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Dynamic Instability of Ducts Conveying Fluid

Final Technical Report

Covering the Period October 1974 to August 1975

NASA Grant NSG 1105

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By

Visiting Professor Yi-Yuan Yu, Principal Investigator

Department of Mechanical Engineering

August 1975

(NASA-CR-143205) DYNAMIC INSTABILITY OF DUCTS CONVEYING FLUID Final Technical Report, 1974 - 1975 (Polytechnic Inst. of New York, Brooklyn.) 13 p HC $3.25 CSCL 20D
N75-28372
G3/34 Unclas 29824

Introduction

This is the Final Technical Report on NASA Grant NSG 1105 entitled "Dynamic Instability of Ducts Conveying Fluid," covering the ten-month period from October 1974 to August 1975, awarded by NASA Langley Research Center to Polytechnic Institute of New York, with Visiting Professor Yi-Yuan Yu as the Principal Investigator.

In this report a summary of the main results is presented first. Presentations and publications based on the results are then itemized. Three of these items are attached to this report as Appendices.

Summary of Results

There have been four separate, although closely related, research tasks on ducts conveying high-speed fluid. The main results are as follows:

1. Cantilevered Curved Ducts. The effect of curvature on the flutter of cantilevered ducts has been examined in great depth by constructing root locus diagrams. Comprehensive results have been obtained for a total of six cases, each including the four lowest modes. Among other things, the results indicate that instability of a curved duct can take place in the third mode as well as in the second. In the case of a straight duct, instability has been found by previous authors to occur in the second mode only.

2. Flexibly Supported Ducts. To provide an understanding of the dynamic instability characteristics of real ducts which are neither truly cantilevered nor simply supported or clamped, flexibly supported end conditions are considered. The critical velocity as a function of the support flexibility has been analyzed. Computer programs for determining the

critical velocity and for constructing root locus diagrams were prepared for very general cases of flexible supports. For the special case of a straight duct with one end rigidly clamped and the other end flexibly supported, comprehensive numerical results were obtained. The flexibility of the second end was found to determine whether initial dynamic instability would take place in the form of divergence or flutter.

3. Finite Element Analysis. The dynamic instability analysis of arbitrarily shaped ducts can be carried out on the basis of the finite element method. Some exploratory work on the finite element formulation of the problem has been carried out for the divergence and flutter of a straight duct. Computer programs have been prepared with the entire duct taken as one or two finite elements. A limited amount of numerical work has also been carried out.

4. Duct Systems. For duct systems composed of straight and circular segments, another approach to the dynamic instability problem is to consider the ducts as composite duct systems and analyze them by solving the appropriate differential equations and matching the boundary conditions between adjacent segments. As an example, a plane system consisting of a circular segment with a straight one attached to each of its two ends was considered. The frequency equation of the problem has been formulated, but computer programs for the problem remain yet to be written.

Presentations and Publications

1. The Principal Investigator presented the paper, "Application of Variational and Galerkin Equations to Linear and Nonlinear Finite Element Analysis," at the 25th Congress of the International Astronautical Federation in Amsterdam, October 1974. The work presented in the paper was

initiated when he was in residence at the NASA Langley Research Center in the summer of 1973. The procedure advocated in the paper has since been applied to the finite element formulation of the problem of dynamic instability of straight ducts, as described above under Task 3.

2. The Principal Investigator presented a Joint Seminar to the Department of Mechanical Engineering and Department of Aerospace Engineering and Applied Mechanics at Polytechnic Institute of New York on October 31, 1974. The seminar was entitled "Some Recent NASA-Supported Research." In addition to a discussion of the above application of variational and Galerkin equations to the finite element analysis, the lecture also covered the problem of dynamic instability of ducts conveying fluid.

3. A presentation entitled "Some Recent NASA-Supported Research in Structural Mechanics" was made by the Principal Investigator at the 14th Midwestern Mechanics Conference at Norman, Oklahoma, March 24-26, 1975. While similar to the seminar described in Item 2, the presentation included a great deal more results on the dynamic instability of ducts conveying fluid. An abstract of the presentation appeared in the Volume of Abstracts published by the Conference and is reprinted in this report in Appendix 1.

