THEORETICAL PREDICTION OF AIRPLANE
STABILITY DERIVATIVES AT
SUBCRITICAL SPEEDS

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This report describes the theoretical development and application of an analysis for predicting the major static and rotary stability derivatives for a complete airplane. The analysis utilizes potential flow theory to compute the surface flow fields and pressures on any configuration that can be synthesized from arbitrary lifting bodies and nonplanar thick lifting panels. The pressures are integrated to obtain section and total configuration loads and moments due side slip, angle of attack, pitching motion, rolling motion, yawing motion, and control surface deflection. Subcritical compressibility is accounted for by means of the Gouthert similarity rule.

Within the scope of predicting the total configuration stability derivatives it was necessary to study the problem of computing the spanwise variation of potential form drag due to panel lift and thickness. Included in appendix F is a solution to this problem. Also, in solving the potential form drag problem the work of Woodward and Wagner was thoroughly analyzed. A complete derivation of Woodard's influence equations is given in appendix D and a comprehensive review of Wagner's lifting surface theory is included in appendix E.
INTRODUCTION

In order to develop a procedure for predicting total configuration stability derivatives, the perturbation flow due to the vehicle had to be represented by a grid of efficient aerodynamic finite elements general enough to satisfy the boundary conditions over the surfaces of diverse shapes and also produce the correct resultant forces and moments. The quadrilateral vortex was selected to represent the perturbation velocity due to the bodies because of its numerical efficiency, net force producing capability, limited range of influence, and relationship to the horseshoe vortex which has demonstrated amazing accuracy in predicting loads on wings of arbitrary shape.

The nonplanar thick lifting panels are divided into two sections, (1) the outboard section which is defined by a locus of chord lines, and (2) the root section which is a transition region from the outboard section to the juncture of the panel and a body. If the panel is attached to another panel there is no root section. The perturbation velocity due to panel lift is represented by quadrilateral vortices in the root section and skewed horseshoe vortices in the outboard section. The perturbation velocity due to panel thickness is represented by a source lattice. The panel aerodynamic finite elements were selected because of their numerical efficiency and proven accuracy in predicting flow fields over wings of general shape.

The panel singularities are placed on a mean surface instead of the actual external surface of the panel to maintain computing efficiency. This will sacrifice surface pressure accuracy at the juncture between two panels or a panel and a body, but for this type of general analysis the savings in computer time makes the compromise practical. Second order corrections to account for the interference between lift and thickness and to account for blunt leading edge airfoil sections are included.

The source and vortex lattice influence equations are formulated in terms of the same quantities, which allows the perturbation velocity due to lift and thickness to be computed simultaneously and thereby save computing effort. Also, due to the limited range of significant influence of the quadrilateral vortex the influence of any quadrilateral vortex is only computed at those points within a given area of influence. This can save considerable computing time in developing the aerodynamic influence matrix.

Computer time is also saved by reducing the number of unknowns by transforming the aerodynamic influence matrix by constraint matrices. The constraint matrices constrain the body and panel vorticity, thereby reducing the number of unknowns from that of the number of vortex elements to the number of constraint functions. This is an option in the program and can be
applied in just the longitudinal direction, lateral direction, both directions, or not at all.

The pressures are integrated by numerical means and the panel section drag is computed by means of the Kutta-Joukowsky theorem. The total configuration induced drag is computed by means of a Trefftz plane analysis.
LIST OF SYMBOLS

\( A_R \)  
aspect ratio

\( A_{1c} \)  
reference area

\([A_x], [A_y], [A_z]\)  
aerodynamic influence matrix components

\( a_{R_i} \)  
body constraint function coefficient

\( a_{p_k} \)  
panel constraint function coefficient

\( b \)  
panel span

\( C \)  
chord or aerodynamic coefficient

\( C_{c} \)  
reference chord

\( C_{s} \)  
chord line between \((X_j, Y_j, Z_j)\) and \((X_R, Y_R, Z_R)\)

\( C_f \)  
trailing edge flap chord

\( C_{\infty} \)  
leading edge flap chord

\( C_p \)  
pressure coefficient

\( [E] \)  
two dimensional influence matrix

\( \hat{n} \)  
unit vector tangent to control surface hinge line

\( h \)  
body height

\( \hat{i}, \hat{j}, \hat{k} \)  
unit vectors in the \(x, y,\) and \(z\) directions, respectively

\( K \)  
quadrilateral vortex strength

\( M_{\infty} \)  
reference mach number

\( \hat{N} \)  
equivalent incompressible surface normal unit vector
\([\hat{N}_x], [\hat{N}_y], [\hat{N}_z]\) \hspace{1cm} \text{equivalent incompressible surface normal unit vector component matrices}

\([N_z], [N_y], [N_z]\) \hspace{1cm} \text{actual surface normal unit vector component matrices}

\(\vec{N}_p\) \hspace{1cm} \text{panel surface normal unit vector}

p \hspace{1cm} \text{roll rate}

q \hspace{1cm} \text{pitch rate}

r \hspace{1cm} \text{yaw rate}

\(R_c, R_c, R_c\) \hspace{1cm} \text{subarea centroid position vector components}

\(R_b\) \hspace{1cm} \text{body radius}

\([R]\) \hspace{1cm} \text{transformation matrix between quadrilateral and horseshoe vortex strengths}

\(\vec{R}\) \hspace{1cm} \text{general position vector}

\(S_B\) \hspace{1cm} \text{body circumferential distance}

\([S_x], [S_y], [S_z]\) \hspace{1cm} \text{source influence matrix components}

\(\vec{T}_M\) \hspace{1cm} \text{equivalent incompressible surface longitudinal tangent unit vector}

\([\vec{T}_M], [\vec{T}_M], [\vec{T}_M]\) \hspace{1cm} \text{equivalent incompressible surface longitudinal tangent unit vector component matrices}

\(\vec{T}_T\) \hspace{1cm} \text{equivalent incompressible surface lateral tangent unit vector}

\([\vec{T}_T], [\vec{T}_T], [\vec{T}_T]\) \hspace{1cm} \text{equivalent incompressible surface lateral tangent unit vector component matrices}

\(\vec{T}_P\) \hspace{1cm} \text{panel surface lateral tangent unit vector}

\([T_{Bi}\] \hspace{1cm} \text{body aerodynamic constraint transformation matrix}
panel aerodynamic constraint transformation matrix

actual surface longitudinal tangent unit vector component matrices

actual surface lateral tangent unit vector component matrices

total velocity in x direction

total velocity in y direction

onset flow components

reference velocity

total velocity in z direction

body width

global coordinates

body coordinates

panel coordinates

points along intersection of panel root section and outboard section

points along intersection of panel root section and a body

center of gravity position vector components

trailing edge of leading edge control surface

leading edge of trailing edge control surface

hinge line location

body multiplication factor in y direction

body multiplication factor in z direction
$Z_c$  
Perpendicular distance between chord line and mean camber line

$Z_t$  
airfoil thickness

**GREEK SYMBOLS**

$\alpha$  
angle of attack

$\beta$  
$\sqrt{1-M^2_{\infty}}$ or angle of yaw

$\gamma$  
vorticity or ratio of specific heats

$\delta$  
control surface deflection

$\Gamma$  
horseshoe vortex strength

$\Delta A_x, \Delta A_y, \Delta A_z$  
directed subareas

$\Delta Y_B$  
displacement of body cross-section in y direction

$\Delta Z_B$  
displacement of body cross-section in z direction

$\Delta S$  
incremental vector tangent to constant percent chord line

$\Delta Y$  
panel surface increment in y direction

$\Delta Z$  
panel surface increment in z direction

$\epsilon$  
panel twist

$\eta$  
fraction of lateral or circumferential distance

$\theta_B$  
body polar coordinate

$\theta$  
general lateral polar coordinate

$\Lambda$  
sweep

$\rho$  
density
source strength

general longitudinal polar coordinate

SUBSCRIPTS

AVG.
average

B
body

C
camber

C_y, C_y, C_Z
centroid components

C.P.
center of pressure

C.G.
center of gravity

edge

f
trailing edge control surface or final

h
hinge line

i
summation index or induced

j
summation index

J
root section and outboard panel juncture

k
summation index

l
lower surface or lift

L.E.
leading edge

m
longitudinal direction

MAX.
maximum

M
longitudinal subpanel number

n
lateral subpanel number

O
origin
P
r
R
S
T
t
T.E.
u
w
X, Y, Z
η
∞

panel
summation index
panel-body juncture
lateral or circumferential direction
lateral direction
airfoil thickness
trailing edge
upper surface or number of longitudinal constraint function
number of lateral constraint function
directions
lateral or circumferential direction
infinity
THEORETICAL DEVELOPMENT

Configuration Representation

The theory discussed in this report is capable of predicting the surface pressures and integrated loads on any configuration which can be synthesized from lifting bodies and thick lifting panels of arbitrary shape. These two basic elements can be attached along longitudinal subpanel edges in order to represent complete airplane configurations of arbitrary shape.

Lifting Body. - The arbitrary lifting body, as shown in figure 1, can be of the solid or flow-through type. The body external surface is divided into a grid, which represents the edges of the body subpanels. The body bound vortex lines are placed at the quarter chord point of the subpanels and the fixed trailing vortex lines along the longitudinal edges of the subpanels. The trailing vortex lines are shed from the aft end of the body and extend in the X direction to infinity unless the free wake option of the program is used, in which case the locations of the trailing vortices aft of the body are determined such that they are force-free. If the body is closed to a point at the aft end, all of the body trailing vortices aft of the body cancel and are, therefore, neglected.

Figure 1.- Arbitrary lifting body.
The body vortex strengths are assumed constant around closed paths defined by the bound vortex lines of two subpanels, adjacent to each other in the longitudinal direction, and that portion of the longitudinal edges of these adjacent panels between the two bound vortex lines. The contribution from the vortex loop, or quadrilateral vortex, is defined as positive when the Biot Savart line integral is taken in the clockwise direction by an observer looking at the external surfaces of the two subpanels. The body subpanels and quadrilateral vortices are shown in figure 1 along with the location of the body control points.

The body control points are located at the three-quarter chord of the body subpanels. At these points the total flow is summed and forced to be a minimum in the direction normal to the body surface. If the body vorticity is not constrained by functions with unknown coefficients, a control point is placed at each subpanel and a discrete solution for the unknown body vortex strengths is obtained. If the body vorticity is constrained, control points are placed as many subpanels as is necessary to obtain a good representation of the body shape and to insure that there is sufficient control of the constraint functions. In this case the unknown coefficients of the constraint functions are determined by the method of least squares.

The body subpanels are further subdivided in the lateral direction so that the vortex grid is mapped to the body surface more accurately. This also allows the use of only one quadrilateral vortex in the lateral direction for the case of a body of revolution in a uniform flow at zero angle of attack. In this case the two side edges of the subpanel are coincident, and therefore, the fixed trailing vortices cancel leaving a longitudinal distribution of ring vortices located at the quarter chord of each subpanel.

Figure 2.- Body subpanel defined by points 1, 4, 5, and 8 with subdivision points 2, 3, 6, and 7.
The coordinates of the body subpanel subdivisions, as shown in figure 2, are used to compute unit vectors tangent to the body surface in both the longitudinal and lateral directions, unit vectors normal to the body surface, directed subareas to be used in the integration of the body surface pressures, and the centroids of the subareas to compute moments about the configuration center of gravity.

The unit vector tangent to the equivalent incompressible body subpanel in the longitudinal direction is given by,

\[ \overline{T}_M = \overline{T}_{M_X} \hat{i} + \overline{T}_{M_Y} \hat{j} + \overline{T}_{M_Z} \hat{k} \]  \hspace{1cm} (1)

\[ \overline{T}_{M_X} = \frac{X_7 - X_2 + X_6 - X_3}{\sqrt{(X_7 - X_2)^2 + \beta(Y_7 - Y_2)^2 + \beta(Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta(Y_6 - Y_3)^2 + \beta(Z_6 - Z_3)^2}} \]  \hspace{1cm} (2)

\[ \overline{T}_{M_Y} = \frac{\beta(Y_7 - Y_2 + Y_6 - Y_3)}{\sqrt{(X_7 - X_2)^2 + \beta(Y_7 - Y_2)^2 + \beta(Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta(Y_6 - Y_3)^2 + \beta(Z_6 - Z_3)^2}} \]  \hspace{1cm} (3)

\[ \overline{T}_{M_Z} = \frac{\beta(Z_7 - Z_2 + Z_6 - Z_3)}{\sqrt{(X_7 - X_2)^2 + \beta(Y_7 - Y_2)^2 + \beta(Z_7 - Z_2)^2 + (X_6 - X_3)^2 + \beta(Y_6 - Y_3)^2 + \beta(Z_6 - Z_3)^2}} \]  \hspace{1cm} (4)

The unit vector tangent to the equivalent incompressible body subpanel in the lateral direction is given by,

\[ \overline{T}_T = \overline{T}_{T_X} \hat{i} + \overline{T}_{T_Y} \hat{j} + \overline{T}_{T_Z} \hat{k} \]  \hspace{1cm} (5)

\[ \overline{T}_{T_X} = \frac{X_3 - X_2 + X_6 - X_7}{\sqrt{(X_3 - X_2)^2 + \beta(Y_3 - Y_2)^2 + \beta(Z_3 - Z_2)^2 + (X_6 - X_7)^2 + \beta(Y_6 - Y_7)^2 + \beta(Z_6 - Z_7)^2}} \]  \hspace{1cm} (6)
\[ \overline{T}_Y = \frac{\beta (Y_3 - Y_2 + Y_6 - Y_7)}{\sqrt{(x_3 - x_2)^2 + \beta^2 (y_3 - y_2)^2 + \beta^2 (z_3 - z_2)^2} + \sqrt{(x_6 - x_7)^2 + \beta^2 (y_6 - y_7)^2 + \beta^2 (z_6 - z_7)^2}} \]  
\[ \overline{T}_Z = \frac{\beta (Z_3 - Z_2 + Z_6 - Z_7)}{\sqrt{(x_3 - x_2)^2 + \beta^2 (y_3 - y_2)^2 + \beta^2 (z_3 - z_2)^2} + \sqrt{(x_6 - x_7)^2 + \beta^2 (y_6 - y_7)^2 + \beta^2 (z_6 - z_7)^2}} \]  

The unit vector normal to the equivalent incompressible body subpanel is given by,

\[ \overline{N} = \overline{N}_X \hat{i} + \overline{N}_Y \hat{j} + \overline{N}_Z \hat{k} \]  

\[ \overline{N}_X = \frac{\overline{T}_M - \overline{T}_Y}{\sqrt{(\overline{T}_M \overline{T}_Z - \overline{T}_M \overline{T}_Y)^2 + (\overline{T}_M \overline{T}_Y - \overline{T}_M \overline{T}_X)^2 + (\overline{T}_M \overline{T}_X - \overline{T}_M \overline{T}_X)^2}} \]  

\[ \overline{N}_Y = \frac{\overline{T}_T - \overline{T}_Z}{\sqrt{(\overline{T}_M \overline{T}_Z - \overline{T}_M \overline{T}_Y)^2 + (\overline{T}_M \overline{T}_Y - \overline{T}_M \overline{T}_X)^2 + (\overline{T}_M \overline{T}_X - \overline{T}_M \overline{T}_X)^2}} \]  

\[ \overline{N}_Z = \frac{\overline{T}_M \overline{T}_Z - \overline{T}_T}{\sqrt{(\overline{T}_M \overline{T}_Z - \overline{T}_M \overline{T}_Y)^2 + (\overline{T}_M \overline{T}_Y - \overline{T}_M \overline{T}_X)^2 + (\overline{T}_M \overline{T}_X - \overline{T}_M \overline{T}_X)^2}} \]
The directed subareas for the middle lateral subdivision for the actual body are given by:

\[ \Delta A_X = \frac{1}{2} [(Y_6-Y_2) (Z_3-Z_7) - (Z_6-Z_2) (Y_3-Y_7)] \]  

\[ \Delta A_Y = \frac{1}{2} [(X_3-X_7) (Z_6-Z_2) - (Z_3-Z_7) (X_6-X_2)] \]  

\[ \Delta A_Z = \frac{1}{2} [(X_6-X_2) (Y_3-Y_7) - (Y_6-Y_2) (X_3-X_7)] \]  

The components of the position vector to the centroid of this subdivision are given by,

\[ R_{Cx} = \frac{1}{4} (X_2 + X_3 + X_6 + X_7) \]  

\[ R_{Cy} = \frac{1}{4} (Y_2 + Y_3 + Y_6 + Y_7) \]  

\[ R_{Cz} = \frac{1}{4} (Z_2 + Z_3 + Z_6 + Z_7) \]

Analogous directed subarea and centroidal position vector expressions are obtained for the other lateral subdivisions.

The body description is defined independent of the subpanel grid and can be input to the program in a number of different ways. Both cartesian and polar coordinates can be used to describe the body cross-sections at a set of chord stations. After the basic cross-sections have been described they can be scaled and then translated in planes perpendicular to the body chord or mean camber line. This provides a means of inputting the correct side and top views.
of an arbitrary body with a minimum of input data to obtain a preliminary estimate of the loads on the body. It also facilitates in inputting the exact shape of some bodies which can be represented by longitudinal segments, over which the cross-sections are mathematically similar. The actual arbitrary shape can be input directly without the use of the scaling and translation options if it is more convenient.

If polar coordinates are used to describe the body cross-sections, the lateral location of body subpanel side edges or fixed trailing vortex lines are also given in terms of angles measured from the local section Z_B axis, in a plane parallel to the (Y_B-Z_B) plane, at an independent set of chordwise stations. These subpanel lateral edges are either specified at a given set of angles or defined to be at equally spaced angle locations. The subpanel longitudinal edges are specified at given X_B stations, evenly spaced in terms of X_B, or evenly spaced in terms of \( \phi_B \), where \( \phi_B = \cos^{-1} (1 - 2 \frac{X_B}{C_B}) \).

The following procedure is used to determine the coordinates \( X_{BE} \), \( (Y_{BE} - \Delta Y_{BE})/Y_M \), \( (Z_{BE} - \Delta Z_{BE})/Z_M \) of the subpanel corners.

1. \( R_{Bi} \) versus \( \theta_{Bi} \) is input at X_{Bi}.

2. \( \theta_{BE} \) (subpanel lateral edge location) versus X_B\( \theta \) is given as input.

3. Interpolation on \( \theta_{BE} \) versus X_B\( \theta \) is done to obtain \( \theta_{BE} \) versus X_BE (subpanel longitudinal edge location) and X_{Bi}.

4. Interpolation on R_{Bi} versus \( \theta_{Bi} \) is done at each X_{Bi} to obtain R_B versus X_{Bi} at each \( \theta_{BE} \).

5. Interpolation on R_B versus X_{Bi} is done to obtain R_{BE} versus X_{BE} at each \( \theta_{BE} \).

6. X_{BE}, \( (Y_{BE} - \Delta Y_{BE})/Y_M \), \( (Z_{BE} - \Delta Z_{BE})/Z_M \) are computed from X_{BE}, R_{BE}, and \( \theta_{BE} \).

If the subpanels are subdivided the coordinates for the corners of these sub-division are computed using the same procedure. There is always an odd number of subdivisions in both the longitudinal and lateral directions.
If cartesian coordinates are used to describe the body cross-sections, the lateral location of the subpanel side edges are defined by the percent circumferential length \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{\text{MAX}_i}}} \) at independent chordwise stations. The lateral location of the subpanel side edges are defined by the percent circumferential length \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{\text{MAX}_i}}} \) at independent chordwise stations.

Tables of \( \frac{(Y_{B_i} - AY_{B_i})}{Y_{B_{\text{MAX}_i}}} \) and \( \frac{(Z_{B_i} - AZ_{B_i})}{Z_{B_{\text{MAX}_i}}} \) versus percent circumferential length \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{\text{MAX}_i}}} \) are developed at the input longitudinal stations \( X_{B_i} \).

The same procedure that was used to obtain \( X_{B_E}, \frac{(Y_{B_E} - AY_{B_E})}{Y_{B_{\text{MAX}_E}}}, \frac{(Z_{B_E} - AZ_{B_E})}{Z_{B_{\text{MAX}_E}}} \) when the body was defined by polar coordinates is also used for this case, except that \( \eta_{B_i} \) is replaced by \( \eta_{B_i} = \frac{S_{B_i}}{S_{B_{\text{MAX}_i}}} \) and \( \theta_{B_i} \) replaced by \( \frac{S_{B_i}}{S_{B_{\text{MAX}_i}}} \). Also, step 6 is unnecessary. The procedure is cycled through twice, first for \( \frac{(Y_{B_E} - AY_{B_E})}{Y_{B_{\text{MAX}_E}}} \) and then for \( \frac{(Z_{B_E} - AZ_{B_E})}{Z_{B_{\text{MAX}_E}}} \).

After \( X_{B_E}, \frac{(Y_{B_E} - AY_{B_E})}{Y_{B_{\text{MAX}_E}}}, \frac{(Z_{B_E} - AZ_{B_E})}{Z_{B_{\text{MAX}_E}}} \) have been computed, \( Y_{B_E} \) is determined by multiplying \( \frac{(Y_{B_E} - AY_{B_E})}{Y_{B_{\text{MAX}_E}}} \) by the multiplication factor \( Y_{B_{\text{MAX}_E}} \) and then adding the translation increment \( AY_{B_E} \). \( Z_{B_E} \) is determined by multiplying \( \frac{(Z_{B_E} - AZ_{B_E})}{Z_{B_{\text{MAX}_E}}} \) by the multiplication factor \( Z_{B_{\text{MAX}_E}} \) and then adding the translation increment \( AZ_{B_E} \).

**Thick lifting panel.** The thick lifting panel, as shown in figure 3, can be warped in any manner laterally, have an arbitrary distribution of chord length, thickness, twist, and camber. It can be used to represent a wing, canard, fin, pylon, horizontal or vertical tail, or be wrapped around to represent a flow through nacelle. The panels can be attached to other panels, such as in the case of a pylon on a wing, or attached to bodies. The panels can have plain leading or trailing edge flaps, ailerons, rudders, or elevators. These control surfaces can be of the full or partial span type and their hinge lines are not restricted to constant percent chord lines.

The panel is divided into two sections; (1) the outboard section where the subpanel longitudinal edges are assumed to be straight lines in the \( X \) direction, and (2) the root section which is a transition region from the outboard section to the intersection of the panel and a body. The line of intersection between the panel and a body does not have to be a straight line. Therefore the subpanel longitudinal edge lines will change in shape from that of the line of intersection at the side of the body to a straight line in the \( X \) direction at the outboard section. If the panel is not attached to a body, there is no root section.
Figure 3.- Thick lifting panel.

The root section and outboard section can have any distribution of subpanel lengths and widths. The subpanel lateral edges are specified as a list of percent chord stations, evenly spaced in terms of percent chord, or evenly spaced in terms of $\phi$, where

$$\phi = \cos^{-1} \left[ 1 - 2 \left( \frac{X - X_{LE}}{C} \right) \right].$$

The subpanel side edges are specified as a list of percent of surface semi-span $\eta$, evenly spaced in terms of percent of surface semi-span, or evenly spaced in terms of $\theta$, where $\theta = \cos^{-1} \eta$. 
The percent of surface semi-span $\eta$ is defined as the percent of length of the line projected into the (y-Z) plane by the panel leading edge. A constant $\eta$ line is in general curved in the root section. In the outboard section, lines of constant $\eta$ are straight and in the X direction. The curved constant $\eta$ lines associated with the longitudinal edges of subpanels in the root section are computed such that the sweep of the leading and trailing edges of the subpanels vary linearly from that of the sweep of the panel at the leading edge to that of the sweep of the panel at the trailing edge. Also, the corner points of the subpanels are equally spaced in the lateral direction along lines of constant percent chord. The chord at a lateral station in the root section is the length of the curved constant $\eta$ line at that station.

If the lateral distance along a constant percent chord line between two corner points is defined by $|\Delta S|$, the unit vector tangent to the leading or trailing edge of a subpanel at any percent chord station is given by:

$$\left(\frac{\Delta S}{|\Delta S|}\right)_k = \frac{\Delta S_{L.E.} K}{|\Delta S_{L.E.} K|} + \left(\frac{\Delta S_{T.E.} K}{|\Delta S_{T.E.} K|} - \frac{\Delta S_{L.E.} K}{|\Delta S_{L.E.} K|}\right)_{(percent chord)}$$

(19)

where

$$|\Delta S_K| = \frac{c_s^2}{\sum_{k=1}^{kJ} \left(\frac{\Delta S}{|\Delta S|}\right)_k \cdot c_s}$$

(20)
and \( K_J \) equals the number of subpanels in the lateral direction in the root section.

\[
\vec{C}_S = (X_J - X_R) \hat{i} + (Y_J - Y_R) \hat{j} + (Z_J - Z_R) \hat{k}
\]  

(21)

where \((X_R, Y_R, Z_R)\) and \((X_J, Y_J, Z_J)\) are the points on the curved constant percent chord line at the line of intersection of the panel and the body and at the juncture of the root and outboard sections, respectively. The equally spaced corner points along a constant percent chord line are then computed by

\[
R (X,Y,Z) \eta = R (X_R,Y_R,Z_R) + \sum_{K=1}^{K=K_\eta} \left( \frac{\Delta S}{|\Delta S|} \right)_K |\Delta S_K|
\]  

(22)

where \( K_\eta \) is the number of subpanels, in the lateral direction, the constant \( \eta \) line is located from the line of intersection of the panel and the body. Since constant \( \eta \) lines in the outboard section are straight and in the \( X \) direction the subpanel corner points are defined directly from the input list of subpanel edge locations in terms of percent of chord and lateral \( \eta \) station.

The coordinates of the leading edge point \((X_{L,E.}, Y_{L,E.}, Z_{L,E.})_K\) associated with a constant \( \eta \) line are obtained from the panel perimeter description by converting the leading edge input points to tables of \( X_{L,E.i} \) versus \( \eta \), \( Y_{L,E.i} \) versus \( \eta \), and \( Z_{L,E.i} \) versus \( \eta \), and then interpolating in these tables to determine the leading edge coordinates at a desired lateral \( \eta \) station. Unit vectors tangent and normal to the panel at any \( \eta \) station can also be determined from these tables by evaluating the \( \Delta Y_{L,E.} \) and \( \Delta Z_{L,E.} \) about the lateral station \( \eta \). The unit vectors are then defined by;

\[
\vec{N}_{P,K} = -\frac{\Delta Z_{L,E.}}{\sqrt{\Delta Y_{L,E.}^2 + \Delta Z_{L,E.}^2}} \hat{j} + \frac{\Delta Y_{L,E.}}{\sqrt{\Delta Y_{L,E.}^2 + \Delta Z_{L,E.}^2}} \hat{k}
\]  

(23)
With the coordinates of the subpanel corner points known, the coordinates and sweep of the vortex and source lattices used to represent the perturbation velocities due to lift and thickness, respectively, can be defined. The bound vortex lines and control points are placed at the quarter and three-quarter chord points of the subpanels, respectively. The fixed trailing vortices are placed along the subpanel side edges. In the root section the vortex lattice is a quadrilateral system, whereas in the outboard section the vortex lattice is a skewed horseshoe system. The skewed source lines are placed at both the quarter and three-quarter chord points of the subpanels.

The section geometry is input in terms of a percent thickness, percent camber, and twist. Where the percent thickness and camber are based on the local chord and measured in the $\tilde{N}_{PK}$ direction. Twist is defined in the plane described by the unit vectors $\hat{i}$ and $\tilde{N}_{PK}$. The $X$ component of the unit vector normal to the section mean camber line and in the plane defined by $\hat{i}$ and $\tilde{N}_{PK}$ is then given by;

$$
\tilde{N}_{PK} = \beta \left(-\frac{dZ_C}{dx} + \epsilon + \tan \delta \right)
$$

where $Z_C$ is the perpendicular distance between the section chord line and the mean camber line, $\epsilon$ is the angle of twist, and $\delta$ the deflection of any control surface. All three of these quantities are functions of both percent chord and $\eta$. The thickness is defined in the same manner as camber.

The trailing vortices aft of the trailing edge of the outboard section are straight lines in the $X$ direction going off to infinity. The trailing vortices aft of the trailing edge of the root section lie along curved constant $\eta$ lines to the end of the body and then go off to infinity in the $X$ direction. These curved constant $\eta$ lines are determined in the same manner as the constant $\eta$ lines in the root section. If the free wake option of the program is utilized the location of the panel free trailing vortices are iterated for such that the wake is force free.
Discrete Influence Equations

The perturbation velocity due to the arbitrary lifting bodies is represented by quadrilateral vortices on the external surface of the bodies. The perturbation velocity due to lift on the panels is represented by quadrilateral vortices in the root section of the panel and by skewed horseshoe vortices in the outboard section of the panel. The thickness is represented by skewed source lines in both the root and outboard sections of the panel. All the panel singularities are placed on the panel chordal surface. The source strengths $\Sigma$ are defined by the change in thickness over that portion of the subpanel it represents, so that:

$$\frac{\Sigma}{V_\infty} = \frac{2 \beta \Delta Z_t}{\sqrt{1 + (\tan \Lambda)^2/\beta^2}} \frac{\vec{V}_X}{V_\infty}$$

(26)

where $\Lambda$ is the sweep of the source line and $\vec{V}_X$ is the total onset velocity in the $X$ direction. The quadrilateral vortex strengths $K$ and the skewed horseshoe vortex strengths $F_{\text{st}}$ must be solved for utilizing the boundary condition that a minimum of flow passes through the external surface of the bodies and the chordal surface of the thick lifting panels at a finite number of control points. In order to satisfy this boundary condition the total flow due to all singularities and onset flow is summed at each control point and the scalar product of this sum and the surface unit normal is minimized. This results in a set of linear aerodynamic influence equations which are solved for the unknown vortex strengths by means of Householder's method, described in appendix A.
The influence equations for the \( j \) th equivalent incompressible body are given by:

\[
\sum_{i=1}^{N_B} \left[ \begin{array}{c}
-N_{x_{B_j}} A_{y_{B_i}} \\
-N_{y_{B_j}} A_{z_{B_i}} \\
-N_{z_{B_j}} A_{x_{B_i}}
\end{array} \right] \frac{K}{V_\infty} + \sum_{K=1}^{N_P} \left[ \begin{array}{c}
-N_{x_{B_j}} A_{z_{B_i}} \\
-N_{y_{B_j}} A_{z_{B_i}} \\
-N_{z_{B_j}} A_{z_{B_i}}
\end{array} \right] \frac{\Gamma}{V_\infty} = e_{B_j}
\]

where

\( N_{x_{B_j}}, N_{y_{B_j}}, \text{ and } N_{z_{B_j}} \) are the components of the \( j \) th equivalent incompressible body surface unit normal vectors,

\( A_{x_{B_i}}, A_{y_{B_i}}, \text{ and } A_{z_{B_i}} \) are the components of the perturbation velocity induced by the \( i \) th equivalent incompressible body unit strength vortices onto the \( j \) th equivalent incompressible body,

\( A_{x_{B_i}} P_{B_i}, A_{y_{B_i}} P_{B_i}, \text{ and } A_{z_{B_i}} P_{B_i} \) are the components of the perturbation velocity induced by the \( k \) th equivalent incompressible panel unit strength vortices onto the \( j \) th equivalent incompressible body, and

\( \Sigma \) and \( \Gamma \) are the surface and panel strength coefficients.
are the components of the perturbation velocity induced by the kth equivalent incompressible panel unit strength sources onto the jth equivalent incompressible body.

Also,

\[
\left\{ \frac{\vec{V}_X}{V_\infty} \right\}, \left\{ \frac{\vec{V}_Y}{V_\infty} \right\}, \text{ and } \left\{ \frac{\vec{V}_Z}{V_\infty} \right\}
\]

are the components of the onset flow divided by the reference velocity \( V_\infty \).

\( \{K\}_i \) are the quadrilateral vortex strengths on the i th equivalent incompressible body, and

\( \{K\}_p, \{\Gamma\}_p, \text{ and } \{\Sigma\}_p \) are the root section quadrilateral vortex strengths, the outboard section skewed vortex strengths, and the skewed source strengths on the kth equivalent incompressible panel, respectively. The vector \( \{e\}_i \) is the flow through the surface of the jth equivalent incompressible body at the control points, for a discrete body solution this vector is zero.

The influence equations for the jth equivalent incompressible panel are given by:

\[
\begin{align*}
\sum_{i=1}^{N_B} & \left[ \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} A_{X_i,j} \\ A_{Y_i,j} \\ A_{Z_i,j} \end{bmatrix} + \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} A_{X_i,k} \\ A_{Y_i,k} \\ A_{Z_i,k} \end{bmatrix} + \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} A_{X_i,j} \\ A_{Y_i,j} \\ A_{Z_i,j} \end{bmatrix} \right] \left\{ \frac{K}{V_\infty} \right\}_i \\
+ & \sum_{k=1}^{N_p} \left[ \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} S_{X_i,k} \\ S_{Y_i,k} \\ S_{Z_i,k} \end{bmatrix} + \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} S_{X_i,k} \\ S_{Y_i,k} \\ S_{Z_i,k} \end{bmatrix} + \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} S_{X_i,k} \\ S_{Y_i,k} \\ S_{Z_i,k} \end{bmatrix} \right] \left\{ \frac{\Gamma}{V_\infty} \right\}_k \\
+ & \sum_{k=1}^{N_p} \left[ \begin{bmatrix} \vec{N}_{X_i} \\ \vec{N}_{Y_i} \\ \vec{N}_{Z_i} \end{bmatrix} \begin{bmatrix} \vec{V}_X \\ \vec{V}_Y \\ \vec{V}_Z \end{bmatrix} \right] = \{e\}_j \\
\end{align*}
\]

(28)
Where

\( \mathbf{N}_j \), \( \mathbf{N}_j \), and \( \mathbf{N}_j \) are the components of the \( j \)th equivalent incompressible panel mean camber surface unit normal vectors,

\( \mathbf{A}_j \), \( \mathbf{A}_j \), and \( \mathbf{A}_j \) are the components of the perturbation velocity induced by the \( i \)th equivalent incompressible body unit strength vortices onto the \( j \)th equivalent incompressible panel,

\( \mathbf{A}_j \), \( \mathbf{A}_j \), and \( \mathbf{A}_j \) are the components of the perturbation velocity induced by the \( k \)th equivalent incompressible panel unit strength vortices onto the \( j \)th equivalent incompressible panel, and

\( \mathbf{S}_j \), \( \mathbf{S}_j \), and \( \mathbf{S}_j \) are the components of perturbation velocity induced by the \( k \)th equivalent incompressible panel unit strength sources onto the \( j \)th equivalent incompressible panel. The vector \( \mathbf{c}_{ij} \) is the flow through the mean camber surface of the \( j \)th equivalent incompressible panel at the control points. This vector is also zero for a discrete panel solution.

The onset flow velocity ratios

\( \frac{V_x}{V_\infty}, \frac{V_y}{V_\infty}, \) and \( \frac{V_z}{V_\infty} \) are given by the following expressions,

\[
\frac{V_x}{V_\infty} = 1 - q^* \frac{2\beta (Z-Z_{C.G.})}{c} - \gamma^* \frac{2\beta (Y-Y_{C.G.})}{b} \\
\frac{V_y}{V_\infty} = -\beta \beta - p^* \frac{2\beta (Z-Z_{C.G.})}{b} + \gamma^* \frac{2(X-X_{C.G.})}{b} \\
\frac{V_z}{V_\infty} = \alpha \beta + p^* \frac{2\beta (Y-Y_{C.G.})}{b} + q^* \frac{2(X-X_{C.G.})}{c}
\]  

(29) (30) (31)

Where \( p^*, q^*, \) and \( \gamma^* \) are the nondimensional roll, pitch, and yaw rates, respectively. These are defined as;
\[ p^* = p/\frac{2V}{b}, \quad q^* = q/\frac{2V}{c}, \quad \text{and} \quad \gamma^* = \gamma/\frac{2V}{b} \]  

(32)

The above onset flow equations assume that angle of attack \( \alpha \) and angle of yaw \( \beta \) are small. The coordinates \( X_{C.G.}, Y_{C.G.}, \) and \( Z_{C.G.} \) define the location of the center of gravity.

Equations (27) and (28) can be combined into a single matrix aerodynamic influence equation. For a completely discrete type solution the influence equation is,

\[
\begin{bmatrix}
[A]_{B_i B_j} & [A]_{B_j P_k} \\
[A]_{P_j B_i} & [A]_{P_j P_k}
\end{bmatrix}
\begin{bmatrix}
[K]_B \\
[K]_P
\end{bmatrix}
= \begin{bmatrix}
\{\nabla S\}_B_i \\
\{\nabla S\}_P_k
\end{bmatrix}
\]

(33)

where

\( B_j = B_1, B_2, \ldots, B_{N_B} \)

\( B_i = B_1, B_2, \ldots, B_{N_B} \)

\( P_j = P_1, P_2, \ldots, P_{N_P} \)

\( P_k = P_1, P_2, \ldots, P_{N_P} \)

\( N_B = \text{Number of bodies} \)

and

\( N_P = \text{Number of panels} \)
The matrices

$$[A]_{B_j B_i}, [A]_{B_j P_k}, [A]_{P_j B_i}, \text{ and } [A]_{P_j P_k}$$

are defined as:

$$[A]_{B_j B_i} = \begin{bmatrix} \bar{N}_{X_{B_j}} & A_{X_{B_j B_i}} & \bar{N}_{Y_{B_j}} & A_{Y_{B_j B_i}} & \bar{N}_{Z_{B_j}} & A_{Z_{B_j B_i}} \end{bmatrix}$$ (34)

$$[A]_{B_j P_k} = \begin{bmatrix} \bar{N}_{X_{B_j}} & A_{X_{B_j P_k}} & \bar{N}_{Y_{B_j}} & A_{Y_{B_j P_k}} & \bar{N}_{Z_{B_j}} & A_{Z_{B_j P_k}} \end{bmatrix}$$ (35)

$$[A]_{P_j B_i} = \begin{bmatrix} \bar{N}_{X_{P_j}} & A_{X_{P_j B_i}} & \bar{N}_{Y_{P_j}} & A_{Y_{P_j B_i}} & \bar{N}_{Z_{P_j}} & A_{Z_{P_j B_i}} \end{bmatrix}$$ (36)

$$[A]_{P_j P_k} = \begin{bmatrix} \bar{N}_{X_{P_j}} & A_{X_{P_j P_k}} & \bar{N}_{Y_{P_j}} & A_{Y_{P_j P_k}} & \bar{N}_{Z_{P_j}} & A_{Z_{P_j P_k}} \end{bmatrix}$$ (37)

The known quantities in the influence equation $\{\nabla S\}_{B_j}$ and $\{\nabla S\}_{P_j}$ are defined as:

$$\{\nabla S\}_{B_j} = - \sum_{k=1}^{N_P} \begin{bmatrix} \bar{N}_{X_{B_j}} & S_{X_{B_j P_k}} \end{bmatrix} + \begin{bmatrix} \bar{N}_{Y_{B_j}} & S_{Y_{B_j P_k}} \end{bmatrix}$$

$$+ \begin{bmatrix} \bar{N}_{Z_{B_j}} & S_{Z_{B_j P_k}} \end{bmatrix} \begin{bmatrix} \Sigma \end{bmatrix} - \begin{bmatrix} \bar{N}_{X_{B_j}} \end{bmatrix} \begin{bmatrix} \frac{V_X}{V_\infty} \end{bmatrix}$$

$$- \begin{bmatrix} \bar{N}_{Y_{B_j}} \end{bmatrix} \begin{bmatrix} \frac{V_Y}{V_\infty} \end{bmatrix} - \begin{bmatrix} \bar{N}_{Z_{B_j}} \end{bmatrix} \begin{bmatrix} \frac{V_Z}{V_\infty} \end{bmatrix}$$ (38)
The elements of the above matrices are computed using the influence equations derived in Appendices B and C. Each element is associated with the influence of a singularity on a control point. The singularities and control points are ordered such that all of the longitudinal stations for the first lateral station are cycled through first and then all of the longitudinal stations for the second lateral station. This process is continued until all stations have been cycled through. The longitudinal stations start at the leading edge of the panel, the nose of a solid body, and the tail end of the inside surface of a flow through body. The lateral stations start at the inside edge of the panel and go toward the tip of the panel. The lateral stations on the body start at the top of the body and progress in a clockwise direction when looking at the body from the tail to the nose.

All bodies are cycled through first and then the panels. Both the bodies and the panels are cycled in the order that they are input. The columns of the influence matrices are associated with singularities and the rows with control points. The elements of the matrices and the submatrices of the combined matrix influence equation are sequenced, for both the columns and the rows, in the same order that the singularities, points, bodies, and panels are cycled.

The influences of quadrilateral vortices in the panel root section and on the body surface are computed by equations derived in Appendix C. The influences of skewed horseshoe vortices on the panel outboard section are computed by equations derived in Appendix B. The influences of skewed source lines on the panel are computed by equations derived in Appendix B.

If the force free wake option of the program is used the influences of the free trailing vortices are computed using the influence equations in Appendix C. The locations of the force free trailing vortices are computed using the following iteration procedure.
1. A solution for the vortex strengths will be obtained first by assuming the location of the free vortices to be in the longitudinal direction and to be straight except in a thick lifting panel-body juncture region.

2. The total velocities $U$, $V$, and $W$ in the $X$, $Y$, and $Z$ directions, respectively, will be computed at the midpoint of each free trailing vortex division.

3. The actual mean camber surfaces of the thick lifting panels will be computed so that the correct relationship between fixed vortices and control points on the thick lifting panels and the free trailing vortices is maintained.

4. The ratios $V/U$ and $W/U$ are integrated in the $X$ direction from the edges of the actual thick lifting panel mean camber surfaces and body aft ends where the free vortices are assumed to be shed, in order to obtain their new locations.

5. The influence of the free vortices on the actual thick lifting panel mean camber and body surfaces is computed and the difference between this influence and that from the free vortices at their previous location is added to the influence matrices for the complete configuration.

6. A new solution for the vortex strengths is determined and a new set of total velocities $U$, $V$, and $W$ along the new free vortex lines is computed using quadrilateral vortices and source lines on the actual thick lifting panel mean camber surfaces and quadrilateral vortices on the body surfaces.

The above procedure is iterated between steps (4) and (6) until the surface pressures converge. When the influence matrix of any individual body or panel is no longer significantly changed due to a new positioning of the free vortices, the calculation of the perturbation to that matrix is terminated.

Constrained Influence Equations

The vortex strengths on any of the bodies or panels can be constrained by representing the vorticity by a finite series. This uncouples the number of unknowns, equations, and vortices used to represent the perturbation velocity. The series used must be capable of producing the type of perturbation velocity needed to satisfy the boundary conditions at the control points to an acceptable degree. If such a series can be found for a given body or panel the number of equations or control points can be reduced from that of the number of
vortices or subpanels to that necessary to describe the geometry. This substantially reduces the computer time, since in general the number of vortices needed to represent the perturbation velocity is far more than the number of control points needed to represent the geometry of the body or panel.

This reduction in the number of required control points results primarily in reducing the computational effort necessary to set up the aerodynamic influence equations. A further reduction in computer time can be realized in the solution of the influence equations, if constraint functions are used, since the number of unknowns is reduced from that of the number of vortices to the number of terms in the constraint series. This results in a system of equations which is overdetermined and is solved by the method of least squares. In this process the vectors \( \{e\}_B \) and \( \{e\}_p \) are minimized. The body constraint equation is given by:

\[
\begin{pmatrix}
\frac{k}{V_\infty} \\
\frac{V_\infty}{k}
\end{pmatrix}
B_i = \begin{bmatrix}
T_{B_i}
\end{bmatrix}
\begin{bmatrix}
a_B
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
T_{B_i}
\end{bmatrix} = \begin{bmatrix}
R
\end{bmatrix} \begin{bmatrix}
\mathcal{M}_{mn} \\
\gamma
\end{bmatrix} \begin{bmatrix}
\frac{V_\infty}{V_\infty}
mn
\end{bmatrix}
\]

The matrix \([R]\) is a transformation matrix used to obtain the quadrilateral vortex strengths \( K \) from the bound horseshoe vortex strengths \( \Gamma_B \).

\[
\begin{bmatrix}
R
\end{bmatrix} = \begin{bmatrix}
[R]_n
\end{bmatrix}
\]

where \([R]_n\) is made up of ones along the diagonal and in the lower triangle. The upper triangle is filled with zeros.

The indices \( m \) and \( n \) refer to longitudinal and lateral subpanel locations on the equivalent incompressible body, respectively. The vorticity ratio \( (V/V_\infty)_{mn} \) is the value of the vorticity series at the \( m \)th subpanel from the nose of the body and the \( n \)th subpanel in the lateral direction around the body. The longitudinal length \( \bar{L}_{mn} \) is given by:

\[
\bar{L}_{mn}' = \frac{1}{2} \left[ \frac{3}{4} \bar{L}_{M(m-1)n} + \bar{L}_{Mmn} + \frac{1}{4} \bar{L}_{M(m+1)n} \right]
\]
where $\overline{I}_m^{\text{mn}}$ is the length of the $m^{th}$ subpanel from the nose of the equivalent incompressible body at the $n^{th}$ lateral station.

The vorticity series $(\gamma/V_\infty)_{mn}$ is the product of a longitudinal series and a lateral series. The elements of the matrix $\{a_B\}$ are the unknown coefficients associated with the terms in $(\gamma/V_\infty)_{mn}$ produced by the product of the longitudinal and lateral series. The terms in $(\gamma/V_\infty)_{mn}$ are ordered such that the products of all of the longitudinal constraint functions and the first lateral constraint function are first, then the products of all of the longitudinal constraint functions and the second lateral constraint functions are second, and etcetera. This process is continued until all combinations of longitudinal and lateral constraint functions have been cycled through.

Both the longitudinal and lateral constraint functions are defined over segments of the body. The same functions are used in all segments. The origin of the segment is designated by the subscript $o$ and the end by $f$. The longitudinal and lateral constraint function segments are given in the data input array.

The longitudinal constraint functions for the body are defined as:

\[
\frac{\gamma}{V_\infty} (X_B/C_B)_1 = 1
\]

\[
\frac{\gamma}{V_\infty} (X_B/C_B)_2 = \frac{1}{\left\{ 1 + \left[ Y_B\left( \frac{N_X}{N_Y}\right) + Z_B\left( \frac{N_X}{N_Z}\right) \right]^2 \right\}^{1/2}}
\]

\[
\frac{\gamma}{V_\infty} (\phi_B)_3 = \cot \frac{\phi_B}{2}
\]

\[
\frac{\gamma}{V_\infty} (\phi_B)_4 = \cot \left( \frac{\pi}{2} - \frac{\phi_B}{2} \right)
\]

\[
\frac{\gamma}{V_\infty} (\phi_B)_5 = \sin \left[ \pi \left( \frac{\phi_B}{\phi_f} - \phi_o \right) \right]
\]

\[
\frac{\gamma}{V_\infty} (\phi_B)_6 = \cos \left[ \pi \left( \frac{\phi_B}{\phi_f} - \phi_o \right) \right]
\]
\[
\frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{7} = \begin{bmatrix}
\frac{X_B}{C_B} \times \frac{X_B}{C_B} \\
\frac{X_B}{C_B} \times \frac{X_B}{C_B} \\
\frac{X_B}{C_B} \times \frac{X_B}{C_B} \\
\end{bmatrix}
\]

(50)

\[
\frac{\gamma}{V_\infty} (\phi_B)_{8} = \sin \left[ 2\pi \left( \phi_B - \phi_0 \right) \right]
\]

(51)

\[
\frac{\gamma}{V_\infty} (\phi_B)_{9} = \cos \left[ 2\pi \left( \phi_B - \phi_0 \right) \right]
\]

(52)

\[
\frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{10} = \left[ \left( \frac{X_B}{C_B} \right) - \left( \frac{X_B}{C_B} \right) \right]^2
\]

(53)

\[
\frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{3270} = L_{S_1} \left( \frac{X_B}{C_B} \right)
\]

(54)

\[
\frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{3320} = L_{S_2} \left( \frac{X_B}{C_B} \right)
\]

(55)

\[
\frac{\gamma}{V_\infty} \left( \frac{X_B}{C_B} \right)_{3370} = L_{S_2} \left( \frac{X_B}{C_B} \right)
\]

(56)
The functions to be used on the body are designated by their number in the above sequence. The last three functions listed above are special functions and are input at the locations in the data array, designated by their subscripts. The independent variable $\phi_B$ is given by, $\phi_B = \cos^{-1} [1 - 2i(N_a/C_a)]$

The lateral constraint functions for the body are defined as:

\[
\frac{\gamma}{V_\infty} (\theta_B)_1 = 1
\]

\[
\frac{\gamma}{V_\infty} (\theta_B)_2 = \frac{N_{\theta}}{N_B} \sqrt{N_{\phi}^2 + N_{\chi}^2}
\]

\[
\frac{\gamma}{V_\infty} (\theta_B)_3 = \frac{N_{\chi}}{N_B} \sqrt{N_{\phi}^2 + N_{\chi}^2}
\]

\[
\frac{\gamma}{V_\infty} (\theta_B)_4 = \sin \left[ \frac{\pi (\theta_B - \theta_0)}{\theta_f - \theta_o} \right]
\]

\[
\frac{\gamma}{V_\infty} (\theta_B)_5 = \cos \left[ \frac{\pi (\theta_B - \theta_0)}{\theta_f - \theta_o} \right]
\]

\[
\frac{\gamma}{V_\infty} (\theta_B)_6 = \left[ \frac{\pi (\theta_B - \theta_0)}{\theta_f - \theta_o} \right] \text{ or } \left[ \frac{\eta_B - \eta_0}{\gamma_f - \eta_o} \right]
\]
Here again, the lateral functions to be used on the body are designated by their number in the above sequence. If no constraint functions are specified the program will do a discrete solution.

The panel constraint equation is given by:

\[
\begin{bmatrix}
\frac{K}{V_\infty} \\
\frac{\Gamma}{V_\infty}
\end{bmatrix}
\begin{bmatrix}
P_K
\end{bmatrix}
= 
\begin{bmatrix}
T_{pK}
\end{bmatrix}
\begin{bmatrix}
ap_K
\end{bmatrix}
\]  

(65)

The \([T_{pK}]\) transformation matrix condenses all rows of the discrete aerodynamic influence matrix by the following procedure.

1. That portion of the row which is associated with the vortices on panel \(P_K\) is divided into the elements due to each lateral station. A new temporary matrix \([A_T]\) is developed with all of the elements due to the vortices at the first lateral station in row one, all of the elements due to the vortices at the second lateral station in row two, and etcetera.
2. This temporary matrix is then premultiplied by the spanwise constraint function transformation matrix \([T(\eta)]\) and postmultiplied by the chordwise constraint function transformation matrix \([T(\eta^{\prime})]\).

3. The transformed temporary matrix \([A_T] = [T(\eta)][A_T][T(\eta^{\prime})]\) is then opened up into a row again and replaces the old row in the original discrete aerodynamic influence matrix. The new row is formed by placing all of the elements from the first row of \([A_T]\) in the portion of the row, from the discrete matrix, due to the \(P_K\) panel first, then by placing all of the elements from the second row of \([A_T]\) next to those elements from the first row of \([A_T]\), and et cetera. The result will be the reduction of the number of elements in a given row due to the \(P_K\) panel from that of the number of vortices or subpanels on panel \(P_K\) to the number of chordwise constant functions times the number of spanwise constraint functions.

If only the chordwise constraint functions are used, then the number of elements in the row due to the \(P_K\) panel will be reduced from the number of vortices on the \(P_K\) panel to the number of chordwise constraint functions times the number of subpanels in the lateral direction. On panels with freestream edges, at span stations other than the tip, only the chordwise functions should be used, since the available spanwise constraint functions are not sufficient to produce the necessary perturbation velocity to satisfy the boundary conditions on a panel of this type.

The above described transformation constrains the skewed horseshoe vortex strengths by the following series developed in appendices F. and G. of reference (26).

\[
\frac{\Gamma}{V_k} (\eta) = \sqrt{1 - \eta^2} \sum_{w = 1}^{N_W - P_W} \left\{ \begin{array}{cc}
\left( \frac{\Gamma_{om}}{V_{\infty}} \right) a_{ow} + \sum_{f = 1}^{N_f} \left( \frac{\Gamma_{fm}}{V_{\infty}} \right) a_{fw} + \sum_{k = 1}^{N_k} \left( \frac{\Gamma_{km}}{V_{\infty}} \right) a_{kw} \\
+ \sum_{n = 1}^{N_u} \left( \frac{\Gamma_{nm}}{V_{\infty}} \right) a_{nw} \end{array} \right\} \eta^w + \sum_{w = (N_W - P_W + 1)}^{N_W} \left\{ \begin{array}{cc}
\left( \frac{\Gamma_{om}}{V_{\infty}} \right) a_{ow} \end{array} \right\}
\]

\[
+ \sum_{f = 1}^{N_f} \left( \frac{\Gamma_{fm}}{V_{\infty}} \right) a_{fw} + \sum_{k = 1}^{N_k} \left( \frac{\Gamma_{km}}{V_{\infty}} \right) a_{kw} + \sum_{n = 1}^{N_u - 1} \left( \frac{\Gamma_{nm}}{V_{\infty}} \right) a_{nw} \right \} P_w (\eta) \]

(66)
Where the indices \( m, f, k, o, h, \) and \( w \) indicate the number of the subpanel aft of the panel leading edge \( \Gamma / V_\infty \) is defined on, the number of the trailing edge flap, the number of the leading edge flap, the first term of the Birnbaum series, the number of the sine term in the Birnbaum series, and the number of the spanwise constraint function, respectively. The quantities \( N_{ul}, N_{f}, N_{k}, P_{w}, \) and \( N_{w} \) are the number of terms in the Birnbaum series, the number of trailing edge flaps with unique hinge line locations, the number of leading edge flaps with unique hinge line locations, the number of special spanwise constraint functions, and the total number of spanwise constraint functions, respectively.

The columns of the matrix \([T(X/C)]\) are made up of the \((\Gamma'/V_\infty)_m\) values, where \( m \) indicates the number of the element in the column. There is one column for each set of \((\Gamma'/V_\infty)_m\)'s associated with a unique chordwise constraint function. The \([T(X/C)]\) matrix is filled such that the \((\Gamma'_om/V_\infty)\) values are in column one, then the \((\Gamma'_on/V_\infty)\) values are in the next set of \(N_{ul}-1\) columns, then the \((\Gamma'_fm/V_\infty)\) values are in the next set of \(N_f\) columns, and then the \((\Gamma'_km/V_\infty)\) values are in the next set of \(N_k\) columns.

The \([T(X/C)]\) matrix is solved here in the same manner as is shown in appendix G. of reference (26). The \((\Gamma'/V_\infty)_m\) values are solved for such that the same downwash is obtained at the three quarter chord point of each subpanel, except at the last subpanel, due to a distribution of discrete vortices as would be obtained by integrating the vorticity distribution in the Blot-Savart integral for each chordwise constraint function. The additional condition, that the sum of the discrete vortex strengths must be equal to the integral of the vorticity distribution is also used. These conditions result in the following matrix equation which is solved for \([T(x/c)]\).

\[
[E] [T(x/c)] = [W] \tag{67}
\]

The matrix \([E]\) is the two-dimensional discrete vortex influence matrix and is defined as follows.

\[
[E] = \begin{bmatrix}
\frac{1}{[(x/c)_j - (x/c)_i]} \\
1, 1, \ldots, 1
\end{bmatrix} \tag{68}
\]
Where 0 ≤ (x/c) ≤ 1.0 and the indices j and i as used here indicate the location of the three quarter and quarter chord points, respectively.

There is one column in the [w] matrix for each unique chordwise constraint function. In these columns is the downwash due to evaluating the vorticity for each chordwise constraint function in the two-dimensional Biot-Savart integral. The columns in [w] are ordered such that the column due to a given chordwise constraint function is in the same location in [w] as its corresponding (T'/Y=0)_m column is in the [T(x/c)] matrix. The five basic types of columns in [w] are defined as follows.

Due to the first term of the Birnbaum series;

\[
\begin{align*}
\{ w \} \begin{pmatrix} \cot \phi/2 \end{pmatrix} &= \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ 1/2 \end{pmatrix} \\
\end{align*}
\]

Due to the second term of the Birnbaum series;

\[
\begin{align*}
\{ w \} \begin{pmatrix} \sin \phi \\ -\cos \phi_1 \\ -\cos \phi_2 \\ \vdots \\ -\cos \phi(N_1-1) \\ \vdots \\ 1/4 \end{pmatrix} &= \begin{pmatrix} \end{align*}
\]

\[
\begin{align*}
\end{align*}
\]
Due to the third and higher order terms of the Birnbaum series:

\[
\begin{vmatrix}
\cos n \phi_1 \\
\cos n \phi_2 \\
\vdots \\
\cos n \phi_{(N_1-1)} \\
\end{vmatrix}
\begin{vmatrix}
w \\
\sin(n\phi) \\
\end{vmatrix}
\]

(71)

where \( \phi_j = \cos^{-1} \left[ 1 - 2 \left( \frac{x}{c} \right)_j \right] \)

Due to the trailing edge flap term:

\[
\begin{vmatrix}
w_f(\phi_1) \\
w_f(\phi_2) \\
\vdots \\
w_f(\phi_{(N_1-1)}) \\
\end{vmatrix}
\begin{vmatrix}
\log \left| \frac{\sin \frac{1}{2}(\phi + \phi_f)}{\sin \frac{1}{2}(\phi - \phi_f)} \right| \\
\end{vmatrix}
\]

(72)

where \( w_f(\phi_j) = -(\pi - \phi_f) \) for \( 0 \leq \phi_j < \phi_f \), \( w_f(\phi_j) = \phi_f \) for \( \phi_f < \phi_j \leq \pi \),

is the polar coordinate at the flap leading edge, and \( N_1 \) equals the number of subpanels per chord.
due to the leading edge flap term;

\[
\left\{ \begin{array}{c}
w_k(\phi_i) \\
w_k(\phi_j) \\
\vdots \\
w_k(\phi_{N_w}) \\
\end{array} \right\} = \begin{bmatrix}
w_k(\phi_1) \\
w_k(\phi_2) \\
\vdots \\
w_k(\phi_{N_w-1}) \\
1/2 \sin \phi_k
\end{bmatrix}
\]

(73)

where \( w_k(\phi) = -(\pi - \phi) \) for \( 0 \leq \phi \leq \phi_k \), \( w_k(\phi) = \phi \) for \( \phi_k < \phi \leq \pi \), and \( \phi_k \) is the polar coordinate at the flap leading edge.

In the root section the \([T \, X/C]\) matrix is transformed to \([\overline{T} \, (X/C)]\), where each column of \([T \, X/C]\) is transformed as follows;

\[
\overline{T} \, (X/C)_M = \sum_{i=1}^{m} T \, (X/C)_i
\]

(74)

where \( i \) and \( m \) indicate the number of the element in the column in the \([T \, (X/C)]\) and \([\overline{T} \, (X/C)]\) matrices, respectively.

The rows of the \([T \, (\eta)]\) matrix are made up of the spanwise constraint functions. Each element in a row is equal to the value of the constraint function at the lateral subpanel which has the same lateral index as the number of the column in the \([T \, (\eta)]\) matrix. There are \( N_W \) rows in the \([T \, (\eta)]\) matrix, with the standard functions described by \( \eta^W \sqrt{1 - \eta^2} \) in the first \( N_W \) \( P_W \) rows, and then the special \( P_W \, (\eta)_{P} \) functions in the last \( P_W \) rows.

The special spanwise constraint functions are needed to account for flow induced by discontinuities in the sweep of the constant percent chord lines, for body-panel juncture induced flow, and for partial semi-span flaps. There are two basic special spanwise constraint functions, (1) polygonal functions which account for the discontinuities in the sweep of the constant percent chord lines and the body-panel junctures, and (2) flap functions which account for the partial semi-span flaps. These special functions are derived in Appendix F of Reference (26).
If the range of influences inboard and outboard of a discontinuity in the sweep of the constant percent chord line are defined as $\Delta \eta_i$ and $\Delta \eta_o$, respectively, and the discontinuity station $\eta_b$, then the following expressions define the special spanwise polygonal constraint function.

For $0 < |\eta_b| < \Delta \eta_i$

$$P(\theta)_S = (1 - \eta_b/\Delta \eta_i) M(\theta, \frac{\pi}{2}) + \frac{1}{\Delta \eta_i} P(\theta, \frac{\pi}{2}) - \left[ \frac{(\Delta \eta_i + \Delta \eta_o)(1 - \eta_b)}{\Delta \eta_i - \Delta \eta_o} \right] P(\theta, \theta_b)$$

$$+ \left[ \frac{\Delta \eta_i + \Delta \eta_o}{\Delta \eta_i \Delta \eta_o} - \frac{1}{\Delta \eta_i} \right] \left[ 1 - \Delta \eta_o - \eta_b \right] P(\theta, \cos^{-1}(\eta_b + \Delta \eta_o))$$

(75)

For $\Delta \eta_i \leq |\eta_b| < (1 - \Delta \eta_o)$

$$P(\theta)_S = (1 - \eta_b + \Delta \eta_i)(1/\Delta \eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta \eta_i))$$

$$- \left[ \frac{\Delta \eta_o + \Delta \eta_i}{\Delta \eta_i \Delta \eta_o} \right] \left[ 1 - \eta_b \right] P(\theta, \theta_b)$$

$$+ (1 - \eta_b - \Delta \eta_o)(1/\Delta \eta_o) P(\theta, \cos^{-1}(\eta_b + \Delta \eta_o))$$

(76)

And for $(1 - \Delta \eta_o) \leq |\eta_b| < 1.0$

$$P(\phi)_S = (1 - \eta_b + \Delta \eta_i)(1/\Delta \eta_i) P(\theta, \cos^{-1}(\eta_b - \Delta \eta_i))$$

$$- \left[ 1 + (1 - \eta_b)/\Delta \eta_i \right] P(\theta, \theta_b)$$

(77)
The special spanwise flap constraint function is given by:

\[ P(\theta, \theta^*) = \frac{1}{2\pi(1 - \cos \theta^*)} \left\{ \begin{array}{l} (\cos \theta^* - \cos \theta) \log_e \frac{\sin 1/2 (\theta^* - \theta)}{\sin 1/2 (\theta^* + \theta)} \\ + (\cos \theta^* + \cos \theta)^2 \log_e \frac{\cos 1/2 (\theta^* + \theta)}{\cos 1/2 (\theta^* - \theta)} \\ + 4 \theta^* \cos \theta^* - 2 \sin \theta^* \sin \theta \end{array} \right\} \]

(78)

And

\[ \theta' = \cos^{-1} \eta \]

The special spanwise flap constraint function is given by;

\[ P(\theta)_S = M(\theta, \theta_i) - M(\theta, \theta_o) \]

(79)

Where \( \theta_i = \cos^{-1} \eta_i \), \( \theta_o = \cos^{-1} \eta_o \), \( \eta_i \) is the inboard station where the control surface begins, and \( \eta_o \) is the outboard station where the control surface ends.

\[ M(\theta, \theta^*) = \frac{1}{\pi} \left\{ \begin{array}{l} (\cos \theta^* - \cos \theta) \log_e \frac{\sin 1/2 (\theta^* - \theta)}{\sin 1/2 (\theta^* + \theta)} \\ + (\cos \theta^* + \cos \theta) \log_e \frac{\cos 1/2 (\theta^* + \theta)}{\cos 1/2 (\theta^* - \theta)} + 2 \theta^* \sin \theta \end{array} \right\} \]

(80)

The discrete aerodynamic influence equation (33) is transformed by substituting equations (40) and (65) into equation (33).
The constrained aerodynamic influence equation is then given by:

\[
\begin{bmatrix}
[A]_B B_i & [A]_B P_{KC} \\
[A]_P B_i & [A]_P P_{KC}
\end{bmatrix}
\begin{bmatrix}
[T_B] & [0] \\
[0] & [T_P]
\end{bmatrix}
\begin{bmatrix}
[a]_B_i \\
[a]_P_K
\end{bmatrix} = 
\begin{bmatrix}
[\nabla S]_B_j \\
[\nabla S]_P_j
\end{bmatrix} + 
\begin{bmatrix}
[e]_B_j \\
[e]_P_j
\end{bmatrix}
\]

(81)

Where the arrays \( \{ a \}_B_i \) and \( \{ a \}_P_K \) are solved for such that the sum of the squares of the elements of the arrays \( \{ e \}_B_j \) and \( \{ e \}_P_j \) are a minimum.
Surface Velocities and Pressures

The velocity tangent to the surface of the jth body in the longitudinal direction is given by:

\[
\begin{pmatrix}
\left( \frac{\nu}{\rho_{mn}} \right)_{B_j} \\
\end{pmatrix}
= \sum_{i=1}^{N_p} \left[ \frac{1}{\rho^2} \begin{pmatrix}
T_{i,j} \\
A_{i,j} \\
A_{i,j} B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{i,j} B_1 \\
A_{i,j} B_1 \\
A_{i,j} B_1 B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{i,j} B_1 B_1 \\
A_{i,j} B_1 B_1 \\
A_{i,j} B_1 B_1 B_1 \\
\end{pmatrix} \right] \begin{pmatrix}
K \\
\frac{V_e}{V_o} \\
\end{pmatrix}
\]

\[
+ \sum_{K=1}^{N_p} \left[ \frac{1}{\rho^2} \begin{pmatrix}
T_{j,k} \\
S_{j,k} \\
S_{j,k} B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{j,k} B_1 \\
S_{j,k} B_1 \\
S_{j,k} B_1 B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{j,k} B_1 B_1 \\
S_{j,k} B_1 B_1 \\
S_{j,k} B_1 B_1 B_1 \\
\end{pmatrix} \right] \begin{pmatrix}
T \\
\frac{V_e}{V_o} \\
\end{pmatrix}
\]

\[
+ \sum_{M=1}^{N_p} \left[ \frac{1}{\rho^2} \begin{pmatrix}
T_{j,M} \\
S_{j,M} \\
S_{j,M} B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{j,M} B_1 \\
S_{j,M} B_1 \\
S_{j,M} B_1 B_1 \\
\end{pmatrix} + \frac{1}{\beta} \begin{pmatrix}
T_{j,M} B_1 B_1 \\
S_{j,M} B_1 B_1 \\
S_{j,M} B_1 B_1 B_1 \\
\end{pmatrix} \right] \begin{pmatrix}
T_{M} \\
\frac{V_e}{V_o} \\
\end{pmatrix}
\]

\[
\left( \frac{\Delta V_M}{V_{co}} \right)_{mn} = \left[ \frac{T_{mn}}{V_{co}} \right] \left( \frac{\frac{3}{4} I_{M(m-1)n} + \frac{1}{4} I_{M(m+1)n}}{3 I_{M(m-1)n} + I_{M(m+1)n}} \right) \left[ \frac{2 K_{mn}}{3 K_{mn} + I_{M(m+1)n}} \right] \left[ \frac{2 K_{mn}}{3 K_{mn} + I_{M(m+1)n}} \right]
\]

And

\[
\frac{T_{mn}}{V_{co}} = \frac{K_{mn}}{V_{co}} - \frac{K_{m-1,n}}{V_{co}}
\]
\[
\frac{\tau_b(m + 1)N}{V_\infty} = \frac{K(m + 1)N}{V_\infty} - \frac{K_{mN}}{V_\infty}
\]  

The onset flow ratios \(V_X/V_\infty\), \(V_Y/V_\infty\), and \(V_Z/V_\infty\) are given by the following expressions.

\[
\frac{V_X}{V_\infty} = 1 - q^* \frac{2(Z - Z_{C.G.})}{c} - \frac{2(Y - Y_{C.G.})}{b}
\]  

\[
\frac{V_Y}{V_\infty} = \beta + p^* \frac{2(Z - Z_{C.G.})}{b} + \frac{2(X - X_{C.G.})}{b}
\]  

\[
\frac{V_Z}{V_\infty} = \alpha + p^* \frac{2(Y - Y_{C.G.})}{b} + q^* \frac{(X - X_{C.G.})}{c}
\]

The components of the unit vectors tangent to the actual body subpanels in the longitudinal direction are given by;

\[
T_{M_X} = \beta \frac{T_{M_X}}{\sqrt{\beta^2 T_{M_X} + T_{M_Y}^2 + T_{M_Z}^2}}
\]  

\[
T_{M_Y} = T_{M_Y} / \sqrt{\beta^2 T_{M_X} + T_{M_Y}^2 + T_{M_Z}^2}
\]  

\[
T_{M_Z} = T_{M_Z} / \sqrt{\beta^2 T_{M_X} + T_{M_Y}^2 + T_{M_Z}^2}
\]

and in the lateral direction by

\[
T_{T_X} = \beta \frac{T_{T_X}}{\sqrt{\beta^2 T_{T_X}^2 + T_{T_Y}^2 + T_{T_Z}^2}}
\]  

\[
T_{T_Y} = T_{T_Y} / \sqrt{\beta^2 T_{T_X}^2 + T_{T_Y}^2 + T_{T_Z}^2}
\]  

\[
T_{T_Z} = T_{T_Z} / \sqrt{\beta^2 T_{T_X}^2 + T_{T_Y}^2 + T_{T_Z}^2}
\]
The velocity tangent to the surface of the jth body in the lateral direction is given by:

\[
\begin{align*}
\left( \begin{array}{c} \frac{V_T}{V_{\infty}} \end{array} \right)_{mn} &= \left( \begin{array}{c} \frac{V_T}{V_{\infty}} \end{array} \right)_{B_j} \\
&= \sum_{i=1}^{N_B} \left[ \frac{1}{\rho^2} \left[ \frac{T_{x_{B_j}}}{\rho^2} \right] \left[ \frac{A_{x_{B_j,1}}}{\rho^2} \right] + \frac{1}{\rho^2} \left[ \frac{T_{y_{B_j}}}{\rho^2} \right] \left[ \frac{A_{y_{B_j,1}}}{\rho^2} \right] + \frac{1}{\rho^2} \left[ \frac{T_{z_{B_j}}}{\rho^2} \right] \left[ \frac{A_{z_{B_j,1}}}{\rho^2} \right] \right] \left( \frac{V_T}{V_{\infty}} \right)_{B_i} \\
&= \sum_{k=1}^{N_P} \left[ \frac{1}{\rho^2} \left[ \frac{T_{x_{B_j}}}{\rho^2} \right] \left[ \frac{S_{x_{B_j,p_k}}}{\rho^2} \right] + \frac{1}{\rho^2} \left[ \frac{T_{y_{B_j}}}{\rho^2} \right] \left[ \frac{S_{y_{B_j,p_k}}}{\rho^2} \right] + \frac{1}{\rho^2} \left[ \frac{T_{z_{B_j}}}{\rho^2} \right] \left[ \frac{S_{z_{B_j,p_k}}}{\rho^2} \right] \right] \left( \frac{V_T}{V_{\infty}} \right)_{P_k} \\
&= \left( \frac{\Delta V_T}{V_{\infty}} \right)_{mn} \left[ \frac{V_x}{V_{\infty}} \right] \left[ \frac{V_y}{V_{\infty}} \right] \left[ \frac{V_z}{V_{\infty}} \right] \\
&= \left( \frac{\Delta V_T}{V_{\infty}} \right)_{mn} \left[ \frac{T_{x_{mn}}(n - \Delta n/2)}{V_{\infty}} \left( T_{x_{mn}} + T_{x_{m(n-1)}} \right) + \frac{T_{x_{m(n+\Delta n/2)}}}{V_{\infty}} \left( T_{x_{mn}} + T_{x_{m(n+1)}} \right) \right] \\
\end{align*}
\]

Where

\[
\begin{align*}
\left( \frac{\Delta V_T}{V_{\infty}} \right)_{mn} &= -\frac{1}{2} \left[ \frac{T_{m(n - \Delta n/2)}}{V_{\infty}} \left( T_{m(n - \Delta n/2)} + T_{m(n - \Delta n/2)} \right) + \frac{T_{m(n + \Delta n/2)}}{V_{\infty}} \left( T_{m(n + \Delta n/2)} + T_{m(n + \Delta n/2)} \right) \right] \\
\end{align*}
\]
\[
\frac{\Gamma_{m}(n - \Delta n/2)}{V_{\infty}} = \frac{K_{m}(n - 1)}{V_{\infty}} - \frac{K_{mn}}{V_{\infty}} - \sum_{r = 1}^{N_{a}} \left[ \frac{K_{m(n - \Delta n/2)}}{V_{\infty}} \right]_{p_r}
\]

\[
\frac{\Gamma_{m}(n + \Delta n/2)}{V_{\infty}} = \frac{K_{mn}}{V_{\infty}} - \frac{K_{m(n + 1)}}{V_{\infty}} - \sum_{r = 1}^{N_{a}} \left[ \frac{K_{m(n + \Delta n/2)}}{V_{\infty}} \right]_{p_r}
\]

Where

\[
\sum_{r = 1}^{N_{a}} \left[ \frac{K_{m(n - \Delta n/2)}}{V_{\infty}} \right]_{p_r} \quad \text{and} \quad \sum_{r = 1}^{N_{a}} \left[ \frac{K_{m(n + \Delta n/2)}}{V_{\infty}} \right]_{p_r}
\]

are the contributions from the trailing legs of the panel vortices along the juncture line of the jth body and rth panel to be attached at the \((n - \Delta n/2)\) and \((n + \Delta n/2)\) lateral stations, respectively. \(N_{a}\) is the total number of panels attached at any one point along the line.

The velocity tangent to the surface of the jth panel in the longitudinal direction is given by;

\[
\left( \frac{V_{M}}{V_{\infty}} \right)_{mn}^{p_j} = \left( \frac{1}{1 + (1 + \tan^{2} \alpha)_{mn} (dz_{T}/dx + dz_{C}/dx)^{2}} \right)^{1/2} \left[ \sum_{i = 1}^{N_{B}} \frac{1}{\beta^{2}} \left[ A_{x_{p_{j}B_{1}}} \left( \frac{K}{V_{\infty}} \right) \right]_{B_{1}} \right]
\]

\[
+ \sum_{k = 1}^{N_{p}} \frac{1}{\beta^{2}} \left[ A_{T_{j,p_{K}}} \left( \frac{K}{V_{\infty}} \right) \right]_{p_{K}} + \left[ S_{x_{p_{j}p_{K}}} \left( \frac{\Sigma}{V_{\infty}} \right) \right]_{p_{K}}
\]

\[
+ \frac{1}{\beta^{2}} \left[ \frac{\Delta V_{M}}{V_{\infty}} \right]_{mn}^{p_j} \left[ \frac{1}{\sqrt{1 + \tan^{2} \alpha_{mn}}} \right]_{p_{j}} + \left[ S_{x_{p_{j}p_{j}}} \left( \frac{\Sigma}{V_{\infty}} \right) \right]_{p_{j}} - \left[ \frac{\Sigma}{V_{\infty}} \right]_{p_{j}}
\]

\[
\ast \left( \frac{V_{X}}{V_{\infty}} \right)_{p_{j}}
\]

(97)
The velocity tangent to the surface of the \( j \)th panel in the lateral direction is given by:

\[
\begin{align*}
\n\left( \begin{array}{c}
\frac{\partial V}{\partial n}
\end{array} \right)_{mn}^p &= \left( \begin{array}{c}
\frac{\partial V}{\partial n}
\end{array} \right)_{mn}^p \\
&= \left. \frac{1}{1 + (1 + \tan^2 \Lambda)_{mn}} \left( \frac{\partial z_t}{\partial x} \right) \left( \frac{\partial z_c}{\partial x} \right) \right| \frac{1}{1 + \left( \frac{\partial z_t}{\partial x} \right)^2 + \left( \frac{\partial z_c}{\partial x} \right)^2}^{1/2}
\end{align*}
\]

and where \( (\Delta V_M/V_\infty)_{mn} \) is computed using equation (84) and \( X/C \) is the local percent chord.

\[
\begin{align*}
\left( \begin{array}{c}
\frac{\partial V}{\partial n}
\end{array} \right)_{mn}^p &= \left. \frac{1}{1 + \tan^2 \Lambda} \right| \frac{1}{1 + \left( \frac{\partial z_t}{\partial x} \right)^2 + \left( \frac{\partial z_c}{\partial x} \right)^2}^{1/2}
\end{align*}
\]

(98)
where

\[
\left( \frac{\Delta V_T}{V_\infty} \right)_{mn} = \frac{1}{2} \left[ \frac{\Gamma \Gamma_m(n-\Delta n)}{V_\infty} + \frac{\Gamma \Gamma_m(n+\Delta n)}{V_\infty} \right] + \left[ \left( \frac{\Delta V_M}{V_\infty} \right) \tan \alpha \right]_{mn}
\]

(100)

and where

\[
\frac{\Gamma \Gamma_m(n-\Delta n)}{V_\infty}
\]

and

\[
\frac{\Gamma \Gamma_m(n+\Delta n)}{V_\infty}
\]

are computed the same as for the body except

\[
\sum_{r=1}^{N_a} \left[ \frac{K_m(n-\Delta n)}{V_\infty} \right]_{pr}
\]
represent the contributions from the trailing legs of the panel vortices along the juncture line of the jth panel and the rth panel to be attached at the lateral stations,

\[(n - \frac{\Delta n}{2})\] and \[(n + \frac{\Delta n}{2})\], respectively.

The surface pressure coefficients at each of the control points on the bodies and panels are then computed using the following expression.

\[
C_{p_{mn}} = \frac{2}{\gamma M_{\infty}^2} \left[ 1 + \frac{\gamma - 1}{2} M_{\infty}^2 \left[ 1 - \left( \frac{V_M}{V_{\infty}} \right)_{mn}^2 - \left( \frac{V_T}{V_{\infty}} \right)_{mn}^2 \right] \right]^{\gamma/(\gamma-1)}
\]

(101)

where \(\gamma\) is the ratio of specific heats.

Note, in equations (97) and (100) the top sign is used to compute the velocity on the upper surface and the bottom sign the lower surface in those terms which have a plus and minus in front.
Section and Total Loads and Moments

The section loads are computed in coefficient form on each of the bodies by interpolating for the surface pressure coefficients at the centroid of the subareas, defined by the corner points of the divisions of the body subpanels, and then using the following equations to sum the product of the pressure coefficients and directed subareas.

\[
\left( \frac{C_X W}{W_{AVG}} \right)_K = - \frac{1}{A_Z} \Delta \left( \frac{X_B}{C_B} \right)_K \sum_i C_{p_{iK}} \Delta A_{X_{iK}}
\]

(102)

\[
\left( \frac{C_Y h}{h_{AVE}} \right)_K = - \frac{1}{A_Y} \Delta \left( \frac{X_B}{C_B} \right)_K \sum_i C_{p_{iK}} \Delta A_{Y_{iK}}
\]

(103)

\[
\left( \frac{C_Z W}{W_{AVG}} \right)_K = - \frac{1}{A_Z} \Delta \left( \frac{X_B}{C_B} \right)_K \sum_i C_{p_{iK}} \Delta A_{Z_{iK}}
\]

(104)

where \( i \) is indexed over all subareas in the longitudinals segment \( \Delta (X_B/C_B)_K \).

\[
A_X = \frac{1}{2} \sum_K \sum_i |\Delta A_{X_{iK}}|
\]

(105)

\[
A_Y = \frac{1}{2} \sum_K \sum_i |\Delta A_{Y_{iK}}|
\]

(106)
\[ A_Z = \frac{1}{2} \sum_{K} \sum_{i} |\Delta A_{z_{iK}}| \]  \( \text{(107)} \)

Also;

\[ W_{AVG} = \frac{A_Z}{C_B} \]  \( \text{(108)} \)

and

\[ h_{AVG} = \frac{A_Y}{C_B} \]  \( \text{(109)} \)

The total loads on a body are then obtained by summing in the longitudinal direction.

\[ C_{X_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_{K} \left( \frac{C_X W}{W_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \]  \( \text{(110)} \)

\[ C_{Y_{B_j}} = \frac{A_Y}{A_{B_j}} \sum_{K} \left( \frac{C_Y h}{h_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \]  \( \text{(111)} \)

\[ C_{Z_{B_j}} = \frac{A_Z}{A_{B_j}} \sum_{K} \left( \frac{C_Z W}{W_{AVG}} \right)_K \Delta \left( \frac{X_B}{C_B} \right)_K \]  \( \text{(112)} \)
The total moments on a body are summed about the center of gravity.

\[ C_{Mx_{Bj}} = - \frac{1}{A_{Bj} C} \sum K \sum i \left[ (Y_{iK} - Y_{C.G.}) \Delta A_{z_{iK}} - (Z_{iK} - Z_{C.G.}) \Delta A_{y_{iK}} \right] C_{piK} \]  

(113)

\[ C_{My_{Bj}} = - \frac{1}{A_{Bj} C} \sum K \sum i \left[ (Z_{iK} - Z_{C.G.}) \Delta A_{x_{iK}} - (X_{iK} - X_{C.G.}) \Delta A_{z_{iK}} \right] C_{piK} \]  

(114)

\[ C_{Mz_{Bj}} = - \frac{1}{A_{Bj} C} \sum K \sum i \left[ (X_{iK} - X_{C.G.}) \Delta A_{y_{iK}} - (Y_{iK} - Y_{C.G.}) \Delta A_{x_{iK}} \right] C_{piK} \]  

(115)

The body center of pressure position vector components divided by \( C \) are given by;

\[
 \left( \frac{X}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{Y_{Bj}} C_{M_{Z_{Bj}}} - C_{Z_{Bj}} C_{M_{Y_{Bj}}}}{C_{X_{Bj}}^2 + C_{Y_{Bj}}^2 + C_{Z_{Bj}}^2} \]  

(116)

\[
 \left( \frac{Y}{C} \right)_{C.P.} = \left( \frac{Y}{C} \right)_{C.G.} + \frac{C_{Z_{Bj}} C_{M_{X_{Bj}}} - C_{X_{Bj}} C_{M_{Z_{Bj}}}}{C_{X_{Bj}}^2 + C_{Y_{Bj}}^2 + C_{Z_{Bj}}^2} \]  

(117)
The panel section loads, moments, and centers of pressure relative to the leading edge are obtained by numerically evaluating the following integrals.

\[
\left( \frac{z}{C} \right)_{\text{C.P.}} = \left( \frac{z}{C} \right)_{\text{C.G.}} + \frac{C_{X_B}}{C_{Y_B}} + \frac{C_{Y_B}}{C_{X_B}} + \frac{C_{Z_B}}{C_{Z_B}} \tag{118}
\]

\[
\frac{C_X}{C_{\text{AVG}}} = \frac{C b_S \cdot \text{AR}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}^2} \left[ C_{P_L} \tan\beta_{L} - C_{P_U} \tan\beta_{U} \right] \sin\phi \, d\phi - \left( \frac{C_T}{C_{\text{AVG}}} \right) \tag{119}
\]

\[
\frac{C_Y}{C_{\text{AVG}}} = \frac{C b_S \cdot \text{AR} \cdot N_{P}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}^2} \left( C_{P_L} - C_{P_U} \right) \sin\phi \, d\phi \tag{120}
\]

\[
\frac{C_Z}{C_{\text{AVG}}} = \frac{C b_S \cdot \text{AR} \cdot N_{P}}{2 b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{\text{L.E.}}^2} \left( C_{P_L} - C_{P_U} \right) \sin\phi \, d\phi \tag{121}
\]

where

\[
b_S = \sum_{N=1}^{N_N} \sqrt{\Delta Y_{\text{L.E.},N}^2 + \Delta Z_{\text{L.E.},N}^2} \tag{122}
\]

\[
\Delta S = \sqrt{\Delta Y_{\text{mN}}^2 + \Delta Z_{\text{mN}}^2} = \text{local panel width} \tag{123}
\]
\[ \Delta S_{L.E.} = \sqrt{\Delta Y_{L.E.}^2 + \Delta Z_{L.E.}^2} = \text{panel width at leading edge.} \]  

and \( \phi = \cos^{-1} [1 - 2 \left( \frac{X}{C} \right)] \) where \( \left( \frac{X}{C} \right) \) is the local percent chord.

Also;

\[
\tan \sigma = \frac{\frac{dZ}{dX} - \frac{dZ}{dX} + \epsilon}{1 - \left( \frac{dZ}{dX} - \frac{dZ}{dX} + \epsilon \right) \tan \delta}
\]  

where the top sign is used with the upper surface and the bottom sign is used with the lower surface. The control surface angle is equal to \( -\delta_K \) along the leading edge flap and equal to \( \beta_f \) along a trailing control surface. \( \delta \) is equal to zero at other points on the chord.

The section normal force coefficient is obtained by taking the following scalar product.

\[
\frac{C_N}{C_{AVG}} = \frac{C_Y}{C_{AVG}} N_{p_j} + \frac{C_Z}{C_{AVG}} N_{p_j}
\]  

The section moment coefficients about the leading edge are computed as follows.

\[
\frac{C_{M_{L.E.}x}}{C_{AVG}} = \frac{2 b_s^2}{\pi} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( Y_{L.E.} \right) N_{p_j} \left( x \cdot Y_{L.E.} \right) N_{p_j} \left[ C_{p_L} - C_{p_U} \right] \sin \phi \, d\phi
\]

\[
\frac{C_{M_{L.E.}y}}{C_{AVG}} = \frac{2 b_s^2}{\pi} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( Z_{L.E.} \right) \left( C_{p_L} \tan \phi - C_{p_U} \tan \phi \right) \left( x \cdot X_{L.E.} \right) N_{p_j} \left[ C_{p_L} - C_{p_U} \right] \sin \phi \, d\phi
\]
The section center of pressure relative to the leading edge is given by:

\[
\frac{c_{M, E. Z}}{c_{AVG}} = \frac{AR \Phi_s}{2 b^2} \int_{x_{L.E.}}^{x_{L.E.}} \left[ (x - x_{L.E.}) \frac{c_p}{c_{P_0}} - (y - y_{L.E.}) \frac{c_p \tan \alpha - c_p \tan \alpha_0}{c_{P_0}} \right] \sin \alpha d \phi
\]  

(129)

\[
\frac{X}{C_{C.P.}} = \frac{\left( \frac{C_Y}{C_{AVG}} \right) \left( \frac{C_{M, E. Z}}{C_{AVG}} \right) - \left( \frac{C_Z}{C_{AVG}} \right) \left( \frac{C_{M, E. Y}}{C_{AVG}} \right)}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2}
\]  

(130)

\[
\frac{Y}{C_{C.P.}} = \frac{\left( \frac{C_Z}{C_{AVG}} \right) \left( \frac{C_{M, E. X}}{C_{AVG}} \right) - \left( \frac{C_X}{C_{AVG}} \right) \left( \frac{C_{M, E. Z}}{C_{AVG}} \right)}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2}
\]  

(131)

\[
\frac{Z}{C_{C.P.}} = \frac{\left( \frac{C_X}{C_{AVG}} \right) \left( \frac{C_{M, E. Y}}{C_{AVG}} \right) - \left( \frac{C_Y}{C_{AVG}} \right) \left( \frac{C_{M, E. X}}{C_{AVG}} \right)}{\left( \frac{C_X}{C_{AVG}} \right)^2 + \left( \frac{C_Y}{C_{AVG}} \right)^2 + \left( \frac{C_Z}{C_{AVG}} \right)^2}
\]  

(132)

The section zero percent suction drag coefficient

\[
\frac{C_{d_{T=0}}}{C_{AVG}}
\]
the section induced drag due to lift coefficient
\[ \frac{C_{dL}}{C_{AVG}} \]
the section leading edge thrust coefficient \( C_T \) \( C/CAVG \), and the section induced drag due to thickness coefficient
\[ \frac{C_{dT}}{C_{AVG}} \]
are derived in appendix C.

The panel total force coefficients are obtained by numerically integrating the section force coefficients in the spanwise direction.

\[ C_{X_{pj}} = \int_{0}^{\pi/2} \frac{C_X C}{C_{AVG}} \sin \theta \, d\theta \]  \hspace{1cm} (133)

\[ C_{Y_{pj}} = \int_{0}^{\pi/2} \frac{C_Y C}{C_{AVG}} \sin \theta \, d\theta \]  \hspace{1cm} (134)

\[ C_{Z_{pj}} = \int_{0}^{\pi/2} \frac{C_Z C}{C_{AVG}} \sin \theta \, d\theta \]  \hspace{1cm} (135)

where
\[ \theta = \cos^{-1} \eta \]  \hspace{1cm} (136)
The panel total moment coefficients about the center of gravity are numerically integrated as follows.

\[
\begin{align*}
C_m_{p_j} &= \frac{AR b_3}{2 \omega^2 c} \int_0^{\pi/2} d\theta \int_0^{\pi/2} \frac{\Delta S}{\omega_2 L E} \left[ (z_{C.G.}) N_{p_j} \cdot (z_{C.G.}) N_{p_j} \right] \left( c_{L L} c_{U U} \right) \sin \theta \sin \theta d\theta d\theta \\
&= \frac{AR b_3}{2 \omega^2 c} \int_0^{\pi/2} d\theta \int_0^{\pi/2} \frac{\Delta S}{\omega_2 L E} \left[ (z_{C.G.}) N_{p_j} \cdot (z_{C.G.}) N_{p_j} \right] \left( c_{L L} c_{U U} \right) \sin \theta \sin \theta d\theta d\theta
\end{align*}
\]

(137)

The panel total zero suction drag, near field induced drag due to lift, leading edge thrust, and near field induced drag due to thickness are given by:

\[
\begin{align*}
C_{D_{T=0}} &= \int_0^{\pi/2} \frac{C_D}{C_{AVG}} \sin \theta d\theta \\
C_{D_{L_i}} &= \int_0^{\pi/2} \frac{C_D}{C_{AVG}} \sin \theta d\theta \\
C_{T_p} &= \int_0^{\pi/2} \frac{C_T}{C_{AVG}} \sin \theta d\theta
\end{align*}
\]

(140) (141) (142)
\[ C_{DT_{ip_j}} = \int_0^{\pi/2} \frac{C_{d_{Ti}}}{C_{AVG}} \sin \theta \, d\theta \quad (143) \]

The panel center of pressure position vector components divided by \( \bar{C} \) are computed as follows.

\[
\left( \frac{X}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{z_{pj}} C_{M_{pj}} - C_{z_{pj}} C_{M_{pj}}}{C_{x_{pj}}^2 + C_{y_{pj}}^2 + C_{z_{pj}}^2} \quad (144)
\]

\[
\left( \frac{Y}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{C.G.} + \frac{C_{y_{pj}} C_{M_{pj}} - C_{y_{pj}} C_{M_{pj}}}{C_{x_{pj}}^2 + C_{y_{pj}}^2 + C_{z_{pj}}^2} \quad (145)
\]

\[
\left( \frac{Z}{C} \right)_{C.P.} = \left( \frac{Z}{C} \right)_{C.G.} + \frac{C_{x_{pj}} C_{M_{pj}} - C_{x_{pj}} C_{M_{pj}}}{C_{x_{pj}}^2 + C_{y_{pj}}^2 + C_{z_{pj}}^2} \quad (146)
\]
The section loads, moments, and center of pressure for the control surfaces are computed as follows.

The equation for the trailing edge of leading edge control surface:

\[ X_k = X_{k_i} + \left( \frac{X_{k_o} - X_{k_i}}{\eta_{k_o} - \eta_{k_i}} \right) (\eta - \eta_{k_i}) \]  \hspace{1cm} (147)

The equation for the leading edge of trailing edge control surface:

\[ X_f = X_{f_i} + \left( \frac{X_{f_o} - X_{f_i}}{\eta_{f_o} - \eta_{f_i}} \right) (\eta - \eta_{f_i}) \]  \hspace{1cm} (148)

The equation for a control surface hinge line:

\[ X_h = X_{h_i} + \left( \frac{X_{h_o} - X_{h_i}}{\eta_{h_o} - \eta_{h_i}} \right) (\eta - \eta_{h_i}) \]  \hspace{1cm} (149)
The unit vector in the direction of the hinge line is given by:

\[ \hat{n} = \left\{ \begin{array}{l} \left[ x_h (\eta + \Delta \eta) - x_h (\eta - \Delta \eta) \right] \hat{i} + \left[ y_h (\eta + \Delta \eta) - y_h (\eta - \Delta \eta) \right] \hat{j} \\
+ \left[ z_h (\eta + \Delta \eta) - z_h (\eta - \Delta \eta) \right] \hat{k} \right\} \left[ \left[ x_h (\eta + \Delta \eta) - x_h (\eta - \Delta \eta) \right]^2 + \left[ y_h (\eta + \Delta \eta) - y_h (\eta - \Delta \eta) \right]^2 \right]^{1/2} \]

(150)

The section loads on a leading edge control surface are obtained by numerically evaluating the following integrals:

\[ \frac{C_{h,x}}{C_{AVG}} = \frac{C_{x,AR}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{pL} \tan \alpha_L - C_{pU} \tan \alpha_U \right) \sin \phi_k \, d\phi_k - \left( \frac{C_{n,C}}{C_{AVG}} \right) \]

(151)

\[ \frac{C_{h,y}}{C_{AVG}} = \frac{C_{k,AR}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{pL} - C_{pU} \right) \sin \phi_k \, d\phi_k \]

(152)

\[ \frac{C_{h,z}}{C_{AVG}} = \frac{C_{k,AR}}{2b^2} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{pL} - C_{pU} \right) \sin \phi_k \, d\phi_k \]

(153)
where

\[ \phi_k = \cos^{-1} \left[ 1 - 2 \left( \frac{C_k}{C} \right) \left( \frac{X}{C} \right) \right] \]  

(154)

and \( C_k \) is the chord of the leading edge control surface. \((X/C)\) is the local percent chord of the panel.

The section loads on a trailing edge control surface are obtained by numerically evaluating the following integrals.

\[ \frac{C_{h_x}}{C_{AVG}} = \frac{C_f AR b_s}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) \sin \phi_f \, d\phi_f \]  

(155)

\[ \frac{C_{h_y}}{C_{AVG}} = \frac{C_f AR b_s N_{Y_{P_i}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_f \, d\phi_f \]  

(156)

\[ \frac{C_{h_z}}{C_{AVG}} = \frac{C_f AR b_s N_{Z_{P_i}}}{2b^2} \int_0^{\pi} \frac{\Delta S}{\Delta S_{L.E.}} \left( C_{P_L} - C_{P_U} \right) \sin \phi_f \, d\phi_f \]  

(157)

where

\[ \phi_f = \cos^{-1} \left[ 1 - 2 \left( \frac{C_f}{C} \right) \left( \frac{X + C_f - C}{C} \right) \right] \]  

(158)

and \( C_f \) is the chord of the trailing edge control surface.
The section normal load on the control surface is given by:

\[
\frac{C_{h_N}}{C_{AVG}} = \left(\frac{C_{h_Y}}{C_{AVG}}\right) N_{y_{P_j}} + \left(\frac{C_{h_z}}{C_{AVG}}\right) N_{z_{P_j}}
\]  

(159)

The section moments about the hinge line for the leading edge control surface are obtained by numerically evaluating the following integrals:

\[
\frac{C_{h_x}}{C_{AVG}} = \frac{C_k}{2b^2 c} \int_0^{\Delta S_{L.E.}} \left[ (Y - Y_h) N_{z_{P_j}} - (Z - Z_h) N_{y_{P_j}} \right] \left( C_{p_L} - C_{p_U} \right) \sin \phi_k \, d\phi_k
\]  

(160)

\[
\frac{C_{h_y}}{C_{AVG}} = \frac{C_k}{2b^2 c} \int_0^{\Delta S_{L.E.}} \left[ (Z - Z_h) \left( C_{p_L} \tan \phi_L - C_{p_j} \tan \phi_j \right) - (X - X_h) N_{p_{j}} \left( C_{p_L} - C_{p_U} \right) \sin \phi_k \, d\phi_k \right]
\]  

(161)

\[
\frac{C_{h_z}}{C_{AVG}} = \frac{C_k}{2b^2 c} \int_0^{\Delta S_{L.E.}} \left[ (X - X_h) N_{y_{P_j}} \left( C_{p_L} - C_{p_U} \right) - (Y - Y_h) \left( C_{p_L} \tan \phi_L - C_{p_j} \tan \phi_j \right) \sin \phi_k \, d\phi_k \right]
\]  

(162)

The section moments about the hinge line for the trailing edge control surface are obtained by numerically evaluating the following integrals:

\[
\frac{C_{h_x}}{C_{AVG}} = \frac{C_E}{2b^2 c} \int_0^{\Delta S_{L.E.}} \left[ (Y - Y_h) N_{z_{P_j}} - (Z - Z_h) N_{y_{P_j}} \right] \left( C_{p_L} - C_{p_U} \right) \sin \phi_k \, d\phi_k
\]  

(163)
The section hinge moment is computed by the following equation.

\[ \frac{C_{M_h}}{C_{AVG}} = \frac{C_f \cdot AR \cdot b \cdot s}{2b^2 C} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (2 - Z_h) \left( C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) - (X - X_h) N_{\phi_j} \left( C_{P_L} - C_{P_U} \right) \right] \sin \phi \, d\phi \]  

(164)

\[ \frac{C_{h_x}}{C_{AVG}} = \frac{C_f \cdot AR \cdot b \cdot s}{2b^2 C} \int_0^\pi \frac{\Delta S}{\Delta S_{L.E.}} \left[ (X - X_h) N_{\phi_j} \left( C_{P_L} - C_{P_U} \right) - (Y - Y_h) \left( C_{P_L} \tan \alpha_L - C_{P_U} \tan \alpha_U \right) \right] \sin \phi \, d\phi \]  

(165)

The section center of pressure, due to the control surface loading, relative to the leading edge is given by the following expressions.

\[ \frac{C_{M_h}}{C_{AVG}} = \left( \frac{C_{M_h}}{C_{AVG}} \right)_{h_x} \frac{C_x}{C} + \left( \frac{C_{M_h}}{C_{AVG}} \right)_{h_y} \frac{C_y}{C} + \left( \frac{C_{M_h}}{C_{AVG}} \right)_{h_z} \frac{C_z}{C} \]  

(166)

\[ \left( \frac{X}{C} \right)_{C.P.} = \left( \frac{X}{C} \right)_{h_x} + \frac{\left( \frac{C_{h_y}}{C_{AVG}} \right) \left( \frac{C_{M_h}}{C_{AVG}} \right)_{h_y} \left( \frac{C_{h_z}}{C_{AVG}} \right) \left( \frac{C_{M_h}}{C_{AVG}} \right)_{h_z}}{\left( \frac{C_{h_x}}{C_{AVG}} \right)^2 + \left( \frac{C_{h_y}}{C_{AVG}} \right)^2 + \left( \frac{C_{h_z}}{C_{AVG}} \right)^2} \]  

(167)

53
\[
\left( \frac{Y}{C} \right)_{C.P.} = \left( \frac{Y}{C} \right)_h + \bar{C} \left( \frac{c_{h_x} C}{C_{AVG}} \right) \left( \frac{C_{h_y}}{C_{AVG}} \right)^2 + \left( \frac{c_{h_y} C}{C_{AVG}} \right) \left( \frac{C_{h_x}}{C_{AVG}} \right)^2 - \left( \frac{c_{h_z} C}{C_{AVG}} \right) \left( \frac{C_{h_z}}{C_{AVG}} \right)^2
\] (168)

\[
\left( \frac{Z}{C} \right)_{C.P.} = \left( \frac{Z}{C} \right)_h + \bar{C} \left( \frac{c_{h_x} C}{C_{AVG}} \right) \left( \frac{C_{h_y}}{C_{AVG}} \right)^2 + \left( \frac{c_{h_y} C}{C_{AVG}} \right) \left( \frac{C_{h_x}}{C_{AVG}} \right)^2 - \left( \frac{c_{h_z} C}{C_{AVG}} \right) \left( \frac{C_{h_z}}{C_{AVG}} \right)^2
\] (169)

The total loads and hinge moment on the control surface are given by:

\[
C_{h_x} = \int_{\eta_i}^{\eta_o} \frac{c_{h_x} C}{C_{AVG}} d\eta
\] (170)

\[
C_{h_y} = \int_{\eta_i}^{\eta_o} \frac{c_{h_y} C}{C_{AVG}} d\eta
\] (171)

\[
C_{h_z} = \int_{\eta_i}^{\eta_o} \frac{c_{h_z} C}{C_{AVG}} d\eta
\] (172)
The total loads and moments for the complete configuration are then given by the following equations.

\[
C_X = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{Bj} + \sum_{j=1}^{N_P} \frac{A_{pj}}{A_R} F_{Sj} C_{pj}
\]  
(174)

\[
C_Y = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{Bj} + \sum_{j=1}^{N_P} \frac{A_{pj}}{A_R} F_{Sj} C_{pj}
\]  
(175)

\[
C_Z = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} C_{Bj} + \sum_{j=1}^{N_P} \frac{A_{pj}}{A_R} F_{Sj} C_{pj}
\]  
(176)

\[
C_{M_X} = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} \frac{\bar{C}_{Bj}}{C} C_{M_{Bj}} + \sum_{j=1}^{N_P} \frac{A_{pj}}{A_R} \frac{\bar{C}_{pj}}{C} F_{Sj} C_{M_{pj}}
\]  
(177)

\[
C_{M_Y} = \sum_{j=1}^{N_B} \frac{A_{Bj}}{A_R} \frac{\bar{C}_{Bj}}{C} C_{M_{Bj}} + \sum_{j=1}^{N_P} \frac{A_{pj}}{A_R} \frac{\bar{C}_{pj}}{C} F_{Sj} C_{M_{pj}}
\]  
(178)
The induced drag for the total configuration $C_{D_{1}}$ is computed in the Trefftz plane with equations derived in Appendix F. The center of pressure for the total configuration is given by the following equations.

\[
C_{M_z} = \sum_{j=1}^{N_B} \frac{A_{B_j}}{A_R} \frac{C_{B_j}}{C} C_{M_z} + \sum_{j=1}^{N_p} \frac{A_{P_j}}{A_R} \frac{C_{P_j}}{C} F_{S_j} C_{M_z_{P_j}} \quad (179)
\]

The induced drag for the total configuration $C_{D_{1}}$ is computed in the Trefftz plane with equations derived in Appendix F. The center of pressure for the total configuration is given by the following equations.

\[
\begin{align*}
\left( \frac{X}{C_R} \right)_{C.P.} &= \left( \frac{X}{C_R} \right)_{C.G.} + \frac{C_Y \frac{C_{M_z}}{C_X^2 + C_Y^2 + C_Z^2}}{} \\
\left( \frac{Y}{C_R} \right)_{C.P.} &= \left( \frac{Y}{C_R} \right)_{C.G.} + \frac{C_Z \frac{C_{M_z} - C_X}{C_X^2 + C_Y^2 + C_Z^2}}{} \\
\left( \frac{Z}{C} \right)_{C.P.} &= \left( \frac{Z}{C_R} \right)_{C.G.} + \frac{C_X \frac{C_{M_y} - C_Y}{C_X^2 + C_Y^2 + C_Z^2}}{}
\end{align*}
\]

$F_{S_j}$ is a symmetry indicator which is equal to 2.0 in equations (174), (176), and (178) and equal to 0.0 in equations (175), (177), and (179) when the panel has an image.
COMPUTER PROGRAM RESULTS

Airfoil Section Velocities

In order to establish the degree of accuracy that can be obtained by the use of a source-vortex lattice procedure with second order corrections to account for the interference between lift and thickness and to account for the fact that the boundary conditions and the perturbation velocities are satisfied and computed, respectively, on the chordal plane, the results from the program have been compared with two dimensional exact solutions for a Karman-Trefftz airfoil in figures (4) and (5). Also, comparisons are made with data for a forty-five degree swept wing with an aspect ratio of five and a taper ratio of one. The airfoil section on this wing is a twelve percent thick R.A.E. 101. These comparisons are in figure (6), (7), and (8) for zero angle of attack and in figures (9), (10), and (11) at 4.2 degrees angle of attack. For all of the above cases discrete solutions were obtained with twenty subpanels in the chordwise direction and ten subpanels in the spanwise direction. Both the upper and lower surface pressures were plotted for the zero angle of attack data to indicate the degree of accuracy involved with the test data. The wing section was symmetrical.

