PROCEDURES FOR THE DESIGN
OF LOW-PITCHING-MOMENT AIRFOILS

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Three approaches to the design of low-pitching-moment airfoils are treated. The first method decreases the pitching moment of a given airfoil by specifying appropriate modifications to its pressure distribution. The second procedure designs an airfoil of desired pitching moment by prescribing parameters in a special formula for the Theodorsen \( \epsilon \)-function. The third method involves appropriate camber-line design with superposition of a thickness distribution and subsequent tailoring. Advantages and disadvantages of the three methods are discussed.
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SUMMARY

Three approaches to the design of low-pitching-moment airfoils are treated. The first method decreases the pitching moment of a given airfoil by specifying appropriate modifications to its pressure distribution. The second procedure designs an airfoil of desired pitching moment by prescribing parameters in a special formula for the Theodorsen ε-function. The third method involves appropriate camber-line design with superposition of a thickness distribution and subsequent tailoring. Advantages and disadvantages of the three methods are discussed.

INTRODUCTION

Low-pitching-moment airfoils find application primarily as helicopter rotor blades; but more recently some attention has been given to the advantages of low-pitching-moment sections for a "span-loader" vehicle. For such applications a symmetric airfoil could conceivably be employed, but cambered airfoils can offer significant advantages. The usual difficulty that is encountered in the design process stems from the fact that the airfoil shape and performance are sensitive to the parameters that control the pitching moment. For example, an airfoil with zero pitching moment but with moderately small positive lift at zero angle of attack deviates significantly from a symmetric section. Similarly if one attempts to modify the lower surface of an airfoil to reduce the pitching moment, while retaining the upper-surface shape and pressure distribution, he generally finds that substantial modification is required even for small reductions in pitching moment.

In this paper three approaches to the problem of designing low-pitching-moment airfoils are treated. Generally these methods utilize equations or procedures that are already in the literature but that have apparently not been heretofore applied in a systematic manner, with the required modifications, to the specific problem of designing low-pitching-moment airfoils.

SYMBOLS

\( A_0, A_1, A_2 \quad \text{real coefficients} \)

\( a = \text{Re}^{-\psi_0} \)
\( b^2 \quad \) modulus of a complex quantity (see eq. (7))

\( c \quad \) chord length

\( c_l \quad \) section lift coefficient

\( c_{l,0} \quad \) section lift coefficient at zero angle of attack

\( c_m \quad \) pitching-moment coefficient about the quarter-chord point

\( c_{m,ac} \quad \) pitching-moment coefficient about the aerodynamic center

\( c_p \quad \) airfoil pressure coefficient

\( c_1, c_2 \quad c_1/R \) and \( c_2/R^2 \) are coefficients in the complex Fourier expansion of \( \psi(\phi) \)

\( g \quad \) complex quantity (see eq. (6))

\( M \quad \) local Mach number

\( M_\infty \quad \) free-stream Mach number

\( P_b \quad \) basic lift distribution

\( R \quad \) radius of circle into which an airfoil is mapped by the Theodorsen transformation

\( t \quad \) maximum thickness

\( x, y \quad \) Cartesian coordinates

\( y_b \quad \) mean-line ordinate

\( \alpha \quad \) section angle of attack

\( \beta \quad \) negative of the angle of zero lift
$2\gamma$ amplitude of a complex quantity (see eq. (7))

$\delta_1, \delta_2$ phase angle

$\epsilon$ function relating angular coordinates of near-circle and exact-circle airfoil transformations

$\eta = \gamma - \beta$

$\theta^*$ transformation variable (see eq. (10))

$\phi$ angular coordinate for points mapped from airfoil surface onto a circle

$\psi$ function relating radial coordinates of near-circle and exact-circle airfoil transformations

$\psi_0$ average value of $\psi$, $\frac{1}{2\pi} \int_0^{2\pi} \psi(\phi) \, d\phi$

GENERAL CONSIDERATIONS

Inasmuch as three distinct approaches to the low-pitching-moment airfoil design problem are discussed in this paper, an initial comparison is perhaps appropriate to provide orientation and avoid possible confusion. The first method is an application to this specific design problem of the design technique that has been described in reference 1. This technique is applicable both to subcritical and supercritical airfoils. The design is effected by modifying an initial airfoil and providing an analysis of the modification on each iteration.