4. A note entitled "Flutter of Cantilevered Curved Ducts Conveying Fluid," authored by the Principal Investigator, has been accepted for publication in Mechanics Research Communications. A preprint is included in this report in Appendix 2.

5. The Principal Investigator prepared the paper, "Effect of Curvature on the Flutter of Cantilevered Ducts Conveying Fluid," for presentation at the 11th International Symposium on Space Technology and Science in Tokyo, July 1975. Since the work has not been published in the U.S., permission to present the paper was not granted. A copy of it is attached as Appendix 3.

Acknowledgement

In addition to the financial support from NASA Langley Research Center, computer programming as well as computer time was provided by the Rocketdyne Division of Rockwell International. Programming was ably carried out by Bob Berggren of Rocketdyne.

The Principal Investigator is also indebted to Polytechnic Institute of New York for accommodating him during his leave of absence from Wichita State University.

Appendix 1

(Reprinted from the "Volume of Abstracts" presented at the 14th Midwestern Mechanics Conference at Norman, Oklahoma, March 24-26, 1975)

Some Recent NASA-Supported Research in Structural Mechanics*

By Yi-Yuan Yu

Visiting Professor of Mechanical Engineering, Polytechnic Institute of New York, and Distinguished Professor of Aeronautical Engineering, Wichita State University

Some recent research carried out by the author under the direct or indirect support of NASA is briefly reviewed.

The first topic being investigated concerns the Galerkin and variational formulation of the finite element analysis. As is known, the advantage of the Galerkin method lies with the fact that its use does not depend upon the existence of a variational principle. Recently the method has received much attention in connection with finite element formulation. This unfortunately can lead to some difficulties, as illustrated by considering the problem of two-dimensional elasticity. When a variational principle does exist, such difficulties can be removed by resorting to the variational equation.

The second topic deals with the finite element analysis of nonlinear vibrations based upon the variational equation. The finite element approximation may be considered as a variational approximation with respect to the space coordinate. A second variational approximation involves that with respect to time. The resulting nonlinear algebraic equations can then be solved by an iteration procedure. To demonstrate the method, vibrations of beams with large deflection are discussed.

The last topic is on the effect of high-velocity fluid flow on the dynamic instability of ducts. When a certain critical velocity of the fluid in a duct is reached, instability of the duct can develop in the form of divergence or flutter. Previous authors have treated mostly straight ducts. There appears to be only one publication on curved-duct fluid systems, and only divergence is discussed in that publication. In the present work the flutter of cantilevered ducts with constant but nonzero curvature is investigated. Results show that the effect of curvature can be very severe.

Briefly mentioned is the relation which the above topics of research bear with the Space Shuttle Main Engine.

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*The first two topics of research described in this abstract were initiated when the author was in residence at the Institute for Computer Applications to Science and Engineering of NASA Langley Research Center. The third topic represents work initiated by the author when he was with Rocketdyne. The work is being continued under the support of NASA Langley Research Center.

Appendix 2

(To be published in Mechanics Research Communications)

FLUTTER OF CANTILEVERED CURVED DUCTS CONVEYING FLUID

Yi-Yuan Yu
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Introduction

When a certain critical velocity of the fluid flow in an elastic duct is reached, instability of the duct can develop in the form of divergence or flutter, depending mainly on the end conditions. Much has been done on the divergence and flutter of straight ducts; a comprehensive review was recently given by Paidoussis and Issis [1]. Some publications on curved ducts have also appeared, but mostly dealing with divergence. Only Chen [2] further analyzed the out-of-plane flutter of a cantilevered curved duct. In this work the in-plane flutter of a cantilevered curved duct is investigated.

Analysis

The duct has a length L and radius a and contains a fluid flow with a steady velocity V. From Hamilton's principle we find the complete system of non-linear equations of motion and associated boundary conditions. By writing from these the equations governing the initial and final states and taking the difference, linearized equations governing the perturbed state are deduced. These are further simplified by assuming inextensional deformation and neglecting the tangential inertia. The assumption of inextension also eliminates the effect of initial stresses. The simplified equations are then used in the stability analysis of the duct-fluid system. As exact methods of solution are not feasible, the equations are solved by an iterative procedure on the digital computer.

