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INELASTIC INTERACTION MEAN FREE PATH OF NEGATIVE PIONS IN TUNGSTEN*

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~~ABSTRACT~~

The inelastic interaction mean free paths λ of 5, 10, and 15 GeV/c pions have been measured by determining the distribution of first-interaction locations in a modular tungsten-scintillator ionization spectrometer. In addition to commonly used interaction signatures of a few (2-5) particles in two or three consecutive modules, we have used a χ^2 distribution to calculate the probability that the first interaction occurred at a specific depth in the spectrometer. This latter technique seems to be more reliable than use of the simpler criteria. No significant dependence of λ on energy has been observed. In tungsten, λ for pions is 206 ± 6 g/cm^2.

I. INTRODUCTION

Studies of hadron-nucleus interactions have long been used indirectly to infer information about the fundamental nucleon-nucleon interaction. A prime example of these endeavors is the consideration of certain subsets of hadron interactions in emulsions, such as those with a small number of accompanying black tracks, as being associated with the interaction between two nucleons. Recently, it has become increasingly evident that studies of hadron-nucleus interactions may play a direct role in differentiating among proposed models of nucleon-nucleon interactions.[1-3]

Apart from its role, direct or indirect, in relation to fundamental interactions, the study of hadron-nucleus interactions has an important application in reliable energy measurements of very high energy particles with ionization spectrometers (calorimeters). In fact, because of the limited past effort in studying interactions in heavy absorbers, it has been necessary to extract information from studies of hadron-nucleon interactions in order to construct models of hadron-nucleus interactions, which can be applied to the complex physical processes that occur in ionization spectrometers.[4-6]

Although extensive studies have been made of the probability for interactions of hadrons in hydrogen, relatively little information is available on the interaction lengths of hadrons in heavy absorbers. This is especially true for tungsten. Accurate knowledge of the interaction lengths of nucleons, pions, and heavy nuclei is essential for constructing models of nuclear cascades which can be used to extrapolate the calibrated response of ionization spectrometers at accelerator energies to the much higher energies to be measured in cosmic rays.

A tungsten-scintillator ionization spectrometer has been exposed to 5, 10, and 15 GeV/c pion and electron beams at the Stanford Linear Accelerator Center (SLAC). Cascade curves for pions and electrons, and the energy resolution of the spectrometer as a function of incident particle energy, interaction location, and spectrometer depth, have been presented in a previous publication.[7] We report here results of measurements of the inelastic interaction mean free paths of pions at these energies in tungsten and discuss techniques for determining the interaction location.

II. EXPERIMENTAL TECHNIQUE

The experimental configuration is shown schematically in Fig. 1. (See Refs. 7 and 8 for a more detailed description of the apparatus and general experimental technique.) Detectors CsI_1- CsI_5 were each 2 cm (9 g/cm^2) thick CsI(Tℓ) crystals. Tungsten modules T_6- T_{16} each contained four layers of tungsten (total thickness 79 g/cm^2) and three 0.65 cm thick plastic scintillators. The ionization was sampled every 3.7 radiation lengths (r.l.), but the signal from a module was a measure of the ionization over the entire module (11.1 r.l.). Modules T_1- T_5, which are called high resolution modules (HRM), each consisted of 13 g/cm^2 of tungsten followed by a 0.65 cm thick plastic scintillator. These modules were used primarily to study the early development of cascades, particularly those initiated by electrons. For the determination of the pion mean free path, signals from T_1, T_3, and T_5 were added to produce, effectively, another thick tungsten module.

The apparatus was calibrated by determining the response of each module to individual non-interacting 15 GeV/c pions. For each event the pulse height of the signal from each module was expressed in units of equivalent particles, i.e., the number of non-interacting 15 GeV/c pions necessary to produce a signal of the same pulse height. The following criteria were used to select for analysis only those events in which the beam particle did not interact before reaching the tungsten modules:

a) The response of the upstream scintillator TOF_1 was required to be between 0.1 and 1.5 equivalent particles.

b) The response of each CsI module was required to be less than 2 equivalent particles.

Various criteria were then applied to these events to determine the distribution of depths corresponding to points of first interaction. The pion mean free path λ was determined from the exponential tail of this distribution at large depths in the spectrometer.

III. INTERACTION CRITERIA

The first interaction was considered to have occurred at the starting point of the nuclear cascade initiated by an incident pion. The parameter measured is the number of the module in which the first interaction occurred. Under the assumption that the probability for an interaction is independent of energy, the probability of an interaction in any module is exponentially dependent on the location of the module in the spectrometer.

