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EFFECT OF SPANWISE VARIATION OF TURBULENCE ON
THE NORMAL ACCELERATION OF AIRPLANES WITH
SMALL SPAN RELATIVE TO TURBULENCE SCALE

Kermit G. Pratt



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**NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LANGLEY RESEARCH CENTER, HAMPTON, VIRGINIA 23665**

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RELATIVE TO TURBULENCE SCALE (NASA) 37 p HC
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INTRODUCTION

The methodology for determining the theoretical statistical averages that describe the symmetric responses of an airplane flying in continuous turbulence was originally developed on the basis of a one-dimensional turbulence model; i.e., turbulence velocities that vary in the direction of flight and are invariant along the span (See bibliography in reference 1). The procedure is currently in wide use and provides useful information for a large number of problems. There is, however, an inherent awkwardness in that in some instances (e.g., for an essentially rigid airplane) the calculation of the average rate of zero crossing requires the truncation of the upper limit of an integral (ref. 2). This injects the effect of engineering judgment and so causes some uncertainty in the results.

Methods for determining the effects of spanwise variations in turbulence (two-dimensional turbulence) have been developing for many years. The earliest investigators, references 3 - 9, were primarily concerned with the mathematical concepts for treating the two-dimensional problem. Attention was given mostly to mean-square responses; particularly of the lift due directly to the turbulence and of normal acceleration to a lesser degree. In view of the then newly-acquired ability, it is not surprising that emphasis was placed on fairly large values of the ratio of wing span to the turbulence scale (up to unity and beyond). There was a general tendency to expect the results from the two-dimensional turbulence model to approach those for the one-dimensional model as the ratio of span to turbulence scale approaches zero.

In the past few years, however, there has been a growing appreciation of the significant effects of the two-dimensional turbulence field on the symmetric responses of airplanes having spans that are less than the order of ten percent of the turbulence scale. (This figure is representative of the great majority of airplanes at cruising altitudes if a value of 762 meters (2500 feet) is accepted.) In reference 10, spectra of normal acceleration and symmetrical wing bending moment for an airplane having a span-to-scale ratio of five percent were shown to be attenuated at the higher frequencies by the two-dimensional turbulence model relative to those for the one-dimensional model. The attenuation was to such an extent that when combined with the unsteady effects of gust penetration, the values of the average rate of zero crossing could be calculated without recourse to the truncation of integrals required by the one-dimensional model. The mean square values of the responses were only slightly smaller than those from the one-dimensional model. At about the same time similar results and conclusions were reached independently in the work reported in reference 11. Also, the effects of the two-dimensional turbulence model on the acceleration of the rigid body and elastic modes of the Concorde supersonic transport were examined in reference 12. A more recent paper, reference 13, emphasizes the importance of the role of two-dimensional turbulence in calculations of the average rate of zero crossings. Presumably, there are many others in the world-wide aerospace industry whose unpublished data from computer calculations have made them aware of these effects. Nevertheless, there are still numerous

engineers who are not familiar with the differences in effects of one- and two-dimensional turbulence models. One of the purposes of this paper is to further the dissemination of knowledge of these differences.

The principal purpose, however, is to describe some insights into the way in which the two-dimensional turbulence model generates the differences from the one-dimensional model and the method by which these insights were obtained. Basic relationships are frequently lost in an avalanche of numbers generated by a complicated computer program. Such results are usually computed with high accuracy and are desirable for good design (e.g., ref. 14), but generalized effects of changes in airplane or turbulence parameters are often obscured.

A rigid airplane with an unswept wing having a constant chord is analyzed. The results obtained show that the aspect ratio of the wing, which often has not been explicitly expressed in past work, is the single source of the significant difference between the results of the one- and two-dimensional models for the relatively small span-to-turbulence-scale ratios. It is shown too that these differences exist for arbitrarily small values of the span-to-scale ratio. The differences are accounted for by a single weighting function of frequency, in contrast to the families of functions presented in references 11 and 13. Finally, the results indicate that two-dimensional gust effects may significantly affect the estimated average rate of zero crossings even for very small values of span-to-scale ratio.

SYMBOLS

$\bar{A}_{\Delta n}$	gust sensitivity $\sigma_{\Delta n} / \sigma_w$
AR	aspect ratio
b	span
c	chord
\bar{c}	mean chord
C_{L_g}	lift coefficient due to gust
$\frac{C_{L_g}(k_1)}{C_{L_g}(0)}$	unsteady gust lift function
C_{L_0}	lift coefficient for steady level flight

$C_{L\alpha}$	lift curve slope
g	acceleration of gravity
$H_{\Delta n}^{w_g}(k_1', k_2')$	normal acceleration frequency response for two-dimensional turbulence
$H_{\Delta n}^{C_L g}(k_1')$	frequency response of acceleration to gust lift
$H_{C_L g}^{w_g}(k_1', k_2')$	gust lift coefficient frequency response for two-dimensional turbulence
k_1	reduced frequency in direction of flight, $\frac{\omega_1 \bar{c}}{2V}$
k_1'	dimensionless wave number in direction of flight $\frac{\omega_1}{V} L = \Omega_1 L$
k_2'	dimensionless wave number in direction of span, $\Omega_2 L$
L	integral scale of turbulence
Δn	normal acceleration increment, g units
N_0	average frequency of zero crossing with positive slope
$R(k_1)$	weighting function for two-dimensional turbulence
S	wing area
\bar{S}	$\frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2}$
t	time
V	airspeed
w_g	gust velocity
$\overline{w_g}$	modulus of sinusoidal gust velocity

W	weight
x	$k_2' \frac{b}{2L}$
y	distance along span
z	normal acceleration
y*	$y/(b/2)$
α_g	angle of attack due to gust
K	mass parameter $\frac{8W}{\rho g S_e C_{L\alpha}} = 4\mu_g$
μ_g	mass ratio
ρ	density of the atmosphere
σ	root mean square
$\tilde{\phi}_{wg}(k_1', k_2')$	two-dimensional power spectrum of turbulence, velocity-squared
$\check{\phi}_{wg}(k_1')$	power spectrum of turbulence, velocity-squared
$\hat{\phi}_{wg_e}(k_1')$	effective power spectrum of turbulence, velocity-squared
$\phi_{\Delta n}(k_1')$	power spectrum of normal acceleration increment
ω_1	circular frequency in direction of flight
Ω_1	wave number in direction of flight, radians/unit distance
Ω_2	wave number in direction of span, radians/unit distance
Subscripts	
Δn	incremental normal acceleration
w _g	gust velocity
B	breakpoint of asymptotic lines

APPROACH

The present approach was originated to check the results obtained during the work reported in reference 10. These results, which were obtained from a digital computer, indicated that, in general, the mean-square values of the responses of the example airplane in two-dimensional turbulence did not approach those for the one-dimensional case as the span-to-scale ratio $\frac{b}{2L}$ approached zero. An analysis of the phenomenon by the present method indicated that, indeed, this characteristic was a reasonable result.

The approach utilizes an atmospheric turbulence spectrum in terms of wave numbers in the direction of flight and along the span, as featured in reference 3. This spectrum is more revealing of the response characteristics than the averaged and transformed cross-correlation functions or the cross-spectra used in references 4 - 7. The present procedure also utilizes simple strip theory for approximating the spanwise distribution of lift due directly to the vertical turbulence velocities. There is evidence in reference 15 that strip theory provides spanwise distributions of gust forces with acceptable accuracy. The combination of this form of turbulence spectrum and strip theory, together with the constraint that the ratio of wing span to turbulence scale be small ($b/L \ll 1.0$) permits spectral integrations with respect to spanwise wave numbers to be obtained in closed form for asymptotic conditions attending small and large wave numbers in the direction of flight. The closed form results provide identification of the turbulence and airplane parameters that are involved and their relationships.

This procedure is applied to a study of the differences noted in the past between statistical averages for normal accelerations calculated on the basis of one- and two-dimensional turbulence fields for airplanes with wing spans that are a small percentage of the turbulence scale. The statistical averages involved are the gust normal acceleration sensitivity,

$$\bar{A}_{\Delta n} = \frac{\sigma_{\Delta n}}{\sigma_{wg}} = \left[\int_0^{\infty} \frac{\Phi_{\Delta n}(k_i)}{\sigma_{wg}^2} dk_i \right]^{1/2} \quad (1)$$

and the average frequency of zero crossing with positive slope

$$N_{0\Delta n} = \frac{V}{2\pi L} \left[\int_0^{\infty} k_i^2 \frac{\Phi_{\Delta n}(k_i)}{\sigma_{wg}^2 \bar{A}_{\Delta n}^2} dk_i \right]^{1/2} \quad (2)$$

The spectrum of normal acceleration, $\Phi_{\Delta n}(k_1')$ is common to both quantities and is the subject of the following analysis.

ANALYSIS

General Relationships.— The acceleration spectrum is expressed in terms of the dual-wave-number atmospheric turbulence spectrum, $\Phi_{\Delta w_g}(k_1', k_2')$, as

$$\frac{\Phi_{\Delta n}}{\sigma_{w_g}^2}(k_1') = \int_0^{\infty} \frac{\Phi_{\Delta w_g}(k_1', k_2')}{\sigma_{w_g}^2} |H_{\Delta n}^{w_g}(k_1', k_2')|^2 dk_2' \quad (3)$$

where $H_{\Delta n}^{w_g}(k_1', k_2')$ is the acceleration frequency response function. This function can be expressed as the product

$$H_{\Delta n}^{w_g}(k_1', k_2') = H_{\Delta n}^{C_L}(k_1') H_{C_{Lg}}^{w_g}(k_1', k_2') \quad (4)$$

where the function $H_{\Delta n}^{C_L}$ is the acceleration due to a sinusoidal lift coefficient C_L and is a function of k_1' only. The function $H_{C_{Lg}}^{w_g}$ is the sinusoidal lift coefficient due solely to the two-dimensional field of sinusoidal gust velocities

$$w_g = \bar{w}_g e^{i(k_1' \frac{y}{L} + k_2' \frac{y}{L})} \quad (5)$$

The spanwise distribution of the aerodynamic lift coefficient due directly to the gust field is approximated by strip theory. The distribution of gust lift coefficient for an unswept wing having a constant chord is

$$\frac{\partial C_{Lg}}{\partial y^*} = \frac{C_{L\alpha}}{2} \frac{C_{Lg}(k_1')}{C_{Lg}(0)} \alpha_g(y^*) \quad (6)$$

where $\frac{C_{Lg}(k_1)}{C_{Lg}(0)}$ is the unsteady lift function due to gust penetration and wake effects and where from equation (5)

$$a_g(y^*) = \frac{\bar{w}_g}{V} e^{i k_1' \frac{Vt}{L}} \left(\cos k_2' \frac{y}{L} + i \sin k_2' \frac{y}{L} \right) \quad (7)$$

The imaginary term in the parentheses in equation (7) is antisymmetric with respect to the wing centerline and so does not contribute to the total lift.