Numerical Results

Comprehensive numerical results have been obtained for the dimensionless critical velocity $v_c = \sqrt{M/EI}\, V_c L$ and frequency $\omega_c = \sqrt{(M+m)/EI}\, \Omega_c L^2$, where M is the mass per unit length of the fluid flow, m that of the duct, EI the flexural rigidity of the duct, and Ω the circular frequency. These are shown in Figs. 1 and 2 as functions of the mass ratio $\beta = M/(M+m)$ and the curvature parameter $\Theta_L = L/a$. The results for a straight duct for which $\Theta_L = 0$ agree with those given by previous authors. Results for $\Theta_L > 0$ show that the effect of curvature can be very severe. The dimensionless complex frequency ω has also been computed for a number of cases. The results show that the third mode can become dynamically unstable first, in contrast to the case of a straight cantilevered duct for which the second mode is known to become unstable first.

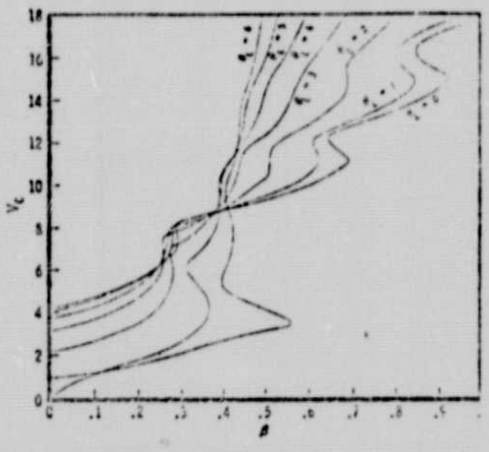

FIG. 1
Dimensionless Critical Velocity

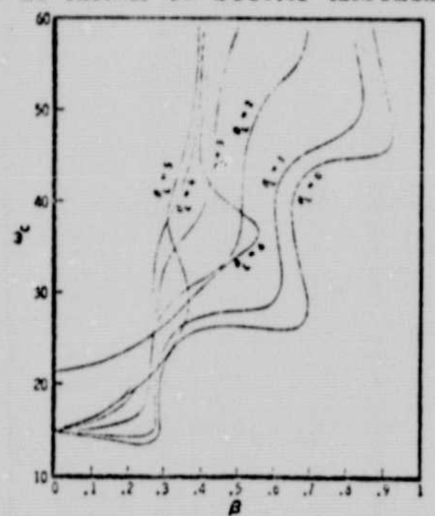

FIG. 2
Dimensionless Critical Frequency

References

1. M. P. Paidoussis and N. T. Issis, J. Sound and Vibration, 33, 267 (1974).
2. S. S. Chen, J. Applied Mechanics, 40, 362 (1973).

Acknowledgement

This work was initiated when the author was a consultant to Rocketdyne and was subsequently supported by NASA Langley Research Center under NASA Grant NSG 1105. A more complete version will be presented at the Eleventh International Symposium on Space Technology and Sciences in Tokyo, July 1975.

Appendix 3

EFFECT OF CURVATURE ON THE FLUTTER OF CANTILEVERED DUCTS CONVEYING FLUID[*]

Yi-Yuan YU[**]

Abstract

High-velocity fluid flow in duct can lead to dynamic instability. The present study deals with the effect of curvature on the flutter of cantilevered ducts. It was motivated by the severely curved configuration of the liquid fuel and oxygen ducts installed on the space shuttle main engine.

1. Introduction

When a certain critical velocity of the fluid in an elastic duct is reached, instability of the duct can develop in the form of divergence or flutter, depending on its end conditions. Ashley and Haviland (Ref.1) were the first to investigate the bending vibrations of a simply supported duct. Housner (Ref.2) re-examined the problem and showed that divergence can take place. His results were verified experimentally by Dodds and Runyan (Ref.3). For cantilevered ducts conveying fluid instability can occur in the form of flutter. This was first studied both analytically and experimentally by Benjamin (Ref.4) by considering a system of articulated rigid ducts. His study was later extended by Gregory and Paidoussis (Ref.5) to a continuous elastic cantilevered duct. A comprehensive review on the dynamic instability of straight ducts was recently given by Paidoussis and Issid (Ref.6).