A semi-logarithmic plot was made of N(i) vs. i, where N(i) is the total number of first interactions occurring in the i^{th} module. A measurement of λ was obtained from this distribution using the maximum-liklihood method described by Crannell, et al.[9] For each event the probability P for an interaction to occur in a particular module is given by the expression

$$P = e^{-t/\lambda}(1 - e^{-\Delta t/\lambda})/(1 - e^{-L/\lambda}) \quad , \tag{1}$$

where Δt is the module thickness, t is the total thickness up to the module in which the interaction occurs, and L is the length of the entire spectrometer. The liklihood function $\mathcal{L}$ is defined as the product of the individual probabilities P_i, determined using Eq. (1), for all N events in the data set, i.e.,

$$\mathcal{L} = \prod_{i=1}^{N} P_i \quad . \tag{2}$$

Substitution of Eq. (1) into Eq. (2) leads to the relation

$$\ln \mathcal{L} = \sum_{i=1}^{N} \left[\frac{-t_i}{\lambda} + \ln(1-e^{-\Delta t/\lambda}) - \ln(1-e^{-L/\lambda}) \right] . \tag{3}$$

For each data set in the experiment, λ was determined by maximizing ln $\mathcal{L}$ (which is equivalent to maximizing the liklihood function $\mathcal{L}$) as a function of λ. Corrections were made for interactions occurring in the scintillator layers of the modules.

The difficulty with this method for measuring λ lies in finding criteria that provide an unambiguous location of the first interaction. The interaction of a high energy hadron results in the production of secondary particles and a rapid buildup of electromagnetic cascades if neutral pions are produced. One usually requires the equivalent of a single particle traversing each module upstream of the interaction, plus the equivalent of more than one particle in the module in which the interaction occurred. The additional requirement that there be more than one equivalent particle in at least one module downstream of the interaction reduces the number of erroneous assignments which would be made as a result of Landau fluctuations of ionization energy loss in a single module.

The specific signature of an interaction, i.e., the threshold number of particles in the interaction module and in subsequent modules, selects the physical process to be observed. The use of a high threshold, e.g., 10 particles, would not permit detection of low multiplicity interactions, unless there were a rapid buildup of electromagnetic cascades following the decay of a neutral pion into gamma rays. Since high multiplicity events occur with low probability at the energies considered here, it is obvious that a very high threshold will result in an excessively large value for the interaction length. On the other hand, the use of a very low threshold, e.g., approaching one particle, will result in a value for the interaction length which is unquestionably too small, because interactions will be simulated by fluctuations in the ionization energy losses of single particles.

Since multiplicities are both material dependent and energy dependent, the threshold most suitable for detection of inelastic interactions will depend on the material composition of the spectrometer modules and on the energy of the interacting hadron. The threshold will also depend on the module thickness, since this determines the extent of electromagnetic cascade development in a single module. We are not aware of any previous measurements of the interaction length using a tungsten spectrometer. However, several such measurements with energies near the ones we consider have been made with iron spectrometers. Jones, et al.[10] have reported using a threshold requirement of at least two particles in two consecutive iron modules 55 g/cm^2 thick. Kurz, et al.[11] have used a threshold of at least two particles in three consecutive iron modules 50 g/cm^2 thick. Crannell, et al.[9] have tested several criteria and reported best results with the requirement of three particles in two consecutive iron modules 50 g/cm^2 thick.

We have tested several thresholds, including those just mentioned. Since the tungsten modules used for our interaction length measurements were 11.1 r.l. thick, electromagnetic cascades reached maximum development in the module in which they originated. Therefore, we obtain better results by using a threshold requirement in only two consecutive modules. Specifically, we have tested the criterion that an interaction occurs in the first of two consecutive modules which indicate the equivalent of at least two particles. This signature is designated as the 2-2 criterion, meaning that there were at least 2 particles in the interaction module and at least 2 particles in the following module. Similarly, we have tested the use of 3-3, 4-4, and 5-5 criteria for interaction signatures.

In addition to these simple interaction signatures, we have tried a more elaborate approach in an attempt to improve the analysis of the measurements. This approach used the probability from a χ^2 distribution that an interaction did not occur before or in the j^{th} module. The procedure consisted of the following steps:

1. For each tungsten module j (counting from the upstream end of the spectrometer):

a) Calculate χ_j^2 on the hypothesis that the incident pion has not interacted through the j^{th} module, using the expression

$$\chi_j^2 = \sum_{i=1}^{j} \left(\frac{N_i - 1}{\sigma_i}\right)^2 \quad . \tag{4}$$

The symbol N_i represents the number of equivalent particles in the i^{th} module, and σ_i is the standard deviation of the normal distribution of signals from single particles traversing that module.

b) From the χ^2 distribution with j degrees of freedom, calculate the probability p_j that $\chi^2 > \chi_j^2$, i.e., the probability that the incident particle has not interacted by the j^{th} module.

2. Starting with j = 1, check successive modules until a module is found in which the non-interaction probability p_j is less than a designated threshold p_o. If $N_j \leq 1$, disregard this module as an interaction module and continue Step 2 for larger values of j. If $N_j > 1$, proceed with Step 3.

