Solar electric propulsion is certainly not a new concept. Indeed, it has been with us since the early 1900's. But what is new is a growing awareness that solar electric propulsion offers a rather interesting alternative approach to study the earth and its environment and the solar system.

This paper will cover some problems that we face in low-thrust mission analysis. After some preliminary comments about hardware and performance parameters, concern will be devoted to the development of a nominal low-thrust trajectory and to the guidance and navigation problem.

To put things into perspective, we should first discuss the major components of a solar electric propulsion system. It can be broken down into three major subsystems: One is a primary power source, which could be made up of batteries, solar cells, and reactors. Its function is to convert thermal or solar energy into electrical power. The second subsystem is a power conditioner and electrical control, which supplies specified levels of voltage and power to the heaters, valves, and thruster electrodes. In effect, it is an electrical power distribution center. The third subsystem is the engine, in which are included the thruster, fuel tanks, and propellant and control system.

Figure 1 shows a comparison of some performance parameters of both chemical and ion propulsion systems. It can be seen that the chemical systems have a thrust-to-weight ratio ranging from about 10 to $10^{-3}$ g, operating with a specific impulse ($I_{sp}$) in a range of perhaps 90 to 250 seconds. The ion propulsion systems, on the other hand, operate with a thrust-to-weight ratio of about $10^{-4}$ to about $10^{-6}$ g, with a specific impulse ranging from 2000 or 3000 seconds up to as high as about 11,000 seconds.

The nuclear propulsion systems fit in this range of 1 to 10 g. Arc jets also fit into this range and overlap the chemical systems and ion propulsion systems as far as thrust-to-weight ratio is concerned.

The low thrust system then provides a very high total velocity increment, and it does this by providing $\Delta V$ at a very low acceleration over a long period of time at very high specific impulse. The high specific impulse in effect translates to a smaller amount of propellant that has to be carried on board.
Figure 2 shows what might be expected from a low-thrust propulsion system. The terminal mass-to-initial mass ratio is plotted against flight time in days; parameters are given for specific impulse and also for input power to the thruster-to-initial mass ratio ($P/M_0$).

The Solar Electric Rocket Test-C (SERT-C) mission is a study to place a spacecraft into a 3100-km circular orbit and slowly spiral out to geosynchronous orbit. The spacecraft will lift off with roughly 821 kg (1810 lb), with a specific impulse of roughly 3000 seconds for the transfer orbit engines, which are about 30-millipound thrusters, and with an expected flight time of about 290 days. The SERT-C has a $P/M_0$ ratio of roughly 4.2 and a terminal mass-to-initial mass ratio of about 0.85. It will get into synchronous orbit with roughly 703 kg (1550 lb) after lifting off with about 821 kg (1810 lb), a fairly high payload ratio.

At the beginning of prelaunch analysis is the task of generating a nominal trajectory, which is usually optimal in some sense. This is where we encounter our first set of problems.

There are certain phenomena peculiar to low-thrust problems, which must be modeled if we are to simulate a low-thrust trajectory with any semblance of accuracy: The first three items—geopotential, N-body, and solar radiation pressure—are not really peculiar to low-thrust systems but certainly must be included in a nominal trajectory algorithm. We have some experience with ballistic high-thrust-type missions with these three items.

The next item, solar array interactions with the environment, are peculiar to low-thrust systems, since the array is the source of power to all spacecraft systems. A model of the radiation belt is required here, and the development of such a model will be akin to the development of the atmospheric density models we have had over the last 15 years. In both cases, we try to construct models for stochastic processes. Atmospheric scientists may
Figure 2. Parameters of low-thrust propulsion system.

have expertise in the development of radiation belt models, but those of us who are concerned with guidance and control and spacecraft systems are relatively unfamiliar with these models. For instance, what kind of assumptions must be made to develop a working model to be inserted into a trajectory generator algorithm.