Section Induced Drag Due to Thickness

The section induced drag \( C_{\text{dt}} \), or potential form drag due to thickness, is computed in the present program by means of a source lattice with equation (24) of Appendix F. The source lattice as shown in figure (12) agrees quite well with the exact solution by R. T. Jones for a sixty degree swept, ten percent thick biconvex section, taper ratio one, and aspect ratio six wing given in reference (51). It should be noted that Woodward's equations, derived in Appendix D, for the constant and linearly varying distributed source density panels can be superimposed to obtain the same solution, for the biconvex section, as obtained by Jones. Even though Woodward, in reference (40), only computes induced velocities at the centroid of trapezoidal panels, it was shown that the correct solution for the longitudinal perturbation velocity is also obtained at any spanwise location, including the edges.

If two taper ratio one semi-infinite swept panels are joined at their side edges to form a wing center section or kink the Woodward distributed source equations will give the same center section solution as obtained by Kuchemann and Weber in reference (2). In addition, they will also give the correct spanwise variation of the kink effect due to thickness, which must be obtained by semi-empirically determined interpolation curves in the solution by Kuchemann and Weber. The Woodward equations also treat the spanwise variation of thickness and other planform induced effects such as taper ratio, tip, and crank effects equally well. These same effects are also correctly treated by the distributed constant density trapezoidal panel equations.
Figure 5. Karman-trefftz airfoil pressure distribution at 1.0 degree angle of attack.
Figure 6.- Swept wing alone pressure distribution at 24.5 percent semi-span, 0.0 degree angle of attack.
Figure 7.- Swept wing alone pressure distribution at 65.3 percent semi-span, 0.0 degree angle of attack.
Figure 8. - Swept wing alone pressure distribution at 89.8 percent semi-span, 0.0 degree angle of attack.
Figure 9.- Swept wing alone pressure distribution at 24.5 percent semi-span, 4.2 degrees angle of attack.
Figure 10.- Swept wing alone pressure distribution at 65.3 percent semi-span, 4.2 degrees angle of attack.
Figure 11. - Swept wing alone pressure distribution at 89.8 percent semi-span, 4.2 degrees angle of attack.
Figure 12.- Spanwise distribution of potential form drag due to thickness.
derived by Hess and Smith in reference (22). In fact Woodward's constant
density source panel influence equations are identical to those derived by
Hess and Smith.

As pointed out by Kuchmann and Weber the perturbation velocity due to
thickness for a taper ratio one finite aspect ratio swept wing can be divided
into two parts; 1) the two dimensional infinite sheared solution, and 2) the
kink and tip effects. The two dimensional solution does not produce any
section potential form drag provided the airfoil sections are closed. How-
ever, the kink and tip effects do produce a section drag and thrust, respec-
tively, which when integrated across the span will give zero drag for the
complete wing. Other planform effects such as taper ratio and cranks will
also produce a section drag due to thickness which also integrates to zero for
the complete wing.

All of the above methods will give the correct spanwise distribution
of section potential form drag due to thickness. However, as also pointed
out by Kuchmann and Weber the source lattice does not give the correct edge
effect right at the tip, kink, or crank for a finite number of source lines
in the chordwise direction. However, since this effect is only a function
of the chordwise component of the thickness distribution gradient and the
value of the inverse guedemannian function, with its argument being the sweep
of the source line, at the point where the perturbation velocity is being
computed, this effect is easily added. The region on either side of the tip,
kink, or crank which is not properly handled by the source lattice is a
function of the number of source lines in the chordwise direction and for
practical solutions, which require about twenty source lines per chord to
represent the distribution of thickness, this region is of no significance.
Therefore, due to the superior numerical efficiency associated with the
source lattice influence equations the source lattice appears to be the
best aerodynamic finite element for predicting the perturbation velocity
and potential form drag due to thickness.

Section Induced Drag Due to Lift

There are two basic approaches that have been tried in the past to
solve the problem of predicting the spanwise distribution of induced drag
or section potential form drag due to lift, 1) to accurately solve for the
thin wing net pressure distribution, including the strength of the leading
edge singularity, utilizing precise integration techniques to solve the
aerodynamic influence integral equation, and 2) to utilize a vortex lattice
procedure in conjunction with the Kutta-Joukowsky theorem. Both of these
approaches have failed to predict a spanwise distribution of induced drag due
to lift which when integrated is equal to the induced drag computed in the
Trefftz plane. The reason for this is that in these attempts the assumption
that the vorticity is constant in the spanwise direction along constant percent chord lines, even for only an infinitesimal distance, leads to a nonanalytic influence function for which no finite value exists for the induced velocity at span stations where the gradients of the constant percent chord lines are discontinuous.

Therefore, at span stations where the constant percent chord lines are kinked or cranked the constant vorticity distributed panel procedures, such as Woodward's, give a logarithmic singularity in the downwash. The lifting surface theories, such as Multhopp's or Wagner's, also produce a logarithmic singularity in the downwash which cannot be handled. Wagner's theory, as given in reference (50), has been investigated in great detail and is commented on in appendix E.

In the case of the skewed vortex-lattice the downwash at the control point is not singular, however, the Cauchy principal value does not exist for the downwash on the vortex line at a span station where the vortex lines have discontinuous sweeps. Therefore, the Kutta-Joukowsky theorem will give an infinite section drag at these stations.

All of these problems can be eliminated by using an unswept horseshoe vortex lattice. It is proven in appendix F that if the bound vortex lines are all parallel and the horseshoe lattice is evenly spaced in the lateral direction, the integral of the spanwise distribution of induced drag and the induced drag computed in the Trefftz plane are identical for all planform shapes. This is also true for multiple lifting surfaces, provided they are all parallel, and for lifting surfaces with jet flaps.

It is not proposed that only unswept horseshoe vortex-lattice procedures be used to compute the net pressures or loads on wings of arbitrary shape. However, this appears to be the only numerical integration procedure known at this time which will always give the same value for the induced drag in the near and far fields. During studies of the error involved in using a skewed lattice it was determined that the error in the section induced drag was limited to a very small region on either side of the discontinuity in the sweep of the vortex lines. Also, since most of the wing is represented better by skewed vortex lines (because the lines of constant pressure do coincide with constant percent chord lines over most of the wing) a good overall answer can probably be obtained at less expense, (since fewer skewed vortices are needed in general to represent a wing than unswept vortices) if a skewed vortex lattice is used to compute the net pressures and the unswept vortex lattice is used to compute the drag once the net pressures are known.
Figure 13. Spanwise distribution of potential form drag due to lift.
A comparison of the section induced drag divided by the section lift for four different procedures is shown in figure (13).

Sphere Surface Velocity

The velocity over the surface of a sphere is compared to the exact solution in figure (14).

X-15 Wing-Fuselage-Horizontal Tail-Vertical Tail

Surface velocities and pressure coefficients, section force and moment coefficients, and total configuration force and moment coefficients are computed for the X-15 wing-fuselage-horizontal tail-vertical tail shown in figure (15). The program input for this configuration plus the ventral is given in the sample input section. The program output for this configuration is given in the sample output section. In the program output the fuselage, wing, horizontal tail and vertical tail are designated as components 1, 2, 3, and 4, respectively.

Some results for this configuration are shown in figures (16), (17), (18), and (19). The data for these comparisons were obtained from References (59), (60), and (61). The force data is at .6 Mach number and the pressure coefficient data is at .2 Mach number. All of the theoretical results are for zero Mach number.

The total configuration $C_{Lx}$ was determined experimentally to be .061. The program predicts .0617.
Figure 14 - Velocity ratio around a sphere.
Figure 16. - Normal Load Distribution on X-15 Fuselage at Five Degrees Angle of Attack
Figure 17. Unit Span Load Distribution on X-15 Wing
Figure 18: Unit Span Load Distribution on X-15 Horizontal Tail
COMPUTER PROGRAM USAGE

Program Setup

There are two types of data decks which can be used in this program: 1) the NASA Langley input format described in reference (52) and 2) the NR input format described in the next two sections of this report. Each of these input formats requires a different program setup.

Program setup for NASA format. - In this form of data setup, routine "OUTIN" preprocesses data from Langley's format and creates a card image file for use by the rest of the subroutines. Changes or additions to the data associated with bodies and panels in the Langley form of input created by "OUTIN" are made by body and panel "INFO" decks. There should be a body and panel "INFO" deck for each body and panel in the Langley input array, respectively. These body and panel "INFO" decks utilize the NR data array format.

Additional configuration bodies and panels not described in the Langley data array are added by means of additional body and panel input decks utilizing the NR data array format.

Most of the input from the Langley data array can be directly converted to the NR data array format. The only exception is a nacelle which is converted from a body of revolution to a ring wing. This occurs if the pod data has a nonzero value at the nose of the pod. If the radius at the nose of the pod is zero, the pod is considered a solid body of revolution.

The input data in the NR data array format ahead of the body and panel descriptions pertain to the total configuration and must always be input as an "INFO" deck. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed in figure 21. Figure 20 contains the entire deck setup.
Figure 20.—Deck setup for use with NASA Langley data array format.
Figure 22. - Deck setup for use with NR data array format.
Program setup for NR format. - In this form of data setup all input decks utilize the NR data array format. The total configuration or universal "INFO" deck is followed by an input deck for each body and then a deck for each panel. Control cards for the C.D.C. 6000 series using Scope 3.1 are listed for this form of data input in figure 21. Figure 22 contains the entire deck setup.

Card No.

1. SEQUENCE CARD
2. CHARGE CARD
3. PFSTAD, CM31000, T7777, P6
4. RUN, S.
5. SET, O.
6. LGO.

Figure 21.- Control card deck sequence

Input Format

The NR data array format is given in this section. Data locations of all input data with limits on size of inputs are designated. A more detailed description of the input is given in the next section.
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<td>(If the list is the same as in loc 41-89 it can be omitted)</td>
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<td>(deg)</td>
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<td>$R_i \text{ or } \left( \frac{z_i - z_{i-1}}{z_{i} - z_{i-1}} \right)$ values at $X_i$ stations</td>
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<td>$R_i \text{ or } \left( \frac{z_i - z_{i-1}}{z_{i} - z_{i-1}} \right)$</td>
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<td>(Input along longitudinal direction at first lateral station, then second lateral station, and etc)</td>
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<td>(max 49)</td>
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<td>$X_{E1}$</td>
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<td>37</td>
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<td>(units)</td>
</tr>
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# FORTRAN FIXED 10 DIGIT DECIMAL DATA

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<td>Lateral vortex grid at first ( X_{BE} ) station</td>
<td>( \phi_E ) or ( S/S_{max} ) _1 ( \text{deg} )</td>
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<td>( \phi_E ) or ( S/S_{max} ) _2 ( \text{deg} )</td>
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<td>( \phi_E ) or ( S/S_{max} ) _2 ( \text{deg} )</td>
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<td>( J_{M1} )</td>
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<td>( J_{M2} )</td>
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<td>( J_{N2} )</td>
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<tr>
<td>61</td>
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</table>

| 1      | 3 1 9 0        | Lateral constraint segments |
| 13     |                | $J_{SN1}$   |                 |
| 25     |                | $J_{SN2}$   |                 |
| 37     |                |             |                 |
| 49     |                |             |                 |
| 61     |                |             |                 |

<p>| 1      | 3 2 1 0        | Longitudinal constraint function sequence No. |
| 13     |                | 1. 1.0 |
| 25     |                | 2. $1.0 / \left[ 1 + \left( \frac{2B(N_B/N_B) - 2B(N_B/N_B)}{N_B/N_B} \right)^2 / (N_B/N_B) \right]^{1/2}$ |
| 37     |                | 3. cot($\phi_B/2$) |
| 49     |                | 4. cot[$(\pi/2) - (\phi_B/2)$] |
| 61     |                | 5. $\sin[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$ |
| 1      | 3 2 1 5        | 6. $\cos[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$ |
| 13     |                | 7. $[(X_B/C_B) - (X_B/C_B)_o] / [(X_B/C_B)_f - (X_B/C_B)_o]$ |
| 25     |                | 8. $\sin[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$ |
| 37     |                | 9. $\cos[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$ |
| 49     |                | 10. $[(X_B/C_B) - (X_B/C_B)_o] / [(X_B/C_B)_f - (X_B/C_B)_o]^2$ |
| 61     |                |             |                 |</p>
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<td>$L_{S_1}(x_B/C_B)$</td>
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<tr>
<td>3320</td>
<td>$L_{S_2}(x_B/C_B)$</td>
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<td>3370</td>
<td>$L_{S_3}(x_B/C_B)$</td>
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3250

Lateral constraint function sequence No.

1. $1.0$

2. $N_{ZB}/[N_3/Z + N_{YB}Z]^{1/2}$

3. $N_{YB}/[N_{ZB}Z + N_{YB}Z]^{1/2}$

4. $\sin[\pi(\phi_E - \phi_0)/(\phi_f - \phi_0)]$

5. $\cos[\pi(\phi_E - \phi_0)/(\phi_f - \phi_0)]$

3255

6. $[\pi(x_B - \phi_0)/(\phi_f - \phi_0)]$ or $[(\eta_B - \eta_0)/(\eta_f - \eta_0)]$  

7. $\sin[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$

8. $\cos[2\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]$

9. $[\pi(\phi_B - \phi_0)/(\phi_f - \phi_0)]^2$ or $[(\eta_B - \eta_0)/(\eta_f - \eta_0)]^2$

3270

First special longitudinal constraint function

$(x_B/C_B)_1$

$L_{S_1}(x_B/C_B)_1$

$(x_B/C_B)_2$

$L_{S_2}(x_B/C_B)_2$
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<td>( L_{S_2} \frac{X_B}{C_B} ) (_1)</td>
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<td>Reference span</td>
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<td>(1.) if just panel (-1.) no image (2.) antisymmetric</td>
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<td>normal side (1.) negative normal side</td>
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<td>Trailing vortex indicator (0.) straight (1.) force free</td>
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<td>(1.) percent deflection relative to chord line</td>
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### FORTRAN FIXED IO DIGIT DECIMAL DATA

<table>
<thead>
<tr>
<th>DECK NO.</th>
<th>PROGRAMMER</th>
<th>DATE</th>
<th>PAGE</th>
<th>JOB NO.</th>
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</thead>
</table>

<table>
<thead>
<tr>
<th>NUMBER</th>
<th>IDENTIFICATION</th>
<th>DESCRIPTION</th>
<th>DO NOT KEY PUNCH</th>
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<tbody>
<tr>
<td>1</td>
<td>3.4.3.0</td>
<td>Panel longitudinal vortex indicator (0.) even ( \Delta \phi ) (-1.) even ( \Delta (X/C) ) (1.) given ( X/C ) Panel lateral vortex indicator (0.) even ( \Delta n ) (-1.) even ( \Delta \Theta ) (1.) given ( n ) ( Y_p ) ( (\text{units}) ) ( Y_{p_0} ) Panel Origin ( (\text{units}) ) ( Z_{p_0} ) ( (\text{units}) )</td>
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<td>1</td>
<td>3.4.3.5</td>
<td>Area of influence in longitudinal direction</td>
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</tr>
<tr>
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<td>3.4.4.0</td>
<td>Area of influence in lateral direction</td>
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</tr>
<tr>
<td>1</td>
<td>3.4.4.0</td>
<td>No. of longitudinal subpanels</td>
<td>(max 40)</td>
</tr>
<tr>
<td>1</td>
<td>3.4.4.0</td>
<td>No. of lateral subpanels</td>
<td>(max 40)</td>
</tr>
<tr>
<td>1</td>
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<td>No. of longitudinal constraint functions</td>
<td>(max 15)</td>
</tr>
<tr>
<td>1</td>
<td>3.4.4.0</td>
<td>No. of lateral constraint functions</td>
<td>(max 20)</td>
</tr>
<tr>
<td>1</td>
<td>3.4.4.5</td>
<td>No. of longitudinal control points</td>
<td>(max 40)</td>
</tr>
<tr>
<td>1</td>
<td>3.4.4.5</td>
<td>No. of lateral control points</td>
<td>(max 40)</td>
</tr>
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<td>No. of points defining panel perimeter</td>
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<td>No. of trailing vortex to which panel inboard trailing vortex is attached</td>
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<td>No. of panel to which outboard trailing vortex is attached</td>
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<tr>
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<tr>
<td>1</td>
<td>3.4.4.5</td>
<td>No. of leading edge control surfaces</td>
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</tr>
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<td>1</td>
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<td>No. of trailing edge control surfaces</td>
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# FORTRAN FIXED 10 DIGIT DECIMAL DATA

<table>
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<th>DECK NO.</th>
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<th>DATE</th>
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<td>Panel perimeter description</td>
<td>(units)</td>
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<tr>
<td></td>
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<td>$x_1$ (Note: The chord is not given in the root section)</td>
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</tr>
<tr>
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<td>3455</td>
<td>$y_1$</td>
<td>(units)</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>$z_1$</td>
<td>(units)</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td>$c_1$</td>
<td>(units)</td>
</tr>
<tr>
<td>49</td>
<td></td>
<td>$x_2$</td>
<td>(units)</td>
</tr>
<tr>
<td>61</td>
<td></td>
<td>$y_2$</td>
<td>(units)</td>
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<tr>
<td></td>
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<td>$z_2$</td>
<td>(units)</td>
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<td>(units)</td>
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<td>Longitudinal stations where camber is defined</td>
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<tr>
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<tr>
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<td>13</td>
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<td>$\eta_1$</td>
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<td>25</td>
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*Diagram of data representation*
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<th>DESCRIPTION</th>
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<td>Mean camber surface definition</td>
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<tr>
<td>25</td>
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<td>((x/c)_2, \eta_1) or (Z_c/c) ([(x/c)_2, \eta_1])</td>
<td>(Rad)</td>
</tr>
<tr>
<td>37</td>
<td></td>
<td>(\alpha [\alpha (x/c)_1, \eta_2]) or (Z_c/c) ([\alpha (x/c)_1, \eta_2])</td>
<td>(Rad)</td>
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<tr>
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<td>75</td>
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<td>(Rad)</td>
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<td>(Rad)</td>
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<td>(\epsilon (\eta_3))</td>
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<td>73</td>
<td>(\epsilon (\eta_9))</td>
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<td>(\epsilon (\eta_{10}))</td>
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<td>((x/c)_9)</td>
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<td>((x/c)_{10})</td>
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</tr>
<tr>
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<td>73</td>
<td>(\eta_9)</td>
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<td>(\eta_{10})</td>
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<td>( z_t/c [(x/c)_2, \eta_1] )</td>
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</tr>
<tr>
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<td>( \vdots )</td>
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<td></td>
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<td>( z_t/c [(x/c)_1, \eta_2] )</td>
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<td>( x/c)_1 )</td>
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<td>( (x/c)_2 )</td>
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</tr>
<tr>
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<td></td>
<td>( (x/c)_2 )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( \eta_2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \vdots )</td>
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<td>( J_{M1} )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( J_{M2} )</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>( \vdots )</td>
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<tr>
<td>NUMBER</td>
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<td>DESCRIPTION</td>
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<td>4760</td>
<td>List of lateral constraint functions</td>
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<td>4780</td>
<td>Special lateral constraint functions</td>
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<td>4880</td>
<td>Special longitudinal constraint functions</td>
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### FORTRAN FIXED 10 DIGIT DECIMAL DATA

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<td>4 8 8 5</td>
<td>Special Longitudinal Constraint Functions Continued</td>
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<td>13</td>
<td></td>
<td>$x_0$ (Outboard L.E. or T.E. Corner of Control Surface) (Units)</td>
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</tr>
<tr>
<td>25</td>
<td></td>
<td>$x_{\eta_1}$ (Inboard Edge - Hinge Line Intersection) (Units)</td>
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</tr>
<tr>
<td>37</td>
<td></td>
<td>$x_{\eta_0}$ (Outboard Edge - Hinge Line Intersection) (Units)</td>
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</tbody>
</table>

Note: Use same format for special longitudinal constraint functions due to additional control surfaces. Start data sets at 4890, 4900, and etc.

---

*Form 114-C-17 REV. 7-86*
### Input Description

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<thead>
<tr>
<th>Location</th>
<th>Description</th>
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<tbody>
<tr>
<td>1</td>
<td>Total number of bodies used to represent the configuration.</td>
</tr>
<tr>
<td>2</td>
<td>Total number of panels used to represent the configuration. This number does not include those due to images when location 3426 has a zero in it.</td>
</tr>
<tr>
<td>3</td>
<td>Mach number</td>
</tr>
<tr>
<td>4</td>
<td>Total configuration reference area. (Items not needed if data format from reference (52) is used.)</td>
</tr>
<tr>
<td>5</td>
<td>Total configuration longitudinal reference length.</td>
</tr>
<tr>
<td>6</td>
<td>Total configuration lateral reference length.</td>
</tr>
<tr>
<td>7</td>
<td>X component of center of gravity position vector.</td>
</tr>
<tr>
<td>8</td>
<td>Y component of center of gravity position vector.</td>
</tr>
<tr>
<td>9</td>
<td>Z component of center of gravity position vector.</td>
</tr>
<tr>
<td>10</td>
<td>Angle of attack.</td>
</tr>
<tr>
<td>11</td>
<td>Angle of sideslip.</td>
</tr>
<tr>
<td>12</td>
<td>Nondimensional roll rate ( P/(2V_\infty/b) ).</td>
</tr>
<tr>
<td>13</td>
<td>Nondimensional pitch rate ( q/(2V_\infty/c) ).</td>
</tr>
<tr>
<td>14</td>
<td>Nondimensional yaw rate ( \gamma/(2V_\infty/b) ).</td>
</tr>
<tr>
<td>15</td>
<td>The number of the component. The bodies are numbered first and then the panels.</td>
</tr>
<tr>
<td>16</td>
<td>The body reference area. (Items not needed if data format from reference (52) is used.)</td>
</tr>
<tr>
<td>17</td>
<td>The body chord.</td>
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* Items not needed if data format from reference (52) is used.
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<tr>
<th>Location</th>
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<tbody>
<tr>
<td>18</td>
<td>This is an indicator to bypass the calculation of the aero-dynamic influence matrices due to the vortices of this body. Input a zero to compute new matrices and a minus one if the calculation of the matrices is to be bypassed.</td>
</tr>
<tr>
<td>*19</td>
<td>This is a symmetry indicator used to take advantage of both symmetrical geometries and loadings. If both the body geometry and the vortex strength are symmetrical about the X-Z plane input a zero, if only the body geometry is symmetrical and not the vortex strengths input a one, and if neither symmetry exists input a minus one.</td>
</tr>
<tr>
<td>*20</td>
<td>This is an indicator used to signify whether cartesian or polar coordinates are used to input the body cross-sections. If polar coordinates are used input a zero and if cartesian coordinates are used input a one.</td>
</tr>
<tr>
<td>21</td>
<td>This is an indicator used to signify whether the lateral scaling factors are equal in the Y and Z directions. Input a zero if they are equal and a one if they are not equal. The $Z_{BM}$ array is not input if $Y_{BM} = Z_{BM}$.</td>
</tr>
<tr>
<td>*22</td>
<td>This indicator is used to signify whether the body cross-sections are placed perpendicular to the mean camber line or the X axis. Input a zero if the cross-section is to be placed perpendicular to the mean camber line or a one if it is to be placed perpendicular to the X axis.</td>
</tr>
<tr>
<td>23</td>
<td>This indicator signifies the type of subpanel spacing used on the body in the longitudinal direction. Input a zero if the subpanels are to be spaced at even increments of $\phi$, where $\phi = \cos^{-1} [1-2(x/c)]$, input a minus one if the subpanels are to be spaced at even increments of $X$, and input a one if the spacing is specified at a given set of $X$ stations. The $X$ stations are to be given in locations 1850 - 2149.</td>
</tr>
<tr>
<td>24</td>
<td>This indicator signifies the type of subpanel spacing used on the body in the lateral direction. Input a zero if the subpanels are to be spaced at even increments of $\theta$ or $n_B = (S/S_{max})$, a one if the subpanel side edges are given at a set of $\theta$ or $n_B$ stations, and a minus one if the subpanel edges are at the same lateral stations as the body geometry is defined. The subpanel side edges are input in locations 2200 - 2799.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>25</td>
<td>This is the longitudinal distance, in terms of the length of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a longitudinal distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.</td>
</tr>
<tr>
<td>26</td>
<td>This is the lateral distance, in terms of the width of the influencing quadrilateral, at which the contribution of the quadrilateral vortex to the perturbation velocity is considered small enough to be neglected. Points at a lateral distance, away from the centroid of the vortex, larger than this value are not cycled through the influence equations in order to save computer time.</td>
</tr>
<tr>
<td>*27</td>
<td>This is the X component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>*28</td>
<td>This is the Y component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>*29</td>
<td>This is the Z component of the origin of the body coordinate frame.</td>
</tr>
<tr>
<td>30</td>
<td>This is the number of subpanels in the longitudinal direction on the body.</td>
</tr>
<tr>
<td>31</td>
<td>This is the number of subpanels in the lateral direction on the body.</td>
</tr>
<tr>
<td>32</td>
<td>This is the number of equally spaced divisions the body subpanels are divided into in the longitudinal direction. These divisions are used to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>This is the number of equally spaced divisions the body subpanels are divided into in the lateral direction. These divisions are used to map the vortex grid closer to the actual body surface and to compute the subareas which in turn are used to integrate the surface pressures to obtain loads and moments. This number must be an odd integer.</td>
</tr>
<tr>
<td>34</td>
<td>This is the number of functions used to constrain the body surface vorticity in the longitudinal direction. The list of functions used is given in locations 3210-3249. These same functions are used over each longitudinal constraint function segment. The list defining these segments is given in locations 3140-3189. The maximum number of longitudinal constraint functions is 50.</td>
</tr>
<tr>
<td>35</td>
<td>This is the number of functions used to constrain the body surface vorticity in the lateral direction. The list of functions used is given in locations 3250-3269. These same functions are used over each lateral constraint function segment. The list defining these segments is given in locations 3190-3209. The maximum number of lateral constraint functions is 20.</td>
</tr>
<tr>
<td>36</td>
<td>This is the number of segments in the longitudinal direction over which the body longitudinal constraint functions are defined. The list of subpanels defining the segments is given in locations 3140-3189. The maximum number of segments is 50.</td>
</tr>
<tr>
<td>37</td>
<td>This is the number of segments around the circumference of the body over which the lateral constraint functions are defined. The list of subpanels defining the segments is given in locations 3190-3209. The maximum number of segments is 20.</td>
</tr>
<tr>
<td>38</td>
<td>This is the number of control points in the longitudinal direction on the body. The maximum number of points in the longitudinal direction is 299. The list of body control points in the longitudinal direction is given in location 2800, 3099.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
<td>This is the number of control points in the lateral direction on the body. The maximum number of points in the lateral direction is 40. This list of control points is given in locations 3100 - 3139.</td>
</tr>
<tr>
<td>*40</td>
<td>This is the number of longitudinal stations where the body cross-sections are defined. The maximum number is 44.</td>
</tr>
<tr>
<td>*41-89</td>
<td>This is the list of longitudinal stations where the body cross-sections are defined. This list starts at the nose and ends at the aft end if the body is solid. For a flow through body this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.</td>
</tr>
<tr>
<td>*85</td>
<td>This is the number of longitudinal stations where the body cross-section lateral stations are defined. The maximum number of these longitudinal stations is 44.</td>
</tr>
<tr>
<td>*86-129</td>
<td>This is the list of longitudinal stations where the body cross-section lateral stations are defined. The body cross-section lateral stations are constant between each longitudinal station in this list. If this list is the same as that in locations 41-89, it can be omitted. If the body is solid, this list starts at the nose and ends at the aft end. If the body is a flow through type, this list begins at the aft end, continues along the inner surface to the nose of the body, and then along the outer surface to the aft end.</td>
</tr>
<tr>
<td>*130</td>
<td>This is the number of lateral stations at the first longitudinal station in the list at locations 91-129. The maximum number of these lateral stations is 39.</td>
</tr>
<tr>
<td>*131</td>
<td>This is the list of lateral stations at the first longitudinal station in the list at locations 91-129. These lateral stations where the body is defined can be given by the angle $\theta$, the lateral fraction $[(\Delta x)/Y]$, or fraction of lateral circumferential length $S$. Note: Lists of lateral stations at the other longitudinal stations, given in the list in locations 91-129, follow this data as shown in the input format.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>*800-1599</td>
<td>In these locations the body cross-section radii or fractional distances ([C_{B_1} - \Delta Z_B] / Z_M), for each of the (X_{i_1}) longitudinal stations and (\theta_i) or ([C_{B_1} - \Delta Z_B] / Y_M) lateral stations, are input.</td>
</tr>
<tr>
<td>*1600</td>
<td>This is the number of longitudinal stations where the multiplication factors (Y_{BM}) and (Z_{BM}) and the translation increments (\Delta Y) and (\Delta Z) are given. The maximum number of stations that can be used here is 49.</td>
</tr>
<tr>
<td>1601-1649</td>
<td>This is the list of longitudinal stations where the multiplication factors (Y_{BM}) and (Z_{BM}) and the translation increments (\Delta Y) and (\Delta Z) are given.</td>
</tr>
<tr>
<td>1650-1699</td>
<td>This is the list of multiplication factors (Z_{BM}) at the longitudinal stations given in locations 1601-1649.</td>
</tr>
<tr>
<td>1750-1799</td>
<td>This is the list of translation increments (\Delta Y) at the longitudinal stations given in locations 1601-1649.</td>
</tr>
<tr>
<td>*1800-1849</td>
<td>This is the list of translation increments (\Delta Z_B) at the longitudinal stations given in locations 1601-1649.</td>
</tr>
<tr>
<td>*1850-2149</td>
<td>This is the list of longitudinal stations where the body subpanel edges are defined.</td>
</tr>
<tr>
<td>2150</td>
<td>This is the number of longitudinal stations where the body lateral subpanel edges are defined. The maximum number of these longitudinal stations is 49.</td>
</tr>
<tr>
<td>2151-2199</td>
<td>This is the list of longitudinal stations where the body lateral subpanel edges are defined. This list can be omitted if it is the same as that in locations 1850-2149.</td>
</tr>
<tr>
<td>2200</td>
<td>This is the list of lateral subpanel edges at the first longitudinal station given in the list at locations 2151-2199. These stations can be defined by (\theta)'s or fractions of body circumference (n_B = (S / S_{max})). Note; Lists of lateral subpanel edges at the other longitudinal stations, given in the list at locations 2151-2199, follow this input as shown in the input format.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2800-3099</td>
<td>This is the list of the number of the subpanels where the control points are located in the longitudinal direction on the body. The integer in this list indicates the number of the subpanel, aft of the nose of a solid body or aft of the tail end on the inner surface and around to the outer surface of a flow through body, at which a control point is placed. If no longitudinal constraint functions are used, this input can be omitted.</td>
</tr>
<tr>
<td>3100-3139</td>
<td>This is the list of control point locations in the lateral direction on the body. The integer in this list indicates the number of the subpanel in the lateral direction from the top of the body at which a control point is placed. If no lateral constraint functions are used, this input can be omitted.</td>
</tr>
<tr>
<td>3140-3189</td>
<td>This is the list of longitudinal constraint segment boundaries. The integers in this list indicate the subpanels between which the longitudinal constraint functions are applied. For example, if this list included the longitudinal number of every tenth subpanel, the longitudinal constraint functions would be applied over a range of ten subpanels and repeated over the segments defined by every tenth subpanel.</td>
</tr>
<tr>
<td>3190-3209</td>
<td>This is the list of lateral constraint segment boundaries. This list is utilized in the same manner as the longitudinal constraint segment boundaries.</td>
</tr>
<tr>
<td>3210-3249</td>
<td>This is the list of longitudinal constraint functions used on the body. The functions to be used are signified by entering here the sequence number of the function as given in the input format. If special functions are to be used the data location where the function is described is input here.</td>
</tr>
<tr>
<td>3250-3269</td>
<td>This is the list of lateral constraint functions used on the body. The same procedure is used here to signify the desired functions as is used in the definition or the longitudinal constraint functions.</td>
</tr>
<tr>
<td>3270-3419</td>
<td>In these locations the special longitudinal constraint functions are described as shown in the input format. These are described in tabular form.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3420</td>
<td>This is the component number for the panels.</td>
</tr>
<tr>
<td>*3421</td>
<td>This is the panel reference area.</td>
</tr>
<tr>
<td>3422</td>
<td>This is the panel reference chord.</td>
</tr>
<tr>
<td>3423</td>
<td>This is the panel lateral reference length.</td>
</tr>
<tr>
<td>3425</td>
<td>This is an indicator to bypass the calculation of the aerodynamic influence matrices due to the vortices of this panel. Input a zero if a new matrix is computed and a minus one if the calculation of the matrix is to be bypassed.</td>
</tr>
<tr>
<td>*3426</td>
<td>This is a symmetry indicator used to take advantage of a configuration's symmetry to save input effort and computer time. Input a zero if both the panel geometry and the vortex strengths have images on the port side of the X-Z plane, a one if just the panel geometry has an image, a minus one if neither have an image, and a two if the vortex strengths on the panel and its image are antisymmetric.</td>
</tr>
<tr>
<td>3427</td>
<td>This indicator is used to signify whether the panel is attached to the positive normal or negative normal side of another panel. Input a zero if it is attached to the positive normal side and a one if it is attached to the negative normal side.</td>
</tr>
<tr>
<td>3428</td>
<td>This indicator is used to signify whether the wake is force free or fixed. Input a zero if the wake is fixed and a one if it is force free. If any panel has a force free wake, the wake or a body it is attached to is also assumed to be force free.</td>
</tr>
<tr>
<td>*3429</td>
<td>This indicator signifies whether the mean camber surface of the panel is described in terms or local angles or attack or as fractions of chord. Input a zero if local angles of attack are input and a one if fractions of chord are input.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3430</td>
<td>This input indicates the type of longitudinal subpanel spacing that is used. If the subpanels are spaced at even increments of ϕ input a zero, if they are at equal increments of percent chord input a minus one, and if they are input as a given set of percent chord use a one.</td>
</tr>
<tr>
<td>3431</td>
<td>This input indicates the type of lateral subpanel spacing that is used on a panel. Input a zero if the spacing is at equal increments of η, a minus one if it is at equal increments of θ, and a one if the spacing is specified at a given set of η stations.</td>
</tr>
<tr>
<td>*3432</td>
<td>This is the X component of the panel origin point.</td>
</tr>
<tr>
<td>*3433</td>
<td>This is the Y component of the panel origin point.</td>
</tr>
<tr>
<td>*3434</td>
<td>This is the Z component of the panel origin point.</td>
</tr>
<tr>
<td>3435</td>
<td>This is the area of influence of a quadrilateral vortex on a panel in the longitudinal direction. This input is in terms of fraction of quadrilateral vortex length.</td>
</tr>
<tr>
<td>3436</td>
<td>This is the area of influence of a vortex on a panel in the lateral direction. This input is in terms of fraction of vortex width.</td>
</tr>
<tr>
<td>3437</td>
<td>This is the number of subpanels in the longitudinal direction on the panel. The maximum number of longitudinal subpanels is 40.</td>
</tr>
<tr>
<td>3438</td>
<td>This is the number of subpanels in the lateral direction on the panel. The maximum number of lateral subpanels is 40.</td>
</tr>
<tr>
<td>3439</td>
<td>This is the number of longitudinal constraint functions on the panel. This number does not include special functions due to control surfaces. The maximum number is 15.</td>
</tr>
<tr>
<td>3440</td>
<td>This is the number of lateral constraint functions on the panel. This number does include special functions due to control surfaces. The maximum number is 20.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3441</td>
<td>This is the number of control points in the longitudinal direction on the panel. The maximum number is 40.</td>
</tr>
<tr>
<td>3442</td>
<td>This is the number of control points in the lateral direction on the panel. The maximum number is 40.</td>
</tr>
<tr>
<td>3443</td>
<td>This is the number of points used to define the panel perimeter. In the root section the perimeter is defined by X, Y, and Z components of points along the leading and trailing edges. The outboard section is defined by X, Y, and Z components of points along the leading edge and the local chord. The root section leading edge is input first, then the outboard section, and then the root section trailing edge. The maximum number of points is 50.</td>
</tr>
<tr>
<td>3444</td>
<td>This input is the number of the body or panel to which this panel's inboard trailing vortex is attached.</td>
</tr>
<tr>
<td>3445</td>
<td>This is the number of the trailing vortex leg or subpanel side edge to which this panel's inboard trailing vortex is attached. The subpanel side edges are numbered consecutively starting at the top of a body or at the inboard edge of a panel and going in the clockwise direction when an observer is looking in the negative X direction.</td>
</tr>
<tr>
<td>3446</td>
<td>This is the number of the panel to which the outboard trailing vortex of this panel is attached.</td>
</tr>
<tr>
<td>3447</td>
<td>This is the number of the trailing vortex leg or subpanel side edge to which this panel's outboard trailing vortex is attached.</td>
</tr>
<tr>
<td>3448</td>
<td>This is the number of leading edge control surfaces on the panel.</td>
</tr>
<tr>
<td>3449</td>
<td>This is the number of trailing edge control surfaces on the panel.</td>
</tr>
<tr>
<td>3450-3599</td>
<td>In these locations the panel perimeter is described.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 3600</td>
<td>This is the number of longitudinal stations where the panel camber is described. The maximum number is 30.</td>
</tr>
<tr>
<td>*3601-3629</td>
<td>This is the list of chord fractions which define the chordwise locations where the panel camber is defined.</td>
</tr>
<tr>
<td>* 3630</td>
<td>This is the number of lateral stations where the panel camber and twist is defined. The maximum number is 30.</td>
</tr>
<tr>
<td>*3631-3659</td>
<td>This is the list of lateral surface length fraction ( \eta ) where the panel camber and twist is defined.</td>
</tr>
<tr>
<td>*3660-4099</td>
<td>This is the table of local angle of attack or deflection, in terms of fraction of chord, of the panel mean camber surface. The table is input as shown in the input format.</td>
</tr>
<tr>
<td>4100-4129</td>
<td>This is the table of panel twist.</td>
</tr>
<tr>
<td>* 4130</td>
<td>This is the number of longitudinal stations where the panel thickness is described. The maximum number is 30.</td>
</tr>
<tr>
<td>*4131-4159</td>
<td>This is the list of chord fractions which define the chordwise locations where the panel thickness is defined.</td>
</tr>
<tr>
<td>* 4160</td>
<td>This is the number of lateral stations where the panel thickness is defined. The maximum number is 30.</td>
</tr>
<tr>
<td>*4161-4189</td>
<td>This is the list of lateral surface length fraction ( \eta ) where the panel thickness is defined.</td>
</tr>
<tr>
<td>*4190-4599</td>
<td>This is the table of panel thickness in terms of fraction of chord. The table is input as shown in the input format.</td>
</tr>
<tr>
<td>4600-4639</td>
<td>This is the list of longitudinal subpanel edge locations in terms of fraction of chord.</td>
</tr>
<tr>
<td>4640-4679</td>
<td>This is the list of lateral subpanel side edge locations in terms of ( \eta ).</td>
</tr>
<tr>
<td>4680-4719</td>
<td>This is the list of panel control points in the longitudinal direction.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
<table>
<thead>
<tr>
<th>Location</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>4720-4759</td>
<td>This is the list of panel control points in the lateral direction.</td>
</tr>
<tr>
<td>4760-4779</td>
<td>This is the list of exponents ( W ) for the standard lateral constraint functions or data locations of the special lateral constraint functions.</td>
</tr>
<tr>
<td>4780-4879</td>
<td>In these locations the special panel lateral constraint functions are described. The input is as shown in the input format.</td>
</tr>
<tr>
<td>4880 - ...</td>
<td>In these locations the special panel longitudinal constraint functions are described. The input is as shown in the input format.</td>
</tr>
</tbody>
</table>

* Items not needed if data format from reference (52) is used.
SAMPLE INPUT
<table>
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<th>1.1.0</th>
<th>4.0</th>
<th>20.0</th>
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<th>0.0</th>
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</tr>
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## PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPONENT 3

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(X/CR) CP  (Y/CR) CP  (Z/CR) CP
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The theory and program described herein are capable of predicting surface velocities and pressure coefficients and section, control surface, and total configuration loads and moments for a diverse class of airplanes. In particular the source - vortex lattice with second order corrections was shown to agree well with the exact solution for a Karman-Trefftz airfoil section. Also, the quadrilateral vortex element predicted surface velocities which agreed well with the exact solution for a sphere.

Because of the options built into the program simple configurations can be input with a minimum of effort while the capability exists to input complicated configurations in significant detail. The program is general enough to analyze complete configurations at angle of attack and in side slip, pitching motion, rolling motion, yawing motion, and with control surfaces deflected. The configuration can be run in each of these modes to obtain static and rotary stability derivatives or in a combination of the modes to predict the loads and moments on the configuration while in a quasi-steady maneuver. The program also has the capability to account for a free wake while operating in any of the above modes.

Within the scope of this study a significant effort was devoted toward understanding the limitations of and the relationships between the different types of aerodynamic finite elements and lifting surface theories available at present. It has been concluded that those used in this analysis are presently the most efficient and generally available. It was also shown that the spanwise variation of potential form drag due to both thickness and lift are correctly computed by the program described in this report.
Appendix A

NUMERICAL PROCEDURES

Discussions of the prime numerical procedures used within the program are given in this appendix. There are essentially three such procedures; (1) straight line interpolation and extrapolation, (2) controlled deviation interpolation, and (3) Householder's simultaneous equation solution.

For straight line interpolation and extrapolation about two given points \((X_1, Y_1)\) and \((X_2, Y_2)\);

\[
Y = \alpha Y_2 + (1-\alpha) Y_1 \tag{1}
\]

where

\[
\alpha = \frac{X-X_1}{X_2-X_1} \tag{2}
\]

The slope \(\frac{dY}{dX}\) for this case is given by;

\[
\frac{dY}{dX} = \alpha' (Y_2-Y_1) \tag{3}
\]

where

\[
\alpha' = \frac{1}{X_2-X_1} \tag{4}
\]

In the case of the controlled deviation interpolation method (CODIM) parabolae are used to curve fit a set of four given points \((X_{N-1}, Y_{N-1}), (X_N, Y_N), (X_{N+1}, Y_{N+1}),\) and \((X_{N+2}, Y_{N+2})\) to obtain interpolated \(Y\) and \(dY/dX\) values for \(X_N \leq x \leq X_{N+1}\). Only that information, relative to this method, which is necessary to judiciously pick input points will be discussed here. A complete derivation is given in reference (20).
One parabola \( P_1 \) is fit through \((X_{N-1}, Y_{N-1}), (X_N, Y_N), \) and \((X_{N+1}, Y_{N+1})\). The other parabola \( P_2 \) is fit through \((X_N, Y_N), (X_{N+1}, Y_{N+1}), \) and \((X_{N+2}, Y_{N+2})\). This curve fitting process involves the solution of two sets of simultaneous equations. If,

\[
P_1 = A_1 X^2 + B_1 X + C_1
\]

and

\[
P_2 = A_2 X^2 + B_2 X + C_2
\]

Then,

\[
\begin{align*}
\begin{bmatrix} A_1 \\ B_1 \\ C_1 \end{bmatrix} &= \begin{bmatrix} X_{N-1}^2 & X_{N-1} & 1 \\
X_N^2 & X_N & 1 \\
X_{N+1}^2 & X_{N+1} & 1 \end{bmatrix}^{-1} \begin{bmatrix} Y_{N-1} \\ Y_N \\ Y_{N+1} \end{bmatrix} \\
&= \begin{bmatrix} Y_{N+1} \\ Y_N \\ Y_{N-1} \end{bmatrix}
\end{align*}
\]

and

\[
\begin{align*}
\begin{bmatrix} A_2 \\ B_2 \\ C_2 \end{bmatrix} &= \begin{bmatrix} X_N^2 & X_N & 1 \\
X_{N+1}^2 & X_{N+1} & 1 \\
X_{N+2}^2 & X_{N+2} & 1 \end{bmatrix}^{-1} \begin{bmatrix} Y_N \\ Y_{N+1} \\ Y_{N+2} \end{bmatrix} \\
&= \begin{bmatrix} Y_{N+2} \\ Y_{N+1} \\ Y_N \end{bmatrix}
\end{align*}
\]

The interpolated values of \( Y \) and \( dY/dX \) between \( X_N \) and \( X_{N+1} \) are defined by either \( P_1 \), \( P_2 \), or a linear combination of \( P_1 \) and \( P_2 \). The amount of \( P_1 \) and \( P_2 \) used in the linear combination is determined by comparing both of the parabolae to the straight line.

\[
S = \alpha Y_{N+1} + (1- \alpha) Y_N
\]

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Therefore;

\[
\frac{dD}{dx} = \frac{d\alpha}{dx} E_1 + \alpha \frac{dE_1}{dx} - \frac{d\alpha}{dx} E_2 + (1-\alpha) \frac{dE_2}{dx} \tag{18}
\]

\[
Y = \begin{cases} 
  P_1 & \text{for } X = X_N \\
  \frac{\alpha E_1 P_2 + (1-\alpha)E_2 P_1}{\alpha E_1 + (1-\alpha)E_2} & \text{for } X_N < X < X_{N+1} \\
  P_2 & \text{for } X = X_{N+1} 
\end{cases} 
\tag{19}
\]

and

\[
\frac{dY}{dx} = \begin{cases} 
  \frac{dP_1}{dx} & \text{for } X = X_N \\
  \frac{dN}{dx} /D - N \frac{dD}{dx} /D^2 & \text{for } X_N < X < X_{N+1} \\
  \frac{dP_2}{dx} & \text{for } X = X_{N+1} 
\end{cases} 
\tag{20}
\]

In the case of an end interval \(X_{N-1} \leq x \leq X_N\); \(P_1\) is set equal to;

\[
P_1 = S + K(P_2 - S) \tag{21}
\]

where

\[
K = 1 - \frac{|M_1| - |M_2|}{|M_1| + |M_2|} \tag{22}
\]
where

\[ \alpha = \frac{Y_{N+1} - Y_N}{X_{N+1} - X_N} \quad (10) \]

The parabola which has the least deviation from the straight line is given the greatest weight. The weighting factors \( E_1 \) and \( E_2 \) are determined as follows:

\[ E_1 = \left| P_1 - S \right| \quad (11) \]
\[ E_2 = \left| P_2 - S \right| \quad (12) \]

The weighted expression for \( Y \) in the range \( X_N \leq x \leq X_{N+1} \) is then;

\[ Y = \frac{\alpha E_1 P_2 + (1 - \alpha) E_2 P_1}{E_1 + (1 - \alpha) E_2} \quad (13) \]

The derivative \( dY/dX \) in the range \( X_N \leq x \leq X_{N+1} \) is then;

\[ \frac{dY}{dX} = \frac{dN}{dx}/D - N \frac{dD}{dx}/D^2 \text{ for } D \neq 0 \quad (14) \]

where

\[ N = \alpha E_1 P_2 + (1 - \alpha) E_2 P_1 \quad (15) \]
\[ D = \alpha E_1 + (1 - \alpha) E_2 \quad (16) \]

And then

\[ \frac{dN}{dx} = \frac{d\alpha}{dx} E_1 P_2 + \alpha \frac{dE_1}{dx} P_2 + \alpha E_1 \frac{dP_2}{dx} - \frac{d\alpha}{dx} E_2 P_1 + (1 - \alpha) \frac{dE_2}{dx} P_1 E_2 \frac{dP_1}{dx} \quad (17) \]
and

\[ M_1 = \frac{Y_N - Y_{N-1}}{X_N - X_{N-1}} \]  \hspace{1cm} (23)

\[ M_2 = \frac{Y_N - Y_{N+1}}{X_N - X_{N+1}} \]  \hspace{1cm} (24)

A similar procedure is followed for the other end interval \( X_{N+1} \leq x \leq X_{N+2} \).

Householder's method for solving simultaneous equations is used in the solution of the aerodynamic influence equations. The method is applicable to both square and rectangular influence matrices. In the case of rectangular matrices it is not necessary to least square the equations first, since Householder's procedure least squares and triangularizes simultaneously. Also, the influence matrix is triangularized by means of orthogonal transformation matrices, which preserve the conditioning of the matrix. The combination of these two advantages, along with a reduction in the number of required computer operations, greatly improves the numerical accuracy and stability of the solution over that of the standard Gaussian reduction method.

A complete, but rather abstract, derivation of the method is given in reference (21). The method in the subroutine has been altered from the original to allow the operation on a single row of the matrix at a time. This reduces the required core allocation necessary to triangularize the matrix.

A derivation of the method, developed by the writer, will be given here in order to describe the basic philosophy of the method.

If \([A]\) is the rectangular influence matrix, the upper triangle is given by;

\[ [R] = [W] [A] \]  \hspace{1cm} (25)

where \([W]\) is the combined orthogonal transformation matrix used by Householder to triangularize \([A]\).
The relationship between Householder's triangularized matrix \([R]\) and that obtained by Gaussian elimination of the least squared influence matrix \([A]^T[A]\), is

\[
[G] = [T][A]^T[A] = [T][R]^T[R] = [D][R]
\] (26)

where \([G]\) is the triangular matrix obtained by Gaussian elimination of \([A]^T[A]\). The matrix \([T]\) is the Gaussian transformation matrix used to triangularize \([A]^T[A]\). And, \([D]\) is a diagonal matrix with the same diagonal as \([R]\). It can be seen from equation (26) that a nonsquare matrix must be least squared first, before applying the Gaussian transformation \([T]\). Whereas, the Householder transformation matrix \([W]\) can be applied directly. The least squared matrix \([A]^T[A]\) is usually more illconditioned than \([A]\), and therefore, less accurate results are obtained.

In the Householder method \([W]\) is equal to the product of \(N+1\) individual orthogonal transformation matrices, where \(n\) equals the number of unknowns. There are \(N+1\) transformations because the augmented influence matrix, made up of the influence matrix itself plus the boundary condition matrix, added on as the last column, has \(N+1\) columns. Each transformation results in reducing all elements below the diagonal to zero for one column. The columns are reduced from left to right.

The individual transformation matrices \([W]_m\) are defined by;

\[
[W]_m = ([I] - 2 \{u\}_m \{u\}_m^T)
\] (27)

where \([I]\) is a unit diagonal matrix and \([u]_m\) is a column matrix defined by the unit vector \(\bar{u}_m = (\bar{a}_m - \bar{\alpha}_m \bar{v}_m) / \mu_m\). The vector \(\bar{a}_m\) is defined by the \(m\)th column of \([A]\) where the elements on rows less than \(m\) are replaced by zeros. The unit vector \(\bar{v}_m\) is defined by a column matrix \([v]_m\) with all zeros except for the \(m\)th row, which is equal to one. The constants \(\alpha_m\) and \(\mu_m\) are defined as,

\[
\alpha_m = |\{a_m\}|
\] (28)

\[
\mu_m = \sqrt{2 \alpha_m (\alpha_m - \bar{v}_m \cdot \bar{a}_m)}
\] (29)

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It can be shown that \([a_m]\) is reduced to \(|a_m||v_m|\) if \([a_m]\) is premultiplied by 
\([(I) - 2|u_m||u_m|^T].\) Also, that the first \(m-1\) rows of 

\[ [W_{m-1}] [W_{m-2}] \ldots [W_1][A] \]  

remain unchanged by the \(m\)th transformation. The result after \(m\) transformations is then zeros below the diagonal for the first \(m\) columns and \(|a_1|, |a_2|, \ldots, |a_{m-2}|, |a_{m-1}|, |a_m|\) on the diagonal. The elements above the diagonal have been defined by the \(m\) preceding transformations and will remain unchanged for the \(N+1-m\) remaining transformations.

\[(I) - 2|u_m||a_m|^T \\{a_m\} = |a_m||v_m| \]  

\[ \hat{u}_m = (\tilde{a}_m - \alpha_m \hat{v}_m) / \mu_m \] or \[ \{u_m\} = (|a_m| - \alpha_m |v_m|) / \mu_m \]  

\[ \alpha_m = |a_m| \]  

and

\[ \mu_m = \sqrt{2 \alpha_m (\alpha_m - \hat{v}_m \cdot \tilde{a}_m)} \] or \[ \mu_m = \sqrt{2 \alpha_m (\alpha_m - |a_m|^T |v_m|)} \]  

remain to be proved. It is helpful in the derivation of equation (31) if the vector identity

\[ |\tilde{a}_m| \hat{v} + 2(\tilde{u}_m \cdot \tilde{a}_m) \tilde{u}_m = \tilde{a}_m \]  

is observed from the following vector diagram.
Then from equation (35);

\[ \hat{\alpha}_m \cdot \hat{v}_m + 2 \hat{u}_m \cdot (\hat{u}_m \cdot \hat{a}_m) = \hat{a}_m \]  
\[ (36) \]

\[ \hat{\alpha}_m \cdot \hat{v}_m + 2 \hat{u}_m \cdot \hat{a}_m = \hat{a}_m \]  
\[ (37) \]

Therefore;

\[ \hat{\alpha}_m \cdot \hat{v}_m = (\hat{\alpha}_m \cdot \hat{v}_m - 2 \hat{u}_m \cdot \hat{u}_m) \hat{a}_m \]  
\[ (38) \]

where \( \hat{\alpha}_m \) and \( \hat{u}_m \) are dyadics. The unit vector \( \hat{\alpha}_m \) is in the direction of \( \hat{a}_m \).

Equation (38) can then be written in matrix notation as follows;

\[ [I] - 2 |u|_m |u|_m^T |a|_m = |a|_m |v|_m \]  
\[ (39) \]

which is equal to equation (31). In matrix or tensor notation it becomes evident that the dimensions of \( |a|_m, |v|_m, \) and \( |u|_m \) are not limited to three.

\[ a_m = |a_m| \]  
\[ (40) \]

and

\[ \mu_m = 2 |a_m|^T a_m \]  
\[ (41) \]

Then if equation (31) is premultiplied by \( |a_m|^T \)

\[ |a_m|^T a_m - 2 |a_m|^T u_m |u_m|^T a_m = |a_m| |a_m|^T v_m \]  
\[ (42) \]
And substituting $\alpha_m$ and $\mu_m$ into equation (42).

$$\alpha_m^2 - \frac{1}{2} \mu_m^2 = \alpha_m [a_m]^T \{v_m\} \tag{43}$$

or

$$\mu_m = \sqrt{2 \alpha_m (\alpha_m - [a_m]^T \{v_m\})} \tag{44}$$

In vector notation equation (44) is seen to be equal to;

$$\mu_m = \sqrt{2 \alpha_m (\alpha_m - \hat{v}_m \cdot \hat{a}_m)} \tag{45}$$

Also, if equation (44) is substituted back into equation (31)

$$\{a_m\} - \sqrt{2 \alpha_m (\alpha_m - [a_m]^T \{v_m\})} \{a_m\} = \alpha_m \{v_m\} \tag{46}$$

Therefore;

$$\{u_m\} = \frac{\{a_m\} - \alpha_m \{v_m\}}{\sqrt{2 \alpha_m (\alpha_m - [a_m]^T \{v_m\})}} \tag{47}$$

or in vector notation

$$\vec{u}_m = \frac{\vec{a}_m - \alpha_m \vec{v}_m}{\sqrt{2 \alpha_m (\alpha_m - \hat{v}_m \cdot \hat{a}_m)}} \tag{48}$$
Appendix B

SKewed Source-Vortex Lattice Influence Equations

The general form of the skewed source and vortex lattice influence equations was derived in appendices B and C of reference (26), respectively. These equations will be specialized to the case where the inboard and outboard sweep angles for a given horseshoe are equal. The perturbation velocity due to a skewed vortex is then given by the following equations.

\[
\frac{U_L}{V_\infty} = \frac{U_V}{V_\infty} \quad (1)
\]

\[
\frac{V_L}{V_\infty} = \frac{\Delta Y_V \left( \frac{V_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} - \frac{\Delta Z_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (2)
\]

\[
\frac{W_L}{V_\infty} = \frac{\Delta Z_V \left( \frac{V_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} + \frac{\Delta Y_V \left( \frac{W_V}{V_\infty} \right)}{\sqrt{\Delta Y_V^2 + \Delta Z_V^2}} \quad (3)
\]

Where \(\Delta Y_V\) and \(\Delta Z_V\) are the changes in \(Y\) and \(Z\) across the width of the horseshoe vortex, respectively.

\[
\frac{U_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_U V}{4\pi} \quad (4)
\]
\[
\frac{V_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_{V_V}}{4\pi} \tag{5}
\]

\[
\frac{W_V}{V_\infty} = \frac{(\Gamma/V_\infty)E_{W_V}}{4\pi} \tag{6}
\]

Where

\[
F_{V_V} = \frac{Z}{R_2^2} (I_2 + I_3) \tag{7}
\]

\[
E_{V_V} = \frac{Z}{2} \left[ \frac{(I_1 + 1)}{R_1^2} - \frac{(I_4 + 1)}{R_3^2} - \frac{1}{R_2^2} (I_2 + I_3) \right] \tag{8}
\]

\[
\Gamma_{W_V} = \frac{(\bar{Y} - \beta Y_V) (I_4 + 1)}{R_3^2} - \frac{(\bar{Y} + \beta Y_V) (I_1 + 1)}{R_1^2} - \frac{(\bar{X} - \bar{Y} \bar{T}) (I_2 + I_3)}{R_2^2} \tag{9}
\]

Let

\[
term 1 = \frac{(I_2 + I_3)}{R_2^2} \tag{10}
\]

\[
term 2 = \frac{I_1 + 1}{R_1^2} \tag{11}
\]
\[ \text{term 3} = \frac{I_4 + 1}{R_3^2} \]  

Then

\[ E_u = \bar{Z} \text{ (term 1)} \]  

\[ E_v = \bar{Z} \left[-\bar{T} \text{(term 1)} + \text{(term 2)} - \text{(term 3)} \right] \]  

\[ E_{uu} = - (\bar{X} - \bar{T} \bar{Y}) \text{(term 1)} - (\bar{Y} + \beta \gamma \nu) \text{(term 2)} + (\bar{Y} - \beta \gamma \nu) \text{(term 3)} \]  

Where

\[ I_1 = \frac{\bar{X} + T \gamma \nu}{\left[ (\bar{X} + T \gamma \nu)^2 + (\bar{Y} + \beta \gamma \nu)^2 + Z^2 \right]^{1/2}} = \frac{\bar{X} + T \gamma \nu}{R_5} \]  

\[ I_2 = \frac{\bar{Y} + T \bar{X} + \beta \gamma \nu (1 + T^2)}{\left[ (\bar{X} + T \gamma \nu)^2 + (\bar{Y} + \beta \gamma \nu)^2 + Z^2 \right]^{1/2}} = \frac{\bar{Y} + T \bar{X} + \beta \gamma \nu (1 + T^2)}{R_5} \]  

\[ I_3 = \frac{\bar{Y} + T \bar{X} - \beta \gamma \nu (1 + T^2)}{\left[ (\bar{X} - T \gamma \nu)^2 + (\bar{Y} - \beta \gamma \nu)^2 + Z^2 \right]^{1/2}} = \frac{\bar{Y} + T \bar{X} - \beta \gamma \nu (1 + T^2)}{R_4} \]  

\[ I_4 = \frac{\bar{X} - T \gamma \nu}{\left[ (\bar{X} - T \gamma \nu)^2 + (\bar{Y} - \beta \gamma \nu)^2 + Z^2 \right]^{1/2}} = \frac{\bar{X} - T \gamma \nu}{R_4} \]
\[ R_1^2 = (\bar{Y} + \beta y_v)^2 + \bar{Z}^2 \] (19)

\[ R_2^2 = (\bar{X} - TY)^2 + \bar{Z}^2(1 + T^2) \] (20)

\[ R_3^2 = (\bar{Y} - \beta y_v)^2 + \bar{Z}^2 \] (21)

\[ R_4^2 = (\bar{X} - Ty_v)^2 + (\bar{Y} - \beta y_v)^2 + \bar{Z}^2 \] (22)

\[ R_5^2 = (\bar{X} + Ty_v)^2 + (\bar{Y} + \beta y_v)^2 + \bar{Z}^2 \] (23)

and

\[ \bar{X} = X_q - X_v \] (24)

\[ \bar{Y} = \beta \frac{\Delta Y_v (Y_q - Y_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} + \beta \frac{\Delta Z_v (Z_q - Z_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} \] (25)

\[ \bar{Z} = -\beta \frac{\Delta Y_v (Y_q - Y_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} + \beta \frac{\Delta Z_v (Z_q - Z_v)}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} \] (26)

\[ y_v = 1/2 \sqrt{\Delta Y_v^2 + \Delta Z_v^2} \] (27)

\[ \bar{T} = \frac{\tan \Lambda}{\beta} \] (28)
where \((X_v, Y_v, Z_v)\) and \((X_q, Y_q, Z_q)\) are the locations of the influencing point and the point being influenced respectively.

The elements of \([A_X]\), \([A_Y]\), and \([A_Z]\) are computed for a unit strength of \(\Gamma/V_\infty\).

Similarly, the perturbation velocity due to a skewed source line is given by;

\[
\frac{U_t}{V_\infty} = \frac{U_s}{V_\infty} \tag{29}
\]

\[
\frac{V_t}{V_\infty} = \frac{V_s}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} - \frac{W_s}{\sqrt{\Delta Y_v^2 + \Delta Z_v^2}} \tag{30}
\]

\[
\frac{W_t}{V_\infty} = \frac{\Delta Z_v V_s}{V_\infty} + \frac{\Delta Y_v W_s}{V_\infty} \tag{31}
\]

where

\[
\frac{U_s}{V_\infty} = \frac{(\Sigma / V_\infty) E_{U_s}}{4\pi} \tag{32}
\]

\[
\frac{V_s}{V_\infty} = \frac{(\Sigma / V_\infty) E_{V_s}}{4\pi} \tag{33}
\]

\[
\frac{W_s}{V_\infty} = \frac{(\Sigma / V_\infty) E_{W_s}}{4\pi} \tag{34}
\]
and

$$E_{Us} = \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} \text{(term 4)} + \frac{1}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY)(\text{term 1})$$

$$E_{Vs} = \frac{1}{\sqrt{1 + \bar{T}^2}} \text{(term 4)} - \frac{\bar{T}}{\sqrt{1 + \bar{T}^2}} (\bar{X} - TY)(\text{term 1})$$

$$E_{Ws} = \sqrt{1 + \bar{T}^2} \bar{Z} \text{(term 1)}$$

and where

$$\text{term 4} = \frac{1}{R_4} - \frac{1}{R_5}$$

The elements of $[S_X], [S_Y], \text{ and } [S_Z]$ are computed for a unit strength of $\Sigma / V_{ac}$. The influences of both the vortices and sources at the quarter chord of the subpanel are computed simultaneously due to the similarity in the vortex and source influence equations.

For the case where $|\bar{Z}| \leq 2 y_{*}$, set $\bar{Z} = 0$ and if both

$$\frac{(\bar{X} - TY)^2}{[(X + T y_{*})^2 + (\bar{Y} + \beta y_{*})^2]}$$

and

$$\frac{(\bar{X} - TY)^2}{[(X - T y_{*})^2 + (\bar{Y} - \beta y_{*})^2]}$$
are less than \((0.08716)^2\), and \(\left| \bar{Y} \right| > \beta y_v\), then use;

\[
\frac{1}{2\sqrt{1 + T^2}} \quad \frac{1}{(\bar{X} - T y_v)^2 + (\bar{Y} - \beta y_v)^2} \quad \frac{1}{(\bar{X} + T y_v)^2 + (\bar{Y} + \beta y_v)^2}
\]

(41)

in place of \((\text{term 1})\)

If \(\left| \bar{Y} \right| \leq \beta y_v\) and \(\left| \bar{X} - T Y \right| < \bar{t}_M/4\) set \((\text{term 1})\) equal to zero.
Appendix C

QUADRILATERAL VORTEX INFLUENCE EQUATIONS

The Biot-Savart law can be used to calculate the influence of a finite vortex segment on a point in three-dimensional space. The incremental change in induced velocity at a point in space due to an incremental change in length of a finite vortex is given by the following expression.

\[ dq = \frac{K \cos \phi \, d\phi}{4\pi h} \]  

where;

- \( K \) = Vortex strength
- \( h \) = Perpendicular distance from the vortex segment to the point in space.
- \( \phi \) = Angle between the line formed by \( h \) and a line from the field point to a point on the vortex segment.
- \( q \) = Velocity induced by the finite vortex segment perpendicular to the plane formed by \( h \) and the vortex segment.

A vector expression for \( q \) can be determined from figure (D-1):

\[ \vec{q} = \frac{d\vec{S}}{d\phi} \times \frac{(X_f, Y_f, Z_f) - (X_q, Y_q, Z_q)}{h} \]

Figure C-1: Velocity induced by finite vortex segment.
The magnitude of the velocity \( \hat{\mathbf{q}} \) induced at \((X_q, Y_q, Z_q)\) by the vortex segment \( \mathbf{s} \) is given by the following equation after equation (1) has been integrated from \( \phi_i \) to \( \phi_f \).

\[
|\hat{\mathbf{q}}| = \frac{k}{4\pi h} (\cos \beta - \cos \alpha) \quad (2)
\]

where

\[
\cos \beta = \frac{\hat{\mathbf{s}} \cdot \hat{\mathbf{R}_f}}{|\hat{\mathbf{s}}| |\hat{\mathbf{R}_f}|} 
\]

\[
\cos \alpha = \frac{\hat{\mathbf{s}} \cdot \hat{\mathbf{R}_i}}{|\hat{\mathbf{s}}| |\hat{\mathbf{R}_i}|} 
\]

The vector \( \hat{\mathbf{h}} \) is determined such as it satisfies the conditions of being perpendicular to \( \hat{\mathbf{s}} \) and equal to the vector sum \( \hat{\mathbf{h}} = \hat{\mathbf{R}_f} - a \hat{\mathbf{s}} \) where "a" defines the length of \( \hat{\mathbf{h}} \).

Since;

\[
\hat{\mathbf{h}} = \hat{\mathbf{R}_f} - a \hat{\mathbf{s}} \quad (3)
\]

and

\[
\hat{\mathbf{h}} \cdot \hat{\mathbf{s}} = 0 \quad (4)
\]

then

\[
\hat{\mathbf{h}} \cdot \hat{\mathbf{s}} = \hat{\mathbf{R}_f} \cdot \hat{\mathbf{s}} - a \hat{\mathbf{s}} \cdot \hat{\mathbf{s}} = 0 \quad (5)
\]

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therefore;

\[ a = \frac{R_f \cdot s}{s \cdot s} \]  

(6)

After substituting "a" into equation (3), \( \hat{h} \) is defined as;

\[ \hat{h} = R_f - \frac{R_f \cdot s}{s \cdot s} s \]  

(7)

Also, a unit vector \( \hat{q} \) in the direction of \( \hat{q} \) is seen to be equal to;

\[ \hat{q} = \frac{R_f \times s}{|R_f \times s|} \]  

(8)

The magnitude and direction of \( \hat{q} \) are then expressed in terms of the coordinates of the control point \( (X_q, Y_q, Z_q) \) and the endpoints of the vortex segment \( (X_i, Y_i, Z_i) \) and \( (X_f, Y_f, Z_f) \). If \( \hat{i}, \hat{j}, \) and \( \hat{k} \) are defined as unit vectors in the \( X, Y, \) and \( Z \) directions respectively, then;

\[ \hat{s} = \hat{s}_f - \hat{s}_i = (X_f - X_i) \hat{i} + \beta (Y_f - Y_i) \hat{j} + \beta (Z_f - Z_i) \hat{k} \]  

(9)

\[ \hat{R}_i = \hat{s}_i - \hat{Q} = (X_i - X_q) \hat{i} + \beta (Y_i - Y_q) \hat{j} + \beta (Z_i - Z_q) \hat{k} \]

and

\[ \hat{R}_f = \hat{s}_f - \hat{Q} = (X_f - X_q) \hat{i} + \beta (Y_f - Y_q) \hat{j} + \beta (Z_f - Z_q) \hat{k} \]  

(10)
The value of "a" is then expressed as:

\[ a = \frac{\vec{R}_f \cdot \vec{s}}{\vec{s} \cdot \vec{s}} = \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \]  

(11)

and the components of \( \vec{h} \) by,

\[ h_x = (X_f - X_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \right] (X_f - X_i) \]  

(12)

\[ h_y = (Y_f - Y_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \right] (Y_f - Y_i) \]  

\[ h_z = (Z_f - Z_q) - \left[ \frac{(X_f - X_q)(X_f - X_i) + \beta^2(Y_f - Y_q)(Y_f - Y_i) + \beta^2(Z_f - Z_q)(Z_f - Z_i)}{(X_f - X_i)^2 + \beta^2(Y_f - Y_i)^2 + \beta^2(Z_f - Z_i)^2} \right] (Z_f - Z_i) \]  

The magnitude of \( \vec{q} \) is found by substituting

\[ h = |\vec{h}| = \sqrt{h_x^2 + h_y^2 + h_z^2} \]

and the following expressions for \( \cos\alpha \) and \( \cos\beta \) into equation (2).
\[
\cos \alpha = \frac{(X_f - X_i) (X_i - X_q) + \beta^2 (Y_f - Y_i) (Y_i - Y_q) + \beta^2 (Z_f - Z_i) (Z_i - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2 (Y_f - Y_i)^2 + \beta^2 (Z_f - Z_i)^2}} \frac{(X_f - X_i) (X_i - X_q) + \beta^2 (Y_f - Y_i) (Y_i - Y_q) + \beta^2 (Z_f - Z_i) (Z_i - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2 (Y_f - Y_i)^2 + \beta^2 (Z_f - Z_i)^2} \sqrt{(X_i - X_q)^2 + \beta^2 (Y_i - Y_q)^2 + \beta^2 (Z_i - Z_q)^2}}
\]

\[
\cos \beta = \frac{(X_f - X_i) (X_f - X_q) + \beta^2 (Y_f - Y_i) (Y_f - Y_q) + \beta^2 (Z_f - Z_i) (Z_f - Z_q)}{\sqrt{(X_f - X_i)^2 + \beta^2 (Y_f - Y_i)^2 + \beta^2 (Z_f - Z_i)^2} \sqrt{(X_f - X_q)^2 + \beta^2 (Y_f - Y_q)^2 + \beta^2 (Z_f - Z_q)^2}}
\]

The components of the vector $\hat{q}$ are then given by the multiplication of the components of equation (8) by $|q|$.

\[
q_x = \frac{|\hat{q}| \left[ (Y_f - Y_q) (Z_f - Z_i) - (Z_f - Z_q) (Y_f - Y_i) \right] \beta^2}{|\vec{RXS}|}
\]

\[
q_x = \frac{|\hat{q}| \left[ (X_f - X_i) (Z_f - Z_q) - (Z_f - Z_i) (X_f - X_q) \right] \beta^2}{|\vec{RXS}|}
\]

\[
q_x = \frac{|\hat{q}| \left[ (X_f - X_i) (Y_f - Y_i) - (Y_f - Y_q) (X_f - X_q) \right] \beta^2}{|\vec{RXS}|}
\]
where

\[
|\mathbf{R}_x| = \left| \begin{pmatrix}
  (Y_f - Y_q)(Z_f - Z_1) - (Z_f - Z_q)(Y_f - Y_1) \\
  (X_f - X_q)(Y_f - Y_1) - (Y_f - Y_q)(X_f - X_1)
\end{pmatrix} \right|^2 \beta^2 + \left[ \begin{pmatrix}
  (X_f - X_q)(Z_f - Z_q) - (Z_f - Z_q)(X_f - X_q)
\end{pmatrix} \right]^2 \\
+ \left[ \begin{pmatrix}
  (X_f - X_q)(Y_f - Y_1) - (Y_f - Y_q)(X_f - X_1)
\end{pmatrix} \right]^2 \beta \left( \frac{1}{2} \right)
\]  

(15)

The velocity induced at a control point by a vortex segment is then given by equation (14). Since a curved vortex can be represented by a number of straight segments, this equation can be used to compute the induced flow produced by a vortex of arbitrary shape.

The components of velocity induced by a quadrilateral vortex can be written as ratios computed by the product of influence matrices and the vortex strengths.

\[
\begin{pmatrix}
  \frac{u}{V_\infty} \\
  \frac{v}{V_\infty} \\
  \frac{w}{V_\infty}
\end{pmatrix} = \begin{bmatrix}
  A_X & A_Y & A_Z
\end{bmatrix} \begin{pmatrix}
  \frac{K}{V_\infty}
\end{pmatrix}
\]  

(16)

\[
\begin{pmatrix}
  \frac{v}{V_\infty} \\
  \frac{w}{V_\infty}
\end{pmatrix} = \begin{bmatrix}
  A_Y & A_Z
\end{bmatrix} \begin{pmatrix}
  \frac{K}{V_\infty}
\end{pmatrix}
\]  

(17)

and

\[
\begin{pmatrix}
  \frac{w}{V_\infty}
\end{pmatrix} = \begin{bmatrix}
  A_Z
\end{bmatrix} \begin{pmatrix}
  \frac{K}{V_\infty}
\end{pmatrix}
\]  

(18)

where the elements of \( A_X \), \( A_Y \), and \( A_Z \), are computed from the following equations.
\[ A_x = \sum \frac{\beta^2 [\cos \beta - \cos \alpha] \left[ (Y_f - Y_q)(Z_f - Z_i) - (Z_f - Z_q)(Y_f - Y_i) \right]}{4\pi |\vec{h}| |\vec{RS}|} \]  
(19)

\[ A_y = \sum \frac{\beta [\cos \beta - \cos \alpha] \left[ (X_f - X_i)(Z_f - Z_q) - (Z_f - Z_i)(X_f - X_q) \right]}{4\pi |\vec{h}| |\vec{RS}|} \]  
(20)

and

\[ A_z = \sum \frac{\beta [\cos \beta - \cos \alpha] \left[ (X_f - X_q)(Y_f - Y_i) - (Y_f - Y_q)(X_f - X_i) \right]}{4\pi |\vec{h}| |\vec{RS}|} \]  
(21)

The \( \Sigma \) sign indicates that the contributions from all of the sides of the quadrilateral vortex are summed.
Appendix D

WOODWARD'S DISTRIBUTED PANEL
INFLUENCE EQUATIONS

The equations will be derived for supersonic flow first, then for subsonic flow in subappendix A.

Preliminaries

Generalized potential function. If

\[ \beta^2 \Omega_{xx} = \Omega_{yy} + \Omega_{zz} \]

Then

\[ \Omega(x, y, z) = -\frac{1}{2\pi} \frac{1}{\partial x} \int \int \left( \frac{\partial \Omega}{\partial \nu} + \frac{\partial \Omega'}{\partial \nu'} \right) \sigma \, dS \]

\[ + \frac{1}{2\pi} \frac{1}{\partial x} \int \int (\Omega - \Omega') \frac{\partial \sigma}{\partial \nu} \, dS \]  

(1)

where

\[ \sigma = \cos h^{-1} \frac{x - \xi}{\beta \sqrt{(y-\eta)^2 + (z-\zeta)^2}} \]

and

\[ \vec{\nu} = \left( -\beta^2 n_1, n_2, n_3 \right) = -\vec{\nu}' \]

with \( n = (n_1, n_2, n_3) \) the unit normal to surface S

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Region of integration. - In the \((\xi, \eta, \zeta)\) coordinate system the plane of the semi-infinite triangular surface is determined by \(\xi = a_\xi\). The lines \(\eta = 0\) and \(\eta = m_\xi\) are the projections in the \(\xi, \eta\) plane of the triangle edges. The area of integration in equation (1) lies on the semi-infinite triangle and is within the Mach forecone from the point \((x, y, z)\) given by \(\xi < x\) and 
\[(x - \xi)^2 > \beta^2(y - \eta)^2 + \beta^2(z - \zeta)^2\]. The surface integral is carried out by integrating first over \(\xi\) and then over \(\eta\).

The \(\xi\) integration goes from the leading edge \(\xi = \xi_1 = \eta/m\) to the intersection of the Mach forecone with the semi-infinite triangle, \(\xi = \xi_2(\eta)\), where since \(\xi = a_\xi\), 
\[(x - \xi_2)^2 = \beta^2(y - \eta)^2 + \beta^2(z - a_\xi \eta)^2\]. The \(\eta\) integration goes from \(\eta = 0\) to the intersection of the Mach forecone the leading edge where \(\eta = \eta_3\). Thus,

\[
\left( x - \frac{\eta_3}{m} \right)^2 = \beta^2(y - \eta_3)^2 + \beta^2\left( z - \frac{a_\eta_3}{m} \right)^2.
\]

Looking down on \(\xi\) axis

Thus

\[
\xi_1 = \frac{\eta}{m} \tag{2}
\]

\[
(x - \xi_2)^2 = \beta^2(y - \eta)^2 + \beta^2(z - a_\xi \eta)^2 \tag{3}
\]
\[(\eta_3 - mx)^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 (m\eta_3 - mz)^2 \]  \hspace{1cm} (4)

These relations may be manipulated into other forms.

\[\left[(\eta_3 - y) - (mx - y)\right]^2 = \beta^2 m^2 (\eta_3 - y)^2 + \beta^2 \left[a(\eta_3 - y) + (ay - mz)\right]^2\]

or

\[(\eta_3 - y)^2 \left[1 - \beta^2 (a^2 + m^2)\right] - 2(\eta_3 - y) \left[(mx - y) + \beta^2 a(ay - mz)\right] + (mx - y)^2 - \beta^2 (ay - mz)^2 = 0\]

Thus on the forecone

\[\left[1 - \beta^2 (a^2 + m^2)\right] (\eta_3 - y) = (mx - y) + \beta^2 a(ay - mz)\]

\[\sqrt{\left[(mx - y) + \beta^2 a(ay - mz)\right]^2 - \left[1 - \beta^2 (a^2 + m^2)\right] (mx - y)^2 - \beta^2 (ay - mz)^2}\]

The following relation may be used to manipulate this further.

\[\left[m(x - \beta^2 az) - y(1 - \beta^2 a^2)\right]^2 \equiv \left[(mx - y) + \beta^2 a(ay - mz)\right]^2\]

\[\equiv \beta^2 m^2 (z - ax)^2 + (1 - \beta^2 a^2) \left[(mx - y)^2 - \beta^2 (ay - mz)^2\right]\]

\hspace{1cm} (5)
Thus

\[ 1 - \beta^2(a^2 + m^2)\eta_3 = m(x - \beta^2 az) - \beta^2 m^2 y \]

\[ - \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \]

(6)

Or starting from (4) again after multiplying by \((1 - \beta^2 a^2)\)

\[(1 - \beta^2 a^2)(\eta_3 - mx)^2 = \beta^2 m^2 (1 - \beta^2 a^2)(\eta_3 - y)^2 \]

\[+ \beta^2 (1 - \beta^2 a^2) [a(\eta_3 - mx) - m(z - ax)]^2 \]

which becomes

\[(1 - \beta^2 a^2)(\eta_3 - mx)^2 + 2 \beta^2 am(z - ax)(1 - \beta^2 a^2)(\eta_3 - mx) \]

\[+ \left[ \beta^2 am \right]^2 (z - ax)^2 = \beta^2 m^2 \left[(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2 \right] \]

or

\[\left[(1 - \beta^2 a^2)(\eta_3 - mx) + \beta^2 am(z - ax) \right]^2 \]

\[= \beta^2 m^2 \left[(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2 \right] \]
or

\[ \left[ \eta_3 (1 - \beta^2 a^2) - m(x - \beta^2 az) \right]^2 = \beta^2 m^2 \left[ 1 - \beta^2 a^2 \right] (\eta_3 - y)^2 + (z - ax)^2 \]

(7)

Surface Distribution of Sources

Boundary conditions. - For a distribution of sources on the plane \( \xi = a \), we will assume

(a) \( \phi = \phi' \)

(b) \( \frac{\partial \phi}{\partial \nu} + \frac{\partial \phi'}{\partial \nu'} = \frac{\partial \phi}{\partial \xi} \nu_1 + \frac{\partial \phi}{\partial \xi'} \nu_3 + \frac{\partial \phi'}{\partial \xi'} \nu_1 + \frac{\partial \phi'}{\partial \xi} \nu_3 \)

\[ = (u - u') \nu_1 + (w - w') \nu_3 \]

\[ = \frac{1}{\sqrt{1 + a^2}} \left[ \beta^2 a (u - u') + (w - w') \right] \]

\[ = \frac{2}{\sqrt{1 + a^2}} \left[ \frac{1}{\bar{w} + \beta^2 a \bar{u}} \right] = \text{const} \]

In equation (1), if we set \( \Omega = \phi \), then (a) says the second integral vanishes and (b) means the quantity \( \partial \phi/\partial \nu + \partial \phi'/\partial \nu' \) may be removed from the integral. These assumptions may be checked after the integration is performed. The statement Woodward makes on the bottom of page 17, of reference (55), \( u' = -u \) and \( w' = -w \) is not true and not necessary.
Evaluation of the integral over $\xi$. - The integral which results for a surface distribution of sources is,

$$\varphi(x, y, z) = \frac{w + \beta^2 a_0^2}{\pi} \int_0^\eta \int_0^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}}$$

But

$$\int \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}}$$

$$= \frac{-1}{2\sqrt{1 - \beta^2 a^2}} \log \frac{(x - s - \beta^2 a (z - a s)) + \sqrt{(1 - \beta^2 a^2)(x - s)^2 + \beta^2 (z - a s)^4}}{(x - s - \beta^2 a (z - a s)) - \sqrt{(1 - \beta^2 a^2)(x - s)^2 + \beta^2 (z - a s)^4}}$$

Which may be verified by differentiating

First we let

$$A = (x - s) - \beta^2 a (z - a s)$$

$$B^2 = \beta^2 [(1 - \beta^2 a^2)(y - s)^2 + (z - a x)^2]$$

$$C^2 = (1 - \beta^2 a^2)(x - s)^2 - \beta^2 (y - s)^2 - \beta^2 (z - a s)^2$$

and we note that

$$A^2 = B^2 + C^2$$
To verify the indefinite integral we must evaluate

\[
\frac{2}{3} \log \frac{A + C}{A - C} = \frac{A' + C'}{A + C} - \frac{A' - C'}{A - C}
\]

\[
= \frac{2(A'C' - A'C)}{A^2 - C^2}
\]

where \( \frac{\partial}{\partial s} \) denotes \( \frac{2}{3} \).

But since \( B' = 0 \)

\( AA' = CC' \)

or

\( C' = \frac{AA'}{C} \)

Therefore

\[
\frac{-1}{2\sqrt{1-\beta^2a^2}} \frac{\partial}{\partial s} \log \frac{A + C}{A - C} = -\frac{A'(A^2 - C^2)}{C (A^2 - C^2) \sqrt{1-\beta^2a^2}}
\]

\[
= \frac{\sqrt{1-\beta^2a^2}}{C}
\]

\[
= \frac{1}{\sqrt{(x-3)^2 - \beta^2((y-7)^2 - \beta^2(z-a)^2)}}
\]

Q.E.D.
\[ (x - \xi_2)^2 = \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi_2)^2 \]

and from Eq (2)

\[ \xi_1 = \frac{\eta}{m} \]

\[
\int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}} = \frac{1}{2 \sqrt{1 - \beta^2 \alpha^2}} \log \frac{(mx - \eta) - \beta^2 \alpha (mz - \eta) + \sqrt{(1 - \beta^2 \alpha^2) [(mx - \eta)^2 - \beta^2 \alpha (mz - \eta)^2]}}{(mx - \eta) - \beta^2 \alpha (mz - \eta) - \sqrt{(1 - \beta^2 \alpha^2) [(mx - \eta)^2 - \beta^2 \alpha (mz - \eta)^2]}}
\]

Therefore from Eq (9)

\[ \phi(x, y, z) = \]

\[
- \frac{w + \beta^2 \alpha u}{2 \pi \sqrt{1 - \beta^2 \alpha^2}} \int_{0}^{\eta} \frac{\log \frac{(mx - \eta) - \beta^2 \alpha (mz - \eta) + \sqrt{(1 - \beta^2 \alpha^2) [(mx - \eta)^2 - \beta^2 \alpha (mz - \eta)^2]}}{(mx - \eta) - \beta^2 \alpha (mz - \eta) - \sqrt{(1 - \beta^2 \alpha^2) [(mx - \eta)^2 - \beta^2 \alpha (mz - \eta)^2]}}}{(mx - \eta) - \beta^2 \alpha (mz - \eta) - \sqrt{(1 - \beta^2 \alpha^2) [(mx - \eta)^2 - \beta^2 \alpha (mz - \eta)^2]}} d\eta
\]
Evaluation of the Integral over $\eta$. To evaluate equation (11) we must first integrate by parts. If we let,

$$ f = (mx - 7) - \beta^2 a (m - a \eta) $$

$$ g^2 = \beta^2 m^2 \left[ (1 - \beta^2 a^2)(7 - \eta)^2 + (2 - ax)^2 \right] $$

$$ h^2 = (1 - \beta^2 a^2) \left[ (mx - 7)^2 - \beta^2 m^2 (7 - \eta)^2 - \beta^2 (m - a \eta)^2 \right] $$

then

$$ f^2 = g^2 + h^2 $$

Now let

$$ u = \frac{1}{2} \log \frac{f + h}{f - h} \quad \text{dv} = d\eta $$

$$ du = \frac{fh' - h' f}{f^2 - h^2} d\eta = \frac{fhh' - f'h^2}{h(f^2 - h^2)} d\eta $$

$$ = \frac{1}{h g^2} \left[ f' g^2 - f g g' \right] d\eta \quad (13) $$

and

$$ v = \eta $$

but

$$ gg' = \beta^2 m^2 (1 - \beta^2 a^2) (7 - \eta) $$

$$ f' = - (1 - \beta^2 a^2) $$
Now we can write

\[- \eta \left[ g f' - g'gf \right] \]

\[= \eta (1 - \beta^2a^2) \beta^2m^2 \left( z - ax \right)^2 + \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] (\eta - y) \]

\[= \beta^2 \beta^2m^2 \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] \left[ 1 - \beta^2a^2 \right] (\eta - y)^2 + (z - ax)^2 \]

\[+ \beta^2 \beta^2m^2 y (1 - \beta^2a^2) \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] (\eta - y) \]

\[+ \beta^2m^2 (z - ax)^2 \left[ \eta (1 - \beta^2a^2) - \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] \right] \]

\[= \beta^2 \beta^2m^2 \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] \left[ 1 - \beta^2a^2 \right] (\eta - y)^2 + (z - ax)^2 \]

\[+ \beta^2 \beta^2m^2 (1 - \beta^2a^2) \left[ y \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] + (z - ax)^2 \right] (\eta - y) \]

\[+ \beta^2m^2 (z - ax)^2 \left[ y (1 - \beta^2a^2) - \left[ m (x - \beta^2az) - y (1 - \beta^2a^2) \right] \right] \]

And since from the integration by parts

\[\frac{1}{2} \int \log \frac{f + k}{f - k} \, d\eta = \frac{1}{2} \eta \log \frac{f + k}{f - k} + \int \frac{\eta (g^2f' - gg'f)}{g^2 \sqrt{f^2 - g^2}} \, d\eta \]

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We get for the integration by parts

\[ \frac{1}{2} \int \log \left( \frac{(mx - y) - \beta a(mx - y) + \sqrt{(1 - \beta^2)(mx - y)^2 - \beta^2(mx - y)^2}}{(mx - y) - \beta a(mx - y) - \sqrt{(1 - \beta^2)(mx - y)^2 - \beta^2(mx - y)^2}} \right) dy \]

\[ = \frac{\gamma}{2} \log \frac{(mx - y) - \beta a(mx - y) + \sqrt{(1 - \beta^2)(mx - y)^2 - \beta^2(mx - y)^2}}{(mx - y) - \beta a(mx - y) - \sqrt{(1 - \beta^2)(mx - y)^2 - \beta^2(mx - y)^2}} \]

\[ \times \left[ a \left( 1 - \beta^2 \right) y (1 - \beta^2) \right] \int \frac{d}{\sqrt{(a(1 - \beta^2) - \gamma (1 - \beta^2)^2) - \beta^2(1 - \beta^2)^2}} \]

\[ \int \frac{d}{\sqrt{(a(1 - \beta^2) - \gamma (1 - \beta^2)^2) - \beta^2(1 - \beta^2)^2}} \]

\[ = \frac{1}{2} \log \left( \frac{-(A7 + \Theta) + \sqrt{A[A7^2 + 2B7 + C]}}{-(A7 + \Theta) - \sqrt{A[A7^2 + 2B7 + C]}} \right) \]

(14)

The first integral in equation (14) can be evaluated using the relation

\[ \int \frac{d}{\sqrt{A\gamma^2 + 2B\gamma + C}} = -\frac{1}{2\sqrt{A}} \log \left( \frac{-(A7 + \Theta) + \sqrt{A[A7^2 + 2B7 + C]}}{-(A7 + \Theta) - \sqrt{A[A7^2 + 2B7 + C]}} \right) \]

(15)
In our case expanding the denominator gives

\[
A = (1 - \beta^2 a^2) \left[ 1 - \beta^2 (a^2 + m^2) \right]
\]
\[
B = \left[ - \eta (x - \beta^2 az) + \beta^2 m^2 (x - \beta^2 az) \right] (1 - \beta^2 a^2)
\]
\[
C = \beta^2 (x - \beta^2 az) \cdot \beta^2 m^2 \left[ (1 - \beta^2 a^2) + (z - ax)^2 \right]
\]

and since we can write

\[
-(A\eta + \Theta) = \left[ (m\eta - \gamma) - \beta^2 a (m^2 - a\gamma) + \beta^2 m^2 (\gamma - \eta) \right] (1 - \beta^2 a^2)
\]
\[
A[A\eta + 2B\eta + C] = (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] \left[ \left[ m(x - \beta^2 az - \eta (1 - \beta^2 a^2)) \right] \beta^2 m^2 [ (1 - \beta^2 a^2) (\gamma - \eta) + (z - ax)^2 ] \right]
\]
\[
= [1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2) \left[ (m\eta - \gamma) - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m^2 - \gamma)^2 \right]
\]

Therefore

\[
\int \frac{d\eta}{\sqrt{[m(x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2)^2 (\gamma - \eta)^2 + (z - ax)^2 \right]}}
\]

\[
= \frac{-1}{2\sqrt{[1 - \beta^2 (a^2 + m^2)] (1 - \beta^2 a^2)}} \log \frac{- (A\eta + \Theta) + \sqrt{A[A\eta + 2B\eta + C]}}{- (A\eta + \Theta) - \sqrt{A[A\eta + 2B\eta + C]}} \quad (16)
\]

The last integral in equation (14) may be evaluated using the method described in Appendix A of reference (55).

From Appendix A of reference (55);

If

\[
\gamma^4 + (c - ae^2) \gamma^2 - b^2 e^2 = 0 \quad (17)
\]
then

\[
\int \frac{(A + Bv) \, dv}{(v^2 + e^2) \sqrt{a v^2 + 2bv + c}}
\]

\[
= \frac{A b Y + B v^3}{\gamma^4 + b^2 e^2} \tan^{-1} \frac{\gamma \sqrt{a v^2 + 2bv + c}}{\gamma^2 - bv}
\]

\[
+ \frac{A \gamma^3}{e} + B b e Y
\]

\[
\frac{3}{2(\gamma^4 + b^2 e^2)} \log \frac{-(v \gamma^2 + b e^2) + \gamma e \sqrt{a v^2 + 2bv + c}}{-(v \gamma^2 + b e^2) - \gamma e \sqrt{a v^2 + 2bv + c}}
\]

(18)

From equation (14) we can write

\[
[m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2]
\]

\[
= (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] (\eta - y)^2
\]

\[-2[m (x - \beta^2 az) - \eta (1 - \beta^2 a^2)] (\eta - y) (1 - \beta^2 a^2)
\]

\[+ [m (x - \beta^2 az) - \gamma (1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2\]
Therefore if we factor out a \((1 - \beta^2 a^2)\) we can write the denominator as

\[
\sqrt{\left[ m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right] - \beta^2 a^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right]}
\]

\[
= (1 - \beta^2 a^2)^{3/2} (\eta^2 + e^2) \sqrt{\bar{a}^2 + 2bv + c}
\]

where

\[
\hat{a} = [1 - \beta^2 (a^2 + m^2)]
\]

\[
b = - [m (x - \beta^2 az) - y (1 - \beta^2 a^2)]
\]

\[
c = \left| [m (x - \beta^2 az) - y (1 - \beta^2 a^2)] - \beta^2 m^2 (z - ax)^2 \right| (1 - \beta^2 a^2)^{-1}
\]

\[
e = (z - ax) (1 - \beta^2 a^2)
\]

\[
v = \eta - y
\]

Referring to equation (17)

\[
c - \hat{a} e^2 = \frac{[m (x - \beta^2 az) - y (1 - \beta^2 a^2)]^2 - \beta^2 m^2 (z - ax)^2 - [1 - \beta^2 (a^2 + m^2)] (z - ax)^2}{(1 - \beta^2 a^2)}
\]

\[
= \frac{b^2}{(1 - \beta^2 a^2)} - e^2 (1 - \beta^2 a^2)
\]

\[
\gamma^4 + (c - \hat{a} e^2) - b^2 e^2 = \left[ \gamma^2 + \frac{B^2}{(1 - \beta^2 a^2)} \right] \left[ \gamma^2 - e^2 (1 - \beta^2 a^2) \right] = 0
\]

Therefore we can choose

\[
\gamma^2 = e^2 (1 - \beta^2 a^2)
\]

or

\[
\gamma = (z - ax)
\]

(20)
Using this the numerator of the second integral in (12) becomes

\[
(1 - \alpha x)^2 \left| y \left( 1 - \beta^2 a^2 \right) - y (x - \beta^2 a) \right| - (1 - \beta^2 a^2) \right| y (x - \beta^2 a) - (z - ax)^2 \left| y \right.
\]

\[
- y^2 \left| y \left( 1 - \beta^2 a^2 \right) \cdot b \right| \cdot (1 - \beta^2 a^2) \cdot \gamma b + \gamma^2 \right| y
\]

Therefore from (18) the coefficient of \( \tan^{-1} \) is

\[
\frac{b \gamma^3 \left| y \left( 1 - \beta^2 a^2 \right) \cdot b \right| \cdot \left( 1 - \beta^2 a^2 \right)^2 \gamma^3 + \gamma b + \gamma^2 \right| y \left( 1 - \beta^2 a^2 \right) \gamma^4 + b^2 \gamma^2 \right| y \left( 1 - \beta^2 a^2 \right) \gamma^4 + b^2 \gamma^2 \right| 1 - \beta^2 a^2 \right|}
\]

\[
\frac{1}{\sqrt{1 - \beta^2 a^2}}
\]

\[
\frac{z - ax}{\sqrt{1 - \beta^2 a^2}}
\]

since \( \gamma = (z - ax) \) and \( \gamma^2 = c^2 \left( 1 - \beta^2 a^2 \right) \)

Using (19) and (18) again the coefficient of \( \frac{1}{2} \log \frac{z - ax}{a} \)

\[
- \gamma^5 \left[ y \left( 1 - \beta^2 a^2 \right) + b \right] + b e^2 \gamma \left( 1 - \beta^2 a^2 \right) \left[ -\gamma b + \gamma^2 \right]
\]

\[
e \left( \gamma^4 + b^2 \gamma^2 \right) \left( 1 - \beta^2 a^2 \right)^{3/2}
\]

\[
= - \frac{1}{(z - ax) \left( \gamma^4 + b^2 \gamma^2 \right) \left( 1 - \beta^2 a^2 \right)} \gamma
\]
And from (19)

\[(1 - \beta^2 \alpha^2) [\hat{a}v^2 + 2bv + c] \]

\[= [m (x - \beta^2 \alpha z) - (v + y) (1 - \beta^2 \alpha^2)]^2 - \beta^2 m^2 [1 - \beta^2 \alpha^2] v^2 + (z - ax)^2 \]

\[\gamma^2 - bv = (z - ax)^2 + v [m (x - \beta^2 \alpha z) - y (1 - \beta^2 \alpha^2)] - (v \gamma^2 + be^2) \]

\[-(v \gamma^2 + be^2) = \left| [m (x - \beta^2 \alpha z) - y (1 - \beta^2 \alpha^2)] - v (1 - \beta^2 \alpha^2) \right| \frac{(z - ax)^2}{(1 - \beta^2 \alpha^2)} \]

Therefore

\[\tan^{-1} \frac{\gamma \sqrt{\hat{a}v^2 + 2bv + c}}{v^2 - bv} = \tan^{-1} (z - ax) \sqrt{\frac{[m (x - \beta^2 \alpha z) - y (1 - \beta^2 \alpha^2)]^2 - \beta^2 m^2 [(1 - \beta^2 \alpha^2) (\eta - y)^2 + (z - ax)^2]}{1 - \beta^2 \alpha^2 (z - ax)^2 + (\eta - y) [m (x - \beta^2 \alpha z) - y (1 - \beta^2 \alpha^2)]}} \]

\[\log \frac{-(v \gamma^2 + be^2) + \gamma e \sqrt{\hat{a}v^2 + 2bv + c}}{-(v \gamma^2 + be^2) - \gamma e \sqrt{\hat{a}v^2 + 2bv + c}} \]

\[= \log \frac{m(x - \beta^2 \alpha z) - \gamma (1 - \beta^2 \alpha^2) + \sqrt{[m(x - \beta^2 \alpha z) - \gamma (1 - \beta^2 \alpha^2)]^2 - \beta^2 m^2 [(1 - \beta^2 \alpha^2) (\gamma - \eta)^2 + (z - ax)^2]}}{m(x - \beta^2 \alpha z) - \gamma (1 - \beta^2 \alpha^2) - \sqrt{[m(x - \beta^2 \alpha z) - \gamma (1 - \beta^2 \alpha^2)]^2 - \beta^2 m^2 [(1 - \beta^2 \alpha^2) (\gamma - \eta)^2 + (z - ax)^2]}} \]

\[= \log \frac{(mx - \gamma) - \beta^2 a(m \beta - \gamma) + \sqrt{(1 - \beta^2 \alpha^2) [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m \beta - \gamma)^2]}}{(mx - \gamma) - \beta^2 a(m \beta - \gamma) - \sqrt{(1 - \beta^2 \alpha^2) [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m \beta - \gamma)^2]}} \]

Because from (12)

\[\left[ m(x - \beta^2 \alpha z) - \gamma (1 - \beta^2 \alpha^2) \right]^2 - \beta^2 m^2 [(1 - \beta^2 \alpha^2) (\gamma - \eta)^2 + (z - ax)^2] \]

\[= (1 - \beta^2 \alpha^2) [(mx - \gamma)^2 - \beta^2 m^2 (\gamma - \eta)^2 - \beta^2 (m \beta - \gamma)^2] \]

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Therefore combining terms

\[ \frac{1}{2} \int \log \frac{(m-x)(-\beta^2(m-z-a))^2 + \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}}{(m-x)(-\beta^2(m-z-a) - \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}} \, dz \]

\[ = \frac{1}{2} \log \frac{(m-x)(-\beta^2(m-z-a) + \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}}{(m-x)(-\beta^2(m-z-a) - \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}} \]

\[ + \frac{y(1-\beta^2z) - m(1-x^2)}{2\sqrt{(1-\beta^2z)(1-\beta^2z)}} \log \frac{(m-x)(-\beta^2(m-z-a)^2 + \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}}{(m-x)(-\beta^2(m-z-a)^2 - \sqrt{(1-\beta^2z)^2 - \beta^2(m-z-a)^2}} \]

\[ \frac{(z-x) \tan^{-1} (z-x) \sqrt{m(x-\beta^2z) - \eta(1-\beta^2z)^2 - \beta^2m^2 (1-\beta^2z)(\eta-\gamma)^2 + (z-x)^2}}{\sqrt{1-\beta^2z^2} \sqrt{(z-x)^2 + (\eta-\gamma) (m(x-\beta^2z) - \gamma(1-\beta^2z))}} \]

\[ (21) \]

Differentiation of the indefinite integral over \( \eta \). Equation (21) may be checked by differentiation.