3. Check whether the non-interaction probabilities for the next three modules are also less than p_o. If so, then module j is designated as the interaction module. If not, then disregard the module as an interaction module and continue with Step 2 for larger values of j.

The non-interaction probability is required to remain below threshold for just three modules following the interaction module, rather than for all subsequent modules, in order to avoid biasing the interaction location distribution in favor of late interactions. (A criterion involving all subsequent modules might be more easily satisfied for modules farther downstream in the spectrometer, because the non-interaction probability would have to remain below threshold for fewer modules.)

A basic difficulty with the technique is that the χ_j^2 are not distributed according to the true χ^2- distribution, because the single-particle distributions for each module are not Gaussian, as is assumed in the technique. This results in arbitrarily using rather low probability thresholds in order to obtain reasonable results. This problem is not so important for our spectrometer, since the added signals from the three scintillators in each module tend to make the Landau distribution, prominent for a single scintillator, approach a Gaussian distribution. An empirical test of the technique can be made by examining the dependence of the interaction lengths obtained on the value chosen for p_o. If the technique has merit, the value obtained for the interaction length should be independent of p_o over a rather broad range.

None of the criteria we have used for an interaction signature can be expected to indicate for all interactions the correct module in which the interaction occurred. Fluctuations in the properties of an interaction affect the accuracy of the interaction location assignment. Back-scattered particles may result in assignments being made in modules upstream of the true location, while events with low or zero charged multiplicity may result in assignments being made downstream. Arguments have been made to show that these competing processes have a negligible effect on the relative number of interactions assigned t the central modules of a uniform array of identical modules.[12] These arguments are valid only if a sufficient number of modules at the two ends of the spectrometer are disregarded. The number of modules which must be disregarded depends on the material and thickness of the modules. We have in all cases disregarded the high resolution modules T_1- T_5 (equivalent to one thick module) at the upstream end of the spectrometer as well as the last two modules (T_{15},T_{16})

at the downstream end. This is believed to be sufficient, based on the smoothness of the distributions of the number of interactions occurring in the various modules.

A typical distribution, obtained for 15 GeV/c pions using the χ^2 technique with a probability threshold of 0.002, is shown in Fig. 2. The number of interactions occurring in each of the modules T_6- T_{13} was used to determine λ. When the χ^2 technique is used, T_{13} is the last possible interaction module, because the non-interaction probability is required to remain below threshold for three modules following the interaction module. (T_{16} is the last module of the spectrometer.)

IV. RESULTS AND DISCUSSION

The interaction mean free paths λ of 5, 10, and 15 GeV/c pions in tungsten, obtained using the interaction criteria discussed in Sec.III, are presented in Table I. For each value the quoted error $\delta\lambda$ indicates the change in λ associated with a decrease of 0.5 in $\ln\mathcal{L}$ from its maximum value. (If $\ln\mathcal{L}$ is normally distributed in λ, $\delta\lambda$ is one standard deviation.[9]) If the non-interaction probability threshold p_o is high ($\gtrsim .5$), small fluctuations in the number of equivalent particles in a module are identified as interactions. Consequently, a shorter mean free path is obtained using such a threshold. On the other hand, if p_o is very low, many real interactions are not detected, resulting in higher values for λ. This increase in λ as p_o is decreased from 0.001 to 0.0001 can be seen in Table I for pions of all three energies. However, the results for each energy are relatively independent of the threshold over a considerable range ($0.001 \lesssim p_o \lesssim 0.1$), as required if the technique is to have any validity.

The values of λ for 5 and 15 GeV/c pions are consistent with each other, but are significantly higher than those for 10 GeV/c pions. We believe that the lower values for 10 GeV/c pions may be due to some beam contamination and that it does not indicate a real dependence of the mean free path on the incident energy.

For an appreciable number of events the beam particle was apparently a muon from a pion decay somewhere upstream of the apparatus. If the interaction criteria are not fairly strict, these events, because of statistical fluctuations, may occasionally be designated as interactions. This is equally likely throughout the tungsten stack. However, the <u>relative</u> number of interactions which are simulated by muons is greater for the downstream modules. Consequently, the tail of the interaction location distribution is raised, and a longer mean free path is obtained. In order to eliminate events in which an interaction was simulated by a muon, a response of three or more equivalent particles was required in at least one tungsten module. When this was done, the resulting values for λ were about 1-2% lower, for probability thresholds of 0.05 or less, than the values obtained with no discrimination against muons. However, as expected, the differences were greater for less stringent interaction criteria (higher p_o).

For the 10 and 15 GeV/c data, the values of λ obtained using the simpler two-counter criteria exhibit the expected threshold dependence, i.e., the measured values of λ increase as more particles are required for the interaction signature. However, this trend is not observed for the 5 GeV/c data. The average multiplicity and, consequently, the number of cascade particles and longitudinal extent of a cascade increase with the energy of the incident particle. Such interaction signatures as the 4-4 and 5-5 criteria are not easily satisfied by cascades initiated by 5 GeV/c pions. The requirement of such large numbers of particles, therefore, results in most events being designated as non-interacting, and the interaction location distributions so obtained are not a good measure of λ.

