EVALUATION OF A LINEAR WASHOUT FOR SIMULATOR MOTION CUE PRESENTATION DURING LANDING APPROACH

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SUMMARY

The comparison of a fixed-base versus a five-degree-of-freedom motion base simulation (the heave cue was not presented) of a 737 conventional take-off and landing (CTOL) aircraft performing instrument landing system (ILS) landing approaches has been used to evaluate a linear motion washout technique. The fact that the pilots felt that the addition of motion increased the pilot workload and this increase was not reflected in the objective data results, indicates that motion cues, as presented, are not a contributing factor to root-mean-square (rms) performance during the landing approach task. Subjective results from standard maneuvering about straight-and-level flight for specific motion cue evaluation revealed that the longitudinal channels (pitch and surge) and possibly the yaw channel produce acceptable motions. The roll cue representation, involving both roll and sway channels, was found to be inadequate for large roll inputs, as used for example, in turn entries.

INTRODUCTION

The major factor affecting the quality of a motion simulation in comparison with a fixed-base simulation of an aircraft, aside from the physical characteristics of the hardware, is the washout scheme. The washout scheme is used to constrain the motion drives to be within the physical capabilities of the motion base and still maintain the fidelity of the motion cues provided to the simulator pilot.

The comparison of a fixed-base versus a five-degree-of-freedom motion base simulation of a 737 CTOL aircraft during landing approach has been used to evaluate a linear motion washout technique. The motion software utilized in the study is described in reference 1 and the hardware (the motion base) in references 2 and 3. The linear washout scheme was the Langley adapted version of Schmidt and Conrad's linear coordinated washout circuitry (refs. 4 and 5). The evaluation process consisted of the collection of objective data results, indicates that motion cues, as presented, are not a contributing factor to root-mean-square (rms) performance during the landing approach task. Subjective results from standard maneuvering about straight-and-level flight for specific motion cue evaluation revealed that the longitudinal channels (pitch and surge) and possibly the yaw channel produce acceptable motions. The roll cue representation, involving both roll and sway channels, was found to be inadequate for large roll inputs, as used for example, in turn entries.
and subjective data from 90 simulated landing approaches, as well as subjective data from standard maneuvering about straight-and-level flight for specific motion cue evaluation.

The landing approach task was selected for the collection of the objective data, despite the suggestion in reference 4 that the task might be a poor one for motion evaluation, for several reasons:

1. Objective performance measures are readily available
2. Landing approach simulation with motion is widespread in use
3. Statistical data could provide verification of the previous suggestion of reference 4

SYMBOLS

Values are given in both SI and U.S. Customary Units. The measurements and calculations were made in U.S. Customary Units.

A,B,C  main effects in analysis of variance

AB,AC,BC  two factor interactions in analysis of variance

ABC  three factor interaction in analysis of variance

A1,A2,A3  acceleration lead parameters for translational channel lag compensation, sec^2

a1,a2,a3  damping parameters for second-order translational washout filters, rad/sec

B1,B2,B3  velocity lead parameters for translational channel lag compensation, sec

b1,b2,b3  frequency parameters for second-order translational washout filters, rad/sec^2

C1,C2,C3  translational acceleration braking parameters, sec^{-1}

\bar{c}  mean aerodynamic chord, m (ft)

f  scale factor, deg/rad
\begin{align*}
  f_{c,x}, f_{c,y} & \quad \text{body-axis longitudinal and lateral accelerations at centroid location after low-pass filtering, m/sec}^2 \text{ (ft/sec}^2) \\
f_{c,z} & \quad \text{body-axis vertical acceleration (referenced about 1g) at centroid location after high-pass filtering, m/sec}^2 \text{ (ft/sec}^2) \\
f_{i,x}, f_{i,y}, f_{i,z} & \quad \text{inertial axis translational acceleration commands prior to translational washout, m/sec}^2 \text{ (ft/sec}^2) \\
f_{i,x}^*, f_{i,y}^* & \quad \text{inertial axis specific force error signals, m/sec}^2 \text{ (ft/sec}^2) \\
f_{i,x}^*, f_{i,y}^*, f_{i,z}^* & \quad \text{components in inertial axis of filtered body-axis vertical acceleration centroid location, m/sec}^2 \text{ (ft/sec}^2) \\
f_{i,z} & \quad \text{artificial yaw error signal, m/sec}^2 \text{ (ft/sec}^2) \\
f_{s,x}, f_{s,y} & \quad \text{body-axis longitudinal and lateral accelerations at centroid location, m/sec}^2 \text{ (ft/sec}^2) \text{ or g units; } F_{sx} \text{ and } F_{sy} \text{ in the computer plots} \\
f_{s,z} & \quad \text{body-axis vertical acceleration (referenced about 1g) at centroid location, m/sec}^2 \text{ (ft/sec}^2) \text{ or g units; } F_{sz} \text{ in the computer plots} \\
f_{x}, f_{y}, f_{z} & \quad \text{aircraft-body-axis translational accelerations, m/sec}^2 \text{ (ft/sec}^2) \\
f_{z,c} & \quad \text{body-axis translational acceleration at centroid location, m/sec}^2 \text{ (ft/sec}^2) \\
G_1 & \quad \text{glide-slope error gain} \\
g & \quad \text{acceleration due to gravity, } 1g = 9.8 \text{ m/sec}^2 \text{ (32.2 ft/sec}^2) \\
h & \quad \text{altitude, m (ft)} \\
h_c & \quad \text{commanded altitude, m (ft)} \\
o & \quad \text{height of aircraft c.g. at touchdown, m (ft)} \\
o & \quad \text{heading error gain}
\end{align*}
\( K_1 \)  
Gain parameter of roll flight director filter