To differentiate the first log term we can refer back to the integration by parts of the integral over \( \gamma \). As before let

\[ f = (m-x)(-\beta^2(m-z-a)) \]

\[ g^2 = \beta^2m^2 [(1-\beta^2z)(\gamma-\gamma)^2 + (z-x)^2] \]

\[ h^2 = (1-\beta^2z) [(m-x)^2 - \beta^2m^2(\gamma-\gamma)^2 - \beta^2(m-z-a)^2] \]

then

\[ f^2 = g^2 + h^2 \]

and

\[ \frac{d}{d\gamma} \frac{1}{2} \log \frac{f + h}{f - h} = \frac{1}{h^2} \left[ f^2g^2 - f^2g^2' \right] = \frac{1}{h^2} \left[ f^2g^2 - f^2g^2' \right] = \frac{1}{h^2} \left[ f^2g^2 - f^2g^2' \right] = \frac{1}{h^2} \left[ f^2g^2 - f^2g^2' \right] \]
Then since

\[
q' = -(1 - a^2)
\]

\[
gg' = \beta \, m^2 \, (1 - a^2) \, (\eta - \gamma)
\]

we can write

\[
\frac{1}{2} \, \frac{2}{\sqrt{7}} \, (\gamma - \eta) \, \log \left( \frac{(mx - \eta) - \beta a (m \eta - \eta)}{mx - \eta - \beta a (m \eta - \eta)} \right) = \frac{1}{2} \, \log \left( \frac{(mx - \eta) - \beta a (m \eta - \eta)}{mx - \eta - \beta a (m \eta - \eta)} \right)
\]

\[
- (\eta - \gamma) \, (1 - a^2)
\]

\[
\sqrt{|n (x - a^2) - \eta (1 - a^2)| - \beta m^2 ([1 - a^2] (\eta - \gamma)^2 + (z - ax)^2) [(1 - a^2) (\eta - \gamma)^2 + (z - ax)^2]}
\]

\[
\sqrt{|n (x - a^2) - \eta (1 - a^2)| - \beta m^2 ([1 - a^2] (\eta - \gamma)^2 + (z - ax)^2)}
\]

(22)

So differentiate the second log term in (21) we will

first perform the following manipulation,
\[
\left\{ m(x - \beta^2 az) - \beta^2 m^2 y - \left[ 1 - \beta^2 (a^2 + m^2) \right] \eta \right\}^2
\]
\[
= \left\{ m(x - \beta^2 az) - \beta^2 a \left[ (mx - y)^2 - \beta^2 (ay - mz)^2 \right] - 2 \left[ (mx - y) - \beta^2 a (mz - ay) \right] \left[ 1 - \beta^2 (a^2 + m^2) \right] (7 - \eta) \right\}^2
\]
\[
= \beta^2 m^2 \left[ (xz - y)^2 - (x - ay)^2 - \beta^2 (ay - mz)^2 \right]
\]
\[
+ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left\{ (mx - y)^2 - 2(7 - \eta)(mx - y) + (7 - \eta)^2 \right\}
\]
\[
- \beta^2 \left[ (ay - mz)^2 - 2a (mz - ay)(7 - \eta) + a^2 (7 - \eta) \right] \left[ (7 - \eta)^2 \right]
\]
\[
\]
\[
= \beta^2 m^2 \left[ (xz - y)^2 - (x - ay)^2 - \beta^2 (ay - mz)^2 \right]
\]
\[
+ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left\{ (mx - y)^2 - \beta^2 m^2 (7 - \eta)^2 - \beta^2 (mz - az)^2 \right\}
\]

Therefore, if we let
\[
f = m(x - \beta^2 az) - \beta^2 m^2 y - \left[ 1 - \beta^2 (a^2 + m^2) \right] \eta
\]
\[
g^2 = \beta^2 m^2 \left[ (xz - y)^2 - (x - ay)^2 - \beta^2 (ay - mz)^2 \right]
\]
\[
h^2 = \left[ 1 - \beta^2 (a^2 + m^2) \right] \left\{ (mx - y)^2 - \beta^2 m^2 (7 - \eta)^2 - \beta^2 (mz - az)^2 \right\}
\]

then
\[
f^2 = g^2 + h^2
\]

(23)
Since in this case \[ \frac{1}{2} \log \frac{f + h}{f - h} = \frac{f'}{h} \] (using (13))

we can write

\[ f' = -\left[ 1 - \beta^2 (a^2 + m^2) \right] \]

and

\[ \frac{1}{2} \frac{d}{d \eta} \log \frac{(mx - \gamma) - \beta^2 (m x - \gamma) + \beta^2 \eta (m x - \gamma) + \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \gamma)^2 - \beta^2 (m x - \gamma) - \beta^2 (m x - \gamma)^2]}}{(mx - \gamma) - \beta^2 (m x - \gamma) + \beta^2 \eta (m x - \gamma) - \sqrt{[1 - \beta^2 (a^2 + m^2)] [(mx - \gamma)^2 - \beta^2 (m x - \gamma) - \beta^2 (m x - \gamma)^2]}} \]

\[ \frac{-\left[ 1 - \beta^2 (a^2 + m^2) \right]}{\sqrt{\left[ m (x - \beta^2 x) - \eta (1 - \beta^2 a^2) \right] \left[ \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right] \right]}} \]

This is because, using (5)

\[ (1 - \beta^2 a^2) \left\{ \left[ n (x - \beta^2 x) - \eta (1 - \beta^2 a^2) \right] \left[ \beta^2 n^2 \left[ \eta (\eta - y)^2 + (z - ax)^2 \right] \right] \right\} \]

\[ + (1 - \beta^2 a^2) \left\{ \left[ n (x - \beta^2 x) - y (1 - \beta^2 a^2) \right] \left[ \beta^2 n^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 \right] \right] \right\} \]

\[ - \left[ 1 - \beta^2 (a^2 + n^2) \right] \left\{ \left[ n (x - \beta^2 x) - y (1 - \beta^2 a^2) \right] \left[ \beta^2 n^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 \right] \right] \right\} \]

We can also write

\[ \frac{d}{d \eta} \tan^{-1} \frac{f}{g} = \frac{f'}{g} - \frac{\frac{g'f}{g^2}}{\left( \frac{f^2}{g^2} \right) + 1} = \frac{g f' - g' f^2}{f (f^2 + g^2)} \]
\[ f = (z - ax) \sqrt{[m (x - \beta^2 a^2) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2]} \]

\[ f^2 - 1 = -(z - ax)^2 (1 - \beta^2 a^2) \left| [m (x - \beta^2 a^2) - \eta (1 - \beta^2 a^2)] + \beta^2 m^2 (\eta - y) \right| \]

\[ g = \sqrt{1 - \beta^2 a^2} [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)] \]

\[ g^2 = \sqrt{1 - \beta^2 a^2} [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)] \]

\[ f^2 + g^2 = (z - ax)^2 \left[ [m (x - \beta^2 a^2) - \eta (1 - \beta^2 a^2)]^2 - \beta^2 m^2 [(1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2] \right] \]

\[ + (z - ax)^2 \left[ [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)]^2 + (\eta - y)^2 (1 - \beta^2 a^2) [1 - \beta^2 (a^2 + m^2)] - \beta^2 m^2 (z - ax)^2 \right] \]

\[ + (z - ax)^2 \left[ [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)]^2 + (\eta - y)^2 [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)] \right] \]

\[ + (z - ax)^2 \left[ [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right] \]

\[ + (1 - \beta^2 a^2) (\eta - y)^2 [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \]

\[ = (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \]

\[ \left[ [m (x - \beta^2 a^2) - y (1 - \beta^2 a^2)]^2 + [1 - \beta^2 (a^2 + m^2)] (z - ax)^2 \right] \]
Therefore

\begin{align*}
\frac{d}{d\eta} \tan^{-1} (z \cdot \omega) \sqrt{\left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] - \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]} \\
\sqrt{1 - \beta^2 \omega^2 (z \cdot \omega)^2 + (\eta - y)^2 \left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] + \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]}
\end{align*}

\begin{align*}
\left( \frac{z \cdot \omega}{\eta} \right) \sqrt{1 - \beta^2 \omega^2 \left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] + \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]}
\end{align*}

\begin{align*}
\left[ (1 - \beta^2 \omega^2)(\eta - y)^2 + (z \cdot \omega)^2 \right] \sqrt{1 - \beta^2 \omega^2 \left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] + \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]}
\end{align*}

\begin{align*}
\left( \frac{z \cdot \omega}{\eta} \right) \tan^{-1} (z \cdot \omega) \sqrt{\left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] - \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]} \\
\sqrt{1 - \beta^2 \omega^2 \left[ n(x, \beta^2 \omega) \cdot n(1 - \beta^2 \omega) \right] + \beta^2 m^2 \left[ n(\eta, \beta^2 \omega) \cdot (\eta - y)^2 + (z \cdot \omega)^2 \right]}
\end{align*}

(25)

Combining (22), (24), and (25)

\begin{align*}
\frac{d}{d\eta} \left\{ \frac{1}{2} (\eta - y) \log \frac{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) + \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) - \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2} \right\}
\end{align*}

\begin{align*}
+ \frac{1}{2} (\eta - y) \left( \frac{1}{1 - \beta \omega^2} \right) \left\{ \frac{1}{2} (\eta - y) \log \frac{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) + \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) - \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2} \right\}
\end{align*}

\begin{align*}
= \frac{1}{2} \log \frac{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) + \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) - \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}
\end{align*}

\begin{align*}
+ \left\{ \frac{1}{2} (\eta - y) \left( \frac{1}{1 - \beta \omega^2} \right) \left\{ \frac{1}{2} (\eta - y) \log \frac{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) + \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) - \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2} \right\} \right\}
\end{align*}

\begin{align*}
= \frac{1}{2} \log \frac{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) + \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}{(mx - \eta \cdot \beta \omega)(mz - \eta \cdot \beta \omega) - \sqrt{1 - \beta \omega^2} (mx - \eta \cdot \beta \omega)^2 - \beta \omega m^2 (\eta - y)^2 - \beta \omega (mz - \eta \cdot \beta \omega)^2}
\end{align*}
Evaluation of the definite integral. - From (6) and (7)

\[ m \left( x - \beta^2 az \right) - \eta_3 \left( 1 - \beta^2 a^2 \right) = \beta m \sqrt{(1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2} \]  
\[ (6) \]

\[ m \left( x - \beta^2 az \right) - \beta^2 m^2 y - \left[ 1 - \beta^2 (a^2 + m^2) \right] \eta_3 = \beta m \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \]  
\[ (7) \]

Therefore using (6),(7) and (23) both of the log terms in equation (21) are zero when \( \eta = \eta_3 \). However the \( \tan^{-1} \) term can be either zero or \( \pm \pi \) when \( \eta = \eta_3 \), depending upon whether the denominator term is greater or less than zero. If it is greater than zero the limit is zero as \( \eta \to \eta_3 \). In (21) when \( \eta = \eta_3 \) using (6) and then (5) and (7),

Now if we examine the numerator of (26) and use (5)

\[ (1 - \beta^2 a^2)^2 \left[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 \right] - \beta^2 m^2 \left[ m(x - \beta^2 az) - y \left( 1 - \beta^2 a^2 \right) \right]^2 \]

\[ \times \left( 1 - \beta^2 a^2 \right) \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 - \beta^2 (ay - mz)^2 \right] - \left[ 1 - \beta^2 a^2 \right]^2 - \beta^4 m^4 \] \( (z - ax)^2 \)

\[ \times \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (1 - \beta^2 a^2) \left[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 \right] + \beta^2 m^2 (z - ax)^2 \right] \]

\[ (27) \]
because
\[(1 - \beta^2 a^2)^2 - \beta^4 m^4 = (1 - \beta^2 a^2 - \beta^2 m^2)(1 - \beta^2 a^2 + \beta^2 m^2)\]

But if \(1 - \beta^2 (a^2 + m^2) > 0\) then from (27)
\[(1 - \beta^2 a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} > \beta m \mid m(x - \beta^2 az) - y(1 - \beta^2 a^2)\mid\]

and (26) shows that the denominator of the \(\tan^{-1}\) term is \(> 0\). But if \(1 - \beta^2 (a^2 + m^2) < 0\) then (27) shows
\[(1 - \beta^2 a^2) \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} < \beta m \mid m(x - \beta^2 az) - y(1 - \beta^2 a^2)\mid\]

and the denominator of the \(\tan^{-1}\) term is \(> 0\) if and only if
\[m(x - \beta^2 az) - y(1 - \beta^2 a^2) > 0\]

But
\[m(x - \beta^2 az) - y(1 - \beta^2 a^2) = m[x - \beta \sqrt{y^2 + z^2}] + \beta m \sqrt{y^2 + z^2} - \beta^2 m z - y(1 - \beta^2 a^2)\]

But we can also write
\[\beta^2 m^2 (y^2 + z^2) - [\beta^2 m z + y(1 - \beta^2 a^2)]^2\]

\[= [\beta^2 (a^2 + m^2) - 1]y^2 + \beta^2 m^2 \beta^2 a^2 z^2 - 2 \beta^2 (1 - \beta^2 a^2) w m z + \beta^2 (1 - \beta^2 a^2) y^2\]

\[= [\beta^2 (a^2 + m^2) - 1]y^2 + \beta^2 (1 - \beta^2 a^2) (m^2 z^2 - 2 w m z + a^2 y^2)\]

\[= [\beta^2 (a^2 + m^2) - 1]y^2 + \beta^2 (1 - \beta^2 a^2) (m z - a y)^2 > 0 \text{ if } 1 - \beta^2 (a^2 + m^2) < 0\]
And therefore

$$\beta m \sqrt{y^2 + z^2} - \beta^2 amz - y(1 - \beta^2 a^2) > 0$$

and

$$m(x - \beta^2 az) - y(1 - \beta^2 a^2) > 0$$

if

$$1 - \beta^2(a^2 + m^2) < 0$$

and

$$x > \beta(y^2 + z^2)$$

Therefore inside the Mach cone from the origin $$x^2 > \beta^2(y^2 + z^2)$$ we have

$$(z - ax)^2 + (\eta_3 - y) [m(x - \beta^2 az) - y(1 - \beta^2 a^2)] > 0 \quad (28)$$

and therefore because of (6) and (28)

$$\tan^{-1} \left[ \frac{(z - ax) \sqrt{[m(x - \beta^2 az) - \eta_3(1 - \beta^2 a^2)]^2 - \beta^2 m^2 (1 - \beta^2 a^2)(\eta_3 - y)^2 + (z - ax)^2}}{\sqrt{1 - \beta^2 a^2} \left| (z - ax)^2 + (\eta_3 - y)[m(x - \beta^2 az) - y(1 - \beta^2 a^2)] \right|} \right] = 0$$

If we change the sign of the denominator we would get $$\pi$$ for $$z > ax$$ and $$-\pi$$ for $$z < ax$$. However when we evaluate the term when $$\eta = 0$$ and subtract, the result is the same.
Therefore if we evaluate (21) for $\eta = 0$ we get for (11)

$$\phi (x, y, z) = - \frac{\tilde{w} + \beta^2 u}{2\pi \sqrt{1 - \beta^2 a^2}} \int_0^{\eta_3} \frac{\sqrt{(m - \eta) \beta - \beta (m - \eta) \beta (m - \eta)}}{\sqrt{(m - \eta) \beta - \beta (m - \eta) \beta (m - \eta)}} \, d\eta$$

$$= \frac{\tilde{w} + \beta^2 u}{\pi} \left\{ \frac{(z - ax)}{1 - \beta^2 a^2} \tan^{-1} \frac{m(z - ax)}{y[(y - mx) - \beta^2 a (ay - mz)] + (z - ax)^2} \right\}$$

$$+ \frac{y(1 - \beta^2 a^2) - (x - \beta^2 az)}{2(1 - \beta^2 a^2) \sqrt{1 - \beta^2 (a^2 + m^2)}} \left( \frac{\log \frac{x - \beta^2 a z - m^2}{x - \beta^2 a z + m^2} + \sqrt{[x^2 - \beta^2 (a^2 + m^2)](1 - \beta^2 a^2)}}{x - \beta^2 a z + \sqrt{[x^2 - \beta^2 (a^2 + m^2)](1 - \beta^2 a^2)}} \right)$$

$$- \frac{y}{2 \sqrt{1 - \beta^2 a^2}} \left( \frac{\log \frac{x - \beta^2 a z + \sqrt{[x^2 - \beta^2 (a^2 + m^2)](1 - \beta^2 a^2)}}{x - \beta^2 a z - \sqrt{[x^2 - \beta^2 (a^2 + m^2)](1 - \beta^2 a^2)}}}{x - \beta^2 a z - \sqrt{[x^2 - \beta^2 (a^2 + m^2)](1 - \beta^2 a^2)}} \right)$$

(29)

**Evaluation of the velocity components.** - The velocity components may now be obtained by differentiation of the velocity potential. However, whoever attempts to take partial derivatives of (29) is in for a long day's work. But the partial derivatives of (29) may be obtained quite simply if it is noted that all terms in the expressions for the velocity components must have coefficients involving either \(\log\) or \(\tan^{-1}\). This is easily shown by differentiating the expression for \(\phi\) given by (11).
\[ u = \frac{\partial \phi}{\partial x} = -\frac{\bar{w} + \beta^2 \bar{u}}{2\pi \sqrt{1 - \beta^2 a^2}} + \frac{m}{2\sqrt{1 - \beta^2 a^2}} \log \frac{x - \beta'(a^2 + 1) + \sqrt{\left(1 - \beta'(a^2 + 1)^2 \right) \left(x - \beta'(a^2 + 1)^2 \right)}}{x - \beta'(a^2 + 1) - \sqrt{\left(1 - \beta'(a^2 + 1)^2 \right) \left(x - \beta'(a^2 + 1)^2 \right)}} \]

However, this integral expression for \( u \) involves integrals of the same form as in (13), (16), or (18). A closer examination shows that the resulting \( \log \) or \(\tan^{-1}\) terms will have arguments which are identical with those for \( \phi \), although the coefficients may be different. Therefore all expressions must involve either \( \log \) or \(\tan^{-1}\) terms. The same arguments can be made for the \( y \) and \( w \) velocity components.

With this in mind the partial derivatives of \( \phi \) can be obtained from (29) by differentiating only the coefficients of the \( \log \) and \(\tan^{-1}\) terms. Differentiation of the \(\tan^{-1}\) or \( \log \) terms will not yield other \(\tan^{-1}\) or \( \log \) terms and therefore the sum of these terms must give zero. Therefore

\[ u = \frac{\partial \phi}{\partial x} = -\frac{\bar{w} + \beta^2 \bar{u}}{2\pi \sqrt{1 - \beta^2 a^2}} + \frac{m}{2\sqrt{1 - \beta^2 a^2}} \log \frac{x - \beta'(a^2 + 1) + \sqrt{\left(1 - \beta'(a^2 + 1)^2 \right) \left(x - \beta'(a^2 + 1)^2 \right)}}{x - \beta'(a^2 + 1) - \sqrt{\left(1 - \beta'(a^2 + 1)^2 \right) \left(x - \beta'(a^2 + 1)^2 \right)}} \]
Constant Pressure Surface

Boundary Conditions. - In (1) we can set $\Omega = u$

Assume that on $S$

(a) \[ u - u' = \text{const} \quad (30) \]

(b) \[ \left[ \frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} \right] = 0 \]

Now since

\[
\frac{\partial u}{\partial \nu} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{1}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi}
\]

\[
\frac{\partial u'}{\partial \nu'} = \frac{-\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial u}{\partial \xi} + \frac{-1}{1 + a^2} \frac{\partial u}{\partial \xi}
\]

and from (b)

\[
\frac{\partial u}{\partial \nu} + \frac{\partial u'}{\partial \nu'} = \frac{\beta^2 a}{\sqrt{1 + a^2}} \frac{\partial}{\partial \xi} (u - u') + \frac{1}{\sqrt{1 + a^2}} \frac{\partial}{\partial \xi} (u - u') = 0
\]

Now from (a), $u - u' = \text{const}$ on $S$ implies that on $S$

\[
\frac{\partial}{\partial \eta} (u - u') = 0
\]

and

\[
\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0
\]
This means that since

$$\beta^2 a \frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0$$

and

$$\frac{\partial}{\partial \xi} (u - u') + a \frac{\partial}{\partial \xi} (u - u') = 0$$

then if

$$1 - \beta^2 a \neq 0$$

$$\frac{\partial}{\partial \xi} (u - u') = \frac{\partial}{\partial \eta} (u - u') = \frac{\partial}{\partial \xi} (u - u') = 0 \text{ on } S$$

Note that assumptions (a) and (b) in (30) are independent assumptions.

(a) Says that \((u - u')\) is constant on \(S\)

(b) Effectively says that the derivative of \(u - u'\) in a direction normal to \(S\) is zero.

Woodward assumes, in reference (55), that (a) implies (b), which is not true.

If we examine the normal velocity on \(S\)

$$u_n = \frac{-au + w}{\sqrt{1 + a^2}} \quad u'_n = \frac{au' - w}{\sqrt{1 + a^2}}$$

then since due to irrotationality \(\frac{\partial w}{\partial \xi} = \frac{\partial u}{\partial \xi}\)

$$\sqrt{1 + a^2} \frac{\partial}{\partial \xi} u_n = -a \frac{\partial u}{\partial \xi} + \frac{\partial w}{\partial \xi} = -a \frac{\partial u}{\partial \xi} + \frac{\partial u}{\partial \xi}$$
\[ \sqrt{1 + a^2 \frac{\partial}{\partial \xi} (u_\xi + u_\eta)} = -a \frac{\partial}{\partial \xi} (u - u') + \frac{\partial}{\partial \eta} (u - u') = 0 \]

and therefore the source strength, if any, is constant.

**Evaluation of the integral over \( \xi \).** - The integral to be evaluated is,

\[ \phi(x,y,z) = \frac{\Delta P}{4\pi \rho \omega} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{\beta^2 a \cdot \frac{(x - \xi)(z - a\xi)}{(y - \eta)^2 + (z - a\xi)^2} \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a\xi)^2}}{\xi - \xi_1 d\xi \eta} \]

\[ = \frac{\Delta P}{4\pi \rho \omega \omega} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{\beta^2 a \cdot \frac{(x - \xi)(z - \omega\xi) + (y - \eta)^2}{(z - \omega\xi)^2 + (y - \eta)^2} \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - \omega\xi)^2}}{\xi - \xi_1 d\xi \eta} \]

\[ = \frac{(1 - \beta^2 a^2) \Delta P}{4\pi \rho \omega \omega} \int_0^{\eta_3} \int_{\xi_1}^{\xi_2} \frac{d\xi}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - \omega\xi)^2}} d\eta \]

\[ \text{182} \]
Except for the coefficient, the second integral is identical to the integral obtained for a surface distribution of sources. It has already been evaluated [see (9)]. The first integral over $\xi$ is easy to evaluate,

$$
\int \frac{[\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2} - \beta^2 (z - a \xi)^2]}{\sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2} - \beta^2 (z - a \xi)^2} d\xi
$$

This can be verified by differentiating. Let,

$$
f = x - 3
$$

$$
g^2 = \beta^2 [(z - a \xi)^2 + (y - \eta)^2]
$$

$$
h^2 = (x - 3)^2 - \beta^2 [(z - a \xi)^2 + (y - \eta)^2]
$$

and

$$
\frac{3}{2} \frac{1}{g^2} \log \frac{f + h}{f - h} = \frac{1}{h^2} \left[ f' g^2 - f g^2' \right] \quad \text{from (13)}
$$

$$
\frac{f'}{g^2} = -1
$$

$$
g g' = -\beta^2 a (z - a \xi)
$$

Therefore

$$
\frac{1}{2} \frac{3}{g^2} \log \frac{\sqrt{(x - 3)^2 - \beta^2 [(z - a \xi)^2 + (y - \eta)^2]} - \beta^2 (z - a \xi)^2}{\sqrt{(x - 3)^2 - \beta^2 [(z - a \xi)^2 + (y - \eta)^2]} - \beta^2 (z - a \xi)^2}
$$

$$
= \frac{(z - a \xi)^2 + (y - \eta)^2 - a (x - \xi) (z - a \xi)}{[(z - a \xi)^2 + (y - \eta)^2] \sqrt{(x - \xi)^2 - \beta^2 (z - a \xi)^2 - \beta^2 (y - \eta)^2}}
$$

$$
\frac{1}{2} \frac{3}{g^2} \log \frac{(z - a \xi) (z - a \xi) + (y - \eta)^2}{[(z - a \xi)^2 + (y - \eta)^2] \sqrt{(x - \xi)^2 - \beta^2 (z - a \xi)^2 - \beta^2 (y - \eta)^2}}
$$

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Since from (3)

\[ (x - \xi_2)^2 = \beta^2 \left( (z - a \xi_2)^2 + (y - \eta)^2 \right) \]

\[ \log \frac{(x - 3_1) + \sqrt{(x - 3_1)^2 - \beta^2 [(z - a_3)^2 + (y - \eta)^2]}}{(x - 3_1) - \sqrt{(x - 3_1)^2 - \beta^2 [(z - a_3)^2 + (y - \eta)^2]}} = 0 \]

and from (2)

\[ \xi_1 = \frac{\eta}{m} \]

\[ \int_{\xi_1}^{\xi_2} \frac{(z - ax)(z - a \xi) + (y - \eta)^2}{\left[ (y - \eta)^2 + (z - a \xi)^2 \right] \sqrt{(x - \xi)^2 - \beta^2 (y - \eta)^2 - \beta^2 (z - a \xi)^2}} \, d\xi \]

\[ = \frac{1}{2} \log \frac{(m x - \eta) + \sqrt{(m x - \eta)^2 - \beta^2 (m \eta - m \xi)^2 + m^2 (\eta - \xi)^2}}{(m x - \eta) - \sqrt{(m x - \eta)^2 - \beta^2 (m \eta - m \xi)^2 + m^2 (\eta - \xi)^2}} \]

d therefore

\[ \phi(x, \eta, z) = \frac{1}{8\pi a \frac{\Delta P}{q_{\infty}}} \int_0^{\gamma_3} \frac{\gamma_3}{(m x - \eta) + \sqrt{(m x - \eta)^2 - \beta^2 (m \eta - m \xi)^2 + m^2 (\eta - \xi)^2}} \, d\eta \]

\[ - \frac{1}{8\pi a \frac{\Delta P}{q_{\infty}}} \sqrt{1 - \beta^3 a^2} \int_0^{\gamma_3} \frac{\gamma_3}{(m x - \eta) - \beta a (m x - a \eta) + \sqrt{(1 - \beta^2 a^2) [(m x - \eta)^2 - \beta a (m x - a \eta)^2 - \beta^2 (m x - a \eta)^2]}} \, d\eta \]

The second integral was evaluated previously for a surface distribution of sources.
Evaluation of the indefinite integral over $\eta$. The first integral in (32) must first be integrated by parts.

Let $f = mx - \eta$

$$g^2 = \beta m^2 (\eta - m)^2 + \beta^2 (m^2 - \eta^2)^2$$

$$h^2 = (mx - \eta)^2 - \beta m^2 (\eta - m)^2 - \beta^2 (m^2 - \eta^2)^2$$

Then let

$$u = \frac{1}{2} \log \frac{f + h}{f - h} \quad dv = d\eta$$

$$du = \frac{1}{g^2} [f' g^2 - gg' f] d\eta \quad v = \eta$$

$$f' = -1$$

$$gg' = \beta^2 [a \eta - mz + m^2 (\eta - y)]$$

$$[f' g^2 - gg' f] = \beta^2 \left\{ (a \eta - mz)^2 + m^2 (\eta - y)^2 + (mx - \eta) \right\} [a \eta - mz + m^2 (\eta - y)]$$

$$= \beta^2 \left\{ (a \eta - mz) \left[ (a \eta - mz) + a \left( mx - \eta \right) \right] + m^2 (\eta - y) \left[ (\eta - y) + (mx - \eta) \right] \right\}$$

$$== \beta^2 \left\{ a \eta - mz \left( a \eta - mz - ax \right) + m^2 (\eta - y) \left( \eta - y - mx \right) \right\}$$

$$185$$
\[
- \eta \left( y^2 - gg^e \right) = \delta^2 \left[ m (mx - y) - a (z - ax) \right] \eta^2 + m^2 \left[ z (z - ax) - y (mx - y) \right] \eta
\]

\[
= \delta^2 \left[ \frac{m (mx - y) - a (z - ax)}{(a^2 + m^2)} \right] \left[ (\eta - mz)^2 + \eta (\eta - y)^2 \right]
\]

\[
= \frac{2m^2 \left[ m (mx - y) - a (z - ax) \right] (az + my) \eta + m^2 \left( a^2 + m^2 \right) \left[ z (z - ax) - y (mx - y) \right] \eta}{(a^2 + m^2)}
\]

\[
= \frac{-m \left[ m (mx - y) - a (z - ax) \right] m^2 (y^2 + z^2)}{(a^2 + m^2)}
\]

But

\[
\left[ m (mx - y) - a (z - ax) \right] (az + my) + (a^2 + m^2) \left[ z (z - ax) - y (mx - y) \right]
\]

\[
= \left[ m (az + my) - y (z^2 + m^2) \right] (mx - y) + \left[ z (z^2 + m^2) - a \left( az + my \right) \right] (z - ax)
\]

\[
= -m \left( ay - mz \right) (mx - y) - m \left( ay - mz \right) (z - ax)
\]

\[
= (ay - mz)^2
\]
Therefore

\[ \frac{1}{2} \int \log \frac{(m x - \eta) + \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}}{(m x - \eta) - \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}} \, d \gamma \]

\[ = \frac{1}{2} \gamma \log \frac{(m x - \eta) + \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}}{(m x - \eta) - \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}} \]

\[ - \frac{n [(a \eta - m \eta) \cdot (a^2 + a^2)]}{(a^2 + m^2)} \int \frac{d \eta}{\sqrt{(m x - \eta)^2 + \beta^2 (a \eta - m \eta)^2 + \beta^2 m^2 (\eta - y)^2}} \]

\[ + \frac{\eta^2}{\beta^2 \eta^2} \int \frac{[(a \eta - m \eta) \cdot (m x - y) \cdot (z - ax)] \cdot \eta - m (\eta - y)^2}{[(a \eta - m \eta)^2 + m^2 (\eta - y)^2] \sqrt{(m x - \eta)^2 + \beta^2 (a \eta - m \eta)^2 + \beta^2 m^2 (\eta - y)^2}} \, d \eta \]

The first integral in (34) may be evaluated using (16) because,

\[ \left[ m (x - \beta^2 a z) - \eta (1 - \beta^2 a^2) \right]^2 - \beta^2 m^2 \left[ (1 - \beta^2 a^2) (\eta - y)^2 + (z - ax)^2 \right] \]

\[ = (1 - \beta^2 a^2) \left| (m x - \eta) \right|^2 - \beta^2 (\eta - m \eta)^2 - \beta^2 m^2 (\eta - y)^2 \]

Therefore using (16) and (35)

\[ \int \frac{d \eta}{\sqrt{(m x - \eta)^2 + \beta^2 (a \eta - m \eta)^2 + \beta^2 m^2 (\eta - y)^2}} = \]

\[ \frac{1}{2 \sqrt{1 - \beta^2 m^2}} \log \frac{(m x - \eta) + \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}}{(m x - \eta) - \sqrt{(m x - \eta)^2 - \beta^2 m^2 (\eta - \gamma)^2 - \beta^2 (m \eta - \alpha \gamma)^2}} \]

(36)
The last term in equation (34) must be changed to the form of equation (18) in order to be evaluated. We can write

\[(a \eta - mz)^2 + m^2 (\eta - y)^2 = (a^2 + m^2) \eta^2 - 2m (az + my) \eta + m^2 (y^2 + z^2)\]

\[= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{m^2 [(a^2 + m^2)(y^2 + z^2) - (az + my)^2]}{a^2 + m^2}\]

\[= (a^2 + m^2) \left[ \eta - \frac{m (az + my)}{a^2 + m^2} \right]^2 + \frac{m^2 (ay - mz)^2}{(a^2 + m^2)}\]

Now we introduce a change of variables. Let

\[u = \eta - \frac{m (az + my)}{a^2 + m^2}\]

Therefore

\[(a \eta - mz)^2 + m^2 (\eta - y)^2 = (a^2 + m^2) \left\{ u^2 + \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\}\]  \hspace{1cm} (37)

\[\eta - mx = u + \frac{m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)}\]  \hspace{1cm} (38)
\[(\eta - mx)^2 - \beta^2 (a \eta - mz)^2 - \beta^2 m^2 (\eta - y)^2\]

\[= \left[1 - \beta^2 (a^2 + m^2)\right] u^2 + \frac{2 m (az + my) - mx (a^2 + m^2)}{(a^2 + m^2)} u\]

\[-\frac{\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x (a^2 + m^2)\right]^2}{(a^2 + m^2)}\]

\[= \frac{\left[(az + my) - x (a^2 + m^2)\right]^2}{(a^2 + m^2)}\]

\[(39)\]

**Note:** (18) can be used. Let

\[\hat{a} = 1 - \beta^2 (a^2 + m^2)\]

\[u = \frac{m \left[(az + my) - x (a^2 + m^2)\right]}{(a^2 + m^2)} = \frac{m \left[a (z - ax) - m (mx - y)\right]}{(a^2 + m^2)}\]

\[b = \frac{m \left[(az + my) - x (a^2 + m^2)\right]}{(a^2 + m^2)}\]

\[c = \frac{-\beta^2 m^2 (ay - mz)^2 (a^2 + m^2) + m^2 \left[(az + my) - x (a^2 + m^2)\right]^2}{(a^2 + m^2)}\]

\[e = \frac{m (ay - mz)}{(a^2 + m^2)}\]

\[(40)\]
and referring to (17)

\[ \begin{align*}
    c - \hat{a}e^2 &= \frac{-m^2 (ay - mz)^2 + m^2 \left[ m (mx - y) - a (z - ax) \right]^2}{(a^2 + m^2)^2} \\
    b^2 e^2 &= \frac{m^2 \left[ m (mx - y) - a (z - ax) \right]^2 m^2 (ay - mz)^2}{(a^2 + m^2)^4}
\end{align*} \]

\[ \gamma^4 + (c - \hat{a}e^2) \gamma^2 - b^2 e^2 = \]

\[ = \left\{ \gamma^2 - \frac{m^2 (ay - mz)^2}{(a^2 + m^2)^2} \right\} \left\{ \gamma^2 + \frac{m^2 \left[ m (mx - y) - a (z - ax) \right]^2}{(a^2 + m^2)^2} \right\} = 0 \]

Therefore we can choose

\[ \gamma = \frac{m (ay - mz)}{(a^2 + m^2)} = e \quad \text{(41)} \]
The numerator of the second integral in equation (34) can be written,

\[
\left\{ (a^2 + m^2) \gamma^2 - m (az + my)b \right\} \left\{ u + \frac{m (az + my)}{(a^2 + m^2)} \right\} + m^2 (y^2 + z^2)b
\]

\[
= \left\{ (a^2 + m^2) \gamma^2 - m (az + my)b \right\} \left\{ u + m (az + my) \gamma^2 + \frac{m^2 b (ay - mz)^2}{(a^2 + m^2)} \right\}
\]

\[
= \left\{ (a^2 + m^2) \gamma^2 - m (az + my)b \right\} \left\{ u + \gamma^2 \left[ m (az + my) + b (a^2 + m^2) \right] \right\}
\]

Now referring to (18) we can write

\[
A = \frac{\gamma^2 \left[ m (az + my) + b (a^2 + m^2) \right]}{a^2 + m^2}
\]

\[
B = \frac{(a^2 + m^2) \gamma^2 - m (az + my) b}{a^2 + m^2}
\]

The \((a^2 + m^2)\) in the denominator of \(A\) and \(B\) comes from (37) which is in the denominator of (34). From (18) and (41)

\[
\frac{A b \gamma + B \gamma^3}{\gamma^4 + b^2 e^2} = \frac{b \gamma \left[ m (az + my) + b (a^2 + m^2) \right] + \left[ (a^2 + m^2) \gamma^2 - m (az + my)b \right] \gamma}{(a^2 + m^2)(\gamma^2 + b^2)}
\]

\[
= \frac{\gamma (a^2 + m^2)}{(a^2 + m^2)} = \frac{m (ay - mz)}{(a^2 + m^2)}
\]

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and

\[ \frac{-A \frac{\gamma^3}{e} + B \beta \gamma}{\gamma^4 + b^2 e^2} = -\gamma^2 \left[ m \left( az + my \right) + b \left( a^2 + m^2 \right) \right] + \left[ (a^2 + m^2) \gamma^2 - b m \left( az + my \right) \right] \]

\[ \frac{b}{(a^2 + m^2)(\gamma^2 + b^2)} \]

\[ = \frac{-m \left( az + my \right)}{a^2 + m^2} \]

From (39) and (40),

\[ \hat{u}^2 + 2b \hat{u} + c = \left( mx - \eta \right)^2 - \beta^2 \left( a \eta - mz \right)^2 - \beta^2 m^2 \left( \eta - y \right)^2 \]

\[ \gamma^2 - b \hat{u} = \gamma^2 \cdot b \left[ \eta - \frac{m \left( az + my \right)}{a^2 + m^2} \right] = \frac{m \left[ m \left( mx - y \right) - a \left( z - ax \right) \right] \eta}{a^2 + m^2} + \left[ \gamma^2 \cdot b \frac{m \left( az + my \right)}{a^2 + m^2} \right] \]

\[ = \frac{m \left[ m \left( mx - y \right) - a \left( z - ax \right) \right] \eta + m \left[ a \left( z - ax \right) - m \left( mx - y \right) \right]}{a^2 + m^2} \]

\[ u + b = \eta - mx \]

from (38) and (40).
Therefore using (18), and (41) we get from (34)

\[
\frac{2}{a^2 + m^2} \int \frac{(ay - mz)^2 + (ny + az) \left[ m (nx - y) - a (z - ax) \right]}{(a \eta - mz)^2 + m^2 (\eta - y)^2} \left[ m (nx - y) - a (z - ax) \right] (y^2 + z^2) \frac{d \eta}{\sqrt{m (nx - y) - a (z - ax) + m (z - ax) - y (mx - y)}}
\]

\[= \left( ay - mz \right) \tan^{-1} \frac{(ay - mz) \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2}{m (nx - y) - a (z - ax) + m (z - ax) - y (mx - y)} \]

Therefore substituting (36), and (42) in (34)

\[
\frac{1}{2} \int \log \frac{(mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2} d \eta = \]

\[= \frac{1}{2} \left\{ \log \frac{mx - \eta + \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2}{mx - \eta - \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2} \right\} \]

\[+ \frac{m (az + mz) - (a^2 + m^2) \eta}{2 (a^2 + m^2)} \log \frac{mx - \eta - \beta \eta (mz - ay) + \sqrt{1 - \beta (a^2 + m^2)}}{mx - \eta - \beta \eta (mz - ay) - \sqrt{1 - \beta (a^2 + m^2)}} \]

\[+ \frac{m (mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay) \sqrt{(mx - \eta)^2 - \beta^2 (a \eta - mz)^2 + m^2 (\eta - y)^2}}{m (mx - y) - a (z - ax) + m (z - ax) - y (mx - y)} \]

\[= \frac{N (a \eta + m z) \log (mx - \eta) + \sqrt{(mx - \eta)^2 - \beta^2 (\eta - y)^2} - \beta^2 (mz - ay)^2}{2 (a^2 + m^2)} \]

\[\text{(42)}\]
Differentiation of the indefinite integral over \( \eta \). From (33)

\[
\frac{1}{z} \frac{d}{d\eta} \log \frac{(xm - \eta) + \sqrt{(mx - \eta)^2 - \beta^2(m^2 - \eta^2)^2} - \beta^2(mz - \eta)^2}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2(m^2 - \eta^2)^2} - \beta^2(mz - \eta)^2} = 
\]

\[
m^2 \left[ y(mx - y) - z(z - ax) \right] + m \left[ a(z - ax) - m(mx - y) \right] \eta
\]

\[
\left[ (a\eta - mz)^2 + m^2(\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2(\eta - y)^2 \right]}
\]

From (24) using (35)

\[
\frac{1}{z} \frac{d}{d\eta} \log \frac{(mx - \eta) - \beta^2(mz - \eta)^2}{(mx - \eta) - \beta^2(mz - \eta)^2} + \sqrt{\beta^2(mz - \eta)^2 - \beta^2(mz - \eta)^2} - \beta^2(mz - \eta)^2}
\]

\[
= -\frac{\sqrt{1 - \beta^2(a^2 + m^2)}}{\sqrt{(mx - \eta)^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2(\eta - y)^2 \right]}}
\]

and

\[
\frac{d}{d\eta} \tan^{-1} \frac{f}{g} = \frac{g'f - gff'}{g^2 \left( \frac{f}{g} \right)^2 + 1} = \frac{gff' - g'f^2}{f (f^2 + g^2)}
\]

where

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\[ f = (mz - ay) \sqrt{\left(mx - \eta \right)^2} \cdot (a \eta - mz)^2 + m^2 \left( \eta - y \right)^2 \]  

(46)

\[ g = \left[ m \left(mx - y\right) - a \left(z - ax\right) \right] \eta \cdot m \left[ z \left(z - ax\right) + y \left(mx - y\right) \right] \]

\[ ff' = (mz - ay)^2 \left[ \eta \left(mx\right) + a \left(\eta - mx\right) + \beta^2 m^2 \left(\eta - y\right) \right] \]

\[ g' = \left[ m \left(mx - y\right) - a \left(z - ax\right) \right] \]

\[ f^2 + g^2 = (mz - ay)^2 \left[ \left(\eta - mx\right)^2 + \beta^2 \left(a \eta - mz\right)^2 + m^2 \left(\eta - y\right)^2 \right] \]

\[ + \left[ m \left(mx - y\right) - a \left(z - ax\right) \right] \eta \cdot m \left[ z \left(z - ax\right) + y \left(mx - y\right) \right]^2 \]

\[ = -\beta^2 (mz - ay)^2 \left[ \left(a \eta - mz\right)^2 + m^2 \left(\eta - y\right)^2 \right] \]

\[ + \left[ \eta \left(mx\right) + a \left(mx - y\right) \right] \left(\eta - mx\right)^2 \]

\[ + \left[ m \left(mx - y\right) - a \left(z - ax\right) \right] \left(\eta - y\right) \cdot \left(z - ax\right) \left(a \eta - mz\right) \]

\[ = -\beta^2 (mz - ay)^2 \left[ \left(a \eta - mx\right)^2 + m^2 \left(\eta - y\right)^2 \right] \]

\[ + \left[ a^2 \left(mx - y\right)^2 + 2 am \left(mx - y\right) \left(z - ax\right) + m^2 \left(z - ax\right)^2 \right] \left(\eta - mx\right)^2 \]

\[ + m^2 \left(mx - y\right)^2 \left(\eta - y\right)^2 - 2m \left(mx - y\right) \left(z - ax\right) \left(\eta - y\right) \left(a \eta - mz\right) + (2 - ax)^2 \left(a \eta - mz\right)^2 \]
\[
\begin{align*}
\mathbf{u}^T \mathbf{p}^T \sigma^2 \frac{\mathbf{p}}{\mathbf{u}} &= \left( \mathbf{u} (\mathbf{mx} - \mathbf{my}) (\mathbf{py} - \mathbf{py}) + \mathbf{m} \left( \mathbf{mx} \mathbf{y} \right) \right)
\end{align*}
\]
Therefore

\[
\frac{d}{d\eta} \tan^{-1} \left( \frac{(ay - mz) \sqrt{\eta - mx}^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]}{m (mx - y) - a (z - ax)} \right) \eta + m \left[ z (z - ax) - y (mx - y) \right]
\]

\[
= \frac{m (ay - mz) (\eta - mx)}{(a\eta - mz)^2 + m^2 (\eta - y)^2} \sqrt{\eta - mx}^2 - \beta^2 \left[ (a\eta - mz)^2 + m^2 (\eta - y)^2 \right]
\]

(48)

Therefore combining terms in (43), (46), (47), and (48)

\[
\frac{1}{2} \frac{d}{d\eta} \int \frac{\log \left( \frac{mx - \eta}{\eta - mx} + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - \eta)^2} \right)}{(mx - \eta) - \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - \eta)^2}} \, d\eta
\]

\[
= \frac{1}{2} \log \left( \frac{mx - \eta}{\eta - mx} + \sqrt{(mx - \eta)^2 - \beta^2 m^2 (\eta - y)^2 - \beta^2 (mz - \eta)^2} \right)
\]

\[
\times \left[ \frac{n^2 \left[ y (mx - \eta) + z (z - ax) \right] + n \left[ a (z - ax) - n (mx - y) \right]}{(a\eta - mz)^2 + n^2 (\eta - y)^2} \sqrt{(mx - \eta)^2 - \beta^2 (mz - \eta)^2} \right]
\]

\[
- \frac{n \left[ (sz - zm) - (s^2 + n^2)x \right]}{(s^2 + n^2) \sqrt{(mx - \eta)^2 - \beta^2 (mz - \eta)^2} \sqrt{(mx - \eta)^2 - \beta^2 (mz - \eta)^2}}
\]

\[
+ \frac{n^2 (ay - mz) (\eta - mx)}{(s^2 + n^2) \left[ (a\eta - mz)^2 + n^2 (\eta - y)^2 \right] \sqrt{(mx - \eta)^2 - \beta^2 (mz - \eta)^2} \sqrt{(mx - \eta)^2 - \beta^2 (mz - \eta)^2}}
\]

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Arranging all terms over a common denominator

\[ \frac{mn}{(mx-y)\sqrt{\Delta}} = \frac{m}{(\Delta \pm \eta)^2} \]

The numerator can be written as:

\[ \eta^2 \left\{ \left( a^2 + m^2 \right) \left[ m \left( z - ay \right) \right] - a \left( z - ax \right) \cdot m \left( mx - y \right) \right\} + \eta \left\{ \left( a^2 + m^2 \right) \left[ n \left( z - ay \right) \right] - a \left( z - ax \right) \cdot m \left( mx - y \right) \right\} \\
+ 2 \left[ a \left( z - ax \right) \cdot m \left( mx - y \right) \right] \left[ iz + my \right] n + m \left( ay - mz \right)^2 \]

Therefore (43) is verified.
Evaluation of the definite integral over \( \eta \). - Because of (4) and (6) and analogous to the section in the evaluation of the definite integral for the surface distribution of sources, all of the terms in (43) will be zero when \( \eta = \eta_3 \) provided the denominator of the \( \tan^{-1} \) term is greater than zero. At \( \eta = \eta_3 \), using (6), and then (5),

\[ m (mx - y) - a (z - ax) \]

\[ \eta_3 - \eta [z (z - ax) - y (mx - y)] \]

\[
\frac{m \left[ m (mx - y) - a (z - ax) \right] \left( 1 - \beta^2 a^2 + \beta^2 (ay - mz)^2 \right)}{1 - \beta^2 (a^2 + m^2)} \]

\[
\beta \left[ m (mx - y) - a (z - ax) \right] \frac{\sqrt{(mx - y)^2 + (z - ax)^2} - \beta^2 (ay - mz)^2}{1 - \beta^2 (a^2 + m^2)} \]

\[
\frac{m \sqrt{(mx - y)^2 + (z - ax)^2} - \beta^2 (ay - mz)^2}{1 - \beta^2 (a^2 + m^2)} \frac{\sqrt{(mx - y)^2 + (z - ax)^2} - \beta^2 (ay - mz)^2}{1 - \beta^2 (a^2 + m^2)} \]

But

\[
\frac{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2}{1 - \beta^2 (a^2 + m^2)} = \beta^2 \left[ m (mx - y) - a (z - ax) \right] \]

\[
\frac{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2}{1 - \beta^2 (a^2 + m^2)} = \beta^2 \left[ m (mx - y) - a (z - ax) \right] \]

\[
\left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] + \beta^2 a^2 (mx - y)^2 + \beta^2 m^2 (z - ax)^2 \]

\[ 2 \beta^2 am (mx - y) (z - ax) - \beta^2 (ay - mz)^2 \]

\[ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] + \beta^2 \left[ a (mx - y) + m (z - ax) \right]^2 - \beta^2 (ay - mz)^2 \]

\[ \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ (mx - y)^2 + (z - ax)^2 \right] \]

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Therefore if \(1 - \beta^2 (a^2 + m^2) > 0\)

\[
\sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} > \beta |m(mx - y) - a(z - ax)|
\]

and from (49)

\[
[m(mx - y) - a(z - ax)]a_3 + m[z(z - ax) - y(mx - y)] > 0
\]

However also,

\[
m(mx - y) - a(z - ax) = (a^2 + m^2)x - (my + az)
\]

and

\[
(a^2 + m^2)x^2 - (my + az)^2 = (a^2 + m^2)x^2 - (a^2 + m^2)(y^2 + z^2) + (ay - mz)^2
\]

\[
= \frac{a^2 + m^2}{\beta^2} \left[ \beta^2 (a^2 + m^2) - 1 \right]x^2 + \left[ x^2 - \beta^2 (y^2 + z^2) \right] + (ay - mz)^2 > 0
\]

if

\[
1 - \beta^2 (a^2 + m^2) < 0
\]

and

\[
x^2 > \beta^2 (y^2 + z^2)
\]
Therefore if \( x > 0 \) also

\[
(a^2 + mx^2) > |my + az|
\]

and

\[
m (mx - y) - a (z - ax) > 0
\]

if

\[
1 - \beta^2 (a^2 + m^2) < 0
\]

\[
\chi^2 > \beta^2 (y^2 + z^2)
\]

Therefore from (50) if \( 1 - \beta^2 (a^2 + m^2) < 0 \)

\[
\left[ \beta^2 (a^2 + m^2) \right]^{-1} \left[ \beta [m (mx - y) - a (z - ax)] \sqrt{(mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2} \right] > 0
\]

and from (49), if \( \chi^2 > \beta^2 (y^2 + z^2) \)

\[
[m (mx - y) - a (z - ax)] \eta_3 + m [z (z - ax) - y (mx - y)] > 0
\]

and therefore, using (4) and (54), the \( \tan^{-1} \) term in (45) is zero when

\( \eta = \eta_3 \).
Therefore substituting $\eta = 0$ into (43), and from (32),

$$
\phi(x, y, z) = \frac{\Delta \rho}{q_{\infty}} \left\{ \begin{align*}
\int_{0}^{\eta_{3}} \log \frac{(m \cdot x \cdot \gamma + \sqrt{(m \cdot x \cdot \gamma)^{2} - \beta \cdot (a \cdot z + m \cdot y)^{2}})}{(m \cdot x \cdot \gamma - \sqrt{(m \cdot x \cdot \gamma)^{2} - \beta \cdot (a \cdot z + m \cdot y)^{2}})} \, dz \\
- \sqrt{1 - \beta^{2} a^{2}} \int_{0}^{\eta_{3}} \log \frac{(m \cdot x \cdot \gamma - \beta a (m \cdot x - \gamma) \cdot \beta^{2} (a \cdot z + m \cdot y)^{2} + \sqrt{[1 - \beta^{2} (a \cdot z + m \cdot y)^{2}] [1 - \beta^{2} (a \cdot z + m \cdot y)^{2}]}}{(m \cdot x \cdot \gamma - \beta a (m \cdot x - \gamma) \cdot \beta^{2} (a \cdot z + m \cdot y)^{2} - \sqrt{[1 - \beta^{2} (a \cdot z + m \cdot y)^{2}] [1 - \beta^{2} (a \cdot z + m \cdot y)^{2}]}} \, dz \\
= \frac{1}{4 \pi a} \frac{\Delta \rho}{q_{\infty}} \left\{ \begin{align*}
\frac{m (a \cdot z + m \cdot y)}{2 (a^{2} + m^{2})} \log \frac{x + \sqrt{x^{2} - \beta^{2} (y^{2} + z^{2})}}{x - \sqrt{x^{2} - \beta^{2} (y^{2} + z^{2})}} \\
- \frac{m [(a \cdot z + m \cdot y) - (a^{2} + m^{2})]}{2 (a^{2} + m^{2}) \sqrt{1 - \beta^{2} (a^{2} + m^{2})}} \log \frac{x - \beta (a \cdot z + m \cdot y) + \sqrt{[1 - \beta^{2} (a^{2} + m^{2})] [x^{2} - \beta^{2} (y^{2} + z^{2})]}}{x - \beta (a \cdot z + m \cdot y) - \sqrt{[1 - \beta^{2} (a^{2} + m^{2})] [x^{2} - \beta^{2} (y^{2} + z^{2})]}} \\
+ \frac{m (a \cdot z - m \cdot y)}{a^{2} + m^{2}} \tan^{-1} \frac{(m \cdot z - m \cdot y) \sqrt{x^{2} - \beta^{2} (y^{2} + z^{2})}}{z (z - ax) - y (m x - y)} \\
+ \frac{(z - ax) \tan^{-1} \frac{m (z \cdot ax) \sqrt{x^{2} - \beta^{2} (y^{2} + z^{2})}}{z [ (y - m x) \cdot \beta^{2} (a \cdot z + m \cdot y) + (z - ax)^{2}]} \\
- \frac{m (x \cdot \beta^{2} a^{2}) - m (1 - \beta^{2})}{2 \sqrt{1 - \beta^{2} (a^{2} + m^{2})}} \log \frac{x - \beta^{2} (a \cdot z + m \cdot y) + \sqrt{[1 - \beta^{2} (a^{2} + m^{2})] [x^{2} - \beta^{2} (y^{2} + z^{2})]}}{x - \beta^{2} (a \cdot z + m \cdot y) - \sqrt{[1 - \beta^{2} (a^{2} + m^{2})] [x^{2} - \beta^{2} (y^{2} + z^{2})]}} \\
- \frac{1}{2} \sqrt{1 - \beta^{2} a^{2}} \log \frac{(x - \beta^{2} a^{2}) + \sqrt{[1 - \beta^{2} a^{2}] [x^{2} - \beta^{2} (y^{2} + z^{2})]}}{(x - \beta^{2} a^{2}) - \sqrt{[1 - \beta^{2} a^{2}] [x^{2} - \beta^{2} (y^{2} + z^{2})]}} \right\} \right\}
\end{align*} \right\}
\end{align*} \right\}
\end{align*}}

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Therefore combining terms

\[ \phi(x, y, z) = \frac{1}{4\pi} \left( \frac{\Delta p}{\rho_0} \right) \left\{ \frac{m(az + my)}{2(a^2 + m^2)} \log \frac{x + \sqrt{x^2 - \beta^2(y^2 + z^2)}}{x - \sqrt{x^2 - \beta^2(y^2 + z^2)}} + \frac{\alpha(a z + m y) \sqrt{1 - \beta^2(a^2 + m^2)}}{2(a^2 + m^2)} \log \frac{x - \beta^2(a z + m y) + \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2(a z + m y) - \sqrt{[1 - \beta^2(a^2 + m^2)][x^2 - \beta^2(y^2 + z^2)]}} \right\} \\
\frac{-m(mz - ay)}{a^2 + m^2} \tan^{-1} \left( \frac{(mz - ay)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{z(z - ax) - y(mx - y)} \right) + (z - ax) \tan^{-1} \left( \frac{m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)}}{y[y - mx] - \beta^2 a(yz - yz)} + (z - ax)^2 \right) \\
- \frac{1}{2} y \sqrt{1 - \beta^2 a^2} \log \frac{x - \beta^2 a z + \sqrt{[1 - \beta^2 a^2][x^2 - \beta^2(y^2 + z^2)]}}{x - \beta^2 a z - \sqrt{[1 - \beta^2 a^2][x^2 - \beta^2(y^2 + z^2)]}} \right\} 
\]

Evaluation of the velocity components. - Using the arguments presented in the section covering the evaluation of the velocity components for the source distribution, the partial derivatives of (51) may be obtained by differentiating only the coefficients of each term.

Modifications and Regions of Validity of the Velocity Potential Functions

Region of validity of \( \phi \). - All of the integrations were performed without regard to the existence of the limits of integration, negative square roots, etc. Therefore the functions in (29) and (51) must be examined and possibly modified for some regions of \((x, y, z)\) space.

Since all of the terms contain \( \sqrt{x^2 - \beta^2(y^2 + z^2)} \) the formula for \( \phi(x, y, z) \) given by (51) is only valid when \( x^2 > \beta^2(y^2 + z^2) \), which means inside the Mach cone from the origin.
If \( 1 - \beta^2 (a^2 + m^2) < 0 \) (supersonic leading edge), the log term which contains the square root of this quantity must be modified. We can write

\[
\frac{i}{2} \log \frac{f+iq}{f-iq} = -\tan^{-1} \frac{q}{f} \tag{52}
\]

and therefore if \( 1 - \beta^2 (a^2 + m^2) < 0 \)

\[
\frac{1}{2} \sqrt{1 - \beta^2 (a^2 + m^2)} \log \frac{x - \beta^3 (a^2 + my) + \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^3 (y^2 + z^2)]}}{x - \beta^3 (a^2 + my) - \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^3 (y^2 + z^2)]}} = -\sqrt{\beta^2 (a^2 + m^2) - 1} \tan^{-1} \frac{\sqrt{[\beta^3 (a^2 + m^2) - 1][x^2 - \beta^3 (y^2 + z^2)]}}{x - \beta^3 (a^2 + my)} \tag{53}
\]

If \( 1 - \beta^2 (a^2 + m^2) = 0 \) there is a sonic leading edge.

For the case where \( 1 - \beta^2 (a^2 + m^2) \to 0 \) we can write

\[
\lim_{1 - \beta^2 (a^2 + m^2) \to 0} \frac{1}{2} \sqrt{1 - \beta^2 (a^2 + m^2)} \log \frac{x - \beta^3 (a^2 + my) + \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^3 (y^2 + z^2)]}}{x - \beta^3 (a^2 + my) - \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^3 (y^2 + z^2)]}} = \frac{\sqrt{x^2 - \beta^3 (y^2 + z^2)}}{x - \beta^3 (a^2 + my)} \tag{54}
\]
All of the log terms appear to potentially involve logarithms of negative numbers. This difficulty may be avoided by taking the absolute value of the arguments. This is allowable since any log of -1 would have canceled in the definite integral over T. The argument could not have passed through zero and therefore must have been always positive or always negative.

Since certain functions occur repeatedly we define

\[ F_1 = \tan^{-1} \frac{m(z - ax)}{y[(y - mx) + (z - ax)]^2} \]

\[ F_2 = \frac{1}{2\sqrt{1 - \beta^2(s^2 + m^2)}} \log \frac{x - \rho^1(\rho^3 + m^2) + \sqrt{[1 - \rho^1(\rho^3 + m^2)]}[x - \rho^1(\rho^3 + m^2)]}{x - \rho^1(\rho^3 + m^2) - \sqrt{[1 - \rho^1(\rho^3 + m^2)]}[x - \rho^1(\rho^3 + m^2)]} \]

\[ F_3 = \tan^{-1} \frac{(m - ax)}{z(z - ax) - y(mx - y)} \]

\[ F_6 = \sqrt{1 - \beta^2} \frac{1}{2} \log \frac{x - \rho^1a^2 + \sqrt{[1 - \rho^1a^2]}[x - \rho^1a^2]}{x - \rho^1a^2 - \sqrt{[1 - \rho^1a^2]}[x - \rho^1a^2]} \]

Evaluation of the \( \tan^{-1} \) functions \( F_1 \) and \( F_3 \), - The terms \( F_1 \) or \( F_3 \) are always real inside the Mach cone from the origin,

\[ x^2 - \beta^2(y^2 + z^2) \geq 0 \]

However as the argument of these functions go to zero the functions may take on different values depending upon how zero is approached. Corresponding to each of the four quadrants, if

\[ \theta = \tan^{-1} \frac{f}{g} \]
then

\[ f \geq 0 \quad g > 0 \quad \text{means} \quad 0 \leq \theta < \frac{\pi}{2} \]

\[ f \geq 0 \quad g < 0 \quad \text{means} \quad \frac{\pi}{2} < \theta < \pi \]

\[ f < 0 \quad g > 0 \quad \text{means} \quad 0 \geq \theta > -\frac{\pi}{2} \]

\[ f < 0 \quad g < 0 \quad \text{means} \quad -\frac{\pi}{2} > \theta \geq -\pi \]  

(57)

Therefore from (52) and (57) as \( z \to ax \)

\[
\begin{array}{c|c}
\text{0} & \text{y < 0 or y > mx} \\
\hline
\lim_{z \to ax} F_1 = \pi & 0 < y < mx, z > ax \\
\lim_{z \to ax} F_3 = \pi & 0 < y < mx, z > ay \\
\end{array}
\]

and as \( mz \to ay \)

\[
\begin{array}{c|c}
\text{0} & \text{y < 0 or y > mx} \\
\hline
\lim_{mz \to ay} F_1 = \pi & 0 < y < mx, mz > ax \\
\lim_{mz \to ay} F_3 = \pi & 0 < y < mx, mz > ay \\
\end{array}
\]

(58)
Velocity potential on \( x^2 = \beta^3 (y^2 + z^2) \) is \( 1 - \beta^3 (a^2 + m^2) > 0 \). Referring to (52) \( F_2 \) and \( F_6 \) are zero if \( x^2 - \beta^2 (y^2 + z^2) = 0 \). The functions \( F_1 \) and \( F_3 \) will be zero if and only if the denominators of their arguments are greater than zero [see (57)]. First we note the following

\[
x^2 - [\beta^3 (m + az)]^2 = [x^2 - \beta^2 (y^2 + z^2)] + \beta^2 [y^2 - \beta^2 (z^2 + x^2)] - \beta^2 [z^2 - \beta^2 (y^2 + x^2)] + \beta^4 (m + az)^2
\]

\[
= [x - \beta^2 (my + az)] [x + \beta^2 (my + az)]
\]

which means \( x > |\beta^2 (my + az)| \)

\[
\therefore \quad x^2 - \beta^3 (y^2 + z^2) \geq 0 \quad 1 - \beta^3 (a^2 + m^2) > 0
\]

For \( F_1 \) the denominator is

\[
y [(y - mx) - \beta^2 a (ay - mz)] + (z - ax)^2
\]

\[
= y^2 [1 - \beta^2 (a^2 + m^2)] - my [x - \beta^2 (az + my)] + (z - ax)^2
\]

\( > 0 \) if \( y < 0 \) from (60)
\[ y \left[(y - mx) - \beta^2 a (ay - mz)\right] + (z - ax)^2 \]

- \[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mx (y - mx) - \beta^2 mz (ay - mz) \]

- \[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + mxy - m^2 (x^2 - \beta^2 z^2) - \beta^2 anyz \]

- \[ (mx - y)^2 + (z - ax)^2 - \beta^2 (ay - mz)^2 + my [x - \beta^2 (az + my)] \]

\[ - m^2 [x^2 - \beta^2 (y^2 + z^2)] > 0 \]

if \[ x^2 - \beta^2 (y^2 + z^2) = 0, y > 0 \] due to (50) and (60).

Therefore

\[ y \left[(y - mx) - \beta^2 a (ay - mz)\right] + (z - ax)^2 > 0 \]

on

\[ x^2 = \beta^2 (y^2 + z^2) \]

if

\[ 1 - \beta^2 (a^2 + m^2) > 0 \]

Therefore due to (57) and (61)

\[ \lim F_1 = 0 \]

as \[ x^2 - \beta^2 (y^2 + z^2) \to 0 \]

if

\[ 1 - \beta^2 (a^2 + m^2) > 0 \]
\[(z - ax) - y (mx - y) = y^2 + z^2 - x (az + my) \]

\[= - \left[ x^2 - \beta^2 \right] \left( y^2 + z^2 \right) \frac{1}{\beta^2} + \frac{x}{\beta^2} \left[ x - \beta^2 \left( az + my \right) \right] > 0 \]

if

\[x^2 - \beta^2 \left( y^2 + z^2 \right) = 0, \text{ and } 1 - \beta^2 \left( a^2 + m^2 \right) > 0 \text{ due to (54c) if } x > 0\]

Therefore examining (29) and (56) shows that for a surface distribution of sources or a constant pressure surface

\[\lim_{r \to 0} \phi = 0\]

as \(x^2 - \beta^2 \left( y^2 + z^2 \right) \to 0\)

if

\[1 - \beta^2 \left( a^2 + m^2 \right) \to 0 \quad (63)\]

Therefore \(\phi\) will be continuous, as it must be, if we define \(\phi\) to be zero outside the Mach cone from the origin.

\[\phi = 0\]

if

\[1 - \beta^2 \left( a^2 + m^2 \right) > 0 \text{ and } x^2 < \beta^2 \left( y^2 + z^2 \right) \quad (64)\]
Supersonic leading edge and the mach cone envelope. With a supersonic leading edge, \( 1 - \beta^2 (a^2 + m^2) < 0 \), all functions in the equations for \( \phi \) will be shown to go to zero for points on the Mach cone from the origin, except for a region on this Mach cone which borders the envelope of Mach cones from the supersonic leading edge. Inside this envelope of Mach cones the functions \( F_1 \), \( F_2 \) and \( F_3 \) will be shown to have constant values. However on the outer boundary of this envelope of Mach cones \( \phi \) will go to zero, and therefore all functions may be defined to be zero outside this envelope.

The envelope of Mach cones from the leading edge is illustrated on p-29 of Reference (55). The Mach cone from any point \( x_0, y_0, z_0 \) can be written,

\[
(x - x_0)^2 = \beta^2 [(y - y_0)^2 + (z - z_0)^2]
\]

on the leading edge \( mx_0 = y_0 \) and \( ax_0 = z_0 \).
Therefore

\[(mx - y_o)^2 = \beta^2 [m^2 (y - y_o)^2 + (mz - ay_o)^2]\]  \hspace{1cm} (65)

The Mach cone envelope is determined by the maximum values for \(z\), at a given \(x\) and \(y\), obtained by a variation of \(y\). Therefore differentiating (65) with respect to \(y_o\), holding \(x\) and \(y\) constant, and setting \(dz/dy_o = 0\):

\[gives\]

\[(mx - y_o) = \beta^2 m^2 (y - y_o) + \beta^2 a (mz - ay_o)\]

or

\[y_o [1 - \beta^2 (a^2 + m^2)] = m [x - \beta^2 (my + az)]\]  \hspace{1cm} (66)

If \(y_o = 0\) (65) and (66) give

\[x^2 = \beta^2 (y^2 + z^2)\] and \(x - \beta^2 (my + az) = 0\]  \hspace{1cm} (67)

or

\[x^2 = \beta^2 y^2 + \beta^2 \left[ \frac{x - \beta^2 my}{\beta^2 a} \right]^2\]

\[\beta^2 a^2 x^2 = \beta^4 a^2 y^2 + x^2 - 2 \beta^2 mxy + \beta^4 m^2 y^2\]

\[\beta^4 (a^2 + m^2) y^2 - 2 \beta^2 mxy + x^2 (1 - \beta^2 a^2) = 0\]
or, if $y_o = 0$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\}$$  \hspace{1cm} (68)$$

where

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m - a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \quad z > 0$$

$$y = \frac{x}{\beta^2 (a^2 + m^2)} \left\{ m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right\} \quad z < 0$$

For $y_o > 0$ we can eliminate $y_o$ from (65) and (66).

From (65)

$$\left[ 1 - \beta^2 (a^2 + m^2) \right] y_o^2 - 2m \left[ x - \beta^2 (az + my) \right] y_o + m^2 \left[ x^2 - \beta^2 (y^2 + z^2) \right] = 0$$

or

$$y_o \left[ 1 - \beta^2 (a^2 + m^2) \right] = m \left[ x - \beta^2 (az + my) \right]$$

$$\pm m \sqrt{\left[ x - \beta^2 (az + my) \right]^2 - \left[ 1 - \beta^2 (a^2 + m^2) \right] \left[ x^2 - \beta^2 (y^2 + z^2) \right]}$$

Therefore from (66)

$$\left| x - \beta^2 (az + my) \right|^2 - \left| 1 - \beta^2 (a^2 + m^2) \right| \left| x^2 - \beta^2 (y^2 + z^2) \right| = 0$$

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or

\[ \beta^2 (a^2 + m^2) x^2 - 2x \beta^2 (az + my) - \beta^4 (ay - mz)^2 + \beta^2 (y^2 + z^2) = 0 \]

and solving for \( x \)

\[ x = \frac{(az + my) \pm \sqrt{(az + my)^2 + (a^2 + m^2) \left[ (ay - mz)^2 \beta^2 - (y^2 + z^2) \right]}}{(a^2 + m^2)} \]

\[ = \frac{(az + my) \pm \sqrt{(ay - mz)^2 \left[ \beta^2 (a^2 + m^2) - 1 \right]}}{(a^2 + m^2)} \]

\[ = \frac{(az + my) \pm (mz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \]

If we write

\[ x = \frac{(az + my) \pm (mz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} \]

or

\[ \frac{\partial z}{\partial y} \left[ a + m \sqrt{\beta^2 (a^2 + m^2) - 1} \right] = \left[ -m + a \sqrt{\beta^2 (a^2 + m^2) - 1} \right] \]
\[
\frac{\partial z}{\partial y} (a^2 + m^2) \left[1 - \beta^2 m^2\right]
\]

\[
= \left(-a + m \sqrt{\beta^2 (a^2 + m^2) - 1}\right) \left(-m + a \sqrt{\beta^2 (a^2 + m^2) - 1}\right)
\]

\[
= (a^2 + m^2) \left[-\beta^2 m + \sqrt{\beta^2 (a^2 + m^2) - 1}\right]
\]

Therefore use:

+ for \(mz > ay\)

- for \(mz < ay\)

or

\[
x = \frac{(az + my) + \left|mz + ay\right| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)}
\]

This is presented on page 29 of reference (55).

If we set

\[
z = y \tan \theta \quad \text{on} \quad x^2 = \beta^2 \left(y^2 + z^2\right)
\]

then

\[
x^2 = \beta^2 y^2 \left(1 + \tan^2 \theta\right)
\]
or

\[ x = \frac{\beta y}{\cos \theta}, \quad \beta y = x \cos \theta \quad (70) \]

Therefore (67) gives for the two points where the envelope of Mach cones from the leading edge is on the Mach cone from the origin,

\[ (y^2 + z^2) = \beta^2 (my + az)^2 \]

or

\[ \left[ 1 + \tan^2 \theta \right] = \beta^2 m^2 + 2 \beta^2 am \tan \theta + \beta^2 a^2 \tan^2 \theta \]

which means

\[ (1 - \beta^2 a^2) \tan \theta = \beta^2 am + \sqrt{\beta^4 a^2 m^2 - (1 - \beta^2 a^2) (1 - \beta^2 m^2)} \]

or

\[ (1 - \beta^2 a^2) \tan \theta = \beta^2 am + \sqrt{\beta^2 (a^2 + m^2)} - 1 \quad (71) \]

but

\[ x - \beta^2 (my + az) = 0 \]

means

\[ \frac{\beta a}{\cos \theta} - \beta^2 a (m + a \tan \theta) = 0 \quad (72) \]
Therefore the two points corresponding to (68) are

\[ \tan \theta_1 = \frac{\beta a}{\cos \theta_1} - \sqrt{\beta^2 (a^2 + m^2) - 1} \]

\[ \tan \theta_2 = \frac{\beta a}{\cos \theta_2} + \sqrt{\beta^2 (a^2 + m^2) - 1} \]  

(73)

(assume \( a > 0 \) and \( \theta_2 > \theta_1 \))

From (56) and (57) the value of \( F_1 \) or \( F_3 \) as we approach the Mach cone from the origin depends on the sign of the denominator of its argument.

For

\[ F_1 \text{ on } x^2 = \beta^2 (y^2 + z^2) \]

we can write

\[ D_1 = \frac{1}{y^2} \left| y (y - mx) - \beta^2 a (ay - mz) + (z - ax)^2 \right| \]

\[ = (1 - \beta^2 a^2) - \frac{\beta m}{\cos \theta} + \beta^2 a \tan \theta + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2 \]

\[ = 1 - \beta^2 (a^2 + m^2) + \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right]^2 - \beta m \left[ \beta m + \frac{1}{\cos \theta} - \beta a \tan \theta \right] \]

\[ = 0 \text{ if } \theta = \theta_1, \text{ or } \theta = \theta_2 \text{ from (72) and (73)} \]
If we differentiate this with respect to $\theta$ we get

$$\frac{dD_1}{d\theta} = 2 \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ \sec^2 \theta - \frac{\beta a \sin \theta}{\cos^2 \theta} \right] + \beta m \left[ \beta a \sec^2 \theta - \frac{\sin \theta}{\cos^2 \theta} \right]$$

$$= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ 2 \left( 1 - \beta a \sin \theta \right) - \beta m \cos \theta \right]$$

$$= \frac{1}{\cos^2 \theta} \left[ \tan \theta - \frac{\beta a}{\cos \theta} \right] \left[ 2 \left( 1 - \beta a \sin \theta - \beta m \cos \theta \right) + \beta m \cos \theta \right]$$

From (72)

$$1 - \beta a \sin \theta - \beta m \cos \theta = 0 \quad \text{at} \quad \theta = \theta_1, \theta_2$$

and since

$$\cos \theta_1, > 0, \cos \theta_2 > 0 \quad \text{and} \quad \beta m > 0$$

and using (73) we get

$$\frac{dD_1}{d\theta} < 0 \quad \theta = \theta_1$$

$$\frac{dD_1}{d\theta} > 0 \quad \theta = \theta_2$$
and therefore using (57) if

$$1 - \beta^2 (a^2 + m^2) < 0$$

$$\lim_{x^2 \to \beta^2 (y^2 + z^2)} F_1 = \begin{array}{ccc}
0 & \theta > \theta_2 & \theta < \theta_1 \\
\pi & z > ax & \theta_1 < \theta < \theta_2 \\
\pi & z < ax & \theta_1 < \theta < \theta_2
\end{array}$$ (74)

For

$$F_3 \text{ on } x^2 = \beta^2 (y^2 + z^2)$$

let

$$D_3 = z (z - ax) - y (mx - y)$$

$$= y^2 (1 + \tan^2 \theta) - yx (m + a \tan \theta)$$

$$= \frac{x^2}{\beta^2} \left[ 1 - \beta \cos \theta (m + a \tan \theta) \right]$$

$$= \frac{x^2}{\beta^2} \left[ 1 - (\beta m \cos \theta + \beta a \sin \theta) \right]$$

$$= 0 \text{ if } \theta = \theta_1 \text{ or } \theta = \theta_2 \text{ from (72)}$$
\[
\frac{d}{d\theta} D^3 = \frac{x^2}{\beta} \left[ m \sin \theta - a \cos \theta \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left[ m (1 - \beta^2 a^2) \tan \theta - a (1 - \beta^2 a^2) \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left[ \beta^2 m^2 - a (1 - \beta^2 a^2) \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \right]
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \left\{ \begin{array}{l}
- a \left[ 1 - \beta^2 (a^2 + m^2) \right] \pm m \sqrt{\beta^2 (a^2 + m^2) - 1} \\
\end{array} \right.
\]

\[
= \frac{x^2 \cos \theta}{\beta (1 - \beta^2 a^2)} \frac{\sqrt{\beta^2 (a^2 + m^2) - 1}}{1} \left\{ a \sqrt{\beta^2 (a^2 + m^2) - 1} \pm m \right\}
\]

> 0 for \( \theta_2 (m) \) (see (73))

< 0 for \( \theta_1 (-m) \) (assuming \( a > 0 \))

because

\[
m^2 > a^2 \left[ \beta^2 (a^2 + m^2) - 1 \right]
\]

since

\[
(a^2 + m^2) (1 - \beta^2 a^2) > 0
\]
Therefore since
\[
\frac{d}{d\theta} D3 > 0 \quad \theta = \theta_2 \\
< 0 \quad \theta = \theta_1
\]
and therefore
\[
D3 > 0 \quad \theta > \theta_2 \text{ or } \theta < \theta_1 \\
< 0 \quad \theta_1 < \theta < \theta_2
\] (75)

We can say
\[
\lim_{x^2 \to \beta^2 \ (y^2 + z^2)} F 3 = \begin{cases} 
0 & \theta > \theta_2 \text{ or } \theta < \theta_1 \\
\pi & mz > ay \\
-\pi & mz < ay
\end{cases} \\
\text{or } \theta_1 < \theta < \theta_2
\] (76)

Also on the plane
\[mz = ay\]

\[D3 = z (z - ax) - y (mx - y) = y^2 \left[ 1 + \frac{a^2}{m^2} \right] - xy \left[ m + \frac{a^2}{m} \right] = \frac{y (y-mx) (a^2 + m^2)}{m^2}\]

and therefore
\[
\lim_{(ay-mz) \to 0} F 3 = \begin{cases} 
0 & y < 0 \quad y > mx \\
\pi & 0<y<mx \\
-\pi & 0<y<mx
\end{cases} \\
\text{or } mz > ay \\
\text{or } mz < ay
\] (77)
To find the limit of $F_2$ as $x^2 \to \beta^2 (y^2 + z^2)$ when $1 - \beta^2 (a^2 + m^2) < 0$ [see (53) for $F_2$] we note that

$$
\lim_{x^2 \to \beta^2 (y^2 + z^2)} \left| \frac{x - \beta^2 (my + az)}{\beta \sqrt{(mx-y)^2 + (z-ax)^2} - \beta^2 (ay-mz)^2} \right| = 1
$$

because

$$
\beta^2 \left[ (mx - y)^2 + (z - ax)^2 - \beta^2(ay - mz)^2 \right] = \beta^2 (a^2 + m^2) x^2 + \beta^2 (y^2 + z^2) + \beta^2 (my + az)^2 + \beta^2 (ay - mz)^2
$$

$$
= \beta^2 \left[ x^2 + \beta^2 (y^2 + z^2) \right] + \beta^2 (my + az)^2 + \beta^2 (ay - mz)^2 + \beta^2 (ay - mz)^2
$$

Now

$$
x - \beta^2 (my + az) = x \left[ 1 - \beta^2 m \frac{y}{x} - \beta^2 a \frac{y}{z} \tan \theta \right]
$$

$$
= x \left[ 1 - \beta m \cos \theta - \beta a \sin \theta \right] = \frac{\beta^2}{x^2} \mathcal{D}_3
$$

where $\mathcal{D}_3$ is defined by (75). Therefore from (75)

$$
\lim_{x^2 \to \beta^2 (y^2 + z^2)} F_2 = \begin{cases} 0 & \text{if } \theta_1 < \theta < \theta_2 \\ \frac{\beta^2}{x^2} \mathcal{D}_3 & \text{if } \theta_1 < \theta \leq \theta_1 \end{cases} \quad (78)
$$

Therefore combining these results

$$
F_1 = \tan^{-1} \frac{m (z-ax) \sqrt{x^2 - \beta^2 (y^2 + z^2)}}{y \left[ (y-mx) - \beta^2 a (ay-mz) \right] + (z-ax)^2} \quad x^2 > \beta^2 (y^2 + z^2)
$$
If
\[ 1 - \beta^2 (a^2 + m^2) > 0 \]
\[ F_1 = 0 \quad x^2 < \beta^2 (y^2 + z^2) \]
(79)

\[ \lim_{z \to ax} F_1 = \begin{array}{c|c|c|c}
0 & y < 0 & y > mx \\
\pi & 0 < y < mx & z > ax \\
\pi & 0 < y < mx & z < ax
\end{array} \]

Then from (69) and (74) if

\[ 1 - \beta^2 (a^2 + m^2) < 0 \]

and

\[ x^2 < \beta^2 (a^2 + m^2) \]
\[ F_1 = 0 \quad \theta > \theta_2 \quad \theta < \theta_1 \]
(80)

\[ F_1 = \pi \text{ sgn} (z-ax) \]

if
\[ \theta_1 < \theta < \theta \]

and

\[ x > \frac{(az + my) + |mz + ay| \sqrt{\beta^2 (a^2 + m^2) - 1}}{(a^2 + m^2)} = 0 \]
and \( F_1 = 0 \)

if \( \theta_1 < \theta < \theta_2 \)

and

\[
x > \frac{(az + my) + mz + ay}{\sqrt{\beta^2 (a^2 + m^2) - 1}}
\]

and where from (71)

\[
(1 - \beta^2 a^2) \tan \theta_1 = \beta^2 am - \sqrt{\beta^2 (a^2 + m^2) - 1}
\]

\[
(1 - \beta^2 a^2) \tan \theta_2 = \beta^2 am + \sqrt{\beta^2 (a^2 + m^2) - 1}
\]

For

\[
x^2 > \beta^2 (y^2 + z^2)
\]

from (53), (54), and (56)

\[
F_2 = \begin{cases} \frac{\sqrt{x^2 - \beta^2 (a^2 + m^2)}}{x - \beta^2 (a^2 + m^2)} & 1 - \beta^2 (a^2 + m^2) > 0 \\ \log \frac{x - \beta^2 (a^2 + m^2) + \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^2 (a^2 + m^2)]}}{x - \beta^2 (a^2 + m^2) - \sqrt{[1 - \beta^2 (a^2 + m^2)][x^2 - \beta^2 (a^2 + m^2)]}} & 1 - \beta^2 (a^2 + m^2) = 0 \\ \frac{-1}{\sqrt{\beta^2 (a^2 + m^2) - 1}} \tan^{-1} \frac{\sqrt{[\beta^2 (a^2 + m^2) - 1][x^2 - \beta^2 (a^2 + m^2)]}}{x - \beta^2 (a^2 + m^2)} & 1 - \beta^2 (a^2 + m^2) < 0 \end{cases}
\]
and from (78)

\[ F_2 = 0 \] 
\[ x^2 \leq \beta^2 (y^2 + z^2) \quad 1 - \beta^2 (a^2 + m^2) > 0 \]
\[ \theta > \theta_2, \quad \theta > \theta_1 \]
\[ \pi \]
\[ x^2 \leq \beta^2 (y^2 + z^2) \quad \theta_1 < \theta < \theta_2 \quad 1 - \beta^2 (a^2 + m^2) < 0 \]
\[ = 0 \quad \text{outside envelope of Mach cones.} \tag{82} \]

For

\[ x^2 > \beta^2 (y^2 + z^2) \]

From (56) and (58)

\[ F_3 = \tan^{-1} \left( \frac{(mz - ay)}{\sqrt{x^2 - \beta^2 (y^2 + z^2)}} \right) \frac{1}{z (z - ax) - y (mx - y)} \tag{83} \]

and

\[ \lim_{(mz - ay) \to 0} F_3 = \begin{cases} 0 & y < 0 \quad y > mx \\ \pi & 0 < y < mx \quad mz > ay \\ -\pi & 0 < y < mx \quad mz < ay \end{cases} \]

and for

\[ x^2 \leq \beta^2 (y^2 + z^2) \]

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from (76)

\[ F3 = 0 \quad \theta < \theta_1 \quad \theta > \theta_2 \]

\[ = \pi \quad mz > ay \quad \theta_1 < \theta < \theta_2 \]

\[ = -\pi \quad mz < ay \quad \theta_1 < \theta < \theta_2 \]

\[ = 0 \quad \text{outside envelope of Mach cones.} \quad (84) \]

Value of \( \phi \) on the envelope of mach cones. For a supersonic leading edge \( [1 - \beta^2 (a^2 + m^2) < 0] \), in the region inside the envelope of mach cones from the leading edge and outside the mach cone from the origin, we have for a surface distribution of sources [from (29), (79), (80), and (81)]

\[
\frac{1}{\sqrt{\beta^2 (a^2 + m^2)} - 1} \frac{(1 - \frac{\beta^2 a^2}{\beta^2 m^2}) \frac{\delta^2}{\delta z} + \frac{\beta^2 a^2}{\delta \omega}}{\sqrt{\beta^2 (a^2 + m^2) - 1}} \phi_s = (z - ax) \text{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} + y (1 - \beta^2 a^2) \cdot m (x - \beta^2 ax)
\]

\[
= \left[ n + a \text{sgn} (z - ax) \right] \sqrt{\beta^2 (a^2 + m^2) - 1} + z \text{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1} + y (1 - \beta^2 a^2) \cdot \beta^2 amz
\]

\[
= \left[ n + a \text{sgn} (z - ax) \right] \sqrt{\beta^2 (a^2 + m^2) - 1} \quad (1 - \beta^2 a^2) \left[ x (a^2 + m^2) + (my + nx) \right] \text{sgn} (z - ax) (nz - ay) \sqrt{\beta^2 (a^2 + m^2) - 1} + \frac{m - a \text{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1}}{m - a \text{sgn} (z - ax) \sqrt{\beta^2 (a^2 + m^2) - 1}}
\]
On the lines

\[ x = \frac{(my + az) + |mz - ay|}{(a^2 + m^2)} \sqrt{\beta^2 \left( \frac{a^2 + m^2}{a^2 + m^2} \right) - 1} \]

\[ \text{sgn} \ (z - ax) \ (mz - ay) = |mz - ay| \] [See figure in Woodward]

Therefore on the envelope of mach cones from the leading edge

\[ \phi_s = 0 \]

In this same region for a constant pressure surface [from (71)] and

\[ \phi_p \left[ \frac{1}{4 \pi a} \left( \frac{\Delta p}{q_{\infty}} \right) \right]^{-1} \ (a^2 + m^2) \ x^{-1} \]

\[ = - a \ (ay - mz) \ \sqrt{\beta^2 \left( \frac{a^2 + m^2}{a^2 + m^2} \right) - 1 + m \ (ay - mz) \ \text{sgn} \ (mz - ay) + (a^2 + m^2) \ (z - ax) \ \text{sgn} \ (z - ax)} \]
and in the region where

\[ \text{sgn} (z - ax) = \text{sgn} (mz - ay) \]

\[ + a |mz - ay| \sqrt{b^2(a^2 + m^2) - 1 + m (ay - mz) + (a^2 + m^2) (z - ax)} \]

\[ = \text{sgn} (z - ax) a \left| - x (a^2 + m^2) + (my + az) + |mz - ay| \sqrt{b^2 (a^2 + m^2) - 1} \right| \]

\[ = 0 \text{ on the lines } x = \frac{(my + az) + |mz - ay| \sqrt{b^2 (a^2 + m^2) - 1}}{a^2 + m^2} \]

Therefore \( \phi_p = 0 \) on the envelope of Mach cones from the leading edge.

Verification of the imposed boundary conditions. Now (8) can be verified for the case of a surface distribution of sources. From (29) and (79), on \( z = ax \), since the \( \text{cosh}^{-1} \) terms are continuous

\[ \phi = \phi' \text{ or } \phi - \phi' = 0 \]
For (8b) using (29) and (79) and the results of 2.6

\[ u = -\frac{\bar{w} + \beta^2 a u}{\pi(1 - \beta^2 a^2)} \left| \pi a + m F^2 \right| \quad 0 < y < m x \quad z > a x \]

\[ u' = -\frac{\bar{w} + \beta^2 a u}{\pi(1 - \beta^2 a^2)} \left| -\pi a + m F^2 \right| \quad 0 < y < m x \quad z < a x \]

\[ w = \frac{\bar{w} + \beta^2 a u}{\pi(1 - \beta^2 a^2)} \left| \pi + \beta^2 a m F^2 \right| \quad 0 < y < m x \quad z > a x \]

\[ w' = \frac{\bar{w} + \beta^2 a u}{\pi(1 - \beta^2 a^2)} \left| -\pi + \beta^2 a m F^2 \right| \quad 0 < y < m x \quad z < a x \]

Therefore

\[(\bar{w} - w') + \beta^2 a (u - u') = 2 \frac{\bar{w} + \beta^2 a u}{\pi(1 - \beta^2 a^2)} \pi [1 - \beta^2 a^2] = 2[\bar{w} + \beta^2 a u] = \text{const}\]

which agrees with (8b)

For

\[ y < 0 \]

or

\[ y > m x \]
where

\[ f = m(z - ax)\sqrt{x^2 - \beta^2(y^2 + z^2)} \]

\[ g = y[(y - mx) - \beta^2 a(ay - mz)] + (z - ax)^2 \]

and since

\[ f = 0 \text{ on } S \]

when

\[ z = ax \]

\[ \frac{\partial u}{\partial x} = \frac{-am\sqrt{x^2(1 - \beta^2a^2)} - \beta^2y^2}{y(y - mx)(1 - \beta^2a^2)} \]

and likewise on

\[ z = ax \]

\[ \frac{\partial u}{\partial y} = 0 \]

\[ \frac{\partial u}{\partial z} = \frac{m\sqrt{x^2(1 - \beta^2a^2)} - \beta^2y^2}{y(y - mx)(1 - \beta^2a^2)} \]

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Therefore

\[ \frac{\partial}{\partial x} \frac{\partial}{\partial y} \frac{\partial}{\partial z} \]

are continuous on \( S \) (except possibly at the edges) and therefore

\[ \frac{\partial}{\partial x} (u - u') = \frac{\partial}{\partial y} (u - u') = \frac{\partial}{\partial z} (u - u') = 0 \]

and (30b) is verified.

**Surface velocities on a constant pressure surface.** - Flow normal to surface \( U_n \)

\[ u_n = \frac{1}{\sqrt{1 + a^2}} (-au + w) \]

\[ u'_n = \frac{1}{\sqrt{1 + a^2}} (au' - w') \]

Flow through surface \[ = \frac{1}{2} (u_n - u'_n) = \frac{1}{2} \frac{1}{\sqrt{1 + a^2}} \left| -a(u + u') + (w + w') \right| \]

on

\[ (z = ax) \ u + u' = 0 \]
\[
\frac{1}{2} (w + w') = \frac{1}{4\pi a} \left( \frac{\Delta P}{q_{\infty}} \right) \left( \frac{am}{a^2 + m^2} \right) \frac{1}{2} \log \left[ \frac{x + \sqrt{x^2(1 - \beta^2 a^2)} - \beta^2 a^2}{x - \sqrt{x^2(1 - \beta^2 a^2)} - \beta^2 a^2} \right]
\]

\[
= \frac{-am}{2(a^2 + m^2)} \sqrt{1 - \beta^2 (a^2 + m^2)} \log \left( \frac{x(1 - \beta^2 a^2) - \beta^2 m^2 + \sqrt{[x^2(1 - \beta^2 a^2) - \beta^2 m^2]}}{x(1 - \beta^2 a^2) - \beta^2 m^2 - \sqrt{[x^2(1 - \beta^2 a^2) - \beta^2 m^2]}} \right)
\]

\[
= \frac{m^2}{a^2 + m^2} \tan^{-1} \left( \frac{(mz - ay) \sqrt{x^2(1 - \beta^2 a^2) - \beta^2 y^2}}{-y(mx - y)} \right)
\]

\[
= \frac{1}{2} \sqrt{1 + a^2} (u_n - u'_n)
\]

Source strength = \( u_n + u'_n = \frac{1}{\sqrt{1 + a^2}} \left[ a(u - u') + (w - w') \right] \)

\[
= \frac{(1 + a^2)}{\sqrt{1 + a^2}} \frac{2\Delta P}{4\pi a q_{\infty} \pi}
\]

\[
= 2 \frac{\sqrt{1 + a^2} \Delta P}{4 a q_{\infty}}
\]

(86)
Velocity discontinuity in the leading edge wake.

Constant Pressure Surface

\[ V_T = \frac{v + \frac{a}{m}w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}} \]

\[ V_n = \frac{-\frac{a}{m}v + w}{\sqrt{1 + \left(\frac{a}{m}\right)^2}} \]

\[ V_n - V_n' = \frac{-\frac{a}{m}(v - v') + (w - w')}{\sqrt{1 + \left(\frac{a}{m}\right)^2}} \]

\[ = \frac{1}{4\pi a\sqrt{1 + \left(\frac{a}{m}\right)^2}} \left(\frac{\Delta P}{q_{\infty}}\right) 2\pi \left[ \left(\frac{a}{m}\right) a - \left(\frac{2}{m^2}\right) \right] \left(a^2 + m^2\right)^{-1} \]

\[ = \frac{-2m}{4a\sqrt{a^2 + m^2}} \left(\frac{\Delta P}{q_{\infty}}\right) \quad \text{Source Distribution! [see (86)]} \]

\[ V_T - V_T' = \frac{2}{4\pi a\sqrt{1 + \left(\frac{a}{m}\right)^2}} \left(\frac{\Delta P}{q_{\infty}}\right) \pi \left[ a - \frac{m^2 a}{m} \right] \left(a^2 + m^2\right)^{-1} = 0 \]
Alternate sign choice of $\tan^{-1}$ denominator.

$$\frac{m(ay - mz) tan^{-1} (mz - ay) \sqrt{(mx - \eta)^2 - \beta^2[(a\eta - mz)^2 + m^2(\eta - y)^2]}}{(a^2 + m^2) [a(z - ax) - m(mx - y)] \eta + m[y(mx - y) - z(z - ax)]}$$

[see (45) and note sign change of denominator and since]

$$[a(z - ax) - m(mx - y)]\eta_3 + m[y(mx - y) - z(z - ax)] < 0$$

The above term when evaluated at

$$\eta = \eta_3$$

is

$$\frac{m(ay - mz) \pi \text{sgn}(mz - ay)}{(a^2 + m^2)}$$

Evaluated at $\eta = 0$ the term becomes

$$\frac{m(ay - mz) tan^{-1} (mz - ay) \sqrt{x^2 - \beta^2(y^2 + z^2)}}{[y(mx - y) - z(z - ax)]}$$

If

$$mz = ay \quad y(mx - y) - z(z - ax) = \frac{y(mx - y)(a^2 + m^2)}{m^2}$$

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which means that on \( mz = ay \) and \( \eta = 0 \)

\[
\tan^{-1} \eta = \begin{cases} 
\pi \text{sgn}(mz - ay) & y < 0 \quad y > mx \\
0 & 0 < y < mx
\end{cases}
\]

\[
\tan^{-1} \eta = \begin{cases} 
0 & y < 0 \quad y > mx \\
\pi \text{sgn}(mz - ay) & 0 < y < mx
\end{cases}
\]
Subappendix A - Woodward's Subsonic Equations

If \( \Omega(x, y, z) \) satisfies

\[
\left\{ \beta^2 \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\} \Omega(x, y, z) = 0 \quad \beta^2 = 1 - M^2
\]

Then we can write the solution for \( \Omega(x, y, z) \) as

\[
\Omega(x, y, z) = \iiint_{S} \left[ \Omega(\xi, \eta, \tau) \frac{\partial}{\partial \nu} - \frac{\partial \sigma(\xi, \eta, \tau)}{\partial \nu} \right] \frac{1}{4 \pi r} \, dS \tag{A1}
\]

where

\[
r = \sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - \tau)^2}
\]

and

\[
\frac{\partial}{\partial \nu} = \left\{ \beta^2 \eta_\xi \frac{\partial}{\partial \xi} + \eta_\eta \frac{\partial}{\partial \eta} + \eta_\tau \frac{\partial}{\partial \tau} \right\}
\]

\( \vec{n} = (\eta_\xi, \eta_\eta, \eta_\tau) \) is the unit normal on \( S \)

We will assume \( S \) is the surface \( \tau = a \xi \) and
Surface distribution of sources. - Let $\Omega = \phi$ and as for supersonic flow assume that on $S \sqrt{1 + a^2} \vec{V} = (-\beta a, 0, 1)$

a) $\phi = \phi'$

b) $\frac{\partial \phi}{\partial \nu} + \frac{\partial \phi}{\partial \nu'} = \frac{2}{\sqrt{1 + a^2}} (\bar{w} - \beta a \bar{u}) = \text{const.} \bar{w} = w - w^1$

the primed quantities refer to $z < ax$ or $\xi < a \xi$

Therefore since on $S \xi = a \xi$ we get

$$\phi(x, y, z) = \frac{\bar{w} - \beta a \bar{u}}{2\pi} \int_0^y \int_{\frac{\eta}{m}}^{\frac{\eta}{m}} \frac{d\xi}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2}} \, d\eta$$

(A2)
This is the same as (9) for the supersonic case except for the limits, a factor of 1/2, and the fact that \( \beta^2 \) is replaced by \(-\beta^2\). Therefore the integral over \( \xi \) may be performed by using the equation below (9) for the supersonic case, and replacing \( \beta^2 \) by \(-\beta^2\).

\[
\int \frac{d\xi}{\sqrt{(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2}} = 
\]

\[
\frac{-1}{2\sqrt{1 + \beta^2 a^2}} \log \frac{\sqrt{(1 + \beta^2 a^2) [(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2] + (x - \xi) + \beta^2 a(z - a\xi)}}{\sqrt{(1 + \beta^2 a^2) [(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2] - (x - \xi) - \beta^2 a(z - a\xi)}}
\]

A negative sign was included in the argument of the logarithm. This was possible since the derivative of \( \log(-1) \) is zero. Therefore

\[
\phi(x, y, z) = \frac{\bar{w} - \beta^2 a \bar{u}}{4\pi \sqrt{1 + \beta^2 a^2}} \int \int \int \frac{\sqrt{(1 + \beta^2 a^2) [(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2] + (x - \xi) + \beta^2 a(z - a\xi)}}{\sqrt{(1 + \beta^2 a^2) [(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a\xi)^2] - (x - \xi) - \beta^2 a(z - a\xi)}} d\eta d\xi d\zeta
\]

The second integral is the same as the first except that \( x, z \) and \( m \) are replaced by \( x - x_3, z - z_3 \) and \( m_1 \). These integrals are the same as for supersonic flow, given by equation (21), if \( \beta^2 \) is replaced by \(-\beta^2\) and a factor of 1/2 is added.
When the limit \( \eta = Y_2 \) is calculated it can be seen that it will be the same as when \( \eta = 0 \) if \( x, y \) and \( z \) are replaced by \( x - x_2, y - y_2 \) and \( z - z_2 \).

Therefore

\[
\phi (x, y, z, m) = \frac{\bar{w} \cdot \beta^2 a}{2 \pi} \left\{ \frac{z - az}{1 + \beta^2 a^2} \tan^{-1} \frac{m(z - az)\sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 (y - ax) + \beta^2 (ay - mz)} + (z - ax)^2 \right\}
\]

\[
- \frac{\gamma (1 + \beta^2 a^2)^2 - m(x + \beta^2 az)}{2(1 + \beta^2 a^2)^2 \sqrt{1 + \beta^2 (a^2 + m^2)}} \log \sqrt{1 + \beta^2 (a^2 + m^2)} \left[ x^2 + \beta^2 (y^2 + z^2) \right] + (x + \beta^2 az)
\]

\[
- \frac{\gamma}{2 \sqrt{1 + \beta^2 a^2}} \log \sqrt{1 + \beta^2 (a^2 + m^2)} \left[ x^2 + \beta^2 (y^2 + z^2) \right] - (x + \beta^2 az)
\]

Now we can write the result of (A4) as

\[
\phi (x, y, z) = \phi_0 (x, y, z, m) - \phi_0 (x - x_2, y - y_2, z - z_2, m)
\]

\[- \phi_0 (x - x_3, y - y_3, z - z_3, m_1) + \phi_0 (x - x_4, y - y_4, z - z_4, m_1)
\]

This means that (A5) may be interpreted as the velocity potential for an infinite panel with leading edge slope \( y = mx \), and at angle of attack \( a \).
Constant pressure surface. - In (A1) we will use \( u(x, y, z) = \Omega(x, y, z) \) and

\[
\text{a)} \quad \frac{\partial u}{\partial v} + \frac{\partial u'}{\partial v'} = 0 \quad \text{on } S
\]

\[
\text{b)} \quad u - u' = \text{const} = \Delta u \quad \text{on } S
\]

and where \( S \) is the same as previously defined.

