THEORETICAL PREDICTION OF AIRPLANE
STABILITY DERIVATIVES AT
SUBCRITICAL SPEEDS

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SUMMARY

This report describes the theoretical development and application of an analysis for predicting the major static and rotary stability derivatives for a complete airplane. The analysis utilizes potential flow theory to compute the surface flow fields and pressures on any configuration that can be synthesized from arbitrary lifting bodies and nonplanar thick lifting panels. The pressures are integrated to obtain section and total configuration loads and moments due side slip, angle of attack, pitching motion, rolling motion, yawing motion, and control surface deflection. Subcritical compressibility is accounted for by means of the Gothert similarity rule.

Within the scope of predicting the total configuration stability derivatives it was necessary to study the problem of computing the spanwise variation of potential form drag due to panel lift and thickness. Included in appendix F is a solution to this problem. Also, in solving the potential form drag problem the work of Woodward and Wagner was thoroughly analyzed. A complete derivation of Woodard's influence equations is given in appendix D and a comprehensive review of Wagner's lifting surface theory is included in appendix E.
INTRODUCTION

In order to develop a procedure for predicting total configuration stability derivatives, the perturbation flow due to the vehicle had to be represented by a grid of efficient aerodynamic finite elements general enough to satisfy the boundary conditions over the surfaces of diverse shapes and also produce the correct resultant forces and moments. The quadrilateral vortex was selected to represent the perturbation velocity due to the bodies because of its numerical efficiency, net force producing capability, limited range of influence, and relationship to the horseshoe vortex which has demonstrated amazing accuracy in predicting loads on wings of arbitrary shape.

The nonplanar thick lifting panels are divided into two sections, (1) the outboard section which is defined by a locus of chord lines, and (2) the root section which is a transition region from the outboard section to the juncture of the panel and a body. If the panel is attached to another panel there is no root section. The perturbation velocity due to panel lift is represented by quadrilateral vortices in the root section and skewed horseshoe vortices in the outboard section. The perturbation velocity due to panel thickness is represented by a source lattice. The panel aerodynamic finite elements were selected because of their numerical efficiency and proven accuracy in predicting flow fields over wings of general shape.

The panel singularities are placed on a mean surface instead of the actual external surface of the panel to maintain computing efficiency. This will sacrifice surface pressure accuracy at the juncture between two panels or a panel and a body, but for this type of general analysis the savings in computer time makes the compromise practical. Second order corrections to account for the interference between lift and thickness and to account for blunt leading edge airfoil sections are included.

The source and vortex lattice influence equations are formulated in terms of the same quantities, which allows the perturbation velocity due to lift and thickness to be computed simultaneously and thereby save computing effort. Also, due to the limited range of significant influence of the quadrilateral vortex the influence of any quadrilateral vortex is only computed at those points within a given area of influence. This can save considerable computing time in developing the aerodynamic influence matrix.

Computer time is also saved by reducing the number of unknowns by transforming the aerodynamic influence matrix by constraint matrices. The constraint matrices constrain the body and panel vorticity, thereby reducing the number of unknowns from that of the number of vortex elements to the number of constraint functions. This is an option in the program and can be
applied in just the longitudinal direction, lateral direction, both directions, or not at all.

The pressures are integrated by numerical means and the panel section drag is computed by means of the Kutta-Joukowsky theorem. The total configuration induced drag is computed by means of a Trefftz plane analysis.
LIST OF SYMBOLS

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<th>Symbol</th>
<th>Description</th>
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<tr>
<td>AR</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>A„</td>
<td>reference area</td>
</tr>
<tr>
<td>a{x}, a{y}, a{z}</td>
<td>aerodynamic influence matrix components</td>
</tr>
<tr>
<td>a_{R_j}</td>
<td>body constraint function coefficient</td>
</tr>
<tr>
<td>a_{P_k}</td>
<td>panel constraint function coefficient</td>
</tr>
<tr>
<td>b</td>
<td>panel span</td>
</tr>
<tr>
<td>c</td>
<td>chord or aerodynamic coefficient</td>
</tr>
<tr>
<td>\bar{c}</td>
<td>reference chord</td>
</tr>
<tr>
<td>C_S</td>
<td>chord line between ((X_J, Y_J, Z_J)) and ((X_R, Y_R, Z_R))</td>
</tr>
<tr>
<td>C_f</td>
<td>trailing edge flap chord</td>
</tr>
<tr>
<td>C_k</td>
<td>leading edge flap chord</td>
</tr>
<tr>
<td>C_P</td>
<td>pressure coefficient</td>
</tr>
<tr>
<td>[M]</td>
<td>two dimensional influence matrix</td>
</tr>
<tr>
<td>\hat{h}</td>
<td>unit vector tangent to control surface hinge line</td>
</tr>
<tr>
<td>h</td>
<td>body height</td>
</tr>
<tr>
<td>\hat{i}, \hat{j}, \hat{k}</td>
<td>unit vectors in the x, y, and z directions, respectively</td>
</tr>
<tr>
<td>K</td>
<td>quadrilateral vortex strength</td>
</tr>
<tr>
<td>M_{\infty}</td>
<td>reference mach number</td>
</tr>
<tr>
<td>\vec{N}</td>
<td>equivalent incompressible surface normal unit vector</td>
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\[ \mathbf{[N_x]}, \mathbf{[N_y]}, \mathbf{[N_z]} \] equivalent incompressible surface normal unit vector component matrices

\[ \mathbf{[N_z]}, \mathbf{[N_y]}, \mathbf{[N_z]} \] actual surface normal unit vector component matrices

\[ \mathbf{\vec{N}_p} \] panel surface normal unit vector

\[ p \] roll rate

\[ q \] pitch rate

\[ r \] yaw rate

\[ R_{c_x}, R_{c_y}, R_{c_z} \] subarea centroid position vector components

\[ R_B \] body radius

\[ \mathbf{[R]} \] transformation matrix between quadrilateral and horseshoe vortex strengths

\[ \mathbf{\vec{R}} \] general position vector

\[ S_B \] body circumferential distance

\[ \mathbf{[S_x]}, \mathbf{[S_y]}, \mathbf{[S_z]} \] source influence martix components

\[ \mathbf{\vec{T}_M} \] equivalent incompressible surface longitudinal tangent unit vector

\[ \mathbf{[\vec{T}_M_x]}, \mathbf{[\vec{T}_M_y]}, \mathbf{[\vec{T}_M_z]} \] equivalent incompressible surface longitudinal tangent unit vector component matrices

\[ \mathbf{\vec{T}_T} \] equivalent incompressible surface lateral tangent unit vector

\[ \mathbf{[\vec{T}_T_x]}, \mathbf{[\vec{T}_T_y]}, \mathbf{[\vec{T}_T_z]} \] equivalent incompressible surface lateral tangent unit vector component matrices

\[ \mathbf{\vec{T}_P} \] panel surface lateral tangent unit vector

\[ \mathbf{[T_{B_i}]} \] body aerodynamic constraint transformation matrix
panel aerodynamic constraint transformation matrix

\[
\begin{bmatrix}
T_{P_{X \_K}} \\
T_{M_{X \_K}} \\
T_{M_{Y \_K}} \\
T_{M_{Z \_K}}
\end{bmatrix},
\begin{bmatrix}
T_{P_{X \_K}} \\
T_{M_{X \_K}} \\
T_{M_{Y \_K}} \\
T_{M_{Z \_K}}
\end{bmatrix},
\begin{bmatrix}
T_{P_{X \_K}} \\
T_{M_{X \_K}} \\
T_{M_{Y \_K}} \\
T_{M_{Z \_K}}
\end{bmatrix}
\]

actual surface longitudinal tangent unit vector component matrices

actual surface lateral tangent unit vector component matrices

U total velocity in x direction

V total velocity in y direction

\(V_x, V_y, V_z\) onset flow components

\(V_{w, o}\) reference velocity

W total velocity in z direction

w body width

X, Y, Z global coordinates

\(X_B, Y_B, Z_B\) body coordinates

\(X_P, Y_P, Z_P\) panel coordinates

\(X_J, Y_J, Z_J\) points along intersection of panel root section and outboard section

\(X_R, Y_R, Z_R\) points along intersection of panel root section and a body

\(X_{C.G.}, Y_{C.G.}, Z_{C.G.}\) center of gravity position vector components

X_k trailing edge of leading edge control surface

X_f leading edge of trailing edge control surface

X_h hinge line location

\(Y_{P_{M \_K}}\) body multiplication factor in y direction

\(Z_{P_{M \_K}}\) body multiplication factor in z direction
<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>( Z_c )</td>
<td>Perpendicular distance between chord line and mean camber line</td>
</tr>
<tr>
<td>( Z_t )</td>
<td>Airfoil thickness</td>
</tr>
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**GREEK SYMBOLS**

<table>
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<tr>
<td>( \alpha )</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>( \beta )</td>
<td>( \sqrt{1-M_{\infty}^2} ) or angle of yaw</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Vorticity or ratio of specific heats</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Control surface deflection</td>
</tr>
<tr>
<td>( \Gamma )</td>
<td>Horseshoe vortex strength</td>
</tr>
<tr>
<td>( \Delta A_x, \Delta A_y, \Delta A_z )</td>
<td>Directed subareas</td>
</tr>
<tr>
<td>( \Delta Y_B )</td>
<td>Displacement of body cross-section in ( y ) direction</td>
</tr>
<tr>
<td>( \Delta Z_B )</td>
<td>Displacement of body cross-section in ( z ) direction</td>
</tr>
<tr>
<td>( \vec{\Delta S} )</td>
<td>Incremental vector tangent to constant percent chord line</td>
</tr>
<tr>
<td>( \Delta Y )</td>
<td>Panel surface increment in ( y ) direction</td>
</tr>
<tr>
<td>( \Delta Z )</td>
<td>Panel surface increment in ( z ) direction</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Panel twist</td>
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<tr>
<td>( \eta )</td>
<td>Fraction of lateral or circumferential distance</td>
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<td>( \theta_B )</td>
<td>Body polar coordinate</td>
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<tr>
<td>( \theta )</td>
<td>General lateral polar coordinate</td>
</tr>
<tr>
<td>( \Lambda )</td>
<td>Sweep</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Density</td>
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source strength

general longitudinal polar coordinate

SUBSCRIPTS

AVG.
average

B
body

C
camber

G, C, y, C z
centroid components

C.P.
center of pressure

center of gravity

edge

trailing edge control surface or final

hinge line

i
summation index or induced

summation index

J
root section and outboard panel juncture

k
summation index

l
lower surface or lift

leading edge

m
longitudinal direction

maximum

M
longitudinal subpanel number

n
lateral subpanel number

O
origin
P  panel
r  summation index
R  panel-body juncture
S  lateral or circumferential direction
T  lateral direction
t  airfoil thickness
T.E.  trailing edge
u  upper surface or number of longitudinal constraint function
w  number of lateral constraint function
X, Y, Z  directions
η  lateral or circumferential direction
∞  infinity
THEORETICAL DEVELOPMENT

Configuration Representation

The theory discussed in this report is capable of predicting the surface pressures and integrated loads on any configuration which can be synthesized from lifting bodies and thick lifting panels of arbitrary shape. These two basic elements can be attached along longitudinal subpanel edges in order to represent complete airplane configurations of arbitrary shape.

Lifting Body. - The arbitrary lifting body, as shown in figure 1, can be of the solid or flow-through type. The body external surface is divided into a grid, which represents the edges of the body subpanels. The body bound vortex lines are placed at the quarter chord point of the subpanels and the fixed trailing vortex lines along the longitudinal edges of the subpanels. The trailing vortex lines are shed from the aft end of the body and extend in the X direction to infinity unless the free wake option of the program is used, in which case the locations of the trailing vortices aft of the body are determined such that they are force-free. If the body is closed to a point at the aft end, all of the body trailing vortices aft of the body cancel and are, therefore, neglected.

Figure 1. - Arbitrary lifting body.
The body vortex strengths are assumed constant around closed paths defined by the bound vortex lines of two subpanels, adjacent to each other in the longitudinal direction, and that portion of the longitudinal edges of these adjacent panels between the two bound vortex lines. The contribution from the vortex loop, or quadrilateral vortex, is defined as positive when the Biot Savart line integral is taken in the clockwise direction by an observer looking at the external surfaces of the two subpanels. The body subpanels and quadrilateral vortices are shown in figure 1 along with the location of the body control points.

The body control points are located at the three-quarter chord of the body subpanels. At these points the total flow is summed and forced to be a minimum in the direction normal to the body surface. If the body vorticity is not constrained by functions with unknown coefficients, a control point is placed at each subpanel and a discrete solution for the unknown body vortex strengths is obtained. If the body vorticity is constrained, control points are placed as many subpanels as is necessary to obtain a good representation of the body shape and to insure that there is sufficient control of the constraint functions. In this case the unknown coefficients of the constraint functions are determined by the method of least squares.

The body subpanels are further subdivided in the lateral direction so that the vortex grid is mapped to the body surface more accurately. This also allows the use of only one quadrilateral vortex in the lateral direction for the case of a body of revolution in a uniform flow at zero angle of attack. In this case the two side edges of the subpanel are coincident, and therefore, the fixed trailing vortices cancel leaving a longitudinal distribution of ring vortices located at the quarter chord of each subpanel.

Figure 2. Body subpanel defined by points 1, 4, 5, and 8 with subdivision points 2, 3, 6, and 7.
The coordinates of the body subpanel subdivisions, as shown in figure 2, are used to compute unit vectors tangent to the body surface in both the longitudinal and lateral directions, unit vectors normal to the body surface, directed subareas to be used in the integration of the body surface pressures, and the centroids of the subareas to compute moments about the configuration center of gravity.

The unit vector tangent to the equivalent incompressible body subpanel in the longitudinal direction is given by,

\[ \vec{T}_M = \vec{T}_{M_X} \hat{i} + \vec{T}_{M_Y} \hat{j} + \vec{T}_{M_Z} \hat{k} \]  

\[ \vec{T}_{M_X} = \frac{X_7 - X_2 + X_6 - X_3}{\sqrt{(X_7 - X_2)^2 + \beta^2 (Y_7 - Y_2)^2 + \beta^2 (Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta^2 (Y_6 - Y_3)^2 + \beta^2 (Z_6 - Z_3)^2}} \]  

\[ \vec{T}_{M_Y} = \frac{\beta (Y_7 - Y_2 + Y_6 - Y_3)}{\sqrt{(X_7 - X_2)^2 + \beta^2 (Y_7 - Y_2)^2 + \beta^2 (Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta^2 (Y_6 - Y_3)^2 + \beta^2 (Z_6 - Z_3)^2}} \]  

\[ \vec{T}_{M_Z} = \frac{\beta (Z_7 - Z_2 + Z_6 - Z_3)}{\sqrt{(X_7 - X_2)^2 + \beta^2 (Y_7 - Y_2)^2 + \beta^2 (Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta^2 (Y_6 - Y_3)^2 + \beta^2 (Z_6 - Z_3)^2}} \]  

The unit vector tangent to the equivalent incompressible body subpanel in the lateral direction is given by,

\[ \vec{T}_T = \vec{T}_{T_X} \hat{i} + \vec{T}_{T_Y} \hat{j} + \vec{T}_{T_Z} \hat{k} \]  

\[ \vec{T}_{T_X} = \frac{X_3 - X_2 + X_6 - X_7}{\sqrt{(X_3 - X_2)^2 + \beta^2 (Y_3 - Y_2)^2 + \beta^2 (Z_3 - Z_2)^2 + (X_6 - X_7)^2 + \beta^2 (Y_6 - Y_7)^2 + \beta^2 (Z_6 - Z_7)^2}} \]
The unit vector normal to the equivalent incompressible body subpanel is given by,

\[
\vec{N} = \vec{N}_X \hat{i} + \vec{N}_Y \hat{j} + \vec{N}_Z \hat{k}
\]  
(9)

\[
\vec{N}_X = \frac{\overline{T}_{MY} \overline{T}_{TZ} - \overline{T}_{MZ} \overline{T}_{TY}}{\sqrt{(\overline{T}_{MY} \overline{T}_{TZ} - \overline{T}_{MZ} \overline{T}_{TY})^2 + (\overline{T}_{TX} \overline{T}_{MZ} - \overline{T}_{TZ} \overline{T}_{MX})^2 + (\overline{T}_{MX} \overline{T}_{TY} - \overline{T}_{MY} \overline{T}_{TX})^2}}
\]  
(10)

\[
\vec{N}_Y = \frac{\overline{T}_{TX} \overline{T}_{MZ} - \overline{T}_{TZ} \overline{T}_{MX}}{\sqrt{(\overline{T}_{MY} \overline{T}_{TZ} - \overline{T}_{MZ} \overline{T}_{TY})^2 + (\overline{T}_{TX} \overline{T}_{MZ} - \overline{T}_{TZ} \overline{T}_{MX})^2 + (\overline{T}_{MX} \overline{T}_{TY} - \overline{T}_{MY} \overline{T}_{TX})^2}}
\]  
(11)

\[
\vec{N}_Z = \frac{\overline{T}_{MX} \overline{T}_{TY} - \overline{T}_{MY} \overline{T}_{TX}}{\sqrt{(\overline{T}_{MY} \overline{T}_{TZ} - \overline{T}_{MZ} \overline{T}_{TY})^2 + (\overline{T}_{TX} \overline{T}_{MZ} - \overline{T}_{TZ} \overline{T}_{MX})^2 + (\overline{T}_{MX} \overline{T}_{TY} - \overline{T}_{MY} \overline{T}_{TX})^2}}
\]  
(12)
The directed subareas for the middle lateral subdivision for the actual body are given by:

\[
\Delta A_X = \frac{1}{2} \left[ (Y_6 - Y_2) (Z_3 - Z_7) - (Z_6 - Z_2) (Y_3 - Y_7) \right] 
\]  

\[
\Delta A_Y = \frac{1}{2} \left[ (X_3 - X_7) (Z_6 - Z_2) - (Z_3 - Z_7) (X_6 - X_2) \right] 
\]  

\[
\Delta A_Z = \frac{1}{2} \left[ (X_6 - X_2) (Y_3 - Y_7) - (Y_6 - Y_2) (X_3 - X_7) \right] 
\]

The components of the position vector to the centroid of this subdivision are given by,

\[
R_{C_X} = \frac{1}{4} (X_2 + X_3 + X_6 + X_7) 
\]

\[
R_{C_Y} = \frac{1}{4} (Y_2 + Y_3 + Y_6 + Y_7) 
\]

\[
R_{C_Z} = \frac{1}{4} (Z_2 + Z_3 + Z_6 + Z_7) 
\]

Analogous directed subarea and centroidal position vector expressions are obtained for the other lateral subdivisions.

The body description is defined independent of the subpanel grid and can be input to the program in a number of different ways. Both cartesian and polar coordinates can be used to describe the body cross-sections at a set of chord stations. After the basic cross-sections have been described they can be scaled and then translated in planes perpendicular to the body chord or mean camber line. This provides a means of inputting the correct side and top views.
of an arbitrary body with a minimum of input data to obtain a preliminary estimate of the loads on the body. It also facilitates in inputting the exact shape of some bodies which can be represented by longitudinal segments, over which the cross-sections are mathematically similar. The actual arbitrary shape can be input directly without the use of the scaling and translation options if it is more convenient.

If polar coordinates are used to describe the body cross-sections, the lateral location of body subpanel side edges or fixed trailing vortex lines are also given in terms of angles measured from the local section ZB axis, in a plane parallel to the (YB-ZB) plane, at an independent set of chordwise stations. These subpanel lateral edges are either specified at a given set of angles or defined to be at equally spaced angle locations. The subpanel longitudinal edges are specified at given XB stations, evenly spaced in terms of XB, or evenly spaced in terms of \( \phi_B \), where \( \phi_B = \cos^{-1} \left( 1 - 2 \frac{X_B}{C_B} \right) \).

The following procedure is used to determine the coordinates \( X_{BE}, \frac{(Y_{BE} - \Delta Y_{BE})}{(Y_M)}, \frac{(Z_{BE} - \Delta Z_{BE})}{(Z_M)} \) of the subpanel corners.

1. \( R_{Bi} \) versus \( \theta_{Bi} \) is input at \( X_{Bi} \).
2. \( \theta_{BE} \) (subpanel lateral edge location) versus \( X_B \) is given as input.
3. Interpolation on \( \theta_{BE} \) versus \( X_B \) is done to obtain \( \theta_{BE} \) versus \( X_{BE} \) (subpanel longitudinal edge location) and \( X_{Bi} \).
4. Interpolation on \( R_{Bi} \) versus \( \theta_{Bi} \) is done at each \( X_{Bi} \) to obtain \( R_{BE} \) versus \( X_{BE} \).
5. Interpolation on \( R \) versus \( \theta_{BE} \) is done to obtain \( R_{BE} \) versus \( X_{BE} \) at each \( \theta_{BE} \).
6. \( X_{BE}, \frac{(Y_{BE} - \Delta Y_{BE})}{(Y_M)}, \frac{(Z_{BE} - \Delta Z_{BE})}{(Z_M)} \) are computed from \( X_{BE}, R_{BE} \), and \( \theta_{BE} \).

If the subpanels are subdivided the coordinates for the corners of these subdivision are computed using the same procedure. There is always an odd number of subdivisions in both the longitudinal and lateral directions.
If cartesian coordinates are used to describe the body cross-sections, the lateral location of the subpanel side edges are defined by the percent circumferential length \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{MAX_i}}} \) at independent chordwise stations.

Tables of \( \left( Y_{B_i} - \Delta Y_{B_i} \right) / Y_{B_i} \) and \( \left( Z_{B_i} - \Delta Z_{B_i} \right) / Z_{B_i} \) versus percent circumferential length \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{MAX_i}}} \) are developed at the input longitudinal stations \( X_{B_i} \). The same procedure that was used to obtain \( X_{BE} \), \( \left( Y_{BE} - \Delta Y_{BE} \right) / Y_{BM} \) and \( \left( Z_{BE} - \Delta Z_{BE} \right) / Z_{BM} \), when the body was defined by polar coordinates is also used for this case, except that \( B_{Bin} \) is replaced by \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{MAX_i}}} \). Also, step 6 is unnecessary. The procedure is cycled through twice, first for \( \left( Y_{BE} - \Delta Y_{BE} \right) / Y_{BM} \) and then for \( \left( Z_{BE} - \Delta Z_{BE} \right) / Z_{BM} \).

After \( X_{BE} \), \( \left( Y_{BE} - \Delta Y_{BE} \right) / Y_{BM} \) and \( \left( Z_{BE} - \Delta Z_{BE} \right) / Z_{BM} \) have been computed, \( Y_{BE} \) is determined by multiplying \( \left( Y_{BE} - Y_{BM} \right) / Y_{BM} \) by the multiplication factor \( Y_{BM} \) and then adding the translation increment \( \Delta Y_{BE} \). \( Z_{BE} \) is determined by multiplying \( \left( Z_{BE} - Z_{BM} \right) / Z_{BM} \) by the multiplication factor \( Z_{BM} \) and then adding the translation increment \( \Delta Z_{BE} \).

Thick lifting panel. - The thick lifting panel, as shown in figure 3, can be warped in any manner laterally, have an arbitrary distribution of chord length, thickness, twist, and camber. It can be used to represent a wing, canard, fin, pylon, horizontal or vertical tail, or be wrapped around to represent a flow through nacelle. The panels can be attached to other panels, such as in the case of a pylon on a wing, or attached to bodies. The panels can have plain leading or trailing edge flaps, ailerons, rudders, or elevators. These control surfaces can be of the full or partial span type and their hinge lines are not restricted to constant percent chord lines.

The panel is divided into two sections: (1) the outboard section where the subpanel longitudinal edges are assumed to be straight lines in the \( X \) direction, and (2) the root section which is a transition region from the outboard section to the intersection of the panel and a body. The line of intersection between the panel and a body does not have to be a straight line. Therefore the subpanel longitudinal edge lines will change in shape from that of the line of intersection at the side of the body to a straight line in the \( X \) direction at the outboard section. If the panel is not attached to a body, there is no root section.
Figure 3.- Thick lifting panel.

The root section and outboard section can have any distribution of subpanel lengths and widths. The subpanel lateral edges are specified as a list of percent chord stations, evenly spaced in terms of percent chord, or evenly spaced in terms of $\phi$, where

$$\phi = \cos^{-1} \left[ 1 - 2 \left( \frac{X - X_{LE}}{C} \right) \right].$$

The subpanel side edges are specified as a list of percent of surface semi-span $\eta$, evenly spaced in terms of percent of surface semi-span, or evenly spaced in terms of $\theta$, where $\theta = \cos^{-1} \eta$. 
The percent of surface semi-span \( \eta \) is defined as the percent of length of the line projected into the (y-Z) plane by the panel leading edge. A constant \( \eta \) line is in general curved in the root section. In the outboard section, lines of constant \( \eta \) are straight and in the X direction. The curved constant \( \eta \) lines associated with the longitudinal edges of subpanels in the root section are computed such that the sweep of the leading and trailing edges of the subpanels vary linearly from that of the sweep of the panel at the leading edge to that of the sweep of the panel at the trailing edge. Also, the corner points of the subpanels are equally spaced in the lateral direction along lines of constant percent chord. The chord at a lateral station in the root section is the length of the curved constant \( \eta \) line at that station.

If the lateral distance along a constant percent chord line between two corner points is defined by \( |\Delta S| \), the unit vector tangent to the leading or trailing edge of a subpanel at any percent chord station is given by;

\[
\left( \frac{\Delta S}{|\Delta S|} \right)_K = \frac{\Delta S_{L,E,K}}{|\Delta S_{L,E,K}|} + \left( \frac{\Delta S_{T,E,K}}{|\Delta S_{T,E,K}|} - \frac{\Delta S_{L,E,K}}{|\Delta S_{L,E,K}|} \right) \text{(percent chord)} \tag{19}
\]

where

\[
|\Delta S_K| = \frac{c_s^2}{\sum_{K=1}^{K_j} \left( \frac{\Delta S}{|\Delta S|} \right)_K \cdot c_s} \tag{20}
\]

*ORIGINAL PAGE IS OF POOR QUALITY*
and \( K_J \) equals the number of subpanels in the lateral direction in the root section.

\[
\vec{c}_S = (X_J - X_R) \hat{i} + (Y_J - Y_R) \hat{j} + (Z_J - Z_R) \hat{k}
\]  

(21)

where \((X_R, Y_R, Z_R)\) and \((X_J, Y_J, Z_J)\) are the points on the curved constant percent chord line at the line of intersection of the panel and the body and at the juncture of the root and outboard sections, respectively. The equally spaced corner points along a constant percent chord line are then computed by

\[
R (X, Y, Z) \eta = R (X_R, Y_R, Z_R) + \sum_{K=1}^{K=K_J} \left( \frac{\Delta S}{|\Delta S|} \right)_K \bigg| \Delta S_K \bigg|
\]

(22)

where \( K \eta \) is the number of subpanels, in the lateral direction, the constant \( \eta \) line is located from the line of intersection of the panel and the body. Since constant \( \eta \) lines in the outboard section are straight and in the \( X \) direction the subpanel corner points are defined directly from the input list of subpanel edge locations in terms of percent of chord and lateral \( \eta \) station.

The coordinates of the leading edge point \((X_{L.E.}, Y_{L.E.}, Z_{L.E.})_K\) associated with a constant \( \eta \) line are obtained from the panel perimeter description by converting the leading edge input points to tables of \( X_{L.E.} \) versus \( \eta \), \( Y_{L.E.} \) versus \( \eta \), and \( Z_{L.E.} \) versus \( \eta \), and then interpolating in these tables to determine the leading edge coordinates at a desired lateral \( \eta \) station. Unit vectors tangent and normal to the panel at any \( \eta \) station can also be determined from these tables by evaluating the \( \Delta Y_{L,E.} \) and \( \Delta Z_{L,E.} \) about the lateral station \( \eta \). The unit vectors are then defined by:

\[
\vec{N}_{P,K} = -\frac{\Delta Z_{L,E.}}{\sqrt{\Delta Y_{L,E.}^2 + \Delta Z_{L,E.}^2}} \hat{j} + \frac{\Delta Y_{L,E.}}{\sqrt{\Delta Y_{L,E.}^2 + \Delta Z_{L,E.}^2}} \hat{k}
\]

(23)
With the coordinates of the subpanel corner points known, the coordinates and sweep of the vortex and source lattices used to represent the perturbation velocities due to lift and thickness, respectively, can be defined. The bound vortex lines and control points are placed at the quarter and three-quarter chord points of the subpanels, respectively. The fixed trailing vortices are placed along the subpanel side edges. In the root section the vortex lattice is a quadrilateral system, whereas in the outboard section the vortex lattice is a skewed horseshoe system. The skewed source lines are placed at both the quarter and three-quarter chord points of the subpanels.

The section geometry is input in terms of a percent thickness, percent camber, and twist. Where the percent thickness and camber are based on the local chord and measured in the \( \bar{N}_{PK} \) direction. Twist is defined in the plane described by the unit vectors \( \hat{i} \) and \( \bar{N}_{PK} \). The X component of the unit vector normal to the section mean camber line and in the plane defined by \( \hat{i} \) and \( \bar{N}_{PK} \) is then given by:

\[
\bar{N}_{PK} = \beta \left( -\frac{dz}{dx} + \epsilon + \tan \delta \right)
\]

where \( z_C \) is the perpendicular distance between the section chord line and the mean camber line, \( \epsilon \) is the angle of twist, and \( \delta \) the deflection of any control surface. All three of these quantities are functions of both percent chord and \( \epsilon \). The thickness is defined in the same manner as camber.

The trailing vortices aft of the trailing edge of the outboard section are straight lines in the X direction going off to infinity. The trailing vortices aft of the trailing edge of the root section lie along curved constant \( \eta \) lines to the end of the body and then go off to infinity in the X direction. These curved constant \( \eta \) lines are determined in the same manner as the constant \( \eta \) lines in the root section. If the free wake option of the program is utilized the location of the panel free trailing vortices are iterated for such that the wake is force free.
Discrete Influence Equations

The perturbation velocity due to the arbitrary lifting bodies is represented by quadrilateral vortices on the external surface of the bodies. The perturbation velocity due to lift on the panels is represented by quadrilateral vortices in the root section of the panel and by skewed horseshoe vortices in the outboard section of the panel. The thickness is represented by skewed source lines in both the root and outboard sections of the panel. All the panel singularities are placed on the panel chordal surface. The source strengths $\Sigma$ are defined by the change in thickness over that portion of the subpanel it represents, so that;

$$\frac{\Sigma}{V_\infty} = \frac{2 \beta \Delta Z_t}{\sqrt{1 + (\tan \Lambda)^2/\beta^2}} \frac{\bar{V}_X}{V_\infty}$$

where $\Lambda$ is the sweep of the source line and $\bar{V}_X$ is the total onset velocity in the X direction. The quadrilateral vortex strengths $K$ and the skewed horseshoe vortex strengths $\Gamma$ must be solved for utilizing the boundary condition that a minimum of flow passes through the external surface of the bodies and the chordal surface of the thick lifting panels at a finite number of control points. In order to satisfy this boundary condition the total flow due to all singularities and onset flow is summed at each control point and the scalar product of this sum and the surface unit normal is minimized. This results in a set of linear aerodynamic influence equations which are solved for the unknown vortex strengths by means of Householder's method, described in appendix A.
The influence equations for the $j^{th}$ equivalent incompressible body are given by:

$$
\sum_{i=1}^{N_B} \left[ \begin{array}{c}
-\bar{N}_{X_{B_i}} \\
A_{X_{B_i}B_j}
\end{array} \right] B_j + \left[ \begin{array}{c}
-\bar{N}_{Y_{B_i}} \\
A_{Y_{B_i}B_j}
\end{array} \right] B_j + \left[ \begin{array}{c}
-\bar{N}_{Z_{B_i}} \\
A_{Z_{B_i}B_j}
\end{array} \right] B_j \right] \left\{ \frac{K}{V_\infty} \right\}_{B_i}
$$

$$
+ \sum_{K=1}^{N_P} \left[ \begin{array}{c}
-\bar{N}_{X_{B_j}} \\
A_{X_{B_j}P_K}
\end{array} \right] P_K + \left[ \begin{array}{c}
-\bar{N}_{Y_{B_j}} \\
A_{Y_{B_j}P_K}
\end{array} \right] P_K + \left[ \begin{array}{c}
-\bar{N}_{Z_{B_j}} \\
A_{Z_{B_j}P_K}
\end{array} \right] P_K \right] \left\{ \frac{\sigma}{V_\infty} \right\}_{P_K}
$$

$$
+ \left[ \begin{array}{c}
-\bar{N}_{X_{B_j}} \\
\frac{\bar{V}_X}{V_\infty}
\end{array} \right] + \left[ \begin{array}{c}
-\bar{N}_{Y_{B_j}} \\
\frac{\bar{V}_Y}{V_\infty}
\end{array} \right] + \left[ \begin{array}{c}
-\bar{N}_{Z_{B_j}} \\
\frac{\bar{V}_Z}{V_\infty}
\end{array} \right] = \{ e \}_{B_j}
$$

(27)

where

$\bar{N}_{X_{B_j}}$, $\bar{N}_{Y_{B_j}}$, and $\bar{N}_{Z_{B_j}}$ are the components of the $j^{th}$ equivalent incompressible body surface unit normal vectors,

$A_{X_{B_i}B_j}$, $A_{Y_{B_i}B_j}$, and $A_{Z_{B_i}B_j}$ are the components of the perturbation velocity induced by the $i^{th}$ equivalent incompressible body unit strength vortices onto the $j^{th}$ equivalent incompressible body,

$A_{X_{B_j}P_K}$, $A_{Y_{B_j}P_K}$, and $A_{Z_{B_j}P_K}$ are the components of the perturbation velocity induced by the $k^{th}$ equivalent incompressible panel unit strength vortices onto the $j^{th}$ equivalent incompressible body.
$S_{X_{P_k}j}$, $S_{Y_{P_k}j}$, and $S_{Z_{P_k}j}$ are the components of the perturbation velocity induced by the kth equivalent incompressible panel unit strength sources onto the jth equivalent incompressible body.

Also,

$$\left\{ \frac{\bar{V}_X}{V_{\infty}} \right\}, \left\{ \frac{\bar{V}_Y}{V_{\infty}} \right\}, \text{and} \left\{ \frac{\bar{V}_Z}{V_{\infty}} \right\}$$

are the components of the onset flow divided by the reference velocity $V_{\infty}$,

$\{K\}_{B_i}$ are the quadrilateral vortex strengths on the i$^{th}$ equivalent incompressible body, and

$\{K\}_{P_k}$, $\{\Gamma\}_{P_k}$, and $\{\Sigma\}_{P_k}$ are the root section quadrilateral vortex strengths, the outboard section skewed vortex strengths, and the skewed source strengths on the kth equivalent incompressible panel, respectively. The vector $\{e\}_{B_i}$ is the flow through the surface of the jth equivalent incompressible body at the control points, for a discrete body solution this vector is zero.

The influence equations for the jth equivalent incompressible panel are given by;

$$\sum_{i=1}^{N_B} \left[ \begin{array}{c} \bar{N}_{X_{P_i}j} \\ \bar{N}_{Y_{P_i}j} \\ \bar{N}_{Z_{P_i}j} \end{array} \right] \begin{bmatrix} A_{X_{P_i}jB_i} \\ A_{Y_{P_i}jB_i} \\ A_{Z_{P_i}jB_i} \end{bmatrix} + \sum_{K=1}^{N_P} \left[ \begin{array}{c} \bar{N}_{X_{P_K}j} \\ \bar{N}_{Y_{P_K}j} \\ \bar{N}_{Z_{P_K}j} \end{array} \right] \begin{bmatrix} A_{X_{P_KjP_K}} \\ A_{Y_{P_KjP_K}} \\ A_{Z_{P_KjP_K}} \end{bmatrix} \left( \frac{K}{V_{\infty}} \right)_{B_i}$$

$$+ \sum_{K=1}^{N_P} \left[ \begin{array}{c} \bar{N}_{X_{P_K}j} \\ \bar{N}_{Y_{P_K}j} \\ \bar{N}_{Z_{P_K}j} \end{array} \right] \begin{bmatrix} S_{X_{P_KjP_K}} \\ S_{Y_{P_KjP_K}} \\ S_{Z_{P_KjP_K}} \end{bmatrix} \left( \frac{K}{V_{\infty}} \right)_{P_k}$$

$$+ \left[ \begin{array}{c} \bar{N}_{X_{P_K}j} \\ \bar{N}_{Y_{P_K}j} \\ \bar{N}_{Z_{P_K}j} \end{array} \right] \left\{ \frac{\bar{V}_X}{V_{\infty}} \right\}_{P_k} + \left[ \begin{array}{c} \bar{N}_{Y_{P_K}j} \end{array} \right] \left\{ \frac{\bar{V}_Y}{V_{\infty}} \right\}_{P_k} + \left[ \begin{array}{c} \bar{N}_{Z_{P_K}j} \end{array} \right] \left\{ \frac{\bar{V}_Z}{V_{\infty}} \right\}_{P_k} = \{e\}_{P_j}$$

(28)
Where

\[ N_x, N_y, \text{ and } N_z \] are the components of the \( j \)th equivalent incompressible panel mean camber surface unit normal vectors,

\[ A_{x_j^{-,i}}, A_{y_j^{-,i}}, \text{ and } A_{z_j^{-,i}} \] are the components of the perturbation velocity induced by the \( i \)th equivalent incompressible body unit strength vortices onto the \( j \)th equivalent incompressible panel,

\[ A_{x_j^{-,k}}, A_{y_j^{-,k}}, \text{ and } A_{z_j^{-,k}} \] are the components of the perturbation velocity induced by the \( k \)th equivalent incompressible panel unit strength vortices onto the \( j \)th equivalent incompressible panel, and

\[ S_{x_j^{-,k}}, S_{y_j^{-,k}}, \text{ and } S_{z_j^{-,k}} \] are the components of perturbation velocity induced by the \( k \)th equivalent incompressible panel unit strength sources onto the \( j \)th equivalent incompressible panel. The vector \( \{c\}_{pj} \) is the flow through the mean camber surface of the \( j \)th equivalent incompressible panel at the control points. This vector is also zero for a discrete panel solution.

The onset flow velocity ratios

\[ \frac{V_x}{V_\infty}, \frac{V_y}{V_\infty}, \text{ and } \frac{V_z}{V_\infty} \] are given by the following expressions,

\[ \frac{V_x}{V_\infty} = 1 - q^* \frac{\beta (Z-Z_{C.G.})}{c} - \gamma^* \frac{\beta (Y-Y_{C.G.})}{b} \] (29)

\[ \frac{V_y}{V_\infty} = -\beta \beta - p^* \frac{\beta (Z-Z_{C.G.})}{b} + \gamma^* \frac{2 (X-X_{C.G.})}{b} \] (30)

\[ \frac{V_z}{V_\infty} = \alpha \beta + p^* \frac{2 \beta (Y-Y_{C.G.})}{b} + q^* \frac{2 (X-X_{C.G.})}{c} \] (31)

Where \( P^*, q^*, \) and \( \gamma^* \) are the nondimensional roll, pitch, and yaw rates, respectively. These are defined as;
\[ p^* = p/ \frac{2V}{b}, \quad q^* = q/ \frac{2V}{c}, \quad \text{and} \quad \gamma^* = \gamma/ \frac{2V}{b} \quad (32) \]

The above onset flow equations assume that angle of attack \( \alpha \) and angle of yaw \( \beta \) are small. The coordinates \( X_{C.G.}, Y_{C.G.}, \) and \( Z_{C.G.} \) define the location of the center of gravity.

Equations (27) and (28) can be combined into a single matrix aerodynamic influence equation. For a completely discrete type solution the influence equation is,

\[
\begin{bmatrix}
[A]_{B_i B_j} & [A]_{B_j P_k} \\
[A]_{B_j B_i} & [A]_{B_i P_k}
\end{bmatrix}
\begin{bmatrix}
\{ \frac{K}{V} \}_{B_i} \\
\{ \frac{K}{V} \}_{P_k}
\end{bmatrix}
= \begin{bmatrix}
\{ \nabla S \}_{B_j} \\
\{ \nabla S \}_{P_j}
\end{bmatrix}
\quad (33)
\]

where

\[ B_j = B_1, B_2, \ldots, B_{N_B} \]
\[ B_i = B_1, B_2, \ldots, B_{N_B} \]
\[ P_j = P_1, P_2, \ldots, P_{N_p} \]
\[ P_k = P_1, P_2, \ldots, P_{N_p} \]
\[ N_B = \text{Number of bodies} \]
and
\[ N_p = \text{Number of panels} \]
The matrices

\[
[A]_{B_jB_1}, \ [A]_{B_jP_k}, \ [A]_{P_jP_k}, \text{ and } [A]_{P_jB_1}
\]

are defined as:

\[
[A]_{B_jB_1} = \begin{bmatrix}
\vec{N}_{X_{B_j}} \\
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix}
= \begin{bmatrix}
\vec{A}_{X_{B_jB_1}} \\
\vec{A}_{Y_{B_jB_1}} \\
\vec{A}_{Z_{B_jB_1}}
\end{bmatrix} + \begin{bmatrix}
\vec{S}_{X_{B_jB_1}} \\
\vec{S}_{Y_{B_jB_1}} \\
\vec{S}_{Z_{B_jB_1}}
\end{bmatrix} \tag{34}
\]

\[
[A]_{B_jP_k} = \begin{bmatrix}
\vec{N}_{X_{B_j}} \\
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix}
= \begin{bmatrix}
\vec{A}_{X_{B_jP_k}} \\
\vec{A}_{Y_{B_jP_k}} \\
\vec{A}_{Z_{B_jP_k}}
\end{bmatrix} + \begin{bmatrix}
\vec{S}_{X_{B_jP_k}} \\
\vec{S}_{Y_{B_jP_k}} \\
\vec{S}_{Z_{B_jP_k}}
\end{bmatrix} \tag{35}
\]

\[
[A]_{P_jB_1} = \begin{bmatrix}
\vec{N}_{X_{B_j}} \\
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix}
= \begin{bmatrix}
\vec{A}_{X_{P_jB_1}} \\
\vec{A}_{Y_{P_jB_1}} \\
\vec{A}_{Z_{P_jB_1}}
\end{bmatrix} + \begin{bmatrix}
\vec{S}_{X_{P_jB_1}} \\
\vec{S}_{Y_{P_jB_1}} \\
\vec{S}_{Z_{P_jB_1}}
\end{bmatrix} \tag{36}
\]

\[
[A]_{P_jP_k} = \begin{bmatrix}
\vec{N}_{X_{B_j}} \\
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix}
= \begin{bmatrix}
\vec{A}_{X_{P_jP_k}} \\
\vec{A}_{Y_{P_jP_k}} \\
\vec{A}_{Z_{P_jP_k}}
\end{bmatrix} + \begin{bmatrix}
\vec{S}_{X_{P_jP_k}} \\
\vec{S}_{Y_{P_jP_k}} \\
\vec{S}_{Z_{P_jP_k}}
\end{bmatrix} \tag{37}
\]

The known quantities in the influence equation \(\{\nabla S\}_{B_j}\) and \(\{\nabla S\}_{P_j}\) are defined as:

\[
\{\nabla S\}_{B_j} = - \sum_{k=1}^{N_p} \begin{bmatrix}
\vec{N}_{X_{B_j}} \\
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix} \begin{bmatrix}
\vec{S}_{X_{B_jP_k}} \\
\vec{S}_{Y_{B_jP_k}} \\
\vec{S}_{Z_{B_jP_k}}
\end{bmatrix} + \begin{bmatrix}
\vec{N}_{Y_{B_j}} \\
\vec{N}_{Z_{B_j}}
\end{bmatrix} \begin{bmatrix}
\vec{V}_X \\
\vec{V}_Y \\
\vec{V}_Z
\end{bmatrix} \tag{38}
\]

\[
+ \begin{bmatrix}
\vec{N}_{Z_{B_j}} \\
\vec{N}_{X_{B_j}}
\end{bmatrix} \begin{bmatrix}
\vec{V}_X \\
\vec{V}_Y
\end{bmatrix} \tag{38}
\]
The elements of the above matrices are computed using the influence equations derived in Appendices B and C. Each element is associated with the influence of a singularity on a control point. The singularities and control points are ordered such that all of the longitudinal stations for the first lateral station are cycled through first and then all of the longitudinal stations for the second lateral station. This process is continued until all stations have been cycled through. The longitudinal stations start at the leading edge of the panel, the nose of a solid body, and the tail end of the inside surface of a flow through body. The lateral stations start at the inside edge of the panel and go toward the tip of the panel. The lateral stations on the body start at the top of the body and progress in a clockwise direction when looking at the body from the tail to the nose.

All bodies are cycled through first and then the panels. Both the bodies and the panels are cycled in the order that they are input. The columns of the influence matrices are associated with singularities and the rows with control points. The elements of the matrices and the submatrices of the combined matrix influence equation are sequenced, for both the columns and the rows, in the same order that the singularities, points, bodies, and panels are cycled.

The influences of quadrilateral vortices in the panel root section and on the body surface are computed by equations derived in Appendix C. The influences of skewed horseshoe vortices on the panel outboard section are computed by equations derived in Appendix B. The influences of skewed source lines on the panel are computed by equations derived in Appendix B.

If the force free wake option of the program is used the influences of the free trailing vortices are computed using the influence equations in Appendix C. The locations of the force free trailing vortices are computed using the following iteration procedure.

\[
\begin{bmatrix} \nabla S \end{bmatrix}_{P_j} = - \sum_{k=1}^{N_p} \left[ \begin{bmatrix} N_{X_{P_j}} \\ N_{Y_{P_j}} \end{bmatrix} \begin{bmatrix} S_{X_{P_j}} \\
S_{Y_{P_j}} \end{bmatrix} + \begin{bmatrix} S_{X_{P_j}} \\
S_{Y_{P_j}} \end{bmatrix} \begin{bmatrix} N_{X_{P_k}} \\
N_{Y_{P_k}} \end{bmatrix} \right] \begin{bmatrix} \frac{V}{V_\infty} \end{bmatrix}_{P_k} - \left[ \begin{bmatrix} N_{X_{P_j}} \\
N_{Y_{P_j}} \end{bmatrix} \begin{bmatrix} \frac{V_x}{V_\infty} \\
\frac{V_y}{V_\infty} \end{bmatrix} - \begin{bmatrix} N_{Z_{P_j}} \\
\frac{V_z}{V_\infty} \end{bmatrix} \right] (39)
\]
1. A solution for the vortex strengths will be obtained first by assuming the location of the free vortices to be in the longitudinal direction and to be straight except in a thick lifting panel-body juncture region.

2. The total velocities \( U, V, \) and \( W \) in the \( X, Y, \) and \( Z \) directions, respectively, will be computed at the midpoint of each free trailing vortex division.

3. The actual mean camber surfaces of the thick lifting panels will be computed so that the correct relationship between fixed vortices and control points on the thick lifting panels and the free trailing vortices is maintained.

4. The ratios \( V/U \) and \( W/U \) are integrated in the \( X \) direction from the edges of the actual thick lifting panel mean camber surfaces and body aft ends where the free vortices are assumed to be shed, in order to obtain their new locations.

5. The influence of the free vortices on the actual thick lifting panel mean camber and body surfaces is computed and the difference between this influence and that from the free vortices at their previous location is added to the influence matrices for the complete configuration.

6. A new solution for the vortex strengths is determined and a new set of total velocities \( U, V, \) and \( W \) along the new free vortex lines is computed using quadrilateral vortices and source lines on the actual thick lifting panel mean camber surfaces and quadrilateral vortices on the body surfaces.

The above procedure is iterated between steps (4) and (6) until the surface pressures converge. When the influence matrix of any individual body or panel is no longer significantly changed due to a new positioning of the free vortices, the calculation of the perturbation to that matrix is terminated.

Constrained Influence Equations

The vortex strengths on any of the bodies or panels can be constrained by representing the vorticity by a finite series. This uncouples the number of unknowns, equations, and vortices used to represent the perturbation velocity. The series used must be capable of producing the type of perturbation velocity needed to satisfy the boundary conditions at the control points to an acceptable degree. If such a series can be found for a given body or panel the number of equations or control points can be reduced from that of the number of
vortices or subpanels to that necessary to describe the geometry. This sub-
stantially reduces the computer time, since in general the number of vortices
needed to represent the perturbation velocity is far more than the number of
control points needed to represent the geometry of the body or panel.

This reduction in the number of required control points results
primarily in reducing the computational effort necessary to set up the aero-
dynamic influence equations. A further reduction in computer time can be
realized in the solution of the influence equations, if constraint functions
are used, since the number of unknowns is reduced from that of the number
of vortices to the number of terms in the constraint series. This results in
a system of equations which is overdetermined and is solved by the method of
least squares. In this process the vectors \( \{e\}_B_i \) and \( \{e\}_p_j \) are minimized.

The body constraint equation is given by:

\[
\begin{pmatrix}
\frac{k}{V_\infty} \\
\end{pmatrix}
_{B_i} = 
\begin{bmatrix}
T_{B_i} \\
\end{bmatrix}
_{B_i} \begin{pmatrix}
a_{B_i} \\
\end{pmatrix}
\tag{40}
\]

where

\[
\begin{bmatrix}
T_{B_i} \\
\end{bmatrix} = 
\begin{bmatrix}
R \\
\end{bmatrix} \begin{bmatrix}
\Gamma_{Mmn} \\
\end{bmatrix} \begin{pmatrix}
\gamma \\
\end{pmatrix} 
\tag{41}
\]

The matrix \([R]\) is a transformation matrix used to obtain the quadrilaterial
vortex strengths \( K \) from the bound horseshoe vortex strengths \( \Gamma_b \).

\[
\begin{bmatrix}
R \\
\end{bmatrix} = \begin{pmatrix}
[R]_n \\
\end{pmatrix}
\tag{42}
\]

where \([R]_n\) is made up of ones along the diagonal and in the lower triangle.
The upper triangle is filled with zeros.

The indices \( m \) and \( n \) refer to longitudinal and lateral subpanel locations
on the equivalent incompressible body, respectively. The vorticity ratio
\( (V/V_\infty)_{mn} \) is the value of the vorticity series at the \( m \)th subpanel from the
nose of the body and the \( n \)th subpanel in the lateral direction around the body.
The longitudinal length \( \Gamma_{Mmn} \) is given by:

\[
\Gamma_{Mmn} = \frac{1}{2} \left[ \frac{3}{4} \Gamma_{M(m-1)n} + \Gamma_{Mmn} + \frac{1}{4} \Gamma_{M(m+1)n} \right]
\tag{43}
\]
where $l_M^{mn}$ is the length of the $n^{th}$ subpanel from the nose of the equivalent incompressible body at the $n^{th}$ lateral station.

The vorticity series $(\gamma / V_\infty)_{mn}$ is the product of a longitudinal series and a lateral series. The elements of the matrix $\{a_{Bi}\}$ are the unknown coefficients associated with the terms in $(\gamma / V_\infty)_{mn}$ produced by the product of the longitudinal and lateral series. The terms in $(\gamma / V_\infty)_{mn}$ are ordered such that the products of all of the longitudinal constraint functions and the first lateral constraint function are first, then the products of all of the longitudinal constraint functions and the second lateral constraint functions are second, and etcetera. This process is continued until all combinations of longitudinal and lateral constraint functions have been cycled through.

Both the longitudinal and lateral constraint functions are defined over segments of the body. The same functions are used in all segments. The origin of the segment is designated by the subscript $o$ and the end by $f$. The longitudinal and lateral constraint function segments are given in the data input array.

The longitudinal constraint functions for the body are defined as:

$$\frac{\gamma}{V_\infty} (X_B / C_B) = 1$$  \hspace{1cm} (44)

$$\frac{\gamma}{V_\infty} (X_B / C_B) = \frac{1}{\sqrt{1 + \frac{Y_B}{N_B} \left( \frac{N_B}{Y_B} \right)^2 \left( \frac{Z_B}{N_B} \right)^2}}$$  \hspace{1cm} (45)

$$\frac{\gamma}{V_\infty} (\phi_B) = \cot \theta B / 2$$  \hspace{1cm} (46)

$$\frac{\gamma}{V_\infty} (\phi_B) = \cot (\pi / 2 - \theta B / 2)$$  \hspace{1cm} (47)

$$\frac{\gamma}{V_\infty} (\phi_B) = \sin \left[ \frac{\pi}{2} \left( \frac{\theta B - \phi o}{\phi f - \phi o} \right) \right]$$  \hspace{1cm} (48)

$$\frac{\gamma}{V_\infty} (\phi_B) = \cos \left[ \frac{\pi}{2} \left( \frac{\theta B - \phi o}{\phi f - \phi o} \right) \right]$$  \hspace{1cm} (49)

Original page is of poor quality.
\[ \frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_7 = \left[ \begin{array}{c} \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \\ \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \\ \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \end{array} \right] \]  

(50)

\[ \frac{\gamma}{V_\infty} (\phi_B)_{18} = \sin \left[ 2\pi \left( \frac{\phi_B - \phi_0}{\phi_f - \phi_0} \right) \right] \]  

(51)

\[ \frac{\gamma}{V_\infty} (\phi_B)_{9} = \cos \left[ 2\pi \left( \frac{\phi_B - \phi_0}{\phi_f - \phi_0} \right) \right] \]  

(52)

\[ \frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{10} = \left[ \begin{array}{c} \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \\ \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \\ \frac{X_B}{C_B} - \left( \frac{X_B}{C_B} \right)_0 \end{array} \right]^2 \]  

(53)

\[ \frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{3270} = L_{S_1} \left( \frac{X_B}{C_B} \right) \]  

(54)

\[ \frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{3320} = L_{S_2} \left( \frac{X_B}{C_B} \right) \]  

(55)
The functions to be used on the body are designated by their number in the above sequence. The last three functions listed above are special functions and are input at the locations in the data array, designated by their subscripts. The independent variable $\phi_B$ is given by, $\phi_B = \cos^{-1} \left[ 1 - 2(X_B/C_B) \right]$

The lateral constraint functions for the body are defined as:

$$\frac{\gamma}{V_{\infty}} (\theta_B)_1 = 1$$ (57)

$$\frac{\gamma}{V_{\infty}} (\theta_B)_2 = \frac{N_Y}{N_B} / \left[ \frac{N_Y}{N_B} + N_Y \right]^{1/2}$$ (58)

$$\frac{\gamma}{V_{\infty}} (\theta_B)_3 = \frac{N_Y}{N_B} / \left[ \frac{N_Y}{N_B} + N_Y \right]^{1/2}$$ (59)

$$\frac{\gamma}{V_{\infty}} (\theta_B)_4 = \sin \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right]$$ (60)

$$\frac{\gamma}{V_{\infty}} (\theta_B)_5 = \cos \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right]$$

$$\frac{\gamma}{V_{\infty}} (\theta_B)_6 = \left[ \pi \left( \frac{\theta_B - \theta_o}{\theta_f - \theta_o} \right) \right] \text{ or } \left[ \frac{\eta_B - \eta_o}{\eta_f - \eta_o} \right]$$ (61)
\[ \frac{\gamma}{V_\infty} (\theta_B)_7 = \sin \left[ 2\pi \left( \frac{\theta_B - \theta_0}{\theta_f - \theta_0} \right) \right] \]  
(62)

\[ \frac{\gamma}{V_\infty} (\theta_B)_8 = \cos \left[ 2\pi \left( \frac{\theta_B - \theta_0}{\theta_f - \theta_0} \right) \right] \]  
(63)

\[ \frac{\gamma}{V_\infty} (\theta_B)_9 = \pi \left( \frac{\theta_B - \theta_0}{\theta_f - \theta_0} \right)^2 \text{ or } \left( \frac{\eta_B - \eta_0}{\eta_f - \eta_0} \right)^2 \]  
(64)

Here again, the lateral functions to be used on the body are designated by their number in the above sequence. If no constraint functions are specified the program will do a discrete solution.

The panel constraint equation is given by;

\[
\begin{bmatrix}
\frac{K}{V_\infty} \\
\frac{\Gamma}{V_\infty}
\end{bmatrix}
\begin{bmatrix}
K \\
\Gamma
\end{bmatrix}
= 
\begin{bmatrix}
T_{pK}
\end{bmatrix}
\begin{bmatrix}
apK
\end{bmatrix}
\]  
(65)

The \([T_{pK}]\) transformation matrix condenses all rows of the discrete aerodynamic influence matrix by the following procedure.

1. That portion of the row which is associated with the vortices on panel \(P_K\) is divided into the elements due to each lateral station. A new temporary matrix \([A_T]\) is developed with all of the elements due to the vortices at the first lateral station in row one, all of the elements due to the vortices at the second lateral station in row two, and etcetera.
2. This temporary matrix is then premultiplied by the spanwise constraint function transformation matrix \([T(\eta)]\) and postmultiplied by the chordwise constraint function transformation matrix \([T(\sigma)]\).

3. The transformed temporary matrix \([\bar{A}_T] = [T(\eta)][A_T][T(\sigma)]\) is then opened up into a row again and replaces the old row in the original discrete aerodynamic influence matrix. The new row is formed by placing all of the elements from the first row of \([\bar{A}_T]\) in the portion of the row, from the discrete matrix, due to the \(P_K\) panel first, then by placing all of the elements from the second row of \([\bar{A}_T]\) next to those elements from the first row of \([\bar{A}_T]\), and etcetera. The result will be the reduction of the number of elements in a given row due to the \(P_K\) panel from that of the number of vortices or subpanels on panel \(P_K\) to the number of chordwise constant functions times the number of spanwise constraint functions.

If only the chordwise constraint functions are used, then the number of elements in the row due to the \(P_K\) panel will be reduced from the number of vortices on the \(P_K\) panel to the number of chordwise constraint functions times the number of subpanels in the lateral direction. On panels with freestream edges, at span stations other than the tip, only the chordwise functions should be used, since the available spanwise constraint functions are not sufficient to produce the necessary perturbation velocity to satisfy the boundary conditions on a panel of this type.

The above-described transformation constrains the skewed horseshoe vortex strengths by the following series developed in appendices F. and G. of reference (26).

\[
\frac{\Gamma}{V_{\infty}}(\eta)_m = \sqrt{1 - \eta^2} \sum_{w=1}^{N_w-P_w} \left\{ \left( \frac{\Gamma'_{om}}{V_{\infty}} \right)_{a_{ow}} + \sum_{f=1}^{N_f} \left( \frac{\Gamma'_{fm}}{V_{\infty}} \right)_{a_{fw}} + \sum_{k=1}^{N_K} \left( \frac{\Gamma'_{km}}{V_{\infty}} \right)_{a_{kw}} \right\} 
\]

\[
+ \sum_{n=1}^{N_u-1} \left( \frac{\Gamma'_{nm}}{V_{\infty}} \right)_{a_{nw}} \right\} \eta^w + \sum_{w=(N_w-P_w+1)}^{N_w} \left\{ \left( \frac{\Gamma'_{om}}{V_{\infty}} \right)_{a_{ow}} 
\]

\[
+ \sum_{f=1}^{N_f} \left( \frac{\Gamma'_{fm}}{V_{\infty}} \right)_{a_{fw}} + \sum_{k=1}^{N_K} \left( \frac{\Gamma'_{km}}{V_{\infty}} \right)_{a_{kw}} + \sum_{n=1}^{N_u-1} \left( \frac{\Gamma'_{nm}}{V_{\infty}} \right)_{a_{nw}} \right\} \eta^w 
\]

\[
= \left( \frac{\Gamma'_{om}}{V_{\infty}} \right)_{a_{ow}} + \left( \frac{\Gamma'_{fm}}{V_{\infty}} \right)_{a_{fw}} + \left( \frac{\Gamma'_{km}}{V_{\infty}} \right)_{a_{kw}} + \left( \frac{\Gamma'_{nm}}{V_{\infty}} \right)_{a_{nw}} \right\} P_w(\eta)_S 
\] (66)
Where the indices m, f, k, o, n, and w indicate the number of the subpanel aft of the panel leading edge \( \Gamma / V_\infty \) is defined on, the number of the trailing edge flap, the number of the leading edge flap, the first term of the Birnbaum series, the number of the sine term in the Birnbaum series, and the number of the spanwise constraint function, respectively. The quantities \( N_u, N_f, N_k, P_w, \) and \( N_w \) are the number of terms in the Birnbaum series, the number of trailing edge flaps with unique hinge line locations, the number of leading edge flaps with unique hinge line locations, the number of special spanwise constraint functions, and the total number of spanwise constraint functions, respectively.

The columns of the matrix \([T(X/C)]\) are made up of the \((\Gamma'/V_\infty)_m\) values, where \( m \) indicates the number of the element in the column. There is one column for each set of \((\Gamma'/V_\infty)_m\)'s associated with a unique chordwise constraint function. The \([T(X/C)]\) matrix is filled such that the \((\Gamma'_{om}/V_\infty)\) values are in column one, then the \((\Gamma'N_{Nf}/V_\infty)\) values are in the next set of \( N_u - 1 \) columns, then the \((\Gamma'_{fN}/V_\infty)\) values are in the next set of \( N_f \) columns, and then the \((\Gamma'_{km}/V_\infty)\) values are in the next set of \( N_k \) columns.

The \([T(X/C)]\) matrix is solved here in the same manner as is shown in appendix G. of reference (26). The \((\Gamma'/V_\infty)_m\) values are solved for such that the same downwash is obtained at the three quarter chord point of each subpanel, except at the last subpanel, due to a distribution of discrete vortices as would be obtained by integrating the vorticity distribution in the Blot-Savart integral for each chordwise constraint function. The additional condition, that the sum of the discrete vortex strengths must be equal to the integral of the vorticity distribution is also used. These conditions result in the following matrix equation which is solved for \([T(x/c)]\).

\[
[E] [T(x/c)] = [W]
\]

The matrix \([E]\) is the two-dimensional discrete vortex influence matrix and is defined as follows.

\[
[E] = \begin{bmatrix}
1 \\
\frac{1}{(x/c)_j - (x/c)_i} \\
\vdots \\
1, 1, \ldots, 1
\end{bmatrix}
\]

\[
(68)
\]

\(E\) is the two-dimensional discrete vortex influence matrix and is defined as follows.
Where \( 0 \leq (x/c) \leq 1.0 \) and the indices \( j \) and \( i \) as used here indicate the location of the three quarter and quarter chord points, respectively.

There is one column in the \([w]\) matrix for each unique chordwise constraint function. In these columns is the downwash due to evaluating the vorticity for each chordwise constraint function in the two-dimensional Biot-Savart integral. The columns in \([w]\) are ordered such that the column due to a given chordwise constraint function is in the same location in \([w]\) as its corresponding \((\mathbf{r}'/V_\infty)\) column is in the \([T(x/c)]\) matrix. The five basic types of columns in \([w]\) are defined as follows.

Due to the first term of the Birnbaum series;

\[
\begin{pmatrix} \cot \phi / 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\cos \phi} \\
1 \\
1 \\
\vdots \\
1 \\
1/2 \end{pmatrix}
\]  \hspace{1cm} (69)

Due to the second term of the Birnbaum series;

\[
\begin{pmatrix} w \end{pmatrix} = \begin{pmatrix} \frac{-\cos \phi_1}{\sin \phi} \\
\frac{-\cos \phi_2}{\sin \phi} \\
\vdots \\
\frac{-\cos \phi (N_1-1)}{\sin \phi} \\
1/4 \end{pmatrix}
\]  \hspace{1cm} (70)
Due to the third and higher order terms of the Birnbaum series;

\[
\begin{bmatrix}
    w \\
    \sin(n\phi)
\end{bmatrix} = \begin{bmatrix}
    -\cos n\phi_1 \\
    -\cos n\phi_2 \\
    \vdots \\
    -\cos n\phi_{(N_1-1)} \\
    0
\end{bmatrix} \tag{71}
\]

where \( \phi_j = \cos^{-1}\left[1 - 2(\xi/c)_j\right] \)

Due to the trailing edge flap term;

\[
\begin{bmatrix}
w \\
\log \left| \frac{\sin 1/2(\phi + \phi_f)}{\sin 1/2(\phi - \phi_f)} \right| 
\end{bmatrix} = \begin{bmatrix}
w_f(\phi_1) \\
w_f(\phi_2) \\
\vdots \\
w_f(\phi_{N_1-1}) \\
1/2 \sin \phi_f
\end{bmatrix} \tag{72}
\]

where \( w_f(\phi_j) = -(\pi - \phi_f) \) for \( 0 < \phi_j < \phi_f \), \( w_f(\phi_j) = \phi_f \) for \( \phi_f \leq \phi_j \leq \pi \).

is the polar coordinate at the flap leading edge, and \( N_1 \) equals the number of subpanels per chord.
due to the leading edge flap term;

\[
\begin{bmatrix}
    \sin \frac{1}{2}(\phi + \phi_k) \\
    \log \left| \frac{\sin \frac{1}{2}(\phi + \phi_k)}{\sin \frac{1}{2}(\phi - \phi_k)} \right|
\end{bmatrix}
\]

\[
\begin{bmatrix}
    w_k(\phi_1) \\
    w_k(\phi_2) \\
    \vdots \\
    w_k(\phi_{N_w-1}) \\
    \frac{1}{2} \sin \phi_k
\end{bmatrix}
\]

(73)

where \( w_k(\phi_j) = -\pi - \phi_k \) for \( 0 \leq \phi_j < \phi_k \), \( w_k(\phi_j) = \phi_k \) for \( \phi_k < \phi_j \leq \pi \), and \( \phi_k \) is the polar coordinate at the flap leading edge.

In the root section the \([T \ X/C]\) matrix is transformed to \([\bar{T} \ (X/C)]\), where each column of \([T \ X/C]\) is transformed as follows;

\[
\bar{T} \ (X/C)_m = \sum_{i=1}^{m} T \ (X/C)_i
\]

(74)

where \( i \) and \( m \) indicate the number of the element in the column in the \([T \ (X/C)]\) and \([\bar{T} \ (X/C)]\) matrices, respectively.

The rows of the \([T \ (\eta)]\) matrix are made up of the spanwise constraint functions. Each element in a row is equal to the value of the constraint function at the lateral subpanel which has the same lateral index as the number of the column in the \([T \ (\eta)]\) matrix. There are \( N_w \) rows in the \([T \ (\eta)]\) matrix, with the standard functions described by \( \eta^N \sqrt{1 - \eta^2} \) in the first \( N_w \) rows, and then the special \( P_w \ (\eta)\) functions in the last \( P_w \) rows.

The special spanwise constraint functions are needed to account for flow induced by discontinuities in the sweep of the constant percent chord lines, for body-panel juncture induced flow, and for partial semi-span flaps. There are two basic special spanwise constraint functions, (1) polygonal functions which account for the discontinuities in the sweep of the constant percent chord lines and the body-panel junctures, and (2) flap functions which account for the partial semi-span flaps. These special functions are derived in Appendix F of Reference (26).
If the range of influences inboard and outboard of a discontinuity in the sweep of the constant percent chord line are defined as $\Delta \eta_i$ and $\Delta \eta_o$, respectively, and the discontinuity station $\eta_b$, then the following expressions define the special spanwise polygonal constraint function.

For $0 < |\eta_b| < \Delta \eta_i$

$$P(\theta)_S = (1 - \frac{\eta_b}{\Delta \eta_i}) M(\theta, \frac{\pi}{2}) + \frac{1}{\Delta \eta_i} P(\theta, \frac{\pi}{2}) - \left[ \frac{(\Delta \eta_i + \Delta \eta_o)(1 - \eta_b)}{\Delta \eta_i - \Delta \eta_o} \right] P(\theta, \theta_b)$$

$$+ \left[ \frac{\Delta \eta_i + \Delta \eta_o}{\Delta \eta_i - \Delta \eta_o} - \frac{1}{\Delta \eta_i} \right] \left[ 1 - \Delta \eta_o - \eta_b \right] P(\theta, \cos^{-1}(\eta_b + \Delta \eta_o))$$

(75)

For $\Delta \eta_i \leq |\eta_b| < (1 - \Delta \eta_o)$

$$P(\theta)_S = (1 - \frac{\eta_b + \Delta \eta_i}{\Delta \eta_i}) (1/\Delta \eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta \eta_i))$$

$$- \left[ \frac{\Delta \eta_o + \Delta \eta_i}{\Delta \eta_o - \Delta \eta_i} \right] \left[ 1 - \eta_b \right] P(\theta, \theta_b)$$

$$+ (1 - \eta_b - \Delta \eta_o) (1/\Delta \eta_o) P(\theta, \cos^{-1}(\eta_b + \Delta \eta_o))$$

(76)

And for $(1 - \Delta \eta_o) \leq |\eta_b| < 1.0$

$$P(\phi)_S = (1 - \frac{\eta_b + \Delta \eta_i}{\Delta \eta_i}) (1/\Delta \eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta \eta_i))$$

$$- \left[ 1 + (1 - \eta_b) / \Delta \eta_i \right] P(\theta, \theta_b)$$

(77)
where

\[ P(\theta, \theta^*) = \frac{1}{2\pi (1 - \cos \theta^*)} \left\{ \begin{array}{c}
(\cos \theta^* - \cos \theta) \log_e \sin 1/2 (\theta^* - \theta) \\
\sin 1/2 (\theta^* + \theta) \\
+ (\cos \theta^* + \cos \theta)^2 \log_e \cos 1/2 (\theta^* + \theta) \\
\cos 1/2 (\theta^* - \theta) \\
+ (4 \theta^* \cos \theta^* - 2 \sin \theta^*) \sin \theta \end{array} \right. \]

(78)

And

\[ \theta = \cos^{-1} \eta \]

The special spanwise flap constraint function is given by;

\[ P(\theta)_S = M(\theta, \theta_i) - M(\theta, \theta_o) \]

(79)

Where \( \theta_i = \cos^{-1} \eta_i \), \( \theta_o = \cos^{-1} \eta_o \), \( \eta_i \) is the inboard station where the control surface begins, and \( \eta_o \) is the outboard station where the control surface ends.

\[ M(\theta, \theta^*) = \frac{1}{\pi} \left\{ \begin{array}{c}
(\cos \theta^* - \cos \theta) \log_e \sin 1/2 (\theta^* - \theta) \\
\sin 1/2 (\theta^* + \theta) \\
+ (\cos \theta^* + \cos \theta) \log_e \cos 1/2 (\theta^* + \theta) \\
\cos 1/2 (\theta^* - \theta) \\
+ 2 \theta^* \sin \theta \end{array} \right. \]

(80)

The discrete aerodynamic influence equation (33) is transformed by substituting equations (40) and (65) into equation (33).
The constrained aerodynamic influence equation is then given by:

\[
\begin{bmatrix}
[A]_{B_i} & [A]_{B_j} P_K \\
[A]_{P_i} & [A]_{P_j} P_K
\end{bmatrix}
\begin{bmatrix}
T_{B_i} \\
0
\end{bmatrix}
\begin{bmatrix}
a_{B_i} \\
a_{P_j}
\end{bmatrix} =
\begin{bmatrix}
\nabla S_{B_i} \\
\nabla S_{P_j}
\end{bmatrix}
+ \begin{bmatrix}
e_{B_j} \\
e_{P_j}
\end{bmatrix}
\]

(81)

Where the arrays \{a_{B_i}\} and \{a_{P_j}\} are solved for such that the sum of the squares of the elements of the arrays \{e_{B_j}\} and \{e_{P_j}\} are a minimum.
Surface Velocities and Pressures

The velocity tangent to the surface of the jth body in the longitudinal direction is given by:

\[
\left( \begin{array}{c}
\frac{\Delta v_n}{V_e} \\
\frac{\Delta v_m}{V_e}
\end{array} \right)_{b_j} = \sum_{i=1}^{N_p} \left[ \frac{1}{\rho^2} \left[ \begin{array}{c}
T_{x,i} \beta_j \gamma_j \beta_i \\
T_{y,i} \beta_j \gamma_j \beta_i \\
T_{z,i} \beta_j \gamma_j \beta_i \\
N_{x,i} \gamma_j \beta_i \\
N_{y,i} \gamma_j \beta_i \\
N_{z,i} \gamma_j \beta_i \\
K_{b} \gamma_j \beta_i
\end{array} \right] \right] \frac{K_{b}}{V_e}
\]

\[
+ \sum_{K=1}^{N_p} \left[ \frac{1}{\rho^2} \left[ \begin{array}{c}
T_{x,K} \beta_j \gamma_j \beta_K \\
T_{y,K} \beta_j \gamma_j \beta_K \\
T_{z,K} \beta_j \gamma_j \beta_K \\
S_{x,K} \gamma_j \beta_K \\
S_{y,K} \gamma_j \beta_K \\
S_{z,K} \gamma_j \beta_K \\
K_{K} \gamma_j \beta_K
\end{array} \right] \right] \frac{K_{K}}{V_e}
\]

\[
= \left[ \begin{array}{c}
T_{x,j} \beta_j \gamma_j \beta_j \\
T_{y,j} \beta_j \gamma_j \beta_j \\
T_{z,j} \beta_j \gamma_j \beta_j \\
S_{x,j} \gamma_j \beta_j \\
S_{y,j} \gamma_j \beta_j \\
S_{z,j} \gamma_j \beta_j \\
K_{j} \gamma_j \beta_j
\end{array} \right] \frac{K_{j}}{V_e} \left( \begin{array}{c}
\frac{\Delta v_n}{V_e} \\
\frac{\Delta v_m}{V_e}
\end{array} \right)_{b_j}
\]

(83)

Where

\[
\left( \frac{\Delta v_n}{V_e} \right)_{mn} = \frac{T_{b,mn}}{V_{eo}} \left( \frac{3}{4} \bar{M}_{M(m+1)n} + \frac{1}{4} \bar{M}_{M(m-1)n} \right) \left[ \frac{2 \bar{M}_{mn} + \bar{M}_{M(m+1)n}}{3 \bar{M}_{mn} + \bar{M}_{M(m+1)n}} \right]
\]

\[
+ \frac{F_{b}(m+1)l}{V_{eo}} \left( \frac{3}{4} \bar{M}_{mn} + \frac{1}{4} \bar{M}_{M(m+1)n} \right) \left[ \frac{2 \bar{M}_{mn} + \bar{M}_{M(m+1)n}}{3 \bar{M}_{mn} + \bar{M}_{M(m+1)n}} \right]
\]

(84)

And

\[
\frac{T_{b,mn}}{V_{eo}} = \frac{K_{mn}}{V_{eo}} - \frac{K_{(m-1)n}}{V_{eo}}
\]

(85)
\[
\frac{T_b(m + 1)N}{V_\infty} = \frac{K(m + 1)N}{V_\infty} - \frac{K m N}{V_\infty}
\]  

The onset flow ratios \(V_x/V_\infty\), \(V_y/V_\infty\), and \(V_z/V_\infty\) are given by the following expressions.

\[
\frac{V_x}{V_\infty} = 1 - q* \frac{2(Z - Z_{C,G.})}{c} - \gamma* \frac{2(Y - Y_{C,G.})}{b}
\]  

\[
\frac{V_y}{V_\infty} = \beta + p* \frac{2(Z - Z_{C,G.})}{b} + \gamma* \frac{2(X - X_{C,G.})}{b}
\]  

\[
\frac{V_z}{V_\infty} = \alpha + p* \frac{2(Y - Y_{C,G.})}{b} + q* \frac{(X - X_{C,G.})}{c}
\]

The components of the unit vectors tangent to the actual body subpanels in the longitudinal direction are given by:

\[
T_{M_x} = \beta \frac{T_{M_x}}{\sqrt{\beta^2 T_{M_x}^2 + T_{M_y}^2 + T_{M_z}^2}}
\]  

\[
T_{M_y} = \frac{T_{M_y}}{\sqrt{\beta^2 T_{M_x}^2 + T_{M_y}^2 + T_{M_z}^2}}
\]  

\[
T_{M_z} = \frac{T_{M_z}}{\sqrt{\beta^2 T_{M_x}^2 + T_{M_y}^2 + T_{M_z}^2}}
\]

and in the lateral direction by

\[
T_{T_x} = \beta \frac{T_{T_x}}{\sqrt{\beta^2 T_{T_x}^2 + T_{T_y}^2 + T_{T_z}^2}}
\]  

\[
T_{T_y} = \frac{T_{T_y}}{\sqrt{\beta^2 T_{T_x}^2 + T_{T_y}^2 + T_{T_z}^2}}
\]

\[
T_{T_z} = \frac{T_{T_z}}{\sqrt{\beta^2 T_{T_x}^2 + T_{T_y}^2 + T_{T_z}^2}}
\]
The velocity tangent to the surface of the $j$th body in the lateral direction is given by:

$$
\left( \frac{V_T}{V_w} \right)_{mn} = \sum_{i=1}^{N_B} \left\{ \begin{array}{c} \frac{1}{\rho} \left[ T_X B_j \right] \left[ A_{B,j,B_1} \right] \cdot \frac{1}{\rho} \left[ T_Y B_j \right] \left[ A_{B,j,B_1} \right] \cdot \frac{1}{\rho} \left[ T_Z B_j \right] \left[ A_{B,j,B_1} \right] \end{array} \right\} \left[ \frac{F}{V_w} \right]_{B_i}
$$

$$
\sum_{k=1}^{N_p} \left\{ \begin{array}{c} \frac{1}{\rho} \left[ T_X B_j \right] \left[ A_{B,j,P_k} \right] \cdot \frac{1}{\rho} \left[ T_Y B_j \right] \left[ A_{B,j,P_k} \right] \cdot \frac{1}{\rho} \left[ T_Z B_j \right] \left[ A_{B,j,P_k} \right] \end{array} \right\} \left[ \frac{F}{V_w} \right]_{P_k}
$$

$$
\sum_{k=1}^{N_p} \left\{ \begin{array}{c} \frac{1}{\rho} \left[ T_X B_j \right] \left[ S_{B,j,P_k} \right] \cdot \frac{1}{\rho} \left[ T_Y B_j \right] \left[ S_{B,j,P_k} \right] \cdot \frac{1}{\rho} \left[ T_Z B_j \right] \left[ S_{B,j,P_k} \right] \end{array} \right\} \left[ \frac{F}{V_w} \right]_{P_k}
$$

$$
\left( \frac{\Delta V_T}{V_w} \right)_{mn} = -\frac{1}{2} \left[ \frac{r_{T_m(n-\Delta n/2)}}{V_w} \left( \frac{T_{mn}}{T_{n,(n-1)}} \right) + \frac{r_{T_m(n+\Delta n/2)}}{V_w} \left( \frac{T_{mn}}{T_{n,(n+1)}} \right) \right]
$$

(93)

(94)
And
\begin{equation}
\frac{\Gamma_m(n - \Delta n/2)}{V_\infty} = \frac{K_m n - 1}{V_\infty} - \frac{K_m n}{V_\infty} - \sum_{r = 1}^{N_a} \frac{K_m(n - \Delta n/2)}{V_\infty} \tag{95}
\end{equation}
\begin{equation}
\frac{\Gamma_m(n + \Delta n/2)}{V_\infty} = \frac{K_m n}{V_\infty} - \frac{K_m n + 1}{V_\infty} - \sum_{r = 1}^{N_a} \frac{K_m(n + \Delta n/2)}{V_\infty} \tag{96}
\end{equation}

Where
\begin{equation}
\sum_{r = 1}^{N_a} \left[ \frac{K_m(n - \Delta n/2)}{V_\infty} \right] \text{ and } \sum_{r = 1}^{N_a} \left[ \frac{K_m(n + \Delta n/2)}{V_\infty} \right] \tag{97}
\end{equation}

are the contributions from the trailing legs of the panel vortices along the juncture line of the \( j \)th body and \( r \)th panel to be attached at the \((n - \Delta n/2)\) and \((n + \Delta n/2)\) lateral stations, respectively. \( N_a \) is the total number of panels attached at any one point along the line.

The velocity tangent to the surface of the \( j \)th panel in the longitudinal direction is given by:
\begin{equation}
\frac{\left( \frac{v_M}{V_\infty} \right)_{mn}}{p_j} = \frac{1}{\left[ 1 + (1 + \tan^2 \alpha) \right]_{mn} (dz_T/dx + (dz_C/dx)^2)_{mn}^{1/2}} \left[ \sum_{i = 1}^{N_B} \frac{1}{\beta^2} \left[ \frac{\alpha_i}{V_\infty} \right] \right] + \sum_{k = 1}^{N_P} \left[ \alpha_{Tk} \right] \left[ \frac{V_\infty}{P_k} \right] + \left[ \sqrt{1 + \tan^2 \Delta_M} \right]_{mn} \left[ \frac{\Sigma}{V_\infty} \right]_{p_j} \tag{97}
\end{equation}
\[
\frac{\Sigma'}{V_\infty} = \left[ \frac{(Z/c)}{(X/C) (1 - X/C)} \right] \left[ \frac{V_x/V_\infty}{\sqrt{1 + \tan^2 \Lambda}} \right] \frac{\bar{f}}{\bar{\rho}}
\]

and where \((\Delta V_M/V_\infty)_{mn}\) is computed using equation (84) and X/C is the local percent chord.

The velocity tangent to the surface of the jth panel in the lateral direction is given by:

\[
\begin{bmatrix}
\frac{V_{ij}}{V_\infty}
\end{bmatrix}_{p_j} = \frac{1}{\left(1 + (1 + \tan^2 \Lambda)_{mn} \frac{dZ_i}{dX} \times \frac{dZ_j}{dX}\right)^{1/2}} \left\{ \sum_{i=1}^{N_B} \frac{1}{\beta} \begin{bmatrix}
\tau_{p_j i} \\
\nu_{p_j i}
\end{bmatrix} + \sum_{k=1}^{N_p} \begin{bmatrix}
\frac{\Sigma}{V_\infty} \\
\frac{\Delta V_x}{V_\infty}
\end{bmatrix}_{kj} \begin{bmatrix}
\begin{bmatrix}
\tau_{p_j i} \\
\nu_{p_j i}
\end{bmatrix} \\
\begin{bmatrix}
\begin{bmatrix}
\tau_{p_j i} \\
\nu_{p_j i}
\end{bmatrix} \\
\begin{bmatrix}
\begin{bmatrix}
\tau_{p_j i} \\
\nu_{p_j i}
\end{bmatrix} \\
\begin{bmatrix}
\begin{bmatrix}
\tau_{p_j i} \\
\nu_{p_j i}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\]

\[
(99)
\]
where

\[
\begin{align*}
\left( \frac{\Delta V_T}{V_\infty} \right)_{mn} &= \frac{1}{2} \left[ \frac{\Gamma_{Tm}(n-\Delta n)}{V_\infty} \left( \frac{\bar{v}_{Tmn} + \bar{v}_{Tm(n-1)}}{V_\infty} \right) + \frac{\Gamma_{Tm}(n+\Delta n)}{V_\infty} \left( \frac{\bar{v}_{TmN} + \bar{v}_{Tm(n+1)}}{V_\infty} \right) \right] + \left[ \frac{\Delta V_M}{V_\infty} \right]_{mn} \\
&= (100)
\end{align*}
\]

and where

\[
\frac{\Gamma_{Tm}(n-\Delta n)}{V_\infty}
\]

and

\[
\frac{\Gamma_{Tm}(n+\Delta n)}{V_\infty}
\]

are computed the same as for the body except

\[
\sum_{r=1}^{N_a} \left[ \frac{K_m(n-\Delta n)}{V_\infty} \right]_p
\]

38
represent the contributions from the trailing legs of the panel vortices along the juncture line of the jth panel and the rth panel to be attached at the lateral stations, 

\[(n - \frac{\Delta n}{2})\) and \((n + \frac{\Delta n}{2})\), respectively.

The surface pressure coefficients at each of the control points on the bodies and panels are then computed using the following expression.

\[
C_{p_{mn}} = \frac{2}{\gamma M_\infty^2} \left[ \left\{ 1 + \frac{\gamma-1}{2} M_\infty^2 \left[ 1 - \left( \frac{V_M}{V_\infty} \right)_{mn}^2 \left( \frac{V_T}{V_\infty} \right)_{mn}^2 \right] \right\}^{\gamma/(\gamma-1)} \right]
\]

where \(\gamma\) is the ratio of specific heats.

Note, in equations (97) and (100) the top sign is used to compute the velocity on the upper surface and the bottom sign the lower surface in those terms which have a plus and minus in front.
Section and Total Loads and Moments

The section loads are computed in coefficient form on each of the bodies by interpolating for the surface pressure coefficients at the centroid of the subareas, defined by the corner points of the divisions of the body subpanels, and then using the following equations to sum the product of the pressure coefficients and directed subareas.

\[
\begin{align*}
\left( \frac{C_x W}{W_{AVG}} \right)_K &= -\frac{1}{A_Z \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{p_iK} \Delta A_{X_{iK}} \\
\left( \frac{C_y h}{h_{AVE}} \right)_K &= -\frac{1}{A_Y \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{p_iK} \Delta A_{Y_{iK}} \\
\left( \frac{C_z W}{W_{AVG}} \right)_K &= -\frac{1}{A_Z \Delta \left( \frac{X_B}{C_B} \right)_K} \sum_i C_{p_iK} \Delta A_{Z_{iK}}
\end{align*}
\]

(102)

(103)

(104)

where \( i \) is indexed over all subareas in the longitudinals segment \( \Delta (X_B/C_B)_K \).

\[
A_X = \frac{1}{2} \sum_K \sum_i |\Delta A_{X_{iK}}| 
\]

(105)

\[
A_Y = \frac{1}{2} \sum_K \sum_i |\Delta A_{Y_{iK}}| 
\]

(106)
\[ A_z = \frac{1}{2} \sum_{K} \sum_{i} |\Delta A_{ziK}| \]  

(107)

Also;

\[ W_{AVG} = \frac{A_z}{C_B} \]  

(108)

and

\[ h_{AVG} = \frac{A_y}{C_B} \]  

(109)

The total loads on a body are then obtained by summing in the longitudinal direction.

\[ C_{X_Bj} = A_z \sum_{K} J \left( \frac{C_x W}{W_{AVG}} \right) K \left( \frac{X_B}{C_B} \right) K \]  

(110)

\[ C_{Y_Bj} = A_y \sum_{K} J \left( \frac{C_y h}{h_{AVG}} \right) K \left( \frac{X_B}{C_B} \right) K \]  

(111)

\[ C_{Z_Bj} = A_z \sum_{K} J \left( \frac{C_z W}{W_{AVG}} \right) K \left( \frac{X_B}{C_B} \right) K \]  

(112)
The total moments on a body are summed about the center of gravity.

\[
C_{X_{B_{j}}} = - \frac{1}{A_{B_{j}} C} \sum_{K} \sum_{i} \left[ (Y_{iK} - Y_{C.G.}) \Delta A_{z_{ik}} - (Z_{iK} - Z_{C.G.}) \Delta A_{y_{ik}} \right] C_{P_{ik}}
\]

(113)

\[
C_{Y_{B_{j}}} = - \frac{1}{A_{B_{j}} C} \sum_{K} \sum_{i} \left[ (Z_{iK} - Z_{C.G.}) \Delta A_{x_{ik}} - (X_{iK} - X_{C.G.}) \Delta A_{z_{ik}} \right] C_{P_{ik}}
\]

(114)

\[
C_{Z_{B_{j}}} = - \frac{1}{A_{B_{j}} C} \sum_{K} \sum_{i} \left[ (X_{iK} - X_{C.G.}) \Delta A_{y_{ik}} - (Y_{iK} - Y_{C.G.}) \Delta A_{x_{ik}} \right] C_{P_{ik}}
\]

(115)

The body center of pressure position vector components divided by \( C \) are given by;

\[
\left( \frac{X}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{Y_{B_{j}}} C_{M_{Z_{B_{j}}}} - C_{Z_{B_{j}}} C_{M_{Y_{B_{j}}}}}{C_{X_{B_{j}}}^2 + C_{Y_{B_{j}}}^2 + C_{Z_{B_{j}}}^2}
\]

(116)

\[
\left( \frac{Y}{C} \right)_{C.P.} = \left( \frac{Y}{C} \right)_{C.G.} + \frac{C_{Z_{B_{j}}} C_{M_{X_{B_{j}}}} - C_{X_{B_{j}}} C_{M_{Z_{B_{j}}}}}{C_{X_{B_{j}}}^2 + C_{Y_{B_{j}}}^2 + C_{Z_{B_{j}}}^2}
\]

(117)
\[
\left( \frac{Z}{C} \right)_{\text{C.P.}} = \left( \frac{Z}{C} \right)_{\text{C.G.}} + \frac{C_{X_B} C_{M_Y_B_j} - C_{Y_B} C_{M_X_B_j}}{C_{X_B}^2 + C_{Y_B}^2 + C_{Z_B}^2} \]  

(118)

The panel section loads, moments, and centers of pressure relative to the leading edge are obtained by numerically evaluating the following integrals.

\[
\frac{C_X}{C_{\text{AVG}}} = \frac{C b_S AR}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}} \left[ C_{P_L} \tan \phi - C_{P_U} \tan \phi \right] \sin \phi \, d\phi - \left( \frac{C_T}{C_{\text{AVG}}} \right) \]  

(119)

\[
\frac{C_Y}{C_{\text{AVG}}} = \frac{C b_S AR N_{Y_p}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}} \left( C_{P_L} - C_{P_U} \right) \sin \phi \, d\phi \]  

(120)

\[
\frac{C_Z}{C_{\text{AVG}}} = \frac{C b_S AR N_{Z_p}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}} \left( C_{P_L} - C_{P_U} \right) \sin \phi \, d\phi \]  

(121)

where

\[
b_S = \sum_{N=1}^{N_N} \sqrt{\Delta Y_{N_{L.E.}}^2 + \Delta Z_{N_{L.E.}}^2} \]  

(122)

\[
\Delta S = \sqrt{\Delta Y_{mN}^2 + \Delta Z_{mN}^2} = \text{local panel width} \]  

(123)
\[ \Delta S_{L.E.} = \sqrt{\Delta Y_{L.E.}^2 + \Delta Z_{L.E.}^2} = \text{panel width at leading edge.} \] (124)

and \( \phi = \cos^{-1} [1 - 2 (X/C)] \) where \((X/C)\) is the local percent chord.

Also;

\[
\tan \sigma = \frac{\tan \delta - \frac{dZ_t}{dX} - \frac{dZ_c}{dX} + \epsilon}{1 - \left( \frac{dZ_t}{dX} - \frac{dZ_c}{dX} + \epsilon \right) \tan \delta}
\] (125)

where the top sign is used with the upper surface and the bottom sign is used with the lower surface. The control surface angle is equal to \(-\delta_K\) along the leading edge flap and equal to \(\delta_F\) along a trailing control surface. \(\delta\) is equal to zero at other points on the chord.

The section normal force coefficient is obtained by taking the following scalar product.

\[
\frac{C_N}{C_{AVG}} = \frac{C_Y}{C_{AVG}} N_Y + \frac{C_Z}{C_{AVG}} N_Z
\] (126)

The section moment coefficients about the leading edge are computed as follows.

\[
\frac{C_{M_x}}{C_{AVG}} = \frac{AR b_s^2}{2 b^2} \int_0^\infty \frac{\Delta S}{\Delta S_{L.E.}} \left( (Y - Y_{L.E.}) N_{Y_{p_j}} - (Z - Z_{L.E.}) N_{Z_{p_j}} \right) \left[ c_{p_L} - c_{p_U} \right] \sin \phi \, d\phi
\] (127)

\[
\frac{C_{M_y}}{C_{AVG}} = \frac{AR b_s^2}{2 b^2} \int_0^\infty \frac{\Delta S}{\Delta S_{L.E.}} \left[ (Z - Z_{L.E.}) \left( c_{p_L \tan \sigma_L - c_{p_U \tan \sigma_U}} \right) - (X - X_{L.E.}) N_{Z_{p_j}} \left( c_{p_L} - c_{p_U} \right) \right] \sin \phi \, d\phi
\] (128)
The section center of pressure relative to the leading edge is given by;

\[
\begin{align*}
\left( \frac{X}{C} \right)_{C.P.} &= \frac{\left( \frac{C_Y}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,Z}}{C_{AVG}} \right) C - \left( \frac{C_Z}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,Y}}{C_{AVG}} \right) C}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2} \\
\left( \frac{Y}{C} \right)_{C.P.} &= \frac{\left( \frac{C_Z}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,X}}{C_{AVG}} \right) C - \left( \frac{C_X}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,Z}}{C_{AVG}} \right) C}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2} \\
\left( \frac{Z}{C} \right)_{C.P.} &= \frac{\left( \frac{C_X}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,Y}}{C_{AVG}} \right) C - \left( \frac{C_Y}{C_{AVG}} \right) \left( \frac{C_{M,L.E.,X}}{C_{AVG}} \right) C}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2}
\end{align*}
\]

The section zero percent suction drag coefficient

\[
\frac{C_{d_{T=0}}}{C_{AVG}}
\]
the section induced drag due to lift coefficient

\[
\frac{C_{d_L}}{C_{AVG}}
\]

the section leading edge thrust coefficient \( C_T \) \( C/C_{AVG} \), and the section induced drag due to thickness coefficient

\[
\frac{C_{d_T}}{C_{AVG}}
\]

are derived in appendix C.

The panel total force coefficients are obtained by numerically integrating the section force coefficients in the spanwise direction.

\[
C_{x_{P_j}} = \int_0^{\pi/2} \frac{C_x}{C_{AVG}} \sin \theta \ d\theta \quad (133)
\]

\[
C_{y_{P_j}} = \int_0^{\pi/2} \frac{C_y}{C_{AVG}} \sin \theta \ d\theta \quad (134)
\]

\[
C_{z_{P_j}} = \int_0^{\pi/2} \frac{C_z}{C_{AVG}} \sin \theta \ d\theta \quad (135)
\]

where

\[
\theta = \cos^{-1} \eta \quad (136)
\]
The panel total moment coefficients about the center of gravity are numerically integrated as follows.

\[ C_{M_{p_j}} = -\frac{AR b_s}{2 \pi b_c^2} \int_{0}^{\pi/2} \int_{0}^{\pi} \frac{\Delta S}{\Delta S_{L,E}} \left[ \frac{N_{p_j}}{C_{p_j}} \right] \left[ (Z_{C.G.})_p - (Z_{C.G.})_u \right] \left( C_{p_L} - C_{p_U} \right) \sin \theta \sin \phi \, d\phi \, d\theta \]  

(137)

\[ C_{N_{p_j}} = -\frac{AR b_s}{2 \pi b_c^2} \int_{0}^{\pi/2} \int_{0}^{\pi} \frac{\Delta S}{\Delta S_{L,E}} \left[ (Z_{C.G.})_p \tan \phi \tan \theta - (X_{C.G.})_p \right] \left( C_{p_L} - C_{p_U} \right) \sin \phi \cos \theta \left( (Y_{C.G.})_p \right) \left( C_{F_{C_{AVG}}} \right) \sin \theta \, d\phi \]  

(138)

\[ C_{L_{p_j}} = -\frac{AR b_s}{2 \pi b_c^2} \int_{0}^{\pi/2} \int_{0}^{\pi} \frac{\Delta S}{\Delta S_{L,E}} \left[ (X_{C.G.})_p \left( C_{p_L} - C_{p_U} \right) - (Y_{C.G.})_p \sin \phi \right] \sin \phi \cos \theta \left( (X_{C.G.})_p \right) \left( C_{F_{C_{AVG}}} \right) \sin \theta \, d\phi \]  

(139)

The panel total zero suction drag, near field induced drag due to lift, leading edge thrust, and near field induced drag due to thickness are given by:

\[ C_{D_{T=0} p_j} = \int_{0}^{\pi/2} C_{d_{T=0}} \frac{C}{C_{AVG}} \sin \theta \, d\theta \]  

(140)

\[ C_{D_{L_{i} p_j}} = \int_{0}^{\pi/2} C_{d_{L_{i}}} \frac{C}{C_{AVG}} \sin \theta \, d\theta \]  

(141)

\[ C_{T_{p_j}} = \int_{0}^{\pi/2} C_{T} \frac{C}{C_{AVG}} \sin \theta \, d\theta \]  

(142)
\[ C_{DTi,p_j} = \int_0^{\pi/2} \frac{C_{d_{Ti}} C}{C_{AVG} \sin \theta} \, d\theta \]  

The panel center of pressure position vector components divided by \( \overline{C} \) are computed as follows.

\[
\left( \frac{X}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{Y_{P_j}} C_{M_{Z_{P_j}}} - C_{Z_{P_j}} C_{M_{Y_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} 
\]

\[
\left( \frac{Y}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{Z_{P_j}} C_{M_{X_{P_j}}} - C_{X_{P_j}} C_{M_{Z_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} 
\]

\[
\left( \frac{Z}{C} \right)_{C.P.} = \left( \frac{Z}{C} \right)_{C.G.} + \frac{C_{X_{P_j}} C_{M_{Y_{P_j}}} - C_{Y_{P_j}} C_{M_{X_{P_j}}}}{C_{X_{P_j}}^2 + C_{Y_{P_j}}^2 + C_{Z_{P_j}}^2} 
\]

\[
48
\]
The section loads, moments, and center of pressure for the control surfaces are computed as follows.

The equation for the trailing edge of leading edge control surface:

\[ X_k = X_{k_i} + \left( \frac{X_{k_o} - X_{k_i}}{\eta_{k_o} - \eta_{k_i}} \right) (\eta - \eta_{k_i}) \]  \hspace{1cm} (147)

The equation for the leading edge of trailing edge control surface:

\[ X_f = X_{f_i} + \left( \frac{X_{f_o} - X_{f_i}}{\eta_{f_o} - \eta_{f_i}} \right) (\eta - \eta_{f_i}) \]  \hspace{1cm} (148)

The equation for a control surface hinge line:

\[ X_h = X_{h_i} + \left( \frac{X_{h_o} - X_{h_i}}{\eta_{h_o} - \eta_{h_i}} \right) (\eta - \eta_{h_i}) \]  \hspace{1cm} (149)
The unit vector in the direction of the hinge line is given by:

\[
\hat{h} = \left[ X_h \left( \eta + \frac{\Delta \eta}{2} \right) - X_h \left( \eta - \frac{\Delta \eta}{2} \right) \right] \hat{i} + \left[ Y_h \left( \eta + \frac{\Delta \eta}{2} \right) - Y_h \left( \eta - \frac{\Delta \eta}{2} \right) \right] \hat{j} + \left[ Z_h \left( \eta + \frac{\Delta \eta}{2} \right) - Z_h \left( \eta - \frac{\Delta \eta}{2} \right) \right] \hat{k}
\]

\[
+ \left[ X_h \left( \eta + \frac{\Delta \eta}{2} \right) - X_h \left( \eta - \frac{\Delta \eta}{2} \right) \right]^2 \left[ Y_h \left( \eta + \frac{\Delta \eta}{2} \right) - Y_h \left( \eta - \frac{\Delta \eta}{2} \right) \right]^2 \left[ Z_h \left( \eta + \frac{\Delta \eta}{2} \right) - Z_h \left( \eta - \frac{\Delta \eta}{2} \right) \right]^2 \right]^{1/2}
\]

(150)

The section loads on a leading edge control surface are obtained by numerically evaluating the following integrals:

\[
\frac{C_{h_x}}{C_{AVG}} = \frac{C_{AR} b_s}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{p_L} \tan \alpha_L - C_{p_U} \tan \alpha_U \right) \sin \phi_k \, d\phi_k - \frac{C_{TC}}{C_{AVG}}
\]

(151)

\[
\frac{C_{h_y}}{C_{AVG}} = \frac{C_{AR} b_s N_{y_p}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{p_L} - C_{p_U} \right) \sin \phi_k \, d\phi_k
\]

(152)

\[
\frac{C_{h_y}}{C_{AVG}} = \frac{C_{AR} b_s N_{z_p}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{p_L} - C_{p_U} \right) \sin \phi_k \, d\phi_k
\]

(153)
where

$$\phi_k = \cos^{-1} \left[ 1 - 2 \left( \frac{C_k}{C} \right) \left( \frac{X}{C} \right) \right]$$  \hspace{1cm} (154)$$

and $C_k$ is the chord of the leading edge control surface. $(X/C)$ is the local percent chord of the panel.