The substitution of equation (7) in (6) yields

$$\frac{\partial C_{Lg}}{\partial y^*} = \frac{C_{L\alpha}}{2} \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \frac{\bar{w}_g}{V} e^{i k_1' \frac{Vt}{L}} \cos\left(k_2' \frac{b}{2L} y^*\right) \quad (8)$$

The total gust lift coefficient is then

$$C_{Lg} = C_{L\alpha} \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \frac{\bar{w}_g}{V} e^{i k_1' \frac{Vt}{L}} \int_0^1 \cos\left(k_2' \frac{b}{2L} y^*\right) dy^* \quad (9)$$

$$= C_{L\alpha} \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \frac{\bar{w}_g}{V} e^{i k_1' \frac{Vt}{L}} \frac{\sin k_2' \frac{b}{2L}}{k_2' \frac{b}{2L}} \quad (10)$$

With

$$\left| \frac{C_{Lg}}{\bar{w}_g} \right| \equiv \left| H_{C_{Lg}}^{w_g}(k_1', k_2') \right|$$

the modulus squared for equation (10) is

$$|H_{C_{L\alpha}}^{wg}(k_1', k_2')|^2 = \left| \frac{C_{L\alpha}(k_1')}{C_{L\alpha}(0)} \right|^2 \frac{C_{L\alpha}^2}{V^2} \frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2} \quad (11)$$

The expression for the spectrum of airplane acceleration is obtained by substituting equation (11) in equation (4), and substituting in turn equation (4) in equation (3).

$$\frac{\Phi_{\Delta n}(k_1')}{\sigma_{wg}^2} = |H_{\Delta n}^{C_L}(k_1')|^2 \left| \frac{C_{L\alpha}(k_1')}{C_{L\alpha}(0)} \right|^2 \times \frac{C_{L\alpha}^2}{V^2} \int_0^{\infty} \frac{\Phi_{wg}(k_1', k_2')}{\sigma_{wg}^2} \frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2} dk_2' \quad (12)$$

The integral

$$\int_0^{\infty} \frac{\Phi_{wg}(k_1', k_2')}{\sigma_{wg}^2} \frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2} dk_2' \equiv \frac{\Phi_{wge}^1(k_1', \frac{b}{2L})}{\sigma_{wg}^2} \quad (13)$$

is an effective turbulence spectrum.

The results from the two-dimensional gust field are related to those for the one-dimensional gust field by

$$\frac{\Phi_{wge}^1(k_1', \frac{b}{2L})}{\sigma_{wg}^2} = R(k_1', \frac{b}{2L}) \frac{\Phi_{wg}(k_1')}{\sigma_{wg}^2} \quad (14)$$

where $R(k_1', \frac{b}{2L})$ is a weighting function which contains the effects of the two-dimensional turbulence, and $\frac{\overline{\Phi}_{wg}}{\sigma_{wg}^2}(k_1')$ is the turbulence spectrum

for the one-dimensional field. In terms of the spectrum for the two-dimension turbulence:

$$\frac{\overline{\Phi}_{wg}(k_1')}{\sigma_{wg}^2} = \int_0^{\infty} \frac{\overline{\Phi}_{wg}(k_1', k_2')}{\sigma_{wg}^2} dk_2' \quad (15)$$

The combination of equations (13), (14), and (15) yields the following expression for R.

$$R = \frac{\overline{\Phi}_{wg}(k_1', \frac{b}{2L})}{\overline{\Phi}_{wg}(k_1')} = \frac{\int_0^{\infty} \overline{\Phi}_{wg}(k_1', k_2') \frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2} dk_2'}{\int_0^{\infty} \overline{\Phi}_{wg}(k_1', k_2') dk_2'} \quad (16)$$

where the integrands in equation (16) are identified by

$$\frac{\overline{\Phi}_{wg}(k_1', k_2')}{\sigma_{wg}^2} \equiv I_1 \quad (17)$$

and

$$\frac{\overline{\Phi}_{wg}(k_1', k_2') \frac{\sin^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2}}{\sigma_{wg}^2} \equiv I_1 \bar{S} \equiv I_2 \quad (18)$$

where, of course,

$$\bar{S}(k_2', \frac{b}{2L}) = \frac{517^2 k_2' \frac{b}{2L}}{(k_2' \frac{b}{2L})^2} \quad (19)$$

In the following analysis, the characteristics of the weighting function R are determined from an examination of these integrands, I_1 and I_2 , over the range of values of k_1' . The Dryden turbulence spectrum is used for illustrative purposes. The von Karman spectrum can be utilized equally well. Results from both spectra are presented subsequently. The Dryden spectrum for two-dimensional turbulence (from ref. 10) is

$$\frac{\tilde{\Phi}_{Wg}(k_1', k_2')}{\sigma_{Wg}^2} = I_1 = \frac{3}{\pi} \frac{(k_1'^2 + k_2'^2)}{(1 + k_1'^2 + k_2'^2)^{5/2}} \quad (20)$$

Log Frequencies. - Consider first the frequency range $k_1' \leq 1.0$. The two components of integrand I_2 (viz., I_1 , the spectra for $k_1' = 0, .5$, and 1.0 , and the function \bar{S}) are shown individually as log-log plots in figure 1 for a value of $b/2L$ arbitrarily chosen to be $\pi/20$. The low and high frequency asymptotic lines are indicated for all functions. The salient feature of the functions is the break frequency between the asymptotes.

The break frequency for \bar{S} is

$$k_{2B,5}' = \frac{20}{\pi} \quad \text{for} \quad \frac{b}{2L} = \frac{\pi}{20}$$

so

$$k_{2B,5}' \frac{b}{2L} = 1.0$$

and

$$k_{2B,5}' = \frac{2L}{b}$$

For

$$k_2' \ll k_{2B,5}'$$

the value of \bar{S} is nearly unity. The maximum break frequency for the spectra I_1 is

$$k'_{2,B,I} = 2^{5/6} = 1.78 \quad (\text{for } k'_1 = 1.0)$$

If $k'_{2,B,I} \ll k'_{2,B,\bar{S}}$

$$\text{or } 1.78 \ll \frac{2L}{b}$$

$$\text{or } b/2L \ll .56$$

$$\text{then } \bar{S} \approx 1.0$$

$$\text{and } I_2 \approx I_1$$

Thus the one- and two-dimensional results are essentially the same and

$$R \approx 1.0 \quad (21)$$

provided that $b/2L \ll .56$ and $k'_1 \lesssim 1.0$

The wave number k'_1 must be expressed in terms of airplane temporal frequency and is commonly related to the reduced frequency

$k'_1 = \frac{\omega_1 \bar{c}}{2V}$ (particularly for calculations based on a one-dimensional turbulence model) as follows

$$k'_1 = k_1 \frac{2L}{c} = \Omega_1 L \quad (22)$$

As indicated in previous equations; e.g., equation (16), the parameter $\frac{b}{2L}$ is introduced by the two-dimensional turbulence model. Thus both chord \bar{c} and span b are involved and, hence, aspect ratio should be explicitly identified, lest its value be inadvertently changed as variations in $\frac{b}{2L}$ are considered. Therefore, equation (22) is written as

$$k'_1 = k_1 \frac{2L}{b} AR \quad (23)$$

For $k'_1 < 1.0$ and $\frac{b}{2L} \ll .56$, Equation (23) becomes

$$k_1 AR \lesssim \frac{b}{2L} \ll .56 \quad (24)$$

This indicates the range of k_1 for which $R \approx 1.0$.

The relationship in equation (24) was refined by a numerical evaluation of the integral of I_2 . It was found that

$$.95 \bar{R} < R < 1.0$$

for values of $b/2L$ up to $\pi/20$ or 0.16. Equation (24), therefore, can be expressed as

$$k_1 \text{ AR} \bar{R} < .16$$

An example of the relationship between the exact integrand I_2 , and the approximate integrand I_1 is given in figure 2, with linear scales, for $k_1' = .5$ and $b/2L = \pi/20$. It can be seen that the effect of \bar{S} is very small for this case.

High frequencies. - Consider now the frequency range $k_1' \gg 1.0$. From equation (20), the two-dimensional gust spectrum in this frequency range is

$$\frac{\tilde{\Phi}(k_1', k_2')}{\sigma_w^2} = I_1 \approx \frac{3}{\pi} \frac{1}{(k_1'^2 + k_2'^2)^{3/2}} \quad (25)$$

A typical spectrum and the \bar{S} function are shown individually in figure 3 as a log-log plot. The shape of the spectrum as a function of k_2' is dependent only on the value of k_1' . The break frequency is

$$k_{2B,I}' = k_1' = .01 \frac{2L}{b} \text{ AR}$$

For

$$k_2' \ll k_{2B,I}'$$

$$I_1 \approx \frac{3}{\pi} \frac{1}{k_1'^3}$$

Therefore

$$I_2 \approx \frac{3}{\pi} \frac{1}{k_1'^3} \bar{S}$$

The integral of \bar{S} with respect to k_2' converges rapidly, reaching 90 percent of the maximum value at an upper limit of

$$k_2' = 2\pi \frac{2L}{b}$$

It follows, then, that if

$$k_2' = k_1 \frac{2L}{b} AR \gg 2\pi \frac{2L}{b}$$

or

$$k_1 AR \gg 2\pi$$

(26)

then

$$\frac{1}{\Phi_{wg_e}}(k_1', \frac{b}{2L}) \approx \frac{3}{\pi} \int_0^\infty \frac{\bar{S}}{k_1'^3} dk_2'$$

(27)

The exact expression (from equations (13) and (20)) is

$$\frac{1}{\Phi_{wg_e}}(k_1', \frac{b}{2L}) = \frac{3}{\pi} \int_0^\infty \frac{(k_1'^2 + k_2'^2)}{(1 + k_1'^2 + k_2'^2)^{5/2}} \bar{S} dk_2'$$

(28)

The integrands of equations (27) and (28) are plotted with linear scales in figure 4 for $k_1' = 100$ and $\frac{b}{2L} = \frac{\pi}{20}$. It is seen that the differences are

very small and so equation (27) is valid for large values of k_1' .

The combination of equations (27) and (16) (with a change in variable of integration) yields

$$R \approx \frac{\frac{3}{\pi} \frac{1}{k_1'^3} \frac{2L}{b} \int_0^\infty \bar{S} dk_2' \frac{b}{2L}}{\frac{\Phi_{wg}}{\sigma_{wg}^2}(k_1')}$$

where $\int_0^\infty \bar{S} dk_2' \frac{b}{2L} = \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$

From equation (34), for $k_1' = k_1 \frac{2L}{b} AR \gg \frac{3}{2}$,

$$\frac{\Phi_{wg}}{\sigma_{wg}^2}(k_1') \approx \frac{3}{\pi} \frac{1}{k_1'^2}$$

The weighting function R is therefore

$$R \approx \frac{2L}{b} \frac{\pi}{2k_1'}$$

In terms of k_1

$$R(k_1) \approx \frac{\pi}{2 AR k_1} \quad (29)$$

provided that $\frac{b}{2L} < 2\pi$ (This restriction on $\frac{b}{2L}$ is due to the initial requirement that

$$k_1' \gg 1.0$$

$$\text{or } k_1 AR \gg \frac{b}{2L}$$

together with the requirement in equation (26). It is of no consequence to the present study, inasmuch as only values of $b/2L$ less than sixteen percent are of interest).