\( K_2 \)  
Damping parameter of roll flight director filter, deg/sec

\( K_3 \)  
Frequency parameter of roll flight director filter, deg/sec^2

\( k_p, k_q, k_r \)  
Scaling parameters for angular rates

\( k_{p,T,1}, k_q, T, 1, k_{r,1} \)  
Parameters of signal-shaping network, m\(^{-1}\) (ft\(^{-1}\))

\( k_{p,T,2}, k_q, T, 2, k_{r,2} \)  
Parameters of signal-shaping network, sec

\( k_{p,T,3}, k_q, T, 3, k_{r,3} \)  
Parameters of signal-shaping network, sec\(^{-1}\)

\( k_z, 1, k_z, 2 \)  
Gain parameters of vertical channel high-pass filter

\( k_{\theta,1}, k_{\theta,2} \)  
Gain parameters of longitudinal channel low-pass filter

\( k_{\phi,1}, k_{\phi,2} \)  
Gain parameters of lateral channel low-pass filter

\( k_{\psi, l}, k_{\theta, l}, k_{\phi, l} \)  
Lead parameters for rotational channel lag compensation, sec

\( L_y \)  
Localizer error rate limit, deg/sec

\( L_{\phi} \)  
Flight director roll limit, deg/sec

\( p, q, r \)  
Body-axis angular velocity commands, rad/sec or deg/sec; \( P, Q, \) and \( R \) in computer plots

\( p', q', r' \)  
Body-axis angular tilt velocity, rad/sec

\( p'', q'', r'' \)  
Scaled body-axis aircraft angular velocities, rad/sec

\( p_a, q_a, r_a \)  
Body-axis aircraft angular velocities, rad/sec

\( \dot{p}_a \)  
Body-axis aircraft roll acceleration, rad/sec

\( \dot{p}_m \)  
Body-axis roll acceleration as measured by instrument mounted on motion simulator, rad/sec^2 or deg/sec^2
**R**
range from runway, m (ft)

**R_x,R_y,R_z**
centroid location with respect to c.g., m (ft)

**S_1**
pitch input to pitch flight director, deg

**S_2**
output of first order pitch flight director, deg

**s**
Laplace operator

**t**
time, sec

**t_0**
starting time for roll flight director operation

**V_\ell**
velocity limit, m/sec (ft/sec)

**x_\ell**
Earth-axis longitudinal coordinate of aircraft c.g., m (ft)

**x,y,z**
commanded inertial translational position of motion simulator, m (ft)

**\hat{x},\hat{y},\hat{z}**
commanded translational positions after compensation, m (ft)

**x_{LF},y_{LF},z_{LF}**
scale factors on position limits

**\ddot{x}_b,\ddot{y}_b,\ddot{z}_b**
intermediate inertial axis translational acceleration commands, m/sec^2 (ft/sec^2)

**\dot{x}_d,\dot{y}_d,\dot{z}_d**
inertial-axis translational position commands, m (ft)

**\dot{x}_z,\dot{y}_z,\dot{z}_z**
inertial-axis position limits for translational channels, m (ft)

**\ddot{x}_\ell,\ddot{y}_\ell,\ddot{z}_\ell**
inertial-axis acceleration limits for translational channels, m/sec^2 (ft/sec^2)

**x_0**
longitudinal coordinate of runway touchdown point, m (ft)

**x_p,y_p,z_p**
coordinates of pilot's station with respect to c.g. in body-axis system, m (ft)
\( x_{p,c}, y_{p,c}, z_{p,c} \) coordinates of centroid location with respect to pilot's station in the body-axis system, m (ft)

\( x_r \) x-distance of aircraft c.g. from runway, m (ft)

\( y_l \) Earth-axis lateral coordinate of aircraft c.g., m (ft)

\( y_b \) distance behind runway touchdown point of localizer beam origin, m (ft)

\( y_o \) lateral coordinate of runway touchdown point, m (ft)

\( y_r \) y-distance of aircraft c.g. from runway, m (ft)

\( z_{neut} \) actuator extension for selected neutral point, m (ft)

\( \gamma_c \) commanded glide-slope angle, deg

\( \Delta t \) time step size, sec

\( \delta_a \) aileron deflection angle, rad

\( \epsilon_h \) vertical glide-slope error, m (ft)

\( \epsilon_y \) localizer error, deg

\( \epsilon_{y,l} \) localizer error lag, deg

\( \epsilon_{y,l} \) rate limited localizer error, deg

\( \epsilon_{\gamma} \) glide-slope error, deg

\( \epsilon_{\psi} \) heading error, deg

\( \epsilon_{\psi,1} \) scaled heading error, deg

\( \theta_a \) actual aircraft pitch angle, deg

\( \theta_c \) commanded pitch angle, deg

\( \theta_s \) pitch command signal, deg
A dot over a variable indicates the time derivative of the variable.