The section loads on a trailing edge control surface are obtained by numerically evaluating the following integrals.

$$\frac{C_h}{C_{AVG}} = \frac{C_fAR b_s}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( c_{PL} \tan \alpha_L - c_{PU} \tan \alpha_U \right) \sin \phi_f \, d\phi_f$$  \hspace{1cm} (155)$$

$$\frac{C_h}{C_{AVG}} = \frac{C_fAR b_s N_y}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( c_{PL} - c_{PU} \right) \sin \phi_f \, d\phi_f$$  \hspace{1cm} (156)$$

$$\frac{C_h}{C_{AVG}} = \frac{C_fAR b_s N_z}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( c_{PL} - c_{PU} \right) \sin \phi_f \, d\phi_f$$  \hspace{1cm} (157)$$

where

$$\phi_f = \cos^{-1} \left[ 1 - 2 \left( \frac{C_f}{C} \right) \left( \frac{X + C_f - C}{C} \right) \right]$$  \hspace{1cm} (158)$$

and $C_f$ is the chord of the trailing edge control surface.
The section normal load on the control surface is given by:

\[
\frac{C_{hN}}{C_{AVG}} = \left(\frac{C_{hY}}{C_{AVG}}\right)N_{h_p} + \left(\frac{C_{hZ}}{C_{AVG}}\right)N_{h_p} \tag{159}
\]

The section moments about the hinge line for the leading edge control surface are obtained by numerically evaluating the following integrals.

\[
\frac{C_{hL}}{C_{AVG}} = \frac{C_{k AR b_s}}{2b_s^2 C_{AVG}} \int_0^\pi \frac{\Delta S_{L,E.}}{\Delta S_{L,E.}} \left[ (Y - Y_h) N_{h_p} - (Z - Z_h) N_{h_p} \right] \left(C_{p_L} - C_{p_U}\right) \sin \phi_k \, d\phi_k \tag{160}
\]

\[
\frac{C_{hL}}{C_{AVG}} = \frac{C_{k AR b_s}}{2b_s^2 C_{AVG}} \int_0^\pi \frac{\Delta S_{L,E.}}{\Delta S_{L,E.}} \left[ (Z - Z_h) \left(C_{p_L} \tan \alpha_L - C_{p_U} \tan \alpha_U\right) - (X - X_h) N_{h_p} \left(C_{p_L} - C_{p_U}\right) \right] \sin \phi_k \, d\phi_k \tag{161}
\]

\[
\frac{C_{hL}}{C_{AVG}} = \frac{C_{k AR b_s}}{2b_s^2 C_{AVG}} \int_0^\pi \frac{\Delta S_{L,E.}}{\Delta S_{L,E.}} \left[ (X - X_h) N_{h_p} \left(C_{p_L} - C_{p_U}\right) - (Y - Y_h) \left(C_{p_L} \tan \alpha_L - C_{p_U} \tan \alpha_U\right) \right] \sin \phi_k \, d\phi_k \tag{162}
\]

The section moments about the hinge line for the trailing edge control surface are obtained by numerically evaluating the following integrals.

\[
\frac{C_{hT}}{C_{AVG}} = \frac{C_{k AR b_s}}{2b_s^2 C_{AVG}} \int_0^\pi \frac{\Delta S_{L,E.}}{\Delta S_{L,E.}} \left[ (Y - Y_h) N_{h_p} - (Z - Z_h) N_{h_p} \right] \left(C_{p_L} - C_{p_U}\right) \sin \phi_k \, d\phi_k \tag{163}
\]
The section hinge moment is computed by the following equation.

\[
\frac{C_{M_h}}{C_{AVG}} = \left(\frac{C_{M_{h_x}}}{C_{AVG}}\right)_{h_x} + \left(\frac{C_{M_{h_y}}}{C_{AVG}}\right)_{h_y} + \left(\frac{C_{M_{h_z}}}{C_{AVG}}\right)_{h_z}
\]  

(166)

The section center of pressure, due to the control surface loading, relative to the leading edge is given by the following expressions.

\[
\left(\frac{X}{C}\right)_{C.P.} = \left(\frac{X}{C}\right)_{h} + \frac{C}{C_{AVG}} \left(\frac{C_{h_y}}{C_{AVG}}\right)^2 \left(\frac{C_{M_{h_x}}}{C_{AVG}}\right) + \left(\frac{C_{h_y}}{C_{AVG}}\right)^2 \left(\frac{C_{M_{h_z}}}{C_{AVG}}\right)
\]  

(167)
The total loads and hinge moment on the control surface are given by;

\[
\frac{y}{C}_{\text{C.P.}} = \left( \frac{y}{C} \right)_h + \frac{C}{C} \left[ \frac{C_{h_x} C}{C_{x_{AVG}}} - \frac{C_{h_y} C}{C_{y_{AVG}}} \right] + \frac{C}{C} \left[ \frac{C_{h_z} C}{C_{z_{AVG}}} \right]
\]

(168)

\[
\frac{z}{C}_{\text{C.P.}} = \left( \frac{z}{C} \right)_h + \frac{C}{C} \left[ \frac{C_{h_x} C}{C_{x_{AVG}}} + \frac{C_{h_y} C}{C_{y_{AVG}}} + \frac{C_{h_z} C}{C_{z_{AVG}}} \right]
\]

(169)

\[
C_{h_x} = \int_{\eta_i}^{\eta_f} \frac{C_h C}{C_{x_{AVG}}} d\eta
\]

(170)

\[
C_{h_y} = \int_{\eta_i}^{\eta_f} \frac{C_h C}{C_{y_{AVG}}} d\eta
\]

(171)

\[
C_{h_z} = \int_{\eta_i}^{\eta_f} \frac{C_h C}{C_{z_{AVG}}} d\eta
\]

(172)
The total loads and moments for the complete configuration are then given by the following equations.

\[ C_X = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{X_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_{j}} C_{X_{P_j}} \]  
(174)

\[ C_Y = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{Y_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_{j}} C_{Y_{P_j}} \]  
(175)

\[ C_Z = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} C_{Z_{B_j}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} F_{S_{j}} C_{Z_{P_j}} \]  
(176)

\[ C_{M_X} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{\bar{C}_{B_j}}{C} C_{M_{X_{B_j}}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} \frac{\bar{C}_{P_j}}{C} F_{S_{j}} C_{M_{X_{P_j}}} \]  
(177)

\[ C_{M_Y} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{\bar{C}_{B_j}}{C} C_{M_{Y_{B_j}}} + \sum_{j=1}^{N_P} \frac{A_{P_j}}{A_R} \frac{\bar{C}_{P_j}}{C} F_{S_{j}} C_{M_{Y_{P_j}}} \]  
(178)
The induced drag for the total configuration $C_D$, is computed in the Treffitz plane with equations derived in Appendix F. The center of pressure for the total configuration is given by the following equations.

\[
C_M = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \overline{C}_{B_j} + \sum_{j=1}^{N_p} \frac{A_{P_j}}{A_R} \overline{C}_{P_j} F_{S_j} C_{M_{P_j}}
\]  

(179)

\[
\begin{align*}
\left( \frac{X}{C_R} \right)_{C.P.} &= \left( \frac{X}{C_R} \right)_{C.G.} + \frac{C_Y C_{M_Z} - C_Z C_{M_Y}}{C_X^2 + C_Y^2 + C_Z^2} \\
\left( \frac{Y}{C_R} \right)_{C.P.} &= \left( \frac{Y}{C_R} \right)_{C.G.} + \frac{C_Y C_{M_X} - C_X C_{M_Y}}{C_X^2 + C_Y^2 + C_Z^2} \\
\left( \frac{Z}{C_R} \right)_{C.P.} &= \left( \frac{Z}{C_R} \right)_{C.G.} + \frac{C_Y C_{M_X} - C_X C_{M_Y}}{C_X^2 + C_Y^2 + C_Z^2}
\end{align*}
\]

(180) 

(181) 

(182)

$F_{S_j}$ is a symmetry indicator which is equal to 2.0 in equations (174), (176), and (178) and equal to 0.0 in equations (175), (177), and (179) when the panel has an image.
COMPUTER PROGRAM RESULTS

Airfoil Section Velocities

In order to establish the degree of accuracy that can be obtained by the use of a source-vortex lattice procedure with second order corrections to account for the interference between lift and thickness and to account for the fact that the boundary conditions and the perturbation velocities are satisfied and computed, respectively, on the chordal plane, the results from the program have been compared with two dimensional exact solutions for a Karman-Trefftz airfoil in figures (4) and (5). Also, comparisons are made with data for a forty-five degree swept wing with an aspect ratio of five and a taper ratio of one. The airfoil section on this wing is a twelve percent thick R.A.E. 101. These comparisons are in figure (6), (7), and (8) for zero angle of attack and in figures (9), (10), and (11) at 4.2 degrees angle of attack. For all of the above cases discrete solutions were obtained with twenty subpanels in the chordwise direction and ten subpanels in the spanwise direction. Both the upper and lower surface pressures were plotted for the zero angle of attack data to indicate the degree of accuracy involved with the test data. The wing section was symmetrical.

Section Induced Drag Due to Thickness

The section induced drag \( C_{d_{s_{t}}} C/C_{avg} \), or potential form drag due to thickness, is computed in the present program by means of a source lattice with equation (24) of Appendix F. The source lattice as shown in figure (12) agrees quite well with the exact solution by R. T. Jones for a sixty degree swept, ten percent thick biconvex section, taper ratio one, and aspect ratio six wing given in reference (51). It should be noted that Woodward's equations, derived in Appendix D, for the constant and linearly varying distributed source density panels can be superimposed to obtain the same solution, for the biconvex section, as obtained by Jones. Even though Woodward, in reference (40), only computes induced velocities at the centroid of trapezoidal panels, it was shown that the correct solution for the longitudinal perturbation velocity is also obtained at any spanwise location, including the edges.

If two taper ratio one semi-infinite swept panels are joined at their side edges to form a wing center section or kink the Woodward distributed source equations will give the same center section solution as obtained by Kuchemann and Weber in reference (2). In addition, they will also give the correct spanwise variation of the kink effect due to thickness, which must be obtained by semi-empirically determined interpolation curves in the solution by Kuchemann and Weber. The Woodward equations also treat the spanwise variation of thickness and other planform induced effects such as taper ratio, tip, and crank effects equally well. These same effects are also correctly treated by the distributed constant density trapezoidal panel equations.
Figure 4.- Karman-trefftz airfoil pressure distribution at 0.0 degree angle of attack.
Figure 5.- Karman-trefftz airfoil pressure distribution at 1.0 degree angle of attack.
Figure 6. Swept wing alone pressure distribution at 24.5 percent semi-span, 0.0 degree angle of attack.
Figure 7.- Swept wing alone pressure distribution at 65.3 percent semi-span, 0.0 degree angle of attack.
Figure 8. Swept wing alone pressure distribution at 89.8 percent semi-span, 0.0 degree angle of attack.
Figure 9 - Swept wing alone pressure distribution at 24.5 percent semi-span, 4.2 degrees angle of attack.
Figure 10. - Swept wing alone pressure distribution at 65.3 percent semi-span, 4.2 degrees angle of attack.
Figure 11.- Swept wing alone pressure distribution at 89.8 percent semi-span, 4.2 degrees angle of attack.
Figure 12: Spanwise distribution of potential form drag due to thickness.
derived by Hess and Smith in reference (22). In fact Woodward's constant density source panel influence equations are identical to those derived by Hess and Smith.

As pointed out by Kuchmann and Weber the perturbation velocity due to thickness for a taper ratio one finite aspect ratio swept wing can be divided into two parts; 1) the two dimensional infinite sheared solution, and 2) the kink and tip effects. The two dimensional solution does not produce any section potential form drag provided the airfoil sections are closed. However, the kink and tip effects do produce a section drag and thrust, respectively, which when integrated across the span will give zero drag for the complete wing. Other planform effects such as taper ratio and cranks will also produce a section drag due to thickness which also integrates to zero for the complete wing.

All of the above methods will give the correct spanwise distribution of section potential form drag due to thickness. However, as also pointed out by Kuchmann and Weber the source lattice does not give the correct edge effect right at the tip, kink, or crank for a finite number of source lines in the chordwise direction. However, since this effect is only a function of the chordwise component of the thickness distribution gradient and the value of the inverse gudermannian function, with its argument being the sweep of the source line, at the point where the perturbation velocity is being computed, this effect is easily added. The region on either side of the tip, kink, or crank which is not properly handled by the source lattice is a function of the number of source lines in the chordwise direction and for practical solutions, which require about twenty source lines per chord to represent the distribution of thickness, this region is of no significance. Therefore, due to the superior numerical efficiency associated with the source lattice influence equations the source lattice appears to be the best aerodynamic finite element for predicting the perturbation velocity and potential form drag due to thickness.

Section Induced Drag Due to Lift

There are two basic approaches that have been tried in the past to solve the problem of predicting the spanwise distribution of induced drag or section potential form drag due to lift, 1) to accurately solve for the thin wing net pressure distribution, including the strength of the leading edge singularity, utilizing precise integration techniques to solve the aerodynamic influence integral equation, and 2) to utilize a vortex lattice procedure in conjunction with the Kutta-Joukowski theorem. Both of these approaches have failed to predict a spanwise distribution of induced drag due to lift which when integrated is equal to the induced drag computed in the Träfftz plane. The reason for this is that in these attempts the assumption
that the vorticity is constant in the spanwise direction along constant percent chord lines, even for only an infinitesimal distance, leads to a nonanalytic influence function for which no finite value exists for the induced velocity at span stations where the gradients of the constant percent chord lines are discontinuous.

Therefore, at span stations where the constant percent chord lines are kinked or cranked the constant vorticity distributed panel procedures, such as Woodward's, give a logarithmic singularity in the downwash. The lifting surface theories, such as Multhopp's or Wagner's, also produce a logarithmic singularity in the downwash which cannot be handled. Wagner's theory, as given in reference (50), has been investigated in great detail and is commented on in appendix E.

In the case of the skewed vortex-lattice the downwash at the control point is not singular, however, the Cauchy principal value does not exist for the downwash on the vortex line at a span station where the vortex lines have discontinuous sweeps. Therefore, the Kutta-Joukowsky theorem will give an infinite section drag at these stations.

All of these problems can be eliminated by using an unswept horseshoe vortex lattice. It is proven in appendix F that if the bound vortex lines are all parallel and the horseshoe lattice is evenly spaced in the lateral direction, the integral of the spanwise distribution of induced drag and the induced drag computed in the Trefftz plane are identical for all planform shapes. This is also true for multiple lifting surfaces, provided they are all parallel, and for lifting surfaces with jet flaps.

It is not proposed that only unswept horseshoe vortex-lattice procedures be used to compute the net pressures or loads on wings of arbitrary shape. However, this appears to be the only numerical integration procedure known at this time which will always give the same value for the induced drag in the near and far fields. During studies of the error involved in using a skewed lattice it was determined that the error in the section induced drag was limited to a very small region on either side of the discontinuity in the sweep of the vortex lines. Also, since most of the wing is represented better by skewed vortex lines (because the lines of constant pressure do coincide with constant percent chord lines over most of the wing) a good overall answer can probably be obtained at less expense, (since fewer skewed vortices are needed in general to represent a wing than unswept vortices) if a skewed vortex lattice is used to compute the net pressures and the unswept vortex lattice is used to compute the drag once the net pressures are known.
Figure 13.- Spanwise distribution of potential form drag due to lift.
A comparison of the section induced drag divided by the section lift four different procedures is shown in figure (13).

Sphere Surface Velocity

The velocity over the surface of a sphere is compared to the exact solution in figure (14).

X-15 Wing-Fuselage-Horizontal Tail-Vertical Tail

Surface velocities and pressure coefficients, section force and moment coefficients, and total configuration force and moment coefficients are computed for the X-15 wing-fuselage-horizontal tail-vertical tail shown in figure (15). The program input for this configuration plus the ventral is given in the sample input section. The program output for this configuration is given in the sample output section. In the program output the fuselage, wing, horizontal tail and vertical tail are designated as components 1, 2, 3, and 4, respectively.

Some results for this configuration are shown in figures (16), (17), (18), and (19). The data for these comparisons were obtained from References (59), (60), and (61). The force data is at .6 Mach number and the pressure coefficient data is at .2 Mach number. All of the theoretical results are for zero Mach number.

The total configuration $C_{L\alpha}$ was determined experimentally to be .061. The program predicts .0617.
Figure 14 - Velocity ratio around a sphere.
Figure 16. - Normal Load Distribution on X-15 Fuselage at Five Degrees Angle of Attack
Figure 17. - Unit Span Load Distribution on X-15 Wing
Figure 18.- Unit Span Load Distribution on X-15 Horizontal Tail
Figure 19. - Chordwise Pressure Distribution on X-15 Wing at 17.8 Percent Semispan at Five Degrees Angle of Attack


COMPUTER PROGRAM USAGE

Program Setup

There are two types of data decks which can be used in this program. 1) the NASA Langley input format described in reference (52) and 2) the NR input format described in the next two sections of this report. Each of these input formats requires a different program setup.

Program setup for NASA format. - In this form of data setup, routine "OUTIN" preprocesses data from Langley's format and creates a card image file for use by the rest of the subroutines. Changes or additions to the data associated with bodies and panels in the Langley form of input created by "OUTIN" are made by body and panel "INFO" decks. There should be a body and panel "INFO" deck for each body and panel in the Langley input array, respectively. These body and panel "INFO" decks utilize the NR data array format.

Additional configuration bodies and panels not described in the Langley data array are added by means of additional body and panel input decks utilizing the NR data array format.

Most of the input from the Langley data array can be directly converted to the NR data array format. The only exception is a nacelle which is converted from a body of revolution to a ring wing. This occurs if the pod data has a nonzero value at the nose of the pod. If the radius at the nose of the pod is zero, the pod is considered a solid body of revolution.

The input data in the NR data array format ahead of the body and panel descriptions pertain to the total configuration and must always be input as an "INFO" deck. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed in figure 21. Figure 20 contains the entire deck setup.
Figure 20.- Deck setup for use with NASA Langley data array format.
Figure 22. - Deck setup for use with NR data array format.
Program setup for NR format. - In this form of data setup all input decks utilize the NR data array format. The total configuration or universal "INFO" deck is followed by an input deck for each body and then a deck for each panel. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed for this form of data input in figure 21. Figure 22 contains the entire deck setup.

Card No.

1. SEQUENCE CARD
2. CHARGE CARD
3. PFSTAD, GM31000, T7777, P6
4. RUN, S.
5. SET, O.
6. LGO.

Figure 21.- Control card deck sequence

Input Format

The NR data array format is given in this section. Data locations of all input data with limits on size of inputs are designated. A more detailed description of the input is given in the next section.
### FORTRAN FIXED 10 DIGIT DECIMAL DATA

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<td><em>C</em> (reference longitudinal length)</td>
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<td><em>V.C.G.</em></td>
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<td><em>Z.C.G.</em></td>
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<td><em>Z.C.G.</em></td>
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<td><em>α</em> (angle of attack)</td>
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<td><em>θ</em> (angle of sideslip)</td>
<td>(deg)</td>
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<td><em>P</em> = <em>P</em>/(2<em>V</em>/<em>b</em>) (nondimensional roll rate)</td>
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<td><em>q</em> = <em>q</em>/(2<em>V</em>/<em>c</em>) (nondimensional pitch rate)</td>
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<td><em>r</em> = <em>r</em>/(2<em>V</em>/<em>b</em>) (nondimensional yaw rate)</td>
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<td><em>C</em> (Chord)</td>
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<td>Indicator for [A] matrix: (0.) compute new matrix and save on</td>
<td>tape; (-1.) bypass matrix calculation and read from tape</td>
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<td>Symm indicator: (0.) body and vortex strengths are symm about x-z plane; (1.) if only body is symm; (-1.) no symm</td>
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### FORTRAN FIXED 10 DIGIT DECIMAL DATA

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<td>Body coordinate table input indicator: (U) for ( \theta_i ) vs ( \xi_i ); ( \ell ) for ( \left( \frac{Z_{B1} - \Delta Z_{B1}}{Z_{B2} - \Delta Z_{B2}} \right) / \left( \frac{Y_{B1} - \Delta Y_{B1}}{Y_{B2} - \Delta Y_{B2}} \right) ) vs ( \xi_i ); Body multiplication factor indicator (0.) for ( Y_{BM} = Y_{BM} = \frac{\gamma_{BM}}{Y_{BM}} ) (1.) ( Y_{MB} = Y_{MB} ) ( \gamma_{MB} ) ( \gamma_{MB} ). Body camber indicator (0.) body section put 1 to camber line (1.) body section put 1 to body x axis. Body longitudinal subpanel indicator: (0.) even ( \Delta \phi ); (1.) even ( \Delta X ); (1.) given ( x ). Body lateral vertex indicator: (0.) even ( \Delta \phi ); (1.) even ( \Delta X ); given ( \theta ) or ( S/S_{\text{max}} ); (1.) same as body definition stations.</td>
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<td>(max 39)</td>
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<td>( JS_{N_1} )</td>
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<td>3 1 9 0</td>
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<td>( JS_{N_2} )</td>
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<td>[ 1. \frac{1}{1 + \left[ Y_B (N_X B/N_Y B) + Z_B (N_X B/N_Z B) \right]^2 / (Y_B^2 + Z_B^2) }^{1/2} ]</td>
<td>[ 2. \frac{1}{1 + \left[ Y_B (N_X B/N_Y B) + Z_B (N_X B/N_Z B) \right]^2 / (Y_B^2 + Z_B^2) }^{1/2} ]</td>
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<td>3 2 1 5</td>
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<td>[ 3. \cot (\phi_B / 2) ]</td>
<td>[ 4. \cot \left[ (\pi/2) - (\phi_B / 2) \right] ]</td>
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<td>[ 5. \sin \left[ \pi (\phi_B - \phi_0) / (\phi_f - \phi_0) \right] ]</td>
<td>[ 6. \cos \left[ \pi (\phi_B - \phi_0) / (\phi_f - \phi_0) \right] ]</td>
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<td>[ 7. \left[ (X_B / C_B) - (X_B / C_B)_o \right] / [ (X_B / C_B)_f - (X_B / C_B)_o ] ]</td>
<td>[ 8. \sin \left[ 2 \pi (\phi_B - \phi_0) / (\phi_f - \phi_0) \right] ]</td>
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<td>[ 9. \cos \left[ 2 \pi (\phi_B - \phi_0) / (\phi_f - \phi_0) \right] ]</td>
<td>[ 10. \left[ (X_B / C_B) - (X_B / C_B)_o \right] / [ (X_B / C_B)_f - (X_B / C_B)_o ] ]</td>
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<td>$N_{ZB}/[N_{-3}^2 + N_{YB}^2]^{1/2}$</td>
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<td>49</td>
<td>$N_{YB}/[N_{ZB}^2 + N_{YB}^2]^{1/2}$</td>
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<td>$\sin[\pi (\phi - \phi_0) / (\phi_f - \phi_0)]$</td>
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<td>61</td>
<td>$\cos[\pi (\phi - \phi_0) / (\phi_f - \phi_0)]$</td>
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<td>3255</td>
<td>$[\pi (\phi_B - \phi_0) / (\phi_f - \phi_0)]$ or $[(\eta_B - \eta_0) / (\eta_f - \eta_0)]$</td>
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<td>7.</td>
<td>$\sin[2\pi (\phi_B - \phi_0) / (\phi_f - \phi_0)]$</td>
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<td>9.</td>
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**Note:** Use the same format for special longitudinal constraint functions due to additional control surfaces. Start data sets at 4890, 4900, and etc.}
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<td>Total number of bodies used to represent the configuration.</td>
</tr>
<tr>
<td>2</td>
<td>Total number of panels used to represent the configuration. This number does not include those due to images when location 3426 has a zero in it.</td>
</tr>
<tr>
<td>3</td>
<td>Mach number</td>
</tr>
<tr>
<td>4</td>
<td>Total configuration reference area.</td>
</tr>
<tr>
<td>5</td>
<td>Total configuration longitudinal reference length.</td>
</tr>
<tr>
<td>6</td>
<td>Total configuration lateral reference length.</td>
</tr>
<tr>
<td>7</td>
<td>X component of center of gravity position vector.</td>
</tr>
<tr>
<td>8</td>
<td>Y component of center of gravity position vector.</td>
</tr>
<tr>
<td>9</td>
<td>Z component of center of gravity position vector.</td>
</tr>
<tr>
<td>10</td>
<td>Angle of attack.</td>
</tr>
<tr>
<td>11</td>
<td>Angle of sideslip.</td>
</tr>
<tr>
<td>12</td>
<td>Nondimensional roll rate $P/(2V_\infty/b)$.</td>
</tr>
<tr>
<td>13</td>
<td>Nondimensional pitch rate $q/(2V_\infty/c)$.</td>
</tr>
<tr>
<td>14</td>
<td>Nondimensional yaw rate $\gamma/(2V_\infty/b)$.</td>
</tr>
<tr>
<td>15</td>
<td>The number of the component. The bodies are numbered first and then the panels.</td>
</tr>
<tr>
<td>16</td>
<td>The body reference area.</td>
</tr>
<tr>
<td>17</td>
<td>The body chord.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>This is an indicator to bypass the calculation of the aero-dynamic influence matrices due to the vortices of this body. Input a zero to compute new matrices and a minus one if the calculation of the matrices is to be bypassed.</td>
</tr>
<tr>
<td>*19</td>
<td>This is a symmetry indicator used to take advantage of both symmetrical geometries and loadings. If both the body geometry and the vortex strength are symmetrical about the X-Z plane input a zero, if only the body geometry is symmetrical and not the vortex strengths input a one, and if neither symmetry exists input a minus one.</td>
</tr>
<tr>
<td>*20</td>
<td>This is an indicator used to signify whether cartesian or polar coordinates are used to input the body cross-sections. If polar coordinates are used input a zero and if cartesian coordinates are used input a one.</td>
</tr>
<tr>
<td>21</td>
<td>This is an indicator used to signify whether the lateral scaling factors are equal in the Y and Z directions. Input a zero if they are equal and a one if they are not equal. The $Z_{BM}$ array is not input if $Y_{BM} = Z_{BM}$.</td>
</tr>
<tr>
<td>*22</td>
<td>This indicator is used to signify whether the body cross-sections are placed perpendicular to the mean camber line or the X axis. Input a zero if the cross-section is to be placed perpendicular to the mean camber line or a one if it is to be placed perpendicular to the X axis.</td>
</tr>
<tr>
<td>23</td>
<td>This indicator signifies the type of subpanel spacing used on the body in the longitudinal direction. Input a zero if the subpanels are to be spaced at even increments of $\phi$, where $\phi = \cos^{-1} [1-2(x/c)]$, input a minus one if the subpanels are to be spaced at even increments of $X$, and input a one if the spacing is specified at a given set of X stations. The X stations are to be given in locations 1850 - 2149.</td>
</tr>
<tr>
<td>24</td>
<td>This indicator signifies the type of subpanel spacing used on the body in the lateral direction. Input a zero if the subpanels are to be spaced at even increments of $\theta$ or $n_B = (S/S_{max})$, a one if the subpanel side edges are given at a set of $\theta$ or $n_B$ stations, and a minus one if the subpanel edges are at the same lateral stations as the body geometry is defined. The subpanel side edges are input in locations 2200 - 2799.</td>
</tr>
</tbody>
</table>
* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>This is the longitudinal distance, in terms of the length of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a longitudinal distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.</td>
</tr>
<tr>
<td>26</td>
<td>This is the lateral distance, in terms of the width of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a lateral distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.</td>
</tr>
<tr>
<td>*27</td>
<td>This is the X component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>*28</td>
<td>This is the Y component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>*29</td>
<td>This is the Z component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>30</td>
<td>This is the number of subpanels in the longitudinal direction on the body.</td>
</tr>
<tr>
<td>31</td>
<td>This is the number of subpanels in the lateral direction on the body.</td>
</tr>
<tr>
<td>32</td>
<td>This is the number of equally spaced divisions the body subpanels are divided into in the longitudinal direction. These divisions are used to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
**Location** | **Description**
---|---
33 | This is the number of equally spaced divisions the body subpanels are divided into in the lateral direction. These divisions are used to map the vortex grid closer to the actual body surface and to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.
34 | This is the number of functions used to constrain the body surface vorticity in the longitudinal direction. The list of functions used is given in locations 3210-3249. These same functions are used over each longitudinal constraint function segment. The list defining these segments is given in locations 3140-3189. The maximum number of longitudinal constraint functions is 50.
35 | This is the number of functions used to constrain the body surface vorticity in the lateral direction. The list of functions used is given in locations 3250-3269. These same functions are used over each lateral constraint function segment. The list defining these segments is given in locations 3190-3209. The maximum number of lateral constraint functions is 20.
36 | This is the number of segments in the longitudinal direction over which the body longitudinal constraint functions are defined. The list of subpanels defining the segments is given in locations 3140-3189. The maximum number of segments is 50.
37 | This is the number of segments around the circumference of the body over which the lateral constraint functions are defined. The list of subpanels defining the segments is given in locations 3190-3209. The maximum number of segments is 20.
38 | This is the number of control points in the longitudinal direction on the body. The maximum number of points in the longitudinal direction is 299. The list of body control points in the longitudinal direction is given in location 2800-3099.

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>This is the number of control points in the lateral direction on the body. The maximum number of points in the lateral direction is 40. This list of control points is given in locations 3100 - 3139.</td>
</tr>
<tr>
<td>*40</td>
<td>This is the number of longitudinal stations where the body cross-sections are defined. The maximum number is 44.</td>
</tr>
<tr>
<td>*41-89</td>
<td>This is the list of longitudinal stations where the body cross-sections are defined. This list starts at the nose and ends at the aft end if the body is solid. For a flow through body this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.</td>
</tr>
<tr>
<td>*85</td>
<td>This is the number of longitudinal stations where the body cross-section lateral stations are defined. The maximum number of these longitudinal stations is 44.</td>
</tr>
<tr>
<td>*86-129</td>
<td>This is the list of longitudinal stations where the body cross-section lateral stations are defined. The body cross-section lateral stations are constant between each longitudinal station in this list. If this list is the same as that in locations 41-89, it can be omitted. If the body is solid, this list starts at the nose and ends at the aft end. If the body is a flow through type, this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.</td>
</tr>
<tr>
<td>*130</td>
<td>This is the number of lateral stations at the first longitudinal station in the list at locations 91-129. The maximum number of these lateral stations is 39.</td>
</tr>
<tr>
<td>*131</td>
<td>This is the list of lateral stations at the first longitudinal station in the list at locations 91-129. These lateral stations where the body is defined can be given by the angle $\theta$, the lateral fraction $[\frac{C_B X}{B_1}]$, or fraction of lateral circumferential length $S$. Note: Lists of lateral stations at the other longitudinal stations, given in the list in locations 91-129, follow this data as shown in the input format.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
In these locations the body cross-section radii or fractional distances \([Z_B - \Delta B Z_B]^N^M\), for each of the \(X_i\) longitudinal stations and \(\theta_i\) or \([C_B - \Delta B C_B]^N^M\) lateral stations, are input.

This is the number of longitudinal stations where the multiplication factors \(Y_B^M\) and \(Z_B^M\) and the translation increments \(\Delta Y\) and \(\Delta Z\) are given. The maximum number of stations that can be used here is 49.

This is the list of longitudinal stations where the multiplication factors \(Y_B^M\) and \(Z_B^M\) and the translation increments \(\Delta Y_b\) and \(\Delta Z_b\) are given.

This is the list of multiplication factors \(Z_B^M\) at the longitudinal stations given in locations 1601-1649.

This is the list of translation increments \(\Delta Y_b\) at the longitudinal stations given in locations 1601-1649.

This is the list of translation increments \(\Delta Z_b\) at the longitudinal stations given in locations 1601-1649.

This is the list of longitudinal stations where the body subpanel edges are defined. This list can be omitted if it is the same as that in locations 1850-2149.

This is the list of longitudinal stations where the body lateral subpanel edges are defined. This list can be defined by \(\theta_s\) or fractions of body circumference \(n_b = (S/S_{max})\).

Note: Lists of lateral subpanel edges at the other longitudinal stations, given in the list at locations 2151-2199, follow this input as shown in the input format.

* Items not needed if data format from reference (52) is used.
Location | Description
--- | ---
2800-3099 | This is the list of the number of the subpanels where the control points are located in the longitudinal direction on the body. The integer in this list indicates the number of the subpanel, aft of the nose of a solid body or aft of the tail end on the inner surface and around to the outer surface of a flow through body, at which a control point is placed. If no longitudinal constraint functions are used, this input can be omitted.

3100-3139 | This is the list of control point locations in the lateral direction on the body. The integer in this list indicates the number of the subpanel in the lateral direction from the top of the body at which a control point is placed. If no lateral constraint functions are used, this input can be omitted.

3140-3189 | This is the list of longitudinal constraint segment boundaries. The integers in this list indicate the subpanels between which the longitudinal constraint functions are applied. For example, if this list included the longitudinal number of every tenth subpanel, the longitudinal constraint functions would be applied over a range of ten subpanels and repeated over the segments defined by every tenth subpanel.

3190-3209 | This is the list of lateral constraint segment boundaries. This list is utilized in the same manner as the longitudinal constraint segment boundaries.

3210-3249 | This is the list of longitudinal constraint functions used on the body. The functions to be used are signified by entering here the sequence number of the function as given in the input format. If special functions are to be used the data location where the function is described is input here.

3250-3269 | This is the list of lateral constraint functions used on the body. The same procedure is used here to signify the desired functions as is used in the definition or the longitudinal constraint functions.

3270-3419 | In these locations the special longitudinal constraint functions are described as shown in the input format. These are described in tabular form.

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3420</td>
<td>This is the component number for the panels.</td>
</tr>
<tr>
<td>*3421</td>
<td>This is the panel reference area.</td>
</tr>
<tr>
<td>3422</td>
<td>This is the panel reference chord.</td>
</tr>
<tr>
<td>3423</td>
<td>This is the panel lateral reference length.</td>
</tr>
<tr>
<td>3425</td>
<td>This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this panel. Input a zero if a new matrix is computed and a minus one if the calculation of the matrix is to be bypassed.</td>
</tr>
<tr>
<td>*3426</td>
<td>This is a symmetry indicator used to take advantage of a configuration's symmetry to save input effort and computer time. Input a zero if both the panel geometry and the vortex strengths have images on the port side of the X-Z plane, a one if just the panel geometry has an image, a minus one if neither have an image, and a two if the vortex strengths on the panel and its image are antisymmetric.</td>
</tr>
<tr>
<td>3427</td>
<td>This indicator is used to signify whether the panel is attached to the positive normal or negative normal side of another panel. Input a zero if it is attached to the positive normal side and a one if it is attached to the negative normal side.</td>
</tr>
<tr>
<td>3428</td>
<td>This indicator is used to signify whether the wake is force free or fixed. Input a zero if the wake is fixed and a one if it is force free. If any panel has a force free wake, the wake or a body it is attached to is also assumed to be force free.</td>
</tr>
<tr>
<td>*3429</td>
<td>This indicator signifies whether the mean camber surface of the panel is described in terms or local angles or attack or as fractions of chord. Input a zero if local angles of attack are input and a one if fractions of chord are input.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3430</td>
<td>This input indicates the type of longitudinal subpanel spacing that is used. If the subpanels are spaced at even increments of ( \phi ) input a zero, if they are at equal increments of percent chord input a minus one, and if they are input as a given set of percent chord use a one.</td>
</tr>
<tr>
<td>3431</td>
<td>This input indicates the type of lateral subpanel spacing that is used on a panel. Input a zero if the spacing is at equal increments of ( \eta ), a minus one if it is at equal increments of ( \theta ), and a one if the spacing is specified at a given set of ( \eta ) stations.</td>
</tr>
<tr>
<td>*3432</td>
<td>This is the X component of the panel origin point.</td>
</tr>
<tr>
<td>*3433</td>
<td>This is the Y component of the panel origin point.</td>
</tr>
<tr>
<td>*3434</td>
<td>This is the Z component of the panel origin point.</td>
</tr>
<tr>
<td>3435</td>
<td>This is the area of influence of a quadrilateral vortex on a panel in the longitudinal direction. This input is in terms of fraction of quadrilateral vortex length.</td>
</tr>
<tr>
<td>3436</td>
<td>This is the area of influence of a vortex on a panel in the lateral direction. This input is in terms of fraction of vortex width.</td>
</tr>
<tr>
<td>3437</td>
<td>This is the number of subpanels in the longitudinal direction on the panel. The maximum number of longitudinal subpanels is 40.</td>
</tr>
<tr>
<td>3438</td>
<td>This is the number of subpanels in the lateral direction on the panel. The maximum number of lateral subpanels is 40.</td>
</tr>
<tr>
<td>3439</td>
<td>This is the number of longitudinal constraint functions on the panel. This number does not include special functions due to control surfaces. The maximum number is 15.</td>
</tr>
<tr>
<td>3440</td>
<td>This is the number of lateral constraint functions on the panel. This number does include special functions due to control surfaces. The maximum number is 20.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
3441 This is the number of control points in the longitudinal direction on the panel. The maximum number is 40.

3442 This is the number of control points in the lateral direction on the panel. The maximum number is 40.

3443 This is the number of points used to define the panel perimeter. In the root section the perimeter is defined by X, Y, and Z components of points along the leading and trailing edges. The outboard section is defined by X, Y, and Z components of points along the leading edge and the local chord. The root section leading edge is input first, then the outboard section, and then the root section trailing edge. The maximum number of points is 50.

3444 This input is the number of the body or panel to which this panel's inboard trailing vortex is attached.

3445 This is the number of the trailing vortex leg or subpanel side edge to which this panel's inboard trailing vortex is attached. The subpanel side edges are numbered consecutively starting at the top of a body or at the inboard edge of a panel and going in the clockwise direction when an observer is looking in the negative X direction.

3446 This is the number of the panel to which the outboard trailing vortex of this panel is attached.

3447 This is the number of the trailing vortex leg or subpanel side edge to which this panel's outboard trailing vortex is attached.

3448 This is the number of leading edge control surfaces on the panel.

3449 This is the number of trailing edge control surfaces on the panel.

*3450-3599 In these locations the panel perimeter is described.

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 3600</td>
<td>This is the number of longitudinal stations where the panel camber is described. The maximum number is 30.</td>
</tr>
<tr>
<td>*3601-3629</td>
<td>This is the list of chord fractions which define the chordwise locations where the panel camber is defined.</td>
</tr>
<tr>
<td>* 3630</td>
<td>This is the number of lateral stations where the panel camber and twist is defined. The maximum number is 30.</td>
</tr>
<tr>
<td>*3631-3659</td>
<td>This is the list of lateral surface length fraction ( \eta ) where the panel camber and twist is defined.</td>
</tr>
<tr>
<td>*3660-4099</td>
<td>This is the table of local angle of attack or deflection, in terms of fraction of chord, of the panel mean camber surface. The table is input as shown in the input format.</td>
</tr>
<tr>
<td>4100-4129</td>
<td>This is the table of panel twist.</td>
</tr>
<tr>
<td>* 4130</td>
<td>This is the number of longitudinal stations where the panel thickness is described. The maximum number is 30.</td>
</tr>
<tr>
<td>*4131-4159</td>
<td>This is the list of chord fractions which define the chordwise locations where the panel thickness is defined.</td>
</tr>
<tr>
<td>* 4160</td>
<td>This is the number of lateral stations where the panel thickness is defined. The maximum number is 30.</td>
</tr>
<tr>
<td>*4161-4189</td>
<td>This is the list of lateral surface length fraction ( \eta ) where the panel thickness is defined.</td>
</tr>
<tr>
<td>*4190-4599</td>
<td>This is the table of panel thickness in terms of fraction of chord. The table is input as shown in the input format.</td>
</tr>
<tr>
<td>4600-4639</td>
<td>This is the list of longitudinal subpanel edge locations in terms of fraction of chord.</td>
</tr>
<tr>
<td>4640-4679</td>
<td>This is the list of lateral subpanel side edge locations in terms of ( \eta ).</td>
</tr>
<tr>
<td>4680-4719</td>
<td>This is the list of panel control points in the longitudinal direction.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4720-4759</td>
<td>This is the list of panel control points in the lateral direction.</td>
</tr>
<tr>
<td>4760-4779</td>
<td>This is the list of exponents W for the standard lateral constraint functions or data locations of the special lateral constraint functions.</td>
</tr>
<tr>
<td>4780-4879</td>
<td>In these locations the special panel lateral constraint functions are described. The input is as shown in the input format.</td>
</tr>
<tr>
<td>4880 - ...</td>
<td>In these locations the special panel longitudinal constraint functions are described. The input is as shown in the input format.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>NO</th>
<th>X0</th>
<th>YQ</th>
<th>ZQ</th>
<th>VM/V</th>
<th>VT/V</th>
<th>CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>.885863</td>
<td>.887939</td>
<td>.590518</td>
<td>-.068040</td>
<td>.646659</td>
</tr>
<tr>
<td>2</td>
<td>5.909044</td>
<td>2.742717</td>
<td>2.642261</td>
<td>1.036237</td>
<td>-.096102</td>
<td>-.083023</td>
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<tr>
<td>3</td>
<td>14.023102</td>
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<td>4.410988</td>
<td>1.059708</td>
<td>-.100525</td>
<td>-.133086</td>
</tr>
<tr>
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<td>25.607729</td>
<td>6.823918</td>
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<td>-.099997</td>
<td>-.156986</td>
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<td>-.091717</td>
<td>-.141862</td>
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<tr>
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<td>58.564146</td>
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<td>11.314363</td>
<td>1.025605</td>
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<td>-.057008</td>
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<tr>
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<td>79.521490</td>
<td>13.655558</td>
<td>16.967221</td>
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<tr>
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<td>23.280504</td>
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<tr>
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<td>1.099374</td>
<td>-.170262</td>
<td>-.237512</td>
</tr>
<tr>
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<td>186.707727</td>
<td>23.454137</td>
<td>20.871473</td>
<td>1.047057</td>
<td>-.177496</td>
<td>-.127833</td>
</tr>
<tr>
<td>12</td>
<td>217.656459</td>
<td>26.221138</td>
<td>19.430254</td>
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<td>-.146017</td>
<td>-.129392</td>
</tr>
<tr>
<td>13</td>
<td>249.514952</td>
<td>28.514243</td>
<td>19.323167</td>
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</tr>
<tr>
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<td>281.682568</td>
<td>30.266977</td>
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<td>1.075833</td>
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<td>-.175454</td>
</tr>
<tr>
<td>15</td>
<td>314.352265</td>
<td>30.869521</td>
<td>19.323167</td>
<td>1.082484</td>
<td>-.113275</td>
<td>-.184570</td>
</tr>
<tr>
<td>16</td>
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(X/CR) CP  (Y/CR) CP  (Z/CR) CP
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CONCLUSIONS

The theory and program described herein are capable of predicting surface velocities and pressure coefficients and section, control surface, and total configuration loads and moments for a diverse class of airplanes. In particular the source - vortex lattice with second order corrections was shown to agree well with the exact solution for a Karman-Trefftz airfoil section. Also, the quadrilateral vortex element predicted surface velocities which agreed well with the exact solution for a sphere.

Because of the options built into the program simple configurations can be input with a minimum of effort while the capability exists to input complicated configurations in significant detail. The program is general enough to analyze complete configurations at angle of attack and in side slip, pitching motion, rolling motion, yawing motion, and with control surfaces deflected. The configuration can be run in each of these modes to obtain static and rotary stability derivatives or in a combination of the modes to predict the loads and moments on the configuration while in a quasi-steady maneuver. The program also has the capability to account for a free wake while operating in any of the above modes.

Within the scope of this study a significant effort was devoted toward understanding the limitations of and the relationships between the different types of aerodynamic finite elements and lifting surface theories available at present. It has been concluded that those used in this analysis are presently the most efficient and generally available. It was also shown that the spanwise variation of potential form drag due to both thickness and lift are correctly computed by the program described in this report.
Appendix A

NUMERICAL PROCEDURES

Discussions of the prime numerical procedures used within the program are given in this appendix. There are essentially three such procedures; (1) straight line interpolation and extrapolation, (2) controlled deviation interpolation, and (3) Householder's simultaneous equation solution.

For straight line interpolation and extrapolation about two given points \((X_1, Y_1)\) and \((X_2, Y_2)\);

\[
Y = \alpha Y_2 + (1 - \alpha) Y_1 \tag{1}
\]

where

\[
\alpha = \frac{X - X_1}{X_2 - X_1} \tag{2}
\]

The slope \(\frac{dY}{dX}\) for this case is given by;

\[
\frac{dY}{dX} = \alpha' (Y_2 - Y_1) \tag{3}
\]

where

\[
\alpha' = \frac{1}{X_2 - X_1} \tag{4}
\]

In the case of the controlled deviation interpolation method (CODIM) parabolae are used to curve fit a set of four given points \((X_{N-1}, Y_{N-1}), (X_N, Y_N), (X_{N+1}, Y_{N+1}),\) and \((X_{N+2}, Y_{N+2})\) to obtain interpolated \(Y\) and \(dY/dX\) values for \(X_N \leq x \leq X_{N+1}\). Only that information, relative to this method, which is necessary to judiciously pick input points will be discussed here. A complete derivation is given in reference (20).
One parabola \( P_1 \) is fit through \((X_{N-1}, Y_{N-1}), (X_N, Y_N), \) and \((X_{N+1}, Y_{N+1})\). The other parabola \( P_2 \) is fit through \((X_N, Y_N), (X_{N+1}, Y_{N+1}), \) and \((X_{N+2}, Y_{N+2})\). This curve fitting process involves the solution of two sets of simultaneous equations. If,

\[
P_1 = A_1 X^2 + B_1 X + C_1 \tag{5}
\]

and

\[
P_2 = A_2 X^2 + B_2 X + C_2 \tag{6}
\]

Then

\[
\begin{bmatrix}
A_1 \\
B_1 \\
C_1
\end{bmatrix} =
\begin{bmatrix}
X_{N-1}^2 & X_{N-1} & 1 \\
X_N^2 & X_N & 1 \\
X_{N+1}^2 & X_{N+1} & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
Y_{N-1} \\
Y_N \\
Y_{N+1}
\end{bmatrix} \tag{7}
\]

and

\[
\begin{bmatrix}
A_2 \\
B_2 \\
C_2
\end{bmatrix} =
\begin{bmatrix}
X_N^2 & X_N & 1 \\
X_{N+1}^2 & X_{N+1} & 1 \\
X_{N+2}^2 & X_{N+2} & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
Y_N \\
Y_{N+1} \\
Y_{N+2}
\end{bmatrix} \tag{8}
\]

The interpolated values of \( Y \) and \( \frac{dY}{dX} \) between \( X_N \) and \( X_{N+1} \) are defined by either \( P_1 \), \( P_2 \), or a linear combination of \( P_1 \) and \( P_2 \). The amount of \( P_1 \) and \( P_2 \) used in the linear combination is determined by comparing both of the parabolae to the straight line.

\[
S = \alpha Y_{N+1} + (1- \alpha) Y_N \tag{9}
\]
Therefore;

\[ \frac{dD}{dx} = \frac{d\alpha}{dx} E_1 + \alpha \frac{dE_1}{dx} - \frac{d\alpha}{dx} E_2 + (1-\alpha) \frac{dE_2}{dx} \quad (18) \]

\[ Y = \begin{cases} P_1 & \text{for } X = X_N \\ \frac{\alpha E_1 P_2 + (1-\alpha)E_2 P_1}{\alpha E_1 + (1-\alpha)E_2} & \text{for } X_N < X < X_{N+1} \\ P_2 & \text{for } X = X_{N+1} \end{cases} \quad (19) \]

and

\[ \frac{dY}{dx} = \begin{cases} \frac{dP_1}{dx} & \text{for } X = X_N \\ \frac{dN}{dx}/D - N \frac{dD}{dx}/D^2 & \text{for } X_N < X < X_{N+1} \\ \frac{dP_2}{dx} & \text{for } X = X_{N+1} \end{cases} \quad (20) \]

In the case of an end interval \( X_{N-1} \leq x \leq X_N; P_1 \) is set equal to;

\[ P_1 = S + K(P_2 - S) \quad (21) \]

where

\[ K = 1 - \frac{|M_1| - |M_2|}{|M_1| + |M_2|} \quad (22) \]
where

\[ \alpha = \frac{Y_{N+1} - Y_N}{X_{N+1} - X_N} \]  \hspace{1cm} (10)

The parabola which has the least deviation from the straight line is given the greatest weight. The weighting factors \( E_1 \) and \( E_2 \) are determined as follows:

\[ E_1 = \left| P_1 - S \right| \]  \hspace{1cm} (11)

\[ E_2 = \left| P_2 - S \right| \]  \hspace{1cm} (12)

The weighted expression for \( Y \) in the range \( X_N \leq x \leq X_{N+1} \) is then;

\[ Y = \frac{\alpha E_1 P_2 + (1 - \alpha) E_2 P_1}{E_1 + (1 - \alpha) E_2} \]  \hspace{1cm} (13)

The derivative \( dY/dX \) in the range \( X_N \leq x \leq X_{N+1} \) is then;

\[ \frac{dY}{dX} = \frac{dN}{dx} / D - N \frac{dD}{dx} / D^2 \text{ for } D \neq 0 \]  \hspace{1cm} (14)

where

\[ N = \alpha E_1 P_2 + (1 - \alpha) E_2 P_1 \]  \hspace{1cm} (15)

\[ D = \alpha E_1 + (1 - \alpha) E_2 \]  \hspace{1cm} (16)

And then

\[ \frac{dN}{dx} = \frac{d\alpha}{dx} E_1 P_2 + \alpha \frac{dE_1}{dx} P_2 + \alpha E_1 \frac{dP_2}{dx} - \alpha \frac{d\alpha}{dx} E_2 P_1 + (1 - \alpha) \frac{dE_2}{dx} P_1 + E_2 \frac{dP_1}{dx} \]  \hspace{1cm} (17)
and

\[ M_1 = \frac{Y_N - Y_{N-1}}{X_N - X_{N-1}} \]  

(23)

\[ M_2 = \frac{Y_N - Y_{N+1}}{X_N - X_{N+1}} \]  

(24)

A similar procedure is followed for the other end interval \( X_{N+1} \leq x \leq X_{N+2} \).

Householder's method for solving simultaneous equations is used in the solution of the aerodynamic influence equations. The method is applicable to both square and rectangular influence matrices. In the case of rectangular matrices it is not necessary to least square the equations first, since Householder's procedure least squares and triangularizes simultaneously. Also, the influence matrix is triangularized by means of orthogonal transformation matrices, which preserve the conditioning of the matrix. The combination of these two advantages, along with a reduction in the number of required computer operations, greatly improves the numerical accuracy and stability of the solution over that of the standard Gaussian reduction method.

A complete, but rather abstract, derivation of the method is given in reference (21). The method in the subroutine has been altered from the original to allow the operation on a single row of the matrix at a time. This reduces the required core allocation necessary to triangularize the matrix.

A derivation of the method, developed by the writer, will be given here in order to describe the basic philosophy of the method.

If \([A]\) is the rectangular influence matrix, the upper triangle is given by;

\[ [R] = [W][A] \]  

(25)

where \([W]\) is the combined orthogonal transformation matrix used by Householder to triangularize \([A]\).
The relationship between Householder's triangularized matrix \([R]\) and that obtained by Gaussian elimination of the least squared influence matrix \([A]^T[A]\), is

\[
[G] = [T][A]^T[A] = [T][R]^T[R] = [D][R] 
\]  

(26)

where \([G]\) is the triangular matrix obtained by Gaussian elimination of \([A]^T[A]\). The matrix \([T]\) is the Gaussian transformation matrix used to triangularize \([A]^T[A]\). And, \([D]\) is a diagonal matrix with the same diagonal as \([R]\). It can be seen from equation (26) that a nonsquare matrix must be least squared first, before applying the Gaussian transformation \([T]\). Whereas, the Householder transformation matrix \([W]\) can be applied directly. The least squared matrix \([A]^T[A]\) is usually more illconditioned than \([A]\), and therefore, less accurate results are obtained.

In the Householder method \([W]\) is equal to the product of \(N+1\) individual orthogonal transformation matrices, where \(n\) equals the number of unknowns. There are \(N+1\) transformations because the augmented influence matrix, made up of the influence matrix itself plus the boundary condition matrix, added on as the last column, has \(N+1\) columns. Each transformation results in reducing all elements below the diagonal to zero for one column. The columns are reduced from left to right.

The individual transformation matrices \([W]_m\) are defined by;

\[
[W]_m = ([I] - 2 [u]_m [u]^T_m) 
\]  

(27)

where \([I]\) is a unit diagonal matrix and \([u]_m\) is a column matrix defined by the unit vector \(\vec{u}_m = (\vec{a}_m - \alpha_m \vec{v}_m)/\mu_m\). The vector \(\vec{a}_m\) is defined by the \(m\)th column of \([A]\) where the elements on rows less than \(m\) are replaced by zeros. The unit vector \(\vec{v}_m\) is defined by a column matrix \([v]_m\) with all zeros except for the \(m\)th row, which is equal to one. The constants \(\alpha_m\) and \(\mu_m\) are defined as,

\[
\alpha_m = |a_m| 
\]  

(28)

\[
\mu_m = \sqrt{2\alpha_m(\alpha_m - \vec{v}_m \cdot \vec{a}_m)} 
\]  

(29)
It can be shown that \([a_m]\) is reduced to \(|a_m|v_m\) if \([a_m]\) is premultiplied by 
\((I - 2u_m\bar{u}_m)\). Also, that the first \(m-1\) rows of 

\[
[W_{m-1}] [W_{m-2}] \ldots [W_1][A]
\]  

remain unchanged by the \(m\)th transformation. The result after \(m\) transformations 
is then zeros below the diagonal for the first \(m\) columns and \(|a_1|, |a_2|, \ldots, |a_{m-2}|, |a_{m-1}|, |a_m|\) on the diagonal. The elements above the diagonal have 
been defined by the \(m\) preceding transformations and will remain unchanged for 
the \(N+1-m\) remaining transformations.

\[
([I] - 2u_m[a_m\bar{u}_m]) a_m = |a_m|v_m
\]  

\[
\hat{u}_m = (\bar{a}_m - \alpha_m \bar{v}_m)/\mu_m \quad \text{or} \quad u_m = (|a_m| - \alpha_m|v_m|)/\mu_m
\]  

\[
|a_m| = |\hat{a}_m|
\]  

and

\[
\mu_m = \sqrt{2\alpha_m(\bar{a}_m - \bar{v}_m \bar{a}_m)} \quad \text{or} \quad \mu_m = \sqrt{2\alpha_m(\bar{a}_m - |a_m\bar{v}_m|)}
\]

remain to be proved. It is helpful in the derivation of equation (31) if the 
vector identity

\[
|\bar{a}_m|\bar{v} + 2(\bar{u}_m \cdot \bar{a}_m) \bar{u}_m = \bar{a}_m
\]

is observed from the following vector diagram.
Then from equation (35);

\[ |\hat{a}_m| \hat{v}_m + 2 \hat{u}_m (\hat{u}_m \cdot \hat{a}_m) = \hat{a}_m \]  

(36)

\[ |\hat{a}_m| \hat{v}_m + 2 \hat{u}_m \hat{u}_m \cdot \hat{a}_m = \hat{a}_m \]  

(37)

Therefore;

\[ |\hat{a}_m| \hat{v}_m = (\hat{1}_m \hat{1}_m - 2 \hat{u}_m \hat{u}_m) \hat{a}_m \]  

(38)

where \( \hat{1}_m \hat{1}_m \) and \( \hat{u}_m \hat{u}_m \) are dyadics. The unit vector \( \hat{1}_m \) is in the direction of \( \hat{a}_m \).

Equation (38) can then be written in matrix notation as follows;

\[ ([I] - 2 |u|_m |u|_m^T) |a|_m = ||a|_m|v|_m \]  

(39)

which is equal to equation (31). In matrix or tensor notation it becomes evident that the dimensions of \( |a|_m, |v|_m, \) and \( |u|_m \) are not limited to three.

\[ a_m = ||a_m|| \]  

(40)

and

\[ \mu_m = 2 |a_m|^T |a_m| \]  

(41)

Then if equation (31) is premultiplied by \( |a_m|^T \)

\[ |a_m|^T |a_m| - 2 |a_m|^T |u_m| |u_m|^T |a_m| = ||a_m|| |a_m|^T |v_m| \]  

(42)
And substituting $\alpha_m$ and $\mu_m$ into equation (42).

$$\alpha_m^2 - \frac{1}{2} \mu_m^2 = \alpha_m \{a_m\}^T \{v_m\}$$  \hspace{1cm} (43)

or

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \{a_m\}^T \{v_m\})}$$  \hspace{1cm} (44)

In vector notation equation (44) is seen to be equal to;

$$\mu_m = \sqrt{2\alpha_m(\alpha_m - \bar{\nu}_m \cdot \bar{a}_m)}$$  \hspace{1cm} (45)

Also, if equation (44) is substituted back into equation (31)

$$\{a_m\} - \sqrt{2\alpha_m(\alpha_m - \{a_m\}^T \{v_m\})} \{a_m\} = \alpha_m \{v_m\}$$  \hspace{1cm} (46)

Therefore;

$$\{u_m\} = \frac{\{a_m\} - \alpha_m \{v_m\}}{\sqrt{2\alpha_m(\alpha_m - \{a_m\}^T \{v_m\})}}$$  \hspace{1cm} (47)

or in vector notation

$$\bar{u}_m = \frac{\bar{a}_m - \alpha_m \bar{v}_m}{\sqrt{2\alpha_m(\alpha_m - \bar{\nu}_m \cdot \bar{a}_m)}}$$  \hspace{1cm} (48)

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Appendix B

SKewed SOURCE-VORTEX LATTICE INFLUENCE EQUATIONS

The general form of the skewed source and vortex lattice influence equations was derived in appendices B and C of reference (26), respectively. These equations will be specialized to the case where the inboard and outboard sweep angles for a given horseshoe are equal. The perturbation velocity due to a skewed vortex is then given by the following equations.

\[ \frac{U_L}{V_\infty} = \frac{U_v}{V_\infty} \]  \hspace{1cm} (1)

\[ \frac{V_L}{V_\infty} = \frac{\Delta Y_V \left( \frac{V_v}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} - \frac{\Delta Z_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \]  \hspace{1cm} (2)

\[ \frac{W_L}{V_\infty} = \frac{\Delta Z_V \left( \frac{V_v}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \frac{\Delta Y_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \]  \hspace{1cm} (3)

Where \( \Delta Y_V \) and \( \Delta Z_V \) are the changes in \( Y \) and \( Z \) across the width of the horseshoe vortex, respectively.

\[ \frac{U_v}{V_\infty} = \frac{(\Gamma/V_\infty)E_{uv}}{4\pi} \]  \hspace{1cm} (4)
Where

\[ F_{UV} = \frac{Z}{R_2^2} (I_2 + I_3) \]  

(7)

\[ E_{VV} = \frac{Z}{R_2} \left[ \frac{(I_1 + 1)}{R_1^2} - \frac{(I_4 + 1)}{R_3^2} - \frac{1}{R_2^2} (I_2 + I_3) \right] \]  

(8)

\[ \Gamma_{wV} = \frac{(\bar{Y} - \beta \gamma_V) (I_4 + 1)}{R_3^2} - \frac{\bar{Y} + \beta \gamma_V (I_1 + 1)}{R_1^2} - \frac{(\bar{X} - \bar{Y} \bar{V}) (I_2 + I_3)}{R_2^2} \]  

(9)

Let

\[ \text{term 1} = \frac{(I_2 + I_3)}{R_2^2} \]  

(10)

\[ \text{term 2} = \frac{I_1 + 1}{R_1^2} \]  

(11)
\[
\text{term 3} = \frac{I_4 + 1}{R_3^2} \quad (12)
\]

Then

\[
E_{u,v} = \overline{Z} \text{ (term 1)} \quad (13)
\]

\[
E_{v,v} = \overline{Z} \left[ -\overline{T} \text{ (term 1)} + \text{ (term 2)} - \text{ (term 3)} \right] \quad (13)
\]

\[
E_{\nu,\nu} = -(\overline{X} - \overline{T} \overline{Y}) \text{ (term 1)} - (\overline{Y} + \beta \, \overline{y}_\nu) \text{ (term 2)} + (\overline{Y} - \beta \, \overline{y}_\nu) \text{ (term 3)} \quad (14)
\]

Where

\[
I_1 = \frac{\overline{X} + T \overline{y}_\nu}{\left[ (\overline{X} + T \overline{y}_\nu)^2 + (\overline{Y} + \beta \, \overline{y}_\nu)^2 + \overline{z}^2 \right]^{1/2}} = \frac{\overline{X} + T \overline{y}_\nu}{R_5} \quad (15)
\]

\[
I_2 = \frac{\overline{Y} + \overline{T} \overline{X} + \beta \, \overline{y}_\nu \left(1 + \overline{T}^2 \right)}{\left[ (\overline{X} + T \overline{y}_\nu)^2 + (\overline{Y} + \beta \, \overline{y}_\nu)^2 + \overline{z}^2 \right]^{1/2}} = \frac{\overline{Y} + \overline{T} \overline{X} + \beta \, \overline{y}_\nu \left(1 + \overline{T}^2 \right)}{R_5} \quad (16)
\]

\[
I_3 = \frac{\overline{Y} + \overline{T} \overline{X} - \beta \, \overline{y}_\nu \left(1 + \overline{T}^2 \right)}{\left[ (\overline{X} - T \overline{y}_\nu)^2 + (\overline{Y} - \beta \, \overline{y}_\nu)^2 + \overline{z}^2 \right]^{1/2}} = -\frac{\overline{Y} + \overline{T} \overline{X} - \beta \, \overline{y}_\nu \left(1 + \overline{T}^2 \right)}{R_4} \quad (17)
\]

\[
I_4 = \frac{\overline{X} - T \overline{y}_\nu}{\left[ (\overline{X} - T \overline{y}_\nu)^2 + (\overline{Y} - \beta \, \overline{y}_\nu)^2 + \overline{z}^2 \right]^{1/2}} = \frac{\overline{X} - T \overline{y}_\nu}{R_4} \quad (18)
\]
\[
R_1^2 = (Y + \beta Y \nu)^2 + \bar{Z}^2
\]  
(19)

\[
R_2^2 = (X - T Y)^2 + \bar{Z}^2(1 + \bar{T}^2)
\]  
(20)

\[
R_3^2 = (\bar{Y} - \beta Y \nu)^2 + \bar{Z}^2
\]  
(21)

\[
R_4^2 = (X - T Y \nu)^2 + (\bar{Y} - \beta Y \nu)^2 + \bar{Z}^2
\]  
(22)

\[
R_5^2 = (X + T Y \nu)^2 + (\bar{Y} + \beta Y \nu)^2 + \bar{Z}^2
\]  
(23)

and

\[
\bar{X} = X_q - X_v
\]  
(24)

\[
\bar{Y} = \beta \frac{\Delta Y_v (Y_q - Y_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} + \beta \frac{\Delta Z_v (Z_q - Z_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}}
\]  
(25)

\[
\bar{Z} = -\beta \frac{\Delta Y_v (Y_q - Y_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} + \beta \frac{\Delta Z_v (Z_q - Z_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}}
\]  
(26)

\[
y_v = 1/2 \sqrt{\Delta Y_v^2 + \Delta Z_v^2}
\]  
(27)

\[
\bar{T} = \frac{\tan \Lambda}{\beta}
\]  
(28)
where \((X_v, Y_v, Z_v)\) and \((X_q, Y_q, Z_q)\) are the locations of the influencing point and the point being influenced respectively.

The elements of \([A_X]\), \([A_Y]\), and \([A_Z]\) are computed for a unit strength of \(\Gamma/V_\infty\).

Similarly, the perturbation velocity due to a skewed source line is given by:

\[
\frac{U_t}{V_\infty} = \frac{U_S}{V_\infty} \tag{29}
\]

\[
\frac{V_t}{V_\infty} = \frac{\Delta Y_{V}}{\Delta Z_{V}} \frac{V_S}{V_\infty} - \frac{\Delta Z_{V}}{\Delta Y_{V}} \frac{W_S}{V_\infty} \tag{30}
\]

\[
\frac{W_t}{V_\infty} = \frac{\Delta Z_{V}}{\Delta Y_{V}} \frac{V_S}{V_\infty} + \frac{\Delta Y_{V}}{\Delta Z_{V}} \frac{W_S}{V_\infty} \tag{31}
\]

where

\[
\frac{U_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{U_S}}{4 \pi} \tag{32}
\]

\[
\frac{V_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{V_S}}{4 \pi} \tag{33}
\]

\[
\frac{W_S}{V_\infty} = \frac{(\Sigma / V_\infty) E_{W_S}}{4 \pi} \tag{34}
\]

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and

\[ E_{WS} = \frac{1}{\sqrt{1 + \frac{1}{T^2}}} \frac{1}{T} (\text{term 4}) + \frac{1}{\sqrt{1 + \frac{1}{T^2}}} (X - TY)(\text{term 1}) \]  \hspace{1cm} (35)

\[ E_{VS} = \frac{1}{\sqrt{1 + \frac{1}{T^2}}} \frac{1}{T} (\text{term 4}) - \frac{1}{\sqrt{1 + \frac{1}{T^2}}} (X - TY)(\text{term 1}) \]  \hspace{1cm} (36)

\[ E_{WS} = \sqrt{1 + \frac{1}{T^2}} \overline{Z} \ (\text{term 1}) \]  \hspace{1cm} (37)

and where

\[ \text{term 4} = \frac{1}{R_4} - \frac{1}{R_5} \]  \hspace{1cm} (38)

The elements of \([S_X], [S_Y], \text{and } [S_Z]\) are computed for a unit strength of \( \Psi / V_\infty \). The influences of both the vortices and sources at the quarter chord of the subpanel are computed simultaneously due to the similarity in the vortex and source influence equations.

For the case where \( |\overline{Z}| \leq 2 \beta \), set \( \overline{Z} = 0 \) and if both

\[ (X - TY)^2 / \left[ (X + T \beta Y)^2 + (Y + \beta Y)^2 \right] \]  \hspace{1cm} (39)

and

\[ (X - TY)^2 / \left[ (X - T \beta Y)^2 + (Y - \beta Y)^2 \right] \]  \hspace{1cm} (40)
are less than \((0.08716)^2\), and \(|Y| > \beta y_v\), then use:

\[
\frac{1}{2\sqrt{1 + T^2}} \left| \frac{1}{(\bar{X} - Ty_v)^2 + (\bar{Y} - \beta y_v)^2} - \frac{1}{(\bar{X} + Ty_v)^2 + (\bar{Y} + \beta y_v)^2} \right|
\]

(41)

in place of (term 1)

If \(|Y| \leq \beta y_v\) and \(|\bar{X} - Ty| < \ell_M/4\) set (term 1) equal to zero.
The Biot-Savart law can be used to calculate the influence of a finite vortex segment on a point in three-dimensional space. The incremental change in induced velocity at a point in space due to an incremental change in length of a finite vortex is given by the following expression.

\[ dq = \frac{K \cos \phi \, d\phi}{4\pi h} \]  

where:

- \( K \) = Vortex strength
- \( h \) = Perpendicular distance from the vortex segment to the point in space.
- \( \phi \) = Angle between the line formed by \( h \) and a line from the field point to a point on the vortex segment.
- \( q \) = Velocity induced by the finite vortex segment perpendicular to the plane formed by \( h \) and the vortex segment.

A vector expression for \( q \) can be determined from figure (D-1):
The magnitude of the velocity vector $\mathbf{q}$ induced at $(X_q, Y_q, Z_q)$ by the vortex segment $s$ is given by the following equation after equation (1) has been integrated from $\phi_i$ to $\phi_f$.

$$|\mathbf{q}| = \frac{k}{4\pi h} (\cos \beta - \cos \alpha)$$  \hspace{1cm} (2)

where

$$\cos \beta = \frac{\mathbf{s} \cdot \mathbf{R}_f}{|\mathbf{s}| \left| \mathbf{R}_f \right|}$$

$$\cos \alpha = \frac{\mathbf{s} \cdot \mathbf{R}_i}{|\mathbf{s}| \left| \mathbf{R}_i \right|}$$

The vector $\mathbf{h}$ is determined such as it satisfies the conditions of being perpendicular to $\mathbf{s}$ and equal to the vector sum $\mathbf{h} = \mathbf{R}_f - a \mathbf{s}$ where "a" defines the length of $\mathbf{h}$.

Since;

$$\mathbf{h} = \mathbf{R}_f - a \mathbf{s}$$  \hspace{1cm} (3)

and

$$\mathbf{h} \cdot \mathbf{s} = 0$$  \hspace{1cm} (4)

then

$$\mathbf{h} \cdot \mathbf{s} = \mathbf{R}_f \cdot \mathbf{s} - a \mathbf{s} \cdot \mathbf{s} = 0$$  \hspace{1cm} (5)
therefore;

\[ a = \frac{\mathbf{R}_f \cdot \mathbf{s}}{\mathbf{s} \cdot \mathbf{s}} \]  

(6)

After substituting "a" into equation (3), \( \mathbf{h} \) is defined as;

\[ \mathbf{h} = \mathbf{R}_f - \frac{\mathbf{R}_f \cdot \mathbf{s}}{\mathbf{s} \cdot \mathbf{s}} \mathbf{s} \]  

(7)

Also, a unit vector \( \mathbf{q} \) in the direction of \( \mathbf{q} \) is seen to be equal to;

\[ \mathbf{q} = \frac{\mathbf{R}_f \times \mathbf{s}}{\left| \mathbf{R}_f \times \mathbf{s} \right|} \]  

(8)

The magnitude and direction of \( \mathbf{q} \) are then expressed in terms of the coordinates of the control point \((X_q, Y_q, Z_q)\) and the endpoints of the vortex segment \((X_1, Y_1, Z_1)\) and \((X_f, Y_f, Z_f)\). If \( \mathbf{i} \), \( \mathbf{j} \), and \( \mathbf{k} \) are defined as unit vectors in the \( X \), \( Y \), and \( Z \) directions respectively, then;

\[ \mathbf{s} = \mathbf{s}_f - \mathbf{s}_i = (X_f - X_i) \mathbf{i} + \beta (Y_f - Y_i) \mathbf{j} + \beta (Z_f - Z_i) \mathbf{k} \]  

(9)

\[ \mathbf{R}_i = \mathbf{s}_i - \mathbf{Q} = (X_i - X_q) \mathbf{i} + \beta (Y_i - Y_q) \mathbf{j} + \beta (Z_i - Z_q) \mathbf{k} \]

and

\[ \mathbf{R}_f = \mathbf{s}_f - \mathbf{Q} = (X_f - X_q) \mathbf{i} + \beta (Y_f - Y_q) \mathbf{j} + \beta (Z_f - Z_q) \mathbf{k} \]  

(10)
The value of "a" is then expressed as;

\[ a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} = \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \]

and the components of \( \vec{h} \) by,

\[ h_x = (X_f - X_q) - \frac{\left( \left( \frac{X_f - X_q}{X_f - X_i} \right) + \left( \frac{Y_f - Y_q}{Y_f - Y_i} \right) \beta^2 + \left( \frac{Z_f - Z_q}{Z_f - Z_i} \right) \beta^2 \right) (X_f - X_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \]

\[ h_y = (Y_f - Y_q) - \frac{\left( \left( \frac{X_f - X_q}{X_f - X_i} \right) + \left( \frac{Y_f - Y_q}{Y_f - Y_i} \right) \beta^2 + \left( \frac{Z_f - Z_q}{Z_f - Z_i} \beta^2 \right) \beta^2 \right) (Y_f - Y_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \]

\[ h_z = (Z_f - Z_q) - \frac{\left( \left( \frac{X_f - X_q}{X_f - X_i} \right) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i) \right) \beta^2 (Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \]

The magnitude of \( \vec{q} \) is found by substituting

\[ h = |\vec{h}| = \sqrt{h_x^2 + h_y^2 + h_z^2} \]

and the following expressions for cos\( \alpha \) and cos\( \beta \) into equation (2).
\[
\cos \alpha = \frac{(X_f - X_i)(X_f - X_i) + \beta^2(Y_f - Y_i)(Y_f - Y_i) + \beta^2(Z_f - Z_i)(Z_f - Z_i)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2}}
\]

\[
\cos \beta = \frac{(X_f - X_i)(X_f - X_i) + \beta^2(Y_f - Y_i)(Y_f - Y_i) + \beta^2(Z_f - Z_i)(Z_f - Z_i)}{\sqrt{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2}}
\]

(13)

The components of the vector \( \vec{q} \) are then given by the multiplication of the components of equation (8) by \( |\vec{q}| \).

\[
q_x = \frac{|\vec{q}| \left[ (X_f - X_i)(Z_f - Z_i) - (Z_f - Z_i)(X_f - X_i) \right]}{|\vec{RXS}|} \beta^2
\]

\[
q_y = \frac{|\vec{q}| \left[ (Y_f - Y_i)(Z_f - Z_i) - (Z_f - Z_i)(Y_f - Y_i) \right]}{|\vec{RXS}|} \beta^2
\]

\[
q_z = \frac{|\vec{q}| \left[ (Y_f - Y_i)(X_f - X_i) - (X_f - X_i)(Y_f - Y_i) \right]}{|\vec{RXS}|} \beta^2
\]

(14)
where

$$|\vec{R}_{x5}| = \left[ (Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i) \right]^2 + \left[ (X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q) \right]^2$$

$$+ \left[ (X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i) \right]^2 \right]^{1/2} \beta^2$$

The velocity induced at a control point by a vortex segment is then given by equation (14). Since a curved vortex can be represented by a number of straight segments, this equation can be used to compute the induced flow produced by a vortex of arbitrary shape.

The components of velocity induced by a quadrilateral vortex can be written as ratios computed by the product of influence matrices and the vortex strengths.

$$\begin{align*}
\begin{bmatrix} u \\ v \\ w \\ \end{bmatrix} &= \begin{bmatrix} A_x \\ A_y \\ A_z \\ \end{bmatrix} \begin{bmatrix} K \\ V_\infty \\ \end{bmatrix} \\
\frac{u}{V_\infty} &= \begin{bmatrix} A_x \\ \end{bmatrix} \begin{bmatrix} K \\ V_\infty \\ \end{bmatrix} \\
\frac{v}{V_\infty} &= \begin{bmatrix} A_y \\ \end{bmatrix} \begin{bmatrix} K \\ V_\infty \\ \end{bmatrix} \\
\frac{w}{V_\infty} &= \begin{bmatrix} A_z \\ \end{bmatrix} \begin{bmatrix} K \\ V_\infty \\ \end{bmatrix}
\end{align*}$$

and where the elements of $A_x$, $A_y$, and $A_z$, are computed from the following equations.
\[ A_x = \sum \frac{\beta^2 [\cos \beta - \cos \alpha] [(Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i)]}{4\pi |\vec{h}| |\vec{RS}|} \]  

(19) 

\[ A_y = \sum \frac{\beta [\cos \beta - \cos \alpha] [(X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q)]}{4\pi |\vec{h}| |\vec{RS}|} \]  

(20) 

and

\[ A_z = \sum \frac{\beta [\cos \beta - \cos \alpha] [(X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i)]}{4\pi |\vec{h}| |\vec{RS}|} \]  

(21) 

The \( \Sigma \) sign indicates that the contributions from all of the sides of the quadrilateral vortex are summed.
Appendix D

WOODWARD'S DISTRIBUTED PANEL
INFLUENCE EQUATIONS

The equations will be derived for supersonic flow first, then for subsonic flow in subappendix A.

Preliminaries

Generalized potential function. If

\[ \beta^2 \Omega_{xx} = \Omega_{yy} + \Omega_{zz} \]

Then

\[ \Omega(x, y, z) = -\frac{1}{2\pi} \frac{\partial}{\partial x} \iint \left( \frac{\partial \Omega}{\partial \nu} + \frac{\partial \Omega'}{\partial \nu'} \right) \sigma \, dS \]

\[ + \frac{1}{2\pi} \frac{\partial}{\partial x} \iint (\Omega - \Omega') \frac{\partial \sigma}{\partial \nu} \, dS \]

(1)

where

\[ \sigma = \cos \theta^{-1} \frac{x - \xi}{\beta \sqrt{(y - \eta)^2 + (z - \xi)^2}} \]

and

\[ \nu = \left( -\beta^2 n_1, n_2, n_3 \right) = -\nu' \]

with \( n = (n_1, n_2, n_3) \) the unit normal to surface \( S \)
Region of integration. - In the \((\xi, \eta, \zeta)\) coordinate system the plane of the semi-infinite triangular surface is determined by \(\xi = a\xi\). The lines \(\eta = 0\) and \(\eta = m\xi\) are the projections in the \(\xi, \eta\) plane of the triangle edges. The area of integration in equation (I) lies on the semi-infinite triangle and is within the Mach forecone from the point \((x, y, z)\) given by \(\xi < x\) and \((x-z)^2 > \beta^2(y-\eta)^2 + \beta^2(z-\xi)^2\). The surface integral is carried out by integrating first over \(\xi\) and then over \(\eta\).

The \(\xi\) integration goes from the leading edge \(\xi = \xi_1 = \eta/m\) to the intersection of the Mach forecone with the semi-infinite triangle, \(\xi = \xi_2(\eta)\), where since \(\xi = a\xi\), \((x-x_2)^2 = \beta^2(y-\eta)^2 + \beta^2(z-a\xi_2)^2\). The \(\eta\) integration goes from \(\eta = 0\) to the intersection of the Mach forecone the leading edge where \(\eta = \eta_3\). Thus,

\[
\left( \frac{x - \eta_3}{m} \right)^2 = \beta^2(y - \eta_3)^2 + \beta^2\left( z - \frac{a\eta_3}{m} \right)^2.
\]

Looking down on \(\xi\) axis

Thus

\[
\xi_1 = \frac{\eta}{m} \quad \text{(2)}
\]

\[
(x - \xi_2)^2 = \beta^2(y - \eta)^2 + \beta^2(z - a\xi_2)^2 \quad \text{(3)}
\]
\[ (\eta_3 - mx)^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 (a \eta_3 - mz)^2 \] (4)

These relations may be manipulated into other forms.

\[ [(\eta_3 - y) - (mx - y)]^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 [a (\eta_3 - y) + (ay - mz)]^2 \]

or

\[ (\eta_3 - y)^2 [1 - \beta^2 (a^2 + m^2)] - 2 (mx - y + \beta^2 a (ay - mz)) (\eta_3 - y) + (mx - y)^2 - \beta^2 (ay - mz)^2 = 0 \]

Thus on the forecone

\[ \sqrt{[(mx - y) + \beta^2 a (ay - mz)]^2 - [1 - \beta^2 (a^2 + m^2)] [(mx - y)^2 - \beta^2 (ay - mz)^2]} \]

The following relation may be used to manipulate this further.

\[ m(x - \beta^2 az) - y (1 - \beta^2 a^2) \]

\[ \equiv \left| mx - y + \beta^2 a (ay - mz) \right|^2 = \beta^2 m^2 (z - ax)^2 + (1 - \beta^2 a^2) [mx - y]^2 - \beta^2 (ay - mz)^2 \]

(5)
Thus

\[
\left| 1 - \beta^2(a^2 + m^2) \right| \eta_3 = m(x - \beta^2 ax) - \beta^2 m^2 y - \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2}
\]

(6)

Or starting from (4) again after multiplying by \((1 - \beta^2 a^2)\n\)

\[
(1 - \beta^2 a^2)(\eta_3 - mx)^2 = \beta^2 m^2 (1 - \beta^2 a^2)(\eta_3 - y)^2 + \beta^2 (1 - \beta^2 a^2) \left[ a(\eta_3 - mx) - m(z - ax) \right]^2
\]

which becomes

\[
(1 - \beta^2 a^2) (\eta_3 - mx)^2 + 2 \beta^2 am(z - ax)(1 - \beta^2 a^2)(\eta_3 - mx)
\]

\[
+ \left| \beta^2 am \right|^2 (z - ax)^2 = \beta^2 m^2 \left| 1 - \beta^2 a^2 \right| (\eta_3 - y)^2 + (z - ax)^2
\]

or

\[
\left| (1 - \beta^2 a^2)(\eta_3 - mx) + \beta^2 am(z - ax) \right|^2
\]
or

\[ \left[ \eta^3 (1 - \beta^2 a^2) - m(x - \beta^2 az) \right]^2 = \beta^2 m^2 \left(1 - \beta^2 a^2 \right) \eta^3 + (z - ax)^2 \]

(7)

Surface Distribution of Sources

Boundary conditions. - For a distribution of sources on the plane \( \xi = a \xi \) we will assume

(a) \( \phi = \phi' \)

(b) \( \frac{\partial \phi}{\partial \nu} + \frac{\partial \phi'}{\partial \nu'} = \frac{\partial \phi}{\partial \xi} \nu_1 + \frac{\partial \phi}{\partial \xi} \nu_3 = \frac{\partial \phi'}{\partial \xi} \nu_1 + \frac{\partial \phi'}{\partial \xi} \nu_3 \)

\[ = (u - u') \nu_1 + (w - w') \nu_3 \]

\[ = \frac{1}{\sqrt{1 + a^2}} \left[ \beta^2 a (u - u') + (w - w') \right] \]

\[ = \frac{2}{\sqrt{1 + a^2}} \left[ \frac{w}{w + \beta^2 a u} \right] = \text{const} \]

In equation (1), if we set \( \Omega = \phi \), then (a) says the second integral vanishes and (b) means the quantity \( \partial \phi / \partial \nu + \partial \phi' / \partial \nu' \) may be removed from the integral. These assumptions may be checked after the integration is performed. The statement Woodward makes on the bottom of page 17, of reference (55), \( u' = -u \) and \( w' = -w \) is not true and not necessary.
Evaluation of the integral over $\xi$. The integral which results for a surface distribution of sources is,

$$
\phi(x, y, z) = -\frac{w + \beta^2 a}{\pi} \int_0^\eta \int_\xi^2 \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} d\eta
$$

But

$$
\int \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} = \frac{-1}{2 \sqrt{1 - \beta^2 a^2}} \log \frac{(x - 3) - \beta^2 a (z - a3) + \sqrt{(1 - \beta^2 a^2) [(x - 3)^2 - \beta^2 (z - a3)^2]}}{(x - 3) - \beta^2 a (z - a3) - \sqrt{(1 - \beta^2 a^2) [(x - 3)^2 - \beta^2 (z - a3)^2]}}
$$

Which may be verified by differentiating

First we let

$$
A = (x - 3) - \beta^2 a (z - a3)
$$

$$
B^2 = \beta^2 [(1 - \beta^2 a^2) (z - a3)^2 + (z - ax)^2]
$$

$$
c^2 = (1 - \beta^2 a^2) [(x - 3)^2 - \beta^2 (z - a3)^2 - \beta^2 (z - a3)^2]
$$

and we note that

$$
A^2 = B^2 + c^2
$$
To verify the indefinite integral we must evaluate

\[
\frac{d}{d \beta} \log \frac{A + C}{A - C} = \frac{A' + C'}{A + C} - \frac{A' - C'}{A - C}
\]

\[
= \frac{2(A'C' - A'C)}{A'^2 - C^2}
\]

where ' denotes \( \frac{d}{d \beta} \)

But since \( B' = 0 \)

\( AA' = CC' \)

or \( C' = \frac{AA'}{C} \)

Therefore

\[
\frac{-1}{2 \sqrt{1 - \beta^2 a^2}} \frac{d}{d \beta} \log \frac{A + C}{A - C} = - \frac{A'(A^2 - C^2)}{C(A^2 - C^2) \sqrt{1 - \beta^2 a^2}}
\]

\[
= \frac{\sqrt{1 - \beta^2 a^2}}{C}
\]

\[
= \frac{1}{\sqrt{(X-3)^2 - \beta^2(\tau-\xi)^2 - \beta^2(\tau-\alpha)^2}}
\]

Q.E.D.
Now, since from Eq (3)

\[(x - \xi_2)^2 = \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi_2)^2\]

and from Eq (2)

\[\xi_1 = \frac{\eta}{m}\]

\[
\int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a \xi)^2}} = \frac{1}{2 \sqrt{1 - \beta^2 a^2}} \log \frac{(mx - \eta - \beta^2 a (mz - a \eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 a^2 (mz - a \eta)^2]}}{(mx - \eta - \beta^2 a (mz - a \eta) - \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 a^2 (mz - a \eta)^2]}}
\]

Therefore from Eq (9)

\[\phi(x, y, z) = \frac{\eta^3}{8 \pi \sqrt{1 - \beta^2 a^2}} \int_0^\infty \frac{d\eta}{\sqrt{(mx - \eta - \beta^2 a (mz - a \eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 a^2 (mz - a \eta)^2]}}} \log \frac{(mx - \eta - \beta^2 a (mz - a \eta) + \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 a^2 (mz - a \eta)^2]}}{(mx - \eta - \beta^2 a (mz - a \eta) - \sqrt{(1 - \beta^2 a^2)[(mx - \eta)^2 - \beta^2 a^2 (mz - a \eta)^2]}}
\]
Evaluation of the Integral over \( \eta \). To evaluate equation (11) we must first integrate by parts. If we let,

\[
\begin{align*}
    f &= (m\eta - \gamma) - \beta^2 a (m \eta - a \gamma) \\
    g^2 &= \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - \gamma)^2 + (2 - ax)^2 \right] \\
    h^2 &= (1 - \beta^2 a^2) \left[ (m\eta - \gamma)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - a \gamma)^2 \right]
\end{align*}
\]

then

\[ f^2 = g^2 + h^2 \]

Now let

\[
\begin{align*}
    u &= \frac{1}{2} \log \frac{f + h}{f - h} \\
    dv &= d\eta \\
    du &= \frac{fh' - h't}{f^2 - h^2} \, d\eta = \frac{fh' - h't}{h (f^2 - h^2)} \, d\eta \\
    &= \frac{fh' - f'(t^2 - g^2)}{h g^2} \, d\eta = \frac{f [hh' - fg'] + fg'}{h g^2} \, d\eta \\
    &= \frac{1}{h g^2} \left[ f'g^2 - fg'g' \right] \, d\eta
\end{align*}
\]

and

\[ v = \gamma \]

but

\[
\begin{align*}
    g'g' &= \beta^2 m^2 (1 - \beta^2 a^2) (\eta - \gamma) \\
    f' &= - (1 - \beta^2 a^2)
\end{align*}
\]
Now we can write

\[ \eta [\beta' \beta'' f' - \beta' \beta'' f] \]

\[ = \eta (1 - \beta'^2 \beta^2) \beta''^2 \left[ (z - ax)^2 + [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] (\eta - y) \right] \]

\[ = \beta''^2 [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \left[ 1 - \beta^2 a^2 \right] (\eta - y)^2 + (z - ax)^2 \]

\[ + \beta''^2 y (1 - \beta^2 a^2) \left[ m (x - \beta^2 az) - y (1 - \beta^2 a^2) \right] (\eta - y) \]

\[ + \beta''^2 (z - ax)^2 \left[ \eta (1 - \beta^2 a^2) - [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \right] \]

\[ = \beta''^2 [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \left[ 1 - \beta^2 a^2 \right] (\eta - y)^2 + (z - ax)^2 \]

\[ + \beta''^2 (1 - \beta^2 a^2) \left[ y [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] + (z - ax)^2 \right] (\eta - y) \]

\[ + \beta''^2 (z - ax)^2 \left[ y (1 - \beta^2 a^2) - [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] \right] \]

And since from the integration by parts

\[ \frac{1}{2} \int \log \frac{f + k}{f - k} \, d\eta = \frac{1}{2} \eta \log \frac{f + k}{f - k} + \int \frac{-\eta (g^2 f' - gg' f)}{g^2 \sqrt{f^2 - g^2}} \, d\eta \]
We get for the integration by parts

\[
\frac{1}{2} \int \log \frac{(mx - y) - \beta'^2(m - y) + \sqrt{(m - y)^2 - \beta'^2(m - y)^2 - \beta'^2(m - y)^2}}{(mx - y) - \beta'^2(m - y) - \sqrt{(m - y)^2 - \beta'^2(m - y)^2 - \beta'^2(m - y)^2}} \, dy
\]

\[
\frac{1}{2} \log \frac{(mx - y) - \beta'^2(m - y) + \sqrt{(m - y)^2 - \beta'^2(m - y)^2 - \beta'^2(m - y)^2}}{(mx - y) - \beta'^2(m - y) - \sqrt{(m - y)^2 - \beta'^2(m - y)^2 - \beta'^2(m - y)^2}}
\]

\[
\int_{a (x - s_0)}^{1} \frac{1}{\sqrt{\left[ (x - s_0)^2 - y (1 - s_0^2) \right] \cdot \left[ y (1 - s_0^2) - (1 - s_0)^2 \right] \cdot \left[ (1 - s_0^2)^2 + (1 - s_0)^2 \right]}} \, dy
\]

\[
\int_{a (x - s_0)}^{1} \frac{1}{\sqrt{\left[ (x - s_0)^2 - y (1 - s_0^2) \right] \cdot \left[ y (1 - s_0^2) - (1 - s_0)^2 \right] \cdot \left[ (1 - s_0^2)^2 + (1 - s_0)^2 \right]}} \, dy
\]

(14)

The first integral in equation (14) can be evaluated using the relation

\[
\int \frac{d\gamma}{\sqrt{A \gamma^2 + 2B\gamma + C}} = -\frac{1}{2\sqrt{A}} \log \frac{-A\gamma + B + \sqrt{A[A\gamma^2 + 2B\gamma + C]}}{-A\gamma + B - \sqrt{A[A\gamma^2 + 2B\gamma + C]}}
\]

(15)
In our case expanding the denominator gives:

\[ A = (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] \]

\[ B = \{-n (x - \beta^2 a z) \cdot \beta m^2 \} (1 - \beta^2 a^2) \]

\[ C = n^2 (x - \beta^2 a z)^2 \cdot \beta^2 m^2 \{n^2 (1 - \beta^2 a^2) + (z - ax)^2 \} \]

and since we can write

\[ -(A^2 + B) = \left[ (mx - \eta) - \beta^2 a (mz - n \eta) + \beta m^2 (n - \eta) \right] (1 - \beta a^2) \]

\[ A[A^2 + 2B + C] = (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] \left\{ [m (x - \beta a z) - \eta (1 - \beta a^2)] \cdot \beta m^2 [(1 - \beta a^2) (n - \eta) + (z - ax)^2] \right\} \]

\[ = [1 - \beta^2 (a^2 + m^2)] (1 - \beta a^2) \left\{ (mx - \eta)^2 - \beta^2 m^2 (n - \eta)^2 - \beta^2 (mz - n \eta)^2 \right\} \]

Therefore

\[
\int \frac{d \eta}{\sqrt{[m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (n - \eta)^2 + (z - ax)^2]}}
\]

\[
= -\frac{1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2)}} \cdot \frac{-\log \left[ -\frac{1}{\beta m^2} \frac{A^2 + B + \sqrt{A[A^2 + 2B + C]}}{A^2 + B - \sqrt{A[A^2 + 2B + C]}} \right]}{\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta a^2)}}
\]

\[
= -\frac{1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta a^2)}} \cdot \lim_{\eta \to 0} \frac{(mx - \eta) - \beta a (mz - n \eta) + \beta m^2 (n - \eta) + \sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta a^2) \cdot \beta m^2 [(n - \eta) + (z - ax)^2]}}{\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta a^2) \cdot \beta m^2 [(n - \eta) + (z - ax)^2]}}
\]

The last integral in equation (14) may be evaluated using the method described in Appendix A of reference (55).

From Appendix A of reference (55);

If

\[ \gamma^4 + (c - \alpha e z) \gamma^2 - b^2 e^2 = 0 \]

(17)
then

\[ \int \frac{(A + Bv) \, dv}{(v^2 + e^2) \, \sqrt{av^2 + 2bv + c}} = \frac{Ab \, Y + By^3}{\gamma^4 + b^2 e^2} \tan^{-1} \frac{\gamma \sqrt{av^2 + 2bv + c}}{\gamma^2 - bv} \]

\[ + \frac{A}{e} \frac{Y^3}{2(\gamma^4 + b^2 e^2)} \log \frac{-(v Y^2 + be^2) + Ye \sqrt{av^2 + 2bv + c}}{-(v Y^2 + be^2) - Ye \sqrt{av^2 + 2bv + c}} \]

(18)

From equation (14) we can write

\[ [m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \]

\[ = (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] (\eta - y)^2 \]

\[ - 2[m (x - \beta^2 az) - y (1 - \beta^2 a^2)] (\eta - y) (1 - \beta^2 a^2) \]

\[ + [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 \beta^2 m^2 (z - ax)^2 \]

165.
Therefore if we factor out a \((1 - \beta^2 a^2)\) we can write the denominator as

\[
\frac{(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2}{\sqrt{\eta (x - \beta^2 ax) - \eta (1 - \beta^2 a^2)^2 + \beta^2 m^2 (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2}}
\]

\[
= (1 - \beta^2 a^2)^{3/2} (\nu^2 + e^2) \sqrt{\nu^2 + 2b\nu + c}
\]

where

\[
\hat{a} = [1 - \beta^2 (a^2 + m^2)]
\]

\[
b = - [m (x - \beta^2 ax) - y (1 - \beta^2 a^2)]
\]

\[
c = \left[ m (x - \beta^2 ax) - y (1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 (z - ax)^2 (1 - \beta^2 a^2)^{-1}
\]

\[
e = (z - ax) (1 - \beta^2 a^2)
\]

\[
\nu = \eta - y
\]

Referring to equation (17)

\[
c - \hat{a} e^2 = \frac{[m (x - \beta^2 ax) - y (1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 - [1 - \beta^2 (a^2 + m^2)] (z - ax)^2}{(1 - \beta^2 a^2)}
\]

\[
= \frac{b^2}{(1 - \beta^2 a^2)} - e^2 (1 - \beta^2 a^2)
\]

\[
\gamma^4 + (c - \hat{a} e^2) - b^2 e^2 = \left[ \gamma^2 + \frac{B^2}{(1 - \beta^2 a^2)} \right] [\gamma^2 - e^2 (1 - \beta^2 a^2)] = 0
\]

Therefore we can choose

\[
\gamma^2 = e^2 (1 - \beta^2 a^2)
\]

or

\[
\gamma = (z - ax)
\]

(20)
Using this the numerator of the second integral in (12) becomes

\[
(z - ax)^2 \left[ y (1 - \beta^2 a^2) \cdot \left[ m (x - \beta^2 ax) \cdot y (1 - \beta^2 a^2) \right] + (z - ax)^2 \right] (y - y)
- y^2 \left[ y (1 - \beta^2 a^2) \cdot b \right] (1 - \beta^2 a^2) | \cdot yb \cdot y^2 | y
\]

Therefore from (18) the coefficient of \( \tan^{-1} \) is

\[
\frac{b \gamma^3 \left[ y (1 - \beta^2 a^2) + b \right] (1 - \beta^2 a^2) \gamma \left[ -yb + y^2 \right]}{(1 - \beta^2 a^2)^{3/2} \left( \gamma^4 + b^2 e^2 \right)} - \frac{y (1 - \beta^2 a^2) \gamma^4 + y^2}{1 - \beta^2 a^2}
\]

\[
\frac{y}{\sqrt{1 - \beta^2 a^2}}
\]

\[
\frac{z - ax}{\sqrt{1 - \beta^2 a^2}}
\]

since \( \gamma = (z - ax) \) and \( \gamma^2 = e^2 (1 - \beta^2 a^2) \)

Using (19) and (18) again the coefficient of \( \frac{1}{2} \log_2 z_a \)

\[
- \gamma^5 \left[ y (1 - \beta^2 a^2) + b \right] + be^2 \gamma (1 - \beta^2 a^2) \left[ -yb + y^2 \right]
\]

\[
e (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)^{3/2}
\]

\[
= -\gamma^5 - b^2 e^2 \gamma \left( 1 - \beta^2 a^2 \right)
\]

\[
(z - ax) (\gamma^4 + b^2 e^2) (1 - \beta^2 a^2)
\]

\[
= -\gamma
\]
And from (19)

\[(1 - \beta^2a^2) [\tilde{a}v^2 + 2bv + c]\]

\[= [m (x - \beta^2az) - (v + y) (1 - \beta^2a^2)]^2 - \beta^2m^2 [1 - \beta^2a^2]v^2 + (z - ax)^2\]

\[\gamma^2 - bv = (z - ax)^2 + v [m (x - \beta^2az) - y (1 - \beta^2a^2)] - (v \gamma^2 + be^2)\]

\[ -(\gamma^2 + be^2) = \frac{[m (x - \beta^2az) - y (1 - \beta^2a^2)] - v (1 - \beta^2a^2)}{(1 - \beta^2a^2)}\]

Therefore

\[\tan^{-1} \frac{\gamma}{\gamma^2 - bv} = \tan^{-1} (z - ax) \frac{\sqrt{m (x - \beta^2az) - \eta (1 - \beta^2a^2)}^2 - \beta^2m^2 [1 - \beta^2a^2] (\eta - y)^2 + (z - ax)^2}{\sqrt{1 - \beta^2a^2} (z - ax)^2 + (\eta - y) [m (x - \beta^2az) - y (1 - \beta^2a^2)]}\]

\[\log \frac{-(\gamma^2 + be^2)}{-(\gamma^2 + be^2) - ye^\sqrt{\tilde{a}v^2 + 2bv + c}}\]

\[= \log \frac{m(x - \beta^2az) - \eta(1 - \beta^2a^2) + \sqrt{[m(x - \beta^2az) - \eta(1 - \beta^2a^2)]^2 - \beta^2m^2 [1 - \beta^2a^2] (\eta - y)^2 + (z - ax)^2}}{m(x - \beta^2az) - \eta(1 - \beta^2a^2) - \sqrt{[m(x - \beta^2az) - \eta(1 - \beta^2a^2)]^2 - \beta^2m^2 [1 - \beta^2a^2] (\eta - y)^2 + (z - ax)^2}}\]

\[= \log \frac{(m - \gamma) - \beta^2a(m2 - \gamma) + \sqrt{(1 - \beta^2a^2)[(m - \gamma)^2 - \beta^2m^2 (\gamma - \beta)(\gamma - z)^2]}}{(m - \gamma) - \beta^2a(m2 - \gamma) - \sqrt{(1 - \beta^2a^2)[(m - \gamma)^2 - \beta^2m^2 (\gamma - \beta)(\gamma - z)^2]}}\]

Because from (12)

\[\frac{[m(x - \beta^2az) - \gamma(1 - \beta^2a^2)]^2 - \beta^3m^2 [1 - \beta^2a^2] (\gamma - \beta)(\gamma - z)^2 + (z - ax)^2}{(m - \gamma)^2 - \beta^3m^2 (\gamma - \beta)(\gamma - z)^2 - \beta^3(m2 - \gamma)^2}\]

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Therefore combining terms

\[
\frac{1}{2} \int \log \frac{(mx-\gamma) - \beta^2 a (mz-a\gamma) + \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]}{(mx-\gamma) - \beta^2 a (mz-a\gamma) + \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]} \, d\gamma
\]

\[= \frac{1}{2} (m-a) \log \frac{(mx-\gamma) - \beta^2 a (mz-a\gamma) + \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]}{(mx-\gamma) - \beta^2 a (mz-a\gamma) - \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]} \]

\[+ \frac{y (1-\beta^2 a^2) - m (x-\beta^2 a^2)}{2 \sqrt{(1-\beta^2 a^2) (1-\beta^2 (a^2 + m^2))}} \log \frac{(mx-\gamma) - \beta^2 a (mz-a\gamma) + \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]}{(mx-\gamma) - \beta^2 a (mz-a\gamma) + \sqrt{(1-\beta^2 a^2)} [(mx-\gamma)^2 - \beta^2 (mz-a\gamma)^2]} \]

\[
+ \frac{(z-a\gamma)}{\sqrt{1 - \beta^2 a^2}} \frac{(z-a\gamma) \sqrt{(m (x + \beta^2 a^2) - \eta (1 - \beta^2 a^2)^2 + \beta^2 \eta^2} [(1 - \beta^2 a^2) \eta^2 + (z - ax)^2]}{\sqrt{1 - \beta^2 a^2}} \frac{(z-a\gamma) \sqrt{(m (x + \beta^2 a^2) - \eta (1 - \beta^2 a^2)^2 + (z - ax)^2)}}{\sqrt{1 - \beta^2 a^2}}
\]

\[(21)\]

Differentiation of the indefinite integral over \( \eta \). - Equation (21) may be checked by differentiation.