It is noteworthy that R in equation (29) does not depend on $b/2L$ at all; it applies for all values of $b/2L < 2\pi$. However, aspect ratio is an important parameter and R varies inversely with k_1 .

Two-dimensional Gust Weighting Function.- The expression for R in equation (29) for the high frequency asymptotic condition ($k_1 \gg \frac{2\pi}{AR}$) together with the results from consideration of the low frequency asymptotic condition (equation (21)), $R \approx 1.0$ for $k_1 < \frac{b}{2L AR}$, allows the construction of a function that approximates R over the entire range of k_1 . The following equation satisfies these asymptotes.

$$R(k_1) \approx \frac{1}{1 + \frac{2AR k_1}{\pi}} \quad (30)$$

This function is plotted on figure 5 and is identified by the label "Dryden." It is of interest to observe that equation (30) is completely independent of $b/2L$ for values of $b/2L$ less than 0.16. Equation (30) is applicable, therefore, to very small airplanes. The amplitude of R is nearly unity at low frequencies and is attenuated inversely with frequency at the high frequencies. The attenuation is governed, otherwise, only by aspect ratio as indicated by the break frequency of the asymptotic lines at $k_{1B} = \frac{\pi}{2AR}$. The attenuation at a given frequency increases with an increase in aspect ratio.

Equation (30) provides results similar to those obtained by numerical integration in reference 11 and by asymptotic analysis of cross spectra in

reference 13. However, in both references, the results are described by families of curves rather than by a single function. In reference 11, aspect ratio is not explicitly identified, and in reference 13, a separate function for each aspect ratio is calculated. Thus, the fundamental role of aspect ratio is not described. Furthermore, although calculations in both references are made for values of $b/2L$ as low as 0.05, evidence is not presented to indicate that the attenuation at high frequencies exists for all small values of $b/2L$, including indefinitely small values.

The similarity of results obtained by the method described herein to those in references 11 and 13 was verified by generating comparable families of curves from a weighting function based on the von Karman turbulence spectrum, used in the references.

$$R = \frac{1}{1 + \frac{3 \cdot AR}{1.339\pi} k_1} \quad (31)$$

This function is also plotted on figure 5. It differs from equation (30) by a slightly lower break frequency. In spite of the fact that the weighting function is exact only for the very low and the very high frequencies, the results differ from those in reference 11 by a little more than 10 percent at the most; the differences are even smaller with respect to results in reference 13.

The weighting function R is useful primarily as a tool for revealing both the characteristic attenuation of lift at high frequencies for arbitrarily small values of $b/2L$ and the role of aspect ratio. While R may be suitable to obtain quick approximate results for conceptual design purposes (for unswept wings), more accurate results can be obtained by use of computer solutions for the aerodynamic forces on lifting surfaces of various shapes (e.g., kernel function, ref. 16; and doublet-lattice, ref. 17). The turbulence spectrum in terms of dual wave numbers, used herein, is particularly well suited for use in such calculations.

The procedure used herein can be applied to tapered unswept wings, although more effort is required to obtain closed-form integrals. From qualitative considerations, the effect of taper is similar to a reduction in aspect ratio of an untapered wing (due to loss of effectiveness of the outboard portion of the surface). The result is an increase in the break frequency of the weighting function R . Quantitative evidence of this is found in reference 11. Triangular span loading (a taper ratio of zero) increases the break frequency by about 25 percent over that for constant span loading (taper ratio of one). This is equivalent to a 20-percent reduction in aspect ratio.

NORMAL ACCELERATION SPECTRA

The full significance of the ratio R is brought out by consideration

of the spectrum of the acceleration response of an airplane. This spectrum is described by combining equations (12), (13), (14), and (30) which yields

$$\frac{\Phi_{\Delta n}(k_1')}{\sigma_{w_g}^2} = |H_{\Delta n}^{C_L}(k_1)|^2 \left| \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \right|^2 \frac{C_{L\alpha}^2}{V^2} R(k_1) \frac{\sqrt{V} \Phi_{w_g}(k_1')}{\sigma_{w_g}^2} \quad (32)$$

or in terms of k_1

$$\frac{\Phi_{\Delta n}(k_1)}{\sigma_{w_g}^2} = |H_{\Delta n}^{C_L}(k_1)|^2 \left| \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \right|^2 \frac{C_{L\alpha}^2}{V^2} R(k_1) \frac{2L}{b} AR \frac{\sqrt{V} \Phi_{w_g}(k_1')}{\sigma_{w_g}^2} \quad (33)$$

The weighting function $R(k_1)$ is shown in the above equations as a separate entity. It could be logically combined with the turbulence spectrum to define an effective spectrum for the two-dimensional turbulence model as in equation (14). However, R is a function of airplane parameters only, notably aspect ratio and so, also logically, it could be combined with the airplane frequency response function.

Functions. - The important properties of the airplane acceleration spectrum can be illustrated through the use of rather simple approximations for the several functions of frequency in equation (33).

The Dryden spectrum for the one-dimension turbulence model is

$$\frac{\sqrt{V} \Phi_{w_g}(k_1')}{\sigma_{w_g}^2} = \frac{1}{\pi} \frac{1 + 3 k_1^2 \left(\frac{2L}{b} AR\right)^2}{\left[1 + k_1^2 \left(\frac{2L}{b} AR\right)^2\right]^2} \quad (34)$$

This is further simplified by use of asymptotic functions to

$$\frac{\sqrt{V} \Phi_{w_g}(k_1')}{\sigma_{w_g}^2} \approx \frac{3}{\pi} \frac{1}{\left[9/4 + k_1^2 \left(\frac{2L}{b} AR\right)^2\right]} \quad (35)$$

The airplane frequency response function considered is for a rigid or quasi-flexible airplane with a single degree of freedom in plunging motion (Pitching motion is suppressed). The unsteady aerodynamic forces due to motion are ignored. The equation of motion is

$$\frac{W}{g} \ddot{z} + \frac{\rho}{2} V^2 S C_{L\alpha} \frac{\dot{z}}{V} = \frac{\rho}{2} V^2 S C_L e^{i\omega t}$$

The modulus-squared of the frequency response function can be written as

$$|H_{\Delta n}^{C_L}(k_1)|^2 = \frac{1}{C_{L0}^2} \left[\frac{k_1^2}{k_1^2 + \frac{4}{X^2}} \right] \quad (36)$$

where $C_{L0} = \frac{2W}{\rho V^2 S}$

and $X = \frac{8W}{\rho g S C_{L\alpha}}$

The frequency response function for this condition contains the significant features of the more general condition.

The unsteady gust function selected is

$$\left| \frac{C_{Lg}(k_1)}{C_{Lg}(0)} \right|^2 = \frac{1}{1 + 2\pi k_1} \quad (37)$$

This is an approximation of the unsteady gust force for a lifting surface having an infinite aspect ratio in incompressible flow, as used in reference 3. Different approximations, but also based on infinite aspect ratio, were used in the studies of responses to two-dimensional turbulence in references 10 and 11. This practice carries the tacit assumption that the variation of total lift with frequency k_1 is independent of the shape of the spanwise distribution of lift, which, of course, is a function of frequency k_2 .

This assumption is consistent with others inherent in a simple strip theory approximation for the aerodynamic forces, but apparently the validity has not been assessed. It would seem desirable to investigate the characteristics of unsteady gust forces by calculating them by means of the doublet-lattice or similar computer program.

The four functions of frequency in equation (33) as described by equations (30) and (35) - (37) are illustrated in figure 6 as log-log plots of the asymptotic functions. The airplane acceleration spectrum is obtained from the sum of these logarithmic plots and is illustrated in figure 7. The slopes of the logarithmic lines, the maximum amplitude, and the break-points of the lines are identified.

The high frequency asymptote is of particular significance, and it is notable that for an aspect ratio of π^2 (2.10), the break-points of the R weighting function and the unsteady gust lift function coincide; thus, each

function contributes equally to the high frequency characteristic. The significance of the high frequency asymptote is associated with its influence on the second moment of the response spectrum that in turn influences the value of the average rate of zero crossings N_0 as indicated by equation (2). The second moment of the acceleration spectrum is shown as a log-log plot of asymptotic lines in figure 8. The corresponding function for the one-dimensional turbulence model (with $R = 1.0$) is shown by the dashed line.

The integral for the one-dimensional turbulence model is infinite due to the asymptotic slope of minus one. Calculations of N_0 for this case have required truncation of the integration at an upper frequency limit corresponding to some stipulated percentage (close to 100%) of the mean-square value of the response as suggested in reference 2. In contrast, the integral of the function described by the solid lines is finite, thus offering the promise of determining N_0 without truncating the integration. This promise is evaluated by an application to a representative example.

Application to Example Airplane

The example airplane is closely akin to a short-haul transport in current service. The airplane features an unswept, nearly constant-chord wing. The pertinent aircraft and atmospheric parameters are listed in table I for the cruise condition. The pitching motion is suppressed. Note that the aspect ratio is 10 which results in about equal attenuation by the two-dimensional turbulence and the gust unsteady lift.

$\bar{A}_{\Delta n}$ -. The spectra for normal acceleration at the center of gravity are presented in log-log plots together with the asymptotic lines in figure 9 and in linear plots in figure 10. The figures include results from the simplified method for the two-dimensional turbulence model and from the conventional one-dimensional model. The difference in associated values of \bar{A} is about 8 percent.

$N_{0\Delta n}$ -. The spectra for the second moment are presented in like forms in figures 11 and 12. The value of N_0 for the two-dimensional turbulence model is .68 per second which is believed to be a reasonable value. Of course, for the one-dimensional turbulence model the value of N_0 is infinite unless the integration of the second moment of the spectrum is truncated.

Vanishing $b/2L$

Turning now from the characteristics related to an airplane at a low but fixed value of $b/2L$, attention is given to the effects of $b/2L$ approaching arbitrarily small values. Much useful information is obtained by reconsidering the properties of the several frequency functions illustrated in

figures 6 - 8. It is seen that the parameter $b/2L$ appears only in the turbulence spectrum function. Consequently, it is convenient to combine the other functions into a single effective acceleration response function and to examine its relation to the turbulence spectrum for various values of $b/2L$. This is illustrated in figure 13. The acceleration spectra at the bottom of the figure indicate that the maximum amplitude decreases directly with a decrease in $b/2L$ and so approaches zero as $b/2L$ approaches zero. Although the bandpass characteristics broaden toward the lower frequencies with decreasing values of $b/2L$, recall that this plot is logarithmic, and the asymptotic lower frequency is zero. The area under the acceleration spectrum, hence, the value of \bar{A} , must approach zero as $b/2L$ approaches zero, provided that κ and AR are held constant.

In contrast to \bar{A} , N_0 approaches a constant value as $b/2L$ approaches zero. N_0 is proportional to the radius of gyration of the response spectrum with respect to the zero frequency axis based on linear scales (See equation (2)). From the log-log plot on figure 13 it can be seen that the shape of the acceleration spectrum remains constant, except at low frequencies, as $b/2L$ decreases. It follows that the radius of gyration (for linear scales), hence N_0 will decrease somewhat and approach a constant value as $b/2L$ becomes indefinitely small.