Another phenomenon to be considered is the solar array degradation, which is closely related to the radiation belt model. For example, silicon solar cells can withstand a radiation dosage of perhaps $10^{14}$ 1-MeV electrons. However, the radiation that might be experienced is perhaps two orders of magnitude larger than that. Such a dose, $10^{16}$ 1-MeV electrons, will have dire consequences on the available power to the thrusters. In particular, it could degrade the solar array power by as much as 40 to 50 percent of its beginning-of-life power, which, for the SERT-C mission, for example, is about 9 kW. Consequently, we are forced to protect the solar cells with a thin coating of material about 76 to 152 µm (0.003 to 0.006 inch) thick. Even so, the degradation must be modeled and will be a strong function of the thickness of the protective coating and of the type of material used in this coating.

The last item to be considered is shadowing, which has been encountered before with regard to solar radiation pressure. No... we must be concerned about it to determine the solar array and thruster performance during passage through shadow regions, in particular, during the thruster on/off times. The question is how long it takes the system to get up to full power after passage through shadow, and it happens that we do not have an answer at this time. Upon exit from shadow, for example, we know that it is going to take at least 10 minutes to go through a preheat and controlled loop sequence. It is after this time that the thruster will operate at full power, which is expected to be a function of duration in shadow.

The prediction here is the behavior of the a priori thruster biases after restarting the thrusters are such that such biases remain the same after restart, but detailed investigations are highly warranted. The implication is that if such biases do change, then, for
the earth orbiting mission with solar occultation and thruster restarts quite frequent, the
orbit determination process must reestimate these biases. Such frequent estimations could
lead to significant orbit uncertainty, and this, in turn, has serious implications on the
guidance policy.

Having made these comments about the trajectory generation problem, we now turn our
attention to the guidance and navigation aspects. I think it is safe to say that, of the small
amount of work that has been done in the past in low-thrust mission analysis, little has been
devoted to guidance and navigation. The problems here are difficult, and the opportunities
for optimization studies abound.

Environment and degradation models have already been mentioned. The same models which
reside in the trajectory generator algorithm could certainly be used in the guidance and
navigation algorithm. It must be emphasized that, in missions of this type, guidance and
navigation are strongly coupled together because of the presence of a stochastic, continuously
acting force.

Another aspect of guidance and navigation is the thrust vector model, which can be structured
as a constant, as a constant plus noise, as a first- or second-order Markov process, or as a
fully stochastic phenomenon. The first option is quite unrealistic. The fourth option leads
us to extremely difficult mathematical problems, since it forces us to integrate random non-
linear differential equations—nonlinear differential equations are difficult enough. Thrust
magnitude depends on ion beam current, total accelerating potential, mass utilization
efficiency, effective specific impulse, and numerous other parameters, each possessing a bias
and time-varying components, which, when combined, may yield a standard deviation of
about 5 percent of nominal thrust magnitude. But this is just a preliminary estimate. It
could possibly get worse than 5 percent. The pointing error, on the other hand, is a function
of launch vibration, thermal distortion of the grids, and accelerated grid wear, in addition
to improper knowledge or measurement of the thruster misalignments, gyro drift, misalign-
ment, and other parameters. Therefore, modeling the thrust vector as a constant is not
realistic.

The next item is thruster orientation with respect to the solar array. If the engines are not
gimballed, then the optimal orientation of the solar panels will not induce an optimum
orientation of the thrust vector and vice versa. This problem is one that really warrants
many trade-off studies. Even if the engines are gimballed and some freedom for the thrus-
ters is allowed, optimization and trade-off studies must be carried out with respect to the
relative orientation of the thrusters with respect to the solar array at various points of the
mission.

The last point to be discussed is the type and number of observations, two factors strongly
affecting the navigation accuracy, which significantly impacts the guidance policy. One
particular data type is that obtained from accelerometers. Because a low-thrust vehicle is
thrusting over long periods of time, and because small deviations in the thrust direction and
magnitude significantly alter the trajectory over these long periods, it becomes important
to evaluate the influence such data have on navigation error. Implied here is another
problem: When extremely sensitive accelerometers (sensing $10^{-10}$ to $10^{-11}$ g with 2-arc-
second accuracy) become flight-ready, a heavy burden is going to be placed upon attitude
control sensor accuracy and measurement process.