So differentiate the first log term we can refer back to the integration by parts of the integral over \( \gamma \). As before let

\[
f = (mx-\gamma) - \beta^2 a (mz-a\gamma)
\]

\[
g^2 = \beta^2 m^2 [(1-\beta^2 a^2) (x-\gamma)^2 + (z-ax)^2]
\]

\[
h^2 = (1-\beta^2 a^2) [(mx-\gamma)^2 - \beta^2 m^2 (x-\gamma)^2 - \beta^2 (mz-a\gamma)^2]
\]

Then

\[
f^2 = g^2 + h^2
\]

and

\[
\frac{d}{d\gamma} \frac{1}{2} \log \frac{f + h}{f - h} = \frac{1}{h^2} \left[ \frac{1}{g^2} \left[ f' g^2 - f g^2' \right] \right] = \frac{f'}{h} - \frac{f g^2'}{h^2}
\]
To differentiate the second log term in (21), we will first simplify the following manipulation:

\[
\ln(x - \alpha) \cdot \ln((1 - \rho^2)x^2) = \frac{1}{2} \ln(x - \alpha) - \frac{1}{2} \ln((1 - \rho^2)x^2) + \ln((1 - \rho^2) - \alpha^2) - \ln(1 - \rho^2)x^2 - \alpha^2
\]

we can write

\[
g(x) = \sqrt{(1 - \rho^2) x^2} + \rho x^2 - \ln(x - \alpha) - \frac{1}{2} \ln((1 - \rho^2)x^2) - \ln((1 - \rho^2) - \alpha^2)
\]
\[
\left\{ m(x - \beta^2 a z) - \beta^2 m_y - \left[ 1 - \beta^2 (a^2 + m^2) \right] \gamma \right\}^2
\]

\[
= \left\{ m(x - \beta^2 a z) - y(1 - \beta^2 a^2) - \left[ 1 - \beta^2 (a^2 + m^2) \right] \gamma \right\}^2
\]

\[
= \beta^2 m^2 (x - ax)^2 + \left(1 - \beta^2 a^2\right) \left(\frac{m x - y}{-a} - \frac{\beta^2 (m^2 y - m^2 z)}{y} \right)^2
\]

\[-2 \left[ (m x - y) - \beta^2 (m^2 - a y) \right] \left[ 1 - \beta^2 (a^2 + m^2) \right] \gamma \]

\[+ \left(1 - \beta^2 a^2\right) \left[ 1 - \beta^2 (a^2 + m^2) \right] \gamma \]

\[= \beta^2 m^2 \left[ (m x - y)^2 + (x - ax)^2 - \beta^2 (a y - m z)^2 \right]
\]

\[+ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (m x - y)^2 - 2 \gamma (m x - y) + \gamma^2 \right]
\]

\[-\beta^2 \left[ (a y - m z)^2 - 2 a (m^2 - a y) \gamma + a^2 (\gamma)^2 \right]
\]

\[-\beta^2 m^2 (\gamma - y)^2 \]

\[= \beta^2 m^2 \left[ (m x - y)^2 + (x - ax)^2 - \beta^2 (a y - m z)^2 \right]
\]

\[+ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (m x - y)^2 - \beta^2 (\gamma - y)^2 - \beta^2 (m^2 a y)^2 \right]
\]

Therefore if we let

\[ t = m(x - \beta^2 a z) - \beta^2 m_y - \left[ 1 - \beta^2 (a^2 + m^2) \right] \gamma \]

\[g^2 = \beta^2 m^2 \left[ (m x - y)^2 + (x - ax)^2 - \beta^2 (a y - m z)^2 \right]
\]

\[h^2 = \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (m x - y)^2 - \beta^2 (\gamma - y)^2 - \beta^2 (m^2 a y)^2 \right]
\]

then \[ t^2 = g^2 + h^2 \]

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Since in this case \( \frac{1}{2} \frac{d}{d\eta} \log \frac{f+h}{f-h} = \frac{f'}{h} \) (using (19)) we can write

\[
f' = -[1 - \beta^2 (a^2 + m^2)]
\]

and

\[
\frac{1}{2} \frac{d}{d\eta} \log \frac{(mx-\gamma) - \beta a (m \omega - \gamma) + \beta m^2 (\eta - \gamma) + \sqrt{[1 - \beta^2 (a^2 + m^2)](mx-\gamma)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \omega - \gamma)^2}}{(mx-\gamma) - \beta a (m \omega - \gamma) + \beta m^2 (\eta - \gamma) - \sqrt{[1 - \beta^2 (a^2 + m^2)](mx-\gamma)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \omega - \gamma)^2}}
\]

\[
\frac{-[1 - \beta^2 (a^2 + m^2)]}{\sqrt{[m (x - \beta^2 a) - \eta (1 - \beta^2 a)]^2 - \beta^2 m^2 [(1 - \beta^2 a) (\eta - \gamma)^2 + (z - ax)^2 - \beta^2 (ay - mx)^2]}}
\]

This is because, using (5)

\[
(1 - \beta^2 a^2) \left[ \left| n (x - \beta^2 a) - y (1 - \beta^2 a^2) \right| \right]^2 - \beta^2 m^2 \left[ (m, x, y)^2 + (z - ax)^2 - \beta^2 (ay - mx)^2 \right]
\]

\[
\times (1 - \beta^2 a^2) \left[ \left| n (x - \beta^2 a) - y (1 - \beta^2 a^2) \right| \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - \gamma)^2 + (z - ax)^2 \right]
\]

\[
\times \beta^2 m^2 \left[ (x - \beta^2 a) - y (1 - \beta^2 a^2) \right]^2 + \beta^2 m^2 (z - ax)^2
\]

\[
\times [-1 + \beta^2 (n^2)] \left[ \left| n (x - \beta^2 a) - y (1 - \beta^2 a^2) \right| \right]^2 - 2 \left(1 - \beta^2 a^2\right) \left[ n (x - \beta^2 a) - y (1 - \beta^2 a^2) \right] \left| (n, \eta - \gamma)^2 + \beta^2 m^2 (z - ax)^2 \right]
\]

\[
\times [1 - \beta^2 (a^2 + n^2)] \left| n (x - \beta^2 a) - y (1 - \beta^2 a^2) \right| \left| (n, \eta - \gamma)^2 + \beta^2 m^2 (z - ax)^2 \right|
\]

We can also write,

\[
\frac{d}{d\eta} \tan^{-1} \frac{f}{g} = \frac{f' - g'f}{g^2} \frac{f^2}{(f^2 + g^2)} = \frac{g f' - g' f^2}{f (f^2 + g^2)}
\]

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\[ f = (z - ax) \sqrt{[m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 \left[(1 - \beta^2 m^2) (\eta - y)^2 + (z - ax)^2\right]} \]

\[ f^1 = -(z - ax)^2 \left[(1 - \beta^2 a^2) \left[m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2)\right] + \beta^2 m^2 (\eta - y)\right] \]

\[ g = \sqrt{1 - \beta^2 a^2} \left[(z - ax)^2 + (\eta - y) \left[m (x - \beta^2 a z) - y (1 - \beta^2 a^2)\right]\right] \]

\[ g^1 = \sqrt{1 - \beta^2 a^2} \left[m (x - \beta^2 a z) - y (1 - \beta^2 a^2)\right] \]

\[ \frac{f^2 + g^2}{(z - ax)^2} = (z - ax)^2 \left[m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2)\right] \left[1 + \beta^2 m^2 (\eta - y)^2 + (z - ax)^2\right] \]
Therefore

\[
\frac{d}{d\eta} \tan^{-1} \left( \frac{z - ax}{\sqrt{1 - \beta^2 z^2}} \right) = \frac{1}{\sqrt{1 - \beta^2 z^2}} \left[ \frac{\eta(1 - \beta^2 z^2)}{\eta(1 - \beta^2 z^2) - \eta(z - ax)^2} \right]^{1/2} \left[ \frac{1 - \beta^2 z^2}{\eta(1 - \beta^2 z^2) - \eta(z - ax)^2} \right]^{1/2}
\]

Combining (22), (24), and (25)

\[
\frac{d}{d\eta} \left\{ \frac{1}{2}(\eta - \gamma) \log \frac{(mx - \beta^2 (m + ax) + \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}{(mx - \beta^2 ax) - \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}} + \frac{1}{2}(\eta - \gamma) \frac{1}{2(1 - \beta^2 ax)} \left[ \log \frac{(mx - \beta^2 (m + ax) + \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}{(mx - \beta^2 ax) - \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}} \right] \right\}
\]

\[
= \frac{1}{2} \log \frac{(mx - \beta^2 (m + ax) + \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}{(mx - \beta^2 ax) - \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}
\]

\[
= \left\{ \frac{1}{2}(\eta - \gamma) (1 - \beta^2 ax) \left[ (1 - \beta^2 ax) (\eta - \gamma)^2 + (z - ax)^2 \right] \right\}
\]

\[
= \frac{1}{2} \log \frac{(mx - \beta^2 (m + ax) + \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}{(mx - \beta^2 ax) - \sqrt{(1 - \beta^2) [(mx - \eta)^2 - \beta^2 (m^2 + ax)^2]}}
\]

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Evaluation of the definite integral. From (6) and (7)

\[ m(x - \beta^2 a z) - \eta_3 (1 - \beta^2 a^2) \cdot \beta m \sqrt{(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2} \]  

(6)

\[ m(x - \beta^2 a z) - m^2 \eta_3 (1 - \beta^2 (a^2 + m^2)) \eta_3 = \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \]  

(7)

Therefore using (6), (7), and (23), both of the log terms in equation (21) are zero when \( \eta = \eta_3 \). However, the tan^{-1} term can be either zero or \( \pm \pi \) when \( \eta = \eta_3 \), depending upon whether the denominator is greater or less than zero. If it is greater than zero the limit is zero as \( \eta \to \eta_3 \). In (21) when \( \eta = \eta_3 \), using (6) and then (5) and (7),

\[
\frac{(x - ax)^2 - \eta_3 (x - y)}{(x + \beta^2 a z) - y(1 - \beta^2 a^2)}
\]

\[
\frac{(1 - \beta^2 a^2)^2 ((x - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2) - \beta^2 m^2 (m(x - \beta^2 a z) - y (1 - \beta^2 a^2))}{(1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] [(mx - y)^2 - \beta^2 (ay - mz)^2] + [(1 - \beta^2 a^2)^2 - \beta^4 m^4] (z - ax)^2}
\]

\[
\frac{[1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2) [(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2] + \beta^2 m^2 (z - ax)^2}{(1 - \beta^2 a^2) [(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2] + \beta^2 m^2 (z - ax)^2}
\]  

(27)
because

\[(1 - \beta^2a^2)^2 - \beta^4m^4 = (1 - \beta^2a^2 - \beta^2m^2)(1 - \beta^2a^2 + \beta^2m^2)\]

But if \(1 - \beta^2(a^2 + m^2) > 0\) then from (27)

\[(1 - \beta^2a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} > \beta m \mid m(x - \beta^2az) - y(1 - \beta^2a^2)\]

and (26) shows that the denominator of the tan\(^{-1}\) term is \(> 0\). But if

\(1 - \beta^2(a^2 + m^2) < 0\) then (27) shows

\[(1 - \beta^2a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2} < \beta m \mid m(x - \beta^2az) - y(1 - \beta^2a^2)\]

and the denominator of the tan\(^{-1}\) term is \(> 0\) if and only if

\[m(x - \beta^2az) - y(1 - \beta^2a^2) > 0\]

But

\[m(x - \beta^2az) - y(1 - \beta^2a^2) = m[x - \beta \sqrt{y^2 + z^2}] + \beta m \sqrt{y^2 + z^2} - \beta^2 amz - y(1 - \beta^2a^2)\]

But we can also write

\[\beta^2m^2(y^2 + z^2) - [\beta^2amz + y(1 - \beta^2a^2)]^2\]

\[= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2m^2 \beta^2a^2z^2 - 2 \beta^2(1 - \beta^2a^2)amz + \beta^2a^2(1 - \beta^2a^2)y^2\]

\[= [\beta^2(a^2 + m^2) - 1]y^2 + \beta^2(1 - \beta^2a^2)(mz - ay)^2 > 0 \text{ if } 1 - \beta^2(a^2 + m^2) < 0\]
And therefore
\[\beta m \sqrt{y^2 + z^2 - \beta^2amz - y(1 - \beta^2a^2)} > 0\]

and
\[m(x - \beta^2az) - y(1 - \beta^2a^2) > 0\]

if
\[1 - \beta^2(a^2 + m^2) < 0\]

and
\[x > \beta(y^2 + z^2)\]

Therefore inside the Mach cone from the origin \(x^2 > \beta^2(y^2 + z^2)\) we have
\[(z - ax)^2 + (\eta_3 - y) [m(x - \beta^2az) - y(1 - \beta^2a^2)] > 0 \quad (28)\]

and therefore because of (6) and (28)
\[
\tan^{-1}\left(\frac{(z - ax) \sqrt{[m(x - \beta^2az) - \eta_3(1 - \beta^2a^2)]^2 - \beta^2m^2[(1 - \beta^2a^2)(\eta_3 - y)^2 + (z - ax)^2]} \right) = 0
\]

If we change the sign of the denominator we would get \(\pi\) for \(z > ax\) and \(-\pi\) for \(z < ax\). However when we evaluate the term when \(\eta = 0\) and subtract, the result is the same.
Therefore if we evaluate (21) for \( \eta = 0 \) we get for (11)

\[
\varphi (x, y, z) = - \frac{\bar{w} + \beta^2 \bar{u}}{2\pi\sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \frac{(m \cdot \bar{\eta}) \cdot \bar{a}(m \cdot \bar{\eta}) + \sqrt{1 - \beta^2 (m^2 + \bar{\eta}^2)}}{(m \cdot \bar{\eta}) \cdot \bar{a}(m \cdot \bar{\eta}) - \sqrt{1 - \beta^2 (m^2 + \bar{\eta}^2)}} d\bar{\eta}
\]

\[
= \frac{\bar{w} + \beta^2 \bar{u}}{\pi} \left\{ \frac{(z - ax) \tan^{-1} \frac{m(z - ax)}{\sqrt{x^2 - \beta^2 (y^2 + z^2)}}}{1 - \beta^2 a^2} + \frac{y(1 - \beta^2 a^2) - m(x - \beta^2 az)}{2(1 - \beta^2 a^2) \sqrt{1 - \beta^2 (a^2 + m^2)}} \log \frac{x - \beta^2 (a^2 + m^2) + \sqrt{(x^2 - \beta^2 (a^2 + m^2))(1 - \beta^2 (a^2 + m^2))}}{x - \beta^2 (a^2 + m^2) - \sqrt{(x^2 - \beta^2 (a^2 + m^2))(1 - \beta^2 (a^2 + m^2))}} \right. \\
\left. - \frac{y}{2\sqrt{1 - \beta^2 a^2}} \log \frac{x - \beta^2 az + \sqrt{(x^2 - \beta^2 (y^2 + z^2))(1 - \beta^2 a^2)}}{x - \beta^2 az - \sqrt{(x^2 - \beta^2 (y^2 + z^2))(1 - \beta^2 a^2)}} \right\}
\]

(29)

**Evaluation of the velocity components.** - The velocity components may now be obtained by differentiation of the velocity potential. However, whoever attempts to take partial derivatives of (29) is in for a long days work. But the partial derivatives of (29) may be obtained quite simply if it is noted that all terms in the expressions for the velocity components must have coefficients involving either \( \log \) or \( \tan^{-1} \). This is easily shown by differentiating the expression for \( \varphi \) given by (11).
\[ u = \frac{\partial \phi}{\partial x} = -\frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{2\pi \sqrt{1 - \beta^2 a^2}} \left[ \int_{-\gamma}^{\gamma} \frac{\log \left( \frac{(mx - \gamma - \beta a(mz - \gamma) + \sqrt{(1 - \beta^2 a^2)(x - \gamma)^2 + z^2})}{(mx - \gamma - \beta a(mz - \gamma) - \sqrt{(1 - \beta^2 a^2)(x - \gamma)^2 + z^2})} \right)}{\partial x} \right] \]

However, this integral expression for \( u \) involves integrals of the same form as in (13), (16), or (18). A closer examination shows that the resulting \( \log \) or \( \tan^{-1} \) terms will have arguments which are identical with those for \( \phi \), although the coefficients may be different. Therefore all expressions must involve either \( \log \) or \( \tan^{-1} \) terms. The same arguments can be made for the \( v \) and \( w \) velocity components.

With this in mind the partial derivatives of \( \phi \) can be obtained from (29) by differentiating only the coefficients of the \( \log \) and \( \tan^{-1} \) terms. Differentiation of the \( \tan^{-1} \) or \( \log \) terms will not yield other \( \tan^{-1} \) or \( \log \) terms and therefore the sum of these terms must give zero. Therefore

\[ u = \frac{\partial \phi}{\partial x} = -\frac{\bar{w} + \beta^2 \bar{a} \bar{u}}{\pi (1 - \beta^2 a^2)} \left\{ \tan^{-1} \frac{m(z - ax)}{y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2} \right. \]

because from (6) and (23a) the \( \log \) term is zero when \( \gamma = \gamma_3 \).
Constant Pressure Surface Boundary Conditions. - In (1) we can set $\Omega = u$

Assume that on $S$

(a) $u - u' = \text{const}$

(b) $\left[ \frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} \right] = 0 \quad (30)$

Now since

$$
\frac{\partial u}{\partial \nu} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{1}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi}
$$

$$
\frac{\partial u'}{\partial \nu'} = \frac{-\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{-1}{1 + a^2} \frac{\partial u}{\partial \xi}
$$

and from (b)

$$
\frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial}{\partial \xi} (u - u') + \frac{1}{\sqrt{1 + a^2}} \frac{\partial}{\partial \xi} (u - u') = 0
$$

Now from (a), $u - u' = \text{const}$ on $S$ implies that on $S$

$$
\frac{\partial}{\partial \eta} (u - u') = 0
$$

and

$$
\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0
$$

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This means that since

$$\beta^2 a \frac{\partial}{\partial \xi} (u - u') + \frac{\partial}{\partial \xi} (u - u') = 0$$

and

$$\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0$$

then if

$$1 - \beta^2 a \neq 0$$

$$\frac{\partial}{\partial \xi} (u - u') = \frac{\partial}{\partial \eta} (u - u') = \frac{\partial}{\partial \xi} (u - u') = 0 \text{ on } S$$

Note that assumptions (a) and (b) in (30) are independent assumptions.

(a) Says that \((u - u')\) is constant on \(S\)

(b) Effectively says that the derivative of \(u - u'\) in a direction normal to \(S\) is zero.

Woodward assumes, in reference (55), that (a) implies (b), which is not true.

If we examine the normal velocity on \(S\)

$$u_n = \frac{-au + w}{\sqrt{1 + a^2}} \quad u'_n = \frac{au' - w}{\sqrt{1 + a^2}}$$

then since due to irrotationality

$$\frac{\partial w}{\partial \xi} = \frac{\partial u}{\partial \xi}$$

$$\sqrt{1 + a^2} \frac{\partial}{\partial \xi} u_n = -a \frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \xi} = -a \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \xi}$$
\[
\sqrt{1 + a^2 \frac{\partial}{\partial \xi}(u_n + u'_n)} = -a \frac{\partial}{\partial \xi}(u - u') + \frac{\partial}{\partial \xi}(u - u') - 0
\]

and therefore the source strength, if any, is constant.

**Evaluation of the integral over \( \xi \).** - The integral to be evaluated is,
Except for the coefficient, the second integral is identical to the integral obtained for a surface distribution of sources. It has already been evaluated [see (9)]. The first integral over \( \xi \) is easy to evaluate,

\[
\int \frac{[(z - ax)(z - a \xi) + (y - \eta)^2]}{(y - \eta)^2 + (z - a \xi)^2} \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a \xi)^2} \, d\xi
\]

\[
= - \frac{1}{2} \log \frac{(x-3) + \sqrt{(x-3)^2 - \beta^2[(z-a\xi)^2 + (y-\eta)^2]}}{(x-3) - \sqrt{(x-3)^2 - \beta^2[(z-a\xi)^2 + (y-\eta)^2]}}
\]

This can be verified by differentiating. Let,

\[
\begin{align*}
 f &= x - 3 \\
 g^2 &= \beta^2 [(z-a\xi)^2 + (y-\eta)^2] \\
 k^2 &= (x-3)^2 - \beta^2 [(z-a\xi)^2 + (y-\eta)^2] \\
 \frac{\partial}{\partial \xi} \left( \frac{1}{2} \log \frac{f + k}{f - k} \right) &= \frac{1}{kg^2} \left[ (g' g - f g g') - \frac{1}{k} \right] \quad \text{from (13)}
\end{align*}
\]

\[
\begin{align*}
 f' &= -1 \\
 g g' &= -\beta^2 a (z-a\xi)
\end{align*}
\]

Therefore,

\[
\begin{align*}
 - \frac{1}{2} \frac{\partial}{\partial \xi} \log \frac{(x-3) + \sqrt{(x-3)^2 - \beta^2 [(z-a\xi)^2 + (y-\eta)^2]}}{(x-3) - \sqrt{(x-3)^2 - \beta^2 [(z-a\xi)^2 + (y-\eta)^2]}} &= \\
= \frac{(z-a\xi)^2 + (y-\eta)^2 - a(x-\xi)(z-a\xi)}{[(z-a\xi)^2 + (y-\eta)^2] \sqrt{(x-\xi)^2 - \beta^2 (z-a\xi)^2 - \beta^2 (y-\eta)^2}} \\
&= \frac{(z-ax)(z-a\xi) + (y-\eta)^2}{[(z-a\xi)^2 + (y-\eta)^2] \sqrt{(x-\xi)^2 - \beta^2 (z-a\xi)^2 - \beta^2 (y-\eta)^2}}
\end{align*}
\]
Since from (3)

\[ (x - \xi_2)^2 = \beta^2 \left[ (z - a \xi_2)^2 + (y - \eta)^2 \right] \]

\[
\log \frac{(x - 3_2) + \sqrt{(x - 3_2)^2 - \beta^2 [(z - a_2)^2 + (y - \eta)^2]}}{(x - 3_2) - \sqrt{(x - 3_2)^2 - \beta^2 [(z - a_2)^2 + (y - \eta)^2]}} = 0
\]

and from (2)

\[ \xi_1 = \frac{\eta}{m} \]

\[
\int_{\xi_1}^{\xi_2} \frac{(z - ax)(z - a \xi) + (y - \eta)^2}{[(y - \eta)^2 + (z - a \xi)^2] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a \xi)^2}} d\xi
\]

\[
= \frac{1}{2} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 [(m \gamma - m a)^2 + m^2 (\gamma - \eta)^2]}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 [(m \gamma - m a)^2 + m^2 (\gamma - \eta)^2]}}
\]

therefore

\[ \phi(x, y, z) = \frac{1}{8\pi a} \frac{\Delta P}{q_{\infty}} \int_0^{\gamma_3} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m \gamma - ma)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m \gamma - ma)^2}} d\gamma
\]

\[ - \frac{1}{8\pi a} \frac{\Delta P}{q_{\infty}} \sqrt{1 - \beta^2 a^2} \int_0^{\gamma_3} \log \frac{(mx - \eta - \beta a (m \gamma - ma) + \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 (m \gamma - ma)^2]}}{(mx - \eta - \beta a (m \gamma - ma) - \sqrt{(1 - \beta^2 a^2) [(mx - \eta)^2 - \beta^2 (m \gamma - ma)^2]}} d\gamma
\]

The second integral was evaluated previously for a surface distribution of sources.
Evaluation of the indefinite integral over $\eta$. - The first integral in (32) must first be integrated by parts.

Let $f = mx - \gamma$

$$g^2 = \beta^2 (\gamma - \eta)^2 + \beta^2 (m^2 - a^2)$$

$$h^2 = (mx - \gamma)^2 - \beta^2 (\gamma - \eta)^2 - \beta^2 (m^2 - a^2)$$

Then let

$$u = \frac{1}{2} \log \frac{f + h}{f - h} \quad dv = d\eta$$

$$du = \frac{1}{h g^2} \left[ f g' - g g' f \right] d\eta \quad v = \eta$$

$$f' = -1$$

$$gg' = \beta^2 \left[ a (a \eta - mz) + m^2 (\eta - y) \right]$$

$$\left[ f' g^2 + gg' f \right] = \beta^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 + (mx - \eta) \left[ a (a \eta - mz) + m^2 (\eta - y) \right] \right]$$

$$= \beta^2 \left[ (a \eta - mz) \left[ (a \eta - mz) + a (mx - \eta) \right] + m^2 (\eta - y) \left[ (\eta - y) + (mx - \eta) \right] \right]$$

$$= \beta^2 \left[ a (a \eta - mz) (z - ax) + m^2 (\eta - y) (mx - y) \right]$$

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\[ - \eta \left[ f \gamma^2 - g \xi \gamma \right] = \eta^2 \left[ \alpha \left( m (x - y) - a (z - ax) \right) \right] \eta^2 + m^2 \left[ z (z - ax) - y (mx - y) \right] \eta \]

\[ = \eta^2 \left[ m (mx - y) - \eta (z - ax) \right] \left[ (\alpha \eta - m \eta)^2 + m^2 (\eta - y)^2 \right] \]

\[ = \frac{2m^2 \left[ m (mx - y) - \eta (z - ax) \right] (nz + my) \eta + m^2 (\alpha^2 + m^2) \left[ z (z - ax) - y (nx - y) \right] \eta}{(\alpha^2 + m^2)} \]

\[ = \frac{-2m \left[ m (mx - y) - \eta (z - ax) \right] m^2 (y^2 + z^2)}{(\alpha^2 + m^2)} \]

But

\[ \left[ m (mx - y) - \eta (z - ax) \right] (nz + my) + \left[ m^2 + m^2 \right] \left[ z (z - ax) - y (mx - y) \right] \]

\[ = \left[ m (az + my) - y (z^2 + m^2) \right] (mx - y) - \left[ m (z^2 + m^2) - \eta (nz + my) \right] (z - ax) \]

\[ = -m (az - my) (mx - y) - m (ay - mz) (z - ax) \]

\[ = (ay - mz)^2 \]
Therefore

\[
\frac{1}{2} \int \log \left( \frac{(mx - \eta) - \beta^2 m^2 (\eta - y)^2 - \beta^2 (m z - \eta)^2}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (m z - \eta)^2}} \right) d\eta
\]

\[
= \frac{1}{2} \eta \log \left( \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (m z - \eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (m z - \eta)^2}} \right)
\]

\[
\quad \quad \quad - \frac{m}{(a z - m y) \cdot (a^2 + m^2)} \int \frac{d\eta}{\sqrt{(mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2}}
\]

\[
= \frac{m}{a^2 + m^2} \int \frac{(a y - m z)^2 + (m x - y) \cdot a (z - ax)}{(a \eta - m z)^2 + m^2 (\eta - y)^2} \frac{d\eta}{\sqrt{(mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2}}
\]

(34)

The first integral in (34) may be evaluated using (16) because,

\[
[m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2]
\]

\[
= (1 - \beta^2 a^2) \left| (mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2 \right|
\]

(35)

Therefore using (16) and (35)

\[
\int \frac{d\eta}{\sqrt{(mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2}} =
\]

\[
\frac{1}{2 \sqrt{1 - \beta^2 (a^2 + m^2)}} \log \frac{(mx - \eta) - \beta^2 (a \eta - m z)^2 + \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2]}}{(mx - \eta) - \beta^2 (a \eta - m z)^2 - \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \eta)^2 - \beta^2 (a \eta - m z)^2 - \beta^2 m^2 (\eta - y)^2]}}
\]

(36)
The last term in equation (34) must be changed to the form of equation (18) in order to be evaluated. We can write

\[(a \eta - mz)^2 + m^2 (\eta - y)^2 = (a^2 + m^2) \eta^2 - 2m (az + my) \eta + m^2 (y^2 + z^2)\]

\[= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{m^2 \left[ (a^2 + m^2) (y^2 + z^2) - (az + my)^2 \right]}{a^2 + m^2}\]

\[= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{2m (ay - mz)^2}{(a^2 + m^2)}\]

Now we introduce a change of variables. Let

\[u = \eta - \frac{m (az + my)}{a^2 + m^2}\]

Therefore

\[(a \eta - mz)^2 + m^2 (\eta - y)^2 = (a^2 + m^2) \left\{ u^2 + \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\}\]

\[\eta - mx = u + \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)}\]
\[(\eta - mx)^2 - \beta^2 (a\eta - mz)^2 - \beta^2 m^2 (\eta - y)^2\]

\[= \left[1 - \beta^2 (a^2 + m^2)\right] u^2 + 2 \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)} u\]

\[\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x (a^2 + m^2)\right]^2 \]

\[\frac{2}{(a^2 + m^2)^2}\]

\[= \left[1 - \beta^2 (a^2 + m^2)\right] u^2 + 2 \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)} u\]

\[\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x (a^2 + m^2)\right]^2 \]

\[\frac{2}{(a^2 + m^2)^2}\]

(39)

Note: (18) can be used. Let

\[a = 1 - \beta^2 (a^2 + m^2)\]

\[b = \frac{m [(az + my) - x (a^2 + m^2)]}{(a^2 + m^2)} = \frac{m [(a - ax) - m (mx - y)]}{(a^2 + m^2)}\]

\[c = \frac{-\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x (a^2 + m^2)\right]^2}{(a^2 + m^2)^2}\]

\[e = \frac{m (ay - mz)}{(a^2 + m^2)}\]

(40)
and referring to (17)

\[ c - \hat{a}e^2 = \frac{-m^2 (ay - mz)^2 + m^2 \left[ m (mx - y) - a(z - ax) \right]^2}{(a^2 + m^2)^2} \]

\[ b^2 e^2 = \frac{m^2 \left[ m (mx - y) - a(z - ax) \right]^2 m^2 (ay - mz)^2}{(a^2 + m^2)^4} \]

Therefore we can choose

\[ \gamma^4 + (c - \hat{a}e^2) \gamma^2 - b^2 e^2 \]

\[ = \left\{ \gamma^2 - \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\} \left\{ \gamma^2 + \frac{m^2 \left[ m (mx - y) - a(z - ax) \right]^2}{(a^2 + m^2)^2} \right\} = 0 \]

\[ \gamma = \frac{m (ay - mz)}{(a^2 + m^2)} = e \]

(41)
The numerator of the second integral in equation (34) can be written,

\[
\left\{ \frac{a^2 + m^2}{(a^2 + m^2)^2} \right\} \frac{\gamma^2 - m(az + my)b}{u + \frac{m(az + my)}{(a^2 + m^2)}} + m^2 \left( \gamma^2 + z^2 \right) b
\]

\[
= \left\{ \frac{a^2 + m^2}{(a^2 + m^2)^2} \right\} \left( u + m(az + my) \gamma^2 + \frac{m^2 b (ay - mz)^2}{(a^2 + m^2)} \right)
\]

\[
= \left\{ \frac{a^2 + m^2}{(a^2 + m^2)^2} \right\} \left( u + \gamma^2 [m(az + my) + b(a^2 + m^2)] \right)
\]

Now referring to (18) we can write

\[
A = \frac{\gamma^2 \left[ m(az + my) + b(a^2 + m^2) \right]}{a^2 + m^2}
\]

\[
B = \frac{(a^2 + m^2) \gamma^2 - m(az + my) b}{a^2 + m^2}
\]

The \((a^2 + m^2)\) in the denominator of \(A\) and \(B\) comes from (37) which is in the denominator of (34). From (18) and (41)

\[
\frac{Ab \gamma + B \gamma^3}{\gamma^4 + b^2 e^2} = \frac{b \gamma \left[ m(az + my) + b(a^2 + m^2) \right] + \left[ (a^2 + m^2) \gamma^2 - m(az + my)b \right] \gamma}{(a^2 + m^2) (\gamma^2 + b^2)}
\]

\[
= \frac{\gamma(a^2 + m^2)}{(a^2 + m^2)} = \frac{m(ay - mz)}{(a^2 + m^2)}
\]
and

\[
-\frac{A \gamma^3}{e} + B b e \gamma = -\gamma^2 \left[ m (az + my) + b (a^2 + m^2) \right] + \left[ (a^2 + m^2) \gamma^2 - b m (az + my) \right] b
\]

\[
= \frac{-m (az + my)}{a^2 + m^2}
\]

From (39) and (40),

\[
\hat{u} (u^2 + 2bu + c) = (mx - \eta)^2 - \beta^2 (a \eta - mz)^2 - \beta^2 m^2 (\eta - y)^2
\]

\[
\gamma^2 - bu = \gamma^2 \cdot b \left[ \frac{\eta - m (az + my)}{a^2 + m^2} \right] = \frac{m (mx - y) - a (z - ax)}{a^2 + m^2} \eta + \left[ \gamma^2 + b \frac{m (az + my)}{a^2 + m^2} \right]
\]

\[
= \frac{m (mx - y) - a (z - ax)}{a^2 + m^2} \eta + \frac{m^2 (a^2 y^2 + 2amy + m^2 z^2) + m^2 (a (z - ax) - m (mx - y)) (az + my)}{a^2 + m^2}
\]

\[
= \frac{m}{a^2 + m^2} \left[ m (mx - y) - a (z - ax) \right] \eta + m \left[ z (z - ax) - y (mx - y) \right]
\]

\[
u + b = \eta - mx \quad \text{from (38) and (40).}
\]
Therefore using (18), and (41) we get from (34)

\[\int \frac{(ay - mz)^2 + (ax + az) [m (nx - y) - a (z - ax)]}{\eta - m [m (nx - y) - a (z - ax)]} \left(\frac{y^2 + z^2}{(a \eta - mz)^2 + m^2 \eta (\eta - y)^2}\right) d\eta\]

\[= \frac{m (ay - mz)}{a^2 + m^2} \tan^{-1} \left(\frac{(ay - mz)}{m (nx - y) - a (z - ax)}\right) \eta + m \left[z (z - ax) - y (mx - y)\right]\]

\[= \frac{m (a^2 + m^2)}{2 (a^2 + m^2)} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}\]

\[+ \frac{m (a^2 + m^2) - (a^2 + m^2)}{2 (a^2 + m^2)} \log \frac{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}\]

\[+ \frac{m (mz - ay)}{a^2 + m^2} \tan^{-1} \left(\frac{mz - ay}{m (nx - y) - a (z - ax)}\right) \left(\frac{(a \eta - mz)^2 + m^2 (\eta - y)^2}{[m (nx - y) - a (z - ax)] \eta + m [z (z - ax) - y (mx - y)]}\right)\]

Therefore substituting (36), and (42) in (34)

\[\int \frac{\log [(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}]}{\log [(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}] d\eta = \frac{1}{2} \frac{m (a^2 + m^2)}{2 (a^2 + m^2)} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}\]

\[+ \frac{m (a^2 + m^2) - (a^2 + m^2)}{2 (a^2 + m^2)} \log \frac{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (m^2 - \eta)^2}}\]

\[+ \frac{m (mz - ay)}{a^2 + m^2} \tan^{-1} \left(\frac{mz - ay}{m (nx - y) - a (z - ax)}\right) \left(\frac{(a \eta - mz)^2 + m^2 (\eta - y)^2}{[m (nx - y) - a (z - ax)] \eta + m [z (z - ax) - y (mx - y)]}\right)\]
Differentiation of the indefinite integral over $\eta$. From (33)

\[ \frac{1}{2} \frac{d}{d\eta} \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta m^2(\gamma - \eta)^2 - \beta^2(m\eta - \gamma)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta m^2(\gamma - \eta)^2 - \beta^2(m\eta - \gamma)^2)} = \]

\[ \frac{m^2 [y (mx - y) - z (z - ax)] + m [a (z - ax) - m (mx - y)] \eta}{\left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right]}} \]

From (24) using (35)

\[ \frac{1}{2} \frac{d}{d\eta} \log \frac{(mx - \eta) + \sqrt{1 - \beta m^2[\eta - \beta(\eta \eta - \eta^2) - m^2(\eta \eta - \eta^2)]}}{(mx - \eta) - \sqrt{1 - \beta m^2[\eta - \beta(\eta \eta - \eta^2) - m^2(\eta \eta - \eta^2)]}} = \frac{-\sqrt{1 - \beta^2 (a^2 + m^2)}}{\sqrt{(mx - \eta)^2 - \beta^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right]}} \]

(44)

\[ \frac{d}{d\eta} \tan^{-1} \frac{f}{g} = \frac{f' \frac{g'f}{g^2} - g \frac{f'f}{g^2}}{(f'f + g^2) + 1} = \frac{gff' - g'f^2}{f (f^2 + g^2)} \]

where
\[ f = (mz - ay) \sqrt{(mx - \eta)^2 + \beta^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right]} \]  
\[ g = \left[ m (mx - y) - a (z - ax) \right] \eta + m \left[ z (z - ax) - y (mx - y) \right] \]  
\[ f' = (mz - ay)^2 \left[ (\eta - mx)^2 - \beta^2 a (a \eta - mz) - \beta^2 m^2 (\eta - y)^2 \right] \]  
\[ g' = m (mx - y) - a (z - ax) \]  
\[ f^2 + g^2 = (mz - ay)^2 \left\{ (\eta - mx)^2 - \beta^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right] \right\} \]  
\[ = -\beta^2 (mz - ay)^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right] \]  
\[ + \left\{ m (mx - y) - a (z - ax) \right\} \eta + m \left[ z (z - ax) - y (mx - y) \right] \]  
\[ = -\beta^2 (mz - ay)^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right] \]  
\[ + \left[ m (z - ax) + a (mx - y) \right] \eta \]  
\[ = -\beta^2 (mz - ay)^2 \left[ (a \eta - mz)^2 + m^2 (\eta - y)^2 \right] \]  
\[ + \left[ a^2 (mx - y)^2 + 2 a m (mx - y) (z - ax) + m^2 (z - ax)^2 \right] (\eta - mx)^2 \]  
\[ + m^2 (mx - y)^2 (\eta - y)^2 - 2 m (mx - y) (z - ax) (\eta - y) (a \eta - mz) \]  
\[ + (2 - ax)^2 (a \eta - mz)^2 \]
\[ \begin{array}{c}
\text{Original page is of poor quality.}
\end{array} \]
\[
\frac{d}{d \eta} \tan^{-1} \left( \frac{(ay - mz) \sqrt{(\eta - mx)^2 - \beta^2 [(a \eta - mz)^2 + m^2 (\eta - y)^2]}}{m (mx - y) - a (z - ax)} \right) \eta + m \left[ z (z - ax) - y (mx - y) \right]
\]

\[
= \frac{m (ay - mz) (\eta - mx)}{[(a \eta - mz)^2 + m^2 (\eta - y)^2] \sqrt{(\eta - mx)^2 - \beta^2 [(a \eta - mz)^2 + m^2 (\eta - y)^2]}}
\]

(48)

Therefore combining terms in (43), (46), (47), and (48)

\[
\frac{1}{2} \frac{d}{d \eta} \int \frac{\log \left( \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2 - \beta^2 (m \eta - ax)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2 - \beta^2 (m \eta - ax)^2}} \right)}{(\eta - mx) - \sqrt{(\eta - mx)^2 - \beta^2 (\eta - y)^2 - \beta^2 (m \eta - ax)^2}} \ d\eta
\]

\[
= \frac{1}{2} \log \left( \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2 - \beta^2 (m \eta - ax)^2}}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2 - \beta^2 (m \eta - ax)^2}} \right)
\]

\[
\cdot \left[ \eta \cdot \left( \frac{a \eta - mz - n}{a^2 + n^2} \right) \right] \cdot \frac{\eta^2 \left[ \eta (mx - \eta) - z (z - ax) \right] - \eta \left[ a (z - ax) - n (mx - \eta) \right]}{\left( a \eta - mz \right)^2 + \eta^2 (\eta - y)^2} \sqrt{(\eta - mx)^2 - \beta^2 \left( (a \eta - mz)^2 + \eta^2 (\eta - y)^2 \right)}
\]

\[
- \frac{\eta \left( (a \eta - mz) \cdot (n^2 + n^2) \right) \eta}{\left( a^2 + n^2 \right) \sqrt{(mx - \eta)^2 - \beta^2 \left( (a \eta - mz)^2 + \eta^2 (\eta - y)^2 \right)} \sqrt{(mx - \eta)^2 - \beta^2 \left( (a \eta - mz)^2 + \eta^2 (\eta - y)^2 \right)}}
\]

\[
\cdot \frac{\eta^2 \left( ay - mz \right)^2 \left( \eta - mx \right)}{\left( a^2 + n^2 \right) \left[ (a \eta - mz)^2 + \eta^2 (\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 \left( (a \eta - mz)^2 + \eta^2 (\eta - y)^2 \right)}}
\]

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Arranging all terms over a common denominator

\[
N^2 \left\{ (a^2 + m^2) \left[ m (z - ax) \right] - (a^2 + m^2) \left[ a (z - ax) - m (mx - y) \right] \right. \\
+ m \left\{ -m (az + my) \left[ a (z - ax) - m (mx - y) \right] + (a^2 + m^2) \left[ y (mx - y) - z (z - ax) \right] \\
+ 2 [a (z - ax) - m (mx - y)] \left[ iz + my \right] + m (ay - mz)^2 \right. \\
- m^2 (az + my) \left[ y (mx - y) - z (z - ax) \right] - m^2 \left[ a (z - ax) - m (mx - y) \right] (y^2 + z^2) - m^2 (ay - mz)^2 \\
= mn \left\{ -a (az + my) + (a^2 + m^2)z + 2a [az + my] - m (ay - mz) \right. \\
+ (mx - y) \left[ m (az + my) + (a^2 + m^2)y - 2m [az + my] - a (ay - mz) \right] \right. \\
+ m^2 \left\{ (z - ax) \left[ z (az + my) - a (y^2 + z^2) + mx (ay - mz) \right] \\
+ (mx - y) \left[ -y (az + my) + m (y^2 + z^2) + ax (ay - mz) \right] \right. \\
- m^2 \left\{ (z - ax) \left[ zm (mx - y) + ay (mx - y) \right] \\
+ (mx - y) \left[ ay (z - ax) + mz (z - ax) \right] \right) = 0
\]

Therefore (43) is verified.
Evaluation of the definite integral over \( \eta \). - Because of (4) and (6) and analogous to the section in the evaluation of the definite integral for the surface distribution of sources, all of the terms in (43) will be zero when \( \eta = \eta_3 \) provided the denominator of the \( \tan^{-1} \) term is greater than zero. At \( \eta = \eta_3 \), using (6), and then (5),

\[
\left[ m \left( m - y \right) - a \left( z - ax \right) \right] \eta_3 - \eta \left[ \left( z - ax \right) - y \left( m - y \right) \right]
\]

\[
m \left[ m \left( m - y \right) - a \left( z - ax \right) \right] \left[ (m - y)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2 \right] - \frac{1}{\left[ \left( m - y \right)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2 \right]}
\]

\[
m \left[ (m - y)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2 \right] - \beta^2 \left[ m \left( m - y \right) - a \left( z - ax \right) \right] \sqrt{\left( m - y \right)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2}
\]

\[
\frac{1}{\left[ \left( m - y \right)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2 \right]}
\]

But

\[
\left[ (m - y)^2 - \left( z - ax \right)^2 - \beta^2 \left( ay - mz \right)^2 \right] - \beta^2 \left[ m \left( m - y \right) - a \left( z - ax \right) \right]^2
\]

\[
= \left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] \left( m - y \right)^2 + \beta^2 a^2 \left( m - y \right)^2 + \beta^2 m^2 \left( z - ax \right)^2
\]

\[
+ 2 \beta^2 m \left( m - y \right) \left( z - ax \right) - \beta^2 \left( ay - mz \right)^2
\]

\[
= \left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] \left( m - y \right)^2 + \beta^2 \left[ a \left( m - y \right) \left. + m \left( z - ax \right) \right] - \beta^2 \left( ay - mz \right)^2
\]

\[
= \left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] \left( m - y \right)^2 + \left( z - ax \right)^2
\]
Therefore if \(1 - \beta^2 (a^2 + m^2) > 0\)

\[
\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} > \beta |m (mx - y) - a (z - ax)|
\]

and from (49)

\[
[m (mx - y) - a (z - ax)] \eta_3 + m [z (z - ax) - y (mx - y)] > 0
\]

However also,

\[
m (mx - y) - a (z - ax) = (a^2 + m^2)x - (my + az)
\]

and

\[
(a^2 + m^2) x^2 - (my + az)^2 = (a^2 + m^2) x^2 - (a^2 + m^2) (y^2 + z^2) + (ay - mz)^2
\]

\[
= \frac{a^2 + m^2}{\beta^2} \left\{ \left[ \beta^2 (a^2 + m^2) - 1 \right] x^2 + \left[ x^2 - \beta^2 (y^2 + z^2) \right] \right\} + (ay - mz)^2 > 0
\]

if

\[
1 - \beta^2 (a^2 + m^2) < 0
\]

and

\[
x^2 > \beta^2 (y^2 + z^2)
\]
Therefore if $x > 0$ also

$$(a^2 + m^2 x) > |my + az|$$

and

$$m (mx - y) - a (z - ax) > 0$$

if

$$1 - \beta^2 (a^2 + m^2) < 0$$

$$\chi^2 > \beta^2 (y^2 + z^2)$$

Therefore from (50) if $1 - \beta^2 (a^2 + m^2) < 0$

$$\left[ \beta^2 (a^2 + m^2) - 1 \right]^{-1} \left[ \beta \left[ m (mx - y) - a (z - ax) \right] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 ay - mz)^2} \right] > 0$$

and from (49), if $x^2 > \beta^2 (y^2 + z^2)$

$$\left[ m (mx - y) - a (z - ax) \right] \eta_3 + m \left[ z (z - ax) - y (mx - y) \right] > 0$$

and therefore, using (4) and (54), the tan$^{-1}$ term in (45) is zero when $\eta = \eta_3$. 

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Therefore substituting \( \eta = 0 \) into (43), and from (32),

\[
\phi(x, y, z) = \frac{\Delta P}{q_{\infty}} \int_0^\infty \frac{d\gamma}{8 \pi a} \left\{ \gamma_3 \right. \\
- \sqrt{1 - \beta^2} \left[ \log \left( \frac{(m-x) + \sqrt{(m-x)^2 - \beta a^2}(y-z)^2 - \beta^2(\gamma^2 - 2\beta^{1/2}\gamma^{1/2}z)}{(m-x) - \sqrt{(m-x)^2 - \beta a^2}(y-z)^2 - \beta^2(\gamma^2 - 2\beta^{1/2}\gamma^{1/2}z)} \right) \right. \\
\left. \right\} \\
= \frac{1}{4\pi a} \frac{\Delta P}{q_{\infty}} \left\{ \frac{m(a^2 + m^2)}{2(a^2 + m^2)} \log \frac{x + \sqrt{x^2 - \beta^2(y^2 + z^2)}}{x - \sqrt{x^2 - \beta^2(y^2 + z^2)}} \\
- \frac{m(a^2 + m^2)}{2(a^2 + m^2)\sqrt{1 - \beta^2(a^2 + m^2)}} \log \frac{x - \beta a^2 + \sqrt{1 - \beta^2(a^2 + m^2)}\left[ x^2 - \beta^2(y^2 + z^2) \right]}{x - \beta a^2 - \sqrt{1 - \beta^2(a^2 + m^2)}\left[ x^2 - \beta^2(y^2 + z^2) \right]} \\
+ \frac{m(a^2 + m^2)}{a^2 + m^2} \tan^{-1} \frac{(m-a)x}{\sqrt{x^2 - \beta^2(y^2 + z^2)}} \\
+ (z-ax) \tan^{-1} \frac{m(z-ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{\sqrt{x^2 - \beta^2(y^2 + z^2)} - \sqrt{(z-ax)^2 - \beta^2(a^2 + m^2)}} \\
- \frac{m(x-\beta^2a^2) - 3(1-\beta^2a^2)}{2\sqrt{1 - \beta^2(a^2 + m^2)} \log \frac{x - \beta^2(a^2 + m^2) + \sqrt{1 - \beta^2(a^2 + m^2)}\left[ x^2 - \beta^2(y^2 + z^2) \right]}{x - \beta^2(a^2 + m^2) - \sqrt{1 - \beta^2(a^2 + m^2)}\left[ x^2 - \beta^2(y^2 + z^2) \right]} \\
- \frac{1}{2} \gamma \sqrt{1 - \beta^2a^2} \log \frac{(x-\beta^2a^2) + \sqrt{(1-\beta^2a^2)\left[ x^2 - \beta^2(y^2 + z^2) \right]}}{(x-\beta^2a^2) - \sqrt{(1-\beta^2a^2)\left[ x^2 - \beta^2(y^2 + z^2) \right]}} \right\} 
\]
Therefore combining terms

\[
\phi(x, y, z) = \frac{1}{4\pi a G}\left\{ \frac{m(a^2 + my)}{2(a^2 + m^2)} \log \frac{x + \sqrt{x^2 - \beta^4(y^2 + z^2)}}{x - \sqrt{x^2 - \beta^4(y^2 + z^2)}} + \frac{\alpha(a^2 + m^2) \sqrt{1 - \beta^4(a^2 + m^2)}}{2(a^2 + m^2)} \log \frac{x - \beta^4(a^2 + my) + \sqrt{[1 - \beta^4(a^2 + m^2)](x^2 - \beta^4(y^2 + z^2))}}{x - \beta^4(a^2 + my) - \sqrt{[1 - \beta^4(a^2 + m^2)](x^2 - \beta^4(y^2 + z^2))}} \right. \\
- \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \left( \frac{mz - ay}{z(z - ax) - y(mx - y)} \right) \\
+ (z - ax) \tan^{-1} \left( \frac{m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[\beta^2(y^2 + \beta^2z^2)] + (z - ax)^2} \right) \\
- \frac{1}{2} \beta^2 \frac{a^2}{2} \log \frac{x - \beta^4 az + \sqrt{[1 - \beta^4 a^2](x^2 - \beta^4(y^2 + z^2))}}{x - \beta^4 az - \sqrt{[1 - \beta^4 a^2](x^2 - \beta^4(y^2 + z^2))}} \right.
\]

Evaluation of the velocity components. - Using the arguments presented in the section covering the evaluation of the velocity components for the source distribution, the partial derivatives of (51) may be obtained by differentiating only the coefficients of each term.

Modifications and Regions of Validity of the Velocity Potential Functions

Region of validity of \( \phi \). - All of the integrations were performed without regard to the existence of the limits of integration, negative square roots, etc. Therefore the functions in (29) and (51) must be examined and possibly modified for some regions of \((x, y, z)\) space.

Since all of the terms contain \( \sqrt{x^2 - \beta^4(y^2 + z^2)} \) the formula for \( \phi(x, y, z) \) given by (51) is only valid when \( x^2 > \beta^4(y^2 + z^2) \), which means inside the null cone from the origin.
\[ 1 - \beta^2(a^2 + m^2) < 0 \quad \text{(supersonic leading edge)} \]

the log term which contains the square root of this quantity must be modified. We can write

\[
\frac{i}{2} \log \frac{f + ig}{f - ig} = - \tan^{-1} \frac{g}{f} \quad (52)
\]

and therefore if \[ 1 - \beta^2(a^2 + m^2) < 0 \]

\[
\frac{1}{2} \sqrt{1 - \beta^2(a^2 + m^2)} \frac{1}{\log \frac{x - \beta^2(a^2 + m^2) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(a^2 + m^2) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}
\]

\[= -\sqrt{\beta^2(a^2 + m^2) - 1} \tan^{-1} \frac{\sqrt{\beta^2(a^2 + m^2) - 1}[x^2 - \beta^2(y^2 + z^2)]}{x - \beta^2(a^2 + m^2)} \quad (53)\]

\[ 1 - \beta^2(a^2 + m^2) = 0 \quad \text{there is a sonic leading edge.} \]

For the case where \[ 1 - \beta^2(a^2 + m^2) \to 0 \] we can write

\[
\lim_{1 - \beta^2(a^2 + m^2) \to 0} \frac{1}{2} \sqrt{1 - \beta^2(a^2 + m^2)} \frac{1}{\log \frac{x - \beta^2(a^2 + m^2) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(a^2 + m^2) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}
\]

\[= \frac{1}{2} \frac{1 - \beta^2(a^2 + m^2)}{\sqrt{x^2 - \beta^2(y^2 + z^2)}} \quad (54)\]
All of the log terms appear to potentially involve logarithms of negative numbers. This difficulty may be avoided by taking the absolute value of the arguments. This is allowable since any log of -1 would have canceled in the definite integral over 1. The argument could not have passed through zero and therefore must have been always positive or always negative.

Since certain functions occur repeatedly, we define

\[
F_1 = \tan^{-1} \frac{m(z - ax) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{\gamma[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2}
\]

\[
F_2 = \frac{1}{2\sqrt{1 - \beta^2(a^2 + m^2)}} \log \frac{x - \beta'(ay + bz) + \sqrt{(1 - \beta^2(a^2 + m^2))(x^2 - \beta^2(y^2 + z^2))}}{x - \beta'(ay + bz) - \sqrt{(1 - \beta^2(a^2 + m^2))(x^2 - \beta^2(y^2 + z^2))}}
\]

\[
F_3 = \tan^{-1} \frac{(mz - ay) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)}
\]

\[
F_6 = \sqrt{1 - \beta^2} \left( \frac{1}{2} \log \frac{x - \beta'a_2 + \sqrt{(1 - \beta^2)(x^2 - \beta^2(y^2 + z^2))}}{x - \beta'a_2 - \sqrt{(1 - \beta^2)(x^2 - \beta^2(y^2 + z^2))}} \right)
\]

Evaluation of the \( \tan^{-1} \) functions \( F_1 \) and \( F_3 \). The terms \( F_1 \) or \( F_3 \) are always real inside the Mach cone from the origin,

\[ x^2 - \beta^2(y^2 + z^2) \geq 0 \]

However as the argument of these functions go to zero the functions may take on different values depending upon how zero is approached. Corresponding to each of the four quadrants, if

\[ \theta = \tan^{-1} \frac{f}{g} \]
then

\[ f \geq 0 \quad g > 0 \quad \text{means} \quad 0 \leq \theta < \frac{\pi}{2} \]

\[ f \geq 0 \quad g < 0 \quad \text{means} \quad \frac{\pi}{2} < \theta < \pi \]

\[ f < 0 \quad g > 0 \quad \text{means} \quad 0 \geq \theta > -\frac{\pi}{2} \]

\[ f < 0 \quad g < 0 \quad \text{means} \quad -\frac{\pi}{2} > \theta \geq -\pi \]

(57)

Therefore from (52) and (57) as \( z \to ax \)

\[ 0 \quad \text{y} < 0 \quad \text{or} \quad \text{y} > \text{mx} \]

\[ \lim_{z \to ax} F_1 = \pi \quad 0 < \text{y} < \text{mx}, \; z > ax \]

\[ -\pi \quad 0 < \text{y} < \text{mx}, \; z > ax \]

(58)

and as \( mz \to ay \)

\[ 0 \quad \text{y} < 0 \quad \text{or} \quad \text{y} > \text{mx} \]

\[ \lim_{mz \to ay} F_3 = \pi \quad 0 < \text{y} < \text{mx}, \; mz > ay \]

\[ -\pi \quad 0 < \text{y} < \text{mx}, \; mz < ay \]

(59)
Velocity potential on $x^2 = \beta^2(y^2 + z^2)$ is $1 - \beta^3(a^2 + m^2) > 0$. Referring to (52) $F_2$ and $F_6$ are zero if $x^2 - \beta^2(y^2 + z^2) = 0$. The functions $F_1$ and $F_3$ will be zero if and only if the denominators of their arguments are greater than zero[see (57)]. First we note the following

\[ x^2 - \beta^2(y^2 + z^2) = [x^2 - \beta^2(y^2 + z^2)] \cdot \beta^2(y^2 + z^2) \cdot \beta^4(a^2y^2 + 2zmys + z^4) \]

\[ = [x^2 - \beta^2(y^2 + z^2) - \beta^2(1 - \beta^2(a^2 + m^2))(y^2 + z^2) - \beta^4(a^2y^2 + 2zmys + z^4)] \]

\[ = [x - \beta^2(my + az)][x + \beta^2(my + az)] \]

which means $x > |\beta^2(my + az)|$.

\[ x^2 - \beta^2(y^2 + z^2) > 0 \quad \Rightarrow \quad 1 - \beta^3(a^2 + m^2) > 0 \]

For $F_1$ the denominator is

\[ y \cdot [(y - mx) - \beta^2a(ay - mz)] + (z - ax)^2 \]

\[ = y^2 [1 - \beta^2(a^2 + m^2)] - my [x - \beta^2(az + my)] + (z - ax)^2 \]

$> 0$ if $y < 0$ from (60)
\[ y \left[ (y - mx) - \beta^2 a (ay - mz) \right] + (z - ax)^2 \]
\[ = (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mx (y - mx) - \beta^2 mz (ay - mz) \]
\[ = (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mxy - m^2 (x^2 - \beta^2 z^2) - \beta^2 m^2 z \]
\[ = (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + my [x - \beta^2 (az + my)] \]
\[ - m^2 \left[ x^2 - \beta^2 (y^2 + z^2) \right] > 0 \]

if \( x^2 - \beta^2 (y^2 + z^2) = 0 \), \( y > 0 \) due to (50) and (60)

Therefore

\[ y \left[ (y - mx) - \beta^2 a (ay - mz) \right] + (z - ax)^2 > 0 \]

on

\[ x^2 = \beta^2 (y^2 + z^2) \quad (61) \]

if

\[ 1 - \beta^2 (a^2 + m^2) > 0 \]

Therefore due to (57) and (61)

\[ \lim F1 = 0 \]

as \( x^2 - \beta^2 (y^2 + z^2) \to 0 \)

if

\[ 1 - \beta^2 (a^2 + m^2) > 0 \]

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or F3

\[(z - ax) - y (mx - y) = y^2 + z^2 - x (az + my)\]  \hspace{1cm} (62)

\[- [x^2 - \beta^2 (y^2 + z^2)] \frac{1}{\beta^2} + \frac{x}{\beta^2} [x - \beta^2 (az + my)] > 0\]

if

\[x^2 - \beta^2 (y^2 + z^2) = 0, \text{ and } 1 - \beta^2 (a^2 + m^2) > 0 \text{ due to (54c) if } x > 0\]

Therefore examining (29) and (56) shows that for a surface distribution of sources or a constant pressure surface

\[\lim \phi = 0\]

as \(x^2 - \beta^2 (y^2 + z^2) \to 0\)

if

\[1 - \beta^2 (a^2 + m^2) \to 0\]  \hspace{1cm} (63)

Therefore \(\phi\) will be continuous, as it must be, if we define \(\phi\) to be zero outside the Mach cone from the origin.

\[\phi = 0\]

if

\[1 - \beta^2 (a^2 + m^2) > 0 \text{ and } x^2 < \beta^2 (y^2 + z^2)\]  \hspace{1cm} (64)
Supersonic leading edge and the mach cone envelope. With a supersonic leading edge, \(1 - \beta^2 (a^2 + m^2) < 0\), all functions in the equations for \(\phi\) will be shown to go to zero for points on the Mach cone from the origin, except for a region on this Mach cone which borders the envelope of Mach cones from the supersonic leading edge. Inside this envelope of Mach cones the functions \(F_1\), \(F_2\) and \(F_3\) will be shown to have constant values. However on the outer boundary of this envelope of Mach cones \(\phi\) will go to zero, and therefore all functions may be defined to be zero outside this envelope.

The envelope of Mach cones from the leading edge is illustrated on p-29 of Reference (55). The Mach cone from any point \(x_o, y_o, z_o\) can be written,

\[
(x - x_o)^2 = \beta^2 [(y - y_o)^2 + (z - z_o)^2]
\]

on the leading edge \(m x_o = y_o\) and \(a x_o = z_o\).
Therefore

\[(mx - y_o)^2 = \beta^2 \left[ m^2 (y - y_o)^2 + (mz - ay_o)^2 \right] \]  \(65\)

The Mach cone envelope is determined by the maximum values for \(z\), at a given \(x\) and \(y\), obtained by a variation of \(y^o\). Therefore differentiating (65) with respect to \(y^o\), holding \(x\) and \(y\) constant, and setting \(dz/dy_o = 0\) gives

\[(mx - y_o) = \beta^2 m^2 (y - y_o) + \beta^2 a (mz - ay_o) \]

or

\[y_o [1 - \beta^2 (a^2 + m^2)] = m [x - \beta^2 (my + az)] \]  \(66\)

If \(y_o = 0\) (65) and (66) give

\[x^2 = \beta^2 (y^2 + z^2) \text{ and } x - \beta^2 (my + az) = 0 \]  \(67\)

or

\[x^2 = \beta^2 y^2 + \beta^2 \left[ \frac{x - \beta^2 my}{\beta^2 a} \right]^2 \]

\[\beta^2 a^2 x^2 = \beta^4 a^2 y^2 + x^2 - 2 \beta^2 mxy + \beta^4 m^2 y^2 \]

\[\beta^4 (a^2 + m^2) y^2 - 2 \beta^2 mxy + x^2 (1 - \beta^2 a^2) = 0 \]
or, if \( y_o = 0 \)

\[
y = \frac{x}{\beta^2 \left( a^2 + m^2 \right)} \begin{cases} m + a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \\ m - a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \end{cases} \quad (68)
\]

where

\[
y = \frac{x}{\beta^2 \left( a^2 + m^2 \right)} \begin{cases} m - a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \\ m + a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \end{cases} \quad z > 0
\]

\[
y = \frac{x}{\beta^2 \left( a^2 + m^2 \right)} \begin{cases} m - a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \\ m + a & \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1} \end{cases} \quad z < 0
\]

For \( y_o > 0 \) we can eliminate \( y_o \) from (65) and (66).

From (65)

\[
\left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] y_o^2 - 2m \left[ x - \beta^2 \left( az + my \right) \right] y_o + m^2 \left[ x^2 - \beta^2 (y^2 + z^2) \right] = 0
\]

or

\[
y_o \left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] = m \left[ x - \beta^2 \left( az + my \right) \right] + m \sqrt{\left[ x - \beta^2 (az + my) \right]^2 - \left[ 1 - \beta^2 \left( a^2 + m^2 \right) \right] \left[ x^2 - \beta^2 (y^2 + z^2) \right]}
\]

Therefore from (66)

\[
\left| x - \beta^2 (az + my) \right|^2 - \left| 1 - \beta^2 \left( a^2 + m^2 \right) \right| \left| x^2 - \beta^2 (y^2 + z^2) \right| = 0
\]

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\[ \beta^2 (a^2 + m^2) x^2 - 2x \beta^2 (az + my) - \beta^4 (ay - mz)^2 + \beta^2 (y^2 + z^2) = 0 \]

and solving for \( x \)

\[ x = \frac{(az + my) \pm \sqrt{(az + my)^2 + (a^2 + m^2) [(ay - mz)^2 \beta^2 - (y^2 + z^2)]}}{(a^2 + m^2)} \]

\[ = \frac{(az + my) \pm \sqrt{(ay - mz)^2 \beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \]

If we write

\[ x = \frac{(az + my) \pm (mz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \]

or

\[ \frac{\partial z}{\partial y} \left[ a + m \sqrt{\beta^2 (a^2 + m^2) - 1} \right] = \left[ -m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right] \]
\[
\frac{\partial z}{\partial y} \left( a^2 + m^2 \right) \left[ 1 - \beta^2 m^2 \right]
\]
\[
= \left( -a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right) \left( -a \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right)
\]
\[
= \left( a^2 + m^2 \right) \left[ -\beta^2 m \pm \sqrt{\beta^2 (a^2 + m^2) - 1} \right]
\]

Therefore use:

+ for \( m^2 > a^2 + m^2 \)

- for \( m^2 < a^2 + m^2 \)

or

\[
x = \frac{(az + my) + |mz + ay|}{(a^2 + m^2)} \sqrt{\beta^2 (a^2 + m^2) - 1}
\]

(69)

This is presented on page 29 of reference (55).

If we set

\[
z = y \tan \theta \quad \text{on} \quad x^2 = \beta^2 (y^2 + z^2)
\]

then

\[
x^2 = \beta^2 y^2 \left( 1 + \tan^2 \theta \right)
\]

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or
\[ x = \frac{\beta y}{\cos \theta} \quad \beta y = x \cos \theta \quad (70) \]

Therefore (67) gives for the two points where the envelope of Mach cones from the leading edge is on the Mach cone from the origin,

\[ (y^2 + z^2) = \beta^2 (my + az)^2 \]

or

\[ \left[ 1 + \tan^2 \theta \right] = \beta^2 m^2 + 2 \beta^2 am \tan \theta + \beta^2 a^2 \tan^2 \theta \]

which means

\[ (1 - \beta^2 a^2) \tan \theta = \beta^2 am + \sqrt{\beta^4 a^2 m^2 - (1 - \beta^2 a^2) (1 - \beta^2 m^2)} \]

or

\[ (1 - \beta^2 a^2) \tan \theta = \beta^2 am + \sqrt{\beta^2 (a^2 + m^2) - 1} \quad (71) \]

but

\[ x - \beta^2 (my + az) = 0 \]

means

\[ \frac{\beta a}{\cos \theta} - \beta^2 a (m + a \tan \theta) = 0 \quad (72) \]
Therefore the two points corresponding to (68) are

\[
\tan \theta_1 = \frac{\beta a}{\cos \theta_1} - \sqrt{\beta^2 (a^2 + m^2) - 1}
\]

\[
\tan \theta_2 = \frac{\beta a}{\cos \theta_2} + \sqrt{\beta^2 (a^2 + m^2) - 1}
\]

(assume \(a > 0\) and \(\theta_2 > \theta_1\))

From (56) and (57) the value of \(F_1\) or \(F_3\) as we approach the Mach cone from the origin depends on the sign of the denominator of its argument.

For

\[
F_1 \text{ on } x^2 = \beta^2 (y^2 + z^2) \text{ we can write}
\]

\[
D_1 = \frac{1}{y^2} \left[ y (y - mx) - \beta^2 a (ay - mz) + (z - ax)^2 \right]
\]

\[
= (1 - \beta^2 a^2) - \frac{\beta m}{\cos \theta} + \beta^2 m \tan \theta + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2
\]

\[
= 1 - \beta^2 (a^2 + m^2) + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2 - \beta m \left[ \frac{\beta m + \frac{1}{\cos \theta}}{\cos \theta} - \beta a \tan \theta \right]
\]

\[
= 0 \text{ if } \theta = \theta_1 \text{, or } \theta = \theta_2 \text{ from (72) and (73)}
\]
If we differentiate this with respect to $\theta$ we get

\[
\frac{dD_1}{d\theta} = 2 \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ \sec^2 \theta - \frac{\beta a \sin \theta}{\cos^2 \theta} \right] + \beta m \left[ \beta a \sec^2 \theta - \frac{\sin \theta}{\cos^2 \theta} \right]
\]

\[
= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ 2 \left( 1 - \beta a \sin \theta \right) - \beta m \cos \theta \right]
\]

\[
= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ 2 \left( 1 - \beta a \sin \theta - \beta m \cos \theta \right) + \beta m \cos \theta \right]
\]

From (72)

\[ 1 - \beta a \sin \theta - \beta m \cos \theta = 0 \quad \text{at} \quad \theta = \theta_1, \theta_2 \]

and since

\[ \cos \theta_1, > 0, \quad \cos \theta_2 > 0 \quad \text{and} \quad \beta m > 0 \]

and using (73) we get

\[ \frac{dD_1}{d\theta} < 0 \quad \theta = \theta_1 \]

\[ \frac{dD_1}{d\theta} > 0 \quad \theta = \theta_2 \]
and therefore using (57) if

\[ 1 - \beta^2 (a^2 + m^2) < 0 \]

\[
\lim_{x^2 \to \beta^2 (y^2 + z^2)} F_1 = \begin{cases} 
0 & \theta > \theta_2 \quad \theta < \theta_1 \\
\pi & \pi < \theta < \theta_2 \\
-\pi & \theta_1 < \theta < \theta_2 
\end{cases}
\]  

For

\[ F_3 \text{ on } x^2 = \beta^2 (y^2 + z^2) \]

let

\[ D_3 = z (z - ax) - y (mx - y) \]

\[ = y^2 (1 + \tan^2 \theta) - yx (m + a \tan \theta) \]

\[ = \frac{x^2}{\beta^2} \left[ 1 - \beta \cos \theta (m + a \tan \theta) \right] \]

\[ = \frac{x^2}{\beta^2} \left[ 1 - (\beta m \cos \theta + \beta a \sin \theta) \right] \]

\[ = 0 \text{ if } \theta = \theta_1 \text{ or } \theta = \theta_2 \text{ from (72)} \]
\[
\frac{d}{d\theta} \beta^3 = \frac{x^2}{\beta} \left[ m \sin \theta - a \cos \theta \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left[ m (1 - \beta^2 a^2) \tan \theta - a (1 - \beta^2 a^2) \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left[ a \beta^2 m^2 - a (1 - \beta^2 a^2) + m \sqrt{\beta^2 (a^2 + m^2) - 1} \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left[ a \frac{1 - \beta^2 (a^2 + m^2)}{1 - \beta^2 a^2} + m \sqrt{\beta^2 (a^2 + m^2) - 1} \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \sqrt{\beta^2 (a^2 + m^2) - 1} \left[ a \sqrt{\beta^2 (a^2 + m^2) - 1} + m \right]
\]

> 0 for \( \theta_2 \) (+) [see (73)]

< 0 for \( \theta_1 \) (-) (assuming \( a > 0 \))

because

\[
m^2 > a^2 \left[ \beta^2 (a^2 + m^2) - 1 \right]
\]

since

\[
(a^2 + m^2) (1 - \beta^2 a^2) > 0
\]
Therefore since

\[ \frac{d}{d\theta} D3 > 0 \quad \theta = \theta_2 \]
\[ \frac{d}{d\theta} D3 < 0 \quad \theta = \theta_1 \]

and therefore

\[ D3 > 0 \quad \theta > \theta_2 \text{ or } \theta < \theta_1 \]
\[ D3 < 0 \quad \theta_1 < \theta < \theta_2 \]

We can say

\[ \lim_{x^2 - \beta^2 (y^2 + z^2)} x^2 \rightarrow \beta^2 (y^2 + z^2) F3 = \begin{cases} 0 & \theta > \theta_2 \text{ or } \theta < \theta_1 \\ \pi & \text{mz > ay } \theta_1 < \theta < \theta_2 \\ -\pi & \text{mz < ay } \theta_1 < \theta < \theta_2 \end{cases} \]

Also on the plane

\[ mz = ay \]

\[ D3 = z(z - ax) - y(mx - y) = y^2 \left[ 1 + \frac{a^2}{m^2} \right] - xy \left[ m + \frac{a^2}{m} \right] \]
\[ = \frac{y(y - mx) (a^2 + m^2)}{m^2} \]

and therefore

\[ \lim_{(ay - mz) \rightarrow 0} F3 = \begin{cases} 0 & y < 0 \text{ or } y > mx \\ \pi & 0 < y < mx \text{ mz > ay} \\ -\pi & 0 < y < mx \text{ mz < ay} \end{cases} \]
To find the limit of $F_2$ as $x^2 \rightarrow \beta^2 \left( y^2 + z^2 \right)$ when $1 - \beta^2 \left( a^2 + m^2 \right) < 0$ [see (53) for $F_2$] we note that

$$\lim_{x^2 \rightarrow \beta^2 \left( y^2 + z^2 \right)} \left| \frac{x - \beta^2 \left( my + az \right)}{\beta \sqrt{(mx - y)^2 + (a - ax)^2} - \beta^2 \left( ay - mz \right)^2} \right| = 1$$

because

$$\beta^2 \left( (mx - y)^2 + (a - ax)^2 - \beta^2 (ay - mz)^2 \right) = \beta^2 (a^2 + m^2) x^2 + \beta^2 \left( y^2 + z^2 \right) - 2 \beta^2 (my + az) \cdot \beta^2 (ay - mz) - \beta^2 \left( ay - mz \right)^2$$

$$= \left[ \beta^2 \left( a^2 + m^2 \right) - 1 \right] \left[ x^2 - \beta^2 (y^2 + z^2) \right] + \left[ x - \beta^2 (my + az) \right] - \beta^2 \left( ay - mz \right)^2$$

Now

$$x - \beta^2 \left( my + az \right) = x \left[ 1 - \beta^2 \frac{y}{x} - \beta^2 \frac{a}{z} \tan \theta \right]$$

$$= x \left[ 1 - \beta m \cos \theta - \beta a \sin \theta \right] = \frac{\beta^2}{x} D_3$$

where $D_3$ is defined by (75). Therefore from (75)

$$\lim_{x^2 \rightarrow \beta^2 \left( y^2 + z^2 \right)} F_2 = \begin{cases} 0 & \text{if} \ 0 < \theta < \theta_1 \\ \frac{\beta^2}{x} D_3 & \text{if} \ \theta_1 < \theta \leq \theta_2 \end{cases} \quad (78)$$

Therefore combining these results

$$F_1 = \tan^{-1} \frac{m (z-ax)}{y [\left( y - mx \right) - \beta^2 a (ay - mz)] + (z-ax)^2} \quad x^2 > \beta^2 \left( y^2 + z^2 \right)$$
If

\[ 1 - \beta^2 (a^2 + m^2) > 0 \]

\[ F_1 = 0 \quad x^2 > \beta^2 (y^2 + z^2) \]

(79)

\[
\lim_{z \to a} F_1 = \begin{array}{ccc}
0 & y < 0 & y > mx \\
\pi & 0 < y < mx & z > ax \\
\pi & 0 < y < mx & z < ax
\end{array}
\]

Then from (69) and (74) if

\[ 1 - \beta^2 (a^2 + m^2) < 0 \]

and

\[ x^2 < \beta^2 (a^2 + m^2) \]

\[ F_1 = 0 \quad \theta > \theta_2 \quad \theta < \theta_1 \]

(80)

\[ F_1 = \pi \text{sgn} (z-ax) \]

if

\[ \theta_1 < \theta < \theta \]

and

\[
x > \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} = 0
\]
and  \( F_1 = 0 \)

if  \( \theta_1 < \theta < \theta_2 \)

and

\[
x > (az + m\gamma) + m\zeta + ay \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1 \over \left( a^2 + m^2 \right)}
\]

and where from (71)

\[
(1 - \beta^2 a^2) \tan \theta_1 = \beta^2 am - \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1}
\]

\[
(1 - \beta^2 a^2) \tan \theta_2 = \beta^2 am + \sqrt{\beta^2 \left( a^2 + m^2 \right) - 1}
\]

For

\[
x^2 > \beta^2 (y^2 + z^2)
\]

from (53), (54), and (56)

\[
F_2 = {-1 \over \sqrt{\beta(a^2 + m^2)}} \tan^{-1} \left( \sqrt{\beta(a^2 + m^2) - 1} \over x - \beta^2 (az + m\gamma) \left[ x^2 - \beta^2 (a^2 + m^2) \right] \right)
\]

\[
1 - \beta^2(a^2 + m^2) = 0
\]
and from (78)

\[ F_2 = 0 \quad x^2 \leq \beta^2 (y^2+z^2) \quad 1 - \beta^2 (a^2+m^2) > 0 \]

\[ = 0 \quad x^2 \leq \beta^2 (y^2+z^2) \quad \theta > \theta_2, \quad \theta > \theta_1 \]

\[ = \pi \quad x^2 \leq \beta^2 (y^2+z^2) \quad \theta_1 < \theta < \theta_2 \quad 1 - \beta^2 (a^2+m^2) < 0 \]

\[ = 0 \quad \text{outside envelope of Mach cones.} \quad (82) \]

For

\[ x^2 > \beta^2 (y^2+z^2) \]

From (56) and (58)

\[ F_3 = \tan^{-1} \left( \frac{(mz-ay) \sqrt{x^2 - \beta^2 (y^2+z^2)}}{z (z-ax) - y (mx-y)} \right) \quad (83) \]

and

\[ \lim_{(mz-ay) \to 0} F_3 = \begin{array}{ccc} 0 & y < 0 & y > mx \\ \pi & 0 < y < mx & mz > ay \\ -\pi & 0 < y < mx & mz < ay \end{array} \]

and for

\[ x^2 \leq \beta^2 (y^2+z^2) \]

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from (76)

\[ F_3 = 0 \quad \theta < \theta_1 \quad \theta > \theta_2 \]

\[ = \pi \quad mz > ay \quad \theta_1 < \theta < \theta_2 \]

\[ = -\pi \quad mz < ay \quad \theta_1 < \theta < \theta_2 \]

\[ = 0 \quad \text{outside envelope of Mach cones.} \quad (84) \]

**Value of \( \phi \) on the envelope of mach cones.** For a supersonic leading edge \([1 - \beta^2 (a^2 + m^2) < 0]\), in the region inside the envelope of mach cones from the leading edge and outside the mach cone from the origin, we have for a surface distribution of sources \([\text{from (29), (79), (80), and (81)}]\)

\[
\phi_s = (z - ax) \text{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} \cdot \left( \frac{\gamma}{T} \right) \sqrt{\frac{\rho^2 (a^2 + m^2) - 1}{\rho^2 (a^2 + m^2) - 1 + y (1 - \beta^2 a^2) \cdot \rho^2 amz}}
\]
On the lines

\[ x = \frac{(my + az) + |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \]

\[ \text{sgn} (z - ax) (mz - ay) = |mz - ay| \text{ [See figure in Woodward]} \]

Therefore on the envelope of mach cones from the leading edge

\[ \phi_s = 0 \]

In this same region for a constant pressure surface [from (71)] and

\[ \phi_p \left[ \frac{1}{4\pi a} \left( \frac{\Delta P}{q_m} \right) \right]^{-1} (a^2 + m^2) \tau^{-1} \]

\[ = -a (ay - mz) \sqrt{\beta^2 (a^2 + m^2) - 1 + m (ay - mz) \text{sgn} (mz - ay) + (a^2 + m^2) (z - ax) \text{sgn} (z - ax)} \]
and in the region where

\[
\text{sgn} (z - ax) = \text{sgn} (mz - ay)
\]

\[
= \text{sgn} (z - ax) a \left| - \frac{a |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1} + m (ay - mz) + (a^2 + m^2) (z - ax)}{\text{sgn} (z - ax)} \right|
\]

\[
= 0 \text{ on the lines } x = \frac{(my + az) + |mz - ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{a^2 + m^2}
\]

Therefore \( \phi_p = 0 \) on the envelope of Mach cones from the leading edge.

Verification of the imposed boundary conditions. Now (8) can be verified for the case of a surface distribution of sources. From (29) and (79), on \( z = ax \), since the \( \cosh^{-1} \) terms are continuous

\[
\phi = \phi' \text{ or } \phi - \phi' = 0
\]
For (8b) using (29) and (79) and the results of 2.6

\[ u = \frac{-w + \beta^2 au}{\pi (1 - \beta^2 a^2)} \left| \pi a + mF2 \right| \quad 0 < y < mx \quad z > ax \]

\[ u' = -\frac{-w + \beta^2 au}{\pi (1 - \beta^2 a^2)} \left| -\pi a + mF2 \right| \quad 0 < y < mx \quad z < ax \]

\[ w = \frac{w}{\pi (1 - \beta a^2)} \left| \pi + \beta^2 amF2 \right| \quad 0 < y < mx \quad z > ax \]

\[ w' = \frac{w}{\pi (1 - \beta a^2)} \left| -\pi + \beta^2 amF2 \right| \quad 0 < y < mx \quad z < ax \]

Therefore

\[ (w - w') + \beta^2 a (u - u') = 2 \frac{-w + \beta^2 au}{\pi (1 - \beta^2 a^2)} \pi [1 - \beta^2 a^2] = 2[w + \beta^2 au] = \text{const} \]

which agrees with (8b)

For

\[ y < 0 \]

or

\[ y > mx \]

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where

\[ f = m(z - ax)\sqrt{x^2 - \beta^2 (y^2 + z^2)} \]

\[ g = y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2 \]

and since

\[ f = 0 \text{ on } S \]

when

\[ z = ax \]

\[ \frac{\partial u}{\partial x} = \frac{-am\sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{y(y - mx)(1 - \beta^2 a^2)} \]

and likewise on

\[ z = ax \]

\[ \frac{\partial u}{\partial y} = 0 \]

\[ \frac{\partial u}{\partial z} = \frac{m\sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{y(y - mx)(1 - \beta^2 a^2)} \]
Therefore

\[ \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \text{ and } \frac{\partial u}{\partial z} \]

are continuous on \( S \) (except possibly at the edges) and therefore

\[ \frac{\partial}{\partial x} (u - u') = \frac{\partial}{\partial y} (u - u') = \frac{\partial}{\partial z} (u - u') = 0 \]

and (30b) is verified.

**Surface velocities on a constant pressure surface.** - Flow normal to surface \( U_n \)

\[ u_n = \frac{1}{\sqrt{1 + a^2}} (-au + w) \]

\[ u'_n = \frac{1}{\sqrt{1 + a^2}} (au' - w') \]

Flow through surface = \( \frac{1}{2} (u_n - u'_n) = \frac{1}{2} \frac{1}{\sqrt{1 + a^2}} \left| -a(u + u') + (w + w') \right| \)

on

\[ (z = ax) \ u + u' = 0 \]
from (51) and (79)

\[
\frac{1}{2} (w + w') = \frac{1}{4\pi a} \left( \frac{\Delta P}{q_\infty} \right) \left\{ \frac{am}{(a^2 + m^2)} \right\} \frac{1}{2} \log \frac{x + \sqrt{x^2(1 - \beta^2a^2) - \beta^2 \theta^2}}{x - \sqrt{x^2(1 - \beta^2a^2) - \beta^2 \theta^2}}
\]

\[
\frac{-am}{2(a^2 + m^2)} \sqrt{1 - \beta^2(a^2 + m^2)} \log \frac{x(1 - \beta^2a^2) - \beta^2m^2 + \sqrt{(1 - \beta^2(a^2 + m^2))[x^2(1 - \beta^2a^2) - \beta^2 \theta^2]}}{x(1 - \beta^2a^2) - \beta^2m^2 - \sqrt{(1 - \beta^2(a^2 + m^2))[x^2(1 - \beta^2a^2) - \beta^2 \theta^2]}}
\]

\[
\tan^{-1} \left( \frac{mz - ay}{-y(mx - y)} \right)
\]

\[
= \frac{1}{2} \sqrt{1 + a^2} (u_n - u'_n)
\]

Source strength = \( u_n + u'_n = \frac{1}{\sqrt{1 + a^2}} \left[ a(u - u') + (w - w') \right] \)

\[
= \frac{(1 + a^2)}{\sqrt{1 + a^2}} \frac{2\Delta P}{4\pi a q_\infty \pi}
\]

\[
= 2 \frac{\sqrt{1 + a^2} \Delta P}{4a q_\infty}
\]

\( \text{(86)} \)
Velocity discontinuity in the leading edge wake.

Constant Pressure Surface

\[ V_T = \frac{v + \frac{a}{m}w}{\sqrt{1 + (\frac{a}{m})^2}} \]

\[ V_n = \frac{\frac{-a}{m}v + w}{\sqrt{1 + (\frac{a}{m})^2}} \]

\[ V_n - V'_n = \frac{-\frac{a}{m}(v - v') + (w - w')}{\sqrt{1 + (\frac{a}{m})^2}} \]

\[ = \frac{1}{4\pi a\sqrt{1 + (\frac{a}{m})^2}} \frac{\Delta P}{q_{\infty}} 2\pi \left[ -\frac{a}{m} \frac{a}{m} + (-m^2) \right] (a^2 + m^2)^{-1} \]

\[ = \frac{-2m}{4a\sqrt{a^2 + m^2}} \frac{\Delta P}{q_{\infty}} \text{ Source Distribution! [see (86)]} \]

\[ V_T - V'_T = \frac{2}{4\pi a\sqrt{1 + (\frac{a}{m})^2}} \frac{\Delta P}{q_{\infty}} \pi \left| am - m^2 \frac{a}{m} \right| (a^2 + m^2)^{-1} = 0 \]
Alternate sign choice of $\tan^{-1}$ denominator.

\[
\frac{m(ay - mz)}{(a^2 + m^2)} \tan^{-1} \left( \frac{mz - ay}{a(z - ax) - m(mx - y)} \right) [\eta + m(y(mx - y) - z(z - ax)) - 2B2 \frac{[mz - ay]^2}{[a(z - ax) - m(mx - y)]\eta + m(y(mx - y) - z(z - ax))} ]
\]

[see (45) and note sign change of denominator and since

\[
[a(z - ax) - m(mx - y)]\eta + m[y(mx - y) - z(z - ax)] < 0
\]

The above term when evaluated at

\[
\eta = \eta_3
\]

is

\[
\frac{m(ay - mz)}{(a^2 + m^2)} \pi \text{sgn}(mz - ay)
\]

Evaluated at $\eta = 0$ the term becomes

\[
\frac{m(ay - mz)}{a^2 + m^2} \tan^{-1} \left( \frac{mz - ay}{[y(mx - y) - z(z - ax)](x^2 - \beta^2(y^2 + z^2))} \right)
\]

If

\[
mz = ay \quad y(mx - y) - z(z - ax) = \frac{y(mx - y)(a^2 + m^2)}{m^2}
\]

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which means that on $mz = ay$ and $\eta = 0$

$$\tan^{-1} = \begin{cases} \pi \text{sgn}(mz - ay) & y < 0 \quad y > mx \\ 0 & 0 < y < mx \end{cases}$$

$$\tan^{-1} |_{\eta} = \begin{cases} 0 & y < 0 \quad y > mx \\ \pi \text{sgn}(mz - ay) & 0 < y < mx \end{cases}$$
If $\Omega(x, y, z)$ satisfies

$$\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\} \Omega(x, y, z) = 0 \quad \beta^2 = 1 - M^2$$

Then we can write the solution for $\Omega(x, y, z)$ as

$$\Omega(x, y, z) = \iint_S \left[ \Omega(\xi, \eta, \xi) \frac{\partial}{\partial \nu} - \frac{\partial \gamma(\xi, \eta, \xi)}{\partial \nu} \right] \frac{1}{4\pi r} \, dS$$

where

$$r = \sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - \xi)^2}$$

and

$$\frac{\partial}{\partial \nu} = \left\{ \beta^2 n_\xi \frac{\partial}{\partial \xi} + n_\eta \frac{\partial}{\partial \eta} + n_\xi \frac{\partial}{\partial \xi} \right\}$$

$\bar{n} = (n_\xi, n_\eta, n_\xi)$ is the unit normal on S.

We will assume S is the surface $\tau = a_\xi$ and...
Surface distribution of sources. - Let $\Omega = \phi$ and as for supersonic flow assume that on $S \sqrt{1 + a^2} \bar{v} = (-\beta^2 a, 0, 1)$

a) $\phi = \phi'$

b) $\frac{\partial \phi}{\partial \nu} + \frac{\partial \phi}{\partial \nu'} = \frac{2}{\sqrt{1 + a^2}} (\bar{w} - \beta^2 a \bar{u} ) = \text{const.} \quad \bar{w} = w - w^1$

the primed quantities refer to $z < ax$ or $\xi < a \xi$

Therefore since on $S \xi = a \xi$ we get

$$
\phi(x, y, z) = \frac{\bar{w} - \beta^2 a \bar{u}}{2\pi} \int \int \int_{S} \frac{\eta}{m_1} x_3 \left( \frac{d\xi}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2}} \right) d\eta
d\xi

(A2)$$
This is the same as \((9)\) for the supersonic case except for the limits, a factor of \(1/2\), and the fact that \(\beta^2\) is replaced by \(-\beta^2\). Therefore the integral over \(\xi\) may be performed by using the equation below \((9)\) for the supersonic case, and replacing \(\beta^2\) by \(-\beta^2\).

\[
\int \frac{d\xi}{\sqrt{(x-\xi)^2 + \beta^2(y-\eta)^2 + \beta^2(z-a\xi)^2}} = \frac{-1}{2\sqrt{1 + \beta^2a^2}} \log \frac{\sqrt{\left(1 + \beta^2a^2\right)\left((x-\xi)(\gamma-3) + \beta^2(z-a\xi)^2\right) + (x-\xi) + \beta^2a(z-a\xi)}}{\sqrt{\left(1 + \beta^2a^2\right)\left((x-\xi)(\gamma-3) + \beta^2(z-a\xi)^2\right) - (x-\xi) - \beta^2a(z-a\xi)}}
\]

A negative sign was included in the argument of the logarithm. This was possible since the derivative of \(\log(\cdot)\) is zero. Therefore

\[
\phi(x, y, z) = \frac{w - \beta^2au}{4\pi \sqrt{1 + \beta^2a^2}} \int_0^{y_2} \int_0^{y_2} \frac{d\eta}{\sqrt{\left(1 + \beta^2a^2\right)\left((x-\xi)(\gamma-3) + \beta^2(z-a\xi)^2\right) + (x-\xi) + \beta^2a(z-a\xi)}} - \frac{d\eta}{\sqrt{\left(1 + \beta^2a^2\right)\left((x-\xi)(\gamma-3) + \beta^2(z-a\xi)^2\right) - (x-\xi) - \beta^2a(z-a\xi)}}
\]

The second integral is the same as the first except that \(x, z\) and \(m\) are replaced by \(x-x_3\), \(z-z_3\) and \(m_1\). These integrals are the same as for supersonic flow, given by equation \((21)\), if \(\beta^2\) is replaced by \(-\beta^2\) and a factor of \(1/2\) is added.
When the limit \( \eta = \frac{y}{2} \) is calculated it can be seen that it will be the same as when \( \eta = 0 \) if \( x, y \) and \( z \) are replaced by \( x - x_2 \), \( y - y_2 \) and \( z - z_2 \).

Therefore

\[
\phi_0(x, y, z, m) = \frac{w - \beta^2 a_x}{2z} \left\{ \frac{z - ax}{1 + \beta^2 a_x^2} \tan^{-1} \frac{m(z - ax)\sqrt{x^2 + \beta^2(y^2 + z^2)} - y(\sqrt{(y - ax)^2 + \beta^2(ay - mz)} - (z - ax)^2 \sqrt{(x - ax)^2 + \beta^2(ay - mz)})}{2(1 + \beta^2 a_x^2)\sqrt{1 + \beta^2(a_x^2 + m^2)} \left[ x^2 + \beta^2(y^2 + z^2) \right] - (x + \beta^2 a_x + my)} \right\}
\]

\[
\phi_0(x, y, z, m) = \frac{w - \beta^2 a_x}{2z} \left\{ \frac{z - ax}{1 + \beta^2 a_x^2} \tan^{-1} \frac{m(z - ax)\sqrt{x^2 + \beta^2(y^2 + z^2)} - y(\sqrt{(y - ax)^2 + \beta^2(ay - mz)} - (z - ax)^2 \sqrt{(x - ax)^2 + \beta^2(ay - mz)})}{2(1 + \beta^2 a_x^2)\sqrt{1 + \beta^2(a_x^2 + m^2)} \left[ x^2 + \beta^2(y^2 + z^2) \right] - (x + \beta^2 a_x + my)} \right\}
\]

Now we can write the result of (A4) as

\[
\phi(x, y, z) = \phi_0(x, y, z, m) - \phi_0(x - x_2, y - y_2, z - z_2, m)
\]

\[
- \phi_0(x - x_3, y - y_3, z - z_3, m_1) + \phi_0(x - x_4, y - y_4, z - z_4, m_1)
\]

This means that (A5) may be interpreted as the velocity potential for an infinite panel with leading edge slope \( y = mx \), and at angle of attack \( a \).
Constant pressure surface. In (A1) we will use \( u(x, y, z) \)

\[ \Omega (x, y, z) \] and

a) \( \frac{\partial u}{\partial n} + \frac{\partial u'}{\partial n'} = 0 \) on \( S \)

b) \( u - u' = \text{const} = \Delta u \) on \( S \)

and where \( S \) is the same as previously defined.

Therefore (A1) becomes

\[
\begin{align*}
    u(x, y, z) &= \frac{\Delta u}{4\pi} \int_S \int_S \frac{-\beta^2 a(x - \xi) + \beta^2 (z - a \xi)}{\left[ (x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2 \right]^{3/2}} \, d\xi \, d\eta \\
    &= \frac{\Delta u}{4\pi} \frac{\partial}{\partial x} \int_S \int_S \frac{-\beta^2 a - (x - \xi)(z - a \xi)}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2}} \, d\xi \, d\eta \quad (A6)
\end{align*}
\]
Now we can write, using \( \Delta p/2 q_\infty = -\Delta u \)

\[
\phi(x, y, z) = \int_{-\infty}^{x} u(x', y, z) \, dx
\]

\[
= \frac{\Delta p}{8\pi q_\infty} \int_{0}^{y_2} \int_{\eta/\Delta}^{y_2 + x_3} \int_{n/\eta}^{n/\Delta} \frac{-\beta^2 a - \frac{(x - a \xi)(z - a \xi)}{(y - \eta)^2 + (z - a \xi)^2}}{\sqrt{(x - \eta)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a \xi)^2}} \, d\xi \, d\eta
\]

(A7)

The first term, except for the limits of integration and an additional factor of 1/2, is the same as the result for supersonic flow, which is given by the first equation of the section covering the evaluation of the integral over \( \xi \) for the constant pressure surface, with \( -\beta \) in place of \( \beta \). Therefore, as in the case of a constant distribution of sources, we can use the supersonic result for subsonic flow if we substitute \( -\beta \) for \( \beta \).