The second and third methods represent the full-thick-airfoil theory and the thin-airfoil superposition theory, respectively, applied systematically to the low-pitching-moment design problems. They are both essentially incompressible, and in both cases the design is initiated by specifying a set of parameters that determine certain airfoil characteristics. The pressure distributions are obtained by an independent analysis program.

All three design procedures are inviscid, but in each case an allowance for boundary-layer effects can be made. This problem has been discussed for the first procedure in reference 1. For the other two methods, a rough estimate of the displacement thickness
effect can be obtained by a judicious use of the thickness distribution controls in the design process. However, if the primary boundary-layer considerations are loss of lift and increase in pitching moment, it is a simple matter to overestimate the lift and underestimate the pitching moment in specifying the design parameters.

**DESIGN BY PRESCRIBING PRESSURE DISTRIBUTION VARIATION**

The first method to be described is applicable to airfoils that have approximately the desired characteristics but require a reduction in the magnitude of the pitching moment. The designer prescribes a change in the known pressure distribution of the original airfoil in such a way that the pitching moment will be changed in the desired manner without destroying the favorable characteristics of the airfoil. The usual procedure is to shift some of the loading from the rear forward by prescribing changes to the pressure distribution along the lower surface and possibly to the rear of the upper surface.

This method has the advantage that by working directly with the pressure distribution the designer can avoid those adverse forms of pressure distribution that are conducive, say, to flow separation or to shock formation. Furthermore, he can indirectly control the results of a specified change in the pressure distribution; that is, whether it will decrease the pitching moment, the lift, etc. However, the effects of a prescribed change on \( c_m \), \( c_l \), \( t/c \), etc. are more difficult to control in a precise manner; and consequently a number of attempts may be required to obtain closely specified values for these parameters. If the thickness is altered slightly in the design process, it can be adjusted by the incompressible method of reference 2.

Figure 1 shows two variations of a basic airfoil that were obtained by this procedure with the use of the design technique of reference 1. The lower-surface pressure distribution was altered so as to unload the airfoil near the rear. To compensate for the consequent loss of lift the loading was increased in the middle part of the lower surface. In the first variation (fig. 1(b)), the pitching-moment coefficient remained constant at -0.025 while the lift was increased by more than 25 percent. In the second variation (fig. 1(c)), the lift remained virtually constant while the pitching moment was reduced in magnitude to the more acceptable value of -0.010.

In this example, the decrease in thickness ratio represents a significant alteration to the geometry of the original airfoil. It is generally true that moderate changes in pitching moment are associated with relatively large changes in airfoil shape especially if the lift coefficient is held constant. For this reason, it is often preferable to start with a design that has near zero, or even positive, pitching moment, and then, if necessary, tailor that design. Such a procedure is discussed in the next section.
DESIGN BY SPECIFYING $\epsilon$-FUNCTION PARAMETERS

A second procedure for the design of low-pitching-moment airfoils is based on a formula used in reference 3 for the $\epsilon$-function of a class of airfoils. Basically, this formula

$$\epsilon(\phi) = A_1 \sin (\phi - \delta_1) + A_2 \sin (2\phi - \delta_2)$$ \hspace{1cm} (1)

represents a simplified $\epsilon$-function with only two Fourier components specified in terms of the amplitudes and phase angles. For this $\epsilon$-function, the angle of zero lift $-\beta$ is approximately determined from the relation

$$\beta = \epsilon(\pi) = A_1 \sin \delta_1 - A_2 \sin \delta_2$$ \hspace{1cm} (2)

The conjugate function to $\epsilon(\phi)$ (see ref. 3, eqs. 11 and 12, where the notation is slightly different) is

$$\psi - \psi_0 = A_1 \cos (\phi - \delta_1) + A_2 \cos (2\phi - \delta_2)$$

where $\psi_0$ is the average value of $\psi$. In order to compute the pitching moment, two complex numbers are needed:

$$c_1 = \frac{R}{\pi} \int_0^{2\pi} \psi(\phi) e^{i\phi} d\phi = A_1 R \cos \delta_1 + iA_1 R \sin \delta_1$$ \hspace{1cm} (3)

and

$$c_2 = \frac{R^2}{\pi} \int_0^{2\pi} \psi(\phi) e^{2i\phi} d\phi = A_2 R^2 \cos \delta_2 + iA_2 R^2 \sin \delta_2$$ \hspace{1cm} (4)