Curved ducts conveying fluid have been investigated by Svetlitskii (Ref.7), Unny, Martin, and Dubey (Ref.8), Chen (Refs.9, 10), and Hill and Davis (Ref.11). All of these authors discussed the divergence of curved ducts, and Svetlitskii and Hill and Davis further included the effect of initial stresses. Only Chen (Ref.10) also analyzed the out-of-plane flutter of a cantilevered duct.

In this work the in-plane flutter of a cantilevered curved duct is investigated. Complete equations including the effect of initial stresses are first derived from Hamilton's principle. These are next simplified. Based on the simplified equations the flutter analysis is then carried out. Numerical results are finally presented.

2. Equations of Motion and Boundary Conditions

The cantilevered circular duct under consideration is shown in Fig.1, where L and a are the length and uniform radius of curvature of the duct, respectively, and V is the steady velocity of the fluid in the duct. The coordinates s, θ are related by $s = a\theta$. Applied to the total length of the duct this gives $L = a\theta_L$. For a duct with given L and a the total angle θ_L may be considered to be the dimensionless curvature.

To derive the equations of motion and the associated boundary conditions for the duct-fluid system we use Hamilton's principle in the form

$$\delta\int_{t_0}^{t_1}(T-U)\,dt + \int_{t_0}^{t_1}\delta W\,dt = 0 \qquad (1)$$

The duct is taken to be the basic free body whose kinetic and strain energies are T and U. The term δW then consists of the virtual work of all external forces, which in this case are those exerted by the fluid on the duct. The fluid is assumed to

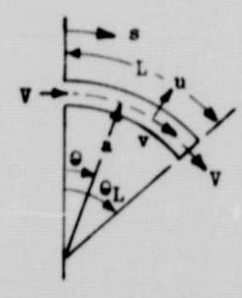

Fig. 1 Curved Cantilever Duct-Fluid System

[*]This work was initiated when the author was a consultant to Rocketdyne. It was subsequently supported by NASA Langley Research Center under NASA Grant NSG 1105.

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be incompressible so that no internal energy is associated with it.

Only in-plane motion of the duct will be considered. The non-zero displacement components at an arbitrary point in the duct are thus those in the radial and circumferential directions. Following the Winkler theory (Ref.12) we take these in the form

$$u_r = u, \qquad u_s = -z u' + (1 + z/a) v \tag{2}$$

where u and v are the radial and circumferential components of the displacement of a point on the centroidal axis of the duct, from which z is measured. A prime denotes differentiation with respect to s.

The kinetic energy of the duct is

$$T = \frac{1}{2}\int_0^L \int_A \rho(\dot{u}_r^2 + \dot{u}_s^2)\, ds\, dA$$

where ρ is the density of the duct material, A the cross-sectional area of the duct, and a dot denotes differentiation with respect to the time t. Substitution from Eqs.(2) yields

$$T = \frac{1}{2}\int_0^L m(\dot{u}^2 + \dot{v}^2)\, ds$$

where m is now the mass per unit length of the duct. Taking the variation in the usual manner we find

$$\int_{t_0}^{t_1} \delta T\, dt = -\int_{t_0}^{t_1} dt \int_0^L m(\ddot{u}\,\delta u + \ddot{v}\,\delta v)\, ds \tag{3}$$

With the assumed u_r and u_s the only non-zero linear strain and rotation components are

$$e_{\theta\theta} = \frac{u}{r} + v' - z\frac{a}{r}u'', \qquad \omega_z = \frac{v}{a} - u' \tag{4}$$

and the corresponding nonlinear strain component is

$$\varepsilon_{\theta\theta} = \frac{u}{r} + v' - z\frac{a}{r}u'' + \frac{1}{2}\left(\frac{v}{a} - u'\right)^2$$