The second integral is new and does not occur for supersonic flow. This integral will now be evaluated.

\[
\int \frac{(z - a \xi) \, d\xi}{(y - \eta)^2 + (z - a \xi)^2} = -\frac{1}{2a} \log \left[ (y - \eta)^2 + (z - a \xi)^2 \right]
\]

(A8)
Therefore

\[ \int_0^{y_2} \int_0^{\eta} \frac{(z - a) \, d\eta}{(y - \eta)^2 + (z - a)^2} \, dz = \frac{1}{2a} \int_0^{y_2} \log \left\{ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right\} \, d\eta \]

\[ = \frac{1}{2a} \int_0^{y_2} \log \left\{ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right\} \, d\eta \]

note that:

\[ mz - a\eta = (mz - ay) - a(\eta - y) = m(z - ax) + a(mx - y) - a(\eta - y) \quad (A9) \]

\[ \int \log \left[ \frac{m^2 (y - \eta)^2 + (a\eta - mz)^2}{m^2} \right] \, d\eta \]

\[ = \left[ \frac{m(mx + az)}{a^2 + m^2} \right] \log \left[ \frac{m^2 (y - \eta)^2 + (a\eta - mz)^2}{m^2} \right] - 2 \]

\[ = \frac{2m(mx - ay)}{a^2 + m^2} \tan^{-1} \frac{m(mx - ay)}{(y - \eta)(a^2 + m^2) - a(mx - ay)} \quad (A10) \]
When this is evaluated at \( \eta = 0 \) we get from (A7), (A9), and (A10)

\[
\phi_{oo}(x, y, z, m) = \frac{\Delta P}{8\pi \rho \infty a} \left\{ \frac{-m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log(y^2 + z^2) - 1 \right] 
\right. \\
- \frac{m(mz - ay)}{(a^2 + m^2)} \tan^{-1} \left( \frac{(mz - ay)m}{-a(mz - ay) - (a^2 + m^2)y} \right) \right\}
\]

(A11)

Since all of the terms in (A10) may be written using only the variables, \((mx - y), (z - ax), m, \) and \((\eta - y)\), the additional term which must be added for subsonic flow is,

\[
\phi_1(x, y, z) = \phi_{oo}(x, y, z, m) - \phi_{oo}(x_2, y_2, z - z_2, m) \\
- \phi_{oo}(x_3, y_3, z - z_3, m_1) \\
+ \phi_{oo}(x_4, y_4, z - z_4, m_1)
\]

(A12)
Therefore using (51) we can write:

\[
\varphi(x, y, z) = \frac{\Lambda \mathcal{P}}{8 \pi q_{\text{ref}}} \left\{ \frac{m(az + my)}{a^2 + m^2} \frac{1}{z} \log \frac{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x} + \frac{a(ay - mz)}{z(a^2 + m^2)} \sqrt{1 + \beta^2(a^2 + m^2)} \log \frac{\sqrt{[1 + \beta^2(a^2 + m^2)][x^2 + \beta^2(y^2 + z^2)]} + [x + \beta^2(ax + mz)]}{\sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]} - [x + \beta^2(ax + mz)]} \right. \\
- \frac{m(az - my)}{a^2 + m^2} \tan^{-1} \left( \frac{(mz - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{z(ax) - y(mx - y)} \right) \\
+ (z - ax) \tan^{-1} \left( \frac{m(z - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{y[(y - mx) + \beta^2(a\gamma - mz)] + (z - ax)^2} \right) \\
- y \sqrt{1 + \beta^2a^2} \frac{1}{2} \log \frac{\sqrt{[1 + \beta^2a^2]} \left[ x^2 + \beta^2(y^2 + z^2) \right] + (x + \beta^2az)}{\sqrt{[1 - \beta^2a^2]} \left[ x^2 + \beta^2(y^2 + z^2) \right] - (x + \beta^2az)} \\
- \frac{m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log (y^2 + z^2) - 1 \right] + (mz - ay) \tan^{-1} \left( \frac{(mz - ay)m}{a(ay - mz) - y(a^2 + m^2)} \right) \right\}
\]
So find the limiting form of $\phi$ as $a \to 0$.

some of the terms must be combined. First, consider the terms

$$\lim_{a \to 0} \frac{y}{2a} \left\{ \log \frac{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x} - \sqrt{1 + \beta a^2} \log \frac{(1 + \beta a^2)[x^2 + \beta^2(y^2 + z^2)] + (x + \beta a^2)}{(1 + \beta a^2)[x^2 + \beta^2(y^2 + z^2)] - (x + \beta a^2)} \right\} \right.$$

and since only terms up to the first power in $a$ must be considered inside the brackets, this becomes

$$\lim_{a \to 0} \frac{y}{2a} \left\{ \log \left| \frac{\beta a^2}{\sqrt{x^2 + \beta^2(y^2 + z^2)} + x} \right| - \frac{\beta a^2}{\sqrt{x^2 + \beta^2(y^2 + z^2)} - x} \right\} = -y \frac{2}{y^2 + z^2} \sqrt{x^2 + \beta^2(y^2 + z^2)}$$

since for $a \ll 1, \log (1 + a) = a$

Next consider

$$\lim_{a \to 0} 2 \left\{ \tan^{-1} \frac{m(x - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{2 \left[ (x - mx)^2 + \beta^2(a y - my) + (a - ax)^2 \right]} - \tan^{-1} \frac{(m x - ay) \sqrt{x^2 + \beta^2(y^2 + z^2)}}{2(a - ax) - (m x - ay)} \right\}$$

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For small $\epsilon$, we can expand

$$
\tan^{-1} \frac{f(\epsilon)}{g(\epsilon)} = \tan^{-1} \frac{f(0)}{g(0)} + \frac{\epsilon}{g(0)} \frac{d}{d\epsilon} \tan^{-1} \frac{f(\epsilon)}{g(\epsilon)} \bigg|_{\epsilon=0}
$$

$$
= \tan^{-1} \frac{f(0)}{g(0)} + \frac{\epsilon}{g(0)} \left( \frac{f'(0)}{g'(0)} - \frac{f(0)g'(0) - f'(0)g(0)}{g(0)^2 + f(0)^2} \right) \bigg|_{\epsilon=0}
$$

Now let

$$
f = m(z-ax) \sqrt{x^2 + \beta^2(y^2 + z^2)}
$$

$$
g = \sqrt{(y-mx + \beta^2z - mx^2)} + (z-ax)^2
$$

$$
f(0) = m(z) \sqrt{x^2 + \beta^2(y^2 + z^2)}
$$

$$
f'(0) = -mx \sqrt{x^2 + \beta^2(y^2 + z^2)}
$$

$$
g(0) = \sqrt{(y-mx) + z^2}
$$

$$
g'(0) = -\beta^2 m y z - 2xz
$$

$$
f_0^2 + g_0^2 = m^2 \left[ x^2 + \beta^2(y^2 + z^2) \right] + \left[ (x-m)^2 + 2xy(y-mx) + z^4 \right]$$

$$
= \left( x^2 + y^2 \right) \left[ (x-m)^2 + (1+\beta^2)z^2 \right] + z^2 \left[ m^2 x^2 + \beta^2 z^2 + y^2 - 2mx y + z^2 \right]
$$

$$
= (x^2 + z^2) \left[ (x-m)^2 + (1+\beta^2)z^2 \right]
$$

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\[ f'g - fg' = \sqrt{x^2 + \beta^2(y^2 + z^2)} \left\{ -\left[ z^2 + y(z - mx) \right] mx + m z^2 \left[ \beta^2 m^2 + 2z \right] \right\} \]

Now for the second term let

\[ f = (mx - ax) \sqrt{x^2 + \beta^2(y^2 + z^2)} \]
\[ g = z(mx - ax) - y(mx - y) \]

\[ f(0) = mx \sqrt{x^2 + \beta^2(y^2 + z^2)} \]
\[ g(0) = z^2 - y(mx - y) \]
\[ f'(0) = -y \sqrt{x^2 + \beta^2(y^2 + z^2)} \]
\[ g'(0) = -xz \]

\[ f^2 + g^2 = (y^2 + z^2) \left[ (mx - ax)^2 + (1 + \beta^2 z^2) z^2 \right] \]

\[ f'g - fg' = \sqrt{x^2 + \beta^2(y^2 + z^2)} \left\{ -y \left[ z^2 - y(mx - y) \right] + m z^2 x \right\} \]

Therefore

\[
\lim_{a \to 0} z \left\{ \tan^{-1} \frac{mx - ax - \beta(z + j)}{\beta(1 + \beta^2 z^2) - y(mx - y)} - \tan^{-1} \frac{mx - ax + \beta(z - j)}{\beta(1 + \beta^2 z^2) - y(mx - y)} \right\} \]

\[ = z \left\{ \frac{(z - mx) [z^2 + y(z - mx)] + m z^2 (\beta^2 m^2 + x)}{(z^2 + z^2) \left[ (mx - y)^2 + (1 + \beta^2 z^2) z^2 \right]} \right\} \sqrt{x^2 + \beta^2(y^2 + z^2)} \]

\[ = z \left\{ \frac{y \sqrt{x^2 + \beta^2(y^2 + z^2)}}{(z^2 + z^2)} \right\} \]
To evaluate the additional term $\phi_1$, as $a \to 0$, we must combine points 1 and 3 and 2 and 4 from (15).

$$\lim_{a \to 0} \phi_{00} (x, y, z, m) = \phi_{00} (x-x_3, y, z-ax_3, m)$$

$$= \lim_{a \to 0} \left[ - \frac{a(z-ax_3) + m_1 y}{m} \frac{1}{2} \log (y^2 + (z-ax_3)^2) - 1 \right] + \frac{a(z-ax_3) + m_1 y}{m} \frac{1}{2} \log (y^2 + (z-ax_3)^2) - 1$$

$$- \frac{mz-ay}{m} \tan^{-1} \frac{mz-ay}{(my+2z)} - \frac{m_1 (z-ax_3) - ay}{m_1} \tan^{-1} \frac{m_1 (z-ax_3) - ay}{[m_1 y + a(z-ax_3)]}$$

$$\frac{\Delta P}{8\pi q_\infty a}$$

$$= \Delta P \frac{8\pi q_\infty a}{8\pi q_\infty a} \left[ \frac{\frac{az + az}{m} \frac{1}{2} \log (y^2 + z^2) - 1}{m} \frac{ax_3}{y^2 + z^2} \right]$$

$$+ \left[ \frac{ay}{m} \frac{a(y + m_1 x_3)}{m_1} \right] \tan^{-1} \frac{z}{m (1 + \frac{z^2}{y^2}) - \frac{az}{m} \frac{1 + \frac{z^2}{y^2} + \frac{m_1 x_3}{y}}{m_1 (1 + \frac{z^2}{y^2})}$$

$$\frac{\Delta P}{8\pi q_\infty} \left[ \frac{- \frac{z}{m} + \frac{z}{m_1}}{\frac{1}{2} \log (y^2 + z^2)} - \frac{[m(x-y)]}{m} \frac{[\frac{m_1 (x-x_3) - y]}{m_1}}{\tan^{-1} \frac{z}{-y}} \right]$$

where we have used

$$\tan^{-1} (a-\Delta a) = \tan^{-1} a + \frac{\Delta a}{1+a^2}$$

ORIGINAL PAGE IS OF POOR QUALITY
Since we can write
\[
\tan^{-1} \frac{z}{-y} = \pi - \tan^{-1} \frac{-y}{z} = \pi + \tan^{-1} \frac{y}{z}
\]

Therefore if we let
\[
\hat{c}_{\infty} (x, y, z, m) = \frac{-\Delta P}{8\pi \rho \omega} \left\{ \frac{(mx-y)}{m} \tan^{-1} \frac{y}{z} + \frac{z}{m} \frac{1}{z} \log (y^2+z^2) \right\}
\]

Then since \(\pi(mx-y)\) will cancel for a complete panel,
\[
\lim_{a \to o} \hat{c}_1 (x,y,z) = \hat{c}_{\infty} (x,y,z,m) - \hat{c}_{\infty} (x-x_2, y-y_2, z,m)
\]
\[
- \hat{c}_{\infty} (x-x_3, y,z,m_1) + \hat{c}_{\infty} (x-x_4, y-y_4,z,m_1)
\]

And therefore for subsonic flow with \(a = o\)
\[
\phi(x,y,z) = \frac{-\Delta P}{8\pi \rho \omega} \left\{ \frac{(mx-y)}{m} \tan^{-1} \frac{mx+y^2}{z} + \frac{z}{m} \frac{1}{z} \log \left( \frac{\sqrt{x^2+y^2+z^2}}{y} \right) \right\}
\]
\[
- \frac{z}{y^2+z^2} \frac{z}{m} \frac{1}{z} \log \left[ \frac{(1+\beta^2 m^2) [x^2+\beta^2 (y^2+z^2)] + (x+\beta^2 y^2)}{(1+\beta^2 m^2) [x^2+\beta^2 (y^2+z^2)] - (x+\beta^2 y^2)} \right]
\]
\[
+ \frac{z}{y^2+z^2} \frac{\sqrt{x^2+y^2+z^2}}{m} \frac{1}{z} \log \left( \frac{\sqrt{x^2+y^2+z^2}}{y} \right)
\]
In the form the above equation is written, derivatives with respect to $x$, $y$, or $z$ of all but the last two terms may be obtained by differentiating only the coefficients of each term. This was discussed previously when obtaining the velocity components for supersonic flow. If we use $\beta^2 = 1-M^2$ the above equation can also be used for $M > 1$. If we multiply the expression by $z$, take absolute values of the log arguments, and omit the last two terms. Therefore

$$
\phi(x, y, z) = \frac{-\Delta p}{8\pi \rho_0} \left\{ \begin{array}{c}
(mx-y) \tan^{-1} \frac{mz}{\sqrt{x^2 + \beta^2 (y^2+z^2)}} - \frac{z}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2+z^2) + x}}{\sqrt{x^2 + \beta^2 (y^2+z^2) - x}} \\
+ z \sqrt{1+\beta^2} \left[ \frac{1}{2} \log \frac{\sqrt{(1+\beta^2)(x^2+\beta^2)(y^2+z^2)}}{\sqrt{(1+\beta^2)(x^2+\beta^2)(y^2+z^2)} - (x+\beta y)} \right] + \frac{x \sqrt{1+\beta^2(y^2+z^2)}}{y^2+z^2} \left[ (mx-y) \tan^{-1} \frac{y}{z} + \frac{z}{2} \log (y^2+z^2) \right] \end{array} \right\}
$$

where we take only the real part and

$$
l = \begin{cases} 
1 & M > 1 \\
2 & M < 1 \end{cases} \quad l_0 = \begin{cases} 
1 & M > 1 \\
0 & M < 1 \end{cases} \quad \beta^2 = M^2-1
$$
This is equivalent to the form derived by Woodward if it is noted that

\[
\log \frac{x + \beta_m^2 y + \sqrt{(1 + \beta_m^2) [x^2 + \beta_m^2 (y^2 + z^2)]}}{\beta \sqrt{(mx - y)^2 + (1 + \beta_m^2) z^2}}
\]

\[= \frac{1}{2} \log \frac{\sqrt{(1 + \beta_m^2) [x^2 + \beta_m^2 (y^2 + z^2)]} + (x + \beta_m^2 y)}{\sqrt{(1 + \beta_m^2) [x^2 + \beta_m^2 (y^2 + z^2)]} - (x + \beta_m^2 y)}
\]

and

\[
\frac{1}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} = \log \frac{x + \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{\beta \sqrt{y^2 + z^2}}
\]

where \( \beta = \sqrt{\beta^2} \)
If we set \( a = 0 \) in (A10), the additional term which occurs in subsonic flow may be easily obtained. For \( a = 0 \),

\[
\phi_1 (x, y, z) = \frac{\Delta P}{8\pi a_\infty} \int_0^\eta \int_{m_1}^{x_3} \frac{z}{(\eta - y)^2 + z^2} \, d\xi \, d\eta
\]

\[
\int_0^2 \int_{m_1}^{x_3} \frac{z}{(\eta - y)^2 + z^2} \, d\xi \, d\eta = -\frac{1}{m} \int_0^2 \frac{\eta z}{(\eta - y)^2 + z^2} \, d\eta + \frac{1}{m_1} \int_0^2 \frac{(\eta + m_1 x_3) z}{(\eta - y)^2 + z^2} \, d\eta
\]

\[
= -\frac{z}{m} \left[ \frac{1}{2} \log [(\eta - y)^2 + z^2] \cdot \frac{y}{z} \tan^{-1} \frac{1}{\eta - y} \right]^{y_2}_{y_1} + \frac{z}{m_1} \left[ \frac{1}{2} \log [(\eta - y)^2 + z^2] \cdot \frac{y + m_1 x_3}{z} \tan^{-1} \frac{1}{\eta - y} \right]^{y_2}_{y_1}
\]

\[
= \left[ \frac{z}{2m} \log (y^2 + z^2) - \frac{y}{m} \tan^{-1} \frac{z}{y} \right]
\]
\[ \phi_1(x,y,z) = \left\{ \begin{array}{l}
\frac{z}{2m_1} \log \left( (y^2 + z^2) \right) - \frac{y + mx_3}{m_1} \tan^{-1} \frac{z}{y} \\
- \frac{z}{2m_1} \log \left( (y - y_2)^2 + z^2 \right) - \frac{y + mx_3}{m_1} \tan^{-1} \frac{z}{(y - y_2)} \\
\frac{z}{m_1} \log \left( (y - y_2)^2 + z^2 \right) - \frac{y + mx_3}{m_1} \tan^{-1} \frac{z}{(y - y_2)} \\
\frac{1}{m} \left\{ \frac{z}{2} \log \left( (y^2 + z^2) + (mx - y) \right) \tan^{-1} \frac{z}{y} \\
- \frac{1}{m_1} \left\{ \frac{z}{2} \log \left( (y_2 - y_2)^2 + z^2 \right) + [m_1 (x - x_3) - y_2] \tan^{-1} \frac{z}{y_2} \right\} \\
- \frac{1}{m} \left\{ \frac{z}{2} \log \left( (y - y_2)^2 + z^2 \right) + [m_1 (x - x_4)] - (y - y_4) \tan^{-1} \frac{z}{(y - y_4)} \right\} \\
\end{array} \right. \]

The above is true since

\[ y_2 = mx_2 \]

\[ y_4 = y_2 = m_1 (x_4 - x_3) \]

**ORIGINAL PAGE IS OF POOR QUALITY.**
Therefore, analogous to (A14) or (A18) when $a = 0$

\[
\hat{\phi}_{oo}(x,y,z) = -\frac{\Delta p}{8\pi q_{\infty}} \left[ (mx-y) \tan^{-1} \frac{z}{-y} + \frac{z}{2} \log (y^2 + z^2) \right],
\]

which is equivalent to [A18] since we can replace

\[
\tan^{-1} \frac{z}{-y} \text{ by } \pi + \tan^{-1} \frac{y}{z} \text{ or by } \tan^{-1} \frac{y}{z}
\]

since $\pi(mx-y)$, when the contributions from each corner are added, gives zero contribution.

Total Source Strength

\[
v_1 - v'_1 = k \frac{1}{a}
\]
\[ v_2 - v'_2 = \frac{K}{a} \left\{ -\frac{a}{[a^2 + m^2]^{1/2}} \frac{am}{a^2 + m^2} + \frac{m}{[a^2 + m^2]^{1/2}} \frac{-m^2}{(a^2 + m^2)} \right\} = \frac{K m}{a (a^2 + m^2)^{1/2}} \]

\[ L_1 = \gamma_0 \]

\[ L_2 = \gamma_0 \left( \frac{a^2 + m^2}{m} \right)^{1/2} \frac{1}{m} \]

\[ (v_1 - v'_1) L_1 + (v_2 - v'_2) L_2 = \gamma_0 K \left[ \frac{1}{a} - \frac{1}{a} \right] = 0 \]

Point Source at \( \xi, \eta, \zeta \)

\[ r^2 = (x-\xi)^2 + \beta^2 [(y-\eta)^2 + (z-\zeta)^2] \]

\[ \phi(x,y,z) = \frac{m}{4\pi r} \]

Doublet at \( \xi, \eta, \zeta \) in \( \hat{e} \) direction

\[ \phi(x,y,z) = \hat{e} \cdot \nabla \left[ \frac{A}{4\pi r} \right] \]

\[ \nabla \beta = \beta^2 \left( \frac{\partial}{\partial \xi} \hat{e}_\xi + \frac{\partial}{\partial \eta} \hat{e}_\eta + \frac{\partial}{\partial \zeta} \hat{e}_\zeta \right) \]

If we integrate a row of these doublets in the direction of \( \hat{e} \) we get a source at one end and a sink at the other. Now let \( \hat{e} \) be
and integrate in the $x$ direction.

\[
\frac{1}{\sqrt{1+a^2}} \left[ -a \vec{e}_x + \vec{e}_x \right]
\]

If we integrate this over $S$ we obtain (A.7). Therefore (A.7) corresponds to a volume of doublets which means there will be a surface of sinks on one side and sources on the other.
WAGNER'S LIFTING SURFACE EQUATIONS

\[ u(x,y,z) = \frac{\partial}{\partial x} \Phi(x,y,z) \]
\[ v(x,y,z) = \frac{\partial}{\partial y} \Phi(x,y,z) \]
\[ w(x,y,z) = \frac{\partial}{\partial z} \Phi(x,y,z) \]

\[ \Phi(x,y,z) = V_\infty \iint_S \frac{z \ k(x',y')}{4\pi \left[(y-y')^2 + z^2\right]} \left[ 1 + \frac{x - x'}{\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \right] \, dx' \, dy' \quad (1) \]

Change of variables

\[ x_h(\eta') - x_v(\eta') = c(\eta') \]
\[ \eta' = \frac{y'}{\frac{1}{2} b} \]
\[ y' = \frac{1}{2} \eta'b \]
\[ \xi' = \frac{x' - x_v(\eta')}{x_h(\eta') - x_v(\eta')} \]
\[ x' = \xi' \ c(\eta') + x_v(\eta') \]
\[
\begin{align*}
\frac{\partial x'}{\partial \xi'} \frac{\partial y'}{\partial \eta'} = \left| \begin{array}{cc}
\frac{\partial x'}{\partial \xi'} & \frac{\partial x'}{\partial \eta'} \\
\frac{\partial y'}{\partial \xi'} & \frac{\partial y'}{\partial \eta'} 
\end{array} \right| \\
\frac{1}{2} bc(\eta') \, d\xi' \, d\eta' =
\end{align*}
\]

Therefore

\[
\Phi(x,y,z) = \frac{V_{\infty} b^2}{8\pi} \int_{-1}^{1} \int_{0}^{1} \frac{z \, c(\eta') \, k \left[ k'(\xi',\eta'), y'(\eta') \right]}{(y - \frac{1}{2} \eta' b)^2 + z^2} \left\{ 1 + \frac{x - x'_y(\eta') - \xi' \, c(\eta')}{\sqrt{(x - x'_y(\eta') - \xi' \, c(\eta'))^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right\} \, d\xi' \, d\eta'
\] (2)

Now if we define

\[
\tilde{g}(x,y,z,\eta') = \frac{c(\eta')}{b} \int_{0}^{1} k \left[ k'(\xi',\eta'), y'(\eta') \right] \left\{ 1 + \frac{x - x'_y(\eta') - \xi' \, c(\eta')}{\sqrt{(x - x'_y(\eta') - \xi' \, c(\eta'))^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right\} \, d\xi'
\]

Then

\[
\Phi(x,y,z) = \frac{V_{\infty} b^2}{8\pi} \int_{-1}^{1} \frac{z \, \tilde{g}(x,y,z,\eta')}{(y - \frac{1}{2} \eta' b)^2 + z^2} \, d\eta'
\]

Near \( z = 0 \) we can write

\[
\Phi(x,y,z) = \Phi(x,y,0) + z \frac{\partial}{\partial z} \Phi(x,y,z) \bigg|_{z=0}
\]

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\[ \tilde{g}(x,y,z,\eta') = \tilde{g}(x,y,0,\eta') + O(z^2) \]

Therefore if we change variables and let

\[ \xi = \frac{x - x_V(\frac{2y}{b})}{c(\frac{2y}{b})} \]

\[ \eta = \left(\frac{2y}{b}\right) \]

\[ \tau = \left(\frac{2z}{b}\right) \]

\[ \tilde{g}(x,y,0,\eta') = g(\xi,\eta,\eta') \]

We can say

\[ \frac{1}{V} \Phi(x,y,0) = \varphi(\xi,\eta) = \frac{b}{2\pi} \lim_{\xi \to 0} \int_{-1}^{1} \frac{\xi g(\xi,\eta,\eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \]

and

\[ \lim_{z \to 0} \frac{-1}{V} \frac{\partial}{\partial z} \Phi(x,y,z) = \alpha(\xi,\eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \int_{-1}^{1} \frac{\xi g(\xi,\eta,\eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \]
Now integrate (6) by parts $[\text{Provided } \frac{\partial}{\partial \eta} g (\xi, \eta, \eta') \text{ is continuous} ]$

$$u = g (\xi, \eta, \eta')$$

$$dv = \frac{t \, d \eta'}{(\eta' - \eta)^2 + t^2}$$

$$du = \frac{\partial}{\partial \eta} g (\xi, \eta, \eta')$$

$$v = -\tan^{-1} \frac{t}{\eta' - \eta}$$

$$\int_{-1}^{1} \frac{t \, g (\xi, \eta, \eta')}{(\eta' - \eta)^2 + t^2} \, d \eta' = - g (\xi, \eta, \eta') \tan^{-1} \left[ \frac{t}{\eta' - \eta} \right] \bigg|_{-1}^{1}$$

$$+ \int_{-1}^{1} \tan^{-1} \left[ \frac{t}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta} g (\xi, \eta, \eta') \right] \, d \eta'$$

$$= \int_{-1}^{1} \tan^{-1} \left[ \frac{t}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta} g (\xi, \eta, \eta') \right] \, d \eta'$$

where the fact that $g(\xi, \eta, \pm 1) = 0$, because the loading goes to zero at $\eta' = \pm 1$, was used and since

$$\lim_{\xi \to 0} \tan^{-1} \frac{t}{\eta' - \eta} = \pi \text{ sgn } t \quad \eta' - \eta < 0$$

$$0 \quad \eta' - \eta > 0$$

(9)
\[ \varphi(\xi, \eta) = \frac{b}{4} \sgn \xi \ g(\xi, \eta, \eta) \] (10)

and

\[ \alpha(\xi, \eta) = \frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \tan^{-1} \left[ \frac{\xi - \eta}{\eta - \eta} \right] \left[ \frac{\partial}{\partial \eta} g(\xi, \eta, \eta') \right] d\eta' \]

\[ = -\frac{1}{2\pi} \lim_{\xi \to 0} \int_{-1}^{1} \frac{(\eta' - \eta) \frac{\partial}{\partial \eta} g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \] (12)

and from A1

\[ = -\frac{1}{2\pi} \int_{-1}^{1} \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \frac{\eta' - \eta}{\eta' - \eta} d\eta' = \frac{1}{2\pi} \left\{ \frac{2g(\xi, \eta, \eta)}{\epsilon} - \int_{-1}^{1} \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \right\} \]

If \( \frac{\partial}{\partial \eta} g(\xi, \eta, \eta') \) is not continuous we define

\[ \alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi' g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' = -\frac{1}{2\pi} \int_{-1}^{1} \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \] (13)

Now let

\[ \frac{c(\eta')}{b} k \left[ x'(\xi', \eta'), y'(\eta') \right] = -\sum_{n=0}^{N} \int_{n}^{N} \phi_{n}(\eta') h_{n}(\xi') \]
which means

\[ g(\xi, \eta, \eta') = - \sum_{n=0}^{N} f_n(\eta') H_n(\xi, \eta, \eta') \]

where

\[ h_n(\xi') = \frac{1}{\pi} \left[ \frac{1 - \xi'}{\xi'} \right]^{1/2} \left[ \frac{T_n(1 - 2 \xi') + T_{n+1}(1 - 2 \xi')}{1 - \xi'} \right] \]

or

\[ \hat{h}_n(\Psi) = h_n \left[ \frac{1}{2} (1 - \cos \Psi) \right] = \frac{2}{\pi} \left[ \cos n\Psi + \cos (n+1)\Psi \right] = \frac{2}{\pi} \frac{\cos \left( n\frac{1}{2} \right) \Psi}{\sin \frac{\Psi}{2}} \]

and

\[ H_n(\xi, \eta, \eta') = \int_{0}^{1} h_n(\xi') \left\{ 1 + \frac{x - x_V(\eta') - \xi'c(\eta')}{\sqrt{[x-x_V(\eta') - \xi'c(\eta')]^2 + \left[ y - \frac{1}{2} \eta'b \right]^2}} \right\} d\xi' \]

\[ = \int_{0}^{1} h_n(\xi') \left\{ 1 + \frac{x - x_V(\eta')}{c(\eta') - \xi'} \right\} \frac{1}{\sqrt{\left[ \frac{x-x_V(\eta')}{c(\eta')} - \xi' \right]^2 + \left[ \frac{\eta - \eta'}{b} \right]^2}} d\xi' \quad (15) \]
and from (5)

\[
\frac{1}{h_n(\xi')} \left\{ 1 + \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi' \right. \\
\left. \sqrt{\left[ \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi' \right]^2 + \left[ \frac{(\eta - \eta') \frac{1}{2} b}{c(\eta')} \right]^2} \right\} d\xi'
\]

Now, from (7), for any \(g(\xi, \eta, \eta')\) differentiable or not we can write

\[
\alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta'
\]

\[
= \frac{1}{2\pi} \sum_{n=0}^{N} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta') \left[ H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta') \right]}{(\eta - \eta)^2 + \xi^2} d\eta'
\]

(16)

where we define

\[
H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta')
\]

(17)

or

\[
\alpha(\xi, \eta) = \frac{1}{2\pi} \sum_{n=0}^{N} \left\{ H_n(\xi, \eta, \eta) \int_{-1}^{1} \frac{\partial}{\partial \eta'} f_n(\eta') d\eta' + \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta') K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \right\}
\]

(18)
where if \( \frac{\partial}{\partial \eta}, K_n(\xi, \eta, \eta') \) is continuous, using (11) and (12)

\[
\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta')K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' = \int_{-1}^{1} \frac{\frac{\partial}{\partial \eta} \left[ f_n(\eta')K_n(\xi, \eta, \eta') \right]}{(\eta - \eta)} \, d\eta'
\]

\[
= \int_{-1}^{1} \frac{f_n(\eta')K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2} \, d\eta'
\]

since \( f_n(+1) = 0 \)

Now referring to (15), let

\[
\tilde{H_n}(p,q) = \int_{-1}^{1} h_n(\xi') \left[ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right] \, d\xi'
\]

Then, from (15) for small \( (p - p_o) \) and small \( q \) \( p_o \neq 0,1 \)

\[
\tilde{H_n}(p,q) = \tilde{H_n}(p_o,0) + 2 h_n(p_o)(p - p_o) - h'_n(p_o)q^2 \ln |q| \quad (20)
\]

Now set \([\text{See (15)}]\)

\[
p = \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')}
\]

\[
p_o = \xi
\]

\[
q = \frac{1/2 \, b(\eta' - \eta)}{c(\eta')}
\]

(21)
when
\[ q = 0 \]
\[ \eta' = \eta \]
\[ p = p_0 \]

Then for \( (\eta' - \eta) \ll 1 \)

\[
p - p_0 = \frac{a}{\partial \eta'} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta' = \eta} = -\frac{1}{2} b \left( \eta' - \eta \right) \tan \phi
\]

(21a)

where we have defined \( \phi(\xi, \eta) \) as the sweep of the constant percent chord lines

\[
\tan \phi(\xi, \eta) = -\frac{c(\eta)}{\frac{1}{2} b} \frac{a}{\partial \eta'} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta' = \eta}
\]

(22)

Therefore for \( \xi \neq 0,1 \)

\[
\left| \frac{1}{2} \frac{b}{c(\eta')} \left( \eta' - \eta \right) \right| \ll 1
\]

\[
H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) - 2 \left[ \frac{\frac{1}{2} b}{c(\eta)} \left( \eta' - \eta \right) \right] h_n(\xi) \tan \phi
\]

\[
- h_n'(\xi) \left[ \frac{\frac{1}{2} b}{c(\eta)} \left( \eta' - \eta \right) \right]^2 \ln \left| \frac{\frac{1}{2} b}{c(\eta)} \left( \eta' - \eta \right) \right|
\]
Then for small \((\eta' - \eta)\) and \(\xi \neq 0, 1\)

\[
K_n(\xi, \eta, \eta') = -2 \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right] h_n(\xi) \tan \phi - h_n'(\xi) \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right] \ln \left[ \frac{\frac{1}{2}b(\eta' - \eta)}{c(\eta)} \right]
\]

(23)

For \(\xi = 0\), from (C2)

\[
\tilde{H}_n(p, q) = \tilde{H}_n(0, 0) + \frac{8 \cos^2 \phi}{\pi} \left[ I_1(\tilde{\phi})(p^2 + q^2)^{1/4} \left( \frac{1}{3} (1 + 4n + 4n^2) I_2(\tilde{\phi})(p^2 + q^2)^{3/4} \right) \right]
\]

where \(I_1(\tilde{\phi})\) and \(I_2(\tilde{\phi})\) are defined in (C3) and C4) and

\[
\tilde{\phi} = \tan^{-1} \frac{-p}{q}
\]

At the leading edge \(\theta = 0\) and, from (21a)

\[
p = \frac{1}{2} \frac{b(\eta' - \eta)}{c(\eta)} \tan \phi_{L.E.} = q \tan \phi_{L.E.}
\]

and therefore

\[
p^2 + q^2 = \frac{\frac{1}{2}b}{c(\eta)} \left[ 1 + \tan^2 \phi_{L.E.} \right] (\eta' - \eta)^2 = \frac{1}{\cos^2 \phi_{L.E.}} \left[ \frac{1}{2} \frac{b}{c(\eta)} \right]^2 (\eta' - \eta)^2
\]

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\[ \phi = \tan^{-1} \left[ \tan \phi_{L.E.} \right] = \phi_{L.E.} \quad q > 0 \text{ or } (\eta' - \eta) > 0 \]

\[ = \pi + \phi_{L.E.} \quad (\eta' - \eta) < 0 \]

Therefore for \((\eta' - \eta) \ll 1\)

\[ K_n(0, \eta, \eta') = \varphi_1(\phi) \left[ \frac{1}{2} b \frac{|\eta' - \eta|}{c(\eta)} \right]^{1/2} - \frac{1}{2} (1 + 4n + 4n^2) \varphi_2(\phi) \left[ \frac{1}{2} b \frac{|\eta' - \eta|}{c(\eta)} \right]^{3/2} \]  

(24)

where

\[ \varphi_1(\phi) = \frac{8}{\pi} \frac{|\cos \phi|}{3^{3/2}} I_1(\phi) \]

\[ \varphi_2(\phi) = \frac{8}{3\pi} \frac{1}{2} I_2(\phi) \]

and

\[ \phi = \phi_{L.E.} \quad (\eta' - \eta) > 0 \]

\[ \phi = \pi + \phi_{L.E.} \quad (\eta' - \eta) < 0 \]
Using a similar analysis for the trailing edge, \( \xi = 1 \), we get

\[
K_n(1, \eta, \eta') = (-1)^n \varphi_z(\phi) (1 + 2n) \left[ \frac{1}{2} b \frac{\eta'-\eta}{c(\eta)} \right]^{5/2} (\eta'-\eta) \ll 1 \quad (25)
\]

\[
\varphi = \phi_{T.E.} + \pi \quad (\eta'-\eta) > 0
\]

\[
\varphi = \phi_{T.E.} \quad (\eta'-\eta) < 0
\]

For \( \xi = 0 \)

\[
\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')
\]

is not continuous at \( \eta' = \eta \) due to the term involving \((\eta'-\eta)^{1/2}\)

Therefore, referring to (18), we must evaluate

\[
\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta')K_n(0, \eta, \eta')}{(\eta'-\eta)^2 + \xi^2} d\eta'
\]

Since the only discontinuity in

\[
\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')
\]

occurs at \( \eta' = \eta \) we can write, using (19)
\[
\lim_{\xi \to 0} \frac{8}{\partial \xi} \int_{-1}^{1} \frac{\frac{\partial f_n(\eta') K_n(0, \eta, \eta')}{(\eta'-\eta)^2 + \xi^2}}{d \eta'} = \lim_{\xi \to 0} \frac{8}{\partial \xi} \int_{\eta-\delta}^{\eta+\delta} \frac{\frac{\partial f_n(\eta') K_n(0, \eta, \eta')}{(\eta'-\eta)^2 + \xi^2}}{d \eta'}
\]

\[
\int_{-1}^{\eta-\delta} \frac{f_n(\eta') K_n(0, \eta, \eta')}{(\eta'-\eta)^2} d \eta' + \int_{1}^{\eta+\delta} \frac{f_n(\eta') K_n(0, \eta, \eta')}{(\eta'-\eta)^2} d \eta' + \int_{-1}^{1} \frac{1}{\xi} \frac{\partial f_n(\eta') K_n(0, \eta, \eta')}{(\eta'-\eta)^2} d \eta'
\]

The only term in \( K_n(0, \eta, \eta') \) which causes trouble is the term

\[
\varphi_1(\Phi_{L.E.}) \left[ \frac{1}{2} \frac{b(\eta'-\eta)}{c(\eta)} \right]^{1/2}
\]

and if \( \delta \) is small enough we can approximate \( f_n(\eta') \) by \( f_n(\eta) \)

Therefore we must evaluate the following expressions

\[
\int_{\delta}^{\delta + \xi/2} \frac{s \sqrt{2}}{s^2 + 1} ds - \int_{\delta}^{\delta + \xi/2} \frac{s \sqrt{2}}{s^2 + 1} ds = \sqrt{\frac{\pi}{2}} \left[ \frac{\xi}{s^2 + 1} \right]_{\delta}^{\delta + \xi/2} - \int_{\delta}^{\delta + \xi/2} \frac{s \sqrt{2}}{s^2 + 1} ds
\]

\[
= \sqrt{\frac{\pi}{2}} \left[ s^{-1/2} \left[ 1 - \frac{1}{s^2} - \frac{1}{s^4} - \frac{1}{s^6} - \ldots \right] ds \right] = \sqrt{\frac{\pi}{2}} \left[ \frac{\xi^{1/2}}{s^2 + 1} + \frac{\xi^{3/2}}{5} + \frac{\xi^{5/2}}{9} + \ldots \right]
\]

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and,

\[ 2 \frac{d}{dx} \int_0^\delta \frac{t}{S^2 + x^2} \, dS = \left\{ \frac{x}{\sqrt{2x}} + \frac{2}{\sqrt{2}} \left[ -2 + \frac{2}{5} \frac{t^3}{x^3} - \frac{10}{9} \frac{t^4}{x^4} \right] \right\} \]

\[ \lim_{x \to 0} \frac{2}{x} \int_0^\delta \frac{t \sqrt{S}}{S^2 + x^2} \, dS = \infty \quad (26a) \]

Subappendices E and F consider the case where \( g(\xi, \eta, \eta') \) is considered to be a function of \( \xi \), as in (3), but the expression (26a) is still infinite.

If \( \tan \phi \) is not continuous at some \( \eta = \eta' \) then, from (23) and (18) or (26), it can be seen that the integral of

\[ K_n(\xi, \eta, \eta') \]

\[ \frac{(\eta' - \eta)^2}{(\eta' - \eta)^2} \]

will give a logarithmic infinity. If \( \tan \phi \) is continuous the first term of (23) will give an odd function of \( \eta' - \eta \) which will give a finite value in the Cauchy principle value sense. Therefore kinks or cranks in the planform cannot be permitted because discontinuities in the angles of the constant percent chord lines will result. In fact subappendix G of this appendix shows that a logarithmic infinity occurs if the leading edge kink is considered as the limit of a hyperbola whose radius of curvature goes to zero. If the term in (18) involving \( \tan \phi \) is neglected for small \( (\eta' - \eta) \), this has the effect of rounding the constant percent chord lines.

Now let \( \eta' = \cos \theta' \) and

\[ f_n(\eta') = \sum_{m=1}^M f_{nm} \overline{s_m}(\theta) \quad (27) \]
where

\[ \bar{S}_m(\theta') = \frac{2}{M+1} \sum_{\mu=1}^{M} \sin \mu \theta_m \sin \mu \theta' = \begin{cases} 1 & \theta' = \theta_m \\ \frac{(-1)^{M+1}}{M+1} \sin \theta_m \sin(M+1) \theta' & \theta' \neq \theta_m \end{cases} \]

\[ \theta_m = \frac{m \pi}{M+1} \quad \text{discrete points} \]

Now the first part of (18) may be performed in closed form

\[ \frac{1}{2\pi} \int_{-1}^{1} \frac{1}{\partial \eta'} \frac{\partial f_n(\eta')}{\partial \eta} \, d\eta' \quad \eta' = \sum_{m=1}^{M} f_{nm} b_{\nu m} \quad \text{(28)} \]

where

\[ b_{\nu m} = \frac{M+1}{4 \sin \theta_\nu} \begin{cases} 1 & \theta_\nu = \theta_m \\ \frac{1 - (-1)^{\nu+M}}{M+1} \frac{\sin \theta_m}{\cos \theta_\nu - \cos \theta_m} & \theta_\nu \neq \theta_m \end{cases} \quad \text{(29)} \]
Referring to (18), (19), (26), and (27) we define

\[
A_{np \nu \nu} = \lim_{\xi \to 0} \frac{1}{2\pi} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{\xi k_n(\xi p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2 + \xi^2} \, d\eta' \quad p = 0
\]

\[
= \frac{1}{2\pi} \int_{\eta_\nu - \delta}^{\eta_\nu + \delta} \frac{K_n(\xi p, \eta_\nu, \eta')}{(\eta_\nu - \eta')^2} \, d\eta' \quad p \neq 0 \quad (30)
\]

\[
B_{np \nu \mu m} = \frac{1}{2\pi} \int_{\eta_\nu + \delta}^{1} \frac{K_n(\xi p, \eta_\nu, \eta') \overline{S}_m \left[ \cos^{-1} \eta' \right]}{(\eta_\nu - \eta')^2} \, d\eta' \quad (31)
\]

and letting

\[
\eta' = \frac{1 + \eta_\nu + \delta}{2} + \frac{1 - \eta_\nu \delta}{2} \cos \theta = \eta_1(\theta)
\]

\[
= \frac{1 - \eta_\nu - \delta}{4} \int_{0}^{\pi} \frac{K_n(\xi p, \eta_\nu, \eta_1(\theta)) \overline{S}_m \left[ \cos^{-1} \eta_1(\theta) \right]}{\left[ \eta_\nu - \eta_1(\theta) \right]^2} \sin \theta \, d\theta
\]

\[
= \frac{1 - \eta_\nu - \delta}{4(M_1 + 1)} \sum_{i=1}^{M_1} \frac{K_n(\xi p, \eta_\nu, \eta_1(\theta_i)) \overline{S}_m \left[ \cos^{-1} \eta_1(\theta_i) \right]}{\left[ \eta_\nu - \eta_1(\theta_i) \right]^2} \sin \theta_i
\]

where

\[
\theta_i = \frac{i\pi}{M_1 + 1} \quad i = 1, 2, \ldots, M_1
\]
\[ C_{np\nu m} = \frac{1}{2\pi} \int_{-1}^{\eta_{\nu}} K_n(\xi, \eta_{\nu}, \eta') \overline{S}_m \left[ \cos^{-1} \eta' \right] \frac{\cos \theta}{(\eta_{\nu} - \eta')^2} \, d\eta' \]

and letting

\[ \eta' = \eta_2(\theta) = \frac{-1 + \eta_{\nu} - \delta}{2} + \frac{1 + \eta_{\nu} - \delta}{2} \cos \theta \]

\[ C_{np\nu m} = \frac{1 - \eta_{\nu} - \delta}{4\pi} \int_0^\pi K_n(\xi, \eta_{\nu}, \eta_2(\theta)) \overline{S}_m(\theta) \sin \theta \frac{\cos \theta}{(\eta_{\nu} - \eta_2(\theta))^2} \, d\theta \]

\[ = \frac{1 - \eta_{\nu} - \delta}{4(M_2 + 1)} \sum_{i=1}^{M_2} K_n(\xi, \eta_{\nu}, \eta_2(\theta_i)) \overline{S}_m(\theta_i) \sin \theta_i \frac{\cos \theta_i}{(\eta_{\nu} - \eta_2(\theta_i))^2} \]

\[ \theta_i = \frac{i\pi}{M_2 + 1} \quad (32) \]

Therefore referring to (18), (19), (26), (28), (29), (30), (31), (32)

\[ \alpha(\xi, \eta_{\nu}) = \sum_{n=0}^{N} \sum_{m=1}^{M} f_{nm} \left[ H_{np\nu n} b_{nm} + \delta_{nm} A_{np\nu} + B_{np\nu m} + C_{np\nu m} \right] \]

which is a set of linear equations to be solved for \( f_{nm} \)

\[ p = 0, 1, \ldots, N \quad M \times (N+1) \]

\[ \nu = 1, 2, \ldots, M \]

\[ H_{np\nu n} = H_n(\xi, \eta_{\nu}, \eta_{\nu}) \]

all of the \( H_n \)'s and \( K_n \)'s are computed numerically.
Consider

\[
\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta) F(\eta')}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= \lim_{z \to 0} \left\{ F(\eta) \int_{-1}^{1} \frac{(\eta' - \eta)}{(\eta' - \eta)^2 + z^2} \, d\eta' + \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta' \right\}
\]

\[
= \lim_{z \to 0} \left\{ \frac{1}{2} F(\eta) \log \left[ (\eta' - \eta)^2 + z^2 \right] \right\} + \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= F(\eta) \left\{ \log \frac{1 - \eta}{\epsilon} + \log \frac{\epsilon}{1 + \eta} \right\} + \lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

If \( F(\eta') \) is differentiable at \( \eta' = \eta \) then \( F(\eta') - F(\eta) = O(\eta' - \eta) \) and we can write

\[
\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= \lim_{\epsilon \to 0} \left\{ \int_{\eta + \epsilon}^{1'} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} \, d\eta' + \int_{-1}^{\eta - \epsilon} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} \, d\eta' \right\}
\]

(A1)
Therefore

\[
\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta) F(\eta')}{(\eta' - \eta)^2 + z^2} \, d\eta' = \int_{-1}^{1} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta'
\]

\[
= \lim_{\epsilon \to 0} \left\{ \int_{\eta + \epsilon}^{1} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta' + \int_{1}^{\eta - \epsilon} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta' \right\}
\]
Subappendix B

\[ \tilde{H}_n(p, q) = \tilde{H}_n(p_0, 0) + \int \nabla \tilde{H}_n(p, q) \cdot d\mathbf{l} \]

\[ = H_n(p_0, 0) + \oint \left( \frac{\partial \tilde{H}_n(p, q)}{\partial \mathbf{n}} \right) d\mathbf{n} + \frac{\partial \tilde{H}_n}{\partial q} dq \]

where \( C \) is a contour integral in the \( p, \xi \) plane from \((p_0, 0)\) to \((p, q)\)

\[ \tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left( 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right) d\xi' \]

\[ \frac{\partial \tilde{H}_n}{\partial p} = \int_0^1 h_n(\xi') \frac{q^2}{[(p - \xi')^2 + q^2]^{3/2}} d\xi' \]

if

\[ p \neq 0, 1 \]

we can write

\[ h_n(\xi') = h_n(p) - h_n(p)(p - \xi') + \frac{1}{2!} h''_n(p)(p - \xi')^2 + r_n(\xi')(p - \xi')^3 \]

(B2)
and since

\[\int \frac{x^3}{r^3} \, dx = -\frac{1}{r} \int \frac{x^2}{r^3} \, dx = \frac{xr}{2} - \frac{a^2}{2} \log (x+r)\]

and

\[\int \frac{a^2}{r^3} \, dx = \frac{x}{r}\]

where \( r = \sqrt{x^2 + a^2} \)

\[\frac{\partial}{\partial q} \tilde{h}_n(p, q) = -h_n(p) \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \bigg|_{\xi' = 0} + o(q^2)\]

or

\[\lim_{q \to 0} \frac{\partial}{\partial p} \tilde{h}_n(p, q) = 2 h_n(p) \quad p \neq 0, 1 \quad (B3)\]

\[\frac{\partial}{\partial q} \tilde{h}_n(p, q) = -q \int_0^1 h_n(\xi') \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}^{3/2}} \, d\xi'\]

now write \((p \neq 0, 1)\)

\[h_n(\xi') = h_n(p) - h_n'(p)(p - \xi') + r_n(\xi')(p - \xi')^2\]
Then as \( q \to 0 \)

\[
q \int_0^1 \frac{(p - \xi')}{[(p - \xi')^2 + q^2]^{3/2}} \, d\xi' = \left. \frac{q}{[(p - \xi')^2 + q^2]^{3/2}} \right|_0^1 = O(q)
\]

and

\[
q \int_0^1 \frac{r_n (\xi') (p - \xi')^3}{[(p - \xi')^2 + q^2]^{3/2}} \, d\xi' = O(q)
\]

but

\[
q \int_0^1 \frac{(p - \xi')^2}{[(p - \xi')^2 + q^2]^{3/2}} \, d\xi' = q \left. \left\{ \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \log \left[ (\xi' - p) + \sqrt{(p - \xi')^2 + q^2} \right] \right\} \right|_0^1
\]

\[
= q \left\{ -\frac{(1-p)}{\sqrt{(1-p)^2 + q^2}} - \frac{p}{\sqrt{p^2 + q^2}} + \log \left[ (1-p) + \sqrt{(1-p)^2 + q^2} \right] - \log \left[ p + \sqrt{p^2 + q^2} \right] \right\}
\]

\[
= O(q) - q \log \left[ -p + \sqrt{p^2 + q^2} \right] = O(q) - q log \left[ -1 + \sqrt{1 + \frac{p^2}{q^2}} \right]
\]

\[
= O(q) - 2q \log |q|
\]
Therefore as $q \to 0$

$$\frac{\partial}{\partial q} \tilde{H}_n(p, q) = -2 h'_n(p) q \ln |q| + O(q) \quad (B4)$$

and since $h_n(p)$ and $h'_n(p)$ are slowly varying for $p \neq 0,1$ referring to (B1) and using (B3) and (B4) for $|p - p_0| << 1$ and $q << 1$

$$\int \int \frac{\partial}{\partial p} \tilde{H}_n(p, q) \ dp = \int \int \frac{\partial}{\partial p} \tilde{H}_n(p, q) \ dp = 2 h_n(p_0)(p - p_0)$$

and

$$\int \int \frac{\partial}{\partial q} \tilde{H}_n(p, q) \ dq = -2 h'_n(p_0) \int_0^q q \ln q \ dq = -h'_n(p_0) q^2 \ln q$$

or to lowest order in $p - p_0$ and $q$ (for $p_0 \neq 0,1$)

$$\tilde{H}_n(p, q) = \tilde{H}_n(p_0, 0) + 2 h_n(p_0)(p - p_0) - h'_n(p_0) q^2 \ln q \quad (B5)$$
Subappendix C

\[ \mathcal{H}_n(p,q) = \int_0^1 h_n(\xi') \left[ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right] d\xi' \]

Leading Edge

Let

\[ p = -\delta^2 \sin \tilde{\phi} \]
\[ q = \delta^2 \cos \tilde{\phi} \]

or

\[ \delta^2 = \sqrt{p^2 + q^2} \]
\[ \tilde{\phi} = \tan^{-1} \frac{-p}{q} \]

and introduce a change of variables

\[ \xi' = \sin^2 \sigma \]
\[ d\xi' = 2 \sin \sigma \cos \sigma \, d\sigma \]
\[ h_n \left( \sin^2 \sigma \right) = h_n \left[ \frac{1}{2} \left( 1 - \cos 2\sigma \right) \right] = \hat{h}_n \left( 2\sigma \right) = \frac{2}{\pi} \frac{\cos 2n\sigma + \cos 2(n+1)\sigma}{\sin 2\sigma} \]

\[ = \frac{2}{\pi} \frac{\cos \left( [2n+1] - 1 \right) \sigma + \cos \left( [2n+1] + 1 \right) \sigma}{2 \sin \sigma \cos \sigma} = \frac{2}{\pi} \frac{\cos (2n+1)\sigma}{\sin \sigma} \]

\[
\left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right\} = 1 + \frac{- (\delta^2 \sin \phi + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}}
\]

and

\[ \tilde{H}_n(p, q) = \tilde{H}_n \left( \delta, \phi \right) \]

\[ = 4 \int_{0}^{\frac{\pi}{2}} \left\{ 1 + \frac{- (\delta^2 \sin \phi + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}} \right\} \cos (2n+1) \sigma \cos \sigma \, d\sigma \]

\[ \frac{\partial \tilde{H}_n}{\partial \delta} = 4 \int_{0}^{\frac{\pi}{2}} \frac{- 2 \sin \phi \cos [(2n+1) \sigma] \cos \sigma \, d\sigma}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}} \, d\sigma \]

\[ + 4 \int_{0}^{\frac{\pi}{2}} \frac{2\delta \left[ \sin \phi \sin^2 \sigma + \delta^2 \right] (\delta^2 \sin \phi + \sin^2 \sigma) \cos [(2n+1) \sigma] \cos \sigma \, d\sigma}{\left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}} \]

\[ = \frac{8 \delta^3 \cos^2 \phi}{\pi} \int_{0}^{\frac{\pi}{2}} \frac{\cos [(2n+1) \sigma] \cos \sin^2 \sigma}{\left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}} \, d\sigma \]

as \( \delta \to 0 \) the only portion of the integrand which is important is in the region of \( \sigma = 0 \)
Therefore let $x = \sin \sigma$ $\, dx = \cos \sigma \, d\sigma$ and expand

\[
\cos [(2n+1) \sigma] = \cos [(2n+1) (x - \frac{3}{6})]
\]

\[
= 1 - \frac{1}{2} (2n+1)^2 x^2 = 1 - \frac{x^2}{2} (1 + 4n + 4n^2) + O(x^4)
\]

Therefore to first order in $x^2$

\[
\frac{\partial}{\partial \delta} \hat{H}_n (\delta, \tilde{\phi}) = \frac{8}{\pi} \cos^2 \tilde{\phi} \int_0^1 \left[ 1 - \frac{x^2}{2} (1 + 4n + 4n^2) \right] x^2
\]

\[
\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + \delta^4 \right]^{3/2} \, dx
\]

\[
= \frac{8}{\pi} \cos^2 \tilde{\phi} \left\{ \int_0^{1/\delta} \frac{x^2 \, dx}{\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} - \delta^2 \int_0^{1/\delta} \frac{(1 + 4n + 4n^2) \, x^4 \, dx}{\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \right\}
\]

Since

\[
\int_0^{1/\delta} \frac{x^2 \, dx}{\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} - \int_0^{1/\delta} \frac{dx}{x^4}
\]

\[
= \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} + O(\delta^3)
\]

\[
\text{281}
\]
Near $\delta = 0$ we can say

$$
\frac{\partial}{\partial \delta} \tilde{H}_n(\delta, \phi) = \frac{8 \cos^2 \tilde{\phi}}{\pi} \int_0^\infty \frac{x^2 \left[ 1 - \frac{\delta^2 x^2}{2} (1 + 4n + 4n^2) \right]}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \, dx + O(\delta^3)
$$

integrating from 0 to $\delta$ we get

$$
\tilde{H}_n(\delta, \phi) = \tilde{H}_n(0, \phi) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left[ I_1(\tilde{\phi}) \delta - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \delta^3 \right] \quad (C1)
$$

or

$$
\tilde{H}_n(p, q) = \tilde{H}_n(0, 0) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left[ I_1(\tilde{\phi}) \left( p^2 + q^2 \right)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \left( p^2 + q^2 \right)^{3/4} \right] \quad (C2)
$$

where

$$
\phi = \tan^{-1} \left( \frac{-p}{q} \right)
$$

and where

$$
I_1(\tilde{\phi}) = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} = \frac{1}{\cos \phi} \frac{d}{d\phi} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \quad (C3)
$$
\[ I_2(\phi) = \int_0^\infty \frac{x^4 \, dx}{[x^4 + 2x^2 \sin \phi + 1]^{3/2}} \]

\[ = -\sin \phi \, I_1(\phi) + \int_0^\infty \frac{(x^4 + x^2 \sin \phi)}{[x^4 + 2x^2 \sin \phi + 1]^{3/2}} \, dx \quad (C4) \]

\[ u = x \]

\[ du = dx \]

\[ dv = \frac{(x^3 + x \sin \phi) \, dx}{[x^4 + 2x^2 \sin \phi + 1]^{3/2}} \]

\[ v = -\frac{1}{2} \frac{1}{\sqrt{x^4 + 2x^2 \sin \phi + 1}} \]

Therefore

\[ I_2(\phi) = -\sin \phi \, I_1(\phi) + \frac{1}{2} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \phi + 1}} \quad (C5) \]
Subappendix D

\[ \tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left[ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right] d\xi' \tag{D1} \]

**Trailing Edge, let**

\[ p = 1 - \delta^2 \sin \theta \]
\[ q = \delta^2 \cos \theta \]

or

\[ \delta^2 = \sqrt{(1 - p)^2 + q^2} \]
\[ \theta = \tan^{-1}\left(\frac{1 - p}{q}\right) \tag{D2} \]

and change variables in (D1)

\[ \xi' = \sin^2 \sigma \]
\[ d\xi' = 2 \sin \sigma \cos \sigma d\sigma \]

\[ h_n(\sin^2 \sigma) = \frac{2}{\pi} \frac{\cos (2n + 1) \sigma}{\sin \sigma} \]

(see leading edge expansion)
\[ \hat{\mathbf{H}}_n(p, q) = \hat{\mathbf{H}}_n(\delta, \theta) = \frac{4}{\pi} \int_0^{\pi/2} \left[ 1 + \frac{\cos^2 \sigma - \delta^2 \sin \theta}{\sqrt{(\cos^2 \sigma - \delta^2 \sin \theta)^2 + \delta^4 \cos^2 \theta}} \right] \cos (2n + 1) \sigma \cos \sigma \ d\sigma \]

\[ = \frac{4}{\pi} \int_0^{\pi/2} \left[ 1 + \frac{\sin^2 \sigma - \delta^2 \sin \theta}{\sqrt{\sin^4 \sigma - 2\delta^2 \sin^3 \sigma \sin \theta + \delta^4}} \right] \cos \left[ (2n + 1) \left( \frac{\pi}{2} - \sigma \right) \right] \sin \sigma \ d\sigma \]

Now

\[ \cos \left[ (2n + 1) \left( \frac{\pi}{2} - \sigma \right) \right] = \cos \left[ (2n + 1) \frac{\pi}{2} \right] \cos \left[ (2n + 1) \sigma \right] + \sin \left[ (2n + 1) \frac{\pi}{2} \right] \sin \left[ (2n + 1) \sigma \right] \]

\[ = \sin \left[ (2n + 1) \frac{\pi}{2} \right] \sin \left[ (2n + 1) \sigma \right] = (-1)^{n-1} \sin \left[ (2n + 1) \sigma \right] \]

The differentiation of the integrand with respect to \( \delta \) may be performed easily if it is compared with the leading edge case.

\[ \frac{\partial}{\partial \delta} \hat{\mathbf{H}}_n(\delta, \theta) = (-1)^n 8\delta^3 \cos^2 \theta \int_0^{\pi/2} \frac{\sin \left[ (2n + 1) \sigma \right] \sin^3 \sigma \ d\sigma}{\left[ \sin^4 \sigma - 2\delta^2 \sin \theta \sin^2 \sigma + \delta^4 \right]^{3/2}} \]

Changing variables of integration once again

\[ x = \sin \sigma \]

\[ dx = \cos \sigma \ d\sigma \]

\[ \sin \left[ (2n + 1) \sigma \right] = (1 + 2n)x + O(x^3) \]
\[ \frac{\partial}{\partial \delta} H_n(\delta, \theta) = (-1)^n (1 + 2n) \frac{8 \delta^2 \cos^2 \theta}{\pi} \int_0^1 \frac{x^4 \, dx}{\left[ x^4 - 2 \delta^2 x^2 \sin \theta + \delta^4 \right]^{3/2}} \]

\[ = (-1)^n (1 + 2n) \frac{8 \delta^2 \cos^2 \theta}{\pi} \int_0^\infty \frac{x^4 \, dx}{\left[ x^4 - 2 \delta^2 x^2 \sin \theta + 1 \right]^{5/2}} + O(\delta^4) \]

\[ = (-1)^n (1 + 2n) \frac{8 \delta^2 \cos^2 \theta}{\pi} I_1(\theta + \pi) \]

where

\[ I_1(\theta) = \int_0^\infty \frac{x^4 \, dx}{\left[ x^4 + 2 x^2 \sin \theta + 1 \right]^{3/2}} \]

and integrating from 0 to \( \delta \) we get

\[ \tilde{H}_n(p, q) = \tilde{H}_n(1, q) + (-1)^n (1 + 2n) \frac{8 \cos^2 \theta}{\pi} I_1(\theta + \pi) \left[ (1 - p)^2 + q^2 \right]^{3/4} \]

where

\[ \theta = \tan^{-1} \left( \frac{1 - p}{q} \right) \]
Subappendix E

Suppose that \( k(x', y') \) may be written (as before)

\[
\frac{c(n')}{b} k [x'(\xi', \eta'), y(\eta')] = \sum_{n=0}^{N} \frac{f_n(\eta')}{h_n(\xi')}
\]

where

\[
h_n(\xi') = \frac{1}{\pi} \left[ \frac{1 - \xi'}{\xi'} \right]^{1/2} \left[ \frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'} \right]
\]

then

\[
h_n(\Psi) = h_n \left[ \frac{1}{2} (1 - \cos \Psi) \right]
\]

\[
= \frac{2}{\pi} \left[ \frac{\cos n\Psi + \cos (n + 1)\Psi}{\sin \Psi} \right] = \frac{2}{\pi} \frac{\cos \left( n + \frac{1}{2} \right) \Psi}{\sin \frac{\Psi}{2}}
\]
Then with

\[ \xi = \frac{x - x_v^2 y}{v b} \]

\[ \eta = \frac{2y}{b} \]

\[ \zeta = \frac{2z}{b} \]

and from (3), (2) and (7) with \( \xi \) kept in \( g \)

\[ \Phi(x, y, z) = \phi(\xi, \eta, \zeta) = \frac{b}{4\pi} \int_{-1}^{1} \frac{\xi \hat{g}(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} \, d\eta' \]

\[ \alpha(\xi, \eta) = \lim_{\xi \to 0} \frac{-1}{2\pi} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{g(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} \, d\eta' \]

where

\[ g(\xi, \eta, \eta', \zeta) = \sum_{n=0}^{N} \hat{f}_n(\eta') \hat{H}_n(\xi, \eta, \eta', \zeta) \]
and

\[
\hat{h}_n(t, \eta, n', \xi) = \int_0^1 h_n(\xi') \left\{ 1 + \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta') - \xi'}{c(\eta')} \right\} d\xi'
\]

We can also define

\[
\hat{n}_n(p, q, \xi') = \int_0^1 h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2 + \xi'^2}} \right\} d\xi'
\]

For small \( n' - \eta \)

\[
p - p_o = -\frac{1}{2}b (n' - \eta) \tan \phi
\]

\( \phi = \) slope of constant percent chord lines

\[
q = \frac{1}{2}b (n' - \eta) \quad \frac{1}{2}b (n' - \eta)
\]

\[
c(\eta') = c(\eta) + \frac{\partial}{\partial \eta} c(\eta') \bigg|_{\eta' = \eta}
\]
\[ \hat{z} = \frac{\xi\left(\frac{1}{2} b\right)}{c(\eta')} = \frac{z}{c(\eta')} = \frac{z}{c(\eta)} \]

\[ \zeta = \frac{z}{\frac{1}{2} b} \]

**Leading Edge**, \( P_0 = 0 \), \( \phi = \phi_{LE} = \) leading edge sweep

\[ p^2 + q^2 + \xi^2 = \left[ \frac{1}{2} b \right]^2 \left( \eta' - \eta \right)^2 \left[ 1 + \tan^2 \phi \right] + \left( \frac{z}{\frac{1}{2} b} \right)^2 \]

\[ = \frac{1}{\cos^2 \phi} \left[ \frac{1}{2} b \right]^2 \left( \eta' - \eta \right)^2 + \xi^2 \cos^2 \phi \]

Therefore at the leading edge since \( \tilde{\phi} = \tan^{-1} \left( \frac{-p}{q} \right) = \phi_{LE} \eta' > \eta \)

\[ \phi_{LE} \quad \eta' < \eta \]

\[ \hat{k}_n (\alpha, \eta, \eta', \xi) = \frac{8(1 - \sin^2 \phi_{LE} \sin^2 \phi)}{\pi} \left[ I_1 (\bar{\alpha}, \phi) \right] \left( p^2 + q^2 + \xi^2 \right)^{\frac{1}{2}} \cdot \frac{1}{b} \left( 1 + 4n + 4n^2 \right) I_2 (\bar{\alpha}, \phi) \left( p^2 + q^2 + \xi^2 \right)^{\frac{1}{2}} \]

\[ = \frac{8(1 - \sin^2 \phi \sin^2 \phi)}{\pi} \left[ I_1 (\bar{\alpha}, \phi) \cos^2 \phi \right] \left( \frac{1}{2} b \right)^{\frac{1}{2}} \left( \frac{1}{c(\eta)} \right)^{\frac{3}{2}} \left( \xi' - \eta \right)^2 + \xi^2 \cos^2 \phi \left( \frac{1}{2} \right)^{\frac{1}{2}} \]

\[ - \frac{1}{b} \left( 1 + 4n + 4n^2 \right) I_2 (\bar{\alpha}, \phi) \left[ \frac{1}{2} b \right]^{\frac{3}{2}} \left( \frac{1}{c(\eta)} \right)^{\frac{3}{2}} \left( \xi' - \eta \right)^2 + \xi^2 \cos^2 \phi \left( \frac{1}{2} \right)^{\frac{1}{2}} \]
\[
\sin \psi = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + z^2}} = \frac{|\eta' - \eta|}{\sqrt{(\eta' - \eta)^2 + z^2 \cos^2 \phi}} = \frac{|u|}{\sqrt{u^2 + \cos^2 \phi}}
\]

where

\[
u = \frac{\eta' - \eta}{\xi}
\]

letting \(s = \eta' - \eta\) we must evaluate an expression of the form

\[
\lim_{t \to 0} \frac{1}{\delta} \int_{-\delta}^{\delta} \frac{(1 - \sin^2 \psi \sin^2 \phi) I_1 (\xi, \psi) t \left[ z^2 + t^2 \cos^2 \phi \right]}{s^2 + t^2} ds
\]

\[
= \lim_{t \to 0} \left[ \sqrt{t} \int_{0}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du \right]
\]

\[
= \lim_{t \to 0} \left[ \sqrt{t} \int_{0}^{-\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du \right]
\]

\[
- \sqrt{t} \int_{0}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du
\]

\[
- \sqrt{t} \int_{0}^{-\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du
\]

\[
= \frac{1}{\xi^2} \int_{0}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du
\]

\[
= \frac{1}{\xi^2} \int_{0}^{-\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du
\]

\[
= \frac{1}{\xi^2} \int_{-\delta}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) [I_1 (\phi, \psi) + I_1 (\phi + \pi, \psi)] [u^2 + \cos^2 \phi]}{u^2 + 1} du
\]

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where

\[ \sin \theta = \frac{u}{\sqrt{u^2 + \cos^2 \phi}} \quad \text{(no } \phi \text{ dependence!)} \]

and

\[ I_1(\phi, \psi) = \int_0^\infty \frac{x^2 \, dx}{[x^4 + 2x^2 \sin \phi \sin \psi + 1]^{3/2}} \]

The second integral goes to zero like \( \sqrt{\xi} \) and the first is independent of \( \xi \). Therefore

\[ \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \sim \frac{1}{\sqrt{\xi}} \to \infty \]
Subappendix F

\[ \hat{H}_n(p, q, \xi) = \int_0^1 h_n(\xi') \left[ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + \xi^2}} \right] d\xi' \]

Leading edge, let

\[ p = -\delta^2 \sin \theta \sin \psi \]
\[ q = \delta^2 \cos \theta \sin \psi \]
\[ \xi = \delta^2 \cos \psi \]
\[ \delta^2 = \sqrt{p^2 + q^2 + \xi^2} \]
\[ \hat{\theta} = \tan^{-1} \frac{p}{q} \]
\[ \psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\xi} \]

introduce a change of variables in the integral

\[ \xi' = \sin^2 \sigma \]
\[ d\xi' = 2 \sin \sigma \cos \sigma \, d\sigma \]
\[
\begin{align*}
\hat{h}_n (\sin^2 \sigma) &= h_n \left[ \frac{1}{2} (1 - \cos 2\sigma) \right] = \hat{h}_n (2\sigma) = \frac{2}{\pi} \frac{\cos 2\sigma + \cos 2(n + 1)\sigma}{\sin 2\sigma} \\
&= \frac{2}{\pi} \cos \left[ (2n + 1) - 1 \right] \sigma + \cos \left[ (2n + 1) + 1 \right] \sigma \frac{2 \cos (2n + 1)\sigma}{2 \sin \sigma \cos \sigma} \\
\end{align*}
\]

and

\[
\begin{align*}
\hat{h}_n (p, q, t) &= \hat{h}_n (\theta, \phi, \theta) = \frac{4}{\pi} \int_0^{\pi/2} \left\{ 1 + \frac{\epsilon^2 \sin \theta \sin \phi + \sin^2 \theta}{\sqrt{\sin^4 \sigma + 2 \epsilon^2 \sin \theta \sin \phi \sin^2 \sigma + \epsilon^4}} \right\} \cos \left[ (2n + 1) \sigma \right] \cos \sigma \, d\sigma \\
\end{align*}
\]
as $\delta \rightarrow 0$ the only portion of the integrand which is important is in the region of $\sigma = 0$.

Therefore let $x = \sin \sigma$, $dx = \cos \sigma \ d\sigma$ and expand

$$\cos \left[ (2n + 1) \sigma \right] = \cos \left[ (2n + 1) \left( x - \frac{x^3}{6} \right) \right] = 1 - \frac{1}{2} (2n + 1)^2 x^2$$

Therefore to first order in $x^2$

$$\mathbf{H}_{n} (\theta, \phi, \sigma) = 8 \delta^3 \left[ 1 - \sin^2 \theta \sin^2 \phi \right] \int_0^1 \frac{1}{x^2} \frac{[1 - \frac{x^2}{2} (1 + 4n + 4n^2)] x^2 \sin \phi}{[x^4 + 2 x^2 \sin \theta \sin \phi + 1^{3/2}] \theta \sin \phi} \ dx$$

$$= 8 \left[ 1 - \sin^2 \theta \sin^2 \phi \right] \int_0^1 \frac{x^2 - \frac{1}{2} \delta^2 x^4 (1 + 4n + 4n^2)}{[x^4 + 2 x^2 \sin \theta \sin \phi + 1]^{3/2}} \ dx + 0(\delta^3)$$

$$\mathbf{H}_{n} (\theta, \phi, \sigma) = \mathbf{H}_{n} (0, \theta, \phi) \left( 1 - \sin^2 \theta \sin^2 \phi \right) \left[ I_1 (\phi, \theta) \delta \frac{1}{2} (1 + 4n + 4n^2) I_2 (\phi, \theta) \delta^3 \right]$$

Integrating from 0 to $\delta$ we get

(C1)
\[ \Pi_n(p, q, \phi) = \Pi_n'(0, 0, 0) \cdot \frac{8 (1 - \sin^2 \phi \sin^2 \theta)}{r} \cdot I_1(\phi, \psi) \left( I_2 + \frac{1}{4} \right) \quad \text{(C2)} \]

where

\[ \phi = \tan^{-1} \frac{p}{q} \]

\[ \psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\xi} \]

\[ I_1(\phi, \psi) = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2 x^2 \sin \phi \sin \psi + 1 \right]^{3/2}} \quad \text{(C3)} \]

\[ I_2(\phi, \psi) = \int_0^\infty \frac{x^4 \, dx}{\left[ x^4 + 2 x^2 \sin \phi \sin \psi + 1 \right]^{3/2}} \quad \text{(C4)} \]

\[ \sin \psi = \frac{\tan \psi}{\sqrt{1 + \tan^2 \psi}} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \xi^2}} \]

\[ \sin \phi = \frac{-p}{\sqrt{p^2 + q^2}} \]

\[ \sin \phi \sin \psi = \frac{-p}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{-p}{\sqrt{p^2 + q^2 + \xi^2}} \]

\[ \frac{1}{\xi^2 + 1} \]

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Subappendix G

\[
\tilde{H}_n(p,q) = \tilde{H}_n(p_0,0) + 2 h_n(p_0)(p - p_0) - h'_n(p_0) q^2 \ln |q|
\]

Hyperbolic leading and trailing edges, constant chord.

\[
\left[ X_v(\eta') \right]^2 = c \tan^2 \theta \left[ \eta'^2 + \rho^2 \tan^2 \theta \right]
\]

\[
X_v(\eta') = c \tan \theta \sqrt{\eta'^2 + \rho^2 \tan^2 \theta}
\]

\[
\eta' > 1 \quad \frac{1}{c} \frac{dX_v}{d\eta} = \tan \theta
\]

Curvature at \( \eta' = 0 \)

\[
\frac{1}{c} \left. \frac{d^2X_v}{d\eta'^2} \right|_{\eta' = 0} = \frac{1}{\rho}
\]

From (21) at \( \eta' = 0 \)

\[
p - p_0 = \frac{X_v(0) - X_v(\eta')}{c}
\]

or

\[
p - p_0 = -\tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right]
\]
Therefore from (15) and (20), for small $\eta'$

$$H_n (\xi, 0, \eta') = H_n (\xi, 0, 0) - 2 h_n (\xi) \tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right]$$

$$+ h'_n (\xi) \eta'^2 \ln |\eta'|$$

Therefore referring to (17) and (18) we must evaluate

$$\lim_{\delta \to 0} \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} \, ds$$

$$= \left. \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} \, ds \right| \right|_0^\delta$$

$$= \left. \left\{ \log \left[ s + \sqrt{s^2 + a^2} \right] + \frac{\sqrt{a^2 - \xi^2}}{\xi} \tan^{-1} \frac{s \sqrt{a^2 - \xi^2}}{\xi \sqrt{s^2 + a^2}} - \frac{a}{\xi} \tan^{-1} \frac{s}{\xi} \right\} \right|_0^\delta$$

$$= \left. \left\{ \log \left[ \delta + \sqrt{\delta^2 + a^2} \right] + \frac{\sqrt{a^2 - \delta^2}}{\delta} \tan^{-1} \frac{\delta \sqrt{a^2 - \delta^2}}{\delta \sqrt{\delta^2 + a^2}} - \frac{a}{\delta} \tan^{-1} \frac{\delta}{\delta} - \log a \right\} \right|_0^\delta$$
and

\[ \lim_{a \to 0} \lim_{\epsilon \to 0} \int_0^\delta \frac{\iota \left[ \sqrt{s^2 + \alpha^2} - \alpha \right]}{s^2 + \epsilon^2} \, ds = \infty. \]

and since \( a = \rho \tan \theta \), there is a logarithmic infinity as the radius of curvature goes to zero and the leading edge becomes kinked.
APPENDIX F

POTENTIAL FORM DRAG

The section load can be obtained by means of the Blasius theorem as follows;

\[
F_X - iF_Y = \frac{1}{2} \rho i \int_C \left( \frac{dw}{dz} \right)^2 dz
\]  

(1)

where

\[
\left( \frac{dw}{dz} \right)^2 = u^2 - 2uvi + (iv)^2 = u^2 - v^2 - 2uvi
\]  

(2)

Therefore;

\[
F_X - iF_Y = \frac{1}{2} \rho \int_C \left[ 2uv i + i (u^2 - v^2) \right] [dx + idy]
\]  

(3)

then

\[
F_X = \frac{1}{2} \rho \int_C \left[ 2uv dx - (u^2 + v^2) dy \right]
\]  

(4)

\[
F_Y = -\frac{1}{2} \rho \int_C \left[ 2uv dy + (u^2 - v^2) dx \right]
\]  

(5)
Since all of the singularities are on the chordal plane, for the panels, equations (4) and (5) reduce to:

\[ F_x = -\rho \sum_{i=1}^{N_i} \left( u_{u_i} v_{u_i} - u_{L_i} v_{L_i} \right) \Delta x_i \]  

(6)

\[ F_y = \frac{\rho}{2} \sum_{i=1}^{N_i} \left[ \left( u_{u_i}^2 + v_{u_i}^2 \right) - \left( u_{L_i}^2 + v_{L_i}^2 \right) \right] \Delta x_i \]  

(7)

where the subscripts \( u \) and \( L \) indicate upper and lower surfaces, respectively.

For the two-dimensional lifting case:

\[ u_{u_i} = \frac{1}{2} \gamma_i + V_\infty \cos \alpha \]  

(8)

\[ u_{L_i} = -\frac{1}{2} \gamma_i + V_\infty \cos \alpha \]  

(9)

\[ v_{u_i} = v_{L_i} = -\frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Gamma_K}{x_i - x_K} + V_\infty \sin \alpha \]  

(10)

Therefore,

\[ F_x = -\rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \sin \alpha + \frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_K} \frac{\Gamma_i \Gamma_K}{x_i - x_K} \]  

(11)
Since

\[ \frac{\rho}{2\pi} \sum_{i=1}^{N_1} \sum_{k=1}^{N_k} \frac{\Gamma_i \Gamma_k}{x_i - x_k} = 0 \]  

(12)

\[ F_x = -\rho V_\infty \sin \alpha \sum_{i=1}^{N_1} \Gamma_i = -L \sin \alpha \]  

(13)

where \( L \) is the lift and \( \Gamma = \gamma \Delta x \) is the local vortex strength.

Also; from equations (7), (8), (9), and (10)

\[ F_y = \rho \sum_{i=1}^{N_1} \Gamma_i V_\infty \cos \alpha = L \cos \alpha \]  

(14)

which demonstrates that the discrete vortex lattice always gives the correct chord force \( F_x \) and normal force \( F_y \), provided the correct circulations is obtained.

Similarly, for the two-dimensional thickness case;

\[ u_{u_i} = u_{L_i} = \frac{1}{2\pi} \sum_{K=1}^{N_1} \frac{\Sigma_k}{x_i - x_K} + V_\infty \]  

(15)

\[ v_{u_i} = V_\infty \left( \frac{dz}{dx} \right)_i \]  

(16)
\[ v_{L_i} = - V_\infty \left( \frac{dz_t}{dx} \right)_i \] (17)

and

\[ \Sigma_i = 2 V_\infty \left( \frac{dz_t}{dx} \right)_i \Delta x_i \] (18)

Therefore;

\[ F_x = - \frac{\rho}{2\pi} \sum_{i=1}^{N_1} \sum_{K=1}^{N_1} \frac{\Sigma_i \Sigma_K}{x_i - x_K} - \rho V_\infty \sum_{i=1}^{N_1} \Sigma_i = 0 \] (19)

since

\[ \sum_{i=1}^{N_1} \Sigma_i = 0 \]

if the airfoil is closed.

Also; \( F_y = 0 \) from equation (7), (15), (16), and (17). This demonstrates that the discrete source lattice also gives the correct chord force \( F_x \) and normal force \( F_y \).

The above equations can be generalized to compute the section potential form drag on a finite wing due to lift and thickness by evaluating the component of force in the free stream direction and by using the three dimensional influence equations.
The section potential form drag due to lift is computed as follows;

\[ d_{L_i} = -\rho \sum_{i=1}^{N_i} \Gamma_i w_i \]

(20)

where \( i \) is summed over the section of the panel and \( w_i \) is the total velocity normal to the panel chordal surface at the quarter chord of the \( i \)th subpanel. The section induced drag coefficient

\[ C_{dL_i} = \frac{C}{C_{AVG.}} \]

is then given by;

\[ \frac{C_{dL_i}}{C_{AVG.}} = \frac{2AR}{b} \sum_{i=1}^{N_i} \left( \frac{\Gamma_i}{V_\infty} \right) \left( \frac{w_i}{V_\infty} \right) \]

(21)

If \( w_i \) is computed at the three-quarter chord of the \( i \)th subpanel, instead of the quarter chord of the \( i \)th subpanel as is done in equation (21), the section zero percent suction drag coefficient

\[ C_{dT=0} = \frac{C}{C_{AVG.}} \]

is obtained. The section leading edge thrust coefficient \((C_T C)/(C_{AVG.})\) is equal to;

\[ \frac{C_T C}{C_{AVG.}} = \frac{C_{dT=0}}{C_{AVG.}} - \frac{C_{dL_i}}{C_{AVG.}} \]

(22)
The section potential form drag due to thickness is computed by a similar procedure.

\[ d_{T_i} = - \rho \sum_{i=1}^{2N_i} \Sigma_i u_i - \rho V_\infty \sum_{i=1}^{2N_i} \left( \frac{V_{x_i}}{V_\infty} \right) \Sigma_i \]  

(23)

where \( i \) is summed over the section for both the quarter and three-quarter chord stations of each subpanel. \( \Sigma_i \) and \( u_i \) are the source strength and total velocity in the free stream direction, respectively, at either the quarter or three-quarter chord point of the subpanel. The section induced drag coefficient

\[ C_{T_i} = \frac{C_i}{C_{AVG}} \]

is then given by;

\[ \frac{C_{d_{T_i}}}{C_{AVG}} = - \frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left( \frac{\Sigma_i}{V_\infty} \right) \left( \frac{U_i}{V_\infty} \right) \frac{2AR}{b} \sum_{i=1}^{2N_i} \sqrt{1 + \tan^2 \Lambda_i} \left( \frac{V_{x_i}}{V_\infty} \right) \left( \frac{\Sigma_i}{V_\infty} \right) \]  

(24)

For the special case where the chordal surfaces of the panels are planar and parallel to each other the integral of the section induced drag

\[ \frac{C_{d_{L_i}}}{C_{AVG}} \]

over the span can be shown to be identical with the value of induced drag as computed in the far field, provided all of the lifting elements (bound vortices) are parallel and the lateral widths of the horseshoe vortices are equal for the complete system.
The total drag of a wing as computed in the near field is given by:

\[ C_{D_i} = -\frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{w}{V_\infty} \right) \left( \frac{r}{r_\infty} \right) \Delta \eta_j \]  

(25)

where

\[ \left( \frac{w}{V_\infty} \right)_{jk} = \frac{\Gamma K}{4\pi} \left\{ \frac{\beta^2 Y_{jk} + X_{jk} + (T^2 + \beta^2) Y_\nu}{(X_{jk} - T Y_{jk}) \sqrt{(X_{jk} + T Y_\nu)^2 + \beta^2 (Y_{jk} + Y_\nu)^2}} \right\} \]

(26)

\[ X_{jk} = X_j - X_K \]

\[ Y_{jk} = (Y_j - Y_K) \]
\((X_K, Y_K)\) is the influencing point

\((X_j, Y_j)\) is the point being influenced

\(\gamma_k\) is half of the spanwise lattice spacing

\(T\) is the tangent of the vortex line sweep

\(\beta^2\) is \(1 - M_o^2\)

Equation (26) can be divided into two parts; that due to the near field stagger of the lifting elements and that due to the limit of integration at infinity.

Therefore;

\[
\left( \frac{W}{V_\infty} \right)_{jK} = \frac{(-\Gamma_k)}{4\pi} \left[ F_{S_{jk}} + F_{\infty_{jk}} \right]
\]

(27)

The contribution to equation (26) from the near field limits of integration or lifting element stagger is given by \(F_{S_{jk}}\).

\[
F_{S_{jk}} = \left\{ \begin{array}{l}
\frac{\beta^2 Y_{jk} + X_{jk} + (\gamma^2 + \beta^2) Y_v}{(X_{jk} - Y_{jk}) \sqrt{(X_{jk} + Y_{jk})^2 + \beta^2 (Y_{jk} + Y_v)^2}} \\
\frac{\beta^2 Y_{jk} + X_{jk} - (\gamma^2 + \beta^2) Y_v}{(X_{jk} - Y_{jk}) \sqrt{(X_{jk} + Y_{jk})^2 + \beta^2 (Y_{jk} - Y_v)^2}}
\end{array} \right.
\]

\[
= \left\{ \begin{array}{l}
\frac{\gamma_{jk} + T Y_v}{(Y_{jk} + Y_v) \sqrt{(X_{jk} + Y_{jk})^2 + \beta^2 (Y_{jk} + Y_v)^2}} \\
\frac{\gamma_{jk} - T Y_v}{(Y_{jk} - Y_v) \sqrt{(X_{jk} - Y_{jk})^2 + \beta^2 (Y_{jk} - Y_v)^2}}
\end{array} \right.
\]

(28)
That due to the limits of integration at infinity is given by \( E_{\infty jK} \)

\[
E_{\infty jK} = \left\{ \frac{1}{Y_{jK} + \gamma^\nu} - \frac{1}{Y_{jK} - \gamma^\nu} \right\} \tag{29}
\]

The contribution to the total drag \( C_{D_1} \), given by equation (25), from \( E_{SjK} \) is seen to be exactly zero for all planar wings and loadings provided both \( T \) and \( \gamma^\nu \) are constant everywhere on the wing. This is due to the fact that there is no contribution to the drag from \( E_{SjK} \) when \( X_{jj} = Y_{jj} = 0 \) or when \( X_{KK} = Y_{KK} = 0 \) due to taking the Cauchy principal value. Also, when \( j \neq K \) the drag from \( E_{SjK} \) is zero because the mutual interference drag due to the stagger is zero. This is seen by interchanging the influencing point and the point being influenced and observing that \( E_{SjK} = -E_{SKj} \).

Therefore;

\[
C_{D_1} = \frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left[ \frac{1}{4 \pi} E_{\infty jK} \left( \frac{\Gamma}{V_{\infty}} \right) \frac{1}{\gamma^\nu} \right] \Delta \eta_j \tag{30}
\]

\[
C_{D_1} = \frac{AR}{4 \pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_{\infty}} \right) \left( \frac{\Gamma}{V_{\infty}} \right) \left( \frac{1}{Y_{jK} + \gamma^\nu} - \frac{1}{Y_{jK} - \gamma^\nu} \right) \Delta \eta_j \tag{31}
\]

This equation is identical to that obtained from the standard Trefftz plane analysis. From reference (47);

\[
D_1 = -\rho V_{\infty}^2 \frac{1}{2} \int \int_{S_{\text{wake}}} \phi \frac{\partial \phi}{\partial N} dS \tag{32}
\]

where \( \phi \) is the velocity potential in the far field.
In the case of a planar wing the vorticity trace in the Trefftz plane can be replaced by a slit and equation (32) replaced by:

\[ D_1 = -\frac{\rho V^2}{2} \int_{-b/2}^{b/2} \Delta \phi(Y) \frac{\partial \phi}{\partial N}(Y) \, dY \]  

(33)

where

\[ \Delta \phi(Y) = \frac{K}{V} \]  

(34)

\[ \frac{\partial \phi}{\partial N}(Y) = \frac{1}{2\pi} \int_{-b/2}^{b/2} \frac{d}{dY_1} \frac{K}{V} \frac{1}{V_{\infty}} \left( \frac{1}{V_1} \right) \frac{1}{(Y - Y_1)} \, dY_1 \]  

(35)

and \( K/V_{\infty}(Y) \) is the total circulation at a given lateral station. Therefore,

\[ \frac{K}{V_{\infty}} = \sum_{i=1}^{N_i} \left( \frac{1}{V_\infty} \right)_i \]  

(36)

and \( N_i \) is the number of vortices per chord. Also,

\[ \frac{\partial \phi}{\partial Y}(Y) = \frac{1}{2\pi} \sum_{n=1}^{N_N} \left( \frac{K}{V_\infty} \right)_n \left( \frac{1}{Y - Y_n + \nu} - \frac{1}{Y - Y_n - \nu} \right) \]  

(37)

where \( N_n \) is the number of vortices per span.
After substituting equations (34), (35), (36), and (37) into equation (33) and letting \( N = N_i \times N_n \),

\[
D_i = \frac{\rho V_\infty^2}{4\pi} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_\infty K} \right) \left( \frac{\Gamma}{V_\infty j} \right) \left( \frac{1}{Y_{jK} + Y_\nu} - \frac{1}{Y_{jK} - Y_\nu} \right) \Delta Y_j \tag{38}
\]

Since \( \Delta Y_j = b/2 \Delta \eta_j \),

\[
C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_\infty K} \right) \left( \frac{\Gamma}{V_\infty j} \right) \left( \frac{1}{Y_{jK} + Y_\nu} - \frac{1}{Y_{jK} - Y_\nu} \right) \Delta \eta_j \tag{39}
\]

which is the same as equation (31).

The far field calculation of induced drag for a complete configuration composed of lifting bodies and thick lifting panels is done by representing the wake, from all of the bodies and panels, by an equivalent horseshoe vortex system where the bound segment of the horseshoe vortex is tangent to the trace of the wake in the Trefftz plane and the trailing legs are in the free stream direction. The section drag associated with the equivalent system, which in general is not equal to the actual configuration section induced drag is given by;

\[
\overrightarrow{d}_j = \rho \overrightarrow{w}_j \times \overrightarrow{\Gamma}_j \Delta S_j \tag{40}
\]

where

\[
\overrightarrow{w}_j = V_j \hat{j} + W_j \hat{k} \tag{41}
\]

and

\[
\overrightarrow{\Gamma}_j = \frac{\Gamma_j}{T_{Y_j}} \left( T_{Y_j} \hat{j} + T_{Z_j} \hat{k} \right) \tag{42}
\]

where \( T_{Y_j} \) and \( T_{Z_j} \) are components of the unit vector tangent to the trace of the wake at the \( j \)th section.
\[ V_j = \sum_{k=1}^{N} \frac{K_k}{4\pi} \left( E_{V,jk} T_{Y,k} + E_{W,jk} N_{Y,k} \right) \]  

(43)

\[ W_j = \sum_{k=1}^{N} \frac{K_k}{4\pi} \left( E_{V,jk} T_{Z,k} + E_{W,jk} N_{Y,k} \right) \]  

(44)

where \( T_{Y,k} \) and \( T_{Z,k} \) are the components of the unit vector tangent to the trace of the wake at \( \text{th} \) station. Also, \( N_{Y,k} \) and \( N_{Z,k} \) are components of the unit vector normal to the trace of the wake at the \( \text{th} \) section. \( N \) is the total number of sections along the trace of the wake for the complete configuration.

\[ \frac{\overline{Z}_{jk}}{\overline{Z}_{jk}^2 + (\overline{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} - \frac{\overline{Z}_{jk}}{\overline{Z}_{jk}^2 + (\overline{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} \]  

(45)

\[ \frac{\overline{Y}_{jk} - \frac{1}{2} \Delta S_k}{\overline{Z}_{jk}^2 + (\overline{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} - \frac{\overline{Y}_{jk} + \frac{1}{2} \Delta S_k}{\overline{Z}_{jk}^2 + (\overline{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} \]  

(46)

where

\[ \overline{Y}_{jk} = (Y_k - Y_j) N_{Z,k} - (Z_k - Z_j) N_{Y,k} \]  

(47)

\[ \overline{Z}_{jk} = -(Y_k - Y_j) T_{Z,k} + (Z_k - Z_j) T_{Y,k} \]  

(48)

and the indices \( j \) and \( k \) refer to the section being influenced and the influencing section, respectively.
therefore;
\[ d_j = \rho(V_j \Gamma_j T_{\gamma_j} - W_j \Gamma_j T_{\nu_j}) \Delta S_j \]  

(49)

the total configuration induced drag is then given by;
\[ D_i = \frac{\rho}{4\pi} \sum_{j=1}^{N} \sum_{k=1}^{N} \Gamma_j \Gamma_k \left( (E_{V_j k} T_{\gamma_k} + E_{W_j k} N_{\nu_k}) T_{Z_j} \right) - \left( E_{V_j k} Z_k = E_{W_j k} N_{\nu_k} ) T_{Y_j} \right) \Delta S_j \]  

(50)

therefore;
\[ C_{D_1} = \frac{AR}{2mb^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{\Gamma_j}{V_{\infty j}} \right) \left( \frac{\Gamma_k}{V_{\infty k}} \right) \left( E_{W_j k} \right) \Delta S_j \]  

(51)

NOTE: For the planar wing equation (51) reduces to
\[ C_{D_1} = \frac{AR}{2mb^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{\Gamma_j}{V_{\infty j}} \right) \left( \frac{\Gamma_k}{V_{\infty k}} \right) \left( \frac{1}{Y_{\nu_j} + Y_{\nu_k}} - \frac{1}{Y_{\nu_j} - Y_{\nu_k}} \right) \Delta \eta_j \]  

(52)

which is the same as equations (31) and (30). This demonstrates that the same induced drag is obtained whether the equality of work and kinetic-energy increment, the equivalent far field horseshoe system, or the near field horseshoe vortices with the Kutta-Joukowsky theorem is used.
APPENDIX G

COMPUTER PROGRAM LISTING
PROGRAM DERIV (INPUT=201, OUTPUT = TAPE5=INPUT, TAPE6=OUTPUT, 0 0010
1 TAPE18, TAPE19, TAPE20=1002, TAPE21=1002, TAPE23, 0 0020
2 TAPE24, TAPE10=1002, TAPE11, TAPE12 }
COMMON DA(5100)
COMMON/PANATT/DUM1(230)
COMMON/NUMBER/DUM2(135)
COMMON/PANEL/DUM3(34)
COMMON/BODY/DUM4(31550)
COMMON/CONPTS/DUM5(7920)
COMMON/SCRT/DUM7(25000)
COMMON/INDEX/DUM10(7)
COMMON/PANINF/PANSYM(10), DUM11(600), PANREF(10), PCHORD(10)
COMMON/CONTRV/DUM13(240)
COMMON/SLOPE/DUM14(2000)
COMMON/COMPR/P/BETAM
CALL FTNBIN(1,0,0)
DO 1 I=1,5100
DA(I)=0.0
OVL=3HOVL
CALL ATTACH(DUM1(1),DUM1(2),DUM1(3),DUM1(4),123)
CALL OVERLAY(OVL,1,0)
CALL OVERLAY(OVL,2,0)
CALL OVERLAY(OVL,3,0)
CALL OVERLAY(OVL,4,0)
CALL OVERLAY(OVL,5,0)
CALL OVERLAY(OVL,6,0)
CALL OVERLAY(OVL,7,0)
1 END
SUBROUTINE ATTACH(XV,YV,ZV,NR,KODE)
  C FOR KODE = -1, THIS SUBROUTINE IS SETTING XATT FROM NOXY INPUTS.
  C FOR KODE = 0, THIS SUBROUTINE IS SETTING XATT FROM PANEL INPUTS.
  C FOR KODE = 1, THIS SUBROUTINE SETS XV,YV,ZV FROM XATT ARRAY.
  DIMENSION XV(NR,1), YV(NR,1), ZV(NR,1)
  COMMON DA(5600)
  1 NX,NXTH,LNVR,LTVOR,NTVV,NRNV,NTV,NTXTH,NRV,NTH(49)
  2 LNDIV,LTDIV,LMPTS,LTPTS
  COMMON/PANATT, NATT(33), XATT(200)
  COMMON/NUMBER, NPPTS(7), NCPTS(7), NLN(7), VLT(7), LTC(7), LNC(7)
  1 NC, NB, NBODS, NPANS, NVL(7), "VT(7), "TAPE, NTAPE, NTV, ITAPE 
  2 LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)
  3 LNDIVB(7), LTDIVB(7), NSPP(7), ROO'T(7), OUTEP(7), SY""(7)
  COMMON/PANEL, ISP, IPSY, INC, NVVP, NTVVP, LNCFP, LTVFP, LNTFP, LNPFP
  1 NPERPT, NSPACE, NATTCH, NTRATT, MP, PRCLN, MPCLT, NCTXC, NCTET, NTHXC
  2 NTHT, NTIP, CHTIP, ROOT, OUTEP, NATT
  3 NP1, MP2, NP3, MP4, NP5, MP6, MP7, MP8, MP9, MP10
  EQUIVALENCE (DA(12), PANS)
  DATA L/0/,L1/0/,L2/0/
  IF(KODE.EQ.123) RETURN
  NPANS=PANS
  NB1=NBVV+1
  IF(KODE) 1,2,200
  1 NSA=NB
  GO TO 3
  2 NSA=NPANS+NLND
  CONTINUE
  DO 100 I=1, NPANS
  11=(I-1)*3+2
  IF(NATT(I).EQ.NSA) GO TO 5
  GO TO 100
  5 IZ=NATT(I1+1)
  GO TO 70
  10 L=L+1
  XATT(L)=-NATT(I)
  XATT(L+1)=I2
  L=L+2
  XATT(L)=NB1
  NA=(I2-1)*LTDIV+1
305 FORMAT (35HCH) ATTACHMENT LINE FOUND FOR PANEL 12 IN SUB ATTACH
RETURN
END
SUBROUTINE DECOD DATA IUNIT
DIMENSION DATA(1)ADATA(5)IDATA(17)IIDATA(8)
DATA IBLANK/10H/
15 READ(IUNIT,16)IIDATA
16 FORMAT(810)
C
IF (EOF(IUNIT).EQ.0) GOTO 19
IF (EOF(IUNIT) 199 19
19 CONTINUE
IUNIT = -IUNIT
RETURN
19 DECODE(72 I7 IIDATA)IADDADATA
17 FORMAT(1125G120)
DECODE(80 I18 IIDATA)IDATA
18 FORMAT(12X17A4)
J = IADD
IF (IADD) 2240 24
22 J = -J
24 DO 30 I = 1, 5
L = 3*I
K = L - 2
DO 26 M = K L
26 CONTINUE
IF (IIDATA(M) IBLANK) 28 26 28
28 CONTINUE
GO TO 30
28 DATA(J) = ADATA(I)
30 J = J + 1
IF (IADD) 100 40 15
40 WRITE(6501 IADD IIDATA
50 FORMAT(17HODECRD ER CARD=(111217A42H))
CALL EXIT
100 RETURN
END

SUBROUTINE CODES (XI,YI,H,T,ANS,NA)

C
C*****
A CONTROLLED DEVIATION INTERPOLATION METHOD
C
DIMENSION XI(1), YI(1), H(1), ANS(1)
C
XK=1.0
N=NI
DO 910 IE=1,NA
X=T(IE)
100 IF(N-2)110,120,200
110 Y = YI(N)
GO TO 900
120 Y = (YI(2)-YI(1))/(XI(2)-XI(1))*(X-XI(1))+YI(1)
GO TO 900
200 J = 1
210 IF(XI(J)-X)230,220,250
220 Y = YI(J)
GO TO 900
230 J = J+1
IF(J=N)210,210,250
250 IF(J-2)120,155,260
155 J = 3
JJ = 1
GO TO 285
260 IF(J-N)280,265,270
265 J = N-1
JJ = 2
GO TO 285
270 Y = (YI(N)-YI(N-1))/(XI(N)-XI(N-1))*(X-XI(N-1))+YI(N-1)
GO TO 900
280 JJ = 3
285 IF(N-3)290,290,295
290 J = 3
295 K = J-1
M = K-1
L = J+1
A1 = X-XI(M)

```
A2 = X-XI(K)
A3 = X-XI(J)
AL = (X-XI(K))/(XI(J)-XI(K))
S = AL*YI(J)+(1.0-AL)*YI(K)
C1 = A3*A2/((XI(M)-XI(K))*(XI(M)-XI(J)))
C2 = A1*A3/((XI(K)-XI(M))*(XI(K)-XI(J)))
C3 = A2*A1/((XI(J)-XI(K))*(XI(J)-XI(K)))
P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)
IF(N-3)305*305*310
305 P2 = P1
GO TO 315
310 A4 = X-XI(L)
C4 = A4*A3/((XI(K)-XI(J))*(XI(K)-XI(L)))
C5 = A2*A4/((XI(J)-XI(K))*(XI(J)-XI(L)))
C6 = A3*A2/((XI(L)-XI(K))*(XI(L)-XI(J)))
P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)
315 GO TO (320*330*350)*JJ
320 P2 = P1
AL = (X-XI(1))/(XI(2)-XI(1))
S = AL*YI(2)+(1.0-AL)*YI(1)
P1 = S+XK*(P2-S)
GO TO 350
330 P1 = P2
AL = (X-XI(N-1))/(XI(N)-XI(N-1))
S = AL*YI(N)+(1.0-AL)*YI(N-1)
P2 = S+XK*(P1-S)
350 E1 = ABS(P1-S)
E2 = ABS(P2-S)
IF(E1+E2)400*400*410
400 Y = S
GO TO 900
410 BT = (E1*AL)/(E1*AL+(1.0-AL)*E2)
Y = BT*P2+(1.0-BT)*P1
900 ANS(IE)=Y
910 CONTINUE
RETURN
END
```
FUNCTION COSD(X)
Y=0.017453293*X
COSD=COS(Y)
RETURN
END
SUBROUTINE DATAWR(DA)
DIMENSION DA(1)
DO 200 I=1,1000
ID=(I-1)*5
LDA=0
DO 100 J=1,5
IJ=ID+J
IF(DA(IJ)*NE.0.0) LDATA=1
CONTINUE
IF(LDATA.EQ.0) GO TO 200
IDL=ID+1
WRITE(6,150) IDL,IJ,DA(K),K=IDL,IJ
150 FORMAT(5HO,DA(*I4,*13H) THRU DA(*I4,*4H)=1PE15.6)
200 CONTINUE
WRITE(6,250)
250 FORMAT(52HO,ALL VALUES OF DA NOT GIVEN ABOVE ARE EQUAL TO ZERO.)
RETURN
END

SUBROUTINE NORM(X,Y,Z)
D=SQRT(X**2 + Y**2 + Z**2)
X=X/D
Y=Y/D
Z=Z/D
RETURN
END
FUNCTION DOT(X,Y,Z1,X2,Y2,Z2)
D0T=X1*Y2+Y1*Z2+Z1*X2
RETURN
END

FUNCTION ARCOS(X)
ARCOS=ACOS(X)
RETURN
END

FUNCTION COTAN4(X)
COTAN=cos(X)/sin(X)
RETURN
END
FUNCTION CODIM1 (X*I, Y*I, N, XK)

CALLING SEQUENCE...