Therefore (A1) becomes

\[
u(x, y, z) = \frac{\Delta u}{4\pi} \int \int_{S} \frac{\beta^2 a(x - \xi) + \beta^2 (z - a \xi)}{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2} \, d\xi \, d\eta
\]

\[
a \frac{\partial u}{\partial x} \int \int_{S} \frac{-\beta^2 a - \frac{(x - \xi)(z - a \xi)}{(y - \eta)^2 + (z - a \xi)^2}}{\sqrt{(x - \xi)^2 + \beta^2 (y - \eta)^2 + \beta^2 (z - a \xi)^2}} \, d\xi \, d\eta \quad (A6)
\]
Now we can write, using $\Delta p / 2 q_\infty = -\Delta u$

\[
\phi(x, y, z) = \int_{-\infty}^{\infty} u(x', y, z) \, dx
\]

\[
= \frac{\Delta p}{8\pi q_\infty} \int_0^{y_2} \int_{\eta_1}^{\eta_2} \int_{\xi_1}^{\xi_2} \frac{-\beta^2 \alpha - \frac{(x - \xi)(z - a \xi)}{(y - \eta)^2 + (z - a \xi)^2}}{\sqrt{(x - \xi)^2 + \beta^2(y - \eta)^2 + \beta^2(z - a \xi)^2}} \, d\xi \, d\eta
\]

\[\text{(A7)}\]

\[
= \frac{\Delta p}{8\pi q_\infty} \int_0^{y_2} \int_{\eta_1}^{\eta_2} \int_{\xi_1}^{\xi_2} \frac{(z - a \xi)}{(y - \eta)^2 + (z - a \xi)^2} \, d\xi \, d\eta
\]

The first term, except for the limits of integration and an additional factor of 1/2, is the same as the result for supersonic flow, which is given by the first equation of the section covering the evaluation of the integral over $\xi$ for the constant pressure surface, with $-\beta$ in place of $\beta$. Therefore, as in the case of a constant distribution of sources, we can use the supersonic result for subsonic flow if we substitute $-\beta$ for $\beta$.

The second integral is new and does not occur for supersonic flow. This integral will now be evaluated.

\[
\int \frac{(z - a \xi) \, d\xi}{(y - \eta)^2 + (z - a \xi)^2} = -\frac{1}{2a} \log \left[ (y - \eta)^2 + (z - a \xi)^2 \right]
\]

\[\text{(A8)}\]
Therefore

\[
\int_{0}^{y_2} \int_{\eta}^{x_3} \frac{(z - a \eta) \cdot \xi}{(y - \eta)^2 + (z - a \xi)^2} \, d\eta = \frac{1}{2\pi} \int_{0}^{y_2} \log \left[ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right] \, d\eta
\]

\[
\frac{1}{2\pi} \int_{0}^{y_2} \log \left[ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right] \, d\eta
\]

\[
\int_{0}^{y_2} \log \left[ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right] \, d\eta
\]

\[
\left[ \eta - m(z + mz - a\eta) \right] \left[ \log \left[ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right] - 2 \right]
\]

\[
\frac{2m(z - a\eta)}{a^2 + m^2} \tan^{-1} \frac{m(z - a\eta)}{(y - \eta)(a^2 + m^2) - a(mz - ay)}
\]

\[
\frac{m(z - ay)}{(a^2 + m^2) - a(mz - ay)}
\]

note that:

\[mz - a\eta = (mz - ay) - a(\eta - y) = m(z - ax) + a(mx - y) - a(\eta - y) \quad (A9)\]

\[
\int_{0}^{y_2} \log \left[ \frac{m^2 (y - \eta)^2 + (mz - a\eta)^2}{m^2} \right] \, d\eta
\]
When this is evaluated at \( \eta = 0 \) we get from (A7), (A9), and (A10)

\[
\phi_{oo}(x, y, z, m) = \frac{\Delta P}{8\pi q_\infty a} \left\{ \frac{-m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log(y^2 + z^2) - 1 \right] \right. \\
\left. - \frac{m(mz - ay)}{(a^2 + m^2)} \tan^{-1} \frac{(mx - ay)m}{-a(mx - ay) - (a^2 + m^2)y} \right\} 
\]

(A11)

Since all of the terms in (A10) may be written using only the variables, \((mx - y), (z - ax), m, \) and \((\eta - y)\), the additional term which must be added for subsonic flow is,

\[
\phi_1(x, y, z) = \phi_{oo}(x, y, z, m) - \phi_{oo}(x - x_2, y - y_2, z - z_2, m) \\
- \phi_{oo}(x - x_3, y - y_3, z - z_3, m_1) \\
+ \phi_{oo}(x - x_4, y - y_4, z - z_4, m_1)
\]

(A12)
Therefore using (51) we can write:

\[ p(x, y, z) = \frac{\Lambda_p}{8 \pi q a} \left\{ \frac{\pi (az + my)}{a^2 + m^2} \frac{1}{z} \log \frac{x^2 + \beta^2 (y^2 + z^2)}{x^2 + \beta^2 (y^2 + z^2)} + x \right. \]

\[ + \frac{a(ay - mz)}{z(a^2 + m^2)} \sqrt{1 + \beta^2 (a^2 + m^2)} \log \frac{\sqrt{\left[1 + \beta^2 (a^2 + m^2)\right] \left[ x^2 + \beta^2 (y^2 + z^2) \right]} + [x + \beta^2 (az + mz)]}{\sqrt{\left[1 + \beta^2 (a^2 + m^2)\right] \left[ x^2 + \beta^2 (y^2 + z^2) \right]} - [x + \beta^2 (az + mz)]} \]

\[ - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \left( \frac{mz - ay}{z - ax} \right) \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)}}{z(z - ax) - y(mx - y)} \]

\[ + (z - ax) \tan^{-1} \left( \frac{m(z - ax)}{y[my + \beta^2 (az + mz)] + (z - ax)^2} \right) \]

\[ - y \sqrt{1 + \beta^2 a^2} \frac{1}{2} \log \frac{\sqrt{\left[1 + \beta^2 a^2\right] \left[ x^2 + \beta^2 (y^2 + z^2) \right]} + (x + \beta^2 az)}{\sqrt{\left[1 + \beta^2 a^2\right] \left[ x^2 + \beta^2 (y^2 + z^2) \right]} - (x + \beta^2 az)} \]

\[ - \frac{m(az + my)}{(a^2 + m^2)} \left[ \frac{1}{2} \log (y^2 + z^2) - 1 \right] \]

\[ - \frac{m(mz - ay)}{a^2 + m^2} \tan^{-1} \frac{(mz - ay)m}{a(ay - mz) - y(a^2 + m^2)} \]
To find the limiting form of $\phi$ as $a \to 0$

some of the terms must be combined. First,

consider the terms

$$
\lim_{a \to 0} \frac{\gamma}{2a} \left\{ \log \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} - \sqrt{1 + \beta^2 a^2} \log \frac{\sqrt{(1 + \beta^2 a^2) [x^2 + \beta^2 (y^2 + z^2)]} + (x + \beta^2 a z)}{\sqrt{(1 + \beta^2 a^2) [x^2 + \beta^2 (y^2 + z^2)]} - (x + \beta^2 a z)} \right\}
$$

and since only terms up to the first power in $a$ must
be considered inside the brackets, this becomes

$$
\lim_{a \to 0} \frac{\gamma}{2a} \left\{ \log \frac{\beta^2 a}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} \right\} = - \gamma \frac{x \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 + z^2}
$$

since for $a \ll 1$ $\log (1 + a) = a$

Next consider

$$
\lim_{a \to 0} \frac{\tau}{2} \left\{ \tan^{-1} \frac{m(E_{-ax}) \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{\frac{1}{2} \left( (y - mx) + \beta^2 (y - mx)^2 + (z - ax)^2 \right)}
 - \tan^{-1} \frac{(mE_{-ax}) \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{2(y - ax) - y(m - \gamma)} \right\}
$$
For small \( \theta \) we can expand

\[
\frac{-\theta}{g} = -\frac{\theta}{g(0)} + \frac{\theta}{g(0)} \frac{g'(0)}{g(0)} + \frac{1}{2} \left( \frac{g'(0)}{g(0)} \right)^2 \frac{g''(0)}{g(0)} + \frac{1}{6} \left( \frac{g'(0)}{g(0)} \right)^3 \frac{g'''(0)}{g(0)} + \cdots
\]

Now let

\[
f(x) = m(z-ax) \sqrt{x^2 + \beta^2 (y^2 + z^2)}
\]

\[
g(x) = \gamma [(y-mx) + \beta^2 (ag-mz)] + (z-ax)^2
\]

\[
f(0) = \gamma (y-mx) + z^2
\]

\[
g(0) = -\beta^2 (y-mx) + 2xz
\]

\[
f'(0) = -mx \sqrt{x^2 + \beta^2 (y^2 + z^2)}
\]

\[
g'(0) = -x \beta^2 (y-mx) + 2xz
\]

\[
f''(0) = -m^2 \sqrt{x^2 + \beta^2 (y^2 + z^2)}
\]

\[
g''(0) = -\gamma (y-mx) + z^2
\]

\[
f'''(0) = -2 \gamma m y z + 2xz
\]

\[
g'''(0) = \beta^2 m y z - 2xz
\]

\[
f''(0) + g''(0) = \gamma^2 \left( (y-mx)^2 + (1+\beta^2)z^2 \right) + \beta^2 \left( y^2 - 2mxz + 2z \right)
\]

\[
= \beta^2 \left( (y-mx)^2 + (1+\beta^2)z^2 \right) + \beta^2 \left( m^2 z^2 + y^2 - 2mxz + 2z \right)
\]

\[
= \beta^2 \left( (y-mx)^2 + (1+\beta^2)z^2 \right)
\]
\[ f'g - fg' = \sqrt{x^2 + \beta^2 (y^2 + z^2)} \left\{ - [x^2 + (y - mz)^2] \right\} \]
To evaluate the additional term $\phi_1$, as $a \to 0$, we must combine points 1 and 3 and 2 and 4 from (A15):

$$
\lim_{a \to 0} \phi_{oo} (x, y, z, m) = c_{oo} (x-x_3, y, z-ax_3, m_1)
$$

$$
= \lim_{a \to 0} \left\{ -\frac{az+my}{m} \log \left( \frac{1}{2} (y^2+z^2) \right) + \frac{a(z-ax_3)+m_1y}{m_1} \left[ \frac{1}{2} \log \left( y^2+(z-ax_3)^2 \right) - 1 \right] 
- \frac{mn-ay}{m} \tan^{-1} \frac{m_1(z=ax_3)-ay}{m_1} \tan^{-1} \frac{m_1(z=ax_3)-ay}{[m_1y+a(z-ax_3)]} \right\} \frac{\Delta P}{8\pi q_{oo}a}
$$

$$
= \frac{\Delta P}{8\pi q_{oo}a} \left\{ -\frac{az+az}{m} \left[ \frac{1}{2} \log \left( y^2+z^2 \right) - 1 \right] - \frac{az}{y+z} 
+ \left[ \frac{ay}{m} - \frac{a(y+mx_3)}{m_1} \right] \tan^{-1} \frac{z}{-y} - \frac{az}{m} \left( \frac{1+z^2}{y^2} \right) + \frac{az}{m_1} \left( \frac{1+z^2}{y^2} \right) \left( 1+\frac{z^2}{y^2} \right) \right\}
$$

$$
= \frac{\Delta P}{8\pi q_{oo}} \left\{ -\frac{z+\frac{z}{m}}{m_1} \left[ \frac{1}{2} \log \left( y^2+z^2 \right) \right] - \frac{m_1(mx-y)}{m} - \frac{m_1(x-x_3)-y}{m_1} \right\} \tan^{-1} \frac{z}{-y}
$$

where we have used

$$
\tan^{-1}(\alpha-\Delta\alpha) = \tan^{-1}\alpha + \frac{\Delta\alpha}{1+\alpha^2}
$$
Since we can write

$$\tan^{-1} \frac{z}{-y} = \pi - \tan^{-1} \frac{-y}{z} = \pi + \tan^{-1} \frac{y}{z}$$

Therefore if we let

$$\hat{\phi}_{oo}(x, y, z, m) = -\frac{\Delta p}{8\pi q_\infty} \left\{ \frac{(m\pi-y)}{m} \tan^{-1} \frac{y}{z} + \frac{z}{m} \frac{1}{2} \log (y^2 + z^2) \right\}$$

(A15)

Then since \(\pi(m\pi-y)\) will cancel for a complete panel,

$$\lim_{a \to 0} \phi_1(x, y, z) = \hat{\phi}_{oo}(x, y, z, m) - \hat{\phi}_{oo}(x-x_2, y-y_2, z, m)$$

$$- \hat{\phi}_{oo}(x-x_3, y, z, m_1) + \hat{\phi}_{oo}(x-x_4, y-y_4, z, m_1)$$

And therefore for subsonic flow with \(a = 0\)

$$\phi(x, y, z) = -\frac{\Delta p}{8\pi q_m} \left\{ \frac{(m\pi-y)}{m} \tan^{-1} \frac{mz \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y (y - m) + z^2} - \frac{z}{2} \log \frac{\sqrt{x^2 + y^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta y^2 (y^2 + z^2)} - x} \right\}$$

$$\left[ \frac{x \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 + z^2} + z \sqrt{1+\beta^2 m^2} \frac{1}{2} \log \frac{(1+\beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)] + (x+\beta^2 y)}{(1+\beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)] - (x+\beta^2 y)} \right]$$

$$+ \frac{y \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{y^2 + z^2} + (m\pi-y) \tan^{-1} \frac{y}{z} + \frac{z}{2} \log (y^2 + z^2)$$
In the form the above equation is written, derivatives with respect to \( x, y, \) or \( z \) of all but the last two terms may be obtained by differentiating only the coefficients of each term. This was discussed previously when obtaining the velocity components for supersonic flow. If we use \( \beta^2 = 1 - M^2 \) the above equation can also be used for \( M > 1 \) if we multiply the expression by \( 2, \) take absolute values of the log arguments, and omit the last two terms. Therefore

\[
\varphi(x, y, z) = \frac{-\Delta p}{8\pi q_m} k \left\{ (mx-y) \tan^{-1} \frac{mz \sqrt{x^2 + \beta^2 (y^2+z^2)}}{y (y-mx) + z^2} - \frac{z}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2+z^2) + x}}{\sqrt{x^2 + \beta^2 (y^2+z^2) - x}} \right. \\
+ z \sqrt{1+\beta^2 m^2} \frac{1}{2} \log \frac{\sqrt{(1+\beta^2 m^2)[x^2+\beta^2 (y^2+z^2)]} + (x+\beta^2 my)}{\sqrt{(1+\beta^2 m^2)[x^2+\beta^2 (y^2+z^2)]} - (x+\beta^2 my)} \\
+ \frac{z y\sqrt{x^2+\beta^2 (y^2+z^2)}}{y^2+z^2} + \left[ (mx-y) \tan^{-1} \frac{y}{z} + \frac{z}{2} \log (y^2+z^2) \right] \left. \right\}
\]

where we take only the real part and

\[
k = \begin{cases} 1 & M = 1 \\ 2 & M > 1 \end{cases} \quad \beta = M^2 - 1
\]
This is equivalent to the form derived by Woodward if it is noted that

$$\log \frac{x + \beta^2 mx + \sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]}}{\beta \sqrt{(mx-y)^2 + (1+\beta^2 m^2) z^2}}$$

$$= \frac{1}{2} \log \frac{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} + (x + \beta^2 my)}{\sqrt{(1 + \beta^2 m^2) [x^2 + \beta^2 (y^2 + z^2)]} - (x + \beta^2 my)}$$

and

$$\frac{1}{2} \log \frac{\sqrt{x^2 + \beta^2 (y^2 + z^2)} + x}{\sqrt{x^2 + \beta^2 (y^2 + z^2)} - x} = \log \frac{x + \sqrt{x^2 + \beta^2 (y^2 + z^2)}}{\beta \sqrt{y^2 + z^2}}$$

where $$\beta = \sqrt{\beta^2}$$
If we set \( a = 0 \) in (A10), the additional term which occurs in subsonic flow may be easily obtained. For \( a = 0 \),

\[
\phi_1(x, y, z) = \frac{\Delta P}{8\pi \rho a} \int_0^y \int_0^{\eta/m} \frac{y_2}{m_1 + x_3} \frac{z}{(\eta - y)^2 + z^2} \, d\xi \, d\eta
\]

\[
= -\frac{z}{m} \left[ \frac{1}{2} \log \left[ (\eta - y)^2 + z^2 \right] \cdot \frac{Y}{z} \tan^{-1} \frac{z}{\eta - y} \right]_{0}^{y_2} - \frac{z}{m_1} \left[ \frac{1}{2} \log \left[ (\eta - y)^2 + z^2 \right] \cdot \frac{Y + m_1 x_3}{z} \tan^{-1} \frac{z}{\eta - y} \right]_{0}^{y_2} = \frac{z}{2m} \log \left( y^2 + z^2 \right) - \frac{Y}{m} \tan^{-1} \frac{z}{-y}
\]
\[ \phi_1 (x,y,z) = \left[ \frac{z}{zm_1} \log \left( y^2 + z^2 \right) - \frac{ym_1 x_3}{m_1} \tan^{-1} \frac{z}{y} \right] \]

\[ - \frac{z}{m_1} \log \left( (y-y_2)^2 + z^2 \right) - \frac{ym_1 x_3}{m_1} \tan^{-1} \frac{z}{(y-y_2)} \]

\[ - \frac{1}{m_1} \left[ \frac{z}{2} \log \left( (y-y_2)^2 + z^2 \right) + \left[ m_1 (x-x_2) - y \right] \tan^{-1} \frac{z}{(y-y_2)} \right] \]

\[ - \frac{1}{m} \left[ \frac{z}{2} \log \left( (y-y_2)^2 + z^2 \right) + \left[ m_1 (x-x_2) - y \right] \tan^{-1} \frac{z}{(y-y_2)} \right] \]

\[ - \frac{1}{m} \left[ \frac{z}{2} \log \left( (y-y_2)^2 + z^2 \right) + \left[ m_1 (x-x_4) - (y-y_4) \right] \tan^{-1} \frac{z}{(y-y_4)} \right] \]

The above is true since

\[ y_2 = mx_2 \]

\[ y_4 = y_2 = m_1 (x_4 - x_3) \]
Therefore, analogous to (A14) or (A18) when $a = 0$

$$
\hat{\phi}_{00}(x,y,z) = \frac{-\Delta P}{8\pi q_{\infty} m} \left[ (mx-y) \tan^{-1} \frac{x}{y} + \frac{x}{y} \log \left( y^2 + x^2 \right) \right].
$$

which is equivalent to [A18] since we can replace

$$
\tan^{-1} \frac{x}{y} \text{ by } \pi + \tan^{-1} \frac{x}{z} \text{ or by } \tan^{-1} \frac{y}{z}
$$

since $\pi(mx-y)$, when the contributions from each corner are added, gives zero contribution.

Total Source Strength

\[ v_1 \]

\[ z - ax = 0 \]

\[ m - ay = 0 \]

\[ \frac{v_1}{v_1'} = K \frac{1}{a} \]
\[ \nu_2 - \nu'_2 = \frac{K}{a} \left\{ \frac{-a}{a^2 + m^2} \frac{a m}{2} + \frac{m}{2} \frac{-m^2}{(a^2 + m^2)^{1/2}} \right\} = K \frac{m}{a (a^2 + m^2)^{1/2}} \]

\[ L_1 = \gamma_0 \]

\[ L_2 = \gamma_0 \frac{a^2 + m^2^{1/2}}{a} \frac{1}{m} \]

\[ (\nu_1 - \nu'_1) L_1 + (\nu_2 - \nu'_2) L_2 = \gamma_0 K \left[ \frac{1}{a} - \frac{1}{a} \right] = 0 \]

Point Source at \( \xi, \eta, \zeta \)

\[ r^2 = (x-\xi)^2 + \beta^2 [(y-\eta)^2 + (z-\zeta)^2] \]

\[ \phi(x, y, z) = \frac{m}{4\pi r} \]

Doublet at \( \xi, \eta, \zeta \) in \( \vec{e} \) direction

\[ \phi(x, y, z) = \vec{e} \cdot \nabla \left[ \frac{A}{4\pi r} \right] \quad \nabla \beta = \beta^2 \frac{\partial}{\partial \xi} \vec{e}_\xi + \frac{\partial}{\partial \eta} \vec{e}_\eta + \frac{\partial}{\partial \zeta} \vec{e}_\zeta \]

If we integrate a row of these doublets in the direction of \( \vec{e} \), we get a source at one end and a sink at the other. Now let \( \vec{e} \) be
\[
\frac{1}{\sqrt{1+a^2}} \left[-a \, e^\xi + \bar{e}^\xi\right]
\]

and integrate in the x direction.

\[
\frac{\phi(x,y,z)}{4\pi \sqrt{1+a^2}} \int_{\xi_0}^{\infty} \frac{e^2 (z-t)}{[(x-t)^2 + \beta^2 (y-t)^2 + (z-t)^2]^{3/2}} - \beta^2 \frac{\partial}{\partial \xi} \frac{1}{\sqrt{(x-t)^2 + \beta^2 [(y-t)^2 + (z-t)^2]}} \, d\xi
\]

\[
= \frac{-A}{4\pi \sqrt{1+a^2}} \frac{(z-t)}{(y-t)^2 + (z-t)^2} \left[ 1 + \frac{x^2 - \xi_0^2}{\sqrt{(x-t)^2 + \beta^2 (y-t)^2 + \beta^2 (z-t)^2}} \right] \frac{A}{4\pi \sqrt{1+a^2}} \frac{\beta^2}{\sqrt{(x-t)^2 + \beta^2 (y-t)^2 + \beta^2 (z-t)^2}}
\]

If we integrate this over S we obtain (A.7). Therefore (A.7) corresponds to a volume of doublets which means there will be a surface of sinks on one side and sources on the other.
WAGNER'S LIFTING SURFACE EQUATIONS

\[
\begin{align*}
  u(x,y,z) &= \frac{\partial}{\partial x} \Phi(x,y,z) \\
  v(x,y,z) &= \frac{\partial}{\partial y} \Phi(x,y,z) \\
  w(x,y,z) &= \frac{\partial}{\partial z} \Phi(x,y,z)
\end{align*}
\]

\[
\Phi(x,y,z) = V_\infty \iint_S \frac{z k(x',y')} {4\pi [(y-y')^2 + z^2]} \left[ 1 + \frac{x - x'} {\sqrt{(x-x')^2 + (y-y')^2 + z^2}} \right] dx'dy' \quad (1)
\]

Change of variables

\[
\begin{align*}
  x_h(\eta') - x_v(\eta') &= c(\eta') \\
  \eta' &= \frac{y'} {\frac{1}{2} b} \\
  y' &= \frac{1}{2} \eta'b \\
  \xi' &= \frac{x' - x_v(\eta')} {x_h(\eta') - x_v(\eta')} \\
  x' &= \xi' c(\eta') + x_v(\eta')
\end{align*}
\]
\[
\begin{align*}
\text{d}x' \text{d}y' = \left| \begin{array}{cc}
\frac{\partial x'}{\partial \xi'} & \frac{\partial x'}{\partial \eta'} \\
\frac{\partial y'}{\partial \xi'} & \frac{\partial y'}{\partial \eta'} \\
\end{array} \right| & \quad \text{d} \xi' \text{d} \eta' = \frac{1}{2} bc(\eta') \text{d} \xi' \text{d} \eta' \\
\end{align*}
\]

Therefore

\[
\phi(x,y,z) = \frac{V_\infty b^2}{8\pi} \int_{-1}^{1} \int_{0}^{1} \frac{\frac{1}{z} c(\eta') k \left[ x'(\xi', \eta'), y'(\eta') \right]}{c(\eta')} \left[ 1 + \frac{x - x_0(\eta') - \xi' c(\eta')}{\sqrt{[x - x_0(\eta') - \xi' c(\eta')]^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right] \text{d} \xi' \text{d} \eta' \\
\]

Now if we define

\[
\tilde{g}(x,y,z, \eta') = \frac{c(\eta')}{b} \int_{0}^{1} k \left[ x'(\xi', \eta'), y'(\eta') \right] \left[ 1 + \frac{x - x_0(\eta') - \xi' c(\eta')}{\sqrt{[x - x_0(\eta') - \xi' c(\eta')]^2 + (y - \frac{1}{2} \eta' b)^2 + z^2}} \right] \text{d} \xi' \\
\]

Then

\[
\phi(x,y,z) = \frac{V_\infty b^2}{8\pi} \int_{-1}^{1} \int_{0}^{1} \frac{z \tilde{g}(x,y,z, \eta')}{(y - \frac{1}{2} \eta' b)^2 + z^2} \text{d} \eta' \\
\]

Near z = 0 we can write

\[
\phi(x,y,z) = \phi(x,y,0) + z \frac{\partial}{\partial z} \phi(x,y,z) \bigg|_{z=0} \\
\]

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\[ \tilde{g}(x,y,z,\eta') = \tilde{g}(x,y,0,\eta') + O(z^2) \]

Therefore if we change variables and let

\[ \xi = \frac{x - x_V(2y)}{c(2y/b)} \]
\[ \eta = \left(\frac{2y}{b}\right) \]
\[ \tau = \left(\frac{2z}{b}\right) \]

\[ \tilde{g}(x,y,0,\eta') = g(\xi,\eta,\eta') \]

We can say

\[ \frac{1}{V} \Phi(x,y,0) = \varphi(\xi,\eta) = \frac{b}{2\pi} \lim_{\xi \to 0} \int_{-1}^{1} \frac{\xi g(\xi,\eta,\eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \] (6)

and

\[ \lim_{z \to 0} \frac{-1}{V} \frac{\partial}{\partial z} \Phi(x,y,z) = \alpha(\xi,\eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi g(\xi,\eta,\eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \] (7)
Now integrate (6) by parts [Provided $\frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')$ is continuous]

$$u = g(\xi, \eta, \eta')$$

$$dv = \frac{t \, d \eta'}{(\eta' - \eta)^2 + \xi^2}$$

$$du = \frac{\partial}{\partial \eta} g(\xi, \eta, \eta')$$

$$v = -\tan^{-1} \frac{\xi}{\eta' - \eta}$$

$$\int_{-1}^{1} \frac{t \, g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' = - g(\xi, \eta, \eta') \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right]_{-1}^{1}$$

$$+ \int_{-1}^{1} \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \, d\eta'$$

$$= \int_{-1}^{1} \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \, d\eta'$$

(8)

where the fact that $g(\xi, \eta, \pm 1) = 0$, because the loading goes to zero at $\eta' = \pm 1$, was used and since

$$\lim_{\xi \to 0} \tan^{-1} \frac{\xi}{\eta' - \eta} = \pi \text{sgn} \xi \quad \eta' - \eta < 0$$

$$0 \quad \eta' - \eta > 0$$

(9)
\[
\varphi(\xi, \eta) = \frac{b}{4} \sgn \xi \ g(\xi, \eta, \eta) \tag{10}
\]

and

\[
\alpha(\xi, \eta) = \frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \tan^{-1} \left[ \frac{\xi}{\eta' - \eta} \right] \left[ \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \right] d\eta' \\
= -\frac{1}{2\pi} \lim_{\xi \to 0} \int_{-1}^{1} \frac{(\eta' - \eta) \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' \tag{12}
\]

and from A1

\[
\frac{1}{2\pi} \int_{-1}^{1} \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \frac{1}{\eta' - \eta} d\eta' = \frac{1}{2\pi} \left\{ \frac{2g(\xi, \eta, \eta)}{\epsilon} - \int_{-1}^{1} \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \right\}
\]

If \( \frac{\partial}{\partial \eta'} g(\xi, \eta, \eta') \) is not continuous we define

\[
\alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta' = -\frac{1}{2\pi} \int_{-1}^{1} \frac{g(\xi, \eta, \eta')}{(\eta' - \eta)^2} d\eta' \tag{13}
\]

Now let

\[
\frac{c(\eta')}{b} k [x'(\xi', \eta'), y'(\eta')] = \sum_{n=0}^{N} f_n(\eta') h_n(\xi')
\]
which means

\[
g(\xi, \eta, \eta') = - \sum_{n=0}^{N} f_n(\eta') H_n(\xi, \eta, \eta')
\]

where

\[
h_n(\xi') = \frac{1}{\pi} \left[\frac{1 - \xi'}{\xi}\right]^{1/2} \left[\frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'}\right]
\]

or

\[
\hat{h}_n(\Psi) = h_n \left[\frac{1}{2} (1 - \cos \Psi)\right] = \frac{2}{\pi} \left[\frac{\cos n\Psi + \cos (n+1)\Psi}{\sin \Psi}\right] = \frac{2}{\pi} \frac{\cos \left(n+\frac{1}{2}\right)\Psi}{\sin \frac{\Psi}{2}}
\]

and

\[
H_n(\xi, \eta, \eta') = \int_{0}^{1} h_n(\xi') \left\{1 + \frac{x - x_v(\eta') - \xi' c(\eta')}{\sqrt{\left[x - x_v(\eta') - \xi' c(\eta')\right]^2 + \left[y - \frac{1}{2} \eta' b\right]^2}}\right\} d\xi'
\]

\[
= \int_{0}^{1} h_n(\xi') \left\{1 + \frac{x - x_v(\eta')}{c(\eta')} - \xi'}{\sqrt{\left[x - x_v(\eta')\right]^2 + \left[\frac{(\eta - \eta') \frac{1}{2} b}{c(\eta')}\right]^2}}\right\} d\xi' \quad (15)
\]
and from (5)

\[
\begin{aligned}
&= \int_0^1 h_n(\xi') \left\{ 1 + \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')} - \xi' \right. \\
&\quad \left. \sqrt{\frac{\left[ x_v(\eta) + \xi c(\eta) - x_v(\eta') \right]^2 - \xi'} + \left[ \frac{(\eta - \eta')}{c(\eta')} \right]^2} \right\} d\xi'
\end{aligned}
\]

Now, from (7), for any \( g(\xi, \eta, \eta') \) differentiable or not we can write

\[
\alpha(\xi, \eta) = -\frac{1}{2\pi} \lim_{\xi \to 0} \frac{\theta}{\theta \xi} \int_{-1}^{1} \frac{\xi g(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \]

\[
= \frac{1}{2\pi} \sum_{n=0}^{N} \lim_{\xi \to 0} \frac{\theta}{\theta \xi} \int_{-1}^{1} \frac{\xi f_n(\eta') [H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta')]}{(\eta - \eta)^2 + \xi^2} \, d\eta'
\]

where we define

\[
H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) + K_n(\xi, \eta, \eta')
\]

or

\[
\alpha(\xi, \eta) = \frac{1}{2\pi} \sum_{n=0}^{N} \left\{ H_n(\xi, \eta, \eta) \int_{-1}^{1} \frac{\theta f_n(\eta')}{(\eta - \eta')^2} \, d\eta' + \lim_{\xi \to 0} \frac{\theta}{\theta \xi} \int_{-1}^{1} \frac{\xi f_n(\eta')K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \right\}
\]

(18)
where if \( \frac{\partial}{\partial \eta}, K_n(\xi, \eta, \eta') \) is continuous, using (11) and (12)

\[
\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta')K_n(\xi, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' = \int_{-1}^{1} \frac{\frac{\partial}{\partial \eta} \left[ f_n(\eta')K_n(\xi, \eta, \eta') \right]}{(\eta' - \eta)^2} \, d\eta'
\]

since \( f_n(+1) = 0 \)

Now referring to (15), let

\[
\tilde{H}_n(p, q) = \int_{0}^{1} h_n(\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right\} d\xi'
\]

Then, from (B5) for small \((p - p_o)\) and small \(q, p_o \neq 0, 1\)

\[
\tilde{H}_n(p, q) = \tilde{H}_n(p_o, 0) + 2 h_n(p_o)(p - p_o) - h_n'(p_o) q^2 \ln |q| \quad (20)
\]

Now set [See (15)]

\[
p = \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta')}
\]

\[
p_o = \xi
\]

\[
q = \frac{1/2 b(\eta' - \eta)}{c(\eta')}
\]

(21)
when
\[ q = 0 \]
\[ \eta' = \eta \]
\[ p = p_0 \]

Then for \((\eta' - \eta) \ll 1\)

\[
p - p_0 = \frac{a}{\eta'} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta' = \eta} \left( \eta' - \eta \right) \equiv - \frac{1}{2} b \left( \eta' - \eta \right) \tan \phi
\]

(21a)

where we have defined \(\phi(\xi, \eta)\) as the sweep of the constant percent chord lines

\[
\tan \phi(\xi, \eta) = - \frac{c(\eta)}{\frac{1}{2} b} \left[ \frac{x_v(\eta) - \xi c(\eta) - x_v(\eta')}{c(\eta')} \right]_{\eta' = \eta}
\]

(22)

Therefore for \(\xi \neq 0, 1\)

\[
\left| \frac{1}{2} b \left( \eta' - \eta \right) \right| \ll 1
\]

\[
H_n(\xi, \eta, \eta') = H_n(\xi, \eta, \eta) - 2 \left[ \frac{1}{2} b \left( \eta' - \eta \right) \right] h_n(\xi) \tan \phi
\]

\[
- h_n'(\xi) \left[ \frac{1}{2} b \left( \eta' - \eta \right) \right]^2 \ln \left| \frac{1}{2} b \left( \eta' - \eta \right) \right|
\]
Then for small $(\eta' - \eta)$ and $\xi \neq 0, 1$

$$K_n(\xi, \eta, \eta') = -2 \left[ \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right] h_n(\xi) \tan \phi - h_n'(\xi) \left[ \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right] \ln \left[ \frac{\frac{1}{2} b (\eta' - \eta)}{c(\eta)} \right]$$

(23)

For $\xi = 0$, from (C2)

$$\tilde{H}_n(p, q) = \tilde{H}_n(0, 0) + \frac{8 \cos^2 \phi}{\pi} \left[ I_1(\tilde{\phi})(p^2 + q^2)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi})(p^2 + q^2)^{3/4} \right]$$

where $I_1(\tilde{\phi})$ and $I_2(\tilde{\phi})$ are defined in (C3) and C4) and

$$\tilde{\phi} = \tan^{-1} \frac{p}{q}$$

At the leading edge $p_0 = 0$ and, from (21a)

$$-p = \frac{1}{2} b \left( \frac{\eta' - \eta}{c(\eta)} \right) \tan \phi_{L.E.} = q \tan \phi_{L.E.}$$

and therefore

$$p^2 + q^2 = \frac{1}{2} b \left[ c(\eta) \left[ 1 + \tan^2 \phi_{L.E.} \right] (\eta' - \eta)^2 = \frac{1}{\cos^2 \phi_{L.E.}} \left[ \frac{1}{2} b \left( \frac{\eta' - \eta}{c(\eta)} \right)^2 \right] \right]$$

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\[ \tilde{\phi} = \tan^{-1}\left[\tan \phi_{\text{L.E.}}\right] = \phi_{\text{L.E.}} \quad q > 0 \text{ or } (\eta' - \eta) > 0 \]

\[ = \pi + \phi_{\text{L.E.}} \quad (\eta' - \eta) < 0 \]

Therefore for \((\eta' - \eta) << 1\)

\[ K_n(0, \eta, \eta') = \varphi_1(\phi) \left[\frac{1}{2} b \frac{|\eta' - \eta|}{c(\eta)}\right]^{1/2} - \frac{1}{2} (1+4n+4n^2) \varphi_2(\phi) \left[\frac{1}{2} b \frac{|\eta' - \eta|}{c(\eta)}\right]^{3/2} \]

where

\[ \varphi_1(\phi) = \frac{8 \left|\cos \phi\right|}{\pi} \left[\frac{\cos \phi}{c(\eta)}\right] \]

\[ \varphi_2(\phi) = \frac{8 \left|\cos \phi\right|}{3\pi} \left[\frac{\cos \phi}{c(\eta)}\right] \]

and

\[ \phi = \phi_{\text{L.E.}} \quad (\eta' - \eta) > 0 \]

\[ \phi = \pi + \phi_{\text{L.E.}} \quad (\eta' - \eta) < 0 \]
Using a similar analysis for the trailing edge, \( \xi = 1 \), we get

\[
K_n(1, \eta, \eta') = (-1)^n \varphi_2(\varphi) (1 + 2n) \left[ \frac{1}{2} b \frac{|\eta' - \eta|}{c(\eta)} \right]^{5/2} (\eta' - \eta) \ll 1 \quad (25)
\]

\[
\varphi = \phi_{T.E.} + \pi \quad (\eta' - \eta) > 0
\]

\[
\varphi = \phi_{T.E.} \quad (\eta' - \eta) < 0
\]

For \( \xi = 0 \)

\[
\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')
\]

is not continuous at \( \eta' = \eta \) due to the term involving \((\eta' - \eta)^{1/2}\)

Therefore, referring to (18), we must evaluate

\[
\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\xi f_n(\eta')K_n(0, \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} d\eta'
\]

Since the only discontinuity in

\[
\frac{\partial}{\partial \eta'} K_n(\xi, \eta, \eta')
\]

occurs at \( \eta' = \eta \) we can write, using (19)
\[
\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{\mathcal{E}_{n}(\eta', \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \\
= \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_{\eta - \delta}^{\eta + \delta} \frac{\mathcal{E}_{n}(\eta', \eta, \eta')}{(\eta' - \eta)^2 + \xi^2} \, d\eta' \\
+ \int_{-1}^{\eta - \delta} \frac{\mathcal{E}_{n}(\eta', \eta, \eta')}{(\eta' - \eta)^2} \, d\eta' + \int_{\eta + \delta}^{1} \frac{\mathcal{E}_{n}(\eta', \eta, \eta')}{(\eta' - \eta)^2} \, d\eta' \\
\quad (26)
\]

The only term in \( \mathcal{E}_{n}(0, \eta, \eta') \) which causes trouble is the term

\[
\varphi_{1}(\phi_{L.E.}) \left[ \frac{1}{2} b (\eta' - \eta) \right]^{1/2} c(\eta)
\]

and if \( \delta \) is small enough we can approximate \( \mathcal{E}_{n}(\eta') \) by \( \mathcal{E}_{n}(\eta) \)

Therefore we must evaluate the following expressions

\[
\int_{0}^{\delta} \frac{\xi \sqrt{s}}{s^2 + \xi^2} \, ds = \sqrt{\xi} \int_{0}^{\delta/\sqrt{\xi}} \frac{\sqrt{s}}{s^2 + 1} \, ds = \sqrt{\xi} \left\{ \int_{0}^{\infty} \frac{\sqrt{s}}{s^2 + 1} \, ds - \int_{\delta/\sqrt{\xi}}^{\infty} \frac{\sqrt{s}}{s^2 + 1} \, ds \right\} = \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - \int_{\delta/\sqrt{\xi}}^{\infty} \sqrt{s} \frac{s^2 + 1}{s^2} \, ds \right\}
\]

\[
= \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - \int_{0}^{\infty} s^{-3/2} \left[ 1 - \frac{1}{s^2} + \frac{1}{s^4} - \frac{1}{s^6} + \ldots \right] \, ds \right\} = \sqrt{\xi} \left\{ \frac{\pi}{\sqrt{2}} - 2 \left[ \frac{\xi}{\delta} \right]^{1/2} + \frac{2}{5} \left[ \frac{\xi}{\delta} \right]^{3/2} - \frac{2}{9} \left[ \frac{\xi}{\delta} \right]^{5/2} + \ldots \right\}
\]

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\[ 2 \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \left[ \frac{\eta}{\sqrt{2} \xi} + \frac{\eta'}{\sqrt{\delta}} \left( -2 + \frac{\eta^2}{5 \delta} - \frac{10}{9} \frac{\eta^4}{\delta^4} \ldots \right) \right] \]

\[ \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \sqrt{S}}{S^2 + \xi^2} dS = \infty \quad (26a) \]

Subappendices E and F consider the case where \( g(\xi, \eta, \eta') \) is considered to be a function of \( \xi \), as in (3), but the expression (26a) is still infinite.

If \( \tan \phi \) is not continuous at some \( \eta = \eta' \) then, from (23) and (18) or (26), it can be seen that the integral of

\[ \frac{K_n(\xi, \eta, \eta')}{(\eta'-\eta)^2} \]

will give a logarithmic infinity. If \( \tan \phi \) is continuous the first term of (23) will give an odd function of \( (\eta'-\eta) \) which will give a finite value in the Cauchy principle value sense. Therefore kinks or cranks in the planform cannot be permitted because discontinuities in the angles of the constant percent chord lines will result. In fact subappendix G of this appendix shows that a logarithmic infinity occurs if the leading edge kink is considered as the limit of a hyperbola whose radius of curvature goes to zero. If the term in (18) involving \( \tan \phi \) is neglected for small \( (\eta'-\eta) \), this has the effect of rounding the constant percent chord lines.

Now let \( \eta' = \cos \theta' \) and

\[ f_n(\eta') = \sum_{m=1}^M f_{nm} \overline{s}_m(\theta) \quad (27) \]
where

\[
\bar{S}_m(\theta') = \frac{2}{M+1} \sum_{\mu=1}^{M} \sin \mu \theta_m \sin \mu \theta' = \begin{cases} 
1 & \theta' = \theta_m \\
\frac{1}{M+1} (\frac{-1}{M+1} \sin \theta_m \sin(M+1) \theta') \sin \left( \frac{\cos \theta'}{\cos \theta_m} \right) & \theta' \neq \theta_m 
\end{cases}
\]

\[
\theta_m = \frac{m \pi}{M + 1} \text{ discrete points}
\]

Now the first part of (18) may be performed in closed form

\[
\frac{1}{2\pi} \int_{-1}^{1} \frac{\delta f_n(\eta')}{\delta \eta - \eta'} d\eta' = \sum_{m=1}^{M} f_{nm} b_{\nu m}
\]

where

\[
b_{\nu m} = \frac{M+1}{4} \sin \theta_{\nu_m} \frac{1 - (-1)^{\nu+m}}{2} \frac{\sin \theta_m}{\left[ \cos \theta_{\nu} - \cos \theta_m \right]^2} \quad \theta_{\nu} = \theta_m \\
\]

\[
b_{\nu m} = \frac{1}{M+1} \frac{1 - (-1)^{\nu+m}}{2} \frac{\sin \theta_m}{\left[ \cos \theta_{\nu} - \cos \theta_m \right]^2} \quad \theta_{\nu} \neq \theta_m
\]
Referring to (18), (19), (26), and (27) we define

\[ A_{np\nu} = \frac{1}{2\pi} \lim_{\xi \to 0} \int_{\eta_{\nu} - \delta}^{\eta_{\nu} + \delta} \frac{\xi K_n(\xi p', \eta_{\nu}, \eta')}{(\eta_{\nu} - \eta')^2 + \xi^2} \, d\eta' \quad p = 0 \]

\[ = \frac{i}{2\pi} \int_{\eta_{\nu} - \delta}^{\eta_{\nu} + \delta} \frac{K_n(\xi p', \eta_{\nu}, \eta')}{(\eta_{\nu} - \eta')^2} \, d\eta' \quad p \neq 0 \]  

(30)

\[ B_{np\nu} = \frac{1}{2\pi} \int_{\eta_{\nu} + \delta}^{1} \frac{K_n(\xi p', \eta_{\nu}, \eta')}{(\eta_{\nu} - \eta')^2} \overline{S}_m \left[ \cos^{-1} \eta' \right] \, d\eta' \]  

(31)

and letting

\[ \eta' = \frac{1 + \eta_{\nu} \delta}{2} + \frac{1 - \eta_{\nu} \delta}{2} \cos \theta = \eta_1(\theta) \]

\[ = \frac{1 - \eta_{\nu} - \delta}{4} \int_{0}^{\pi} K_n \left[ \xi p', \eta_{\nu}, \eta_1(\theta) \right] \overline{S}_m \left[ \cos^{-1} \eta_1(\theta) \right] \sin \theta \, d\theta \]

\[ = \frac{1 - \eta_{\nu} - \delta}{4(M_{1} + 1)} \sum_{i=1}^{M_{1}} K_n \left[ \xi p', \eta_{\nu}, \eta_1(\theta_i) \right] \overline{S}_m \left[ \cos^{-1} \eta_1(\theta_i) \right] \sin \theta_i \]

where

\[ \theta_i = \frac{i\pi}{M_1 + 1} \quad i = 1, 2, \ldots, M_1 \]
\[
C_{np\nu m} = \frac{1}{2\pi} \int_{-1}^{\eta_{\nu} - \delta} \frac{K_n(\xi_p, \eta_{\nu}, \eta')}{\eta_{\nu} - \eta'} \frac{\bar{S}_m}{d\eta'}
\]

and letting

\[
\eta' = \eta_2(\theta) = \frac{-1 + \eta_{\nu} - \delta}{2} + \frac{1 + \eta_{\nu} - \delta}{2} \cos \theta
\]

\[
C_{np\nu m} = \frac{1 - \eta_{\nu} - \delta}{4\pi} \int_{0}^{\pi} \frac{K_n[\xi_p, \eta_{\nu}, \eta_2(\theta)] \bar{S}_m(\theta) \sin \theta}{[\eta_{\nu} - \eta_2(\theta)]^2} d\theta
\]

\[
\frac{1 - \eta_{\nu} - \delta}{4(M_2 + 1)} \sum_{i=1}^{M_2} \frac{K_n[\xi_p, \eta_{\nu}, \eta_2(\theta_i)] \bar{S}_m(\theta_i) \sin \theta_i}{[\eta_{\nu} - \eta_2(\theta_i)]^2}
\]

\[
\theta_i = \frac{i\pi}{M_2 + 1}
\]

Therefore referring to (18), (19), (26), (28), (29), (30), (31), (32)

\[
\alpha(\xi_p, \eta_{\nu}) = \sum_{n=0}^{N} \sum_{m=1}^{M} f_{nm} [H_{np\nu m} b_{nm} + \delta_{nm} A_{np\nu} + B_{np\nu m} + C_{np\nu m}]
\]

which is a set of linear equations to be solved for \( f_{nm} \)

\[
p = 0, 1, \ldots, N \quad M \times (N+1)
\]

\[
\nu = 1, 2, \ldots, M
\]

\[
H_{np\nu m} = H_n(\xi_p, \eta_{\nu}, \eta_{\nu})
\]

all of the \( H_n \)'s and \( K_n \)'s are computed numerically
Consider

\[
\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta) F(\eta')}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= \lim_{z \to 0} \left\{ F(\eta) \int_{-1}^{1} \frac{(\eta' - \eta)}{(\eta' - \eta)^2 + z^2} \, d\eta' + \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta' \right\}
\]

\[
= \lim_{z \to 0} \left\{ \frac{1}{2} F(\eta) \log \left[ \left( \eta' - \eta \right)^2 + z^2 \right] \right\}^{1}_{-1} + \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= F(\eta) \left\{ \log \frac{1 - \eta}{\varepsilon} + \log \frac{\varepsilon}{1 + \eta} \right\} + \lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

If \( F(\eta') \) is differentiable at \( \eta' = \eta \) then \( F(\eta') - F(\eta) = O(\eta' - \eta) \) and we can write

\[
\lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)[F(\eta') - F(\eta)]}{(\eta' - \eta)^2 + z^2} \, d\eta'
\]

\[
= \lim_{\varepsilon \to 0} \left\{ \int_{\eta + \varepsilon}^{1} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} \, d\eta' + \int_{-1}^{\eta - \varepsilon} \frac{F(\eta') - F(\eta)}{(\eta' - \eta)} \, d\eta' \right\}
\]

(A1)
Therefore

\[ \lim_{z \to 0} \int_{-1}^{1} \frac{(\eta' - \eta)F(\eta')}{(\eta' - \eta)^2 + z^2} \, d\eta' = \int_{-1}^{1} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta' \]

\[ = \lim_{\epsilon \to 0} \left\{ \int_{\eta + \epsilon}^{1} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta' + \int_{-1}^{\eta - \epsilon} \frac{F(\eta')}{(\eta' - \eta)} \, d\eta' \right\} \]
Subappendix B

\[ \tilde{H}_n (p, q) = \tilde{H}_n (p_0, 0) + \oint \nabla \tilde{H}_n (p, q) \cdot dl \]

\[ = H_n (p_0, 0) + \oint \left( \frac{\partial \tilde{H}_n (p, q)}{\partial p} \right) dp + \frac{\partial \tilde{H}_n (p, q)}{\partial q} dq \]

(B1)

where \( C \) is a contour integral in the \( p, q \) plane from \((p_0, 0) \) to \((p, q) \)

\[ \tilde{H}_n (p, q) = \int_0^1 h_n (\xi') \left[ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right] d\xi' \]

\[ \frac{\partial \tilde{H}_n (p, q)}{\partial p} = \int_0^1 k_n (\xi') \frac{q^2}{\left[ (p - \xi')^2 + q^2 \right]^{3/2}} d\xi' \]

if

\[ p \neq 0, 1 \]

we can write

\[ h_n (\xi') = h_n (p) - h'_n (p) (p - \xi') + \frac{1}{2} h''_n (p) (p - \xi')^2 + r_n (\xi') (p - \xi')^3 \]

(B2)
and since

\[\int \frac{x^2}{r^3} = -\frac{1}{r} \quad \int \frac{x^2}{r^3} \, dx = \frac{xr}{2} - \frac{a^2}{2} \log (x+r)\]

and

\[\int \frac{a^2}{r^3} = \frac{x}{r}\]

where \( r = \sqrt{x^2 + a^2} \)

\[\frac{\partial}{\partial q} \tilde{h}_n(p, q) = -h_n(p) \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \left| \begin{array}{c}
\xi' = 1 \\
\xi' = 0
\end{array} \right. + o(q^2)\]

or

\[\lim_{q \to 0} \frac{\partial}{\partial p} \tilde{h}_n(p, q) = 2 h_n(p) \quad p \neq 0, 1 \quad (B3)\]

\[\frac{\partial}{\partial q} \tilde{h}_n(p, q) = -q \int_0^1 h_n(\xi') \frac{(p - \xi')}{\left[(p - \xi')^2 + q^2\right]^{3/2}} \, d\xi'\]

now write (\( p \neq 0, 1 \)) \( h_n(\xi') = h_n(p) - h_n(p)(p - \xi') + r_n(\xi')(p - \xi')^2 \)
Then as $q \to 0$

$$
q \int_0^1 \frac{(p - \xi')}{[(p - \xi')^2 + q^2]^{3/2}} \, d \xi' = \frac{q}{[1 - (p - \xi')^2 + q^2]^{3/2}} \bigg|_0^1 = O(q)
$$

and

$$
q \int_0^1 \frac{r_n(\xi')(p - \xi')^3}{[(p - \xi')^2 + q^2]^{3/2}} \, d \xi' = O(q)
$$

but

$$
q \int_0^1 \frac{(p - \xi')^2}{[(p - \xi')^2 + q^2]^{3/2}} \, d \xi' = q \left\{ \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \log \left( \frac{(1 - p) + \sqrt{(p - \xi')^2 + q^2}}{p \log \left( \frac{1 - p^2 + q^2}{p^2 + q^2} \right)} \right) \right\} \bigg|_0^1
$$

$$
= q \left\{ \frac{(1 - p)}{\sqrt{(1 - p)^2 + q^2}} - \frac{p}{\sqrt{p^2 + q^2}} \log \left[ (1 - p) + \sqrt{(1 - p)^2 + q^2} \right] - \log \left[ -p + \sqrt{p^2 + q^2} \right] \right\}
$$

$$
= O(q) - q \log \left[ -p + \sqrt{p^2 + q^2} \right]
$$

$$
= O(q) - q \log \left[ 1 + \sqrt{1 + \frac{p^2}{q^2}} \right]
$$

$$
= O(q) - 2q \log |q|
$$
Therefore as \( q \to 0 \)

\[
\frac{\partial}{\partial q} \tilde{H}_n(p,q) = -2 h_n'(p) q \ln |q| + o(q) \tag{B4}
\]

and since \( h_n(P) \) and \( h_n'(P) \) are slowly varying for \( p \neq 0,1 \) referring to (B1) and using (B3) and (B4) for \((p-p_0) << 1 \) and \( q << 1 \)

\[
\int_p \frac{\partial}{\partial p} \tilde{H}_n(p,q) \, dp = \int_{p_0}^p \frac{\partial}{\partial p} \tilde{H}_n(p,q) \, dp = 2 h_n(p_0)(p-p_0)
\]

and

\[
\int_q \frac{\partial}{\partial q} \tilde{H}_n(p,q) \, dq = -2 h_n'(p_0) \int_0^q q \ln q \, dq = -h_n'(p_0)q^2 \ln q
\]

or to lowest order in \( p-p_0 \) and \( q \) (for \( p_0 \neq 0,1 \))

\[
\tilde{H}_n(p,q) = \tilde{H}_n(p_0,0) + 2 h_n(p_0)(p-p_0) - h_n'(p_0)q^2 \ln q \tag{B5}
\]
Subappendix C

\[ \tilde{H}_n(p,q) = \int_0^1 h_n(\xi') \left[ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2}} \right] d\xi' \]

**Leading Edge**

Let

\[ p = -\delta^2 \sin \bar{\phi} \]

\[ q = \delta^2 \cos \bar{\phi} \]

or

\[ \delta^2 = \sqrt{p^2 + q^2} \]

\[ \bar{\phi} = \tan^{-1} \frac{-p}{q} \]

and introduce a change of variables

\[ \xi' = \sin^2 \sigma \]

\[ d\xi' = 2 \sin \sigma \cos \sigma \, d\sigma \]
\[ h_n (\sin^2 \sigma) = h_n \left[ \frac{1}{2} (1 - \cos 2\sigma) \right] = \hat{h}_n (2\sigma) = \frac{2}{\pi} \frac{\cos 2n\sigma + \cos 2(n+1)\sigma}{\sin 2\sigma} \]

\[ = \frac{2}{\pi} \frac{\cos \left( [2n+1] - 1 \right) \sigma + \cos \left( [2n+1] + 1 \right) \sigma}{2 \sin \sigma \cos \sigma} = \frac{2}{\pi} \frac{\cos (2n+1)\sigma}{\sin \sigma} \]

\[
\left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right\} = 1 + \frac{- (\delta^2 \sin \phi + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}}
\]

and

\[ \tilde{H}_n (p, q) = \hat{H}_n (\delta, \phi) \]

\[ \frac{4}{\pi} \int_0^{\pi/2} \left\{ 1 + \frac{- (\delta^2 \sin \phi + \sin^2 \sigma)}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}} \right\} \cos (2n+1)\sigma \cos \sigma \, d\sigma \]

\[ \frac{\partial \tilde{H}_n}{\partial \delta} = \frac{4}{\pi} \int_0^{\pi/2} \frac{- 2 \sin \phi \cos \left( (2n+1) \sigma \right) \cos \sigma \, d\sigma}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4}} \, d\sigma \]

\[ + \frac{4}{\pi} \int_0^{\pi/2} \frac{2\delta \left[ \cos \phi \sin^2 \sigma + \delta^2 \right] (\delta^2 \sin \phi + \sin^2 \sigma) \cos \left( (2n+1) \sigma \right) \cos \sigma \, d\sigma}{\left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}} \]

\[ - 8 \delta^3 \cos^2 \phi \int_0^{\pi/2} \frac{\cos \left( (2n+1) \sigma \right) \cos \sigma \sin^2 \sigma}{\left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}} \, d\sigma \]

As \( \delta \to 0 \) the only portion of the integrand which is important is in the region of \( \sigma = 0 \)

\[ C = \delta \]

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Therefore let \( x = \sin \sigma \) \( dx = \cos \sigma \ d\sigma \) and expand

\[
\cos [(2n+1) \sigma] = \cos \left[ (2n+1) \left( x - \frac{x^3}{6} \right) \right]
\]

\[
= 1 - \frac{1}{2} (2n+1)^2 x^2 = 1 - \frac{x^2}{2} (1 + 4n + 4n^2) + O(x^4)
\]

Therefore to first order in \( x^2 \)

\[
\frac{\partial^2}{\partial \delta^2} \mathcal{H}_n \left( \delta, \tilde{\phi} \right) = \frac{8}{\pi} \cos^2 \tilde{\phi} \int_0^1 \left[ 1 - \frac{x^2}{2} (1 + 4n + 4n^2) \right] x^2 \left[ x^4 + 2\delta x^2 \sin \tilde{\phi} + \delta^2 \right]^{3/2} dx
\]

\[
= \frac{8}{\pi} \cos^2 \tilde{\phi} \left\{ \int_0^{1/\delta} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} - \frac{1}{2} \int_0^{1/\delta} \frac{(1 + 4n + 4n^2) x^4 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \right\}
\]

Since

\[
\int_0^{1/\delta} \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} = \int_0^\infty \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \int_0^{1/\delta} \frac{dx}{x^4} = \int_0^\infty \frac{x^2 dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} + O(\delta^3)
\]
Near $\delta = 0$ we can say

$$
\frac{\partial}{\partial \delta} \tilde{H}_n(\delta, \phi) = \frac{8 \cos^2 \tilde{\phi}}{\pi} \int_0^\infty \frac{x^2 \left[ 1 - \frac{\delta^2 x^2}{2} (1 + 4n + 4n^2) \right]}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} \, dx + O(\delta^3)
$$

integrating from 0 to $\delta$ we get

$$
\tilde{H}_n(\delta, \tilde{\phi}) = \tilde{H}_n(0, \tilde{\phi}) + \frac{8 \cos^2 \tilde{\phi}}{\pi} \left[ I_1(\tilde{\phi}) \delta - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \delta^3 \right]
$$

or

$$
\tilde{H}_n(p, q) = \tilde{H}_n(0, 0)
$$

$$
+ \frac{8 \cos^2 \tilde{\phi}}{\pi} \left[ I_1(\tilde{\phi}) \left( p^2 + q^2 \right)^{1/4} - \frac{1}{6} (1 + 4n + 4n^2) I_2(\tilde{\phi}) \left( p^2 + q^2 \right)^{3/4} \right]
$$

where

$$
\tilde{\phi} = \tan^{-1} \left( \frac{-p}{q} \right)
$$

and where

$$
I_1(\tilde{\phi}) = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2x^2 \sin \tilde{\phi} + 1 \right]^{3/2}} = \frac{1}{\cos \tilde{\phi}} \frac{d}{d\tilde{\phi}} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}}
$$

(C3)
\[ I_2(\tilde{\phi}) = \int_0^\infty \frac{x^4 \, dx}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} \]

\[ = - \sin \tilde{\phi} \, I_1(\tilde{\phi}) + \int_0^\infty \frac{(x^4 + x^2 \sin \tilde{\phi})}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} \, dx \] (C4)

\[ u = x \]

\[ du = dx \]

\[ dv = \frac{(x^3 + x \sin \tilde{\phi}) \, dx}{[x^4 + 2x^2 \sin \tilde{\phi} + 1]^{3/2}} \]

\[ v = -\frac{1}{2} \frac{1}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \]

Therefore

\[ I_2(\tilde{\phi}) = - \sin \tilde{\phi} \, I_1(\tilde{\phi}) + \frac{1}{2} \int_0^\infty \frac{dx}{\sqrt{x^4 + 2x^2 \sin \tilde{\phi} + 1}} \] (C5)
Subappendix D

\[ \tilde{H}_n(p, q) = \int_0^1 h_n(\xi') \left( 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2}} \right) d\xi' \]  \hspace{1cm} (D1)

**Trailing Edge, let**

\[
\begin{align*}
p & = 1 - \delta^2 \sin \theta \\
q & = \delta^2 \cos \theta \\
\delta^2 & = \sqrt{(1 - p)^2 + q^2} \\
\theta & = \tan^{-1} \left( \frac{1 - p}{q} \right) \hspace{1cm} (D2)
\end{align*}
\]

and change variables in (D1)

\[
\begin{align*}
\xi' & = \sin^2 \sigma \\
d\xi' & = 2 \sin \sigma \cos \sigma \, d\sigma \\
& \hspace{1cm} h_n(\sin^2 \sigma) = \frac{2}{\pi} \frac{\cos (2n + 1) \sigma}{\sin \sigma}
\end{align*}
\]

(see leading edge expansion)
\[ \tilde{h}_n (p, q) = \tilde{h}_n (\delta, \theta) = \frac{4}{\pi} \int_0^{\pi/2} \left( 1 + \frac{\cos^2 \sigma - \delta^2 \sin \theta}{\sqrt{(\cos^2 \sigma - \delta^2 \sin \theta)^2 + \delta^4 \cos^2 \theta}} \right) \cos (2n + 1) \sigma \cos \sigma \, d\sigma \]

\[ = \frac{4}{\pi} \int_0^{\pi/2} \left( 1 + \frac{\sin^2 \sigma - \delta^2 \sin \theta}{\sqrt{\sin^4 \sigma - 2\delta^2 \sin^2 \sigma \sin \theta + \delta^4}} \right) \cos (2n + 1) \left( \frac{\pi}{2} - \sigma \right) \sin \sigma \, d\sigma \]

Now

\[ \cos \left[ (2n + 1) \left( \frac{\pi}{2} - \sigma \right) \right] = \cos \left( (2n + 1) \frac{\pi}{2} \right) \cos \left[ (2n + 1) \sigma \right] + \sin \left( (2n + 1) \frac{\pi}{2} \right) \sin \left[ (2n + 1) \sigma \right] \]

\[ = \sin \left[ (2n + 1) \frac{\pi}{2} \right] \sin \left[ (2n + 1) \sigma \right] = (-1)^{n-1} \sin \left[ (2n + 1) \sigma \right] \]

The differentiation of the integrand with respect to \( \delta \) may be performed easily if it is compared with the leading edge case.

\[ \frac{\partial}{\partial \delta} \tilde{h}_n (\delta, \theta) = (-1)^n \frac{8\delta^3 \cos^2 \theta}{\pi} \int_0^{\pi/2} \left[ \sin \left[ (2n + 1) \sigma \right] \sin^3 \sigma \, d\sigma \right] \times \]

\[ \left[ \sin^4 \sigma - 2\delta^2 \sin \theta \sin^2 \sigma + \delta^4 \right]^{3/2} \]

Changing variables of integration once again

\[ x = \sin \sigma \]

\[ dx = \cos \sigma \, d\sigma \]

\[ \sin \left[ (2n + 1) \sigma \right] = (1 + 2n)x + O(x^3) \]
\[
\frac{\partial}{\partial \delta} H_n(\delta, \theta) = (-1)^n (1 + 2n) \frac{8 \delta^3 \cos^2 \theta}{\pi} \int_0^{1/\delta} \frac{x^4 \, dx}{[x^4 - 2 \delta^2 x^2 \sin \theta + \delta^4]^{3/2}}
\]

\[
= (-1)^n (1 + 2n) \frac{8 \delta^2 \cos^2 \theta}{\pi} \int_0^{\infty} \frac{x^4 \, dx}{[x^4 - 2 x^2 \sin \theta + 1]^{5/2}} + o(\delta^4)
\]

\[
= (-1)^n (1 + 2n) \frac{8 \delta^2 \cos^2 \theta}{\pi} I_1(\theta + \pi)
\]

where

\[
I_1(\theta) = \int_0^\infty \frac{x^4 \, dx}{[x^4 + 2 x^2 \sin \theta + 1]^{3/2}}
\]

and integrating from 0 to \(\delta\) we get

\[
\tilde{H}(p, q) = \tilde{H}(1, q) + (-1)^n (1 + 2n) \frac{8 \cos^2 \theta}{3\pi} \frac{I_1(\theta + \pi)}{(1 - p)^2 + q^2}^{3/4}
\]

where

\[
\theta = \tan^{-1} \left( \frac{1 - p}{q} \right)
\]
Suppose that \( k(x', y') \) may be written (as before)

\[
 \frac{c(n')}{b} k \left[ x'(\xi', \eta'), y(\eta') \right] = \sum_{n=0}^{N} f_n(\eta') h_n(\xi')
\]

where

\[
 h_n(\xi') = \frac{1}{\pi} \left[ \frac{1 - \xi'}{\xi'} \right]^{1/2} \left[ \frac{T_n(1 - 2\xi') + T_{n+1}(1 - 2\xi')}{1 - \xi'} \right]
\]

then

\[
 h_n(\Psi) = h_n \left[ \frac{1}{2} (1 - \cos \Psi) \right]
\]

\[
 = \frac{2}{\pi} \left[ \cos n \Psi + \cos (n + 1) \Psi \right] = \frac{2}{\pi} \frac{\cos \left( n + \frac{1}{2} \right) \Psi}{\sin \frac{\Psi}{2}}
\]
Then with

\[ \xi = \frac{x - x_v - \frac{2y}{b} \frac{v}{c}}{2y/b} \]

\[ \eta = \frac{2y}{b} \]

\[ \zeta = \frac{2z}{b} \]

and from (3), (2) and (7) with \( t \) kept in \( g \)

\[ \Phi(x, y, z) = \phi(\xi, \eta, \zeta) = \frac{b}{4\pi} \int_{-1}^{1} \frac{\xi \hat{g}(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} d\eta' \]

\[ \alpha(\xi, \eta) = \lim_{\zeta \to 0} \frac{-1}{2\pi} \frac{\partial}{\partial \xi} \int_{-1}^{1} \frac{g(\xi, \eta, \eta', \zeta)}{(\eta' - \eta)^2 + \zeta^2} d\eta' \]

where

\[ g(\xi, \eta, \eta', \zeta) = \sum_{n=0}^{N} f_n(\eta') \hat{H}_n(\xi, \eta, \eta', \zeta) \]

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and

\[ \hat{h}(t, \eta, \eta', t) = \int_{0}^{1} h_n(\xi') \left\{ 1 + \frac{x_v(\eta) + \xi c(\eta) - x_v(\eta')}{c(\eta') - \xi'} \right\} \frac{1}{\sqrt{\left[ x_v(\eta) + \xi c(\eta) - x_v(\eta') \right]^2 + \left[ \frac{\eta - \eta'}{c(\eta')} \right]^2 + \left[ \frac{\xi - \eta'}{c(\eta')} \right]^2}} \, d\xi' \]

We can also define

\[ \tilde{h}_n(p, q, \xi) = \int_{0}^{1} h_n(\xi') \left\{ 1 + \frac{p - \xi'}{\sqrt{(p - \xi')^2 + q^2 + \xi'^2}} \right\} \, d\xi' \]

For small \( \eta' - \eta \)

\[ p - p_o = \frac{1}{2} b (\eta' - \eta) \tan \phi \]

\( \phi \) = slope of constant percent chord lines

\[ q = \frac{1}{2} b (\eta' - \eta) \frac{1}{c(\eta')} \quad \frac{1}{2} b (\eta' - \eta) \frac{1}{c(\eta)} \]

\[ c(\eta') = c(\eta) + \frac{\partial}{\partial \eta} c(\eta') \bigg| \eta' = \eta \]
\[ \hat{\xi} = \frac{\xi(\frac{1}{2} b)}{c(\eta')} = \frac{z}{c(\eta')} = \frac{z}{c(\eta)} \]

\[ \xi = \frac{z}{\frac{1}{2} b} \]

**Leading Edge**, \( P_0 = 0 \), \( \phi = \phi_{\text{LE}} = \) leading edge sweep

\[ p^2 + q^2 + \xi^2 = \left[ \frac{\frac{1}{2} b}{c(\eta)} \right]^2 \left\{ (\eta' - \eta)^2 [1 + \tan^2 \phi] + \left( \frac{z}{\frac{1}{2} b} \right)^2 \right\} \]

\[ = \frac{1}{\cos^2 \phi} \left[ \frac{\frac{1}{2} b}{c(\eta)} \right]^2 \left\{ (\eta' - \eta)^2 + \xi^2 \cos^2 \phi \right\} \]

Therefore at the leading edge since \( \tilde{\phi} = \tan^{-1}\left(\frac{-p}{q}\right) = \begin{cases} \phi_{\text{LE}} & \eta' > \eta \\ \phi_{\text{LE}} + \pi & \eta' < \eta \end{cases} \)

\[ \hat{x}_n (\xi, \eta, \eta', t) = \frac{8(1 - \sin^2 \phi_{\text{LE}} \sin^2 \phi)}{v} \left\{ I_1 (\tilde{\phi}, \phi) \left[ \frac{1}{2} \frac{b}{c(\eta)} \right]^2 \left[ (\eta' - \eta)^2 + \xi^2 \cos^2 \phi \right] \right\} \]

\[ = \frac{8(1 - \sin^2 \phi_{\text{LE}} \sin^2 \phi)}{v} \left\{ I_1 (\tilde{\phi}, \phi) \left[ \frac{1}{2} \frac{b}{c(\eta)} \right]^2 \left[ (\eta' - \eta)^2 + \xi^2 \cos^2 \phi \right] \right\} \]

\[ = \frac{1}{6} (1 + 4\eta + 4\eta^2) I_2 (\tilde{\phi}, \phi) \left[ \frac{1}{2} \frac{b}{c(\eta)} \right]^2 \left[ (\eta' - \eta)^2 + \xi^2 \cos^2 \phi \right] \]

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\[
\sin \psi = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \xi^2}} = \frac{|\eta' - \eta|}{\sqrt{(\eta' - \eta)^2 + \xi^2 \cos^2 \phi}} = \frac{|u|}{\sqrt{u^2 + \cos^2 \phi}}
\]

where

\[
u = \frac{\eta' - \eta}{\xi}
\]

letting \( s = \eta' - \eta \) we must evaluate an expression of the form

\[
\lim_{t \to 0} \frac{\partial}{\partial t} \int_{-\delta}^{\delta} \frac{(1 - \sin^2 \phi \sin^2 \psi) I_1 (x, \phi) t [s^2 + t^2 \cos^2 \phi]^{1/2}}{s^2 + t^2 \cos^2 \phi} ds = \begin{cases} \phi_{\text{LE}} & s > 0 \\ \phi_{\text{LE} + \pi} & s < 0 \end{cases}
\]

\[
\lim_{t \to 0} \frac{\partial}{\partial t} \left\{ \sqrt{t} \int_{0}^{4t} (1 - \sin^2 \phi \sin^2 \psi) \left[ I_1 (\phi_{\text{LE}}, \phi) + I_1 (\phi_{\text{LE} + \pi}, \phi) \right] \frac{u^2 + \cos^2 \phi}{u^2 + 1} \frac{1}{du} \right\}
\]

\[
\lim_{t \to 0} \frac{\partial}{\partial t} \left\{ \sqrt{t} \int_{0}^{4t} (1 - \sin^2 \phi \sin^2 \psi) \left[ I_1 (\phi_{\text{LE}}, \phi) + I_1 (\phi_{\text{LE} + \pi}, \phi) \right] \frac{u^2 + \cos^2 \phi}{u^2 + 1} \frac{1}{du} \right\}
\]

\[
- \sqrt{t} \int_{4t}^{\delta} (1 - \sin^2 \phi \sin^2 \psi) \left[ I_1 (\phi_{\text{LE}}, \phi) + I_1 (\phi_{\text{LE} + \pi}, \phi) \right] \frac{u^2 + \cos^2 \phi}{u^2 + 1} \frac{1}{du}
\]

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where

\[ \sin \dot{\phi} = \frac{u}{\sqrt{u^2 + \cos^2 \phi}} \quad (\text{no } \xi \text{ dependence!}) \]

and

\[ I_1 (\phi, \psi) = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2x^2 \sin \phi \sin \psi + 1 \right]^{3/2}} \]

The second integral goes to zero like \( \sqrt{\xi} \) and the first is independent of \( \xi \). Therefore

\[ \lim_{\xi \to 0} \frac{\partial}{\partial \xi} \sim \frac{1}{\sqrt{\xi}} \to \infty \]
Subappendix F

\[ \tilde{H}_n (p, q, \xi) = \int_0^1 h_n (\xi') \left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + \xi^2}} \right\} \, d\xi' \]

Leading edge, let

\[ p = -\delta^2 \sin \phi \sin \psi \]
\[ q = \delta^2 \cos \phi \sin \psi \]
\[ \xi = \delta^2 \cos \psi \]
\[ \delta^2 = \sqrt{p^2 + q^2 + \xi^2} \]
\[ \phi = \tan^{-1} \frac{-p}{q} \]
\[ \psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\xi} \]

introduce a change of variables in the integral

\[ \xi' = \sin^2 \sigma \]
\[ d\xi' = 2 \sin \sigma \cos \sigma \, d\sigma \]
\[
\hat{h}_n (\sin^2 \sigma) = h_n \left[ \frac{1}{2} (1 - \cos 2\sigma) \right] = \hat{h}_n (2\sigma) = \frac{2}{\pi} \frac{\cos 2n \sigma + \cos (2(n + 1)) \sigma}{\sin 2\sigma}
\]

\[
= \frac{2}{\pi} \frac{\cos [(2n + 1) - 1] \sigma + \cos [(2n + 1) + 1] \sigma}{2 \sin \sigma \cos \sigma} \quad = \frac{2}{\pi} \frac{\cos (2n + 1) \sigma}{\sin \sigma}
\]

and

\[
\left\{ 1 + \frac{(p - \xi')}{\sqrt{(p - \xi')^2 + q^2 + t^2}} \right\} = \left\{ 1 + \frac{\delta^2 \sin \phi \sin \phi + \sin^2 \sigma}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4}} \right\}
\]

\[
\hat{H}_n (p, q, t) = \hat{H}_n (\phi, \theta, \psi) = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \left\{ 1 + \frac{\delta^2 \sin \phi \sin \phi + \sin^2 \sigma}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4}} \right\} \cos [(2n + 1) \sigma] \cos \sigma \cos \sigma \, d\sigma
\]

\[
\frac{\partial \hat{H}_n (\phi, \theta, \psi)}{\partial \theta} = \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{2 \delta \sin \phi \sin \phi \cos [(2n + 1) \sigma]}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4}} \cos [(2n + 1) \sigma] \cos \sigma \cos \sigma \, d\sigma
\]

\[
= \frac{4}{\pi} \int_0^{\frac{\pi}{2}} \frac{2 \delta \sin \phi \sin \phi \sin^2 \sigma + \delta^2}{\sqrt{\sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4}} \left( \delta^2 \sin \phi \sin \phi + \sin^2 \sigma \right) \cos [(2n + 1) \sigma] \cos \sigma \cos \sigma \, d\sigma
\]

\[
\quad \cdot \left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}
\]

\[
= 8 \delta^3 \left[ 1 - \sin^2 \phi \sin^2 \phi \right] \int_0^{\frac{\pi}{2}} \frac{\cos [(2n + 1) \sigma] \cos \sigma \sin^2 \sigma \, d\sigma}{\left[ \sin^4 \sigma + 2 \delta^2 \sin \phi \sin \phi \sin^2 \sigma + \delta^4 \right]^{3/2}}
\]

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as \( \delta \to 0 \) the only portion of the integrand which is important is in the region of \( \sigma = 0 \).

Therefore let \( x = \sin \sigma \), \( dx = \cos \sigma \, d\sigma \) and expand

\[
\cos \left[ (2n + 1) \sigma \right] = \cos \left[ (2n + 1) \left( x - \frac{x^3}{6} \right) \right] = 1 - \frac{1}{2} (2n + 1)^2 x^2
\]

\[
1 - \frac{x^2}{2} (1 + 4n + 4n^2) + O(x^4)
\]

Therefore to first order in \( x^2 \)

\[
\hat{h}_n (\theta, \phi, \Phi) = 8 \delta^3 \left[ 1 - \sin^2 \theta \sin^2 \phi \right] \int_0^1 \frac{\left[ 1 - 2 \delta^2 (1 + 4n + 4n^2) \right] x^2 \, dx}{\left[ x^4 + 2 \delta^2 x^2 \sin \theta \sin \phi + 1 \right]^{3/2}}
\]

\[
= 8 \left[ 1 - \sin^2 \theta \sin^2 \phi \right] \int_0^{\frac{1}{2}} \frac{x^2 - \frac{1}{2} \delta^2 x^4 (1 + 4n + 4n^2)}{\left[ x^4 + 2 x^2 \sin \theta \sin \phi + 1 \right]^{3/2}} \, dx + O(\delta^5)
\]

\[
\hat{h}_n (\hat{\theta}, \hat{\phi}, \hat{\Phi}) = \hat{h}_n (\theta, \phi, \Phi) \delta \left( 1 - \sin^2 \theta \sin^2 \phi \right) \left[ t_1 (\Phi, \phi) \delta - \frac{1}{6} (1 + 4n + 4n^2) t_2 (\Phi, \phi) \delta^3 \right]
\]

Integrating from 0 to \( \delta \) we get

\[\text{(C1)}\]
\[ \Pi_n(p, q, \tau) = \Pi_n(0, 0, 0) \cdot \frac{8(1 - \sin^2 \phi \sin^2 \theta)}{\tau^2} \left[ I_1(\phi, \psi) \left( \frac{p^2 + q^2 + \tau^2}{\tau} \right)^{1/2} \right] \]

where

\[ \phi = \tan^{-1} \frac{p}{q} \]
\[ \psi = \tan^{-1} \frac{\sqrt{p^2 + q^2}}{\tau} \]

\[ I_1(\phi, \psi) = \int_0^\infty \frac{x^2 \, dx}{\left[ x^4 + 2 x^2 \sin \phi \sin \psi + 1 \right]^{3/2}} \]  

\[ I_2(\phi, \psi) = \int_0^\infty \frac{x^4 \, dx}{\left[ x^4 + 2 x^2 \sin \phi \sin \psi + 1 \right]^{3/2}} \]  

\[ \sin \psi = \frac{\tan \psi}{\sqrt{1 + \tan^2 \psi}} = \frac{\sqrt{p^2 + q^2}}{\sqrt{p^2 + q^2 + \tau^2}} = \frac{\sqrt{\frac{p^2 + q^2}{\tau^2}}}{\sqrt{\frac{p^2 + q^2}{\tau^2} + 1}} \]

\[ \sin \phi = \frac{-p}{\sqrt{p^2 + q^2}} \]

\[ \sin \phi \sin \psi = \frac{-p}{\sqrt{p^2 + q^2 + \tau^2}} = \frac{-p}{\tau} \]
Subappendix G

\[ \tilde{H}_n (p, q) = \tilde{H}_n (p_o, 0) + 2 h_n (p_o) (p - p_o) - h''_n (p_o) q^2 \ln |q| \]

Hyperbolic leading and trailing edges, constant chord.

\[ \left[ X_v (\eta') \right]^2 = c \tan^2 \theta \left[ \eta'^2 + \rho^2 \tan^2 \theta \right] \]

\[ X_v (\eta') = c \tan \theta \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} \]

\[ \eta' >> 1 \quad \frac{1}{c} \frac{dX_v}{d\eta} = \tan \theta \]

Curvature at \( \eta' = 0 \)

\[ \left. \frac{1}{c} \frac{d^2X_v}{d\eta'^2} \right|_{\eta'=0} = \frac{1}{\rho} \]

From (21) at \( \eta' = 0 \)

\[ p - p_o = \frac{X_v (0) - X_v (\eta')}{c} \]

or

\[ p - p_o = -\tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right] \]
Therefore from (15) and (20), for small $\eta'$:

$$H_n(\xi, 0, \eta') = H_n(\xi, 0, 0) - 2 h_n(\xi) \tan \theta \left[ \sqrt{\eta'^2 + \rho^2 \tan^2 \theta} - \rho \tan \theta \right]$$

$$+ h'_n(\xi) \eta'^2 \ln |\eta'|$$

Therefore referring to (17) and (18) we must evaluate

$$\lim_{\xi \to 0} \frac{\partial}{\partial \xi} \int_0^\delta \frac{\xi \left[ \sqrt{s^2 + a^2} - a \right]}{s^2 + \xi^2} \, ds$$

$$= \left. \xi \left\{ \log \left[ s + \sqrt{s^2 + a^2} \right] + \frac{\sqrt{a^2 - \xi^2}}{\xi} \tan^{-1} \frac{s \sqrt{a^2 - \xi^2}}{\xi} \right. - \frac{a}{\xi} \tan^{-1} \frac{s}{\xi} \right|_0^\delta$$

$$= \left. \xi \left\{ \log \left[ \delta + \sqrt{\delta^2 + a^2} \right] + \frac{\sqrt{a^2 - \delta^2}}{\delta} \tan^{-1} \frac{\delta a - \delta^2}{\delta \sqrt{a^2 - \delta^2}} - \frac{a}{\delta} \tan^{-1} \frac{\delta}{\delta} - \log a \right\} \right|_0^\delta$$
and

$$\lim_{\alpha \to 0} \lim_{\epsilon \to 0} \int_{0}^{\delta} \frac{\epsilon \left[ \frac{\sqrt{s^2 + a^2} - a}{s^2 + t^2} \right]}{ds = \infty.}$$

and since $a = \rho \tan \theta$, there is a logarithmic infinity as the radius of curvature goes to zero and the leading edge becomes kinked.
The section load can be obtained by means of the Blasius theorem as follows:

\[ F_X - iF_Y = \frac{1}{2} \rho \int_C \left( \frac{dw}{dz} \right)^2 dz \]  

(1)

where

\[ \left( \frac{dw}{dz} \right)^2 = u^2 - 2uvi + \left( iv \right)^2 = u^2 - v^2 - 2uvi \]  

(2)

Therefore;

\[ F_x - iF_y = \frac{1}{2} \rho \int_C \left[ 2uv + i \left( u^2 - v^2 \right) \right] [dx + idy] \]  

(3)

then

\[ F_x = \frac{1}{2} \rho \int_C \left[ 2uv dx - \left( u^2 + v^2 \right) dy \right] \]  

(4)

\[ F_y = -\frac{1}{2} \rho \int_C \left[ 2uv dy + \left( u^2 - v^2 \right) dx \right] \]  

(5)
Since all of the singularities are on the chordal plane, for the panels, equations (4) and (5) reduce to:

\[ F_x = -\rho \sum_{i=1}^{N_i} \left( u_{i1} v_{i1} - u_{L1} v_{L1} \right) \Delta x_i \]  

(6)

\[ F_y = \rho/2 \sum_{i=1}^{N_i} \left[ \left( u_{i1}^2 - u_{L1}^2 \right) - \left( u_{L1}^2 - v_{L1}^2 \right) \right] \Delta x_i \]  

(7)

where the subscripts \( u \) and \( L \) indicate upper and lower surfaces, respectively.

For the two-dimensional lifting case:

\[ u_{i1} = \frac{1}{2} \gamma_i + V_\infty \cos \alpha \]  

(8)

\[ u_{L1} = -\frac{1}{2} \gamma_i + V_\infty \cos \alpha \]  

(9)

\[ v_{u1} = v_{L1} = -\frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Gamma K}{x_i - x_K} + V_\infty \sin \alpha \]  

(10)

Therefore,

\[ F_x = -\rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \sin \alpha + \rho/2\pi \sum_{i=1}^{N_i} \sum_{K=1}^{N_K} \frac{\Gamma_i \Gamma_K}{x_i - x_K} \]  

(11)
Since

$$\frac{\rho}{2\pi} \sum_{i=1}^{N_i} \sum_{K=1}^{N_i} \frac{\Gamma_i \Gamma_K}{x_i - x_K} = 0$$  \hspace{1cm} (12)$$

$$F_x = -\rho V_\infty \sin \alpha \sum_{i=1}^{N_i} \Gamma_i = -L \sin \alpha$$  \hspace{1cm} (13)$$

where $L$ is the lift and $\Gamma = \gamma \Delta x$ is the local vortex strength.

Also, from equations (7), (8), (9), and (10)

$$F_y = \rho \sum_{i=1}^{N_i} \Gamma_i V_\infty \cos \alpha = L \cos \alpha$$  \hspace{1cm} (14)$$

which demonstrates that the discrete vortex lattice always gives the correct chord force $F_x$ and normal force $F_y$, provided the correct circulations is obtained.

Similarly, for the two-dimensional thickness case;

$$u_{u_i} = u_{L_i} = \frac{1}{2\pi} \sum_{K=1}^{N_i} \frac{\Sigma_K}{x_i - x_K} + V_\infty$$  \hspace{1cm} (15)$$

$$v_{u_i} = V_\infty \left(\frac{dz_t}{dx}\right)_i$$  \hspace{1cm} (16)$$
\[ v_{L_i} = -V_\infty \left( \frac{dz}{dx} \right)_i \]  

(17)

and

\[ \Sigma_i = 2V_\infty \left( \frac{dz}{dx} \right)_i \Delta x_i \]  

(18)

Therefore;

\[ F_x = -\frac{\rho}{2\pi} \sum_{i=1}^{N_1} \sum_{K=1}^{N_1} \frac{\Sigma_i \Sigma_K}{x_i - x_K} - \rho V_\infty \sum_{i=1}^{N_1} \Sigma_i = 0 \]  

(19)

since

\[ \sum_{i=1}^{N_1} \Sigma_i = 0 \]

if the airfoil is closed.

Also; \( F_y = 0 \) from equation (7), (15), (16), and (17). This demonstrates that the discrete source lattice also gives the correct chord force \( F_x \) and normal force \( F_y \).

The above equations can be generalized to compute the section potential form drag on a finite wing due to lift and thickness by evaluating the component of force in the free stream direction and by using the three dimensional influence equations.
The section potential form drag due to lift is computed as follows:

\[
\delta_{L_i} = -\rho \sum_{i=1}^{N_i} \Gamma_i w_i
\]  

(20)

where \( i \) is summed over the section of the panel and \( w_i \) is the total velocity normal to the panel chordal surface at the quarter chord of the \( i \)th subpanel. The section induced drag coefficient

\[
\frac{C_{dL_i}}{C_{AVG}} C
\]

is then given by;

\[
\frac{C_{dL_i}}{C_{AVG}} = -\frac{2AR}{b} \sum_{i=1}^{N_i} \left( \frac{\Gamma_i}{V_\infty} \right) \left( \frac{w_i}{V_\infty} \right)
\]  

(21)

If \( w_i \) is computed at the three-quarter chord of the \( i \)th subpanel, instead of the quarter chord of the \( i \)th subpanel as is done in equation (21), the section zero percent suction drag coefficient

\[
\frac{C_{dL,0}}{C_{AVG}}
\]

is obtained. The section leading edge thrust coefficient \((C_T C)/(C_{AVG})\) is equal to;

\[
\frac{C_T C}{C_{AVG}} = \frac{C_{dL,0}}{C_{AVG}} - \frac{C_{dL_i}}{C_{AVG}}
\]  

(22)
The section potential form drag due to thickness is computed by a similar procedure.

\[ d_{T_i} = -\rho \sum_{i=1}^{2N_1} \Sigma_{i} u_{i} - \rho V_{\infty} \sum_{i=1}^{2N_1} \left( \frac{V_{x_i}}{V_{\infty}} \right) \Sigma_{i} \]  

(23)

where \( i \) is summed over the section for both the quarter and three-quarter chord stations of each subpanel. \( \Sigma_{i} \) and \( u_{i} \) are the source strength and total velocity in the free stream direction, respectively, at either the quarter or three-quarter chord point of the subpanel. The section induced drag coefficient

\[ C_{T_i} = \frac{C}{C_{AVG}}. \]

is then given by;

\[ \frac{C_{d_{T_i}}}{C_{AVG}} = -\frac{2AR}{b} \sum_{i=1}^{2N_1} \sqrt{1 + \tan^2 \Lambda_{i} \left( \frac{U_{i}}{V_{\infty}} \right)} - \frac{2AR}{b} \sum_{i=1}^{2N_1} \sqrt{1 + \tan^2 \Lambda_{i} \left( \frac{V_{x_i}}{V_{\infty}} \right)} \]  

(24)

For the special case where the chordal surfaces of the panels are planar and parallel to each other the integral of the section induced drag

\[ \frac{C_{d_{L_i}}}{C_{AVG}}. \]

over the span can be shown to be identical with the value of induced drag as computed in the far field, provided all of the lifting elements (bound vortices) are parallel and the lateral widths of the horseshoe vortices are equal for the complete system.
The total drag of a wing as computed in the near field is given by:

\[ C_{D_i} = -\frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{w}{V_{\infty}} \right)^2 \left( \frac{r}{V_{\infty}} \right) \Delta \eta_j \]  

where

\[
\left( \frac{w}{V_{\infty}} \right)_{jK} = \left( \frac{\Gamma}{V_{\infty}} \right)_K \frac{1}{4\pi} \left\{ \frac{\beta^2 Y_{jK} + X_{jK}T + (T^2 + \beta^2) Y_{\nu}}{(X_{jK} - TY_{jK})\sqrt{(X_{jK} + TY_{\nu})^2 + \beta^2 (Y_{jK} + Y_{\nu})^2}} \right. 
\]

\[ \left. \left[ 1 + \frac{X_{jK} + TY_{\nu}}{\sqrt{(X_{jK} + TY_{\nu})^2 + \beta^2 (Y_{jK} + Y_{\nu})^2}} \right] \right\} \]  

and

\[ X_{jK} = X_j - X_K \]

\[ Y_{jK} = (Y_j - Y_K) \]
$$(x_K, y_K)$$ is the influencing point

$$(x_j, y_j)$$ is the point being influenced

$$y_j$$ is half of the spanwise lattice spacing

$$T$$ is the tangent of the vortex line sweep

$$
\alpha^2 \text{ is } 1 - \alpha^2
$$

Equation (26) can be divided into two parts; that due to the near field stagger of the lifting element and that due to the limit of integration at infinity.

Therefore,

$$
\left( \frac{W}{V_\infty} \right)_{jk} = \frac{( \Gamma_\infty )}{4\pi} \left[ E_{jk} + E_{\infty} \right] \tag{27}
$$

The contribution to equation (26) from the near field limits of integration or lifting element stagger is given by $E_{Sjk}$.

$$
E_{Sjk} = \left\{ \begin{array}{c}
\frac{\beta^2 y_{jk} + T x_{jk} + (T^2 + \beta^2) y_\nu}{(x_{jk} - T y_{jk}) \sqrt{(x_{jk} + T y_\nu)^2 + \beta^2 (y_{jk} + y_\nu)^2}} - \frac{\beta^2 y_{jk} + T x_{jk} + (T^2 + \beta^2) y_\nu}{(x_{jk} - T y_{jk}) \sqrt{(x_{jk} + T y_\nu)^2 + \beta^2 (y_{jk} + y_\nu)^2}} \\
\frac{x_{jk} + T y_\nu}{(y_{jk} + y_\nu) \sqrt{(x_{jk} + T y_\nu)^2 + \beta^2 (y_{jk} + y_\nu)^2}} - \frac{x_{jk} + T y_\nu}{(y_{jk} - y_\nu) \sqrt{(x_{jk} + T y_\nu)^2 + \beta^2 (y_{jk} - y_\nu)^2}}
\end{array} \right. \tag{28}
$$
That due to the limits of integration at infinity is given by $E_{\infty jK}$

$$E_{\infty jK} = \left\{ \frac{1}{Y_{jK} + \gamma_{\nu}} - \frac{1}{Y_{jK} - \gamma_{\nu}} \right\}$$

(29)

The contribution to the total drag $C_{D_i}$, given by equation (25), from $E_{SjK}$ is seen to be exactly zero for all planar wings and loadings provided both $T$ and $\gamma_{\nu}$ are constant everywhere on the wing. This is due to the fact that there is no contribution to the drag from $E_{SjK}$ when $X_{jj} = Y_{jj} = 0$ or when $X_{KK} = Y_{KK} = 0$ due to taking the Cauchy principal value. Also, when $j \neq K$ the drag from $E_{SjK}$ is zero because the mutual interference drag due to the stagger is zero. This is seen by interchanging the influencing point and the point being influenced and observing that $E_{SjK} = -E_{SKj}$.

Therefore;

$$C_{D_i} = \frac{AR}{b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left[ \frac{(\Gamma_{jK})}{4\pi} E_{\infty jK} \right] \left( \frac{1}{V_{\infty jK}} \right) \Delta \eta_j$$

(30)

$$C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_{\infty jK}} \right) \left( \frac{\Gamma}{V_{\infty j}} \right) \left( \frac{1}{Y_{jK} + \gamma_{\nu}} - \frac{1}{Y_{jK} - \gamma_{\nu}} \right) \Delta \eta_j$$

(31)

This equation is identical to that obtained from the standard Trefftz plane analysis. From reference (47);

$$D_i = -\frac{\rho V_{\infty}^2}{2} \int \int_{S_{WAKE}} \phi \frac{\partial \phi}{\partial N} \, dS$$

(32)

where $\phi$ is the velocity potential in the far field.
In the case of a planar wing the vorticity trace in the Trefftz plane can be replaced by a slit and equation (32) replaced by:

\[ D_1 = -\frac{\rho V^2}{2} \int_{-b/2}^{b/2} \Delta \phi(Y) \frac{\partial \phi}{\partial N}(Y) \, dY \]  

(33)

where

\[ \Delta \phi(Y) = \frac{K}{V_\infty}(Y) \]  

(34)

\[ \frac{\partial \phi}{\partial N}(Y) = \frac{1}{2\pi} \int_{-b/2}^{b/2} \frac{dK}{dy_1 V_\infty}(Y_1) \frac{dY_1}{(Y - Y_1)} \]  

(35)

and \( K/V_\infty(Y) \) is the total circulation at a given lateral station. Therefore,

\[ \frac{K}{V_\infty} = \sum_{i=1}^{N_1} \left( \frac{1}{V_\infty} \right)_i \]  

(36)

and \( N_1 \) is the number of vortices per chord. Also;

\[ \frac{\partial \phi}{\partial Y}(Y) = \frac{1}{2\pi} \sum_{n=1}^{N_N} \left( \frac{K}{V_\infty} \right) \frac{1}{Y - Y_n + \gamma_\nu} - \frac{1}{Y - Y_n - \gamma_\nu} \]  

(37)

where \( N_n \) is the number of vortices per span.
After substituting equations (34), (35), (36), and (37) into equation (33) and letting \( N = N_1 \times N_n \):

\[
D_i = \frac{\rho V_\infty^2}{4\pi} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_\infty} \right)_K \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu} \right) \Delta Y_j
\]

(38)

Since \( \Delta Y_j = \frac{b}{2} \Delta \eta_j \),

\[
C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{K=1}^{N} \left( \frac{\Gamma}{V_\infty} \right)_K \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{1}{Y_{jK} + y_\nu} - \frac{1}{Y_{jK} - y_\nu} \right) \Delta \eta_j
\]

(39)

which is the same as equation (31).