If only one accelerometer is placed on board along the nominal thrust axis, then information
about mass flow rate becomes available (provided thrust magnitude is known fairly well); however, no information about thrust misalignment is available. On the other hand, it is
expected that the navigation problems can be alleviated somewhat by placement of three
highly sensitive accelerometers on board to reduce thrust vector misalignments. Such an
alleviation is contingent upon precision alignment with respect to attitude control sensors.
Thrust direction with respect to the accelerometer axis can then be accurately determined,
depending upon accelerometer accuracy, and referenced to inertial coordinates by the
attitude control system. This is an area where very little work has been done and numerous
studies must be made using not only earth-based data but also onboard navigation sensors.

I would like to close by saying that the overall problem of low-thrust mission analysis is
quite fascinating with new and nontrivial aspects requiring the development of new tech-
nology. This is an area where it is necessary to reconsider a lot of the concepts that might
have been formed in studying ballistic high-thrust-type mission analysis. The problems
are not insurmountable, but they are going to be very time-consuming to overcome.

This paper has discussed some of the mathematical models which are needed. In addition,
there is the orbit determination problem where data types must be evaluated and used in
combination with an optimal filter. Deciding upon a particular filter is not as easy as it
might seem, taking into account the thrust-vector-related biases and time-varying compon-
ents that must be estimated. The strong coupling between navigation and guidance and the
problems it poses to the attitude control system are crucial.

DISCUSSION

VOICE: What kind of funding is available to study these types of problems? They sound
very interesting.

BARBIERI: The funding right now is nebulous, at best. At this time we are not quite sure
where we stand with regard to funding for low-thrust mission analysis.

VOICE: Is anyone in particular interested in pushing this concept further?

HOUNGH: Yes. There is a possibility of a new start for solar electric research and develop-
ment in the 1976 budget, but there is a very low probability of it actually coming into being.
If it does not happen in the 1976 new start, it will probably be continued as a low level
technology-type effort. The primary person who would be supporting it, should it stay at
low level technology, would be Jim Lazar at the Office of Aeronautics and Space Technology.
From our point of view, there is a wide degree of uncertainty as to where we are going right
now. We have to wait until we get a reading from the Office of Management and Budget
on how they feel about it.
Low thrust propulsion appears to have useful application as a means of satellite maneuvering in a strong gravity field. This thesis investigates the usefulness of one possible guidance scheme for such applications by means of a computer simulation. The guidance scheme uses some of the recent optimal trajectory theory applied to a particular class of orbit transfers. These transfers, between inclined circular orbits, are considered because they typify many mission objectives and have a relatively simple optimal solution. The optimal solution is presented here along with a mathematical approach to solving it on a computer. The simulation program, which investigates the effects of an oblate gravity field on the guidance, is also presented. However, oblateness was found to cause relatively small errors and "closed-loop" guidance offered no significant improvement over "open-loop."
Performance characteristics of optimal low thrust rendezvous missions to the comets Giacobini-Zinner, Borrelly, and Tempel (2) with launches in the 1981-1986 time period are discussed.

Also discussed are performance characteristics of optimal low thrust extra-ecliptic missions, including launch declination effects and the importance of optimizing the launch date.
GEOMETRIES DESCRIBING AN ORBITER'S RELATIVE MOTION

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Analytical solutions to a set of modified Euler-Hill equations lead to interesting geometric descriptions of a relative motion. Traces on the displacement and hodograph planes, defining a time history of the motion state, tell much of what can be expected from the solution to any relative motion problem.

The neoclassic solution of Clohessy and Wiltshire (for intercept) has been extended to include effects of forces and general initial values. These results are depicted on both the "local rotating" frame of reference and the companion "inertially oriented" one.

General results for the relative motion state will be described, some special cases will be noted, and examples of uses of these results will be mentioned.
STABILITY OF RELATIVE MOTION

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The equations of the relative motion of two bodies in a given force-field are formulated, and it is shown that the conventional methods of representation lead to instability at rendezvous in the Earth's gravitational field. A method for selecting new dependent and independent variables is offered in order to stabilize the equations of relative motion.