X  INDEPENDENT VARIABLE...ABSCISSA REQUESTED
XI ARRAY OF GIVEN ABSCISSAS
YI ARRAY OF GIVEN ORDINATES
N NUMBER OF GIVEN POINTS DESCRIBING THE CURVE
XK END INTERVAL INTERPOLATION CONTROL CONSTANT

XK=0  STRAIGHT LINE INTERPOLATION
XK=+1  FULL PARABOLIC INTERPOLATION
XK BETWEEN FUNCTION THAT LIES BETWEEN A STRAIGHT
0 AND 1 LINE AND A PARABOLIC INTERPOLATION
XK=-1  PROGRAM WILL COMPUTE END INTERPOLATION CONTROL CONSTANT

DIMENSION XI(1), YI(1)

W = X
N1 = N

DETERMINE THE NUMBER OF POINTS GIVEN
IF (N1-2)100,200,300

ONE POINT GIVEN
100 IF (XI(1)-W)130,175,130
130 WRITE ( 6,150)
150 FORMAT (99H- ONLY ONE POINT WAS GIVEN FOR ARRAY XI IN CODIM1....THE
1E ABSCISSA ARGUMENT IS NOT THE SAME AS THE /16HO GIVEN ABSCISSA)
165 CALL DUMP

175 CODIM1 = YI(1)
GO TO 1700

TWO POINT STRAIGHT LINE COMPUTATION
200 N1 = 2

TEST IF ABSCISSAS ARE IDENTICAL
225 IF (XI(N1-1)-XI(N1))250,275,250
STRAIGHT LINE COMPUTATION

250 CODIM1 = (Y1(1)-Y1(1-1))/(X1(1)-X1(1-1))*(-X1(1-1)+Y1(1-1))
GO TO 1700

ERROR...ASCIGNAS ARE IDENTICAL
275 WRITE ( 6,325)X1(N1)
285 FORMAT (90H- A STRAIGHT LINE COMPUTATION IN CODIM1 IS ATTEMPTED
1 BUT THE TWO ASCIGNAS ARE IDENTICAL /28/0 THE ASCIGNA VALUE = 2817.8)
GO TO 165

THERE ARE MORE THAN TWO POINTS...TEST IF THEY ARE INCREASING OR
DECREASING ALGEBRACALLY
300 IF (X1(1)-X1(2))A400*325*61
C
C XI(1) EQUALS XI(2)...ERROR
325 WRITE ( 6,335)X1(1)
350 FORMAT (89H- THE FIRST TWO ASCIGNAS IN ARRAY XI IN CODIM ARE THE
1E SAME...THIS IS A ERROR... ASC =E17.8)
GO TO 165

C ASCIGNA VALUES ARE INCREASING IN VALUE ALGEBRACALLY
C FIND NEXT LARGEST ASCIGNA AFTER XI
400 GO 450 J=1,N1
410 IF (W-X1(J))A800*475*450
450 CONTINUE
C 470 IF W IS GREATER THAN XI(1)
GO TO 225

W IS EQUAL TO XI(J)
C CHECK IF XI(J) IS LAST GIVEN ASCIGNA
475 IF (U-J1)A500*575*575
C CHECK IF XI(J) = XI(J+1) IF SO THE FUNCTION IS NOT SINGLE VALUED
C AND THE ORGINATE OF XI(J) IS RETURNED AND A STATEMENT IS PRINTED
500 IF (XI(J)-XI(J+1))A75*525*575
525 WRITE ( 6,550)XI(J)
550 FORMAT (43H- THERE IS MORE THAN ONE ASCIGNA EQUAL TO E17.8/77H0
THE ORGINATE VALUE OF THE FIRST SUCH ASCIGNA HAS BEEN RETURNED.
2 CODIM1.
)
575 CODIM1 = Y1(J)
GO TO 1700

C  ABSCISSAS ARE DECREASING IN VALUE ALGEBRAICALLY
C  FIND NEXT SMALLEST ABSCISSA AFTER W
600 DO 650 J=1,N1
   IF (XI(J)-W)800,475,650
500 CONTINUE
C  W IS LESS THAN XI(N)
   GO TO 225
650 CONTINUE

C  TEST IF W LIES BETWEEN XI(1) AND XI(2)
800 IF (J-2)200,825,850
C  W LIES BETWEEN XI(1) AND XI(2)
825 J = 3
   JJ = 1
   GO TO 925

C  W OCCURS AFTER XI(2).....TEST IF W IS BETWEEN XI(N-1) AND XI(N)
850 IF (J-N1)900,875,225
C  W LIES BETWEEN XI(N-1) AND XI(N)
875 J = N1-1
   JJ = 2
   GO TO 925
C  W LIES BETWEEN XI(2) AND XI(N-1)
900 JJ = 3
C  SETUP SUBSCRIPTS
925 K = J-1
   M = K-1
   L = J+1
XIM = XI(M)
XIK = XI(K)
XIJ = XI(J)
XIL = XI(L)
C  TEST IF N=3.....IF SO ALTER M AND JT SO THAT DO LOOPS 1040 AND
C 1120 TEST ONLY 3 POINTS VICE 4
   IF (XI-3)970,940,970
940 IF (JJ-2)970,940,970
950 JT = 2
   GO TO 1000
960 M = 1
970 JT = J
C C TEST IF ABSCISSA VALUES ARE INCREASING OR DECREASING ALGEBRAICALLY
1000 IF (XI(1)-XI(2))1920,325,1100
C C TEST IF ABSCISSA VALUES ARE ALL INCREASING ALGEBRAICALLY
1020 DO 1040 IB=M*JT
   IF (XI(IB)-XI(IB+1))1040,1060,1060
1040 CONTINUE
   GO TO 1175
1160 IB1 = IB+1
   WRITE (6,1070)IB,XI(IB),IB1,XI(IB1)
1070 FORMAT (7F-10.6)
   1/DECREASING ALGEBRAICALLY / 10H0 ABSCISSA17.2H =E17.6H*E17.8H
   2SA17.2H =E17.8H*E17.8H
   GO TO 165
C C TEST IF ABSCISSA VALUES ARE ALL DECREASING ALGEBRAICALLY
1100 DO 1120 IB=M*JT
   IF (XI(IB)-XI(IB+1))1120,1060,1120
1120 CONTINUE
C 1175 A1 = W-XI M
   A2 = W-XI K
   A3 = W-XI J
   A4 = W-XI L
   AL = (W-XIK)/(XIJ-XIK)
   C1 = A3*A2/((XI'M-XIK)*(XIM-XIJ))
   C2 = A1*A3/((XIK-XIM)*(XIK-XIJ))
   C3 = A2*A1/((XI-J-XIM)*(XI-J-XIK))
   C4 = A4*A3/((XIK-XIJ)*(XIK-XIL))
   C5 = A2*A4/((XIJ-XIK)*(XIJ-XIL))
   C6 = A3*A2/((XIL-XIK)*(XIL-XIJ))
S = AL*YI(J)+(1-AL)*YI(K)
P1 = C1*YI(J)+C2*YI(K)+C3*YI(J)
P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)
GO TO (1200,1400,1500),JJ

1200 P2 = P1
IF (XK)1230,1220,1220
1220 XE = XK
GO TO 1260

1230 SLOPE1 = ABS((YI(K)-YI(J))/(XIX-XI))
SLOPE2 = ABS((YI(J)-YI(J))/(XIX-XI))
XE = 1-(ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))

1260 AL = (W-XI1)/(XI2-XI1)
S = AL*YI(2)+(1-AL)*YI(1)
P1 = S+XE*(P2-S)
GO TO 1500

1400 P1 = P2
IF (XK)1430,1420,1420
1420 XE = XK
GO TO 1460

1430 SLOPE1 = ABS((YI(J)-YI(L))/(XIJ-XIL))
SLOPE2 = ABS((YI(J)-YI(J))/(XIJ-XI))
XE = 1-(ABS(SLOPE1-SLOPE2)/(SLOPE1+SLOPE2))

1460 AL = (W-XIJ)/(XIL-XIJ)
S = AL*YI(L)+(1-AL)*YI(J)
P2 = S+XE*(P1-S)

1500 E1 = ABS(P1-S)
E2 = ABS(P2-S)
IF (E1+E2)1530,1530,1560
1530 CODIM1 = S
GO TO 1700
PROGRAM BODYG
COMMON DA(5000)
1 NX,NXTH,LNVOR,LTVOR,NTVV,NBV,NV,NXTHV,NBV,NTH(49).
2 LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 NCT,NB,NBODS,NPANS,NVL(7),NVT(7),MTAPE,MTAPE,NCTV,ITAPE,ITAPE
2 LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)
EQUIVALENCE(DA(1),BODIES),(DA(15),BODYNO)
EQUIVALENCE(DA(10),ALPHA),(DA(11),BETA)
COMMON /COMPRS/BETAM
EQUIVALENCE(DA(3),FMACH)

C***************************************************************
C***************************************************************
COMMON/PANATT,NATT(30),XATT(200)
COMMON/SCRAT/ ATACH(50)
CALL NARDAP
REIND 21
READ(21) ATACH
DO 5 I=1,10
K=(I-1)*5
J=(I-1)*3
NATT(J+1)=ATACH(K+1)
NATT(J+2)=ATACH(K+2)
5 NATT(J+3)=ATACH(K+3)
WRITE(6,205) NATT
205 FORMAT(*ONATT*/(3I5))
C***************************************************************
C***************************************************************
DO 1 I=1,14
1 DA(1)=0.0
MTAPE=19
NTAPE=20
C UNIT 18 - VORTEX COORDINATES
REIND 18
C UNITS 19,20 - AX,AY,AZ MATRICES (COMPUTED IN SUBROUTINE INFL)
REIND 19
REIND 20
C UNIT 21 - COMPLETE INFLUENCE MATRIX *A* (COMPUTED IN SUB. MATA) 1 0.00
REWRIND 21 1 0.00
C UNIT 23 - PHITHEGA MATRIX.
REWRIND 23 1 0.00
C REWRIND UNIT 12. UNIT 12 WILL HAVE DATA USED FOR FORCES.
REWRIND 12 1 0.00
CALL DECRD(DA) 1 0.00
NBODS=3DIES 1 0.40
ALPHA=ALPHA/57.29577951 1 0.50
BETA=BETA/57.29577951 1 0.60
BETAM=SQRT(1.0-FMACH**2) 1 0.70
IF(NBODS.EQ.0) GO TO 105 1 0.70
C NCT IS COUNTER FOR CONTROL POINTS. NCT IS INCREMENTED IN SUB. VPTS. 1 0.70
NCT=0 1 0.70
NCTV=0 1 0.70
KCON=0 1 0.70
KLT=0 1 0.70
DO 100 I=1,NBODS 1 0.70
DO 10 J=1,3419 1 0.70
10 DA(J)=0.0 1 0.70
CALL DECRD(DA) 1 0.70
NB=BODYNO 1 0.70
CALL SETDAT 1 0.70
CALL XYZVB 1 0.70
CALL MALT(KL) 1 0.70
CALL MATL(KLT) 1 0.70
CALL VPTS 1 0.70
CALL ATTACH(XV1,YV,ZV,151,-1) 1 0.70
NSEG=TSEG(I)+LSEG(I) 1 0.70
KCON=KCON+NSEG 1 0.70
IF(NSEG.EQ.0) GO TO 100 1 0.70
CALL MATPT 1 0.70
100 CONTINUE 1 0.70
C VORTEX POINTS (INCLUDING DIVISION PTS.) ARE ON UNIT 18. (ALL BODIES) 1 0.70
C UNIT 23 HAS PHITHEGA MATRIX FOR ALL BODIES. 1 0.70
C ALNTH HAS L FOR ALL BODIES 1 0.70
105 CONTINUE
END
SUBROUTINE MATPT

COMPUTE PHI*THETA CONSTRAINT TRANSFORMATION MATRIX

COMMON DA(5000)

1  *NX*NXTH*LNVOR*LTQV*NTV*NBV*NTV*NXTHV*NBV*NTH(49)
2  *INDEX*LTQIV*LNPTS*LTPT

EQUIVALENCE (DA(17)*CHORD)

1  *(DA(3140)*CSL(10)*CSL(20)*CSL(30)*CSL(40)*CSL(50))
2  *(DA(3140)*CSL(10)*CSL(20)*CSL(30)*CSL(40)*CSL(50))
3  *(DA(3140)*CSL(10)*CSL(20)*CSL(30)*CSL(40)*CSL(50))

COMMON/NUMBER/ NVPTS(7)*NCPTS(7)*NLN(7)*NLT(7)*LTC(7)*LNC(7)

1  *NCT*NB*NBODS*NPARS*NVL(7)*NVT(7)*MTAPE*NTAPE*NTAPE*NTAPE*NTAPE*NTAPE*NTAPE
2  *LSEG(7)*TSEG(7)*TSEG(7)*TSEG(7)*TSEG(7)*TSEG(7)*TSEG(7)
3  *LNDIVS(7)*LTDIVR(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)

COMMON/SCRT/ XVV(2000)*TVV(15131)

1  *XVOR(100)

DIMENSION CFLT(40)*CFLT(40)*CFLT(40)*CFLT(40)*CFLT(40)

DIMENSION PHTH(5000)

COMMON/BODY/ ARRAY(50000)

EQUIVALENCE (ARRAY(15000)*PHTH)

C DIVLDA IS NO. LONG DIV. GIVEN IN DATA.

EQUIVALENCE (DIVLDA*DA(321))

LNF=FUNCLN
LTF=FUNCLT
LNG=SEGLN
LTG=SEGLT
PI=3.1415926

C DETERMINE IF Y*Z AND NORMALS ARE REQUIRED FOR CONSTRAINT FUNCTIONS.
C SET KODE=1 IF THEY ARE REQUIRED.
KODE=0
DO 1 I=1,2
IF(CFLT(I)*NE.2) GO TO 1
KODE=1
DO TO 3
GO TO 3
1 CONTINUE
DO 2 I=1,3
IF(CFLT(I)*NE.2*AND.CFLT(I)*NE.3) GO TO 2
KODE=1
GO TO 3
2 CONTINUE
1 CONTINUE
1 0810
1 0820
1 0830
1 0840
1 0850
1 0860
1 0870
1 0880
1 0890
1 0900
1 0910
1 0920
1 0930
1 0940
1 0950
1 0960
1 0970
1 0980
1 0990
1 1000
1 1010
1 1020
1 1030
1 1040
1 1050
1 1060
1 1070
1 1080
1 1090
1 1100
1 1110
1 1120
1 1130
1 1140
1 1150
1 1160
1 1170
1 1180
1 1190
2 CONTINUE
3 CONTINUE
10 IF(LIFUNC)=10+10+11
11 JTLIM=NTVV
12 GO TO 12
13 JTLIM=LIFUNC
14 IF(LIFUNC)=13+13+14
15 JNLIM=NBVV
16 GO TO 15
17 CONTINUE
18 KFUNC=0
19 IF(LIFUNC .NE. 0) GO TO 25
20 IF(LIFUNC .EQ. 0) GO TO 25
21 KFUNC=1
22 JTS=JTLIM
23 JTLIM=JNLIM
24 JNLIM=JTS
25 CONTINUE
26 ITAPE=0
27 DO 5000 JTT=1,JTLIM
28 JT=JTT
29 IF(KFUNC .NE. 1) GO TO 26
30 JN=JTT
31 DO 5000 JNN=1,JNLIM
32 ITAPE=ITAPE+1
33 IF(KFUNC .EQ. 0) GO TO 261
34 JT=JNN
35 GO TO 27
36 JN=JNN
37 IF(LIFUNC .EQ. 0) GO TO 28
38 LTFCT=CFLT(JT)
39 IF(LIFUNC .EQ. 0) GO TO 29
40 LNCF=CFLN(JN)
41 CONTINUE
42 KC=0
43 DO 1000 J=1,NTVV
44 DO 1000 I=1,NBVV
45 KC=KC+1
46 1 1200
47 1 1210
48 1 1220
49 1 1230
50 1 1240
51 1 1250
52 1 1260
53 1 1270
54 1 1280
55 1 1290
56 1 1300
57 1 1310
58 1 1320
59 1 1330
60 1 1340
61 1 1350
62 1 1360
63 1 1370
64 1 1380
65 1 1390
66 1 1400
67 1 1410
68 1 1420
69 1 1430
70 1 1440
71 1 1450
72 1 1460
73 1 1470
74 1 1480
75 1 1490
76 1 1500
77 1 1510
78 1 1520
79 1 1530
80 1 1540
81 1 1550
82 1 1560
83 1 1570
84 1 1580
IF(LNCF.EQ.1) GO TO 75
IF(LNCF.EQ.0) GO TO 75
DO 50 II=1,LNSEG
LNCS=CSLN(II)
IF(II.LE.LNCS) GO TO 55
50 CONTINUE
55 IF(II.NE.1) GO TO 60
X0=X$1)-XBO
58 PHI0=ARCOS(1.0-2.0*X0/CHORD)
GO TO 65
60 II1=CSLN(II-1)
IX1=II1
IX2=IX1+1
X0=0.75*XVOR(IX2)-0.25*XVOR(IX1)
X0=X0-XBO
GO TO 58
65 II1=CSLN(II)
IF(II1.LT.NBVV) GO TO 66
XF=CHORD
GO TO 68
66 IX1=II1
IX2=IX1+1
XF=0.75*XVOR(IX2)-0.25*XVOR(IX1)
XF=XF-XBO
IF(XF.GT.CHORD) XF=CHORD
68 PHIF=ARCOS(1.0-2.0*XF/CHORD)
X=XVOR(1)
X=X-XBO
PHI=ARCOS(1.0-2.0*X/CHORD)
PRAT=P1*(PHI-PHI0)/(PHIF-PHI0)
75 IF(LTCF.EQ.1) GO TO 85
IF(LTCF.EQ.0) GO TO 85
DO 80 JJ=1,LTSEG
LTCS=CSLT(JJ)
IF(J.LE.LTCS) GO TO 81
80 CONTINUE
81 JX1=(J-1)*LTDIV+1
JX2=J*LTDIV+1
II=(I-1)*DIVLDA+1
THETA=0.5*(THV(I1,JX1)+THV(I1,JX2)) 1 1980
IF(JJ*NE.1) GO TO 82
THETO=THV(I1) 1 1990
GO TO 83
82 JX1=(CSTT(JJ-1))*LTDIV+1.01 1 2000
THETO=THV(I1,JX1) 1 2000
83 JX2=LTDV*LTDIV+1 1 2010
THETO=THV(I1,JX2) 1 2010
TRAT=P1*(THETA-THETO)/(THEO-THETO) 1 2020
85 CONTINUE 1 2030
IF(KODE*EQ.0) GO TO 100 1 2040
CALL VYZN(J,I,Y,Z,ANORM) 1 2050
100 CONTINUE 1 2060
LATF=1 1 2070
LONF=1 1 2080
C IF LATF=0, DO NOT COMPUTE GT* 1 2090
C IF LONF=0, DO NOT COMPUTE GN* 1 2100
IF(LIFUNC*NE.0) GO TO 105 1 2110
LATF=0 1 2120
IF(JT*NE.J) GO TO 30 1 2130
GT=1.0 1 2140
GO TO 105 1 2150
30 GT=0.0 1 2160
LONF=0 1 2170
105 CONTINUE 1 2180
IF(LIFUNC*NE.0) GO TO 106 1 2190
LONF=0 1 2200
IF(JN*NE.1) GO TO 251 1 2210
GN=1.0 1 2220
GO TO 106 1 2230
251 GN=0.0 1 2240
LATF=0 1 2250
106 CONTINUE 1 2260
IF(LATF*EQ.O) GO TO 300 1 2270
GO TO (110*120*130*140*150*160*170*180*190*200,LTCF 1 2280
110 GT=1.0 1 2290
GO TO 300 1 2300
120 GT=ANORM(3)/SQRT(ANORM(3)**2+ANORM(2)**2) 1 2310
GO TO 300 1 2320
120  GT=ANORM(1)/SQR(ANORM(1)**2*ANORM(2)**2)
    GO TO 300
140  GT=SIN(TRAT)
    GO TO 300
150  GT=COS(TRAT)
    GO TO 300
160  GT=TRAT
    GO TO 300
170  GT=SIN(2*O*TRAT)
    GO TO 300
180  GT=COS(2*O*TRAT)
    GO TO 300
190  GT=TRAT**2
    GO TO 300
200  GT=O*O
    GO TO 500
300  CONTINUE
    IF(LONF.EQ.0) GO TO 500
    GO TO (310,320,330,340,350,360,370,380,390,400
    1  *403,404,405,406,407,408,409,410,411,412,413,414,415)*LNCF
310  GN=1.0
    GO TO 500
320  T1=Y*ANORM(1)/ANORM(2)+Z*ANORM(1)/ANORM(3)
    GN=1.0/SQRT(1.0+(T1**2)/(Y**2+Z**2))
    GO TO 500
330  GN=COTAN(0.5*PHI)
    GO TO 500
340  GN=COTAN(0.5*(PI-PHI))
    GO TO 500
350  GN=SIN(PRAT)
    GO TO 500
360  GN=COS(PRAT)
    GO TO 500
370  GN=(X-XO)/(XF-XO)
    GO TO 500
380  GN=SIN(2.0*PRAT)
    GO TO 500
390  GN=COS(2.0*PRAT)
    GO TO 500
400  GN=((X-XO)/(XF-XO))**2
SUBROUTINE VPTS
COMMON DA(5000)
1. *NX=NXTH; LTVOR=LTV; NTVM=NEV; NT=MTV; NHV=NETH(49)
2. *LNW = LTV = LNPTS = LTPTS
COMMON / NUMBER/ NTVM(7); NLNI(7); NLN(7); NL(7); LT(7); LTVC(7); LNC(7); 
. *NC1, *NG00; NN = 00; NV1(7); NV1; NTV1; NTV = NT1; NT2; NTAP; NTV = NTAP; NTAP(7); NTAP1(7)
2. *LSG(7); TSEG(7); FNFC(7); FNC(7);
3. *LNR = LNV = LV = LVTS = TSQ; NSP(7); R00PT(7); OQRT(7); SYMM(7)
DIMENSION XAREA(5000); YAREA(5000); ZAREA(5000)
1. * XPC(5000); YPC(5000); ZPC(5000); XTL(100)
EQUIVALENCE; XAREA*XPC.D(15001); YAREA*YPC.D(20001)
1. * (ZAREA*ZPC.D(25001)); (XTL.D(14100))
2. * (CHORD=DA(17)); (BREF=DA(16))
2. * (DA(7); XCGY; DA(13); YCGY; DA(2); ZCGY; DA(2); ALPHA)
COMMON; BODY/XV(151); YV(151); ZV(151)
1. * TMX(1320); TMY(1320); TMZ(1320)
2. * TTX(1320); TTY(1320); TTV(1320)
4. * DUMB(5000)
COMMON; CONPTS/X0(1320); X0(1320); Z0(1320)
1. * XN(1320); YN(1320); ZN(1320)
DIMENSION D(31000)
EQUIVALENCE; D*XY(1)
EQUIVALENCE (DA(2800); FLNC(1); DA(3100); FLTC)
DIMENSION FLNC(150); FLTC(40)
COMMON; SCRAT/XV(200); DMNV(4681); XVOR(100); XCON(100)
DIMENSION T1(300); T2(300); T3(300)
1. * T1(300); T2(300); T3(300)
EQUIVALENCE (XN(1); T1); (XN(301); T2); (XN(601); T3)
1. * (YN(1); T1); (YN(301); T2); (YN(601); T3)
EQUIVALENCE (BODYWR1; XV(1, 1))
DIMENSION BODYWR(31000)
31000 WORDS OF /BODY/ WILL BE WRITTEN ON UNIT 18.

IF (FLNC(1); NE=0.0) GO TO 2
DO 1 I=1, LNPTS
1 FLNC(I)=I
2 IF (FLTC(I); NE=0.0) GO TO 4
DO 3 J=1, LTPTS
3 FLTC(J)=J
CONTINUE
NB1=NBVV+1
SUBAX=0.0
SUBAY=0.0
SUBAZ=0.0
K=0
DO 50 J=1,NTV
J1=J+1
DO 50 I=1,NBV
I1=I+1
K=K+1
X31=XV(I1,J1)-XV(I,J)
Y31=YV(I1,J1)-YV(I,J)
Z31=ZV(I1,J1)-ZV(I,J)
X24=XV(I,J1)-XV(I1,J)
Y24=YV(I,J1)-YV(I1,J)
Z24=ZV(I,J1)-ZV(I1,J)
XAREA(K) = (Y31*Z24-Z31*Y24)*0.5
YAREA(K) = (Z31*X24-X31*Z24)*0.5
ZAREA(K) = (X31*Y24-Y31*X24)*0.5
SUBAX=SUBAX+XAREA(K)
SUBAY=SUBAY+YAREA(K)
SUBAZ=SUBAZ+ZAREA(K)
WRITE(6,55) SUBAX,SUBAY,SUBAZ
55 FORMAT(1HO/(1P8E15.6))
DO 100 I=1,NBV
K1=(I-1)*LNDIV+1
K2=K1+LNDIV
100 XVOR(I)=0.75*XVV(K1)+0.25*XVV(K2)
WRITE(6,55) (XVOR(I),I=1,NBV)
N3=NBV+1
N2=NTV+1
DO 300 I=1,LNPTS
KC=FLNC(I)
KC=(KC-1)*LNDIV+1
300 XCON(I)= 0.25*XVV(KC)+0.75*XVV(KC+LNDIV)
C
C WRITE AREAS ON UNIT 12 FOR USE IN CALCULATING FORCES.
C
WRITE(12) XAREA, YAREA, ZAREA
K=0
DO 60 J=1,NTV
    J1=J+1
DO 60 I=1,NBV
    I1=I+1
    K=K+1
    XPC(K1)=0.25*(XV(I, J)+XV(I, J1)+XV(I1, J1))
    YPC(K1)=0.25*(YV(I, J)+YV(I, J1)+YV(I1, J1))
60    ZPC(K1)=0.25*(ZV(I, J)+ZV(I, J1)+ZV(I1, J1))
    DO 65 K=1,NBV
        K1=(K-1)*LNDIV+1
65    XTL(K)=XV(K1)
WRITE SUB-PANEL CENTERS AND X FOR PANEL EDGES ON UNIT 12 FOR FORCES.
ALSO WRITE XCOM=X COORDS FOR CONTROL POINTS AND OTHER NEEDED TERMS.
WRITE(12) XPC, YPC, ZPC, XTL, XCOM, FLNC, FLTC, NBV, NTV, LNPTS, LTPTS
WRITE(6551)(XCON(I), I=1, LNPTS)
NLAT1=LTDIV/2 + 1
JS=NC1
DO 500 J=1,LTPT.
    K1=LTDIV*(FLTC(J)-1.0)+NLAT1
    K2=K1+1
DO 400 I=1,NBV
    I2=I+1
    XQ(I1)=0.25*(XV(I2, K1)+XV(I1, K1)+XV(I2, K2)+XV(I1, K2))
    T11(I1)=XV(I2, K1)-XV(I1, K1)+XV(I2, K2)-XV(I1, K2)
    T21(I1)=YV(I2, K1)-YV(I1, K1)+YV(I2, K2)-YV(I1, K2)
400    T31(I1)=ZV(I2, K1)-ZV(I1, K1)+ZV(I2, K2)-ZV(I1, K2)
CALL INTER(XQ, T11, NBV, XCOM, T12, LNPTS)
CALL INTER(XQ, T21, NBV, XCOM, T22, LNPTS)
CALL INTER(XQ, T31, NBV, XCOM, T32, LNPTS)
DO 410 I=1, LNPTS
DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
T12(I)=T12(I)/DENOM
T22(I)=T22(I)/DENOM
T32(I)=T32(I)/DENOM
JS1=JS+1
TMX(JSI)=T12(I)
TMX(JSI)=T22(I)

410 TMZ(JSI)=T32(I)
DO 450 I=1*NBV
I2=I1+1
T11(I1)=XV(I1*K2)-XV(I1*K1)+XV(I2*K2)-XV(I2,K1)
T21(I1)=YV(I1*K2)-YV(I1*K1)+YV(I2*K2)-YV(I2,K1)
450 T31(I1)=ZV(I1*K2)-ZV(I1,K1)+ZV(I2,K2)-ZV(I2,K1)
CALL INTER(XQ,T11*NBV*XCON+T12*LNPTS)
CALL INTER(XQ,T21*NBV*XCON+T22*LNPTS)
CALL INTER(XQ,T31*NBV*XCON+T32*LNPTS)
DO 460 I=1*LNPTS
DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
T12(I)=T12(I)/DENOM
T22(I)=T22(I)/DENOM
T32(I)=T32(I)/DENOM
JSI=JS+1
TTX(JSI)=T12(I)
TTY(JSI)=T22(I)
460 TTZ(JSI)=T32(I)
JS=JS+LNPTS
500 CONTINUE
JS=NCT
DO 600 K=1*LTPTS
K1=LTDIV*(FLTC(K)-1.0)+NLAT1
K2=K1+1
CALL INTER(XV(1,K1)*YV(1,K1)*N3*XCON+T11*LNPTS)
CALL INTER(XV(1,K2)*YV(1,K2)*N3*XCON+T12*LNPTS)
CALL INTER(XV(1,K1)*ZV(1,K1)*N3*XCON+T21*LNPTS)
CALL INTER(XV(1,K2)*ZV(1,K2)*N3*XCON+T22*LNPTS)
DO 550 I=1*LNPTS
JS=JS+1
XQ(JSI)=XCON(I)
YQ(JSI)=0.5*(T11(I)+T12(I))
550 ZQ(JSI)=0.5*(T21(I)+T22(I))
500 CONTINUE
600 DO 250 J=1*N2
CALL INTER(XV(1,J)*YV(1,J)*N3*XVOR+T11*NBVV)
CALL INTER(XV(1,J)*ZV(1,J)*N3*XVOR+T12*NBVV)
DO 250 I=1,NBVV
YV(I,J)=T11(I)
ZV(I,J)=T12(I)
250 CONTINUE
DO 650 I=1,NBVV
DO 650 J=1,N2
650 XV(I,J)=XVOR(I)
DO 700 J=1,N2
XV(NB1+J)=XV+V(N3)
DO 700 J=1,N2
YV(NB1+J)=YV(N3+J)
700 ZV(NB1+J)=ZV(N3+J)
DO 800 J=1,LTPTS
DO 800 I=1,LNPTS
NCT=NCT+1
XN1=TMY(NCT)*TTZ(NCT)-TMZ(NCT)*TTY(NCT)
YN1=TMZ(NCT)*TTX(NCT)-TMX(NCT)*TTZ(NCT)
ZN1=TMX(NCT)*TTY(NCT)-TMY(NCT)*TTX(NCT)
DENOM=XN1**2+YN1**2+ZN1**2
XN(NCT)=XN1/DENOM
YN(NCT)=YN1/DENOM
ZN(NCT)=ZN1/DENOM
800 CONTINUE
WRITE(18) BODYWR
WRITE(6,55) (XQ(I),I=1,NCT)
WRITE(6,55) (YQ(I),I=1,NCT)
WRITE(6,55) (ZQ(I),I=1,NCT)
WRITE(6,55) (XN(I),I=1,NCT)
WRITE(6,55) (YN(I),I=1,NCT)
WRITE(6,55) (ZN(I),I=1,NCT)
C
RETURN
END
SUBROUTINE MLTT(K)

BLNGTH GIVE LATERAL LENGTHS OF PANELS

FIRST LONGITUDINALLY -- THEN LATERALLY

COMMON DA(5000)
1  *NX,NTH,LNVOR,LTOR,NTV,NBVV,NTV,NTH,V,NBV,NTH(49)
2  *LNDIV,LTDIV,LNPTS,LPTS
COMMON/BODY/ ARRAY(31000)
DIMENSION XV1(151,31),YV(151,31),ZV(151,31),BLNGTH(5000)
EQUIVALENCE(BLNGTH,ARRAY(15001)),(XV1,ARRAY(I1))
1  *(YV,ARRAY(4682)),(ZV,ARRAY(9363))

REAL L2
LN=LNDIV/2+1
DO 100 J=1,NTV
JT=(J-1)*LTDIV
DO 100 I=1,NBVV
II1=(I-1)*LNDIV+LN
II2=II1+1
L2=0.0
DO 50 JJ=1,LTDIV
JJ1=JT+JJ
JJ2=JJ1+1
X1=0.5*(XV1(II1,JJ1)+XV1(II2,JJ2))
X2=0.5*(XV1(II1,JJ2)+XV1(II2,JJ1))
DX=X2-X1
Y1=0.5*(YV(II1,JJ1)+YV(II2,JJ2))
Y2=0.5*(YV(II1,JJ2)+YV(II2,JJ1))
DY=Y2-Y1
Z1=0.5*(ZV(II1,JJ1)+ZV(II2,JJ2))
Z2=0.5*(ZV(II1,JJ2)+ZV(II2,JJ1))
DZ=Z2-Z1
50 L2=SQRT(DX**2+DY**2+DZ**2)+L2
K=K+1
100 BLNGTH(K)=L2
WRITE(6,200)(BLNGTH(I),I=1,K)
200 FORMAT(*OBLNGTH*/(1P8E15.6))
WRITE(18) BLNGTH
RETURN

344
SUBROUTINE MATL(K)
COMMON DA(5000)
1 *NX*NXTH*LNVOR*LTVOV*NTVV*NBVV*NTV*NXTH*NBV*NXTH(49)
2 *LNDIV*LTDIV*LNPST*LPTST
COMMON/NUMBER/ NVPTS(7)*NCPTS(7)*NLN(7)*LTC(7)*LNC(7)
1 *NCT*NB*NBODS*NPAVS*VLT(7)*NVIT(7)*TAPA*TAPENCTV*ITAPE*JTAPE
2 *LSSEG(7)*TSSEG(7)*LFUNC(7)*TFUNC(7)
3 *LNDIVB(7)*LTDIVB(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)
COMMON/BODY/ ARRAY(31000)
DIMENSION XV(151,31),YV(151,31),ZV(151,31),ALNGTH(5000)
EQUIVALENCE(ALNGTH,ARRAY(15001),(XV,ARRAY(1))
1 * (YV,ARRAY(4682),(ZV,ARRAY(9363))
1 DO 10 J=1,NTVV
  JJ1=(J-1)*LTDIV+LTDIV/2+1
  JJ2=JJ1+1
1 DO 10 I=1,NBVV
  K=K+1
  VL=0.0
1 DO 4 II=1,LNDIV
  II1=LNDIV*(I-1)+II
  II2=II1+1
  X1=0.5*(XV(II1,II1)+XV(II1,II2))
  X2=0.5*(XV(II2,II1)+XV(II2,II2))
  DX=X2-X1
  Y1=0.5*(YV(II1,II1)+YV(II1,II2))
  Y2=0.5*(YV(II2,II1)+YV(II2,II2))
  DY=Y2-Y1
  Z1=0.5*(ZV(II1,II1)+ZV(II1,II2))
  Z2=0.5*(ZV(II2,II1)+ZV(II2,II2))
  DZ=Z2-Z1
4 VL=SORT(DX**2+DY**2+DZ**2)+VL
10 ALNGTH(K)=VL
WRITE(6*Z200)(ALNGTH(I),I=1,K)
200 FORMAT(*,ALNGTH*(/1P8E15.6))
WRITE(18) ALNGTH
RETURN
END
SUBROUTINE XYZVB

COMMON DA(5000)
1 NXNXTHLNVORLTMOV1NTV1NBVVNTVNTXNVBNB1NTH(49)
COMMON/NUMB/NTVPS71NCPT51NLN1NLT7LTC7LNC7
1 +NCTNBNBODSNPANSNVL7NVT7XTAPEXTAPENTCTVITAPEJTAPE
2 +LSEG71TSEG71LFUNC71TFUNC7
3 + LNDIVB71LTDIVB71NSPP71ROTHP7OUTERP7SYMM7
EQUIVALENCD(A131FMACH)

COMMON/CMPS/BETAM

EQUIVALENCD(A171CHORD1)
1 *(DA20)BDTAB1*(DA19)SYM1*(DA2151)XTHV1*(DA131)TH
2 *(DA800)R1*(DA141)XS1*(DA806)XTH
3 *(DA1600)CPT51*(DA1601)XMF1*(DA1605)YMF1*(DA1700)ZMF
4 *(DA1750)YCAM1*(DA1800)ZCAM1*(DA2200)THYS1*(DA22)CAMBI

EQUIVALENCD(A211BMULT)

COMMON/CRAT/XVX2001THVV15131

DIMENSION THV131311THSS131311RS131311S13131
1 ST131311RR131311YCM2001ZCM200
2 YCP2001ZCP2001SINCY2001COSYC200
3 SINZC2001COSZC2001SLAT11001TH1100
4 XTHV11501TH11691R18001XS49
5 XTH491XMIF491YMIF491ZMF49
6 YCAM11501THV8001XV11511ZCAM50

DIMENSION D25001THET12001Y11501Z150
1 YM2001ZM2001YM2001ZC200
2 YY151311ZZ151311Y1151311Z115131
3 XM2001XP200

EQUIVALENCDARRAY)

EQUIVALENCD1THV1D14821THSS1D193631RS
1 *(D110324)1Y1*(D15905)1ST1*(D15966)1RR
2 *(D120671YCM1D120841ZCM1D121041YCP
3 *(D121471ZCP1D121471SINCY1D121471COSYC
4 *(D1218471SINZC1D1220471COSZC1D1222471SLAT1
5 *(D122471TH11D1225471THE11D1227471Y11
6 *(D1227971Z11D1228471YM1D1230471ZM1
7 *(D1232471Y11D1234471ZC1
8 *(D1236471XM1D1238471XP1

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6000
6010
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6090
COMMON/BOXY/ ARRAY(31000)
EQUIVALENCE (XV, ARRAY(146821), YV, ARRAY(1363))
1   = (YO0, ARRAY(26964)), (YBO0, ARRAY(26969))
2   = (ZBO0, ARRAY(26974))
DIMENSION XV(151, 31), YV(151, 31), ZV(151, 31), XBO0(5), YBO0(5)
1   = ZBO0(5)
COMMON/CONPTS/XQ(1320), YQ(1320), ZQ(1320)
EQUIVALENCE (YY(1, 1), THSS(1, 1), ZZ(1, 1), RS(1, 1))
1   = (Y(1, 1), YV(1, 1)), (Z(1, 1), ZV(1, 1))

C
MCPTS=CPTS
C TEST FOR (R, THETA) INPUT OR (Y, Z) INPUT
C
IF (BODTAB .NE. 0) GO TO 110
C R, THETA INPUT
C
C
C
SET UP THV ARRAY
C TEMPORARY CHANGE SET GRIDL=1.
C THIS IS USED FOR EVEN DELTA THETA VORTEX INPUT,
C AND ONLY ONE SET OF THETA'S CREATED IN SUBROUTINE SETDAT.
C
GRIDL=1.0
C
N1=NTVV+1
LOC=0
DO 65 J=1, NXTHV
DO 60 J=1, N1
LOC=LOC+1
TH1(J)=THVS(LOC)
CONTINUE
60 IF (GRIDL.GT.0.0) LOC=0
65 CALL FILLDV(TH1, THV(1, 1)+N1, DIVLAT)
WRITE(6, 550) (TH1(J), J=1, N1)
400 FORMAT (1H1/(1P8E15.6))
550 FORMAT (*0XYZVB*/(5X, 1P6E15.6))
IF (BODTAB.EQ.1.0) GO TO 150
C
C A ROW OF THV IS FOR A TRAILING VORTEX.
C
C NOW INTERPOLATE IN THE THV ARRAY TO GET THVV AT XVV STATIONS.
C ALSO INTERPOLATE FOR THSS'S AT XS'S.

N2=NTV+1.
N3=NBV+1
WRITE(6,500)(XTHV(J),J=1,NXTHV)
WRITE(6,500) (XV(I),I=1,NXTHV)
WRITE(6,500)(XV(I),I=1,N3)
WRITE(6,500)(XS(I),I=1,NX)
DQ 70 J=1,N2
DO 68 J=1,NXTHV
68 TH1(J)=THV(I,J)
CALL CODIM(XV,TH1,NXTHV,XVV,THV(1,I),N3)
70 CALL CODIM(XV,TH1,NXTHV,XS,THSS(1,I),NX)
C
C INTERPOLATE ON R VS. THETA AT EACH XS STATION TO
C OBTAIN R'S AT THETA'S.
C
L=2
IREGNO=1
JSUB=1
KSUB=1
DO 90 I=1,NX
71 IF(XTH(L)-XTH(L-1),GE.0.0) GO TO 72
72 IF(XS(I),LT,XTH(L)) GO TO 80
GO TO 75
73 FORMAT(*0IREGNO GREATER THAN NXTH*)
STOP
75 JSUB=JSUB+NTH(IREGNO)+1
KSUB=KSUB+NTH(IREGNO)
IREGNO=IREGNO+1
L=L+1
IF(IREGNO,LE,NXTH) GO TO 71
WRITE(6,73)
71 WRITE(6,73)
70 WRITE(6,73)
72 WRITE(6,73)
73 WRITE(6,73)
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87 WRITE(6,73)
88 WRITE(6,73)
89 WRITE(6,73)
90 WRITE(6,73)
C
C
WRITE (6*4000) ((RS(I,J)*1=NX), I=1,N2)
4000 FORMAT (1HO/(1P10E12.4))

C
C COLUMNS OF RS ARE R'S AT THSS'S.
C EACH COLUMN CORRESPONDS TO ONE XS STATION.
C
C ROWS OF RS ARE R VS. XS FOR SUCCESSIVE TRAILING VORTICES.
C THERE ARE N2 ROWS AND NX COLUMNS IN RS.  (N2 * NX)
C
C INTERPOLATE ON RS VS. XS CURVES DEVELOPED IN ABOVE STEP
C TO OBTAIN R AT XVV STATIONS.
C
C DO 100 I=1,N2
C DO 95 J=1,NX
C 95 THETA(J)=RS(I,J)
C 100 CALL INTER(XS, THETA,NX,XVV, RR(I+1),N3)

C ARRAYS THVV(I,J) AND RR(I,J)  I=1,N3  J=1,N2
C DESCRIBE LOCATION OF VORTEX POINTS.
C ROWS CORRESPOND TO BOUND VORTEX LINES.
C COLUMNS CORRESPOND TO TRAILING VORTEX LINES.
C
C GO TO 180
C
C Y,Z INPUT OR DELTA-S INPUT
C
C 110 CONTINUE
C DEVELOP Z VS. S AND Y VS. S AT INPUT STATIONS.
C KSUB=1-NTH(I)
C JSUB=KSUB-1
C DO 120 I=1,NXTH
C KSUB=KSUB+NTH(I)
C JSUB=JSUB+NTH(I)+1
C N=NTH(I)
C IF(SYM*LT*0.) N=N+1
C Y(I+1)=TH(JSUB)
C Z(I+1)= R(KSUB)
C DELS=0*0
S(1:1) = 0.0
DO 120 J = 2: N
IF (J = EQ. N .AND. SYM .LT. 0.1) GO TO 115
K1 = JSUB + J - 1
K2 = KSUB + J - 1
Y(J, I) = TH(K1)
Z(J, I) = RI(K2)
DELY = TH(K1) - TH(K1 - 1)
DELZ = RI(K2) - RI(K2 - 1)
DELS = SORT(DELY**2 + DELZ**2) + DELS
S(J, I) = DELS
GO TO 120
115 S(J, I) = SORT((Y(J - 1, I) - Y(1, I))**2 + (Z(J - 1, I) - Z(1, I))**2) + DELS
Y(N, I) = Y(1, I)
Z(N, I) = Z(1, I)
120 CONTINUE
IF (THVS(1) .NE. 0.1) GO TO 56
C Y.Z INPUT
C
DO 130 I = 1, N
NTH(I) = NTH(I)
CALL FILLDV(S(1, I), SLAT, N, DIVLAT)
NLAT = DIVLAT * N + 1.01
CALL CODIM(S(1, I), Y(1, I), SLAT, YY(1, I), NLAT)
130 CALL CODIM(S(1, I), Z(1, I), SLAT, ZZ(1, I), NLAT)
C A COLUMN OF YY (OR ZZ) ARE VALUES OF Y (OR Z) AT INPUT XS STATIONS.
C NOW INTERPOLATE TO OBTAIN Y AND Z AT XVV'S.
C
132 N3 = NBV + 1
N2 = NTV + 1
DO 140 I = 1, N2
DO 135 J = 1, N
Y1(J) = YY(I, J)
135 Z1(J) = ZZ(I, J)
CALL INTER(X5, Y1, NXTH, XVV, YY(1, I), N3)
140 CALL INTER(X5, Z1, NXTH, XVV, ZZ(1, I), N3)
GO TO 220
150 CONTINUE
C NOW A COLUMN OF THV CONTAINS S'S AT AN INPUT X-THETA STATION.
C A ROW OF THV GIVES S'S FOR A TRAILING VORTEX (AT ALL INPUT X-THETA'S)
C INTERPOLATE IN THE S VS. XTH PLOTS FOR S'S AT INPUT XS STATIONS.
C
N2=NTV+1
DO 160 I=1,N2
DO 155 J=1,NXTH
155 THETA(J)=THV(I,J)
160 CALL INTER(XTH,THETA,NXTH,XS,ST(I,1)*NX)
C INTERPOLATE IN THE Y VS. S AND Z VS. S ARRAYS AT XS STATIONS
C FOR Y'S AND Z'S AT ST'S FOUND ABOVE.
C
DO 170 I=1,NX
DO 165 J=1,N2
165 THETA(J)=ST(I,J)
170 CALL CODIM(S(I,1)*Y(I,1)*N,THETA,YY(I,1)*N2)
180 CONTINUE
C COMPUTE YY,ZZ FROM THVV AND RR.
C
DO 200 I=1,N3
DO 200 J=1,N2
THSS(I,J)=RR(I,J)*SIND(THVV(I,J))
200 S(I,J)=RR(I,J)*COSD(THVV(I,J))
C THSS IS THE Y ARRAY.  S IS THE Z ARRAY.
C THESE WILL BE MULTIPLIED BY THE MULT. FACTORS TO GET YV,ZV.
C
C 220 CONTINUE
C PERFORM MULTIPLICATION AND REVISION OF VORTEX POINT
C COORDINATES DUE TO CAMBER
C
C INTERPOLATE FOR MULTIPLICATION FACTORS AND CAMBER AT XVV'S.
IF(MCPTS.NE.0) GO TO 223
DO 222 I=1,N3
YC(I)=0.0
ZC(I)=0.0
YM(I)=1.0

222 ZM(I)=1.0
GO TO 236

223 CONTINUE
CALL INTER(XMF, YMF, MCPTS, XVV, YM, N3)
IF(BMULTEQ.1.0) GO TO 230
DO 225 I=1, N3
225 ZM(I)=YM(I)
GO TO 235

230 CONTINUE
CALL INTER(XMF, ZMF, MCPTS, XVV, ZM, N3)

235 CONTINUE
CALL INTER(XMF, YCAM, MCPTS, XVV, YC, N3)
CALL INTER(XMF, ZCAM, MCPTS, XVV, ZC, N3)

236 CONTINUE

C FOR CAMBI=0, COMPUTE ANGLES IN X-Z AND X-Y PLANES WHICH CAMBER LINES
C MAKE AT XVV STATIONS*
C

IF(CAMBI NE 0.) GO TO 280
DX=CHORD*0.001
DO 240 I=1, N3
XM(I)=XVV(I)+DX
240 XP(I)=XVV(I)-DX

CALL INTER(XVV, YC, N3, XM, YCM, N3)
CALL INTER(XVV, ZC, N3, XM, ZCM, N3)
CALL INTER(XVV, YC, N3, XP, YCP, N3)
CALL INTER(XVV, ZC, N3, XP, ZCP, N3)
DO 260 I=1, N3
DY=YCP(I)-YCM(I)
DZ=ZCP(I)-ZCM(I)
YCA=ATAN(DY/(2.*DX))
ZCA=ATAN(DZ/(2.*DX))
SINYC(I)=SIN(YCA)
COSYC(I)=COS(YCA)
SINZC(I)=SIN(ZCA)
COSZC(I)=COS(ZCA)

260 CONTINUE
GO TO 290
280 DO 285 I=1,N3
   SINCY(I)=0.0
   COSY(I)=1.0
   SINZC(I)=0.0
285  COSZC(I)=1.0
290 DO 300 J=1,N2
   DO 300 I=1,N3
      Y2=YM(I)*THSS(I,J)
      Z2=ZM(I)*S(I,J)
   XV1(I,J)=XV1(I,J)-Y2*SINCY(I)*COSZC(I)-Z2*SINZC(I)+XBOO(NB)
   YV(I,J)=(YC(I)+Y2*COSY(I)+YBOO(NB))*BETAM
300  ZV(I,J)=(ZC(I)-Y2*SINCY(I)*SINZC(I)+Z2*COSZC(I)+ZPOO(NB))*BETAM
      WRITE(6,301) N2,N3
      WRITE(12)(YV(N3,J),ZV(N3,J),J=1,N2)
      WRITE(6,7003)(J,YV(N3,J),ZV(N3,J),J=1,N2)
7003 FORMAT(*0BODY DRAG COORDS */(15,2F15.5))
301 FORMAT(*0XYZV8 N2,N3=*2I5)
      RETURN
      END
SUBROUTINE SETDAT
COMMON DA(5000)
 1 XNX,NXTH,LTOR,NTOR,TVV,NBV,NV,NXTHV,NBV,NTH(1)
 2 LNIV+V,LIV+V,NPTLS,LIV+V
COMMON/NR/NVPT(7),NCPT(7),NLN(7),LNL(7),LVC(7),NVENT(7)
 1 NCT,NBOLA,NPN,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,NTAPE,NTAPE
 2 LSEG(7),TSGL(7)+V,NFLN(7),VALC(7)
COMMON/BO/ DUMMY(26953),XBOO(5),YBOO(5),ZBOO(5)
 1 EQUIVALENCE(1,4),1,85),FNNTH(30),FLONGV1
 1 (DA131),FLTV1(32),DIVLON),1,33),DIVLON),1,36),XTH)
 2 (DA141),XSL(1215),FNNTHV1),123),VORLN),1,24),VORLT)
 3 (DA1215),XTHV),1,130),THN),1,1950),XV),1,17),CHORD)
 4 (DA138),PSL),1,391),PTSL)
 5 (DA127),XBO),1,28),YBO),1,29),ZBO)
EQUIVALENCE(DA36),SEG),1,37),SEG)
 1 1 (DA134),FUNCL),1,35),FUNCL)
DIMENSION XTH(99),XSL(99),XTHV(149),THN(760),XV(150)
COMMON/SCRAT/ XTV(200)
DIMENSION NPP(48)
EQUIVALENCE(NPP(1),NVPT(1))
EQUIVALENCE(DA19),SYM)
EQUIVALENCE(DA12200),THV)
DIMENSION THV(799)
NX=FNX
NXTH=FNNTH
IF( (NXTH.EQ.0) ) NXTH=NX

LOC=1
10 1 I=1,NXTH
 1 NTH(I)=THN(LOC)
 1 LOC=LOC+NTH(I)+1
 1 IF(DIVLON.EQ.0) DIVLON=1
 1 IF(DIVLAT.EQ.0) DIVLAT=1
 1 LNDIV=DIVLON
 1 LTDIV=DIVLAT
 1 LNDIVB(NB)=LNDIV
 1 LTDIVB(NB)=LTDIV
 1 LTPTLS=PTSL
LNPTS = PTSLN
LTC(NB) = LTPTS
LNC(NB) = LNPTS
LSNG(NB) = SEGLN
TSEG(NB) = SEGLT
LFUNC(NB) = FUNCLN
TFUNC(NB) = FUNCJT
SYM(NB) = SYM
NTVV = FLATV
C NBVV EQUALS NO. OF LONGITUDINAL PANELS.
C NTVV EQUALS NO. OF LATERAL PANELS.
NBVV = FLONGV
LNVOR = VORLN
LTOR = VORTL
NXTHV = FNXTHV
XBOO(NB) = XBO
YBOO(NB) = YBO
ZBOO(NB) = ZBO
C C SET UP LONGITUDINAL VORTEX GRID (XVV ARRAY)
NB1 = NBVV + 1
15 IF (LNVOR) 15, 20, 30
   DEL = CHORD / NBVV
   XV(1) = 0.0
   GO TO 24
20   DEL = 3.1415926 / NBVV
   XV(1) = 0.0
24   DO 25 I = 2, NB1
25      XV(I) = XV(I - 1) + DEL
C C FOR LNIVOR = 1, XV IS GIVEN IN INPUT
NB1 = NBVV + 1
30   IF (LNIVOR .NE. 1) GO TO 40
   DO 35 I = 1, NB1
35      XVV(I) = XV(I)
   GO TO 45
40   CALL FILLDV(XV, XVV, NB1, DOVLON)
45   CONTINUE
311 TRANGE=360.0
312 GO TO 312
313 DTH=TRANGE/NTVV
314 DO 314 I=1,NTVV
315 THVS(I)=(I-1)*DTH
C FOR LT vor = 1, THVS ARE GIVEN IN DATA.
320 CONTINUE
321 IF(NXTHV.EQ.0) NXTHV=LONGV+1.*J1
322 IF(LNVOR.NE.0) GO TO 58
323 DO 54 I= 1,NXTHV
324 XV(I)=C2*(1.0-COS(XV(I))
325 IF(FNXTHV.NE.0) GO TO 58
326 DO 60 I=1,NXTH
327 XTH(I)=XS(I)
328 CONTINUE
C CALL DTAWR(DA)
329 RETURN
330 END
SUBROUTINE VTVN(J,1*Y*Z+AN)
COMMON /A(31000)/
1 2*NTV*NTH+LMVQ+LTVQ+NTVV+NBVV+NTV+NTHV+NBV+ANTH(491)
*LNIV=LTDIV*LNPTS*LTPTS
COMMON /A(31000)/
DIMENSION XV(151+31)*YV(151+31)*ZV(151+31)
EQUIVALENCE(XV*A(11)),(YV*A(4682)),(ZV*A(9363))
DIMENSION AN(1)
J1=LTDIV*(J-1)+LTDIV/2+1
J2=J1+1
IF(I*NE*1) GO TO 100
X3=XV(2,J1)
X4=XV(2,J2)
X1=2.0*XV(1,J1)-X3
X2=2.0*XV(1,J2)-X4
Y3=YV(2,J1)
Y4=YV(2,J2)
Y1=2.0*YV(1,J1)-Y3
Y2=2.0*YV(1,J2)-Y4
Z3=ZV(2,J1)
Z4=ZV(2,J2)
Z1=2.0*ZV(1,J1)-Z3
Z2=2.0*ZV(1,J2)-Z4
GO TO 200
100 I1=I-1
I2=I1+2
X1=XV(I1,J1)
X2=XV(I1,J2)
X3=XV(I2,J1)
X4=XV(I2,J2)
Y1=2.0*YV(I1,J1)
Y2=2.0*YV(I1,J2)
Y3=2.0*YV(I2,J1)
Y4=2.0*YV(I2,J2)
Z1=2.0*ZV(I1,J1)
Z2=2.0*ZV(I1,J2)
Z3=2.0*ZV(I2,J1)
Z4=2.0*ZV(I2,J2)
200 Y=0.25*(Y1+Y2+Y3+Y4)
\[ Z = 0.25 \left( (x_1 + x_2 + z_3 + z_4 ) \right) \]
\[ TX = x_3 - x_1 + x_4 - x_2 \]
\[ TY = y_3 - y_1 + y_4 - y_2 \]
\[ TZ = z_3 - z_1 + z_4 - z_2 \]
\[ D = \sqrt{ (TX^2 + TY^2 + TZ^2) } \]
\[ T1 = TX / D \]
\[ T2 = TY / D \]
\[ T3 = TZ / D \]
\[ TX = x_2 - x_1 + x_4 - x_3 \]
\[ TY = y_2 - y_1 + y_4 - y_3 \]
\[ TZ = z_2 - z_1 + z_4 - z_3 \]
\[ D = \sqrt{ (TX^2 + TY^2 + TZ^2) } \]
\[ AN(1) = (T2*Z-T3*Y)/D \]
\[ AN(2) = (T3*TX-T1*Z)/D \]
\[ AN(3) = (T1*TY-T2*TX)/D \]
RETURN
END
SUBROUTINE INTER (X,Y,N,X1,Y1,N1)
DIMENSION X(1)*Y(1)*X1(1)*Y1(1)
IF(X(2)-X(1)*GT.0.0) GO TO 20
IM=1
XMIN=X1(1)
DO 5 I=2*N1
IF(XMIN-X1(I)*LE.0.0) GO TO 6
XMIN=X1(I)
IM=1
5 CONTINUE
6 NU=IM
IM=1
XMIN=X(1)
DO 10 I=2*N
IF(XMIN-X(I)*LE.0.0) GO TO 11
XMIN=X(I)
IM=1
10 CONTINUE
11 CONTINUE
CALL REVERS(X*IM)
CALL REVERS(Y*IM)
CALL REVERS(X1,NU)
CALL CODIM(X,Y,IM*,X1,Y1,NU)
CALL REVERS(X*IM)
CALL REVERS(Y*IM)
CALL REVERS(X1,NU)
CALL REVERS(Y1,NU)
WRITE(6,100)X(I),Y(I),I=1,IM)*,X1(I),Y1(I),I=1,NU)
100 FORMAT(*0S3B,INTER(1PB'45.6))
CALL CODIM(X1,IM*Y1,IM,IM+1*,X1(IM+1),Y1(IM+1),I=1..N1)
NX=N-IM+1
NY=NU-NU
WRITE(6,100)X(IM-1+I),Y(IM-1+I),I=1,NX*X1(IM+I),Y1(IM+I)
1 ,I=1,NY
RETURN
20 CALL CODIM(X,Y,N,X1,Y1,N1)
RETURN
END
SUBROUTINE FILLDV(X,Y,N,DIV)
DIMENSION X(1),Y(1)
N1=N-1
NDIV=DIV+0.01
DO 10 I=1,N1
DEL=(X(I+1)-X(I))/NDIV
II=(I-1)*NDIV
DO 10 J=1,NDIV
K=II+J
10 Y(K)=X(I)+(J-1)*DEL
Y(K+1)=X(N)
RETURN
END
SUBROUTINE REVERSE(X,N)
DIMENSION X(N)
NL=N/2
DC 100 I=1, NL
XHOLD=X(I)
NI=N-I+1
X(I)=X(NI)
X(NI)=XHOLD
RETURN
END

100
PROGRAM PANEL00
COMMON DA(5000)
COMMON/BOXY/XVR(10,20)*YVR(10,20)*ZVR(10,20)*XVO(10,20)
1 *YVO(10,20)*ZVO(10,20)*PLL(500)*PLT(500)*YSUBV(100)*CHORD(100)
2 *XVO(20)*XCCO(20)*XLE(20)*ZLE(20)
3 *XTE(20)*YTE(20)*LCE(20)*PLE(20)*XJ(20)*YJ(20)*ZJ(20)
4 *ETLE(20)*XVT(50)*YVT(50)*ZVT(50)*XRI(20)*YRI(20)*ZRI(20)
5 *BDHHY(12130)*XYZN(3030)
COMMON/PANELY/NPAN*IPSYH*IMC*NTWVP*NTWVP*NLCEP*LTCEP*NLCEP*LTCEP
1 *APPRT*SPACE*ATTCH*INTATT*NPRLN*NPRLN*CTX*CTX*CTX*CTX
2 *NTT*NTIP*CTIP*CTIP*CTIP*CTIP*CTIP*CTIP
3 *MP1*MP2*MP3*MP4*MP5*MP6*MP7*MP8*MP9*MP10
COMMON/SCRAT/ DUMB(6000)
EQUIVALENCE(DA(2),PANS)
COMMON/PANAF*/PANSY(10),DUMB(400),PANREF(10),PCHORD(10)
EQUIVALENCE(DA(3422),PREF), (DA(3422),PCH)
IF(PANS*EQ*0.0) GO TO 2
10 NPANS=PANS
DO 1 I=1,NPANS
DO 10 J=3420,4000
DA(J)=.08
1 CALL DECRED(DA)
CALL PANDAT
PANSY(I) = IPSYH
PCHORD(I) = PCH
PANREF(I) = PREF
1 CALL PANEL
1 CALL ATTACH(XVR,YVR,ZVR,10,0)
1 CONTINUE
2 CONTINUE
END
C

WRITE(6,230) I, XLE(I), YLE(I), ZLE(I)+I=1, NTIP)
WRITE(6,230) I, XTE(I), YTE(I), ZTE(I)+I=1, NTIP)
200 FORMAT(1H, 13, 1P3E16.6)
DO 401 I=1, NTIP
401 ETLE(I)=SORT((YLE(I)-YLE(I))**2+(ZLE(I)-ZLE(I))**2)
1 /SQRT((YLE(I,NTIP)-YLE(I))**2+(ZLE(I,NTIP)-ZLE(I))**2)
DELS=0.0
NTIP1=NTIP-1
SLE(I)=0.0
DO 5 I=1, NTIP1
DY=YLE(I)-YLE(I+1)
DZ=ZLE(I)-ZLE(I+1)
DELS=DELS+SQRT(DY**2+DZ**2)
SLE(I+1)=DELS
IF(I.EQ.0 .NSPACE) DSROOT=DELS
5 CONTINUE
CHTIP=PERIM(4*NTIP)
N4=4*NTIP
XTIP=PERIM(N4-3)
YTOP=PERIM(N4-2)
ZTOP=PERIM(N4-1)
IF(ROOT) GO TO 10
C FOR NO ROOT SECTION
XRLE=XTIP
YRLE=YTOP
ZRLE=ZTOP
XRTE=PERIM(N4+1)
YRTE=PERIM(N4+2)
ZRTE=PERIM(N4+3)
GO TO 20
C FOR LIFTING PANEL WITH ROOT SECTION
10 NS4=4*NSPACE
XRLE=PERIM(NS4+1)
YRLE=PERIM(NS4+2)
ZRLE=PERIM(NS4+3)
N1=4*(NPERPT-NSPACE-1)
XRTE=PERIM(N1+1)
YRTE=PERIM(N1+2)
ZRITE = PHEME(N1+3)
C NLROOT EQUA8 NO. OF TR. VORTECS IN ROOT SECTION.
NLROOT = NSPACE + 1
20 XJLE = PHEME(1)
YJLE = PHEME(2)
ZJLE = PHEME(3)
N4 = 4*NPERPT
XJTE = PHEME(N4-3)
YJTE = PHEME(N4-2)
ZJTE = PHEME(N4-1)
IF (ROOT) GO TO 30
C FOR NO ROOT, DIVIDE JUNCTURE TR. VORTEX INTO 2ND VORTEX COORDINATES
CX = XJTE - XJLE
CY = YJTE - YJLE
CZ = ZJTE - ZJLE
XJ(1) = XJLE
YJ(1) = YJLE
ZJ(1) = ZJLE
DO 25 I = 2,NBVP
DPER = PXCV(I) - PXCV(I-1)
XJ(I) = XJ(I-1) + CX*DPER
YJ(I) = YJ(I-1) + CY*DPER
ZJ(I) = ZJ(I-1) + CZ*DPER
25 N1 = NBVP + 1
XJ(N1) = XJTE
YJ(N1) = YJTE
ZJ(N1) = ZJTE
GO TO 50
30 CONTINUE
C
C COMPUTE XJ, YJ, ZJ ARRAYS FOR ROOT-BODY JUNCTURE.
XJ(1) = XJLE
C COMPUTE Y, Z ON TR. VORTEX FOR 20 X'S.
A = 20
DX = (XJTE - XJLE)/A
XAX(1) = XJLE + XPC
DO 45 I = 1,A
45 XAX(I+1) = XAX(I) + DX
NAPI=NA+1
CALL CODIM(XVT,YVT,A,1,XA,YA,NAPI)
CALL CODIM(XVT,ZVT,A,1,XA,ZA,NAPI)
WRITE(6,451) (XA(I),YA(I),ZA(I),I=1,NAPI)

451 FORMAT(*XA,YA,ZA*/ )
SAI=0.0
SA(I)=0.0
DO 46 I=1,NA
DY=YA(I+1)-YA(I)
DZ=ZA(I+1)-ZA(I)
SAI=SAI+SQRT(DX**2+DY**2+DZ**2)

46 SA(I+1)=SAI

DO 47 I=1,NBP1
SJ(I+1)=SJ(I)+(PXCV(I+1)-PXCV(I))*SAI
CALL CODIM(SA,XA,NAPI,5J,XJ,NBP1)
CALL CODIM(SA,YA,NAPI,5J,YJ,NBP1)
CALL CODIM(SA,ZA,NAPI,5J,ZJ,NBP1)

50 CONTINUE
DO 49 I=1,NBP1
XJ(I)=XJ(I)-XPO
YJ(I)=YJ(I)-YPO
ZJ(I)=ZJ(I)-ZPO

49 C COMPLETE XR,YR,ZR ARRAYS
DX=XRTE-XRLE
XR(I)=XRLE
YR(I)=YRLE
ZR(I)=ZRLE

DO 55 I=1,NBVVP
XR(I+1)=XRLE+DX*PXCV(I+1)
YR(I+1)=YRLE
ZR(I+1)=ZRLE

55 WRITE(6,200) (I,XJ(I),YJ(I),ZJ(I),I=1,NBP1)
WRITE(6,200) (I,XR(I),YR(I),ZR(I),I=1,NBP1)

C CALL PANEL2
RETURN
END
SUBROUTINE PANDAT
COMMON/PANEL/HAN,IPSYN,IVC,NBVVP,NTVVP,LNCF,LTCP,LNCPF,LNCPF,LTCPF
1 SNPFRPT,INSPACE,INATT,HATT,INPRCLI,INPRCLT,IVCTXC,INCTET,INTHXC
2 SNHET,INIP,ICHTIP,ROOT,OUTEK,INATT
3 *PI,IP2,IP3,IP5,IP6,IP7,IP8,IP9,IP10
COMMON/ANUGER/HVPTS(7)*CPTS(7)*LH(7)*NLT(7)*LTC(7)*LNC(7)
1 SNCT,HF,NBODS,HPANS,NVL(7)*NVT(7)*TAPE1,TAPE2,CTV,ITAPE,JTAPE
2 SLSEG(7)*TSEG(7)*LFUNC(7)*TFUNC(7)
3 SLNDIVP(7)*LTDIVP(7)*NJFFP(7)*KOOTP(7)*OUTERP(7)*SY50M(7)
COMMON DA(5000)
EQUVALENCE (DA(3429)*PH)* (DA(3426)*PSYN)* (DA(3429)*VICI)
2 EQUVALENCE (DA(3437)*PNBV)* (DA(3438)*NVT)* (DA(3439)*PLNCF)
3 * (DA(3440)*PLTCP)* (DA(3441)*PLTCP)* (DA(3442)*PLTCP)
4 * (DA(3443)*PTSPER)* (DA(3444)*ROOTSP)* (DA(3445)*ATTCH)
5 * (DA(3446)*PCLN)* (DA(3447)*PCLT)* (DA(3600)*PANXC)
6 * (DA(4130)*XCTH)* (DA(4140)*CTATH)* (DA(4150)*CTATH)
EQUVALENCE (DA(3420)*VLNI)* (DA(3421)*VLTI)
1 * (DA(4600)*PXCV)* (DA(4640)*PETV)
EQUVALENCE (DA(3423)*SPAN)
COMMON/CUMPRI/BETAM
DIMENSION PXCV(40)*PETV(40)
2 DIMENSION CPLN(40)*CPLT(40)
3 COMMON/PANATT/NATT(30)*NATT(200)
4 COMMON/CONTRV/LNC(340)*LTT(340)
5 DIMENSION NLOOK(50)
6 EQUVALENCE (NLOOK*HVPTS)
WRITE(6,1) NLOOK
1 FORMAT(*OPANDAT*/(1H10))
CALL DATAVR(DA)
SPAN=SPAN*BETAM
NPA=PH
IPSYN=IPSYN
IVC=IVC
NBVVP=NBVVP
NTVVP=NTVVP
NPP=NPAN+NBODS
NVL(NPP)=NBVVP
NVT(NPP)=NTVVP
ALC(IPP) = NVV*1
LT(IPP) = TVV+1
NPTS(IPP) = NVV*TVV
LCFPP = PLINE
LTCPP = PLTC
LUNK(NP) = LNCPP
TFUNK(NP) = LTCPP
LNCPP = PLINE
LTCP = PLTC
LTCPP = PLTC
LUNK(NP) = LNCPP
NCPTS(NP) = LNCPP/LTCPP
NSRC = PSLET + 0
NSPACE = ROOTSP
NAP = (NPAN-1)*3+2
NATTCH = LATT(NAP)
LTRATT = LATT(NAP+1)
IF(NATTCH.EQ.0) GO TO 101
WRITE(6,100) NPAN, NATTCH, NTRATT
100 FORMAT(*NPANEL*,*IP* IS ATTACHED TO COMPONENT*,*IP* AT TRAILING VOR2, LTEX*,12)
GO TO 103
101 WRITE (6,102) NPAN, NATTCH
102 FORMAT(*NPANEL*,*IP* IS NOT ATTACHED, NATTCH=*,12)
103 CONTINUE
NPRLN = PCLN
NPRLT = PCLT
NXTXC = PAXXC
NXTET = ETALF
TXXC = NXJC
NXET = ETATH
IF(CPLN(1) .EQ. 0) GO TO 6
DO 5 1 = 1,LTCPP
5 CPLN(1) = 1
6 IF(CPLT(1) .EQ. 0) GO TO 6
DO 7 1 = 1,LTCPP
7 CPLT(1) = 1
8 CONTINUE
SAVE CONTROL POINT LOCATIONS FOR LTI IN VELOCITY CALCULATION.

DO 15 I=1,NLCCPP
15 LTI(I) = CPL(I)
DO 16 I=1,LTCCP
16 LTI(I) = CPL(I)

LXI=VLI
IF(LXI) 10,20,25
10 PXCV(I) = 0.0
D=1.0/IVBVP
DO 11 I=1,NBVP
11 PXCV(I+1) = PXCV(I) + D
GO TO 25
20 PXCV(I) = 0.0
D=180.0/NBVVP
DO 21 I=1,NBVVP
21 PXCV(I+1) = D*(1.0-COS(PI*1))
25 CONTINUE
NL=NBVP+1
RITE(6+26) (PXCV(I),I=1,NL)
FORMAT(*3PXCV ARRAY*(1P6E15.6))
LTI=VLI
IF(LTI) 40,30,55
30 PETV(I) = 0.0
D=1.0/TVVP
DO 31 I=1,TVVP
31 PETV(I+1) = PETV(I) + D
GO TO 31
31 PETV(I+1) = 1.0
31 CONTINUE
GO TO 55
40 PETV(I) = 0.0
D=180.0/TVVP
DO 41 I=1,TVVP
41 PETV(I+1) = D*(1.0-COS(PI*1))
GO TO 41
41 PETV(I+1) = 1.0
LOGICAL ROOTP, OUTER
NSPP(1:NPAN)=NSPACE
ROOTP(NPAN)=ROOT
OUTERP(1:NPAN)=OUTER
NU=FN
N=FN
LETV=0
LETC=0
NBPI=NBVVP+1
NLROOT=NSPACE + 1
IF(*NOT*ROOT) GO TO 65

COMPUTE ALL PANEL POINTS BOUNDED BY JLE*RLE*RT*JTE.
N1=NSPACE-1
DO 58 J=1,NBPI
XVR(NLROOT,J)=XR(J)
YVR(NLROOT,J)=YR(J)
ZVR(NLROOT,J)=ZR(J)
XVR(I,J)=XJ(J)
YVR(I,J)=YJ(J)

58 ZVR(I,J)=ZJ(J)
DO 581 I=1,NLROOT
XVR(I,1)=XLE(I)
YVR(I,1)=YLE(I)
ZVR(I,1)=ZLE(I)
XVR(I,NBPI)=XTE(I)
YVR(I,NBPI)=YTE(I)

581 ZVR(I,NBPI)=ZTE(I)
DO 60 J=2,NBVVP
CSX=XR(J)-XJ(J)
CSY=YS(J)-YJ(J)
CSZ=ZR(J)-ZJ(J)
CS2=CSX**2+CSY**2+CSZ**2
PER= PXCV(J)
DENO'=0.0
DO 59 I=1,NSPACE
DXLE=XLE(I+1)-XLE(I)
DYLE=YLE(I+1)-YLE(I)
DZLE=ZLE(I+1)-ZLE(I)
CALL NORM(DXLE,DYLE,DZLE)
DXTE=XTE(I+1)-XTE(I)
DYTE=YTE(I+1)-YTE(I)
DZTE=ZTE(I+1)-ZTE(I)
CALL NORM(DXTE, DYTE, DZTE)
DCX(I)=DXLE+(DXTE-DXLE)*PER
DCY(I)=DYLE+(DYTE-DYLE)*PER
DCZ(I)=DZLE+(DZTE-DZLE)*PER
CALL NORM(DCX(I), DCY(I), DCZ(I))

59
DENOM=DENOM + DCX(I)*DCY(I)*DCZ(I) + CSX*CSY*CSZ
DS=CSZ/DENOM

DO 60 J=1,N1
XVR(I+1,J)=XVR(I,J)+DS*DCX(I)
YVR(I+1,J)=YVR(I,J)+DS*DCY(I)

200
FORMAT(1H13,1P3E16.6)

60
ZVR(I+1,J)=ZVR(I,J)+DS*DCZ(I)
DO 618 I=2,NSPACE
SAI=U*0
SA(I)=0*0

DO 601 J=1,NBVVP
DX= XVR(I,J+1)-XVR(I,J)
DY= YVR(I,J+1)-YVR(I,J)
DZ=ZVR(I,J+1)-ZVR(I,J)
SAI= SAI + SQRT(DX**2 + DY**2 + DZ**2)

601
SA(I,J+1)=SAI
DO 603 J=1,NBP1
XA(I)=XVR(I,J)
YA(I)=YVR(I,J)

603
ZA(J)=ZVR(I,J)
SJ(J)=0*0
DO 604 J=1,NBVVP
SJ(J+1)=SJ(J)+SAI*(PXCV(J+1)-PXCV(J))
CALL CODI*(SA*X*A*1*BP1*SJ*DCX*1*BVVP)
CALL CODI*(SA*YA*1*BP1*SJ*DCY*1*BVVP)
CALL CODI*(SA*ZA*1*BP1*SJ*DCZ*1*BVVP)
DO 618 J=2,NSPACE
XVR(I,J)=DCX(J)
YVR(I,J)=DCY(J)

618
ZVR(I,J)=DCZ(J)

C

C

C
65 CONTINUE
DO 70 I=1,NTVVP
IF(I.NE.1) GO TO 66
IF(ROOT) GO TO 66
CH = XJ(NBP1)-XJ(1)
GO TO 70
66 CH=0.*
DO 67 J=1,NBVVP
DX=XVR(I,J)-XVR(I,J+1)
DY=YVR(I,J)-YVR(I,J+1)
DZ=ZVR(I,J)-ZVR(I,J+1)
67 CH=CH+SQRT(DX**2+DY**2+DZ**2)
GO TO 70
68 IF(ROOT.AND.I.LE.NLROOT) GO TO 66
CH=CHORD(I-1)+(PETV(I)-PETV(I-1))*(CHTIP-CHORD(I-1))/
1 (1.0-PETV(I-1))
70 CHORD(I)=CH
CHORD(NTVVP+1)=CHTIP
NTV1=NTVVP+1
WRITE(6,200)(I,CHORD(I),PETV(I),CHTIP,I=1,NTV1)
NCO=0
NCOO=0
IF(NVODS.EQ.0.AND.KPAN.EQ.1) NCT=0
NCP=NCT
IF(NPAN.GT.1) NCP=NP1
IF(NOT.ROOT) GJ TO 85
COMPUTE VORTEX AND CONTROL POINTS IN ROOT REGION:
LT=1
DO 80 I=1;NSPACE
KLT=FALSE.
LTCP=CPLT(LT)
IF(I.NE.LTCP) GO TO 71
KLT=TRUE.
LT=LT+1
71 CONTINUE
LN=1
DO 75 J=1,NBVVP
KLN=FALSE.
LJCP=CPLN(LT)
75 CONTINUE
IF(J LE LNC) GO TO 72
KLM=.TRUE.
LN=L'+1
72 CONTINUE
 DX1=XVR(I*,J+1)-XVR(I*,J)
 DY1=YVR(I*,J+1)-YVR(I*,J)
 DZ1=ZVR(I*,J+1)-ZVR(I*,J)
 DX2=XVR(I+1*,J+1)-XVR(I+1*,J)
 DY2=YVR(I+1*,J+1)-YVR(I+1*,J)
 DZ2=ZVR(I+1*,J+1)-ZVR(I+1*,J)
 NC0=NC0+1
 PL1=SQRT(DX1**2+DY1**2+DZ1**2)
 PL2=SQRT(DX2**2+DY2**2+DZ2**2)
 PLL00=0.5*(PL1+PL2)
 DX11=XVR(I+1*,J)-XVR(I*,J)
 DY11=YVR(I+1*,J)-YVR(I*,J)
 DZ11=ZVR(I+1*,J)-ZVR(I*,J)
 DX21=XVR(I+1*,J+1)-XVR(I*,J+1)
 DY21=YVR(I+1*,J+1)-YVR(I*,J+1)
 DZ21=ZVR(I+1*,J+1)-ZVR(I*,J+1)
 PL1=SQRT(DX11**2+DY11**2+DZ11**2)
 PL2=SQRT(DX21**2+DY21**2+DZ21**2)
 PLT=0.5*(PL1+PL2)
 T1=DX11/SQRT(DY11**2+DZ11**2)
 T2=DX21/SQRT(DY21**2+DZ21**2)
 FIRST=.FALSE.
 R1=0.25
722 NC0=NC0+1
 X55(NC0)=XP*0.5*(XVR(I*,J)+R1*DX1+XVR(I+1*,J)+R1*DX2)
 Y55(NC0)=YP*0.5*(YVR(I*,J)+R1*DY1+YVR(I+1*,J)+R1*DY2)
 Z55(NC0)=ZP*0.5*(ZVR(I*,J)+R1*DZ1+ZVR(I+1*,J)+R1*DZ2)
 T55(NC0)=T1+R1*(T2-T1)
 Y55(NC0)=DY11+R1*(DY21-DY11)
 Z55(NC0)=DZ11+R1*(DZ21-DZ11)
 IF(FIRST) GO TO 724
 FIRST=.TRUE.
 R1=0.75
 GO TO 722
724 CONTINUE
IF (.NOT. KLT) GO TO 73
IF (.NOT. KLN) GO TO 73
NCP=NCP+1
XCR(NCP)=0.5*(XVR(I+1,J)+0.75*DX1+XVR(I+1,J)+0.75*DX2)+XPO
YCR(NCP)=0.5*(YVR(I+1,J)+0.75*DY1+YVR(I+1,J)+0.75*DY2)+YPO
ZCR(NCP)=0.5*(ZVR(I+1,J)+0.75*DZ1+ZVR(I+1,J)+0.75*DZ2)+ZPO
73 CONTINUE
XVR(I+1,J)=XVR(I,J)+0.25*DX1+XPO
YVR(I+1,J)=YVR(I,J)+0.25*DY1+YPO
ZVR(I+1,J)=ZVR(I,J)+0.25*DZ1+ZPO
IF (.NE. NSPACE) GO TO 75
XVR(I+1,J)=XVR(I+1,J)+0.25*DX2+XPO
YVR(I+1,J)=YVR(I+1,J)+0.25*DY2+YPO
ZVR(I+1,J)=ZVR(I+1,J)+0.25*DZ2+ZPO
75 CONTINUE
LETV=LETV+1
ETV(LETV)=0.5*(PETV(I+1)+PETV(I))
IF (.NOT. KLT) GO TO 80
LETC=LETC+1
ETC(LETC)=ETV(LETV)
80 CONTINUE
DO 82 I=1,NLROOT
XVR(I,NBP1)=XVR(I,NBP1)+XPO
YVR(I,NBP1)=YVR(I,NBP1)+YPO
82 ZVR(I,NBP1)=ZVR(I,NBP1)+ZPO
NCR=NC00
C 85 IF (.NOT. OUTER) GO TO 120
DO 90 J=1,NBVVP
XCVG(I)=0.75*PXCV(I)+0.25*PXCV(I+1)
XCVG(NBVVP+1)=PXCV(NBVVP+1)
DO 90 J=1,L:CPP
J=CLP(I)
90 XCVG(I)=0.25*PXCV(J)+0.75*PXCV(J+1)
C 95 CONTINUE
NTVOUT=NTVVP-1SPACE
COMPUTE VORTEX POINTS AND CONTROL POINTS FOR OUTER PANEL.
C SPACE CPLT ARRAY OUT OF FOOT REGION.
DO 96 I=1,LTCP
   IC=1
   LTCP=CPLT(I)
   IF(LTCP.GT.*:SPACE) GO TO 97
96 CONTINUE
97 CONTINUE
DO 110 I=1,NTVOUT
   INS=I+NSPACE
   LT1=PETV(INS)
   ET2=PETV(INS+1)
   ET=0.5*(ET1+ET2)
   CH1=CHORD(INS)
   CH2=CHORD(INS+1)
   CHAVG=0.5*(CH1+CH2)
DO 99 L=1,NTIP
   IF(ET.LE.ETLE(L+1).*:AND.*:ET.GT.ETLE(L)) GO TO 99
99 CONTINUE
99 DELET=ETLE(L+1)-ETLE(L)
   RATIO=(ET-ETLE(L))/DELET
   XLE1=XLE(L)+(XLE(L+1)-XLE(L))*RATIO
   YLE1=YLE(L)+(YLE(L+1)-YLE(L))*RATIO
   ZLE1=ZLE(L)+(ZLE(L+1)-ZLE(L))*RATIO
   R1=(ET1-ETLLE(L))/DELET
   R2=(ET2-ETLE(L))/DELET
   DX=XLE(L+1)-XLE(L)
   DY=YLE(L+1)-YLE(L)
   DZ=ZLE(L+1)-ZLE(L)
   XLI=XLE(L)+DX*R1
   YLI=YLE(L)+DY*R1
   ZLI=ZLE(L)+DZ*R1
   XLI=XLE(L)+DX*R2
   YLI=YLE(L)+DY*R2
   ZLI=ZLE(L)+DZ*R2
   DX=XTE(L+1)-XTE(L)
   DY=YTE(L+1)-YTE(L)
   DZ=ZTE(L+1)-ZTE(L)
   XTI=XTE(L)+DX*R1
   YTI=YTE(L)+DY*R1
   ZTI=ZTE(L)+DZ*R1
2 5830
2 5840
2 5850
2 5860
2 5870
2 5880
2 5890
2 5900
2 5910
2 5920
2 5930
2 5940
2 5950
2 5960
2 5970
2 5980
2 5990
2 6000
2 6010
2 6020
2 6030
2 6040
2 6050
2 6060
2 6070
2 6080
2 6090
2 6100
2 6110
2 6120
2 6130
2 6140
2 6150
2 6160
2 6170
2 6180
2 6190
2 6200
2 6210
XT2=XTE(L)+DX*R2 2 6220
YT2=YTE(L)+DY*R2 2 6230
ZT2=ZTE(L)+DZ*R2 2 6240

C SAVE VORTEX POINTS ON TRAILING EDGE. (THOSE THAT LIE OUTSIDE ROOT.) 2 6250
XVS(I)=XT1+XPO 2 6260
YVS(I)=YT1+YPO 2 6270
ZVS(I)=ZT1+ZPO 2 6280
IF(I>=NTVOUT) GO TO 990 2 6290
XVS(I+1)=XT2+XPO 2 6300
YVS(I+1)=YT2+YPO 2 6310
ZVS(I+1)=ZT2+ZPO 2 6320

990 CONTINUE 2 6330
DXL=XL2-XL1 2 6340
DYL=YL2-YL1 2 6350
DZL=ZL2-ZL1 2 6360
YSUBV(I)=0.5*SQRT(DYL**2+DZL**2) 2 6370
DXT=XT2-XT1 2 6380
DYT=YT2-YT1 2 6390
DZT=ZT2-ZT1 2 6400
TTLE=DXL/SQRT(DYL**2+DZL**2) 2 6410
TTTE=DXT/SQRT(DYT**2+DZT**2) 2 6420
DO 150 J=1,NBVVP 2 6430
NCO=NCO+1 2 6440
PD=PXCV(J+1)-PXCV(J) 2 6450
PLL(NCO)=0.5*PD*(CH1+CH2) 2 6460
FIRST=.FALSE. 2 6470
XCTERM=XCVO(J) 2 6480
XVO(I,J)=XP0+XLE1+XCTERM*CHAVG 2 6490
YVO(I,J)=YP0+YLE1 2 6500
ZVO(I,J)=ZP0+ZLE1 2 6510

991 NCO=NCO+1 2 6520
IF(FIRST) GO TO 992 2 6530
XSS(NCO)=XVO(I,J) 2 6540
YSS(NCO)=YVO(I,J) 2 6550
ZSS(NCO)=ZVO(I,J) 2 6560
GO TO 993 2 6570

992 XSS(NCO)=XP0+XLE1+XCTERM*CHAVG 2 6580
YSS(NCO)=YP0+YLE1 2 6590

381
ZBS(NCO)=ZPO+ZLE1
993
DYS(NCO)=DYL
DZS(NCO)=DZL
TS(NCO)=TTLTE(TTLE-TTLE)*XCTERM.
IF(FIRST) GO TO 100
FIRST=.TRUE.
XCTERM'=0.25*PSV(J)+0.75*PSV(J+1)
GO TO 991
100 CONTINUE
LETV=LETV+1
ETV(LETV)=ET
LTCP=CPLT(1C)
IF(I+MSPACE NE LTCP) GO TO 110
IC=IC+1
LET=LET+1
ETC(LET)=ET
DO 105 II=1,LPNP
NCP=NC+1
XCR(NCP)=XP0+XLE1+XCC0(I1)*CHAVG
YCR(NCP)=YP0+YLE1
105 XCR(NCP)=ZPO+ZLE1
110 CONTINUE
NCP=NC+1
120 CONTINUE
MP1=NCP
C
C CALL PAMFNC(NCT,ETV,ETC)
NCT=NCP
C
IF(NU+NU+EL,0) GO TO 1111
CALL SUBROUTINES REQUIRED TO SET UP PANEL CONSTRAINT MATRIX.
LOC1=(NPN-1)*1000+1
LOC2=LOC1+400
LOC3=LOC2+400
C ARRAY XA IS USED FOR SCRATCH.
CALL TACC(NU*XA,CRY,VARVPP,
1 SAVEC(LOC1),SAVEC(LOC2),XLE1,XCC0,XAVG)
CALL MATETA(SAVEC(LOC3),...ETV,MP1,MP1,ETV,SLCF)
1111 CONTINUE
SUBROUTINE Perg1( Perim, Array)
DIMENSION Perim(1), Array(1)
1  * NFERPT * NSPACE * IATTC : NTATT * PRCLN * PRCLT * N : CTX : N : CTET : THAC
2  * NTET * NTIP * CHTIP : ROOT : OUTER : MNATT
COMMON/ BODY/XVR(10, 20) : YVR(10, 20) : ZVR(10, 20) : XVR(10, 20)
1  * XVR(10, 20) : ZVR(10, 20) : PLL(500) : PLT(500) : YSV(100) : CHORD(100)
2  * XVR(10, 20) : XCC(50) : YLE(20) : YLE(20) : ZLE(20)
3  * XLE(20) : YLE(20) : ZLE(20) : XLE(20) : YLE(20) : ZLE(20)
4  * XLE(20) : XVT(50) : YVT(50) : ZVT(50) : XP(20) : YP(20) : ZP(20)

LOGICAL ROOT : OUTER
COMMON DA(5000)
EQUIVALENCE (DA(3432), XP0) : (DA(3433), YPO) : (DA(3434), ZPO)
NP=NTIP
N4=4*NTIP
DO 10 I=1, N4
10 ARRAY(I)=PERIM(I)
N=2*NTIP-NFESPT
DO 20 I=1, N
NP=NP+1
11=4*(NTIP+I-1)+1
N2=4*(NTIP-I)+1
ARRAY(N1)=ARRAY(N2)+ARRAY(N2+1)
ARRAY(N1+1)=ARRAY(N2+1)
ARRAY(N1+2)=ARRAY(N2+2)
ARRAY(N1+3)=0.0
IF (ROOT) CALL ATTACH(XVT, YVT, ZVT, 50, 1)
MNATT=MP3
N=NTIP-N
IF (N.EQ.0) GO TO 35
DO 30 I=1, N
NP=NP+1
N1=1+4
N2=1+I+1+4
ARRAY(N1)=PERIM(N2)
ARRAY(N1+1)=PERIM(N2+1)
ARRAY(N1+2)=PERIM(N2+2)
ARRAY(N1+3)=Q: 0
IF (I .NE. N) GO TO 30
IF (.NOT. ROOT) GO TO 30
ARRAY(N1) = ARRAY(N1) + XPO
CALL CODIM(XVT, YVT, NNATT, ARRAY(N1), ARRAY(N1+1), 1)
CALL CODIM(XVT, ZVT, NNATT, ARRAY(N1), ARRAY(N1+2), 1)
ARRAY(N1) = ARRAY(N1) - XPO
ARRAY(N1+1) = ARRAY(N1+1) - YPO
ARRAY(N1+2) = ARRAY(N1+2) - ZPO
30 CONTINUE
35 NPERPT = NP
NP4 = 4 * NP
WRITE(6, 100) NTIP
WRITE(6, 100) NSPACE
WRITE(6, 100) NPERPT, (I, ARRAY(I), I=1, NP4)
100 FORMAT(*OPANEL PERIMETER*/I4/*I4,1PE15.5))
RETURN
END
SUBROUTINE MATELA (TETA, NETA, META, STARAY, ETA, SLCF)
SUBROUTINE TO BUILD TETA MATRIX

TETA = TETA MATRIX
NW = NUMBER OF SPECIAL LATERAL CONSTRAINT FUNCTIONS
META = NUMBER OF ETA STATIONS
STARAY = LIST OF LATERAL CONSTRAINT FUNCTIONS
ETA = ARRAY OF ETAS
SLCF = LIST OF SPECIAL LATERAL CONSTRAINT FUNCTIONS

DIMENSION TETA(NETA, NW), STARAY(NW), ETA(NETA), SLCF(1)
REAL H
INTEGER PTR

EQUVALENCE (RI*ETAI), (RO*ETAO)
DATA PI02/1.5707 96326 79489/
DO 2J INW=1, NW
IF (STARAY(INW)-100.) 1, 21, 21
1 TETA(IETA, INW) = SQRT ( 1.*C - ETA( IETA )**ETA( ETA ) ) * ETA( ETA )
2 CONTINUE
20 CONTINUE
RETURN
21 NWI = INW
DO 80 INW=NWI, NW
PTR = STARAY(INW) - 4779
GET INFO OUT OF SPECIAL LATERAL CONSTRAINT FUNCTION TABLE
ETAB = SLCF(PTR+1)
R1 = SLCF(PTR+2)
R0 = SLCF(PTR+3)
IF (SLCF(PTR))22*31,22
22 THI = ACOS(ETAI)
THO = ACOS(ETAO)
DO 30 IETA=1, NETA
TH = ACOS(ETA(IETA))
30 TETA(IETA, INW) = M(TH, THI) - M(TH, THO)
GOTO 80
31 A21 = 1 - RO
A5 = 1 / RI
A9 = 1 - ETAB
A35 = (A9 + RI)*A5
A35B = ACOS(ETAB - RI)
THB = ACOS(ETAB)
IF(A21-ABS(ETAB))32*32*41
32 A50 = A9 * A5 + 1.0
DO 40 IETA=1* NETA
TH = ACOS(ETA(IETA))
40 TETA(IETA, INW) = A35 * P(TH, A35B) - A50 * P(TH, THB)
GOTO 80
41 A8 = RI+RO
A11 = RI*RO
A18 = AB/A11
A22 = A21-ETAB
A 51 = A18 * A 9
A27B = ACOS(ETAB+RO)
IF(RI-ABS(ETAB))42*42*51
42 A50 = A22 * A5
A26B = ACOS(ETAB-RO)
DO 50 IETA=1* NETA
TH = COS(ETA(IETA))
50 TETA(IETA, INW) = A50 * P(TH, A27B) + A35*P(TH, A35B) - P(TH, THB) * A52
GOTO 80
51 A 50 = A 22 * ( A 18 - A 5)
A 52 = 1.0 - ETAB * A5
DO 60 IETA=1* NETA
TH = COS(ETA(IETA))
60 TETA(IETA, INW) = P(TH, A27B) * A 50 + A52 * M(TH, TH2) + 1 A5 * P(TH, PI02) - P(TH, THR) * A51
80 CONTINUE
RETURN
C BLAINE D. GAITHER 9/72
END
SUBROUTINE TXOC(NU, THETA1, NF, THETAK, NK, NCOL, NXOC,  
1 SCRTCH, SCRCH, NROOT, THET, XCI, X CJ)
   CNU - NU FOR THIS PANEL
   CTHTHETA1 - ARRAY OF THETAS
   CTHETAK - ARRAY OF THETAKS
   CNF - NUMBER OF THETA1 S
   CNK - NUMBER OF THETAK S
   CNCOL - NU+NF+NK
   CNXOC - NUMBER OF X OVER C S
   CSCRCH - T(XOC) RESULT
   CSCRCH - SCRCH ARRAY IF NROOT > 0 MATRIX FOR USE IN ROOT SECTION
   CNROOT - NUMBER OF ELEMENTS IN THE ROOT SECTION
   CTHET - SCRCH MATRIX THAT THETHAS ARE PUT 1 TO
   CXCI - ARRAY OF X OVER C S 1/4 PANEL
   CXCJ - ARRAY OF X OVER C S 3/4 PANEL

   CDIMENSION THETA(NF), THETAK(NK),
   1 SCRTCH(NXOC, NCOL), SCRCH(NXOC, NCOL),
   3 XCI(1), X CJ(1)
   CALL FILLNU(SCRTCH(1,1), NXOC, NXOC*NU, XCI, THET)
   CALL FILNFK(SCRTCH(1, NU+1), NF, NXOC, THETA1, THET)
   CALL FILL FK(SCRTCH(1, NU+NF+1), NK, NXOC, THETAK, THET)
   CALL EFIL(XCI, X CJ, NXOC, SCRCH)
   CALL GELG(SCRTCH, SCRCH, NXOC, NCOL, 1.0F-12, I)
   IF(I .EQ. 11, 19, 11)
11 FORMAT(*0GELG ERROR*, 13)
16 IF(NROOT.LT.1) RETURN
19 DO 20 IUFK=1, NCOL
   TEMP = 0.0
   DO 20 IXOC=1, NXOC
   TEMP = TEMP + SCRCH(IXOC, IUFK)
20 SCRCH(IXOC, IUFK) = TEMP
RETURN
C BLAINE D. GAITHER 9/72
END
SUBROUTINE PANFNC(KL,ETV,ETC)

C IWC=WING-CONTOUR INDICATOR
C =0 FOR LOCAL ANGLES OF ATTACK, ALPHA, GIVEN.
C =1 FOR DEFLECTIONS, Z/C GIVEN.
DIMENSION ETV(1),ETC(1)
COMMON DA(5000)
COMMON/_BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1 *YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)
2 *XCO(20),XLE(20),YLE(20),ZLE(20)
3 *XTT(20),YTT(20),ZTT(20),SLE(20),XJ(20),YJ(20),ZJ(20)
4 *ETL(20),XVT(50),YVT(50),ZVT(50),XR(20),YR(20),ZR(20)
5 *SMT(1000),SYM(1000),SZHT(1000),DYS(1000),DNS(1000)
6 *TS(1000),XSS(1000),YS(1000),ZSS(1000),SIGMA(1000)
COMMON/CONPTS/ XQ(1320),YQ(1320),ZQ(1320)
1 *XN(1320),YN(1320),ZN(1320)
COMMON/PANEL/ NPAN,IPSYM,IWC,NBVP,NTVP,LNCFP,LTCP,LNCP,LTCP
1 *NPERP,NSPACE,NATTCH,TRATT,NPRCLN,NPRCLT,WCXCN,WCET,NTHXC
2 *NHT,NTP,CHTP,RDOT,OUTER,KNATT
3 *MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
EQUIVALENCE(ALPHA,DA(3600)), (XTS,T,D4(4100)), (XR,DA(3600))
1 * (ETAT,DA(3631)), (FNX,DA(3600)), (FNTAI,DA(3630))
2 * (FNX,DA(3437)), (FN3C,DA(3442))
C DUMS(1504) PRESERVES /SCRAT/ FROM PANEL2.
COMMON/SCMAT/ DUMS(1504),ALP(400),TW(20)
1 *DYI(400),DZI(400),THK(820),XC(50),XCTH(51)
2 *DUMX(101),DUMY(202),DUMZ(101),XP(40),ZP(40),CH(21),CH(20)
DIMENSION ALPHA(440),TWIST30,XA(29),ETAI(29)
EQUIVALENCE(DA(4600),PXCV), (DA(4131),XTHCK), (DA(4120),FXTCHK)
1 * (DA(4161),ETATHK), (DA(4160),FTETHK), (DA(4190),THICK)
DIMENSION PXCV(40),XTHICK(29),ETHICK(29),THICK(410)
EQUIVALENCE(DA(4640),PETV)
COMMON /COMPRS/ BETAV
DIMENSION PETV(40)
COMMON/SLOPE/SIGMA(500), DZXT(500),DZNXC(500),TANPB(500)
EQUIVALENCE(MP5,LSIGMA,L5)
EQUIVALENCE(DA(7),XCG), (DA(8),YCG), (DA(9),ZCG)
1 * (DA(11),BETAV), (DA(12),PETAV), (DA(13),QSTAR), (DA(14),RSTAR)
EQUIVALENCE(DA(4720),CPLT), (DA(4580),CPLN)
DIMENSION CPLT(40), CPL2(40)
EQUIVALENCE(B,DA(3423)),(REPA,DA(3421)),(CHRM,DA(3422))
1 ,(SPCF,DA(4880))
DIMENSION SPCF(40)
COMMON/NUMBER, NLOOK(50)
IF (NPA,.EQ.,1) MP5=0
C MP5 WILL BE TOTAL NO. OF SOURCE POINTS.
C
NXA=FNXA
NETAI=FNETHAI
NXC=FNXC
NEC=FNEC
NXC2=2*NXC
NXTHK=FN4XTHK
NETTHK=FETHHK
WRITE(6,100)(ETV(I),I=1,4)
WRITE(6,100)(ETC(I),I=1,4)
100 FORMAT(*0PANFNC*/(1PE15.6))
INO=100
IN1=INO+1
DO 1 I=1,IN1
1 DUMX(I)=(I-1)*.001
NT1=NTVVP+1
K=0
DX=0.01
DO 10 I=1,INCPP
K=K+1
XP(K)=XCO(I)-DX
K=K+1:
10 XP(K)=XCO(I)+DX
IF(NXA.NE.0) GO TO 33
N1=LNCPP*NEC
DO 31 I=1,N1
31 ALP(I)=0.0
DO 32 I=1,NEC
32 TW(I)=0.0
GO TO 34
33 CONTINUE
WRITE(6,100)(XP(I),I=1,20)
DO 2 J=1*NETAI
J1=(J-1)*NXA+1
J2=(J-1)*IN1+1
CALL CODIMX&ALPHA(J1),NXA,DUMX,DUMY(J2),IN1
IF(IWC*EQ.1)GO TO 2
CALL QTFG(DUMX,DUMY(J2),DUMY(J2),IN1)
CONTINUE
WRITE(6,100)(DUMY(I),I=1,202)
CALL XLINE(PETV,CHORD,NT1,ETAI,CH,NETAI)
CALL XLINE(PETV,CHORD,NT1,ETC,CHC,LTCPP)
WRITE(6,100)(CHORD(J),J=1,NT1)
WRITE(6,100)(CH(J),J=1,NETAI)
WRITE(6,100)(CHC(J),J=1,LTCPP)
WRITE(6,100)(ETAI(J),J=1,NETAI)
WRITE(6,100)(DUMX(J),J=1,IN1)
K1=0
L=2
DO 6 I=1,NEC
IF(ETC(I)*GT.ETAI(L))GO TO 5
LM1=L-1
D1=ETC(I)-ETAI(LM1)
D2=ETAI(L)-ETAI(LM1)
R=D1/D2
L1=(LM1-1)*IN1
L2=L1+IN1
K=0
DO 4 J=1,IN1
L1=L1+1
L2=L2+1
ZZZ=CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
K=K+1
4 DUMZ(K)=ZZZ/CHC(I)
LN2=2*LNCPP
WRITE(6,100)CH(LM1),CH(L),R
CALL CODIM(DUMX,DUMZ,IN1,XP,ZP,LN2)
WRITE(6,100)(DUMZ(J),J=1,IN1)
WRITE(6,101)L=2
WRITE(6,100)(XP(J),ZP(J),J=1,LN2)
DO 20 J=1,LC1
20 2
K1=K1+1
J2=2*J

20 ALP(K1)=(2P(J2)-ZP(J2-1))*0.5/DX
WRITE(6*100)(ALP(J),J=1*K1)
GO TO 6

5 L=L+1
GO TO 3

6 CONTINUE
WRITE(6*100),(ETA(I),TWIST(I),I=1,NETAI)
CALL CODIM(ETA,TWIST,NETAI,ETC,TW,NEC)

34 CONTINUE
L=0
DO 9 J=1,NXC
   L=L+1
   XCTH(L)=PXCV(J)
   XC(L)=0.75*PXCV(J)+0.25*PXCV(J+1)
   L=L+1
   XCTH(L)=0.5*(PXCV(J)+PXCV(J+1))
   XC(L)=0.25*PXCV(J)+0.75*PXCV(J+1)
   XCTH(L+1)=PXCV(NXC+1)
   CALL FINDF(XC,NXC2,ETV,NTVVP,DSY,XCUP,L,CPP,ETC,NEC,DYI)
   WRITE(6*100),(DYI(I),I=1,80)
   CALL FINDF(XC,NXC2,ETV,NTVVP,DZI,XXCU,L,CPP,ETC,NEC,DZI)
   WRITE(6*100),(DZI(I),I=1,80)
   CALL XLINE(PTV,CHORD,NT,Etv,CHC,NTVVP)
   IF(NXTHK,NE=0)GO TO 8
   NT=NTVVP*NXC2
   DO 7 I=1;NT
   LSIGMA=LSIGMA+1
   7 SIGMA(LSIGMA)=0.0
   GO TO 81

8 CONTINUE
   CALL XLINE(PTV,CHORD,NT,ETATHK,CH,NETT,HK)
   DO 102 J1=1,NETT+1
      J2=(J1)-1
      CALL CODIM(XTHICK,THICK(J1),XTHK, Conte,NTVVP,NTVVP(J1))
      L2=2
      DO 106 I=1,NTVVP
103 IF (ETV(I) .LT. ETATHK(L)) GO TO 105
   L1 = L - 1
   D1 = ETV(I) - ETATHK(L + 1)
   D2 = ETATHK(L) - ETATHK(L + 1)
   R = D1/D2
   L1 = (L1 - 1) * IN1
   L2 = L1 + IN1
   K = 0
   DC 104 J = 1, IN1
   L1 = L1 + 1
   L2 = L2 + 1
   TH = CH(L + 1) * DUMY(L1) + R * (CH(L) * DUMY(L2) - CH(L + 1) * DUMY(L1))
   K = K + 1
104  DUMZ(K) = TH/CHC(I)
   L3 = (I - 1) * (NXC2 + 1) + 1
   CALL CODIM(DUMX, DUMZ, IN1, XCTHK, THK(L3), NXC2 + 1)
   GO TO 106
105  L = L + 1
   GO TO 103
106  CONTINUE
    K1 = 0
    K = KL
    L2 = 1
    DO 110 I = 1, NEC
      MT = (CPLT(I) - 1.0) * (NXC2 + 1) + 1
      DO 110 J = 1, LNCPP
        N1 = CPLN(U) * 2.0 + C001
        M = T + N1
        ST = (THK(M + 1) - THK(M)) / (XCTHK(M + 1) - XCTHK(M + 1))
        K = K + 1
        TANP1(K) = 1.0 + TS(L2) ** 2 * FETA ** 2
        L2 = L2 + 2
      110 CONTINUE
    31  CONTINUE
    WRITE(6*100)(X(I), I = 1, NXC2)
    WRITE(6*100)(XCTHK(I), I = 1, NXC2)
WRITE(6,100)(XCG(I)*I=1,LNCPP)
WRITE(6,100)(THK(I)*I=1,30)
K1=0
K=K+1
DO 40 I=1,NEC
   TW1=TW(I)
   DO 40 J=1,LNCPP
      K=K+1
      K1=K1+1
      SQ1= SQRT(DY1(K1)**2 + DZ1(K1)**2)
      YN(K) = -DZ1(K1)/SQ1
      ZN(K) = +DY1(K1)/SQ1
C XN = DZ/UX + TWIST * OR ALPHA+TWIST
      TD=0.0
      XN(K) = (-ALP(K1)+TW1+TD)*BETAM
101 FORMAT (1H*31S,J1PE15.5)
40 CONTINUE
   IF(NXT,HK*EQ.0.0) GO TO 23
C C
C TEMPORARY CBAR,BREF
   CBAR=1.0
   BREF=1.0
C
   L2=0
   DO 22 I=1,NTVPV
      DO 21 J=1,NXC2
         LS=LS+1
         L2=L2+1
         YY=YSS(L2)-YCG
         ZZ=ZSS(L2)-ZCG
         VX=1.0-2.0*(QSTAR*ZZ/CBAR - RSTAK*YY/BREF)
         PP = SQRT(1.0+TS(LS)**2)
         SIGMA(LS) = 2.0*CHC(I)*BETAM*(THK(L2+1)-THK(L2))**VX/PP
         AL=0.5*PLL(LS)**2
      21 SIGMAP(LS) = 0.5*BETAM*(THK(L2+1)-THK(L2))**AL/(XC(J)**2-XC(J)**2)
      22 L2=L2+1
   23 CONTINUE
5001 FORMAT(*6, SIGMA(I) = 1:280)
5001 WRITE(6, *6001) (SIGMA(I), I=1:280)
5001 WRITE(6, 1000) (SIGMA(I), I=1:40)
5001 WRITE(6, 6000) NLOOK
5000 FORMAT(*6000) NLOOK
5000 FORMAT(*6000) NLOOK IN PANAC8/(1015)
5000 CALL XLINE(EFLX, YLE, NTIP, PETV, DUMY, NT1)
5000 CALL XLINE(EFLX, ZLE, NTIP, PETV, DUMZ, NT1)
5000 BS = 0.0
5000 DO 200 I = 1, NTVYP
5000 DUX(I) = SQRT((DUMY(I) + 1 - DUMY(I)) * 2 + (DUMZ(I) + 1 - DUMZ(I)) * 2)
5000 BS = BS + DUX(I)
200 CONTINUE
200 CALL XLINE(EFLX, YLE, NTIP, ETC, DUMY, LTPPP)
200 CALL XLINE(EFLX, ZLE, NTIP, ETC, DUMZ, LTPPP)
200 CALL XLINE(EFLX, XLE, NTIP, ETC, XLP, LTPPP)
200 WRITE(12) B, REFA, LTPPP, (ETC(I) = 1:50) LNCPP, XCC, BYI, BZI, T =, CHC
200 , CHC, (XP(I), DUMY(I), DUMZ(I), I = 1: LTPPP), CPLT, SPF, BS, DUX
200 RETURN
200 END
REAL FUNCTION P(T,TS)
1  S = SIN(.5*(TS-T))
   SD= SIN(.5*(TS+T))
   IF(S*SD) 4*3*4
   T = T + 1.0E-6
   GOTO 1
4  C = COS(.5*(TS+T))
   CD= COS(.5*(TS-T))
   IF(C*CD) 8*3*8
   CTS = COS(TS)
   CT = COS(T)
   CTSPCT = CTS + CT
   P = ((CTS-CT)*ALOG(ABS(S/SD)) + CTSPCT*CTSPCT*ALOG(ABS(C/CD)) + 
   1(4.0*TS*CTS-2.0*SIN(TS)*SIN(T)) /6.2921 85207 17958*(1.0-CTS))
   IF(LEGVAR(P)) 3*9*3
9  RETURN
C  BLAINE D. GAITHER 9/72
END
REAL FUNCTION SIN(TH,THS)

1 S = SIN((THS-TH)*5)  
SD= SIN((THS+TH)*5)  
IF(S=SD) 3*2*3

2 TH = TH + 1.0E-6
GOTO 1

3 C = COS((THS+TH)*5)  
CD= COS((THS-TH)*5)  
IF(C*CD) 4*2+4

4 CTHS = COS(THS)
CTH = COS(TH)

M = 3.1830 88861 83790E-1  
((CTHS-CTH)*ALOG(ABS(S/SD)) + C*CD-CTH)*2

1ALOG(ABS(C/CD)) + 2.0*THS*SIN(TH))
IF(LEGVAR(M)) 2*5+2

5 RETURN

C BLAINE D. GAITHER 9/72
END
SUBROUTINE GELG(R,A,M,N,EPS,IER)

SUBROUTINE GELG

PURPOSE
TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS*

USAGE
CALL GELG(R,A,M,N,EPS,IER)

DESCRIPTION OF PARAMETERS
R - THE M BY N MATRIX OF RIGHT HAND SIDES.  (DESTROYED)
    ON RETURN R CONTAINS THE SOLUTION OF THE EQUATIONS
A - THE M BY M COEFFICIENT MATRIX.  (DESTROYED)
M - THE NUMBER OF EQUATIONS IN THE SYSTEM
N - THE NUMBER OF RIGHT HAND SIDE VECTORS
EPS - AN INPUT CONSTANT WHICH IS USED AS RELATIVE
    TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE
IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS
    IER=0 - NO ERROR
    IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
              PIVOT ELEMENT AT ANY ELIMINATION STEP
              EQUAL TO 0
    IER=K - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
              CANCE INDICATED AT ELIMINATION STEP K+1
              WHERE PIVOT ELEMENT WAS LESS THAN OR
              EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
              ABSOLUTELY GREATEST ELEMENT OF MATRIX A

REMARKS
INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE
IN M*N RESP.  M*M SUCCESSIVE STORAGE LOCATIONS.  ON RETURN,
SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
ARE DIFFERENT FROM 0.  HOWEVER WARNING IER=K - IF GIVEN -
INDICATES POSSIBLE LOSS OF SIGNIFICANCE.  IN CASE OF A WELL
SCALING MATRIX A AND APPROPRIATE TOLERANCE
INTERPRETED THAT MATRIX A HAS THE RANK K.
GIVEN IN CASE M=1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE

METHOD
SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION
COMPLETE PIVOTING.