The far field calculation of induced drag for a complete configuration composed of lifting bodies and thick lifting panels is done by representing the wake, from all of the bodies and panels, by an equivalent horseshoe vortex system where the bound segment of the horseshoe vortex is tangent to the trace of the wake in the Trefftz plane and the trailing legs are in the free stream direction. The section drag associated with the equivalent system, which in general is not equal to the actual configuration section induced drag is given by;

\[
\overline{d}_j = \rho \overline{W}_j \times \overline{\Gamma}_j \Delta S_j
\]

(40)

where

\[
\overline{W}_j = V_j \hat{i} + W_j \hat{k}
\]

(41)

and

\[
\overline{\Gamma}_j = \Gamma_j (T_{Y_j} \hat{i} + T_{Z_j} \hat{k})
\]

(42)

where \( T_{Y_j} \) and \( T_{Z_j} \) are components of the unit vector tangent to the trace of the wake at the \( j \)th section.
\[ V_j = \sum_{k=1}^{N} \frac{k_k}{4\pi} \left( E_{Vj_k} T_{Y_k} + E_{Wj_k} N_{V_k} \right) \]  

\[ W_j = \sum_{k=1}^{N} \frac{k_k}{4\pi} \left( E_{Vj_k} T_{Z_k} + E_{Wj_k} N_{Y_k} \right) \]

where \( T_{Y_k} \) and \( T_{Z_k} \) are the components of the unit vector tangent to the trace of the wake at \( k \)th station. Also, \( N_{Y_k} \) and \( N_{Z_k} \) are components of the unit vector normal to the trace of the wake at the \( k \)th section. \( N \) is the total number of sections along the trace of the wake for the complete configuration.

\[ E_{Vj_k} = \frac{\bar{Z}_{jk}}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} - \frac{\bar{Z}_{jk}}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} \]

\[ E_{Wj_k} = \frac{\bar{Y}_{jk} - \frac{1}{2} \Delta S_k}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} - \frac{1}{2} \Delta S_k)^2} - \frac{\bar{Y}_{jk} + \frac{1}{2} \Delta S_k}{\bar{Z}_{jk}^2 + (\bar{Y}_{jk} + \frac{1}{2} \Delta S_k)^2} \]

where

\[ \bar{Y}_{jk} = (Y_k - Y_j) N_{Z_k} - (Z_k - Z_j) N_{Y_k} \]

\[ \bar{Z}_{jk} = - (Y_k - Y_j) T_{Z_k} + (Z_k - Z_j) T_{Y_k} \]

and the indices \( j \) and \( k \) refer to the section being influenced and the influencing section, respectively.
therefore;

\[ d_j = \rho (V_j \Gamma_j T_{z_j} - W_j \Gamma_j T_{y_j}) \Delta S_j \]  

(49)

the total configuration induced drag is then given by;

\[ D_i = \frac{\rho}{4\pi} \sum_{j=1}^{N} \sum_{k=1}^{N} \Gamma_j \Gamma_k \left( (E_{ij} T_{y_i} + E_{kj} N_{y_k}) T_{z_j} \right) \]  

\[ - \left( E_{ij} T_{z_i} + E_{kj} N_{z_k} \right) T_{y_j} \Delta S_j \]  

(50)

therefore;

\[ C_{D_i} = \frac{AR}{2mb^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{\Gamma}{V_\infty} \right)_k \left( (E_{ij} T_{y_i} + E_{kj} N_{y_k}) T_{z_j} \right) \]  

\[ - \left( E_{ij} T_{z_i} + E_{kj} N_{z_k} \right) T_{y_j} \Delta S_j \]  

(51)

NOTE: For the planar wing equation (51) reduces to

\[ C_{D_i} = \frac{AR}{2mb^2} \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{\Gamma}{V_\infty} \right)_k E_{Wj} \Delta S_j \]  

(52)

\[ C_{D_i} = \frac{AR}{4\pi b} \sum_{j=1}^{N} \sum_{k=1}^{N} \left( \frac{\Gamma}{V_\infty} \right)_j \left( \frac{\Gamma}{V_\infty} \right)_k \left( \frac{1}{Y_{jk} + y_\nu} - \frac{1}{Y_{jk} - y_\nu} \right) \Delta \eta_j \]  

(53)

which is the same as equations (31) and (30). This demonstrates that the same induced drag is obtained whether the equality of work and kinetic-energy increment, the equivalent far field horseshoe system, or the near field horseshoe vortices with the Kutta-Joukowsky theorem is used.
SUBROUTINE ATTACH(XV,YV,ZV,NR,KODE)
C FOR KODE = -1, THIS SUBROUTINE IS SETTING XATT FROM NOXY INPUTS.
C FOR KODE = 0, THIS SUBROUTINE IS SETTING XATT FROM PANEL INPUTS.
C FOR KODE = 1, THIS SUBROUTINE SETS XV,YV,ZV FROM XATT ARRAY.
DIMENSION XV(NR+1), YV(NR+1), ZV(NR+1)
COMMON DA(5600)
1 *NX,NXTH,LNVOR,LTVOR,NTVV,RVVV,MTV,MTVTH,RTV,RTVTH(49)
2 *LNDIV,LTDIV,LNPTS,LTPTS
COMMON/PANATT/ NATT(33),XATT(200)
COMMON/NUMBER/ MPPTS(7),NCPPTS(7),NLN(7),XLT(7),XTC(7),XNC(7)
1 *NCT,NB,NBODS,NPANS,NVL(7),XVT(7),XTAPE,XTAPE,XTV,XTAPE
2 *LSEG(7),TSEG(7),LGNC(7),TEDC(7)
3 *LNDIVB(7),LTDIVB(7),NSPP(7),XOUTP(7),XXTAPP(7),SXY**7(7)
COMMON/PANEL/ IVAN,PAN,IPSY,NINC,KBVPP,NTVPP,NLCPP,LXCFP,LXCFP,LXCFP
1 *NPERPT,NSPACE,NATTCH,NTRATT,MPRC,MPRL,MRCL,MRCL,TXMCTC,MTXC
3 *IP8,IP9,IP10
EQUIVALENCE (DA(12),PANS)
DATA L/0/,L1/0/,L2/0/
IF (KODE.EQ.123) RETURN
NPANS=PANS
NB1=NBV+1
IF (KODE) 1,2,200
1 NSA=NB
GO TO 3
2 NSA=NPAN+NBODS
3 CONTINUE
DO 10 I=1,NPANS
11=(I-1)*3+2
IF (NATT(I1).EQ.NSA) GO TO 5
GO TO 10
5 I2=NATT(I1+1)
GO TO 70
10 L=L+1
XATT(L)=-NATT(I1)
XATT(L+1)=I2
L=L+2
XATT(L)=NB1
NA=(I2-1)*LTDIV+1
IF(KODE.EQ.0) GO TO 50
DO 20 K=1,NBL
L=L+1
XATT(L)=XV(K,NA)
XATT(L+1)=YV(K,NA)
L=L+2
20 XATT(L)=ZV(K,NA)
GO TO 65
50 NA=NATT(I1)
DO 60 K=1,NBL
L=L+1
XATT(L)=XV(NA+K)
XATT(L+1)=YV(NA+K)
L=L+2
60 XATT(L)=ZV(NA+K)
65 L1= NATT(I1)
L2= 12
GO TO 100
70 IF(L1.EQ.NATT(I1).AND.L2.EQ.12) GO TO 100
GO TO 10
100 CONTINUE
WRITE(6,101)(XATT(I),I=1,L)
101 FORMAT(2TH0IN SUB. ATTACH, XATT ARRAY/(1P8E13.4))
RETURN
C HERE, SUB. ATTACH FINDS WHICH SET OF X, Y, Z ARE FOR THE ATTACH LINE.
200 AN1=NATTCH
AN2=NTRATT
DO 300 I=1,200
IF(XATT(I).EQ.-AN1.AND.XATT(I+1).EQ.AN2) GO TO 201
GO TO 300
201 J=I+2
NB1=XATT(J)+0.01
DO 205 K=1,NB1
J=J+1
XV(K+1)=XATT(J)
YV(K+1)=XATT(J+1)
J=J+2
205 ZV(K+1)=XATT(J)
MP3=NB1
WRITE(5, 206) (X! (K+1), Y! (K+1), Z! (K+1), C! (K+1), (K=1, 121))

206 FORMAT(437DIN SUB. ATTACH. AT Y/TAT. FOR ATTACH. LINE/*13.4))
300 CONTINUE
WRITE(5, 305) WPAN
305 FORMAT(35HCH0 ATTACHMENT LINE FOUND FOR PANEL 12H13H IN SUB ATTACH.)
RETURN
END
SUBROUTINE DECRD (DATA, IUNIT)
DIMENSION DATA(1), ADATA(5), IDATA(17), IIDATA(8)
DATA IBLANK / 10H /
15 READ (UNIT, 16) IUNIT
16 FORMAT (8A10)
C
IF (EOF (UNIT) .EQ. 0) GOTO 19
IF (EOF (UNIT) .GT. 199) 199, 19
199 CONTINUE
IUNIT = -IUNIT
RETURN
19 DECODE (72, 17, IUNIT) IADD, ADATA
17 FORMAT (112, 5G12.0)
DECODE (80, 18, IUNIT) IUNIT
18 FORMAT (12X, 17A4)
J = IADD
IF (IADD) 22, 40, 24
22 J = -J
24 DO 30 I = 1, 5
      L = 3 * I
      K = L - 2
      DO 26 M = K, L
      IF (IDATA (M) .LT. IBLANK) 28, 26, 28
26 CONTINUE
28 DATA (J) = ADATA (I)
30 J = J + 1
IF (IADD) 100, 40, 15
40 WRITE (6, 50) IADD, IDATA
50 FORMAT (17HODECRD, ER, CARD=(*112, 17A4, 2H)), CALL EXIT
100 RETURN
END
SUBROUTINE CDIX(XI,YI,NI,T,ANS,NA)

A CONTROLLED DEVIATION INTERPOLATION METHOD

DIMENSION XI(1),YI(1),T(1),ANS(1)

XK=1.0
N=NI
DO 910 IE=1,NA
  X=T(IE)
100 IF(N-2)+110,120,200
110 Y=YI(N)
  GO TO 900
120 Y=(YI(2)-YI(1))/(XI(2)-XI(1))* (X-XI(1))+YI(1)
  GO TO 900
200 J=1
210 IF(XI(J)-X)230,220,250
220 Y=YI(J)
  GO TO 900
230 J=J+1
  IF(J-N)210,210,250
250 IF(J-2)120,155,260
155 J=3
  JJ=1
  GO TO 285
260 IF(J-N)280,265,270
265 J=N-1
  JJ=2
  GO TO 285
270 Y=(YI(N)-YI(N-1))/(XI(N)-XI(N-1))* (X-XI(N-1))+YI(N-1)
  GO TO 900
280 JJ=3
285 IF(N-3)290,290,295
290 J=3
295 K=J-1
  M=K-1
  L=J+1
  A1=X-XI(M)
A2 = X-XI(K)
A3 = X-XI(J)
AL = (X-XI(K))/(XI(J)-XI(K))
S = AL*YI(J)+(1.0-AL)*YI(K)
C1 = A3*A2/((XI(M)-XI(K))*(XI(M)-XI(J)))
C2 = A1*A3/((XI(K)-XI(M))*(XI(K)-XI(J)))
C3 = A2*A1/((XI(J)-XI(M))*(XI(J)-XI(K)))
P1 = C1*YI(M)+C2*YI(K)+C3*YI(J)

IF(N=3)305*305*310
305 P2 = P1
GO TO 315
310 A4 = X-XI(L)
C4 = A4*A3/((XI(K)-XI(J))*(XI(K)-XI(L)))
C5 = A2*A4/((XI(J)-XI(K))*(XI(J)-XI(L)))
C6 = A3*A2/((XI(L)-XI(K))*(XI(L)-XI(J)))
P2 = C4*YI(K)+C5*YI(J)+C6*YI(L)
315 GO TO (320*330*350)*JJ
320 P2 = P1
AL = (X-XI(1))/(XI(2)-XI(1))
S = AL*YI(2)+(1.0-AL)*YI(1)
P1 = S+XK*(P2-S)
GO TO 350
330 P1 = P2
AL = (X-XI(N-1))/(XI(N)-XI(N-1))
S = AL*YI(N)+(1.0-AL)*YI(N-1)
P2 = S+XK*(P1-S)
350 E1 = ABS(P1-S)
E2 = ABS(P2-S)
IF(E1+E2)400*400*410
400 Y = S
GO TO 900
410 BT = (E1*AL)/(E1*AL+(1.0-AL)*E2)
Y = BT*P2+(1.0-BT)*P1
900 ANS(IE)=Y
910 CONTINUE
RETURN
END
FUNCTION COSD(X)
Y=0.017453293*X
COSD=COS(Y)
RETURN
END
SUBROUTINE DATAWR(DA)
DIMENSION DA(1)
DO 200 I=1,1000
ID=(I-1)*5
LDATA=0
DO 100 J=1,5
IJ=ID+J
IF(DA(IJ)*NE.0.0) LDATA=1
100 CONTINUE
IF(LDATA.EQ.0) GO TO 200
ID1=ID+1
WRITE(6,150) ID1,IJ,DA(K) K=ID1,IJ
150 FORMAT(5H0DA(14,13H) THRU DA(14,4H) =1P5E15.6)
200 CONTINUE
WRITE(6,250)
250 FORMAT(5H0ALL VALUES OF DA NOT GIVEN ABOVE ARE EQUAL TO ZERO. )
RETURN
END

SUBROUTINE NORM(X,Y,Z)
D=SQRRT(X**2 + Y**2 + Z**2)
X=X/D
Y=Y/D
Z=Z/D
RETURN
END
FUNCTION DOT(X1*Y1+Z1*X2+Y2*Z2)
DOT=X1*X2+Y1*Y2+Z1*Z2
RETURN
END

FUNCTION ARCOS(X)
ARCOS=ACOS(X)
RETURN
END

FUNCTION COTAN(X)
COTAN=COS(X)/SIN(X)
RETURN
END
FUNCTION CODIM1 (X, XI, YI, N, XK)

CALLING SEQUENCE

X INDEPENDENT VARIABLE ABSCISSA REQUESTED
XI ARRAY OF GIVEN ABSCESSAS
YI ARRAY OF GIVEN ORDINATES
N NUMBER OF GIVEN POINTS DESCRIBING THE CURVE
XK END INTERVAL INTERPOLATION CONTROL CONSTANT

XK=0 STRAIGHT LINE INTERPOLATION
XK=+1 FULL PARABOLIC INTERPOLATION
XK BETWEEN FUNCTION THAT LIES BETWEEN A STRAIGHT 0 2910
0 AND 1 LINE AND A PARABOLIC INTERPOLATION 0 2920
XK=-1 PROGRAM WILL COMPUTE END INTERPOLATION 0 2930
CONTROL CONSTANT 0 2940

DIMENSION XI(1), YI(1)

W = X
N1 = N

Determine the number of points given
IF (N1-2)100,200,300

ONE POINT GIVEN

100 IF (XI(1)-W)130,175,130
130 WRITE (6,150)
150 FORMAT (99H- ONLY ONE POINT WAS GIVEN FOR ARRAY XI IN CODIM1 THE 0 3090
1E ABSCESSA ARGUMENT IS NOT THE SAME AS THE 16H0 GIVEN ABSCISSA 0 3100
165 CALL DUMP

CODIM1 = YI(1)
GO TO 1700

TWO POINT STRAIGHT LINE COMPUTATION

200 N1 = 2
225 IF (XI(N1-1)-XI(N1))250,275,250
STRAIGHT LINE COMPUTATION

250 CODI1 = (YI(1)-YI(1+1))/XI(1)-XI(1+1)**(YI(1)-YI(1+1))/XI(1)-XI(1+1)
GO TO 1700

ERROR....ABSCISSAS ARE IDENTICAL

275 WRITE (6,225)XI(1)
285 FORMAT (19H- A STRAIGHT LINE COMPUTATION IN CODI1 WAS ATTEMPTED 0)
BUT THE TWO ABSCISSAS ARE IDENTICAL / 2840 THE ABSCISSA VALUE =
2917.8

300 GO TO 165

THERE ARE MORE THAN TWO POINTS...TEST IF THEY ARE INCREASING OR

DECREASING ALGEBRATICALLY

300 IF (XI(1)-XI(2))*Y107*325*617

XI(1) EQUALS XI(2)....ERROR

325 WRITE (6,350)XI(1)
350 FORMAT (18H- THE FIRST TWO ABSCISSAS IN ARRAY XI IN CODI1 ARE THE

1E SAME... THIS IS AN ERROR. ABSC =E17.8)

360 GO TO 165

ABSCISSA VALUES ARE INCREASING IN VALUE ALGEBRATICALLY

370 FIND NEXT LARGEST ABSCISSA AFTER J

400 DO 450 J=1,N1
450 IF (U-XI(J))*1800*475*450

U IS GREATER THAN XI(J)

475 CON TINUE

GO TO 225

U IS EQUAL TO XI(J)

475 CHECK IF XI(J) IS LAST GIVEN ABSCISSA

475 IF (U-N1)*1800*575*575

CHECK IF XI(J) = XI(J+1).... IF SO THE FUNCTION IS NOT SINGLE VALUED

C
AND THE ORDI INATE OF XI(J) IS RETURNED AND A STATEMENT IS PRINTED

500 IF (XI(J)-XI(J+1))*175*525*575

525 WRITE (6,550)XI(J)
550 FORMAT (43H- THERE IS MORE THAN ONE ABSCISSA EQUAL TO E17.8/770)

575 CODI1 = YI(J)
GO TO 1700
C
C Absolute are decreasing in value algebraically
C Find next smallest abscissa after w
600 DO 650 J=1,N1
IF (XI(J)-W)G75,650
650 CONTINUE
C W is less than XI(N)
GO TO 225
C
C Test if w lies between XI(1) and XI(2)
800 IF ((J-2)200,825,850
C
C W lies between XI(1) and XI(2)
825 J = 3
JJ = 1
GO TO 925
C
C W occurs after XI(2).....Test if w is between XI(N-1) and XI(N)
850 IF ((J-N1)900,875,225
C
C W lies between XI(N-1) and XI(N)
875 J = N1-1
JJ = 2
GO TO 925
C
C W lies between XI(2) and XI(N-1)
900 JJ = 3
C
C Setup subscripts
925 K = J-1
M = K-1
L = J+1
XIM = XI(M)
XIK = XI(K)
XIJ = XI(J)
XIL = XI(L)
C
C Test if N=3.....If so alter M and JT so that do loops 1040 and
C 1120 TEST ONLY 3 POINTS VICE 4
IF (XI-3)970,940,970
940 IF (JJ-2)970,940,970
950 JT = 2
   GO TO 1000
960 M = 1
970 JT = J
C
C TEST IF ABSISSA VALUES ARE INCREASING OR DECREASING ALGEBRAICALLY
1000 IF (XI(1)-XI(2))1920, 325,1100
C
C TEST IF ABSISSA VALUES ARE ALL INCREASING ALGEBRAICALLY
1020 DO 1040 IB=M, JT
   IF (XI(IB)-XI(IB+1))1040, 1060, 1060
1040 CONTINUE
   GO TO 1175
1160 IB1 = IB+1
   WRITE ( 6,1070)IB, XI(IB), IB1, XI(IB1)
1070 FORMAT (' 79H ALL ABSSIAS IN COMMON ARE NOT EITHER ALL INCREASING/
1/DECREASING ALGEBRAICALLY /1040 ABSSISSA17,2H =E17.8,HARSC150
2SA17,2H = E17.8)'
   GO TO 165
C
C TEST IF ABSISSA VALUES ARE ALL DECREASING ALGEBRAICALLY
1100 DO 1120 IB=M, JT
   IF (XI(IB)-XI(IB+1))1060, 1060, 1120
1120 CONTINUE
C
1175 A1 = W-XIM
A2 = W-XIK
A3 = W-XIJ
A4 = W-XIL
AL = (W-XIK)/(XIJ-XIK)
C1 = A3*A2/((XIM-XIK)*(XIM-XIJ))
C2 = A1*A3/((XIK-XIM)*(XIK-XIJ))
C3 = A2*A1/((XIJ-XIM)*(XIJ-XIK))
C4 = A4*A3/((XIK-XIJ)*(XIK-XIL))
C5 = A2*A4/((XIJ-XIK)*(XIJ-XIL))
C6 = A3*A2/((XIL-XIK)*(XIL-XIJ))
$S = AL*YI(J) + (1 - AL)*YI(K)$
$P1 = C1*YI(K) + C2*YI(K) + C3*YI(J)$
$P2 = C4*YI(K) + C5*YI(J) + C6*YI(L)$
GO TO (1200, 1400, 1500), JJ

C

W LIES BETWEEN XI(1) AND XI(2)

1200 P2 = P1
1220 IF (XK) 1230.1220.1220
1220 XE = XK
GO TO 1260
C

COMPUTE XK

1230 SLOPE1 = ABS((YI(K) - YI(J))/(XI-K-XI)M))
SLOPE2 = ABS((YI(K) - YI(J))/(XI-K-XI))
XE = 1 - (ABS(SLOPE1 + SLOPE2)/(SLOPE1 + SLOPE2))
C

W LIES BETWEEN XI(N-1) AND XI(N)

1400 P1 = P2
1420 IF (XK) 1430.1420.1420
1420 XE = XK
GO TO 1460

1430 SLOPE1 = ABS((YI(J) - YI(L))/(XI-J-XI))
SLOPE2 = ABS((YI(J) - YI(K))/(XI-J-XI))
XE = 1 - (ABS(SLOPE1 - SLOPE2)/(SLOPE1 + SLOPE2))
C

AL = (W-XI(J))/(XI-L-XI)
S = AL*YI(L) + (1 - AL)*YI(J)
P2 = S + XE*(P2-S)

C

GO TO 1700
C
PROGRAM BODYGM
COMMON DA(5000)
1 *NX, NXTH+1NVOR+LTWOR+TVV+NBV, NTV, NXTHV, NBV, NTH(49)
2 *LNDIV+LTDIV+LNPTS+LTPTS
COMMON/NUMBER/+TEPS(7)+NCPTS(7)+NLNL(7)+NLT(7)+LTC(7)+LNC(7)
1 *NCT+NB, NBODS+NPANS+NVL(7)+NVT(7)+MTAPE+NTAPE+NTV+ITAPE+ITAPE
2 *LSEG(7)+TSEG(7)+LFUNC(7)+TFUNC(7)
3 *LNDIVB(7)+LTDIVB(7)+NSPP(7)+ROOTP(7)+OUTERP(7)+SYMM(7)
COMMON/BODY/+XV1(151,31)+YV(151,31)+ZV(151,31)
EQUIVALENCE(DA(1)+BODIES)+(DA(15)+BODYNO)
EQUIVALENCE(DA(10)+ALPHA)+(DA(11)+BETA)
COMMON /COMPRS/, BETAM
EQUIVALENCE(DA(3)+FMACH)
C**********************************************************************
C**********************************************************************
COMMON/PNATT/NATT(30)+XATT(200)
COMMON/SCRAT/ ATACH(50)
CALL NARDAP
REWIND 21
READ(21) ATACH
DO 5 I=1,10
K=(I-1)*5
J=(I-1)*3
NATT(J+1)=ATACH(K+1)
NATT(J+2)=ATACH(K+2)
5 NATT(J+3)=ATACH(K+3)
WRITE(6,205) NATT
205 FORMAT(*ONATT*(3I5))
C**********************************************************************
C**********************************************************************
DO 1 I=1,14
1 DA(I)=0.0
MTAPE=19
NTAPE=20
C UNIT 18 - VORTEX COORDINATES
REWIND 18
C UNITS 19,20 - AX,AY,AZ MATRICES (COMPUTED IN SUBROUTINE INFL)
REWIND 19
REWIND 20
C UNIT 21 - COMPLETE INFLUENCE MATRIX \( A' \) (COMPUTED IN SUB. MATA) 0.00
REWRITE 21
C UNIT 23 - PHI+THETA MATRIX 0.00
REWRITE 23
C REWRITE UNIT 12. UNIT 12 WILL HAVE DATA USED FOR FORCES 0.00
REWRITE 12
CALL DECRODA
NBODS=BODIES
ALPHA=ALPHA/57.2957951
BETA=BETA/57.2957951
BETAM = SQRT(1.0-FMACH**2)
IF(NBODS.EQ.0) GO TO 105
C NCT IS COUNTER FOR CONTROL POINTS. NCT IS INCREMENTED IN SUB. VPTS 0.00
NCT=0
NCTV=0
KCON=0
KL=0
KLT=0
DO 100 I=1,NBODS
DO 10 J=15,3419
10 DA(J)=0.0
CALL DECRODA
NB=BODYNO
CALL SETDAT
CALL XYZVB
CALL MATH(KL)
CALL MATHL(KLT)
CALL VT5
CALL ATTACH(XV1,YV,ZV,151,-1)
NSEG=TSEG(I)+LSEG(I)
KCON=KCON+NSEG
IF(NSEG.EQ.0) GO TO 100
CALL MATPT
100 CONTINUE
C VORTEX POINTS (INCLUDING DIVISION PTS.) ARE ON UNIT 18. (ALL BODIES) 0.00
C UNIT 23 HAS PHI+THETA MATRIX FOR ALL BODIES 0.00
C ALNGTH HAS L FOR ALL BODIES
SUBROUTINE MATP

COMPUTE PHI*THETA CONSTRAINT TRANSFORMATION MATRIX.

COMMON DA(5000)
1  *NX,MXTH,LVOR,LTVOR,NTV,NBVV,NTV,NXTHV,NBV,NTH(49)
2  *LNDIV,LTDIV,LNPTS,LTPTS
EQUIVALENCE (DA(17),CHORD)
1  *(DA(3140),CSLN),(DA(3190),CSLT),(DA(3210),CFLN),(DA(3250),CFLT)
1  *(DA(41),XS),(DA(34),FUNCLN),(DA(35),FUNCL),(DA(36),SEGLN)
1  *(DA(37),SEGLT),(DA(27),XBG)
COMMON/N:NUMBER/ NVPTS(7),NCPTS(7),NLN(7),NLT(7),LTC(7),LNC(7)
1  *NCT,NCTB,NOBS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,NTAPE,NTAPE
1  *LSEG(7),LSEG(7),LFUNC(7),TFUNC(7)
3  *LNDIV(7),LTDIV(7),NSPP(7),ROOTP(7),OUTERP(7),SYMM(7)
COMMON/SCRAT/ XVV(200),THVV(151,31)
1  *XVOR(100)
DIMENSION CFLN(40),CFLT(40),CSLN(50),CSLT(20),ANORM(3),XS(49)
DIMENSION PHTH(5000)
COMMON/BODY/ ARRAY(20000)
EQUIVALENCE (ARRAY(15001),PHTH)
C DIVLDA IS NO. LONG. DIV. GIVEN IN DATA.
EQUIVALENCE (DIVLDA,DA(321))
1  *LNDFC,FUNCLN
1  *LTFUNC,FUNCLT
1  *LNSEG,SEGLN
1  *LTSEG,SEGLT
1  *P1=3.1415926

C DETERMINE IF Y-Z AND NORMALS ARE REQUIRED FOR CONSTRAINT FUNCTIONS.
C SET KODE=1 IF THEY ARE REQUIRED.

KODE=0
1  DO 1 I=1,2
1  IF(CFLN(I).LE.2.0) GO TO 1
1  KODE=1
1  GO TO 3
1  CONTINUE
1  DO 2 I=1,3
1  IF(CFLT(I).LE.2.0.AND.CFLT(I).NE.3.0) GO TO 2
1  KODE=1
1  GO TO 3
CONTINUE
CONTINUE
IF(LTFUNC) 10•10•11
JTLIM=NTVV
GO TO 12
JTLIM=LTFUNC
IF(LNFUNC)13•13•14
JNLIM=NBVV
GO TO 15
JNLIM=LNFUNC
CONTINUE
KFUNC=0
IF(LNFUNC•NE•0) GO TO 25
IF(LTFUNC•EQ•0) GO TO 25
KFUNC=1
JTS=JTLIM
JTLIM=JNLIM
JNLIM=JTS
CONTINUE
ITAPE=0
DO 5000 JTT=1•JTLIM
JT=JTT
IF(KFUNC•NE•1) GO TO 26
JN=JTT
DO 5000 JNN=1•JNLIM
ITAPE=ITAPE+1
IF(KFUNC•EQ•0) GO TO 261
JT=JNN
GO TO 27
JN=JNN
IF(LTFUNC•EQ•0) GO TO 28
LTCF=CFLT(JT)
IF(LNFUNC•EQ•0) GO TO 29
LNCF=CFLN(JN)
CONTINUE
KC=0
DO 1000 J=1•NTVV
DO 1000 I=1•NBVV
KC=KC+1
IF(LNCF.EQ.1) GO TO 75
IF(LNCF.EQ.0) GO TO 75
DO 50 II=1:LNSEG
LNCS=CSLN(II)
IF(II.LE.LNCS) GO TO 55
50 CONTINUE
55 IF(II.NE.1) GO TO 60
X0=XS(1)-XBO
58 PH10=ARCOS(1.0-2.0*X0/CHORD)
GO TO 65
60 III=CSLN(II-1)
IX1=III
IX2=1X1+1
X0=0.75*VXOR(IX2)-0.25*VXOR(IX1)
X0=X0-XBO
GO TO 58
65 III=CSLN(II)
IF(III.LT.NBVV) GO TO 66
XF=CHORD
GO TO 68
66 IX1=III
IX2=1X1+1
XF=0.75*VXOR(IX2)-0.25*VXOR(IX1)
XF=XF-XBO
IF(XF.GT.CHORD) XF=CHORD
68 PHIF=ARCOS(1.0-2.0*XF/CHORD)
X=VXOR(1)
X=X-XBO
PHI=ARCOS(1.0-2.0*X/CHORD)
PRAT=PHI-PH10/(PHIF-PH10)
75 IF(LTCF.EQ.1) GO TO 85
IF(LTCF.EQ.0) GO TO 85
DO 80 JJ=1:LTSEG
LTCS=CSLT(JJ)
IF(J.LE.LTCS) GO TO 81
80 CONTINUE
81 JX1=(J-1)*LTDIV+1
JX2=J*LTDIV+1
II=(I-1)*DIVLDA+1
THETA=0.5*(THVV(I1,JX1)+THVV(I1,JX2))
IF(JJ*NE.*1) GO TO 82
THETO=THVV(I1,J1)
GO TO 83
82 JX1=(CSLT(JJ-1))*LTDIV+1*01
THETO=THVV(I1,JX1)
83 JX2=LTCI*LTDIV+1
THETO=THVV(I1,JX2)
TRAT=P1*(THETA-THETO)/(THETO-THETO)
85 CONTINUE
IF(KODE.EQ.0) GO TO 100
CALL VYUZ(N,J1,Y1,ANORM)
100 CONTINUE
LATF=1
LONF=1
C IF LATF=0, DO NOT COMPUTE GT.
C IF LONF=0, DO NOT COMPUTE GN.
IF(LT_FUNC.NE.0) GO TO 105
LATF=0
IF(JT.NE.J) GO TO 30
GT=1.0
GO TO 105
30 GT=0.0
LONF=0
105 CONTINUE
IF(LT_FUNC.NE.0) GO TO 106
LONF=0
IF(JN.NE.I) GO TO 251
GN=1.0
GO TO 106
251 GN=0.0
LATF=0
106 CONTINUE
IF(LATF.EQ.0) GO TO 300
GO TO ((110*120*130*140*150*160*170*180*190*200)*LTCF
110 GT=1.0
GO TO 300
120 GT=ANORM(3)/SQR(T(ANORM(3)**2+ANORM(2)**2))
GO TO 300
GO TO 500
403 GN=SIN(3.0*PRAT)  
GO TO 500
404 GN=SIN(4.0*PRAT)  
GO TO 500
405 GN=SIN(5.0*PRAT)  
GO TO 500
406 GN=SIN(6.0*PRAT)  
GO TO 500
407 GN=SIN(7.0*PRAT)  
GO TO 500
408 GN=SIN(8.0*PRAT)  
GO TO 500
409 GN=SIN(9.0*PRAT)  
GO TO 500
410 GN=SIN(10.0*PRAT)  
GO TO 500
411 GN=SIN(11.0*PRAT)  
GO TO 500
412 GN=SIN(12.0*PRAT)  
GO TO 500
413 GN=SIN(13.0*PRAT)  
GO TO 500
414 GN=SIN(14.0*PRAT)  
GO TO 500
415 GN=SIN(15.0*PRAT)  
500 CONTINUE
1000 PTHH(KC)=GN*GT
5000 WRITE(23)(PTHH(K1)*K1=1*KC)
RETURN
END
SUBROUTINE VTPS
COMMON DA(5000)
1 *NX*NXTH*LNVOR*LTVOR*NTV*NBV*NTV*MAXTHV*NBV*MTH(69)
2 *LNDV,LTV:LNTPS,LTPTS
COMMON/NUMBER/ NVRTS(7),NLN(7),NL(7),LTS(7),LNC(7),
1 *NLN:NLNOPS,NSNPS,NVL(7),NVT(7),NTAPE,NTAPE,NVT,NTAPE,NTAPE
2 *LSEG(7),TSEG(7),LFUNC(7),TFUNC(7),
3 *LNDV(LTV:LNT:LTPT:LNT:1:NRTP(L),OUTERPT(7),SYMM(7),
DIMENSION XAREA(5000),YAREA(5000),ZAREA(5000)
1 * XP(5000), YPC(5000), ZPC(5000), XTL(100)
2 EQUIVALENCE: XAREA*XP*C(15001), YAREA*YPC*D(20001)
1 * ZAREA*ZPC*D(25001), XTL*D(14100)
2 *CHORD*DA(17), BREF*DA(16)
3 *DA(7)*XCG=(DA(7)*YCG=(DA(7)*ZCG)=(DA(10)*ALPHA)
COMMON/BODY, XV(15), YV(151,31), ZV(151,31)
2 *TMX(1320), TMY(1320), TYZ(1320)
3 *TXX(1320), TTY(1320), TTZ(1320)
4 *DUMB(5000)
COMMON/CONPTS/ XQ(1320), YQ(1320), ZQ(1320)
1 *XN(1320), YN(1320), ZN(1320)
DIMENSION D(31000)
2 EQUIVALENCE (D*XV(1,1))
3 EQUIVALENCE (DA(2800),FLNC*(DA(3100)*FTLC)
DIMENSION FLNC(150),FTLC(40)
2 COMMON/SCAT/ XVY(200),DUMIVV(4681), XVOR(100), XCON(100)
3 DIMENSION T1(1000), T2(1000), T3(1000)
1 *T1(300), T2(300), T3(300)
2 EQUIVALENCE (XN(1), T1), (XN(301), T2), (XN(601), T3)
3 * (YN(1), T1), (YN(301), T2), (YN(601), T3)
4 EQUIVALENCE (BODYWR(1), XV(1,1))
DIMENSION BODYWR(31000)
2 31000 WORDS OF /BODY/ WILL BE WRITTEN ON UNIT 13.
3 IF IFLNC(1), NE=0.0 GO TO 2
DO 1 I=1, LNPTS
1 FLNC(I)=1
2 IF (FLTC(I), NE=0.0) GO TO 4
DO 3 J=1, LTPTS
3 FLTC(J)=J
1 3 3 4 3 4
CONTINUE
NBI=NBVV+1
SUBAX=0.0
SUBAY=0.0
SUBAZ=0.0
K=0
DO 50 J=1,NTV
J1=J+1
DO 50 I=1,NBV
I1=I+1
K=K+1
X31=XV (I1,J1)-XV (I,J)
Y31=YV(I1,J1)-YV(I,J)
Z31=ZV(I1,J1)-ZV(I,J)
X24=XV (I,J1)-XV (I1,J)
Y24=YV (I,J1)-YV (I1,J)
Z24=ZV (I,J1)-ZV (I1,J)
XAREA(K) = (Y31*Z24-Z31*Y24)*0.5
YAREA(K) = (Z31*X24-X31*Z24)*0.5
ZAREA(K) = (X31*Y24-Y31*X24)*0.5
SUBAX=SUBAX+XAREA(K)
SUBAY=SUBAY+YAREA(K)
SUBAZ=SUBAZ+ZAREA(K)
WRITE(6,55) SUBAX,SBAY,SBAZ
55 FORMAT(1HO/(1P8E15.6))
DO 100 I=1,NBV
K1=(I-1)*LNDIV+1
K2=K1+LNDIV
100 XVOR(I) = 0.75*XVV(K1)+0.25*XVV(K2)
WRITE(6,55) (XVOR(I),I=1,NBV)
N3=NBV+1
N2=NTV+1
DO 300 I=1,LNPTS
KC=FLNC(I)
KC=(KC-1)*LNDIV+1
300 XCON(I)= 0.25*XVV(KC)+0.75*XVV(KC+LNDIV)
C WRITE AREAS ON UNIT 12 FOR USE IN CALCULATING FORCES.
C
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>3850</td>
<td>WRITE(12) XAREA*YAREA*ZAREA</td>
</tr>
<tr>
<td>3860</td>
<td>K=0</td>
</tr>
<tr>
<td>3870</td>
<td>DO 60 J=1*NTV</td>
</tr>
<tr>
<td>3880</td>
<td>J1=J+1</td>
</tr>
<tr>
<td>3890</td>
<td>DO 60 I=1*NBV</td>
</tr>
<tr>
<td>3900</td>
<td>I1=I+1</td>
</tr>
<tr>
<td>3910</td>
<td>K=K+1</td>
</tr>
<tr>
<td>3920</td>
<td>XPC(K)=0.25*(XV(I,J)) + XV(I,J) + XV(I,J1) + XV(I1,J1))</td>
</tr>
<tr>
<td>3930</td>
<td>YPC(K)=0.25*(YV(I,J)) + YV(I,J) + YV(I,J1) + YV(I1,J1))</td>
</tr>
<tr>
<td>3940</td>
<td>DO 60 K=1*NBV</td>
</tr>
<tr>
<td>3950</td>
<td>K1=(K-1)*LNDIV+1</td>
</tr>
<tr>
<td>3960</td>
<td>XTL(K)=XV(I,J1)</td>
</tr>
<tr>
<td>3970</td>
<td>*LNDIV<em>LTDIV</em>NBV<em>NTV</em>CHORD<em>XCG</em>YCG<em>ZCG</em>ALPHA*BREF</td>
</tr>
<tr>
<td>3980</td>
<td>WRITE(6*55)(XCON(I),I=1*LNPTS)</td>
</tr>
<tr>
<td>3990</td>
<td>NLAT1=LTDIV/2 + 1</td>
</tr>
<tr>
<td>4000</td>
<td>JS=NCT</td>
</tr>
<tr>
<td>4010</td>
<td>DO 500 J=1*LPTPS</td>
</tr>
<tr>
<td>4020</td>
<td>K1=LTDIV*(FLTC(J)-1.0)+NLAT1</td>
</tr>
<tr>
<td>4030</td>
<td>K2=K1+1</td>
</tr>
<tr>
<td>4040</td>
<td>DO 400 I=1*NBV</td>
</tr>
<tr>
<td>4050</td>
<td>I2=I+1</td>
</tr>
<tr>
<td>4060</td>
<td>XQ1(I1)=0.25*(XV(I2,K1)+XV(I1,K1)+XV(I2,K2)+XV(I1,K2))</td>
</tr>
<tr>
<td>4070</td>
<td>T11(I1)=XV(I2,K1)-XV(I1,K1)+XV(I2,K2)-XV(I1,K2)</td>
</tr>
<tr>
<td>4080</td>
<td>T21(I1)=YV(I2,K1)-YV(I1,K1)+YV(I2,K2)-YV(I1,K2)</td>
</tr>
<tr>
<td>4090</td>
<td>T31(I1)=ZV(I2,K1)-ZV(I1,K1)+ZV(I2,K2)-ZV(I1,K2)</td>
</tr>
<tr>
<td>4100</td>
<td>CALL INTER(XQ1,T11,NBV,XCON,T12,LNPTS)</td>
</tr>
<tr>
<td>4110</td>
<td>CALL INTER(XQ1,T21,NBV,XCON,T22,LNPTS)</td>
</tr>
<tr>
<td>4120</td>
<td>CALL INTER(XQ1,T31,NBV,XCON,T32,LNPTS)</td>
</tr>
<tr>
<td>4130</td>
<td>DO 410 I=1*LNPTS</td>
</tr>
<tr>
<td>4140</td>
<td>DENOM=SQR(T12(I)**2+T22(I)**2+T32(I)**2)</td>
</tr>
<tr>
<td>4150</td>
<td>T12(I)=T12(I)/DENOM</td>
</tr>
<tr>
<td>4160</td>
<td>T22(I)=T22(I)/DENOM</td>
</tr>
<tr>
<td>4170</td>
<td>T32(I)=T32(I)/DENOM</td>
</tr>
<tr>
<td>4180</td>
<td>JS1=JS+1</td>
</tr>
</tbody>
</table>
I

TMX(JSI)=T12(I)
TMY(JSI)=T22(I)
TMZ(JSI)=T32(I)
DO 450 I=1*NBV
I2=I1+1
T11(I1)=XV(I1*K2)-XV(I1*K1)+XV(I2*K2)-XV(I2*K1)
T21(I1)=YV(I1*K2)-YV(I1*K1)+YV(I2*K2)-YV(I2*K1)
T31(I1)=ZV(I1*K2)-ZV(I1*K1)+ZV(I2*K2)-ZV(I2*K1)
CALL INTER(XQ*T11*NBV*XCON+T12+LNPTS)
CALL INTER(XQ*T21*NBV*XCON+T22+LNPTS)
CALL INTER(XQ*T31*NBV*XCON+T32+LNPTS)
DO 460 I=1*LNPTS
DENOM=SQRT(T12(I)**2+T22(I)**2+T32(I)**2)
T12(I)=T12(I)/DENOM
T22(I)=T22(I)/DENOM
T32(I)=T32(I)/DENOM
JSI=JS+I
TTX(JSI)=T12(I)
TTY(JSI)=T22(I)
TTTY(JSI)=T32(I)
JS=JS+LNPTS
500 CONTINUE
JS=NCT
DO 600 K=1*LTPTS
K1=LTID*FLTC(K)-1.0+NLAT1
K2=K1+1
CALL INTER(XV(1*K1)*YV(1*K1)*N3*XCON+T11+LNPTS)
CALL INTER(XV(1*K2)*YV(1*K2)*N3*XCON+T12+LNPTS)
CALL INTER(XV(1*K1)*ZV(1*K1)*N3*XCON+T21+LNPTS)
CALL INTER(XV(1*K2)*ZV(1*K2)*N3*XCON+T22+LNPTS)
DO 550 I=1*LNPTS
JS=JS+1
XQ(JSI)=XCON(I)
YQ(JSI)=0.5*(T11(I)+T12(I))
ZQ(JSI)=0.5*(T21(I)+T22(I))
550 CONTINUE
DO 600 J=1*N2
CALL INTER(XV(1,J)*YV(1,J)*N3*XVOR+T11*NBVV)
CALL INTER(XV(1,J)*ZV(1,J)*N3*XVGK+T12*NBVV)
DO 250 I=1,NBVV
YV(I,J)=T11(I)
ZV(I,J)=T12(I)
250 CONTINUE
DO 650 I=1,NBVV
DO 650 J=1,N2
650 XV(I,J)=XVOR(I)
DO 700 J=1,N2
XV(NB1+J)=XVV(N3)
YV(NB1+J)=YV(N3+J)
700 ZV(NB1+J)=ZV(N3+J)
DO 800 J=1,LTPTS
DO 800 I=1,LNPTS
NCT=NCT+1
XN1=TMY(NCT)*TTZ(NCT)-TMZ(NCT)*TTY(NCT)
YN1=TMZ(NCT)*TTX(NCT)-TMX(NCT)*TTZ(NCT)
ZN1=TMX(NCT)*TTY(NCT)-TMY(NCT)*TTX(NCT)
DENOM=XN1**2+YN1**2+ZN1**2
XN(NCT)=XN1/DENOM
YN(NCT)=YN1/DENOM
ZN(NCT)=ZN1/DENOM
800 CONTINUE
WRITE(18) BODWR
WRITE(6,55) (XQ(I),I=1,NCT)
WRITE(6,55) (YQ(I),I=1,NCT)
WRITE(6,55) (ZQ(I),I=1,NCT)
WRITE(6,55) (XN(I),I=1,NCT)
WRITE(6,55) (YN(I),I=1,NCT)
WRITE(6,55) (ZN(I),I=1,NCT)
C
RETURN
END
SUBROUTINE MAUL(k)

BLNGTH GIVE LATERAL LENGTHS OF PANELS ..............
FIRST LONGITUDINALLY -- THEN LATERALLY.

COMMON DA(5000)
1 *NX*NXTH*LNVOR*LTVO**NTVV*NBVV*NTV*NXTHV*NBV*NTH(49)
2 *LNDIV*LTDIV*LNPTS*LTPTS
COMM/ON/ BODY/ ARRAY (31000)
DIMENSION XV1(151*31),YV(151*31),ZV(151*31),BLNGTH(5000)
EQUIVALENCE(BLNGTH-ARRAY(15001)),(XV1-ARRAY(1))
1 
REAL L2
LN=LNDIV/2+1
DO 100 J=1,NTV
JT=(J-1)*LTDIV
DO 100 I=1,NBVV
II1=(I-1)*LNDIV+LN
II2=II1+1
L2=0.0
DO 50 JJ=1,LTDSV
JJ1=JT+JJ
JJ2=JJ1+1
X1=0.5*(XV1(II1,JJ1)+XV1(II2,JJ1))
X2=0.5*(XV1(II1,JJ2)+XV1(II2,JJ2))
DX=X2-X1
Y1=0.5*(YV1(II1,JJ1)+YV1(II2,JJ1))
Y2=0.5*(YV1(II1,JJ2)+YV1(II2,JJ2))
DY=Y2-Y1
Z1=0.5*(ZV1(II1,JJ1)+ZV1(II2,JJ1))
Z2=0.5*(ZV1(II1,JJ2)+ZV1(II2,JJ2))
DZ=Z2-Z1
50 L2*SQRT(DX**2+DY**2+DZ**2)+L2
100 BLNGTH(K)=L2
WRITE(6*200)(BLNGTH(I),I=1,K)
200 FORMAT(*OBLNGTH/*1(P8E15.6)
WRITE(18) BLNGTH
RETURN
344

1 4950
1 4960
1 4970
1 4980
1 4990
1 5000
1 5010
1 5020
1 5030
1 5040
1 5050
1 5060
1 5070
1 5080
1 5090
1 5100
1 5110
1 5120
1 5130
1 5140
1 5150
1 5160
1 5170
1 5180
1 5190
1 5200
1 5210
1 5220
1 5230
1 5240
1 5250
1 5260
1 5270
1 5280
1 5290
1 5300
1 5310
1 5320
1 5330
SUBROUTINE MATL(K)
COMMON DA(5000)
1 LNDIV=LTDIV+LNPTS*LTPTS
COMMON/NVTS(7)*NCPTS(7)*NLT(7)*LTC(7)*LNC(7)
1 NCT*NB*NBOSS*NPANS*VLT(7)*VVT(7)*TAPE*TAPE*NCT*TAPE*TAPE
2 LSEG(7)*TSEG(7)*LFUNC(7)*LFUNC(7)
3 LNDIVB(7)*LTDIVB(7)*NSPP(7)*ROOTP(7)*OUTERP(7)*SYMM(7)
COMMON/BODY/ ARRAY(31000)
DIMENSION XVL(151,31),YVL(151,31),ZVL(151,31),ALNGTH(5000)
EQUIVALENCE(ALNGTH,ARRAY(15001))
1 (YV*ARRAY(46821),(ZV*ARRAY(9363))
DO 10 J=1,NTVV
JJ1=(J-1)*LTDIV+LTDIV/2+1
JJ2=JJ1+1
DO 10 I=1,NBVV
K=K+1
VL=0.0
DO 4 II=1,LNDIV
II1=LNDIV*(I-1)+I
II2=II1+1
X1=0.5*(XVL(II1,JJ1)+XVL(II1,JJ2))
X2=0.5*(XVL(II2,JJ1)+XVL(II2,JJ2))
DX=X2-X1
Y1=0.5*(YVL(II1,JJ1)+YVL(II1,JJ2))
Y2=0.5*(YVL(II2,JJ1)+YVL(II2,JJ2))
DY=Y2-Y1
Z1=0.5*(ZVL(II1,JJ1)+ZVL(II1,JJ2))
Z2=0.5*(ZVL(II2,JJ1)+ZVL(II2,JJ2))
DZ=Z2-Z1
4 VL=SORT(DX**2+DY**2+DZ**2)+VL
10 ALNGTH(K)=VL
WRITE(6,200)(ALNGTH(I),I=1,K)
200 FORMAT(*ALNGTH*/(1P8E15.6))
WRITE(18) ALNGTH
RETURN
END
SUBROUTINE XYZVB

COMMON DA(5000)
  1 *NX*NXTH*LNVR*LTVOR*NTVV*NBV*NTV*NXTHV*NBV*NTH(49)
  1 5720 5730 5740
 COMMON/NNUMBER/ JVPTS(7)*NCPTS(7)*NLN(7)*NLH(7)*LTH(7)*LTH(7)
  1 5750
  1 5760
  1 5770
  1 5780
  1 5790
  1 5800
  1 5810
  1 5820
  1 5830
  1 5840
  1 5850
  1 5860
  1 5870
  1 5880
  1 5890
  1 5900
  1 5910
  1 5920
  1 5930
  1 5940
  1 5950
  1 5960
  1 5970
  1 5980
  1 5990
  1 6000
  1 6010
  1 6020
  1 6030
  1 6040
  1 6050
  1 6060
  1 6070
  1 6080
  1 6090

EQUIVALENCE(DA(3),FMACH)
COMMON/COMPRS/ BETAM
EQUIVALENCE (DA(17),CHORD), (DA(33),DIVALAT)
  1 5900
  1 5910
  1 5920
  1 5930
  1 5940
  1 5950
  1 5960
  1 5970
  1 5980
  1 5990
  1 6000
  1 6010
  1 6020
  1 6030
  1 6040
  1 6050
  1 6060
  1 6070
  1 6080
  1 6090

DIMENSION THV(151,31)
  1 5920
  1 5930
  1 5940
  1 5950
  1 5960
  1 5970
  1 5980
  1 5990
  1 6000

EQUIVALENCE(D*ARRAY)
EQUIVALENCE(D(1),THV), (D(4682),THSS), (D(9363),RS)
  1 6010
  1 6020
  1 6030
  1 6040
  1 6050
  1 6060
  1 6070
  1 6080
  1 6090

DIMENSION D(25000), THETA(200), Y1(150), Z1(50)
  1 6020
  1 6030
  1 6040
  1 6050
  1 6060
  1 6070
  1 6080
  1 6090

DIMENSION D(2000), XP(200)
  1 6010
  1 6020
  1 6030
  1 6040
  1 6050
  1 6060
  1 6070
  1 6080
  1 6090
COMMON/BOY/ ARRAY(31000)
EQUIVALENCE(X1•ARRAY(1)),(YV•ARRAY(4682)),(ZV•ARRAY(9363))
1 (XBOO•ARRAY(26964)),(YBOO•ARRAY(26969))
2 (ZBOO•ARRAY(26974))
DIMENSION X1(151•31),YV(151•31),ZV(151•31),XBOO(5),YBOO(5)
1 ZBOO(5)
COMMON/CONPTS/XQ(1320),YQ(1320),ZQ(1320)
EQUIVALENCE(YY(1•1),THSS(1•1)),(ZZ(1•1),RS(1•1))
1 *(Y(1•1),YV(1•1)),(Z(1•1),ZV(1•1))

C MCPTS=CPTS
C TEST FOR (R,THETA) INPUT OR (Y,Z) INPUT
C
C IF(BODTAB•NE.0.) GO TO 110
C
C R,THETA INPUT
C
C SET UP THV ARRAY
C TEMPORARY CHANGE•••••• SET GRIDL=1.
C THIS IS USED FOR EVEN DELTA THETA VORTEX INPUT,
C AND ONLY ONE SET OF THETA'S CREATED IN SUBROUTINE SETDAT.
GRIDL=1.0
C
56 N1=NTV+1
LOC=0
DO 65 I=1,NXTHV
DO 66 J=1,N1
LOC=LOC+1
58 TH1(J)=THVS(LOC)
60 CONTINUE
IF(GRIDL•GT.0.)LOC=0
65 CALL FILLDV(TH1,THV(1•1),N1,DIVLAT)
WRITE(6,550)(TH1(J),J=1,N1)
400 FORMAT(1H1/(1P8E15.6))
500 FORMAT(*0XZVB*/(5X,1P6E15.6))
IF(BODTAB•EQ.1.0.) GO TO 150
C
C A ROW OF THV IS FOR A TRAILING VORTEX.
C
C NOW INTERPOLATE IN THE THV ARRAY TO GET THV AT XVV STATIONS.
C ALSO INTERPOLATE FOR THSS'S AT XS'S.
C
N2=NTV+1.
N3=NBV+1
WRITE(6,500)(XTHV(J),J=1,NXTHV)
WRITE(6,500) (XV(I),I=1,NXTHV)
WRITE(6,500)(XVV(I),I=1,N3)
WRITE(6,500)(XSV(I),I=1,NX)
DQ 70 I=1,N2
DO 68 J=1,NXTHV
68 TH1(J)=THV(I,J)
CALL CODIM(XV,TH1,NXTHV,XVV,THV(1*I),N3)
70 CALL CODIM(XV,TH1,NXTHV,XS,THSS(1*I),NX)
C
C INTERPOLATE ON R VS. THETA AT EACH XS STATION TO
C OBTAIN R'S AT THETA'S.
C
L=2
IREGNO=1
JSUB=1
KSUB=1
DO 90 I=1,NX
71 IF(XTH(L)-XTH(L-1),GE,0.0) GO TO 72
IF(XS(I),GT,XTH(L)) GO TO 80
GO TO 75
72 IF(XS(I),LT,XTH(L)) GO TO 80
75 JSUB=JSUB+NTH(IREGNO)+1
KSUB=KSUB+NTH(IREGNO)
IREGNO=IREGNO+1
L=L+1
IF(IREGNO,LE,NXTH) GO TO 71
WRITE(6,73)
73 FORMAT(*,IREGNO GREATER THAN NXTH*)
STOP
80 DO 85 K=1,N2
85 THETA(K)=THSS(1*K)
90 CALL CODIM(TH,JSUB)*R(KSUB)*NTH(IREGNO)*THETA*RS(1*I)*N2)
WRITE(6,4000)((RS(I,J)*J=1,NX)*I=1,N2)
4000 FORMAT(1HO/(1P10E12.4))
C
C COLUMNS OF RS ARE R'S AT THSS'S.
C EACH COLUMN CORRESPONDS TO ONE XS STATION.
C
C ROWS OF RS ARE R VS. XS FOR SUCCESSIVE TRAILING VORTICES.
C THERE ARE N2 ROWS AND NX COLUMNS IN RS. (N2 * NX)
C
C INTERPOLATE ON RS VS. XS CURVES DEVELOPED IN ABOVE STEP
C TO OBTAIN R AT XVV STATIONS.
C
DO 100 I=1,N2
DO 95 J=1,NX
95 THETA(J)=RS(I,J)
100 CALL INTER(XS, THETA,NX,XVV, RR(1), N3)
C
C ARRAYS THVV(I,J) AND RR(I,J) I=1,N3 J=1,N2
C DESCRIBE LOCATION OF VORTEX POINTS.
C ROWS CORRESPOND TO BOUND VORTEX LINES.
C COLUMNS CORRESPOND TO TRAILING VORTEX LINES.
C
GO TO 180
C
Y,Z INPUT OR DELTA-S INPUT
C
110 CONTINUE
C DEVELOP Z VS. S AND Y VS. S AT INPUT STATIONS.
KSUB=1-NTH(I)
JSUB=KSUB-1
DO 120 I=1,NXTH
KSUB=KSUB+NTH(I)
JSUB=JSUB+NTH(I)+1
N=NTH(I)
IF(SYM*LT.0.) N=N+1
Y(I+1)=TH(JSUB)
Z(I+1)=R(KSUB)
DELS=0.0
C
S(J+1)=0.0
DO 100 J=2:N
IF(J.EQ.N .AND.SYM.LT.0.) GO TO 115
K1=JSUB+J-1
K2=KSUB+J-1
Y(J+1)=TH(K1)
Z(J+1)=R(K2)
DELY=TH(K1)-TH(K1-1)
DELZ=R(K2)-R(K2-1)
DELS=SORT(DELY**2+DELZ**2)+DELS
S(J+1)=DELS
GO TO 120
115 S(J+1)=SORT((Y(J-1+1)-Y(1+1))**2+(Z(J-1+1)-Z(1+1))**2)+DELS
Y(N+1)=Y(1+1)
Z(N+1)=Z(1+1)
120 CONTINUE
IF(THVS(1).NE.0.) GO TO 56
C Y Z INPUT
C
DO 130 I=1,NXTH
N=TH(1)
CALL FILLDY(S(I+1),SLAT,N,DIVLAT)
NLAT=DIVLAT*N+1.01
CALL CODIM(S(I+1),Y(I+1),N,SLAT,YY(I+1),NLAT)
130 CALL CODIM(S(I+1),Z(I+1),N,SLAT,ZZ(I+1),NLAT)
C A COLUMN OF YY (OR ZZ) ARE VALUES OF Y (OR Z) AT INPUT XS STATIONS.
C NOW INTERPOLATE TO OBTAIN Y AND Z AT XVV S.
C
132 N3=NBV+1
N2=NTV+1
DO 140 I=1,N2
DO 135 J=1,NXTH
Y(I,J)=YY(I,J)
135 Z(I,J)=ZZ(I,J)
CALL INTER(XS,Y1,NXTH,XVV,YV(I,J),N3)
140 CALL INTER(XS,Z1,NXTH,XVV,ZV(I,J),N3)
GO TO 220
150 CONTINUE
NOW A COLUMN OF THV CONTAINS S'S AT AN INPUT X-THETA STATION.
A ROW OF THV GIVES S'S FOR A TRAILING VORTEX (AT ALL INPUT X-THETA'S)
INTERPOLATE IN THE S VS. XTH PLOTS FOR S'S AT INPUT XS STATIONS.

N2=NTV+1
DO 160 I=1,N2
DO 155 J=1,NXTH
155 THETA(J)=THV(I,J)
160 CALL INTER(XTH,THETA,NXTH,XS,ST(1,I),NX)

INTERPOLATE IN THE Y VS. S AND Z VS. S ARRAYS AT XS STATIONS
FOR Y'S AND Z'S AT ST'S FOUND ABOVE.

DO 170 I=1,NX
DO 165 J=1,N2
165 THETA(J)=ST(I,J)
170 CALL CODIM(S(1,I),Y(1,I),N,THETA,YY(1,I),N2)
170 CALL CODIM(S(1,I),Z(1,I),N,THETA,ZZ(1,I),N2)
GO TO 132
180 CONTINUE

COMPUTE YY,ZZ FROM THVV AND RR.
DO 200 I=1,N3
DO 200 J=1,N2
THSS(I,J)=RR(I,J)*SIND(THVV(I,J))
200 THSS IS THE Y ARRAY. S IS THE Z ARRAY.
THSS IS THE Y ARRAY. S IS THE Z ARRAY.
THESE WILL BE MULTIPLIED BY THE MULTI. FACTORS TO GET YV,ZV.

220 CONTINUE

PERFORM MULTIPLICATION AND REVISION OF VORTEX POINT
COORDINATES DUE TO CAMBER

INTERPOLATE FOR MULTIPLICATION FACTORS AND CAMBER AT XVV'S.
IF(MCPTS=NE.0) GO TO 223
DO 222 I=1,N3
YC(I) = 0.0
ZC(I) = 0.0
YM(I) = 1.0
ZM(I) = 1.0

222 CONTINUE
CALL INTER(XMF*YM*MCPTS*XVV*YM*N3)
IF(BMULT*EQ.1.0) GO TO 230
DO 225 I = 1, N3
225 ZM(I) = YM(I)
GO TO 235
230 CONTINUE
CALL INTER(XMF*ZM*MCPTS*XVV*ZM*N3)
235 CONTINUE
CALL INTER(XMF*YC*MCPTS*XVV*YC*N3)
CALL INTER(XMF*ZC*MCPTS*XVV*ZC*N3)
236 CONTINUE

C FOR CAMBI=0, COMPUTE ANGLES IN X-Z AND X-Y PLANES WHICH CAMBER LINES
C MAKE AT XVV STATIONS
C
IF(CAMBI*NE.0.0) GO TO 280
DX = CHORD*0.001
DO 240 I = 1, N3
XM(I) = XVV(I) + DX
XP(I) = XVV(I) - DX
240 CALL INTER(XVV*YC*N3*XM*YCM*N3)
CALL INTER(XVV*ZC*N3*XM*ZCM*N3)
CALL INTER(XVV*YC*N3*XP*YCP*N3)
CALL INTER(XVV*ZC*N3*XP*ZCP*N3)
DO 260 I = 1, N3
DY = YCP(I) - YCM(I)
DZ = ZCP(I) - ZCM(I)
YCA = ATAN(DY/(2.*DX))
ZCA = ATAN(DZ/(2.*DX))
SINYC(I) = SIN(YCA)
COSYC(I) = COS(YCA)
SINZC(I) = SIN(ZCA)
260 COSZC(I) = COS(ZCA)
GO TO 290
280   DO 285  I=1,N3
       SINVC(I)=0.0
       COSYC(I)=1.0
       SINZC(I)=0.0
285   COSZC(I)=1.0
290   DO 300  J=1,N2
       DO 300  I=1,N3
       Y2=YM(I)*THSS(I,J)
       Z2=ZM(I)*S(I,J)
       XV1(I,J)=XV(V(I,J))=Y2*SINVC(I)*COSZC(I)-Z2*SINZC(I) + XBOO(NB)
       YV1(I,J)=(YC(I)+Y2*COSVC(I) + YBOO(NB)) *BETAM
       ZV1(I,J)=(ZC(I)-Z2*COSVC(I) + ZP00(NB)) *BETAM
300   WRITE(6,301) N2,N3
       WRITE(12) YV(N3,J),ZV(N3,J),J=1,N2
       WRITE(6,7003) (J,YV(N3,J),ZV(N3,J),J=1,N2)
7003  FORMAT(*OBODYY DRAG COORDS.*/15*2F15.5)
301  FORMAT(*0XYZVB N2*N3=*2I5)
       RETURN
       END

354
SUBROUTINE SETDAT
COMMON DA(5000)
1  »NX*NXTH*LNVOR*TVQV*NTV*NXTHN*NBV*NTH(49)
2  »LNDIV*LTVL*LNPTS*LTPE
COMMON/NUMBER/ NVPTS(7)*NCPTS(7)*NLN(7)*NL(7)*LNC(7)
1  »NCT*NB*NPANS*NVL(7)*NVT(7)*NTAPE*NTA*SCTA*ITAPE*JTAPE
2  »LSEG(7)*TSEG(7)*LUNEC(7)*TUCE(7)
3  »LNDIV(7)*LTVL(7)*NSPP(7)*ROTP(7)*OUTERP(7)*SYMM(7)
COMMON/BODY/ DUMMY(26953)*XB00(5)*YBJU(5)*ZB00(5)
EQUIVALENCE(DA(40)*FNI*(DA(85)*FNTI)* (DA(30)*FLOCN)
1  »(DA(81)*FLATV)* (DA(122)*DIVLON)* (DA(33)*DIVLAT)* (DA(86)*XTHI)
2  »(DA(41)*XS)* (DA(215)* FNT(I)* (DA(23)*VORNI)* (DA(24)*VORLT)
3  »(DA(215)*XTHV)* (DA(130)*THN)* (DA(185)*XV)* (DA(17)*CHORD)
4  »(DA(38)*PTSLN)* (DA(391)*PTSLT)
5  »(DA(27)*X90)* (DA(28)*Y90)* (DA(29)*Z90)
EQUIVALENCE(DA(36)*SEGLN)* (DA(37)*SEGLT)
1  »(DA(34)*FUNCLN)* (DA(35)*FUNCLT)
DIMENSION XTH(49)*XS(49)*XTHV(149)*THN(670)*XV(150)
COMMON/SCRAT/ XVV(200)
DIMENSION NPP(48)
EQUIVALENCE(NPP(1)*NVPTS(1))
EQUIVALENCE(DA(19)*SYM)
EQUIVALENCE(DA(22000)*THVS)
DIMENSION THVS(799)
NX=FNI
NXTH=FNTI
IF(NXTH.EQ.0) NXTH=NX

LOC=1
DO 10 I=1,NXTH
THI(1)=THN(LOC)
10 LOC=LOC+NTH(I)+1
IF(DIVLON.EQ.0) DIVLON=1.0
IF(DIVLAT.EQ.0) DIVLAT=1.0
LNDIV=DIVLON
LTVL=DIVLAT
LTVL(NB)=LNDIV
LTVL(NB)=LTVL
LTPTS=PTSLT

C
LNPTS=PTSLN
LTC(NB)=LTPTS
LNC(NB)=LNPTS
LSEG(NB)=SEGLN
TSEG(NB)=SEGLT
LFUNC(NB)=FUNCLN
TFUNC(NB)=FUNCLT
SYMM(NB)=SYM
NTVV=FLATV

C NBVV EQUALS NO. OF LONGITUDINAL PANELS.
C NTVV EQUALS NO. OF LATERAL PANELS.

NBVV=FLONGV
LNVOR=VORLN
LT Vor=VORLT
NXTHV=FNXTHV
XB00(NB)=XBO
YB00(NB)=YBO
ZB00(NB)=ZBO

C SET UP LONGITUDINAL VORTEX GRID (XVV ARRAY)

NB1=NBVV+1
IF(LNVOR) 15,20,30
15 DEL=CHORD/NBVV
XV(1)=0.0
GO TO 24
20 DEL=3.1415926/NBVV
XV(1)=0.0
24 DO 25 I=2,NB1
26 XV(I)=XV(I-1)+DEL
C
30 CONTINUE
C FOR LNVOR=1, XV IS GIVEN IN INPUT
NB1=NBVV+1
IF(LNDIV>1.1) GO TO 40
DO 35 I=1,NB1
35 XV(I)=XV(I)
GO TO 45
40 CALL FILLDV(XV,XVV,NB1,DIVLON)
45 CONTINUE
C

NV1(NB)=NV1V
NVT(NB)=NVTV
NI=NI+1
NVPO:1=NLV+1
NCL(NB)=LCLPS+NVLPS
WRITE(6,400)(NP(i),i=1,48)
400 FORMAT(110,10(3X,15))
C

NBV IS TOTAL NO. OF LONGITUDINAL VORICES.
DO 48 I=1,N1
48 XVV(i)=C2*(1.0-COS:XVV1)
50 CONTINUE

C

SET-UP BODY LATERAL VORICE PARAMETERS IF REQUIRED HERE.
IF(LTVOR) 300,310,320
C

THETA'S SAME AS BODY DEFINITION STATIONS
C

FOR THIS OPTION THERE MUST BE (NTVV+1) THETA'S (THN'S) GIVEN AT X'S.
300 THETA1=0
N1=NTVV+1
M1=0
M2=1
DO 301 I=1,N1
M1=M1+1
M2=M2+1
THVS(M1)=THN(M2)
IF (I.EQ.N1) M2=M2+1
301 CONTINUE
GO TO 320
C

THIS OPTION IS FOR EVEN DELTA THETA'S. DETERMINE(NTVV+1) THVS'S.
310 THETA1=1.0
N1=NTVV+1
IF(SYM) 311,312,311
311 TRANGE=360.0
GO TO 312
312 TRANGE=180.0
313 DTH=TRANGE/NTVV
DO 314 I=1,N1
314 THV(I)=(I-1)*DTH
C FOR LTVOR = 1, THVS ARE GIVEN IN DATA.
320 CONTINUE
IF(NXTHV.EQ.0) NXTHV=FLONGV+1.*J1
IF(LNVOR.NE.0) GO TO 58
DO 54 I=1,NXTHV
54 XV(I)=C2*(1.0-COS(XV(I)))
58 IF(FNXTH.NE.0) GO TO 65
DO 60 I=1,NXTH
60 XTH(I)=XS(I)
65 CONTINUE
C CALL DATAWR(DA)
RETURN
END
SUBROUTINE VVZN(J1, J2, Y, Z, AN)
COMMON DA(5000)
1 *NX*NY*NXV*LYV*NYV*NTV*NXHV*NYHV*NTH(49)
2 *LNDIV*LTDIV*LNPTS*LTPTS
COMMON /BODY/ A(31000)
DIMENSION X(151*31), YV(151*31), ZV(151*31)
EQUIVALENCE (XV, A(1)), (YV, A(4682)), (ZV, A(9363))
DIMENSION AN(1)
J1 = LTDIV*(J1-1)*LTDIV/2 + 1
J2 = J1 + 1
IF (I*NE*1) GO TO 100
X3 = XV (2, J1)
X4 = XV (2, J2)
X1 = 2.0*XV (1, J1) - X3
X2 = 2.0*XV (1, J2) - X4
Y3 = YV (2, J1)
Y4 = YV (2, J2)
Y1 = 2.0*YV (1, J1) - Y3
Y2 = 2.0*YV (1, J2) - Y4
Z3 = ZV (2, J1)
Z4 = ZV (2, J2)
Z1 = 2.0*ZV (1, J1) - Z3
Z2 = 2.0*ZV (1, J2) - Z4
GO TO 200
100 I1 = I - 1
I2 = I1 + 2
X1 = XV (I1, J1)
X2 = XV (I1, J2)
X3 = XV (I2, J1)
X4 = XV (I2, J2)
Y1 = YV (I1, J1)
Y2 = YV (I1, J2)
Y3 = YV (I2, J1)
Y4 = YV (I2, J2)
Z1 = ZV (I1, J1)
Z2 = ZV (I1, J2)
Z3 = ZV (I2, J1)
Z4 = ZV (I2, J2)
200 Y = 0.25*(Y1 + Y2 + Y3 + Y4)
SUBROUTINE INTER (X, Y, N, X1, Y1, N1)
DIMENSION X(1), Y(1), X1(1), Y1(1)
IF(X(2) - X(1) .GT. 0.0) GO TO 20
IM = 1
XMIN = X(1)
DO 5 I = 2, N
IF(XMIN - X1(I) .LE. 0.0) GO TO 6
XMIN = X1(I)
IM = I
5 CONTINUE
6 NU = IM
IM = 1
XMIN = X(1)
DO 10 I = 2, N
IF(XMIN - X(I) .LE. 0.0) GO TO 11
XMIN = X(I)
IM = I
10 CONTINUE
11 CONTINUE
CALL REVERS(X, IM)
CALL REVERS(Y, IM)
CALL REVERS(X1, NU)
CALL CODIM(X, Y, IM, X1, Y1, NU)
CALL REVERS(X, IM)
CALL REVERS(Y, IM)
CALL REVERS(X1, NU)
CALL REVERS(Y1, NU)
WRITE(6, 100) (X(I), Y(I), I = 1, IM) , (X1(I), Y1(I), I = 1, NU)
100 FORMAT (*05JB, INTER(I1P8*15*6I))
CALL CODIM(X1, Y1, IM + 1) . . .
NX = N - IM + 1
NY = NU - NU
WRITE(6, 100) (X(IM - 1 + I) . . .
1 , I = 1, NY)
RETURN
20 CALL CODIM(X, Y, N, X1, Y1, N1)
RETURN
END
SUBROUTINE FILLDV(X,Y,N,DIV)
DIMENSION X(1),Y(1)
N1=N-1
NDIV=DIV+0.01
DO 10 I=1,N1
DEL=(X(I+1)-X(I))/DIV
II=(I-1)*NDIV
DO 10 J=1,NDIV
K=II+J
10 Y(K)=X(I)+(J-1)*DEL
Y(K+1)=X(N)
RETURN
END
SUBROUTINE REVERS(X,N)
DIMENSION X(1)
NL=N/2
DO 100 I=1,NL
XHOLD=X(I)
N1=N+1-I
X(I)=X(N1)
100 X(N1)=XHOLD
RETURN
END
PROGRAM PANELP
COMMON DA(5000)
COMMON/COSY/XVR(10,20)+YVR(10,20)+ZVR(10,20)+XVO(10,20)
1+YVR(10,20)+ZVR(10,20)+PLL(50)+PLT(50)+YSUBV(100)+CHORD(100)
2+XVO(20)+XCCO(20)+XLE(20)+XLE(20)
3+XTLE(20)+YTE(20)+LTE(20)+YLE(20)+XJ(20)+YJ(20)+ZJ(20)
4+ETLE(20)+XVT(50)+YVT(50)+ZVT(50)+XR(20)+YR(20)+ZR(20)
5+BDUNHY(12130)+XYZN(3020)
COMMON/PANEL'/NPS+IPSY+IWC+NRWVP+NTWVP+QLC+CLP+LTCEP+LNEP+LTCEP
1+IPERTP+SPACE+ATTCH+TRATT+NPRTCL+PRCLT...CTXC...MCET+NTHXC
2+NTHET+NTIP+CTIP+RTOT+OUTER+EXTATT
3+XMP1+XMP2+XMP3+XMP4+XMP5+XMP6+XMP7+XMP8+XMP9+XMP10
COMMON/SCRAT/ DUM1(6000)
EQUVALENCE(DA(2),PAN6)
COMMON/PANAF/PANSYM(10), DUM1(400), PANREF(100), PCHORD(10)
EQUVALENCE(DA(422),PREF), (DA(422),PCH)
IF(PANS=EQ.0,0) GO TO 2
NPS=NPS
DO 1 I=1,NPS
DO 10 J=3420,3500
DA(J)=.0
1 CALL DECRI(DA)
CALL PANDAT
PANSYM(I)=IPSY
PCHORD(I)=PCH
PANREF(I)=PREF
CALL PANEL1
CALL ATTACH(XVR+YVR+ZVR,10,0)
1 CONTINUE
2 CONTINUE
END
SUBROUTINE PANEL1

C THIS SUBROUTINE STARTS GEOM. OF A PANEL.
COMPUTES XJ,YJ,ZJ POINTS: (COORDINATES OF JUNCTURE)
COMPUTES XR,YR,ZR POINTS: (COORDINATES OF OUTER EDGE OF ROOT OR PANEL
C HAVING NO ROOT SECTION.

COMMON DA(5000)
COMMON /BODY/XVR(10*20),YVR(10*20),ZVR(10*20),XVO(10*20)
1 *XVO(10*20),XVO(10*20),PLL(500),PLT(500),YFUBV(5000),CHORD(1000)
2 *XCVQ(20),XCOI(20),XLEI(20),YLEI(20),ZLEI(20)
3 *XTEI(20),YTEI(20),ZTEI(20),SLEI(20),XJ(20),YJ(20),ZJ(20)
4 *ELTEI(20),XVTI(50),YVTI(50),ZVTI(50),XRI(20),YRI(20),ZRI(20)
5 *SXMTI(1000),SYMTI(1000),SZMTI(1000),DYSI(1000),DZSI(1000)
6 *TSI(1000),XSSI(1000),YSSI(1000),ZSSI(1000),SIGMASHI(1000)
7 *XVSI(100),YVSI(100),ZVSI(100)
COMMON /PANEL/ NPN,IPSYN,IVC,IVVP,NTVVP,LCFCP,LTFCP,LCPP,LTFCP
1 *NPERPT,NSPACE,NATTCH,NTRATT,PNRLIN,PNRLIN,PNCTXC,NCTET,NTHXNC
2 *NDHET,NTIP,CHTIP,ROOT,OUTER,NATT
3 *MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10
COMMON /SCRAT/ XA(21),YA(21),ZA(21),SA(21),SJ(20)
1 *DUMMY(1300)
COMMON /COMPRS/ BETAM,EQUIVALENCE(PERIM,DUMMY)
DIMENSION PERIM(400)
COMMON /NUMBER/ TVPTS(7),VCPTS(7),VLN(7),NLN(7),LTC(7),LNC(7)
1 *NCT,NBNODS,PNANS,NVL(7),NVT(7),NTAPE,NTAPE,NTAPE,NTAPE
2 *LSEG(7),LSEG(7),LFUNC(7),LFUNC(7)
3 *LNIIVS(7),LTDIVS(7),NSPPS(7),KOOTP(7),OUTEXP(7),SYMM(7)
EQUIVALENCE(DA(4500),PERP), (DA(4600),PXCV), (DA(4690),CPLN)
1 * (DA(4720),CPLT), (DA(4640),PETV)
EQUIVALENCE(DA(3432),XPO), (DA(3433),YPO), (DA(3434),ZPO)
DIMENSION PERP(250),PXCV(40),PETV(40),CPLT(40)
LOGICAL ROOT,OUTER
NSPACE = 0
DO 11 I=1,NPERPT
IF(PER(D4*1),NE=0.0) GO TO 12
NSPACE=NSPACE + 1
11 CONTINUE
12 WRITE(6,13) NSPACE
13 FORMAT (HNSPACE = *13)
   ROOT = .FALSE.
   OUTER = .FALSE.
   NBP = NBVVP + 1
   IF (NSPACE .NE. 0) ROOT = .TRUE.
   IF (.NOT. ROOT) OUTER = .TRUE.
   DO 1 I = 1, NPERPT
      I1 = NPERPT + I - 1
      I2 = 1
      IF (PERP(4*I1).NE.0.0) GO TO 2
      CONTINUE
   2 NTIP = NPERPT + 1 - I2
      IF (OUTLR) GO TO 3
      IF (PERP(4*(NTIP - 1)).EQ.0.0) GO TO 3
      OUTER = .TRUE.
   3 CONTINUE
      CALL PERGM(PERP, PERIM)
      DO 4 I = 1, NTIP
         I1 = 4*(NPERPT - I)
         XTE(I) = PERIM(I1 + 1)
         YTE(I) = PERIM(I1 + 2) * BETA
         ZTE(I) = PERIM(I1 + 3) * BETA
         I4 = 4*(I - 1)
         XLE(I) = PERIM(I4 + 1)
         YLE(I) = PERIM(I4 + 2) * BETA
         ZLE(I) = PERIM(I4 + 3) * BETA
      4 CONTINUE
      YPO = YPO * BETA
      ZPO = ZPO * BETA
      C FIND MINIMUM XLE+XPO (ROOT) AND SET=XVS(100).
      X:LE=XLE(I) + XPO
      IF (X:LE.EQ.1) GO TO 100
      IF (X:LE.LT.XMLEH) GO TO 100
      GO TO 110
      100 XMLEH=XMLE
      110 XVS(100)=XMLEH
      WRITE(6,120) HAN, X:LE, XVS(100)
      120 FORMAT (30HOPANEL1: HAN*X:LE*XVS(100)=I5,1P2E2G,6)
C

\begin{verbatim}
WRITE(6,200)I,YLE(I),ZLE(I)
WRITE(6,200)I,XLE(I),YTE(I)
200 FORMAT(1H,13,1P3E16.6)
DO 401 I=1,NTP
401 ETLE(I)=SORT((YLE(I)-YLE(1))**2+(ZLE(I)-ZLE(1))**2)
     /SQR((YLE(I,NTP)-YLE(1))**2+(ZLE(I,NTP)-ZLE(1))**2)

DELS=0.0
NTP1=NTP-1
SLE(1)=0.0
DO 5 I=1,NTP1
   DY=YLE(I)-YLE(I+1)
   DZ=ZLE(I)-ZLE(I+1)
   DELS=DELS+SQR(DY**2+DZ**2)
   SLE(I+1)=DELS
   IF(I.EQ.NSPACE)DSROOT=DELS
5 CONTINUE

CHTIP=PERIM(4*NTP)
N4=4*NTP
XTIP=PERIM(N4-3)
YTOP=PERIM(N4-2)
ZTOP=PERIM(N4-1)
IF(ROOT)GO TO 10
C FOR NO ROOT SECTION:
   XRLE=XTIP
   YRLE=YTE
   ZRLE=ZTOP
   XRLE=PERIM(N4+1)
   YRTE=PERIM(N4+2)
   ZRTE=PERIM(N4+3)
   GO TO 20
C FOR LIFTING PANEL WITH ROOT SECTION:
   N5=4*NSPACE
   XRLE=PERIM(N54+1)
   YRLE=PERIM(N54+2)
   ZRLE=PERIM(N54+3)
   N1=4*(NPERPT-NSPACE-1)
   XRTE=PERIM(N1+1)
   YRTE=PERIM(N1+2)
\end{verbatim}
ZRTE=PERIM(N1+3)
C NROOT EQUALS NO. OF TR. VORTICES IN ROOT SECTION.
NROOT=NSPACE+1
20 XJLE=PERIM(1)
YJLE=PERIM(2)
ZJLE=PERIM(3)
N4=4*NPERPT
XJTE=PERIM(N4-3)
YJTE=PERIM(N4-2)
ZJTE=PERIM(N4-1)
IF (ROOT) GO TO 30
C FOR NO ROOT, DIVIDE JUNCTURE TR. VORTEX INTO 2 NEW VORTEX COORDINATES
21 CX=XJTE-XJLE
CY=YJTE-YJLE
CZ=ZJTE-ZJLE
XJ(1)=XJLE
YJ(1)=YJLE
ZJ(1)=ZJLE
DO 25 I=2,NBVP
DPER=PXCV(I)-PXCV(I-1)
XJ(I)=XJ(I-1)+CX*DPER
YJ(I)=YJ(I-1)+CY*DPER
ZJ(I)=ZJ(I-1)+CZ*DPER
25 CONTINUE
30 CONTINUE
C C COMPUTE XJ,YJ,ZJ ARRAYS FOR ROOT-BODY JUNCTURE.
XJ(1)=XJLE
C C COMPUTE YZ ON TR. VORTEX FOR 20 X'S.
NA=20
DX=(XJTE-XJLE)/NA
XJ(1)=XJLE+XPO
DO 45 I=1,NA
45 XJ(I+1)=XJ(I)+DX
NAP1=NA+1
CALL CODIM(XVT,YVT,YA,NA,NAP1)
CALL CODIM(ZVT,YZT,ZA,NAP1)
WRITE(6,451) (XA(I),YA(I),ZA(I),I=1,NAP1)
FORMAT(*2X,1P6E20.5)
SAI=0.0
SA(I)=0.0
DO 46 I=1,NA
   DY=YA(I+1)-YA(I)
   DZ=ZA(I+1)-ZA(I)
   SAI=SAI+SORT(DX**2+DY**2+DZ**2)
46   SA(I+1)=SAI
   SJ(I)=0.0
   DO 47 I=1,NBP1
      XJ(I)=XJ(I)-XPO
      YJ(I)=YJ(I)-YPO
   47   ZJ(I)=ZJ(I)-ZPO
   C    COMPUTE XR,YR,ZR ARRAYS
   DX=XRITE-XRLE
   XR(I)=XRLE
   YR(I)=YRLE
   ZR(I)=ZRLE
   DO 55 I=1,NBVVP
      XR(I+1)=XRLE+DX*PXCV(I+1)
      YR(I+1)=YRLE
   55   ZR(I+1)=ZRLE
      WRITE(6,200) (I,XJ(I),YJ(I),ZJ(I),I=1,NBP1)
      WRITE(6,200) (I,XR(I),YR(I),ZR(I),I=1,NBP1)
      CALL PANE2
      RETURN
      END
SUBROUTINE PANDAT
COMMON/PANEL/,PAN,IPSYN,INC,NBVVP,NTVVP,LCF,LCPTF,PLCF,PLP,PLCN
1 *NPRT,P,SPS,SMTCH,TRATT,SNPCL,SNPCL,TMCM,MOCTR,TMCT,THR
2 *INT,INTIP,CHTIP,ROOT,OUTER,INATT
3 *PN,PNF,P5,acct,P6,P7,PL,P8,P,P9,P10
COMMON/NUMER/IVPTS(7),NPVTS(7),LN(7),LNT(7),LTC(7),LNC(7)
1 *NCT,NC,MBODS,PAV,TVL(7),IVT(7),TAPE,TAPI,TAPE,TCTV,TAPZ
2 LSEQ(7),TSEQ(7),LFUNC(7),TFUNC(7)
3 *LNDIVP(7),LTDIV(7),IVFP(7),KOUTP(7),OUTERP(7),SYNM(7)
COMMON DA(560)
EQUIVALENCE(DA(3420),PH1),(DA(3426),PSYF),(DA(3429),C)
EQUIVALENCE(DA(3437),PNSVY),(DA(3438),PNTV),(DA(3439),PLNC)
2 *(DA(3440),PLTCF),(DA(3441),PLNC),(DA(3442),PLTCF)
3 *(DA(3443),PSTPER),(DA(3444),ROO],(DA(3445),ATTCH)
4 *(DA(3447),PCLN),(DA(3448),PCLT),(DA(3460),PAV)
5 *(DA(3460),CLP),(DA(3460),CLL),(DA(3460),CLM)
6 *(DA(4630),CPLN),(DA(4720),CPLT)
EQUIVALENCE(DA(3430),VLI1),(DA(3431),VLI)
1 *(DA(4600),PCV),(DA(4640),PETV)
EQUIVALENCE(DA(3423),SPAN)
COMMON/CUMPS/,BETAM
DIMENSION PCV(40),PETV(40)
DIMENSION CPLN(40),CLT(40)
COMMON/PANATT/,ATT(30),ATT(200)
COMMON/CONTRV/,LNC(3,40),LSC(3,40)
DIMENSION NLOOK(50)
EQUIVALENCE(NLOOK,IVPT)
WRITE(6,1)NLOOK
FORMAT(*0PANDAT*/1H1010)
CALL DATAWR(DA)
SPAN=SPAN*BETAM
NPA=PN
IPSYN=PSYN
INC=INC
NBVVP=NzV
NTVVP=PNTV
NPP=NPP+NBP
NVL(NPP)=NBVVP
NVT(NPP)=NTVVP

C

C

C
`REAL (PP) = 36VVR+1
.LT (PP) = TVVR+1
NPTS (PP) = 36VVR+1
LNCPP = LNCFP
LTCPP = LTCFP
LPUNQE (NPAH+1) = LNCPP
TPUNQE (NPAH+1) = LTCPP
NCPTS (NP) = LNCPP
NCPTP = LNCPP
NPERPT = PTSPER
NSPACE = ROOTSP
NAP = (NPAH-1)*3+2
NATTCH = LATT (NAP)
NTRATT = LATT (NAP+1)
IF (NATTCH .EQ. 0) GO TO 101
WRITE (6, 100) NPAH, NATTCH, NTRATT
100 FORMAT (*OPANEL#•••13••• IS ATTACHED TO COMPONENT#•••13••• AT TRAILING END
1TEX#•••12)
GO TO 103
101 WRITE (6, 102) NPAH, NATTCH
102 FORMAT (*OPANEL#•••13••• IS NOT ATTACHED. NATTCH=*•••12)
103 CONTINUE
NPRCL = PCL
NPRCLT = PCLT
NCTXC = PARXC
NCTET = CTALF
NCTXC = XCTK
NCTET = ETATH
IF (CPLN1 .LE. NCL) GO TO 6
DO 3 I = I+1, LNCPP
3 CONTINUE
5 CPLN1 = 1
6 IF (CPLN1 .LE. NCL) GO TO 6
DO 7 I = I+1, LTCPP
7 CPLTI1 = 1
8 CONTINUE`
41 CONTINUE
55 NT1=NTV+1
   WRITE(6,42) (PETV(I)*I+1,NT1)
42 FORMAT(*OPETV ARRAY*/(1P6E15.6))
   RETURN
END
SUBROUTINE PANEL2

COMMON/DA(5300)
COMMON/BODY/XOR(10,20),YOR(10,20),ZOR(10,20)
1 *VO(10,20),ZVO(10,20),PO(10),TO(10),YO(10),ZO(10)
2 *XCO(10),XCC(10),YCC(10),ZCC(10)
3 *XTE(10),YTE(10),ZTE(10),SLE(10),SJ(10)
4 *ETLE(10),XTL(10),YTL(10),ZTL(10)
5 *X,S*(1000),Y,S*(1000),Z,S*(1000)
6 *TS*(1000),XS*(1000),YS*(1000),ZS*(1000)
7 *XS*(1000),YS*(1000),ZS*(1000)

COMMON/PANEL/ XPAR,IPSY,YINC,SWVP,NTVP,LNCFP,LTEFP,LNCPP,LTCPP
1 *PFRPT,SPACE,NATTCH,TRATT,NPCL,TCL,CTC,CLTE,THX
2 *NTET,NTIP,CTHIP,ROO,TSC,HTATT
3 *TP1,TP2,TP3,TP4,TP5,TP6,TP7,TP8,TP9,TP10

COMMON/COMPTS/XQ(1320),YO(1320),ZQ(1320)
DIMENSION XCR(1000),YCR(1000),ZCR(1000)

EQUIVALENCE(XCR,X),YCR,Y),ZCR,Z)
COMMON/OCRAT/XA(21),YA(21),ZA(21),SA(21),LA(21)
1 ,DMY(1300),ETC(50),ETV(50)

COMMON/UMBER/XPTS(7),CPTS(7),LNM(7),ALT(7),LTC(7),LNC(7)
1 *CT,NPV,PS,MAP,SNV,VTAP,TAPE,PT,CTV,TAPE,TAPE
2 *LSG,LSE(7),TFUN(1),TFUN(1)
3 *LSD,IDV,LTIV,NSP,ROTP(7),OUTERP(7),SYM(7)

EQUIVALENCE((DA(3450),PCLTV), (DA(4600),PCLTV)
1 * (DA(4600),PCLTV)
EQUIVALENCE(DA(3450),PCLTV), (DA(4600),PCLTV)

DIMENSION PLOT(50),PCLTV(40),PCLTV(40),PCLTV(40)
DIMENSION CRX(20),CXY(20),CXY(20)
DIMENSION BODYR(31000)

EQUIVALENCE(XMR(10),XYR(10)
DIMENSION STARY(20),SLCF(50),SLCEF(600)

EQUIVALENCE((DA(4840),STARY), (DA(4840),SLCEF)
1 * (DA(4840),STARY), (DA(4840),SLCEF)

LOGICAL ROOT,ROTE,CLT,KE
LOGICAL FIRST

3490
3500
3510
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3870
DXTE = XTE(I+1) - XTE(I)
DYTE = YTE(I+1) - YTE(I)
DZTE = ZTE(I+1) - ZTE(I)
CALL NORM(DXTE, DYTE, DZTE)
DCX(I) = DXLE + (DXTE - DXLE) * PER
DCY(I) = DYLE + (DYTE - DYLE) * PER
DCZ(I) = DZLE + (DZTE - DZLE) * PER
CALL NORM(DCX(I), DCY(I), DCZ(I))
DENOM = DENOM + DCXDCYDCZCSXSYSZCS
DS = CS2/DENOM
DO 60 J = 1, N1
XVR(I+1, J) = XVR(I, J) + DS * DCX(I)
YVR(I+1, J) = YVR(I, J) + DS * DCY(I)
600 FORMAT(1H, 13, 1P3E16.6)
60 ZVR(I+1, J) = ZVR(I, J) + DS * DCZ(I)
DO 618 I = 2, NSPACE
SVA = 0.0
SAI(I) = 0.0
DO 601 J = 1, NBVP
DX = XVR(I, J) + 1, - XVR(I, J)
DY = YVR(I, J) + 1, - YVR(I, J)
DZ = ZVR(I, J) - ZVR(I, J)
SVA = SAI + SQRT(DX**2 + DY**2 + DZ**2)
601 SA(J+1) = SVA
DO 603 J = 1, NBP1
XA(J) = XVR(I, J)
YA(J) = YVR(I, J)
603 ZA(J) = ZVR(I, J)
SJ(1) = 0.0
DO 604 J = 1, NEVP
SJ(J+1) = SJ(J) + SVA * (PXC(J+1) - PXC(J))
CALL COD(i)(SA*XA, NEP1, SJ, DCX, NEVP)
CALL COD(i)(SA*YA, NEP1, SJ, DCY, NEVP)
CALL COD(i)(SA*ZA, NEP1, SJ, DCZ, NEVP)
DO 618 J = 2, NEVP
XVR(I, J) = DCX(J)
YVR(I, J) = DCY(J)
618 ZVR(I, J) = DCZ(J)
CONTINUE
DO 70 I=1,NTVVP
IF(NEQ.1) GO TO 66
IF(ROOT) GO TO 66
CH = XJ(NBP1)-XJ(1)
GO TO 70
CH=0.0
DO 67 J=1,NBVVP
DX=XVR(I,J)-XVR(I,J+1)
DY=YVR(I,J)-YVR(I,J+1)
DZ=ZVR(I,J)-ZVR(I,J+1)
CH=CH+SQRT(DX**2+DY**2+DZ**2)
GO TO 70
IF(ROOT.AND.I.LE.NLROR) GO TO 66
CH=CHORD(I-1)+(PETV(I)-PETV(I-1))*CHTIP-CHORD(I-1))
1 (I,0-PETV(I-1))
CHORD(I)=CH
CHORD(NTVVP+1)=CHTIP
NTV1=NTVVP+1
WRITE(6,200)(I,CHORD(I),PETV(I),CHTIP,=I,NTV1)
NCO=0
NCOO=0
IF(NBODS.EQ.0.AND.NPAN.EQ.1) NCT=0
NCP=NCT
IF(NPAN.GT.1) NCP=NP1
IF(.NOT.ROOT) GO TO 85
COMPUTE VORTEX AND CONTROL POINTS IN ROOT REGION.
LT=1
DO 80 I=1,NSPACE
KLT=.FALSE.
LTCP=CPLT(LT)
IF(NEQ.LTCP) GO TO 71
KLT=.TRUE.
LT=LT+1
CONTINUE
LN=1
DO 75 J=1,NBVVP
KLN=.FALSE.
L:ICP=CPLN(L')
       2 4660
       2 4670
       2 4680
       2 4690
       2 4700
       2 4710
       2 4720
       2 4730
       2 4740
       2 4750
       2 4760
       2 4770
       2 4780
       2 4790
       2 4800
       2 4810
       2 4820
       2 4830
       2 4840
       2 4850
       2 4860
       2 4870
       2 4880
       2 4890
       2 4900
       2 4910
       2 4920
       2 4930
       2 4940
       2 4950
       2 4960
       2 4970
       2 4980
       2 4990
       2 5000
       2 5010
       2 5020
       2 5030
       2 5040
       2 5050
IF(J .NE. LNP) GO TO 72
KLN=.TRUE.
LNP=LNP+1
CONTINUE

72

DX1=XVR(I,J+1)-XVR(I,J)
DY1=YVR(I,J+1)-YVR(I,J)
DZ1=ZVR(I,J+1)-ZVR(I,J)
DX2=XVR(I+1,J+1)-XVR(I+1,J)
DY2=YVR(I+1,J+1)-YVR(I+1,J)
DZ2=ZVR(I+1,J+1)-ZVR(I+1,J)
NCO=NCO+1
PL1=SQRT((DX1**2+DY1**2+DZ1**2)
PL2=SQRT((DX2**2+DY2**2+DZ2**2)
PLL(NCO)=0.5*(PL1+PL2)
DX11=XVR(I+1,J)-XVR(I,J)
DY11=YVR(I+1,J)-YVR(I,J)
DZ11=ZVR(I+1,J)-ZVR(I,J)
DX21=XVR(I+1,J+1)-XVR(I,J+1)
DY21=YVR(I+1,J+1)-YVR(I,J+1)
DZ21=ZVR(I+1,J+1)-ZVR(I,J+1)
PL1=SQRT((DX11**2+DY11**2+DZ11**2)
PL2=SQRT((DX21**2+DY21**2+DZ21**2)
PLT(NCO)=0.5*(PL1+PL2)
T1=DX11/SQRT(DY11**2+DZ11**2)
T2=DX21/SQRT(DY21**2+DZ21**2)
FIRST=.FALSE.
R1=.25

722

NCO=NCO+1
XSS(NCO)=XP0+0.5*(XVP(:,:,:)+R1*DX1+XVP(:,:,:)+R1*DX2)
YSS(NCO)=YP0+0.5*(YVR(:,:,:)+R1*DY1+YVR(:,:,:)+R1*DY2)
ZSS(NCO)=ZP0+0.5*(ZVR(:,:,:)+R1*DZ1+ZVR(:,:,:)+R1*DZ2)
TS(NCO)=T1+R1*(T2-T1)
DYSS(NCO)=DY11+R1*(DY21-DY11)
DZSS(NCO)=DZ11+R1*(DZ21-DZ11)
IF(FIRST) GO TO 724
FIRST=.TRUE.
R1=.75
GO TO 722

724

CONTINUE
IF (.NOT. KLT) GO TO 73
IF (.NOT. KLN) GO TO 73
NCP=NCP+1
XCR(NCP) = 0.5*(XVR(I,J) + 0.75*DX1 + XVR(I+1,J) + 0.75*DX2) + XPO
YCR(NCP) = 0.5*(YVR(I,J) + 0.75*DY1 + YVR(I+1,J) + 0.75*DY2) + YPO
ZCR(NCP) = 0.5*(ZVR(I,J) + 0.75*DVZ1 + ZVR(I+1,J) + 0.75*DVZ2) + ZPO
73
CONTINUE
XVR(I,J)=XVR(I,J)+0.25*DX1 + XPO
YVR(I,J)=YVR(I,J)+0.25*DY1 + YPO
ZVR(I,J)=ZVR(I,J)+0.25*DVZ1 + ZPO
IF (.NOT. NSPACE) GO TO 75
XVR(I+1,J)=XVR(I+1,J)+0.25*DX2 + XPO
YVR(I+1,J)=YVR(I+1,J)+0.25*DY2 + YPO
ZVR(I+1,J)=ZVR(I+1,J)+0.25*DVZ2 + ZPO
75
CONTINUE
LETV=LETV+1
ETV(LETV)=0.5*(PETV(I+1)+PETV(I))
IF (.NOT. KLT) GO TO 80
LETC=LETC+1
ETC(LETC)=ETV(LETV)
80
CONTINUE
DO 82 I=1,NLROOT
XVR(I,NBP1)=XVR(I,NBP1)+XPO
YVR(I,NBP1)=YVR(I,NBP1)+YPO
ZVR(I,NBP1)=ZVR(I,NBP1)+ZPO
NCR=NC00
82
CONTINUE
IF (.NOT. OUTER) GO TO 120
DO 90 J=1,NBVVP
XCVO(I)=0.75*PXCV(I)+0.25*PXCV(I+1)
XCVO(NBVVP+1)=PXCV(NBVVP+1)
DO 92 I=1,LNCP
J=CPLS(I)
90
CONTINUE
XCVO(I)=0.25*PXCV(J)+0.75*PXCV(J+1)
92
CONTINUE
NTVOUT=NTVVP+1
SPACE CPLT ARRAY OUT OF FOOT REGION.
DO 96 I=1*LTCPP
IC=I
LTCPP=CPLT(I)
IF(LTCP*GT*SPACE) GO TO 97
96 CONTINUE
97 CONTINUE
DO 110 I=1*NTVOUT
INS=I+NSPACE
LT1=PETV(INS)
ET2=PETV(INS+1)
ET=0.*((ET1+ET2)
CH1=CHORD(INS)
CH2=CHORD(INS+1)
CHAVG=0.5*(CH1+CH2)
DO 99 L=1*NTIP
IF(ET*LE*ETLE(L+1) AND ET*GT*ETLE(L)) GO TO 99
93 CONTINUE
99 DELET=ETLE(L+1)-ETLE(L)
RATIO=(ET-ETLE(L))/DELET
XLE1=XLE(L)+(XLE(L+1)-XLE(L))*RATIO
YLE1=YLE(L)+(YLE(L+1)-YLE(L))*RATIO
ZLE1=ZLE(L)+(ZLE(L+1)-ZLE(L))*RATIO
R1=(ET1-ETLE(L))/DELET
R2=(ET2-ETLE(L))/DELET
DX=XLE(L+1)-XLE(L)
DY=YLE(L+1)-YLE(L)
DZ=ZLE(L+1)-ZLE(L)
XL1=XLE(L)+DX*R1
YL1=YLE(L)+DY*R1
ZL1=ZLE(L)+DZ*R1
XL2=XLE(L)+DX*R2
YL2=YLE(L)+DY*R2
ZL2=ZLE(L)+DZ*R2
DX=XTE(L+1)-XTE(L)
DY=YTE(L+1)-YTE(L)
DZ=ZTE(L+1)-ZTE(L)
XT1=XTE(L)+DX*R1
YT1=YTE(L)+DY*R1
ZT1=ZTE(L)+DZ*R1
XT2=XTE(L)+DX*R2
YT2=YTE(L)+DY*R2
ZT2=ZTE(L)+DZ*R2

C SAVE VORTEX POINTS ON TRAILING EDGE. (THOSE THAT LIE OUTSIDE ROOT.)
XVS(I)=XT1+XP0
YVS(I)=YT1+YP0
ZVS(I)=ZT1+ZP0
IF(INE+NTVOUT) GO TO 990
XVS(I+1)=XT2+XP0
YVS(I+1)=YT2+YP0
ZVS(I+1)=ZT2+ZP0

990 CONTINUE
DXL=XL2-XL1
DYL=YL2-YL1
DZL=ZL2-ZL1
YSUBV(I)=0.5*SQRT(DYL**2+DZL**2)
DXT=XT2-XT1
DYT=YT2-YT1
DZT=ZT2-ZT1
TTLE=DXL/SQRT(DYL**2+DZL**2)
TTTE=DXT/SQRT(DYT**2+DZT**2)
DO 100 J=1,NBVVP
NCO=NCO+1
PD=PXCV(J+1)-PXCV(J)
PLL(NCO)=0.5*PD*(CH1+CH2)
FIRST=.FALSE.
XCTERM=XCVO(J)
XVO(I,J)=XP0+XLE1+XCTERM*CHAVG
YVO(I,J)=YP0+YLE1
ZVO(I,J)=ZP0+ZLE1

991 NCO=NCO+1
IF(FIRST) GO TO 992
XSS(NCO)=XVO(I,J)
YSS(NCO)=YVO(I,J)
ZSS(NCO)=ZVO(I,J)
GO TO 993

992 XSS(NCO)=XP0+XLE1+XCTERM*CHAVG
YSS(NCO)=YP0+YLE1
ZBS(NCO)=ZP0+ZLE1
993  DYS(NCO)=DYL
  DZS(NCO)=DZL
  TS(NCO)=TTLE+(TTE-TTLE)*XCTER:
  IF(FIRST) GO TO 100
  FIRST=.TRUE.
  XCTER'=0.25*PXCV(J)+0.75*PXCV(J+1)
  GO TO 991
100  CONTINUE
  LETV=LETV+1
  ETV(LETV)=ET
  LTCP=CPLT(IC)
  IF(1+MSPACE*NE_LTCP) GO TO 110
  IC=IC+1
  LETC=LETG+1
  ETC(LETG)=ET
  DO 105 II=1,1NCPP
  NCP=NCP+1
  XCR(NCP)=XP0+XLE1+XCCO(I1)*CHAVG
  YCR(NCP)=YP0+YLE1
105  ZCR(NCP)=ZP0+ZLE1
110  CONTINUE
120  CONTINUE
  MPI=NCP

C
C
  CALL PANFNC(NCT+ETV+ETC)
  NCT=NCP
C
  IF(NU+N+ES0.0) GO TO 1111
CALL 3SUBROUTINES REQUIRED TO SET UP PANEL CONSTRAINT MATRIX.
  LOC1=(NPN-1)*1000+1
  LOC2=LOC1+400
  LOC3=LOC2+400
C
ARRAY XA IS USED FOR SCRATCH.
  CALL TACC(NU+XA+5,XA+NWiVVP,
1  SAVEC(LOC1),SAVEC(LOC2),H:PACE,XA,XCVG,XCCO)
  CALL MATETA(SAVEC(LOC3),...ETV+STAX,ETV,SLCF)
1111  CONTINUE
SUBROUTINE PERM(PERI, ARRAY)
DIMENSION PERI(1),ARRAY(1)
COMMON/Panel/NPA,IPSYN,ICONAVVP,NTVVP,LCFP,LCPP,LCFPP,LTCP,LPAD
1 NPERPT,NSPACE,INATT,NSRCNL,NSRCRT,NCXCN,NTTET,NTHTAC
2 NTHTEN,NTHPT,NHTIP,CHTIP,RROT,OUTER,NNATT
3 NP1,NP2,NP3,NP4,NP5,NP6,NP7,NP8,NP9,NP10
COMMON/BODY/XVR(10,20),YVR(10,20),ZVR(10,20),YVR(10,20),ZVR(10,20),ZVR(10,20)
1 YVO(10,20),ZVO(10,20),PLL(500),PLT(500),YSUEV(100),CHORD(100)
2 XCO(20),XCO(20),XLE(20),XLE(20),XLE(20)
3 YTE(20),YTE(20),YTE(20),YTE(20),YTE(20),YTE(20),YTE(20)
4 XTE(20),XTE(20),XTE(20),XTE(20),XTE(20),XTE(20),XTE(20)
LOGICAL ROOT, OUTER
COMMON DA(5000)
EQUIVALENCE (DA(3432),XPO), (DA(3433),YPO), (DA(3434),ZPO)
NP=NTIP
N4=4*NTIP
DO 10 I=1,N4
10 ARRAY(I)=PERI(I)
N=2*NTIP-NPERPT
DO 20 I=1,N
NP=NP+1
1 N1=4*(NTIP+I-1)+1
2 N2=4*(NTIP-I)+1
3 ARRAY(N1)=ARRAY(N2)+ARRAY(N2+3)
4 ARRAY(N1+1)=ARRAY(N2+1)
5 ARRAY(N1+2)=ARRAY(N2+2)
6 ARRAY(N1+3)=0*0
7 IF(ROOT) CALL ATTACH(XVT,YVT,ZVT,50,1)
8 NNATT=MP3
9 N=NTIP-N
10 IF(N.EQ.0) GO TO 35
11 DO 30 I=1,N
12 NP=NP+1
13 N1=1+4
14 N2=1+4*(I-1)+4+1
15 ARRAY(N1)=PERI(N2)
16 ARRAY(N1+1)=PERI(N2+1)
17 ARRAY(N1+2)=PERI(N2+2)
18 ARRAY(N1+3)=0*0
2 7030
2 7040
2 7050
2 7060
2 7070
2 7080
2 7090
2 7100
2 7110
2 7120
2 7130
2 7140
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2 7210
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2 7280
2 7290
2 7300
2 7310
2 7320
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2 7340
2 7350
2 7360
2 7370
2 7380
2 7390
2 7400
2 7410
IF(I .NE. N) GO TO 30
IF(.NOT. ROOT) GO TO 30
ARRAY(N1)=ARRAY(N1)+XPO
CALL CODIM(XVT,YVT,NNATT,ARRAY(N1),ARRAY(N1+1),1)
CALL CODIM(XVT,ZVT,NNATT,ARRAY(N1),ARRAY(N1+2),1)
ARRAY(N1)=ARRAY(N1)-XPO
ARRAY(N1+1)=ARRAY(N1+1)-YPO
ARRAY(N1+2)=ARRAY(N1+2)-ZPO
30 CONTINUE
35 NPERPT = NP
NP4=4*NP
WRITE(6,100) NTIP
WRITE(6,100) NSPACE
WRITE(6,100) NPERPT,(I,ARRAY(I),I=1,NP4)
100 FORMAT(*0PANEL PERIMETER*/I4/(I4,1PE15.5))
RETURN
END
SUBROUTINE MATELA( TETA, NETA, NETA, STARAY, ETA, SLCF)

SUBROUTINE TO BUILD TETA MATRIX

TETA - TETA MATRIX

NWA - NUMBER OF SPECIAL LATERAL CONSTRAINT FUNCTIONS

META - NUMBER OF ETA STATIONS

STARAY - LIST OF LATERAL CONSTRAINT FUNCTIONS

ETA - ARRAY OF ETAS

SLCF - LIST OF SPECIAL LATERAL CONSTRAINT FUNCTIONS

DIMENSION TETA(NETA, NWA), STARAY(NWA), ETA(NETA), SLCF(1)

REAL H

INTEGER PTR

EQUVALENCE (RI, ETAI), (RO, ETAO)

DATA PI02/1.5707, 98326, 79489/

DO 2 J = 1, NWA

IF (STARAY(INW) == 100) 1, 21, 21

1

TETA(IETA, INW) = SQRT( 1.0 - ETA(IETA) * ETA(IETA)) * ETA(IETA)

