=7 70, for NCHAN=8
MIN	1	Smallest value displayed in histogramming ($MIN \geq 0$)
MAX	100	Largest value displayed in histogramming ($MAX \leq 300$)
SMX	---	Maximum number of points stored, in Algorithms 1 and 2 SMX Defaults to: 800, for NCHAN=1-3 700, for NCHAN=4 500, for NCHAN=5-6 400, for NCHAN=7-8
NSIG	---	Number of signatures to be combined, used only with COMCOV=ON.
WEIGHT	ON	When WEIGHT=ON, combining of signatures is weighted as to the number of points in each signature. Used only with COMCOV=ON.
COMCOV	OFF	When COMCOV=ON, the program will 'cluster' signatures, i.e., combine signatures on the basis of high probability of misclassification
CUT(2)	2	Any cluster with less than this number of points in it is ignored for purposed of combining.
THETA	4	This is the θ of algortihms 1 and 2 (step 1 of description)
RHORST	8.0	This is the $\sigma_{ij}^2(0)$ of algorithms 1 and 2 (step 2 of description)
REPLAC	4	Any cluster with a $\sigma_{ij}^2 <$REPLACE has that σ_{ij}^2 set equal to REPLAC during the computation of the probability of misclassification. It is assumed that for a cluster with a variance less than REPLAC has, the estimate of the variance is poor.
SIZE	---	This is a vector giving the minimum and maximum values of each channel, (SIZE(1)=max value of first channel, SIZE(3)=max value of second channel, etc.), used with algorithm 3 to specify the data space.

103

APPENDIX F

DESCRIPTION OF NINE-POINT PROGRAM (NPM)

This program, written in the projramming language MAD as implemented on
a IBM 7094 computer performs the algorithm of nine-point mixtures.

It uses the output tape of LIMMIX as input, and determines the amount
of each material found in a region. The program NPM is a module of POINT,
a program which transfers data from the input tape to its modules on a
point-by-point basis. POINT calls its modules as follows: STEP(1) of NPM
is transferred to before any processing takes place, STEP(4) after each
scanner line of data has been processed, and STEP(5) after each area in
the region has been processed. A call is made to the internal function
of NPM for each data point to be processed.

The organization of the program is as follows:

In STEP(1), input of control variables, set-up and initialization is
done.

In STEP(4), after each line is processed, the actual decision rule
is implemented, and a running sum of the amount of each material
found is kept.

In STEP(5), after each area of the region is processed, the ID field
of the POINT control card is examined to determine whether or not
the end of the region has been reached, if so, the totals are printed
out, otherwise nothing is done.

In the internal function, the data which is passed by POINT is stored
into the vectors LINE, LINE1, LINE2, LINE3, LINE4, LINE5, for processing
after the end of a scanner line of data.

An outline of the program is as follows:

A. read in control data, initialize storage

B. for each point of data, store DATUM(2) in LINE, DATUM(4) in LINE1,
DATUM(5) in LINE2, DATUM(6) in LINE3, DATUM(7) in LINE4, DATUM(9) in LINE5.
Each DATUM is entered into the appropriate LINE vector in the position
corresponding to the data point's position in the scanner line.

DATUM(2) is the identity of the recognition

DATUM(4) is the chi-squared level of the recognition (x 500)

DATUM(5) is the proportion of material one in the mixture (x 500)

DATUM(6) is the proportion of material two in the mixture (x 500)

DATUM(7) is the code giving the identities of the materials in the mixture

DATUM(9) is the chi-squared level of the mixture (x 500)

DATUM(2) and DATUM(4) are stored only if DATUM(2) $\leq$ CHICUT, where CHICUT is the η_1^2 of nine-point mixtures.

C. After each line, perform the following for each point of the previous data line:

C(1) take a vote of the 9 pixels forming a block around the center pixel with regard to their identity, which is obtained from the LINE vector.

C(2) find the largest and second largest vote totals and store these in HOLD and HOLD2, with the number of the corresponding signature in SAVE and SAVE2.

C(3) if the vote is $\geq$ HOWMNY (HOWMNY is the N_1 of nine-point mixtures), add one to the corresponding signature's total, which is kept in SUM. Go to C.

C(4) if the vote (C(3)) fails, examine LINE1, to see if the center pixel's chi-squared level is $\leq$ CHI2 (CHI2 corresponds to η_2^2 in nine-point mixtures). If this chi-squared level is $\leq$ CHI2, accept the center pixel's recognition and add one to the corresponding signature's total.

C(5) if C(4) fails, examine HOLD and HOLD2, to see if they are both $\geq$ CUT2 (CUT2 corresponds to N_2 in the nine-point mixtures), if this is so, add (HOLD/(HOLD+HOLD2)) to the SAVE signature's total, and (HOLD2/HOLD+HOLD2)) to the SAVE2 signature's total. Go to C.

C(6) if C(5) fails, check LINE5, to see if it is $\leq$ CHICT2 (CHICT2 corresponds to η_3^2 in nine-point mixtures). If this is so, decode the combination number in LINE4 to determine which materials are in the mixture and add the correct proportion to each of these two material's totals (obtained from LINE2, LINE3) after $\geq$ CHICT2, call the pixel alien, and go to C.

D. After all the lines in the area have processed, examine the ID field
of POINT's control card to determine whether or not the next area
is to be added to this one. If so, go to B, otherwise, print out
the totals in SUM, zero all sums and go to B.

E. End of Program.

VARIABLE	DEFAULT	EXPLANATION
CHICUT	--	η_1^2 of nine-point mixtures
CHI2	--	η_2^2 of nine-point mixtures
CHICT2	--	η_3^2 of nine-point mixtures
NUM1	--	N_1 of nine-point mixtures
NUM2	--	N_2 of nine-point mixtures

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