**WASHOUT CIRCUITRY**

The adapted version of Schmidt and Conrad's linear coordinated washout circuitry used in this study is shown in figure 1 in block diagram form. The detailed equations are presented in appendix A. The function of the circuitry is to represent the translational accelerations and the rotational rates of the simulated aircraft while constraining the motion commands to be within the hardware capabilities. The concept of this coordinated
washout circuitry is to represent longitudinal and lateral translational cues by utilizing both translational and rotational motions and to obtain rotational washout through elimination of the false gravitational g cues that would be induced by a rotational movement.

The selection of the parameters for the washout circuitry began with employment of the values suggested in figure A.7 of reference 5. A representative "worst case" ILS approach was made with the fixed-base simulator and the resulting translational accelerations and rotational rates were placed on tape. The tape was then used iteratively to drive the motion software for parameter variation.

Initial modification of the parameters was made to constrain the motions to remain within the motion limits of the hardware. Further modification of the parameters to improve the fidelity of the motion cues, in terms of time history comparisons of aircraft motion cues (simulated flight data) with washout commands to represent these cues, was made next. Final determination of the parameters was then made based on the subjective opinions of three participating research pilots. The major emphasis of this portion of the parameter selection process was placed on the roll channel parameters which will be discussed in a later section.

**TASK CONDITIONS AND DATA BASE**

Figure 2 illustrates the ILS task, which consisted of (1) a transition to the localizer beam, (2) a transition to the glide slope, and (3) the ensuing approach to about 76 m (250 ft). Three approach conditions were provided: the standard approach described above, the standard approach with instantaneous encounter of a weather front (a 10-knot cross wind with moderate turbulence), and the standard approach with the occurrence of an engine failure. Typical time histories of comparisons of aircraft motion cues (simulated flight data) with washout commands to represent those cues are presented in figures 3, 4, and 5 for the three approach conditions. These figures are intended to illustrate the levels of aircraft motion involved in the simulated task as well as the amount provided by the motion base.

The approaches were flown under fixed-base conditions and under moving-base conditions with the washout parameters of reference 1 as shown in table I. Five runs were made for each of the three pilots at each condition. Motion was restricted to five degrees of freedom because (1) extreme hydraulic noise is induced by the heave motion of the synergistic base (all six actuators have to move alike to present a heave cue) and (2) only a small amount of vertical cue was available.

The small amount of vertical cue available is due to a combination of the position limits of the motion base and the short-period frequency of the 737 aircraft in the landing approach configuration. Since the position limits of the synergistic motion base change as
the orientation of the base varies, the position limits used in determining the linear wash-out parameters must be conservative. For the motions involved in this study, the vertical position limits were chosen to be 0.45 m (1.5 ft). The low-frequency content of the normal acceleration of the aircraft (less than 1 rad/sec, neglecting turbulence) is due to the low short-period frequency. (See table II.) The amount of vertical cue available for motion simulation is thus less than 0.05g (the product of amplitude and frequency squared). The participating pilots felt that the vertical cue available was not worth the noise distraction.

During the performance of the landing approach task under the preceding conditions, root-mean-square (rms) data were collected over two regimes. A short-duration regime, intended to reflect the immediate effect of the weather front and the engine-failure conditions, and a long-duration regime, to evaluate total performance, were used. The rms values were obtained for deviations of the glide slope, localizer, pitch command bar, roll command bar, and speed command bar. The equations for the flight director used in this study are presented in appendix B.

Subjective data consisted of revised Cooper-Harper ratings of the ILS task as shown in table III (ref. 6) and pilot comments solicited during objective data collection. In addition, standard maneuvers about straight-and-level flight were also used to generate subjective pilot evaluation data of the motion cues.

OBJECTIVE DATA RESULTS

The design of the experiment for objective data consisted of the $2 \times 3 \times 3$ factorial design (ref. 7). The fixed-effect factors are pilots, approach conditions, and motion versus fixed-base operation. The results of the analysis of variance for each of the 10 separate rms measurements (deviations of glide slope, localizer, pitch command bar, roll command bar, and speed command bar for short and long durations) are shown in table IV. Significance of the one-tailed $F$-tests is indicated by an asterisk for the 5-percent level and a double asterisk for the 1-percent level.

The results indicate significant statistical differences in mean performances between pilots and also between approaches. No significant differences in mean performances are found between motion and fixed-base operation. The occasional statistical significance of the two factor interaction AC indicates that the differences between pilots varied with the approach condition over all motion conditions, or alternately, the differences in performance between approaches varied from pilot to pilot, regardless of the motion condition, for some of the performance measures.
Although the subjective data obtained from the pilots ranged the gamut from complete dislike of the motion used to something approaching delight, two points of agreement were shared by all three pilots: (1) the addition of motion increased the pilot workload (the pilots believed their ability to make the precision inputs required to null the flight director command bars was lessened under motion conditions) and (2) the roll motion is borderline, if not unacceptable, for large roll inputs. While the first point of agreement is not borne out by the objective data results, the revised Cooper-Harper ratings presented in table V clearly illustrate this belief (increased pilot workload is reflected in an increased Cooper-Harper rating), and possibly suggest that the selected ILS task is a poor one for motion evaluation in terms of objective data (substantiating the opinion expressed in ref. 4).