10 H = H + 1

20 CONTINUE

RETURN

11 NWA = INW

DO 20 INW = NWA + 1, NWA

PTR = STARAY(INW) - 4779

GET INFO OUT OF SPECIAL LATERAL CONSTRAINT FUNCTION TABLE

ETAB = SLCF(PTR + 1)

RI = SLCF(PTR + 2)

RO = SLCF(PTR + 3)

IF (SLCF(PTR) == 22) GOTO 21

21 THI = ACOS(ETAI)

THO = ACOS(ETAO)

DO 30 IETA = 1, NETA

TH = ACOS(ETA(IETA))

30 TETA(IETA, INW) = 0.5(THI, THI) - 0.5(TH, THO)

GOTO 80

22 THI = ACOS(ETAI)

THO = ACOS(ETAO)

DO 30 IETA = 1, NETA

TH = ACOS(ETA(IETA))

30 TETA(IETA, INW) = 0.5(THI, THI) - 0.5(TH, THO)

GOTO 80

31 A21 = 1 - RO

A5 = 1 / RI

A9 = 1 - ETAB
A35 = (A9 + RI)*A5
A35B = ACOS(ETAB - RI)
THB = ACOS(ETAB)
IF(A21-ABS(ETAB))32 * 32 * 41
32 A50 = A9 * A5 + 1.0
DO 40 IETA=1* NETA
TH = ACOS(ETA(IETA))
40 TETA(IETA, INW) = A35 * P(TH, A35B) - A50 * P(TH, THB)
GOTO 80
41 A8 = RI+RO
A11 = RI*RO
A18 = A8/A11
A22 = A21-ETAB
A 51 = A18 * A 9
A27B = ACOS(ETAB+RO)
IF(RI-ABS(ETAB))42 * 42 * 51
42 A50 = A22 * A5
A26B = ACOS(ETAB-RO)
DO 50 IETA=1* NETA
TH = COS(ETA(IETA))
50 TETA(IETA, INW) = A50 * P(TH, A27B) + A35 * P(TH, A35B) - P(TH, THB) * A52
GOTO 80
51 A 50 = A 22 * ( A 18 - A 5)
A 52 = 1.0 - ETAB * A 5
DO 60 IETA=1* NETA
TH = COS(ETA(IETA))
60 TETA(IETA, INW) = P(TH, A27B) * A 50 + A52 * M(TH, PI2) + A 5 * P(TH, PI02) - P(TH, THR) * A 51
80 CONTINUE
RETURN
C BLAINE D. GAITHER 9/72
END
SUBROUTINE TXOC(NU, THETAF, NF, THETAK, NK, NCOL, NXOC)

1 SCRCH, SCRCH+NROOT, THET, XCI, XCJ

C NU - NU FOR THIS PANEL
C THETAF - ARRAY OF THETAFS
C THETAK - ARRAY OF THETAKS
C NF - NUMBER OF THETAFS
C NK - NUMBER OF THETAKS
C NCOL - NU+NF+NK
C NXOC - NUMBER OF X OVER C S
C SCRCH - T(XOC) RESULT
C SCRCH - SCRCH ARRAY IF NROOT > 0 MATRIX FOR USE IN ROOT SECTION
C NROOT - NUMBER OF ELEMENTS IN THE ROOT SECTION
C THET - SCRCH MATRIX THAT THETAS ARE PUT IN TO
C XCI - ARRAY OF X OVER C S 1/4 PANEL
C XCJ - ARRAY OF X OVER C S 3/4 PANEL

DIMENSION THETA(NF), THETAK(NK),

1 SCRCH(NXOC, NCOL), SCRCH(NXOC, NCOL),

3, XCI(1), XCJ(1)

CALL FILLNU(SCRCH(1,1), NXOC, NK, NU+NF, XCI, THET)
CALL FILLNK(SCRCH(1,1), NF, NK+NU, XOC, THETAK, THET)
CALL EFIL(XCJ, XCI, NK, XOC, SCRCH)
CALL GELG(SCRCH, SCRCH, NXOC, NCOL, 0.0E-12, I)

IF( I ) 11, 19, 11

11 PRINT 16, I
16 FORMAT(*0GELG ERROR*, 13)
19 IF(NROOT<LT.1) RETURN
DO 20 IUFK=1, NCOL
TEMP = 0.0
DO 20 IXOC=1, NXOC
TEMP = TEMP + SCRCH(IXOC, IUFK)
20 SCRCH(IXOC, IUFK) = TEMP
RETURN
C BLAINE D. GAITHER 9/72
END
SUBROUTINE PANFNC(KL,ETV,ETC)

C IWC=VING-CONTOUR INDICATOR
C =0 FOR LOCAL ANGLES OF ATTACK, ALPHA, GIVEN.
C =1 FOR DEFLECTIONS, Z/C GIVEN.
DIMENSION ETV(1),ETC(1)
COMMON DA(5000)
COMMON BODY, YXR(10*20), YVR(10*20), ZVR(10*20), XVD(10*20)
1 *YV0(10*20), ZV0(10*20), PLL(500), PLT(500), YSUBV(100), CHORD(100)
2 *XCVO(20), XCCO(20), XLE(20), YLE(20), ZLE(20)
3 *XET(20), YET(20), ZET(20), SLE(20), XJ(20), YJ(20), ZJ(20)
4 *ETLE(20), XVT(50), YVT(50), ZVT(50), XR(20), YR(20), ZR(20)
5 *SXMT(1000), SYMT(1000), SZMT(1000), DYS(1000), DNS(1000)
6 *TS(1000), XSS(1000), YSS(1000), ZSS(1000), SIGMA(1000)
COMM/CONPTS/ XQ(1320), YQ(1320), ZQ(1320)
1 *XN(1320), YN(1320), ZN(1320)
COMMON/PANEL/ NPSYM, NPSYM, NPSYM, NPSYM, NPSYM, NPSYM, NPSYM, NPSYM
1 *NPERPT, NSPACE, NATTCH, NTRATT, NPRCLN, NPRCLT, WCTXC, WCTET, WTHXC
2 *NTHET, NTP, CHTIP, ROOT, OUTER, KNATT
3 *MP1, MP2, MP3, MP4, MP5, MP6, MP7, MP8, MP9, MP10
EQUIVALENCE(ALPHA, DA(3650)), (TXST, DA(4100)), (X+A, DA(3601))
1 *(ETA, DA(3631)), (FNX, DA(3600)), (FNETA, DA(3630))
2 *(FNX, DA(3437)), (FNEC, DA(3442))
C DUMS(1504) PRESERVES /SCRAT/ FROM PANEL2.
COMMON/SCRAT/ DUMS(1504), ALP(400), TW(20)
1 *DYI(400), DZI(400), THK(820), XCH(50), XCHTHK(51)
2 *DUMZ(101), DUMY(2020), DUMZ(101), XPY(40), ZP(40), CH(21), CHC(20)
DIMENSION ALPHAD(440), TWIST(30), XA(29), ETAI(29)
EQUIVALENCE(DA(4600), PXCV), (DA(4131), XHTHCK), (DA(4170), FXTHK)
1 *(DA(4161), ETAHCK), (DA(4160), FETHK), (DA(4190), THICK)
DIMENSION PXCV(40), XTHICK(29), ETHICK(29), THICK(410)
EQUIVALENCE(DA(4640), PETV)
COMMON /COMPRS/ ETAV
DIMENSION PETV(40)
COMMON/SLOPE/SIGMAP(500), ZDXT(500), ZDXT(500), TANP1(500)
EQUIVALENCE(MP5, SIGMA, LS)
EQUIVALENCE(DA(7), XCG), (DA(4), YCG), (DA(9), XCG)
1 *(DA(11), ETA), (DA(12), PETAR), (DA(13), OSTAR), (DA(14), RSTAR)
EQUIVALENCE(DA(4720), CPTL), (DA(4820), CPTL)
DIMENSION CPLT(40), CPLX(40)
EQUIVALENCE (B, DA(3423)), (RTFA, DA(3421)), (CHRY, DA(3422))
1  *(SPCF, DA (4880))
DIMENSION SPCF(40)
COMMON/NUMBER/ NLOOK(55)
IF (NPAN. EQ. 1) EP5 = 0

C MP5 WILL BE TOTAL NO. OF SOURCE POINTS.
C
NXA=FNXA
NETAI=FNETHAI
NXC=FNXC
NEC=FNEC
NXC2=2*NXC
NXTHK=FNXTHK
NETTHK=FNTHHK
WRITE (6, 100) (ETV(I), I=1, 4)
WRITE (6, 100) (ETC(I), I=1, 4)
100 FORMAT (*0PANFNC*/(1P8E15.6))
IN0=100
IN1=IN0+1
DO 1 I=1, IN1
1 DUMX(I)=(I-1)*0.01
NT1=NTVVP+1
K=0
DX=0.01
DO 10 I=1, LNCPP
K=K+1
XP(K)=XCCO(I)-DX
K=K+1
10 XP(K)=XCCO(I)+DX
IF (NXA .NE. 0) GO TO 33
N1=LNCPP+NEC
DO 31 I=1, N1
31 ALP(I)=0.0
DO 32 I=1, NEC
32 TW(I)=0.0
GO TO 34
33 CONTINUE
WRITE (6, 100) (XP(I), I=1, 20)
DO 2 J=1*NETAI
J1=(J-1)*NXA+1
J2=(J-1)*IN1+1
CALL CODIM(XA,ALPHA(J1),NXA,DUMX,DUMY(J2),IN1)
IF(IWC.EQ.1) GO TO 2
CALL QTFG(DUMX,DUMY(J2),DUMY(J2),IN1)
CONTINUE
WRITE(6,100)(DUMY(I),I=1,2,2)
        CALL XLINE(PETV,CHORD,NT1,ETAI,CH,NETAI)
        CALL XLINE(PETV,CHORD,NT1,ETC,CHC,LTCPP)
WRITE(6,100)(CHORD(J),J=1,NT1)
WRITE(6,100)(CH(J),J=1,NETAI)
WRITE(6,100)(CHC(J),J=1,LTCPP)
WRITE(6,100)(ETAI(J),J=1,NETAI)
WRITE(6,100)(DUMX(J),J=1,IN1)
K1=0
L=2
DO 6 I=1,NEC
IF(ETC(I).GT.ETAI(L)) GO TO 5
LM1=L-1
D1=ETC(I)-ETAI(LM1)
D2=ETAI(L)-ETAI(LM1)
R=D1/D2
L1=(LM1-1)*IN1
L2=L1+IN1
K=0
DO 4 J=1,IN1
L1=L1+1
L2=L2+1
ZZZ = CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
K=K+1
DUMZ(K) = ZZZ/CHC(I)
LN2=2*LNCPP
WRITE(6,100)CH(LM1)*CH(L),R
CALL CODIM(DUMX,DUMZ*IN1,XP,ZP,LN2)
WRITE(6,100)(DUMZ(J),J=1,IN1)
WRITE(6,101)L:2
WRITE(6,100)(XP(J),ZP(J),J=1,LN2)
DO 20 J=1,LCNP
K1=K1+1
J2=2*J
20 ALP(K1)=(ZP(J2)-ZP(J2-1))/DX
WRITE(6,100)(ALP(J),J=1,K1)
GO TO 6
5 L=L+1
GO TO 3
6 CONTINUE
WRITE(6,100) (ETA1(I),TWIST(I),I=1,NETAI)
CALL CODIM(ETA1,TWIST,NETAI,ETC,TW,NEC)
34 CONTINUE
L=0
DO 9 J=1,NXC
L=L+1
XCTHK(L)=PXCV(J)
XC(L)=0.75*PXCV(J)+0.25*PXCV(J+1)
L=L+1
XCTHK(L)=0.5*(PXCV(J)+PXCV(J+1))
9 XC(L)=0.25*PXCV(J)+0.75*PXCV(J+1)
XCTHK(L+1)=PXCV(NXC+1)
CALL FINDF(XC,NXC2,ETV,NTVVP,DYS,XCCU,LG,CRP,ETC,NEC,DYI)
WRITE(6,100)(DYI(I),I=1,80)
CALL FINDF(XC,NXC2,ETV,NTVVP,DZI,XCCU,LG,CRP,ETC,NEC,DZI)
WRITE(6,100)(DZI(I),I=1,80)
CALL XLINE(PETV,CHRD,NT1,ETV,CHC,TVVP)
IF(NXTHK,NE=0) GO TO 8
NT=NTVVP-NXC2
DO 7 I=1,NT
LSIGMA=LSIGMA+1
7 SIGMA(LSIGMA)=0.0
GO TO 81
8 CONTINUE
CALL XLINE(PETV,CHRD,NT1,TATHK,CH,NETTHK)
DO 102 J=1,NETTHK
J1=(J-1)*TXHK+1
J2=(J-1)*TXHK+1
102 CALL CODIM(XTHK1,THIKK(J1),J2,XTHK2,CRP5,NE1,NE2,TH1)
L=2
DO 106 I=1,NTVVP
103 IF(ETV(I) .LT. ETATHK(L)) GO TO 105
LM1=L-1
D1=ETV(I)-ETATHK(LM1)
D2=ETATHK(L)-ETATHK(LM1)
R=D1/D2
L1=(LM1-1)*IN1
L2=L1+IN1
K=0
DC 104 J=1,IN1
L1=L1+1
L2=L2+1
TH = CH(LM1)*DUMY(L1)+R*(CH(L)*DUMY(L2)-CH(LM1)*DUMY(L1))
K=K+1
104 DUMZ(K) = TH/CHC(I)
L3=(I-1)*(NXC2+1)+1
CALL CODIM(DUMX,DUMZ,IN1,XCTHK,THK(L3),NXC2+1)
GO TO 106
105 L=L+1
GO TO 103
106 CONTINUE
K1=0
K=KL
L2=1
DO 110 I=1,NEC
MT=(CPLT(I)-1.0)*(NXC2+1)+0.01
DO 110 J=1,LCPP
H1=CPLN(J)*2.0+C*0.1
M=.T+.1
ST=(THK(M+1)-THK(M))/((XCTHK(M+1)-XCTHK(M))
K=K+1
TANP1(K) = 1.0 +TS(L2)**2 *FETA**2
L2=L2+2
DZDXT(K)=ST
K1=K1+1
DZDXC(K)=ALP(K1)
110 CONTINUE
31 CONTINUE
WRITE(6,100)(XC(I),I=1,NXC2)
WRITE(6,100)(XCTHK(I),I=1,NXC2)
WRITE(6,100)(XCCU(I)*I=1,LCPP)
WRITE(6,100)(THK(I)*I=1,30)
K1=0
K=KL
DO 40 I=1,NEC
TW1 = TW(I)
DO 40 J=1,LCPP
K=K+1
K1=K+1
SQI = SQRT(DY1(K1)**2 + DZI(K1)**2)
YN(K) = -DZI(K1)/SQI
ZN(K) = +DYI(K1)/SQI
C XN = DZ/DX + TWIST * OR ALPHA+TWIST
TD=0*0
XN(K)=(-ALP(K1)+TW1+TD)*BETAM
101 FORMAT(1H,315,1PE15.5)
40 CONTINUE
IF(NXT,THK*EQ.0.0) GO TO 23
C
C...........................................................................
C TEMPORARY CBAR, BREF
CBAR=1.0
BREF=1.0
C
L2=0
DO 22 I=1,NTVVP
DO 21 J=1,NXC2
LS=LS+1
L2=L2+1
YY=YSS(L2)-YCG
ZZ=ZSS(L2)-ZCG
VX=1.0*2.0*(QSTAR*ZZ/CBAR - RSTAR*YY/BREF)
PP = SQRT(1.0+TS(LS)**2)
SIGNAL=2.0*CHC(I)*BETAM*(THK(L2+1)-THK(L2))**2*VX/PP
AL=0.5*PLL((J+1)/2)
21 SIGNAL=0.5*BETAM*(THK(L2+1)-THK(L2))**2*X**AL/(XC(J)*1.0-XC(J))
22 L2=L2+1
23 CONTINUE
WRITE(6,3001) (SIGMA(I),I=1,290)
3001 FORMAT(*0SIGMA/*((IP1321K*4))
WRITE(6,100)(SIGMAP(I),I=1,40)
WRITE(6,6000) NLOOK
6000 FORMAT(*0NLOOK IN PANCE/*((1015))
CALL XLINE(ETLE,YLE,NTIP,ETC,DUMMY,NT1)
CALL XLINE(ETLE,ZLE,NTIP,ETC,DUMZ,NT1)
BS=0.0
DO 200 I=1,NTVPP
DUMX(I)=SQR((DUMY(I+1)-DUMY(I))**2+(DUMZ(I+1)-DUMZ(I))**2)
BS=BS+DUMX(I)
200 CONTINUE
CALL XLINE(ETLE,YLE,NTIP,ETC,DUMY,LTCPP)
CALL XLINE(ETLE,ZLE,NTIP,ETC,DUMZ,LTCPP)
WRITE(12) B*REFA,LTCPP,ETC(I),I=1,50)*LCPP,XCC,LEY,DTI,TC,CHC
1 1,CHRN(XP(I),DUMY(I),DUMZ(I),I=1,LTCPP)*CPLT,SPCF,BS,DUMX
RETURN
END
REAL FUNCTION P(T, TS)

1 S = SIN(.5*(TS-T))
2 SD = SIN(.5*(TS+T))
3 IF(S*SD) 4 3 4
4 T = T + 1.0E-6
5 GOTO 1
6 C = COS(.5*(TS+T))
7 CD = COS(.5*(TS-T))
8 IF(C*CD) 6 3 8
9 CTS = COS(TS)
10 CT = COS(T)
11 CTSPCT = CTS + CT
12 P = (((CTS-CT)* ALOG(ABS(S/SD)) + CTSPCT*CTSPCT* ALOG(ABS(C/CD))+
13 1(4.0*TS*CTS-2.0*SIN(TS)*SIN(T)) / (6.2832 85207 17958*(1.0-CTS))
14 IF(LEGVAR(P)) 3 9 3
15 RETURN
16 C  BLAINE D. GAITHER 9/72
17 END
REAL FUNCTION SINH(TH,THS)

1 5 = SINH(THS-TH)*6.5
   SD = SINH(THS+TH)*6.5
   IF(S<SD) 3*2.3
2 5H = TH : 1.0E-6
   GOTO 1:
3 5 = COS((THS+TH)*6.5)
   CD = COS((THS-TH)*6.5)
   IF(C*CD) 4*2.4
4 5H = COS(THS)
   CT = COS(TH)
   M = 3.1830 98861 83790E-1 ((CTH - CTH)*AOLS(ABS(S/SD)) + CT)*4.5
   1AOLS(ABS(C/CD))+2.0*THS*SIN(TH)
   IF(LEGVAR(!)) 2*5.2
5 RETURN

C  BLAINE D. GAITHER 9/72
END
SUBROUTINE GELG(R,A,M,N,EPS,IER)

SUBROUTINE GELG

PURPOSE
TO SOLVE A GENERAL SYSTEM OF SIMULTANEOUS LINEAR EQUATIONS.

USAGE
CALL GELG(R,A,M,N,EPS,IER)

DESCRIPTION OF PARAMETERS
R - THE M BY N MATRIX OF RIGHT HAND SIDES. (DESTROYED)
ON RETURN R CONTAINS THE SOLUTION OF THE EQUATIONS.
A - THE M BY M COEFFICIENT MATRIX. (DESTROYED)
M - THE NUMBER OF EQUATIONS IN THE SYSTEM.
N - THE NUMBER OF RIGHT HAND SIDE VECTORS.
EPS - AN INPUT CONSTANT WHICH IS USED AS RELATIVE
TOLERANCE FOR TEST ON LOSS OF SIGNIFICANCE.
IER - RESULTING ERROR PARAMETER CODED AS FOLLOWS
IER=0 - NO ERROR.
IER=-1 - NO RESULT BECAUSE OF M LESS THAN 1 OR
Pivot element at any elimination step
EQUAL TO 0.
IER=k - WARNING DUE TO POSSIBLE LOSS OF SIGNIFI-
CANCE INDICATED AT ELIMINATION STEP K+1,
WHERE PIVOT ELEMENT WAS LESS THAN OR
EQUAL TO THE INTERNAL TOLERANCE EPS TIMES
ABSOLUTELY GREATEST ELEMENT OF MATRIX A.

REMARKS
INPUT MATRICES R AND A ARE ASSUMED TO BE STORED COLUMNWISE.
IN M*N+M+1 SUCCESSIVE STORAGE LOCATIONS. ON RETURN,
SOLUTION MATRIX R IS STORED COLUMNWISE TOO.
THE PROCEDURE GIVES RESULTS IF THE NUMBER OF EQUATIONS M IS
GREATER THAN 0 AND PIVOT ELEMENTS AT ALL ELIMINATION STEPS
ARE DIFFERENT FROM 0. HOWEVER WARNING IER=k - IF GIVEN -
INDICATES POSSIBLE LOSS OF SIGNIFICANCE. IN CASE OF A WELL
SCALING MATRIX A AND APPROPRIATE TOLERANCE INTERPRETED THAT MATRIX A HAS THE RANK K.
GIVEN IN CASE M=1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE.

METHOD
SOLUTION IS DONE BY MEANS OF GAUSS-ELIMINATION AND COMPLETE PIVOTING.

DIMENSION A(1),R(1)
IF(M)23,23:1

1 IER=0
PIV=0.
NM=N*M
MM=N*M
DO 3 L=1,MM
TB=ABS(A(L))
IF(TB-PIV)3,3,2
2 PIV=TB
I=L
3 CONTINUE
TOL=EPS*PIV
A(1) IS PIVOT ELEMENT. PIV CONTAINS THE ABSOLUTE VALUE OF A(1).

START ELIMINATION LOOP
LST=1
DO 17 K=1,M

TEST ON SINGULARITY
IF(PIV)23,23:4
4 IF(IER)7,5,7
5 IF(PIV-TOL)6,6,7
6 IER=K-1
7 PIVI=I*AI(I)
   J=(I-1)/M
   I=I-J*M-K
   J=J+1-K
C
C I+K IS ROW-INDEX, J+K COLUMN-INDEX OF PIVOT ELEMENT
C
C PIVOT ROW REDUCTION AND ROW INTERCHANGE IN RIGHT HAND SIDE R
C
DO 8 L=K+NM+M
LL=L+I
TB=PIVI*R(LL)
R(LL)=R(L)
8 R(L)=TB
C
C IS ELIMINATION TERMINATED
C
IF(K-M)9,18,18
C
9 LEND=LST+M-K
   IF(J)12,12,10
10 II=J*M
   DO 11 L=LST,LEND
   TB=A(L)
   LL=L+II
   A(L)=A(LL)
11 A(LL)=TB
C
C ROW INTERCHANGE AND PIVOT ROW REDUCTION IN MATRIX A
C
12 DO 13 L=LST,MM,1
LL=L+I
TB=PIVI*A(LL)
A(LL)=A(L)
13 A(L)=TB
C
C SAVE COLUMN INTERCHANGE INFORMATION
C
A(LST)=J
C ELEMENT REDUCTION AND NEXT PIVOT SEARCH
PIV=C 
LST=LST+1 
J=0 
DO 16 II=LST,LEND 
PIVI=-A(II)  
IST=II+M 
J=J+1 
DO 15 L=IST,MM,M 
LL=L-J 
A(L)=A(L)+PIVI*A(LL) 
TB=ABS(A(LL)) 
14 IF(TB-PIV)15,15,14 
PIV=TB 
I=L 
15 CONTINUE 
DO 16 L=K,NM,M 
LL=L+J 
16 R(LL)=R(LL)+PIVI*R(L) 
17 LST=LST+M 
C END OF ELIMINATION LOOP 
C C 
C BACK SUBSTITUTION AND BACK INTERCHANGE 
18 IF(M-1)23,22,19 
19 IST=NM+M 
LST=M+1 
DO 21 I=2,M 
II=LST-I  
IST=IST-LST 
L=IST-M 
L=A(L)+I 
20 TB=TB-A(K)*R(LL) 
K=J+L 

\begin{verbatim}
21 \textbf{R(K)}=T8
22 \textbf{ERROR RETURN}
23 \textbf{IER=-1}\textbf{RETURN}
END
\end{verbatim}
SUBROUTINE QTGF(X,Y,Z,NDIM)

SUBROUTINE QTGF

PURPOSE
TO COMPUTE THE VECTOR OF INTEGRAL VALUES FOR A GIVEN
GENERAL TABLE OF ARGUMENT AND FUNCTION VALUES.

USAGE
CALL QTGF (X,Y,Z,NDIM)

DESCRIPTION OF PARAMETERS
X   - THE INPUT VECTOR OF ARGUMENT VALUES.
Y   - THE INPUT VECTOR OF FUNCTION VALUES.
Z   - THE RESULTING VECTOR OF INTEGRAL VALUES. Z MAY BE
      IDENTICAL WITH X OR Y.
NDIM - THE DIMENSION OF VECTORS X,Y,Z.

REMARKS
NO ACTION IN CASE NDIM LESS THAN 1.

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
NONE

METHOD
BEGINNING WITH Z(1)=C, EVALUATION OF VECTOR Z IS DONE BY
MEANS OF TRAPEZOIDAL RULE (SECOND ORDER FORMULA).

FOR REFERENCE, SEE
F.B. HILDEBRAND, INTRODUCTION TO NUMERICAL ANALYSIS,
McGRAW-HILL, NEW YORK/TORONTO/LONDON, 1956, PP.75.

DIMENSION X(1),Y(1),Z(1)
SUM2=0.
IF(NDIM-114,3,1

C
C INTEGRATION LOOP
1 DO 2 I=2*NDIM
SUM1=SUM2
SUM2=SUM2+5*(X(I)-X(I-1))*(Y(I)+Y(I-1))
2 Z(I-1)=SUM1
3 Z(NDIM)=SUM2
4 RETURN
END
SUBROUTINE EFIL(XCJ, XCI, NI, E)

C
FILL E MATRIX
C
C
XCJ = .25 PANEL
XCJ = .75 PANEL
C
DIMENSION E(NI,NI), XCI(1), XCJ(1)
NJJ=NI-1

DO 10 J=1, NJJ
DO 10 I=1, NI
10 E(J,I) = 1.0/(XCJ(J)-XCI(I))
DO 20 I=1, NI
20 E(NI,I) = 1.0
RETURN
C
BLAINE D. GAITHER 9/72
END
SUBROUTINE FILNFK(W, NF, NJ, THETA, FLAG, THET)
SUBROUTINE TO FILL THE NF OR NK SECTION OF THE W MATRIX

C
C W = NF MATRIX
C NF = NUMBER OF COL IN W
C NJ = NUMBER OF ROWS IN NJM
C THETA = ARRAY OF UNIQUE THETA FOR KS
C FLAG = LOGICAL VARIABLE TO WORK ON NF PER WORK ON NK
C THET = ARRAY OF THETA FROM FILLNF

DIMENSION W(NJ,NF), THETA(N), THET(1)
LOGICAL FLAG
DATA PI/3.14159265358979/

IF(NF.LE.0) RETURN
I = 1
NJ = NJ - 1
DO 400 M=1, NF
TH = THETA(M)
THMPI = TH - PI
DO 300 J=1, NJ
IF((FLAG.AND.(THET(J).GE.TH)).OR.((.NOT.FLAG).AND.(THET(J).GT.TH2)),
1)) GOTO 221
W(J,I) = THMPI
GOTO 300
221 W(J,I) = TH
300 CONTINUE
W(NJ+1) = .5 * SIN(TH)
400 I=I+1
RETURN
C BLAINE D. GAITHER 9/72
END
SUBROUTINE FILLNU(NU, NJ, NUE, XC, SCRTCH)

C SUBROUTINE TO FILL THE NU SECTION OF THE W MATRIX

C NUM  = NU MATRIX
C NJ   = NUMBER OF ROWS OF NU
C NUE  = NUMBER OF ELEMENTS IN NUM (NU*NJ)
C XC   = ARRAY OF X OVER C S
C SCRTCH= ARRAY OF THETAS
C
C THIS ROUTINE SHOULD BE CALLED ONCE (FOR THE ROUTINE WITH THE LARGEST NU)

 REAL NUM(NUE), XC(NJ), SCRTCH(NJ)

NU = NUE/NJ
NJ = NJ - 1
DO 10 I = 1, NJ

NUM(I) = 1.0
10 SCRTCH(I) = A * COS(1.0 - 2.0*X(I))

NUM(NJ) = -5
IF (NU .LE. NJ) RETURN

NUE = NUE - 1
RI = 0
DO 30 J = NJ, NUE, NJ
RI = RI + 1.0
DO 40 L = 1, NJ
40 NUM(J+L) = -COS((RI ) * SCRTCH(L))

IF (J = NJ) 20 20 21

20 DO 30 J = NJ, NUE, NJ

20 NUM(J+NJ) = .25

GOTO 30
20 CONTINUE
20 NUM(J+NJ) = .0

30 CONTINUE
RETURN
END
FUNCTION TANDEL(ETA, XOC)
COMMON STATEMENT THAT PUTS FLP(11) ON THE FLAPS
THIS FUNCTION RETURNS TAN(DELTA) IF A CP FALLS ON A FLAP,
 OTHERWISE ZERO
DIMENSION FLP(8,1)
NF = 1
TANDEL = 0.0
7 IF(FLP(1*NF))2*1.2
2 IF(FLP(3*NF)-ETA) 3*3.4
3 IF(FLP(4*NF)-ETA)4* 5*5
5 T = FLP(5*NF) + (FLP(6*NF)-FLP(5*NF))*(ETA - FLP(3*NF)) /
1 (FLP(4*NF) - FLP(3*NF))
IF((XOC - T)* FLP(1*NF)) 6* 6* 4
4 NF = NF +1
GOTO 7
6 TANDEL = TAN( FLP(2*NF) * 0.0174 53292 51994 32957)
1 RETURN
C BLAINE D. GAITHER OCT 72
END
C
PROGRAM TO INTERPOLATE FOR A FUNCTION AT A GIVEN (X/C*ETA) POINT.
C GRID UPON WHICH INTERPOLATION IS MADE IS GIVEN BY
C X = GIVEN VALUES OF X/C,  NX=NUMBER OF X VALUES.
C E = GIVEN VALUES OF ETA,  NE=NUMBER OF ETA VALUES.
C THE INPUT GRID IS DESCRIBED BY ONE SET OF X/C VALUES
C AND ONE SET OF ETA VALUES.
C F = GIVEN VALUES OF A FUNCTION OF (X*E), GIVEN FIRST LONGITUDINALLY
C AND THEN SPANNWISE. (EX: IF NX=10,NE=5, F(42) IS AT X(2) AND E(41))
C (NUMBER OF F VALUES TO BE AVAILABLE MUST EQUAL NX*NE).
C
X = X ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
NX = NUMBER OF X GIVEN, TO MAXIMUM OF 20. (DIMENSION OF F1,F2 = 20)
EC = ETA ARRAY FOR POINTS AT WHICH INTERPOLATED VALUES ARE FOUND.
NEC = NUMBER OF EC GIVEN.
FC = INTERPOLATED FUNCTION VALUES, GIVEN CHORDWISE THEN SPANNWISE.
NUMBER OF OUTPUT VALUES OF FC = NXC*NEC

INTERPOLATION IN THE X DIRECTION IS MADE BY SUBROUTINE CORDL.
INTERPOLATION IN THE ETA DIRECTION IS LINEAR.

DIMENSION X(1),E(1),XC(1),EC(1),F(1),FC(1)
DIMENSION F1(20),F2(20)
NE1=NE-1
L=0
DO 5 J=1,NEC
  EI=EC(J)
  DO 5 IE=1,NE1
    IEL=IE
    IF(EI*GT.E(I+1)) GO TO 5
    ETA1=E(I+1)
    ETA2=E(I+1)
    GO TO 6
5 CONTINUE
6 CONTINUE
DELET=ETA2-ETA1
D = ELE-ETA1
I1=(IEL-1)*NX+1

12=I1+NX
CALL CODIM(X*F(12)*NX*XC+F2*NXC)
CALL CODIM(X*F(I1)*NX*XC+F1*NXC)
DO 50 I=1,NXC
L=L+1
FC(L)=F1(I)+D*(F2(I)-F1(I))/DELET
50 CONTINUE
RETURN
END
SUBROUTINE XLINE(X,Y,N,X1,Y1,N1)
C THIS ROUTINE DOES LINEAR INTERPOLATION AT N1 (X1,Y1) POINTS
C ON SEGMENTS BETWEEN N (X,Y) POINTS (GIVEN).
C ALL X1 MUST BE WITHIN RANGE OF VALUES OF X.
C VALUES OF X MUST BE ALGEBRAICALLY ASCENDING.
C
DIMENSION X(1),Y(1),X1(1),Y1(1)
L2=2
DO 10 I=1,N1
2  IF(X1(I)*GT,X(L2)) GO TO 5
3   L1=L2-1
4   DX=X(L2)-X(L1)
5   D1=X1(I)-X(L1)
6   DY=Y(L2)-Y(L1)
7   Y1(I)=D1*DY/DX+Y(L1)
8   L2=L2+1
9   GO TO 2
10  CONTINUE
11  RETURN
12  END
PROGRAM INFLM
COMMON DA(5000)
1 *NXNXTH, LNVDOR, TVOR, TVS, BVY, ITV, NTHV, BVY, NT4(49)
2 *LNDIV, LTDIV, LNTS, LPTS
COMMON/PANEL/, NPA, IRSY, INC, NRVAP, NTVAP, LUCFR, LTCFR, LNCPP, LTPFR
1 *NPERD, NSPACE, HATTCH, NTRATT, NRCRL, NPRCLT, NUC, LTXC, NCTET, THAC
2 *NHE, NTP, NHTIP, ROOT, OUTER, WATT
3 *NPT, XPT2, XPT3, XPT4, XPT5, XPT6
COMMON/NUMBER/, NTPS(7), NCTPS(7), YLN(7), YLT(7), LTC(7), LUC(7)
1 *NC, *NBOD, NPA, S, V, L(7), VLT(7), TAPE, TAPE, NCT, TAPE, JTAPE
2 *LS6G(7), TLS6(7), LFUNC(7), TFUNC(7)
3 *LNDIV(7), LTDIV(7), NERR(7), ROOTP(7), OUTERP(7), SY,,,,(7)
COMMON /CONS/, /DIMA/
COMMON/BODY/B3(31000)
DIMENSION BODYR(21000)
COMMON/COPTS/, XQ(1320), YQ(1320), ZQ(1320)
1 *XN(1320), YN(1320), ZN(1320)
EQUIVALENCE (B(1), BODYR)
EQUIVALENCE (DA(2), PANS)
EQUIVALENCE (SY, DA(3))
EQUIVALENCE (DA(1), XCG, TQA, YCG, TCA, ZCG, TCA, ALPHA)
1 *(DA(11), BETA), (DA(12), STAR), (DA(13), QSTAR), (DA(14), RSTAR)
COMMON/SCRAT/, X55L, Y55L, Z55L, AX, AY, AZ(5000)
1 , AYP(5000), AZP(5000)
DIMENSION A(5000)
EQUIVALENCE (A, AXB), (B, 23)
DIMENSION B(5000)
EQUIVALENCE (SYM, DA(19))
COMMON/PANINP/, PANSP(19)
LOGICAL ROOT, OUTER
LOGICAL ROOTP, OUTER
C REWIND UNIT 11 IN THIS PROGRAM. UNIT 11 WILL HAVE SLVT, SYNT, SY"
NPANS=NPANS
C
TEMPORARY VALUES FOR BREF,CBAR
BREF=1.0
CBAR=10.0
IF(NBODS.EQ.0) GO TO 1111
C READ 2 RECORDS ON 18 TO SPACE OVER ALGNT1,ALGNT2 ARRAYS
READ(18) B
READ(18) h
C
1111 CONTINUE
C
NBP=NBODS+NPANS
M=0
DO 10 KK=1,NBP
LTPTS=LTC(KK)
LNPTS=LNC(KK)
DO 10 JJ=1,LTPTS
DO 10 II=1,LNPTS
M=M+1
XX=XQ(M)-XCG
YY = YQ(M) -YCG *BETAM
ZZ = ZQ(M) -ZCG *BETAM
VY = -BETA*BETAM-2.0*(PSTAR*ZZ-RSTAR*XX)/BREF
VZ = ALPHA*BETAM+2.0*(PSTAR*YY/BREF-QSTAR*XX/CBAR)
VX=1.0-2.0*(OSTAR*ZZ/CBAR-RSTAR*YY/BREF)
10 BOUND(M)=-XX*(M)*VX-Y*(M)*VY-Z*(M)*VZ
NB=0
NCTV=0
KCON=0
IF(NBODS.EQ.0) GO TO 99
DO 50 I=1,NBODS
NB=NB+1
SY=SY+1(I)
NBV=NBV+1
KTV=KTV+1
N3=NBV+1
N2=NLT(I)
MSEG=LSEG(I)+TSEG(I)
KCON=KCON+1SEG
LNDIVB(I)=1
LNDIV=1
LTDIV=LTDIVB(I)
READ(18) SS
CALL INFL(NSEG,NEP)
REWIND 19
REWIND 20
CONTINUE

C MATRIX FOR INFLUENCE OF BODY VORTICES IS ON UNIT 21.
C AX*AY*AZ MATRICES FOR BODY INFLUENCES IS ON UNIT TAPE (19 OR 20).
C UNIT TAPE (20 OR 19) IS AVAILABLE FOR ANOTHER USE.
CALL MATA(KCON,NCTV,TTAPE,0)
IF(KCON.NE.0),REWIND 23
CONTINUE
IF(NPANS.EQ.0) GO TO 101

NCOLP=0
DO 100 I=1,NPANS
SYM=PANSYM(I)
NB = NB+1
NBVNP=NVL(NB)
NTVNP=NVT(NB)
ROOT=ROOTP(I)
OUTER=OUTERP(I)
WRITE(6,40) I,ROOT,OUTER
WRITE(6,40) I,ROOTP(K),OUTERP(K),K=1,NPANS)
FORMAT(*81=*14/10L5)
NSPACE=NSPP(I)
READ(18) BODYRC
IF(.NOT.ROOT) GO TO 55
NROOT=NSPACE+1
NS=NBVNP+1
55 IF(.NOT.OUTER) GO TO 60
NVTOUT=NTVNP-NSPACE
CONTINUE
IP=1
CALL PANMAT(XG,YG,ZG,NCT,NCOLP,IP)
CONTINUE
XP2 = UNIT FOR STORAGE OF AX*AY*AZ FOR PANEL VORTICES.
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SUBROUTINE PANMAT(XO,YO,ZO,MCP,NCCLP,IP)
COMMON DA(5000)
COMMON BODY/XVR(10,20),YVR(10,20),ZVR(10,20),XVO(10,20)
1  YVR(10,20),ZVR(10,20),XVO(10,20)
2  XVR(20),XCC(20),XLE(20),YVR(20),ZLE(20)
3  XTE(20),YTE(20),ZTE(20),XJF(20),YJ(20),ZJ(20)
4  XTE(20),XTO(50),XTR(50),ZTO(50),XRF(20),YRF(20),ZRF(20)
5  SXMT(1000),SYM(1000),ZXM(1000),YS(1000),ZS(1000)
6  TS(1000),XSS(1000),YSS(1000),ZSS(1000),SIGMA(1000)
7  XVS(100),YVS(100),ZVS(100)
COMMON/CONPTS/ DUMQ(3960)
1  *XN(1320),YN(1320),ZN(1320)
COMMON/SCRAT/XSOL(5000),BSUNW(5000),AXR(5000),AYR(5000),AZR(5000)
EQUAL(AX,AXE),(AY,AYE),(AZ,AZE),(X3,XSOL(1)),(X3,XSOL(101))
1  *(Z3,XSOL(201)),(XT,XSOL(301)),(YT,XSOL(401))
2  *(ZT,XSOL(501)),(SUM,XSOL(601))
DIMENSION AX(5000),AY(5000),AZ(5000),X3(1001),Y3(1001),Z3(1001)
1  XT(100),YT(100),ZT(100),SUM(100)
DIMENSION B(5100)
EQUAL(AX,AXE)
C FREE TRAILING VORTEX VARIABLES...........
C XN,YN,ZN, USE SAME SPACE AS UTV,VTY,VTZ TO BE COMPUTED IN TRAIL.
DIMENSION XTV(1000),YTV(1000),ZTV(1000),XTVI(500)
1  ZTVI(500),YTVI(500),XTVI(1000),V1(100),V2(100)
2  V3(100),NOP(100),NTVE(100)
EQUAL(B15001),UTV, (R16001),VTI, (R17001),VTI
1  *(B18001),XTV, (B19001),YTV, (B20001),ZTV
2  *(B21001),XTVI, (B21001),YTVI, (B22001),ZTVI
3  *(B22501),V1, (B22501),V2, (B22501),V3
4  *(B22001),NOP, (B22701),NTVE
5  *(B23001),NTV
C MP9=NTR=NO. OF FREE TRAILING VORTICES.
C MP10 = NTRV=TOTAL NO. OF INITIAL VORTICES.
EQUAL((MP9,NTP),(MP10,TTR))
COMMON/PANEL/IPANC,IPSYS,ICW4,IVVP,ITVP,LMVCL,LMVCL,LMVCL,LTCPP
1  NPERP,TSPACE,IMATT,IMATT,IPRC,IPRC,IPRC,IPRC
2  NTHET,HTIP,HTIP,HTIP,HTIP,HTIP
3  MP1,MP2,MP3,MP4,MP5,MP6,MP7,MP8,MP9,MP10

COMMON/NONUMBER/ AIVPTS(7) • NCPTS(7) • NLN(7) • NLT(7) • LTB(7) • NLC(7) 3 1630
COMMON /PANNF/ PAPSYN(10)

1. COMMON /PANNF/ PAPSYN(10)
   EQUIVALENCE(DA(3432)*XP01), (DA(3433)*YPO), (DA(3434)*ZP0)
DIMENSION XP(11), YP(11), ZP(11)
DIMENSION SAVEC(6000) • ASSP(1000) • AS(1000)
EQUIVALENCE(R1(15001), SAVEC1, R2(21001), ASSP1, R3(22001), AS)
DIMENSION YPD(2001), ZPD(2001)
EQUIVALENCE(YPD(1), R(30001), ZPD(1), R(30201))
REAL LLEGY, LLEGX, LLEGZ
LOGICAL FLAG

LOGICAL: ROOT, OUTER

DATA LSIG/0/
DATA LDD/0/ IF(IP.GT.1) GO TO 999
NTR=0
NTRV=0
NU=FNU
NV=FNW
999 CONTINUE
NVL1=NBVVP
PI=3.141592654
IC=0
NBV1=NBVVP+1
IF(NOT-ROOT) GO TO 2
C
XVR1=XVR1(NBVI)
IF(XVR1.GE.XVT(1NNATT)) GO TO 2
DO 1 I=1, NNATT
IF(XVT(I).LT.XVR1) IC=IC+1
1 CONTINUE
C
LET IC BE NUMBER OF LONGITUDINAL PANELS IN TRAILING VORTEX SECTION.
N1=NSPACE+1
DO 3 I=1, N1
X3(I)=XVR1(I)• NBPI
3 1980
3 1990
3 2000
3 2010
Y3(I) = YVR(I, NBP1)
Z3(I) = ZVR(I, NBP1)
WRITE(6, 101) (X3(I), Y3(I), Z3(I), I = 1, N1)
101 FORMAT (1HE/1H *1P3E20.6))
CALL TVG:N(I, XVT, YVT, ZVT, NMA1T, IC, X3, Y3, Z3, N1, X, YT, ZT, NT)
WRITE(6, 100) NMA1T, IC, N1, NT
100 FORMAT (7H PANMAT/1015)
WRITE(6, 101) (XT(I), YT(I), ZT(I), I = 1, NT)
IF (IX, EQ, 1) WRITE(6, 502) NCP
502 FORMAT(*QONCP=*, I4))
NTX = NT/N1
DO 503 I = 1, N1
NTR = NTR + 1
503 NIP(NTR) = NTX
K = 0
DO 504 KK = 1, N1
DO 504 KK1 = 1, NTX
K = K + 1
NTRV = NTRV + 1
XTVI(NTRV) = XT(K)
YTVI(NTRV) = YT(K)
ZTVI(NTRV) = ZT(K)
IF (NTX .NE. KK1) GO TO 504
LDD = LDD + 1
YPD(LDD) = YTVI(NTRV)
ZPD(LDD) = ZTVI(NTRV)
504 CONTINUE
2 CONTINUE
NIT = NTVVP + 1 - NSPACE
DO 505 K = 1, NIT
IF (NSPACE .NE. 0. AND. K = EQ. 1) GO TO 505
NTR = NTR + 1
NIP(NTR) = 1
NTRV = NTRV + 1
XTVI(NTRV) = XVS(K)
YTVI(NTRV) = YVS(K)
ZTVI(NTRV) = ZVS(K)
LDD = LDD + 1
YPD(LDD) = YTVI(NTRV)
240
ZPD(LPD)=ZTVI(NTRV)
WRITE(6,7002) (1,YPD(I),ZPD(I)*I=1,LPD)
7002 FORMAT(*PANEL DRAG COORDINATES*/(I5,2F15.5))
505 CONTINUE
WRITE(6,800)XTVI(I),YTVI(I),ZTVI(I),I=1,NTRV
800 FORMAT(26HXTVI*,YTVI*,ZTVI*,IN PADMAT/*(1P?20.,6))
C SET NV=NO. OF FREE TRAILING VORTICES.
NV=NTR
WRITE(6,100) NSPACE,NVL1
DO 1000 IX=1,NCP
X=XQ(IX)
Y=YQ(IX)
Z=ZQ(IX)
MA=0
MS=0
IF(*.NOT.*ROOT) GO TO 105
DO 104 I=1,NSPACE
DO 104 J=1,NVL1
DO 5 K1=1,3
5 SUM(K1)=0.0
DO 50 K=1,4
IF(J.EQ.NVL1.AND.K.EQ.4) GO TO 41
IF(K.GT.1) GO TO 20
X1=XVR(I,J+1)
Y1=YVR(I,J+1)
Z1=ZVR(I,J+1)
X2=XVR(I,J)
Y2=YVR(I,J)
Z2=ZVR(I,J)
GO TO 4C
20 X1=X2
Y1=Y2
Z1=Z2
IF(K-3) 25,30,35
25 X2=XVR(I+1,J)
Y2=YVR(I+1,J)
Z2=ZVR(I+1,J)
GO TO 40
30 X2=X/R(I+1,J+1)
Y2=YVR(I+1,J+1)
Z2=ZVR(I+1,J+1)
GO TO 40
3 2300
3 2310
3 2320
3 2330
3 2340
3 2350
3 2360
3 2370
3 2380
3 2390
3 2400
3 2410
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3 2980
3 2990
3 3000
3 3010
3 3020
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3 3070
3 3080
3 3090
3 3100
3 3110
3 3120
3 3130
3 3140
3 3150
3 3160
3 3170
3 3180
3 3190
409 CONTINUE
C NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR RIGHT LEG.
T1=SQR((Y2-YC)**2+(Z2-ZC)**2)
IF(T1<0.00001) 412,411,411
411 CONTINUE
T2=(X2-XC)/SQR((X2-XC)**2+(Y2-YC)**2+(Z2-ZC)**2)
QT=0.25*(1.0-T2)/(PI*T1)
RLEGY=RLEGY+SYMPS*QT*(Z2-ZC)/T1
RLEGZ=RLEGZ+SYMPS*QT*(Y2-YC)/T1
412 CONTINUE
IF(SYMPS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 471
   Y2=Y2H
   GO TO 472
471 SYMPAS=1.0
   Y2=-Y2
   GO TO 409
472 CONTINUE
IF(I.EQ.1) GO TO 421
   LLEGX=-RLEGX1
   LLEGY=-RLEGY1
   LLEGZ=-RLEGZ1
   GO TO 4721
421 DO 431 K1=1,3
431 SUM(K1)=0.0
   DO 461 K1=1,IC
      IF(K1.EQ.1) GO TO 441
      X2=X1
      Y2=Y1
      Z2=Z1
      GO TO 451
441 X2=XT(I1)
   Y2=YT(I1)
   Z2=ZT(I1)
451 I1=K1+1
   X1=XT(I1)
   Y1=YT(I1)
   Z1=ZT(I1)
461 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,XC,YC,ZC)
   LLEGX=SUM(1)
LLEGY=SUM(?)
LLEGZ=SUM(3)
Y1H=Y1
SYMPAS=-1.0

4091 CONTINUE
C NOW ADD STRAIGHT LINE TRAILING VORTEX CONTRIBUTION FOR LEFT LEG.
T1=SQRT((Y1-YC)**2+(Z1-ZC)**2)
IF(T1<0.00001) 470, 415, 415

415 CONTINUE
T2=(X1-XC)/SQRT((X1-XC)**2+(Y1-YC)**2+(Z1-ZC)**2)
QT=0.25*(1.0-T2)/(PI*T1)
LLEGY=LLEGY+SYMPAS*QT*(Z1-ZC)/T1
LLEGZ=LLEGZ+SYMPAS*QT*(Y1-YC)/T1

470 CONTINUE
IF(SYMPAS.EQ.-1.0.AND.PANSYM(IP).EQ.0) GO TO 4711
Y1=Y1H
GO TO 4721

4711 SYMPAS=1.0
Y1=-Y1
GO TO 4091

4721 CONTINUE
SUM(1)=SUM1+RLEGX+LLEGX
SUM(2)=SUM2+RLEY+LLEY
SUM(3)=SUM3+RLEGZ+LLEGZ

480 CONTINUE
C PROVIDE HERE FOR STR. LINE TR. VORTICES WHERE PANEL TR. EDGE IS
C EITHER AT END OF BODY OR BEHIND IT. (IF=0 CASE)
50 CONTINUE
M4=MA+1
AX(MA)=SUM(1)
AY(MA)=SUM(2)
AZ(MA)=SUM(3)
DO 90 IS=1,2
MS=MS+1
IV=0
JV=0
T=TS(MS)
DZ=DZS(MS)
DY=DYS(MS)
KSOL=1
CALL PVSKIT, DZ, XC, YC, ZC, MS, WA, JV, KSOL

90 CONTINUE
104 CONTINUE
105 CONTINUE
C 20 OUTER PANEL INFLUENCE EQUATIONS AND SOURCE INFLUENCE EQUATIONS.
NVOUT=NTVVP-NSPACE
DO 200 I=1,NVOUT
DO 200 J=1,NTVVP
MS=MS+1
WA=WA+1
KSOL=2
110 T=TS(MS)
DZ=DZS(MS)
DY=DYS(MS)
CALL PVSKIT, DZ, XC, YC, ZC, MS, WA, J, KSOL
IF(KSOL.EQ.1) GO TO 115
KSOL=1
MS=MS+1
GO TO 110
115 CONTINUE
200 CONTINUE
IF(DA(5000).LT.0) GO TO 361
WRITE(6,300) IX(I), AX(I), AX(I), AZ(I), I=1,WA
300 FORMAT(///"INFLUENCE COEFFICIENTS FOR CONTROL PT. I3///
  1 (I3,1PE18.6))
WRITE(6,350) IX(I), SX:I(I), SY:I(I), SZ:I(I), I=1,5S
350 FORMAT(///"SOURCE COEFFICIENTS FOR CONTROL PT. I3///
  1 (I3,1PE18.6))
361 CONTINUE
C IN SUBROUTINE PANKAT, WRITE THE SX'I'T, SY'I'T, SZ'I'T MATRICES ON "WT II"
C THERE WILL BE NO ROWS OF SX'I'T FOR EACH PANEL.
C NCT = TOTAL NUMBER OF CONTROL POINTS
WRITE(11)(SX'I'T(I), SY'I'T(I), SZ'I'T(I), I=1,5S)
XH1=XH(I)
YH1=YH(I)
ZH1=ZH(I)
SN=0.0
DN=0.0
BNY=0.0
J=LSIG
DO 355 I=1,MS
J=J+1
BNX=SIGMA(J)*SXMT(I)+BNX
BYN=SIGMA(J)*SYMT(I)+BYN
355 BNX=SIGMA(J)*(XN1*SXMT(I)+YN1*SYMT(I)+ZN1*SZMT(I))+BN
IF(NBODS.EQ.0) GO TO 358
IF(I*X.LE.NCPTS(I)) GO TO 359
358 BOUND(IX)=BOUND(I)+BN
359 CONTINUE
IF(NU+NW*.EQ.0) GO TO 3060
CALL CONFUN FOR CONSTRAINT MATRIX MULTIPLY.
LOC1=(IP-1)*1000+1
LOC2=LOC1+400
LOC3=LOC2+400
FLAG=.FALSE.
IF(NW.EQ.0)FLAG=.TRUE.
CALL CONFUN(AX,SAVEC(LOC1),SAVEC(LOC2),SAVEC(LOC3),NSPACE
1 ,ASPP,NVVP,TVVP,AS,NW,FLAG,NU)
IF(FLAG) GO TO 1050
MA=NU*NW
DO 1005 I=1,MA
1005 AX(I)=ASSP(I)
GO TO 1060
1050 MA=NU*NW
DO 1055 I=1,MA
1055 AX(I)=ASPP(I)
GO TO 1060
1060 CONTINUE
CALL CONFUN(AY,SAVEC(LOC11),SAVEC(LOC21),SAVEC(LOC31),NSPACE
1 ,ASSP,NVVP,TVVP,AS,MA,FLAG,NU)
IF(FLAG) GO TO 2050
DO 2005 I=1,MA
2005 AY(I)=ASSP(I)
GO TO 2060
2050 DO 2055 I=1,MA
2055 AY(I)=ASPP(I)
2060 CONTINUE
CALL CONFUN(AZ,SAVEC(LOC11),SAVEC(LOC21),SAVEC(LOC31),NSPACE
100 IF(FLAG) GO TO 3050
   DO 3055 I=1,NA
   3055 AZ(I)=ASSP(I)
   GO TO 3060
   3060 CONTINUE
   LSIGH=J
   IF(IP*NE.1) GO TO 400
   IF(IP*NE.1) GO TO 260
   NCOLP=NCOLP+2
   MUNIT=MTAPE
   360 CONTINUE
C MUNIT IS THE FILE TO BE WRITTEN.
C MUPR IS THE FILE TO BE READ.
C FOR IP=1, MUNIT=MTAPE=19 (OR 20).
C FOR IP=2, MUNIT=24
C MUPR=19 (OR 20).
C THEREAFTER, FOR IP EVEN, MUNIT AND MUPR SAME AS FOR IP=2.
C FOR IP ODD, MUNIT AND MUPR VALUES ARE REVERSED.
C
   IF(IP.EQ.1) WRITE(*,500) IP,MTAPE,NCOLP
500 FORMAT(*IP,MTAPE,NCOLP,215)
   WRITE(MTAPE)(AX(I),AY(I),AZ(I),I=1,NCOLP)
   GO TO 900
   400 IF(2*(IP/2).NE.IP) GO TO 401
      MUNIT=24
      GO TO 402
401 MUNIT=MTAPE
402 MC1=MCPRE+1
   MC2=MCPRE+1
   J=0
   DO 424 I=MC1,MC2
      J=J+1
      AX(I)=AX(J)
      AY(I)=AY(J)
424 AZ(I)=AZ(J)

IF (IX.EQ.1) WRITE (6,501) IP,'"MUNPR='UNIT,NCPR+"A
501 FORMAT(40I8,MUNPR,UNIT,NCPR,'A='5,5(5B1))
READ (MUNPR) (AX(I),AY(I),AZ(I),I=1,NCPR)
IF (IX.EQ.1) NCOLP=NCPR + "A
WRITE (UNIT) (AX(I),AY(I),AZ(I),I=1,NCOLP)
900 CONTINUE
1000 CONTINUE
LSIG=LSIGH
NCPR=NCOLP
MUNPR=MUNIT
MP2=MUNIT
MP3=NCOLP
REWIND MUNPR
REWIND MTAPE
REWIND 24
RETURN
END
SUBROUTINE INFL(KCN,NBP)

COMPUTE AX,AY,AZ MATRICIES AND BOUNDARY CONDITIONS.
COMMON DA(5000)
1  NX,XYTH,LXVOR,LTVOR,NTV,NXYV,NXYTV,NXTH,NXYTH,NTH(49)
2  LNDBN,TDIV,LTPTS
COMMON/NUMBRE/ AVPTS(7),NLPTS(7),NLN(7),NL,LT,LC,LN(7)
1  NCT,ND,NNOPS,NNP,NNVL,NTV,NTAPE,NTAPE,NTAPE,NTAPE
2  LSNG(7),TSEG(7),LSEG(7),TSEG(7),TSEG(7)
3  LNDBN(7),TDIV(7),NNPS(7),ROтвер(7),OUTER(7),SYMN(7)

COMMON/SCAY/ XV1(151,31),XY(151,31),XZ(151,31)
1  TIX(1320),TTX(1320),TXZ(1320)
2  TTY(1320),TYX(1320),TYZ(1320)
COMMON/COMPTS/ XX(1320),XY(1320),XZ(1320)
1  XX(1320),XY(1320),XZ(1320)
COMMON/SCRAT/ XSO(5000),YSO(5000),AXS(5000),AYS(5000),BZ(5000)
COMMON/INDEX/ X,Y,Z,111,112,191,112
DIMENSION SUM(3)
EQUIVALENCE (DA(7),XCG), (DA(8),XCG), (DA(9),XCG), (DA(10),XCG), (DA(11),XCG), (DA(12),XCG), (DA(13),XCG), (DA(14),XCG)
1  (DA(15),XCG), (DA(16),XCG), (DA(17),XCG), (DA(18),XCG), (DA(19),XCG), (DA(20),XCG)

EQUIVALENCE (DA(21),XCG)
DIMENSION XSO(1320),AY(1320),AY(1320)
DIMENSION BS(3100)
DIMENSION 3 (5000)
EQUIVALENCE (BS,XYV(1,1))
EQUIVALENCE (S,XYV(1,1))
EQUIVALENCE (S,XYV(1,1))
EQUIVALENCE (XQS,XYV(1,1))
EQUIVALENCE (N,NVTV) (N,NVTV)
EQUIVALENCE (N,NVTV) (N,NVTV)
DIMENSION NLNGTH(5000)
C  IF KCN DOES NOT = 0, ALNGTH IS NEEDED FOR AXYZ.
IF(KCN.LT.0) GO TO 5
C  REIND 18 AND THEN READ ALNGTH ARRAY.
REIND 18
READ(18) ALNGTH
C  NOW SKIP 2 RECORDS SO THAT 18 IS POSITIONED TO READ PANEL DATA.
READ(18) A
READ(18) B
CONTINUE
PI4=12.5663704
NSTART=NCTV
NCT=0
DO 65 KK=1,NBP
LTPTS=LTC(KK)
LNPTS=LNC(KK)
DO 65 JJ=1,LTPTS
DO 65 II=1,LNPTS
NCT=NCT+1
L=M:
X=XQ(K)
Y=YQ(K)
Z=ZQ(K)
N=NSTART
DO 60 J=1,NTVV
DO 60 I=1,NEVV
N=N+1
TOT1=0.0
TOT2=0.0
TOT3=0.0
DO 55 K=1,4
DO 8 KS=1,3
SUM(KS)=0.0
GO TO (10*20*30*40),K
CONTINUE
12 III=LNDIV*(I-1)+1
IF1=III
III2=(J-1)*LTDIV
DO 14 L=1,LTDIV
III2=III2+1
IF2=III2+1
14 CALL VORTEX(SUM)
GO TO 50
20 III2=1+J*LTDIV
IF2=III2
III=(I-1)*LNDIV
DO 24 L=1,LNDIV
III=III+1
IF1=III+1
24 CALL VORTEX(SUM)
GO TO 50
111=1F1
112=112+1
GO 34 L=1•LTDIV
112=112-1
1F2=112-1
IF(I•NE•NBVV)= GO TO 33

C  
C ADD TRAILING VORTEX CONTRIBUTION.
C  
C RIGHT SIDE.
SYML0=1.0
301 CONTINUE
DX=XLV(111•112)-X
DY=SYML0•SYV(111•112)-Y
DZ=ZV(111•112)-Z
TYZ=DY**2+DZ**2
T1=SQRT(TYZ)
T2=DX/SQRT(DX**2+TYZ)
QT=(1.0-T2)/T1
QTT=SYML0•QT
SUM(2)=SUM(2)+QTT*DZ/T1
SUM(3)=SUM(3)-QTT*SY/T1

C  
C LEFT SIDE.
DX=XLV(1F1•1F2)-X
DY=SYML0•SYV(1F1•1F2)-Y
DZ=ZV(1F1•1F2)-Z
TYZ=DY**2+DZ**2
T1=SQRT(TYZ)
T2=DX/SQRT(DX**2+TYZ)
QT=(1.0-T2)/T1
QTT=SYML0•QT
SUM(2)=SUM(2)-QTT*DZ/T1
SUM(3)=SUM(3)+QTT*SY/T1

C  
IF(SYML0•NE•6.0)= GO TO 24
IF(SYML0=12.0•-1.0)= GO TO 34
SYML0=1.0
GO TO 301

C  
CALL VORTEX(SU")

3 6080
3 6090
3 6100
3 6110
3 6120
3 6130
3 6140
3 6150
3 6160
3 6170
3 6180
3 6190
3 6200
3 6210
3 6220
3 6230
3 6240
3 6250
3 6260
3 6270
3 6280
3 6290
3 6300
3 6310
3 6320
3 6330
3 6340
3 6350
3 6360
3 6370
3 6380
3 6390
3 6400
3 6410
3 6420
3 6430
3 6440
3 6450
3 6460
34   CONTINUE
35         GO TO 50
36   CONTINUE
37   I12=1F2
38   I11=I11+1
39   DO 44 L=1,LNDIV
40   I11=I11-1
41   1F1=I11-1
42   CALL_VORTEX(SUM)
43     TOT1=TOT1+SUM(1)
44       TOT2=TOT2+SUM(2)
45       TOT3=TOT3+SUM(3)
46   CONTINUE
47   AXB(N)=TOT1/PI4
48   AYB(N)=TOT2/PI4
49   AZB(N)=TOT3/PI4
50   IF(I.NE.1) GO TO 60
51       AXB(N)=0.0
52       AYB(N)=0.0
53       AZB(N)=0.0
54   CONTINUE
55
      C IF BODY HAS CONSTRAINT FUNCTIONS, CONVERT AX TO AXRL. SAME FOR AY, AZ
56     IF(KCN.EQ.0) GO TO 601
57       NS1=NSTART+1
58       CALL AXYZRL(NS1,ALNGTH,AXB,AYB,AZB)
59   CONTINUE
60     N1=1
61     IF(NB.EQ.1) GO TO 62
62       NB=NB-1
63       N2=0
64       DO 61 J=1,NB1
65           N2=N2+NVL(J)*NVT(J)
66       READ(NTAPE)(AXB(I),AYB(I),AZB(I),I=1,N1,N2)
67     WRITE(MTAPE)(AXB(I),AYB(I),AZB(I),I=1,N1,NCTV)
68     CONTINUE
69   MTAPE=39-MTAPE
70   NTAPE=39-NTAPE
71   RETURN
72   END
SUBROUTINE CONFUN(A, SCRCH, SCRCI, TETA, NROOT, ASSP, NETA, NXOC, 3 6860
1 AS, NW, FLAG, NCOL)
3 6870
3 6880
C A = A MATRIX (ONE ROW FROM FRED)
C SCRCH = T(XOC)
C SCRCI = T(XOC) FOR USE IN ROOT SECTION
C TETA = T(ETA)
C NROOT NUMBER OF ETAS IN ROOT SECTION OF PANEL
C ASSP = OUTPUT A/
C NETA = NUMBER OF ETAS
C NXOC = NUMBER OF XOCs
C AS = CHORDWISE TRANSFORMED MATRIX A/
C NW = NUMBER OF LATERAL CONSTRAINT FUNCTIONS
C FLAG = IF FALSE SOLVE FOR COMPLETELY CONstrained
C NCOL = NU + NF + NK (NUMBER OF COLs OF AS)
DIMENSION A(NXOC, NETA), SCRCH(NXOC, NCOL), SCRCI(NXOC, NCOL)
1 TETA(ETA+NW), AS(NCOL, NETA), ASSP(NCOL, NW)
LOGICAL FLAG
IF(NROOT) 41, 41, 1
1 DO 40 IETA = 1, NROOT
DO 40 IC = 1, NCOL
TEM P=0.0
DO 30 K=1, NXOC
30 TEM P = TEM P + SCRCI(K*IC)*A(K*IETA)
40 AS(IC*IETA) = TEM P
41 IS = NROOT + 1
IF(IS GT NETA) RETURN
DO 60 IETA = IS, NETA
DO 60 IC = 1, NCOL
TEM P = 0.0
DO 50 K = 1, NXOC
50 TEM P = TEM P + SCRCH(K*IC)*A(K*IETA)
60 AS(IC*IETA) = TEM P
IF(FLAG) RETURN
C SUBROUTINE TO SOLVE COMPLETELY CONSTRAINED MATRIX
DO 70 INW = 1, NW
DO 70 IUKF = 1, NCOL
TEM P = 0.0
DO 20 IETA = 1, NETA

20  TEMP = TENV + TETAL + TETALE
30  ASSUMING...
40  RETURN BLA
50  END
SUBROUTINE TVG11(X,Y,Z,N,N5,X1,Y1,Z1,N1,XT,YT,ZT,NT)
CALCULATES FIRST ITERATION TRAILING VORTEX GEOMETRY.
C X,Y,Z = JUNCURE TRAILING VORTEX, N=N0,OF, POINTS.
C X1,Y1,Z1 = PANEL TRAILING EDGE, N1=NO,OF, POINTS.
C XT,YT,ZT = COMPUTED TRAILING VORTEX POINTS, NT=NO,OF, POINTS.
C
DIMENSION X(1),Y(1),Z(1),X1(1),Y1(1),Z1(1),XT(1),YT(1),ZT(1)
DIMENSION XS(21),YS(21),ZS(21),SA(21),SJ(21)
DIMENSION DCX(20),DCY(20),DCZ(20),XTT(21),YTT(21),ZTT(21)
WRITE(5,100)(I,X(I),Y(I),Z(I),I=1,N)
WRITE(6,100)(I,X(I),Y(I),Z(I),I=1,N1)
FORMAT(1004/13,1P3E18.5))
NT=(NS+1)'N1
DX=(X(N)-X1(1))'NS
XS(1)=X(1)
DO 1 I=1,NS
1 XS(I+1)=XS(I)+DX
NS1=NS+1
CALL CODIM(X,Y,N,XS,YS,N51)
CALL CODIM(X,Z,N,XS,ZS,N51)
SAI=0.0
DO 5 I=1,NS
5 DY=YS(I+1)-YS(I)
DZ=ZS(I+1)-ZS(I)
SAI = SAI + SQRT(DX*2 + DY*2 + DZ*2)
SA(I+1)=SAI
SD=SAI/NS
SJ(I)=0.0
DO 10 I=1,NS
10 SJ(I+1)=SJ(I)+SD
CALL CODIM(SA,XS,NS1,SJ,XT,NS1)
CALL CODIM(SA,YS,NS1,SJ,YT,NS1)
CALL CODIM(SA,ZS,NS1,SJ,ZT,NS1)
C DIVIDE LAST BOUND VORTEX INTO EQUAL SPACES.
N1=N1-1
XF=X(N)
YF=Y1(N1)
ZF=Z1(N1)
DX=0.0
DY=(YF-Y(N1))/N1
DZ=(ZF-Z(N1))/N1
DO 15 I=2*N1
IT1=(I-1)*NS1
C (IT1+1) PT. IS ON FIRST BOUND VORTEX.
N3=IT1+1
XT(N3)=X1(I)
YT(N3)=Y1(I)
ZT(N3)=Z1(I)
IT2=I*NS1
XT(IT2)=XT(IT1)
YT(IT2)=YT(IT1)+DY
ZT(IT2)=ZT(IT1)+DZ
15 CALL NORM(DX,DY,DZ)
N2=N1-1
NN=NS1*N11+1
C COMPUTE DX ALONG LAST TRAILING VORTEX.
DXL=(XF-X1(N1))/NS
XT(NN)=X1(N1)
DO 16 I=1*NS
NN=NN+1
XT(NN)=XT(NN-1)+DXL
YT(NN)=Y1(N1)
ZT(NN)=Z1(N1)
16 NP=N11*NS1
N4=N2-1
DO 20 J=2*NS
CSX=XT(NP+J)-XT(J)
CSY=YT(NP+J)-YT(J)
CSZ=ZT(NP+J)-ZT(J)
C52=CSX**2+CSY**2+CSZ**2
PER=(J-1+0)/NS
DENOM=0.0
DO 18 I=1*N2
DXLE=X1(I+1)-X1(I)
DYLE=Y1(I+1)-Y1(I)
```
DZLE=Z1(I+1)-Z1(I)
CALL NORM(DXLE,DYLE,DZLE)
DIRX=DXLE+(DX-DXLE)*PER
DIRY=DYLE+(DY-DYLE)*PER
DIRZ=DZLE+(DZ-DZLE)*PER
18
CALL NORM(DIRX,DIRY,DIRZ)
DCX(I)=DIRX
DCY(I)=DIRY
DCZ(I)=DIRZ
DNM=DENOM + DOT(DIRX,DIRY,DIRZ,CSX,CSY,CSZ)
DS=CS2/DENOM
NN=NS1+J
DO 20 I=1,N4
N3=NN-NS1
XT(NN)=XT(N3)+DS*DCX(I)
YT(NN)=YT(N3)+DS*DCY(I)
ZT(NN)=ZT(N3)+DS*DCZ(I)
20
NN=NN+NS1
DO 30 I=2,N11
NN=(I-1)*NS1
SAI=0.0
435
SAI(1)=0.0
DO 22 J=1,NS
N2=NN+J
N4=N2+1
DX=XT(N2)-XT(N4)
DY YT(N2)-YT(N4)
DZ=ZT(N2)-ZT(N4)
SAI=SAI+SORT(DX**2+DY**2+DZ**2)
22
SAI(J+1)=SAI
SD=SAI/NS
DO 23 J=1,NS1
N2=NN+J
XTT(J)=XT(N2)
YTT(J)=YT(N2)
23
ZTT(J)=ZT(N2)
SJ(I)=0.0
DO 24 J=1,NS
24
SJ(J+1)=SJ(J)+SD
```
SUBROUTINE WATCON

C PERFORM PHI•THETA MATRIX MULTIPLY ON AXB•AYB•AZB MATRICES.
C NO REPLACEMENT IS MADE FOR BODIES HAVING NO CONSTRAINT FUNCTIONS.

COMMON DA(5000)
1 *NXT•NVT•NTV•NVV•NTX•NTH(49)
2 *NLN•LTV•LNV•LNT•NTH(7)
COMMON/NUMBER/ NVPTS(7)•NRPTS(7)•NLN(7)•NLT(7)•LNC(7)
1 •NC•NB•NBODS•NPANS•NVL(7)•NVT(7)•NTAPE•NTAPE•NCTV•NTAPE
2 •LSSEG(7)•TSSEG(7)•LFUNC(7)•TFUNC(7)
3 •LNDIVB(7)•LTDIVB(7)•NSPP(7)•ROOTP(7)•OUTERP(7)•SYMP(7)

COMMON/BODY/ ARRAY(31000)

DIMENSION BX(5000)•BY(5000)•BZ(5000)•PHTH(1000)

EQUIVALENCE (BX•ARFAY(1))•(BY•ARFAY(1001))•(BZ•ARRAY(10001))

COMMON/SCRAT/XSCL(5000)•BOUND(5000)•AXB(5000)•AYB(5000)•AZB(5000)

DO 1000 T=1•NC
READ(NTAPE)(AXB(K)•AYB(K)•AZB(K)•K=1•NCTV)

M=0
IV=0
DO 1000 NB=1•NBODS
NVP=NVL(NB)•NVT(NB)
IF (LSSEG(NB)•EQ.0.0 AND TSEG(NB)•EQ.0.0) GO TO 50

KODE=1
NC=ITAPE
GO TO 70

NC=NVP

KODE=0
DO 60 II=1•NC
M=M+1
IV=IV+1
BX(M)=AXB(IV)
BY(M)=AYB(IV)

BZ(M)=AZB(IV)
GO TO 100

CONTINUE
IS=IV
DO 80 II=1•NC
SX=0.0

80 CONTINUE
GO TO 90

90 STOP
SY=0.0
SZ=0.0
READ(231)(PHTH(KC),KC=1,NVP)
M=M+1
DO 75 I1=1,NVP
IS11=IS+I1
SX=SX+PHTH(I1)*AXR(ISI1)
SY=SY+PHTH(I1)*AYR(ISI1)
75    SZ=SZ+PHTH(I1)*AZR(ISI1)
IV=IV+1
BX(M)=SX
BY(M)=SY
80    BZ(M)=SZ
100 CONTINUE
IF(KODE.EQ.1) REWIND 23
1000 WRITE(MTAPE)(BX(K),BY(K),BZ(K),K=1,N)
C CHANGE MEANING OF NCTV FROM TOT. NO. OF VORTICES ON BODIES
C TO NUMBER OF COLUMNS OF 'A' MATRIX. (FOR DISCRETE CASE, NO CHANGE.)
NCTV=4
NTAPE=39-MTAPE
MTAPE=39-MTAPE
REWIND 19
REWIND 20
RETURN
END
<table>
<thead>
<tr>
<th>Line</th>
<th>Description</th>
<th>Column</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SUBROUTINE MATA(KCON,NCOLS,UNIT,IBP)</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>C KCON = 0 FOR PANEL SIDE. KCON = 0 OR ISSEG FOR BODY SIDE (SEE INFLM)</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>C NCOLS = NO. OF AXB,AYB,AZB (NO. OF COLS OF BODY OR PANEL SIDE)</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>COMMON DA(5000)</td>
<td>3</td>
</tr>
<tr>
<td>5</td>
<td>1  *NX,NXTH,INOR,LIVOR,NTV,NBTV,NTV,NXTH,NBV NTH(49)</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2  *LNDIV,LTDIV,LTPTS,LPTS</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>COMMON/NUMBER/ NVPTS(7),NCPTS(7),NL(7),NLT(7),LTC(7),LNC(7)</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>1  *NCT,NB,NBODS,NPANS,NVL(7),NVT(7),NTAPE,NTAPE,NCTV,NTAPE,NTAPE</td>
<td>3</td>
</tr>
<tr>
<td>9</td>
<td>2  *LSEG(T),TSEG(F),TLSEG,FUNC(T),TFUNC(F)</td>
<td>3</td>
</tr>
<tr>
<td>10</td>
<td>3  *LNDIVB(7),LTDIVB(7),NSPP(7),KROOTP(7),OUTERP(7),SYN(7)</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>COMMON/CONPTS/XG(1320),YG(1320),ZG(1320),XN(1320),YN(1320),ZN(1320)</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>COMMON/SCRAT/XSOL(5000),BOUND(5000),AXB(5000),AYB(5000),AZB(5000)</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>DIMENSION A(5000)</td>
<td>3</td>
</tr>
<tr>
<td>14</td>
<td>EQUIVALENCE(A,AXB)</td>
<td>3</td>
</tr>
<tr>
<td>15</td>
<td>EQUIVALENCE(DA(2),PANS)</td>
<td>3</td>
</tr>
<tr>
<td>16</td>
<td>NPANS = PANS</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>IWR = 21</td>
<td>3</td>
</tr>
<tr>
<td>18</td>
<td>IF(IBP.EQ.1) IWR=10</td>
<td>3</td>
</tr>
<tr>
<td>19</td>
<td>NBP = NBODS + NPANS</td>
<td>3</td>
</tr>
<tr>
<td>20</td>
<td>IF(KCON.EQ.0) GO TO 100</td>
<td>3</td>
</tr>
<tr>
<td>21</td>
<td>CALL MATCON</td>
<td>3</td>
</tr>
<tr>
<td>22</td>
<td>100 L=0</td>
<td>3</td>
</tr>
<tr>
<td>23</td>
<td>DO 300 I=1,NBP</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>LTPTS=LTC(I)</td>
<td>3</td>
</tr>
<tr>
<td>25</td>
<td>LNPTS=LNC(I)</td>
<td>3</td>
</tr>
<tr>
<td>26</td>
<td>DO 300 JJ=1,LTPTS</td>
<td>3</td>
</tr>
<tr>
<td>27</td>
<td>DO 300 II=1,LNPTS</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>L=L+1</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>XNN=XN(L)</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>YNN=YN(L)</td>
<td>3</td>
</tr>
<tr>
<td>31</td>
<td>ZNN=ZN(L)</td>
<td>3</td>
</tr>
<tr>
<td>32</td>
<td>READ(UNIT)(AXB(K),AYB(K),AZB(K),K=1,NCOLS)</td>
<td>3</td>
</tr>
<tr>
<td>33</td>
<td>DO 200 K=1,NCOLS</td>
<td>3</td>
</tr>
<tr>
<td>34</td>
<td>A(K)=XNN<em>AXB(K)+YNN</em>AYB(K)+ZNN*AZB(K)</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>300 WRITE(IWR)(A(K),K=1,NCOLS)</td>
<td>3</td>
</tr>
<tr>
<td>36</td>
<td>REWIND IWR</td>
<td>3</td>
</tr>
<tr>
<td>37</td>
<td>REWIND NTAPE</td>
<td>3</td>
</tr>
<tr>
<td>38</td>
<td>JTAPE=21</td>
<td>3</td>
</tr>
</tbody>
</table>
SUBROUTINE AXYZRL(KSTART, ALNGTH, AXF, AYR, AZR)
DIMENSION ALNGTH(1)
DIMENSION AXS(1), AYR(1), AZR(1)
COMMON /DAT5000/  
1  NX, NXTH, LMVOR, LTVOV, NTWV, NBV, NTWV, NBV, NTW(49)
2  LNIV, LTVIV, LNPTS, LTPTS
COMMON/NUMBER/ NVPETS(7), NCPTS(7), NLN(7), NLT(7), LTC(7), LNC(7)
1  NS, NB, NPANE, NPOL(7), NWK(7), NTPE, NTPE, NTPE, NTPE, NTPE
2  LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)
3  LNIVS(7), LTVIVS(7), NSPP(7), RGCEP(7), OUTERP(7), SYM(7)
DO 100 J1=1, NTWV
   I2=J1*NBV
   I3=KSTART-1+I2
   ADDX=0.0
   ADDY=0.0
   ADDZ=0.0
   DO 100 I1=1, NBV
       ADDX=AXB(I1)+ADDX
       AXB(I1)=ADDX*ALNGTH(I1)
       ADDY=AYB(I1)+ADDY
       AYB(I1)=ADDY*ALNGTH(I1)
       ADDZ=AZB(I1)+ADDZ
       AZB(I1)=ADDZ*ALNGTH(I1)
   I1=I1+1
100 CONTINUE
RTURN
END
SUBROUTINE PVSKIT(DYY,DZZ,XY,YC,ZC,MS,IV,JV,KSOL)
COMMON DA(13000)
EQUIVALENCE(DA(13426),SYM)
COMMON/PANIF/*PANSY:(10)
COMMON/CRAT/XSG(5000)*BOUN(5000)*AX(5000)*AY(5000)*AZ(5000)
COMMON/ODY/VXR(10*20)*YVR(10*20)*ZVR(10*20)*XVO(10*20)
1 *YYO(10*20)*ZVO(10*20)*PLL(500)*PLT(500)*YSUBV(100)*CHORD(100)
2 *XCO(20)*XCCO(20)*XLE(20)*YLE(20)*ZLE(20)
3 *XTE(20)*YTE(20)*ZTE(20)*SLE(20)*XJI(20)*YJ(20)*ZJ(20)
4 *ELE(20)*XVI(50)*YVI(50)*ZVI(50)*XRI(20)*YRI(20)*ZRI(20)
5 *SXMT(1000)*SYMT(1000)*SZMT(1000)*SYS(1000)*DYS(1000)*DZS(1000)
6 *TS(1000)*XXS(1000)*YSS(1000)*ZSS(1000)*SIGMA(1000)
COMMON/PANEL/*NPAN/*IPSX/*IXC/*NBVP/*NTVP/*LNF/*LTCP/*LNCP/*LTCPP
1 *NPMP/*NPSPACE/*NATTCH/*NTRATT/*NPREL/*NPCRLT/*NTX*/NCTET/*NTHXC
2 *NTX/*NRP/*NTP/*ROOT/*OUTER/*ATT
3 *MP1/*MP2/*MP3/*MP4/*MP5/*MP6/*MP7/*MP8/*MP9/*MP10
COMMON/NUMBER/*NPVTS(7)*NCPTS(7)*NLN(7)*NLT(7)*LTC(7)*LNC(7)
1 *NCT/*NB/*NODS/*NPAN/*NVL(7)/NVT(7)/NTE/*NTAPE/*NTAPE/*NCTV/*ITAPE*/JTAPI
2 /LSEG(7)*TSEG(7)*LFCN(7)*TCFN(7)
3 /LNDIVB(7)*LTDIV(7)*NSPP(7)*ROOTP(7)*OUTER(7)*SYMP(7)

C KSOL=1 FOR SOURCE PTS. ONLY
C KSOL=2 FOR BOTH SOURCE PTS. AND VORTEX POINTS.
C MS=SUBSCRIPT OF SOURCE PT.
C IX/YC/ZC --- CONTROL PT.
C IV = LATERAL VORTEX SUBSCRIPT.
C JV = LONGITUDINAL VORTEX SUBSCRIPT.
C INSERT SPECIFICATION STATEMENTS HERE.
REAL 11,12,13,14
YVV=0.5*SORT(DYY**2+DZZ**2)
YVV=2.0*YVV
PI=3.141592654
YV=YVV
GO TO (10,20)*KSOL
10 YK=YSS(MS)
ZK=ZSS(MS)
XK=XSS(MS)
GO TO 25
20 IF(MS-2*(MS/2),EQ.0) GO TO 10

C C C
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>YK=YVO(IV+JV)</td>
</tr>
<tr>
<td>2</td>
<td>ZK=ZVO(IV+JV)</td>
</tr>
<tr>
<td>3</td>
<td>XK=XVO(IV+JV)</td>
</tr>
<tr>
<td>25</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>26</td>
<td>SUMX=0.0</td>
</tr>
<tr>
<td>27</td>
<td>SUMY=0.0</td>
</tr>
<tr>
<td>28</td>
<td>SUMZ=0.0</td>
</tr>
<tr>
<td>29</td>
<td>UTSUM=0.0</td>
</tr>
<tr>
<td>30</td>
<td>VTSUM=0.0</td>
</tr>
<tr>
<td>31</td>
<td>WTSUM=0.0</td>
</tr>
<tr>
<td>32</td>
<td>SIGH=1.0</td>
</tr>
<tr>
<td>33</td>
<td>DZ=ZC-ZK</td>
</tr>
<tr>
<td>50</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>51</td>
<td>DY=YC-SIGN*YK</td>
</tr>
<tr>
<td>52</td>
<td>RY=DYY/YV2</td>
</tr>
<tr>
<td>53</td>
<td>RZ=DZY/YV2</td>
</tr>
<tr>
<td>54</td>
<td>Y=RY<em>DY+RZ</em>DZ</td>
</tr>
<tr>
<td>55</td>
<td>Z=-RZ<em>DY+RY</em>DZ</td>
</tr>
<tr>
<td>56</td>
<td>R12=(Y+YV)<strong>2+Z</strong>2</td>
</tr>
<tr>
<td>57</td>
<td>R22=(X-T<em>Y)<strong>2+Z</strong>2</em>(1.0+T*T)</td>
</tr>
<tr>
<td>58</td>
<td>R32=(Y-YV)<strong>2+Z</strong>2</td>
</tr>
<tr>
<td>59</td>
<td>R4= SQRT((X-T*YV)**2+(Y-YV)<strong>2+Z</strong>2)</td>
</tr>
<tr>
<td>60</td>
<td>R5= SQRT((X+T*YV)**2+(Y+YV)<strong>2+Z</strong>2)</td>
</tr>
<tr>
<td>61</td>
<td>I1=(X+T*YV)/R5</td>
</tr>
<tr>
<td>62</td>
<td>I2=(Y+T<em>X+YV</em>(1.0+T**2))/R5</td>
</tr>
<tr>
<td>63</td>
<td>I3=(Y+T<em>X-YV</em>(1.0+T**2))/R4</td>
</tr>
<tr>
<td>64</td>
<td>I4=(X-T*YV)/R4</td>
</tr>
<tr>
<td>65</td>
<td>IF(ABS(Z) GT YV2) GO TO 42</td>
</tr>
<tr>
<td>66</td>
<td>Z=0.0</td>
</tr>
<tr>
<td>67</td>
<td>R6D=(X+T*YV)**2+(Y+YV)**2</td>
</tr>
<tr>
<td>68</td>
<td>R7D=(X-T*YV)**2+(Y-YV)**2</td>
</tr>
<tr>
<td>69</td>
<td>RXY=(X-T*Y)**2</td>
</tr>
<tr>
<td>70</td>
<td>R6=RXY/R6D</td>
</tr>
<tr>
<td>71</td>
<td>R7=RXY/R7D</td>
</tr>
<tr>
<td>72</td>
<td>IF(R6 GE 0.0075968656) GO TO 41</td>
</tr>
<tr>
<td>73</td>
<td>IF(R7 GE 0.0075968656) GO TO 41</td>
</tr>
<tr>
<td>74</td>
<td>IF(ABS(Y) LE YY) GO TO 41</td>
</tr>
<tr>
<td>75</td>
<td>TERN=ABS(1.0/R7D-1.0/R6D)*U.5/P</td>
</tr>
</tbody>
</table>
GO TO 43
41 IF(ABS(Y) GT YV) GO TO 42
IF(ABS(X-T*Y) GE 0.25*ABS(PLL(NA))) GO TO 42
TERM1=0.0
GO TO 43
42 TERM1=(I2+13)/R22
CONTINUE
TERM2=(I1+1.0)/R12
TERM3=(I3+1.25)/R32
TERM4=1.0/R4-1.0/R5
P=SQRT(1.0+T*T)
EUS=(T*TERM4+(X-T*Y)*TERM1)/P
EVS=(TERM4-T*(X-T*Y)*TERM1)/P
EWS=P*Z*TERM1
US=0.25*EUS/PI
VS=0.25*EVs/PI
WS=0.25*EWS/PI
UT=US
VT=VS*RY-WS*RZ
WT=VS*RZ+WS*RY
UTSUM=UT+UTSUM
VTSUM=VT+VTSUM
WTSUM=WT+WTSUM
IF(KSOL.EQ.1) GO TO 45
EU=Z*TERM1
EV=Z*(-T*TERM1+TERM2-TERM3)
EW=-(X-T*Y)*TERM1-(Y+YV)*TERM2+(Y-YV)*TERM3
UV=0.25*EU/PI
VV=0.25*EV/PI
WW=0.25*EW/PI
VI=UV
VI=RY*VV-RZ*VV
W1=RY*VV+RZ*VV
SUMX=VI+SUMX
SUMY=VI+SUMY
SUMZ=VI+SUMZ
CONTINUE
44 FOR SYMMETRY: GET IMAGE CONTRIBUTION.
C IF(SYM.NE.0.0) GO TO 50
IF (SIGN.LT.0.0) GO TO 60
SIGN=-1.0
DZZ=-DZZ
T=-T
GO TO 50
60 CONTINUE
SXMT(MS)=UTSUM
SYMT(MS)=VTSUM
SZMT(MS)=WTSUM
IF (KSOL.EQ.1) RETURN
AX(MA)=SUMX
AY(MA)=SUMY
AZ(MA)=SUMZ
RETURN
END
SUBROUTINE VORPAN(UM,XI,YI,ZI,XF,YF,ZF,X,Y,Z)
COMMON DA(5000)
EQUIVALENCE(DA(3426),SYM)
DIMENSION SUM(1)
R4PI=0.07957747
YIH=YI
YFH=YF
XFQ=XF-X
ZFQ=ZF-Z
XIQ=XI-X
ZIQ=ZI-Z
SYMLOO=1.0
YFQ=YF-Y
YIQ=YI-Y
DELX=XF-XI
DELY=YF-YI
DELZ=ZF-ZI
RXS1=YFQ*DELZ-ZFQ*DELY
RXS2=ZFQ*DELX-XFQ*DELZ
RXS3=XFQ*DELY-YFQ*DELX
RXS =SORT(RXS1**2+RXS2**2+RXS3**2)
TERM1=SORT(DELX**2+DELY**2+DELZ**2)
TERM2=SORT(XFQ**2+YFQ**2+ZFQ**2)
TERM3=SORT(XIQ**2+YIQ**2+ZIQ**2)
TERM4= XFQ*DELX+YFQ*DELY+ZFQ*DELZ
RATIO = TERM4/TERM1**2
COSA=(DELX*XI+DELY*YI+DELZ*ZI)/TERM1**2
COSB = TERM4/TERM1*TERM2
CC=COSB-COSA
HX=XFQ-RATIO*DELX
HY=YFQ-RATIO*DELY
HZ=ZFQ-RATIO*DELZ
H=SQR1(HX*HX+HY*HY+HZ*HZ)
IF(H=0.00001) 11,12,12
11 COEF=0.0
GO TO 13
12 CONTINUE
HRXS=H*RXS
CCEF=R4PI*SYMLOO*CC/HRXS

3 11220
3 11230
3 11240
3 11250
3 11260
3 11270
3 11280
3 11290
3 11300
3 11310
3 11320
3 11330
3 11340
3 11350
3 11360
3 11370
3 11380
3 11390
3 11400
3 11410
3 11420
3 11430
3 11440
3 11450
3 11460
3 11470
3 11480
3 11490
3 11500
3 11510
3 11520
3 11530
3 11540
3 11550
3 11560
3 11570
3 11580
3 11590
3 11600
CONTINUE
SUM(1)=COEF*RXS1 + SUM(1)
SUM(2)=COEF*RXS2 + SUM(2)
SUM(3)=COEF*RXS3 + SUM(3)
YI=YIH
YF=YFH
IF(SYMLOO.EQ.-1.0 .OR. SYMNE.EQ.0.0) RETURN
SYMLOO=-1.0
YI=-YI
YF=-YF
GO TO 10
END
<table>
<thead>
<tr>
<th>Line</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>COMMON BODY: X(151:31), Y(151:31), Z(151:31)</td>
</tr>
<tr>
<td>3</td>
<td>COMMON CON: XQ(1320), YQ(1320), ZQ(1320)</td>
</tr>
<tr>
<td>3</td>
<td>COMMON DA(5000)</td>
</tr>
<tr>
<td>3</td>
<td>EQUIVALENCE (DA(19)*SYM)</td>
</tr>
<tr>
<td>3</td>
<td>COMMON INDEX: X, Y, Z, I1, I2, IF1, IF2</td>
</tr>
<tr>
<td>3</td>
<td>DIMENSION SJM(1)</td>
</tr>
<tr>
<td>3</td>
<td>XI=XV(I11*I12)</td>
</tr>
<tr>
<td>3</td>
<td>YI=YV(I11*I12)</td>
</tr>
<tr>
<td>3</td>
<td>ZI=ZV(I11*I12)</td>
</tr>
<tr>
<td>3</td>
<td>XF=XV(IF1*IF2)</td>
</tr>
<tr>
<td>3</td>
<td>YF=YV(IF1*IF2)</td>
</tr>
<tr>
<td>3</td>
<td>ZF=ZV(IF1*IF2)</td>
</tr>
<tr>
<td>3</td>
<td>XFO=XF-X</td>
</tr>
<tr>
<td>3</td>
<td>ZFO=ZF-Z</td>
</tr>
<tr>
<td>3</td>
<td>XIQ=XI-X</td>
</tr>
<tr>
<td>3</td>
<td>ZIQ=ZI-Z</td>
</tr>
<tr>
<td>3</td>
<td>SYMLOO=1.0</td>
</tr>
<tr>
<td>3</td>
<td>YFQ=YF-Y</td>
</tr>
<tr>
<td>3</td>
<td>YIQ=YI-Y</td>
</tr>
<tr>
<td>3</td>
<td>DELX=XF-XI</td>
</tr>
<tr>
<td>3</td>
<td>DELY=YF-YI</td>
</tr>
<tr>
<td>3</td>
<td>DELZ=ZF-ZI</td>
</tr>
<tr>
<td>3</td>
<td>RXS1=YFQ<em>DELZ-ZFO</em>DELY</td>
</tr>
<tr>
<td>3</td>
<td>RXS2=ZFO<em>DELX-XFO</em>DELZ</td>
</tr>
<tr>
<td>3</td>
<td>RXS3=XFO<em>DELY-YFO</em>DELX</td>
</tr>
<tr>
<td>3</td>
<td>RXS = SQRT(RXS1<strong>2+RXS2</strong>2+RXS3**2)</td>
</tr>
<tr>
<td>3</td>
<td>TERM1=SQRT(DELX<strong>2+DELY</strong>2+DELZ**2)</td>
</tr>
<tr>
<td>3</td>
<td>TERM2=SQRT(XFO<strong>2+YFO</strong>2+ZFO**2)</td>
</tr>
<tr>
<td>3</td>
<td>TERM3=SQRT(XIQ<strong>2+YIQ</strong>2+ZIQ**2)</td>
</tr>
<tr>
<td>3</td>
<td>TERM4= XFO<em>DELY+YFO</em>DELY+ZFO*DELZ</td>
</tr>
<tr>
<td>3</td>
<td>RATIO = TERM4/TERM1**2</td>
</tr>
<tr>
<td>3</td>
<td>COSA = (DELX<em>XI+YIQ+DELZ</em>ZIQ)/(TERM1*TERM3)</td>
</tr>
<tr>
<td>3</td>
<td>COSB = TERM4/(TERM1*TERM2)</td>
</tr>
<tr>
<td>3</td>
<td>CC = COSB-COSA</td>
</tr>
<tr>
<td>3</td>
<td>HX=XFO-RATIO*DELX</td>
</tr>
<tr>
<td>3</td>
<td>HY=YFO-RATIO*DELY</td>
</tr>
<tr>
<td>3</td>
<td>HZ=ZFQ-RATIO*DELZ</td>
</tr>
<tr>
<td>3</td>
<td>H=SQRTH(HX<strong>2+HY</strong>2+HZ**2)</td>
</tr>
</tbody>
</table>
IF(H-0.00001)11,12,12

11 COEF=0.0
GO TO 13

12 CONTINUE
HRXS=H*RXS
COEF=SYMLOO*CC/HRXS

13 CONTINUE
SUM(1)=COEF*RXS1 + SUM(1)
SUM(2)=COEF*RXS2 + SUM(2)
SUM(3)=COEF*RXS3 + SUM(3)
IF(SYMLOO.EQ.-1.0 OR SYMNE.EQ.0.0) RETURN
SYMLOO=-1.0
YF=-YF
GO TO 10

END
PROGRAM SOL
COMMON/SCRAT/XSOL(5000) , BOUND(5000) , AXB(5000) , AYB(5000) , AZB(5000)
COMMON/PANEL/ NPAN , IPSYM , IWC , NBVP , NTVVP , LNCFP , LTCFP , LNCP , LTCPP
1   * NPERPT , NSPACE , NATTCH , NTRATT , NPRCLN , NPRCLT , NWCTXC , NWCTET , NTHXC
2   * NTHET , NIP , CHIP , ROOT , OUTER , NNATT
3   * MP1 , MP2 , MP3 , MP4 , MP5 , MP6 , MP7 , MP8 , MP9 , MP10
COMMON/ BODY/ AA(31000)
DIMENSION PHTH(5000) , ALNTH(5000) , GA(5000)
EQUIVALENCE (PHTH = AA(1)) , (ALNTH = AA(5001)) , (GA = AA(10001))
COMMON/ NUMBER/ NVPTS(7) , NCPTS(7) , NLN(7) , NLT(7) , LTC(7) , LNC(7)
1   * NCT , NB , NBODS , NPANS , NVL(7) , NVT(7) , MTAPE , NTAPE , NCTV , ITAPE , JTAPE
2   * LSEG(7) , TSEG(7) , TFUNC(7) , LFUN(7)
3   * LNDIVB(7) , LTRIVB(7) , NSPP(7) , ROOTP(7) , OUTERP(7) , SYMP(7)
DIMENSION B1(1000) , P1(5000) , SAVE(6000) , DUM(2000)
DIMENSION SAVE(16000) , ATET(1000)
EQUIVALENCE (B1 , AXB(1)) = (P1 , AXB(1001)) = (SAVE , AYB(1001))
EQUIVALENCE (DUM , AZB(2001)) = (ATET , AZB(4001)) = (DA(2) , PANS)
EQUIVALENCE (SAVE , 1(15001))
COMMON DA(5100)
DIMENSION LT(10) , LN(10) , NP1(10)
COMMON/PANINF/ PANSYM(10) , ASOL(600)
NPANS = PANS
IF (NPANS .EQ. 0) GO TO 10
   DO 8 I = 1 , 6000
      8 SAVE(i) = SAVEC1(i)
10   CONTINUE
C    REWIND 23
C    REWIND 18 IN ORDER TO READ ALNTH ARRAY.
    REWIND 18
    WRITE(6*1) NCTV , MP3 , NCT
1   FORMAT(*OSURROUTINE SOL*/315)
    WRITE(6*3) (BOUND(I) , I = 1 , NCT)
3   FORMAT(20HOBOUNDARY CONDITIONS/(1P10E13.5))
2   FORMAT(*OSOLUTION*/(1P10E13.5))
   NCOLS = NCTV + MP3
   DO 20 I = 1 , NCOLS
      IF (BOUND(I) .LE .0) GO TO 22
20   CONTINUE
   DO 21 I = 1 , NCOLS
   21   CONTINUE
21 XSOL(I)=0.0  
   DO 211 I=1,600  
211 ASOL(I) = 0.0  
   GO TO 200  
22 CONTINUE  
   CALL MSOLX(NCTV*MP3*NCT*BOUND*XSOL*DUM)  
   DO 2305 I=1,600  
2305 ASOL(I)=XSOL(I)  
23 CONTINUE  
   NR=NCOLS  
   KCN=LSEG(1)+TSEG(1)  
   IF(KCN*EQ.0) GO TO 111  
   READ(18) ALNGTH  
   REWIND 18  
   NTVV=NVT(1)  
   NBVV=NVL(1)  
   NR=NTVV*NBVV  
   NC=ITAPE  
   WRITE(6*231) (XSOL(I),I=1,NCOLS)  
231 FORMAT(50H0*SOLUTION FOR COEFFICIENTS OF CONSTRAINT EQUATIONS/  
   1 (1P10E13*5))  
   C  
   CONVERT FIRST NC XSOL TERMS INTO NR K'S.  
   C NOW COMPUTE THE PRODUCT OF PHTHA = GA, A COLUMN MATRIX  
   DO 101 I=1, NR  
101 GA(I)=0.0  
   DO 102 J=1, NC  
   READ(23) (PHTH(I),I=1, NR)  
   DO 102 I=1, NR  
102 GA(I)=GA(I)+PHTH(I)*XSOL(J)  
   C  
   C NOW DO R*L MATRIX PRE-MULTIPLY OF GA VECTOR... RESULT IS K VECTOR.  
   K=0  
   DO 110 J=1, NTVV  
110 TL=0.0  
   DO 110 I=1, NBVV  
   K=K+1  
   AL1=ALNGTH(K-1)  
   AL2=ALNGTH(K)  
   4 0400  
   4 0410  
   4 0420  
   4 0430  
   4 0440  
   4 0450  
   4 0460  
   4 0470  
   4 0480  
   4 0490  
   4 0500  
   4 0510  
   4 0520  
   4 0530  
   4 0540  
   4 0550  
   4 0560  
   4 0570  
   4 0580  
   4 0590  
   4 0600  
   4 0610  
   4 0620  
   4 0630  
   4 0640  
   4 0650  
   4 0660  
   4 0670  
   4 0680  
   4 0690  
   4 0700  
   4 0710  
   4 0720  
   4 0730  
   4 0740  
   4 0750  
   4 0760  
   4 0770  
   4 0780
AL3=ALNGTH(K+1)
IF(I*EQ.1) AL1=0.0
IF(I*EQ.NBVV) AL3=0.0
AL=0.5*(0.75*AL1+AL2+0.25*AL3)
XSOL(K)=AL*GA(K)+TL
TL=XSOL(K)
110 CONTINUE
111 CONTINUE
NB1=NVT(1)*NVL(1)
IF(KCN *NE. 0) NB1 = ITAPE
IF(NBODS*EQ.0) NB1 = 0
IF(NBODS*EQ. 0) GO TO 301
DO 300 I=1,NB1
300 B1(I) = XSOL(I)
301 IF(NPANS*EQ.0) GO TO 700
DO 305 I=1,NPANS
NBPI = NBODS+I
LT(I) = NVT(NBPI)
LN(I) = NVL(NBPI)
IF(LFUNC(I+1) *NE. 0) LN(I) = LFUNC(I+1)
IF(TFUNC(I+1) *NE.0) LT(I) = TFUNC(I+1)
6000 FORMAT(615,3X,2F15.5)
305 NP1(I) = LT(I) * LN(I)
IP1 = NB1
DO 310 I=1,NPANS
IT = (I-1) *1000
M = NP1(I)
WRITE(6,6000) I,NP1(I),IT,M,IP1
DO 308 K=1,M
PL(IT+K) = XSOL(IP1+K)
308 CONTINUE
310 IP1 = IP1 +M
C NOW DO ANY CONVERSION NECESSARY FOR PANEL A'S TO K'S
DO 500 IP=1,NPANS
I=IP
M = NP1(I)
IT = (I-1) *1000
LOC1 = (IP-1) *1000
LOC2 = LOC1 +400
LOC3 = LOC2 +400
NBPI = NBODS + IP