It would appear trivial to compare the characteristics for the two- and one-dimensional turbulence models for $b/2L = 0$. However, there is often some value in interpreting asymptotic conditions, and this has been done in past studies; e.g., references 3 and 10, the latter being the motivation for the study presented here. As can be seen from the high-frequency asymptotes for the acceleration spectra in figure 13, the results for the one- and two-dimensional turbulence models are the same only for aspect ratio equal to zero, inasmuch as the break-point frequency associated with the R weighting function becomes infinite. The usual application of the one-dimensional model results in precisely this. The frequency k_1 is expressed in terms of wing chord, i.e., $k_1 = \frac{\omega_1 c}{2V}$. At the same time the scale is stipulated to be some finite value while the span is assumed to be very small compared to scale and zero in the limit; the aspect ratio is thus forced to zero.

Choice of parameters.- The use of the parameters $b/2L$ and k_1 in the final expressions (i.e., equations (30) and (35)-(37)) is arbitrary. The parameter $b/2L$ was selected because the study was directed toward effects for small values of the quantity. It appears in the results only in the expression for the one-dimensional turbulence spectrum as a function of k_1 . In view of the conclusion that the effects of the two-dimensional gust field are independent of $b/2L$ for small values, the airplane size can be expressed in terms of a reference chord instead of span. The aspect ratio then would appear only in the weighting function R .

It is felt that the choice of k_1 for the frequency expression produces simpler and more easily interpreted results than other alternatives such

as ω . The reduced frequency k_1 is a fundamental parameter widely used with regard to unsteady aerodynamic forces, including those due to gust penetration. The frequency response function for the normal acceleration of the airplane is a simple expression in terms of k_1 . The shape of the function for a plunging airplane is dependent only on the mass parameter κ . The weighting function R is dependent only on aspect ratio. As previously mentioned, the description of the one-dimensional turbulence spectrum in terms of k_1 is dependent only on airplane size and scale of turbulence. In general, if alternate expressions for frequency are utilized (e.g., ω), and if the airplane size is described by a reference chord, the weighting function R remains dependent on aspect ratio.

CONCLUDING REMARKS

The normal acceleration response of a rigid airplane with a constant-chord, unswept wing, flying in turbulence that varies in the spanwise and flight path directions, is analyzed. The analysis is restricted to a range of values of the ratio of wing span to turbulence scale from 30 percent to vanishingly small, a range which includes most airplanes. The analytical approach features an atmospheric turbulence spectrum in terms of wave numbers in both directions. The spanwise distribution of aerodynamic forces is approximated by strip theory. Spectral integrations with respect to spanwise wave numbers are obtained in closed form for asymptotic conditions attending small and large wave numbers in the direction of flight.

Results show that the power spectrum of normal acceleration in the two-dimensional turbulence field is significantly attenuated at the higher frequencies relative to that for a one-dimensional field and that the attenuation persists for values of wing span to turbulence scale approaching zero. The attenuation is accounted for by a simple weighting function of frequency that is dependent only on the aspect ratio of the wing, the attenuation increasing with an increase in aspect ratio.

The attenuation, together with that from the unsteady flow of gust penetration, permits the determination of the average number of zero crossings without recourse to an integral truncation that is often required in one-dimensional turbulence calculations. It is notable that for a wing aspect ratio of about 10, the contributions of the two dimensional gust field and gust penetration to the attenuation are equal.

The above result is based on the use of an unsteady gust force for a lifting surface having an infinite aspect ratio, a frequently used approximation. This practice is based on the tacit assumption that the unsteady flow effects are independent of the shape of the spanwise lift distribution. The adequacy of this assumption needs to be assessed by application of unsteady lifting surface computational programs.

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TABLE I.- AIRCRAFT AND ATMOSPHERE PARAMETERS

Wing area, S	39 m ² , (420 square feet)
Mean wing chord \bar{c}	1.98 m, (6.5 feet)
Wing span	19.8 m, (65 feet)
Weight, W	50,042 Newtons, (11,250 pounds)
Altitude, h	914 m, (3,000 feet)
Cruise Speed, V	80.5 m/s, (264 feet/second)
Turbulence scale, L	762 m, (2,500 feet)
Aspect ratio, AR,	10
Lift-curve slope, $C_{L\alpha}$,	5
Span-to-scale ratio, $\frac{b}{2L}$.013
Cruise lift coefficient, C_{L0} ,	.353
Mass parameter, κ ,	94.1

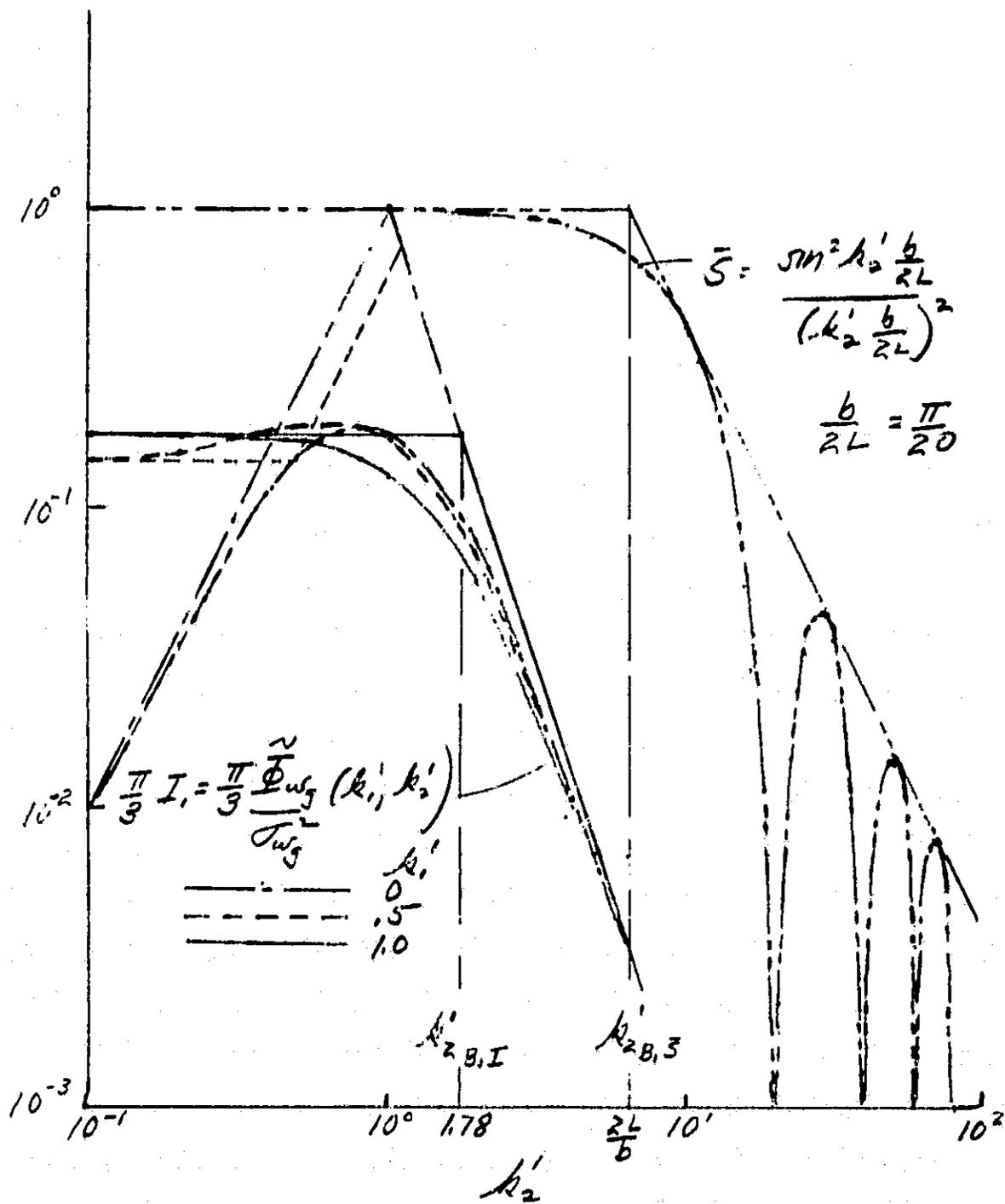


FIGURE 1 TWO DIMENSIONAL TURBULENCE SPECTRA FOR $k_1' \leq 1.0$ AND \bar{S} FUNCTION

$$k_1' = .5 \quad \frac{b}{2L} = \frac{\pi}{20}$$

$$\text{APPROX.} \quad \frac{\pi}{3} I_1 = \frac{\pi}{3} \frac{\int_{-a}^a \Phi_{\text{avg}}}{\sqrt{a^2 - x^2}} (k_1', k_2')$$