The fractional error $\delta\lambda/\lambda$ has been used to evaluate the various criteria for the determination of λ. This quantity, obtained in the maximum liklihood calculation, is shown as a function of p_o if Fig. 3. A well-defined minimum in $\delta\lambda/\lambda$ is seen at a probability threshold of about 0.002 for each of the three incident pion energies. It can be shown using Eq. (3) that the minimum value for $\delta\lambda/\lambda$ is given by the expression[9]

$$(\delta\lambda/\lambda)_{min} = \frac{1}{\sqrt{N}}\left[e^{\Delta t/\lambda}\left(\frac{\Delta t/\lambda}{e^{\Delta t/\lambda}-1}\right)^2 - e^{L/\lambda}\left(\frac{L/\lambda}{e^{L/\lambda}-1}\right)^2\right]^{-1/2} . \tag{5}$$

It is clear that if the total number of events increases, $\delta\lambda/\lambda$ will decrease because of the statistical factor $1/\sqrt{N}$. When the interaction criterion is fairly loose (high p_o), many interactions are assigned to the first module, which, in order to reduce boundary effects, is not used in the determination of λ. As the interaction criterion is made more stringent, fewer interactions are assigned to the first module and more to the subsequent modules. Consequently, $\delta\lambda/\lambda$ in Fig. 3 decreases with decreasing p_o, because of the decrease in $1/\sqrt{N}$. However, when the interaction criterion is made too stringent, many real interactions are missed, so $1/\sqrt{N}$ increases causing $\delta\lambda/\lambda$ to increase. As the threshold p_o is decreased from 1.0 to 0.002 (where the minima in $\delta\lambda/\lambda$ are observed in Fig. 3), N increases monotonically and the $1/\sqrt{N}$ dependence is no doubt responsible for much of the decrease in $\delta\lambda/\lambda$. However, although N continues to increase as p_o is decreased further, $\delta\lambda/\lambda$ begins to increase. This must indicate a poorer fit to the hypothesis of exponential decay. Consequently, we have chosen $p_o = 0.002$ as the best value for the probability threshold.

For the 5 and 10 GeV/c data, the $\delta\lambda/\lambda$ for the two-module criteria are considerably larger than the minimum values obtained using the χ^2 technique. For the 15 GeV/c data, the $\delta\lambda/\lambda$ are comparable or slightly larger for the two-module criteria. Consequently, for the energies considered in this experiment, the χ^2 technique seems to be the best method for determining λ.

The mean free path for pions in tungsten and other heavy absorbers can, of course, be determined by methods which do not employ an ionization spectrometer. The absorption cross section σ_{abs} can be measured by standard techniques and λ calculated according to the relation

$$\lambda = \frac{A}{N_o\,\sigma_{ABS}} \quad , \tag{6}$$

where A is the atomic mass number of the absorber and N_o is Avogadro's number. For the purpose of comparing data for different absorbers, it is more informative to consider σ_{ABS} rather than λ as a function of A. According to a simple opaque sphere or "billard ball" model, σ_{ABS} should be proportional to $A^{2/3}$, if the target nucleus is perfectly opaque to the incident particle. However, absorber nuclei are to some extent transparent, more so for lower A, resulting in a steeper A dependence (more like $A^{3/4}$).

The mean free paths obtained using the χ^2 technique with $p_o = 0.002$ are nearly the same for 5 and 15 GeV/c pions. If these data are combined, our result is a value of 206 $\pm$ 6 g/cm^2 for the pion mean free path in tungsten, or equivalently, a cross section σ = 1480 $\pm$ 45 mb. Allaby, et al.[13] have determined the absorption cross sections for a series of absorbers, using the method of removal of particles from an accelerator beam. A value of λ = 214 $\pm$ 3 g/cm^2 (σ = 1425 $\pm$ 20 mb) is obtained for tungsten by interpolating between their values (20 GeV/c π^-) for tin and lead. A value of λ = 207 $\pm$ 6 g/cm^2 (σ = 1475 $\pm$ 40 mb) is obtained for tungsten by interpolation between the values obtained by Denisov, et al.[14] for 13.3 GeV/c pions in tin and lead. Crannell, et al.[9] quote a value of λ = 164 $\pm$ 9 g/cm^2 (σ = 565 $\pm$ 31 mb) for 17.8 GeV pions in iron. Their experiment involved an ionization spectrometer somewhat similar to ours.

Measured pion cross sections obtained in these and other experiments[15] are shown as a function of A in Fig. 4. Our result for tungsten is consistent with the values obtained for different absorbers in the other experiments. The straight line shown in Fig. 4 was determined by a non-weighted least squares fit to all the data. It can be seen that the data fit the hypothesis $\sigma = aA^b$, where a = 31 millibarns and b = 0.74.