where $R$ is the radius of the circle into which the airfoil is mapped by the Theodorsen transformation. Now the real number $a$ is related to $R$ by

$$R = a e^{\psi_0}$$ \hspace{1cm} (5)

and the complex quantity $g$ defined by

$$g = a^2 + \frac{c_1^2}{2} + c_2$$ \hspace{1cm} (6)
is represented in polar form as
\[ g = b^2 e^{2i\gamma} \]  (7)
in accordance with the procedure of reference 3. Then the pitching moment about the aerodynamic center is given by (see ref. 3, eq. 51)
\[ c_{m,ac} = \frac{4\pi b^2}{c^2} \sin 2(\gamma - \beta) \]  (8)
It is clear that the value of \( c_{m,ac} \) depends on the angle \( \eta = \gamma - \beta \); specifically, \( c_{m,ac} = 0 \) when \( \eta = 0 \). Now one can express \( \eta = \gamma - \beta \) from equations (2) and (7) in terms of \( A_1, A_2, \delta_1, \delta_2, \) and \( \psi_0 \) by means of equations (3) to (6). Thus a unique airfoil can be determined by specifying the five parameters \( A_1, A_2, \delta_1, \psi_0, \) and \( \eta \) in the equation
\[ \gamma - \beta - \eta = 0 \]  (9)
and solving it for \( \delta_2 \). This highly nonlinear equation is solved by interval halving.

Varying each parameter produces a class or family, of airfoils. The value of \( \eta \) chosen controls the pitching moment according to equation (8). The selection of the other parameters requires some care. Although varying any one of these parameters influences to some extent all the airfoil characteristics, each individual parameter has a dominant influence on a particular property of the airfoil.

The value of \( A_1 \) provides the basic thickness distribution, which is then modified by the choice of \( A_2 \). The effect of varying \( A_1 \) can be seen in the example shown in figure 2(a). (For all the airfoils shown in fig. 2 the pitching moment about the aerodynamic center is essentially zero.) Very small values of \( A_1 \) yield a shape very much like an ellipse, whereas large values produce negative thickness near the trailing edge.

When \( A_2 \) is varied, the distribution of thickness is modified, as shown in figure 2(b). The magnitude of \( A_2 \) also influences the extent to which the second term in equation (1) affects the airfoil performance. Since this term is the one that involves \( \delta_2 \), equation (9) may not be solvable for \( \delta_2 \) if \( A_2 \) is too small. On the other hand, large values of \( A_2 \) (relative to \( A_1 \)) tend to produce impractical distorted airfoil shapes. This effect is seen in figure 2(b) where for \( A_1 = 0.1 \) and \( A_2 = 0.06 \), the airfoil becomes too thin in the 75-percent chord region. The parameter \( \psi_0 \) affords a control over the maximum thickness (ref. 2), as is seen in figure 2(c).

The parameter \( \delta_1 \) primarily controls the lift, as indicated by figure 2(d), where varying \( \delta_1 \) from 0.1 to 0.9 radian has the effect of changing \( \beta \) from 0.0027 to
0.03 radian. Notice, however, in figure 2(b), that the variation of $A_2$ has very little effect on the lift.

Since five parameters can be varied in this design procedure, it appears that a wide variety of shapes and characteristics is attainable. However, the fact that the $\epsilon$-function is represented by only two Fourier components is a significant restriction. Furthermore the availability of numerous parameters is in one sense a disadvantage in that the designer might spend a considerable time "toying" with the parameters in an effort to obtain exactly some desired design characteristic.

These difficulties can usually be circumvented in actual practice. For example, the airfoil shown in figure 3(a), which was designed by this method, was too thick near the trailing edge. Its other properties – lift, pitching moment, and maximum thickness – were satisfactory. Therefore a smooth analytic fairing was made, starting at the 0.60 chord station and proceeding to the trailing edge, so as to reduce the thickness in this region while maintaining the same mean line. The resulting airfoil is shown in figure 3(b), together with its pressure distribution. (The viscous pressure distributions in figs. 3 to 5 were computed by the method of ref. 4.) The lift, pitching moment, and maximum thickness are essentially unchanged, but the trailing-edge angle and consequently the pressure distribution near the trailing edge are improved.

Of course, not every arbitrary combination of parameters yields a solution of equation (9). Furthermore, as has been seen, even those combinations that yield a solution do not necessarily correspond to a practical airfoil shape.