The second point of agreement, the poor representation of roll, can be substantiated with objective data. The problem is a combination of hardware weakness and software performance. The hardware weakness, a turn-around bump (a problem common to most motion systems), exists in all degrees of freedom, but was only noticed by the pilots in the roll channel, perhaps due to the frequency and amplitude of the roll inputs. This problem is documented in figure 6 with a time history taken from an accelerometer mounted on the motion-base cockpit to measure $\dot{\phi}_m$. The base was driven with a sine wave of $4^\circ$ amplitude and at a frequency of 1.5 rad/sec.

The software performance problem is a result of the difficulty in presenting a roll cue on a motion base, as discussed in reference 8. A negative roll angle induces a positive sway force $f_{s,y}$ in a motion simulator, due to the gravity vector. As may be seen from the flight data of figures 3, 4, and 5, $f_{s,y}$ and $p$ are in phase, and thus translational acceleration is necessary to (1) offset the misalignment of the gravity vector due to the roll cue and (2) present the side-force cue.

The parameters for the coordinated roll channel, as picked by the pilots involved, yield the results presented in figure 7 for a pulse input on the aileron. The fact that the negative peak of $p$ is larger than the positive peak could give the pilot the impression that the net result of the maneuver was a left bank, rather than a right bank.

The parameters of the roll channel used in the study have an effective natural frequency of about 0.1 rad/sec. Figures 8 and 9 display the washout response for the pulse input of figure 7 for effective natural frequencies of 0.3 and 0.5 rad/sec, respectively. This range of frequency is typically employed in classical circuits (ref. 5). Subjectively, the pilots preferred the frequency of 0.1 rad/sec. All frequencies tried gave the impression of a left bank, rather than a right bank, to two of the three pilots. This impression
was obtained only for large roll inputs, such as used in turn entries, and the false bank
was not noted during the control inputs required for the ILS task.

The lack of sufficient lateral travel for sway representation was not noticed sub­jectively by any of the participating pilots, even though \( f_{s,y} \) is 180° out of phase for the
chosen parameter values. Parameters that bring \( f_{s,y} \) in phase result in p represen­
tation being 180° out of phase, a situation considered by the pilots to be intolerable.

Table VI presents the subjective ratings of the three pilots of the motion cues
encountered during maneuvering about straight-and-level flight. Pilot 1 has had the only
experience to date in the flight version of the simulated aircraft, and his general com­ments were that the motion felt like a long, narrow airplane that wallows, although large
roll inputs felt "mechanical" (artificial). He felt that the yaw cue was the best of any
motion simulator he had flown. Pilot 2 has had little experience in long-bodied airplanes
and felt that generally the motions were jerky and confusing. Pilot 3 felt that the motions
were unnatural and distracting, rather than helpful, and did not feel like an airplane. How­
ever, he would rather fly with motion because the confusion increases the workload level.

CONCLUDING REMARKS

The objective and subjective results of this study lead to two general conclusions,
one of which is concerned with the ILS task and the other with the question of motion vali­
dation of the linear washout. The fact that the pilots all felt that the addition of motion
increased the workload, and yet this increase in workload did not deteriorate the pilot per­
formance of the task, indicates that motion cues, as presented, are not important in terms
of rms performance measures during instrument landing system landing approaches.

The question of motion evaluation can best be discussed in terms of the subjective
ratings of the individual cues. The roll representation must be improved by changes in
either the software, the hardware, or both. The yaw cue may or may not be acceptable,
with the issue of pilot experience in long-bodied aircraft intruding on the evaluation. The
same factor could be in force on the evaluation of the pitch cue. The motion cues with
throttle change were acceptable.

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DETAILED EQUATIONS FOR THE WASHOUT CIRCUIT

The following is a block-by-block list of equations corresponding to figure 1:

Centroid transformation:

\[ R_x = x_p + x_{p,c} \]
\[ R_y = y_p + y_{p,c} \]
\[ R_z = z_p + z_{p,c} \]

\[ f_{s,x} = f_x - \left( q_a^2 + r_a^2 \right) R_x + \left( q_a p_a - \dot{r}_a \right) R_y + \left( r_a p_a + \dot{q}_a \right) R_z \]
\[ f_{s,y} = f_y + \left( p_a q_a + \dot{r}_a \right) R_x - \left( p_a^2 + r_a^2 \right) R_y + \left( r_a q_a - \dot{p}_a \right) R_z \]
\[ f_{z,c} = f_z + \left( p_a q_a - \dot{q}_a \right) R_x + \left( q_a r_a + \dot{p}_a \right) R_y - \left( p_a^2 + q_a^2 \right) R_z \]

Variation about lg:

\[ f_{s,z} = f_{z,c} + g \]

High-pass filter:

\[ f_{c,z} = \frac{k_{z,1} f_{s,z} - 2\xi_{z,1} \omega_{n,z,1} \int f_{c,z} \, dt - \omega^2_{n,z,1} \int \int f_{c,z} \, dt \, dt}{k_{z,2}} \]