The variation of the strain energy is then

$$\delta U = \int_0^{\theta_L}\int_A \sigma_{\theta\theta}\,\delta\varepsilon_{\theta\theta}\, r\, d\theta\, dA$$

$$= \int_0^{\theta_L}\left[N_s\delta u - M_s a\,\delta u'' + (M_s + N_s a)\delta v' + (M_s + N_s a)\left(\frac{v}{a} - u'\right)\left(\frac{\delta v}{a} - \delta u'\right)\right] d\theta$$

where

$$N_s = \int_A \sigma_{\theta\theta}\, dA, \qquad M_s = \int_A \sigma_{\theta\theta}\, z\, dA$$

are the tension and bending moment in the duct. Differentiation by parts finally transforms U into

$$\delta U = \int_0^L \left[\frac{N_s}{a} - M_s'' + \frac{\partial}{\partial s}\left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right)\right]\delta u\, ds$$
$$- \int_0^L \left[\frac{M_s'}{a} + N_s' - \frac{1}{a}\left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right)\right]\delta v\, ds$$
$$+ \left[M_s' - \left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right)\right]\delta u\Big|_0^L - M_s\,\delta u'\Big|_0^L + \left(\frac{M_s}{a} + N_s\right)\delta v\Big|_0^L \tag{5}$$

The fluid action on the duct is due to the inertia forces induced by the fluid acceleration. The components of the fluid acceleration in the radial and circumferential directions have been known (Ref.11). If M is the mass per unit length of the fluid stream, the virtual work of the inertia forces on the duct is simply

$$\delta W = -\int_0^L M\left[\ddot{u} + 2V\left(\dot{u}' - \frac{1}{a}\dot{v}\right) + V^2\left(u'' - \frac{1}{a}v' - \frac{1}{a}\right)\right]\delta u\, ds$$
$$- \int_0^L M\left[\ddot{v} + V\left(\dot{v}' + \frac{1}{a}\dot{u}\right)\right]\delta v\, ds \tag{6}$$

With Eqs.(3), (5), and (6) substituted in Hamilton's principle in Eq.(1), we find

$$\int_{t_0}^{t_1} dt \int_0^L \left[\frac{N_s}{a} - M_s'' + \frac{\partial}{\partial s}\left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right) + (M+m)\ddot{u} + 2MV\left(\dot{u}' - \frac{\dot{v}}{a}\right) + MV^2\left(u'' - \frac{v'}{a} - \frac{1}{a}\right)\right]\delta u\, ds$$

$$-\int_{t_0}^{t_1} dt \int_0^L \left[\frac{M_s'}{a} + N_s' - \left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right)\frac{1}{a} - (M+m)\ddot{v} - MV\left(\dot{v}' + \frac{\dot{u}}{a}\right)\right]\delta v \, ds$$
$$+ \int_{t_0}^{t_1} \left\{\left[M_s' - \left(\frac{M_s}{a} + N_s\right)\left(\frac{v}{a} - u'\right)\right]\delta u\Big|_0^L - M_s \delta u'\Big|_0^L + \left(\frac{M_s}{a} + N_s\right)\delta v\Big|_0^L\right\} dt = 0 \qquad (7)$$

from which the complete system of nonlinear equations of motion and the associated boundary conditions for the cantilevered duct can be written immediately.

3. Initial and Perturbed States

An initial state of stress and deformation of the duct is reached after a steady flow is established in the duct. Perturbation from the initial state will be induced by any further disturbance of the system, which may become unstable. The final state governed by Eq.(7) is thus equal to the sum of the initial and perturbed states. With the subscripts o and a denoting these states we shall write

$$N_s = N_o + N_a, \quad M_s = M_o + M_a, \quad u = u_o + u_a, \quad v = v_o + v_a \qquad (8)$$

Equations governing the initial state can also be written readily from Eq.(7). Equations governing the perturbed state are then derived by substracting the equations for the initial state from those for the final state. The results are further linearized to yield

$$\frac{N_a}{a} - M_a'' + \left[\left(\frac{M_o}{a} + N_o\right)\left(\frac{v_a}{a} - u_a'\right)\right]' + (M+m)\ddot{u}_a + 2MV\left(\dot{u}_a' - \frac{\dot{v}_a}{a}\right) + MV^2\left(u_a'' - \frac{v_a'}{a}\right) = 0$$
$$\frac{M_a'}{a} + N_a' - \left(\frac{M_o}{a} + N_o\right)\left(\frac{v_a}{a} - u_a'\right)\frac{1}{a} - (M+m)\ddot{v}_a - MV\left(\dot{v}_a' + \frac{\dot{u}_a}{a}\right) = 0 \quad (0 \le s \le L) \qquad (9)$$

$$u_a = u_a' = v_a = 0 \qquad (s=0)$$
$$M_a' - \left(\frac{M_o}{a} + N_o\right)\left(\frac{v_a}{a} - u_a'\right) = M_a = \frac{M_a}{a} + N_a = 0 \qquad (s=L) \qquad (10)$$

where M_o and N_o are from the solution of the initial state.