**DESIGN BY GEOMETRIC SUPERPOSITION**

Perhaps the simplest approach to the design of airfoils is to design the mean line and then superimpose a thickness distribution on it. In reference 5 it is shown that, if the variable $\theta^*$ is defined by the relation

$$x = \frac{c}{2}(1 - \cos \theta^*) \quad (10)$$

then the basic lift distribution (that which is dependent only on the mean-line shape and not on the angle of attack) can be represented by a Fourier sine series

$$P_b = 4 \sum_{n=1}^{\infty} A_n \sin n\theta^* \quad (11)$$

Then reference 5 also shows that the distribution of slope of the mean line $dy_b(\theta^*)/dx$ at the ideal angle of attack is the conjugate of $P_b(\theta^*)/4$, provided that both functions are extended to the interval $(\pi, 2\pi)$ with $dy/dx$ symmetric about $\pi$ and $P_b$ antisymmetric.
about \( \pi \). The situation is similar to that in thick-airfoil theory where the \( \epsilon \)-function can be prescribed and its conjugate \( \psi - \psi_o \) can then be calculated to determine the airfoil geometry. Here, a basic lift distribution can be prescribed and the corresponding mean line calculated. For a lift distribution expressed as a sine series as in equation (11), the conjugate of \( \frac{Pb}{4} \) is

\[
\frac{dy_b}{dx} = \sum_{n=1}^{\infty} A_n \cos (n\theta^*)
\]

Naturally some experience would normally be required to design a lift distribution that provided the desired lift and pitching moment as well as a reasonable mean-line shape.

However, a simpler, more direct approach is available. From reference 6, equations (4.7) and (4.8), it is seen that the lift coefficient at zero angle of attack is simply

\[
c_{L,0} = \pi(2A_0 + A_1)
\]

where

\[
A_0 = -\frac{1}{\pi} \int_0^{\pi} \frac{dy(\theta^*)}{dx} d\theta^*
\]

and the pitching-moment coefficient about the quarter-chord point is

\[
c_m = \frac{\pi}{4}(A_2 - A_1)
\]

Here \( A_1 \) and \( A_2 \) are the first two coefficients in the Fourier series of equation (11). Thus, in the design of a mean line, the lift coefficient can be controlled by specifying the value of \( A_1 \) and the pitching-moment coefficient is proportional to the difference between \( A_2 \) and \( A_1 \). Specifically, \( A_2 = A_1 \) gives a pitching-moment coefficient of zero.

Families of mean lines can be derived by specifying various values of \( A_1 \) and \( A_2 \) in a simple 2-component lift distribution. However, it should be noted that large values of \( A_2 \) yield impractical distorted mean lines; consequently, large values of lift cannot be specified if the pitching moment is required to be near zero or positive.

For each mean line so derived, a family of airfoils can be obtained by specifying various thickness distributions. It is in this phase of the design that the superposition procedure of this airfoil theory displays its limitations. These limitations appear whenever the assumptions of thin-airfoil theory are violated; specifically over the entire airfoil if it is sufficiently thick and near the leading edge for any airfoil. The former problem is not as troublesome as the latter.
For a thick airfoil the lift and pitching moment do not appear to be very sensitive to thickness, even though the velocities due to thickness and camber are not simply additive. Furthermore, a thick airfoil generally has a large leading-edge radius and consequently a relatively smooth pressure distribution. Therefore, desired adjustments in the pressure distribution can be made fairly simply with a design method such as that of reference 1.

For thin airfoils, on the other hand, the superposition of velocities is valid except near the nose. Low-pitching-moment cambered airfoils generally have a mean line with considerable slope at the leading edge. This large slope, together with a small leading-edge radius, often results in a lower-surface suction spike near the leading edge. This effect is seen in the example of figure 4, for which the camber line is determined by $A_1 = A_2 = 0.025$ and $A_0 = 0.0106$, which correspond to $C_{L_0} = 0.15$ and $C_m = 0.0$, with the NACA 65A010 thickness distribution (ref. 6, p. 369). The possibility of lower-surface boundary-layer separation at small negative angles of attack is introduced by this type of lower-surface pressure distribution. Furthermore, the modification of an airfoil to eliminate such a suction spike is not a minor modification, inasmuch as the required change in local pressure coefficient is large.