V. SUMMARY AND CONCLUSIONS

The interaction mean free path λ of pions in tungsten has been determined from the exponential distribution of starting points of cascades in a tungsten scintillator ionization spectrometer. The value obtained for λ depends on the specific signature used to locate the depth of the first interaction from which the cascade ensues. If the interaction signature is based upon a low number of particles, fluctuations in the energy loss of single particles may falsely indicate an interaction. If the signature is based upon a large number of particles, cascades initiated by low energy primaries may not contain enough particles to satisfy the requisite signature, and some interactions will not be detected. The technique utilizing the probability from a χ^2 distribution to locate the depths in the spectrometer of the first interactions seems to provide a less biased measurement of λ, since a rather wide range of probability thresholds gives essentially the same value for λ. By selecting the probability threshold which minimizes the fractional error $\delta\lambda/\lambda$, we have obtained a value of $\lambda = 206 \pm 6$ g/cm^2 as the best estimate for the inelastic interaction mean free path of pions in tungsten. This value is in agreement with the absorber dependence of the interaction cross section given by $\sigma = 31\ A^{3/4}$ millibarns.

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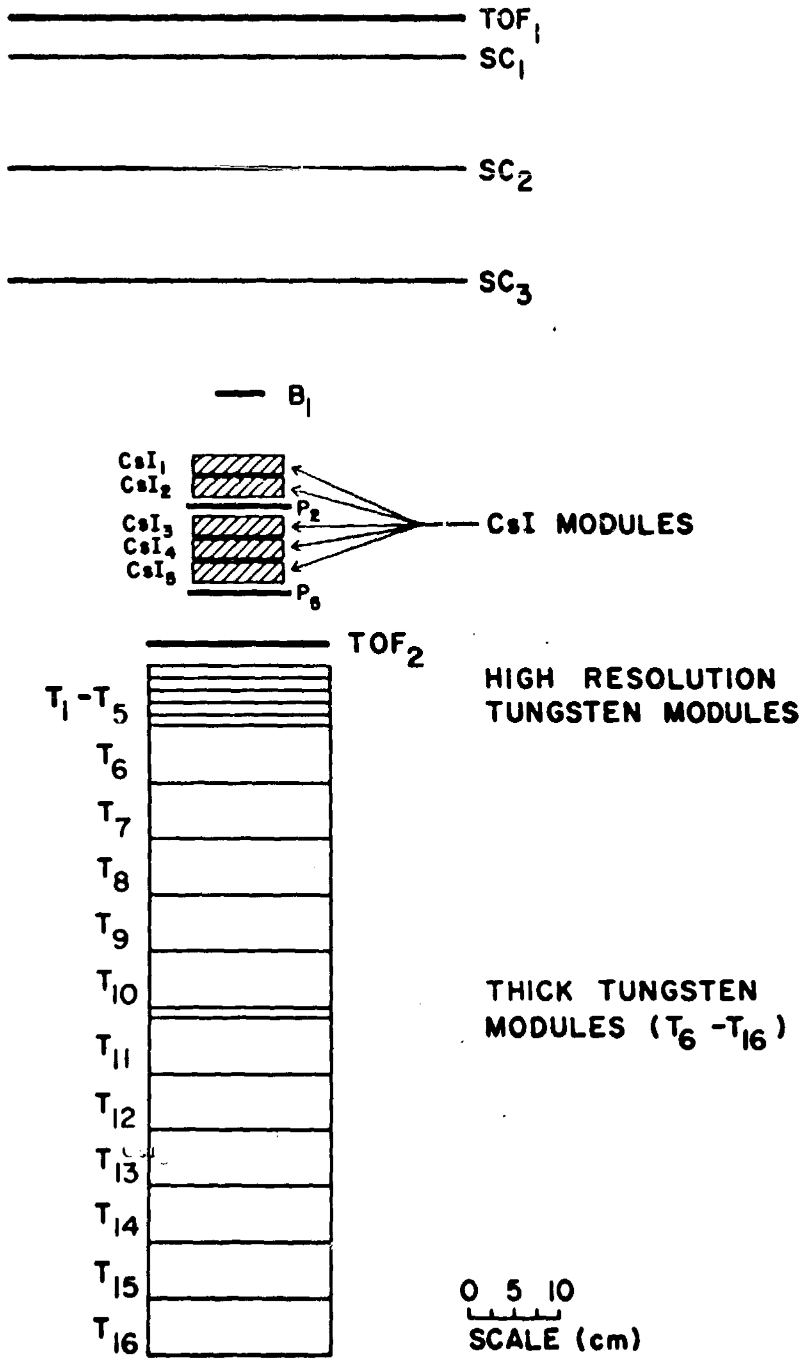
TABLE I

Mean free paths λ of pions in tungsten. The probability thresholds p_o which were used in the χ^2 technique and the two-module criteria, as defined in the text, are given in the first column.