The relations between M_a, N_a and u_a, v_a in the perturbed state are assumed to be the same as in the usual static situation:

$$M_a = -EAZ(u_a + a^2 u_a''), \qquad N_a = \frac{EA}{a}(u_a + a v_a') + \frac{EAZ}{a}(u_a + a^2 u_a'') \qquad (11)$$

where Z is a dimensionless constant of the Winkler theory defined by

$$\int_A z/(a+z)\, dA = -ZA$$

For a duct for which $a \gg z$ we have approximately $Z = I/Aa^2$, I being the moment of inertia of the cross section of the duct.

4. Simplified Equations

For vibration analysis of thin circular rings, Evensen (Ref.13) has shown that accuracy is not impaired by assuming inextensional deformation ($u_a + av_a' = 0$) and by neglecting the tangential inertia ($\ddot{v}_a = 0$). Interestingly, the assumption of inextensional deformation will also eliminate the effect of the initial stresses M_o and N_o. With the adoption of these assumptions the second of Eqs.(9) becomes identically satisfied, and the first of these reduces to

$$\frac{EAZ}{a^4}(u_a + a^2 u_a'' + a^4 u_a'''') + (M+m)\ddot{u}_a + 2MV\left(\dot{u}_a' - \frac{\dot{v}_a}{a}\right) + MV^2\left(u_a'' + \frac{u_a}{a^2}\right) = 0 \quad (0 \le s \le L) \qquad (12)$$

The $\dot{v}_a/a$-term in Eq.(12) is now assumed to be negligible in comparison with the $\dot{u}_a'$-term preceding it by adopting the often used practice for developing equations of cylindrical shells (such as the well-known Donnell equations), which is to approximate the rotation component in Eq.(4) by $\omega_z = -u'$. With the subscript a further dropped we may now write Eq.(12) in the following form:

$$\frac{EAZ}{a^4}(u + a^2 u'' + a^4 u'''') + (M+m)\ddot{u} + 2MV\dot{u}' + MV^2\left(u'' + \frac{u}{a^2}\right) = 0 \quad (0 \le s \le L) \qquad (13)$$

The associated boundary conditions for a cantilevered duct are now, according to Eqs. (10) and (11),

$$u = u' = 0 \quad (s=0), \qquad u' + a^2 u''' = u + a^2 u'' = 0 \quad (s=L) \qquad (14)$$

In the original Eqs.(10) the boundary condition $v_a = 0$ at $s = 0$ is no longer needed, and the condition $M_a/a + N_a = 0$ at $s = L$ is now identically satisfied.

The simplified system of Eqs.(13) and (14) may also be derived directly from Hamilton's principle by taking

$$T = \frac{1}{2}\int_0^L m\dot{u}^2\,ds, \qquad U = \frac{1}{2}\int_0^L EI\left(u'' + \frac{u}{a^2}\right)^2 ds$$

$$\delta W = -\int_0^L M\left[\ddot{u} + 2V\dot{u}' + V^2\left(u'' + \frac{u}{a^2}\right)\right]\delta u\,ds$$

The above expression for U has been used by Timoshenko (Ref.14). For a thin ring ($Z = I/Aa^2$) and without fluid flow ($M = 0$) Eq.(13) reduces to the basic equation used by Evensen. For zero curvature Eqs.(13) and (14) reduce to the standard equations of a straight cantilevered duct conveying fluid.

Upon introducing the dimensionless parameters

$$\xi = s/L, \qquad \eta = u/L$$

$$v = \sqrt{\frac{M}{EI}}\,VL, \quad \tau = \sqrt{\frac{EI}{M+m}}\,\frac{t}{L^2}, \quad \beta = \frac{M}{M+m}$$