DIMENSION A(1), R(1)
IF (M) 23*23+1

SEARCH FOR GREATEST ELEMENT IN MATRIX A
1 IER=0
PIV=0, M=M*M
NM=N*M
DO 3 L=1, MM
TB=ABS(A(L))
IF (TB=PIV) 3, 3+2
2 PIV=TB
I=L
3 CONTINUE
TOL=EPS*PIV
A(1) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(I).

START ELIMINATION LOOP
LST=1
DO 17 K=1, M

TEST ON SINGULARITY
IF (PIV) 23*23+3

17 CONTINUE

1 2 3 4

0 1 2 3 4

5 6 7 8 9
4 IF(IER)7,5,7
5 IF(PIV-TOL)6,6,7
6 IER=K-1
7 PIVI=1./A(I)
   J=(I-1)/M
   I=I-J*M-K
   J=J+1-K
   I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
8 R(L)=TB
C
C IS ELIMINATION TERMINATED
IF(K-M)9,18,18
C
C COLUMN INTERCHANGE IN MATRIX A
9 LEND=LST+M-K
   IF(J)12,12,10
10 II=J*M
   DO 11 L=LST,LEND
      TB=A(L)
      LL=L+II
      A(L)=A(LL)
      A(LL)=TB
11
C ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
12 DO 13 L=LST,M,M
      LL=L+I
      TB=PIVI*A(LL)
      A(LL)=A(L)
13 A(L)=TB
C
C SAVE COLUMN INTERCHANGE INFORMATION
A(LST)=J
C ELEMENT REDUCTION AND NEXT PIVOT SEARCH
PIV=0
LST=LST+1
J=0
DO 16 II=LST*LEND
PIV=-A(II)
IST=II+M
J=J+1
DO 15 L=IST*MM+M
LL=L-J
A(L)=A(L)+PIV*A(LL)
TB=ABS(A(L))
IF(TB-PIV)15,15,14
14 PIV=TB
I=L
15 CONTINUE
DO 16 L=K+NM+M
LL=L+J
16 R(LL)=R(LL)+PIV*R(L)
17 LST=LST+M
C END OF ELIMINATION LOOP
C
C BACK SUBSTITUTION AND BACK INTERCHANGE
18 IF(N-1)23,22,19
19 IST=NM+M
LST=M+1
DO 21 I=2*M
II=LST-I
IST=IST-LST
L=IST-M
L=A(L)+J
DO 21 J=II+NM+M
TB=R(J)
LL=J
DO 20 K=IST*MM+M
LL=LL+1
20 TB=TB-A(K)*R(LL)
K=J+L

2 12710
2 12720
2 12730
2 12740
2 12750
2 12760
2 12770
2 12780
2 12790
2 12800
2 12810
2 12820
2 12830
2 12840
2 12850
2 12860
2 12870
2 12880
2 12890
2 12900
2 12910
2 12920
2 12930
2 12940
2 12950
2 12960
2 12970
2 12980
2 12990
2 13000
2 13010
2 13020
2 13030
2 13040
2 13050
2 13060
2 13070
2 13080
2 13090
R(J)=R(K)
21 R(K)=TB
22 RETURN

C
C  ERROR RETURN
23 IER=-1
RETURN
END
SUBROUTINE QTGF(X,Y,Z,NDIM)

SUBROUTINE QTGF

PURPOSE
TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN
GENERAL TABLE OF ARGUMENT AND FUNCTION VALUES.

USAGE
CALL QTGF (X,Y,Z,NDIM)

DESCRIPTION OF PARAMETERS
X - THE INPUT VECTOR OF ARGUMENT VALUES.
Y - THE INPUT VECTOR OF FUNCTION VALUES.
Z - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE
     IDENTICAL WITH X OR Y.
NDIM - THE DIMENSION OF VECTORS X,Y,Z.

REMARKS
NO ACTION IN CASE NDIM LESS THAN 1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE

METHOD
BEGINNING WITH Z(1)=0, EVALUATION OF VECTOR Z IS DONE BY
MEANS OF TRAPEZOIDAL RULE (SECOND ORDER FORMULA).
FOR REFERENCE, SEE
F. B. HILDORNS, INTRODUCTION TO NUMERICAL ANALYSIS,
MCGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP. 75.

DIMENSION X(1),Y(1),Z(1)

2 13190
2 13200
2 13210
2 13220
2 13230
2 13240
2 13250
2 13260
2 13270
2 13280
2 13290
2 13300
2 13310
2 13320
2 13330
2 13340
2 13350
2 13360
2 13370
2 13380
2 13390
2 13400
2 13410
2 13420
2 13430
2 13440
2 13450
2 13460
2 13470
2 13480
2 13490
2 13500
2 13510
2 13520
2 13530
2 13540
2 13550
2 13560
2 13570
<table>
<thead>
<tr>
<th>SUM2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>I (NDIM-1)</td>
<td>3</td>
</tr>
</tbody>
</table>

**INTRODUCTION LOOP**

1. \( \text{SUM1} = \text{SUM2} \)
2. \( \text{SUM2} = \text{SUM2} + \text{SUM1} \)
3. \( \text{RETURN} \)
4. \( \text{END} \)

\( X(Y-1)(Y-1) \)
SUBROUTINE EFIL( XCJ, XCI, NI, E)

C FILL E MATRIX

C XCI = .25 PANEL
C XCJ = .75 PANEL

DIMENSION E(NI,NI), XCI(1), XCJ(1)

NJJ=NI-1

DO 10 J=1, NJJ
DO 10 I=1, NI

10 E(J,I) = 1.0/(XCJ(J)-XCI(I))

DO 20 I=1, NI

20 E(NI,I) = 1.0

RETURN

C BLAINE D. GAITHER 9/72

END
SUBROUTINE FILNFK(W, NF, NJ, THETA, FLAG, THET)
SUBROUTINE TO FILL THE NF OR NK SECTION OF THE W MATRIX

W  - NF MATRIX
NF  - NUMBER OF COL IN W
NJ  - NUMBER OF RQS IN NF!
THETA - ARRAY OF UNIQUE THETA FOR KS
FLAG  - LOGICAL VARIABLE T(WORK ON NF) FIWORK ON NK
THET  - ARRAY OF THETAS FROM FILNNU

DIMENSION W(NJ*NF), THETA(1), THET(1)
LOGICAL FLAG
DATA PI/ 3.1415 92653 58979/
IF(NF.LE.0) RETURN
I = 1
NJJ = NJ - 1
DO 400 M=1, NF
  TH = THETA(M)
  THMPI = TH - PI
  DO 300 J=1, NJJ
  IF((FLAG.AND.(THET (J).GE.TH)) .OR. (.NOT.FLAG).AND.(THET (J).GT.TH2)) GOTO 221
  111) GOTO 221
  W(J*I) = THMPI
  GOTO 300
221 W(J*I) = TH
300 CONTINUE
  W(NJ*I) = .5 * SIN(TH)
400 I=I+1
RETURN

C BLAINE D. GAITHER 9/72
END
SUBROUTINE FILLNU(NUM, NU, NUE, XC, SCRTCH)
SUBROUTIN TO FILL THE NU SECTION OF THE W MATRIX

NUM  =  NU MATRIX
NJ  =  NUMBER OF ROWS OF NU
NUE  =  NUMBER OF ELEMENTS IN NU (NJ*NJ)
XC  =  ARRAY OF X OVER C S
SCRTCH = ARRAY OF THETAS

THIS ROUTIN SHOULD BE CALLED ONCE (FOR THE ROUTINE WITH THE LARGEST NU)

REAL NUM(NUE), XC(NJ), SCRTCH(NJ)
NUM = NUE/NJ
NJ = NJ - 1
DO 10 I=1, NJ
NUM(I) = 1.0
10 SCRTCH(I) = A COS(1.0 - 2.0*XC(I))
NUM(NJ) = 0.5
IF (NUE LE. NJ) RETURN
NUE = NUE - 1
RI = 0.0
DO 30 J=NJ, NUE, NJ
RI = RI + 1.0
DO 40 L=1, NJ
40 NUM(J+L) = -COS((RI) * SCRTCH(L))
IF(J-NJ)21*21*22
21 NUM(J+NJ) = .25
GOTO 30
22 NUM(J+NJ) = 0.0
GOTO 30
CONTINUE
RETURN
BLAINE D. GAITHER 9/72
END
FUNCTION TANDEL(ETA, XOC)

COMMON STATEMENT THAT PUTS FLP(1*1) ON THE FLAPS

C THIS FUNCTION RETURNS TAN(DELTA) IF A CP FALLS ON A FLAP,

C OTHERWISE ZERO

DIMENSION FLP(8,1)

NF = 1

TANDEL = 0.0

7 IF (FLP(1*NF)) 2*1, 2

2 IF (FLP(3*NF) - ETA) 3*3, 4

3 IF (FLP(4*NF) - ETA) 4*5, 5

5 T = FLP(5*NF) + (FLP(6*NF)-FLP(5*NF))*(ETA - FLP(3*NF)) / 

1 (FLP(4*NF) - FLP(3*NF))

6 IF ((XOC - T) * FLP(1*NF)) 6*6, 4

4 NF = NF + 1

GOTO 7

6 TANDEL = TAN( FLP(2*NF) * 0.0174 53292 51994 32957)

1 RETURN

C BLAINE D. GAITHER OCT 72

END
SUBROUTINE FINDF(X,NX,E,NE,F,XC,EC,NEC,FC)
C PROGRAM TO INTERPOLATE FOR A FUNCTION AT A GIVEN (X/C*ETA) POINT.
C GRID UPON WHICH INTERPOLATION IS MADE IS GIVEN BY
C X = GIVEN VALUES OF X/C.   NX=NUMBER OF X VALUES.
C E = GIVEN VALUES OF ETA.   NE=NUMBER OF ETA VALUES.
C THE INPUT GRID IS DESCRIBED BY ONE SET OF X/C VALUES
C AND ONE SET OF ETA VALUES.
C F = GIVEN VALUES OF A FUNCTION OF (X,E), GIVEN FIRST LONGITUDINALLY
C AND THEN SPANNING (EX. IF NX=10,NE=5, F(42) IS AT X(2) AND E(41) )
C (NUMBER OF F VALUES TO BE AVAILABLE MUST EQUAL NX*NE )
C.
C XC = XC ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
C NXC = NUMBER OF XC GIVEN, TO MAXIMUM OF 20. (DIMENSION OF F1*F2 = 20)
C EC = ETA ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
C NEC = NUMBER OF EC GIVEN.
C FC = INTERPOLATED FUNCTION VALUES, GIVEN CHORDWISE THEN SPANNING.
C NUMBER OF OUTPUT VALUES OF FC = NXC*NEC
C.
C INTERPOLATION IN THE X DIRECTION IS MADE BY SUBROUTINE CORDIN.
C INTERPOLATION IN THE ETA DIRECTION IS LINEAR.
C.
DIMENSION X(1),E(1),XC(1),EC(1),F(1),FC(1)
DIMENSION F1(20),F2(20)
NE1=NE-1
L=0
DO 5 1,J,NEC
E1=EC(J)
DO 5 1,E=1,NE1
IEL=IE
IF(E1*GT.*E(I+1)) GO TO 5
ETA1=E(IEL)
ETA2=E(IEL+1)
GO TO 6
5 CONTINUE
6 CONTINUE
DELET=ETA2-ETA1
D = EL-ETA1
I1=(IEL-1)*NX+1
12=I1+NX
CALL CODIM(X*F(I2)*NX*XC+F2*NXC)
CALL CODIM(X*F(I1)*NX*XC+F1*NXC)
DO 50 I=1,NXC
L=L+1
FC(L)=F1(I)+D*(F2(I)-F1(I))/DELET
50 CONTINUE
RETURN
END
SUBROUTINE XLINE(X,Y,N,X1,Y1,N1)
C THIS ROUTINE DOES LINEAR INTERPOLATION AT N1 (X1,Y1) POINTS
C ON SEGMENTS BETWEEN N (X,Y) POINTS (GIVEN).
C ALL X1 MUST BE WITHIN RANGE OF VALUES OF X.
C VALUES OF X MUST BE ALGEBRAICALLY ASCENDING.
C
DIMENSION X(1),Y(1),X1(1),Y1(1)
L2=2
DO 10 I=1,N1
2 IF(X1(I).GT.X(L2)) GO TO 5
3 L1=L2-1
4 DX=X(L2)-X(L1)
5 DY=Y(L2)-Y(L1)
6 Y1(I)=D1*DY/DX+Y(L1)
GO TO 10
5 IF(L2.GE.N) GO TO 3
 L2=L2+1
GO TO 2
10 CONTINUE
RETURN
END
PROGRAM INFLM
COMMON DA(5000)
1 *NX*NXTH*LNVD*TV*HV*TV*NXTHV*NY*NYT(49)
2 *LNDIV*LTDIV*LNPTS*LPTS
COMMON/PANEL/ NP*IPSY*INC*NB*NPV*NPV*LUCP*LTCP*CRP*CRP
1 *NPERD*SPACE*ATTCH*TRATT*NP*PRCL*PRCL*NL*CT*MC*MTET*THAC
2 *NTHET*NTHIP*CTHIP*ROOT*OUTER*WATT
3 *NL1*NL2*NL3*NL4*NL5*NL6
COMMON/NUMBER/ NVPTS(7)*NCT(7)*NL(7)*LCT(7)*LTC(7)*LUC(7)
1 *NCT(7)**NBODS*NS*NL(7)*VLT(7)**TAPE*TAPE*NCT*TAPE
2 *LSEG(7)*TSF(7)*LUNIC(7)*TUFNC(7)
3 *LNDIV(7)*LTDIV(7)*NSAPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)
COMMON /CO*PR* BETA*
COMMON/BODY/B3(31000)
DIMENSION BODYR(21000)
COMMON/COMPTS/ XQ(1320)*YQ(1320)*ZQ(1320)
1 *XH(1320)*YH(1320)*ZH(1320)
EQUIVALENCE (B(1)*BODYR)
EQUIVALENCE (DA(2)*PANS)
EQUIVALENCE (SYM*DA(34261))
EQUIVALENCE (DA(7)*XC*1A(8)*YC*1A(9)*ZC*1A(10)*ALPHA)
1 *(DA(11)*BETA)*DA(12)*PSTAR*DA(13)*QSTAR*DA(14)*RSTAR
COMMON/SCRAT/ XSL(5000)*BOND(5000)*AXR(5000)
1 * AYR(5000)* AZR(5000)
DIMENSION A(5000)
EQUIVALENCE (A*AXB)(B*23)
DIMENSION B(5000)
EQUIVALENCE (SYM*DA(191))
COMMON/PANIF* PANSYM(10)
LOGICAL ROOTT*OUTER
LOGICAL ROOTP*OUTERP
C REWIND UNIT 11 IN THIS PROGRAM. UNIT 11 WILL HAVE START, SYM*ZC.
REWIND 11
REWIND 18
REWIND 23
REWIND 21
REWIND 19
REWIND 12
NCT=MP1
NPANS=NPANS
C TEMPORARY VALUES FOR BREF,CBAR
BREF=1.0
CBAR=10.0
IF(NBODS.EQ.0) GO TO 1111
C READ 2 RECORDS ON 18 TO SPACE OVER LENGTH LENGTH ARRAYS
READ(18) B
READ(18) b
1111 CONTINUE C
NBP=NBODS+NPANS
M=0
DO 10 KK=1,NBP
LTPTS=LTC(KK)
LNPTS=LNC(KK)
DO 10 JJ=1,LTPTS
DO 10 II=1,LNPTS
M=M+1
XX=XQ(M)-XCG
YY = YQ(M) -YCG *BETAM
ZZ = ZQ(M) -ZCG *BETAM
VY = -BETA*BETAM-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
VZ = ALPHABETAM-2.0*(PSTAR*YY/BREF-QSTAR*XX/CBAR)
VX=1.0-2.0*(OSTAR*ZZ/CBAR-RSTAR*YY/BREF)
10 BOUND(N)=XN(M)*VX-YN(M)*VY-ZN(M)*VZ
NS=0
NCTV=0
KCOM=0
IF(NBODS.EQ.0) GO TO 99
DO 50 I=1,NBODS
NB=NB+1
SY=E-SY(I)
NBVW=NBVW(I)
NTVW=NTVW(I)
N3=NBVW+1
N2=NLT(I)
MSEG=LSEG(I)+TSEG(I)
KCOM=KCOM+1.SEG
LNDIVB(I)=1
LNDIV=1

LT(IV)=LTDIVB(I)
READ(18) 55
CALL INFL(NSEG,NEP)
REWIND 19
REWIND 20

CONTINUE

50

C MATRX FOR INFLUENCE OF BODY VORTICES IS ON UNIT 21.
C AX,AY,AZ MATRICES FOR BODY INFLUENCES IS ON UNIT HTAPE (19 OR 20).
C UNIT HTAPE (20 OR 19) IS AVAILABLE FOR ANOTHER USE.

CALL MATA(KCON,NCT,HTAPE,0)
IF(KCON.NE.0).REWIN 23

99 CONTINUE

IF(NPANS.EQ.0) GO TO 141

C

NCLP=0
DO 100 I=1,NPANS
SYM=PANSY(I)
NB=NB+1
NBBV=NBV(NB)
NVT(VP)=NVTV(NB)
ROOT=ROCP(I)
OUTER=OUTERP(I)
WRITE(6,40) I,ROOT,OUTER
WRITE(6,40) I,ROCP(K),OUTERP(K),K=1,NPANS

40

FORMAT(*01=*,14/I0L5)
NSPACE=NSPP(I)
READ(18) BODYSRC
IF(.NOT.ROOT) GO TO 55
NROOT=NSPACE+1
N3=NBBV+1

55 IF(.NOT.ROOT) GO TO 60
NVTOUT=NVTVP-NSPACE

60 CONTINUE

IP=1
CALL PANMAT(XG,YG,ZG,NCT,NCLP,IP)

100 CONTINUE

C XP2 = UNIT FOR STORAGE OF AX,AY,AZ FOR PANEL VORTICES.
NZERO = 0
CALL MATA(NZERO,NCOLP,MP2,1)
RE:IND 21

101 CONTINUE
RE:IND 11
END
SUBROUTINE PANMAT(XO,YO,ZO,NCPL,NCNP,IP)
COMMON DA(5000)
COMMON/BODY/XYR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1  *YVR(10,20),ZVR(10,20),XVL(500),PLT(500),YSURV(100),CHORD(100)
2  *XVR(10,20),XCC(10,20),YLL(20),YLE(20),YVR(10,20)
3  *XTE(20),YTE(20),ZTE(20),XLE(20),XJ2(20),YJ(20),ZJ(20)
4  *ETLE(20),XVT(50),YVT(50),ZVT(50),XRL(20),YRL(20),ZRL(20)
5  *SXM(1000),SYM(1000),XAM(1000),YAM(1000),ZAM(1000)
6  *TS(1000),XS(1000),YS(1000),ZS(1000),SIGMA(1000)
7  *XVS(100),YVS(100),ZVS(100)
COMMON/CONPTS/ DUMP(3960)
1  *XN(1320),YN(1320),ZN(1320)
COMMON/SCRAT/XSOL(5000),AGUN(5000),AXA(5000),AYA(5000),AZA(5000)
EQUALITY(A,XA),(AYA),(AZA),(X3,XSOL(1)),(Y3,XSOL(101))
1  *(Z3,XSOL(201)),(XT,XSOL(301)),(YT,XSOL(401))
2  *(ZT,XSOL(501)),(SUM,XSOL(601))
DIMENSION AX(5000),AY(5000),AZ(5000),X3(100),Y3(100),Z3(100)
1  *XT(100),YT(100),XT(100),SUM(100)
DIMENSION B(3100)
EQUALITY (B,XVR(1,1))
FREE TRAILING VORTEX VARIABLES
XN,YN,ZN, USE SAME SPACE AS UTV, VTV, WTV TO BE COMPUTED IN TRAIL
DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI(500),YTVI(500)
1  *ZTV(500),UTV(1000),VTV(1000),V1(100),V2(100)
2  *V1(100),NIP(100),NTVE(100)
EQUALITY(B,15001),UTV = (B,16001),VTV = (B,17001),WTV
1  *(B,19001),XTV = (B,18001),XTV = (B,20001),ZTV
2  *(B,21001),XTV = (B,21001),XTVI = (B,22001),ZTVI
3  *(B,22501),V1 = (B,22501),V1 = (B,22701),V3
4  *(B,22901),NIP = (B,22901),NIP = (B,22301),HTV
MP9=NTR=NO. OF FREE TRAILING VORTICES
MP10=NTPL=TOTAL NO. OF INITIAL VORTICES
EQUALITY(MP9,HTP)=(MP10,TRV)
COMMON/PANELY :PANY,IPSY,ITC,BVY,ITP,LMCF,P,LTC,LMCP,LTC
1  *PERVER,PSPACE,TATT,TATT,PRCL,PRCL,TXCTC,MTET,TXHGC
2  NTET,HTP,CHTP,ROET,QTET,ATT
3  MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
3  1250
3  1260
3  1270
3  1280
3  1290
3  1300
3  1310
3  1320
3  1330
3  1340
3  1350
3  1360
3  1370
3  1380
3  1390
3  1400
3  1410
3  1420
3  1430
3  1440
3  1450
3  1460
3  1470
3  1480
3  1490
3  1500
3  1510
3  1520
3  1530
3  1540
3  1550
3  1560
3  1570
3  1580
3  1590
3  1600
3  1610
3  1620
COMMON/NUMBER/ NVPTS(7)*NCPTS(7)*NLNY(7)*NLT(7)*LTC(7)*LNC(7) 3 1630
1 *NCT*NBVDOS*PANS*NLVL(7)*NIT(7)*TAPE*NTAPE*ACTV*ITAPE*JTAPE 3 1640
2 *LSEG(7)*TSEG(7)*LFUNC(7)*TFUNC(7) 3 1650
3 *NLVID(7)*LDIVP(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMP(7)
COMMON/PANINF/ PA;SYMP(10)
EQUIVALENCE(DA(3432)*XP0), (DA(3433)*YPO), (DA(3434)*ZPO) 3 1660
DIMENSION XQ(11), YQ(11), ZQ(11) 3 1670
DIMENSION SAVEC(6000), ASSP(1000), AS(1000) 3 1680
EQUIVALENCE(B(15001), SAVEC), (B(21001), ASSP), (B(22001), AS) 3 1690
1 (DA(3439), FNU), (DA(3440), FNU) 3 1700
DIMENSION YPD(200), ZPD(200) 3 1710
EQUIVALENCE(YPD(1), B(30001)), (ZPD(1), B(30201)) 3 1720
REAL LLEGX,LLEGY,LLEGZ 3 1730
LOGICAL FLAG 3 1740
LOGICAL: ROOT, OUTER 3 1750
DATA LSIG/0/
DATA LDD/0/
IF(IP*GT*1) GO TO 999 3 1760
NTR=0 3 1770
NTRV=0 3 1780
NU=FNU 3 1790
NW=FNW 3 1800
999 CONTINUE 3 1810
NVL1=NBVVP 3 1820
PI=3.141592654 3 1830
IC=0 3 1840
NBP1=NBVVP+1 3 1850
IF(NOT.ROOT) GO TO 2 3 1860
C XVR1=XVR(1)*NBP1) 3 1870
IF(XVR1.GE.XVT(INNATT)) GO TO 2 3 1880
DO 1 I=1,NNATT 3 1890
1 CONTINUE 3 1900
C LET IC BE NUMBER OF LONGITUDINAL PANELS IN TRAILING VORTEX SECTION.
N1=NSPACE+1 3 1910
DO 3 I=1,N1 3 1920
X3(I)=XVR(1)*NBPI 3 1930
3 CONTINUE 3 1940
Y3(I)=YVR(I+NP1)
Z3(I)=ZVR(I+NP1)
WRITE(6,101)(X3(I),Y3(I),Z3(I),I=1,N1)
FORMAT(1H0/1H*1P3E20.6))
CALL TVGHI(XVT,YVT,ZVT,NMAT,IC,X3,Y3,Z3,N1,XT,YT,ZT,NT)
WRITE(6,100)NMAT,IC,N1,NT
FORMAT(7H PANMAT/1015)
WRITE(6,101)(XT(I),YT(I),ZT(I),I=1,NT)
IF(I.EQ.1)WRITE(6,502)NCAP
NCAP=MCP(*,14)
NTX=NT/N1
DO 503 I=1,N1
NTR=NTR+1
503 NIP(NTR)=NTX
K=0
DO 504 KK=1,N1
DO 504 KK=1,NTX
K=K+1
NTRV=NTRV+1
XTVI(NTRV)=XT(K)
YTVI(NTRV)=YT(K)
ZTVI(NTRV)=ZT(K)
IF(NTX.NE.KK)GO TO 504
LDD=LDD+1
YPD(LDD)=YTVI(NTRV)
ZPD(LDD)=ZTVI(NTRV)
504 CONTINUE
2 CONTINUE
NIT=NTVNP+1-NSPACE
DO 505 K=1,NIT
IF(NSPACE.NE.0.AND.K.EQ.1)GO TO 505
NTR=NTR+1
NIP(NTR)=1
NTRV=NTRV+1
XTVI(NTRV)=XVS(K)
YTVI(NTRV)=YVS(K)
ZTVI(NTRV)=ZVS(K)
LDD=LDD+1
YPD(LDD)=YTVI(NTRV)
ZPD(LPD)=ZTVI(NTRV)
WRITE(6,7002) (I,YPD(I),ZPD(I),I=1,LPD)
7002 FORMAT(*PANEL DRAG COORDINATES*/(I5,2E15.5))
505 CONTINUE
WRITE(6,800)(XTVI(I),YTVI(I),ZTVI(I),I=1,NTRV)
800 FORMAT(26HXTVI,YTVI,ZTVI, IN PANNAT/(1P3E20.6))
C SET NV=NO. OF FREE TRAILING VORTICES.
NV=NTR
WRITE(6,100) NSPACENVL1
DO 1000 IX=1,NCP
XC=XQ(IX)
YC=YQ(IX)
ZC=ZQ(IX)
MA=0
MS=0
IF(.NOT.ROOT) GO TO 105
DO 104 I=1,NSPACE
DO 104 J=1,NVL1
DO 5 K1=1,3
5 SUM(K1)=0.0
DO 50 K=1,4
IF(J.EQ.NVL1.AND.K.EQ.4) GO TO 41
IF(K.GT.1) GO TO 20
X1=XVR(I,J+1)
Y1=YVR(I,J+1)
Z1=ZVR(I,J+1)
X2=XVR(I,J)
Y2=YVR(I,J)
Z2=ZVR(I,J)
GO TO 40
20 X1=X2
Y1=Y2
Z1=Z2
IF(K-3) 25,30,35
25 X2=XVR(I+1,J)
Y2=YVR(I+1,J)
Z2=ZVR(I+1,J)
GO TO 40
30 X2=X/R(I+1,J+1)
Y2=YVR(I+1,J+1)
Z2=ZVR(I+1,J+1)
GO TO 40
3 2800
3 2810
3 2820
X2=XVR(I,J+1)
Y2=YVR(I,J+1)
Z2=ZVR(I,J+1)
40 CALL VORPAN(SUM1,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)
GO TO 50
41 CONTINUE
3 2830
3 2840
3 2850
3 2860
3 2870
3 2880
3 2890
3 2900
3 2910
3 2920
3 2930
3 2940
3 2950
3 2960
3 2970
3 2980
3 2990
3 3000
3 3010
3 3020
3 3030
3 3040
3 3050
3 3060
3 3070
3 3080
3 3090
3 3100
3 3110
3 3120
3 3130
3 3140
3 3150
3 3160
3 3170
3 3180
40
409 CONTINUE
C NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR RIGHT LEG.
T1=SQR((Y2-YC)**2+(Z2-ZC)**2)
IF(I1=0.00001) 412,411,411
411 CONTINUE
T2=(X2-XC)/SQR((X2-XC)**2+(Y2-YC)**2+(Z2-ZC)**2)
QT=0.25*(1.0-T2)/(PI*T1)
RLEGY=RLEGY-SYM_PAS*QT*(Z2-ZC)/T1
RLEGZ=RLEGZ+SYM_PAS*QT*(Y2-YC)/T1
412 CONTINUE
IF(SYM_PAS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 471
   Y2=Y2H
   GO TO 472
471 SYMPAS=1.0
   Y2=-Y2
   GO TO 409
472 CONTINUE
IF(I1.EQ.1) GO TO 421
LLEGX=-RLEGX1
LLEGY=-RLEGY1
LLEGZ=-RLEGZ1
GO TO 4721
421 DO 431 K1=1,3
431 SUM(K1)=0.0
   DO 461 K1=1,10
      IF(K1.EQ.1) GO TO 441
         X2=X1
         Y2=Y1
         Z2=Z1
         GO TO 451
      X2=XT(1)
      Y2=YT(1)
      Z2=ZT(1)
   441 I1=K1+1
   X1=XT(I1)
   Y1=YT(I1)
   Z1=ZT(I1)
451 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2+XC,YC,ZC)
   LLEGX=SUM(I1)
LLEGX=SUM(P_1)
LLEGY=SUM(P_2)
LLEGZ=SUM(P_3)
Y1H=Y1
SYMPAS=-1.0
4091 CONTINUE

C NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR LEFT LEG.
T1=SQRT((Y1-YC)**2+(Z1-ZC)**2)
IF(T1<0.00001) 470,415,415

415 CONTINUE
T2=(X1-XC)/SQRT((X1-XC)**2+(Y1-YC)**2+(Z1-ZC)**2)
QT=0.25*(1.0-T2)/(PI*T1)
LLEGX=LLEGX+SYMPAS*QT*(Z1-ZC)/T1
LLEGY=LLEGY+SYMPAS*QT*(Y1-YC)/T1
LLEGZ=LLEGZ+SYMPAS*QT*(X1-XC)/T1

470 CONTINUE
IF(SYMPAS.EQ.-1.0.AND.PANSYM(IP).EQ.C) GO TO 4711
Y1=Y1H
GO TO 4721

4711 SYMPAS=1.0
Y1=-Y1
GO TO 4091

4721 CONTINUE
SUM(1)=SUM1+RLEGX+LLEGX
SUM(2)=SUM2+RLEGY+LLEGY
SUM(3)=SUM3+RLEGZ+LLEGZ

480 CONTINUE

C PROVIDE HERE FOR STR. LINE TR. VORTICES WHERE PANEL TR. EDGE IS
C EITHER AT END OF BODY OR BEHIND IT. (ICEU CASE)
50 CONTINUE
MA=MA+1
AX(MA)=SUM(1)
AY(MA)=SUM(2)
AZ(MA)=SUM(3)
DO 90 IS=1,2
MS=MS+1
IV=0
JV=0
T=TS(MS)
DZ=DZS(MS)
DY=DYS(MS)
90 CONTINUE
KSOL=1
CALL PVSK(T, DY, DZ, XC, YC, ZC, MS, MA, I, J, KSOL)
90 CONTINUE
104 CONTINUE
105 CONTINUE
C 50 OUTER PANEL INFLUENCE EQUATIONS AND SOURCE INFLUENCE EQUATIONS.
NVTOUT = NTVPV-NSPACE
DO 200 I=1,NVTOUT
DO 200 J=1,NTVPV
MS = MS + 1
WA = WA + 1
KSOL = 2
110 T = TS(MS)
DZ = DZS(MS)
DY = DYS(MS)
CALL PVSK(T, DY, DZ, XC, YC, ZC, MS, MA, I, J, KSOL)
IF(KSOL .EQ. 1) GO TO 115
KSOL = 1
MS = MS + 1
GO TO 110
115 CONTINUE
200 CONTINUE
IF(DA(5000).LT.0) GO TO 361
WRITE(6,300) IX(I,1), AZ(I,1), IA(I,1)
300 FORMAT(1010), 'COEFFICIENTS FOR CONTROL PT. I3//
I (13,1P3E18.6))
WRITE(6,350) I, SX(I), SY(I), SZ(I)
350 FORMAT(1010), 'SOURCE COEFFICIENTS FOR CONTROL PT. I3//
I (13,1P3E18.6))
361 CONTINUE
C IN SUBROUTINE PARMAT, WRITE THE SXMT, SYMT, SZMT MATRICES OF XII 11.
C THERE WILL BE "NET" ROLES OF SXMT FOR EACH PANEL.
C NET = TOTAL NUMBER OF CONTROL POINTS.
WRITE(11) SXMT(I), SYMT(I), SZMT(I), I = I, 'T'
XN1 = XN(IX)
YN1 = YN(IX)
ZN1 = ZN(IX)
SN = 0
BN = 0
BNY=0.0
J=LSIG
DO 355 I=1,MS
J=J+1
BNX=SIGMA(J)*SXMT(I)+BNX
BNY=SIGMA(J)*SYM(I)+BNY
355 BNX=SIGMA(J)*(X(I)*SXMT(I)) + YN(I)*SYM(I) + 2*N1*SZMT(I) + BN
IF(NBODS.EQ.0) GO TO 358
IF(I*LE*NCPTS(I)) GO TO 359
358 BOUND(I)=BOUND(I)+BN
359 CONTINUE
. IF(NU+NW.EQ.0) GO TO 3060
CALL CONFUN FOR CONSTRAINT MATRIX MULTIPLY.
LOC1=(IP-1)*1000+1
LOC2=LOC1+400
LOC3=LOC2+400
FLAG=.FALSE.
3060 IF(NU.EQ.0)FLAG=.TRUE.
CALL CONFUN(A1,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
1 ,ASSP,NV,TVVP,NBV,VP,AS,NW,FLAG,NU)
3061 IF(FLAG) GO TO 1050
MA=NU*NW
DO 1005 I=1,MA
1005 AX(I)=ASSP(I)
GO TO 1060
1050 MA=NU*NV
DO 1055 I=1,MA
1055 AX(I)=AS(I)
1060 CONTINUE
CALL CONFUN(A2,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
1 ,ASSP,NV,TVVP,NBV,VP,AS,MA,FLAG,NU)
3062 IF(FLAG) GO TO 2050
DO 2005 I=1,MA
2005 AY(I)=ASSP(I)
GO TO 2060
2050 DO 2055 I=1,MA
2055 AY(I)=AS(I)
2060 CONTINUE
CALL CONFUN(A3,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td></td>
<td>IF(FLAG) GO TO 3090</td>
</tr>
<tr>
<td>201</td>
<td></td>
<td>DO 3055 I=1, I+1</td>
</tr>
<tr>
<td>202</td>
<td></td>
<td>3005 AZ(I)=ASSP(I)</td>
</tr>
<tr>
<td>203</td>
<td></td>
<td>GO TO 3060</td>
</tr>
<tr>
<td>204</td>
<td></td>
<td>3055 AZ(I)=AS(I)</td>
</tr>
<tr>
<td>205</td>
<td></td>
<td>CONTINUE</td>
</tr>
<tr>
<td>206</td>
<td></td>
<td>LSIGH=J</td>
</tr>
<tr>
<td>207</td>
<td></td>
<td>IF(IP*NE.*1) GO TO 400</td>
</tr>
<tr>
<td>208</td>
<td></td>
<td>IF(IP*NE.*1) GO TO 260</td>
</tr>
<tr>
<td>209</td>
<td></td>
<td>NCOLP=NCOLP+1</td>
</tr>
<tr>
<td>210</td>
<td></td>
<td>MUNIT=MTAPE</td>
</tr>
<tr>
<td>211</td>
<td></td>
<td>360 CONTINUE</td>
</tr>
<tr>
<td>212</td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>213</td>
<td></td>
<td>C MUNIT IS THE FILE TO BE WRITTEN.</td>
</tr>
<tr>
<td>214</td>
<td></td>
<td>C MUNIT IS THE FILE TO BE READ.</td>
</tr>
<tr>
<td>215</td>
<td></td>
<td>FOR IP=1, MUNIT=MTAPE=19 (OR 20).</td>
</tr>
<tr>
<td>216</td>
<td></td>
<td>FOR IP=2, MUNIT=24</td>
</tr>
<tr>
<td>217</td>
<td></td>
<td>THEREAFTER, FOR IP EVEN, MUNIT AND NUNIT SAME AS FOR IP=2.</td>
</tr>
<tr>
<td>218</td>
<td></td>
<td>FOR IP ODD, MUNIT AND NUNIT VALUES ARE REVERSED.</td>
</tr>
<tr>
<td>219</td>
<td></td>
<td>C</td>
</tr>
<tr>
<td>220</td>
<td></td>
<td>IF(IX*EQ.*1) WRITE(6,500) IP,MTAPE,NCOLP</td>
</tr>
<tr>
<td>221</td>
<td></td>
<td>FORMAT(<em>9IP,MTAPE,NCOLP,1I5)</em></td>
</tr>
<tr>
<td>222</td>
<td></td>
<td>WRITE(MTAPE)(AX(I),XY(I),AZ(I),1=1,NCOLP)</td>
</tr>
<tr>
<td>223</td>
<td></td>
<td>GO TO 900</td>
</tr>
<tr>
<td>224</td>
<td></td>
<td>400 IF(2*IP/2-NE.*IP) GO TO 401</td>
</tr>
<tr>
<td>225</td>
<td></td>
<td>MUNIT=24</td>
</tr>
<tr>
<td>226</td>
<td></td>
<td>GO TO 402</td>
</tr>
<tr>
<td>227</td>
<td></td>
<td>401 MUNIT=MTAPE</td>
</tr>
<tr>
<td>228</td>
<td></td>
<td>402 MC1=NCPRE+1</td>
</tr>
<tr>
<td>229</td>
<td></td>
<td>MC2=NCPRE+1</td>
</tr>
<tr>
<td>230</td>
<td></td>
<td>J=0</td>
</tr>
<tr>
<td>231</td>
<td></td>
<td>DO 404 I=MC1,MC2</td>
</tr>
<tr>
<td>232</td>
<td></td>
<td>J=J+1</td>
</tr>
<tr>
<td>233</td>
<td></td>
<td>AX(I)=AX(J)</td>
</tr>
<tr>
<td>234</td>
<td></td>
<td>AY(I)=AY(J)</td>
</tr>
<tr>
<td>235</td>
<td></td>
<td>404 AZ(I)=AZ(J)</td>
</tr>
</tbody>
</table>
IF (IX.EQ.1) WRITE(6,501) IP,'MUNPR=UNIT+NCOLP

501 FORMAT (90IP,MUNPR,UNIT,NCOLP,'A=',5I5)
READ(*,MUNPR)(AX(I),AY(I),AZ(I),I=1,NCOLP)
IF (IX.EQ.1) NCOLP=NCOLP + 'A'
WRITE (UNIT)(AX(I),AY(I),AZ(I),I=1,NCOLP)
900 CONTINUE
1000 CONTINUE
LS1G=LS1GH
NCOLP=NCOLP
MUNPR=MUNIT
MP2=MUNIT
MP3=NCOLP
REWIND MUNPR
REWIND MTAPE
REWIND 24
RETURN
END
SUBROUTINE INFL(KCH,*NSP)
COMPUTE AX,AY,AZ MATRICES AND BOUNDARY CONDITIONS
COMMON DA(5000)
1 1 NX,NTH,LIVOR,LTVOR,NTV,NBV,NTH(49)
2 LNDIV,LTDIV,LYPT,LPTP
COMMON/NUMBER/ NVPTS(7),NCTP(7),NLN(7),LCLT(7),LNC(7)
1 NCT,NCTP,NPAN,NVNL,NTV,NVNL,NTV,ITAPE,ITAPE,ITAPE
2 LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 LNDIV(3),LTDIV(3),NSFP(7),ROOTP(7),SORTAR(7),SYMM(7)
COMMON/SCY/ XV(151,31),YV(151,31),ZV(151,31)
1 TX(XV(1320),TY(XY(1320),TZ(XZ(1320))
2 TT(XT(1320),TTY(1320),TZY(1320)
COMMON/SCRT/ XQ(1320),YQ(1320),ZQ(1320)
1 XQ(1320),YQ(1320),ZQ(1320)
COMMON/INX/ XY(1320),ZQ(1320)
DIMENSION SUM(3)
EQUIVALENCE (DA(7),XC),(DA(8),YD),(DA(9),ZC),(DA(10),ALPHA)
1 (DA(11),BETA),(DA(12),PST),(DA(13),QST),(DA(14),RST)
EQUAEMENT (DA(15),SY) 
DIMENSION XQ(1320),YQ(1320),ZQ(1320)
DIMENSION BS(3100)
DIMENSION 3 (5000)
EQUIVALENCE (BS,XY(1,1))
EQUIVALENCE (3 ,XY(1,1))
EQUIVALENCE (XQ,YQ) ,5Q ,ZQ)
EQUIVALENCE (N,NCTV) ,11,NCT)
EQUIVALENCE (NTH,XQOL)
DIMENSION ALTHTH(5000)
C IF KCH DOES NOT = 0, ALTHTH IS NEEDED FOR AXYZRL
IFKCH=0.0) GO TO 5
C REIND 18 AND THEN READ ALTHTH ARRAY
REIND 18
READ(18) ALTH
C NOW SKIP 2 RECORDS SO THAT 18 IS POSITIONED TO READ PANEL DATA
READ(18) BB
READ(18) BB
CONTINUE
PI=12.663704
NSTART=NCTV
NCT=0
DO 65 KK=1,NBP
LTPS=LTC(KK)
LNPTS=LHC(KK)
DO 65 JJ=1,LTPS
DO 65 II=1,LNPTS
NCT=NCT+1
L=M
X=XQ(K)
Y=YQ(K)
Z=ZQ(K)
N=NSTART
DO 60 J=1,NTVV
DO 60 I=1,NBVV
N=N+1
T01=0.0
T02=0.0
T03=0.0
DO 55 K=1,4
DO 8 KS=1,3
8 SUM(KS)=0.0
GO TO (10,20,30,40),K
10 CONTINUE
12 III=LMDIV*(I-1)+1
IFI=III
III2=(J-1)*LTDIV
DO 14 L=1,LTDIV
14 CALL VORTEX(SUM)
GO TO 50
20 III2=1+J*LTDIV
IFI2=III2
III=(I-1)*LMDIV
DO 24 L=1,LMDIV
III=III+1
IFI=III+1
24 CALL VORTEX(SUM)
GO TO 50

30
II1=1F1
II2=II2+1
GO 34 L=1,LTDIV
II2=II2-1
IFZ=II2-1
IF(I.EQ.NBVV) GO TO 33
C
C ADD TRAILING VORTEX CONTRIBUTION
C
RIGHT SIDE
SYMLOO=1.0
301 CONTINUE
DX=XXI(II1+II2)-X
DY=SYMLOO*YY(II1+II2)-Y
DZ=ZZ(II1+II2)-Z
TYZ=DY**2+DZ**2
T1=SQR(TYZ)
T2=DX/SQR(DX**2+TYZ)
QT=(1.0-T2)/T1
QTT=SYMLOO*QT
SUM(2)=SUM(2)+QTT*DZ/T1
SUM(3)=SUM(3)-QTT*DY/T1
C
LEFT SIDE
DX=XXI(IF1+IF2)-X
DY=SYMLOO*YY(IF1+IF2)-Y
DZ=ZZ(IF1+IF2)-Z
TYZ=DY**2+DZ**2
T1=SQR(TYZ)
T2=DX/SQR(DX**2+TYZ)
QT=(1.0-T2)/T1
QTT=SYMLOO*QT
SUM(2)=SUM(2)-QTT*DZ/T1
SUM(3)=SUM(3)+QTT*DY/T1
C
IF(SYMLOO.EQ.0.0) GO TO 24
IF(SYMLOO.EQ.-1.0) GO TO 24
SYMLOO=1.0
GO TO 301
C
CALL VORTEX(SUM)
34 CONTINUE
35 GO TO 50
36 CONTINUE
37 I12=1F2
38 II1=II1+1
39 DO 44 L=1,LNDIV
40 II1=II1-1
41 IF1=IF1-1
42 CALL VORTEX(SUM)
43 TOT1=TOT1+SUM(1)
44 TOT2=TOT2+SUM(2)
45 TOT3=TOT3+SUM(3)
46 CONTINUE
47 AXB(N)=TOT1/PI4
48 AYB(N)=TOT2/PI4
49 AZB(N)=TOT3/PI4
50 IF(I.NE.1) GO TO 60
51 AXB(N)=0.0
52 AYB(N)=0.0
53 AZB(N)=0.0
54 CONTINUE
55 C IF BODY HAS CONSTRAINT FUNCTIONS, CONVERT AX TO AXRL. SAME FOR AY, AZ
56 IF(KCN.EQ.0) GO TO 601
57 NS1=NS1+1
58 CALL AXYZRL(NS1,ALNGTH,AXB,AYB,AZB)
59 CONTINUE
60 IF(NB.EQ.1) GO TO 62
61 NB1=NB1-1
62 N2=0
63 DO 61 J=1,NB1
64 N2=N2+NLJ(NJ)*NVT(J)
65 READ(NTAPE),(AXB(J),AYB(J),AZB(J),I=N1,N2)
66 WRITE(NTAPE),(AXB(I),AYB(I),AZB(I),I=N1,NCTV)
67 CONTINUE
68 MTAPE=39-MTAPE
69 NTape=39-NTape
70 RETURN
71 END
SUBROUTINE CONFUN(A, SCRTCH, SCRCH, TETA, NROOT, ASSP, NETA, NXOC, 3
1 AS, NW, FLAG, NCOL)
C A -- A MATRIX (ONE ROW FROM FRED)
C SCRTCH = T(XOC)
C SCRCH -- T(XOC) FOR USE IN ROOT SECTION
C TETA = T(TETA)
C NROOT = NUMBER OF ETAS IN ROOT SECTION OF PANEL
C ASSP = OUTPUT A/
C NETA = NUMBER OF ETAS
C NXOC = NUMBER OF XOCs
C AS = CHORDWISE TRANSFORMED MATRIX A/
C NW = NUMBER OF LATERAL CONSTRAINT FUNCTIONS
C FLAG = IF FALSE SOLVE FOR COMPLETELY CONSTRAINED
C NCOL = NU + NF + NK (NUMBER OF COLs OF AS)
C DIMENSION A(NXOC, NETA), SCRTCH(NXOC, NCOL), SCRCH(NXOC, NCOL),
1 TETA(NETA+NW), AS(NCOL, NETA), ASSP(NCOL+NW)
C LOGICAL FLAG
IF(NROOT) 41, 41, 1
1 DO 40 IETA=1, NROOT
DO 40 IC = 1, NCOL
TEMP = 0.0
DO 30 K=1, NXOC
30 TEMP = TEMP + SCRCH(K, IC)*A(K, IETA)
40 AST(I, IETA) = TEMP
41 IS = NROOT + 1
IF(IS, 9, NETA) RETURN
DO 50 IETA=IS, NETA
DO 50 IC=1, NCOL
TEMP = 0.0
DO 50 K=1, NXOC
50 TEMP = TEMP + SCRTCH(K, IC)*A(K, IETA)
60 AST(I, IETA) = TEMP
IF(FLAG) RETURN:
C SUBROUTINE TO SOLVE COMPLETELY CONSTRAINED MATRIX
DO 70 INW=1, NW
DO 70 IUKF=1, NCOL
TEMP = 0.0
DO 70 IETA=1, NETA

20 TEMP = TEMP + TETA(IETA*INM) * ASG(IUKF*INM) = TEMP
70 CONTINUE
RETURN
C        BLAINE 0. CAITHER 9/72
END
```plaintext
SUBROUTINE TVG*1(X,Y,Z,N,X1,Y1,Z1,XT,YT,ZT,NT)
C
CALCULATES FIRST ITERATION TRAILING VORTEX GEOMETRY.
C
X,Y,Z = JUNCTURE TRAILING VORTEX, N=NO. OF POINTS.
C
X1,Y1,Z1 = PANEL TRAILING EDGE, N1=NO. OF POINTS.
C
XT,YT,ZT = COMPUTED TRAILING VORTEX POINTS, NT=NO. OF POINTS.
C
DIMENSION X(1),Y(1),Z(1),X1(1),Y1(1),Z1(1),XT(1),YT(1),ZT(1)
DIMENSION XS(21),YS(21),ZS(21),SA(21),SJ(21)
DIMENSION DCX(20),DCY(20),DCZ(20),XTT(21),YTT(21),ZTT(21)

WRITE(5,100)(I,X(1),Y(1),Z(1),I=1,N)
WRITE(6,100)(I,X1(1),Y1(1),Z1(1),I=1,N1)
FORMAT(1HG/13,1P3E18.5))
NT=(NS+1)*N1
DX=(X(N)-X1(1))/NS
XS(1)=X1(1)

DO 1 I=1,NS
1
XS(I+1)=XS(I)+DX
NS1=NS+1
CALL CODIM(X,Y,N,XS,YS,N51)
CALL CODIM(X,Z,N,XS,ZS,N51)
SAI=0.0
DO 5 I=1,NS
5
DY=YS(I+1)-YS(I)
DZ=ZS(I+1)-ZS(I)
SAI = SAI + SQRT(DX**2 + DY**2 + DZ**2)
SA(I+1)=SAI
SD=SAI/NS
SJ(I)=0.0
DO 10 I=1,NS
10
SJ(I+1)=SJ(I)+SD
CALL CODIM(SA,XS,NS1,SJ,XT,NS1)
CALL CODIM(SA,YS,NS1,SJ,YT,NS1)
CALL CODIM(SA,ZS,NS1,SJ,ZT,NS1)

C
C DIVIDE LAST BOUND VORTEX INTO EQUAL SPACES.
N1=N1-1
XF=X(N)
YF=Y1(N1)
```

ZF=Z1(N1)
DX=0.0

DY=(YF-Y(N1))/N11
DZ=(ZF-Z(N1))/N11
DO 15 I=2*N1
IT1=I-1

C (IT1+1) PT. IS ON FIRST BOUND VORTEX.
N3=IT1+1
XT(N3)=X1(I)
YT(N3)=Y1(I)
ZT(N3)=Z1(I)

IT2=I*NS1
XT(IT2)=XT(IT1)
YT(IT2)=YT(IT1)+DY
ZT(IT2)=ZT(IT1)+DZ

15 CALL NORMDX,DX,DI
N2=N1-1
NN=NS1*N11+1

C COMPUTE DX ALONG LAST TRAILING VORTEX.

DNL=(XF-X1(N1))/NS
XT(NN)=X1(N1)
DO 16 I=1*NS
NN=NN+1

XT(NN)=XT(NN-1)+DNL
YT(NN)=Y1(N1)
ZT(NN)=Z1(N1)

16 NP=N11*NS1
N4=N2-1
DO 20 J=2*NS
CSX=XT(NP+J)-XT(J)
CSY=YT(NP+J)-YT(J)
CSZ=ZT(NP+J)-ZT(J)

CS2=CSX**2+CSY**2+CSZ**2
PER=(J-1+0.0)/NS

DENOM=0.0
DO 18 I=1*NS
DXL=X1(I+1)-X1(I)
DYL=X1(I+1)-Y1(I)

3 7700
3 7710
3 7720
3 7730
3 7740
3 7750
3 7760
3 7770
3 7780
3 7790
3 7800
3 7810
3 7820
3 7830
3 7840
3 7850
3 7860
3 7870
3 7880
3 7890
3 7900
3 7910
3 7920
3 7930
3 7940
3 7950
3 7960
3 7970
3 7980
3 7990
3 8000
3 8010
3 8020
3 8030
3 8040
3 8050
3 8060
3 8070
3 8080
\[
\begin{align*}
DZLE &= Z1(I+1)-Z1(I) \\
\text{CALL NORM(DXLE,DX,YLE,ZLE)} \\
DIRX &= DXLE + (DX-DXLE)*PER \\
\text{DIRY} &= DYLE + (DY-DYLE)*PER \\
\text{DIRZ} &= DZLE + (DZ-DZLE)*PER \\
\text{CALL NORM(DIRX,DIRY,DIRZ)} \\
DCX(I) &= DIRX \\
DCY(I) &= DIRY \\
DCZ(I) &= DIRZ \\
18 &\text{DENOM = DENOM + DOT(DIRX,DIRY,DIRZ,CX,CY,CZ)} \\
\text{DS = CS2/DENOM} \\
NN &= NS1 + J \\
\text{DO 20 I = 1, N4} \\
N3 &= NN - NS1 \\
\text{XT(NN) = XT(N3) + DS*DCX(I)} \\
\text{YT(NN) = YT(N3) + DS*DCY(I)} \\
\text{ZT(NN) = ZT(N3) + DS*DCZ(I)} \\
20 &\text{NN = NN + NS1} \\
\text{DO 30 I = 2, N11} \\
\text{NN = (I - 1)*NS1} \\
SAI &= 0\cdot0 \\
\text{SAI(1) = 0\cdot0} \\
\text{DO 22 J = 1, NS} \\
\text{N2 = NN + J} \\
\text{N4 = N2 + 1} \\
DX &= XT(N2) - XT(N4) \\
DY &= YT(N2) - YT(N4) \\
DZ &= ZT(N2) - ZT(N4) \\
\text{SAI = SAI + SORT(DX**2 + DY**2 + DZ**2)} \\
22 &\text{SAI(J + 1) = SAI} \\
\text{SD = SAI/NS} \\
\text{DO 23 J = 1, NS1} \\
\text{N2 = NN + J} \\
\text{XTT(J) = XT(N2)} \\
\text{YTT(J) = YT(N2)} \\
23 &\text{ZTT(J) = ZT(N2)} \\
\text{SJ(J) = 0\cdot0} \\
\text{DO 24 J = 1, NS} \\
\text{SJ(J + 1) = SJ(J) + SD}
\end{align*}
\]
CALL CODIM(SA,XT,NS1,SJ,DCX,NS)
CALL CODIM(SA,YT,NS1,SJ,DCY,NS)
CALL CODIM(SA,ZT,NS1,SJ,DCZ,NS)

DO 30 J=2,NS
   N2=NN+J
   XT(N2)=DCX(J)
   YT(N2)=DCY(J)
   ZT(N2)=DCZ(J)
30
   RETURN

END
SUBROUTINE MATCON

C PERFORM PHI•THETA MATRIX MULTIPLY ON AXB•AYB•AZB MATRICES.
C NO REPLACEMENT IS MADE FOR BODIES HAVING NO CONSTRAINT FUNCTIONS.
COMMON DA(5000)
1 *NX,NXTH,LNVOR,LTVOV,NNTV,NNLVV,NXTHV,NNBV,NTH(49)
2 *LNDIV,LTDIV,LNPTS,LTPTS
COMMON/NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLTC(7),LNLC(7)
1 *NCI,NB,NBODS,NPANS,NVLT(7),NVT(7),ITAPE,NTAPE,NCTV,ITAPE,NTAPE
2 *LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 *LNDIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYM(7)
COMMON/BODY/ ARRAY(31000)
DIMENSION BX(5000),BY(5000),BZ(5000),PHTH(5000)
EQUIVALENCE(BX•ARRAY(1)),(BY•ARRAY(5001)),(BZ•ARRAY(10001))
1 COMMON/SCRAT/XSCL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)
DO 1000 I=1,NCT
READ(NTAPE)(AXB(K),AYB(K),AZB(K),K=1,NCTV)
1 M=0
IV=0
DO 100 NB=1,NBODS
NVP=NVLT(NB)*NVT(NB)
IF(LSEG(NB).EQ.0.0.AND.TSEG(NB).EQ.0.0) GO TO 50
KODE=1
NC=ITAPE
GO TO 70
1 NC=NVP
KODE=0
DO 60 I1=1,NC
M=M+1
IV=IV+1
BX(M)=AXB(IV)
BY(M)=AYB(IV)
60 BZ(M)=AZB(IV)
GO TO 100
70 CONTINUE
IS=IV
DO 80 I2=1,NC
SX=0.0
80
SY=0.0
SZ=0.0
READ(231)(PHTH(KC),KC=1,NVP)
M=M+1
DO 75 I1=1,NVP
IS11=IS+I1
SX=SX+PHTH(I1)*AXR(ISI1)
SY=SY+PHTH(I1)*AYB(ISI1)
75
QZ=QZ+PHTH(I1)*AZR(ISI1)
IV=IV+1
BX(N)=SX
BY(M)=SY
80
BZ(M)=SZ
100 CONTINUE
IF(KODE.EQ.1) REWIND 23
1000 WRITE(MTAPE)(BX(K),BY(K),BZ(K),K=1,N)
C CHANGE MEANING OF NCTV FROM TOT. NO. OF VORTICES ON BODIES
C TO NUMBER OF COLUMNS OF 'A' MATRIX. (FOR DISCRETE CASE, NO CHANGE.)
NCTV=4
NTAPE=39-MTAPE
NTAPE=39-MTAPE
REWIND 19
REWIND 20
RETURN
END
SUBROUTINE MATA(KCON, NCOLS, UNIT, IBP)
C
KCON = 0 FOR PANEL SIDE, KCON = 0 OR 1 SEG FOR BODY SIDE (SEE INFLM*)
C
NCOLS = NO. OF AXB, AYB, AZB* (NO. OF COLS OF BODY OR PANEL SIDE*)

COMMON DA(5000)
1    NX, NXTH, LN10, LIVOR, NTNV, NVBV, NVT, NXTHV, NVBV, NTH(49)
2    LNDIV, LDTIV, LNPTS, LTPTS
     COMMON/NUMBER/ NVPTS(7), NCPTS(7), NLN(7), NLT(7), LTN(7), LNC(7)
1    NCT, NB, NBODS, NPANS, NVL(7), NTV(7), NTAPE, NTAPE, NCTV, ITAPE, JTAPE
2    LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)
3    LNDIVB(7), LDTIVB(7), NSPP(7), KROOT(7), OUTERP(7), SYNN(7)
COMMON/CONPTS/XG(1320), YG(1320), ZG(1320)
1    ,XN(1320), YN(1320), ZN(1320)
COMMON/SCRAT/ XSOL(5000), BOUND(5000), AXB(5000), AYB(5000), AZB(5000)
DIMENSION A(5000)
EQUIVALENCE(A, AXB)
EQUIVALENCE(DA(2), PANS)
NPANS = PANS
IWR = 21
IF(IBP.EQ.1) IWR = 10
NBP = NBODS + NPANS
IF(KCON.EQ.0) GO TO 100

CALL MATCON

100  L = 0
     DO 300 I = 1, NBP
     LTPTS = LTC(I)
     LNPTS = LNC(I)
     DO 300 JJ = 1, LTPTS
     DO 300 II = 1, LNPTS
     L = L + 1
     XNN = XN(L)
     YNN = YN(L)
     ZNN = ZN(L)
     READ(IUNIT)(AXB(K), AYB(K), AZB(K), K = 1, NCOLS)
     DO 200 K = 1, NCOLS
     A(K) = XNN*AXB(K) + YNN*AYB(K) + ZNN*AZB(K)
     200 WRITE(IWR)(A(K), K = 1, NCOLS)
     REWIND IWR
     REWIND NTAPE
     JTAPE = 21
SUBROUTINE AXYZRL(KSTART, ALNGTH, AXB, AYB, AZB)
DIMENSION ALNGTH(1)
DIMENSION AXS(1), AYZ(1), AZB(1)
COMMON DAT(5000)
  1 *NX,NXTH,LMVOR,LTVOV,NTV,NBV,NTH(49)
  2 *LMDIV,LTDIV,LMPTS,LTPTS
COMMON/NUMBER/ NVEPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
  1 *NCT, NB, NNBS, NPANS,NVL(7),NTAPE,NTAPE,NCTV,ITAPE,ITAPE
  2 *LSEG(7),TSSEG(7),TLFNC(7),TTFNC(7)
  3 *LNDIVG(7),LTDIVR(7),NSPP(7),RGT(7),OUTERD(7),SYMP(7)
DO 100 J1=1,NTV
  I2=J1*NBV
  I3=KSTART-1+I2
  ADDX=0.0
  ADDY=0.0
ADDZ=0.0
DO 100 I1=1,NBV
  ADDX=AXB(I3)+ADDX
  AXB(I3)=ADDX*ALNGTH(I3)
  ADDY=AYB(I3)+ADDY
  AYB(I3)=ADDY*ALNGTH(I3)
  ADDZ=AZB(I3)+ADDZ
  AZB(I3)=ADDZ*ALNGTH(I3)
  I3=I3+1
100 CONTINUE
RRETURN
 END
SUBROUTINE PVSKIT+DY+ZZ+XC+YCY+ZC+MS+IV+JIV+KSOL
COMMON DA(1500)
COMMON/DA(13426)*SYM
COMMON/PANINF/PANSY(:110)
COMMON/SCRAT/XSOL(5000)*BOUND(5000)*AX(5000)*AY(5000)*AZ(5000)
COMMON/BODY/XVR(100+20)*YVR(100+20)*ZVR(100+20)*XVO(100+20)
1 *YVO(100+20)*ZVO(100+20)*PL(5000)*PLT(5000)*YSUBV(100)*CHORD(100)
2 *XCOV(20)*XCO(20)*XLE(20)*YLE(20)*ZLE(20)
3 *XTE(20)*YTE(20)*ZTE(20)*SLE(20)*XJ(20)*YJ(20)*ZJ(20)
4 *ETE(20)*XVT(50)*YVT(50)*ZVT(50)*XRT(20)*YRT(20)*ZRT(20)
5 *SXMT(1000)*SYMT(1000)*SZMT(1000)*DYS(1000)*DZS(1000)
6 *TS(1000)*XSS(1000)*YSS(1000)*ZSS(1000)*SIGMA(1000)
COMMON/PANEL/NPAN+IPSYM+IPX+NPBVP+NTVVP+NLCPF+LTCFP+LNCPP+LTCP
1 *NPRT*PSPACE+NATTACH+NRATT+NPRLN+NPRCLN+NACTXC+NACTET+NTHXC
2 *NHTET+NP+ROOT+OUTER+INATT
3 *MP1*MP2*MP3*MP4*MP5*MP7*MP8*MP9*MP10
COMMON/NVPTS(NP)+NCPTS(NP)+NL(NP)+NLT(NP)+LTC(LP)+LNC(LP)
1 *NCT+NB+NBODS+NPANS+NVL(NP)+NTV(NP)+NTAPE+NTAPE+NCTV+NTAPE+NTAPE
2 *LSEG(LP)+TSEG(LP)+LFUNC(LP)+FUNC(LP)
3 *LNDIVB(LP)+LTDIV(LP)*NSPP(LP)*ROOTP(LP)*OUTERP(LP)*SYMM(LP)

C
C KSOL=1 FOR SOURCE PTS. ONLY
C KSOL=2 FOR BOTH SOURCE PTS. AND VORTEX POINTS.
C MS=SUBSCRIPT OF SOURCE PT.
C XC+YC+ZC --- CONTROL PT.
C IV = LATERAL VORTEX SUBSCRIPT.
C JV = LONGITUDINAL VORTEX SUBSCRIPT.
C INSERT SPECIFICATION STATEMENTS HERE.
REAL 11+12+13+14
YVV=0.5*SORT(DYY**2+DZZ**2)
YV2=2.0*YVV
PI=3.141592654
YV=YVV
GO TO (10,20)*KSOL
10 YK=YSS(MS)
ZK=ZSS(MS)
XX=XSS(MS)
GO TO 25
24 IF(MS=2*(MS/2),EO.0) GO TO 10
YK=YVO(IV*JV)
ZK=ZVO(IV*JV)
XK=XVO(IV*JV)

25 CONTINUE
SUMX=0.0
SUMY=0.0
SUMZ=0.0
UTSUM=0.0
VTSUM=0.0

WTSUM=0.0
SIGN=1.0
DZ=ZC-ZK
X=XC-XK

50 CONTINUE
DY=YC-SIGN*YK
RY=DYY/VV2
RZ=DZZ/YV2

Y=RY*DY+RZ*DZ
Z=-RZ*DY+RY*DZ

R12=(Y+VY)**2+Z**2
R22=(X-T*Y)**2+Z**2*(1.0+T*T)

R32=(Y-YV)**2+Z**2
R4= SQRT((X-T*Y)**2+(Y-YV)**2+Z**2)
R5= SQRT((X+T*Y)**2+(Y+YV)**2+Z**2)

II=(X+T*YV)/R5
I2=(Y+T*X+YV*(1.0+T**2))/R5
I3=(Y+T*X-YV*(1.0+T**2))/R4
I4=(X-T*YV)/R4

IF(ABS(Z) GT YV2) GO TO 42
Z=0.0

R6D=(X+T*YV)**2+(Y+YV)**2
R7D=(X-T*YV)**2+(Y-YV)**2
RXTY=(X-T*Y)**2

R6=RXTY/R6D
R7=RXTY/R7D

IF(R6 GE 0.0075968656) GO TO 41
IF(R7 GE 0.0075968656) GO TO 41
IF(ABS(Y) LE YV) GO TO 41

TERM=ABS(1.0/R7D-1.0/R6D)*U.5/P
GO TO 43

41 IF(ABS(Y)*GT*YV) GO TO 42
IF(ABS(X-T*Y)+.25*ABS(PLE(NA))) GO TO 42
TERM1=0.0
GO TO 43

42 TERM1=(L2+13)/R22
CONTINUE
TERM2=(L1+1.0)/R12
TERM3=(L4+1.0)/R32
TERM4=1.0/R4-1.0/R5
P=SQRT(1.0+T*T)
EU=-(T*TERM4+(X-T*Y)*TERM1)/P
EV=(T*TERM4-T*(X-T*Y)*TERM1)/P
EWS=P*Z*TERM1
US=0.25*EUS/P1
VS=0.25*EV3/P1
WS=0.25*EWS/P1
UT=US
VT=VS*RY=XS*RZ
WT=VS*RZ+WS*RY
UTSUM=UT+UTSUM
VTSUM=VT+VTSUM
WTSUM=WT+WTSUM
IF(KSOL.EQ.1) GO TO 45
EU=Z*TERM1
EV=Z*(-T*TERM1+TERM2-TERM3)
EW=-(X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
UV=0.25*EU/P1
VU=0.25*EV/P1
WV=C.25*EW/P1
UI=UV
VI=RY*VV-RZ*VV
WI=RZ*VV+RY*VV
SUMX=UI+SUMX
SUMY=VI+SUMY
SUMZ=WI+SUMZ
CONTINUE
C FOR SYMMETRY: GET IMAGE CONTRIBUTION
IF(SYM*NE.0.0) GO TO 50
IF(SIGN.LT.0.0) GO TO 60
SIGN=-1.0
DZZ=-DZZ
T=-T
GO TO 50
60 CONTINUE
SXMT(MS)=UTSUM
SYMT(MS)=VTSUM
SZMT(MS)=WTSUM
IF(KSOL.EQ.1) RETURN
AX(MA)=SUMX
AY(MA)=SUMY
AZ(MA)=SUMZ
RETURN
END
13 CONTINUE
SUM(1)=COEF*RXS1 + SUM(1)
SUM(2)=COEF*RXS2 + SUM(2)
SUM(3)=COEF*RXS3 + SUM(3)
YI=YIH
YF=YFH
IF(SYMLOO.EQ.-1.0.OR.SYMNE.EQ.0.0) RETURN
SYMLOO=-1.0
YI=-YI
YF=-YF
GO TO 10
END
SUBROUTINE VORTEX(SUM)  
COMMON/BOUD/X(V:151:31),Y(V:151:31),Z(V:151:31)  
COMMON/PRNTS/XQ(1320),YQ(1320),ZQ(1320)  
COMMON/DA15000)  
EQUIVALENCE (DA19:SYM)  
COMMON/INDEX/X,Y,Z,II1,II2,IF1,IF2  
DIMENSION SJM(1)  
X1=XV(II1,II2)  
Y1=YY(II1,II2)  
Z1=ZV(II1,II2)  
X2=XV(IF1,IF2)  
Y2=YY(IF1,IF2)  
Z2=ZV(IF1,IF2)  
XFO=XS-X  
ZFO=ZF-Z  
X=XS-X  
Y=YS-Y  
SYM=SYM+1.0  
SUM=SUM+1.0  

84

DELY=YS-Y  
DELZ=ZF-Z  
RXS1=YS*DELS-ZFO*DELY  
RXS2=ZF*DELS-XF*DELY  
RXS3=ZW*DELS-YF*DELY  
RXS=SQR(4X(S1**2+32**2+3S3**2)  
TERM1=SQR(DLX**2+3E**2+3LZ**2)  
TERM2=SQR(XF**2+3YF**2+3FQ**2)  
TERM3=SQR(XI**2+3YI**2+3IQ**2)  
TERM4=XFDELX+YFDELY+ZFDELZ  
RATIO=TERM4/TERM1**2  
COSA=(DLX*YI+DELY*YI+DELZ*Z10)/(TERM1*TERM3)  
COSB=TERM4/TERM1**2  
CC=COSB-COSA  
HX=XF-RATIO*DELS  
HY=YF-RATIO*DELY  
HZ=ZF-RATIO*DELZ  
H=SQR(HX**2+HY**2+HZ**2).
11 IF(H<0.00001) 11, 12, 12
   COEF=0.0
   GO TO 13
12 CONTINUE
   HRXS=H*RXS
   COEF=SYMLOO*CC/HRXS
13 CONTINUE
   SUM(1)=COEF*RXS1 + SUM(1)
   SUM(2)=COEF*RXS2 + SUM(2)
   SUM(3)=COEF*RXS3 + SUM(3)
   IF(SYMLOO.EQ.-1.0 .OR. SYMNE.EQ.0.0) RETURN
   SYMLOO=-1.0
   YI=-YI
   YF=-YF
   GO TO 10
   END
PROGRAM SOL
COMMON/SCRAT/XSOL(5000)*BOUND(5000)*AXB(5000)*AYB(5000)*AZB(5000)
COMMON/PANEL/ NPAN*IPSYM*WCVNBVP*NTVVP*LNCFP*LTCPF*LNCPP*LTCP
        1 *NPERP*TSPACE*NNATT*NPRCLN*NPRCLT*NWCTXCNWCTET*NTHXC
        2 *NTHE*TNP*CHTP*ROOT*OUTER*NNAT
        3 *MP1*MP2*MP3*MP4*MP5*MP6*MP7*MP8*MP9*MP10
COMMON/BODY/ AA(31000)
DIMENSION PHTH(5000)*ALNGTH(5000)*GA(5000)
EQUIVALENCE (PHTH*AA(1)) (ALNGTH*AA(5001)) (GA*AA(10001))
COMMON/NUMBER/ NVPTS(7)*NCPTS(7)*NLN(7)*NLT(7)*LTC(7)*LNC(7)
        1 *NCT*NB*NRODS*NPANS*NLV(7)*NVT(7)*NTAPE*NTAPE*NTAPE*NTAPE
        2 *LSEG(7)*TSEG(7)*LFUNC(7)*TFUNC(7)
        3 *LNDIVB(7)*LTDIVB(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)
DIMENSION B1(1000)*P1(5000)*SAVEC(6000)*DUM(2000)
DIMENSION SAVEC1(6000)*ATET(1000)
EQUIVALENCE (B1*AXB(1)) (P1*AXB(1001)) (SAVEC*AYB(1001))
EQUIVALENCE (DUM*AZB(2001)) (ATET*AZB(4001)) (DA(2)*PANS)
EQUIVALENCE (SAVEC1*AA(15001))
COMMON DA(5100)
DIMENSION LT(10)*LN(10)*NP1(10)
COMMON/PANINF/PANSYM(10)*ASOL(600)
NPANS = PANS
IF(NPANS*EQ.0) GO TO 10
   DO 8 I=1,6000
     SAVEC(I) = SAVEC1(I)
8 CONTINUE
REWIND 23
C
REWIND 18 IN ORDER TO READ ALNGTH ARRAY.
REWIND 18
WRITE(6,1) NTVP,MP3,NCT
1 FORMAT(*OSURROUTINE SOL*/315)
WRITE(6,3) (BOUND(I),I=1,NCT)
3 FORMAT(20 HBOBOUNDARY CONDITIONS/(1P10E13.5))
2 FORMAT(*OSOLUTION*/(1P10E13.5))
NCOLS=NTVP+MP3
DO 20 I=1,NCOLS
   IF(BOUND(I)*NE.0.0) GO TO 22
20 CONTINUE
DO 21 I=1,NCOLS

21 \text{XSO}(I) = 0.0
\text{DO} 211 I = 1, 600
211 \text{XSO}(I) = 0.0
\text{GO TO} 200
22 \text{CONTINUE}
\text{CALL MSOLX(NCTVMP3, NCTBOUND, XSO, DUM)}
\text{DO} 2305 I = 1, 600
2305 \text{XSO}(I) = \text{XSO}(I)
23 \text{CONTINUE}
NR = NCOLS
KCN = LSEG(1) + TSEG(1)
\text{IF} (KCN = 0) \text{GO TO} 111
\text{READ(18) ALNTH}
\text{REWIND} 18
NTV = NVT(1)
NBV = NVL(1)
NR = NTVV * NBV
NC = ITAPE
\text{WRITE(6, 231) \{XSO(I) \* I = 1, NCOLS\}}
231 \text{FORMAT(5)} \text{HQSOLUTION FOR COEFFICIENTS OF CONSTRAINT EQUATIONS/}
1 \text{(1P10E13, 5)}}
\text{C}
\text{NOW COMPUTE THE PRODUCT OF PHTH} \ast A = GA, \text{ A COLUMN MATRIX}
\text{DO} 101 I = 1, NR
101 GA(I) = 0.0
\text{DO} 102 J = 1, NC
102 \text{READ(23) \{PTH(I) \* I = 1, NR\}}
\text{DO} 102 I = 1, NR
102 \text{GA(I) = GA(I) + PTH(I) \ast XSO(J)}}
\text{C}
\text{NOW DO R\ast L MATRIX PRE-MULTIPLY OF GA VECTOR... RESULT IS K VECTOR.}
K = 0
\text{DO} 110 J = 1, NTVV
110 TL = 0.0
\text{DO} 110 I = 1, NBV
110 K = K + 1
110 AL1 = ALNTH(K - 1)
AL2 = ALNTH(K)
AL3 = ALNTH(K+1)
IF(I.EQ.1) AL1 = 0.0
IF(I.EQ.NBVV) AL3 = 0.0
AL = 0.5*(0.75*AL1 + AL2 + 0.25*AL3)
XSOL(K) = AL*GA(K) + TL
TL = XSOL(K)

110 CONTINUE

111 CONTINUE
NB1 = NVT(1)*NVL(1)
IF(KCN .NE. 0) NB1 = ITAPE
IF(NBODS .EQ. 0) NB1 = 0
IF(NBODS .EQ. 0) GO TO 301
DO 300 I=1,NB1
300 B1(I) = XSOL(I)
301 IF(NPANS .EQ. 0) GO TO 700
   DO 305 I=1,NPANS
   NBPI = NBODS+I
   LT(I) = NVT(NBPI)
   LN(I) = NVL(NBPI)
   IF(LFUNC(I+1) .NE. 0) LN(I) = LFUNC(I+1)
   IF(TFUNC(I+1) .NE. 0) LT(I) = TFUNC(I+1)

6000 FORMAT(6I5,3X,2F15.5)
305 NP1(I) = LT(I) * LN(I)
   IP1 = NB1
   DO 310 I=1,NPANS
   IT = (I-1) * 1000
   M = NP1(I)
   WRITE(6,6000) I,NP1(I),IT,M,IP1
   DO 308 K=1,M
   PL1(IT+K) = XSOL(IP1+K)

308 CONTINUE
310 IP1 = IP1 + M

C NOW DO ANY CONVERSION NECESSARY FOR PANEL A'S TO K'S
   DO 500 IP=1,NPANS
   I=IP
   M = NP1(I)
   IT = (I-1) * 1000
   LOC1 = (IP-1) * 1000
   LOC2 = LOC1 + 400

500 CONTINUE
LOC3 = LOC2 +400
NBPI = NBODS + IP

IF(LN(I) .EQ. NVL(NBPI) .AND. LT(I) .EQ. NVT(NBPI)) GO TO 499
   DO 315 K=1*M
   315 DUM(K) = P1(I+K)
   NTVVP = NVT(NBPI)
   NU = LN(IP)
   NW=LT(IP)
   NBVVP = NVL(NBPI)
   IF(LT(I) .NE. NVT(NBPI)) GO TO 330
   L = 0
   DO 320 J=1,NTVVP
   DO 320 I=1,NBVVP
   L = L + 1
   IF(J .GT. NSPP(IP)) GO TO 317
   SUM = 0.0
   DO 316 K=1,NU
   316 SUM = SUM + SAVEC(LOC2+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
   GO TO 319
   317 SUM = 0.0
   DO 318 K=1,NU
   318 SUM = SUM + SAVEC(LOC1+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
   319 XSOL(L) = SUM
   320 CONTINUE
   GO TO 361
   330 CONTINUE

   L = 0
   DO 355 J=1,NTVVP
   DO 355 I=1,NU
   L = L + 1
   SUM = 0.0
   DO 346 K=1,NW
   346 SUM = SUM +DUM((K-1)*NU+I) *SAVEC((K-1)*NTVVP+J+LOC3)
   ATET(L) = SUM
   WRITE(6,6001) L,ATET(L)
   355 CONTINUE

   L = 0
   DO 360 J=1,NTVVP
DO 360 I=1*NBVVP
L = L+ 1
IF(J .GT. NSPP(IP)) GO TO 357
SUM = 0*0
DO 356 K=1*NU
356 SUM = SUM +SAVEC(LOC2+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
GO TO 359
357 SUM = 0*0
DO 358 K=1*NU
358 SUM = SUM +SAVEC(LOC1+(K-1)*NBVVP+I) *ATET((J-1)*NU+K)
359 XSOL(L) = SUM
WRITE(6*6001) L*XSOL(L)
360 CONTINUE
361 CONTINUE
IT=(IP-1)*1000
WRITE(6*6001) IT
DO 370 I=1,L
370 P1(IT+I) = XSOL(I)
499 CONTINUE
500 CONTINUE
L = 0
IF(NBODS .EQ. 0) GO TO 600
NB1=NVT(1)*NVL(1)
DO 550 I=1*NB1
L = L+ 1
550 XSOL(L) = B1(I)
600 IF(NPANS .EQ. 0) GO TO 700
DO 650 I=1*NPANS
IT = (I-1)*1000
M=NVT(NBODS+I )*NVL(NBODS+I )
DO 625 K=1*M
L = L+ 1
625 XSOL(L) = P1(IT+K)
650 CONTINUE
WRITE(6*50)
50 FORMAT(47HOCONVERT SOLUTION FOR BODY CONSTRAINT FUNCTIONS) NRM=NR+NMOVE
WRITE(6*150) (XSOL(I),I=1,L)
150 FORMAT(16HO SOLUTION MATRIX/(1P10E13.5))
200 CONTINUE
700 CONTINUE
WRITE(6,2)(XSOL(I)*I=1,NCOLS)
END
SUBROUTINE MSOLX(NB, NP, NOT, B, XSOL, IL)
C     NKB = NO. OF COLUMNS IN BODY INFLUENCE MATRIX
C     NKTP = NO. OF COLUMNS IN PANEL INFLUENCE MATRIX
DIMENSION B(11), XSOL(11), IL(11)
COMMON/ BODY/ A(28000), AR(500), ICX(500)
COMMON/ DA(500)
EQUIVALENCE (FUNCL, DA(34)), (FNB, DA(30)), (FNT, DA(31))
NBVV = FNB
NTVV = FNT
WRITE(6, 1000)
1000 FORMAT(1A MATRIX*)
NKT=NB+NP
IF(NB.EQ.0 OR FUNCL.NE.0) GO TO 5
NKT=NKT-NTVV
5 CONTINUE
NKTP=NKT+1
N=(NKT*(NKT+3))/2
DO 10 I=1, N
10 A(I)=0.0
NKT2=NKT+2
DO 60 K=1, NKT
IF(NB.EQ.0) GO TO 1
READ(21)(AR(L), L=1, NB)
IF(FUNCL.NE.0) GO TO 23
L2 = 1
L1 = 1
NBM1 = NBVV - 1
DO 22 J=1, NTVV
DO 21 I=1, NBM1
AR(L1) = AR(L1+L2)
21 L1 = L1 + 1
22 L2 = L2 + 1
23 CONTINUE
IF(K.GT.20) GO TO 20
1001 FORMAT(10ROW*13) BODY ON BODY/(1P10E13.4))
20 CONTINUE
1 IF(NP.EQ.0) GO TO 2
NN=NB+1
IF(NB.EQ.0 OR FUNCL.NE.0) GO TO 24

NN = NN - NTVV
24 CONTINUE
READ(10)(AR(I), I = NN, NKT)
IF(K.GT.20) GO TO 30
1003 FORMAT(1OROW'I3' WING ON BODY'/(1P10E13.4))
30 CONTINUE
2 CONTINUE
AR(NKT+1) = B(K)
IXI = 1
DO 50 I = 1, NKT
R = SQRT(A(IXI)**2 + AR(I)**2)
IF(R.EQ.0.) GO TO 50
C = A(IXI)/R
S = AR(I)/R
IXJ = IXI
DO 40 J = I, NKT
T2 = C*A(IXJ) + S*AR(J)
AR(J) = -S*A(IXJ) + C*AR(J)
A(IXJ) = T2
40 IXJ = IXJ + 1
50 IXI = IXI + NKT2 - 1
60 CONTINUE
REWIND 21
REWIND 10
II = 1
IXI = 1
DO 80 I = 1, NKT
IXC(I) = IXI
XSOL(I) = 0.
IL(I) = 0
IF(A(IXI) .LE. 0.00000001) GO TO 80
IL(I) = II
II = II + 1
80 IXI = IXI + NKT2 - 1
II = NKT
DO 210 J = 1, NKT
IF(IL(II) .LE. 0) GO TO 210
JI = IL(II)
JS = IXC(JI) - JI
4 2390
4 2400.
4 2410
4 2420
4 2430
4 2440
4 2450
4 2460
4 2470
4 2480
4 2490
4 2500
4 2510
4 2520
4 2530
4 2540
4 2550
4 2560
4 2570
4 2580
4 2590
4 2600
4 2610
4 2620
4 2630
4 2640
4 2650
4 2660
4 2670
4 2680
4 2690
4 2700
4 2710
4 2720
4 2730
4 2740
4 2750
4 2760
4 2770
JXI=JS+1
JXN=JS+NKTP
IF(II-NKT) 170*200*220
170  IK=II+1
      JXK=JXI
      DO 180 K=IK,NKT
      JXK=JXK+1
180      XSOL(II)=XSOL(II)-A(JXK)*XSOL(K)
      XSOL(II)=(XSOL(II)+A(JXK))/A(JXI)
      II=II-1
      CONTINUE
      IF(NB.EQ.0.OR.FUNCL.NE.0.0) GOTO 310
      L2 = 1
      L1 = 1
      DO 301 J=1, NTVV
      DO 300 I=1, NBVV
      IF(I.NE.1) GOTO 299
      AR(L1) = 0.0
      GOTO 300
299      AR(L1) = XSOL(L1-L2)
300      L1 = L1 + 1
301      L2 = L2 + 1
      IF(NP.EQ.0) GOTO 304
      DO 303 I=1, NP
      AR(L1) = XSOL(L1-NTVV)
303      L1 = L1 + 1
304      L1 = L1 - 1
      DO 305 I=1, L1
305      XSOL(I) = AR(I)
      CONTINUE
      RETURN
      END
SUBROUTINE SFSOL (M,N)

COEFFICIENT MATRIX ON UNIT 23 BY ROWS
M ROWS...
N COLUMNS
B VECTOR (RIGHT SIDE) IN BOUND
COMMON /BODY/S(150) ,X(1000) , BWORK(150) ,XWORK(150) ,COL(150) ,TOPCOL (150) ,WORK(150), IWORK(150) ,ATA(1000) ,ATB(1000) ,A(1000)
2PAD(600)
COMMON /SCRAT/XSOL(5Q00) ,BOUND(5Q00)
INTEGER UNIT
REWIND 10
CALL ATAN (A,M,N,BOUND,ATB,ATA,UNIT)
NOW SOLVE N X N SYSTEM
DETERMINE NROW, MAXIMUM PARTITION IS 150 ROWS
THERE MUST BE AT LEAST TWO PARTITION ROWS
IF (N - 150 ) 5 , 5 , 7
5 NROW = 2
GO TO 9
7 NROW = N /150
IF (N . (NROW*150) ) 8, 9, 8
8 NROW = NROW + 1
DTERMINE SIZES OF SUBMATRICES
INTERIOR SUBMATRICES
9 NPO = N /NROW
NP1 = N - NPQ * NROW + NPO
STORE SUBMATRICES IN UNIT 10
DO 15 KROW = 1, NROW
SUBMATRIX SIZE FOR KROW . NE. 1
NP = NPO
SUBMATRIX SIZE FOR KROW = 1
IF (KROW .EQ. 1) NP = NP1
ROW NUMBER OF TOP ELEMENT IN SUBMATRIX
KTOP = N - NP - (NROW-KROW)*NPO +1
ROW NUMBER OF BOTTOM ELEMENT IN SUBMATRIX
KBO7 = KTOP + NP - 1
STORE SUBMATRIX COLUMNS
READ ONE COLUMN AT A TIME FROM UNIT, N ELEMENTS
C LOAD JCOL OF COEFFICIENT MATRIX FROM KTOP TO KBOT
DO 10 JCOL = 1,N
READ (UNIT) (A(I), I=1,N)
10 WRITE (10) (A(I), I = KTOP,KBOT)
C WRITE CONSTANT VECTOR FROM KTOP TO KBOT
WRITE (10) (ATB(I), I = KTOP,KBOT)
C REWIND UNIT
REWIND 10
C PUT SOLUTION IN X
18 CALL XSOLVE (1000,150,N, NPO, NP1, NROW, S, XSOL, IWORK, WORK,
IBWORK, XWORK, COL, TOPCOL)
RETURN
END
SUBROUTINE XSOLVE(MX,NPX,N,NPO,NP1,NROW,Z,X,IWORK,WORK,  
*                        *                        *                        *  
4 3660  
4 3660  
4 3660  
4 3670  
4 3680  
4 3690  
4 3700  
4 3710  
4 3720  
4 3730  
4 3740  
4 3750  
4 3760  
4 3770  
4 3780  
4 3790  
4 3800  
4 3810  
4 3820  
4 3830  
4 3840  
4 3850  
4 3860  
4 3870  
4 3880  
4 3890  
4 3900  
4 3910  
4 3920  
4 3930  
4 3940  
4 3950  
4 3960  
4 3970  
4 3980  
4 3990  
4 4000  