IF(LN(I)) .EQ. NVL(NBPI) .AND. LT(I) .EQ. NVT(NBPI)) GO TO 499
   DO 315 K=1*M
      DUM(K) = PIL(I+K)
      NTVVP = NVT(NBPI)
      NU = LN(IP)
      NW=LT(IP)
      NBVVP = NVL(NBPI)
      IF(LT(I)) .NE. NVT(NBPI)) GO TO 330
      L = 0
      DO 320 J=1,NTVVP
         DO 320 I=1, NBVVP
            L = L + 1
            IF(J .GE. NSPP(IP)) GO TO 317
      SUM = 0.0
      DO 316 K=1,NU
         316 SUM = SUM + SAVEC(LOC2+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
      GO TO 319
      317 SUM = 0.0
      DO 318 K=1,NU
      318 SUM = SUM + SAVEC(LOC1+I+(K-1)*NBVVP) *DUM((J-1)*NU+K)
      319 XSOL(L) = SUM
      320 CONTINUE
      GO TO 361
      330 CONTINUE
      L = 0
      DO 355 J=1,NTVVP
         DO 355 I=1,NU
            L = L + 1
            SUM = 0.0
      346 SUM = SUM + DUM((K-1)*NU+I) *SAVEC((K-1)*NTVVP+J+LOC3)
      ATET(L) = SUM
      WRITE(6,6001) L,ATET(L)
      355 CONTINUE
      L = 0
      DO 360 J=1,NTVVP
DO 360 I=1*NBVVP
L = L+ 1
IF(J*GT= NSPP(IP)) GO TO 357
SUM = 0.0
DO 356 K=1*NU
SUM = SUM + SAVEC(LOC2+(K-1)*NBVVP+I) * ATET((J-1)*NU+K)
GO TO 359
357 SUM = 0.0
DO 358 K=1*NU
SUM = SUM + SAVEC(LOC1+(K-1)*NBVVP+I) * ATET((J-1)*NU+K)
359 XSOI(L) = SUM
WRITE(6*6001) L*XSOI(L)
360 CONTINUE
361 CONTINUE
IT=(IP-1)*1000
WRITE(6*6001) IT
DO 370 I=1,L
370 P(I(IT+I) = XSOI(I)
499 CONTINUE
500 CONTINUE
L = 0
IF(NBODS*EQ. 0) GO TO 600
NB1=NVT(1)*NVL(1)
DO 550 I=1,NB1
L = L + 1
550 XSOI(L) = B1(I)
600 IF(NPANS*EQ. 0) GO TO 700
DO 650 I=1,NPANS
IT = (I-1)*1000
M=NVT(NBODS+1)*NVL(NBODS+1)
DO 625 K=1,M
L = L + 1
625 XSOI(L) = P1(IT+K)
650 CONTINUE
WRITE(6*50)
50 FORMAT(47H0CONVERT SOLUTION FOR BODY CONSTRAINT FUNCTIONS)
NRM=NRM+NMVE
WRITE(6*150) (XSOI(I),I=1,L)
150 FORMAT(16H0SOLUTION MATRIX/((1P10E13.5))
200 CONTINUE
700 CONTINUE
WRITE(6,2)(XSOL(I)*I=1,NCOLS)
END

<table>
<thead>
<tr>
<th>Year</th>
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<tr>
<td>1960</td>
<td>1970</td>
<td>1980</td>
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<td>1990</td>
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</tbody>
</table>
SUBROUTINE NSOLX(NB,NP,NOT,B,XSOL,IL)
C
NKBE = NO. OF COLUMNS IN BODY INFLUENCE MATRIX.
C
NKTP = NO. OF COLUMNS IN PANEL INFLUENCE MATRIX.
DIMENSION B(11),XSOL(1),IL(1)
COMMON/BODY/ A(2:8000),AR(500),IXC(500)
COMMON DA(5000)
EQUVALENCB (FUNClDA(34)), (FNB, DA(30)), (FNT,DA(31))
NBV = FNB
NTVV = FNT
WRITE(*,1000)
1000 FORMAT(*1A MATRIX*)
NK = NB+NP
IF(NB.EQ.0 OR FUNCNE.0) GO TO 5
NK = NK - NTVV
5 CONTINUE
NKTP = NK + 1
N = (NK*(NK+3)) / 2
10 A(I) = 0.0
NKTP = NK + 2
DO 60 K = 1,NKTP
IF(NB.EQ.0) GO TO 1
READ(21)(AR(L),L = 1,NB)
IF(FUNCNE.0) GO TO 23
L2 = 1
L1 = 1
NBM1 = NBV - 1
DO 22 J = 1,NTVV
DO 21 I = 1,NBM1
AR(L1+1) = AR(L1+L2)
21 L1 = L1 + 1
22 L2 = L2 + 1
23 CONTINUE
IF(K.GT.20) GO TO 20
1001 FORMAT(*100W13) BODY ON BODY/(1P10E13.4))
20 CONTINUE
1 IF(NP.EQ.0) GO TO 2
NN = NB + 1
IF(NB.EQ.0 OR FUNCNE.0) GO TO 24
NN = NN - NTVV
24 CONTINUE
READ(10)(AR(I),L=NN,NKI)
IF(K.GT.20) GO TO 30
1003 FORMAT(15REW13, 8WING ON BODY/(1P10E13.4))
30 CONTINUE
2 CONTINUE
AR(NKT+1)=B(K)
IXI=1
DO 50 I=1,NKT
R=SQRT(A(IXI)**2+AR(I)**2)
IF(R.EQ.0.) GO TO 50
C=A(IXI)/R
S=AR(I)/R
IXJ=IXI
DO 40 J=1,NKTP
T2=C*A(IXJ)+S*AR(J)
AR(J)=-S*A(IXJ)+C*AR(J)
A(IXJ)=T2
40 IXJ=IXJ+1
50 IXI=IXI+NKT2-1
60 CONTINUE
REWIND 21
REWIND 10
II=1
IXI=1
DO 80 I=1,NKT
IXC(I)=IXI
XSOL(I)=0.
IL(I)=0
IF(A(IXI).LE.0.0000001) GO TO 80
IL(I)=II
II=II+1
80 IXI=IXI+NKT2-1
II=NKT
DO 210 J=1,NKT
IF(IL(I).LE.0) GO TO 210
JI=IL(I)
JS=IXC(JI)-JI
JXI=JS+1
JXN=JS+NKTP
IF(I-I-NKT) 170*200*220
170 IK=IK+1
  JXK=JXI
  DO 180 K=IK,NKT
    JXK=JXK+1
  180 XSOL(I)=XSOL(I)-A(JXK)*XSOL(K)
  200 XSOL(I)=(XSOL(I)+A(JXN))/A(JXI)
  210 II=II-1
  220 CONTINUE
    IF(NB.EQ.0.OR.FUNCT.EQ.0) GOTO 310
    L2 = 1
    L1 = 1
  300 DO 301 J=1,NTVV
        301 L2=L2+1
        IF(NP.EQ.0) GOTO 304
        DO 303 I=1,NP
            AR(L1)=XSOL(L1-NTVV)
            303 L1=L1+1
  304 DO 305 I=1,L1
        305 AR(I)=AR(I)
  310 CONTINUE
    RETURN
END
SUBROUTINE SFSOL (M,N)
COEFFICIENT MATRIX ON UNIT 23 BY ROWS
  M ROWS.
  N COLUMNS
B VECTOR (RIGHT SIDE) IN BOUND
COMMON /BODY/S(150),X(1000), BWORK(150),IWORK(150),COL(150),
TOPCOL(150), WORK(150), A(1000), ATB(1000), A(1000),
2PAD(600)
COMMON /SCRAT/XSOL(5000)*BOUND(5000)
INTEGER UNIT
REWIN 10
CALL ATRAN (A,M,N,BOUND,ATB,ATA,UNIT)
NOW SOLVE N X N SYSTEM
DETERMINE NROW, MAXIMUM PARTITION IS 150 ROWS.
There must be at least two partition rows
IF (N = 150) 5,5,7
5 NROW = 2
GO TO 9
7 NROW = N /150
IF (N = (NROW*150)) 8,9,8
8 NROW = NROW + 1
DETERMINE SIZES OF SUBMATRICES
INTERIOR SUBMATRICES
9 NPO = N /NROW
UPPER LEFT SUBMATRIX
NP1 = N - NPO * NROW + NPO
STORE SUBMATRICES IN UNIT 10
DO 15 KROW = 1,NROW
SUBMATRIX SIZE FOR KROW *NE* 1
NP = NPO
SUBMATRIX SIZE FOR KROW = 1
IF (KROW *EQ* 1) NP = NP1
ROW NUMBER OF TOP ELEMENT IN SUBMATRIX
KTOP = N - NP - (NROW-KROW)*NPO +1
ROW NUMBER OF BOTTOM ELEMENT IN SUBMATRIX
KBOT = KTOP + NP - 1
STORE SUBMATRIX COLUMNS
READ ONE COLUMN AT A TIME FROM UNIT, N ELEMENTS
C LOAD JCOL OF COEFFICIENT MATRIX FROM KTOP TO KBOT
DO 10 JCOL = 1, N
READ (UNIT) (A(I), I = 1, N)
10 WRITE (10) (A(I), I = KTOP, KBOT)
C LOAD CONSTANT VECTOR FROM KTOP TO KBOT
WRITE (10) (ATB(I), I = KTOP, KBOT)
15 REWIND UNIT
REWIND 10
C PUT SOLUTION IN X
18 CALL XSOLVE (1000, 150, N, NP0, NP1, NROW, S, XSOL, IWORK, WORK,
1BWORK, XWORK, COL, TOPCOL)
RETURN
END
SUBROUTINE XSOLVE(MX, NPX, N, NPO, NP1, NROW, Z, X, IWORK, WORK)

* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

* SUBROUTINE FOR SOLUTION BY PARTITIONING USING FILE STORAGE

* WRITTEN BY D.G. ELLIOTT USING SCHEME SUGGESTED BY C.L. LAWSON

* JET PROPULSION LABORATORY, PASADENA, NOVEMBER 1, 1972

INTEGER IWORK(NPX), UNIT1, UNIT2, UNIT3
REAL WORK(NPX)
REAL Z(NPX, NPX), X(MX), BWORK(NPX), XWORK(NPX),

* COL(NPX), TOPCOL(NPX)

* FORTRAN UNITS 10, 11, AND 12 USED FOR STORAGE

* TRIANGULARIZATION

6 NCXS = 0
NCX1 = N+1-NP1

TRIANGULARIZE THE NROW ROWS OF SUBMATRICES

DO 90 KROW = 1, NROW

SET UP UNITS

IF (KROW - 1) 2, 2*3

2 UNIT1 = 10
UNIT2 = 21
UNIT3 = 22
GO TO 4

3 JU = UNIT3
UNIT3 = UNIT2
UNIT2 = UNIT1
UNIT1 = JU

CONTAINS OLD LOWER-RIGHT MATRIX

TO RECEIVE TRANSFORMED TOP ROW

TO RECEIVE NEW LOWER-RIGHT MATRIX

SIZE OF INTERIOR SUBMATRICES

NP = NPO

SIZE OF UPPER-LEFT SUBMATRIX

IF (KROW .EQ. 1) NP = NP1
NUMBER OF COLUMNS BEYOND FIRST MATRIX IN KROW

\[ NCX = N+1-NP1-(KROW-1) \times NPO \]

DO 10 J=1,NP
READ MATRIX IN KROW

10 READ (UNIT1) (Z(K,J)*K=1,NP)
TRIANGULARIZE FIRST MATRIX
CALL DECOMP(NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
TRANSFORM REST OF ROW, INCLUDING RT-SIDE VCT
DO 20 J=1,NCX
NEXT COLUMN IN ROW
READ (UNIT1) (BWORK(K)*K=1,NP)
PUT TRANSFORMED COLUMN IN XWORK
CALL SOLVE (NPX,NP,Z,BWORK,XWORK,WORK,IWORK)
TRIANGULARIZATION COMPLETE
IF(KROW.EQ.NROW) GO TO 100
STORE TRANSFORMED COLUMN OF KROW
20 WRITE (UNIT2) (XWORK(K)*K=1,NP)
NO PREVIOUS ROW TO STACK IN 2
IF(KROW.EQ.1) GO TO 40
NNP = NPO
STACK TRANSFORMED ROWS IN REVERSE ORDER
DO 30 J=1,NCXS
IF((NCXS-J) .LT. NCX1) NNP = NP1
READ PREVIOUSLY TRANSFORMED ROWS
READ (UNIT3) (COL(K)*K=1,NNP)
AND WRITE AFTER NEW ONE
30 WRITE (UNIT2) (COL(K)*K=1,NNP)
REWIND UNIT3
FOR WRITING NEW LOWER-RT MATRIX IN NEXT OPER
40 KROWP1 = KROW + 1
SUBTRACT MULTIPLES OF TOP ROW FROM EACH ROW
DO 80 KKROW=KROWP1,NROW
DO 50 J=1,NP
READ FIRST MATRIX IN KKROW
50 READ (UNIT1) (Z(K,J)*K=1,NP)
REPOSITION AT START OF TOP ROW
REWIND UNIT2
SUBTRACT MULTIPLES OF TOP COLUMNS FROM EA CO
DO 70 J=1,NCX
NEXT COLUMN IN KKROW OF OLD MAT
READ (UNIT1) (COL(K)*K=1,NPO)
NEXT COLUMN IN TOP ROW
READ (UNIT2) (TOPCOL(K)*K=1,NP)
SUBTRACT FROM EACH ELEMENT IN COLUMN
DO 60 K=1,NPO
SUBTRACT FIRST MAT TIMES TOP ROW
DO 60 JJ=1,NP
COL(K) = COL(K) -Z(K,JJ) *TOPCOL(JJ)
60
WRITE (UNIT3) (COL(K)*K=1,NPO)
NEXT COL IN NBW MATRIX
CONTINUE
REWRITE UNIT1
REWRITE UNIT2
REWRITE UNIT3
NCXS = NCXS +NCX
CONTINUE
**** BACK SUBSTITUTION ****
DO 105 K=1,N
105 X(K) = 0
ZERO X-VECTOR
DO 110 K=1,NPO
BOTTOM SUBVECTOR IN X
110 X(N-NPO+K) = XWORK(K)
NROWM1 = NROW -1
BACK SUBSTITUTE FROM BOTTOM
DO 140 KRX=1,NROWM1
KROW = NROWM1 -KRX +1
NP = NPO
NMKROW = NROW -KROW
IF(KPOW .EQ. 1) NP = NP1
KSTART = N-NP-NMKROW *NPO

C SOLVE FOR KROW SUBVECTOR
DO 130 KCOL=1,NMKROW
KCOLM1 = KCOL - 1
DO 120 J=1,NPO

C NEXT MATRIX IN KROW
READ (UNIT3) (Z(K,J),K=1,NP)

C FORM PRODUCT OF MATRICES AND SUBVECTORS
DO 130 K=1,NP
DO 130 JJ=1,NPO
130 X(KSTART+K) = X(KSTART+K) -Z(K, JJ) * X(KSTART+NP+KCOLM14)

C READ RIGHT SIDE VECTOR
READ (UNIT3) (BWORK(K),K=1,NP)

C ADD RIGHT-SIDE VECTOR TO COMPLETE X FOR KROW
140 DO 140 K=1,NP
140 X(KSTART+K) = X(KSTART+K) + BWORK(K)

RETURN
END
SUBROUTINE ATAN \( (A, M, N, B, ATB, ATA, UNIT) \)

M \( A \) TRANSPOSE TIMES \( A \) \( (\text{ATAN}) \)
M \( A \) STORED BY ROWS ON UNIT 23
M \( A \) \( \times \) N COLUMNS \( \times \) M IS GREATER THAN OR EQUAL N
M \( A \) TRANSPOSE \( A \) STORED BY COLUMNS \( (\text{ROWS DUE TO SYMMETRY}) \)
UNIT IS AUXILIARY UNIT CONTAINING \( A \) TRANSPOSE \( A \)
DIMENSION A(N), ATA(N), B(N), ATB(N)
INTEGER UNIT, UNITN, UNITN1

1 UNITN = 21
UNITN1 = 22
IFLAG = -1
REWIND 21
REWIND 22
REWIND 23
DO 5 I = 1, N
DO 4 J = 1, N
4 ATA(J) = 0*0
5 WRITE (21) ATA
REWIND 21

C INITIALIZE STORAGE FOR A TRANSPOSE B VECTOR
DO 6 I = 1, N
6 ATB(I) = 0*0
C READ A ROW OF A
DO 40 I = 1, M
READ (23) A
C READ PARTIAL VALUE OF ELEMENTS IN A COLUMN OF ATA
DO 30 J = 1, N
READ (UNITN) ATA
C GENERATE NEXT TERM FOR ELEMENTS IN COLUMN O ATA
DO 20 L = 1, N
20 ATA(L) = ATA(L) + A(J) * A(L)
WRITE (UNITN1) ATA
C CONTINUE
C GENERATE NEXT TERM FOR ELEMENTS OF ATB
DO 25 K = 1, N
25 ATB(K) = ATB(K) + A(K) * B(I)
IFLAG = IFLAG * (-1)
UNITN = UNITN + IFLAG

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SUBROUTINE DECOMP(MX,N,UL,B,X,SCALES,IPS) 4 5440

C 4 5450
C **** SUBROUTINE FOR SIMULTANEOUS EQUATION SOLVING **** 4 5460
C **** REF: G. FORSYTHE AND C. MOLER, 'COMPUTER SOLUTION OF LINEAR 4 5470
C ALGEBRAIC SYSTEMS', PRENTICE-HALL, 1967 **** 4 5480
C **** NOMENCLATURE: UL=COEFF MATRIX, B=RIGHT-SIDE VCTR, X=UNKNOWN VCT4 5490
C **** CALL TO DECOMP TRIGANULARIZES THE COEFFICIENT MATRIX **** 4 5500
C
REAL UL(MX,MX),B(MX),X(MX),PVT,EM,SM
DIMENSION SCALES(MX),IPS(MX)
C
C INITIALIZE IPS AND SCALES
DO 5 I = 1,N 4 5560
IPS(1) = 1 4 5570
ROWNM = 0.0 4 5580
  DO 2 J = 1,N 4 5590
    ROWNRM = AMAX1( ROWNRM, ABS(UL(I,J)) ) 4 5600
2 IF (ROWNM) 3,4,3 4 5610
3 SCALES(I) = 1.0/ROWNM 4 5620
GO TO 5 4 5630
4 PRINT 20 4 5640
  SCALES(I) = 0.0 4 5650
5 CONTINUE
C
C GAUSSIAN ELIMINATION WITH PARTIAL PIVOTING
NM1 = N-1 4 5660
DO 17 K = 1,NM1 4 5670
BIG = 0.0 4 5680
  DO 11 I = K,N 4 5690
    IP = IPS(I) 4 5700
    SIZE = ABS(UL(IP,K))*SCALES(IP) 4 5710
11 IF (SIZE = BIG) 10,11,10 4 5720
  BIG = SIZE 4 5730
  IDXPIV = I 4 5740
  CONTINUE 4 5750
10 IF (BIG) 13,12,13 4 5760
  PRINT 25 4 5770
  GO TO 17 4 5780
C
13 IF (IDXPIV-K) 14,15,14
14 J = IPS(K)
IPS(K) = IPS(IDXPIV)
IPS(IDXPIV) = J
15 KP = IPS(K)
Pivot = UL(KP*K)
KP1 = K+1
   DO 16 I = KP1*N
      IP = IPS(I)
      EM = -UL(IP*K)/Pivot
      UL(IP*K) = -EM
      DO 161 J= KP1*N
161 UL(IP,J1) = UL(IP,J1) + EM*UL(KP,J1)
   CONTINUE
   CONTINUE
   CONTINUE
   CONTINUE
17 KP = IPS(N)
18 IF (UL(KP*N)) 19,18,19
19 PRINT 20
GO TO 500
20 FORMAT (* MATRIX WITH ZERO ROW IN DECOMP*)
25 FORMAT (* SINGULAR MATRIX IN DECOMP*)
C **** THIS ENTRY BACK-SUBSTITUTES THE RIGHT SIDE ****
C ENTRY SOLVE
NP1 = N + 1
C IP = IPS(1)
X(1) = B(IP)
   DO 30 J = 2,N
      IP = IPS(1)
      SUM = 0*Q
      IM1 = I-1
      DO 27 J= 1,IM1
         SUM = SUM + UL(IP,J)*X(J)
27   X(I) = B(IP) - SUM
   C IP = IPS(N)


\[
X(N) = X(N) / UL(IP,N) \\
\text{DO 40 IBACK = 2*N} \\
\text{I = NP1-IBACK} \\
C I GOES (N-1)*...*1 \\
\text{IP = IPS(I)} \\
\text{IP1 = I+1} \\
\text{SUM = 0.0} \\
\text{DO 35 J = IP1*N} \\
35 \quad \text{SUM = SUM + UL(IP,J)*X(J)} \\
40 \quad X(I) = (X(I)-SUM)/UL(IP,I) \\
500 \quad \text{RETURN} \\
\text{END}
\]
PROGRAM VEL  
REWIND 12  
CALL VELOC  
END  
SUBROUTINE VELOC  
COMMON/SLOPE/SIGMAP(500)* DZDX(500)*DZDC(500)*TANP1(500)  
EQUVALENCE (B(11871)*SIGMA)  
DIMENSION SIGMA(1000)  
COMMON/SCRT/ AK(500)*XG(11*151)*GAMT(11*151)*GAMK(150)  
1 *DLVM(1500)*DELVT(1500)*VT(1500)*VM(1500)*SLOPEE(150)  
2 *XAA(151)*GAMMA1(151)*GAMMA2(151)*XD(150)*GD(150)  
3 *CP(1320)  
EQUVALENCE(XCCO,B(2421))  
DIMENSION XCCO(20)*XOC(20)  
COMMON/BODY/ B(31000)  
EQUVALENCE(B(14044)*TMX)* (B(15364)*TMY)* (B(16584)*TZ)  
1 *(B(18004)*TTX)* (B(19324)*TTY)* (B(20644)*TTZ)  
EQUVALENCE(DA(3)*FMACH)  
DIMENSION TMX(1320)*TMY(1320)*TMZ(1320)  
1 *TTX(1320)*TTY(1320)*TTZ(1320)  
COMMON/CONPTS/ XQ(1320)*YQ(1320)*ZQ(1320)  
1 *XN(1320)*YN(1320)*ZN(1320)  
COMMON DA(5000)  
COMMON /COMPRS/ BETAM  
COMMON/NUMBER/ NVPTE(7)*NCPTS(7)*NLH(7)*NLT(7)*LTC(7)*LNC(7)  
1 *NCT*NB*NBDS*NPTS*NVT(7)*NVT(7)*NTAPE*NTAPE*NTAPE*JTAPE  
2 *SEG(7)*TSEG(7)*JFUNC(7)*JFUNC(7)  
3 *LNDIVB(7)*LTDIVB(7)*NSPP(7)*ROTOP(7)*OUTER(7)*SYMP(7)  
C  
WHEN SOURCE MATRIX IS READ GAMA ARRAYS WILL BE DESTROYED.  
EQUVALENCE (SXX*XG),(SXY*GAMT),(SXX*GAMK)  
DIMENSION SXX(18000),SXY(18000),SZZ(18000)  
C  
DIMENSION VX(1320),VY(1320),VZ(1320)  
EQUVALENCE(VX,B(220001)),(VY,B(232211)),(VZ,B(24641))  
C VX,VY,VZ EQUVALENCE SHOULD NOT DESTROY SIGMA ARRAY.  
C  
EQUVALENCE (DA(7)*XCG),(DA(8)*YCG),(DA(9)*ZCG),(DA(10)*ALPHA)  
1 *(DA(11)*BETA),(DA(12)*PSI),(DA(13)*QSTK),(DA(14)*QSTK)}
COMMON/PANEL/MP4+IPSYM1:GNVVP+NTVVP+LICFPLTCFP+LICFP+LTCPP
1  *PERPT+NSPACE+MATCHE+MRATT+PRLCL+LRICTC+LCTET+LTHXC
2  *NTHET+NTIP+CHTIP+ROOT+CUTER+MMATT
3  *MP1*MP2*MP3*MP4*MP5*MP6*MP7*MP8*MP9*MP10
DIMENSION AX(4681),AY(4681),AZ(4681)
EQUIVALENCE(B1)*AX),(E(S4681)*AY),(B9363)*AZ)
EQUIVALENCE(B1BB1P)
EQUIVALENCE(ALKTB124001),(ALKTP224501),(DA12+PANS)
DIMENSION AKTB1500,AKTP1500

C
C  BB IS READ INTO CORE TO GET TMX - TTZ ARRAYS INTO CORE.
C  BB IS NOT LONG ENOUGH TO DESTR0Y SIGMA ARRAY.
C
C  MP2 IS UNIT FOR PANEL INFLUENCE MATRIX.
C  MP3 IS NUMBER OF AX,AY,AZ TERMS IN EACH ROW OF THE MATRIX ON MP2.
C  NCB = NO. OF CONTROL POINTS ON BODY.
C
COMMON/PANINF/PANSYM(10)+ASOL(600)
DO 1 I=1,20
XOC(I)=XCCO(I)
1
C
C  TEMPORARY BREF,CBAR
BREF=1.0
CBAR=10.0
C
GAM14=1.4
IF(NBODS.NE.0)REWIND NTAPE
IF(PANS.NE.0)REWIND MP2
CALL GAMMA FOR CAPITAL GAMMA ARRAYS.
CALL GAMMA
C
CALL DELV FOR DELVM AND DELV1 ARRAYS.
CALL DELV
WRITE(*,5000) (DELVT(I),I=1,200)
5000 FORMAT(6H0DELVT/1H10F12.6)
WRITE(*,5001) (DELVM(I),I=1,200)
5001 FORMAT(6H0DELVM/1H10F12.6)
IF(NBODS.EQ.0)GO TO 4
C
READ(18)
READ(18)
READ(18) BS
CONTINUE
C
NCB=NCPTS(1)
IF(NBODS*EQ.0) NCB=0
NPANS*PANS
DO 5 I=1,NCT
XX=XQ(I)-XCG
YY=YQ(I)-YCG
ZZ=ZQ(I)-ZCG
VX(I)=1.0-2.0*(QSTAR*ZZ/CBAR-RSTAR*YY/SREF)
VY(I)=-BETA-2.0*(PSTAR*ZZ-RSTAR*XX)/SREF
VZ(I)=ALPHA+2.0*(PSTAR*YY/SREF+QSTAR*XX/CBAR)
CONTINUE
NPWT=(NCT-NCB)*2+NCB
DO 6 I=1,NPWT
VM(I)=0.0
6 VT(I)=0.0
IF(NPANS*EQ.0) GO TO 2005
ISIG=0
C READ PANEL DATA TO GET SIGMA
DO 7 I=1,NPANS
READ(18) DP
REWIND 18
WRITE(6,8000) (ASOL(I),I=1,220)
8000 FORMAT(1H1*,1P10E13.4)
WRITE(6,8000) (SIGMAP(I),I=1,280)
WRITE(6,8000) (TAMP(I),I=1,140)
WRITE(6,8000) (DZDX(I),I=1,140)
WRITE(6,8000) (DZDT(I),I=1,140)
DO 2000 I=1,NPANS
NSIG=NVPTS(NBODS+1)*2
C COMPUTE TERMS INVOLVING SOURCE MATRICES.
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>WRITE(6<em>8003)IC</em>VN(IC)*ZN(IC)*VY(IC)*VZ(IC)</td>
<td>5 1570</td>
</tr>
<tr>
<td>2</td>
<td>WRITE(6<em>8003)IX</em>VM(IX)<em>VT(IX)<em>T3</em>T4</em>T5*T6</td>
<td>5 1580</td>
</tr>
<tr>
<td>3</td>
<td>1XXX=IX+1</td>
<td>5 1590</td>
</tr>
<tr>
<td>4</td>
<td>WRITE(6<em>8003)IXXX</em>VM(IXXX)*VT(IXXX)*DELVM(IC)*DELV(T(IC)</td>
<td>5 1600</td>
</tr>
<tr>
<td>5</td>
<td>CONTINUE</td>
<td>5 1610</td>
</tr>
<tr>
<td>6</td>
<td>600 CONTINUE</td>
<td>5 1620</td>
</tr>
<tr>
<td>7</td>
<td>1000 CONTINUE</td>
<td>5 1630</td>
</tr>
<tr>
<td>8</td>
<td>ISIG=IS</td>
<td>5 1640</td>
</tr>
<tr>
<td>9</td>
<td>2000 CONTINUE</td>
<td>5 1650</td>
</tr>
<tr>
<td>10</td>
<td>2005 NB1=NBODS+1</td>
<td>5 1660</td>
</tr>
<tr>
<td>11</td>
<td>DO 3000 IC=1*NCT</td>
<td>5 1670</td>
</tr>
<tr>
<td>12</td>
<td>FLAG=1.0</td>
<td>5 1680</td>
</tr>
<tr>
<td>13</td>
<td>IF(NPANS.EQ.0) GO TO 2014</td>
<td>5 1690</td>
</tr>
<tr>
<td>14</td>
<td>IF(IC-NCB-NCPTS(NB1)*GT.0) NB1=NB1+1</td>
<td>5 1700</td>
</tr>
<tr>
<td>15</td>
<td>LPTS=LNC(NB1)</td>
<td>5 1710</td>
</tr>
<tr>
<td>16</td>
<td>ICR=IC-((IC-1)/LPTS)*LPTS</td>
<td>5 1720</td>
</tr>
<tr>
<td>17</td>
<td>CONTINUE</td>
<td>5 1730</td>
</tr>
<tr>
<td>18</td>
<td>2014 IX=2*IC-NCB-1</td>
<td>5 1740</td>
</tr>
<tr>
<td>19</td>
<td>IF(NBODS.EQ.0) GO TO 2050</td>
<td>5 1750</td>
</tr>
<tr>
<td>20</td>
<td>READ(INTAPE)(AX(I)*AY(I)*AZ(I)<em>I=1</em>NCTV)</td>
<td>5 1760</td>
</tr>
<tr>
<td>21</td>
<td>IF(IC<em>GT</em>NCB) GO TO 2020</td>
<td>5 1770</td>
</tr>
<tr>
<td>22</td>
<td>TERM=0.0</td>
<td>5 1780</td>
</tr>
<tr>
<td>23</td>
<td>DO 2015 I=1,NCTV</td>
<td>5 1790</td>
</tr>
<tr>
<td>24</td>
<td>TERMTERM+(TMX(IC)*AX(I)+TMY(IC)*AY(I)+TMZ(IC)*AZ(I))*ASOL(I)</td>
<td>5 1800</td>
</tr>
<tr>
<td>25</td>
<td>VM(IC)=VM(IC)+TERM</td>
<td>5 1810</td>
</tr>
<tr>
<td>26</td>
<td>TERM=0.0</td>
<td>5 1820</td>
</tr>
<tr>
<td>27</td>
<td>DO 2016 I=1,NCTV</td>
<td>5 1830</td>
</tr>
<tr>
<td>28</td>
<td>TERMTERM+(TTX(IC)*AX(I)+TTY(IC)*AY(I)+TTZ(IC)*AZ(I))*ASOL(I)</td>
<td>5 1840</td>
</tr>
<tr>
<td>29</td>
<td>VT(IC)=VT(IC)+TERM</td>
<td>5 1850</td>
</tr>
<tr>
<td>30</td>
<td>VT(IC)=VT(IC)+TERM</td>
<td>5 1860</td>
</tr>
<tr>
<td>31</td>
<td>VM(IC)=VM(IC)+BETAM<strong>2*TMX(IC)*VX(IC)+BETAM</strong>2*TMY(IC)*VY(IC)+</td>
<td>5 1870</td>
</tr>
<tr>
<td>32</td>
<td>BETAM**2*TMZ(IC)*VZ(IC)+DELVM(INC)</td>
<td>5 1880</td>
</tr>
<tr>
<td>33</td>
<td>VT(IC)=VT(IC)+BETAM<strong>2*TTX(IC)*VX(IC)+BETAM</strong>2*TTY(IC)*VY(IC)+</td>
<td>5 1890</td>
</tr>
<tr>
<td>34</td>
<td>BETAM**2*TTZ(IC)*VZ(IC)+DELV(T)IC</td>
<td>5 1900</td>
</tr>
<tr>
<td>35</td>
<td>DBETA=SQRT(BETAM**2*(TMX(IC)**2+TMY(IC)**2+TMZ(IC)**2))</td>
<td>5 1910</td>
</tr>
<tr>
<td>36</td>
<td>VM(IC)=VM(IC)*DBETA</td>
<td>5 1920</td>
</tr>
<tr>
<td>37</td>
<td>DBETA=SQRT(BETAM**2*(TTX(IC)**2+TTY(IC)**2+TTZ(IC)**2))</td>
<td>5 1930</td>
</tr>
<tr>
<td>38</td>
<td>VT(IC)=VT(IC)<em>/BETAM</em>DBETA</td>
<td>5 1940</td>
</tr>
<tr>
<td>39</td>
<td>GO TO 2030</td>
<td>5 1950</td>
</tr>
<tr>
<td>40</td>
<td>2020 TERM=0.0</td>
<td>5 1960</td>
</tr>
<tr>
<td>41</td>
<td>DO 2025 I=1,NCTV</td>
<td>5 1970</td>
</tr>
</tbody>
</table>
2025 TERM=TERM+AX(I)*ASOL(I)
WRITE(6,8001) IC,TERM
3001 FORMAT(1H0,1S5,1PE15.5)
VM(IX)=VM(IX)+TERM
VM(IX+1)=VM(IX+1)+TERM
TERM=0.0
DO 2026 I=1,NCTV
2026 TERM=TERM+(ZN(IC)*AY(I)-YN(IC)*AZ(I))*ASOL(I)
WRITE(6,8001) IC,TERM
VT(IX)=VT(IX)+TERM
VT(IX+1)=VT(IX+1)+TERM
2050 CONTINUE
IF(NPANS.EQ.0) GO TO 2900
REA4(MP2)(AX(I),AY(I),AZ(I),I=1,NP2)
IF(IC.GT.NCEL) GO TO 2070
TERM=0.0
DO 2065 I=1,MP3
IK=NCTV+I
2065 TERM=TERM+(THX(IC)*AX(I)+THY(IC)*AY(I)+THZ(IC)*AZ(I))*ASOL(IK)
VM(IC)=VM(IC)+TERM
TERM=0.0
DO 2066 I=1,MP3
IK=NCTV+I
2066 TERM=TERM+(TTX(IC)*AX(I)+TTY(IC)*AY(I)+TTZ(IC)*AZ(I))*ASOL(IK)
VT(IC)=VT(IC)+TERM
GO TO 2900
2070 TERM=0.0
DO 2075 I=1,MP3
IK=I+NCTV
2075 TERM=TERM+AX(I)*ASOL(IK)
WRITE(6,8001) IC,TERM
VM(IC)=VM(IC)+TERM
VM(9X+1)=VM(IX+1)+TERM
TERM=0.0
DO 2076 I=1,MP3
IK=I+NCTV
2076 TERM=TERM+(ZN(IC)*AY(I)-YN(IC)*AZ(I))*ASOL(IK)
WRITE(6,8001) IC,TERM
VT(IX)=VT(IX)+TERM
5 1960
5 1970
5 1980
5 1990
5 2000
5 2010
5 2020
5 2030
5 2040
5 2050
5 2060
5 2070
5 2080
5 2090
5 2100
5 2110
5 2120
5 2130
5 2140
5 2150
5 2160
5 2170
5 2180
5 2190
5 2200
5 2210
5 2220
5 2230
5 2240
5 2250
5 2260
5 2270
5 2280
5 2290
5 2300
5 2310
5 2320
5 2330
5 2340
VT(IX+1) = VT(IX+1) + TERM
IF (IC .LE. NCB) GO TO 2900
ICP = IC - NCB
TOP = 1.0 / SORT(1.0 + TANPI(IX) * (DZDXT(IX) + DZDXYC(IX)) ** 2)
BOTTOM = 1.0 / SORT(1.0 + TANPI(IX) * (DZDXT(IX) - DZDXYC(IX)) ** 2)
VM(IX) = TOP * VM(IX) / BETAM ** 2 + VX(IX)
VM(IX+1) = BOTTOM * VM(IX+1) / BETAM ** 2 + VX(IX)
VT(IX) = TOP * VT(IX) / BETAM
VT(IX+1) = BOTTOM * VT(IX+1) / BETAM
WRITE(6, 8002) IX, VM(IX), VM(IX+1), VT(IX), VT(IX+1)
8002 FORMAT (I4, 1P4E20.5/)
2900 IF (IC .LT. NCB) 2901, 2902, 2903
2901 ICX = IC
GO TO 2903
2902 ICX = IX
2903 A = 1.0 - VT(ICX)**2 - VM(ICX)**2
AM2 = A**2
IF (AM2 .LT. 0.1) 2905, 2905, 2904
2904 CP(IX) = (2.0/(GAM14**2*AM2)) * ((1.0 + (GAM14 - 1.0) * 0.5 * AM2) ** 2)
1 ** (GAM14/(GAM14 - 1.0)) - 1.0
GO TO 2906
2905 CP(IX) = A * (1.0 + 0.25 * AM2 * (1.0 - 0.1 * AM2))
2906 CONTINUE
IF (FLAG .EQ. -1.0) GO TO 2903
IF (IC .LE. NCB) 2908, 2908, 2907
2907 FLAG = -1.0
ICX = ICX + 1
GO TO 2903
2908 CONTINUE
3000 CONTINUE
IXX = IX + 1
IF (NPANS .EQ. 3) IXX = IC - 1
REWIND 11
IF (NBODS .NE. 0) REWIND 25
IF (PANS .NE. 0) REWIND 52
C SET TRAILING EDGE X VALUES.
L = 0
IV = 0
SUBROUTINE DELV
COMMON DA(5000)
COMMON/NUMBER/ NVPTS(7), NCPTS(7), NLN(7), NLT(7), LTC(7), LNC(7)
1 NCT, NB, NBODS, NPANS, NVL(7), NVT(7), NTAPE, NTAPE, NCTV, ITAPE, JTAPE
2 *SEG(7), TFSG(7), *LNFC(7), !FUNC(7)
3 *LNDIVB(7), LDTIVB(7), NSPP(7), ROOTP(7), OUTERP(7), SYMM(7)
COMMON/SCRAT/XSOL(5000), XG(11, 151), GAMT(11, 151), GAMR(1500)
1 *DELVM(1500), *DELVT(1500), *VT(1500), *VM(1500), SLOPEC(1500)
2 *XAA(151), *GAMMA1, (151), *GAMMA2, (151), XD(150), XD(150)
3 *CP(1320), *XGW(50, 20), *GAMTW(50, 20)
EQUIVALENCE (DA(2), PANS)
EQUIVALENCE (DA(2800), FLNC), (DA(3100), FLTC)
DIMENSION FLNC(150), FLTC(40)
COMMON/BODY/B(31000)
EQUIVALENCE (ALNCHR, B(20000)), (ALNCHR, B(25000))
DIMENSION ALNCHR(5000), BLNCHR(5000)
COMMON/CONPTS/XQ(1320), YQ(1320), QO(1320)
1 XN(1320), YN(1320), ZN(1320)
COMMON/CONTR/ LNN(3, 40), LTT(3, 40)
COMMON/AF/ ATTFB(10), ATTFw(50)
IF(NBODS.EQ.0) GO TO 5

REWIND 18
NNV=NVL(1)*NVT(1)
READ(18) (ALNCHR(I), I=1, NNV)
READ(18) (BLNCHR(I), I=1, NNV)
REWIND 18
CONTINUE

C NPANS=PANS
KC=0
WRITE(6, 4998) (GAMBI, I=1, 100)
4998 FORMAT(5H0GAMBI//(1P10E12.4))
IF(NBODS.EQ.0) GO TO 100

C COMPUTE DELVM, DELVT AT BODY CONTROL POINTS
LTC3=LTC(1)
LNC=LNC(1)
DO 50 J=1, LTC3

50 CONTINUE
C IT1, IT2 ARE TRAILING VORTICES ON WHICH INTERPOLATED GAMT ARE FOUND.

IT1 = FLTC(J) + 0.01
IT2 = IT1 + 1
DO 50 I = 1, LNC.G
KC = KC + 1

C IB1, IB2 ARE BOUND VORTICES.

IB1 = FLNC(I) + 0.01
IB2 = IB1 + 1
L2 = (IT1 - 1) * NVL(1) + IB1 - 1
IF (IB1 .NE. 1) GO TO 10
ALM2 = 0.0
GO TO 12

10 ALM2 = ALNGTH(L2)
12 L5 = L2 + 1
ALM5 = ALNGTH(L5)
IF (IB1 .NE. NVL(1)) GO TO 14
ALM8 = 0.0
13 ALM11 = 0.0
GO TO 20
14 L8 = L5 + 1
ALM8 = ALNGTH(L8)
IF (IB2 .GE. NVL(1)) GO TO 13
ALM11 = ALNGTH(L8 + 1)

20 CONTINUE
NGAM1 = NVL(1)
NGAM2 = NGAM1
IF (ATTFB(IT1) .EQ. 99.0) NGAM1 = 20
25 CONTINUE
IF (ATTFB(IT2) .EQ. 99.0) NGAM2 = 20

26 CONTINUE
DO 261 M = 1, NGAM1
XD(M) = XG(IT1 * M)
261 GD(M) = GAMT(IT1 * M)
IF (XG(KC) .GT. XD(NGAM1)) XD(NGAM1) = XG(KC)
CALL CODIM(XD, GD, NGAM1, XG(KC), GAM1 * 1)
DO 262 M = 1, NGAM2
XD(M) = XG(IT2 * M)
262 GD(M) = GAMT(IT2 * M)
```
IF(XQ(KC)*GT.*XD(NGAM'2)) XD(NGAM'2) = XQ(KC)
CALL CODIM(XD*GD*NGAM'2*XQ(KC)*GAM'2*1)
L2=(IT1-1)*NVZ(1)+IB1
L1=L2-NVZ(1)
L3=L2+NVZ(1)
BL2=BLNZH(L2)
IF(IT1*NE*1) GO TO 35
BL1=BL2
BL3=BLNZH(L3)
GO TO 40
35 IF(IT1*NE*NVT(1)) GO TO 36
BL1=BLNZH(L1)
BL3=BLNZH(L3)
40 CONTINUE
D = 0.75*ALM5+0.25*ALM8
IF(IB1-NVZ(1)) 42,41,42
41 GAMZ=0.0
GO TO 43
42 GAMZ=GAMZ(L2+1)
43 CONTINUE
6002 FORMAT(815)
6001 FORMAT(1H0,1P4E20.4)
DELV(KC)=GAMZ(L2)/(0.75*ALM2+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
+GAMZ2/(0.75*ALM5+ALM8+0.25*ALM11)*0.5*ALM5/D
DELVT(KC)=-0.5*(GAMZ1/(BL1+BL2)+GAMZ2/(BL2+BL3))
50 CONTINUE
100 CONTINUE
IF(NPANS*EQ.0) GO TO 300
KTRV=0
IF(NBODS) 101,101,102
101 NBVOR=0
GO TO 103
102 NBVOR=NVZ(1)*NVT(1)
103 CONTINUE
DO 250 II=1,NPANS
NBP=NBODS+II
```

COMPUTE D5LVM, DELVT ON PANELS.
C
LTCC=LTC(NBP)
LNCC=LNC(NBP)
DO 200 J=1,LTCC
IT1=LTT(I*J)+0.01+KTRV
IT2=IT1+1
DO 200 I=1,LNCC
KC=KC+1
IB1=LMN(I*I)+0.01
IB2=IB1+1
IT1L=IT1-KTRV
IT2L=IT1L+1
L2=(IT1L-1)*NVL(NBP)+IB1-1+RVORT
L5=L2+1
L8=L5+1
L11=L8+1
ALM2=(ALNGTH(L2))
ALM5=(ALNGTH(L5))
ALM8=(ALNGTH(L8))
ALM11=(ALNGTH(L11))
IF(IB1*EQ.1) ALM2=ALM5
IF(IB1*EQ.NVL(NBP)) ALM8=ALM5
IF(IB2*EQ.NVL(NBP)) ALM11=ALM8
NGAM1=NVL(NBP)
NGAM2=NGAM1
IF(ATTFW(IT1)*EQ.99.0) NGAM1=20
125 CONTINUE
IF(ATTFW(IT2)*EQ.99.0) NGAM2=20
126 CONTINUE
DO 361 M=1,NGAM1
XD(M)=XGW(IT1*M)
361 GD(M)=GAMTW(IT1*M)
CALL CODIM(XD*GD*NGAM1,XD(KC),CAM1,1)
DO 362 M=1,NGAM2
XD(M)=XGW(IT2*M)
362 GD(M)=GAMTW(IT2*M)
CALL CODIM(XD*GD*NGAM2,XD(KC),CAM2,1)
L2=(IT1L-1)*NVL(NBP)+IB1+RVORT
L1=L2-NVL(NBP)
L3=L2+NVL(NBP)
BL2=BLNGTH(L2)
IF(IT1LNE1) GO TO 135
BL1=BL2
BL3=BLNGTH(L3)
GO TO 140
135 IF(IT1LNENV(NBP)) GO TO 136
BL1=BLNGTH(L1)
BL3=BL2
GO TO 140
136 BL1=BLNGTH(L1)
BL3=BLNGTH(L3)
140 CONTINUE
D=0.75*ALM5+0.25*ALM8
IF(IB1-NVL(NBP)) 142,141,142
141 GAMB2=0.0
GO TO 143
142 GAMB2=GAMB(L2+1)
143 CONTINUE
DELVM(KC)=GAMB(L2)/(0.75*ALM5+ALM5+0.25*ALM8)*0.25*(ALM5+ALM8)/D
1+GAMB2/(0.75*ALM5+ALM8+0.25*ALM11*0.5*ALM57/D
TERMP=-DELVM(KC)*SLOPEC(L2)
C ADD TERMP ONLY FOR PANEL
DELVT(KC)=0.5*(GAM1/(BL1+BL2)+GAM2/(BL2+BL3)) + TERMP
200 CONTINUE
KTRV=KTRV+NV(NBP)+1
NVORT=NVPTS(NBP)+NVORT
250 CONTINUE
300 CONTINUE
RETURN
END
DO 200 I=1,NPANS
KP=KPI
IN8=I+NBODS
CALL PANGAN1(T,AK,NVL(INB),NVT(INB),KPI,NBODS)
WRITE(6,7007) (AK(IX),IX=1,100)
7007 FORMAT(90A1K AFTER PANGA1/(1P10E13.4))
KPI=KPI+NCPTS(INB)
200 CONTINUE
205 CONTINUE
REW9ND 18
IF(NPANS.EQ.0) RETURN
NTRVP=0
NPRE=0
DO 300 I=1,NPANS
NA=NATT((I-1)*3+2)
300   CONTINUE
C NA=COMPONENT (BODY OR PANEL) TO WHICH PANEL I IS ATTACHED.
   IF(NA.EQ.0) GO TO 260
C DETERMINE WHICH TRAILING VORTICES NEED TO HAVE MODIFIED GAMT'S.
C THESE WILL BE THE VORTICES NUMBERED N1 AND N2.
N1=0
   IF(NA.EQ.1) GO TO 190
N11=NA-1
180   DO 190 K=1,N11
190   N1=NVT(K)+N1+1
180
C N1=TRAILING VORTEX LINE TO WHICH PANEL I IS ATTACHED.
C N2=FINAL TRAILING VORTEX LINE OF ATTACHED PANEL I.
C DETERMINE XMIN AND XMAX OF TR. VORTICES N1+N2.
IF(XG(N1+1)-XG(N2+1)) 202,202,203
202 XMIN=XG(N1+1)
   GO TO 204
203 XMIN=XG(N2+1)
204   NVOR1=NVL(NA)
   NVOR2=NVL(NBODS+1)
   IF(NL.EQ.NPPE) NVOR1=20
   IF(XG(N1,NVOR1)-XG(N2,NVOR2)) 206,206,207
206   XMAX=XG(N2,NVOR2)
GO TO 208
207  XMAX=XG(N1*NVOR1)
208  CONTINUE
      N20=19
      XINT=XMAX-XMIN
      DX=XINT/N20
      N21=N20+1
      DO 210 M=1,N21
  210  XAA(M)=XMIN+(M-1)*DX
      DO 211 M=1,NVOR1
         XD(M)=XG(N1*M)
      GD(M)=GAMT(N1*M)
      CALL CODIM(XD*GD,NVOR1,XAA,GAMMA1,N21)
      DO 230 M=1,N21
         IF(XAA(M)-XG(N1+1)) 222,224,224
      222  GAMMA1(M)=0.0
      GO TO 230
  224  IF(XAA(M)-XG(N1+Nовор1)) 228,228,226
  226  GAMMA1(M)=GLAST
      GO TO 230
  228  GLAST=GAMMA1(M)
  230  CONTINUE
      DO 213 M=1,NVOR2
         XD(M)=XGW(N2*M)
      GD(M)=GAMT(N2*M)
      CALL CODIM(XD*GD,NVOR2,XAA,GAMMA2,N21)
      DO 240 M=1,N21
         IF(XAA(M)-XGW(N2+1)) 232,234,234
      232  GAMMA2(M)=0.0
      GO TO 240
  234  IF(XAA(M)-XGW(N2+2)) 238,238,236
  236  GAMMA2(M)=GLAST
      GO TO 240
  238  GLAST=GAMMA2(M)
  240  CONTINUE
      DO 250 ::=1,N21
         XG(N1:M)=XAA(M)
         XGW(N2:M)=XAA(M)
      WRITE(6,261) N1,N2::IPRED
261 FORMAT(*ON1,N2,N1PRE* 315) 5 6070
WRITE(6,262) M,GAMMA1(M),GAMMA2(M) 5 6090
262 IF(N1*NE*N1PRE) GO TO 249 5 6100
IF(N1*NE*N1PRE) GO TO 249 5 6100
GAMT(N1*M)= GAMMA2(M)+GAMT(N1*M) 5 6110
GO TO 250 5 6120
249 GAMT(N1*M)=GAMMA1(M)+GAMMA2(M) 5 6130
250 GAMTW(N2*M)=GAMT(N1*M) 5 6140
5 6150
C
ATTFB(N1)=99.0 5 6160
ATTFW(N2)=99.0 5 6170
WRITE(6,5998)(XG(N1*M),M=1,20) 5 6180
5998 FORMT(*0X-COORDINATES FOR ATTACHMENT LINE*/(1P10E13.4)) 5 6190
WRITE(6,5999) N1,N2 5 6200
5999 FORMT(*0BODY ATTACHMENT LINE IS*13/ 5 6210
1 *OPANEL ATTACHMENT LINE IS*13/ 5 6220
WRITE(6,6000)(GAMT(N1*M),M=1,20) 5 6230
6000 FORMT(*0BODY GAMT AT ATTACHMENT LINE*/(1P10E13.4)) 5 6240
WRITE(6,6001)(GAMTW(N2*M),M=1,20) 5 6250
6001 FORMT(*0PANEL GAMT AT ATTACHMENT LINE*/(1P10E13.4)) 5 6260
260 CONTINUE 5 6270
NTRVP=NTRVP+NVTV(NBODS+I)+1 5 6280
N1PRE=N1 5 6290
300 CONTINUE 5 6300
RETURN 5 6310
END
II=(J-1)*NL+1+KP-1
SUM= SUM+AK(I)
DUMM(I)=SUM
WRITE(6,6000) J*II=SUM,AK(I)
300 AK(I)=SUM
305 CONTINUE
WRITE(6,7007) (DUMM(I)*=1,100)
7007 FORMAT(*0DUMM*/(1P1E13.4))
NS1=NSPACE+1
N1=NT+1
DO 100 J=1,N1
JMNS=J-NSPACE
JT=J+NTBP
DO 100 I=1,NL
IF(J.EQ.N1) GO TO 45
LENGTH=LENGTH+1
KK=NBVORT+LENGTH
L1=L1+1
ALNGTH(KK)=PL(L1)
IF(JMNS) 41,41,42
41 BLENGTH(KK)=PLT(L1)
GO TO 43
42 BLENGTH(KK)=2.0*YSUBV(JMNS)
43 CONTINUE
SLOPEC(LENGTH)=TS(LS)
45 CONTINUE
IF(JMNS) 11,11,21
11 XG(JT+I)=0.5*(XVR(J,I)+XVR(J,I+1))
GO TO 30
21 CONTINUE
IF(J-N1) 22,24,24
22 FACT=-0.5
JTR=JMNS
GO TO 251
24 LS=NL*(NT-1)*2+1
FACT=+0.5
JTR=JMNS-1
251 IF(I-1) 26,26,28
26 X1=XVC(JTR+I)+FACT*TS(LS)*SQR(DYS(LS)**2+DZS(LS)**2)
GO TO 29
28 X1=X2
29 CONTINUE
   IF(I.EQ.NL) GO TO 291
   LS2=LS+2
   X2=VXO(JTR+I+1)+FACT*TS(LS2)*SQRT(DYS(LS2)**2+D2S(LS2)**2)
   GO TO 292
291 X2=X1+DELX
292 XG(XJ+I)=0.5*(X2+X1)
   DELX=X2-X1
30 CONTINUE
   K1=(J-1)*NL+1+KP-1
   LS=LS+2
   IF(J-1) 1,1,5
   1 IF(SYM) 2,9,2
   2 GAMTW(JT+I)=-AK(K1)
   GO TO 10
   5 IF(J-N1) 6,7,6
   6 GAMTW(JT+I)=AK(K1-NL)-AK(K1)
   GO TO 10
   7 GAMTW(JT+I)=AK(K1-NL)
   GO TO 10
   9 IF(NBODS.NE.0) GO TO 2
   GAMTW(JT+I)=0.0
10 CONTINUE
   IF(J.EQ.N1) GO TO 40
   L=L+1
   IF(I-1) 25,20,25
20 GAMB(L)=AK(K1)
   GO TO 40
25 GAMB(L)=AK(K1)-AK(K1-1)
30 CONTINUE
150 CONTINUE
   NTBP=NTBP+NV(T(IP+NBOJS)+1
RETURN
END
SUBROUTINE BODGAM (IB, AK, NL, NT, L)
COMMON DA(5000)
COMMON/NUMBER/NVPTS(7), NCPTS(7), NLN(7), NLT(7), LTC(7), LNC(7)
1  NCT, NBB, NBOOS, NPAIR, NLV(7), NVT(7), NTAPE, NTAPE, NCTV, ITAPE, JTAPE
2  LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)
3  LNDIVB(7), LTDIVB(7), NSPP(7), ROCTEXP(7), OUTER(7), SYM(17)
COMMON/BODY/B(30000)
DIMENSION XV(151, 31)
EQUIVALENCE (XV+8)
COMMON/CR/XTXSL(5000), XG(11, 151), GAMT(11, 151), GAMB(1500)
1  DELVM(1500), DELVT(1500), VT(1500), VM(1500), SLOPEC(1500)
2  XAA(151), GAMMAL(151), GAMMA2(151), XD(150), GD(150)
3  CP(1320)
DIMENSION AK(1)
EQUIVALENCE (SYM, DA(19))
READ(18) N
LTDIV = LTDIVB(IB)
DO 100 J = 1, N1
J1 = (J - 1) * LTDIV + 1
DO 100 I = 1, NL
K1 = (J - 1) * NL + 1
XG (J, I) = 0.5 * (XV(I, J1) + XV(I + 1, J1))
IF (J - 1) LE 1, 5
1  IF (SYM) 2, 4, 2
2  GAMT(J, I) = -AK(K1)
GO TO 10
4  GAMT(J, I) = 0.0
GO TO 10
5  IF (J = N1) 6, 7, 6
6  GAMT(J, I) = AK(K1 - NL) - AK(K1)
GO TO 10
7  IF (SYM) 8, 9, 8
8  GAMT(J, I) = AK(K1 - NL)
GO TO 10
9  GAMT(J, I) = 0.0
10  CONTINUE
IF (J EQ N1) GO TO 40
L = L + 1
IF(I-1) 25, 20, 25
20   GAMB(L) = AK(K1)
     GO TO 40
25   GAMB(L) = AK(K1) - AK(K1-1)
40   CONTINUE
100  CONTINUE
     IF(SYM.EQ.0) GO TO 15
     DO 14 I = 1, NL
         GAMT(1, I) = GAMT(1, I) + GAMT(N1, I)
14     GAMT(N1, I) = GAMT(1, I)
     CONTINUE
     RETURN
END
PROGRAM FORCES

MAIN PROGRAM FOR FORCES.

COMMON/ BODY/ XAREA(5000), YAREA(5000), ZAREA(5000)
1  XCP(5000), YCP(5000), ZCP(5000), XTL(100)
2  FLNC(150), FLTC(40), XCON(100)

COMMON /COMPRS/ BETAM

DIMENSION YPD(200), ZPD(200), YV(100), ZV(100), PINP(10)

SEE EQUIVALENCE IN PANMAT FOR YPD, ZPD

EQUIVALENCE (YPD(11), XTL(11), ZPD(11), FLNC(11))

EQUIVALENCE (YV(11), XAREA(11), ZV(11), YAREA)

COMMON /PANINF/ PSYM(10), DUM11(600), PANREF(10), PCHORD(10)

DIMENSION TPD(10), TPD(10)

EQUIVALENCE (DA(19), BSYM)

DIMENSION AKT(500), AKTP(500)

EQUIVALENCE (AKTB(1), YCP(4001), (AKTP(1), YCP(4501))

DIMENSION CP(1320)

COMMON/SCRAT/ D(25000)

DIMENSION VM(1500), VT(1500)

EQUIVALENCE (VM(1), D(14323), (VT(1), D(12823))

DIMENSION CPBOD(1500)

EQUIVALENCE (D1(18076), CP, D(20000), CPBOD)

DIMENSION E (50), XCCO(20), YDI(400), DZI(400)

1  TWIST(20), CPU(400), CPL(400), DSLE(20), XLE(20), YLE(20)

2  ZLE(20), SCPL(400), CHC(20)

EQUIVALENCE (E*(5001)), (XCCO, D(5101)), (YDI, D(5201)), (DZI, D(5701))

1  TWIST, D(6201), CPU, D(6301), CPL, D(6801)

2  (DSLE, D(7301)), (XLE, D(7401)), (YLE, D(7501))

3  (ZLE, D(7601)), (SCPL, D(7701)), (CHC, D(7801))

4  (VMU, D(8001)), (VML, D(8201)), (VTU, D(8401)), (VTL, D(8601))

DIMENSION VMU(200), VML(200), VTU(200), VTL(200)

COMMON DA(5000)

COMMON/ NUMBER/ NVPTS(7), NPCPTS(7), NMLN(7), NMLT(7), NLT(7), LT(7), LMN(7)
1  NCT, NB, NBOOS, NPS, NVL(7), NVT(7), NTAPE, NTAP, NTAP, NCTV, ITAPE, ITAPE

2  LSEG(7), TSEG(7), LFUNC(7), TFUNC(7)

3  LNDIV(7), TNDIV(7), NSW, (ROTP(7), OUTER(7), SYMM(7))

COMMON/ SLOPE/ SIGMA(500), DZDXT(500), DZDXC(500), TANPI(500)

DIMENSION ZERO(50)
DIMENSION SR(5,8) PS(10,6)
EQUIVALENCE (BR, DA(16)), (PR, DA(3421)), (PSY, DA(3426)),
1 (PCH, DA(3422)), (RCH, DA(17))
COMMON/CONPTS/ XQ(1320), YQ(1320), ZQ(1320)
1 XN(1320), YN(1320), ZN(1320)
DIMENSION BP(13000)
DIMENSION XCC(20), XCV(20), XSS(1000), YSS(1000), ZSS(1000), ET(20)
1 TS(1000), SIGMA(1000)
EQUIVALENCE (RP, XAREA), (XCC, BP(2421)), (XCV, BP(2401))
1 (XSS, BP(8871)), (YSS, BP(9871)), (ZSS, BP(10871)), (TS, BP(7871))
2 (SIGMA, BP(11871))
DIMENSION CPM(1000), CDS(100), CDA(100), CDTH(100)
EQUIVALENCE (SPAN, DA(3423)), (PVL, DA(3431))
EQUIVALENCE (D(23001), CPM), (D(2401), CDS), (D(24201), CDA)
1 (D(24301), CDTH), (D(24401), CDTH)
COMMON/PANEL/ NPANS = 1, ISP = 1, NCV = 1, NVV = 1, NVC = 1, NC = 1, NCT = 1
DATA PIND/10*1.0/
NC = 0
IF(NBODS .EQ. 0) GO TO 50
N2 = LTDLV(1) * NVT(1) + 1
READ (12) (YV(I), ZV(I), I = 1, N2)
50 CONTINUE
NPANS = PANS
IF(PANS .EQ. 0) GO TO 55
WRITE (6, 7002) (1, YPD(1), ZPD(1), I = 1, 20)
7002 FORMAT(*PANEL drag coordinates*/ (15, 2F15.5))
WRITE (6, 7001) (1, AKTP(I), I = 1, 20)
7001 FORMAT(*0 AKTP*/ (15, 1PE20.6))
WRITE (6, 6001) NVT(NBODS + 1), NVT(NBODS + 2)
WRITE (6, 6002) PSYM
WRITE (6, 6002) PIND
55 CONTINUE
NSP = 0
DO 101 IS = 1, LTPP
CDS(IS) = 0
CDA(IS) = 0
CDT(IS) = 0
101 CDTH(IS) = 0
CDST = 0
<table>
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<th>Line</th>
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<td>56</td>
<td>CONTINUE</td>
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<td>58</td>
<td>CONTINUE</td>
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<tr>
<td>59</td>
<td>CPNET(I) = CP(II) - CP(II)</td>
</tr>
<tr>
<td>61</td>
<td>SIGMA(I) = SIGMA(I) / BETA</td>
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<tr>
<td>551</td>
<td>ZV(I) = ZV(I) / BETA</td>
</tr>
<tr>
<td>552</td>
<td>ZP(I) = ZP(I) / BETA</td>
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<td>60</td>
<td>DO 61 I = 1 * NCPT</td>
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<td>61</td>
<td>SIGMA(I) = SIGMA(I) / BETA</td>
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<td>SIGMA(I) = SIGMA(I) / BETA</td>
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<td>62</td>
<td>TS = SIGMA * CPNET * NSP * CDS * CDA * CDT * CDTH * CST * CDAT * COTT * COTT</td>
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<td>TS = SIGMA * CPNET * NSP * CDS * CDA * CDT * CDTH * CST * CDAT * COTT * COTT</td>
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</table>
IF(NBODS.EQ.0.) GO TO 100
READ(12) XAREA,YAREA,ZAREA
READ(12) XCP,YCP,ZCP,XTL,XCO,FMC,FLT,LMBL,NBV,NTV,LPW,LPW,LPW
1 LNDIV,LNDIV,LBVV,NTVV,CHORD,XCG,YCG,ZCG,ALPHA,REF
6000 FORMAT(1P9E12.4)
6001 FORMAT(1015)
NC=NCPTS(I)
DO 60 I=1,NC
60 CPBODY(I)=CP(I)
6002 FORMAT(*0*/(1P1E13.4))
CALL BPRINT(LMW,LMBL,XCP,YCP,ZCP,XTL,XCO,FMC,FLT,LMBL)
CALL REFLE(DA(19),NBV,NTV,XAREA,YAREA,ZAREA,XCP,YCP,ZCP)
1 LMBL,LMBL,FBODY,LNDIV,LNDIV
NPAA = NBV*NTV
DO 30 I=1,NPAA
XAREA(I) = XAREA(I)/BETAM**2
YAREA(I) = YAREA(I)/BETAM
ZAREA(I) = ZAREA(I)/BETAM
YCP(I) = YCP(I)/BETAM
30 ZCP(I) = ZCP(I)/BETAM
CALL BINTEG(XAREA,YAREA,ZAREA,XCP,YCP,ZCP,FLT,LMBL)
1 ,CPBODY,LNDIV,LNDIV,XTL,NBV,NTV,DA(19),XCO,FMC,FLT,LMBL
2 ,DA(19),DA(19),CHORD,D(301),D(401),D(501),D(601),D(701),D(801)
3 ,NBV,NTV,DA(19),XCG,YCG,ZCG,ALPHA,REF
4 SB(1,1),SB(1,2),SB(1,3),SB(1,4),SB(1,5),SB(1,6),SB(1,7),SB(1,8)
3
SB(1,7) = BR
SB(1,8) = BH
6
103 IF(PANS.EQ.0.) GO TO 200
PANS=PANS
NCPI=NC
NDI=1
DO 554 I=1,500
554 DZ(DK) = DZ(DK)/D(LTA)
THE FIRST SUBSCRIPTS OF SB(I,7) SHOULD BE THE BODY PANEL NUMBER
100 IF(PANS.EQ.0.) GO TO 200
PANS=PANS
NCPI=NC
NDI=1
DO 554 I=1,500
554 DZ(DK) = DZ(DK)/D(LTA)
THE FIRST SUBSCRIPTS OF SB(I,7) SHOULD BE THE BODY PANEL NUMBER
YQ(I) = YQ(I) / BETAM
ZQ(I) = ZQ(I) / BETAM
DO 150 K = 1, NPIANS
READ (12) B*REFANLTCCPE, LINCPP, XCCO, DIY, DZI, TWIST, CHC
1     *CHRHM*(XLE(I)), YLE(I), ZLE(I), I = 1, LTCCP, FLT, SPCF, BS, DSLE
DO 105 I = 1, LTCCP
K1 = FLTC(I)
DSLE(I) = DSLE(K1) / BETAM
YLE(I) = YLE(I) / BETAM
ZLE(I) = ZLE(I) / BETAM
NPTS = NCPTS(NBODS + K)
DO 110 I = 1, NPTS
IU = (I - 1) * 2 + 1 + NCP1
IL = IU + 1
VMU(I) = VM(IU)
VTU(I) = VT(IU)
VHL(I) = VM(II)
VTL(I) = VT(II)
CPL(I) = CP(IL)
CPU(I) = CP(IU)
N1 = NC + 1
DO 111 I = 1, 50
ZERO(I) = 0.0
CALL PPRINT(E*XCCO, LTCCP, LINCPP, VMU, VHL, VTU, VTL, CPU, CPL, NBODS + K)
B = B / BETAM
BS = B / BETAM
DO 553 I = 1, 400
DYI(I) = DYI(I) / BETAM
553
DZI(I) = DZI(I) / BETAM
CALL PANNINTB*REFANLTCCPE, LINCPP, XCCO, DIY, DZI, TWIST, BS, SPCF
1     *CPU, CPL, DSLE, XCG, YCG, ZCG, CHC, CHRHM, DZOC(NPD2), DZox(NPD1)
2     *XQ(N1), YQ(N1), ZQ(N1), XLE, YLE, ZLE, D(8000)
3     *TPD(K), CDA, DCS, CSTH, CDT, PS(K), 1, PS(K), 2, PS(K), 3,*
4     PS(K), 4, PS(K), 5, PS(K), 6, K, NBODS, CAH, CSTH, CDTT
NC = NC + NPTS
NCP1 = NCP1 + 2 * NPTS
NPD2 = NP2 + NPTS
100 CONTINUE
200 CONTINUE
CALL CCIINTG(SB(1,1), SB(1,2), SB(1,3), SR(1,4), SP(1,5), SP(1,6))
1  SB(1,7), PS(1,1), PS(1,2), PS(1,3), PS(1,4), PS(1,5), PS(1,6), 6 1970
2  PANREF, PSYM, PCHORD, SB(1,9)
3  STOP 6543
END
USEROUTINE CCINTG (CXB, CYB, CZB, CMXB, CMYB, CMZB, ARB, CXP, CYP) 2030
1, CZP, CHXP, CMYP, CMZP, AP, FS, CP, CB 2040
16 2040
6 2050

THIS ROUTINE INTEGRATES THE TOTAL CONFIGURATION

CXB CYB CZB -ARRAYS OF C & S OF BODIES
CMXB CMYB CMZB -ARRAYS OF CM & S OF BODIES
ARB -ARRAY OF REFERENCE AREAS OF BODIES
CXP CYP CZP -ARRAYS OF C & S OF PANELS
CHXP CMYP CMZP -ARRAYS OF CMS OF PANELS
AP -ARRAY OF PANEL AREAS
CB -ARRAY OF BODY CHORD LENGTHS
CP -ARRAY OF PANEL CHORD LENGTHS
NB -NUMBER OF BODIES
NP -NUMBER OF PANELS

T -TOTAL CONFIGURATION CHORD LENGTH
XCRCG YCRCG ZCRCG -X Y AND Z OF CR CG
AR -TOTAL CONFIGURATION REFERENCE AREA
FS -SYMMETRY INDICATOR 0=0 IS SYMMETRICAL

DIMENSION CXB(1), CYB(1), CZB(1), CMXB(1)
1 CMYB(1), CMZB(1), ARB(1)
2 CXP(1), CYP(1), CZP(1), CHXP(1), CMYP(1)
3 CMZP(1), AP(1), FS(1), CP(1), CB(1)

EQUVALENC E (ARBAR, CBJC)
COMMON DA(5100)
EQUVALENC E (DA(5), A, DA(4), BN), (DA(2), PN)
1 (AR, DA(4)), (XCRCG, DA(7)), (YCRCG, A(8)), (ZCRCG, DA(9))
NB = BN
NP = PN
PRINT 5

6 FORMAT(1H1, 35X, TOTAL CONFIGURATION LOADS)
CX = 0.0
CY = 0.0
CZ = 0.0
CMX = 0.0
CMY = 0.0
CMZ = 0.0

6 2360
6 2370
6 2380
6 2390
6 2400
6 2410
C SUM OVER BODIES
IF(NB .LE. 0) GOTO 300
DO 100 J=1, NB
ARBAR = ARB(J)/AR
CX = CX + ARBAR * CXB(J)
CY = CY + ARBAR * CYB(J)
CZ = CZ + ARBAR * CZB(J)
CBJC = CB(J)/ C * ARBAR
CMX = CMX + CBJC*CMXB(J)
CMY = CMY + CBJC*CMYB(J)
CMZ = CMZ + CBJC*CMZB(J)
100 CONTINUE
C SUM OVER PANELS
300 IF(NP .LE. 0) GOTO 400
DO 200 J=1, NP
FSV = 1.
FS2 = 1.
IF(LS(J)) 102, 101, 102
101 FSV = 0.0
FS2 = 2.0
102 APJ1R = AP(J)7AR
CX = CX + APJAR * FS2 * CXP(J)
CY = CY + APJAR * FS0 * CYP(J)
CZ = CZ + APJAR * FS2 * CZP(J)
CPJC = APJAR * CP(J)/C
CMX = CMX + CPJC*FS0*CMXP(J)
CMY = CMY + CPJC*FS2*CMYP(J)
CMZ = CMZ + CPJC*FS0*CMZP(J)
200 CONTINUE
400 CONTINUE
PRINT 206, CX, CY, CZ, CMX, CMY, CMZ
206 FORMAT (1H0,8X,CX*,13X,CY*,13X,CZ*,13X,CMX*,12X,CMY*,12X,CMZ*)
CPJC = CX*CX + CY*CY + CZ*CZ
XCR = XCRCG + (CY*CZ - CZ*CY)/CPJC
YCR = YCRCG + (CZ*CX - CX*CZ)/CPJC
ZCR = ZCRCG + (CX*CY - CY*CX)/CPJC
PRINT 216, XCR, YCR, ZCR
216 FORMAT(1HO, 3X, *(X/CR) CP*, 6X, *(Y/CR) CP*, 6X, *(Z/CR) CP*/1X, 3F15.76
1)
RETURN
END
<table>
<thead>
<tr>
<th>SUBROUTINE</th>
<th>ARGUMENTS</th>
<th>DESCRIPTION</th>
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<tbody>
<tr>
<td>PAMINTC</td>
<td>SPAN X, ETA X, ETA Y, ETA Z, DZ</td>
<td>The great number of arguments were used to allow greater ease in adding this</td>
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<tr>
<td></td>
<td>EPSILO X, BS X, DA</td>
<td>routine to the others. At a later date these should be replaced by common</td>
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<tr>
<td></td>
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<td>references.</td>
</tr>
<tr>
<td>B</td>
<td>SPAN</td>
<td>This routine integrates panels with control surfaces.</td>
</tr>
<tr>
<td>REF 1</td>
<td>REFERENCE AREA</td>
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<tr>
<td>ETA</td>
<td>ETAS for given CPS</td>
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<tr>
<td>NX03</td>
<td>NUMBER OF X/C STATION FOR GIVEN</td>
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<tr>
<td></td>
<td>CPS</td>
<td></td>
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<tr>
<td>XOC</td>
<td>X/C ARRAY</td>
<td></td>
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<tr>
<td>DY</td>
<td>DELTA Y FOR VORTEX WHERE CP IS</td>
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</tr>
<tr>
<td></td>
<td>GIVEN</td>
<td></td>
</tr>
<tr>
<td>DZ</td>
<td>DELTA Z FOR VORTEX WHERE CP IS</td>
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</tr>
<tr>
<td></td>
<td>GIVEN</td>
<td></td>
</tr>
<tr>
<td>EPSILO</td>
<td>TWIST ARRAY</td>
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<tr>
<td>BS</td>
<td>SURFACE SPAN LENGTH (FOR 2-D</td>
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<tr>
<td></td>
<td>SURFACE BS = B</td>
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<tr>
<td>SPCF</td>
<td>SPECIAL LONGITUDINAL CONTROL</td>
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<tr>
<td>CPU</td>
<td>CP ARRAY FOR UPPER SURFACE</td>
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<tr>
<td>CPL</td>
<td>CP ARRAY FOR LOWER SURFACE</td>
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<tr>
<td>DSL5</td>
<td>DELTA S ON LEADING EDGE FOR</td>
<td></td>
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<tr>
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<td>SPANWISE VORTEX SECTIONS WHERE</td>
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</tr>
<tr>
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<td>CPS ARE GIVEN</td>
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<tr>
<td>XCG</td>
<td>X CENTER OF GRAVITY</td>
<td></td>
</tr>
<tr>
<td>YCG</td>
<td>Y CENTER OF GRAVITY</td>
<td></td>
</tr>
<tr>
<td>ZCG</td>
<td>Z CENTER OF GRAVITY</td>
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<tr>
<td>CHORD</td>
<td>MEAN AERODYNAMIC CHORD (GIVEN IN</td>
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<tr>
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<td>INPUT)</td>
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</tr>
<tr>
<td>DZC</td>
<td>DZ/DX FOR C/A ITEM</td>
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<tr>
<td>DZT</td>
<td>DZ/DX FOR THICKNESS</td>
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<td>X, Y, Z</td>
<td>CO ORDINATES AT CPS</td>
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<tr>
<td>XLE, YLE, ZLE, DS</td>
<td>X, Y, Z ARRAY FOR LEADING EDGE POINTS AT GIVEN ETAS</td>
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<tr>
<td>CDTO</td>
<td>A SCRATCH ARRAY</td>
<td>(CDT = 0 C)/CAVG</td>
</tr>
<tr>
<td>Column 1</td>
<td>Column 2</td>
<td>Column 3</td>
</tr>
<tr>
<td>---------</td>
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<tr>
<td>CDL9C</td>
<td>(CDLIC)/CAVG</td>
<td>6 3240</td>
</tr>
<tr>
<td>CTAVG</td>
<td>(CTC)/CAVG</td>
<td>6 3250</td>
</tr>
<tr>
<td>CDTICA</td>
<td>(CDTIC)/CAVG</td>
<td>6 3260</td>
</tr>
<tr>
<td>CXCAVG</td>
<td>CXC/CAVG</td>
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<tr>
<td>CYCAVG</td>
<td>CYC/CAVG</td>
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<td>CZCZVG</td>
<td>CZC/CZVG</td>
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<td>CMX3AV</td>
<td>CMXC/CAVG</td>
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<td>CMYC/CAVG</td>
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<tr>
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<td>CMZC/CAVG</td>
<td>6 3320</td>
</tr>
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<td>CNCAVG</td>
<td>CNBC/CAVG</td>
<td>6 3330</td>
</tr>
<tr>
<td>CMLEXC</td>
<td>CMLEXC/CAVG</td>
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</tr>
<tr>
<td>CMLEYC</td>
<td>CMLEYC/CAVG</td>
<td>6 3350</td>
</tr>
<tr>
<td>CMLEZC</td>
<td>CMLEZC/CAVG</td>
<td>6 3360</td>
</tr>
<tr>
<td>XCPLE</td>
<td>XCP/C LE</td>
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<tr>
<td>YCPLE</td>
<td>YCP/C LE</td>
<td>6 3380</td>
</tr>
<tr>
<td>ZCPLE</td>
<td>ZCP/C LE</td>
<td>6 3390</td>
</tr>
<tr>
<td>T1 2 3 8</td>
<td>SCRATCH ARRAYS NETA LONG WHERE DEPENDENT VARIABLES STORED TO INTEGRATE ARE STORED</td>
<td>6 3400</td>
</tr>
<tr>
<td>T1 1 2 8</td>
<td>SCRATCH VARIABLES WHERE VALUES OF INTEGRALS ARE STORED</td>
<td>6 3420</td>
</tr>
<tr>
<td>CMX CHY CHZ</td>
<td>CMX CHY CHZ</td>
<td>6 3440</td>
</tr>
<tr>
<td>CX CY C2</td>
<td>CX CY C2</td>
<td>6 3450</td>
</tr>
<tr>
<td>XCP2</td>
<td>XCP/C</td>
<td>6 3460</td>
</tr>
<tr>
<td>YCP2</td>
<td>YCP/C</td>
<td>6 3470</td>
</tr>
<tr>
<td>ZCP2</td>
<td>ZCP/C</td>
<td>6 3480</td>
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<tr>
<td>DS</td>
<td>DELTA S</td>
<td>6 3490</td>
</tr>
<tr>
<td>DSEE</td>
<td>DELTA S L.E.</td>
<td>6 3500</td>
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</table>

 DIMENSION DSLE(30), DS(NXOC,NETA), DYN(NXOC,NETA), 6 3510
 1DZ(NXOC,NETA), CPU(NXOC,NETA), CRL(NXOC,NETA), EPSILON(NETA), 6 3520
 2ETA(NETA), XOC(NXOC), X(NXOC,NETA), Y(NXOC,NETA), Z(NXOC,NETA), 6 3530
 3T1(30), T2(30), T3(30), T4(30), T5(30), T6(30), T7(30), T8(30), 6 3540
 4 CXCAVG(30), CYCAVG(30), C2CAVG(30), CMXCAVG(30), CMYCAVG(30), CMHCAVG(30), CHZCAVG(30), 6 3550
 5V(30), CNCAVG(30), CMLFXC(30), CMLEYC(30), CMLEZC(30), XCple(30), 6 3560
 6YCple(30), ZCPLE(30), CDTG(NETA), CDLIC(NETA), CTAVG(NETA), CDTICA(16), 6 3570
 7 ETA), DA(9,1), XLE(1), YLE(1), ZLE(1), 6 3580
 DIMENSION DZ(NXOC,NETA), ETA(NXOC,NETA), 6 3590
 DIMENSION C(11), XSLC(20), YSLC(20), ZSLC(20), NSLC(20), 6 3600
 1 XSMALL(20), YSMALL(20), ZSMALL(20), XPCPS(20), YPCPS(20), 6 3610
 2 ZPCPS(20), CMHC(20), 6 3620
EQUVALENCE (CXC,AVG*XLCS) * (Y=LC5,CYCAVG)* 6 3630
1 (C2CAVG*ZLCS) * (C1CAVG*XSLCS)* 6 3640
2 (CMXCAY*YSMAHL) * (CMYCAV*YSMAHL) 6 3650
3 (CMZCAY*YSMAHL) * (XCLPE*ZSCPCS)* 6 3660
4 (YCPEL,YSCPCS)* (ZCPEL*ZSCPCS) * (CMLEXC,CMHC) 5 3670
5 (RLINTX1, XO, ETA1, ETA0, AT1) 6 3680
6 PRINT: 26, NC 6 3690
7 FORMAT(1H1, 36X) * PANEL SECTIONAL LOADS FOR COMPONENT*, IS/ 6 3700
8 11HC, 5X, 3HETA, 7X, 6 3710
11 SET UP DELTA S AND DELTA T 6 3740
12 BT = B*B 6 3750
13 AR = BT/REFA 6 3760
14 DO 20 K=1, NETA 6 3770
15 DO 10 I=1, NXOC 6 3780
16 DS(I,K) = SQRT(DY(I,K)**2 + DZ(I,K)**2) 6 3790
17 CONTINUE 6 3800
18 DO 20 CONTINUE 6 3810
19 ARBS82 = AR*BS/(BT) 6 3820
20 C STEP SPAN WISE 6 3830
21 DO 500 K=1, NETA 6 3840
22 DYGIT = DY(I,K)/DS(I,K) 6 3850
23 DZPIT = DZ(I,K)/DS(I,K) 6 3860
24 CONTINUE 6 3870
25 DO 100 I=1, NXOC 6 3880
26 C FILL DEPENDENT VARIABLE ARRAYS 6 3890
27 DSILS = DS(I,K)/DSLE(K) 6 3900
28 TCON = CPL(I,K) 6 3910
29 1TANLF(1, DZT(I,K), DZC(I,K), EPSI00(K), DA, ETA(K), X(I,K), 1.0) 6 3920
30 2 = CPU(I,K) * 6 3930
31 3TANLF(1, DZT(I,K), DZC(I,K), EPSI00(K), DA, ETA(K), X(I,K), -1.0) 6 3940
32 CPLCPU = CPL(I,K) - CPU(I,K) 6 3950
33 Ti(1) = DSILS/TCOMP 6 3960
34 T2(1) = DSILS/CPLCPU 6 3970
35 YIYCG = Y(I,K) - YCG 6 3980
36 ZIKZCG = Z(I,K) - ZCG 6 3990
37 XIKXC = X(I,K) - XCG 6 4000
38 YIYCG = Y(I,K) - YCG 6 4010
T3(I) = DSDSLE*(YIKYCG*DYOPIT + ZIKZCG*DZOPIT)*CPLCPU
T4(I) = DSDSLE*(ZIKZCG*TCON - XIKXCG*DYOPIT*CPLCPU)
T5(I) = DSDSLE*(-ZIKZCG*DZOPIT*CPLCPU - YIKYCG*TCON)
XIKXLE = (X1*K) - XLE(K)
YIKYLE = (Y1*K) - YLE(K)
ZIKZLE = (Z1*K) - ZLE(K)
T6(I) = DSDSLE*(YIKYLE*DYOPIT + ZIKZLE*DZOPIT)*CPLCPU
T7(I) = DSDSLE*(ZIKZLE*TCON - XIKXLE*DYOPIT*CPLCPU)
T8(I) = DSDSLE*(-XIKXLE*DZOPIT*CPLCPU - YIKYLE*TCON)

CONTINUE

C INTEGRATE CHORD WIZE
T1 = POLINT(XOC*T1 - NXOC*0.0 - 0.1*0)
T2 = POLINT(XOC*T2 - NXOC*0.0 - 0.1*0)
T3 = POLINT(XOC*T3 - NXOC*0.0 - 0.1*0)
T4 = POLINT(XOC*T4 - NXOC*0.0 - 0.1*0)
T5 = POLINT(XOC*T5 - NXOC*0.0 - 0.1*0)
T6 = POLINT(XOC*T6 - NXOC*0.0 - 0.1*0)
T7 = POLINT(XOC*T7 - NXOC*0.0 - 0.1*0)
T8 = POLINT(XOC*T8 - NXOC*0.0 - 0.1*0)
CARBB = C(K) + ARBSB2
CTAVRG(K) = CTAVRG(K) * ARBSB2
CXCAGV(K) = CARBB*(T1) - CTAVRG(K)
CYCAVG(K) = -CARBB*DZOPIT*TI2
CZC1VG(K) = CARBB*DYOPIT*TI2
CARBB = CARBB/CHORD
CMLXAV(K) = CARBB*C13
CMYCAV(K) = CARBB*C14 - (Z1*K) - ZCG*CTAVRG(K)/CHORD
CMZAV(K) = CARBB*(T15 + (Y1*K) - YCG*CTAVRG(K)/CHORD
CNCAVG(K) = DZOPIT*CYCAVG(K) + DYOPIT*CZCAVG(K)
TPIT = CXCAVG(K)**2 + CYCAVG(K)**2 + CZCAVG(K)**2
CMLXEC(K) = ARBSB2*T16
CMLEXY(K) = ARBSB2*T17
CMLEZG(K) = ARBSB2*T18
CDTO(K) = CDTO(K) * ARBSB2
CDLIC(K) = CDLIC(K) * ARBSB2
COTICA(K) = COTICA(K) * ARBSB2
IF ABS(TPIT) * ST - 1.0 = 7 GOTO 131
XCPLE(K) = 0.0
YCPLE(K) = 0.0
131 CONTINUE
ZCPL(K)=CYGAV(K)+CMLEZC(K)+ZCAVG(K)*CMLEYC(K)/TPIT
YCPL(K)=ZCAVG(K)+CMLEXC(K)+CYCAVG(K)*CMLEYC(K)/TPIT
XCPL(K)=CYCAVG(K)+CMLEXC(K)+ZCPL(K)*CMLEYC(K)/TPIT

132 CONTINUE
PRINT 156; ETA(K), CXCAVG(K), CYCAVG(K), ZCAVG(K), CNEAVG(K),
1CMLEXC(K), CMLEYC(K), CMLEZC(K)
156 FORMAT(1H1, 110, 7F13.6)
506 CONTINUE
PRINT 506
PRINT 1H0, 5X, 3HETA, 7X,
PRINT 156; ETA(K), XCPL(K), YCPL(K), ZCPL(K), CDT0(K), CDLIC
1(K), CTAVRG(K), CDTICA(K), K=1, NETA)
INTEGRATE SPAN WIZE
CX=POLINT(ETA, CXCAVG, NETA, 0.0, 1.0)
CY=POLINT(ETA, CYCAVG, NETA, 0.0, 1.0)
CZ=POLINT(ETA, ZCAVG, NETA, 0.0, 1.0)
CMX=POLINT(ETA, CMXCAV, NETA, 0.0, 1.0)
CMY=POLINT(ETA, CMYCAV, NETA, 0.0, 1.0)
CMZ=POLINT(ETA, CMZCAV, NETA, 0.0, 1.0)
PRINT 606; CX, CY, CZ, CMX, CMY, CMZ
CDI=CDI/REFA
CDT=CDT/REFA
CDL=CDL/REFA
CT=CT/REFA
CTDI=CTDI/REFA
606 FORMAT(1H10, 44X, *TOTAL PANEL LOADS*/
2/1H0, 6F15.6)
XYZ = CX*CX + CY*CY + CZ*CZ
IF(ABS(XYZ) GT 1.0E-7) GOTO 611
XCPC = 0.0
YCPC = 0.0
ZCPC = 0.0
GOTO 612
611 CONTINUE
   XCPG=XC/CHORD+(CY*CMZ-CZ*CMY)/XYZ
   YCPG=YC/CHORD+(CY*CMX-CZ*CMY)/XYZ
   ZCPG=ZC/CHORD+(CY*CMX-CZ*CMY)/XYZ
612 CONTINUE
   PRINT 616, CDI, CDL, CDI*CT, CDI* XCPG, YCPG, ZCPG
616 FORMAT(1H1,3X*CDI=0.10X*CDI*12X*CDI*13X*CT*12X*CDI*11X)
   1/1H, 5FI5.6/1H, 5X,
   2 *X/C CP*, 9X, *Y/C CP*, 9X, *Z/C CP*/1H, 3FI5.6)
   C
   C THIS SECTION STEPS THROUGH THE CONTROL SURFACES WHILE SOLVING THE
   C CONTROL SURFACE EQUATIONS
   NCS = 1
   709 IF(DA(1,NCS)=EQ.0) RETURN
   PRINT 706, NCS
706 FORMAT(1H1,35X,15HCONTROL SURFACE, 13'16H SECTIONAL LOADS)
   C
   C FIN4 THE ETAS AT WHICH THIS CONTROL SURFACE STARTS AND ENDS
   DO 710 KS=1, NETA
   IF(ETA(KS)-DA(3,NCS)) 710,711,711
710 CONTINUE
   711 DO 720 KE=KS, NETA
   IF(ETA(KE)-DA(4,NCS)) 720,721,722
720 CONTINUE
   722 KE = KE - 1
   IF(KE .LE. 0) KE = KS
   721 NESPN = KE-KS + 1
   TFS = (DA(1,NCS)+1.)/2.0
   ETAD = DA(4,NCS) - DA(3,NCS)
   C
   STEP SPANWISE THROUGH THIS PANEL
   DO 900 K=KS, KE
   T7(K) = ETA(K)-DA(3,NCS)/ETAD
   DYPIT= DY(1,K)/DS (K)
   UZPIT= DZ(1,K)/DS (K)
   C
   FIN4 THE XOCs AT WHICH THIS CONTROL SURFACE STARTS OR ENDS
   XKF= RELINT(DA(5,NCS),DA(6,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
   XH = RELINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS), ETA(K))
   XHP = RELINT(DA(7,NCS),DA(8,NCS), DA(3,NCS), DA(4,NCS), ETA(K)+1.E-7)
XHM = RLINE(Y(I*K), Y(I-1*K), X(I*K), X(I-1*K), XHM)
XZ = RLINE(Z(I*K), Z(I-1*K), X(I*K), X(I-1*K), XHP)
YHP = RLINE(Y(I*K), Y(I-1*K), X(I*K), X(I-1*K), XHP)
YHM = RLINE(Y(I*K), Y(I-1*K), X(I*K), X(I-1*K), XH)
YHD = YHP - YH
ZHP = RLINE(Z(I*K), Z(I-1*K), X(I*K), X(I-1*K), XHP)
ZHM = RLINE(Z(I*K), Z(I-1*K), X(I*K), X(I-1*K), XHM)
T = SORT(YHD*YHD + XHD*XHD + ZHD*ZHD)
HZ = ZHD/T
HY = YHD/T
HX = XHD/T
TT = C(K) / CCS
DO 770 I=1, IS
LDSLE = DS(I*K)/DSLE(K)
XIKXH = X(I*K) - XH
YIKYH = Y(I*K) - YH
ZIKZH = Z(I*K) - ZH
T = TAN(SAM2*KCS))
TCSN = CFE(I*K) + (T + SZT(I*K) - [ZC(I*K) + EPSILON(K)]) /
1. $(1 - \text{DZT}(I,K) - \text{DZC}(I,K) + \text{EPSIL0}(K)) \times T$

2. $(1 - \text{CPU}(I,K) \times (T - \text{DZT}(I,K) - \text{DZC}(I,K) + \text{EPSIL0}(K))) / T$

3. $(1 - T \times (\text{DZT}(I,K) - \text{DZC}(I,K) + \text{EPSIL0}(K)))$

TI(I) = DSLSLE * TCON

CPU = CFU(I,K) - CPU(I,K)

T2(I) = DSLSLE * CPLCPU

T3(I) = DSLSLE * (YIKYH*DYOPIT + ZIKZH*ZDOPIT) * CPLCPU

T4(I) = DSLSLE * (ZIKZH*TCON - YIKYH*DYOPIT * CPLCPU)

T5(I) = DSLSLE * (YIKYH*DYOPIT * CPLCPU - YIKYH*TCON)

IF(TFS) 768, 769, 762

768 TB(I) = TT * (XOC(I) + 1) / TT - 1

GOTO 763

762 TB(I) = TT * XOC(I)

763 CONTINUE

770 CONTINUE

INTEGRATE CHORDIZE

NX = IE - IS + 1

TI1 = POLINT(T8(IS), T1(IS), NX, 0.0, 1.0)

TI2 = POLINT(T6(IS), T2(IS), NX, 0.0, 1.0)

TI3 = POLINT(T8(IS), T3(IS), NX, 0.0, 1.0)

TI4 = POLINT(T6(IS), T4(IS), NX, 0.0, 1.0)

TI5 = POLINT(T8(IS), T5(IS), NX, 0.0, 1.0)

CARBB = CCS * ARBSB2

XSLCS(K) = CARBB * TI1 - CTAVRG(K)

YSLCS(K) = -CARBB * ZDOPIT * TI2

ZSLCS(K) = CARBB * DYOPIT * TI2

CARBB = CARBB / CHORD

NSLCS(K) = -YSLCS(K) * DZOPIT + ZSLCS(K) * DYOPIT

XSMHSL(K) = CARBB * TI3

YSMHSL(K) = CARBB * T14 * TFS * (Z1*K) - ZH * CTAVRG(K) / CHORD

ZSMHSL(K) = CARBB * T15 * TFS * (Y1*K) - YH * CTAVRG(K) / CHORD

T = XSLCS(K) * x2 + YSLCS(K) * x2 + ZSLCS(K) * x2

T = T * C(K) / CHORD

XSCPCS(K) = (YSLCS(K) * ZSMHSL(K) - ZSLCS(K) * YSMHSL(K)) / T

YSCPCS(K) = (ZSLCS(K) * XSMHSL(K) - XSLCS(K) * ZSMHSL(K)) / T

ZSCPCS(K) = (XSLCS(K) * YSMHSL(K) - YSLCS(K) * XSMHSL(K)) / T

CMHC(K) = XSMHSL(K) * HX + YSMHSL(K) * HY + ZSMHSL(K) * HZ

903 CONTINUE

CHX = POLINT(T7(KE), XSLCS(KE), NESPN, 0.0, 1.0)
CHX = POLINT(T7(KE)*YSLCS(KE)*NESP=0.0*1.0) 6 5970
CHY = POLINT(T7(KE)*YSLCS(KE)*NESP=0.0*1.0) 6 5980
CHZ = POLINT(T7(KE)*ZSLS(CS(KE)*NESP=0.0*1.0) 6 5990
PRINT 806 6 6000
806 FORMAT(1HO*7X*31ETA:6X*9HCHXC/CAVG:3X*9HCHYC/CAVG:3X*
1 9HCHZC/CAVG:2X,*
2 34HCHXC/CAVG CMHYC/CAVG CMH2C/CAVG 3X*9HCMHC/CAVG)
816 FORMAT(1HO*10X*27HCONTROL SURFACE TOTAL LOADS;/
1 1HO*48H ETA (X/C) CP (Y/C) CP (Z/C) CP)
826 FORMAT(47HO CHX CHY CHZ CHM) 6 6060
836 FORMAT(1HO*8F12.6)
846 FORMAT(F13.6, 7(12H 0.000000))
K = KS-1
IF(KS=EQ=1) GOTO 841
DO 840 K=1, KK
840 PRINT 846, ETA(K)
841 DO 850 K=KS, KE
850 PRINT 836, ETA(K), XSLCS(K), YSLCS(K), ZSLS(CS(K), XSMALH(K),
1 YSMALH(K), ZSMALH(K), CMHC(K)
IF(KS=EQ=NETA) GO TO 861
KP =K+1
DO 860 K=KP, NETA
860 PRINT 846, ETA(K)
861 PRINT 816
IF(KS=EQ=1) GOTO 871
DO 870 K=1, KK
870 PRINT 876, ETA(K)
876 FORMAT(F13.6, 4(12H 0.000000))
877 DO 880 K=KS, KE
880 PRINT 836, ETA(K), X5CPGCS(K), YSCP(CS(K), ZSCP(CS(K)
IF(KS=EQ=NETA) GOTO 891
DO 890 K=KP, NETA
890 PRINT 876, ETA(K)
891 CONTINUE
NCS = NCS + 1
GOTO 709
END
SUBROUTINE PPRINT(ETA, XOC, NETA, NXOC, VMU, VML, VTU, VTL, CPU, CPL, NC)

PRINT 1, NC
10 FORMAT(1H1, 15X, *PANEL VELOCITY AND PRESSURE COEFFICIENTS FOR COMPO6)
11 NENT*, 14 )
DO 10 I = 1, NETA
PRINT 2, ETA(I)
PRINT 3
DO 10 J = 1, NXOC
CP = CPL(J, I) - CPU(J, I)
100 PRINT 4, XOC(J), VMU(J, I), VML(J, I), VTU(J, I), VTL(J, I), CPU(J, I), CP
2 FORMAT(5HOETA=, F9.6)
4 FORMAT(3F12.6)
1 RETURN
C BLAINE D. GAITHER 11/72
END
SUBROUTINE BINTEG(DAX, DAY, DAZ, XC, YC, ZC, LONPNA, LONLEN, LATPAN)

1) LATLEN, CPD, NSSTAP, NSSECP, XTL, NSTAT, S, X,
2) PANS, SLX, DXC, RCHRDL, CX, CY, CZ, CMX, CMY, CMZ
3) NSSTA, NSSEC, LONPNA, XCG, YCG, ZCG, ALPHA, ARBJ
4) TSXL, TSYL, TSZL, TMX, TMY, TZ, CDI, NC
5)

DAX, DAY, DAZ — AREAS OF SUB PANELS VIEWED FROM X, Y, Z

XC, YC, ZC — X, Y, Z VALS OF CENTROIDS OF SUB PANELS

NSSTA — NUMBER OF SUBSTATIONS

NSSEC — NUMBER OF SUBSECCTIONS

LONPNA — CONTAINS NUMBER LONGITUANALLY OF THE PANELS

WHERE CPS ARE SPECIFIED

LATPNA — IS LATERAL COUNTER PART TO LONPAN

LONLEN — NUMBER OF STATIONS WHERE CPS ARE GIVE

LATLEN — NUMBER OF SECTIONS WHERE CPS ARE GIVE

CPD — CONTAINS CP VALS AT PANELS SPECIFIED BY LONPAN AND LATPAN

NSSECP — NUMBER OF SUBSECCTIONS PER PANEL

NSSTAP — NUMBER OF SUBSTATIONS PER PANEL

XTL — VECTOR OF PANEL BOUNDRIES

NSTAT — NUMBER OF STATIONS

BCHRDL — BODY CHORD LENGTH

ARBJ — REFERENCE AREA FOR THIS BODY

X — ARRAY OF X VALS FOR CP VALS OF CPD

S SS XX — SCRATCH ARRAYS

PANS — SET TO INTEGRAL OF EACH STATION

SLX — XS WHERE SECTION LOADS

DXC — DELTA X OVER C

CX, CY, CZ — SECTION LOADS

CMX, CMY, CMZ — SECTION MOMENTS

LONPAN, LATPAN SCRATCH

DIMENSION DAX(NSSTA, NSSEC), DAY(NSSTA, NSSEC),
1) DAZ(NSSTA, NSSEC), XCG(NSSTA, NSSEC), YCG(NSSTA, NSSEC), YC(NSSTA, NSSEC),
6) 6940
22C(NSSTA,NSSEC) , LONPNA(LONLEN),
3 CPU(LONLEN, LATEN), X(LORLEN), XX(49), LONPAN(LONLEN), LATPAN
4(LATLEN), XTL(1), SLX(NSTAT), DXC(NSTAT), CX(NSTAT), CY(NSTAT),
5SCZ(NSTAT), CMX(NSTAT), CYN(NSTAT), CMZ(NSTAT), PANS(NSTAT),
6 S(NSEC), SS(49), SLSS(SJ), SLSS(S0),
EQUIVALENCE (SLSS(2), SS), (SLXX(2), XX)
REAL LATPAN, LONPNA
NAMELIST /LA/ SX, SY, SZ /LS/ NSTA,NBOT, NTOP /LC/ LT,LL
1 /LD/ AXT, AYT, AZT, ATX, ATY, ATZ
C CONVERT LATPAN * LONPAN SO THEY POINT TO THE CORRECT SUB-PANEL
C INSTEAD OF PANEL
LT = NSSTAP/ 2 + 1
DO 10 I=1, LONLEN
10 LONPAN(I) = (LONPNA(I)-1)*NSSTAP + LT
SX = 0.
SY = 0.
SZ = 0.
DO 21 J=1,NSSEC
DO 21 I=1,NSSTA
SX = SX + ABS(DAX(I,J))    
SY = SY + ABS(DAY(I,J))    
21 SZ = SZ + ABS(DAZ(I,J))    
SX = SX/2.0
SY = SY/2.0
SZ = SZ/2.0
DO 400 NSTA=1, NSTAT
NBOT = (NSTA-1)*NSSTAP + 1
NTOP = NBOT + NSSTAP - 1
PANSUM = 0
AXT = 0.
AYT = 0.
AZT = 0.
ATX = 0.
AZY = 0.
ATY = 0.
C START ON THIS STATION
DO 390 NSTAT=NBOT, NTOP
C FIND GPS FOR THIS RING(SUB-STATION)
DO 315 I=1, LATLEN

C FIND CP FOR A MERIDIAN
LT = LATPAN(I)
942 FORMAT (1H • I5 • G11.4 • 5X • 3G11.4 • 5X • 3G11.4)
IF (LONLEN • GT • 3) GOTO 305
T = XC(NST • LT)
NB = 1
NT = 2
IF (T • LE • X(2) • OR • LONLEN • LT • 3) GOTO 325
NB = 2
NT = 3
325 XX(I) = (T • X(NB)) • (CPD(N3 • I) - CPD(NB • I)) / (X(NT) • X(NB)) + 1 CPD(NB • I)
GOTO 315
305 XX(I) = CODIM1(XC(NST • LT), X • CPD(1 • I) • LONLEN • -1.0)
CONTINUE
941 FORMAT (10G11.4)
31 IF (LATLEN • GT • 1) GOTO 32
PANSUM = PANSUM + NSSEC • XX(1)
DO 500 I = 1, NSSEC
AXT = AXT + XX(1) • DAX(NST • I)
AYT = AYT + XX(1) • DAY(NST • I)
AZT = AZT + XX(1) • DAZ(NST • I)
ATX = ATX + ((YC(NST • I) - YCG) • DAZ(NST • I) - (ZC(NST • I) - ZCG) • DAY(NST • I)
ATY = ATY + ((ZC(NST • I) - ZCG) • DAX(NST • I) - (XC(NST • I) - XCG) • DAZ(NST • I)
500 ATZ = ATZ + ((XC(NST • I) - XCG) • DAX(NST • I) - (YC(NST • I) - YCG) • DAX(NST • I)
GOTO 390
32 LTT = LATPAN(1)
LTP = LTT
IF (LTT • EQ • 1) LTP = 2
SI • LTP = 0 • 0
DO 40 I = LTP, NSSEC
40 SI(I) = SI(I-1) + SQRT((YC(NST • I) - YC(NST • I-1) • 2 + (ZC(NST • I) - ZC(NST • I-1) • 2)
1C(NST • I-1) • 2 • 2)
TSF = SQRT((YC(NSSEC) - YC(1)) • 2 + (ZC(NSSEC) - ZC(1)) • 2)
IF (LTT • EQ • 1) GOTO 46
SI(1) = SI(NSSEC) + TSF
LTTT = LTT - 1
IF(LTTT*EQ.1) GOTO 46
DO 45 I= 2, LTTT
45 S(I) = S(I-1) + SQR((YC(NST,I)-YC(NST,I-1))**2 + (ZC(NST,I))
2(ZC(NST,I-1)**2)
46 XX(LATLEN+1) = XX(I)
DO 50 I=1,LATLEN
LT = LATPAN(I)
50 SS(I) = S(LT)
SS(LATLEN+1) = SS(LATLEN) + SS(LATLEN+1) + SS(LATLEN)
DO 60 I=1, NSSEC
6 T = COSM1(S(I), SS, SLXX, LATLEN+2, 1.0)
PANSUM = PANSUM + T
AXT = AXT + T*DAX(NST,I)
AYT = AYT + T*DAY(NST,I)
AZT = AZT + T*DAZ(NST,I)
ATX = ATX + (YC(NST,I) - YCG) * DAZ(NST,I) - (ZC(NST,I)-ZCG)*DAY(NST,1ST,I)) * T
6 ATY = ATY + (ZC(NST,I)-ZCG)*DAX(NST,I) - (XC(NST,I)-XCG)*DAZ(NST,1ST,I)) * T
6 ATZ = ATZ + (XC(NST,I)-XCG)*DAY(NST,I) - (YC(NST,I)-YCG)*DAX(NST,1ST,I)) * T
390 CONTINUE
PANS(NSTA) = PANSUM
SLX(NSTA) = (XTL(NSTA)+XTL(NSTA+1))/2.0
T = (XTL(NSTA+1)-XTL(NSTA))/ BCHRDL
DXC(NSTA) = T
TS = T * SY
T = T*SZ
CX(NSTA) = -AXT/T
CY(NSTA) = -AYT/T S
CZ(NSTA) = -AZT/T
T = ARBJ * BCHRDL
CMX(NSTA) = -ATX/T
CMY(NSTA) = -AYT/T
400 CMZ(NSTA)= -ATZ/T
C FIND TOTAL MOMENTS * LOADS
TSXL = 0.
TSYL = 0.
TSZL = 0.
TMX = 0.
TMY = 0.
TMZ = 0.
PRINT 923, NC

923 FORMAT(1H1,35X,5*BODY SECTIONAL LOADS FOR COMPONENT*, I5,1H0,10X, 6
1, X/C*,8X,*CXW/WAVG*,7X,CYH/HAVG*,07X,*CZW/WAVG*)

DO 410 I=1, NSTAT
SLX(I)=SLX(I)/BCHRD
PRINT 948, SLX(I), CX(I), CY(I), CZ(I)

948 FORMAT(1H0,4F15.6)
TSXL = TSXL + CX(I)*DXC(I)
TSYL = TSYL + CY(I)*DXC(I)
TSZL = TSZL + CZ(I) * DXC(I)
TMX = TMX + CMX(I)*DXC(I)
TMY = TMY + CMY(I)*DXC(I)
TMZ = TMZ + CMZ(I)*DXC(I)
TSXL = TSXL * SZ/ARBJ
TSYL = TSYL*SY/ARBJ
TSZL = TSZL*SZ/ARBJ
T = TSXL*TSXL + TSYL*TSYL + TSZL*TSZL
XCPC = XCG / BCHRD + (TSXL*TMZ - TSZL*TMY) / T
YCPC = YCG/BCHRD + (TSZL*TMX-TSXL*TMZ) / T
ZCPC = ZCG/BCHRD + (TSXL*TMY-TSYL*TMX) / T
CL = TSZL*COS(ALPHA) - TSXL*SIN(ALPHA)
CD = TSXL * SIN(ALPHA) + TSXL * COS(ALPHA)
PRINT 906

906 FORMAT(1H0,44X,*TOTAL BODY LOADS*)
CDI=CDI/ARBJ
PRINT 901, TSXL, TSYL, TSZL, TMX, TMY, TMZ, CDI

901 FORMAT(1H0,7X,*CX*,13X,*CY*,13X,*CZ*,12X,*CMX*,12X,*CMY*,12X,*CMZ*
1*, 12X, *CDI*/1H,7F15.6)
PRINT 902, XCPC, YCPC, ZCPC, SX, SY, SZ

902 FORMAT(1H0, 5X,*X/C CP*,9X,*Y/C CP*,9X,*Z/C CP*,11X,*SX*,13X,*SY*
1, 15X, *SZ*/1H,6F15.6)
RETURN
C
BLAINE D. GAITHER 9/72
END

6 8510
6 8520
6 8530
SUBROUTINE REFLECT(SYM,LO,LA,AX,AY,AZ,X,Y,Z,LON,LAT,LT,CPD,NSS)  
DIMENSION AX(LO,LA),AY(LO,LA),AZ(LO,LA),X(LO,LA),Y(LO,LA),Z(LO,LA)
 1) LATP(LAT),CPD(LON,LAT) 
REAL LATP  
LL=NSS/2+1  
DO 1 I=1,LAT  
1 LATP(I)=(LATP(I)-1)*NSS+LL  
IF (SYM.EQ.-1.) RETURN

C REFLECT GEOMETRY
ILA=LA  
ILA1=ILA+1  
K=LA  
DO 10 J=1,LA  
JJ=LA+1-J  
10 CONTINUE  
IF (ABS(Y(I,JJ)) .LT. 0.001) GO TO 10  
K=K+1  
DO 5 I=1,LO  
AX(I,K)=AX(I,JJ)  
AY(I,K)=-AY(I,JJ)  
AZ(I,K)=AZ(I,JJ)  
X(I,K)=X(I,JJ)  
Y(I,K)=-Y(I,JJ)  
Z(I,K)=Z(I,JJ)  
5 CONTINUE

C REFLECT CPS
K=LAT  
DO 20 J=1,LAT  
JJ=LAT+1-J  
LT=LATP(JJ)  
IF (ABS(Y(I,LT)) .LT. 0.001) GO TO 20  
K=K+1  
DO 15 I=1,LON  
CPD(I,K)=CPD(I,LT)  
20 CONTINUE

C REFLECT CPS
K=LA  
DO 25 K=1,ILA1,LA  
IF (Z(I,K1).EQ.Z(I,LT) .AND. -Y(I,K1).EQ.Y(I,LT)) GO TO 26  
25 CONTINUE

GO TO 26
26 LATP(K) = K1
20 CONTINUE
   LAT = K
   RETURN
   END
<table>
<thead>
<tr>
<th>Line</th>
<th>Text</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SUBROUTINE BPRINT(NO, NA, X, Y, Z, VM, VT, CP, NC)</td>
</tr>
<tr>
<td>2</td>
<td>DIMENSION X(NO, NA), Y(NO, NA), Z(NO, NA), VM(NO, NA), VT(NO, NA)</td>
</tr>
<tr>
<td>3</td>
<td>PRINT 1, NC</td>
</tr>
<tr>
<td>4</td>
<td>1 FORMAT(1HI, 24X, <em>BODY VELOCITY AND PRESSURE COEFFICIENTS FOR COMP</em>)</td>
</tr>
<tr>
<td>5</td>
<td>IONENT*, I3)</td>
</tr>
<tr>
<td>6</td>
<td>DO 10 I=1, NA</td>
</tr>
<tr>
<td>7</td>
<td>PRINT 2, I</td>
</tr>
<tr>
<td>8</td>
<td>2 FORMAT(*0LATERAL STATION *, I3)</td>
</tr>
<tr>
<td>9</td>
<td>PRINT 3</td>
</tr>
<tr>
<td>11</td>
<td>PRINT 4, (J, X(J, I), Y(J, I), Z(J, I), VM(J, I), VT(J, I), CP(J, I))</td>
</tr>
<tr>
<td>12</td>
<td>J=1, NO</td>
</tr>
<tr>
<td>13</td>
<td>4 FORMAT(4X, I3, 5X, 6F12.6)</td>
</tr>
<tr>
<td>14</td>
<td>10 PRINT 5</td>
</tr>
<tr>
<td>15</td>
<td>5 FORMAT(1HI)</td>
</tr>
<tr>
<td>16</td>
<td>RETURN</td>
</tr>
<tr>
<td></td>
<td>C BLA9NE D. GAITHER 11/72</td>
</tr>
<tr>
<td></td>
<td>END</td>
</tr>
</tbody>
</table>
SUBROUTINE FFDRAAG(NR,NP,YP,ZB,XKB,NPE,BSYM,YP,ZP

1  XKP,NPE,PSYM,PIND,TBD,TPD,NPS)

DIMENSION YP(I),ZB(I),XKB(I),NPE(I),BSYM(1),YP(I)

1  ZP(I),XKP(I),NPE(I),PSYM(1),PIND(1)

2  T2D(I),TPD(I)

COMMON/BODY/ DUM(15000),Y(200),Z(200)

1  S(200),SYM(200),YN(200),ZN(200),XK(200),D(200),NUMB(200)

2  NUMP(51),XP(200),DY(200),NZ(200),NS(200),ETA(200),YNP(200)

3  ZNP(200)

DATA IONE717,MAXVOR7000,NVOR40/

K=0

TNB5=0.