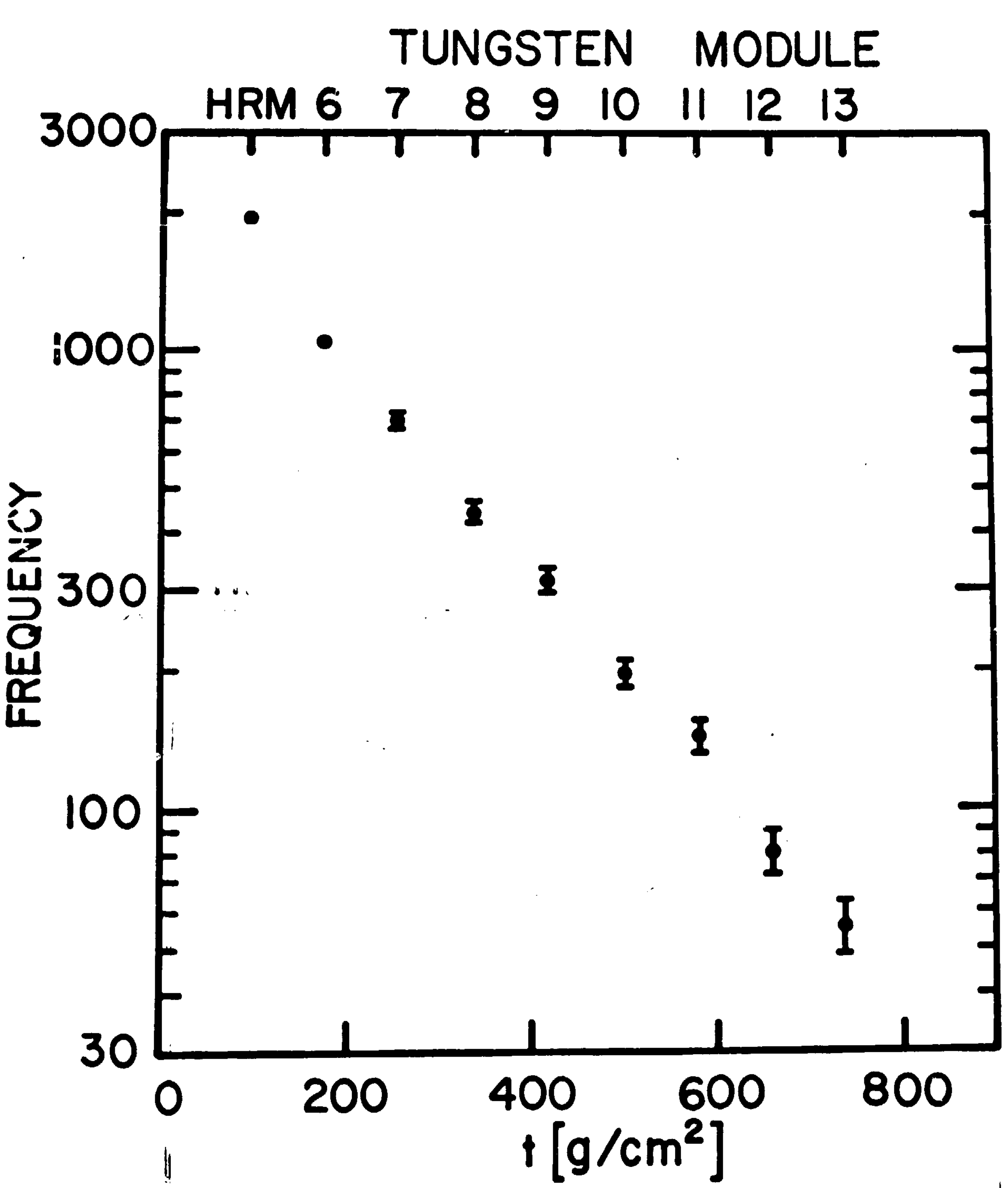
p_o for χ^2 Technique	λ (g/cm^2) 5 GeV/c	10 GeV/c	15 GeV/c
0.5	187 ± 16	176 ± 10	201 ± 8
0.2	192 ± 11	187 ± 11	204 ± 7
0.1	198 ± 12	188 ± 11	205 ± 7
0.05	203 ± 12	186 ± 10	206 ± 7
0.02	202 ± 12	186 ± 10	207 ± 7
0.01	204 ± 12	188 ± 10	206 ± 7
0.005	205 ± 12	187 ± 10	206 ± 7
0.002	207 ± 12	190 ± 10	206 ± 7
0.001	209 ± 12	192 ± 10	207 ± 7
0.0005	211 ± 12	195 ± 11	209 ± 7
0.0002	215 ± 13	199 ± 11	214 ± 7
0.0001	217 ± 13	203 ± 11	216 ± 7
Two-module Criteria			
2-2	198 ± 12	205 ± 12	219 ± 8
3-3	201 ± 13	217 ± 12	228 ± 8
4-4	198 ± 14	225 ± 13	226 ± 8
5-5	188 ± 16	224 ± 14	232 ± 8