** SUBROUTINE FOR SOLUTION BY PARTITIONING USING FILE STORAGE **

**** WRITTEN BY D.G. ELLIOTT USING SCHEME SUGGESTED BY C.L. LAWSON ****

***** JET PROPULSION LABORATORY, PASADENA, NOVEMBER 1, 1972 *****

** INTEGER iwork(NPX),unit1,unit2,unit3 

** REAL work(NPX) 

** REAL z(NPX,NPX),x(mx),bwork(NPX),xwork(NPX), 

** COL(NPX),TOPCOL(NPX) 

***** FORTRAN UNITS 10, 11, AND 12 USED FOR STORAGE *****

***** TRIANGULARIZATION *****

6 NCXS = 0 
NCX1 = N+1-NP1 
TRIANGULARIZE THE NROW ROWS OF SUBMATRICES

DO 90 KROW=1,NROW

C SET UP UNITS

IF (KROW .EQ. 1) 2,2,3

2 UNIT1 = 10
UNIT2 = 21
UNIT3 = 22
GO TO 4

3 JU = UNIT3
UNIT3 = UNIT2
UNIT2 = UNIT1
UNIT1 = JU

C CONTAINS OLD LOWER-RIGHT MATRIX 
TO RECEIVE TRANSFORMED TOP ROW 
TO RECEIVE NEW LOWER-RIGHT MATRIX 
SIZE OF INTERIOR SUBMATRICES 
SIZE OF UPPER-LEFT SUBMATRIX 

4 NP = NPO 

IF(KROW .EQ. 1) NP = NP1
C NUMBER OF COLUMNS BEYOND FIRST MATRIX IN KRO4
NCX = N+1-NP1-(KROW-1)*NPO
4 4010
DO 10 J=1,NCX
4 4020
C READ MATRIX IN KROW
10 READ (UNIT1) (Z(K,J)*K=1,NCX)
4 4040
C TRIANGULARIZE FIRST MATRIX
4 4050
CALL DECOMP(NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
4 4060
C TRANSFORM REST OF ROW, INCLUDING RT-SIDE VCT
4 4070
DO 20 J=1,NCX
4 4080
C NEXT COLUMN IN ROW
20 NEXT COLUMN IN ROW
4 4090
C READ (UNIT1) (BWORK(K)*K=1,NCX)
4 4100
C PUT TRANSFORMED COLUMN IN XWORK
4 4110
CALL SOLVE(NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
4 4120
C TRIANGULARIZATION COMPLETE
4 4130
IF(KROW.EQ. NROW) GO TO 100
4 4140
C STORE TRANSFORMED COLUMN OF KROW
20 WRITE (UNIT2) (XWORK(K)*K=1,NCX)
4 4150
C NO PREVIOUS ROW TO STACK IN 2
4 4160
IF(KROW.EQ. 1) GO TO 40
4 4170
C NNP = NPO
4 4180
C STACK TRANSFORMED ROWS IN REVERSE ORDER
4 4190
DO 30 J=1,NCX
4 4200
IF((NCXS-J) .LT. NCX1) NNP = NP1
4 4210
C READ PREVIOUSLY TRANSFORMED ROWS
4 4220
READ (UNIT3) (COL(K)*K=1,NNP)
4 4230
C AND WRITE AFTER NEW ONE
4 4240
30 WRITE (UNIT2) (COL(K)*K=1,NNP)
4 4250
C REWIND UNIT3
4 4260
C FOR WRITING NEW LOWER-RT MATRIX IN NEXT OPER4
4 4270
40 KROWP1 = KROW +1
4 4280
C SUBTRACT MULTIPLES OF TOP ROW FROM EACH ROW
4 4290
DO 80 KKROW=KROWP1,NROW
4 4300
DO 50 J=1,NCX
4 4310
C READ FIRST MATRIX IN KKROW
4 4320
50 READ (UNIT1) (Z(K,J)*K=1,NPO)
4 4330
C REPOSITION AT START OF TOP ROW
4 4340
REWIND UNIT2

SUBTRACT MULTIPLES OF TOP COLUMNS FROM EA COL

DO 70 J=1,NCX

NEXT COLUMN IN KKROW OF OLD MAT

READ (UNIT1) (COL(K),K=1,NPO)

NEXT COLUMN IN TOP ROW

READ (UNIT2) (TOPCOL(K),K=1,NP)

SUBTRACT FROM EACH ELEMENT IN COLUMN

DO 60 K=1,NPO

SUBTRACT FIRST MAT TIMES TOP ROW

DO 60 JJ=1,NP

COL(K) = COL(K) - Z(K,JJ) * TOPCOL(JJ)

NEXT COL IN NBW MATRIX

WRITE (UNIT3) (COL(K),K=1,NPO)

CONTINUE

REWIND UNIT1
REWIND UNIT2
REWIND UNIT3

TOTAL NUMBER OF COLUMNS STORED IN UNIT2

NCXS = NCXS + NCX

CONTINUE

**** BACK SUBSTITUTION ****

DO 105 K=1,N

X(K) = 0

DO 110 K=1,NPO

BOTTOM SUBVECTOR IN X

X(N-NPO+K) = XWORK(K)

NROWM1 = NROW - 1

BACK SUBSTITUTE FROM BOTTOM

DO 140 KRX=1,NROWM1

KROW = NROWM1 - KRX + 1

NP = NPO

NMKROW = NROW - KROW
IF (KPOW .EQ. 1) NP = NP1
KSTART = N-NP-NMKROW * NPO

C SOLVE FOR KROW SUBVECTOR
   DO 130 KCOL=1,NMKROW
      KCOLM1 = KCOL - 1
      DO 120 J=1,NPO
         C NEXT MATRIX IN KROW
         READ (UNIT3) (Z(K,J), K=1,NP)
         C FORM PRODUCT OF MATRICES AND SUBVECTORS
         DO 130 K=1,NP
            DO 130 JJ=1,NPO
               X(KSTART+K) = X(KSTART+K) - Z(K, JJ) * X(KSTART+NP+KCOLM14)
            130      * *NPO+JJ)
         C READ RIGHT SIDE VECTOR
         READ (UNIT3) (BWORK(K), K=1,NP)
         C ADD RIGHT-SIDE VECTOR TO COMPLETE X FOR KROW
         DO 140 K=1,NP
            X(KSTART+K) = X(KSTART+K) + BWORK(K)
         140   RETURN
      END
   120   CONTINUE
   130 CONTINUE
   RETURN
END
SUBROUTINE ATRAN (A, M, N, B, ATB, ATA, UNIT)

C M A TRANSPOSE TIMES A (ATRAN)
C A STORED BY ROWS ON UNIT 23
C M ROWS * N COLUMNS. M IS GREATER THAN OR EQUAL N
C A TRANSPOSE A STORED BY COLUMNS (ROWS DUE TO SYMMETRY)
C UNIT IS AUXILIARY UNIT CONTAINING A TRANSPOSE A

DIMENSION A(N), ATA(N), B(N), ATB(N)
INTEGER UNIT, UNITN, UNITN1

C PREPARE AUXILIARY STORAGE
1 UNITN = 21
UNITN1 = 22
IFLAG = -1
REWIND 21
REWIND 22
REWIND 23
DO 5 I = 1, N
5 WRITE (21) ATA
REWIND 21

C INITIALIZE STORAGE FOR A TRANSPOSE B VECTOR
6 ATB(1) = 0.0
DO 6 I = 1, N
6 READ A ROW OF A
DO 40 J = 1, M
40 READ (23) A

C READ PARTIAL VALUE OF ELEMENTS IN A COLUMN OF ATA
DO 30 J = 1, N
30 READ (UNITN) ATA

C GENERATE NEXT TERM FOR ELEMENTS IN COLUMN OF ATA
DO 20 L = 1, N
20 ATA(L) = ATA(L) + A(J) * A(L)
WRITE (UNITN1) ATA

C GENERATE NEXT TERM FOR ELEMENTS OF ATB
DO 25 K = 1, N
25 ATB(K) = ATB(K) + A(K) * B(I)
IFLAG = IFLAG * (-1)
UNITN = UNITN + IFLAG
SUBROUTINE DECOMP(MX,N,UL,B,X,SCALES,IPS)

C
C ****** SUBROUTINE FOR SIMULTANEOUS EQUATION SOLVING ******
C ****** REF* G. FORSYTHE AND C. MOLER, 'COMPUTER SOLUTION OF LINEAR
C  ALGEBRAIC SYSTEMS', PRENTICE-HALL, 1967 ******
C
C ****** NOMENCLATURE: UL=COEFF MATRIX, B=RIGHT-SIDE VCTR, X=UNKNOWN VCTR
C ****** CALL TO DECOMP TRIANGULARIZES THE COEFFICIENT MATRIX ******

REAL  UL(MX,MX),B(MX),X(MX),PIVOT,EM,SUM
DIMENSION SCALES(MX),IPS(MX)

C
C INITIALIZE IPS AND SCALES
DO 5 I = 1,N
   IPS(I) = 1
   ROWNRM = 0.0
   DO 2 J = 1,N
      ROWNRM = AMAX1( ROWNRM, ABS(UL(I,J)) )
   2   IF ( ROWNRM ) 3,4,3
   3   SCALES(I) = 1.0/ROWNRM
   G0 TO 5
   PRINT 20
   SCALES(I) = 0.0
   CONTINUE

C
C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
NM1=N-1
DO 17 K=1,NM1
   BIG = 0.0
   DO 11 I = K,N
      IP = IPS(I)
      SIZE = ABS(UL(IP,K))*SCALES(IP)
      IF ( SIZE=BIG ) 11,11,10
   10   BIG = SIZE
   11   IDXPIV = I
   CONTINUE
   IF ( BIG ) 13,12,13
   PRINT 25
   GO TO 17

5440 4
5450 4
5460 4
5470 4
5480 4
5490 4
5500 4
5510 4
5520 4
5530 4
5540 4
5550 4
5560 4
5570 4
5580 4
5590 4
5600 4
5610 4
5620 4
5630 4
5640 4
5650 4
5660 4
5670 4
5680 4
5690 4
5700 4
5710 4
5720 4
5730 4
5740 4
5750 4
5760 4
5770 4
5780 4
5790 4
5800 4
5810 4
5820 4
13 IF (IDXPIV-K) = 14,15,14
14 J = IPS(K)
15 IPS(K1) = IPS(IDXPIV)
IPS(IDXPIV) = J
15 KP = IPS(K)
15 PIVOT = UL(KP+K)
15 KP1 = K+1
4 DO 16 I = KP1*N
15 IM = IPS(I)
15 EM = -UL(IP*K)/PIVOT
15 UL(IP*K) = -EM
15 DO 16 J1 = KP1*N
15 UL(IP*J1) = UL(IP*J1) + EM*UL(KP*J1)
16 CONTINUE
16 CONTINUE
17 KP = IPS(N)
17 IF (UL(KP*N)) = 19,18,19
18 PRINT 20
19 *GO TO 500
20 FORMAT (* MATRIX WITH ZERO ROW IN DECOMP*)
25 FORMAT (* SINGULAR MATRIX IN DECOMP*)
C
C ***** THIS ENTRY BACK-SUBSTITUTES THE RIGHT SIDE *****
C
ENTRY SOLVE
NPI = N + 1
C
IP = IPS(1)
X(1) = B(IP)
DO 30 1 = 2,N
IP = IPS(1)
SUM = 0.0
IM1 = I-1
DO 27 J = 1,IM1
SUM = SUM + UL(IP*J)*X(J)
27 SUM = SUM + UL(IP*J)*X(J)
30 X(I) = B(IP) - SUM
C
IP = IPS(N)
\begin{verbatim}
X(N) = X(N)/UL(IP,N)
    DO 40 IBACK = 2*N - 1
        I = NP1 - IBACK
        C I GOES (N-1)\ldots 1
        IP = IPS(I)
        IP1 = I+1
        SUM = 0.0
            DO 35 J = IP1,N
            SUM = SUM + UL(IP,J)*X(J)
35            X(I) = (X(I)-SUM)/UL(IP,I)
40    X(N) = X(N)/UL(IP,N)
    RETURN
END
\end{verbatim}
```
PROGRAM VEL
REWIND 12
CALL VELOC
END
SUBROUTINE VELOC
COMMON/SLOPE/SIGMA(500)* DZDT(500)*DZDC(500)*TANP1(500)
EQUIVALENCE (B(11871),SIGMA)
DIMENSION SIGMA(1000)
COMMON/SCRAT/ AK(5000)*XG(11*151)*GAMT(11*151)*GAMV(1500)
1 *DELVM(1500)*DELVT(1500)*VT(1500)*VM(1500)*SLOPEC(1500)
2 *XAA(151)*GAMMA1(151)*GAMMA2(151)*XD(150)*GD(150)
3 *CP(1320)
EQUIVALENCE(XCCO,B(2421))
DIMENSION XCCO(20),XOC(20)
COMMON/BODY/ B(131000)
EQUIVALENCE(B(14044),TMX) (B(15364),TMY) (B(16544),TMZ)
1 * (B(18004),TTX) (B(19324),TTY) (B(20644),TTZ)
EQUIVALENCE(DA(3),FMACH)
DIMENSION TMX(1320)*TMY(1320)*TMZ(1320)*
1 *TTX(1320)*TTY(1320)*TTZ(1320)
COMMON/COMPTS/ XQ(1320)*YQ(1320)*ZQ(1320)
1 *XN(1320)*YN(1320)*ZN(1320)
COMMON DA(5000)
COMMON COMPRBS BETAM
COMMON/NUMBER/ NVT(7),NCP(7),NLH(7),NLT(7),LTC(7),LNC(7)
1 *NCT,NB*NBD5,MPNS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,ITAPE,ITAPE
2 *SEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 *LNDIVB(7),LNDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYM(7)
C WHEN SOURCE MATRIX IS READ, GAMMA ARRAYS WILL BE DESTROYED.
EQUIVALENCE (SMX*XG),(SMY*GAMT),(SMZ*GAMV)
DIMENSION SMX(18000),SMY(18000),SMZ(18000)
C
DIMENSION VX(1320)*VY(1320)*VZ(1320)
EQUIVALENCE(VX*B(22001)),(VY*B(23211)),(VZ*B(24411))
C VX,VY,VZ EQUIVALENCING SHOULD NOT DESTROY SIGMA ARRAY.
EQUIVALENCE (DA(7),XCG),(DA(8),YCG),(DA(9),ZCG),(DA(10),ALPHA)
1 *(DA(11),BETA),(DA(12),PSK),(DA(13),PSK),(DA(14),KSTK)
C C C
```
COMMON/ PANEL// NPAr. IPSYM1: C: NEVVP. NTVVP. LNECFP. LTCEF. LNECF. LTCEP 5 0400
1 *NPERPT. NSPACE. NATCH. NTAR. NPRCLN. NFRCLT. LAC. LACET. LTHC 5 0410
2 *NTHET. NTIP. CHTIP. ROOT. COUTER. NMATT 5 0420
DIMENSION AX(4681), AY(4681), AZ(4681) 5 0440
EQUIVALENCE (B(1), AX), (C(4681), AY), (D(9363), AZ) 5 0450
EQUIVALENCE (B*BB, B*BP) 5 0460
EQUIVALENCE (AKTB, B*24001), (AKTP, B*36501), (CA(2), PANS) 5 0470
DIMENSION AKTB(500), AKTP(500) 5 0480
DIMENSION BB(22000), BP(13000) 5 0490

C BB IS READ INTO CORE TO GET TMX - TTO ARRAYS INTO CORE. 5 0510
C BB IS NOT LONG ENOUGH TO DESTROY SIGMA ARRAY. 5 0520
C
C MP2 IS UNIT FOR PANEL INFLUENCE MATRIX. 5 0530
C
C MP3 IS NUMBER OF AX, AY, AZ TERMS IN EACH ROW OF THE MATRIX ON MP2. 5 0540
C NCB = NO. OF CONTROL POINTS ON BODY. 5 0550
C
C COMMON/PANINF. PANSYM(lD), ASOL(600) 5 0560
1 DO 1 I=1,120 5 0570
2 XOC(I) = XCCO(I) 5 0580
1
C TEMPOURARY BREF, CBAR
BREF = 1.0 5 0590
CBAR = 10.0 5 0600
C
GAM14 = 1.4 5 0610
IF(NBODS*NE.0) REWIND NTAP 5 0620
IF(PANS*NE.0) REWIND MP2 5 0630
CALL GAMMA FOR CAPITAL GAMMA ARRAYS. 5 0640
CALL GAMMA 5 0650
C
CALL DELV FOR DELVM AND DELVI ARRAYS. 5 0660
CALL DELV 5 0670
WRITE(6,5000) (DELVT(I), I=1,200) 5 0680
5000 FORMAT (6H0, DELVT(I), 1H, 10F12.6) 5 0690
WRITE(6,5001) (DELVM(I), I=1,200) 5 0700
5001 FORMAT (6H0, DELVM(I), 1H, 10F12.6) 5 0710
IF(NBODS*EO.0) GO TO 4 5 0720
5 0730
5 0740
5 0750
5 0760
5 0770
5 0780
C
READ(18)
READ(18)
C
READ(18) BB
CONTINUE

C
NCB=NCPTS(1)
IF(NBODS.EQ.0) NCB=0
C
NPANS=PANS
DO 5 I=1,NCT
XX=XQ(I)-XC
YY=YQ(I)-YC
ZZ=ZQ(I)-ZC
VX(I)=1.0+2.0*(QSTAR*ZZ/C3AR-R3TAR*YY/REF)
VY(I)=-BETA-2.0*(PSTAR*ZZ-R3TAR*XX)/REF
VZ(I)=ALPHA+2.0*(PSTAR*YY/REF+QSTAR*XX/C3AR)
CONTINUE
5 NPWT=(NCT-NCB)*2+NCB
DO 6 I=1,NPWT
VM(I)=0.0
6 VT(I)=0.0
IF(NPANS.EQ.0) GO TO 2005
ISIG=0
C
READ PANEL DATA TO GET SIGMA
DO 7 I=1,NPANS
7 READ(18) DP
REIND 1B
WRITE(6,B900) (ASOL(I),I=1,220)
8000 FORMAT(1H1,1P10E13.4)
WRITE(6,B900) (SIGMA(I),I=1,290)
WRITE(6,B900) (SIGMAP(I),I=1,280)
WRITE(6,B900) (TAMP1(I),I=1,140)
WRITE(6,B900) (DZDX(I),I=1,140)
WRITE(6,B900) (DZDXC(I),I=1,140)
DO 9000 I=1,NPANS
NSIG=NVPTS(NBODS+1)*2
C
COMPUTE TERMS INVOLVING SOURCE MATRICES.
DO 1000 IC=1*NCT
T1=0.0
T2=0.0
T3=0.0
T4=0.0
T5=0.0
T6=0.0
READ(11)(SMX(J),SMY(J),SMZ(J),J=1*NIG)
DO 50 J=1,NIG
IS=1SIG+J
T1=T1+(SMX(J)*TMX(IC)+SMY(J)*MY(J)+SMZ(J)*TMZ(J))*SIGMA(J)
T2=T2+(SMX(J)*TMX(IC)+SMY(J)*MY(J)+SMZ(J)*TMZ(J))*SIGMA(J)
T3=T3+(SMX(J)*TMX(IC)+SMY(J)*MY(J)+SMZ(J)*TMZ(J))*SIGMA(J)
SIGDIF=SIGMA(J)-SIGMAP(J)
T4=T4+(SMX(J)*SIGDIF)
TERM=ZN(IC)*SMY(J)-YN(J)*SMZ(J)
T5=T5+TERM*SIGMA(J)
T6=T6+TERM*SIGDIF
50 CONTINUE
C NOW ADD MORE TERMS TO THOSE ABOVE.
IF(1.NE.NPANS) GO TO 1000
IF(9.C.GT.NCB) GO TO 500
VM(IC)=T1
VT(IC)=T2
WRITE(6,8003) IC,VM(IC),VT(IC)
8003 FORMAT(14,1P6E20.5)
GO TO 600
C
500 IX=2*IC-NCB-1
COMPUTE SUBSCRIPT FOR TOP CONTROL POINT ON PANEL.
C TOP WILL HAVE VELOCITY(IX)
C BOTTOM WILL HAVE VELOCITY(IX+1).
ICP=IC-NCB
VM(IX)=T3+DELVM(IX)*(1.0+SORT(TMP1(ICP)))*T4)
VM(IX+1)=T3-DELVM(IX)*(1.0+SORT(TMP1(ICP)))*T4)
TERM = T5+ZN(IC)*VY(J)-YN(J)*VZ(J)*BETA*
TERM2=DELVT(IC)*(1.0+SORT(TMP1(ICP)))*T6)
VT(IX)=TERM+TERM2
VT(IX+1)=TERM-TERM2
WRITE(6,8003) IC, VN(IC), ZN(IC), VY(IC), VZ(IC)
WRITE(6,8003) 1X, VM(1X), VT(1X), T3, T4, T5, T6
1XXX=IX+1

WRITE(6,8003) IXXX, VM(1XXX), VT(1XXX), DELVM(IC), DELVT(IC)
CONTINUE
600
1000
CONTINUE
ISIG=IS
2000 CONTINUE
2005 NB1=NBODS+1
DO 3000 IC=1,NCT
FLAG=1.0
IF(MPANS.EQ.0.0) GO TO 3014
IF(IC-NCB-NCPTS(NB1).GT.0) NB1=NB1+1
LPTS=LNC(NB1)
ICR=IC-((IC-1)/LPTS)*LPTS
3000 CONTINUE
3014 IX=2*IC-NCB-1
IF(NBODS.EQ.0.0) GO TO 3025
READ(NTAPE)(AX(I),AY(I),AZ(I),I=1,NCTV)
3020 IF(IC.GT.NCB) GO TO 3030
TERM=0.0
3015 DO 3050 I=1,NCTV
3025 TERM=TERM+AX(I)*TMX(IC)*AY(I)*TMZ(IC)*AZ(I)*ASOL(I)
3030 VM(IC)=VM(IC)*TERM
TERM=0.0
3050 DO 3060 I=1,NCTV
3060 TERM=TERM+AX(I)*TTX(IC)*AY(I)*TTZ(IC)*AZ(I)*ASOL(I)
VT(IC)=VT(IC)*TERM
VM(IC)=VM(IC)+BETAM**2*TMX(IC)*VX(IC)+BETAM**2*TMZ(IC)*VY(IC)+
1*DELVM(IC)
VT(IC)=VT(IC)+BETAM**2*TTX(IC)*VX(IC)+BETAM**2*TTZ(IC)*VY(IC)+
1*DELVT(IC)
DBETA=SQR(BETAM**2*(TMX(IC)**2+TMZ(IC)**2+TMZ(IDC)**2))
VM(IC)=VM(IC)/DBETA
DBETA=SQR(BETAM**2*(TTX(IC)**2+TTZ(IC)**2)/DBETA
3070_go_to 3050
3080 TERM=0.0
3090 DO 3020 I=1,NCTV
3020_go_to 3020
2025 TERM = TERM + AX(I) * ASOL(I)
WRITE(6,8001) IC,TERM
303  FORMAT(1H0, 15, 1PE15.5)      
VM(IX) = VM(IX) + TERM
VM(IX+1) = VM(IX+1) + TERM
TERM = 0.0
DO 2026 I = 1, NCTV
2026 TERM = TERM + (ZM(IC) * AY(I) - YM(IC) * AZ(I)) * ASOL(I)
WRITE(6,8001) IC,TERM
VT(IX) = VT(IX) + TERM
VT(IX+1) = VT(IX+1) + TERM
2050 CONTINUE
IF(NPANS.EQ.0) GO TO 2900
READ(MP2) (AX(I), AY(I), AZ(I), I = 1, MP2)
IF IC.GT. NCL) GO TO 2070
TERM = 0.0
DO 2065 I = 1, MP3
IK = NCTV + I
2065 TERM = TERM + (THX(IC) * AX(I) + THY(IC) * AY(I) + THZ(IC) * AZ(I)) * ASOL(IK)
VM(IC) = VM(IC) + TERM
TERM = 0.0
DO 2066 I = 1, MP3
IK = NCTV + I
2066 TERM = TERM + (TTX(IC) * AX(I) + TTY(IC) * AY(I) + TTZ(IC) * AZ(I)) * ASOL(IK)
VT(IC) = VT(IC) + TERM
GO TO 2900
2070 TERM = 0.0
DO 2075 I = 1, MP3
IK = I + NCTV
2075 TERM = TERM + AX(I) * ASOL(IK)
WRITE(6,8001) IC,TERM
VM(IC) = VM(IC) + TERM
VM(IX) = VM(IX) + TERM
VM(IX+1) = VM(IX+1) + TERM
TERM = 0.0
DO 2076 I = 1, MP3
IK = I + NCTV
2076 TERM = TERM + (ZM(IC) * AY(I) - YM(IC) * AZ(I)) * ASOL(IK)
WRITE(6,8001) IC,TERM
VT(IC) = VT(IC) + TERM
VT(I+1)=VT(I+1)+TERM
5 2350
IF(IC.LE.NCB) GO TO 2900
5 2360
ICP=IC-NCB
5 2370
TOP=1.0/SORT(1.0+TANPI(1CP)*DZDX(T(1CP)+DZDXC(1CP))+2)
5 2380
BOTTOM=1.0/SORT(1.0+TANPI(1CP)*DZDX(T(1CP)-DZDXC(1CP))+2)
5 2390
VM(9X)=TOP*(VM(IX)/BETAM**2+VX(1IC))
5 2400
VM(I+1)=BOTTOM*(VM(I+1)/BETAM**2+VX(1IC))
5 2410
VT(I)=TOP*V(IX)/BETAM
5 2420
VT(9X+1)=BOTTOM*VT(I+1)/BETAM
5 2430
WRITE(6,8002) IX,VM(IX),VM(I+1),VT(I),VT(I+1)
5 2440
8002 FORMAT(14,1P4E20.5//)
5 2450
2900 IF(IC-NCB) 2901,2901,2902
5 2460
2901 ICX=IC
5 2470
GO TO 2903
5 2480
2902 ICX=IX
5 2490
2903 A=1.0-VT(ICX)**2-VM(ICX)**2
5 2500
AM2=A*FMACH**2
5 2510
IF(6MACH-0.1) 2905,2905,2904
5 2520
2904 CP(9CX)=((2.0/(GM14*FMACH**2))*((1.0+(GM14-1.0)*0.5*A**2)
5 2530
**((GM14/(GM14-1.0))-1.0))
5 2540
GO TO 2906
5 2550
2905 CP(ICX)=A*(1.0+0.25*A**2*(1.0-0.1*A**2))
5 2560
2906 CONTINUE
5 2570
IF(FLAG.EQ.-1.0) GO TO 2905
5 2580
IF(IC-NCB) 2908,2908,2907
5 2590
2907 FLAG=-1.0
5 2600
ICX=ICX+1
5 2610
GO TO 2903
5 2620
2908 CONTINUE
5 2630
3000 CONTINUE
5 2640
IXX=IX+1
5 2650
IF(NPANS.EQ.0) IXX=IC-1
5 2660
REWIND 11
5 2670
IF(NEODS.EQ.0) REWIND 11
5 2680
IF(PANS.EQ.0) REWIND 12
5 2690
C SET TRAILING EDGE X VALUES.
5 2700
L=0
5 2710
L=0
5 2720
IF(NRDS.EQ.0) GO TO 310
NT=MVT(1)
NL=NL(1)
IV=NT*NL
DO 305 I=1,NT
J=1*NL
LT0=LT0+IV(1)
DO 305 I=1,LT0
L=L+1
305 AKTB(L)=AK(J)
WRITE(6,7000) (1*AKTB(I)+I=1,L)
7000 FORMAT(#0AKTB#/15.1PE20.5))
310 IF(PANS.EQ.0) GO TO 400
L=0
DO 350 K=1,NPANS
NT=MVT(NRDS+K)
NL=NL(K)
DO 340 I=1,NT
J=1*NL+IV
L=L+1
340 AKTLP(L)=AK(J)
WRITE(6,7001) (1*AKTLP(I)+I=1,L)
7001 FORMAT(#0AKTLP#/15.1PE20.5))
350 IV=IV+LT0*NL
400 CONTINUE
RETURN.
END
SUBROUTINE DELV
COMMON DA(5000)
COMMON/NUSP/ NVPTS(7), NCPTS(7), NLQ(7), LTE(7), LNC(7)
1 NCT, NB, NBDS, NPANS, NLTI(7), NVT(7), NTAP, ITAPE, NCTV, ITAPE, ITAPE
COMMON/SCAT/ XPSL(5000), XG(11, 1151), GAMT(11, 151), GAMR(1500)
1 DELVM(1500), DELVT(1500), VT(1500), VM(1500), SLOPEC(1500)
2 XAI(151), GAMMA1(151), GAMMA2(151), XD(150), GD(150)
3 CP(1320), XGW(50, 20), GAMT(150, 20)
EQUIVALENCE (DA(2), PANS)
EQUIVALENCE (DA(2800), FLNC), (DA(3100), FLTC)
DIMENSION FLNC(150), FLTC(40)
COMMON/BODY/ B(31000)
EQUIVALENCE (ALNCH, B(20000)), (ALNCH, B(25000))
DIMENSION ALNCH(5000), BLNCH(5000)
COMMON/COMPT/ XQ(1320), YQ(1320), ZQ(1320)
1 XN(1320), YN(1320), ZN(1320)
COMMON/CONTRV/ LN(3, 40), LT(3, 40)
COMMON/AF/ ATTFB(10), ATTFW(50)
IF(NBODS.EQ.0) GO TO 5
5 3010
REW 9 ND 18
NNV=NVL(1), NVT(1)
READ(18) (ALNCHI), I=1, NNV
5 3020
READ(18) (BLNCHI), I=1, NNV
5 3030
REWIND 18
5 3040
CONTINUE
5 3050
NPANS=PANS
KC=0
WRITE(6, 4998) (GAMBI), I=1, 100
4998 FORMAT(_5HO, GAMBI) / (1IP10, E12.4)
5 3060
IF(NBODS.EQ.0) GO TO 100
5 3070
C
COMPUTE DELV, DELVT AT BODY CONTROL POINTS
LTC3=LTC(1)
LNC=LNC(1)
DO 50 J=1, LCC
5 3080
C
5 3090
C IT1.IT2 ARE TRAILING VORTICES ON WHICH INTERPOLATED GAMT ARE FOUND.
IT1=FLTC(J)+0.01
IT2=IT1+1
DO 50 I=1,LEN(C
KC=KC+1
C IB1,IB2 ARE BOUND VORTICES.
IB1=FLNC(I)+0.01
IB2=IB1+1
L2=(IT1-1)*NVL(1)+IB1-1
IF(IB1.LE.NVL(1)) GO TO 10
ALM2=0.0
GO TO 12
10 ALM2=ALNGTH(L2)
12 L5=L2+1
ALM5=ALNGTH(L5)
IF(IB1.GE.NVL(1)) GO TO 14
ALM8=0.0
13 ALM11=0.0
GO TO 20
14 L8=L5+1
ALM8=ALNGTH(L8)
IF(IB2.GE.NVL(1)) GO TO 13
ALM11=ALNGTH(L8+1)
20 CONTINUE
NGAM1=NVL(1)
NGAM2=NGAM1
IF(ATTFB(IT1).EQ.99.0) NGAM1=20
25 CONTINUE
IF(ATTFB(IT2).EQ.99.0) NGAM2=20
26 CONTINUE
DO 261 M=1,NGAM1
XD(M)=XG(IT1+M)
261 GD(M)=GAMT(IT1+M)
IF(XD(KC).GT.XD(NGAM1)) XD(NGAM1+1) = XD(KC)
CALL CODIM(XD,GD,NGAM1,XG(KC),GAM1+1)
DO 262 M=1,NGAM2
XD(M)=XG(IT2+M)
262 GD(M)=GAMT(IT2+M)
IF(XQ(KC)<GT.XD(NGAM2)) XD(NGAM2) = XQ(KC)
CALL CODIM(XD*GD*NGAM2*XQ(KC),GAM2*1)
L2=IT1-1*NLV(1)+IB1
L1=L2-NLV(1)
L3=L2+NLV(1)
BL2=BLN/TH(L2)
IF(IT1*NE*1) GO TO 35
BL1=BL2
BL3=BLN/TH(L3)
GO TO 40
35 IF(IT1*NE-NVT(1)) GO TO 36
BL1=BLN/TH(L1)
BL3=BLN/TH(L3)
40 CONTINUE
D = 0.75*ALM5+0.25*ALM8
IF(IB1-NLV(1)) 42,41,42
41 GAMB2=0.0
GO TO 43
42 GAMB2=GAMB(L2+1)
43 CONTINUE
6002 FORMAT(8151)
6001 FORMAT(1H0,1P4E20.4)
DELV(KC)=GAMB(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
+GAMB2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D
DELVT(KC)=-0.5*(GAM1/(BL1+BL2) +GAM2/(BL2+BL3))
50 CONTINUE
100 CONTINUE
IF(NPANS<EQ.0) GO TO 300
KTRV=0
IF(NBODS) 101,101,102
101 NBVORT=0
GO TO 103
192 NBVORT=NLV(1)*NVT(1)
103 CONTINUE
DO 250 II=1,NPANS
NBP=NBODS+II
250 CONTINUE
COMPUTE D5LVM, DELVT ON PANELS.

C

LTCC=LTC(NBP)
LNCC=LNC(NBP)
DO 200 J=1,LTCC
IT1=LTT(I+J)+0.01+KTRV
IT2=IT1+1
DO 200 I=1,LNCC
KC=KC+1
IB1=LNN(I+I)+0.01
IB2=IB1+1
IT1L=IT1-KTRV
IT2L=IT1L+1
L2=(IT1L-1)*NVL(NBP)+IB1-1+NRVORT
L5=L2+1
L8=L5+1
L11=L8+1
ALM2=(ALNGTH(L2))
ALM5=(ALNGTH(L5))
ALM8=(ALNGTH(L8))
ALM11=(ALNGTH(L11))
IF(IB1.EQ.1) ALM2=ALM5
IF(IB1.EQ.NVL(NBP)) ALM8=ALM5
IF(IB2.EQ.NVL(NBP)) ALM11=ALM8
NGAM1=NVL(NBP)
NGAM2=NGAM1
IF(ATTFW(IT1).EQ.99.0) NGAM1=20

125 CONTINUE
IF(ATTFW(IT2).EQ.99.0) NGAM2=20

126 CONTINUE
DO 361 M=1,NGAM1
XD(M)=XGW(IT1*M)
361 GD(M)=GAMTW(IT1*M)
CALL CODIM(XD*GD*NGAM1,XD(KC)*GAM1+1)
DO 362 M=1,NGAM2
XD(M)=XGW(IT2*M)
362 GD(M)=GAMTW(IT2*M)
CALL CODIM(XD*GD*NGAM2,XD(KC)*GAM2+1)
L2=(IT1L-1)*NVL(NBP)+IB1+NRVORT
L1=L2-NVL(NBP)
L3=L2+NVL(NBP)
BL2=BLNGTH(L2)
IF(IT1LNE1) GO TO 135
BL1=BL2
BL3=BLNGTH(L3)
GO TO 140
135 IF(IT1LNE-NVT(NBP)) GO TO 136
BL1=BLNGTH(L1)
BL2=BL2
GO TO 140
136 BL1=BLNGTH(L1)
BL3=BLNGTH(L3)
140 CONTINUE
D=0.75*ALM5+0.25*ALM8
IF(IB1-NVL(NBP)) 142,141,142
141 GAMB2=0.0
GO TO 143
142 GAMB2=GAMB(L2+1)
143 CONTINUE
DELVM(KC)=GAMB(L2)/(0.75*ALM5+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
+GAMB2/(0.75*ALM5+ALM8+0.25*ALM11*0.5*ALM5/D)
TEMP=-DELVM(KC)*SLOPEC(L2)
C
C ADD TEMP ONLY FOR PANEL
DELVT(KC)=-0.5*(GAM1/(BL1+BL2)+GAM2/(BL2+BL3)) + TEMP
200 CONTINUE
KTRV=KTRV+NVT(NBP)+1
NBVORT=NVPTS(NBP)+NBVORT
250 CONTINUE
300 CONTINUE
RETURN
END
**SUBROUTINE GAMMA**

C
C THIS PROGRAM COMPUTES GAMMA'S FOR BOUND AND TRAILING VORTICES.
C
C**************************
C INSERT N Att
C COMMON/PANATT/ NATT(30)*XATT(200)
C**************************
C COMMON/NUMBER/ NVPTS(7)*NCPANTS(7)*NLIN(7)*NLIT(7)*LTC(7)*LNC(7)
1  NCT*NB*NBOIDS*NPANS*NLV(7)*NVT(7)*NTAPE*NTAPE*NTAPE*NTAPE*NTAPE
2  LSEG(7)*TSEG(7)*LFUNC(7)*TFUNC(7)
3  LNDIVB(7)*LTDIVB(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)
C COMMON/SCRAT/ AK(15000)*XG(11151)*GAMT(11151)*GAMB(1500)
1  DFLGM(15000)*DELVT(15000)*VT(15000)*VM(15000)*SLOPEC(1500)
2  XAA(151)*GAMMA1(151)*GAMMA2(151)*XD(150)*GD(150)
3  CP(1320)*XGWM(5020)*GAMTM(5020)
C COMMON DA(5000)
C COMMON/AF/ ATTFB(10)*ATTFW(50)
C EQUIVALENCE (DA(2)*PANS)
C DATA L/9/KBODS/9/NTBODS/0/
C REWIND 18
C IF(NBODS.EQ.0) GO TO 1111
C SKIP ALNGTH AND BLNGTH ARRAYS
C READ(18)
C READ(18)
C 1111 CONTINUE
C NPANS=PANS
C IF(NBODS.EQ.0) GO TO 100
C DC 10 I=1,NBODS
C IF(I-1) 1,1,2
C 1 I=1
C GO TO 3
C 2 M=NCPANTS(I-1)+1
C 3 CONTINUE
C CALL BOLGAM(I,AK(*,NLN(I)),NVT(I),L)L
C KBOIDS=KBOIDS+NCPANTS(I)
C 10 CONTINUE
C 100 IF(NPANS.EQ.0) GO TO 205
C KPI=KBOIDS+1
DO 200 I=1,NPANS
KP=KPI
INB=I+NBODS
CALL PANGAM(I,AK,NVL(INB),NVT(INB),KPI,LNTBODS)
WRITE(6,7007) (AK(IX),IX=1,100)
7007 FORMAT(*OAK AFTER PANGAM*/(1P10E13.4))
KPI=KPI+NCPTS(INB)
200 CONTINUE
205 CONTINUE
READ 18
IF(NPANS.EQ.0) RETURN
NTRVP=0
N1PRE=0
DO 300 I=1,NPANS
NA=NATT((I-1)*3+2)
300 NA=COMPONENT (BODY OR PANEL) TO WHICH PANEL I IS ATTACHED.
IF(NA.EQ.0) GO TO 260
C
C DETERMINE WHICH TRAILING VORTICES NEED TO HAVE MODIFIED GAMS.
C THESE WILL BE THE VORTICES NUMBERED N1 AND N2.
N1=0
IF(NA.EQ.1) GO TO 190
N11=NA-1
DO 180 K=1,N11
180 N1=NVT(K)+N1+1
190 N1=N1+NATT(3*1)
N2=NTRVP+1
C
C N1=TRAILING VORTEX LINE TO WHICH PANEL I IS ATTACHED.
C N2=IN2ORD TRAILING VORTEX LINE OF ATTACHED PANEL I.
C DETERMINE XMIN AND XMAX OF TR. VORTICES N1+N2.
IF(XG(N1,1)-XG(N2,1)) 202,202,203
202 XMIN=XG(N1,1)
GO TO 204
203 XMIN=XG(N2,1)
204 NVOR1=NVL(NA)
NVOR2=NVL(NBODS+1)
IF(NL.EQ.NPRE) NVOR1=20
IF(XG(N1,NVOR1)-XG(N2,NVOR2)) 206,206,207
206 XMAX=XG(N2,NVOR2)
GO TO 208
207 XMAX=XG(N1*NVOR1)
208 CONTINUE
N20=19
XINT=XMAX-XMIN
DX=XINT/N20
N21=N20+1
DO 210 M=1,N21
210 XAA(M)=XMIN+(M-1)*DX
DO 211 M=1,NVOR1
XD(M)=XG(N1*M)
211 GD(M)=GAMT(N1*M)
CALL CODIM(XD*GD,NVOR1,XAA,GAMMA1,N21)
DO 230 M=1,N21
IF(XAA(M)-XG(N1+1)) 222,224,224
222 GAMMA1(M)=0.0
224 IF(XAA(M)-XG(N1+NVOR1)) 228,228,226
226 GAMMA1(M)=GLAST
GO TO 230
228 GLAST=GAMMA1(M)
230 CONTINUE
DO 213 M=1,NVOR2
XD(M)=XGW(N2*M)
213 GD(M)=GAMT(N2*M)
CALL CODIM(XD*GD,NVOR2,XAA,GAMMA2,N21)
DO 240 M=1,N21
IF(XAA(M)-XGW(N2+1)) 232,234,234
232 GAMMA2(M)=0.0
234 IF(XAA(M)-XGW(N2+NVOR2)) 238,238,236
236 GAMMA2(M)=GLAST
GO TO 240
238 GLAST=GAMMA2(M)
240 CONTINUE
DO 250 ::=1,N21
XG(N1,M)=XAA(M)
XGW(N2,M)=XAA(M)
WRITE(6,261) N1,N2,*NPRE
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>261</td>
<td><code>FORMAT(*ON1,N2,NIPRE*315)</code></td>
</tr>
<tr>
<td>262</td>
<td><code>WRITE(6,262)M,GAMMA1(M),GAMMA2(M)</code></td>
</tr>
<tr>
<td></td>
<td>IF(N1.NE.NIPRE) GO TO 249</td>
</tr>
<tr>
<td></td>
<td><code>IF(N1.NE.NIPRE) GO TO 249</code></td>
</tr>
<tr>
<td></td>
<td><code>GAMT(N1,M)=GAMMA2(M)+GAMT(N1,M)</code></td>
</tr>
<tr>
<td>249</td>
<td><code>GAMT(N1,M)=GAMMA1(M)+GAMMA2(M)</code></td>
</tr>
<tr>
<td>250</td>
<td><code>GAMTW(N2,M)=GAMT(N1,M)</code></td>
</tr>
<tr>
<td>C</td>
<td><code>ATTFB(N1)=99.0</code></td>
</tr>
<tr>
<td></td>
<td><code>ATTFW(N2)=99.0</code></td>
</tr>
<tr>
<td></td>
<td><code>WRITE(6,5998)(XG(N1,M),M=1:20)</code></td>
</tr>
<tr>
<td>5998</td>
<td><code>FORMAT(*0X-COORDINATES FOR ATTACHMENT LINE*/(1P10E13.4))</code></td>
</tr>
<tr>
<td></td>
<td><code>WRITE(6,5999)N1,N2</code></td>
</tr>
<tr>
<td>5999</td>
<td><code>FORMAT(*0BODY_ATTACHMENT LINE IS*/13/</code></td>
</tr>
<tr>
<td>I</td>
<td><code>*OPANEL_ATTACHMENT LINE IS*/13/</code></td>
</tr>
<tr>
<td></td>
<td><code>WRITE(6,6001)(GAMT(N1,M),M=1:20)</code></td>
</tr>
<tr>
<td>6000</td>
<td><code>FORMAT(*0BODY GANT AT ATTACHMENT LINE*/(1P10E13.4))</code></td>
</tr>
<tr>
<td></td>
<td><code>WRITE(6,6001)(GAMTW(N2,M),M=1:20)</code></td>
</tr>
<tr>
<td>6001</td>
<td><code>FORMAT(*0PANEL GANT AT ATTACHMENT LINE*/(1P10E13.4))</code></td>
</tr>
<tr>
<td>260</td>
<td><code>CONTINUE</code></td>
</tr>
<tr>
<td></td>
<td><code>NTRVP=NTRVP+NVT(NBODS+I)+1</code></td>
</tr>
<tr>
<td></td>
<td><code>NIPRE=N1</code></td>
</tr>
<tr>
<td>300</td>
<td><code>CONTINUE</code></td>
</tr>
<tr>
<td></td>
<td><code>RETURN</code></td>
</tr>
<tr>
<td></td>
<td><code>END</code></td>
</tr>
</tbody>
</table>
SUBROUTINE PANSAM(IPAK,IP,T,KP,L,NTBP)  5  6330
COMMON/NUMPB/ :VPB(7),NCPTS(7),NLT(7),LTC(7),LNC(7)  5  6340
1  NCT,NB,NOBS,VPAMS,NVL(7),NV1(7),MTAPE,NTAPE,NCTV,ITAPE,TAPE  5  6350
2  LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)  5  6360
3  LNQIVB(7),LTDIVB(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)  5  6370
COMMON/BOV/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)  5  6380
1  YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUBV(100),CHORD(100)  5  6390
2  XCV0(20),XCC0(20),XLE(20),YLE(20),ZLE(20)  5  6400
3  XTE(20),YTE(20),ZTE(20),SLE(20),XJF(20),YJ(20),ZJ(20)  5  6410
4  ETLE(20),XVT(50),YVT(50),ZVT(50),XRT(20),YR(20),ZRT(20)  5  6420
5  XSMT(1000),XTMT(1000),ZMT(1000),DYS(1000),DZS(1000)  5  6430
6  TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)  5  6440
EQUIVALENCE (B,BP,XVR)  5  6450
DIMENSION BP(13000)  5  6460
DIMENSION B(30000),ALNGH(5000),BLNGH(5000),AK(1)  5  6470
EQUIVALENCE (B(20000),ALNGH,B(25000),BLNGH)  5  6480
COMMON/SCRT/DRK(15000),XG(11,151),GAM1(11,151),GAMB(1500)  5  6490
1  DEVM(1500),DEVL(1500),MV(1500),VM(1500),SLOPEC(1500)  5  6500
2  XAM(151),GAMMA1(151),GAMMA2(151),XN(150),XN(150)  5  6510
3  CP(1320),XG(50,20),GAMMA(50,20)  5  6520
COMMON/PANINF,PANSYM(10)  5  6530
DATA LENGTH/0/
LI=0  5  6540
NSPACE = NSPP(IP)  5  6550
SYM=PANSYM(IP)  5  6560
LS=1  5  6570
13 IF(NBODS) 14,13,14  5  6580
14 NBVOR=0  5  6590
GO TO 15  5  6600
14 NBVOR=NLV(1)*NV1(1)  5  6610
15 CONTINUE:
READ(18): BP  5  6620
WRITE(6,6000) IP,NL,NT  5  6630
WRITE(6,6000) KP,L,NTBP  5  6640
6000 FORMAT(315,1P2E16.4)  5  6650
DC 305 J=1,NT  5  6660
IF (J.LE.NSPACE) GO TO 305  5  6670
SUM=0.0  5  6680
DC 305 I=1,I,L  5  6690

I1=(J-1)*NL+1+K-1
SUM = SUM+AK(I1)
DUMM(I1)=SUM
WRITE(*,6000) J*I1,1*SUM,AK(I1)
300 AK(I1)=SUM
305 CONTINUE
WRITE(*,7007) DUMM(I),I=1,100
7007 FORMAT(*,DUMM*,I1P10E13.4)
NS1=NSPACE+1
N1=NT+1
DO 100 J=1,N1
JMNS=J-NSPACE
JT=J+NTBP
DO 100 I=1,NL
IF(J,EQ,N1) GO TO 45
LENGTH=LENGTH+1
KK=NBVORT+LENGTH
L1=L1+1
ALNGTH(KK)=Pll(L1)
IF(JMNS) 41,41,42
41 BNGTH(KK)=PLT(L1)
GO TO 43
42 BNGTH(KK)=2.0*YSUBV(JMNS)
43 CONTINUE
SLOPEC(LENGTH)=TS(LS)
45 CONTINUE
IF(JMNS) 11,11,21
11 XGM(JT*I1)=0.5*(XVR(J,I)+XVR(J,I+1))
GO TO 30
21 CONTINUE
IF(J=N1) 22,24,24
22 FACT=-0.5
JTR=JMNS
GO TO 251
24 LS=NL*(NT-1)*2+1
FACT=+0.5
JTR=JMNS-1
251 IF(I-1) 26,26,28
26 X1=XVCR(JTR*I1)+FACT*TS(LS)*SQR(DYS(LS)**2+DZS(LS)**2)
GO TO 29
28 X1=X2
29 CONTINUE
   IF(I.EQ.NL) GO TO 291
   LS2=LS+2
   X2=XV0(JTR+1)+FACT*TS(LS2)*SQRT(DYS(LS2)*2+DS(LS2)*2)
   GO TO 292
291 X2=X1+DELX
292 XGW(JT+1)=0.5*(X2+X1)
   DELX=X2-X1
30 CONTINUE
   K1=(J-1)*NL+I+KP-1
   LS=LS+2
   IF(J-1) 1,1,5
   1 IF(SYM) 2,9,2
   2 GAMTW(JT+I)=-AK(K1)
   GO TO 10
   5 IF(J-N1) 6,7,6
   6 GAMTW(JT+I)=AK(K1-NL)-AK(K1)
   GO TO 10
   7 GAMTW(JT+I)=AK(K1-NL)
   GO TO 10
   9 IF(NBODS.NE.0) GO TO 2
   GAMTW(JT+I)=0.0
10 CONTINUE
   IF(J.EQ.N1) GO TO 40
   L=L+1
   IF(I-1) 25,20,25
20 GAMB(L)=AK(K1)
   GO TO 40
25 GAMB(L)=AK(K1)-AK(K1-1)
40 CONTINUE
150 CONTINUE
   NTPE=NTB+NT(VT(IP+NBODS)+1
RETURN
END
SUBROUTINE BODGAM(IB,AK,NL,NT,L)
COMMON DA(5000)
COMMON/NVPTS(7),NCPPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1 *NCT,NB,NBODS,NPAN,NVNL(7),NVN(7),NTAPE,NTAPE,NCTV,NTAPE,NTAPE
2 *LSEG(7),TSEG(7),LFUNC(7),TFUNC(7)
3 *LNDIVB(7),LTDIVB(7),NSPP(7),ROOPT(7),OUTERP(7),SYM(7)
COMMON/BODY/B(30000)
DIMENSION XV(151,31)
EQUIVALENCE(XV*8)
COMMON/SCROT/XSOL(5000),XG(11,151),GAMT(11,151),GAMB(1500)
1 *DELVM(1500),DELVT(1500),VT(1500),VM(1500),SLOPEC(1500)
2 *XAA(151),GAMMA1(151),GAMMA2(151),XD(150),GD(150)
3 *CP(1320)
DIMENSION AK(1)
EQUIVALENCE(SYM,DA(19))
READ(18) B
NL=NT+1
LTDIV=LTDIVB(IB)
DO 100 J=1,NL
J1=(J-1)*LTDIV+1
DO 100 I=1,NL
K1=(J-1)*NL+I
XG(J,I)=0.5*(XV(I,J1)+XV(I+1,J1))
IF(J-I) 1,1,5
1 IF(SYM) 2:4:2
2 GAMT(J,I)=-AK(K1)
GO TO 10
4 GAMT(J,I)=0.0
GO TO 10
5 IF(J-NL) 6,7,6
6 GAMT(J,I)=AK(K1-NL)-AK(K1)
GO TO 10
7 IF(SYM) 8:9:8
8 GAMT(J,I)=AK(K1-NL)
GO TO 10
9 GAMT(J,I)=0.0
10 CONTINUE
IF(J-EQ-NL) GO TO 40
L=L+1
```plaintext
IF(I-1) 25,20,25
20 GAMBL = AK(K1)
      GO TO 40
25 GAMBL = AK(K1)-AK(K1-1)
40 CONTINUE
100 CONTINUE
      IF(SYM.EQ.0) GO TO 15
      DO 14 I=1,NL
      GAMT(1,I) = GAMT(1,I)+GAMT(N1+I)
14 GAMT(N1+I) = GAMT(1+I)
15 CONTINUE
      RETURN
END
```
PROGRAM FORCES

MAIN PROGRAM FOR FORCES.

COMMON/BODY/ XAREA(5000), YAREA(5000), ZAREA(5000)
1   XCP(5000), YCP(5000), ZCP(5000), XTL(100)
2   FLNC(150), FLTC(40), XCON(100)

COMMON /COMPRS/ BETAM
DIMENSION YPD(200), ZPD(200), YV(100), ZV(100), PINP(10)

SEE EQUIVALENCE IN PANMAT FOR YPD, ZPD

EQUVALENCE (YPD(1), XTL(1), ZPD(1), FLNC(101))
EQUVALENCE (YV(1), XAREA(1), ZV(1), YAREA)
COMMON /PANINF/ PSYM(10), DUM11(600), PANREF(10), CHORD(10)
DIMENSION TPD(10), TPD(10)
EQUVALENCE (DA(19), BSYM)
DIMENSION AKTB(500), AKTP(500)

EQUVALENCE (AKTB(1), YCP(4001), AKTP(1), YCP(4501))
DIMENSION CP(1320)
COMMON/SCRAT/ D(25000)
DIMENSION VM(1500), VT(1500)

EQUVALENCE (VM(1), D(14323), VT(1), D(12823))
DIMENSION CPBOD(1500)

EQUVALENCE (D18076), CP(1), D(20000), CPBOD)Y
DIMENSION E (50), XCO(20), YDI(400), DZI(400)
1   TWIST(20), CPU(40), CPL(40), DSLE(20), XLE(20), YLE(20)
2   ZLE(20), SPCT(40), CHC(20)

EQUVALENCE (E(D15001), XCCO(D5101), YDI(D5201), DZI(D5701))
1   TWIST(D6011), CPU(D6301), CPL(D6801)
2   DSLE(D7301), XLE(D7401), YLE(D7501)
3   ZLE(D17070), SPCT(D7701), CHC(D7801)
4   XCG(DA(7)), YCG(DA(8)), ZCG(DA(9)), DANS(DA(2))
5   VMU(D8001), VML(D8201), UTV(D8401), UVL(D8601)

DIMENSION VMU(200), VM(200), UTL(200), UT(200)

COMMON DA(15000)

COMMON/NUMBER/ NVPTS(7), QCPTS(7), NLI(7), NLT(7), LEC(7), LNC(7)
1   NCT, NSN, NSN, NPSN, NTV(7), NTAPA, NTAP, NCTV, NTAPA, NTAPA
2   LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)
3   LNDIV(7), LNDIV(7), NSP(7), RODTP(7), OUTREF(7), SYMP(7)

COMMON/SLOPE/ SIGMAP(500), DZDXT(500), DZDXYC(500), TANPI(500)
DIMENSION ZERO(50)
```
DIMENSION SR(5,8), PS(10,6)
EQUIVALENCE (BR, DA(16)), (PR, DA(3421)), (PSY, DA(3426))
1 (PCH, DA(3422)), (BCH, DA(17))
COMMON/CONPTS/ XQ(1320), YQ(1320), ZQ(1320)
1 *XN(1320), YN(1320), ZN(1320)
DIMENSION BP(13000)
DIMENSION XCC(20), XCV(20), XSS(1000), YSS(1000), ZSS(1000), ET(20)
1 *TS(1000), SIGMA(1000)
EQUIVALENCE (TP, XPAR), (XCC, BP(2421)), (XCV, BP(2401))
1 *(XSS, BP(8871)), (YSS, BP(9871)), (ZSS, BP(10871)), (TS, BP(7871))
2 *(SIGMA, BP(11871))
DIMENSION CPNET(1000), CDS(100), CDA(100), CDT(100), CDTH(100)
EQUIVALENCE (SPAN, DA(3423)), (PLVI, DA(3431))
EQUIVALENCE (BP, 23001), CPNET, (D(24101), CDS), (D(24201), CDA)
1 *(D(24301), CDT), (D(24401), CDTH)
COMMON/PANEL/ NPAN, IPXY, IIVC, NVVVP, NTVP, LNFCP, LTCP, LNCPP, LTCPP
DATA PIND/10*1.0/
NC=0
IF(NBODS.EQ.0) GO TO 50
N2=LTDIVB(1)*NVT(1)+1
READ(12)(YV(I), ZV(I), I=1,42)
50 CONTINUE
NPANS=PANS
IF(PANS.EQ.0) GO TO 55
WRITE(6,7002) (1*YPD(I), ZPD(I), I=1,20)
7002 FORMAT(*OPANEL, DRAG COORDINATES/*(I5,2F15.5))
WRITE(6,7001) (1*AKTP(I), I=1,20)
7001 FORMAT(*O AKTP/*(I5,1PE20.6))
WRITE(6,6001) NT, NBODS+1, NVT, NBODS+2
WRITE(6,6002) PSYM
WRITE(6,6002) PIND
55 CONTINUE
NSP=0
DO 101 IS=1, LTCP
CDS(IS)=0.
CDA(IS)=0.
CDT(IS)=0.
101 CDTH(IS)=0.
CDST=0.
```
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>CDAT=0.</td>
</tr>
<tr>
<td>6</td>
<td>CDTT=0.</td>
</tr>
<tr>
<td>6</td>
<td>CDTHT=0.</td>
</tr>
<tr>
<td>6</td>
<td>IF(DA(3424).NE.1.) GO TO 58</td>
</tr>
<tr>
<td>6</td>
<td>IF(NBODS.NE.0) GO TO 58</td>
</tr>
<tr>
<td>6</td>
<td>IF(NPANS.NE.1) GO TO 58</td>
</tr>
<tr>
<td>6</td>
<td>BETAM=1.0</td>
</tr>
<tr>
<td>6</td>
<td>NCPT=NCPTS(1)</td>
</tr>
<tr>
<td>6</td>
<td>DO 56 I=1,NCPT</td>
</tr>
<tr>
<td>56</td>
<td>IF(ZI(I).NE.1.0) GO TO 58</td>
</tr>
<tr>
<td>6</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>6</td>
<td>IF(PLVI.NE.0.0) GO TO 58</td>
</tr>
<tr>
<td>6</td>
<td>REWIND 18</td>
</tr>
<tr>
<td>6</td>
<td>READ(18) 8P</td>
</tr>
<tr>
<td>6</td>
<td>REWIND 18</td>
</tr>
<tr>
<td>6</td>
<td>HSPAN=NTVVP*(YSS(NBVVP*2+1)-YSS(1))</td>
</tr>
<tr>
<td>41</td>
<td>ET(I)=YQ(I-1)*LNCPP+1)/HSPAN</td>
</tr>
<tr>
<td>6</td>
<td>DO 59 I=1,NCPT</td>
</tr>
<tr>
<td>6</td>
<td>IU=(I-1)*2+1</td>
</tr>
<tr>
<td>59</td>
<td>IL=IU+1</td>
</tr>
<tr>
<td>5</td>
<td>CPNET(I)=CP(I*)-CP(IU)</td>
</tr>
<tr>
<td>6</td>
<td>NVPT=NBVVP<em>NTVVP</em>2</td>
</tr>
<tr>
<td>6</td>
<td>DO 61 I=1,NVPT</td>
</tr>
<tr>
<td>6</td>
<td>YSS(I)=YSS(I)/BETAM</td>
</tr>
<tr>
<td>6</td>
<td>ZSS(I)=ZSS(I)/BETAM</td>
</tr>
<tr>
<td>6</td>
<td>TS(I) =BETAM*TS(I)</td>
</tr>
<tr>
<td>61</td>
<td>SIGMA(I)=SIGMA(I)/BETAM</td>
</tr>
<tr>
<td>6</td>
<td>CALL &quot;FDRAK(NBODS,NPANS,YV,ZV,AKTR,N2-1,RSYM,YPD,ZPD,AKTP</td>
</tr>
<tr>
<td>5</td>
<td>1,NVT(NBODS+1),PSY,LEM,TDP,TDP,NSP</td>
</tr>
<tr>
<td>58</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>6</td>
<td>DO 551 I=1,100</td>
</tr>
<tr>
<td>6</td>
<td>YV(I)=YV(I)/BETAM</td>
</tr>
<tr>
<td>551</td>
<td>ZV(I)=ZV(I)/BETAM</td>
</tr>
<tr>
<td>6</td>
<td>DO 552 I=1,200</td>
</tr>
<tr>
<td>6</td>
<td>YPD(I)=YPD(I)/BETAM</td>
</tr>
<tr>
<td>552</td>
<td>ZPD(I)=ZPD(I)/BETAM</td>
</tr>
<tr>
<td>6</td>
<td>CALL FDRAK(NBODS,NPANS,YV,ZV,AKTR,N2-1,RSYM,YPD,ZPD,AKTP</td>
</tr>
<tr>
<td>6</td>
<td>1,NVT(NBODS+1),PSY,LEM,TDP,TDP,NSP</td>
</tr>
</tbody>
</table>
IF(NBODS.EQ.0) GO TO 103
READ(12) XAREA,YAREA,ZAREA
READ(12) XCP,YCP,ZCP,XTL,XCOG,FLMC,FLTC,NBV,NTV,LTPTS,LTPTS
1 *LNDTV,LTDIV,NAV,NBV,NTV,CHORD,XCOG,YCG,ZCG,ALPHA,REF
6200 FORMAT(1P9E12.4)
6301 FORMAT(10I5)
NC=NCPTS(1)
DO 60 I=1,NC
60 CPBODY(I)=CP(1)
6401 FORMAT(*04/(1P10E13.4))
CALL BPRINT(LTPTS,LTPTS,XCP,YCP,ZCO,YM,VT,CP,1)
CALL REFLECT(NAV,NTV,XAREA,YAREA,ZAREA,XCP,YCP,ZCP
1 LNTPTS,LTPTS,FLTC,CPBODY,LTDIV)
NPAA=NBV*NTV
DO 30 I=1,NPAA
XAREA(I)=XAREA(I)/BETAM#2
YAREA(I)=YAREA(I)/BETAM
ZAREA(I)=ZAREA(I)/BETAM
YCP(I)=YCP(I)/BETAM
30 ZCP(I)=ZCP(I)/BETAM
CALL BINTEG(XAREA,YAREA,ZAREA,XCP,YCP,ZCP,FLMC,LTPTS,FLTC,LTPTS
1 *CPBODY,LNDIV,LTDIV,XTL,NAV,D(100),XCOG,D(1))
2 *D(101),D(201),CHORD,D(301),D(401),D(501),D(601),D(701),D(801)
3 *NBV,NTV,D(951),XCOG,YCG,ZCG,ALPHA,BREF,
4 SB(1),SB(2),SB(3),SB(4),SB(5),SB(6),SB(7),SB(8)
5 SB(1),SB(2),SB(3),SB(4),SB(5)
C THE FIRST SUBSCRIPTS OF SB(I,J) SHOULD BE THE LAYER(PANEL) NUMBER
103 IF(PANS.EQ.0) GO TO 200
PANS=PANS
NCPI=NC
NP=1
DO 554 I=1,1500
554 I=I+1500
DZUXC(I)=DZUXC(I)/SLETAN
DZUXT(I)=DZUXT(I)/SLETAN
N1=NC+1
C 555 I=I+1500
YQ(I) = YQ(I) / BETAM
555
ZQ(I) = ZQ(I) / BETAM
DO 150 K = 1, NPIANS
READ(12) * REFA, LTcpp, E, Lncpp, Xcco, Dy1, Dz1, TWIST, CHC
1   * CHRM, (XLE(I), YLE(I), ZLE(I), I = 1, LTcpp), FLTC, SPcF, BS, DSLE
DO 105 I = 1, LTcpp
K1 = FLTC(I)
DSLE(I) = DSLE(K1) / BETAM
YLE(I) = YLE(I) / BETAM
105
ZLE(I) = ZLE(I) / BETAM
NPTS = NCPTS(NBODS + K)
DO 110 I = 1, NPTS
IU = (I - 1) * 2 + 1 + NPC1
IL = IU + 1
VMU(I) = VM(IU)
VTU(I) = VT(IU)
VNL(I) = VM(IU)
VTL(I) = VT(IU)
CPL(I) = CP(IU)
NPC1 = NPC1 + 1
110
DO 111 I = 1, 50
ZERO(I) = 0.0
CALL PPRINT(E, Xcco, LTcpp, Lncpp, Vmu, Vnl, Vtlu, Cpu, CPL, NBODS + K)
B = B / BETAM
BS = B / BETAM
DO 553 I = 1, 400
Dy1(I) = DY1(I) / BETAM
553
Dz1(I) = Dz1(I) / BETAM
CALL PANNINT1(B, REFA, LTcpp, E, Lncpp, Xcco, Dy1, Dz1, TWIST, BS, SPcF
1   * CPU, CPL, DSLE, XCG, YCG, ZCG, CHC, CHRM, Dz0Xc(NPDZ), Dz0XT(NPDZ)
2   * XQ(N1), YQ(N1), ZQ(N1), XLE, YLE, ZLE, D(8000)
3   * TPdK(K), CDA, CDS, COTH, CDT, PS(K, 1), PS(K, 2), PS(K, 3)
4   * PS(K, 4), PS(K, 5), PS(K, 6), K + NBODS, COAT, CST, COTH, CDTT)
NC = NC + NPC1
NPC1 = NPC1 + 2 * NPTS
NPd2 = NPd2 + NPTS
150 CONTINUE
200 CONTINUE
CALL CCINTG(SB(1:2), SB(1:2), SB(1:8))
1  SB(1:7), PS(1:1), PS(1:2), PS(1:3), PS(1:3), PS(1:3), PS(1:3), PS(1:3), PS(1:3)
2  PANREF, PSYM, PCHORD, SB(1:8)
3  STOP 6543
END
USEROUTINE CCI:TG (CXB, CYB, CZB, CMXB, CMYB, CMZB, ARB, CXP, CYP, CYP6)
1, CZP, CMXP, CMYP, CMZP, AP, FS, CP, CB 16 2040
6 2050

THIS ROUTINE INTEGRATES THE TOTAL CONFIGURATION

CXB CYB CZB - ARRAYS OF C S OF BODIES
CMXB CMYB CMZB - ARRAYS OF CMS OF BODIES
ARB - ARRAY OF REFERENCE AREAS OF BODIES
CXP CYP CZP - ARRAYS OF CMS OF PANELS
CMXP CMYP CMZP - ARRAYS OF CMS OF PANELS
AP - ARRAY OF PANEL AREAS
CB - ARRAY OF BODY CHORD LENGTHS
CP - ARRAY OF PANEL CHORD LENGTHS
NB - NUMBER OF BODIES
NP - NUMBER OF PANELS

- TOTAL CONFIGURATION CHORD LENGTH
XRCG YRCG ZRCG-X Y AND Z ( /CR)CG
AR - TOTAL CONFIGURATION REFERENCE AREA
FS - SYMMETRY INDICATOR 0.0 IS SYMMETRICAL

ODIMENSION CXB(1), CYB(1), CZB(1), CMXB(1)
1 CMYB(1), CMZB(1), ARB(1)
2 CXP(1), CYP(1), CZP(1), CMXP(1), CMYP(1)
3 CMZP(1), AP(1), FS(1), CP(1), CB(1)

EQUIVALENCE (ARB, CBJC)
COMMON DA(5100)

EQUIVALENCE (DA(5), CA(1), BN, DA(2), PN)
1 (AR, DA(4), (XRCG, DA(7)), (YRCG, A(8)), (ZRCG, DA(9))
NB = BN
NP = PN
PRINT 6

6 FORMAT(1H1, 35X, *TOTAL CONFIGURATION LOADS*)
CX = 0.0
CY = 0.0
CZ = 0.0
CMX = 0.0
CMY = 0.0
CMZ = 0.0
1)
RETURN
END
<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>PANTIPD·REFA·NETA·ETA·NXOC·XOC·DY·DZ·EPSILO·BS·DA</th>
<th>6</th>
<th>2850</th>
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<tbody>
<tr>
<td>CPU·CPL·DSLE·XCG·YCG·ZCG·C·CHORD·DZC·DZT·X·Y·Z</td>
<td>6</td>
<td>2860</td>
<td></td>
</tr>
<tr>
<td>XLE·YLE·ZLE·DS·CDT·CDTO·CDLC·CTAVRG·CDTCA·CX·CY</td>
<td>6</td>
<td>2870</td>
<td></td>
</tr>
<tr>
<td>CODED BY E. D. GAITHER</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>THE GREAT NUMBER OF ARGUMENTS WERE USED TO ALLOW GREATER EASE IN ADDING THIS ROUTINE TO THE OTHERS. AT A LATER DATE THESE SHOULD BE REPLACED BY COMMON REFERENCES.</td>
<td></td>
<td></td>
<td></td>
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</table>

**THIS ROUTINE INTEGRATES PANELS WITH CONTROL SURFACES**

<table>
<thead>
<tr>
<th>C</th>
<th>B</th>
<th>SPAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>REF1</td>
<td>REFERENCE AREA</td>
</tr>
<tr>
<td>E</td>
<td>NET1</td>
<td>NUMBER OF ETAS WHERE CPS ARE GIVEN</td>
</tr>
<tr>
<td>T</td>
<td>ETA</td>
<td>ETAS FOR GIVEN CPS</td>
</tr>
<tr>
<td>N</td>
<td>NX03</td>
<td>NUMBER OF X/C STATION FOR GIVEN CPS</td>
</tr>
<tr>
<td>X</td>
<td>XOC</td>
<td>X/C ARRAY</td>
</tr>
<tr>
<td>Y</td>
<td>DY</td>
<td>DELTA Y FOR VORTEX WHERE CP IS GIVEN</td>
</tr>
<tr>
<td>Z</td>
<td>DZ</td>
<td>DELTA Z FOR VORTEX WHERE CP IS GIVEN</td>
</tr>
<tr>
<td>E</td>
<td>EPSILO</td>
<td>TWIST ARRAY</td>
</tr>
<tr>
<td>S</td>
<td>BS</td>
<td>SURFACE SPAN LENGTH (FOR 2-D SURFACE; BS=B)</td>
</tr>
<tr>
<td>P</td>
<td>SPCF</td>
<td>SPECIAL LATERAL CONTROL FUNCTIONS</td>
</tr>
<tr>
<td>C</td>
<td>CPU</td>
<td>CP ARRAY FOR UPPER SURFACE</td>
</tr>
<tr>
<td>P</td>
<td>CPL</td>
<td>CP ARRAY FOR LOWER SURFACE</td>
</tr>
<tr>
<td>D</td>
<td>DSL5</td>
<td>DELTA S ON LEADING EDGE FOR SPANWISE VORTEX SECTIONS WHERE CPS ARE GIVEN</td>
</tr>
<tr>
<td>X</td>
<td>XCG</td>
<td>X CENTER OF GRAVITY</td>
</tr>
<tr>
<td>Y</td>
<td>YCG</td>
<td>Y CENTER OF GRAVITY</td>
</tr>
<tr>
<td>Z</td>
<td>ZCG</td>
<td>Z CENTER OF GRAVITY</td>
</tr>
<tr>
<td>C</td>
<td>CHORD</td>
<td>CHORD ARRAY FOR SPANWISE STATIONS WHERE CPS ARE GIVEN</td>
</tr>
<tr>
<td>D</td>
<td>D2C</td>
<td>DZ/DX FOR CAHIER</td>
</tr>
<tr>
<td>T</td>
<td>DZT</td>
<td>DZ/DX FOR THICKNESS</td>
</tr>
<tr>
<td>X</td>
<td>X</td>
<td>COORDINATES AT CPS</td>
</tr>
<tr>
<td>Y</td>
<td>Y</td>
<td>X ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS</td>
</tr>
<tr>
<td>Z</td>
<td>Z</td>
<td>Y ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS</td>
</tr>
<tr>
<td>L</td>
<td>DS</td>
<td>Z ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS</td>
</tr>
<tr>
<td>D</td>
<td>CDT</td>
<td>A SCRATCH ARRAY</td>
</tr>
<tr>
<td>C</td>
<td>CDTO</td>
<td>(CDT=O C)/CAVG</td>
</tr>
</tbody>
</table>

**INCL**
<table>
<thead>
<tr>
<th>EQUVALENCE (CXC,CAVJ,×SLCS)</th>
<th>(Y,SLCS,×CAVJ)</th>
<th>6</th>
<th>3630</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(CYC×SLCS)</td>
<td>(CYC×SLCS)</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>(CXC×SLCS)</td>
<td>(CXC×SLCS)</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>(CXC×SLCS)</td>
<td>(CXC×SLCS)</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>(CXC×SLCS)</td>
<td>(CXC×SLCS)</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>(CXC×SLCS)</td>
<td>(CXC×SLCS)</td>
<td>6</td>
</tr>
<tr>
<td>RLINT(XI, XO, ETAI, ETAO, ATAI)</td>
<td>6</td>
<td>3690</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>= XI + (XO - XI) * (ATA - ETAI) / (ETAO - ETAI)</td>
<td>6</td>
<td>3700</td>
</tr>
<tr>
<td>PRINT 26, NC</td>
<td>6</td>
<td>3710</td>
<td></td>
</tr>
<tr>
<td>26 FORMAT(1H1*, 36X*, <em>PANEL SECTIONAL LOADS FOR COMPONENT</em>, IS/</td>
<td>6</td>
<td>3720</td>
<td></td>
</tr>
<tr>
<td>11HC, 5X, 3ETA, 7X,</td>
<td>6</td>
<td>3730</td>
<td></td>
</tr>
<tr>
<td>2XC×CAVJ, 5X, CYC×CAVJ, 5X, CXC×CAVJ, 5X,</td>
<td>6</td>
<td>3740</td>
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</tr>
<tr>
<td>3XC×CAVJ, 5X, CML×CAVJ, 2X, CML×CAVJ, 2X,</td>
<td>6</td>
<td>3750</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>SET UP DELTA S AND DELTA S LE</td>
<td>6</td>
<td>3760</td>
</tr>
<tr>
<td>BT = B*B</td>
<td>6</td>
<td>3770</td>
<td></td>
</tr>
<tr>
<td>AR = BT/REFA</td>
<td>6</td>
<td>3780</td>
<td></td>
</tr>
<tr>
<td>DO 20 K=1, NETA</td>
<td>6</td>
<td>3790</td>
<td></td>
</tr>
<tr>
<td>DO 10 I=1, NXOC</td>
<td>6</td>
<td>3800</td>
<td></td>
</tr>
<tr>
<td>DS(I,K) = SQRT(DY(I,K)**2 + DZ(I,K)**2)</td>
<td>6</td>
<td>3810</td>
<td></td>
</tr>
<tr>
<td>10 CONTINUE</td>
<td>6</td>
<td>3820</td>
<td></td>
</tr>
<tr>
<td>20 CONTINUE</td>
<td>6</td>
<td>3830</td>
<td></td>
</tr>
<tr>
<td>ARBSB2 = AR×BS/(BT)</td>
<td>6</td>
<td>3840</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>STEP SPAN WISE</td>
<td>6</td>
<td>3850</td>
</tr>
<tr>
<td>DO 500 K=1, NETA</td>
<td>6</td>
<td>3860</td>
<td></td>
</tr>
<tr>
<td>DYOPIT = DY(I,K)/DS(I,K)</td>
<td>6</td>
<td>3870</td>
<td></td>
</tr>
<tr>
<td>DZOPIT = DZ(I,K)/DS(I,K)</td>
<td>6</td>
<td>3880</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>STEP CHORD WISE</td>
<td>6</td>
<td>3890</td>
</tr>
<tr>
<td>DO 100 I=1, NXOC</td>
<td>6</td>
<td>3900</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>FILL DEPENDENT VARIABLE ARRAYS</td>
<td>6</td>
<td>3910</td>
</tr>
<tr>
<td>DSDSL=DS(I,K)/DSLE(K)</td>
<td>6</td>
<td>3920</td>
<td></td>
</tr>
<tr>
<td>TCON = CPL(I,K)</td>
<td>6</td>
<td>3930</td>
<td></td>
</tr>
<tr>
<td>1TANLF( DZT(I,K), DZC(I,K), EPSI(K), DA, ETA(K), X(I,K), 1.0)</td>
<td>6</td>
<td>3940</td>
<td></td>
</tr>
<tr>
<td>2 CPU(I,K) *</td>
<td>6</td>
<td>3950</td>
<td></td>
</tr>
<tr>
<td>3TANLF( DZT(I,K), DZC(I,K), EPSI(K), DA, ETA(K), X(I,K), 1.0)</td>
<td>6</td>
<td>3960</td>
<td></td>
</tr>
<tr>
<td>CPLCPU = CPL(I,K) - CPU(I,K)</td>
<td>6</td>
<td>3970</td>
<td></td>
</tr>
<tr>
<td>T1(I) = DSDSL + TCON</td>
<td>6</td>
<td>3980</td>
<td></td>
</tr>
<tr>
<td>T2(I) = DSDSL + CPLCPU</td>
<td>6</td>
<td>3990</td>
<td></td>
</tr>
<tr>
<td>YIKC= Y(I,K) - YCG</td>
<td>6</td>
<td>4000</td>
<td></td>
</tr>
<tr>
<td>ZIKZCG = Z(I,K) - ZCG</td>
<td>6</td>
<td>4010</td>
<td></td>
</tr>
<tr>
<td>XIKXC= X(I,K) - XCG</td>
<td>6</td>
<td>4020</td>
<td></td>
</tr>
</tbody>
</table>
T3(9) = DS3SLE*(YIKYCG*DYOPIT+ZIKZCG*ZDZOPIT)*CPLCPU
T4(1) = DS3SLE*(ZIKZCG*TCON-XIKXCG*DYOPIT*CPLCPU)
T5(11) = DS3SLE*(-XIKXCG*DYOPIT*CPLCPU-YIKYCG*TCON)
XIKXLE = X(I*K) = XLE(K)
YIKYLE = Y(I*K) = YLE(K)
ZIKZLE = Z(I*K) = ZLE(K)
T6(1) = DS3SLE*(YIKYLE*DYOPIT+ZIKZLE*ZDZOPIT)*CPLCPU
T7(1) = DS3SLE*(ZIKZLE*TCON-XIKXLE*DYOPIT*CPLCPU)
T8(1) = DS3SLE*(-XIKXLE*DYOPIT*CPLCPU-YIKYLE*TCON)

100 CONTINUE
C INTEGRATE CHORD WIZE
T1 = POLINT(XOC*T1+NXOC*0.0+1.0)
T2 = POLINT(XOC*T2+NXOC*0.0+1.0)
T3 = POLINT(XOC*T3+NXOC*0.0+1.0)
T4 = POLINT(XOC*T4+NXOC*0.0+1.0)
T5 = POLINT(XOC*T5+NXOC*0.0+1.0)
T6 = POLINT(XOC*T6+NXOC*0.0+1.0)
T7 = POLINT(XOC*T7+NXOC*0.0+1.0)
T8 = POLINT(XOC*T8+NXOC*0.0+1.0)
CARBB = C(K)*ARBSB2
CTAVRG(K) = CTAVRG(K)*ARBSB2
CXCAVG(K) = CARBB*(T11)-CTAVRG(K)
CYCAVG(K) = CARBB*DZOPIT*T12
CZCAVG(K) = CARBB*DYOPIT*T12
CARBB = CARBB/CHORD
CMXCAV(K) = CARBB*C13
CMYCAV(K) = CARBB*(T14-(Z(1*K)-ZCG)*CTAVRG(K)/CHORD
CMZCAV(K) = CARBB+(T15+Y(1*K)-YCG)*CTAVRG(K)/CHORD
CNCAVG(K) = DZOPIT+CYCAVG(K)+DYOPIT*CZCAVG(K)
TPIT = CXCAVG(K)**2+CYCAVG(K)**2+CZCAVG(K)**2
CMLEXE(K) = ARBSB2*T16
CMLEYC(K) = ARBSB2*T17
CMLEZC(K) = ARBSB2*T18
CDT0(K) = CDT0(K)*ARBSB2
CDLICE(K) = CDLICE(K)*ARBSB2
CDTICA(K) = CDTICA(K)*ARBSB2
IF(AUG*(TPIT) ST.1.0E-7) GOTO 131
XCPLE(K) = 0.0
YCPLE(K) = 0.0
ZCLE(1)=0.0
GOTO 132

131 CONTINUE
XCLE(K)=(CYCAVG(K)*CMLCZ(K)-CZCAVG(K)*CMLYC(K))/TPIT
YCLE(K)=(CZCAVG(K)*CMLCX(K)-CXCAVG(K)*CMLZC(K))/TPIT
ZCLE(K)=(CXCAVG(K)*CMLYC(K)-CYCAVG(K)*CMLEX(K))/TPIT

132 CONTINUE
PRINT 156 ETAF(K),CXCAVG(K),CYCAVG(K),CZCAVG(K),CNCAVG(K),
1CMLCZ(K),CMLYC(K),CMLZC(K)

156 FORMAT(1H*,F10.6,F13.6)
500 CONTINUE
PRINT 506

506 FORMAT(1H0,5X,3HETA,7X,1*XLE/C CP**,5X,SYLE/C CP**,5X,ZLE/C CP**,4X,CTC=0/C/CAVG*)
2.3X*,CMLIC/CAVG*,4X*,CMLIC/CAVG*)
PRINT 156 ETAF(K),XCLE(K),YCLE(K),ZCLE(K),CDO0(K),CMLIC
1(K),CTAVG(K),CDTICA(K),K=1,NETA)

C INTEGRATE SPAN WIZE
CX=POLINT ETA,CXCAVG,NETA,0.0,1.0
CY=POLINT ETA,CYCAVG,NETA,0.0,1.0
CZ=POLINT ETA,CZCAVG,NETA,0.0,1.0
CMX=POLINT ETA,CMXCAVG,NETA,0.0,1.0
CMY=POLINT ETA,CMYCAVG,NETA,0.0,1.0
CMZ=POLINT ETA,CMZCAVG,NETA,0.0,1.0
PRINT 606 CX,CY,CZ,CMX,CMY,CMZ
CDI=CDI/REFA
CDT=CDT/REFA
CDL=CDL/REFA
CT=CT/REFA
CDTI=CDTI/REFA

606 FORMAT(//1H0,44X,*TOTAL PANEL LOADS*/
1 1H0,7X,CX*13X,CY*13X,CZ*13X,CMX*12X,CMY*12X,CMZ
2/1H06F15.6)
XYZ = CX*CX+CY*CY+CZ*CZ
IF(ABS(XYZ) GT 1.0E-7) GOTO 611
XCPK = 0.0
YCPK = 0.0
ZCPK = 0.0
GOTO 612
611 CONTINUE
XPCX=XC/CHORD+(CY*CMZ-CZ*CMY)/XYZ
YPCY=YCW/CHORD+(CX*CMX-CX*CMZ)/XYZ
ZPCZ=ZCW/CHORD+(CX*CMX-CX*CMZ)/XYZ

612 CONTINUE
PRINT 616 CDT, CDL, CDI, CTI, XPCX, YPCY, ZPCZ
616 FORMAT(1H1,3X*CDT=0*10X*CDL*12X*CDI*13X*CTI*12X*CDTI*11X)
T,
1/1H, 5F15.6/1H, 5X,
2 *X/C C(5,*), 9X,*Y/C C(9,*), 9X,*Z/C C(9,*+1H, 3F15.6)

C
691 THIS SECTION STEPS THROUGH THE CONTROL SURFACES WHILE SOLVING THE
692 CONTROL SURFACE EQUATIONS
693 NCS = 1
694 IF(DA(1,NCS) .EQ. 0) RETURN
PRINT 706 NCS
706 FORMAT(1H1,35X,15HCONTROL SURFACE, 13,16H SECTIONAL LOADS)

C
697 FIN4 THE ETAS AT WHICH THIS CONTROL SURFACE STARTS AND ENDS
698 DO 710 KS=1, NETA
699 IF(ETA(KS)-DA(3,NCS)) 710, 711, 711
710 CONTINUE
711 DO 720 KE=KS, NETA
720 CONTINUE

C
695 NESP=KE-KS+1
696 TFS=(DA(1,NCS)+1.)/2.0
697 ETAD=DA(4,NCS)-DA(3,NCS)
698 STEP SPANWISE THROUGH THIS PANEL
699 DO 900 KE=KS, KE
700 T7(K)=ETA(K)-DA(3,NCS)/ETAD
701 DYPIT=DY(1,K)/DS (K)
702 UZPIT=DZ(1,K)/DS (K)

C
690 FIN4 THE XCS AT WHICH THIS CONTROL SURFACE STARTS OR ENDS
691 XKF=RLINT(DA(5,NCS),DA(6,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
692 XH=RLINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
693 XHP=RLINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS), ETA(K)+1.E-7)
**CONTINUATION**

740 DO 740 ISE = 1, NXOC

741 ISE = ISE - 1

IF (ISE .EQ. 1) 742, 744, 743

742 ISE = IE

IF (X(I*IS,K) .NE. XKF) ISE = ISE + 1

IE = NXOC

CCS = X(I*K) + C(K) - X(I*IS,K)

**FIT FOR Y AND Z OF HING LINE**

743 DO 760 I=2, NXOC

IF (X(I*K) - XH) 760, 761, 761.

760 CONTINUE

761 YH = RLINT(Y(I*K), Y(I-1*K), X(I*K), X(I-1*K), XH)

ZH = RLINT(Z(I*K), Z(I-1*K), X(I*K), X(I-1*K), XH)

YHP = RLINT(Y(I*K), Y(I-1*K), X(I*K), X(I-1*K), XHP)

YH = YHP - YH

ZHP = RLINT(Z(I*K), Z(I-1*K), X(I*K), X(I-1*K), XHP)

ZH = ZHP - ZH

T = SQRTH(YH*YH + XH*XH + ZH*ZH)

HZ = ZH/T

HY = YH/T

HX = XH/T

TT = C(K) / CCS

DO 770 I=1S, IE

LD8SLE = DS(I*K)/DSLE(K)

XIKXH = X(I,K) - XH

YIKYH = Y(I,K) - YH

ZIKZH = Z(I,K) - ZH

T = TAN(DA(2*IS))

TCSR = CFL(I,K) * (T + U2T(I,K) - (ZC(I*K) + EPSILO(K))) /
INTEGRATE CHORDIZE

\[ \text{NX} = \text{IE} - \text{IS} + 1 \]

\[ \text{T11} = \text{POLINT}(\text{T8(1S)},\text{T1(1S)}), \text{NX}, 0.0, 1.0) \]
\[ \text{T12} = \text{POLINT}(\text{T8(1S)},\text{T2(1S)}), \text{NX}, 0.0, 1.0) \]
\[ \text{T13} = \text{POLINT}(\text{T8(1S)},\text{T3(1S)}), \text{NX}, 0.0, 1.0) \]
\[ \text{T14} = \text{POLINT}(\text{T8(1S)},\text{T4(1S)}), \text{NX}, 0.0, 1.0) \]
\[ \text{T15} = \text{POLINT}(\text{T8(1S)},\text{T5(1S)}), \text{NX}, 0.0, 1.0) \]

\[ \text{CARBS} = \text{CCS} * \text{ARBRS2} \]
\[ \text{X5LCS(K)} = \text{CARBS} * \text{T11} - \text{CTAVRG(K)} \]
\[ \text{Y5LCS(K)} = \text{CARBS} * \text{T12} - \text{DZOPIT} * \text{T12} \]
\[ \text{Z5LCS(K)} = \text{CARBS} * \text{T13} * \text{T12} + \text{Y5LCS(K)} * \text{DYOPIT} \]
\[ \text{X5MAHL(K)} = \text{CARBS} * \text{T11} \]
\[ \text{Y5MAHL(K)} = \text{CARBS} * \text{T14} - \text{TFS}(\text{T8(1S)}, \text{T15}) - \text{ZH} * \text{CTAVRG(K)} / \text{CHORD} \]
\[ \text{Z5MAHL(K)} = \text{CARBS} * \text{T14} - \text{TFS}(\text{T8(1S)}, \text{T15}) - \text{YH} * \text{CTAVRG(K)} / \text{CHORD} \]
\[ \text{T = X5LCS(K) * T8(1S) + Y5LCS(K) * T8(1S) + Z5LCS(K) * T8(1S)} \]
\[ \text{T = T * C(K)} \]
\[ \text{X5CPCS(K)} = (\text{Y5LCS(K)} * \text{Z5MAHL(K)} - \text{Z5LCS(K)} * \text{Y5MAHL(K)}) / \text{T} \]
\[ \text{Y5CPCS(K)} = (\text{Z5LCS(K)} * \text{X5MAHL(K)} - \text{X5LCS(K)} * \text{Z5MAHL(K)}) / \text{T} \]
\[ \text{Z5CPCS(K)} = (\text{X5LCS(K)} * \text{Y5MAHL(K)} - \text{Y5LCS(K)} * \text{X5MAHL(K)}) / \text{T} \]
\[ \text{CHX = POLINT(T7(KE), X5LCS(KE)), NESP, 0.0, 1.0)} \]
CHY = POLINT(T7(KE) • YSLCS(KE) • NESPN • 0.0 • 1.0)

CHZ = POLINT(T7(KE) • ZSLCS(KE) • NESPN • 0.0 • 1.0)

CMH = POLINT(T7(KE) • CMHC(KE) • NESPN • 0.0 • 1.0)

PRINT 806

806 FORMAT(1H0 • 7X • 3HETA • 6X • 9HCHX/CAVG • 3X • 9HCHYC/CAVG • 3X •
1 9HCHZ/CAVG • 2X ,
2 3SHCCHX/CAVG CMHYC/CAVG CMH2C/CAVG • 3X • 9HCMHC/CAVG )

816 FORMAT(///1H0 • 10X • 27HCONTROL SURFACE TOTAL LOADS/)

1 1H0 • 48H ETA (X/C) CP (Y/C) CP (Z/C) CP

826 FORMAT(47H0 CHX CHY CHZ CMH)

836 FORMAT(1H • 8F12.6)

846 FORMAT(F13.6, 7(12H • 0.000000))

KK = KS-1

IF(KS • EQ • 1) GOTO 841

DO 840 K=1, KK

840 PRINT 846, ETA(K)

841 DO 850 K=K5, KE

850 PRINT 836, ETA(K), XSLCS(K), YSLCS(K), ZSLCS(K), XSMAH(KE),

1 YSMH(KE), ZSMAH(KE), CMHC(KE)

IF(KE • EQ • NETA) GO TO 861

KP =KE+1

DO 860 K=KP, NETA

860 PRINT 846, ET(A(K)

861 PRINT 816

IF(KS • EQ • 1) GOTO 871

DO 870 K=1, KK

870 PRINT 876, ETA(K)

876 FORMAT(F13.6, 12H • 0.000000)

871 DO 880 K=KS, KE

880 PRINT 836, ETA(K), XSCPCS(K), YSCPCS(K), ZSCPCS(K)

IF(KE • EQ • NETA) GOTO 891

DO 890 K=KP, NETA

890 PRINT 876, ETA(K)

891 CONTINUE

NCS = NCS + 1

GOTO 709

END
OSUBROUTINE PPRINT(ETA, XOC, NETA, NXOC, VMU, VML, VTU, VTL, CPU, CPL, NC) 6 6340
1 0DIMENSION ETA(NEA), XOC(NXOC), VMU(NXOC, NETA), VML(NXOC, NETA), VTL(NXOC, NETA), CPU(NXOC, NETA) 6 6350
2 PRINT 1, NC 6 6360
10FORMAT(1H1*20X, *PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPO 6 6370
1NENT*, I4 ) 6 6380
DO 10 I=1, NETA 6 6390
PRINT 2, ETA(I) 6 6400
PRINT 3 6 6410
DO 10 J=1, NXOC 6 6420
CP = CPL(J, I) - CPU(J, I) 6 6430
100PRINT 4, XOC(J), VMU(J, I), VML(J, I), VTU(J, I), VTL(J, I), CPU(J, I) 6 6440
1 0 FORMAT(5HPOETA=, F9.6) 6 6450
2 FORMAT(3F12.6) 6 6460
1 RETURN 6 6480
C C C
BLAINE D. GAITHER 11/72 6 6490
END 6 6500
C
SUBROUTINE DINTEG(DAX, DAY, DAZ, XC, YC, ZC, LONPNA, LATPNA, LONLEN, LATLEN) 6560
1, LATLEN, CPD, NSSTAP, NSSEC, XTL, NSTAT, S, X) 6570
2, PANS, SLX, DX, RC, CHRL, CX, CY, CZ, CMX, CMY, CMZ 6580
3, NSSTTA, NSSEC, LONPNA, XCG, YCG, ZCG, ALPHA, ARRJ, 6590
4, TSXL, TSYL, TSLZ, TMX, TMY, T'M', ZDI, CDI, NC 6600
5

DAX DAY DAZ = AREAS OF SUBPANELS VIEWED FROM X, Y, Z 6650
XC YC ZC = X, Y, Z VALS OF CENTROIDS OF SUB PANELS 6660
NSSTTA = NUMBER OF SUBSTATIONS 6670
NSSEC = NUMBER OF SUBSECTIONS 6680
LONPNA = CONTAINS NUMBER LONGITUDINALLY OF THE PANELS 6690
WHERE CPS ARE SPECIFIED 6700
LATPNA = IS LATERAL COUNTER PART TO LONPAN 6710
LONLEN = NUMBER OF STATIONS WHERE CPS ARE GIVEN 6720
LATLEN = NUMBER OF SECTIONS WHERE CPS ARE GIVEN 6730
CPD = CONTAINS CP VALS AT PANELS SPECIFIED BY LONPAN 6740
AND LATPAN 6750
NSSEC = NUMBER OF SUBSECTIONS PER PANEL 6760
NSSTAP = NUMBER OF SUBSTATIONS PER PANEL 6770
XTL = VECTOR OF PANEL BOUNDARIES 6780
NSTAT = NUMBER OF STATIONS 6790
RC = BODY CHORD LENGTH 6800
ARJ = REFERENCE AREA FOR THIS BODY 6810
X = ARRAY OF X VALS FOR CP VALS OF CPD 6820
S SS XX = SCRATCH ARRAYS 6830
PANS = SET TO INTEGRAL OF EACH STATION 6840
SLX = XS WHERE SECTION LOADS 6850
DX = DELTA X OVER C 6860
CX CY CZ = SECTION LOADS 6870
CMX CMY CMZ = SECTION MOMENTS 6880
LONPNA, LATPNA SCRATCH 6890

DIMENSION DAX(NSSTTA, NSSEC), DAY(NSSTTA, NSSEC), 6900
IDAZ(NSSTTA, NSSEC), XCG(NSSTTA, NSSEC), YCG(NSSTTA, NSSEC), 6910
6920
6930
6940
22C(NSSTA,NSSEC), LONPNA(LONLEN),
3 CPU(LONLEN, LATLEN), X(X(LONLEN)), XX(49), LONPAN(LONLEN), LATPAN
4(LATLEN), XTL(1), SLX(NSTAT), DXC(NSTAT), CX(NSTAT), CY(NSTAT),
5CZ(NSTAT), CMX(INSTAT), CHY(INSTAT), CMZ(INSTAT), PANS(NSTAT),
6 S(NSSEC), SS(49), DLLS(50), LXX(GL0)
EQUIVALENCE (SLXX(2)*SS), (SLXX(2)*XX)
REAL LATPAN, LONPNA
NAMELIST /LA/ SX, SY, SZ /LS/ NSTA, NBOT, NTOP /LC/ LT, LL
1 /LD/ AXT, AYT, AZT, ATX, AYT, ATZ
C
CONVERT LATPAN * LONPAN 50 THEY POINT TO THE CORRECT SUB-PANEL
C INSTEAD OF PANEL
LT = NSSTAP/2 + 1
DO 10 I=1, LONLEN
10 LONPAN(I) = (LONPNA(I)-1)*NSSTAP + LT
SX = 0.
SY = 0.
SZ = 0.
DO 21 J=1,NSSEC
DO 21 J=1,NSSTA
SX = SX + ABS(DAX(I,J))
SY = SY + ABS(DAY(I,J))
21 SZ = SZ + ABS(DAZ(I,J))
SX = SX/2.0
SY = SY/2.0
SZ = SZ/2.0
DO 400 NSTA=1, NSTAT
NBOT = (NSTA-1)*NSSTAP + 1
NTOP = NBOT + NSSTAP - 1
PANSUM = 0
AXT = 0.
AYT = 0.
AZT = 0.
ATZ = 0.
ATX = 0.
C START ON THIS STATION
DO 390 NST=NBOT, NTOP
C FIND GPS FOR THIS RING(SUB-STATION)
DO 315 I=1, LATLEN
C    FIND CP FOR A MERIDIAN
    LT = LATPAN(I)
7340
942  FORMAT(1H,5G11.4,5X,3G11.4,5X,3G11.4)
7350
   IF(LONLEN.GT.3) GOTO305
6
    T = XC(NST,LT)
6
    NB = 1
6
    NT = 2
6
   IF(T.LT.X(2).OR.LONLEN.LT.3) GOTO325
6
    NB = 2
6
    NT = 3
6
325  XX(I) = (T - X(NB)) * (CPD(NT,I) - CPD(NB,I)) / (X(NT) - X(NB)) +
6
     1 CPD(NB,I)
6
   GOTO 315
6
305  XX(I) = CODIM1(XC(NST,LT), XC*CPD(1,I)*LONLEN,-1.0)
6
315  CONTINUE
6

941  FORMAT(10G11.4)
5
31  IF(LATLEN.GT.1) GOTO 32
5
   PANSUM = PANSUM + NSSEC*XX(1)
5
   DO 500  I = 1, NSSEC
5
      AXT = AXT + XX(I)*DAZ(NST,I)
5
      AYT = AYT + XX(I)*DAY(NST,I)
5
      AZT = AZT + XX(I)*DAZ(NST,I)
5
      ATX = ATX + ((Y(NST,I)-YCG)*DAZ(NST,I) - (ZC(NST,I)-ZCG)*DAY(NST,6
5
      LI)) * XX(I)
5
      ATY = ATY + ((ZC(NST,I)-ZCG)*DAZ(NST,I) - (XC(NST,I)-XCG)*DAZ(NST,6
5
      LI)) * XX(I)
5
   500  ATZ = ATZ + ((XC(NST,I)-XCG)*DAY(NST,I) - (Y(NST,I)-YCG)*DAZ(NST,6
5
      LI)) * XX(I)
5
   GOTO 390
5
32  LIT = LATPAN(1)
5
   LTP = LIT
5
   IF(LIT.EQ.1) LTP = 2
5
   SI(LTP -1) = 0.0
5
   DO 40  I = LTP, NSSEC
5
5
40  SI(I) = 5*(I-1) + SQRT((Y(NST,I) - YCG(NST,I-1))**2 + (ZC(NST,I) - 26
5
   ZC(NST,I-1))**2)
5
   TSF = SORT((Y(NST,NSSEC-1) - YCG)**2 + (ZC(NST,NSSEC) - ZC(I))**2)
5
   IF(LIT.EQ.1) GOTO 46
5
   SI(I) = SIN(NSSEC) + TSF
5
5
LTTT = LTT - 1
IF (LTTT.EQ.1) GOTO 46
DO 45 I = 2, LTTT
45 S(I) = S(I-1) + SORT((YC(NST,I)-YC(NST,I-1)**2 + (ZC(NST,I) -
12C(NST,I-1))**2)
46 XX(LATLEN+1) = XX(1)
DO 50 I = 1, LATLEN
LT = LATPAN(I)
50 SS(I) = S(LT)
SS(LATLEN+1) = TSF + S(NSSEC)
IF (LTTT.NE.1) SS(LATLEN+1) = S(LTTT) + SORT((YC(NST,LTTT) - YC(NST,
1) = LTTT)**2 + (ZC(NST,LTTT) - ZC(NST,LTTT))**2)
SLXX(1) = XX(LATLEN)
SLSS(1) = SS(1) - SS(LATLEN+1) + SS(LATLEN)
DO 60 I = 1, NSSEC
T = CODIM1(S(I), SLSS, SLXX, LATLEN+2, 1.0)
PANSUM = PANSUM + T
AXT = AXT + T*DAX(NST,I)
AYT = AYT + T*DAY(NST,I)
AZT = AZT + T*DAZ(NST,I)
ATX = ATX + ((YC(NST,I) - YCG) * DAZ(NST,I) - (ZC(NST,I) - ZCG)*DAY(N6
1ST,I)) * T
1640 ATY = ATY + ((ZC(NST,I) - ZCG)*DAX(NST,I) - (XC(NST,I) - XCG)*DAZ(NST,
11)) * T
60 ATZ = ATZ + ((XC(NST,I) - XCG)*DAY(NST,I) - (YC(NST,I) - YCG)*DAX(NST,
11)) * T
390 CONTINUE
PANS(NSTA) = PANSUM
SLX(NSTA) = (XTL(NSTA)+XTL(NSTA+1))/2.0
T = (XTL(NSTA+1)-XTL(NSTA))/ BCHRDL
DXC(NSTA) = T
TS = T* SY
T = T*SZ
CX(NSTA) = -AXT/T
CY(NSTA) = -AYT/T S
CZ(NSTA) = -AZT/T
T = ARBJ * BCHRDL
CMX(NSTA) = -ATX/T
CMY(NSTA) = -ATY/T
8000 7730
8010 7740
8020 7750
8030 7760
8040 7770
8050 7780
8060 7790
8070 7800
8080 7810
8090 7820
8100 7830
8110
400 CMZ(NSTA) = -ATZ/T
C FIND TOTAL MOMENTS * LOADS
TSXL = 0.
TSYL = 0.
TSZL = 0.
TMX = 0.
TMY = 0.
TMZ = 0.
PRINT 923, NC
6 8120
6 8130
6 8140
6 8150
6 8160
6 8170
6 8180
6 8190
6 8200
923 FORMAT(1H1, 35X*, BODY SECTIONAL LOADS FOR COMPONENT*, I5, /1H0, 10X*, 6 8210
X/C*, 8X*, *CXL/AVG*, 7X*, CYH/AVG*, 07X*, CIZ/AVG*)
1 6 8220
DO 410 I=1, NSTAT
6 8230
SLX(I)=SLX(I)/BCHRD
6 8240
PRINT 948, SLX(I), CX(I), CY(I), CZ(I)
6 8250
948 FORMAT(1H1, *F15.6)
6 8260
TSXL = TSXL + CX(I)*DXC(I)
6 8270
TSYL = TSYL + CY(I)*DXC(I)
6 8280
TSZL = TSZL + CZ(I) * DXC(I)
6 8290
TMX = TMX + CMX(I)*DXC(I)
6 8300
TMY = TMY + CMY(I)*DXC(I)
6 8310
410 TMZ = TMZ + CMZ(I)*DXC(I)
6 8320
TSXL = TSXL * SZ/ARB
6 8330
TSYL = TSYL*SY/ARB
6 8340
TSZL = TSZL*SZ/ARB
6 8350
T = TSXL*TSXL + TSYL*TSYL + TSZL*TSZL
6 8360
XCP = XCG/BCHRD + (TSXL*TMZ - TSZL*TMY)/T
6 8370
YCP = YCG/BCHRD + (TSZL*TMX - TSXL*TMZ)/T
6 8380
ZCP = ZCG/BCHRD + (TSXL*TY-TSYM*TMX)/T
6 8390
CL = TSXL*COS(ALPHA) - TSYL*SIN(ALPHA)
6 8400
CD = TSXL*SIN(ALPHA) + TSXL * COS(ALPHA)
6 8410
PRINT 906
6 8420
906 FORMAT(///1H0, *4X*, TOTAL BODY LOADS*)
6 8430
CDI=CDI/ARB
6 8440
PRINT 901, TSXL, TSYL, TSZL, TMX, TMY, TMZ, CDI
6 8450
901 FORMAT(1H0, 7X*, CX*, 13X*, CY*, 13X*, CZ*, 12X*, CX*, 12X*, CY*, 12X*, CMZ*
6 8460
1*, 12X*, *CDI/=1H, 7F15.6)
6 8470
PRINT 902, XCP, YCP, ZCP, 5X, SY, SZ
6 8480
902 FORMAT(1H0, 5X*, X/C CP*, 9X, Y/C CP*, 9X, Z/C CP*, 11X, 5X, SY*, 6 8490
1*, 15X*, *SZ*/1H, 6F15.6)
SUBROUTINE REFLECT(SYM,LO,LA,AX,AY,AZ,X,Y,Z,LON,LAT,LT,CPD,NSS) 6 8540
DIMENSION AX(LO,LA),AY(LO,LA),AZ(LO,LA),X(LO,LA),Y(LO,LA),Z(LO,LA) 6 8550
1 LATP(LAT),CPD(LON,LAT) 6 8560
REAL LATP
LL=NSS/2+1
DO 1 I=1,LAT
1 LATP(I)=(LATP(I)-1)*NSS+LL
IF(SYM.EQ.-1.0 RETURN 6 8590

C REFLECT GEOMETRY
ILA=LA
ILA1=ILA+1
K=LA
DO 10 J=1,LA
JJ=LA+1-J
IF(ABS(Y(I,JJ)).LE.0001 GO TO 10
K=K+1
10 CONTINUE
LA=K
IF(SYM.EQ.1.0 RETURN 6 8670

C REFLECT CPS
K=LAT
DO 20 J=1,LAT
JJ=LAT+1-J
LT=LATP(JJ)
IF(ABS(Y(I,LT)).LE.0001 GO TO 20
K=K+1
20 CONTINUE

GO TO 20
SUBROUTINE BPRINT(NO, NA, X, Y, Z, VM, VT, CP, NC)
DIMENSION X(NO, NA), Y(NO, NA), Z(NO, NA), VM(NO, NA), VT(NO, NA)
1) CP(NO, NA)
PRINT 1, NC
1 FORMAT(1HI, 24X, *BODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMP*
IONENT*, I3)
DO 10 I=1, NA
PRINT 2, I
2 FORMAT(*OLATERAL STATION *, I3)
PRINT 3
1 PRINT 4, (J, X(J, I), Y(J, I), Z(J, I), VM(J, I), VT(J, I), CP(J, I))
1 J=1, NO
4 FORMAT(4X, I3, 5X, 6F12.6)
10 PRINT 5
5 FORMAT(1HI)
RETURN
C BLA9NE D. GAITHER 11/72
END
SUBROUTINE FFDRAG(NB, NP, YB, ZB, XKB, NPE, BSYM, YP, ZP)
  1 *XKP, NPE, PSYM, PIND, TB, TPD, NPS)
DIMENSION Y(1), ZB(1), XKB(1), NPE(1), BSYM(1), YP(1)
  1 *ZP(1), XKP(1), NPE(1), PSYM(1), PIND(1)
  2 *T2D(1), TPD(1)
COMMON/BODY/ DUM(15000), Y(200), Z(200)
  1 *S(200), SYM(200), YN(200), ZN(200), XK(200), D(200), NUMB(200),
  2 NUMP(51), XP(200), DY(200), NZ(200), NS(200), ETA(200), YNP(200)
  3 *ZNP(200)
DATA IONE/17, MAXVOR/2007, NVOR/40/
K=0
TNB=0.
IF(NB.EQ.0) GO TO 20

CALCULATION OF BODY DATA
DO 5 I=1,NB
  5 TNBE=TNBE+NBE(I)
    L=0
    LKB=0
    DO 15 I=1,NB
      NUB=NBE(I)
      DO 10 J=1,NUB
        NI=L+J
        N2=N1+1
        K=K+1
        XK(K)=XKB(LKB+J)
        DELY=YB(N2)-YB(N1)
        DELZ=ZB(N2)-ZB(N1)
        S(K)=SQRT(DELY**2+DELZ**2)
        EPS=.000001
        IF(S(K).GT.EPS) GO TO 2
        S(K)=0.
        YN(K)=1.
        ZN(K)=1.
        GO TO 3
      10 TN(K)=DELZ/S(K)
      ZN(K)=DELZ/S(K)
    15 K=K+1
  2 YN(K)=-DELZ/S(K)
  3 SYM(K)=BSYM(I)
    Y(K)=5*(YB(N1)+YN(N2))
  10 Z(K)=5*(ZB(N1)+ZN(N2))
NUM2(I)=NBE(I)
LKB=LKB+NUB
15 L=L+NUB+1
20 IF(NP.EQ.0) GO TO 50

CALCULATION OF PANEL DATA

INDP=0
IF(NPS.EQ.0) GO TO 150
INDP=1
GO TO 8
150 NPS=NVOR/NPE(I)
DO 4 I=1,NP
4 TNPE=0.
DO 7 I=1,NP
7 TNPE=TNPE+NPS*NPE(I)
IF(TNPE+TNBE.EQ.MAXVOR) GO TO 8
NPS=NPS-1
GO TO 6
8 L=Q
LKP=0
DO 49 I=1,NP
NUP=NPE(I)
SPAN=0.
DO 45 J=L-1:NUP
41 L=L+J
N2=L+1
DY(J)=YP(N2)-YP(N1)
DZ(J)=ZP(N2)-ZP(N1)
DS(J)=SORT(DY(J)**2+DZ(J)**2)
YKP(J)=-DZ(J)/DS(J)
Z*K(J)=DY(J)/DS(J)
45 SPAN=SPAN+DS(J)
SUM=0.
DO 46 J=1:NUP
41 ETA(J)=(SUM+5*DS(J))/SPAN
46 SUM=SUM+DS(J)
DO 47 J=1:NUP
47 X*K(J)=X*K(LKP+J)
NUP=NUP+1
IF(PIND(I) .NE. 1.) GO TO 44
NUZ=NUP+1
ETA(NUZ)=1.
XP(NUZ)=0.