Low-pass filter:

\[ k_{\theta,2} \ddot{f}_{c,x} = k_{\theta,1} f_{s,x} - 2\xi_{\theta} \omega_{n,\theta} \dot{f}_{c,x} - \omega^2_{n,\theta} f_{c,x} \]

\[ k_{\phi,2} \ddot{f}_{c,y} = k_{\phi,1} f_{s,y} - 2\xi_{\phi} \omega_{n,\phi} \dot{f}_{c,y} - \omega^2_{n,\phi} f_{c,y} \]
APPENDIX A

Body to inertial transformation, high-frequency components:

\[ f_{1,x}' = f_{c,z}(\cos \theta \sin \theta \cos \psi + \sin \phi \sin \psi) \]

\[ f_{1,y}' = f_{c,z}(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \]

\[ f_{1,z}' = f_{c,z}(\cos \phi \cos \theta) \]

Body to inertial transformation, low-frequency components:

\[ f_{1,x}^* = f_{c,x}^*(\cos \theta \cos \psi) + f_{c,y}^*(\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) \]

\[ -g(\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) \]

\[ f_{1,y}^* = f_{c,x}^*(\cos \theta \sin \psi) + f_{c,y}^*(\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) \]

\[ -g(\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) \]

Sum of low- and high-frequency components:

\[ f_{1,x} = f_{1,x}' + f_{1,x}^* \]

\[ f_{1,y} = f_{1,y}' + f_{1,y}^* \]

\[ f_{1,z} = f_{1,z}' \]

Signal-shaping network:

\[ \dot{\theta}_T = k_{q,T,1}k_{q,T,2}f_{1,x}' + k_{q,T,1}f_{1,x}^* + k_{q,T,1}f_{1,x}^* + k_{q,T,1}k_{q,T,3} \int \int f_{1,x}' \, dt \, dt \]

\[ \dot{\phi}_T = -k_{p,T,1}k_{p,T,2}f_{1,y}' - k_{p,T,1}f_{1,y}^* - k_{p,T,1}f_{1,y}^* - k_{p,T,1}k_{p,T,3} \int \int f_{1,y}' \, dt \, dt \]

\[ \dot{\psi}_T = k_{r,1}k_{r,2}f_{1,z}'' + k_{r,1}f_{1,z}'' + k_{r,1}f_{1,z}'' + k_{r,1}k_{r,3} \int \int f_{1,z}'' \, dt \, dt \]
APPENDIX A

Inertial to body transformation:

\[ p' = \dot{\phi}_T (\cos \theta \cos \psi) + \dot{\theta}_T (\cos \theta \sin \psi) - \dot{\psi}_T (\sin \theta) \]

\[ q' = \dot{\phi}_T (\sin \phi \sin \theta \cos \psi - \cos \phi \sin \psi) + \dot{\theta}_T (\sin \phi \sin \theta \sin \psi + \cos \phi \cos \psi) + \dot{\psi}_T (\sin \phi \cos \theta) \]

\[ r' = \dot{\phi}_T (\cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi) + \dot{\theta}_T (\cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi) + \dot{\psi}_T (\cos \phi \cos \theta) \]

Scale airplane angular rates:

\[ p'' = k_p p_a \]

\[ q'' = k_q q_a \]

\[ r'' = k_r r_a \]

Sum of airplane and tilt rates:

\[ p = p'' + p' \]

\[ q = q'' + q' \]

\[ r = r'' + r' \]

Transformation to Euler rates:

\[ \dot{\phi} = p + q \sin \phi \tan \theta + r \cos \phi \tan \theta \]

\[ \dot{\theta} = q \cos \phi - r \sin \phi \]

\[ \dot{\psi} = (q \sin \phi + r \cos \phi) \sec \theta \]
Angular lead compensation:

\[ \hat{\psi} = \psi + k_\psi \dot{\psi} \]
\[ \hat{\theta} = \theta + k_\theta \dot{\theta} \]
\[ \hat{\phi} = \phi + k_\phi \dot{\phi} \]

Translational lead compensation:

\[ \hat{x} = x_d + A_1 \ddot{x}_d + B_1 \dot{x}_d \]
\[ \hat{y} = y_d + A_2 \ddot{y}_d + B_2 \dot{y}_d \]
\[ \hat{z} = z_d + A_3 \ddot{z}_d + B_3 \dot{z}_d \]

Translational washout:

\[ \ddot{x}_d = f_{i,x} - a_1 \dot{x}_d - b_1 x_d \]
\[ \ddot{y}_d = f_{i,y} - a_2 \dot{y}_d - b_2 y_d \]
\[ \ddot{z}_d = f_{i,z} - a_3 \dot{z}_d - b_3 z_d \]

Limit prediction based on current position:

See reference 1 for equations and derivation.
APPENDIX B

FLIGHT DIRECTOR EQUATIONS

The following is a list of the equations for the flight director used in this study:

Input equations:

\[ x_r = (x_t - x_0) \cos \psi_c + (y_t - y_0) \sin \psi_c \]

\[ y_r = -(x_t - x_0) \sin \psi_c + (y_t - y_0) \cos \psi_c \]

\[ R = \left( x_t^2 + y_t^2 \right)^{1/2} \]

\[ h_c = R \tan \gamma_c + h_0 \]

\[ \epsilon_h = h - h_c \]

\[ \epsilon_\gamma = f \tan^{-1} \frac{\epsilon_h}{R} \]

\[ \epsilon_\psi = \psi_a - \psi_c \]

\[ \epsilon_y = f \tan^{-1} \frac{y_r}{R + y} \]

Pitch flight director:

\[ G_1 = \begin{cases} 
0.14 (h-50) & (h < 100) \\
h/15 & (h \geq 100) 
\end{cases} \]

\[ G_1 \text{ limited to } [0, 100] \]

\[ S_1 = -\left( \theta_a + 2 \right) \]

\[ \dot{S}_2 = \frac{S_1 - S_2}{15} \quad \text{(Initial condition: } S_2 = S_1) \]

\[ \theta_c = G_1 \epsilon_\gamma - S_2 \]

\[ \theta_c \text{ limited to } [-12, 12] \]

\[ \theta_s = \theta_c + S_1 \]
APPENDIX B

Roll flight director:

\[ \epsilon_{y,l} = \frac{\epsilon_y - \epsilon_{y,l}}{\Delta t} \]

\( \epsilon_{y,l} \) limited to \([-L_y, L_y]\)

\[ \epsilon_{y,l} = \epsilon_{y,l} + \Delta t \epsilon_{y,l} \]

\[ \epsilon_{\psi,1} = K_0 \epsilon_{\psi} \]

\[ \phi_1 = \epsilon_{\psi} - 65 \epsilon_{y,l} + 1.6 \psi_a \]

\[ \ddot{\phi}_1 = K_1 \phi_1 - K_2 \dot{\phi}_1 - K_3 \phi_1 \]

(Initial conditions: \( \dot{\phi}_1 = 0 \) and \( \phi_1 = 8.499 \phi_f \))

\[ \phi_1 = 21.2 \epsilon_{y,l} + \epsilon_{\psi} - \dot{\phi}_1 \]

\( \phi_1 \) limited to \([-30, 30]\)

\[ \phi_c = \phi_1 - \epsilon_{\psi} \]

\( \phi_c \) limited to \([-L_\phi, L_\phi]\)

\[ \phi_s = \phi_c - \phi_a \]

\[ t - t_o \leq 90 \]

\[ t - t_o > 90 \]

\[ L_y = 0.4 \]
\[ L_y = 0.12 \]
\[ L_\phi = 25 \]
\[ L_\phi = 15 \]
\[ K_0 = 2.8 \]
\[ K_0 = 3.8 \]
\[ K_1 = 2.833 \]
\[ K_1 = 3.5714 \]
\[ K_2 = 2.867 \]
\[ K_2 = 3.534 \]
\[ K_3 = 0.3333 \]
\[ K_3 = 0.7518 \]
REFERENCES


<table>
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<th>Variable</th>
<th>Value in SI units</th>
<th>Program value (U.S. units)</th>
<th>Value in SI units</th>
<th>Program value (U.S. units)</th>
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</thead>
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<td>0.14</td>
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<td>0.14</td>
</tr>
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<td>0.1</td>
<td>0.14</td>
<td>0.14</td>
</tr>
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<td>1.0</td>
<td>0.14</td>
<td>0.14</td>
</tr>
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<td>$k_p, T, 1$, per m (per ft)</td>
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<td>0.001</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
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<td>30.0</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
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<td>0.05</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$k_q, T, 1$, per m (per ft)</td>
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<td>0.001</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$k_q, T, 2$, sec</td>
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<td>30.0</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$k_r, 1$, per m (per ft)</td>
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<td>0.05</td>
<td>0.5</td>
<td>0.5</td>
</tr>
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<td>$k_r, 2$, sec</td>
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<td>3.8</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.05</td>
<td>0.5</td>
<td>0.5</td>
</tr>
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<td>$a_1$, rad/sec</td>
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<td>1.414</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$a_2$, rad/sec</td>
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<td>2.1</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$a_3$, rad/sec</td>
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<td>2.1</td>
<td>0.5</td>
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</tr>
<tr>
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<td>1.0</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>$b_2$, rad/sec$^2$</td>
<td>2.25</td>
<td>2.25</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$b_3$, rad/sec$^2$</td>
<td>2.25</td>
<td>2.25</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\ddot{x}_t$, m/sec$^2$ (ft/sec$^2$)</td>
<td>5.8840</td>
<td>19.3044</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\ddot{y}_t$, m/sec$^2$ (ft/sec$^2$)</td>
<td>5.8840</td>
<td>19.3044</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\ddot{z}_t$, m/sec$^2$ (ft/sec$^2$)</td>
<td>7.8453</td>
<td>25.7392</td>
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<td>0.5</td>
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<td>$A_1$, sec$^2$</td>
<td>0.0069</td>
<td>0.0069</td>
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<td>0.5</td>
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<tr>
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<td>0.0069</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$A_3$, sec$^2$</td>
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<td>0.0069</td>
<td>0.5</td>
<td>0.5</td>
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<td>$B_1$, sec</td>
<td>0.14</td>
<td>0.14</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
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<td>0.14</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$B_3$, sec</td>
<td>0.14</td>
<td>0.14</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{\psi, \eta}$, sec</td>
<td>0.12</td>
<td>0.12</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{\theta, \eta}$, sec</td>
<td>0.12</td>
<td>0.12</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>$k_{\phi, \eta}$, sec</td>
<td>0.12</td>
<td>0.12</td>
<td>0.5</td>
<td>0.5</td>
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<td>$k_r$</td>
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<td>0.4</td>
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<td>0.5</td>
</tr>
<tr>
<td>$k_q$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_r$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$C_1$, per sec</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$C_2$, per sec</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$C_3$, per sec</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{\theta, 1}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$k_{\theta, 2}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\xi_{\theta}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_{n, \theta}$, rad/sec</td>
<td>1.0</td>
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<td>0.5</td>
<td>0.5</td>
</tr>
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<td>$k_{\phi, 1}$</td>
<td>0.5</td>
<td>0.5</td>
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<td>0.5</td>
</tr>
<tr>
<td>$k_{\phi, 2}$</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>$\xi_{\phi}$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega_{n, \phi}$, rad/sec</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>$z_{\text{neut}}$, m (ft)</td>
<td>0.6487</td>
<td>2.128</td>
<td>0.5</td>
<td>0.5</td>
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<td>$V_t$, m/sec (ft/sec)</td>
<td>0.3048</td>
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<td>0.5</td>
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<tr>
<td>$x_{\text{LF}}$</td>
<td>2.5</td>
<td>2.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$y_{\text{LF}}$</td>
<td>2.5</td>
<td>2.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$z_{\text{LF}}$</td>
<td>3.0</td>
<td>3.0</td>
<td>0.5</td>
<td>0.5</td>
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### TABLE II. - 737 FLIGHT CHARACTERISTICS