FIGURE CAPTIONS

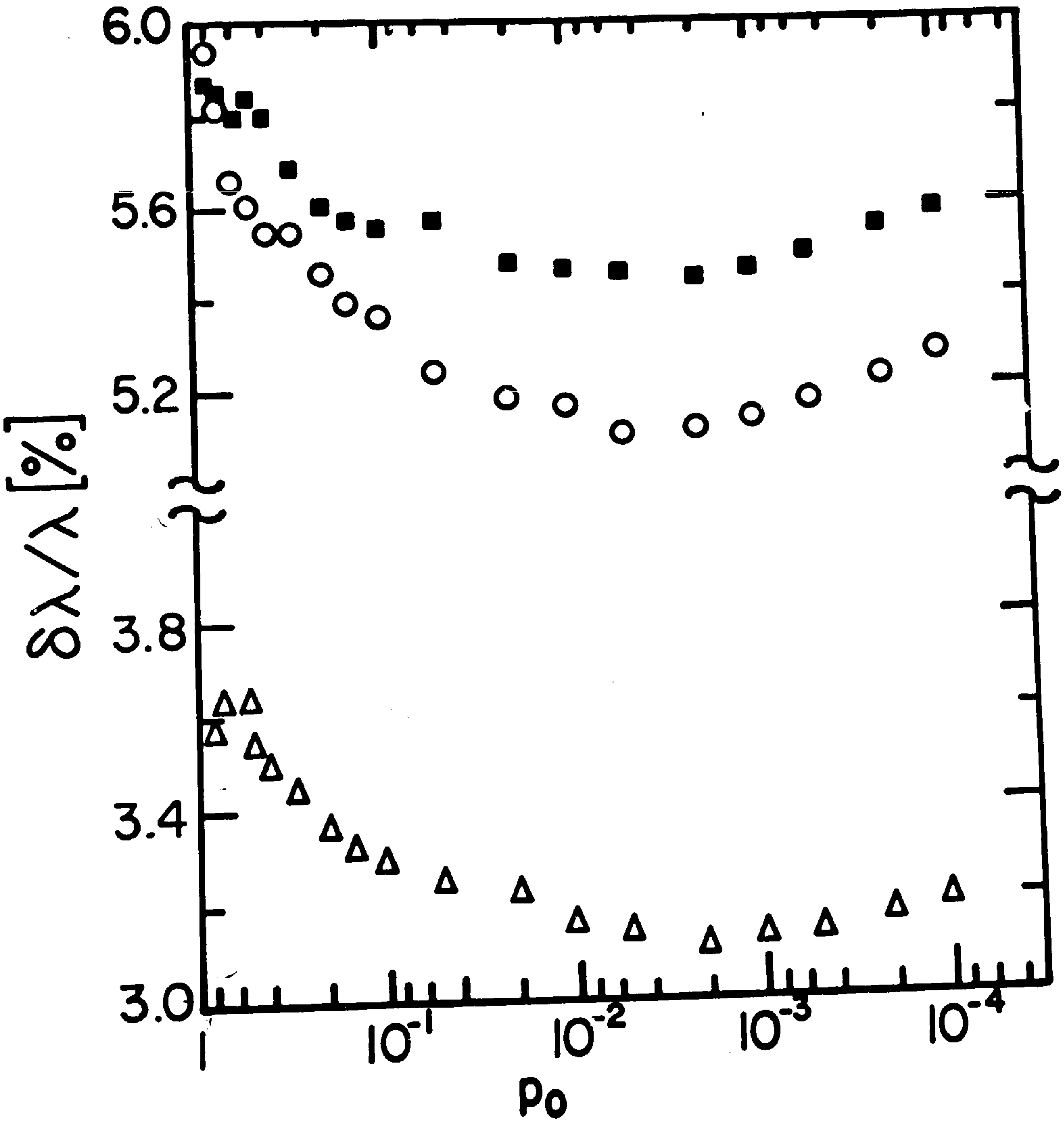
1. Scale drawing of apparatus. SC_1, SC_2, and SC_3 are wire spark chambers; TOF_1, TOF_2, B_1, P_2, and P_5 are plastic scintillators.
2. Frequency of first interactions at various depths t in the spectrometer for 15 GeV/c pions. The χ^2 technique was used with a probability threshold $p_o = 0.002$. The errors are statistical $(\sigma/\sqrt{N})$.
3. Fractional uncertainty $\delta\lambda/\lambda$ in the mean free path of pions in tungsten. The χ^2 technique was used with different probability thresholds p_o. Results at 5 GeV/c: ■. Results at 10 GeV/c: o . Results at 15 GeV/c: Δ.
4. Absorption cross section σ vs. atomic mass number A of the absorber. Results from this experiment: ●. Results from Ref. 9: ■ . Results from Ref. 13: Δ . Results from Ref. 14: ∇ . Results from Ref 15: o . The straight line was determined by a non-weighted least squares fit to all the data.