$$\text{EXACT} \quad \frac{\pi}{3} I_2 = \frac{\pi}{3} I_1$$

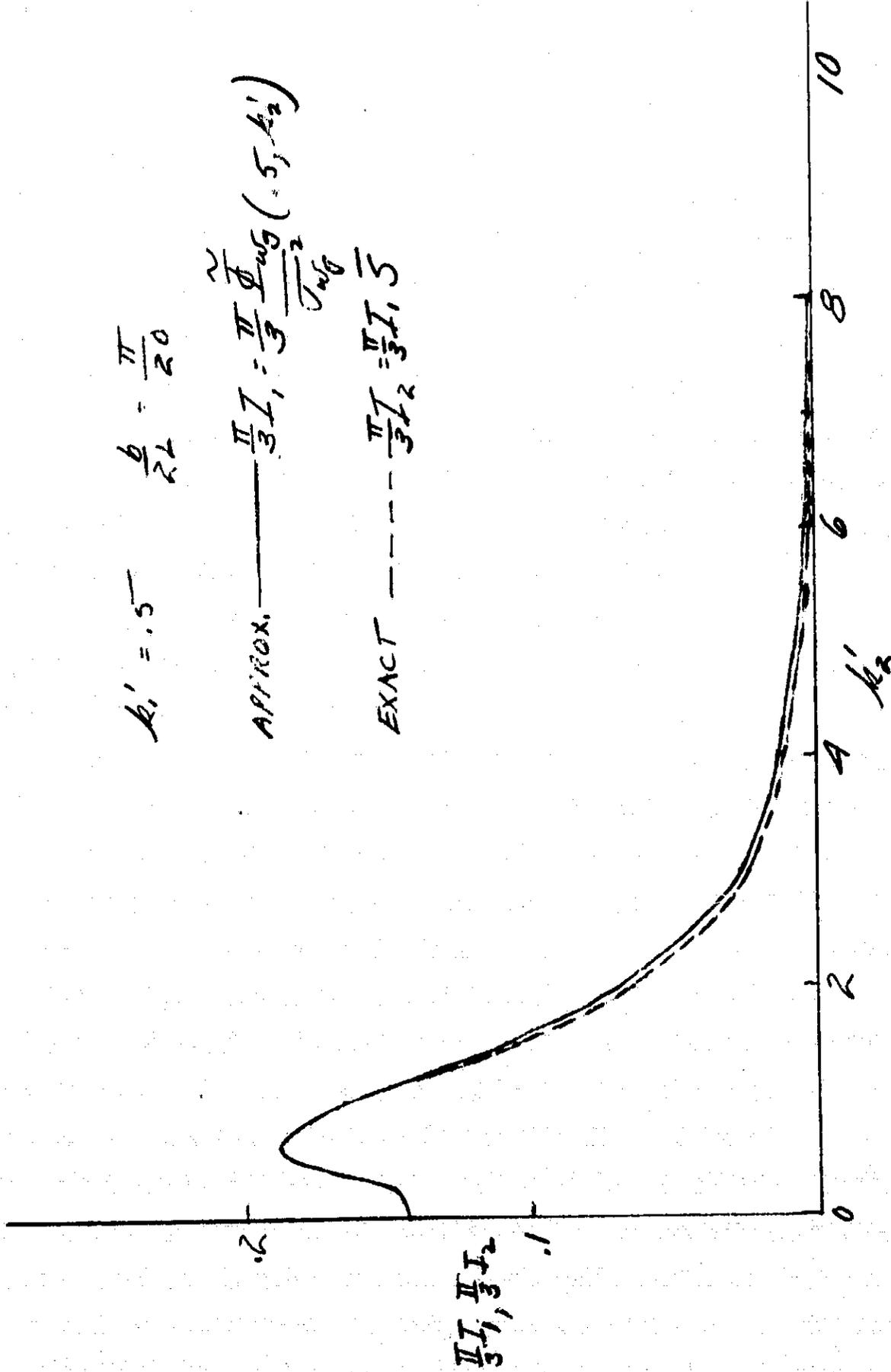


FIGURE 2 EXACT AND APPROXIMATE INTEGRANDS

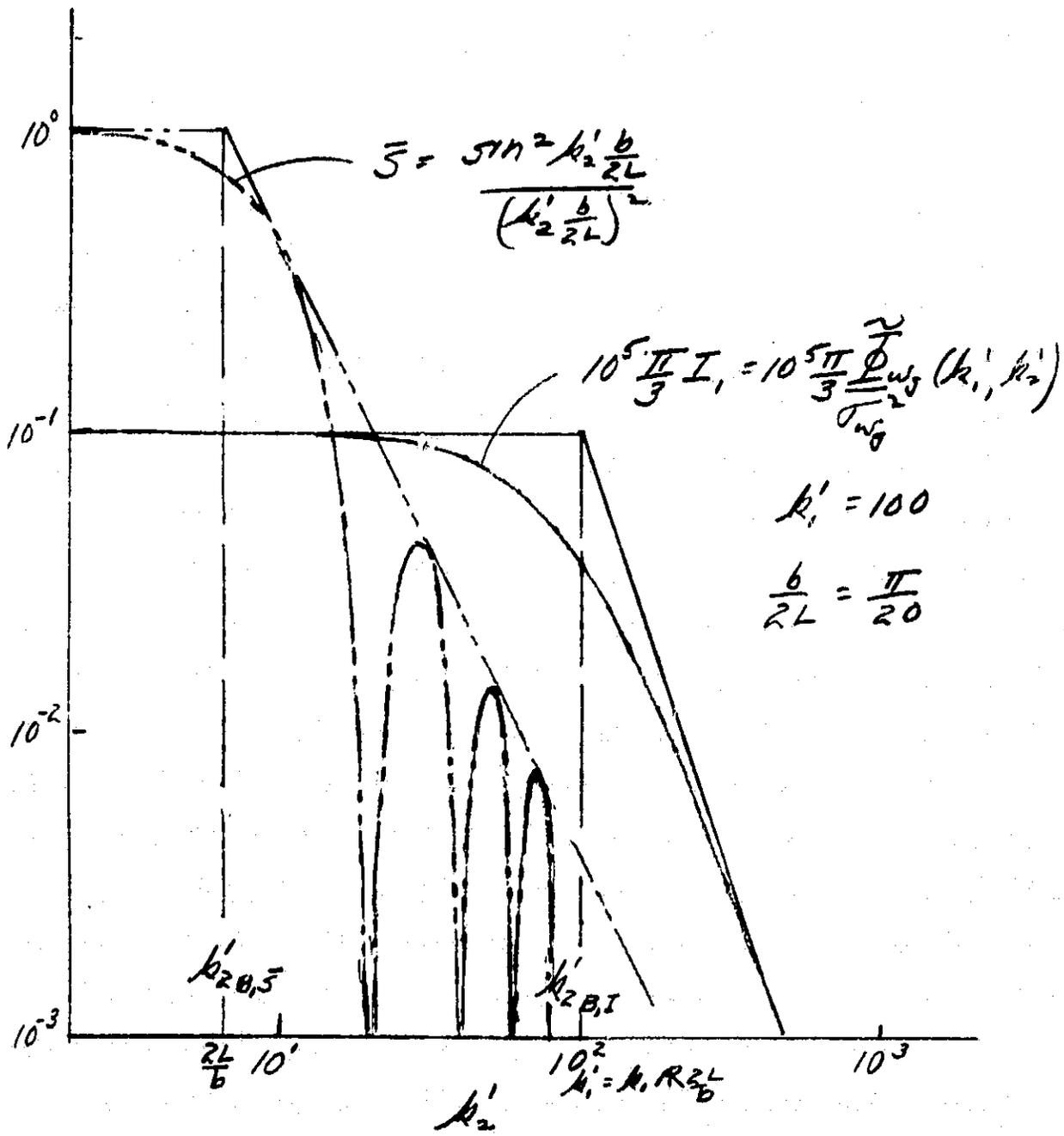


FIGURE 3 TWO DIMENSIONAL TURBULENCE SPECTRUM FOR $k_1' \gg 1.0$ AND \bar{S} FUNCTION

$$k_1' = 100 \quad \frac{q}{rL} = \frac{\pi}{20}$$

$$\frac{5}{k_1' 3}$$

APPROX.