Of course, a certain amount of modification is possible, as shown by the example of figure 5. At an angle of attack of zero, the lower surface of the original airfoil does not display a high leading-edge suction peak, but it does have a kind of pressure distribution that rapidly forms a spike at negative angles of attack. Thus, by making the pressure less negative in this region (fig. 5(b)), the performance at small negative angles of attack is improved. The method of reference 7 was used to make this adjustment.

CONCLUDING REMARKS

Three approaches to the design of low-pitching-moment airfoils have been discussed. The first method is applicable to a wide variety of airfoil types in compressible flow; but control of lift and pitching moment is indirect, by means of specifying appropriate changes in the pressure distribution and consequently several attempts are sometimes required to obtain the desired values of the parameters.

The other two methods which are essentially incompressible provide a closer control over such parameters as maximum thickness, lift, and pitching moment, but the airfoils generated fall within restricted families and often require tailoring. This tailoring, either to the geometry directly or to the pressure distribution, can often be accomplished without significantly altering the values of the airfoil parameters.
The design methods are essentially inviscid, but it is possible to make an allowance for the boundary layer with each method.

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REFERENCES

(a) Original airfoil. $t/c = 0.095; \ c_l = 0.038; \ c_m = -0.025$. 

Figure 1.- Example of changing pitching moment and lift by prescribed changes in the inviscid airfoil pressure distribution at $M_\infty = 0.75$ and $\alpha = 0.8^\circ$. 
Figure 1.- Continued.

(b) First variation. \( c_l = 0.049; \ c_m = -0.025; \ t/c = 0.091. \)
(c) Second variation. $c_l = 0.035; c_m = -0.010; t/c = 0.090$.

Figure 1.- Concluded.
(a) Effect of varying the leading coefficient $A_1$. $A_2 = 0.0; \; \delta_1 = 0.0; \; \psi_0 = 0.1.$

Figure 2.- Examples illustrating the influence of various parameters in the $\epsilon$-function formula on the airfoil shape. $\eta = 0.0.$
(b) Effect of varying \( A_2 \). \( A_1 = 0.1; \ \delta_1 = 0.5 \) radian; \( \psi_0 = 0.1; \) computed values of \( \beta \) of \( 0.0173 \pm 0.0002 \).
\( \psi_0 = 0.14, \ t/c = 0.143 \)

\( \psi_0 = 0.10, \ t/c = 0.108 \)

\( \psi_0 = 0.06, \ t/c = 0.075 \)

(c) Effect of varying \( \psi_0 \) on the thickness ratio. \( A_1 = 0.1; \ A_2 = 0.05; \ \delta_1 = 0.9. \)

Figure 2.- Continued.
(d) Effect of varying $\delta_1$ with computed values of $\beta$. Angles in radians. $A_1 = 0.1$; $A_2 = 0.05$; $\psi_0 = 0.1$.

Figure 2.- Concluded.
Figure 3.- Example of tailoring airfoil by an analytic fairing without altering design parameters. Pressures calculated by method of reference 4 at $M_\infty = 0.1$, $\alpha = 0$, and a Reynolds number of $44.0 \times 10^6$. 

(a) Unmodified airfoil with corresponding pressure distribution. 

$c_l = 0.10$; $c_m = 0.00$; $t/c = 0.20$. 

(b) Airfoil modified by reducing thickness aft of the 0.6 chord station with corresponding pressure distribution. $c_l = 0.10; \ c_m = 0.0; \ t/c = 0.20$.

Figure 3.- Concluded.
Figure 4.- Example illustrating pressure distribution typical of an airfoil designed by superimposing a thin airfoil thickness distribution on a camber line designed for zero pitching moment. $c_{L,0} = 0.15; c_m = 0.0$; NACA 65A010 thickness distribution. Pressures calculated by method of reference 4 at $M_\infty = 0.1$, $\alpha = 0^\circ$, and a Reynolds number of $44.0 \times 10^6$. 
(a) Unmodified airfoil. $c_{l,0} = 0.08$; $c_m = 0.00$; $t/c = 0.12$.

Figure 5.- Example of tailoring airfoil designed by geometric superposition. Pressures calculated by method of reference 4 at $M_\infty = 0.1$, $\alpha = 0^\circ$, and a Reynolds number of $44.0 \times 10^6$. 
(b) Airfoil modified by reducing lower-surface suction near the leading edge.
\[ c_\ell = 0.08; \quad c_m = 0.00; \quad t/c = 0.12. \]

Figure 5.- Concluded.
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—National Aeronautics and Space Act of 1958

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