Eqs.(13) and (14) become

$$\frac{\partial^4\eta}{\partial\xi^4} + (2\theta_L^2 + v^2)\frac{\partial^2\eta}{\partial\xi^2} + \theta_L^2(\theta_L^2 + v^2)\eta + 2\sqrt{\beta}\,v\frac{\partial^2\eta}{\partial\xi\partial\tau} + \frac{\partial^2\eta}{\partial\tau^2} = 0 \qquad (0 \le \xi \le 1) \tag{15}$$

$$\begin{aligned} \eta = \frac{\partial\eta}{\partial\xi} &= 0 && (\xi = 0) \\ \frac{\partial^2\eta}{\partial\xi^2} + \theta_L^2\eta = \frac{\partial^3\eta}{\partial\xi^3} + \theta_L^2\frac{\partial\eta}{\partial\xi} &= 0 && (\xi = 1) \end{aligned} \tag{16}$$

The stability analysis to be presented below is based on the simplified equations. Stability analysis based on the complete equations will be carried out separately.

5. Stability Analysis

For stability analysis consider motion given by the expression

$$\eta = A e^{i\alpha\xi} e^{i\omega\tau} \tag{17}$$

where ω is the dimensionless frequency defined by $\omega = \sqrt{(M+m)/EI}\,\Omega L^2$, Ω being the circular frequency. In general ω is a complex number. The system will be stable or unstable depending upon whether the imaginary part of ω is positive or negative. For neutral stability the imaginary part of ω is zero.

For a given duct-fluid system the mass ratio β and dimensionless curvature θ_L are easily calculated. When a value of the dimensionless flow velocity v is further selected, an infinite set of values ω_n ($n = 1, 2, \ldots$) can be found, since the system possesses an infinite number of degrees of freedom. For $v = 0$, ω_n are real and are the dimensionless natural frequencies of the duct filled with a stationary fluid. As v increases, ω_n will become complex, but the system remains stable as long as the imaginary part of ω_n is positive. This is true until a certain critical value $v = v_{nc}$ is reached, when the imaginary part of ω_n becomes zero again, and a condition of neutral stability is arrived at in the nth mode. When $v > v_{nc}$, the system becomes unstable in the nth mode. The lowest among all v_{nc} is then the dimensionless critical flow velocity v_c, and the corresponding dimensionless critical frequency is denoted by ω_c, which is necessarily real.

The analysis begins with substituting Eq.(17) into (15). This yields the characteristic equation

$$\alpha^4 - (2\theta_L^2 + v^2)\alpha^2 - 2\sqrt{\beta}\,v\omega\alpha + \theta_L^2(\theta_L^2 + v^2) - \omega^2 = 0 \tag{18}$$

which is quartic. Assume that the four roots are all distinct and denoted by

$$\alpha = \alpha_j \qquad (j = 1, 2, 3, 4)$$

The complete solution is then

$$\eta = \sum_{j=1}^{4} A_j e^{i\alpha_j\xi} e^{i\omega\tau} \tag{19}$$

When η is now substituted from Eq.(19) into the boundary conditions (16), we find

$$\sum_{j=1}^{4} A_j = 0, \qquad \sum_{j=1}^{4} \alpha_j A_j = 0$$

$$\sum_{j=1}^{4} (\theta_L^2 - \alpha_j^2)\, A_j\, e^{i\alpha_j} = 0, \qquad \sum_{j=1}^{4} (\theta_L^2 - \alpha_j^2)\, \alpha_j A_j\, e^{i\alpha_j} = 0$$

For a non-trivial solution to exist, the determinant of the coefficients of A_j must vanish. This gives

$$\Delta \equiv \begin{vmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ (\theta_L^2-\alpha_1^2)e^{i\alpha_1} & (\theta_L^2-\alpha_2^2)e^{i\alpha_2} & (\theta_L^2-\alpha_3^2)e^{i\alpha_3} & (\theta_L^2-\alpha_4^2)e^{i\alpha_4} \\ (\theta_L^2-\alpha_1^2)\alpha_1 e^{i\alpha_1} & (\theta_L^2-\alpha_2^2)\alpha_2 e^{i\alpha_2} & (\theta_L^2-\alpha_3^2)\alpha_3 e^{i\alpha_3} & (\theta_L^2-\alpha_4^2)\alpha_4 e^{i\alpha_4} \end{vmatrix} = 0 \qquad (20)$$

Equation (20) is the complex frequency equation, in which α_j are functions of the dimensionless frequency ω as determined by Eq.(18). Since Δ is in general complex, Eq.(20) requires that the real and imaginary parts of Δ vanish separately.