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
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<tbody>
<tr>
<td>Weight, N (lb)</td>
<td>400 341 (90 000)</td>
</tr>
<tr>
<td>Center of gravity</td>
<td>0.31c</td>
</tr>
<tr>
<td>Flap deflection, deg</td>
<td>40</td>
</tr>
<tr>
<td>Landing gear</td>
<td>Down</td>
</tr>
<tr>
<td>Damping ratio for</td>
<td></td>
</tr>
<tr>
<td>Short period</td>
<td>0.562</td>
</tr>
<tr>
<td>Long period</td>
<td>0.089</td>
</tr>
<tr>
<td>Dutch roll</td>
<td>0.039</td>
</tr>
<tr>
<td>Period, sec, for</td>
<td></td>
</tr>
<tr>
<td>Short period</td>
<td>6.30</td>
</tr>
<tr>
<td>Long period</td>
<td>44.3</td>
</tr>
<tr>
<td>Dutch roll</td>
<td>5.12</td>
</tr>
</tbody>
</table>
### TABLE III. - REVISED COOPER-HARPER RATINGS

<table>
<thead>
<tr>
<th>Rating</th>
<th>Description</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Satisfactory</strong></td>
<td>Meets all requirements and expectations; good enough without improvement. Clearly adequate for mission.</td>
<td>A1</td>
</tr>
<tr>
<td><strong>Acceptable</strong></td>
<td>May have deficiencies which warrant improvement, but adequate for mission. Pilot compensation, if required to achieve acceptable performance, is feasible.</td>
<td>A2</td>
</tr>
<tr>
<td><strong>Unsatisfactory</strong></td>
<td>Reluctantly acceptable. Deficiencies which warrant improvement. Performance adequate for mission with feasible pilot compensation.</td>
<td>A3</td>
</tr>
<tr>
<td><strong>Unacceptable</strong></td>
<td>Deficiencies which require improvement. Inadequate performance for mission even with maximum feasible pilot compensation.</td>
<td>A4</td>
</tr>
<tr>
<td><strong>Uncontrollable</strong></td>
<td>Control will be lost during some portion of mission.</td>
<td>A5</td>
</tr>
</tbody>
</table>

**CONTROLLABLE**

Capable of being controlled or managed in context of mission, with available pilot attention.

**Excellent, highly desirable.**

Good, pleasant, well behaved.

Fair. Some mildly unpleasant characteristics.

Good enough for mission without improvement.

Some minor but annoying deficiencies. Improvement is requested. Effect on performance is easily compensated for by pilot.

Moderately objectionable deficiencies. Improvement is needed. Reasonable performance requires considerable pilot compensation.

Very objectionable deficiencies. Major improvements are needed. Requires best available pilot compensation to achieve acceptable performance.

Major deficiencies which require improvement for acceptance. Controllable. Performance inadequate for mission, or pilot compensation required for minimum acceptable performance in mission is too high.

Controllable with difficulty. Requires substantial pilot skill and attention to retain control and continue mission.

Marginally controllable in mission. Requires maximum available pilot skill and attention to retain control.