IF(NB.EQ.0) GO TO 20

CALCULATION OF BODY DATA

DO 5 I=1,NB

5  TNBE=TNBE+NBE(I)

L=0

LKB=0

DO 15 I=1,NB

10  J=1,NUB

N1=L+J

N2=N1+1

K=K+1

XK(K)=XKB(LKB+J)

DELY=YP(N2)-YP(N1)

DELZ=ZB(N2)-ZB(N1)

S(K)=SQRT(DELY**2+DELZ**2)

EPS=.0000001

IF(S(K).GT.EPS) GO TO 2

S(K)=0.

YN(K)=1.

ZN(K)=1.

GO TO 3

2  YN(K)=-DELZ/S(K)

ZN(K)=DELY/S(K)

SYM(K)=BSYM(I)

Y(K)=5*(YP(N1)+YP(N2))

10  Z(K)=5*(ZB(N1)+ZB(N2))
NUM2(I)=NBE(I)
LKB=LKB+NUB
15 L=L+NUB+1
20 IF(NP.EQ.0) GO TO 50

CALCULATION OF PANEL DATA

INDP=0
IF(NP.EQ.0) GO TO 150
INDP=1
GO TO 8

150 NPS=NVOR/NPE(I)
DO 4 I=1,NP
4 IF(NVOR/NPE(I).LT.NPS) NPS=NVOR/NPE(I)
6 TNPE=0.
7 TNPE=TNPE*NPS*NPE(I)
DO 7 I=1,NP
IF(TNPE+TNBE.LE.MAXVOR)GO TO 8
NPS=NPS+1
8 GO TO 6

8 L=0
LKP=0
DO 49 I=1,NP
49 NUP=NPE(I)
SPAN=0.
DO 45 J=1,NUP
45 N1=L+J
N2=N1+1
DY(J)=YP(N2)-YP(N1)
DZ(J)=ZP(N2)-ZP(N1)
DS(J)=SORT(DY(J)**2+DZ(J)**2)
YMP(J)=-DZ(J)/DS(J)
ZMP(J)=DY(J)/DS(J)

45 SPAN=SPAN+DS(J)
SUM=0.
DO 46 J=1,NUP
46 ETA(J)=(SUM+5*DS(J))/SPAN

46 SUM=SUM+DS(J)
DO 47 J=1,NUP
47 XP(J)=XKP(LKP+J)
NUZ=NUZ+1
IF (PIAD(I).NE.1.) GO TO 44
NUZ=NUP+1
ETA(NUZ)=1.
XP(NUZ)=0.
44 SUM=* 
   DO 48 J=1,NUP
   N1=L+J
   N2=N1+1
   DELS=DS(J)/NPS
   DO 48 J1=1,NPS
   K=K+1
   S(K)=DELS
   SYM(K)=PSYM(I)
   YN(K)=YNP(J)
   ZN(K)=ZNP(J)
   SUM=SUM+.5*DELS
   ET=SUM/SPAN
   SUM=SUM+.5*DELS
   IF (INDP.EQ.1) GO TO 488
   CALL COND1(ETA,XP,NUZ,ET,XK(K),IONE)
488 Y(K)=YP(N1)+(J1-.5)*DELS*SYM(K)
48 Z(K)=ZP(N1)+(J1-.5)*DELS*(-YN(K))
   NUPP(I)=NPS*NUP
   LKP=LKP+NUP
49 L=L+NUP+1
C   CALCULATION OF DRAG CN EACH ELEMENT
50 NV=K
   WRITE(6,333)(XK(J),J=1,NV)
333 FORMAT(//(10F12.5))
   DO 56 I=1,NV
   V=0.
   H=0.
   DO 56 J=1,NV
   YY=Y(I)-Y(J)
   ZZ=Z(I)-Z(J)
   CALL HSHOE(YY,ZZ,S(J),Y(J),ZN(J),AV,AA)
   IF (SYM(J).EQ.1) OR (SYM(J).EQ.-1) GO TO 54
   YY=-Y(I)-Y(J)
   CALL HSHOE(YY,ZZ,S(J),Y(J),ZN(J),AV,DELF,DELF)
   C
   C
IF (SYM(J) .EQ. 0.) GO TO 52
DELV = DELV
DElw = DELw
52 AV = AV + DELV
AW = AW + DELw
54 V = V AV * XK(J)
56 W = W + AW * XK(J)
58 D(I) = -2. * XK(I) * S(I) * (V * YN(I) + W * ZN(I))

CALCULATION OF TOTAL DRAG
K = 0
IF (NB .EQ. 0.) GO TO 95
DO 93 I = 1, NB
TBD(I) = 0.
N1 = K + 1
N2 = K + NUMB(I)
DO 92 J = N1, N2
92 TBD(I) = TBD(I) + D(J)
K = N2
IF (SYM(J) .EQ. 0. OR SYM(J) .EQ. -2.) TBD(I) = 2. * TBD(I)
93 CONTINUE
95 IF (NP .EQ. 0.) RETURN
DO 98 I = 1, NP
TPD(I) = 0.
N1 = K + 1
N2 = K + NUMP(I)
DO 96 J = N1, N2
96 TPD(I) = TPD(I) + D(J)
K = N2
98 CONTINUE
RETURN
END
SUBROUTINE NFDRAG( NC, NS, NCP, NSP, XOCPC, XOCV, ETACP, XV, YV, ZV, NV1, NV2 )
DIMENSION XOCPC( 1 ), XOCV( 1 ), ETACP( 1 ), XV( 1 ), YV( 1 ), ZV( 1 ), TANV( 1 )
DIMENSION XOCPC( 1 ), XOCV( 1 ), ETACP( 1 ), XV( 1 ), YV( 1 ), ZV( 1 ), TANV( 1 )
DIMENSION XOCPC( 50 ), XOCV( 50 )
!
COMMON DA( 15000 ), COMMON/BODY/ B( 31550 )
!
![Equivalence Table]

1. (CBAR, DA( 5 )) (SPAN, DA( 6 ))

![Equivalence Table]

EQUIVALENCE ( Q, DA( 13 ) ), ( GAM, DA( 14 ) ), ( YCG, DA( 8 ) ), ( ZCG, DA( 9 ) )

1. (CBAR, DA( 5 )) (SPAN, DA( 6 ))

EQUIVALENCE ( X( 1 ), R( 16201 ) ), ( X( 1 ), R( 19001 ) ), ( Y( 1 ), R( 17001 ) )

1. (Z( 1 ), B( 18001 ) ), ( TAN( 1 ), R( 15001 ) ), ( CP( 1 ), R( 21001 ) )
2. (SIGMA( 1 ), R( 23001 ) ), ( O( 1 ), R( 27001 ) ), ( CCG( 1 ), R( 92001 ) )
3. (CDA( 1 ), B( 29101 ) ), ( CDT( 1 ), B( 29201 ) ), ( DUMI( 1 ), B( 29301 ) )
4. (DUMO( 1 ), B( 29401 ) ), ( P( 1 ), B( 29501 ) ), ( XCPC( 1 ), B( 31501 ) )

333 FORMAT(// (1X, 10F12.5))
332 FORMAT(// (1X, 10I12))
WRITE( 6, 333 ) NC, NS, NCP, NSP, NSPP
WRITE( 6, 333 ) ( XOCPC( I ), I = 1, NC )
WRITE( 6, 333 ) ( XOCV( I ), I = 1, NC )
WRITE( 6, 333 ) ( ETACP( I ), I = 1, NC )
R4P1 = 0.7957747
EPS2 = 0.0075968656
EPS3 = 0.000001
ISYM = DA( 3426 )
NV1 = NS * NC
NV2 = 2 * NV1
WRITE( 6, 333 ) ( XV( I ), I = 1, NV2 )
WRITE( 6, 333 ) ( YV( I ), I = 1, NV2 )
WRITE( 6, 333 ) ( ZV( I ), I = 1, NV2 )
WRITE( 6, 333 ) ( TANV( I ), I = 1, NV2 )
WRITE( 6, 333 ) ( SIGMA( 1 ), I = 1, NV2 )
WRITE( 6, 333 ) ( CP( 1 ), I = 1, NV1 )
CALCULATE NEW VORTEX COORDINATES
NSPP = 3
JK=0
NC2=2*NC
DELY=(YV(NC2+1)-YV(1))/NSPP
DELY1=-DELY*(NSPP/2+1)
DO 100 I=1,NS
DELY=DELY1
DO 100 J=1,NSPP
DELY=DELY+DELY
DO 100 K=1,NC2+2
IK1=(K-1)*NC2+K
IK2=IK1+1
JK=JK+1
X(JK,1)=XV(IK1)+DELY*TANV(IK1)
X(JK,2)=XV(IK2)+DELY*TANV(IK2)
Y(JK)=YV(IK1)+DELY
Z(JK)=ZV(IK1)
TAN(JK,1)=TANV(IK1)
TAN(JK,2)=TANV(IK2)
SIGMA(JK,1)=SIGMAV(IK1)
100 SIGMA(JK,2)=SIGMAV(IK2)
NV=JK
NSS=NS*NSPP
WRITE(6,333)(X(I,1),X(I,2),I=1,NV)
WRITE(6,333)(Y(I),I=1,NV)
WRITE(6,333)(Z(I),I=1,NV)
WRITE(6,333)(TAN(I,1),TAN(I,2),I=1,NV)
WRITE(6,333)(SIGMA(I,1),SIGMA(I,2),I=1,NV)
CALCULATE ETAS OF VORTICES
DELE=1./NSS
SUM=-.5*DELE
DO 110 I=1,NSS
SUM=SUM+DELE
110 ETAV(I)=SUM
WRITE(6,333)(ETAV(I),I=1,NSS)
CALCULATE X/C OF PANEL CENTROIDS
SLOPE=(TANV(2*NC)-TANV(1))/(XV(2*NC)-XV(1))
TANL=TANV(1)-.5*(XV(2)-XV(1))*SLOPE
TANT=TANV(2*NC)+.5*(XV(2*NC)-XV(2*NC-1))*SLOPE
DX=DA(3450)+DA(3432)
XLE = YV(1) * TANL + DX
XTE = YV(1) * TANL + DX + DA(3453)
CHORD = XTE - XLE
J = 0
DO 1000 I = 1, NC2 * 2
J = J + 1
1000 XOCPC(J) = 0.5 * (XV(I) + XV(I + 1)) - XLE / CHORD
CALCULATE CPNET AT VORTICES
DO 120 I = 1, NCP
DO 130 J = 1, NSP
130 DUM(J) = CPNET(NCP * (J - 1) + 1)
CALL POL(ETACP, DUM * NSP, ETAV, DUMO * NSS)
DO 120 J = 1, NSS
NJ = NCP * (J - 1) + 1
120 P(NJ) = DUMO(J)
DO 140 I = 1, NSS
K = NC * (I - 1) + 1
J = NCP * (I - 1) + 1
140 CALL POL(XOCCP, P(J) * NCP, XOCP, CP(K), NC)
WRITE(6, 333) (CP(I) * I = 1, NV)
YVV = 0.5 * DELY
EPS1 = 2.5 * YVV
NV2 = 2 * NV
IF(ISYM + EQ. 1) NSS = 2 * NSS
DO I = 1, NV2
IF(ABS(CP(I)) * GT. EPS3) GO TO 3
1 CONTINUE
GO TO 44
CALCULATE DRAG DUE TO LIFT
3 DO 43 I = 1, 2
WRITE(6, 333) YVV
331 FORMAT(1X * 10E12, 5)
DO 5 JJ = 1, NV2
5 D(JJ) = 0.0
NSKIPI = 0
9 DO 20 J = 1, NV
IPASS = 1
XMF = 1.0
NSKIP2 = 0
20 CONTINUE
GO TO 44
W=0.
DO 22 K=1,NV
DELX=X(J+1)-X(K,1)
YY=Y(J)
IF(ISYM.EQ.1.AND.SKIP1.NE.0) YY=-YY
DELY=YY-Y(K)
DELZ=Z(J)-Z(K)
DELXSQ=DELX**2
DELZSQ=DELZ**2
YP=DELY+YVV
YN=DELY-YVV
YPSQ=YP**2
YNsq=YN**2
TERM1=YPSQ+DELZSQ
TERM2=YNSQ+DELZSQ
TERM3=DELXSQ+YPSQ
TERM4=DELXSQ+YNSQ
TERM5=TERM1+DELXSQ
TERM6=TERM2+DELXSQ
CHECK1=DELXSQ/TERM3
CHECK2=DELXSQ/TERM4
F=0.
IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY).LT.1.EVYV) GO TO 32
1)  GT.YVV) GO TO 32
IF(DELZ.LT.EPS1.AND.ABS(DELY).LE.YVV.AND.ABS(DELX).LT.ABS(0.5*(X(K,6)-X(K,1)))) GO TO 33
F=(YP/SQRT(TERM5)-YN/SQRT(TERM6))*DELX/(DELXSQ+DELZSQ)
GO TO 33
32 F=5*DELX*ABS(1./TERM4-1./TERM3)
33 IF(TERM1.LT.EPS2.AND.DELX.LT.0.) GO TO 34
IF(SQRT(TERM11.LT.EPS3) GO TO 35
F=F+YP/TERM1*(1.+DELX/SQRT(TERM5))
GO TO 35
34 F=F+SQRT(TERM1)/TERM5
35 IF(TERM2.LT.EPS2.AND.DELX.LT.0.) GO TO 36
IF(SQRT(TERM21.LT.EPS3) GO TO 30
F=F+YP/TERM2*(1.+DELX/SQRT(TERM46))
GO TO 30
36 F=F-SQRT(TERM2)/TERM5
30 KK=K+NSkip2
   W=W+XMF*CP(KK)*(X(K,2)-X(K,1))#F
   IF(IPASS.EQ.2 .OR. ISYM.EQ.-1) GO TO 31
   IPASS=2
   DELY=-YY-Y(K)
   IF(ISYM.EQ.0) GO TO 10
   IF(ISYM.EQ.1) GO TO 11
   XMF=-1.
   GO TO 10
11 NSkip2=NV
   GO TO 10
31 IPASS=1
   XMF=1.
   KK=J+NSkip1
   D(KK)=2.*CP(KK)*(XT(J,2)-X(J,1))*W*R4PI
   IF(ISYM.NE.1 .OR. NSkip1.NE.0) GO TO 21
   NSkip1=NV
   GO TO 9
21 IF(II.EQ.2) GO TO 40
   DO 41 I=1*NSS
      CDS(I)=0.
   DO 41 J=1*NC
      K=NC*(I-1)+J
   41 CDS(I)=CDS(I)+D(K)
   GO TO 43
40 DO 42 I=1*NSS
   CDA(I)=0.
   DO 42 J=1*NC
      K=NC*(I-1)+J
   42 CDA(I)=CDA(I)+D(K)
43 CONTINUE
44 DO 45 I=1*NV2
   DO 45 J=1*2
      IF(ABS(SIGMA(I,J)).GT.EPS3) GO TO 39
   45 CONTINUE
GO TO 500
CALCULATE DRAG DUE TO THICKNESS
39 DO 46 I=1*NV2
46 D(I)=0.
NSKIPI1=0
47 DO 50 I=1,NV
J=1,2
IPASS=1
NSKIPI2=0
U=0.
DO 70 K=1,NV
L=1,2
DELX=X(I,J)-X(K,L)
YY=Y(I)
IF(ISYM.EQ.1.AND.NSKIPI1.NE.0) YY=-YY
DELY=YY-Y(K)
DELZ=Z(I)-Z(K)
T=TAN(K,L)
XP=DELX+T*YVV
XN=DELX-T*YVV
YN=DELY-YVV
XY=DELX-T*DELY
TSQ1=1.+.T**2
PP=SQR(TSQ1)
R2SQ=XY**2+DELZ**2*TSQ1
TEST1=XN**2+YN**2
TEST2=XP**2+YP**2
R4=SQR(TEST1+DELZ**2)
R5=SQR(TEST2+DELZ**2)
YX=DELY+T*DELX
YVT=YVV*TSQ1
TERM4=1./R4-1./R5
XYSG=XY**2
CHECK1=XYSG/TEST1
CHECK2=XYSG/TEST2
IF(DELZ.LT.EPS1.AND.CHECK1.LT.EPS2.AND.CHECK2.LT.EPS2.AND.ABS(DELY).LT.EPS1) GO TO 48
IF(D(1).LT.YVV) GO TO 49
IF(D(1).LT.EPS1.AND.ABS(DELY).LT.YVV.AND.ABS(XX).LT.ABS(.5*(X(K,2))') GO TO 49
1=-X(K,1)) GO TO 49
XI2=(YX+YVT)/R5
XI3=-(-YX-YVT)/R4
TERM1=(X12+X13)/R25Q
GO TO 51
48 TERM1=5/PP*ABS(1./TEST1-1./TEST2)
GO TO 51
49 TERM1=0.
51 EUS=T/PP*TERM4+1./PP*XY*TERM1
KK=K+NSKIP2
U=U+SIGMA(KK,L)*EUS
IF(IPASS.EQ.2.OR.ISYM.EQ.1) GO TO 61
IPASS=2
DELY=-YY-Y(K)
IF(ISYM.EQ.0) GO TO 62
NSKIP2=NV
GO TO 62
61 IPASS=1
70 NSKIP2=0
KK=I+NSKIP1
U=U*R4PI
60 D(KK)=D(KK)-((U+1.-Q*2.*Z(I)-ZCG)/CBAR-GAM*2.*Y(I)-YCG)/SPAN)*
1 SIGMA(KK,J)*SQRT(1.+TAN(KK,J)**2)
50 D(KK)=2.*D(KK)
IF(ISYM.NE.1.OR.NSKIP1.NE.0) GO TO 81
NSKIP1=NV
GO TO 47
81 DO 80 I=1,NSS
CDT(I)=0.
DO 80 J=1,NC
K=NC*(I-1)+J
80 CDT(I)=CDT(I)+D(K)
500 CONTINUE
CDST=0.
CDAT=0.
CDTT=0.
CALCULATION OF TOTAL DRAG
DO 200 I=1,NSS
CDST=CDST+CDST(I)
CDAT=CDAT+CDAT(I)
200 CDTT=CDTT+CDTT(I)
CDST=CDST*2.*YVV
CDAT=CDAT#2.*YVV
CDTT=CDTT#2.*YVV
CDTHT=CDAT-CDST

CALCULATE SPANWISE DISTRIBUTION OF DRAG
CALL POL(ETA1,CDS,NSP,ETACP,CDSCP,NSP)
CALL POL(ETA1,CDA,NSP,ETACP,CDACP,NSP)
CALL POL(ETA1,CDT,NSP,ETACP,CDTCP,NSP)
DO 190 I=1,NSP

190 CDTH(I)=CDACP(I)-CDSCP(I)
WRITE(6,333)(CDS(I),I=1,NSS)
WRITE(6,333)(CDA(I),I=1,NSS)
WRITE(6,333)(CDT(I),I=1,NSS)
WRITE(6,333)(CDSCP(I),I=1,NSP)
WRITE(6,333)(CDACP(I),I=1,NSP)
WRITE(6,333)(CDTCP(I),I=1,NSP)
WRITE(6,333)(CDTH(I),I=1,NSP)
WRITE(6,333)CDST,CDAT,CDTT,CDTHT

CALCULATE TRAILING EDGE K VALUES
DO 180 I=1,NSS
XK(I)=0.
DO 180 J=1,NC
K=NC*(I-1)+J

180 XK(I)=XK(I)+CP(K)*((X(K,J)-X(K,1))
WRITE(6,333)(XK(I),I=1,NSS)
RETURN
END
FUNCTION POLINT(X,Y,NPTS,RLLIM,RULIM)

C X - INDEPENDENT VARIABLES
C Y - DEPENDENT VARIABLES
C NPTS - NUMBER OF DEPENDENT AND INDEPENDENT VARIABLES
C O .LT. RLLIM .LT. RULIM .LT. 1
C
DIM5NSION X(NPTS), Y(NPTS), PX(22), PY(22)
IF(NPTS-20) 11, 11, 1

1 PRINT 6
6 FORMAT(*Too Many Points Given to POLINT*)
STOP
11 PX(1) = ACOS(1.0-2.0*RLLIM)
PY(1) = 0.0
PX(NPTS+2) = ACOS(1.0-2.0*RULIM)
PY(NPTS+2) = 0.0
DO 20 I=1, NPTS
PHI = ACOS(1.0-2.0*X(I))
PX(I+1) = PHI
PY(I+1) = Y(I) *.5*SIN(PHI)
20 CONTINUE
CALL INTNC(PX(1),PX(NPTS+2), NPTS+2, PX, PY, POLINT, IERR)
RETURN
END
FUNCTION TANALF(DZT, DZC, EPSILO, DA, ETA, X, TB)  

DZT - DELTA X T  
DZC - DDELTA X C  
ESILO - EPSILON  
X - X VALUE AT THIS POINT  
TB - 1. FOR BOTTOM SURFACE -1. FOR TOP SURFACE  
DA - ARRAY OF CONTROL SURFACE INFO.  

DIMENSION DA(1)  

IS THERE A CONTROL SURFACE AT THIS POINT  
IJ = 1  

IS THIS CONTROL POINT AT OUR ETA  
1 IF(DA(IJ).EQ.0.0) GOTO150  
IF(DA(IJ+2).LE.ETA .AND. DA(IJ+3).GE. ETA) GOTO 30  
2 IJ = IJ + 8  
IF(IJ.LT.80) GOTO 1  
PRINT 156, IJ, DA  
STOP, 445  

156 FORMAT(I3*8G14.6)  

30 T3 = DA(IJ+4)+(DA(IJ+5)*DA(IJ+4))/(DA(IJ+3)-DA(IJ+2))*ETA-DA(IJ+2)  

1 IF(DA(IJ).EQ.1.0 .AND. T3 .GE. X) GOTO 100  
IF(DA(IJ).EQ.-1.0 .AND. T3 .LE. X) GOTO 75  
GOTO 2  

75 TA = DA(IJ+1)  
GOTO 101  

100 TA = -DA(IJ+1)  

101 T = (TB*DZT - DZC) + EPSILO  
TA = TAN(TA)  
TANALF = (TA+T)/(1.0-T*TA)  
RETURN  

150 TANALF = (TB*DZT-DZC) + EPSILO  
RETURN  
END
SUBROUTINE HSHOE(Y,Z,S,YN,ZN,V,W)
DATA R4PI/*07957747/*
DATA EPS/*0000001/*
IF(S.GT.EPS) GO TO 5
V=0.*
W=0.*
RETURN
5 HS=5*S
YP=Y*ZN-Z*YN
ZP=Z*ZN+Y*YN
YPP=YP+HS
YPN=YP-HS
RP=YP**2+ZP**2
RN=YPN**2+ZP**2
EV=ZP*(1./RP-1./RN)*R4PI
EW=(YPN/RN-YP/RP)*R4PI
V=EV*ZN+EW*YN
W=EW*ZN-EV*YN
RETURN
END
SUBROUTINE POL(XI,YI,NI,XO,YO,NO)
DIMENSION XI(1),YI(1),XO(1),YO(1)
DIMENSION TI(50),FI(50),TO(100)
DATA PI/3.1415927/
NI1=NI+1
NI2=NI+2
FI(1)=0.
DO 10 I=2,NI1
   TI(I)=ACOS(1.0-2.0*XI(I-1))
10   FI(I)=YI(I-1)*2.0*SQRT(XI(I-1)*(1.0-XI(I-1)))
   FI(I)=0.
   TI(NI2)=PI
   FI(NI2)=0.
   DO 15 I=1,NO
      TO(I)=ACOS(1.0-2.0*XO(I))
      CALL COSIM(TI,FI,NI2,TO,YO,NO)
   15   DO 20 I=1,NO
      Y0(I)=YO(I)/(2.0*SQRT(XO(I)*(1.0-XO(I))))
   20   RETURN
END
SUBROUTINE INTNC (XL, XU, NX, X, Y, Z, IERR)
CALL INTNC (XL, XU, NX, X, Y, Z, IERR)
XL LOWER LIMIT OF INTEGRATION
XU UPPER LIMIT OF INTEGRATION
NX NUMBER OF ITEMS IN X AND Y ARRAYS
X ARRAY OF ABSCISSAS (INDEPENDENT VARIABLE)
Y ARRAY OF ORDINATES (DEPENDENT VARIABLE)
Z VALUE OF DEFINITE INTEGRAL
XI ARRAY OF ABSCISSAS USED IN INTEGRATION
YI ARRAY OF ORDINATES USED IN INTERPOLATION
IERR ERROR INDICATOR, ZERO VALUE INDICATES PROPER SOLUTION, NONZERO INDICATES ERROR
IXU INDICATOR FOR UPPER LIMIT
DIMENSION X(1), Y(1), XI(5), YI(5)
IXU = 0
IERR = 0
TEST TO BE WITHIN RANGE
IF (X(1) - XL) < 10, 10, 5
WRITE (6, 7, 1) Xl, X(1)
7 FORMAT (1HO E15.8, 66H, LOWER LIMIT OF INTEGRATION IS LESS THAN F6 4850
FIRST ABSCISSA OF ARRAY, E17.8)
GO TO 110
IF (XU - X(NX)) > 15, 15, 12
WRITE (6, 14, 1) XU, X(NX)
14 FORMAT (1HO E15.8, 68H, UPPER LIMIT OF INTEGRATION IS GREATER THAN F6 4900
LAST ABSCISSA OF ARRAY, E17.8)
GO TO 110
Z = 0.0
XK = 1.0
INITIALIZE
FIND FIRST INTERVAL
DO 30 I = 1, NX
IF (X(I) - XL) < 30, 45, 40
30 CONTINUE
SETUP FOR FIRST INTERVAL
DX = X(I) - XL
XI(1) = XL
X(5) = X(I)
Y(5) = Y(I)
N = 4
K1 = 1
GO TO 60

C 45 I = I + 1

C 50 X(I) = X(I-1)
Y(I) = Y(I-1)
X(5) = X(I)
Y(5) = Y(I)
N = 3
K1 = 2
DX = X(I) - X(I-1)

C 60 X(2) = X(1) + .25 * DX
X(3) = X(1) + .5 * DX
X(4) = X(1) + .75 * DX

C 70 CALL CODIS1 (N, X(I), Y(I), X, Y, NX, NK)

C 80 Z = Z + (DX / 90, 0) * (7, 0 * (Y(I) + Y(5)) + 32, 0 * (Y(2) + Y(4)))

C 90 I = I + 1

C 94 IF (NX - I) 100, 94, 94

C 110 IERR = 1

C 130 IXU = 1
X(5) = XU
X(I) = X(I-1)
Y(I) = Y(I-1)
<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>NI = 4</td>
</tr>
<tr>
<td>30</td>
<td>K1 = 2</td>
</tr>
<tr>
<td>70</td>
<td>DX = XU - X(I-1)</td>
</tr>
<tr>
<td>60</td>
<td>GO TO 60</td>
</tr>
<tr>
<td>60</td>
<td>END</td>
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**SUBROUTINE DUMP**

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
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<tr>
<td>6</td>
<td>SUBROUTINE DUMP</td>
</tr>
<tr>
<td>6</td>
<td>PRINT 6</td>
</tr>
<tr>
<td>6</td>
<td>FORMAT(<em>OSUBROUTINE DUMP</em>)</td>
</tr>
<tr>
<td>6</td>
<td>STOP</td>
</tr>
<tr>
<td>6</td>
<td>END</td>
</tr>
</tbody>
</table>

C
SUBROUTINE CODIS1(N1,X,Y,XI,YI,N2,XK)
C   F R ANDERSON 056 291 072 BLDG. 2 STATION 15
C   CONTROLLED DEVIATION INTERPOLATION SUBROUTINE..CODIS1
C   CALLING SEQUENCE
C   CALL CODIS1(N1,X,Y,XI,YI,N2,XK)
C   N1 = NO. OF ARGUMENTS X
C       X = ARGUMENTS-ABSCISSA VALUES.
C   Y = INTERPOLATED ORDINATES.
C   XI = ARRAY OF THE ABSCISSAE.
C   YI = ARRAY OF THE ORDINATES.
C   N2 = NO. OF POINTS ON CURVE.
C   XK = END INTERVAL CONTROL CONSTANT ( 0 TO 1.0 )
C
C  DIMENSION X(1),Y(1),XI(1),YI(1),D(2),A(2),B(2),C(2)
100 IN = 0
   DO 800 N = 1,N1
   IF(N2-2).NE.110,115,120
110 Y(IN) = YI(N2)
   GO TO 800
115 Y(IN) = (YI(2)-YI(1))/(XI(2)-XI(1))* (X(N)-XI(1)) + YI(1)
   GO TO 800
   J = 1
120 IF(XI(J)-X(N)).LE.130,140,150
130 J = J+1
   IF(J-N2).LE.125,125,145
140 Y(IN) = YI(J)
   GO TO 800
145 Y(IN) = (YI(N2)-YI(N2-1))/(XI(N2)-XI(N2-1))* (X(N)-XI(N2-1)) + YI(N2-1)
   GO TO 800
150 IF(J-2).GT.115,155,160
155 K = 2
   JJ= 1
   GO TO 185
160 IF(J-N2).GT.170,165,145
165 K = N2-1
   JJ= 2
   GO TO 185
170 IF(J-IN)180,300,130
180 JJ = 3
K = J
185 DO 200 M = 1,2
X1 = XI(K-1)-XI(K)
X2 = XI(K)-XI(K-2)
X3 = XI(K-2)-XI(K-1)
Y1 = YI(K-1)-YI(K)
Y2 = YI(K)-YI(K-2)
Y3 = YI(K-2)-YI(K-1)
XX1 = XI(K-2)**2
XX2 = XI(K-1)**2
XX3 = XI(K)**2
D(M) = XX1*X1 +XX2*X2 + XX3*X3
A(M) = (YI(K-2)*X1 +YI(K-1)*X2 + YI(K)*X3)/D(M)
B(M) = (XX1*Y1 + XX2*Y2 + XX3*Y3)/D(M)
C(M) = YI(K-2) - A(M)*XX1 - B(M)*XI(K-2)
200 K = K+1
300 P1 = X(N)*(A(1)*X(N)+B(1)) +C(1)
P2 = X(N)*(A(2)*X(N)+B(2)) +C(2)
AL = (X(N)-XI(J-1))/XI(J)-XI(J-1))
S = YI(J)*AL + YI(J-1)*(1-O-AL)
GO TO (320,330,350),JJ
320 P2 = P1
AL = (X(N)-XI(1))/XI(2)-XI(1))
S = AL*YI(2) + (1-O-AL)*YI(1)
P1 = S + XK*(P2-S)
GO TO 350
330 P1 = P2
AL = (X(N)-XI(N2-1))/XI(N2)-XI(N2-1))
S = AL*YI(N2) + (1-O-AL)*YI(N2-1)
P2 = S + XK*(P1-S)
350 E1 = ABS(P1-S)
E2 = ABS(P2-S)
IN = J
IF(E1+E2)700,700,750
700 Y(N) = S
GO TO 800
750 BT = (E1*AL)/(E1*AL+(1-O-AL)*E2)
Y(N) = BT*P2 + (1.0 - BT)*P1
800 CONTINUE
900 RETURN
END
OVERLAY(0VL,070)

PROGRAM TRAIL

C MAIN PROGRAM FOR FREE TRAILING VORTICES

DIMENSION XTVI(1000), YTVI(1000), ZTVI(1000), XTVI(500), YTVI(500)

1 ZTVI(500), YTVI(1000), ZTVI(1000), XTVI(1000), V1(100), V2(100)

2 V3(100), NIP(100), NTVE(100)

EQUIVALENCE(B(15001), UTV), (B(16001), VTV), (B(17001), WTV)

1 (B(18001), XTV), (B(19001), YTV), (B(20001), ZTV)

2 (B(21001), XTVI), (B(21501), YTVI), (B(22001), ZTVI)

3 (B(22501), V1), (B(22601), V2), (B(22701), V3)

4 (B(22801), NIP), (B(22901), NTVE)

5 (B(23001), NV)

EQUIVALENCE(XVS, B(12871))

DIMENSION XVS(100)

COMMON/ BODY/ B(25000)

COMMON/ SCRAT/ XQ(1000), YQ(1000), ZQ(1000)

COMMON DA(5000)

EQUIVALENCE (DA(7), XCG), (DA(8), YCG), (DA(9), ZCG), (DA(10), ALPHA)

1 (DA(11), BETA), (DA(12), PSTAR), (DA(13), QSTAR), (DA(14), RSTAR)

C COMPUTE FREE TRAILING VORTEX POINTS, INCLUDING INITIAL POINTS.

XMIN=XVS(100)
XMAX=XTVI(1)

WRITE(6,70) XMIN, XMAX

70 FORMAT(6HOTRAIL/(1P3E20.6))

WRITE(6,65) (NIP(J), J=1,NV)

65 FORMAT(11H0NIP ARRAY=1015)

K=0
DO 75 J=1,NV
NN=NIP(J)
DO 75 I=1,NN
K=K+1
WRITE(6,70) XTVI(K), YTVI(K), ZTVI(K)

IF(XMAX, GT, XTVI(K)) GO TO 75

XMAX=XTVI(K)

75 CONTINUE

DXX=XMAX-XMIN
DELX=DXX/20.0
XMAX=XMIN+1.*25.*DXX
WRITE(6,5) XMAX,DXN,XMIN,DELX
5 FORMAT(20H0,XMAX,DXN,XMIN,DELX=1P4E20.6)
K=0
DO 50 I=1,NV
M1=0
10 IF(I-1) 15,15,10
IM1=I-1
DO 11 I=1,IM1
11 N1=NIP(I)+N1
15 NP=NIP(I)
DO 20 J=1,NP
K=K+1
NJ=N1+J
XTV(K)=XTV(NJ)
YTV(K)=YTV(NJ)
20 ZTV(K)=ZTV(NJ)
DO 30 J=1,20
J1=J
K=K+1
XTV(K)=XTV(K-1)+DELX
YTV(K)=YTV(K-1)
ZTV(K)=ZTV(K-1)
IF(XTV(K) .GT. XMAX) GO TO 35
30 CONTINUE
35 CONTINUE
50 NTVE(I)=NIP(I)+J1
WRITE(6,80)(XTV(I)+YTV(I)+ZTV(I),I=1,K)
80 FORMAT(19H0,TRAIL,XTV+YTV+ZTV/1P3E20.6)
C COMPUTE POINTS AT WHICH VELOCITIES WILL BE FOUND IN SUBTRAVEL.
C
K=0
DO 100 I=1,NV
M=NTVE(I)
DO 90 J=1,M
K=K+1
K1=K+1
IF(J.EQ.M) GO TO 85
800 0390
801 0400
802 0410
803 0420
804 0430
805 0440
806 0450
807 0460
808 0470
809 0480
810 0490
811 0500
812 0510
813 0520
814 0530
815 0540
816 0550
817 0560
818 0570
819 0580
820 0590
821 0600
822 0610
823 0620
824 0630
825 0640
826 0650
827 0660
828 0670
829 0680
830 0690
831 0700
832 0710
833 0720
834 0730
835 0740
836 0750
837 0760
838 0770
X0(K) = 0.5*(XTV(K1)+XTV(K))  
YQ(K) = 0.5*(YTV(K1)+YTV(K))  
ZQ(K) = 0.5*(ZTV(K1)+ZTV(K))  
GO TO 90  
8 0780
8 0790
8 0800
8 0810
8 0820
8 0830
8 0840
8 0850
8 0860
8 0870
8 0880
8 0890
8 0900
8 0910
8 0920
8 0930
8 0940
8 0950
8 0960
8 0970
8 0980
8 0990
8 1000
8 1010
8 1020
8 1030
8 1040
8 1050
8 1060
8 1070
8 1080
8 1090
8 1100
8 1110
8 1120
8 1130
8 1140
8 1150
8 1160
L1=L2-1
DELX=(X(L2)-X(L1))/U(L1)
Y(L2)=Y(L1)+DELX*V(L1)
Z(L2)=Z(L1)+DELX*W(L1)
CONTINUE
10 K=K+KE
RETURN
END

SUBROUTINE TRAVEL(XI,YI,ZI,UI,VI, WI,NPTS)
DIMENSION XI(1), YI(1), ZI(1), UI(1), VI(1), WI(1)
EQUVALENCE(B(1),XV), (B(1),YV), (B(1),ZV), (B(1), XV),
DIMENSION XV(5000), YV(5000), ZV(5000)
COMMON INDEX, XY, ZI, II, IF1, IF2
COMMON DA(5000)
COMMON MATRIX, NDUM(1000), XSO(200)
COMMON BODY, B(25000)
EQUVALENCE (B(1), XV), (B(201), YV), (B(401), ZV), (B(601), XV),
1   (B(801), YV), (B(1001), ZV), (B(2871), XV),
2   (B(3871), SYMT), (B(4871), SYNT), (B(5871), YSY)
3   (B(6871), DSZ), (B(7871), TS), (B(8871), XSS)
4   (B(9871), YSS), (B(10871), ZSS)
5   (B(11871), SIGMA), (B(12871), XV), (B(12971), YSY)
6   (B(13071), ZSS)
DIMENSION XV(1020), YV(1020), ZV(1020), XV(1020),
1   (B(1020), XV), (B(1000), SYMT), (B(2000), SYNT)
2   (B(3000), DSZ), (B(4000), TS), (B(5000), XSS)
3   (B(6000), YSS), (B(7000), ZSS)
DIMENSION BODYG(15000), PANELE(13170)
EQUVALENCE (B(1), BODYX, PANELE)
COMMON SCRAT, X(Q1(1000), Y(Q1(1000), Z(Q1(1000), AXB(200), AYB(200)
1   (Q1(1000), AXB), (Q1(1000), AYB), (Q1(1000), AZR(200)
2   (Q1(1000), AZR)
COMMON NUMBER, NPTS(5), NCPTS(5), NNLN(5), NLT(5), NLC(5)
1   NLS, NBLD, NVL(5), NVT(5), NTAPE, NTAPE, NCTV, NTAPE, JTAPE
2   NSEG, TSEG(5), TFUNC(5), TFUNC(5)
3   LNDIVB(5), LNDIVB(5), NSPP(5), ROOTP(5), OUTER(5)
4   SYM(5)
COMMON PANSYN, PANSYN(10)
EQUVALENCE (SYMP, PA(3426)), (SYMB, PA(12))
```plaintext
SNEQVALENCE(2A(2)*PANS)
PI=3.1415926
PI=12.5663704
I.PANS=PANS
REXIND 18
LNDIV=1
DO 2 I=1,NPTS
   UI(1)=0.*0
   VJ(1)=0.*0
   NJ(1)=0.*0
   IF(NBODS.EQ.0) GO TO 190
   DO 65 KK=1,NBODS
       READ(18) BODYG
       SYM=SYM1(KK)
       LTDIV=LTDIV3(KK)
       NTVV=NVT(KK)
       NBV=NVL(KK)
       DO 63 M=1,NPTS
           X=XI(M)
           Y=YL(M)
           Z=Z(M)
           U=0.*0
           V=0.*0
           W=0.*0
           N=0
           DO 60 J=1,NTVV
               DO 60 I=1,NBV
                   N=N+1
                   TOT1=0.*0
                   TOT2=0.*0
                   TOT3=0.*0
                   DO 55 K=1,4
                       DO 8 KS=1,3
                           SUM(KS)=0.*0
                           GO TO (10*20*30*39) K
                           IF(I.EQ.1) GO TO 12
                           DO 11 KS=1,3
                               SUM(KS)=-SUM1(KS)
                               GO TO 50
2      65
       60
55
8
11
10
```

12 \text{I11=1}
\text{IF1=1}
\text{I12=(J-1)*LTDIV}
\text{DO 14 \text{L}=1,LTDIV}
\text{I12=I12+1}
\text{IF2=I12+1}
14 \text{CALL VORTEX(SUM)}
\text{GO TO 50}
20 \text{I12=1+J*LTDIV}
\text{IF2=I12}
\text{I11=(I-1)*LNDIV}
\text{DO 24 \text{L}=1,LNDIV}
\text{I11=I11+1}
\text{IF1=I11+1}
24 \text{CALL VORTEX(SUM)}
\text{SX=SUM(1)}
\text{SY=SUM(2)}
\text{SZ=SUM(3)}
\text{GO TO 50}
30 \text{I11=IF1}
\text{I12=I12+1}
\text{DO 34 \text{L}=1,LTDIV}
\text{I12=I12-1}
\text{IF2=I12-1}
\text{IF(I,NE,NBIV) GO TO 33}
\text{DO 32 KS=1,3}
\text{SUM(KS)=0.0}
\text{GO TO 34}
32 \text{CALL VORTEX(SUM)}
34 \text{CONTINUE}
\text{DO 35 KS=1,3}
35 \text{SUM1(KS)=SUM(KS)}
\text{GO TO 50}
39 \text{IF(J.EQ.1) GO TO 42}
\text{SUM(1)=-TRSUMX(1)}
\text{SUM(2)=-TRSUMY(1)}
\text{SUM(3)=-TRSUMZ(1)}
\text{GO TO 50}
42 \text{I12=IF2}
III=III+1
DO 44 L=1,LNDIV
III=III-1
IF1=II-1
44 CALL VORTEX(SUM)
      TOT1=TOT1+SUM(1)
      TOT2=TOT2+SUM(2)
      TOT3=TOT3+SUM(3)
55 CONTINUE
      TRSUMX[I]=SX
      TRSUMY[I]=SY
      TRSUMZ[I]=SZ
      AXB[N]=TOT1/PI4
      AYB[N]=TOT2/PI4
      AZB[N]=TOT3/PI4
60 CONTINUE
   DO 62 II=1,N
      U=U+AXB(II)*XSOL(II)
      V=V+AYB(II)*XSOL(II)
      W=W+AZB(II)*XSOL(II)
      U1(M)=U+U1(M)
      VI(M)=V+VI(M)
63     W1(M)=W+W1(M)
65 CONTINUE
190 IF(NPANS.EQ.0) GO TO 405
   DO 200 KK=1,NPANS
   READ(18) PANELG
   SYMP=PANSYM(KK)
   NSPACE=NSPP(KK)
   NTVV=NVT(NBODS+KK)
   NBVV=NVL(NBODS+KK)
   IF(KK-1) 191,191,192
191 iPpv=0
   GO TO 193
192 KK1=K-1
   NB1=NVL(NBODS+KK1)
   NT1=NVT(NBODS+KK1)
   NPPV=NT1*NB1
193 CONTINUE
DO 198 M=1,NPTS
X=XI(M)
Y=YI(M)
Z=ZI(M)
U=J.*0
V=U.*0
W=0.*0
KVOK=0
DO 195 I=1,NTV
IS=I-NSPACE
DO 195 J=1,NBV
J1=J+1
KVOK=KVOK+1
IF(IS.LE.0) GO TO 75
IT=(I-1)*NBVV*2
IT1=IT+1+(J-1)*2
IT2=IT1+2
T1=TS(IT1)
T2=TS(IT2)
CONTINUE
GO TO 5
75
DO 5 K1=1,3
SUM(K1)=0.*0
DO 41 K=1,4
IF(J.EQ.NBVV.AND.K.EQ.4) GO TO 41
IF(K.GT.1) GO TO 21
IF(IS.LE.0) GO TO 76
X2=XVO(IS,J) -T2*SQRT(DYS(IT1)**2+DZS(IT1)**2)*0.5
Y2=YVO(IS,J) -0.5*DYS(IT1)
Z2=ZVO(IS,J) -0.5*DZS(IT1)
IF(J.EQ.NBVV) GO TO 305
X1=XVO(IS,J1) -T1*SQRT(DYS(IT1)**2+DZS(IT1)**2)*0.5
Y1=YVO(IS,J1) -0.5*DYS(IT1)
Z1=ZVO(IS,J1) -0.5*DZS(IT1)
X3=X1
Y3=Y1
Z3=Z1
GO TO 40
305
X1=XVS(IS)
Y1=YVS(IS)
GO TO 40
38 IF (I.GT.NSPACE) GO TO 351
    X2=XVR(I,J+1)
    Y2=YVR(I,J+1)
    Z2=ZVR(I,J+1)
    GO TO 40
351 X2=X3
    Y2=Y3
    Z2=Z3
40 IF (J.EQ.NSVV.AND.K.EQ.0) GO TO 411
491 CALL VORPAN(SUM,X1,Y1,Z1,X2,Y2,Z2,X,Y,Z)
41 CONTINUE
GO TO 417
411 CONTINUE
    Y2H=Y2
    Y1H=Y1
    SYMPAS=-1.0
C LEFT SEMI-INFINITE VORTEX LINE.
410 T1=SQR2((Y2-Y)**2+(Z2-Z)**2)
    IF (T1-0.00001) 412,411
411 CONTINUE
    T2=(X2-X)/SQR2((X2-X)**2+(Y2-Y)**2+(Z2-Z)**2)
    QT=0.25*(1.0-T2)/(PI*T1)
    SUM(2)=SUM(2)+SYMPAS*QT*(Z2-Z)/T1
    SUM(3)=SUM(3)+SYMPAS*QT*(Y2-Y)/T1
C RIGHT SEMI-INFINITE VORTEX LINE.
412 CONTINUE
    T1=SQR2((Y1-Y)**2+(Z1-Z)**2)
    IF (T1-0.00001) 414,413
413 CONTINUE
    T2=(X1-X)/SQR2((X1-X)**2+(Y1-Y)**2+(Z1-Z)**2)
    QT=0.25*(1.0-T2)/(PI*T1)
    SUM(2)=SUM(2)-SYMPAS*QT*(Z1-Z)/T1
    SUM(3)=SUM(3)+SYMPAS*QT*(Y1-Y)/T1
414 CONTINUE
    IF (SYMPAS.EQ.-1.0.AND.SYMP.EQ.0) GO TO 415
GO TO 416
415 Y1=-Y1
    Y2=-Y2
SY: PAG=1.0
GO TO 410
416 Y1=Y1H
Y2=Y2H
417 IF(IS.GT.0) GO TO 46
TERM=NOMAX(XSOL+XVOR+NPPV)
GO TO 48
46 TERM=0.0
K1=(I-1)/NBV
DO 47 12=1,J
TERM=TERM+XSOL*(I-1)*TERM
47 U=U+SUM(I)*TERM
V=V+SUM(J)*TERM
195 W=W+SUM(3)*TERM
UI(I)=U+I
VI(I)=V+I
198 WI(M)=WI(M)+I
200 CONTINUE
RETURN
DO 400 1=1,NPT
X=X1(K)
Y=Y1(K)
Z=Z1(K)
U=U*K
V=V*K
W=W*K
MA=0
MS=0
DO 300 I=1,NTV
DO 300 J=1,NSV
300 CALL PVSKIT(DY,DZ,X,Y,Z,MS,KV)
DC 510 KV=12
U=U+XS(T(KV)*SIGMA(KV))
V=V+SYM(T(KV))\cdot\sigma(KV)
350 W=V+SYM(T(KV))\cdot\sigma(KV)
U1(K)=U1(K)+U
V1(K)=V1(K)+V
400 W1(K)=W1(K)+W
405 CONTINUE
RETURN
END
SUBROUTINE VORTEX(SUM)
COMMON/BODY/ XV(151,31),YV(151,31),ZV(151,31)
COMMON/COMPTS/ XO(1320),YO(1320),ZO(1320)
COMMON DA(15000)
EQUIVALENCE (DA(19),SYM)
COMMON/INDEX/ X,Y,Z,II1,II2,IF1,IF2
DIMENSION SUM(1)
XI=XV(II1,II2)
YI=YV(II1,II2)
ZI=ZV(II1,II2)
XF=XV(IF1,IF2)
YF=YV(IF1,IF2)
ZF=ZV(IF1,IF2)
XF0=XF-X
ZF0=ZF-Z
XI0=XI-X
ZI0=ZI-Z
SYML00=1.0
10 YFO=YF-Y
Y10=Y1-Y
DELX=XF-XI
DELY=YF-Y1
DELZ=ZF-ZI
RXS1=YFO*DELZ-ZFO*DELY
RXS2=ZFQ*DELX-XFQ*DELZ
RXS3=XFQ*DELY-YFO*DELX
RXS=SQR(RXS1**2+RXS2**2+RXS3**2)
TERM1=SQR(DELX**2+DELY**2+DELZ**2)
TERM2=SQR(XFQ**2+YFO**2+ZFQ**2)
TERM3=SQR(XIQ**2+YIQ**2+ZIQ**2)
TERM4= XFO*DELX+YFO*DELY+ZFC*DELZ
8 4290
8 4300
8 4310
8 4320
8 4330
8 4340
8 4350
8 4360
8 4370
8 4380
8 4390
8 4400
8 4410
8 4420
8 4430
8 4440
8 4450
8 4460
8 4470
8 4480
8 4490
8 4500
8 4510
8 4520
8 4530
8 4540
8 4550
8 4560
8 4570
8 4580
8 4590
8 4600
8 4610
8 4620
8 4630
8 4640
8 4650
8 4660
8 4670
RATIO = TERM4/TERM1**2
COSA = (DELX*X10+DELY*Y10+DELZ*Z10)/(TERM1*TERM3)
COSB = TERM4/(TERM1*TERM2)

CC = COSB-COSA
HX = XFQ-RATIO*DELX
HY = YFQ-RATIO*DELY
HZ = ZFQ-RATIO*DELZ
H = SORT(HX*HX+HY*HY+HZ*HZ)
IF(H=0.00001) 11*12*12

11 COEF=0.0
GO TO 13
12 CONTINUE
HRXS=H*RXS
COEF=SYMLOO*CC/RXSS
13 CONTINUE
SUM(1)=COEF*RXS1 + SUM(1)
SUM(2)=COEF*RXS2 + SUM(2)
SUM(3)=COEF*RXS3 + SUM(3)
IF(SYMLOO.EQ.-1.0.OR.SYMNE.EQ.0.0) RETURN
SYMLOO=-1.0
YI=-YI
YF=-YF
GO TO 10
END

SUBROUTINE VORPAR(SUM,XI,YI,ZI, XF, YF, ZF, X, Y, Z)
COMMON DA(5000)
EQUIVALENCE (DA(3426), SYM)
DIMENSION SUM(1)
R4PI=3.14159265
YIH=YI
YFH=YF
XFQ=XF-X
ZFQ=ZF-Z
X10=XI-X
Z10=ZI-Z
SYMLOO=1.0
10 YFQ=YF-Y
YIQ=YI-Y
DELEX=XF-XI
DELY=YL-YI
DELY=Z=ZI
RXS1=YL*DELY-2*Z=Y*DELY
RXS2=Z=ZI*DELY-2*YL*DELY
RXS3=XZ*DELY-YL*DELY
RXS4=SQRT(RXS1**2+RXS2**2+RXS3**2)
TERM1=SQRT(Z=ZI**2+DELY**2+DELY**2)
TERM2=SQRT(XL*DELY-YL**2+Z=ZI**2)
TERM3=SQRT(X=I**2+YL**2+Z=ZI**2)
TERM4=XL*DELY+YL*DELY+Z=ZI*DELY
RATIO=TERM4/TERM1**2
COSA=(DELY*XI+DELY*YL+DELY*Z=I)/T(ERM1*TERM3)
COSB=TERM4/TERM1**2(CC=COB=COA)
HX=XL=RATIO*DELY
HY=YL=RATIO*DELY
HZ=Z=ZI=RATIO*DELY
H=SQRT(HX*HY+HY*HZ+HZ*HZ)
IF(H=0.00001)11=12=12
COEF=0.0
GO TO 13
12 CONTINUE
HRXS=H*RXS
COEF=R4*P=4SYML=0*CC/HRXS
13 CONTINUE
SUM(1)=COEF*RXS1+SUM(1)
SUM(2)=COEF*RXS2+SUM(2)
SUM(3)=COEF*RXS3+SUM(3)
YI=YH
YF=YL
IF(SYML=0.EQ.-1.0.OR.SYM.EQ.-0.0)RETURN
SYML=0=-1.0
YI=YH
YF=YL
GO TO 10
END
SUBROUTINE PVSK(T,X,H,Q,Y,DS,M,A,B,IV,JO,KSOL)
COMMON DA(500)
EQUVALENCE (DA(3426),SYM)
COMMON/PAN.LE/ PANEL (16)
COMMON/SCAT/YSOL (5000) * ROWNP (5000) * AX (5000) *AY (5000) * AZ (5000)
COMMON/BDY/XVR (10, 20) * YVR (10, 20) * ZVR (10, 20) * VXO (10, 20) 
1 * YV (10, 20) * ZV (10, 20) * PLT (500) * YSP (100) * CHRD (1000) 
2 * XCV (201) * XCC (20) * XLE (20) * YLE (20) * ZLE (20) 
3 * XTE (20) * YTE (20) * ZTE (20) * XJ (20) * YJ (20) * ZJ (20) 
4 * ETLE (20) * XVL (50) * YVL (50) * ZVL (50) * XJ (20) * YJ (20) * ZJ (20) 
5 * SX (1000) * SY (1000) * SZ (1000) * DX (1000) * DZ (1000) 
6 * TS (1000) * XST (1000) * YST (1000) * ZST (1000) * SIGMA (1000) 
COMMON/PANEL/ MPAN*/IPSY */ JNC */ NTVVP */ LCCP */ RTLCP */ LMCPP * RTLCP 
1 * MPRPT * NSPACE * MATTCH * NTATT * MPRLN * MPRCLN * NACTXC * NTETC * NTTHC 
2 * NTREET * NTIP * CHTIP * ROOT * OUTER * NNATT 
3 * MP1 * MP2 * MP3 * MP4 * MP5 * MP6 * MP7 * MP8 * MP9 * MP10 
COMMON/NUMBER / NPPTS (7) * NCPTS (7) * NNLN (7) * NLT (7) * LT (7) 
1 * NCTN * NBODS * MPANS * NVL (7) * NTV (7) * NTAPE * NTAPExNCTV * NTAPJ * NTAPEJ 
2 * LSEG (7) * TSEG (7) * LFUNC (7) * TFUNC (7) 
3 * LNIVB (7) * LTIVB (7) * NSPP (7) * ROOTP (7) * OUTERP (7) * SYMM (7) 
C KSO=1 FOR SOURCE Pts ONLY 
C KSO=2 FOR BOTH SOURCE Pts AND VORTEX POINTS 
C MS=SUBSCRIPT OF SOURCE Pt. 
C XC, YC, ZC --- CONTROL Pt. 
C Y* = LATERAL VORTEX SUBSCRIPT 
C JV* = LONGITUDINAL VORTEX SUBSCRIPT. 
C INSERT SPECIFICATION STATEMENTS HERE. 
REAL I1, I2, I3, I4 
YV=0.5*SORT (YY**2 + D**2) 
YV2=2.0*YY 
PI=3.141592654 
YY=YYV 
GO TO (10, 20) * KSO 
10 YK=YS (MS) 
ZK=ZS (MS) 
XK=XS (MS) 
GO TO 25 
20 IF (MS-2*(MS/2)*.EO.0) GO TO 10 
YK=YS (IV, JV) 
ZK=ZV (IV, JV) 
XK=XV (IV, JV)
<table>
<thead>
<tr>
<th>Line</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>CONTINUE</td>
</tr>
<tr>
<td></td>
<td>SUMX=0.0</td>
</tr>
<tr>
<td></td>
<td>SUMY=0.0</td>
</tr>
<tr>
<td></td>
<td>SUMZ=0.0</td>
</tr>
<tr>
<td></td>
<td>UTSUM=0.0</td>
</tr>
<tr>
<td></td>
<td>VTSUM=0.0</td>
</tr>
<tr>
<td></td>
<td>WTSUM=0.0</td>
</tr>
<tr>
<td></td>
<td>SIGN=1.0</td>
</tr>
<tr>
<td></td>
<td>DZ=ZC-ZK</td>
</tr>
<tr>
<td></td>
<td>X=XC-XK</td>
</tr>
<tr>
<td>50</td>
<td>CONTINUE</td>
</tr>
<tr>
<td></td>
<td>DY=YC<em>SIGN</em>YK</td>
</tr>
<tr>
<td></td>
<td>R3=DYY/YV2</td>
</tr>
<tr>
<td></td>
<td>RZ=DZZ/YV2</td>
</tr>
<tr>
<td></td>
<td>Y=RY<em>DY+RZ</em>DZ</td>
</tr>
<tr>
<td></td>
<td>Z=-RZ<em>DY+RY</em>DZ</td>
</tr>
<tr>
<td></td>
<td>R12=(Y+YV)<strong>2+Z</strong>2</td>
</tr>
<tr>
<td></td>
<td>R22=(X-T<em>Y)<strong>2+Z</strong>2</em>(1.0+T*T)</td>
</tr>
<tr>
<td></td>
<td>R32=(Y-YV)<strong>2+Z</strong>2</td>
</tr>
<tr>
<td></td>
<td>R4=SQRT((X-T*YV)**2+(Y-YV)<strong>2+Z</strong>2)</td>
</tr>
<tr>
<td></td>
<td>R5=SQRT((X+T*YV)**2+(Y+YV)<strong>2+Z</strong>2)</td>
</tr>
<tr>
<td></td>
<td>IL=(X+T*YV)/R5</td>
</tr>
<tr>
<td></td>
<td>I2=(Y+T<em>X+YV</em>(1.0+T**2))/R5</td>
</tr>
<tr>
<td></td>
<td>I3=-(Y+T<em>X-YV</em>(1.0+T**2))/R4</td>
</tr>
<tr>
<td></td>
<td>I4=(X-T*YV)/R4</td>
</tr>
<tr>
<td></td>
<td>IF(ABS(Z)*GT.*YV2) GO TO 42</td>
</tr>
<tr>
<td></td>
<td>Z=0.0</td>
</tr>
<tr>
<td></td>
<td>R6D=(X+T*YV)**2+(Y+YV)**2</td>
</tr>
<tr>
<td></td>
<td>R7D=(X-T*YV)**2+(Y-YV)**2</td>
</tr>
<tr>
<td></td>
<td>RXTY=(X-T*Y)**2</td>
</tr>
<tr>
<td></td>
<td>R6=RXTY/R6D</td>
</tr>
<tr>
<td></td>
<td>R7=RXTY/R7D</td>
</tr>
<tr>
<td></td>
<td>IF(R6.GE.0.0075968656) GO TO 41</td>
</tr>
<tr>
<td></td>
<td>IF(R7.GE.0.0075968656) GO TO 41</td>
</tr>
<tr>
<td></td>
<td>IF(ABS(Y).*LE.*YV) GO TO 41</td>
</tr>
<tr>
<td></td>
<td>TERM1=ABS(1.0/R7D-1.0/R6D)*0.5/P</td>
</tr>
<tr>
<td></td>
<td>GO TO 43</td>
</tr>
<tr>
<td>41</td>
<td>IF(ABS(Y).*GT.*YV) GO TO 42</td>
</tr>
<tr>
<td></td>
<td>IF(ABS(X-T*Y).<em>GE.0.25</em>ABS(PLL(MA))) GO TO 42</td>
</tr>
</tbody>
</table>
TER1=0.0
GO TO 43

42 TER1=(12+13)/R22
CONTINUE

43 TER2=(11+1.01)/R12
TER3=(14+1.0)/R3.2
TER4=1.0/R4-1.0/R5
P=SORT(1.0+T*T)
EUS=(T*TER4+1.0+1.0)*TER11/P
EV=(TER4-T)*1.0*TER11/P
UV=EUS*P*Z*TER11
US=0.25*EUS/PI
VS=0.25*EV/S/PI
WS=0.25*EWS/PI
UT=US
VT=VS*RY=WS*RZ
UT=UV*RY
UTSUM=UT+UTSUM
VTSM=VT+VTSM
WTSUM=WT+WTSUM

IF(KSOL.EQ.1) GO TO 45

EU=Z*TER1

45 CONTINUE

C FOR SYMMETRY, GET IMAGE CONTRIBUTION.

IF(SYM.NE.0.0) GO TO 60
IF(SIG.LT.0.0) GO TO 60
SIGM=-1.0
DZ=-DZ
```
T = -T
GO TO 50
CONTINUE
SXMT(PS) = VTSUM
SYMT(PS) = VTSS
SZMT(NS) = VTSU1
IF(KSOL.EQ.1) RETURN
AX(NA) = SUMX
AY(NA) = SUNY
AZ(NA) = SUMZ
RETURN
END
```
OVERLAY(CVL*10*0)
PROGRAM HARDAP
COMMON/SCRAT/A(20),DA(5000),ATACH(5,10)
INTEGER UNIT,UNIT2
DATA UNIT2/21/
DATA UNIT/10/
REWIND UNIT
REWIND UNIT2
PRINT 3
READ 5,A
IF(A(1).EQ.4.AND.A(2).EQ.4.AND.A(3).EQ.4) GOTO 60
CALL OUTIN(A)
GOTO 99
1 READ 5,A
60 IF.EOF(5) 2,372
372 PRINT 5,A
WRITE(UNIT,5)A
GOTO 1
2 ENDFILE UNIT
REWIND UNIT
CALL DECRD(DA,UNIT)
NB = DA(1)
IF(NB.EQ.0) GOTO 20
DO 10 I=1,NB
10 CALL DECRD(DA,UNIT)
20 NP = DA(2)
IF(NP.EQ.0) GOTO 40
DO 30 I=1,NP
CALL DECRD(DA,UNIT)
ATACH(1,I) = DA(3420)
ATACH(2,I) = DA(3444)
ATACH(3,I) = DA(3445)
ATACH(4,I) = DA(3446)
30 ATACH(5,I) = DA(3447)
40 REWIND UNIT
WRITE(UNIT2) ATACH
WRITE(6,51)UNIT2
51 FORMAT(*UNIT2=813)
REWIND UNIT2
99 CONTINUE
5 FORMAT (20A4)
5 FORMAT (1H1, 20A4)
3 FORMAT (1H1, 30X, LISTING OF INPUT DATA CARDS/1H0)
C
BLAINE D. GAITHER 11/72
END
SUBROUTINE DECRT(DA,IARG,MAX,UNIT)
DIMENSION DA(1)
ISTART = IARG
444 IF(MAX - ISTART - 4) 1, 1, 2
1 MSTART = -ISTART
WRITE(UNIT,111) MSTART, (DA(I), I=ISTART,MAX)
PRINT 111, MSTART, (DA(I), I=ISTART,MAX)
111 FORMAT(112,5G12.5)
RETURN
2 MSTART = ISTART + 4
IF(DA(ISTART).EQ.0.0.AND.DA(ISTART+1).EQ.0.0.AND.DA(ISTART+2).EQ.0.0) GOTO 3
1.0.AND.DA(ISTART+3).EQ.0.0.AND.DA(ISTART+4).EQ.0.0) GOTO 3
WRITE(UNIT,111) ISTART, (DA(I), I=ISTART, MSTART)
PRINT 111, ISTART, (DA(I), I=ISTART, MSTART)
3 ISTART = ISTART + 5
GOTO 444
END
**SUBROUTINE OUTIN(TITLE)**

<table>
<thead>
<tr>
<th>CODED BY</th>
<th>B. D. GAITHER</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT SOME LATER DATE ALL TAPE I/O SHOULD BE DONE BY BUFFER IN AND OUT</td>
<td>OUT 0110</td>
</tr>
<tr>
<td>THEN ALL ZEROING LOOPS AND CALLS TO DECRL AND DECRUT COULD BE REMOVED. AT PRESENT THE MAIN ROUTINES EXPECT DECRL STYLE INPUT/O</td>
<td>OUT 0150</td>
</tr>
</tbody>
</table>

```
INTEGER TITLE(20), NRADX(4), NFORX(4), OUNIT, QUNIT2, SCRATCH
REAL DA(5000), XAF(30), XCHAFL(20), YOAFLE(20), ZCHAFL(20)
IAFSCL(120), TZORN(30), MAFLADD(100), XVXFXF(120), ZLCCL(120)
2, ECSAD(120), YOHS(30,120), ZOHS(30,120)
3, XOP, YOP, ZOP, PX(30), POORAD(30), FINXLY
5FINYL * FINZL * LCHORD * FINXH * FINYM * FINZH
6 HCHORD * XLC * YLC * ZLC * CONCRD * XHC * YHC * ZHC
7 CONCOH * CAMPCL(10) * CAMPLM(10) * CANS(10)
8 PCHORD(10) * FAFHT(10) * ATA(29) * ZCT(40) * PPD(150)
9 PANARE(1580) * BODARE(3405)
A ATACH(150) * MPPDP, NTTXCS
B NXSOBT*RCT(50) * LCT(670) * RNC(800)
C TTT(29) * ETA(29) * NATA
D BMFCX(49)
E ZLCCCL(120) * SR(30) * N(39) * ZCSA(39) * SZ(120,30) * SY(120,30)
```

**DOUBLE PRECISION PI, RAD, RAD 1 DG**

```
COMMON /PATCH/ ATACH
```

```
EQUIVALENCE (REFA,DA(16))(PSI,DA(9))
1 (ATA(1),DA(3631)) (CANS,DA(3630)) (ZC(1),DA(3660))
2 (XAF(1),DA(3601)) (XCNUN,DA(3600))
3 (REFA,DA(3421))
4 (XPO,DA(3432)) (YPO,DA(3432)) (ZPO,DA(3434))
5 (IAFS,DA(3426))
6 (XAF(1),PCHORD(1)) (MAFLADD1,MATF(1)) (YAFRM(1),FAHFT(1))
7 (PANARE(1),DA(3420)) (BODARE(1),DA(15)) (XCH,FINXLY)
8 (YCL,FINYL) (ZCL,FINZL) (CONCRD,LCHORD) (XHC,FINXH)
9 (YHC,FHYM) (ZHC,FHYM) (CONCOH,HCHORD) (CAMPCL(1),PCHORD(OUT))
A (CAMPLM(1),FAHFT(1)) (PPD(1),DA(3450))
B (MPPDP,DA(3443))(C,TTXCS,DA(4130))
```

OUT 0070 OUT 0080 OUT 0090 OUT 0100 OUT 0140 OUT 0150 OUT 0160 OUT 0170 OUT 0190 OUT 0200 OUT 0200 OUT 0200 OUT 0210 OUT 0220 OUT 0230 OUT 0240 OUT 0260 OUT 0270 OUT 0280 OUT 0290 OUT 0300 OUT 0310 OUT 0320 OUT 0330 OUT 0340 OUT 0350 OUT 0360 OUT 0370 OUT 0380 OUT 0390 OUT 0400 OUT 0410 OUT 0420 OUT 0430 OUT 0440 OUT 0450

563
DATA IUNIT/5/, IUNIT/10/, IUNIT/20/, DA/5000*-0.0/, SCRTCH/18/
1, PI/ 3.1415 92653 58979 32384 62643 D+0/, 1
2, RAD 1, DG/0.0174 53292 51994 32957 69237 D+07/ 1
3, ZLCSS/120*-0.0/

NAMELIST /JLIST/ J0, J1, J2, J3, J4, J5, J6, NJAF, NJAFOR, NFUS, NOUT
1RADX, NFORX, NP, NPODOR, NF, NFINOR, NCAN, NCANOR
2/JLIST/ XAF, XOAFLE, YOAFLE, ZOAFLE, AFSWCL, SPAN, NTA, APP
3, /AFORD
4/JLIST/ LVXFXF, ZLCSS, FCBA
5/JLIST/ XOP, YOP, ZOP, PX, PODRAD
6/JLIST/ FINXL, FINYL, FINZL, LCHORD, FINXH, FINYH, FINZH, HCHORD
7/PHORU, FAFHT
8/PLLOTI/ PHI, THETA, PSI
9/CLIST/ XLCL, YLC, ZLC, CCONS, XHC, YHC, ZHC, CONCHO, CANCL
ACANPLN, CAN
E/ATTACH/ATACH

READ IDENTIFICATION CARD AND ECHO
PRINT 6

6 FORMAT(1H1, 30X, *BEGIN CONVERSION OF LANGLEY DATA*)
PRINT 2, TITLE
2 FORMAT(1X*20A4)

READ CONTROL INTEGERS
READ 3, J0, J1, J2, J3, J4, J5, J6, NJAF, NJAFOR, NFUS, (1RADX(1), NOUT
1INFORX(I), I=1, 4, 1), NP, NPODOR, NF, NFINOR, NCAN, NCANOR
3 FORMAT(24I3)
PRINT JLIST

READ REFERENCE AREA CARD
IF(J0 .EQ. 1) READ 4, REFAR

C
C
C
4 FORMAT (10F7.0)
   REFAA=REFA
   REFAF=REFA
C
C PUNCH UNIVERSAL INFO
C CALL DECWRT(DA,1,14,0UNIT)
C
C *********************************************************************
 C READ WING DATA
C IF(IABS(J1) .NE. 1) GOTO 101
C
C NUMBER OF PERCENT CHORD LOCATIONS
XCNUM = NWAFOR
C
C SET SYMMETRY OF WING
AFSIM = 0.0
C
C SET PANEL CONTOUR INDICATOR
PCONT = 1.0
C
C READ PERCENT CHORD LOCATIONS
READ 4, (XAF(I), I=1, NWAFOR)
DO 324 I=1, NWAFOR
XAF(I)= XAF(I)/100.0
324 TT(I) = XAF(I)

C READING DATA CARDS
READ 5, (XOAFLE(I), YOAFLE(I), ZOAFLE(I), AFSLCL(I), I=1, NWAFL)
5 FORMAT (4F7.0)

C NUMBER OF LTA(A) STATIONS
ANAS = NWAFL
NATA = ANAS
C
C FIND AIRFOIL LENGTHS (SPAN IS THEIR SUM)
SPAN = 0.0
ATA(1) = SPAN
C
C OUT 0850
OUT 0860
OUT 0870
OUT 0880
OUT 0900
OUT 0910
OUT 0920
OUT 0930
OUT 0940
OUT 0950
OUT 0960
OUT 0970
OUT 0980
OUT 0990
OUT 1000
OUT 1010
OUT 1020
OUT 1030
OUT 1040
OUT 1050
OUT 1060
OUT 1070
OUT 1080
OUT 1090
OUT 1100
OUT 1110
OUT 1120
OUT 1130
OUT 1140
OUT 1150
OUT 1160
OUT 1170
OUT 1180
OUT 1190
OUT 1200
DO 102 I=1, NWAF
ATA(I)=SQRT((YOAFL(I)^2+ZOAFL(I)^2))
SPAN = ATA(I) + SPAN
102 ATA(I)=SPAN

LOAD(AIRFOIL LENGTH / SPAN) INTO ARRAY ATA
DO 104 I=1, NWAF
ATA(I)=ATA(I)/SPAN
104 ETA(I)=ATA(I)

C  
READ DELTA Z VALUES AND CONVERT TO Z/CHORD LENGTH
IF(J1.NE.1) GOTO 222
I=1
DO 31 J=1, NWAF
IF(AFSWCL(J).NE.0) GOTO 517
PRINT 223, J
228 FORMAT(4THE CHORD LENGTH OF WING AIRFOIL*13** IS ZERO; IT HAS BEEN SET TO .01*/)
AFSWCL(J)=.01
517 READ 4, (TZORD(I), I=1, NWAFOR)
PRINT 400, (TZORD(I), I=1, NWAFOR)
400 FORMAT(4OTZORD(I)*3/(10G12.5))
DO 31 IT = 1, NWAFOR
ZC(I)=TZORD(IT)/AFSWCL(J)
31 I=I+1
C  
C PANEL (WING) ORIGIN
222 XPO = XZ
YPO = XZ
ZPO = XZ
C  
C PANEL PERIMETER DESCRIPTION (LEADING EDGE)
J=1
DO 150 I=1, NWAF
PPD(J)=XCAFL(I)
PPD(J+1)=YOAFL(I)
PPD(J+2)=ZOAFL(I)
PPD(J+3) = AFSWCL(I)
150 J = J + 4
C SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP=NWAF

C SET NUMBER OF X/C STATIONS ENTRIES IN THE THICKNESS TABLE PER AIRFOIL
NTTXCS=NWAFOR

C READ WING AIRFOIL ORDNATE CARDS (HALF THICKNESS)
JJ = 0
DO 22 J=1, NWAF
IT = JJ+1
JJ = NWAFOR+JJ
22 READ 4*, (NWAFOR(I), I=IT, JJ)
DO 516 I=1, JJ
515 NWAFOR(I)=NWAFOR(I)/100.
PRINT :LIST

C START FUSELAGE DATA CARDS

101 IF(J2 .EQ. 0 .OR. NFUS .EQ. 0) GOTO 171

C SET BODY COORDINATE TABLE INPUT INDICATOR TO INDICATE A CIRCULAR
BODY OR ARBITRARY BODY
BCTII =0.0
BSYII = 1.0
IF( J2 .EQ. 1) BCTII= 1.0
LLL = 0
LL = 1
DO 100 IX=1, NFUS
NFO = NFORX(IX)
LLL=LLL+NFO
READ X STATIONS
READ 4*, (XVXF(X), I=LLL, LLL)
XVXF(LLL) = XVXF(X)+1.0E-7
READ CAMPER(IF ANY)
IF(J2.EQ.-1.AND.J6.EQ.0) READ 4, (ZLCCS(I), I=LL, LLL)
IF(J2.NE.-1) GOTO 200

READ CROSS SECTIONAL AREAS (IF ANY)
READ 4, (FCSA(I), I=LL, LLL)
GOTO 100

200 IF(J2.NE.1) GOTO 100

READ Y S AND Z S OF ARBITRARY BODY
NFF=NRAIX(I)
DO 7044 M=LL, LLL
READ 4, (YOHS(I,M), I=1, NFF)
READ 4, (ZOHS(I,M), I=1, NFF)
PRINT 401, (YOHS(I,M), I=1, NFF)
PRINT 402, (ZOHS(I,M), I=1, NFF)

7044 CONTINUE
401 FORMAT(*YOHS*,3(/10G12.5))
402 FORMAT(*ZOHS*,3(/10G12.5))
100 LL=LL+NFO

FIND AND SET TOTAL NUMBER OF X STATIONS
J=NFORXT1+NFORX(2)+NFORX(3)+NFORX(4)
NXS=J

PUNCH FUSELAGE AND WING
CALL DECRF(DA, 15, 5419, OUNIT)
PRINT BLIST

171 IF((ABS(J1).NE.1) GOTO 715
CALL DECWR(DA, 5420, 5000, OUNIT)

ZERO BODY AREA OF STORAGE
715 DO 229 I=1, 3405
229 BODARE(I)=0.0

ZERO PANEL SECTION OF STORAGE
DO 226 I=1, 1580
226 PANARE(I)=-0.0

******************************************************************************
C READ POD DATA
C IF(J3.E.1.OR.NP.EQ.0) GOTO 300
C REWIND SCRATCH UNIT TO PUT PODS ON
REWIND SCRATCH
DO 290 IX=1, NP
READ 4, XOP, YOP, ZOP
READ 4, (PX(I), I=1,NP0DOR)
READ 4, (PODRAD(I), I=1,NP0DOR)
PRINT PLIST
C IS THIS A POD OR A NACELLE
IF(PODRAD(1).LE.0) GOTO 123
C ITS A NACELLE
C SET REFERANCE AREA CARDS
REFAF=REFAA
C SET PANEL CONTOUR INDICATOR
PCONT = 1.0
C SET PANEL ORIGIN:
XPO=XOP
YPO=YOP
ZPO=ZOP
C FIND CHORD LENGTHS
CHORD=PX(NP0DOR)
C PANEL PERIMETER DESCRIPTION:(STEP BY TEN DEGREES)
J=1
DO 764 K=1, 361, 10
RAD = RAD 1 DG * FLOAT(K-1)
PPD(J)=PX(I)
PPD(J+1)=DSIN(RAD)*PODRAD(I)
PPD(J+2)=DCOS(RAD)*PODRAD(I)
PPD(J+3)=CHORD
C OUT 2660
C OUT 2680
C OUT 2690
C OUT 2700
C OUT 2710
C OUT 2720
C OUT 2730
C OUT 2740
C OUT 2750
C OUT 2760
C OUT 2770
C OUT 2780
C OUT 2790
C OUT 2800
C OUT 2810
C OUT 2820
C OUT 2830
C OUT 2840
C OUT 2850
C OUT 2860
C OUT 2870
C OUT 2880
C OUT 2890
C OUT 2900
C OUT 2910
C OUT 2920
C OUT 2930
C OUT 2940
C OUT 2950
C OUT 2960
C OUT 2970
C OUT 2980
C OUT 2990
C OUT 3000
C OUT 3010
764 J=J+4
C SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP=37
C PERCENT CHORD LOCATIONS(X/CHORD LENGTH)
XCNUN=NPDOC
DO 746 I=1, NPDOC
OUT 3040
746 PCHORD(I)=PX(I)/CHORD
C SET ETA(ATA) STATIONS (AS WELL AS THEIR NUMBER)
ANAS=2
ATA(I)=0.0
ATA(2)=1.0
C CAMBER LINE = DELTA Z S (X S) / AIRFOIL STREAMWISE CHORD LENGTH
OUT 3170
DO 476 I=1, NPDOC
ZC(I)=PODRA(D(I))/CHORD
OUT 3190
476 ZC(NPDOC+I)=ZC(I)
C PUNCH PANEL
CALL DECHRT(DA,3420,5005,UNIT)
C ZERO PANEL AREA
DO 227 I=1, 1580
227 PANARE(I)=-0.0
GOTO 290
C ITS A POD
C SET BODY ORIGIN
123 BOX=XOP
BOY=YOP
BOZ=ZOP
C SET BODY COORDINATE TABLE INPUT INDICATOR
3CTII = 0.0
BSYI = 1.0
C SET NUMBER OF X STATIONS IN BODY CO-ORDINATE TABLE
OUT 3290
OUT 3300
OUT 3310
OUT 3320
OUT 3330
OUT 3340
OUT 3350
OUT 3360
OUT 3380
OUT 3390
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>IF(J4, NE, 1.0, OR, NF, EQ, 0.0) GOTO 390</td>
<td>IF condition 1</td>
</tr>
<tr>
<td>310</td>
<td>DO 330 I = 1, NF</td>
<td>Do statement 2</td>
</tr>
<tr>
<td>330</td>
<td>READ 4, FINXH, FINZH, FINXL, FINYL, LCHORD, FINXH, FINYH</td>
<td>READ statement 3</td>
</tr>
<tr>
<td></td>
<td>READ 4, (PCHORD(I), I = 1, NF, IOR)</td>
<td>READ statement 4</td>
</tr>
</tbody>
</table>
READ 4, (F*FHT(I) + I=1, NFINOR)
DO 225 I=1, NFINOR
PCHORD(I)=PCHORD(I)/100.
225 FAFHT(I)=FAFHT(I)/100.0
PERCENT CHORD LOCATIONS (NUMBER OF ORDINATES USED TO DESCRIBE EACH OUT 3860
AIR FOIL SECTION THICKNESS TABLE X/C STATIONS))
XCNUM = NFINOR
NTXCS = XCNUM
DO 379 I=1, NFINOR
379 TT(I)=PCHORD(I)
PRINT FLIST
ASSIGN 380 TO LABEL
GOTO 1000
380 CONTINUE
****************************
READ CANARD DATA
390 IF(J5. NE. 1. OR. NCAN. EQ. 0.) GOTO 999
DO 500 IX = 1, NCAN
READ 4, XLQ, YLC, ZLC, CONCRA, XHC, YHC, ZHC 4000
1 CONCQH
J=JABS(NCANOR)
READ 4, (CANPCL(I) + I=1, J)
READ 4, (CAMPLS(I) + I=1, J)
DO 223 I=1, J
CAMPLN(I)=CAMPLN(I)/100.0
223 CANPCL(I)=CANPCL(I)/100.0
IF(NCANOR * GE. 0) GOTO 7094
C HERE IF CAMPERED(NOT SYMETRICAL)
SET PANEL CC:TOUR INDICATOR
PCONT = 1.0
READ 4, (CANS(I), I=1, J)
DO 712 I=1, J
712 CANS(I)=CANS(I)/100.0
C
C SET CAMBER
DO 1052 I=1, J
1052 ZC(I) = (CAMLN(I) - CAMS(I))/2.0
C ADJUST THICKNESS TO BE RELATIVE TO CAMBER LINE
DO 1050 I=1, J
1050 CANPLN(I) = (CAMLN(I) + CAMS(I))/2.0
C PERCENT CHORD LOCATIONS (NUMBER OF ORDINATES USED TO DESCRIBE EACH OUT 4225
7094 XNUM = J
NTTCS = XNUM
C ZERO UNIVERSAL DATA STORAGE
DO 7093 I=1, J
7093 TT(I) = CANPCL(I)
PRINT CLIST
ASSIGN 500 TO LABEL
GOTO 1000
500 CONTINUE
C ***********************************************
C READ PLOT DATA
999 READ 7, PHII, THETA, PSI
7 FORMAT(7X, 3F5.0)
PRINT PLOTLT
C REWIND OUTPUT FILE FROM COMPUTER CONVERSION
REWIND UNIT
ENDFILE SCRCH
REWIND SCRCH
C ***********************************************
C USER EDIT SECTION
C EDIT UNIVERSAL INFO
ZERO UNIVERSAL AREA OF STORAGE
PRINT 8
8 FORMAT(14H1, 30X, *START EDIT*)
DO 710 I = 1, 14
710 DA(I) = 0.0
CALL DECRD(DA, OUNIT)
CALL DECRD(DA, IUNIT)
CALL DECWRD(DA, IUNIT2)
C ZERO BODY AREA STORAGE
DO 711 I = 1, 3405
711 BODARE(I) = 0.0
C EDIT BODIES IF ANY
NPAD = 0
IF(J2 .EQ. 0) GOTO 505
CALL DECRD(DA, OUNIT)
CALL DECRD(DA, IUNIT)
J = NXS
IF(J .GT. 44) GOTO 201
C FILL IN DATA FOR BODY THAT FITS AS IS
NXSBOT = NXS
BMFCXN = NXSBOT
DO 7030 I = 1, J
BCT(I) = XVXFXF(I)
ZLCT(I) = ZLCCS(I)
7030 BMFCX(K(I)) = BCT(I)
IF(J2 .NE. -1) GOTO 202
C SET ANGLE OF CIRCULAR BODY
LCT(1) = 2.0
LCT(2) = 0.0
LCT(3) = 180.0
L = 1
DO 7040 I = 1, J
C FIND RADIUS OF CIRCULAR BODY
RB(L) = DSORT(FCSA(I)/PI)
RB(L+1) = RB(L)
C
C
7040 L = L + 2
GOTO 205
202 MM = 1
  N = 0
  LLL = 0
  LL = 1
  DO 110 IX=1, NFUS
  NFF = NRADX(IX)
  NZ = NFF
  NF0 = NF0R(X(IX)
  LLL = LLL + NF0
  DO 10 M=LL, LLL
  LCT(MM) = NFF
  DO 7090 I=1, NFF
C
  SET Y AND Z VALUES
  RB(N+I) = ZOHS(NZ,M)
  LCT(MM+I) = YOHS(NZ,M)
7090 NZ = NZ - 1
  N = N + NFF
  10 MM = MM + 1 + NFF
110 LL = LL + NF0
GOTO 205
C
C INTERPOLATE FOR OVER SPECIFIED DATA
201 J = NXS0T
BMFCXN = NXS0T
  DO 217 I=1, J
217 BMFCXS(I) = BCT(I)
  IF( J2 .NE. -1) GOTO 204
C
C SET ANGLE OF CIRCULAR BODY
LCT(1) = 2.0
LCT(2) = 0.0
LCT(3) = 180.0
J=NXS0T
L=1
C
C FIND RADIUS OF CIRCULAR BODY
DO 203 I=1, J
RB(L) = DSQRT(CODIM1(BCT(I), XVXFXF, FCSA, NXS, 0.0) / PI)
RB(L+1) = RB(L)

FIND CAMBER
ZLCT(I) = CODIM1(BCT(I), XVXFXF, ZLCCS, NXS, 0.0)
203 L = L + 2
GOTO 205
204 CONTINUE

FIND 5  BAR(CIRCUMFERENCE)
LL = 1
LLL = 0
N = SN
DO 206 IX=1, NFUS
NFG = NFORX(IX)
LLL = LLL + NFG
NFG = NRADX(IX)

FILL AND NON DIMENSIONALIZE S MATRIX
DO 208 I=LL, LLL
SD(I) = 0.0
DO 207 I=1, NFG
207 CSS(I) = SR(I-1) + SQRT((YCHS(I+M) - YCHS(I-1,M))**2 + 1 + (ZOH5(I,M) - ZOH5(I-1,M))**2)
2 + 1.0E-7
T = SL(NFG)
DO 209 I=2, NFG
209 SS(I) = SS(I) / T
DO 210 I=1, N
FCSA(I) = CODIM1(S(I), SS, YCHS(I,M), NFG, 0.0)
210 ZCSA(I) = CODIM1(S(I), SS, ZOH5(I,M), NFG, 0.0)
IN = N
DO 215 I=1, N
YCHS(I,M) = FCSA(IN)
ZOH5(I,M) = ZCSA(IN)
215 IN = IN - 1
206 CONTINUE
208 LL = LL + NFG
DO 211 I=1, N
DO 211 M=1, NXS
SZ(M,I) = ZOH(S(I,M))
211 SY(M,I) = YOH(S(I,M))
J     = NXSBOT
L     = 1
K     = 0
DO 212 I=1, J
LCT(L) = N
DO 213 K=1, N
LCT(L+M) = CODIM1(BCT(I), XVXF, SY(I,M), NXS, 0.0)
213 RB(K+M) = CODIM1(BCT(I), XVXF, SZ(I,M), NXS, 0.0)
L     = L + 1 + N
212 K     = K + H
DO 214 L=1, J
214 ZLCC(T) = CODIM1(BCT(I), XVXF, ZLCCS, NXS, 0.0)
205 CONTINUE
CALL DECRDA(DA, 15, 3419, OUNIT2)
C ZERO BODY AREA STORAGE
720 DO 721 I=1, 3405
721 BODARE(I) = 0.0
C EDIT PODE WIT ZERO RADIUS OF ROSE
505 CALL DECRDA(DA, SCRTCH)
IF(SCRTCH) 501, 501, 499
499 CALL DECRDA(DA, IUNIT)
CALL DECRWRT(DA, 15, 3419, OUNIT2)
NPOD = NPOD + 1
GOTO 720
C EDIT PANELS (IF ANY)
501 CALL DECRDA(DA, OUNIT)
IF(OUNIT) 502, 502, 503
503 CALL DECRDA(DA, IUNIT)
CALL DECRWRT(DA, 3420, 5003, OUNIT2)
C ZERO PANEL AREA OF STORAGE
DO 713 I=1, 1580
713 PANARE(I) = 0.0
GOTO 901
C
C RELOAD FILES
502 CUNIT = IABS(CUNIT)
RELOAD CUNIT
RELOAD CUNIT2

*ADD ADDITIONAL Routines and pointers TO SUBRON IT IS WANTED-
ADD ADDITIONAL Routines and pointers TO SUBRON IT IS WANTED-

ADDITION SECTION

COPY PAST OLD BODY (IF ANY)
PRINT 9
9 FORMAT(1H1, 3U2"START ADDITIONS")
DO 714 I=1, 14

714 DA(I) = -9.6
CALL DECRT(DA, CUNIT2)
CALL DECRT(DA, 1, 14, CUNIT)
I = DA(I)
IF (J2 .EQ. 3) GOTO 507
I = I - 1
CALL DECRT(DA, CUNIT2)
CALL DECRT(DA, 15, 3419, CUNIT):

C ZERO BODY AREA STORAGE

513 DO 716 J=1, 2403
516 BODARE(J) = -0.6

507 IF(NPOD) 504, 504, 508
508 CALL DECRT(DA, CUNIT2)
CALL DECRT(DA, 15, 3419, CUNIT)
I = I - 1
NPOD = NPOD - 1
GOTO 513

C ADD NEW BODIES (IF ANY)

504 IF(I) 518, 518, 505
505 DO 506 J=1, I
CALL DECRT(DA, IUNIT)
CALL DECRT(DA, 15, 3419, CUNIT)
C ZERO BODY AREA STORAGE

DO 717 K=1, 3403
717 BODARE(K) = -0.6
C
509 CONTINUE

C
C COPY REST OF PANELS

518 IF (DA(2) .EQ. 0) GOTO 514
I = DA(2)
DO 516 J = 1, I
C ZERO PANEL AREA OF STORAGE
DO 718 K = 1, 1580
718 PANELE = 0.0
CALL DECDFD(DA, UUNIT2)
IF (UNIT2) 512, 512, 511
511 CALL DECWRK(DA, 3420, 5000, UUNIT)
ATACH(1, J) = DA(3420)
ATACH(2, J) = DA(3444)
ATACH(3, J) = DA(3445)
ATACH(4, J) = DA(3446)
ATACH(5, J) = DA(3447)
510 CONTINUE
C
C UUNIT NOW CONTAINS FULLY UPDATED DATA, READY FOR FRED
C UUNIT2 IS FREE
C
514 REWIND UUNIT
REWIND UNIT2
PRINT ATTACH
RETURN
C
C ADD NEW PANELS (IF ANY)

512 UNIT2 = IABS (UNIT2)
DO 515 K = 1, J
C ZERO PANEL AREA OF STORAGE
DO 719 L = 1, 1580
719 PANELE = 0.0
CALL DECDFD (DA, UUNIT)
CALL DECWRK (DA, 3420, 5000, UUNIT)
ATACH(1, K) = DA(3420)
ATACH(2, K) = DA(3444)
ATACH(3, K) = DA(3445)
ATACH(4, K) = DA(3446)
ATACH(5, K) = DA(3447)
CONTINUL
GTOC 514

PANEL DATA STORED

CO = ORIGINATE OF LEADING EDGE OF FIRST AIRFOIL
1000 XPO = 0.0
YPO = 0.0
ZPO = 0.0

SET REFERENCE AREA
REFAF=REFAA

PANEL PERIMETER DESCRIPTIONS
PPD(1) = FINXL
PPD(2) = FINYL
PPD(3) = FINZL
PPD(4) = LCHORD
PPD(5) = FINXH
PPD(6) = FINYH
PPD(7) = FINZH
PPD(8) = HCHORD

SET NUMBER OF PANEL PERIMETER DESCRIPTION POINTS
NPPDP=2

NUMBER OF ETA(ATA) STATIONS
ANAS = 2
NATA=ANAS

ETA STATIONS
ETA(1)=0.0
ETA(2)=1.0
ATA(1)=0.0
ATA(2)=1.0
C
C        PUNCH PANEL
C    CALL DECART(DA•3420•5000•UNIT)
C
C        ZERO PANEL SECTION OF STORAGE
C    DO 224 I=1, 1580
C  224   PAMARE(1)=-0.0
C
C        RETURN TO CALLER
C    GOTO LABEL,(380•500)
C    END
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