$$\frac{(k_1'^2 + k_2'^2)}{(1 + k_1'^2 + k_2'^2)^{3/2}} 5$$

EXACT

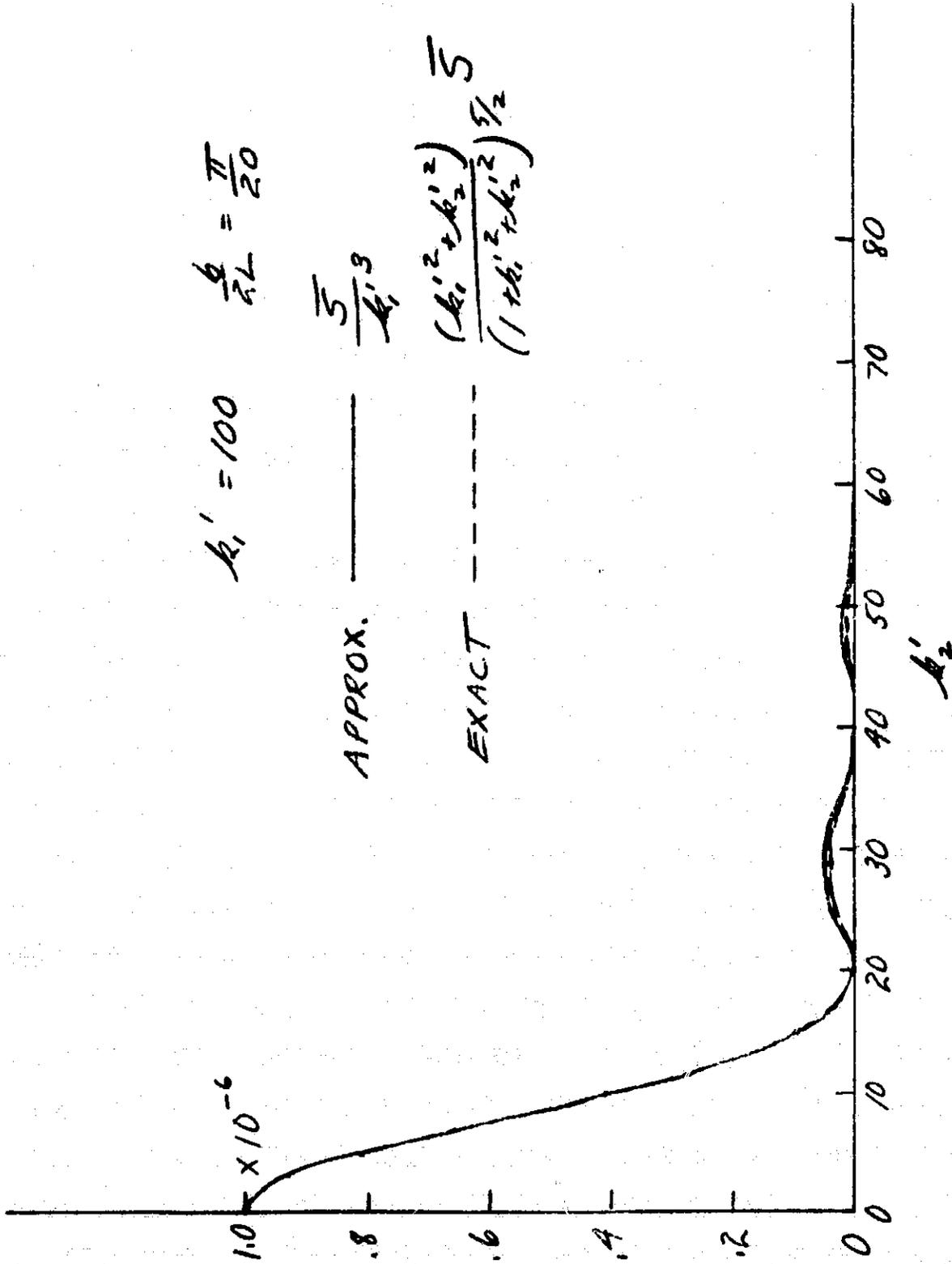


FIGURE 4 EXACT AND APPROXIMATE INTEGRANDS

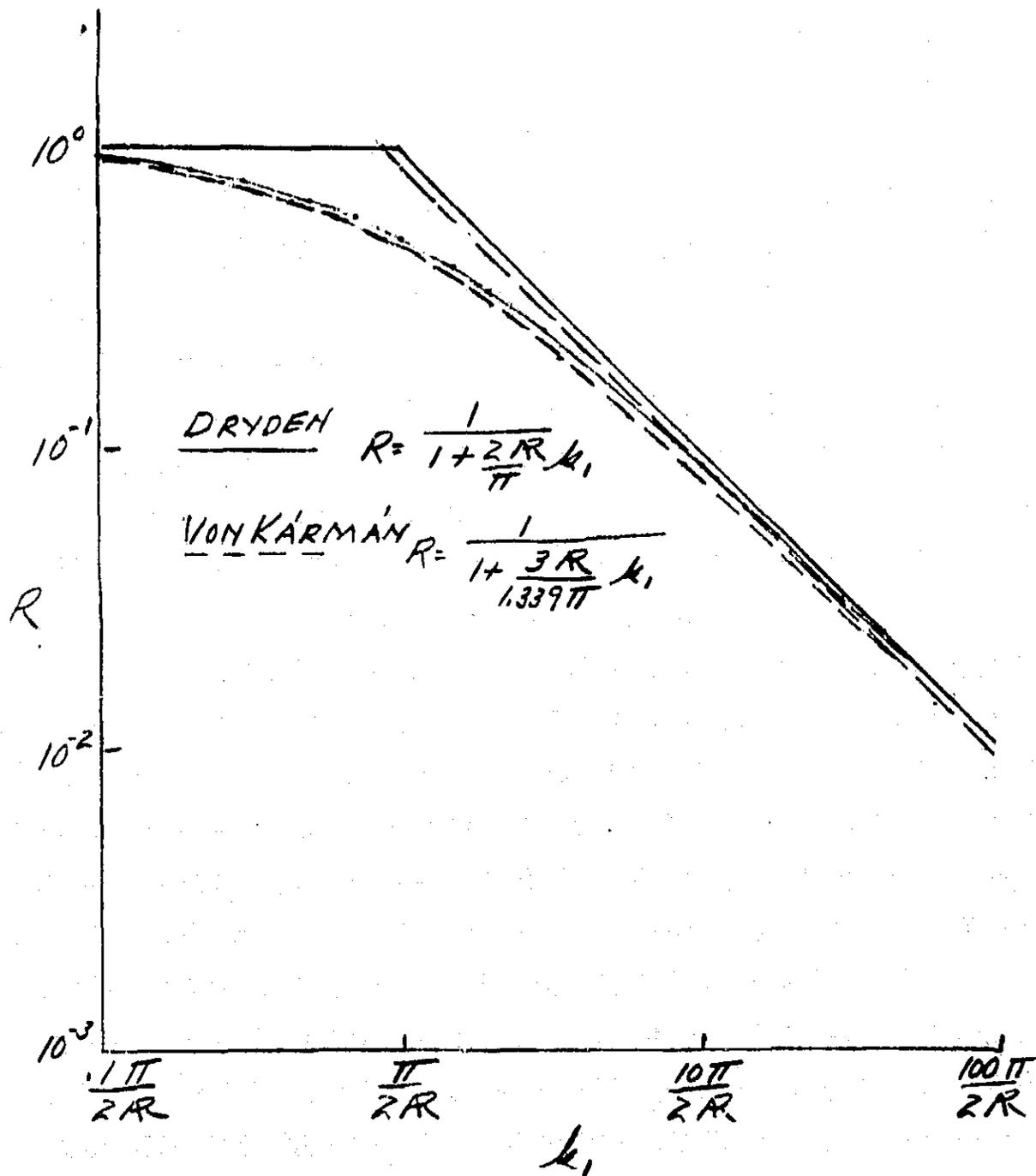


FIGURE 5 TWO DIMENSIONAL
 WEIGHTING FUNCTIONS

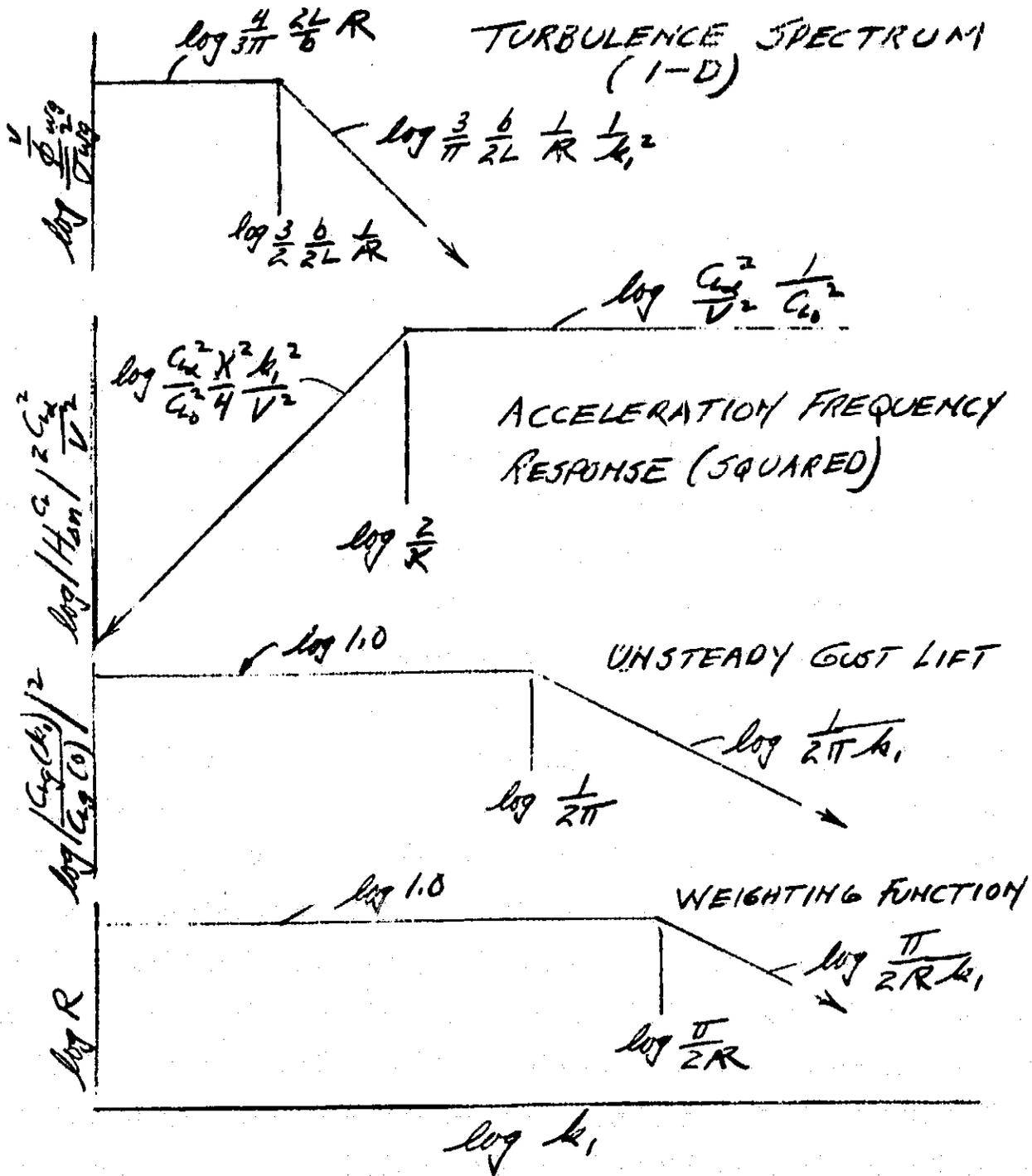


FIGURE 6 FUNCTIONS OF k_1

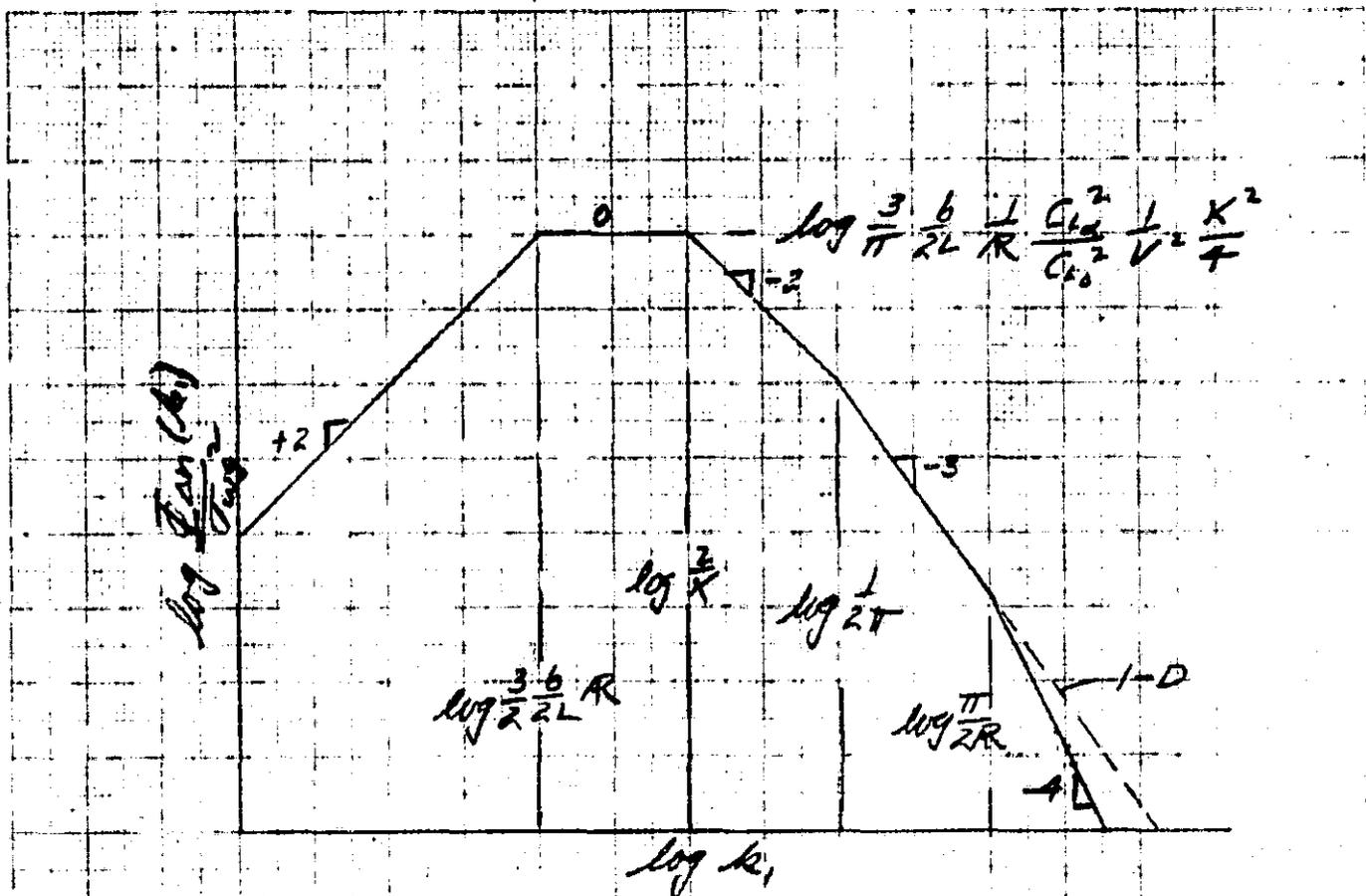