44 SUM=0.
DO 48 J=1,NUP
N1=L+J
N2=N1+1
DELS=DS(J)/NPS
DO 48 J1=1,NPS
K=K+1
S(K)=DELS
SYM(K)=PSYM(I)
YN(K)=YMP(J)
ZN(K)=ZNP(J)
SUM=SUM+.5*DELS
ET=SUM/SPAN
SUM=SUM+.5*DELS
IF(INDP.EQ.1) GO TO 48
CALL CONTM(ETA,XP,NUZ,ET,XK(K),IONE)

48 Y(K)=YP(N1)+(J1-.5)*DELS*SYM(K)
Z(K)=ZP(N1)+(J1-.5)*DELS*(-YN(K))

49 L=L NUP+1
CALCULATION OF DRAG ON EACH ELEMENT
50 NV=K
WRITE(6,333)(XK(J),J=1,NV)
333 FORMAT(//(10F12.5))
DO 56 I=1,NV
Y=0.
W=0.
DO 56 J=1,NV
YY=Y(I)-Y(J)
ZZ=Z(I)-Z(J)
CALL HSHOF(YY,ZZ,S(J),YN(J),ZN(J),NV,A..)
IF(SYM(J).EQ.1.0.OR.SYM(J).EQ.-1.) GO TO 54
YY=-Y(I)-Y(J)
CALL HSHOE(YY,ZZ,S(J),YN(J),ZN(J),DELV,DELW)
IF(SYM(J) .EQ. 0) GO TO 52
DELV = DELV
DELW = DELW
52 AV = AV + DELV
AW = AW + DELW
54 V = V * AV * XK(J)
56 W = W + AW * XK(J)
58 D(I) = -2 * XK(I) * S(I) * (V * YN(I) + W * ZN(I))

CALCULATION OF TOTAL DRAG
K = 0
IF(NB .EQ. 0) GO TO 95
DO 93 I = 1, NB
TBD(I) = 0.
N1 = K + 1
N2 = K + NUMB(I)
DO 92 J = N1, N2
92 TBD(I) = TBD(I) + D(J)
K = N2
IF(SYM(J) .EQ. 0 .OR. SYM(J) .EQ. 2) TBD(I) = 2 * TBD(I)
93 CONTINUE
95 IF(NP .EQ. 0) RETURN
DO 98 I = 1, NP
TPD(I) = 0.
N1 = K + 1
N2 = K + NUMP(I)
DO 96 J = N1, N2
96 TPD(I) = TPD(I) + D(J)
K = N2
98 CONTINUE
RETURN
END
SUBROUTINE NFDRAG(INC,NS,NCP,NSP,XOCCP,XOVC,ETACP,XV,YV,ZV)
1,TANV,SIGMAV,CPSN,NSPP,CPNET,NISP,CDSCP,CDACP,CDTCP,CDTH,CDST,CDAT,CDT,CDTH6
2)
DIMENSION XOCCP(1)*XOVC(1)*ETACP(1)*XV(1)*YV(1)*ZV(1)*TANV(1)
1,SIGMAV(1)*CPNET(1)*CDSCP(1)*CDACP(1)*CTTCP(1)*CDTH(1)
DIMNSION DUMM(100),NUM(100),P(2000),FTAV(100)
DIMENSION X(1000*2),Y(1000),Z(1000),TAN(1000*2),CP(2000)
1*SIGMA(2000*2),N(2000),CPS(100),CDA(100),CDT(100)
DIMENSION XK(200)

COMMON DA(5000)
COMMON BODY/B(31550)
EQUIVALENCE (Q,DA(13)),(GAM,DA(14)),(YCG,DA(8)),(ZCG,DA(9))
1,(CBAR,DA(5)),(SPAN,DA(6))
EQUIVALENCE (XK(1)*R(16201),X(1)*R(19001),Y(1)*R(17001))

1*(Z(1)*B(18001))*(TAN(1)*R(15001))*(CP(1)*R(21001))
2*(SIGMA(1)*R(23001))*(P(1)*R(27001))*(CPS(1)*R(29001))
3*(CDACP(1)*B(29101))*(CDTCP(1)*B(29201))*(DUMM(1)*B(29301))
4*(DUM(1)*B(3901))*(P(1)*B(39501))*(XOCCP(1)*B(31501))

333 FORMAT(/(1X*10F12.5))
332 FORMAT(/(1X*10I12))

WRITE(6,332)INC,NS,NCP,NSP,NSPP
WRITE(6,333)(XOCCP(I),I=1,NCP)
WRITE(6,333)(XOVC(I),I=1,NCP)
WRITE(6,333)(ETACP(I),I=1,NC)
WRITE(6,333)(ETACP(I),I=1,NSP)
R4PI=0.7957747
EPS2=0.079568656
EPS3=0.00001
ISYM=DA(3426)
NV1=NS*NC
NV2=2*NV1
WRITE(6,333)(XV(I),I=1,NV1)
WRITE(6,333)(YV(I),I=1,NV2)
WRITE(6,333)(ZV(I),I=1,NV2)
WRITE(6,333)(TANV(I),I=1,NV2)
WRITE(6,333)(SIGMAV(I),I=1,NV2)
WRITE(6,333)(CPNET(I),I=1,NV1)

CALCULATE NEW VORTEX COORDINATES
NSPP=3
JK=0  
NC2=2*NC  
DELY1=(YV(NC2+1)-YV(1))/NSPP  
DELY1=DELY1*NSPP/2+1  
DO 100 I=1,NS  
DELY=DELY1  
DO 100 J=1,NSPP  
DEL=DELY1  
DO 100 K=1,NC2+2  
IK1=(I-1)*NC2+K  
IK2=IK1+1  
JK=JK+1  
X(JK+1)=XV(IK1)+DEL*TANV(IK1)  
X(JK+2)=XV(IK2)+DEL*TANV(IK2)  
Y(JK)=YV(IK1)+DEL  
Z(JK)=ZV(IK1)  
TAN(JK+1)=TANV(IK1)  
TAN(JK+2)=TANV(IK2)  
SIGMA(JK+1)=SIGMAV(IK1)  
SIGMA(JK+2)=SIGMAV(IK2)  
NV=JK  
NSS=NS*NSPP  
WRITE(6,333)(X(I+1),X(I+2),Y(I+1),Y(I+2),Z(I+1),Z(I+2),TAN(I+1),TAN(I+2),SIGMA(I+1),SIGMA(I+2),1,10,10,15)  
CALCULATE ETAS OF VORTICES  
DELE=1./NSS  
SUM=-.5*DELE  
DO 110 I=1,NSS  
SUM=SUM+DELE  
110 ETAV(I)=SUM  
WRITE(6,333)(ETAV(I),1,NSS)  
CALCULATE X/C OF PANEL CENTROIDS  
SLOPE=(TANV(2*NC)-TANV(1))/(XV(2*NC)-XV(1))  
TANL=TANV(1)-5*(XV(2)-XV(1))*SLOPE  
TANR=TANV(2*NC)+5*(XV(2*NC)-XV(2*NC-1))*SLOPE  
DX=DA(3450)+DA(3432)
XLE = YV(I) * TANLE + DX
XTE = YV(I) * TANTE + DX + D4(A(3453))
CHORD = XTE - XLE
J = 0
DO 1000 I = 1, NC2, 2
J = J + 1
1000 XOCP(J) = (5 * (XV(I) + XV(I+1)) - XLE) / CHORD
CALCULATE CPNET AT VORTICES
   DO 120 I = 1, NCP
   DO 130 J = 1, NSP
   130 DUM(J) = CPNET(NCP*(J-1) + 1)
   CALL POL(ETACP, DUM, NSP, ETAV, DUMO, NSS)
   DO 120 J = 1, NSS
   NJ = NCP*(J-1) + 1
   DO 140 I = 1, NSS
   K = NC*(I-1) + 1
   J = NCP*(I-1) + 1
140 CALL POL(XOCP, P(J), NCP, XOCP, CP(K), NC)
   WRITE(6*331)(CP(I)*I = 1, NV)
   YVV = 5*DELY
   EPS1 = 2.*YVV
   NV2 = 2.*NV
   IF(ISYM.EQ.1) NSS = 2*NSS
   DO 1 I = 1, NV2
   IF(ABS(CP(I)) .GT. EPS3) GO TO 3
   CONTINUE
   GO TO 44
CALCULATE DRAG DUE TO LIFT
   3 DO 43 I = 1, 2
   WRITE(6331) YVV
   331 FORMAT(1X*10E12, 5)
   DO 5 JJ = 1, NV2
   5 D(JJ) = 0.
   NSKIP1 = 0
   9 DO 20 J = 1, NV
   IPASS = 1
   XMF = 1.
   NSKIP2 = 0
   20 CONTINUE
   GO TO 44
W=0.
DO 22 K=1, NV
DELX=X(J+1)-X(K,1)
YY=Y(J)
IF(ISYM.EQ.1.AND.NSK1P1.LE.0) YY=-YY
DELY=YY-Y(K)
DELZ=Z(J)-Z(K)
DELXSQ=DELX**2
DELZSQ=DELZ**2
10 YP=DELY+YYV
YN=DELY-YYV
YPSQ=YP**2
YNSQ=YN**2
TERM1=YPSQ+DELZSQ
TERM2=YNSQ+DELZSQ
TERM3=DELXSQ+YPSQ
TERM4=DELXSQ+YNSQ
TERM5=TERM1+DELXSQ
TERM6=TERM2+DELXSQ
CHECK1=DELXSQ/TERM1
CHECK2=DELXSQ/TERM4
F=0.
IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY).GT.YVV) GO TO 32
IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(DELX).LT.ABS(0.5*(X(K,1))))) GO TO 33
12)
F=(YP/SQRT(TERM1)-YN/SQRT(TERM6))*DELX/(DELXSQ+DELZSQ)
GO TO 33
32 F=+5*DELX*ABS(1./TERM4-1./TERM3)
33 IF(TERM1/TERM5.LT.EPS2.AND.DELX.LT.0.) GO TO 34
IF(SQRT(TERM1).LT.EPS3) GO TO 35
F=F+YP/TERM1*(1.+DELX/SQRT(TERM5))
GO TO 35
34 F=F+SQRT(TERM1)/TERM5
35 IF(TERM2/TERM6.LT.EPS2.AND.DELX.LT.0.) GO TO 36
IF(SQRT(TERM2).LT.EPS3) GO TO 37
F=F+YN/TERM2*(1.+DELX/SQRT(TERM6))
GO TO 37
36 F=F-SQRT(TERM2)/TERM6
30  KK=K+NSKP2
    U=U+XMF*CP(KK)*(X(K+2)-X(K+1))*F
    IF(IPASS.EQ.2.OR.ISYM.EQ.-1) GO TO 31
    IPASS=2
    DELY=-YY-Y(K)
    IF(ISYM.EQ.0) GO TO 10
    IF(ISYM.EQ.1) GO TO 11
    XMF=-1.
    GO TO 10
11  NSKP2=NV
    GO TO 10
31  IPASS=1
    XMF=1.
    KK=J+NSKP1
20  DI(KK)=2.*CP(KK)*(XT(J+2)-X(J+1))*W*R4PI
    IF(ISYM.NE.1.OR.NSKP1.NE.0) GO TO 21
    NSKP1=NV
    GO TO 9
21  IF(I1.EQ.2) GO TO 40
    DO 41  I=1,NSS
         CDS(I)=0.
        DO 41  J=1,NC
         K=NC*(I-1)+J
        CDS(I)=CDS(I)+DI(K)
    GO TO 43
40  DO 42  I=1,NSS
    CDA(I)=0.
    DO 42  J=1,NC
         K=NC*(I-1)+J
        CDA(I)=CDA(I)+DI(K)
43  CONTINUE
44  DO 45  I=1,NV2
    DO 45  J=1,Z
        IF(ABS(SIGMA(J)) GT EPS3) GO TO 39
45  CONTINUE
    GO TO 500
CALCULATE DRAG DUE TO THICKNESS
39  DO 46  I=1,NV2

46 \text{D(I)=0.} \\
\text{NSKIP1=0} \\
47 \text{DO 50 I=1*NV} \\
\text{DO 60 J=1*2} \\
\text{IPASS=1} \\
\text{NSKIP2=0} \\
\text{U=0.} \\
\text{DO 70 K=1*NV} \\
\text{DO 70 L=1*2} \\
\text{DELX=X(I,J)-X(K,L)} \\
\text{YY=Y(I)} \\
\text{IF(ISYM.EQ.1.AND.NSKIP1.NE.0) YY=-YY} \\
\text{DELY=YY-Y(K)} \\
\text{DELZ=Z(I)-Z(K)} \\
\text{T=TAN(K,L)} \\
\text{XP=DELX+T*YVV} \\
\text{XN=DELX-T*YVV} \\
\text{62 YP=DELY+YVV} \\
\text{YN=DELY-YVV} \\
\text{XY=DELX-T*DELY} \\
\text{TSQ1=1.0.T**2} \\
\text{PP=SQRT(TSQ1)} \\
\text{R2SQ=XY**2+DELZ**2*TSQ1} \\
\text{TEST1=XN**2+YN**2} \\
\text{TEST2=XP**2+YP**2} \\
\text{R4=SQRT(TEST1+DELZ**2)} \\
\text{R5=SQRT(TEST2+DELZ**2)} \\
\text{XY=DELY+T*DELX} \\
\text{YVT=YVV*TSQ1} \\
\text{TERM4=1.0/R4-1.0/R5} \\
\text{XYSG=XY**2} \\
\text{CHECK1=XYSG/TEST1} \\
\text{CHECK2=XYSG/TEST2} \\
\text{IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY6.0)) GO TO 48} \\
1.0.GT.YVV) GO TO 48 \\
\text{IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(XY).LT.ABS(.5*(X(K,2)))) GO TO 49} \\
1-X(K,1))} \\
\text{XI2=(XY+YVT)/R5} \\
\text{XI3=-(YX-YVT)/R4}
TERM1=(X12+X13)/R250
GO TO 51
48 TERM1=5/PP*ABS(1./TEST1-1./TEST2)
GO TO 51
49 TERM1=0.
51 EUS=T/PP*TERM4+1./PP*XY*TERM1
KK=K+NSKIP2
U=U+SIGMA(KK,L)*EUS
IF(IPASS.EQ.2.OR.ISYM.EQ.1) GO TO 61
IPASS=2
DELY=-YY-Y(K)
IF(ISYM.EQ.0) GO TO 62
NSKIP2=NV
GO TO 62
61 IPASS=1
70 NSKIP2=0
KK=I+NSKIP1
U=U*R4P1
60 D(KK)=D(KK)-(U+1.*Q*2.*(Z(I)-ZCG)/CBAR-GAM*2.*(Y(I)-YCG)/SPAN)*
1 SIGMA(KK,J)*SQRT(1.+TAN(KK,J)**2)
50 D(KK)=2.*D(KK)
IF(ISYM.NE.1.OR.NSKIP1.NE.0) GO TO 81
NSKIP1=NV
GO TO 47
81 DO 80 I=1,NSS
CDT(I)=0.
DO 80 J=1,NC
K=NC*{(I-1)+J
80 CDT(I)=CDT(I)+D(K)
500 CONTINUE
CDST=0.
CDAT=0.
CDTT=0.
CALCULATION OF TOTAL DRAG
500 CONTINUE
DO 200 I=1,NSS
CDST=CDST+CDTS(I)
CDAT=CDAT+CDAT(I)
200 CDTT=CDTT+CDT(I)
CDST=CDST*2.*YVV
CDAT=CDAT*2.*YVV  
CDTT=CDTT*2.*YVV  
CDTHT=CDAT-CDST  
CALCULATE SPANWISE DISTRIBUTION OF DRAG 
CALL POL(ETAV,CDS,NSS,ETACP,CDSCP,NSP)  
CALL POL(ETAV,CDA,NSS,ETACP,CDACP,NSP)  
CALL POL(ETAV,CDT,NSS,ETACP,CDTCP,NSP) 
DO 190 I=1,NSP  
190 CDTH(I)=CDACP(I)-CDSCP(I)  
WRITE(6,333)(CDS(I),I=1,NSS)  
WRITE(6,333)(CDA(I),I=1,NSS)  
WRITE(6,333)(CDT(I),I=1,NSS)  
WRITE(6,333)(CDSCP(I),I=1,NSP)  
WRITE(6,333)(CDACP(I),I=1,NSP)  
WRITE(6,333)(CDTCP(I),I=1,NSP)  
WRITE(6,333)(CDTH(I),I=1,NSP)  
WRITE(6,333)CDST,CDAT,CDTT,CDTHT  
CALCULATE TRAILING EDGE K VALUES  
DO 180 I=1,NSS  
   XK(I)=0.  
DO 180 J=1,NC  
   K=NC*(I-1)+J  
180 XK(I)=XK(I)+CP(K)*(X(K.J)-X(K.I)) 
WRITE(6,333)(XK(I),I=1,NSS)  
RETURN 
END
FUNCTION POLINT(X, Y, NPTS, RLLIM, RULIM)

X          - INDEPENDENT VARIABLES
Y          - DEPENDENT VARIABLES
NPTS      - NUMBER OF DEPENDENT AND INDEPENDENT VARIABLES
O .LT. RLLIM .LT. RULIM .LT. 1

DIMENSION X(NPTS), Y(NPTS), PX(22), PY(22)
IF(NPTS-20) 11, 1

1 PRINT 6
6 FORMAT(* TOO MANY POINTS GIVEN TO POLINT*)
STOP

11 PX(1) = ACOS(1.0-2.0*RLLIM)
PY(1) = 0.0
PX(NPTS+2) = ACOS(1.0-2.0*RULIM)
PY(NPTS+2) = 0.0
DO 20 I=1, NPTS
   PHI = ACOS(1.0-2.0*X(I))
   PX(I+1) = PHI
   PY(I+1) = Y(I)*0.5*SIN(PHI)
20 CONTINUE

CALL INTNC(PX(1), PX(NPTS+2), NPTS+2, PX, PY, POLINT, IERR)
RETURN
END
FUNCTION TANALF(DZT,DZC,EPsiLO,DA,ETA,X,TB)

DZT = DELTA X T
DZC = DDELTA X C
EPsiLO = EPSILON
X = X VALUE AT THIS POINT
TB = 1 FOR BOTTOM SURFACE -1 FOR TOP SURFACE
DA = ARRAY OF CONTROL SURFACE INFO.

DIMENSION DA(IJ)

IS THERE A CONTROL SURFACE AT THIS POINT
I J = 1

IS THIS CONTROL POINT AT OUR ETA
1 IF(DA(IJ) .EQ. 0.0) GOTO 150
 IF(DA(IJ+2) .LE. ETA .AND. DA(IJ+3) .GE. ETA) GOTO 30

2 I J = I J + 8
 IF(I J .LT. 80) GOTO 1
 PRINT 156, I J, DA

156 FORMAT(I3*8G14.6)
 STOP, 445

30 T 3 = DA(I J + 4) + (DA(I J + 5)*DA(I J + 4))/(DA(I J + 3) - DA(I J + 2)) * ETA - DA(I J + 2)

1
 IF(DA(I J) .EQ. 1.0 .AND. T 3 .GE. ETA) GOTO 100
 IF(DA(I J) .EQ. 1.0 .AND. T 3 .LE. ETA) GOTO 75
 GOTO 2

75 TA = DA(I J + 1)
 GOTO 101

100 TA = -DA(I J + 1)

101 T = (TB*DZT - DZC) + EPSILO
 TA = TAN(TA)
 TANALF = (TA + T)/(1.0 - T* TA)
 RETURN

150 TANALF = (TB*DZT - DZC) + EPSILO
 RETURN
 END
SUBROUTINE HSHOE(Y*Z*S*YN*ZN*V*W)
DATA R4PI/07957747/
DATA EPS/0000001/
IF(S.GT.EPS) GO TO 5
V=0.
W=0.
RETURN
5 HS=5*S
YP=Y*ZN-Z*YN
ZP=Z*ZN+Y*YN
YPP=YP+HS
YPN=YP-HS
RP=YPP**2+ZP**2
RN=YPN**2+ZP**2
EV=ZP*(1./RP-1./RN)*R4PI
EW=(YPN/RN-YPP/RP)*R4PI
V=EV*ZN+EW*YN
W=EW*ZN-EV*YN
RETURN
END
SUBROUTINE POL(XI,YI,NI,XO,YO,NO)
DIMENSION XI(1),YI(1),XO(1),YO(1)
DIMENSION TI(50),FI(50),TO(100)
DATA PI/3.1415927/
NI1=NI+1
NI2=NI+2
TI(1)=0.
DO 10 I=2,NI1
   TI(I)=ACOS(1.-2.*XI(I-1))
   10 FI(I)=YI(I-1)*2.*SQR(TI(I-1)*SQR(XI(I-1)*SQR(1.-XI(I-1))))
   IF(I)=0.
   TI(NI2)=PI
   FI(NI2)=0.
   DO 15 I=1,NO
   TO(I)=ACOS(1.*XO(I))
   15 CALL COSIN(TI,FI,NI2,TO,YO,NO)
   DO 20 I=1,NO
   YO(I)=YO(I)/SQR(XO(I)*SQR(1.-XO(I)))
   20 RETURN
END
SUBROUTINE INTNC (XL, XU, NX, X, Y, Z, IERR)

F R A N 4 E R S O N 0 5 6 2 9 1 0 7 2 B LDL G 2 S T A T I O N 1 5

INTEGRATION ROUTINE, NEWTON-COTES METHOD

CALL INTNC (XL, XU, NX, X, Y, Z, IERR)

XL LOWER LIMIT OF INTEGRATION

XU UPPER LIMIT OF INTEGRATION

NX NUMBER OF ITEMS IN X AND Y ARRAYS

X ARRAY OF ABSCISSAS (INDEPENDENT VARIABLE)

Y ARRAY OF ORDINATES (DEPENDENT VARIABLE)

Z VALUE OF DEFINITE INTEGRAL

XI ARRAY OF ABSCISSAS USED IN INTEGRATION

YI ARRAY OF ORDINATES USED IN INTERPOLATION

IERR ERROR INDICATOR, ZERO VALUE INDICATES PROPER

SOLUTION, NONZERO INDICATES ERROR

IXU INDICATOR FOR UPPER LIMIT

DIMENSION X(1), Y(1), XI(5), YI(5)

IXU = 0

IERR = 0

TEST TO BE WITHIN RANGE

1 IF (X(1) - XL)10,10,5

WRITE (6, 7) XL, X(1)

7 FORMAT (1HO E1 5.8, 66H, LOWER LIMIT OF INTEGRATION IS LESS THAN F6 4850

1 FIRST ABSCISSA OF ARRAY, E17.8 )

GO TO 110

10 IF (XU - X(NX))15,15,12

WRITE (6, 14) XU, X(NX)

14 FORMAT (1HO E1 5.8, 68H, UPPER LIMIT OF INTEGRATION IS GREATER THAN

1 LAST ABSCISSA OF ARRAY, E17.8)

GO TO 110

15 Z = 0.0

XK = 1.0

INITIALIZE

16 DO 30 I = 1, NX

30 IF (X(I) - XL)30,45,40

CONTINUE

SETUP FOR FIRST INTERVAL

DX = X(I) - XL

XI(1) = XL

C
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
<th>Code</th>
</tr>
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<tbody>
<tr>
<td>1-42</td>
<td>Lower Limit Same as a Tabular Value</td>
<td>C</td>
</tr>
<tr>
<td>43</td>
<td>Setup for succeeding intervals</td>
<td>C</td>
</tr>
<tr>
<td>44</td>
<td>XI(1) = X(I-1)</td>
<td>6</td>
</tr>
<tr>
<td>45</td>
<td>XI(5) = X(I)</td>
<td>6</td>
</tr>
<tr>
<td>46</td>
<td>YI(1) = Y(I-1)</td>
<td>6</td>
</tr>
<tr>
<td>47</td>
<td>YI(5) = Y(I)</td>
<td>6</td>
</tr>
<tr>
<td>48</td>
<td>NI = 4</td>
<td>6</td>
</tr>
<tr>
<td>49</td>
<td>K1 = 1</td>
<td>6</td>
</tr>
<tr>
<td>50</td>
<td>GO TO 60</td>
<td>6</td>
</tr>
<tr>
<td>51</td>
<td>I = I + 1</td>
<td>6</td>
</tr>
<tr>
<td>52</td>
<td>DX = X(I) - X(I-1)</td>
<td>C</td>
</tr>
<tr>
<td>53</td>
<td>Intermediate points in interval</td>
<td>C</td>
</tr>
<tr>
<td>54</td>
<td>XI(2) = XI(1) + .25 * DX</td>
<td>6</td>
</tr>
<tr>
<td>55</td>
<td>XI(3) = XI(1) + .5 * DX</td>
<td>6</td>
</tr>
<tr>
<td>56</td>
<td>XI(4) = XI(1) + .75 * DX</td>
<td>6</td>
</tr>
<tr>
<td>57</td>
<td>Obtain interpolated ordinates</td>
<td>C</td>
</tr>
<tr>
<td>58</td>
<td>CALL CODIS1(NI, XI(K1), YI(K1), X, Y*, NX, XK)</td>
<td>C</td>
</tr>
<tr>
<td>59</td>
<td>Newton-Cotes five point formula</td>
<td>C</td>
</tr>
<tr>
<td>60</td>
<td>Z = Z + (DX/90.0)<em>(7.0</em>(YI(1)+YI(5))+32.0*(YI(2)+YI(4))</td>
<td>6</td>
</tr>
<tr>
<td>61</td>
<td>1+12.0*YI(3))</td>
<td>6</td>
</tr>
<tr>
<td>62</td>
<td>Prepare for next interval</td>
<td>C</td>
</tr>
<tr>
<td>63</td>
<td>I = I + 1</td>
<td>6</td>
</tr>
<tr>
<td>64</td>
<td>Test for last tabular value</td>
<td>C</td>
</tr>
<tr>
<td>65</td>
<td>IF (NX - I) 100 &gt; 94</td>
<td>6</td>
</tr>
<tr>
<td>66</td>
<td>Test for last interval</td>
<td>C</td>
</tr>
<tr>
<td>67</td>
<td>IF (XU - X(I)) 120 &gt; 120 &gt; 50</td>
<td>6</td>
</tr>
<tr>
<td>68</td>
<td>IERR = 1</td>
<td>6</td>
</tr>
<tr>
<td>69</td>
<td>Test for completion</td>
<td>C</td>
</tr>
<tr>
<td>70</td>
<td>RETURN</td>
<td>6</td>
</tr>
<tr>
<td>71</td>
<td>IF (IXU) 100 &gt; 130 &gt; 100</td>
<td>6</td>
</tr>
<tr>
<td>72</td>
<td>Setup for last interval</td>
<td>C</td>
</tr>
<tr>
<td>73</td>
<td>IXU = 1</td>
<td>6</td>
</tr>
<tr>
<td>74</td>
<td>XI(5) = XU</td>
<td>6</td>
</tr>
<tr>
<td>75</td>
<td>XI(1) = X(I-1)</td>
<td>6</td>
</tr>
<tr>
<td>76</td>
<td>YI(1) = Y(I-1)</td>
<td>6</td>
</tr>
</tbody>
</table>
NI = 4
K1 = 2
DX = XU - X(I-1)
GO TO 60
END

SUBROUTINE DUMP
PRINT 6
FORMAT(*OSUBROUTINE DUMP*)
STOP
END
SUBROUTINE CODIS1(N1,WX,YXI,YI,N2,XK)
C F R ANDERSON 056 291 072 BLDG. 2 STATION 15
C CONTROLLED DEVIATION INTERPOLATION SUBROUTINE...CODIS1
C CALLING SEQUENCE
C CALL CODIS1(N1,WX,YXI,YI,N2,XK)
C N1 = NO. OF ARGUMENTS WX
C X = ARGUMENTS-ABSCISSA VALUES.
C Y = INTERPOLATED ORDINATES.
C XI = ARRAY OF THE ABSCISSAE.
C YI = ARRAY OF THE ORDNATES.
C N2 = NO. OF POINTS ON CURVE.
C XK = END INTERVAL CONTROL CONSTANT ( 0 TO 1.0 )
C
DIMENSION X(1),Y(1),XI(1),YI(1),D(2),A(2),B(2),C(2)
100 IN = 0
   DO 800 N = 1,N1
   IF(N2-2)110,115,120
   110 Y(N) = YI(N2)
   GO TO 800
   115 Y(N) = (YI(2)-YI(1))/(XI(2)-XI(1))*(X(N)-XI(1))+YI(1)
   GO TO 800
   120 J = 1
   125 IF(XI(J)-X(N))130,140,150
      130 J= J+1
      IF(J-N2)125,125,145
   140 Y(N) = YI(J)
   GO TO 800
   145 Y(N) = (YI(N2)-YI(N2-1))/(XI(N2)-XI(N2-1))*(X(N)-XI(N2-1))+YI(N2-1)
   GO TO 800
   150 IF(J-2)115,155,160
   155 K = 3
      JJ= 1
   GO TO 185
   160 IF(J-N2)170,165,145
   165 K = N2-1
      JJ= 2
   GO TO 185
170 IF(J-IN)180,300,130
180 JJ = 3
K = J
185 DO 200 M = 1,2
X1 = XI(K-1)-XI(K)
X2 = XI(K)-XI(K-2)
X3 = XI(K-2)-XI(K-1)
Y1 = YI(K-1)-YI(K)
Y2 = YI(K)-YI(K-2)
Y3 = YI(K-2)-YI(K-1)
XX1 = XI(K-2)**2
XX2 = XI(K-1)**2
XX3 = XI(K)**2
D(M) = XX1*X1 +XX2*X2+ XX3*X3
A(M) = (YI(K-2)*X1 +YI(K-1)*X2 + YI(K)*X3)/D(M)
B(M) = (XX1*Y1 + XX2*Y2+XX3*Y3)/D(M)
C(M) = YI(K-2) - A(M)*XX1 -B(M)*XI(K-2)
200 K = K+1
300 P1 = X(N)*A(1)*X(N)+B(1) +C(1)
P2 = X(N)*A(2)*X(N)+B(2) +C(2)
AL = (X(N)-XI(J-1))/XI(J)-XI(J-1))
S = YI(J)*AL + YI(J-1)*(1.0-AL)
GO TO (320,330,350),JJ
320 P2 = P1
AL = (X(N)-XI(1))/XI(2)-XI(1))
S = AL*YI(2) + (1.0-AL)*YI(1)
P1= S + X* *(P2-S)
GO TO 350
330 P1 = P2
AL = (X(N)-XI(N-1))/XI(N)-XI(N-1))
S = AL* YI(N) + (1.0-AL)*YI(N-1)
P2 = S + X* *(P1-S)
350 E1 = ABS(P1-S)
E2 = ABS(P2-S)
IN = J
IF(E1+E2)700,700,750
700 Y(N) = S
GO TO 800
750 BT = (E1*AL)/(E1*AL+(1.0-AL)*E2)
OVERLAY(OVL,07,0)
PROGRAM TRAIL

C MAIN PROGRAM FOR FREE TRAILING VORTICES

DIMENSION XT(1000), YT(1000), ZT(1000), XT(500), YT(500)
1 ZT(500), UT(1000), VT(1000), VT(1000), V1(100), V2(100)
2 V3(100), NIP(100), NTVE(100)
3 EQUIVALENCE (B15001, UTV), (B16001, VTV), (B17001, ZTV)
4 (B18001, XT), (B19001, YT), (B20001, ZT)
5 (B21001, XTVI), (B21501, YTVI), (B22001, ZTVI)
6 (B22501, V1), (B22601, V2), (B22701, V3)
7 (B22801, NIP), (B22901, NTVE)
8 (B23001, NV)

EQUIVALENCE (X V5, B(12871))
DIMENSION X VS(100)
COMMON/BODY/ B(2500)
COMMON/SCAT/ XQ(1000), YQ(1000), ZQ(1000)
COMMON DA(5000)
EQUIVALENCE (DA(7), XCG), (DA(8), YCG), (DA(9), ZCG), (DA(10), ALPHA)
1 (DA(11), BETA), (DA(12), PSTAR), (DA(13), QSTAR), (DA(14), RSTAR)
2
C COMPUTE FREE TRAILING VORTEX POINTS, INCLUDING INITIAL POINTS.

XMIN=XVS(100)
XMAX=XTVI(1)
WRITE(6, 70) XMIN, XMAX

70 FORMAT(6HOTAIL/1P3E20.6)
WRITE(6, 65) (NIP(J), J=1, N V)

65 FORMAT(11H0NIP ARRAY=101S)
K=0
DO 75 J=1, N V
NN=NIP(J)
DO 75 I=1, NN
K=K+1
WRITE(6, 70) XTVI(K), YT(VI(K), ZTVI(K))
IF(XMAX, GT, XTVI(K)) GO TO 75
XMAX=XTVI(K)
CONTINUE
DXX=XMAX-XMIN
DELX=DXX/20.0
XMAX=XMIN+1.25*DX
WRITE(6,5) XMAX,DX,A,XMIN,DELX
5 FORMAT(29H0XMAX,DX,A,XMIN,DELX=1P3E20.6)
K=0
DO 50 I=1,NV
N1=0
IF(I-1) 15,15,10
10 IM1=I-1
DO 11 I=1,IM1
11 N1=NIP(I)+N1
15 NP=NIP(I)
DO 20 J=1,MP
K=K+1
NJ=N1+J
XTV(K)=XTV(J)
YTV(K)=YTV(J)
20 ZTV(K)=ZTV(J)
DO 30 J=1,20
X(J)=X(J)+DELX
30 CONTINUE
35 CONTINUE
50 NTVE(I)=NIP(I)+J1
WRITE(6,80)I,XTV(I),YTV(I),ZTV(I),J1
80 FORMAT(12H0TRAIL,XTV,YTV,ZTV/(1P3E20.6))
C
COMPUTE POINTS AT WHICH VELOCITIES WILL BE FOUND IN SUBTRAVEL.
C
K=0
DO 100 I=1,NV
M=NTVE(I)
DO 90 J=1,M
K=K+1
K1=K+1
IF(J.EQ.M) GO TO 85
XQ(K) = 0.5*(XTV(K1)+XTV(K)) 8 0780
YQ(K) = 0.5*(YTV(K1)+YTV(K)) 8 0790
ZQ(K) = 0.5*(ZTV(K1)+ZTV(K)) 8 0800
GO TO 90 8 0810
35 XQ(K) = XTV(K) 8 0820
YQ(K) = YTV(K) 8 0830
ZQ(K) = ZTV(K) 8 0840
90 CONTINUE 8 0850
100 CONTINUE 8 0860
CALL TRAVEL(XQ,YQ,ZQ,UTV,VT2,UTV,K) 8 0870
WRITE(6,105) (XQ(I),YQ(I),ZQ(I),UTV(I),VT2(I),I=1,K) 8 0880
105 FORMAT(28H0TRAIL*,XQ,YQ,ZQ,UTV,VT2/(1P3E20.6)) 8 0890
C TEMPORARY CBAR,BREF* 8 0900
CBAR=10.0 8 0910
BREF=1.0 8 0920
C
DO 110 I=1,K 8 0930
XX=XQ(I)-XCG 8 0940
YY=YQ(I)-YCG 8 0950
ZZ=ZQ(I)-ZCG 8 0960
VX=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/BREF) 8 0970
VY=-(BETA-2.0)*(PSTAR*ZZ-RSTAR*XX)/BREF 8 0980
VZ=ALPHA+2.0*(PSTAR*YY/BREF+QSTAR*XX/CBAR) 8 0990
WRITE(6,109) I,VX,VY,VZ 8 1000
109 FORMAT(15,1P3E16.4) 8 1010
UTV(I)=UTV(I)+VX 8 1020
VT2(I)=VT2(I)+VY 8 1030
110 WTV(I)=#TV(I)+VZ 8 1040
CALL INTEGRATE(XTV,YTV,ZTV,UTV,VT2,TV1,K,N,NTVE) 8 1050
WRITE(6,106) (XTV(I),YTV(I),ZTV(I),I=1,K) 8 1060
106 FORMAT(23H0INTEGRATED XTV,YTV,ZTV/(1P3E20.6)) 8 1070
END 8 1080
SUBROUTINE INTEGRATE(X,Y,Z,U,V,W,N,NTVE) 8 1090
DIMENSION X(1),Y(1),Z(1),U(1),V(1),W(1),NTVE(1) 8 1100
K=0 8 1110
DO 10 I=1,NTV 8 1120
NE=NTVE(I) 8 1130
DO 9 J=2,NE 8 1140
L2=J+K 8 1150
9
L1 = L2 - 1
DELX = (X(L2) - X(L1)) / U(L1)
YL(L2) = Y(L1) + DELX * V(L1)
Z(L2) = Z(L1) + DELX * W(L1)
CONTINUE
K = K + NE
RETURN
END
SUBROUTINE TRAVEL(XI, YI, ZI, UI, VI, WI, NPTS)
DIMENSION XI(1), YI(1), ZI(1), UI(1), VI(1), WI(1)
EQUIVALENCE (B(1), X), (B(1001), Z)
DIMENSION XV(5000), YV(5000), ZV(5000)
COMMON/NODE/ X, Y, Z, I, J, K, IF1, IF2
COMMON DA(5000)
COMMON/HAT/ MCUM(1000), XSOL(200)
COMMON/ BODY/ X(25000)
EQUIVALENCE (B(1), XV), (B(201), YV), (B(401), ZV), (B(601), XO)
DIMENSION XV(10, 20), YV(10, 20), ZV(10, 20), XO(10, 20)
DIMENSION BODYG(15000), PANELG(13170)
COMMON/ SCAT/ XQ(1000), YQ(1000), ZQ(1000), AX(200), AY(200)
COMMON/ NUMBER/ MNPTS(5), NCPTS(5), NL(5), NLN(5), LDT(5), LNC(5)
COMMON/ PANINF/ PANSYN(10)
EQUIVALENCE (SYMP, DA(3426)), (SYMB, DA(12))
12  III=1  
    IF1=1  
    I12=(J-1)*LTDIV  
    DO 14 L=1,LTDIV  
    I12=I12+1  
    IF2=I12+1  
14    CALL VORTEX(SUM)  
    GO TO 50  
    I12=I12+1  
    IF2=I12  
    III=(I-1)*LNDIV  
    DO 24 L=1,LNDIV  
    III=III+1  
    IF1=III+1  
24    CALL VORTEX(SUM)  
    SX=SUM(I)  
    SY=SUM(J)  
    SZ=SUM(K)  
    GO TO 50  
30    III=IF1  
    I12=I12+1  
    DO 34 L=1,LTDIV  
    I12=I12+1  
    IF2=I12-1  
    IF1=NE.NBVV) GO TO 33  
    DO 32 KS=1,3  
    SUM(KS)=0.0  
32    CALL VORTEX(SUM)  
    GO TO 34  
33    CALL VORTEX(SUM)  
    CONTINUE  
34    DO 35 KS=1,3  
35    SUM1(KS)=SUM(KS)  
    GO TO 30  
39    IF(J.EQ.1) GO TO 42  
39    SUM(1)=-TRSUMX(1)  
    SUM(2)=-TRSUMY(1)  
    SUM(3)=-TRSUMZ(1)  
    GO TO 50  
42    I12=IF2
III=III+1
DO 44 L=1, LNDIV
III=III-1
IF1=III-1
44 CALL VORTEX(SUM)
50 TOT1=TOT1+SUM(1)
TOT2=TOT2+SUM(2)
TOT3=TOT3+SUM(3)
55 CONTINUE
TRSUMX(I)=SX
TRSUMY(I)=SY
TRSUMZ(I)=SZ
AXB(N)=TOT1/PI4
AYB(N)=TOT2/PI4
AZB(N)=TOT3/PI4
60 CONTINUE
DO 62 II=1, N
U=U+AXB(II)*XSOL(II)
V=V+AYB(II)*XSOL(II)
w=w+AZB(II)*XSOL(II)
61 U1(N)=U+U1(N)
VI(N)=V+VI(N)
63 w1(N)=W+W1(N)
65 CONTINUE
190 IF(NPANS.EQ.0) GO TO 405
DO 200 KK=1, NPANS
READ(18) PANELG
SYMP=PANSYM(KK)
NSPACE=NSPP(KK)
NTV=NV(NBODS+KK)
NB=NV(NBODS+KK)
IF(KK=1) 191, 191, 192
191 NPPV=0
GO TO 193
192 KK1=K-1
NB1=NV(NBODS+KK1)
NT1=NV(NBODS+KK1)
NPPV=NT1*NB1
193 CONTINUE
```
<table>
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<tr>
<td>198</td>
<td>DO 198 M=1*NPTS</td>
</tr>
<tr>
<td></td>
<td>X=XI(M)</td>
</tr>
<tr>
<td></td>
<td>Y=YI(M)</td>
</tr>
<tr>
<td></td>
<td>Z=ZI(M)</td>
</tr>
<tr>
<td></td>
<td>U=J*0</td>
</tr>
<tr>
<td></td>
<td>V=U*0</td>
</tr>
<tr>
<td></td>
<td>W=0*0</td>
</tr>
<tr>
<td></td>
<td>KVOR=0</td>
</tr>
<tr>
<td>195</td>
<td>DO 195 I=1*NTVV</td>
</tr>
<tr>
<td></td>
<td>IS=I-NSPACE</td>
</tr>
<tr>
<td>195</td>
<td>DO 195 J=1*NBVV</td>
</tr>
<tr>
<td></td>
<td>J1=J+1</td>
</tr>
<tr>
<td></td>
<td>KVOR=KVOR+1</td>
</tr>
<tr>
<td></td>
<td>IF(IS.LE.0) GO TO 75</td>
</tr>
<tr>
<td></td>
<td>IT=(I-1)<em>NBVV</em>2</td>
</tr>
<tr>
<td></td>
<td>IT1=IT+1+(J-1)*2</td>
</tr>
<tr>
<td></td>
<td>I12=IT1+2</td>
</tr>
<tr>
<td></td>
<td>T1=TS(IT1)</td>
</tr>
<tr>
<td></td>
<td>T2=TS(IT1)</td>
</tr>
<tr>
<td></td>
<td>CONTINUE</td>
</tr>
<tr>
<td>75</td>
<td>DO 5 K1=1,3</td>
</tr>
<tr>
<td></td>
<td>SUM(K1)=0.0</td>
</tr>
<tr>
<td>41</td>
<td>DO 41 K=1,4</td>
</tr>
<tr>
<td></td>
<td>IF(J.EQ.NBVV.AND.K.EQ.4) GO TO 41</td>
</tr>
<tr>
<td></td>
<td>IF(K.GT.1) GO TO 21</td>
</tr>
<tr>
<td></td>
<td>IF(IS.LE.0) GO TO 76</td>
</tr>
<tr>
<td></td>
<td>X2=XVO(IS,J) - T2*SQR(SYS(I1)**2+DZS(I1)**2)*0.5</td>
</tr>
<tr>
<td></td>
<td>Y2=YVO(IS,J) - 0.5*DYS(I1)</td>
</tr>
<tr>
<td></td>
<td>Z2=ZVO(IS,J) - 0.5*DZS(I1)</td>
</tr>
<tr>
<td></td>
<td>IF(J.EQ.NBVV) GO TO 305</td>
</tr>
<tr>
<td></td>
<td>X1=XVO(IS,J1) - T1*SQR(SYS(I12)**2+DZS(I12)**2)*0.5</td>
</tr>
<tr>
<td></td>
<td>Y1=YVO(IS,J1) - 0.5*DYS(I12)</td>
</tr>
<tr>
<td></td>
<td>Z1=ZVO(IS,J1) - 0.5*DZS(I12)</td>
</tr>
<tr>
<td></td>
<td>X3=X1</td>
</tr>
<tr>
<td></td>
<td>Y3=Y1</td>
</tr>
<tr>
<td></td>
<td>Z3=Z1</td>
</tr>
<tr>
<td></td>
<td>GO TO 40</td>
</tr>
<tr>
<td>305</td>
<td>X1=XVS(IS)</td>
</tr>
<tr>
<td></td>
<td>Y1=YVS(IS)</td>
</tr>
</tbody>
</table>
```
Z1=ZVS(IS)
X3=X1
Y3=Y1
Z3=Z1
GO TO 40

CONTINUE
X1=XVR(I*J+1)
Y1=YVR(I*J+1)
Z1=ZVR(I*J+1)
X2=XVR(I*J)
Y2=YVR(I*J)
Z2=ZVR(I*J)
GO TO 40

IF(K-3) 25,36,38

IF(I.GT.NSPACE) GO TO 251
X2=XVR(I+1,J)
Y2=YVR(I+1,J)
Z2=ZVR(I+1,J)
GO TO 40

251 X2=XVO(IS+J)+0.5*DS(TI1)**2+DZS(TI1)**2*T2
Y2=YVO(IS+J)+0.5*DYS(TI1)
Z2=ZVO(IS+J)+0.5*DZS(TI1)
GO TO 40

IF(I.GT.NSPACE) GO TO 301
X2=XVR(I+1,J+1)
Y2=YVR(I+1,J+1)
Z2=ZVR(I+1,J+1)
GO TO 40

301 IF(J.EQ.NBVV) GO TO 303
X2=XVO(IS+J1)+0.5*DS(TI12)**2+DZS(TI12)**2*T1
Y2=YVO(IS+J1)+0.5*DYS(TI12)
Z2=ZVO(IS+J1)+0.5*DZS(TI12)
GO TO 40

303 X2=XVS(IS+1)
Y2=YVS(IS+1)
Z2=ZVS(IS+1)
GO TO 40
38 IF(I.GT.NSPACE) GO TO 351
X2=XVR(I,J+1)
Y2=YVR(I,J+1)
Z2=ZVR(I,J+1)
GO TO 40
351 X2=X3
Y2=Y3
Z2=Z3
40 IF(J.EQ.MVX.AND.K.EQ.4) GO TO 411
491 CALL VORPAN(SUM*X1+Y1+Z1*X2+Y2+Z2*X+Y+Z)
41 CONTINUE
GO TO 417
411 CONTINUE
Y2H=Y2
Y1H=Y1
SYMPAS=-1.0
C LEFT SEMI-INFINITE VORTEX LINE
410 T1=SQRT((Y2-Y)**2+(Z2-Z)**2)
IF(T1-0.00001) 412,411
411 CONTINUE
T2=(X2-X)*SQRT((X2-X)**2+(Y2-Y)**2+(Z2-Z)**2)
QT=0.25*(1.0-T2)/(PI*T1)
SUM(2)=SUM(2)+SYMPAS*QT*(Z2-Z)/T1
SUM(3)=SUM(3)+SYMPAS*QT*(Y2-Y)/T1
C RIGHT SEMI-INFINITE VORTEX LINE
412 CONTINUE
T1=SQRT((1-Y)**2+(Z1-Z)**2)
IF(T1-0.00001) 414,413
413 CONTINUE
T2=(X1-X)*SQRT((X1-X)**2+(Y1-Y)**2+(Z1-Z)**2)
QT=0.25*(1.0-T2)/(PI*T1)
SUM(2)=SUM(2)-SYMPAS*QT*(Z1-Z)/T1
SUM(3)=SUM(3)+SYMPAS*QT*(Y1-Y)/T1
414 CONTINUE
IF(SYMPAS.EQ.-1.0.AND.SYMP1.EQ.0) GO TO 415
GO TO 416
415 Y1=-Y1
Y2=-Y2
SY::PAS=1.0
GO TO 410
410 Y1=Y1H
Y2=Y2H
417 IF(IS*GT.0) GO TO 46
TERM=XSOL(KVOR+NPPV)
GO TO 48
46 TERM=0.0
K1=I-J*NBVV
DO 47 I2=1,J
47 TERM=TERM+XSOL(K1+I2+NPPV)
48 U=U+SUM(1)*TERM
V=V+SUM(2)*TERM
195 W=W+SUM(3)*TERM
UI(I)=UI(I)+U
VI(I)=VI(I)+V
198 WI(I)=WI(I)+W
200 CONTINUE
RETURN
DO 400 K=1,MPTS
X=X(K)
Y=Y(K)
Z=Z(K)
U=U+U
V=V+V
W=W+W
MA=0
NS=0
DO 300 I=1,NTVV
DO 300 J=1,NSVV
DO 300 IS=1,2
XS=IS+1
XSOL=1
T=TS(NS)
DZ=DZ(S NS)
BY=BY(S NS)
300 CALL PVSKIT,KV,KSOL
DC 250 X=I+IS
U=U+X*T(KV)+SIGMA(KV)
V=V+SYMT(KV)*SIGMAKV
W=V+SYMT(KV)*SIGMAKV
UI(K)=UI(K)+U
VI(K)=VI(K)+V
400 W(K)=W(K)+W
405 CONTINUE
RETURN
END
SUBROUTINE VORTEX(SUM)
COMMON/ BODY/ XV(151,31),YV(151,31),ZV(151,31)
COMMON/ COMPTS/ XQ(1320),YQ(1320),ZQ(1320)
COMMON DA(5000)
EQUVALENCE (DA(19),SYM)
COMMON/ INDEX/ X,Y,Z,II1,II2,IF1,IF2
DIMENSION SUM(1)
XI=XV(II1,II2)
YI=YV(II1,II2)
ZI=ZV(II1,II2)
XQ=XV(IF1,IF2)
YQ=YV(IF1,IF2)
ZQ=ZV(IF1,IF2)
XFO=XF-X
ZFO=ZF-Z
XIQ=XI-X
ZIQ=ZI-Z
SYMLOO=1.0
10 YFO=YF-Y
YIQ=I1-Y
DELX=XF-XI
DELY=YF-Y1
DELZ=ZF-Z1
RXS1=YFO*DELZ-ZFO*DELY
RXS2=ZFO*DELX-XFO*DELZ
RXS3=XYFO*DELX-YFO*DELX
RXS4=SQRT(RXS1**2+RXS2**2+RXS3**2)
TERM1=SQRT(DELX**2+DELY**2+DELZ**2)
TERM2=SQRT(XFQ**2+YFO**2+ZFQ**2)
TERM3=SQRT(XIQ**2+YIQ**2+ZIQ**2)
TERM4= XFO*DELX+YFO*DELY+ZFQ*DELZ
10
RATIO = TERM4/TERM1*#2
COSA=(DELX*XI0+DELY*YI0+DELZ*ZI0)/(TERM1*TERM3)
COSB =TERM4/(TERM1*TERM2)
CC=COSB-COSA
HX=XFQ-RATIO*DELX
HY=YFQ-RATIO*DELY
HZ=ZFQ-RATIO*DELZ
H=SOR(T(HX*HX+HY*HY+HZ*HZ)
IF(H=0.00001)11*12*12
11 COEF=0.0
GO TO 13
12 CONTINUE
HRXS=H*RXS
COEF=SYML00*CC/HRXS
13 CONTINUE
SUM(1)=COEF*RXS1 + SUM(1)
SUM(2)=COEF*RXS2 + SUM(2)
SUM(3)=COEF*RXS3 + SUM(3)
IF(SYML00.EQ.-1.0*OR SYML00 .EQ.0.0) RETURN
SYML00=-1.0
YI=-YI
YF=-YF
GO TO 10
END
SUBROUTINE VORPAN(SUM,XI,YI, ZI, XF, YF, ZF, X, Y, Z)
COMMON DA(5666)
EQUIVALENCE(FA(3426),SYM)
DIMENSION SUM(1)
R4P1=0.07957747
YIH=YI
YFH=YF
XFQ=XF-X
ZFQ=ZF-Z
XI0=XI-X
ZI0=ZI-Z
SYML00=1.0
YFQ=YF-Y
YIQ=YI-Y
DELX=XF-XI
DELAY = YF - Y1
DELZ = ZF - Z1
RXS1 = YF0 * DELZ - ZF0 * DELY
RXS2 = ZF0 * DELX - XFQ * DELZ
RXS3 = XFQ * DELY - YFQ * DELX
RXS = SQRT(RXS1**2 + RXS2**2 + RXS3**2)
TERM1 = SQRT(DELX**2 + DELY**2 + DELZ**2)
TERM2 = SQRT(XFO**2 + YFQ**2 + ZFO**2)
TERM3 = SQRT(X1O**2 + Y1O**2 + Z1O**2)
TERM4 = XFQ * DELX + YFQ * DELY + ZFO * DELZ
RATIO = TERM4/TERM1**2
COSA = (DELX * X1O + DELY * Y1O + DELZ * Z1O) / (TERM1 * TERM3)
COSB = TERM4 / (TERM1 * TERM2)
CC = COSB - COSA
HX = XFO - RATIO * DELX
HY = YFQ - RATIO * DELY
HZ = ZFO - RATIO * DELZ
H = SQRT(HX*HX + HY*HY + HZ*HZ)
IF (H > 0.000001) 11 * 12 * 12
COEF = 0.0
GO TO 13
11 CONTINUE
HRXS = H * RXS
COEF = 4 * PI * SYML00 * CC / HRXS
12 CONTINUE
SUM(1) = COEF * RXS1 + SUM(1)
SUM(2) = COEF * RXS2 + SUM(2)
SUM(3) = COEF * RXS3 + SUM(3)
YI = Y1H
YF = YF1
IF (SYML00 .EQ. -1.0) OR (SYM .NE. C.0)) RETURN
SYML00 = -1.0
YI = YI
YF = -YF
GO TO 10
END
SUBROUTINE PVSK(T, DYY, DZZ, XCI, YC, ZC, MS, MA, IV, JV, KSOL)
COMMON DR(5600)
EQUIVALENCE (DA(3426), SYM)
COMMON/PANEL/  PANEL(16)
COMMON/SCRAT/KSOL(500)  SCRTD(500)  AX(500)  AY(500)  AZ(500)
COMMON/BODY/YVR(10,20)  ZVR(10,20)  XVR(10,20)  XQO(10,20)  
1  YVQ(10,20)  ZVO(10,20)  PLL(500)  PLT(500)  YVARV(100)  CHORD(100)
2  XCV0(20)  XCCO(20)  XLE(20)  YLE(20)  ZLE(20)  
3  XTE(20)  YTE(20)  ZTE(20)  XE(20)  YJ(20)  ZJ(20)
4  XETE(20)  XVT(50)  YVT(50)  ZVT(50)  XR(20)  YR(20)
5  SX(1000)  SY(1000)  SZNT(1000)  S5YT(1000)  D5Y(1000)  D5S(1000)
6  TS(1000)  TXS(1000)  YSS(1000)  ZSS(1000)  SIGMA(1000)
COMMON/PANEL/  PANEL(16)

1  NPERPT  NSPACE  NATTCH  NTATT  NRPRCLN  NRPRCLT  NACTQC  NACTET  NTHXG
2  ITHET  NTIP  CHTIP  ROOT  OUTER  NNATT
3  MP1  MP2  MP3  MP4  MP5  MP6  MP6  MP8  MP9  MP10

COMMON/NUMBER/  NWPST(7)  NCPST(7)  NLN(7)  NLT(7)  LDC(7)
1  NCT  NNBOD  NPAWS  NVL(7)  NVT(7)  NTAPE  NTAPE  NCTR  NTAPE  NTAPE
2  LSEG(7)  TSEG(7)  LFUNC(7)  TFUNC(7)
3  LSDVB(7)  LTDIVB(7)  NSPP(7)  ROOTP(7)  OUTERP(7)  SYMM(7)

C

KSOL=1 FOR SOURCE PTS. ONLY
KSOL=2 FOR BOTH SOURCE PTS. AND VORTEX POINTS.
MS=SUBSCRIPT OF SOURCE PT.
XC,YC,ZC --- CONTROL PT.
IV = LATERAL VORTEX SUBSCRIPT.
JV = LONGITUDINAL VORTEX SUBSCRIPT.

C INSERT SPECIFICATION STATEMENTS HERE.
REAL I*1,II*2,III*3,IV*4
YVV=0.5*SORT(DYY**2+DZZ**2)
YV2=2.0*YVV
PI=3.141592654
YV=YVV
GO TO (10*20)*KSOL

YK=YSS(MS)
ZK=ZSS(MS)
XK=XSS(MS)
GO TO 25

IF(MS-2*(MS/2)*.EO.0) GO TO 10

YK=YVO(I1*JV)
ZK=ZVO(I1*JV)
XK=XVO(I1*JV)

C C
25 CONTINUE
SUMX=0.0
SUMY=0.0
SUMZ=0.0
UTSUM=0.0
VTSUM=0.0
WTSUM=0.0
SIGN=1.0
DZ=ZC-ZK
X=XC-ZK
50 CONTINUE
DY=YC-SIGN*YK
R3=DYY/YV2
RZ=DZZ/YV2
Y=RY*DY+YZ*DZ
Z=-RZ*DY+RY*DZ
R12=(Y+YV)**2+Z**2
R22=(X-T*Y)**2+Z**2*(1.0+T*T)
R32=(Y-YV)**2+Z**2
R4=SQRT((X-T*YV)**2+(Y-YV)**2+Z**2)
R5=SQRT((X+T*YV)**2+(Y+YV)**2+Z**2)
I1=(X+T*YV)/R5
I2=(Y+T*X+YV*(1.0+T**2))/R5
I3=-(Y+T*X-YV*(1.0+T**2))/R4
I4=(X-T*YV)/R4
IF(ABS(Z).GE.YV2) GO TO 42
Z=0
R6D=(X+T*YV)**2+(Y+YV)**2
R7D=(X-T*YV)**2+(Y-YV)**2
RXY=(X-T*Y)**2
R6=RXY/R6D
R7=RXY/R7D
IF(R6.GE.0.0075968656) GO TO 41
IF(R7.GE.0.0075968656) GO TO 41
IF(ABS(Y).LE.YV) GO TO 41
TERM=ABS(1.0/R7D-1.0/R6D)*0.5/P
GO TO 43
41 IF(ABS(Y).GT.YV) GO TO 42
IF(ABS(X-T*Y).GE.0.25*ABS(PLL(MA))) GO TO 42
TERM1=0.0
GO TO 43
42 TERM1=(I2+I3)/R22
CONTINUE
TERM2=(I1+1.0)/R12
TERM3=(I4+1.0)/R32
TERM4=1.0/R4-1.0/R5
P=SQRT(1.0+T*T)
EUS=(T*TERM4+(X-T*Y)*TERM1)/P
EVS=(TERM4-T*(X-T*Y)*TERM1)/P
EWS=P*Z*TERM1
US=0.25*EUS/PI
VS=0.25*EV5/PI
WS=0.25*EWS/PI
UT=US
VT=VS*RY-RS*RZ
UT=VS*RZ+RS*RY
UTSM=UT+UTSUM
VTSM=VT+VTSUM
WTSUM=WT+WTSUM
IF(KSOL.EQ.1) GO TO 45
EU=Z*TERM1
EV=Z*(-T*TERM1+TERM2-TERM3)
E=-(X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
UV=0.25*EU/PI
VV=0.25*EV/PI
WW=0.25*EW/PI
UI=UV
VI=RY*VV-RZ*WV
WI=RZ*VV+RY*WV
SUMX=UI+SUMX
SUMY=VI+SUMY
SUMZ=WI+SUMZ
CONTINUE
C FOR SYMMETRY, GET IMAGE CONTRIBUTION.
IF(SYMN.EQ.0.0) GO TO 60
IF(SIGN.LT.0.0) GO TO 60
SIGN=-1.0
DZ=-DZZ
C
T = -T
GO TO 50
CONTINUE
SXM1(MS) = UTSU'M
SYMT(NS) = VTSC'M
SZIT(NS) = WTSU'M
IF (KSOL.EQ.1) RETURN
AX(MA) = SUNX
AY(MA) = SUNY
AZ(MA) = SUN2
RETURN
END
OVERLAY(OVL,10,0)
PROGRAM HARDAP
COMMON/SCRAT(A(20),DA(5000),ATACH(5,10),
INTEGER UNIT,UNIT2
DATA UNIT2/21/
DATA UNIT/10/
REWIND UNIT
REWIND UNIT2
PRINT 3
READ 5,A
IF(A(1).EQ.4 .AND.A(2).EQ.4 .AND.A(3).EQ.4) GOTO 60
CALL OUTIN (A)
GOTO 99
1 READ 5,A
60 IF(EOF(5)) 2,372
2 PRINT 6,A
WRITE(UNIT,5) A
GOTO 1
ENDFILE UNIT
REWIND UNIT
CALL DECRD(DA,UNIT)
NB = DA(1)
IF(NB.EQ.0) GOTO 20
DO 10 I=1,NB
10 CALL DECRD(DA,UNIT)
20 NT = DA(2)
IF(NP.EQ.0) GOTO 40
DO 30 I=1,NP
CALL DECRD(DA,UNIT)
ATACH(1,I) = DA(3420)
ATACH(2,I) = DA(3444)
ATACH(3,I) = DA(3445)
ATACH(4,I) = DA(3446)
ATACH(5,I) = DA(3447)
30 GOTO 40
REWIND UNIT
WRITE(UNIT2) ATACH
WRITE(6,51) UNIT2
51 FORMAT(*UNIT2=213)
REWIND UNIT2
SUBROUTINE DECRT(DA, IARG, MAX, IUNIT)

DIMENSION DA(1)
ISTART = IARG

444 IF(MAX - ISTART - 4) 1, 1, 2
1 MSTART = -ISTA
WRITE(IUNIT, 111) MSTART, (DA(I), I = ISTART, MAX)
PRINT 111, MSTART, (DA(I), I = ISTART, MAX)
111 FORMAT(112, 5G12.5)
RETURN

2 MSTART = ISTART + 4
IF(DA(ISTART) .EQ. 0.0 .AND. DA(ISTART+1) .EQ. 0.0 .AND. DA(ISTART+2) .EQ. 0.0) GOTO 3
1.0 .AND. DA(ISTART+3) .EQ. 0.0 .AND. DA(ISTART+4) .EQ. 0.0) GOTO 3
WRITE(IUNIT, 111) ISTART, (DA(I), I = ISTART, MSTART)
PRINT 111, ISTART, (DA(I), I = ISTART, MSTART)
3 ISTART = ISTART + 5
GOTO 444
END
SUBROUTINE OUTIN(TITLE)

Coded by B. D. Gaither
At some later date all tape I/O should be done by buffer in and
out, then all zeroing loops and calls to decrd and decroi could
be removed. At present the main routines expect decrd style input.

OUT  0110
OUT  0120
OUT  0080
OUT  0090
OUT  0100
OUT  0140
OUT  0150
OUT  0160
OUT  0170
OUT  0190
OUT  0200
OUT  0210
OUT  0220
OUT  0230
OUT  0240
OUT  0260
OUT  0270
OUT  0280
OUT  0290
OUT  0300
OUT  0310
OUT  0320
OUT  0330
OUT  0340
OUT  0350
OUT  0360
OUT  0370
OUT  0380
OUT  0390
OUT  0400
OUT  0410
OUT  0420
OUT  0430
OUT  0440
OUT  0450

**INTEGER** TITLE(20), NRADX(4), NFORX(4), OUNIT, OUNIT2, SCRTH
**REAL** DA(5000), XAF(301), XOFLE(20), YOAFLE(20), ZOAFLE(20),
IAFS(120), TZOCC(120), MAFOR(410), ZVXX(120), ZLCCS(120)

2, ECSA(120), YOHS(30,120), ZOHS(30,120)
3, XOP, YOP, ZOP, PX(30), POORD(30), FINXL,
5FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH
6, HCHORD, XL, YLC, ZLC, CONC, XHC, YHC, ZHC
7, CONC, KANTCL(10), KANTML(10), KAN(10)
8, PHCHORD(10), LAFHT(10), ATT(20), ZCT(440), PPD(150)
9, PARE(1580), RODARE(3405)
A, ATACH(1580), NPPDP, NTTXCS
B, NXSAP, BCT(50), LCT(460), RPA(800)
C, TTT(29), ETA(29), NATA
D, BMFCX(49)
E, ZLCT(120), SP(39), SP(39), ZCSA(39), SLC(120,30), SY(120,30)

DOUBLE PRECISION PI, RAD, RAD1, SG

COMMON /ATACH/ ATACH

EQUIVALENCE (REFA, DA(16), (PSI, DA(9)))
1, (ATA(1), DA(3631), (KANS, DA(3630), (ZC(1), DA(3660)))
2, (XAF(1), DA(3601), (XCNM, DA(3600)))
3, (REFA, DA(3421))
4, (XPO, DA(3432), (YPO, DA(3432), (ZPO, DA(3434)))
5, (IAFS, DA(3426))
6, (XAF(1), PHCHORD(1)), (I, XAF(1), DA(4190)), (I, MAFOR(11), XAFH(1)),
7, (PANARE(1), DA(3420), (BODARE(1), DA(15)), (XCL, FINXL),
8, (YCL, FINYL), (ZCL, FINZL), (CONC, LCHORD), (XHC, FINXH),
9, (YHC, FINYH), (ZHC, FINZH), (CONC, HCHORD), (KANTCL(1), PHCHORD(1)),
A, (KANTML(1), XAFH(1)), (PPD(1), DA(3450))
B, (NPPDP, DA(3443)), (S, TTXCS, DA(4130))
C *(POX*DA(27)) *(BOY*DA(28)) *(BOZ*DA(29)) *(IXSBOT*DA(40))*
D *(LCT(1)*DA(130)) *(RRL1)*DA(83)) *(CCT(1)*DA(41))
E *(TTI)*DA(41311) *(ETA(1)*DA(4161)) *(NATA*DA(4162))
F *(BMFXN*DA(1601)) *(BMFX5(1)*DA(1601))
G *(ACTII*DA(201)) *(SN*DA(130)) *(ZLCCT(1)*DA(1300))
H *(S(1)*DA(131)) *(PSYM*DA(19)) *(PCONT*DA(3429))

OUT 0460
OUT 0470
OUT 0480
OUT 0490
OUT 0510
OUT 0520
OUT 0530
OUT 0540
OUT 0550
OUT 0560
OUT 0570
OUT 0580
OUT 0590
OUT 0600
OUT 0610
OUT 0620
OUT 0630
OUT 0640
OUT 0650
OUT 0660
OUT 0670
OUT 0680
OUT 0690
OUT 0700
OUT 0710
OUT 0740
OUT 0750
OUT 0760
OUT 0770
OUT 0780
OUT 0790
OUT 0800
OUT 0810
OUT 0820
OUT 0830
OUT 0840
4 FORMAT(10F7.0)
REFAA=RLFA
REFAF=REFA

C PUNCH UNIVERSAL INFO
CALL DECWRT(DA,14,UNIT)

C ********************************************

C READ WING DATA

C IF(IABS(J1).NE.1) GOTO 101

C NUMBER OF PERCENT CHORD LOCATIONS
XCNUM = NWAFOF

C SET SYMMETRY OF WING
AFSIM = 0.0

C SET PANEL CONTOUR INDICATOR
PCONT = 1.0

C READ PERCENT CHORD LOCATIONS
READ 4, (XAF(I), I=1, NWAFOF)
DO 324 I=1, NWAFOF
XAF(I) = XAF(I)/100.0
324 TT(I) = XAF(I)

C READING DATA CARDS
READ 5, (XOAFLE(I), YOAFLE(I), ZOAFLE(I), AF5CL(I), I=1, NWAFF)

C NUMBER OF LTA(ATA) STATIONS
ANAS = NWAFF
NATA = ANAS

C FIND AIRFOIL LENGTHS (SPAN IS THEIR SUM)
SPAN = 0.0
ATA(1) = SPAN

C OUT 0850 0850 0870
C OUT 0880
C OUT 0900
C OUT 0910
C OUT 0920
C OUT 0930
C OUT 0940 0950 0960
C OUT 0970
C OUT 0980
C OUT 0990
C OUT 1000
C OUT 1010
C OUT 1020
C OUT 1030 1040 1050
C OUT 1060
C OUT 1070
C OUT 1080
C OUT 1090
C OUT 1100
C OUT 1110
C OUT 1120
C OUT 1130
C OUT 1140
C OUT 1150
C OUT 1160
C OUT 1170
C OUT 1180
C OUT 1190
C OUT 1200
DO 102 I=2, NWAF
ATA(I) = SQRT((YOAFLE(I) - YOAFLE(I-1))**2 + (ZOAFLE(I) - ZOAFLE(I-1))**2) OUT 1210
SPAN = ATA(I) + SPAN
102 ATA(I) = SPAN OUT 1240

LOAD(AIRFOIL LENGTH / SPAN) INTO ARRAY ATA
DO 104 I = 1, NWAF
ATA(I) = ATA(I) / SPAN
104 ETA(I) =ATA(I)

READ DELTA Z VALUES AND CONVERT TO Z / CHORD LENGTH
IF(J1 .NE. 1) GOTO 222
I = 1
DO 31 J = 1, NWAF
IF(AF5hCL(J) .NE. 0) GOTO 517
PRINT 223, J

228 FORMAT(*THE CHORD LENGTH OF WING AIRFOIL*13*) IS ZERO, IT HAS BEEN
LEN SET TO .01*/
1EN IF(J1 .LE. 1)
AF5hCL(J) = .01

517 READ 4, (TZORD(I), I = 1, NWAFOR)
PRINT 400, (TZORD(I), I = 1, NWAFOR)
FORMAT(*OTZORD*3/(10612.5))
DO 31 IT = 1, NWAFOR
ZC(I) = TZORD(IT) / AF5hCL(J)
31 I = I + 1

PANEL (WING) ORIGIN
222 XPO = .2
YP0 = 0.0
ZP0 = 0.0

PANEL PERIMETER DESCRIPTION (LEADING EDGE)
J = 1
DO 150 I = 1, NWAF
PPD(J) = XCAFLE(I)
PPD(J+2) = YOAFLE(I)
PPD(J+5) = ZOAFLE(I)
150 J = J + 4
SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP=NWAF

SET NUMBER OF X/C STATIONS ENTRIES IN THE THICKNESS TABLE PER AIRFOIL
NTTXCS=NWAFOR

READ WING AIRFOIL ORDINATE CARDS (HALF THICKNESS)
JJ = 0
DO 22 J=1, NWAF
IT = JJ+1
JJ = NWAFOR+JJ
22 READ 4* (WAFORD(I), I=IT, JJ)
DO 516 I=1, JJ
516 WAFORD(I)=WAFORD(I)/100.
PRINT 'LIST

START FUSELAGE DATA CARDS

IF(J2 EQ 0 OR NFUS EQ 0) GOTO 171

SET BODY COORDINATE TABLE INPUT INDICATOR TO INDICATE A CIRCULAR
BODY OR ARBITRARY BODY
BCTII = 0.0
BSYII = 1.0
IF(J2 EQ 1) BCTII= 1.0
LLL = 0
LL = 1
DO 100 IX=1, NFUS
NFO = NFORX(IX)
LLL=LLL+NFO
READ X STATIONS
READ 4* (XXVXF(I), I=LL, LLL)
XXVXF(LLL) = XXVXF(LL)+1.0E-7
READ CAMBER(IF ANY)
IF(J2.EQ.1.AND.J6.EQ.0) READ 4, (ZLCCS(I)), I=LL, LLL
IF(J2.NE.1) GOTO 200

READ CROSS SECTIONAL AREAS (IF ANY)
READ 4, (FCSA(I)), I=LL, LLL
GOTO 100

200 IF(J2.NE.1) GOTO 100

READ Y S AND Z S OF ARBITRARY BODY
NFF=NRADX(IX)
DO 7044 M=LL, LLL
READ 4, (YOHS(I*M)), I=1, NFF
READ 4, (ZOHs(I*M)), I=1, NFF
PRINT 401, (YOHS(I*M)), I=1, NFF
PRINT 402, (ZOHs(I*M)), I=1, NFF

7044 CONTINUE
401 FORMAT(*YOHS*,3(/10G12.5))
402 FORMAT(*ZOHs*,3(/10G12.5))
100 LL=LL+NFO

568 FIND AND SET TOTAL NUMBER OF X STATIONS
J=NFORXT1+NFORX(2)+NFORX(3)+NFORX(4)
NXS = J

PUNCH FUSELAGE AND WING
CALL DECWRT(DA=15, 3419, OUNIT)
PRINT BLIST

171 IF(IBAB(JI).NE.1) GOTO 715
CALL DECWRT(DA=3420, 5000, OUNIT)

ZERO BODY AREA OF STORAGE
715 DO 229 I=1, 3405
229 BODARE(I)=0.0

ZERO PANEL SECTION OF STORAGE
DO 226 I=1, 1580
226 PANARE(I)=-0.0

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C READ POD DATA
C IF (J3.E.1.OR.NP.EQ.0) GOTO 300
C REWIND SCRATCH UNIT TO PUT PODS ON
REWIND SCRTCH
DO 290 IX=1, NP
READ 4*, XOP  , YOP  , ZOP
READ 4*, (PX(I)  , I=1,NP)OR)
READ 4*, (PODRAD(I)  , I=1,NP)OR)
PRINT PLIST
C IS THIS A POD OR A NACELLE
IF (PODRAD(1).LE.0) GOTO 123
C ITS A NACELLE
C SET REFERANCE AREA CARDS
REFAF=REFAA
C SET PANEL CONTOUR INDICATOR
PCONT = 1.0
C SET PANEL ORIGIN:
XPO=XOP
YPO=YOP
ZPO=ZOP
C FIND CHORD LENGTHS
CHORD=PX(NP)OR)
C PANEL PERIMETER DESCRIPTION (STEP BY TEN DEGREES)
J=1
DO 764 K=1, 361, 10
RAD = RAD 1 DG * FLOAT(K-1)
PPD(J)=PX(I)
PPD(J+1)=DSIN(RAD)*PODRAD(I)
PPD(J+2)=DCOS(RAD)*PODRAD(I)
PPD(J+3)=CHORD
764  J=J + 4
C
C  SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP=37

C  PERCENT CHORD LOCATIONS (X/CHORD LENGTH)
XCNUM=NPORDP
DO 746  I=1, NPORDP

746  PCHORD(I)=PX(I)/CHORD

C  SET ETA (ATA) STATIONS (AS WELL AS THEIR NUMBER)
ANAS=2
ATA(1)=0.0
ATA(2)=1.0

C  CAMBER LINE = DELTA Z S (X S) / AIRFOIL STREAMWISE CHORD LENGTH
DO 476  I=1, NPORDP
ZC(I)=PODRAD(I)/CHORD

476  ZC(NPORDP+1)=ZC(I)

C  PUNCH PANEL
CALL DECWRT(DA, 3420, 500, OUNIT)

C  ZERO PANEL AREA
DO 227  I=1, 1580

227  PANARE(I)=-0.0
GOTO 290

C
C  ITS A POD

C  SET BODY ORIGIN
123  BOX=XOP
BOY=YOP
BOZ=ZOP

C  SET BODY COORDINATE TABLE INPUT INDICATOR
BCTII = 0.0
BSYN=1.

C  SET NUMBER OF X STATIONS IN BODY COORDINATE TABLE

OUT 3020
OUT 3030
OUT 3040
OUT 3050
OUT 3060
OUT 3070
OUT 3080
OUT 3090
OUT 3100
OUT 3110
OUT 3120
OUT 3130
OUT 3140
OUT 3150
OUT 3160
OUT 3170
OUT 3180
OUT 3190
OUT 3200
OUT 3210
OUT 3220
OUT 3230
OUT 3240
OUT 3250
OUT 3260
OUT 3270
OUT 3280
OUT 3290
OUT 3300
OUT 3310
OUT 3320
OUT 3330
OUT 3340
OUT 3350
OUT 3360
OUT 3380
OUT 3390
NX5BOT = NPODOR
BMFCXN = NX5BOT

C
FILL IN BODY CO - ORDINATE TABLE
DO 321 I=1, NPODOR
BMFCX5(I)=PX(I)
321 BCT(I)=PX(I)
K=1
DO 456 I=1, NPODOR

C
FILL IN LATERAL CO - ORDINATE TABLE
C
SET RADI
RB(K)=PODRAD(I)
RB(K+1)=PODRAD(I)

456 K=K+2
LCT(1) = 2.0
LCT(2)=0.0
LCT(3)=180.0

C
SET REFERENCE AREA
REFA=REFAQ
C
PUNCH BODY(POD)
CALL DECRT(11,15,3419,SCRTCH)
C
ZERO BODY AREA
DO 722 I=1, 3405

722 BODARE(I)=-0.0
290 CONTINUE

C
READ FIN DATA
C
300 IF(J4,NE,1,OR,NF,E0,0) GOTO390
DO 380 IX=1, NF
READ 4, FINXL, FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH, HCHORD
1FINYH, FINZH, HCHORD
READ 4, (PCHORD(I), I=1, NF, IOR)
READ 4, (F*FHT(I) , I=1, NFINOR)
DO 225 I=1, NFINOR
   PCHORD(I)=PCHORD(I)/100.
225 FAFHT(I)=FAFHT(I)/100.*
C
C PERCENT CHORD LOCATIONS (NUMBER OF ORDINATES USED TO DESCRIBE EACH)
C AER FOIL SECTION (THICKNESS TABLE X/C STATIONS))
C
XCNUM = NFINOR
NITXCS=XCNUM
DO 379 I=1, NFINOR
379 TT(I)=PCHORD(I)
PRINT FLIST
ASSIGN 380 TO LABEL
GOTO 1000
380 CONTINUE
C
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
C READ CANARD DATA
C
390 IF(J5.0.1 OR NCAN.0.0) GOTO 999
DO 500 IX =1, NCAN
   READ 4, XLC,YLC,ZLC,CONCRD,XHC,YHC,ZHC
1,CONCRD
   J=IABS(NCANOR)
   READ 4, (CAMPCL(I), I=1,J)
   READ 4, (CAMLJ(I), I=1,J)
DO 223 I=1, J
   CAMPNL(I)=CAMPPL(I)/100.*
223 CAMPCL(I)=CAMPCL(I)/100.*
C
C HERE IF CAMPRED (NOT SYMETRICAL)
C
C SET PANEL CONTOUR INDICATOR
PCONT = 1.0
READ 4, (CANS(I), I=1, J)
DO 712 I=1, J
712 CANS(I)=CANS(I)/100.*
C
C
C SET CAMBER
DO 1052 I=1, J
1052 ZC(I)=(CAMLN(I)-CAMS(I))/2.0
C ADJUST THICKNESS TO BE RELATIVE TO CAMBER LINE
DO 1050 I=1, J
1050 CANPLN(I)=(CAMLN(I)+CAMS(I))/2.0
C PERCENT CHORD LOCATIONS(NUMBER OF ORDINATES USED TO DESCRIBE EACH OUT 4260
SECTION(THICKNESS TABLE STATIONS))
7094 XCNUM= J
NNTAXS=XCNUM
C ZERO UNIVERSAL DATA STORAGE
DO 7093 I=1, J
7093 TT(I)=CAMLN(I)
PRINT CLIST
ASSIGN 500 TO LABEL
GOTO 1000
500 CONTINUE
C
C ********************************************
C
C READ PLOT DATA
C
C 999 READ 7,PHI,THETA,PSII
7 FORMAT(7X,3F5.0)
PRINT PLOTLT
C
C REWIND OUTPUT FILE FROM COMPUTER CONVERSION
REWIND UNIT
ENDFILE SCRTH
REWIND SCRTH

C ********************************************
C
C USER EDIT SECTION
C
C EDIT UNIVERSAL INFO
C ZERO UNIVERSAL AREA OF STORAGE
PRINT 8
OUT 4570
8 FORMAT(1H1,30X,*START EDIT*)
OUT 4580
DO 710 I=1, 14
OUT 4590
710 DA(I)=-0.0
OUT 4600
CALL DECRD(DA,OUNIT)
OUT 4610
CALL DECRD(DA,1UNIT)
OUT 4620
CALL DECRD(DA,1,14,OUNIT2)
OUT 4630
C ZERO BODY AREA STORAGE
OUT 4640
DO 711 I=1, 3405
OUT 4650
711 BODARE(I)=-0.0
OUT 4660
C EDIT BODIES IF ANY
OUT 4670
NPOD = 0
OUT 4680
IF(J2 .EQ. 0) GOTO 505
OUT 4690
CALL DECRD(DA,OUNIT)
OUT 4700
CALL DECRD(DA,1UNIT)
OUT 4710
J = NX5
OUT 4720
IF( J .GT. 44) GOTO 201
OUT 4730
C
C FILL IN DATA FOR BODY THAT FITS AS IS
C NX5BOT = NX5
C BMFCXN = NX5BOT
C DO 7030 I=1, J
C BCT(I) = XVXFXF(I)
C ZLCC(S(I) = ZLCCS(I)
C
574

7030 BMFCXS(I) = BCT(I)
C
C IF(J2 .NE. -1) GOTO 202
C
C SET ANGLE OF CIRCULAR BODY
C LCT(1)=2.0
C LCT(2)=0.0
C LCT(3)=180.0
C L=1
C DO 7840 I=1, J
C
C FIND RADIUS OF CIRCULAR BODY
C RB(L) =DSRT(FCSA(I)/PI)
C RB(L+1) = RB(L)
7040 L = L + 2
GOTO 205
202 MM = 1
N = 0
LLL = 0
LL = 1
DO 110 IX=1, NFUS
NFF = NRADX(IX)
NZ = NFF
NFO = NFORX(IX)
LLL = LLL + NFO
DO 10 M=LL, LLL
LCT(MM)= NFF
DO 7090 I=1, NFF

C SET Y AND Z VALUES
RB(N+I) = ZOHS(NZ+M)
LCT(MM+I) = YOHS(NZ+M)

7090 NZ = NZ - 1
N = N + NFF
110 MM = MM + 1 + NFF
110 LL = LL + NFO
GOTO 205

C INTERPOLATE FOR OVER SPECIFIED DATA
201 J = NXSBOT
BMFCXN = NXSBOT
DO 217 I=1, J
217 BMFCXS(I) = BCT(I)
IF( J2 .NE. -1) GOTO 204

C SET ANGLE OF CIRCULAR BODY
LCT(1) = 2.0
LCT(2) = 0.0
LCT(3) = 180.0
J=NXSBOT
L=1

C FIND RADIUS OF CIRCULAR BODY
DO 211 I=1, N
DO 211 M=1, NXS
SZ(M, I) = ZOHS(I, M)

211 SY(M, I) = YOHS(I, M)

J = NXSBOT
L = 1
K = 0
DO 212 I=1, J
LCT(L) = N

DO 213 K=1, N
LCT(L+M) = CODIM1(BCT(I), XVFXF, SY(I, M), NXS, 0.0)

213 RB(K+M) = CODIM1(BCT(I), XVFXF, SZ(I, M), NXS, 0.0)

L = L + 1 + N

212 K = K + 1

DO 214 L=1, J

214 ZLCCT(I) = CODIM1(BCT(I), XVFXF, ZLCCS, NXS, 0.0)

CONTINUE

CALL DECRDT(DA, 15, 3419, OUNIT2)

C ZERO BODY AREA STORAGE

720 DO 721 I=1, 3405

721 BODARE(I) = 0.0

C EDIT POST WITH ZERO RADIUS OF ROSE

505 CALL DECRDT(DA, SCRTCH)

IF(SCRTCH) 501, 501, 499

499 CALL DECRDT(DA, OUNIT)

CALL DECRDT(DA, 15, 3419, OUNIT2)

NPOR = NPOR + 1

GOTO 720

C EDIT PANELS (IF ANY)

501 CALL DECRDT(DA, OUNIT)

IF(OUNIT) 502, 502, 503

503 CALL DECRDT(DA, OUNIT)

CALL DECRDT(DA, 3420, 5000, OUNIT2)

C ZERO PANEL AREA OF STORAGE

DO 713 I=1, 1580

713 PANEI(I) = -0.0

GOTO 901

C
C REINDEX FILES
502 QUNIT = IABS(QUNIT)
REWIND QUNIT
REWIND QUNIT2

* * *

ADDITION SECTION

COPY PAST OLD BODY (IF ANY)
PRINT 9
9 FORMAT(1H1, 3X, #START ADDITIONS#)
DO 714 I=1, 14
714 DA(I)=0.6
CALL DECRT(DA, QUNIT2)
CALL DECRT(DA, I, 14, QUNIT)
1 = DA(I)
IF (J2 *EQ. 0) GOTO 507
I = I - 1
CALL DECRT(DA, QUNIT2)
CALL DECRT(DA, 15, 3419, QUNIT):
C ZERO BODY AREA STORAGE
C
513 DO 716 J=1, 2033
716 BSQARE(J)=0.6
507 IF(NPOD) 504, 504, 508
508 CALL DECRT(DA, QUNIT2)
CALL DECRT(DA, 15, 3419, QUNIT)
I = I - 1
NPOD = NPOD - 1
GOTO 513
C
C ADD NEW BODIES (IF ANY)
504 IF(I) 518, 518, 505
505 DO 506 J=1, I
CALL DECRT(DA, IUNIT)
CALL DECRT(DA, 15, 3419, QUNIT)
C ZERO BODY AREA STORAGE
DO 717 K=1, 3403
717 BSQARE(K)=0.6
509 CONTINUE

510 CONTINUE

511 CALL DECDRT(DA, OUNIT)
      ATACH(1,J) = DA(3420)
      ATACH(2,J) = DA(3444)
      ATACH(3,J) = DA(3445)
      ATACH(4,J) = DA(3446)
      ATACH(5,J) = DA(3447)

512 OUNIT2 = IABS(OUNIT2)
      DO 515 K=J, 1

513 OUNIT NOW CONTAINS FULLY UPDATED DATA, READY FOR READ

514 CONTINUE

515 CONTINUE

516 ZERO PANEL AREA OF STORAGE
      DO 718 K=1, 1580

718 PANARE(K) = 0.0
      CALL DECRD(DA, OUNIT2)
      IF (OUNIT2 .NE. 1) 512, 512, 511

719 PANARE(L) = 0.0
      CALL DECRD(DA, OUNIT)
      CALL DECRD(DA, 3420, 5000, OUNIT)
      ATACH(1*K) = DA(3420)
      ATACH(2*K) = DA(3444)
      ATACH(3*K) = DA(3445)
      ATACH(4*K) = DA(3446)
      ATACH(5*K) = DA(3447)
CONTINUL
GOTO 514

CO - ORDIINATE OF LEADING EDGE OF FIRST AIRFOIL
XPO = 0.0
YP0 = 0.0
ZPO = 0.0

SET REFERENCE AREA
REFAF = REFAA

PANEL PERIMETER DESCRIPTIONS
PPD(1) = FINXL
PPD(2) = FINYL
PPD(3) = FINZL
PPD(4) = LCHORD
PPD(5) = FINXH
PPD(6) = FINYH
PPD(7) = FINZH
PPD(8) = HCHORD

SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP = 2

NUMBER OF ETA(ATA) STATIONS
ANAS = 2
NATA = ANAS

ETA STATIONS
ETA(1) = 0.0
ETA(2) = 1.0
ATA(1) = 0.0
ATA(2)=1.0

C

C PUNCH PANEL
CALL DECART(DA,3420,500,UNIT)

C

C ZERO PANEL SECTION OF STORAGE
DO 224 I=1,150
224 PAMARE(I)=-0.0

C

C RETURN TO CALLER
GOTO LABEL,(380,500)
END
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