**Uncontrollable in mission.**
### TABLE IV. - COMPUTED F-DISTRIBUTION VALUES FOR THE ANALYSES OF VARIANCE

<table>
<thead>
<tr>
<th>Factors</th>
<th>Degrees of freedom</th>
<th>Root-mean-square performance measures</th>
<th>Deviation of -</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Localizer</td>
<td>Glide slope</td>
<td>Speed</td>
<td>Pitch bar</td>
<td>Roll bar</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Short duration</td>
<td>Long duration</td>
<td>Short duration</td>
<td>Long duration</td>
<td>Short duration</td>
<td>Long duration</td>
<td>Short duration</td>
<td>Long duration</td>
</tr>
<tr>
<td>Pilots, A</td>
<td>2</td>
<td>0.0578</td>
<td>0.0619</td>
<td>**11.56</td>
<td>**25.89</td>
<td>**11.08</td>
<td>**13.18</td>
<td>**15.24</td>
<td>**34.50</td>
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<tr>
<td>Motion, B</td>
<td>1</td>
<td>0.1463</td>
<td>0.1065</td>
<td>0.0199</td>
<td>0.0186</td>
<td>0.6034</td>
<td>0.0610</td>
<td>0.6817</td>
<td>0.1202</td>
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<tr>
<td>Approaches, C</td>
<td>2</td>
<td>**42.55</td>
<td>**36.31</td>
<td>**9.414</td>
<td>**5.574</td>
<td>0.2739</td>
<td>3.399</td>
<td>3.943</td>
<td>1.5999</td>
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<td>Replicates</td>
<td>4</td>
<td>0.7615</td>
<td>0.4040</td>
<td>0.6287</td>
<td>0.4790</td>
<td>0.3059</td>
<td>0.2240</td>
<td>1.126</td>
<td>0.9488</td>
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<tr>
<td>AB</td>
<td>2</td>
<td>0.0346</td>
<td>0.3428</td>
<td>0.0334</td>
<td>0.2791</td>
<td>0.0845</td>
<td>0.0689</td>
<td>1.037</td>
<td>1.549</td>
</tr>
<tr>
<td>AC</td>
<td>4</td>
<td>0.4077</td>
<td>0.5740</td>
<td>2.902</td>
<td>2.429</td>
<td>**3.734</td>
<td>**3.940</td>
<td>2.192</td>
<td>1.683</td>
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<tr>
<td>BC</td>
<td>2</td>
<td>0.0718</td>
<td>0.0415</td>
<td>0.5605</td>
<td>1.731</td>
<td>1.518</td>
<td>1.101</td>
<td>0.4292</td>
<td>0.8435</td>
</tr>
<tr>
<td>ABC</td>
<td>4</td>
<td>0.4217</td>
<td>0.6464</td>
<td>0.2776</td>
<td>0.7830</td>
<td>0.6870</td>
<td>0.2644</td>
<td>0.1638</td>
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<tr>
<td>Error</td>
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<td>---</td>
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<td>---</td>
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<td>---</td>
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*Indicates 5% significance level.

**Indicates 1% significance level.
TABLE V.- COOPER-HARPER RATINGS FOR THE ILS TASK

<table>
<thead>
<tr>
<th>Pilot</th>
<th>Rating for -</th>
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<tr>
<td></td>
<td>Fixed base</td>
</tr>
<tr>
<td>1</td>
<td>A4.0</td>
</tr>
<tr>
<td>2</td>
<td>A3.5</td>
</tr>
<tr>
<td>3</td>
<td>A1.5</td>
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</tbody>
</table>

TABLE VI.- SUBJECTIVE PILOT RATINGS OF MOTION CUES

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<tr>
<th>Cue</th>
<th>Excellent</th>
<th>Rating of pilot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Good</td>
</tr>
<tr>
<td>Pitch</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Roll</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Yaw</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Throttle</td>
<td></td>
<td>1,2,3</td>
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</tbody>
</table>
Figure 1.- Detailed block diagram of washout circuitry.
(b) Braking acceleration circuit.

Figure 1.- Concluded.
Figure 2.- Approach conditions.
Figure 3. - Standard approach condition time histories.
Figure 3. - Continued.
Figure 3. - Continued.
Figure 3. - Continued.
+ Flight Data - Washout Commands

Figure 3. - Continued.
Flight Data - Washout Commands

Figure 3. - Continued.
Figure 3.- Continued.
+ Flight Data - Washout Commands

![Graphs of flight data and washout commands with time on the x-axis and various measurements on the y-axis.](image)

Figure 3. - Concluded.
Figure 4. - Weather front approach condition time histories.
Figure 4.- Continued.
Flight Data - Washout Commands

Figure 4 - Continued.
+ Flight Data - Washout Commands

Figure 4.- Continued.
Figure 4. - Continued.
Figure 4.—Continued.
Figure 4.- Continued.
Figure 4. - Concluded.
Figure 5. - Engine-out approach condition time histories.
Figure 5. - Continued.
Figure 5. - Continued.
Figure 5. - Continued.
Figure 5. - Continued.

- Flight Data
- Washout Commands
Figure 5.—Continued.
Figure 5.- Continued.
Figure 5.- Concluded.
Figure 6.- Accelerometer output documenting turn-around bump in roll.
Figure 7. - Response to an aileron pulse input.

(a) Simulated aircraft response.
(b) Washout response with 0.1 rad/sec frequency.

Figure 7. - Concluded.
Figure 8.- Washout response with 0.3 rad/sec frequency.
Figure 9. - Washout response with 0.5 rad/sec frequency.
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