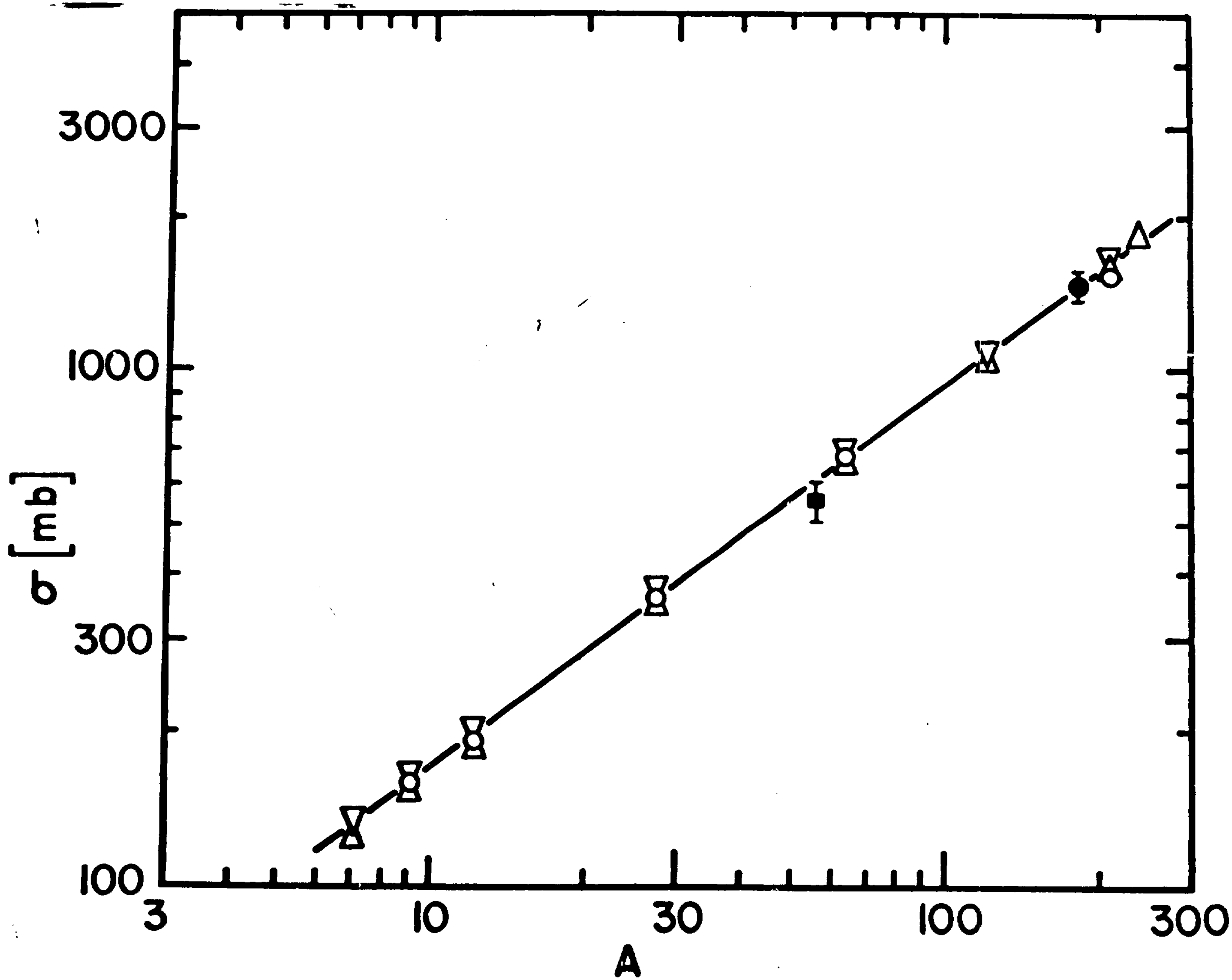


TOF1
SC1
SC2
SC3
B1
CsI1
CsI2
P2
CsI3
CsI4
CsI5
P5
CsI MODULES
TOF2
HIGH RESOLUTION TUNGSTEN MODULES
T1 -T5
T6
T7
T8
T9
T10
T11
T12
T13
T14
T15
T16
THICK TUNGSTEN MODULES (T6 -T16)
0 5 10
SCALE (cm)



TUNGSTEN MODULE
HRM 6 7 8 9 10 11 12 13
3000
1000
300
100
30
FREQUENCY
0 200 400 600 800
t [g/cm²]


6.0
5.6
5.2
3.8
3.4
3.0
δλ/λ [%]
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
p_0



3000
1000
300
100
σ [mb]
3
10
30
100
300
A