It is not possible to solve Eqs.(18) and (20) by direct methods, and iterative methods together with the use of digital computers have been resorted to. The equations involve the four parameters β, θ_L, v, and ω, of which the first two (or any two) can be given and the remaining two are to be determined. In applying the iteration procedure when β and θ_L are given, trial values of v and ω are first assumed. Equation (18) is then solved numerically for α_j, but true results of α_j are obtained only when the real and imaginary parts of Δ in Eq.(20) both vanish. This is achieved by iterating v and ω on the computer.

Numerical Results

Comprehensive numerical results have been obtained for v_c and ω_c for the full range of $0 \leq \beta \leq 1$ and for $\theta_L = 0, 1, 2, \ldots, 6$, as shown in Figs. 2 and 3. The results for a straight duct for which $\theta_L = 0$ agree with those given by previous authors. Results for $\theta_L > 0$ show that the effect of curvature can be very severe. In general, v_c increases with increasing θ_L in the range of large β (heavier fluid). On the other hand, it decreases with increasing θ_L in the range of small β (lighter fluid), but there seems to be a lower bound near $\theta_L = 5, 6$. The result for ω_c for the straight duct always provides a lower bound to the results for curved ducts. The value of ω_c increases monotonically with θ_L until $\theta_L = 4$, beyond which the trend becomes rather irregular.

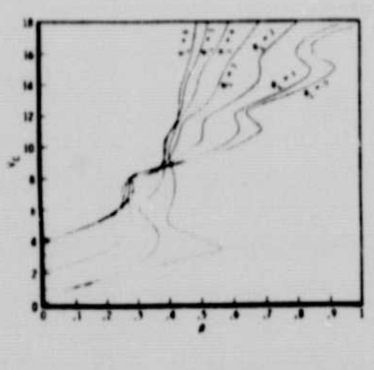

Fig. 2 Dimensionless Critical Velocity v_c as Function of Mass Ratio β and Curvature Parameter θ_L

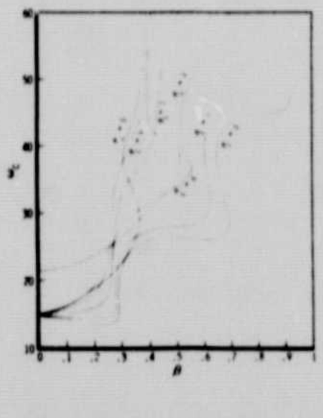

Fig. 3 Dimensionless Critical Frequency ω_c as Function of Mass Ratio β and Curvature Parameter θ_L

The dimensionless complex frequency ω has been computed for a number of cases. The results for the lowest four modes in the case of $\beta = 0.35$ and $\Theta_L = 2$ are shown in Fig. 4, which is essentially a root locus diagram. As mentioned before, when $v = 0$, ω is real and the branches intersect the horizontal axis at points corresponding to the natural frequencies of a duct with a stationary fluid. As v increases ω becomes complex, and all four modes are damped at first. As v approaches 8.5, ω for the third mode becomes real again and is equal to approximately 30 (these values of v and ω being verifiable by the results in Figs. 2 and 3). This is the dimensionless critical velocity v_c. When it is exceeded, the duct becomes unstable. It is interesting to note that the third mode can become dynamically unstable first, in contrast to the case of a straight cantilevered duct for which the second mode has been known to become unstable first.

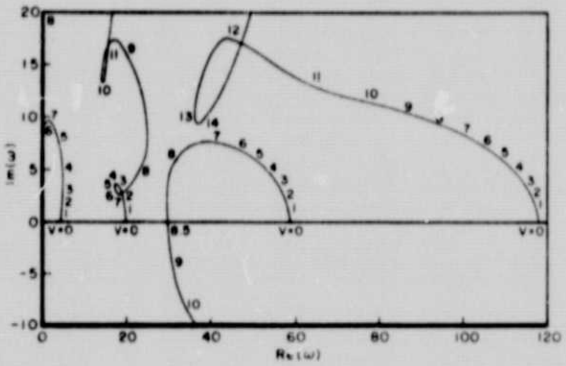

Fig. 4 Root Locus Diagram ($\beta = 0.35$, $\Theta_L = 2$)

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