FIGURE 7 ACCELERATION SPECTRUM

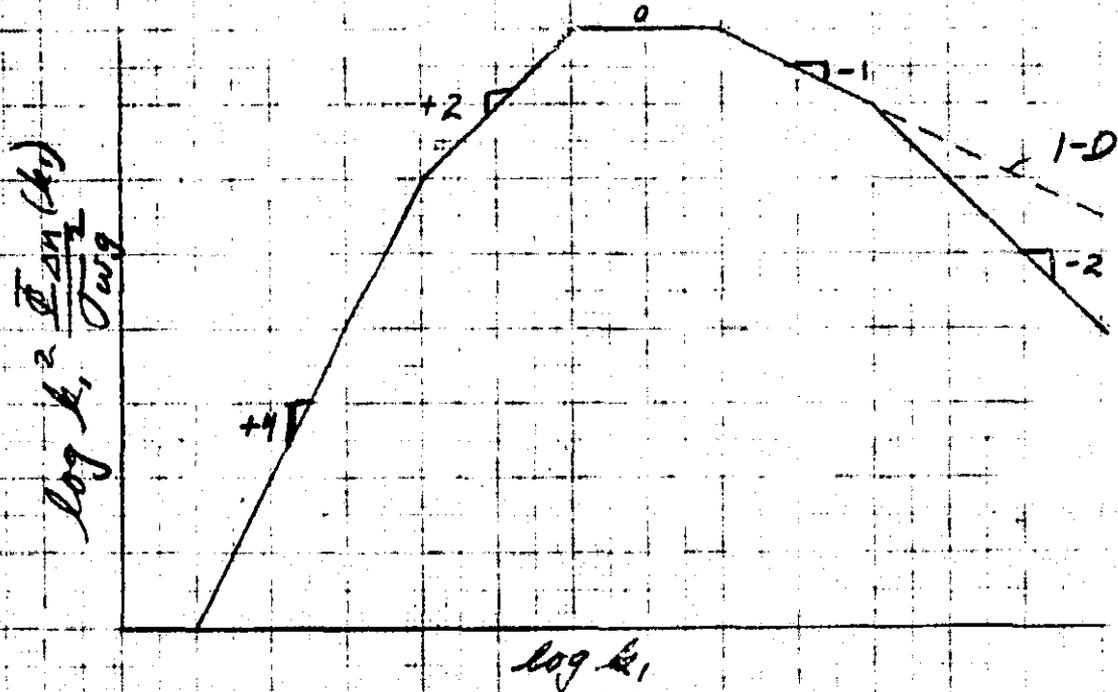


FIGURE 8 SECOND MOMENT OF ACCELERATION SPECTRUM

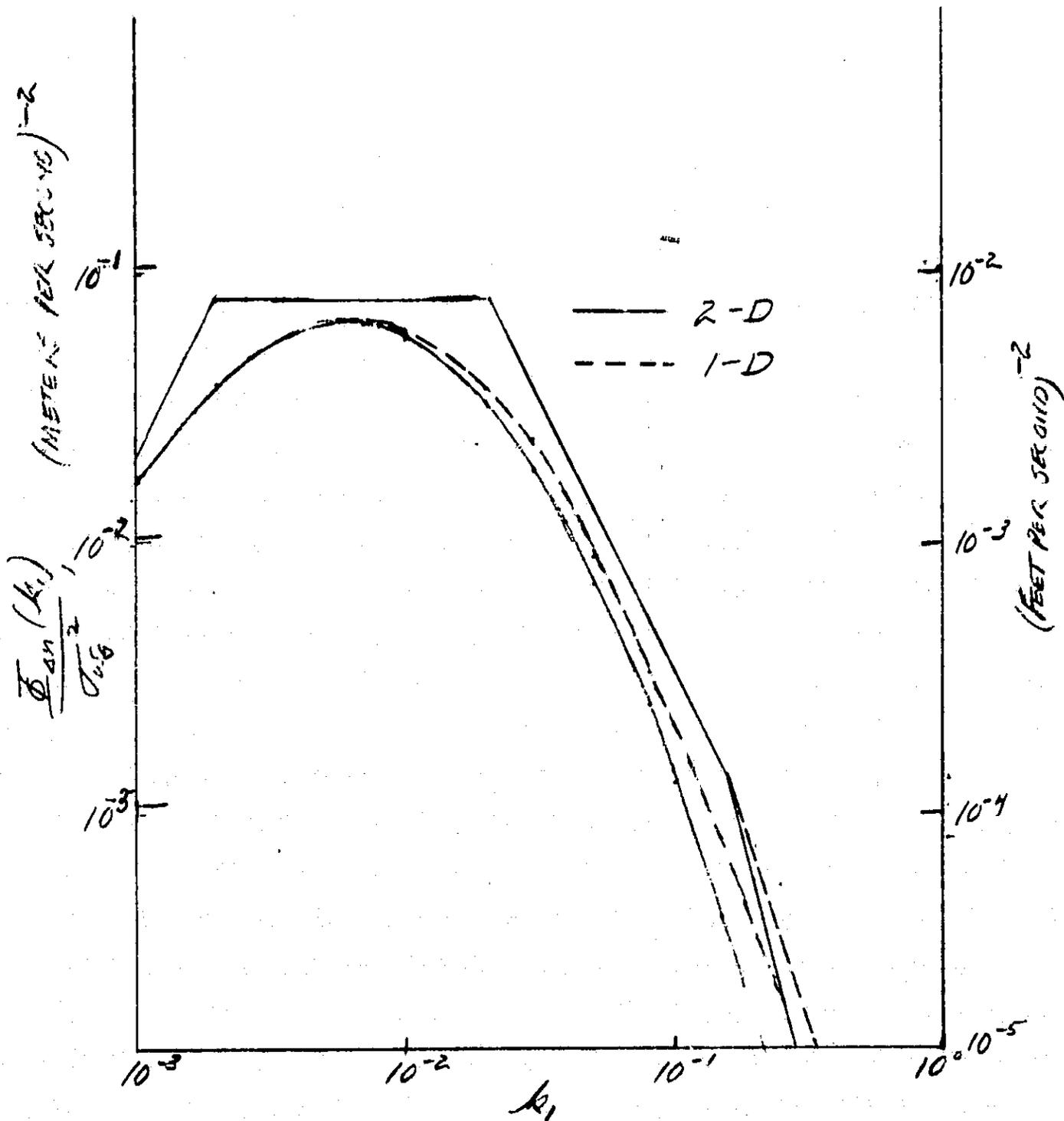


FIGURE 9

NORMAL ACCELERATION SPECTRA
 FOR EXAMPLE AIRPLANE

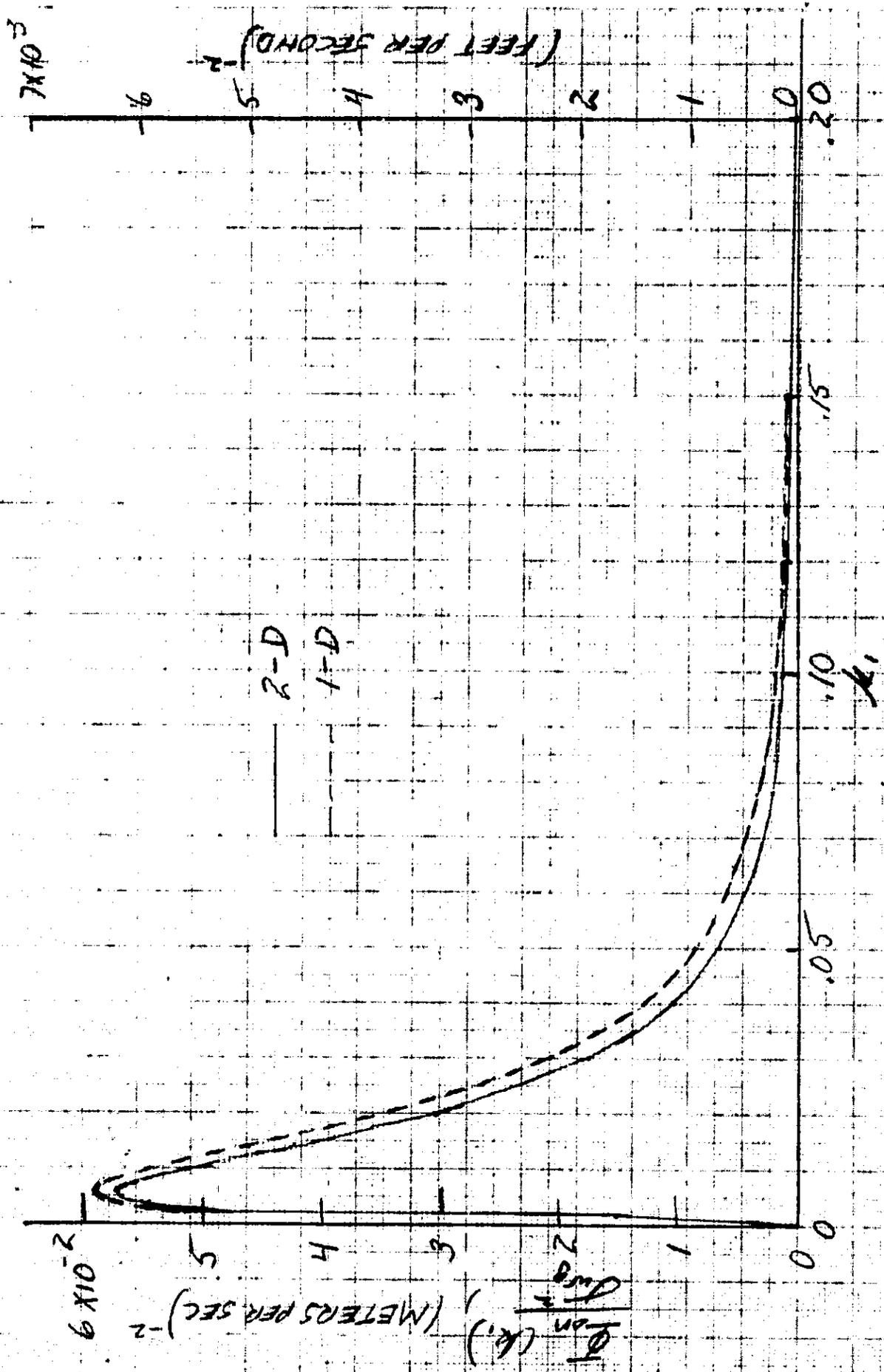


FIGURE 10 NORMAL ACCELERATION SPECTRA FOR EXAMPLE AIRPLANE

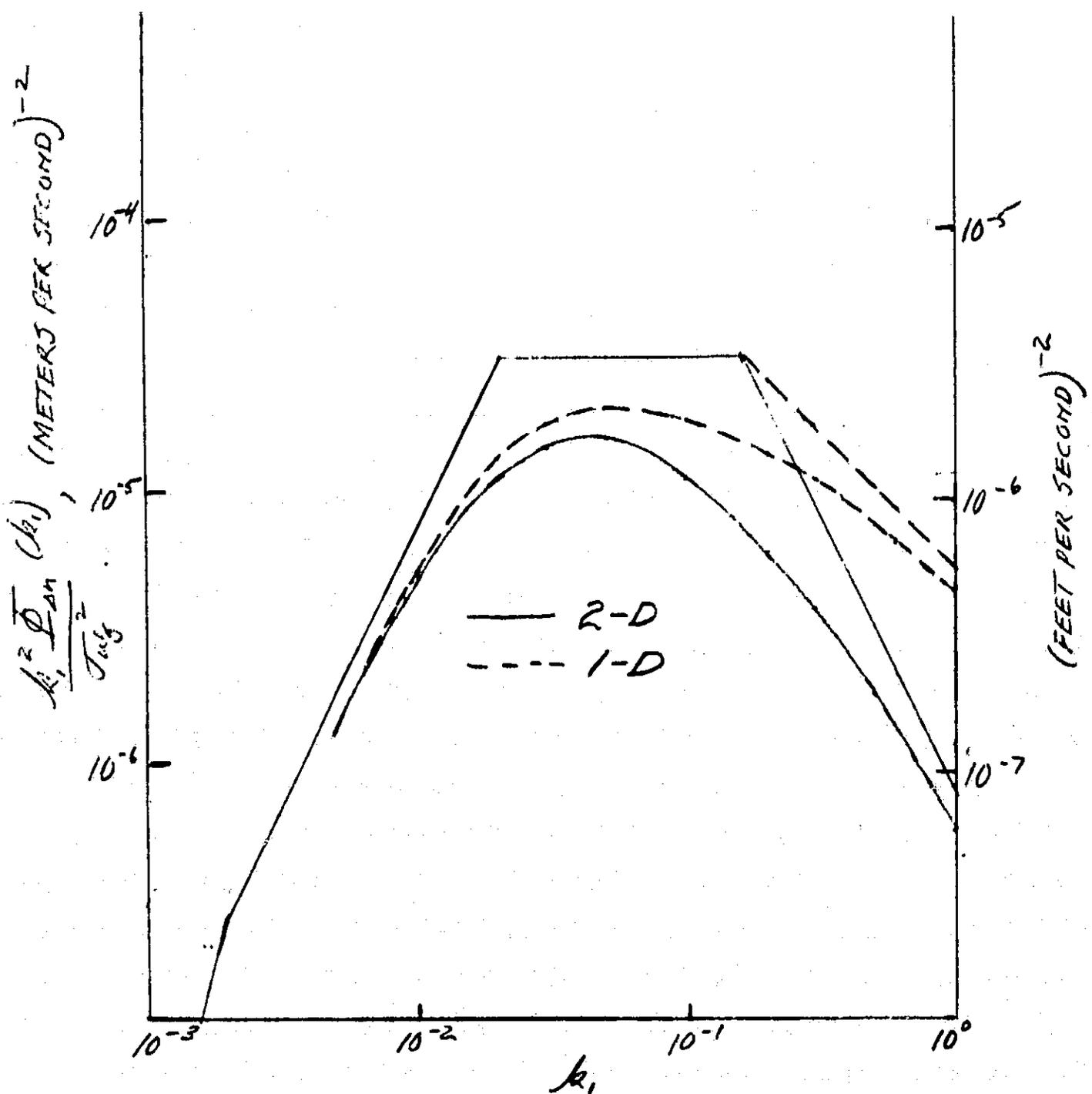


FIGURE 11
 SECOND MOMENT OF NORMAL
 ACCELERATION SPECTRA FOR
 EXAMPLE AIRPLANE

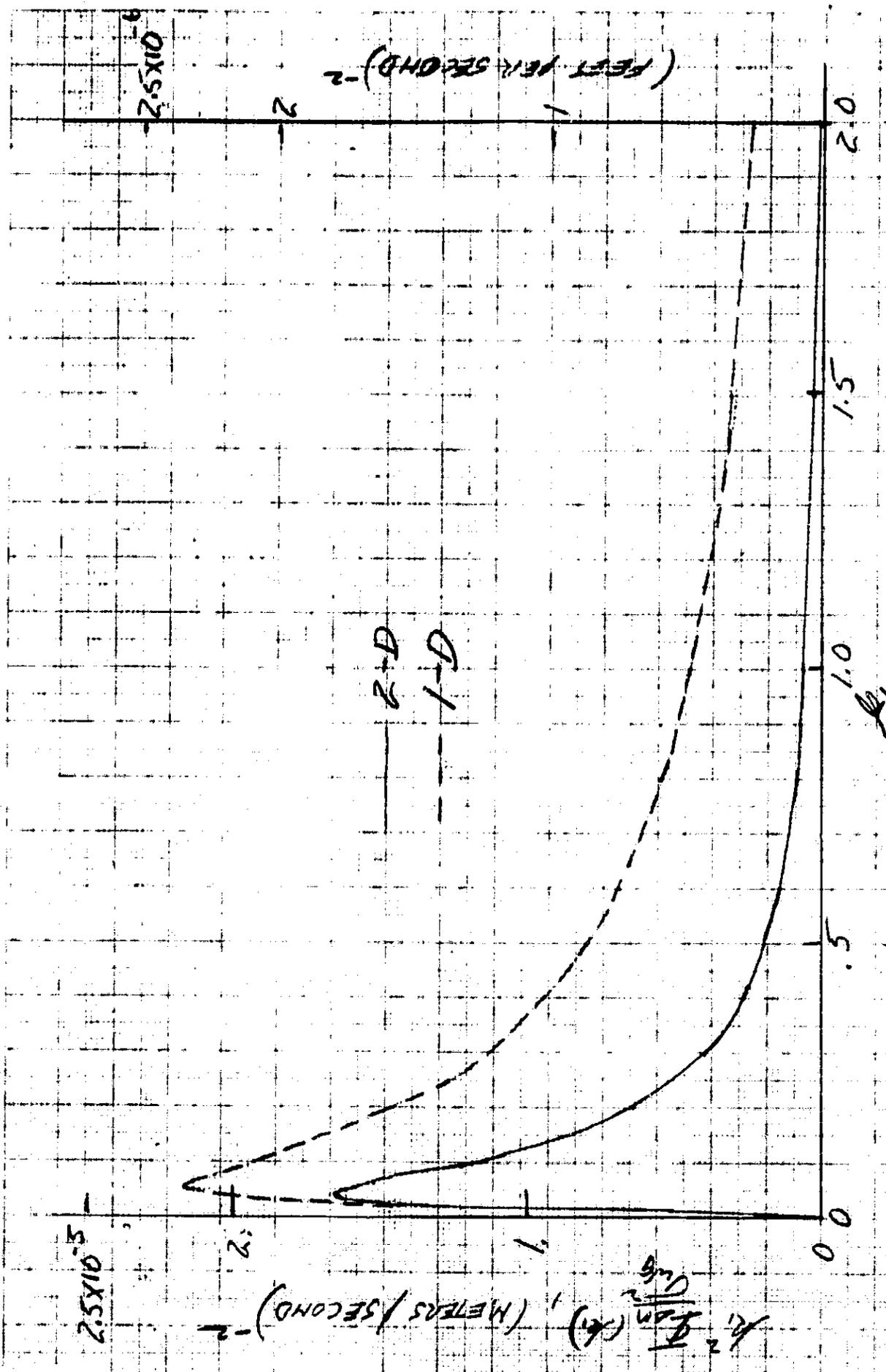


FIGURE 12 SECOND MOMENT OF NORMAL ACCELERATION SPECTRA FOR EXAMPLE AIRPLANE

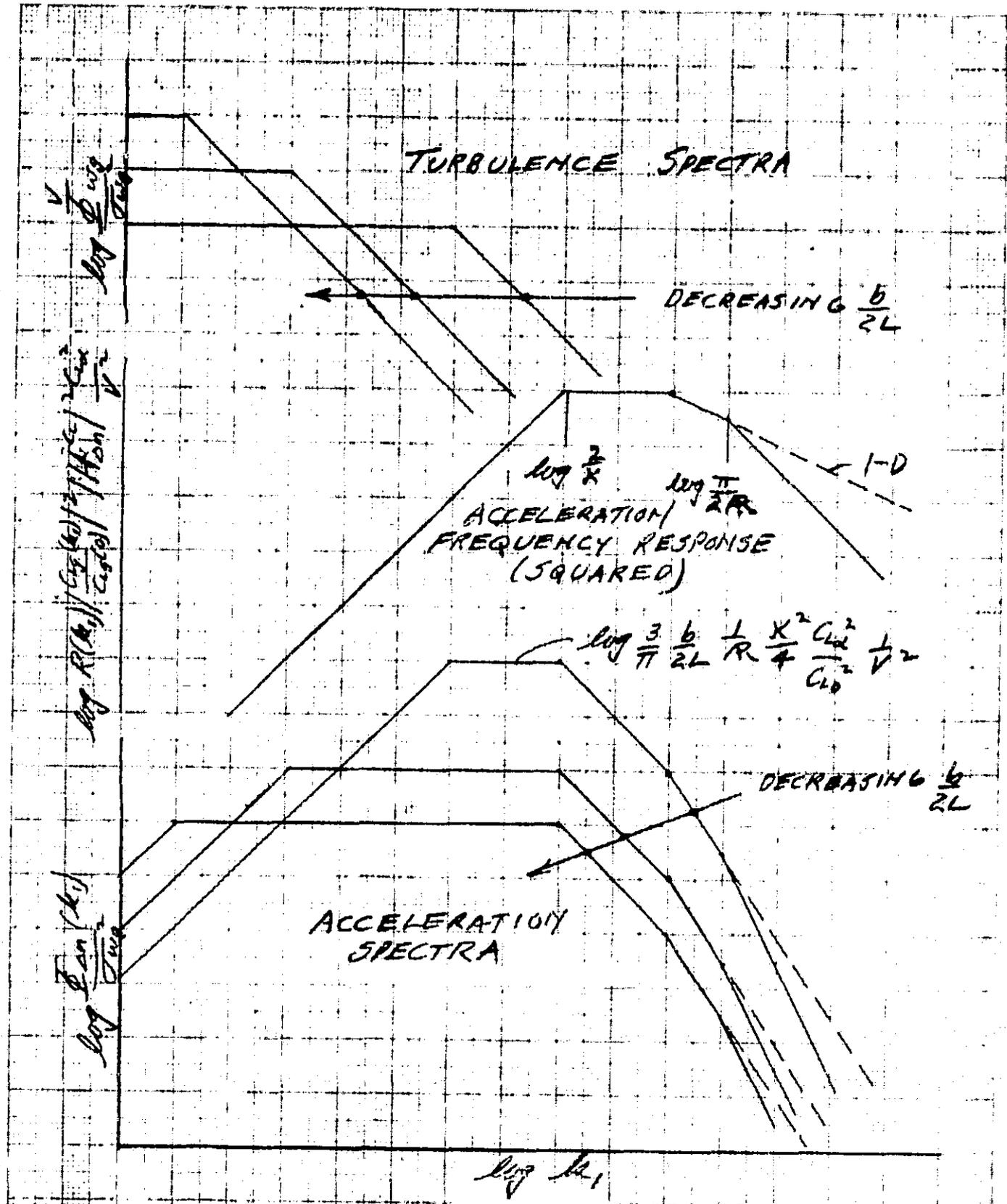


FIGURE 13 RELATION OF TURBULENCE SPECTRA, ACCELERATION FREQUENCY-RESPONSE, & ACCELERATION SPECTRA FOR, DECREASING $\frac{b}{2L}$