N76-10189

DETERMINATION OF INTRACK ORBITAL POSITION FROM EARTH AND SUN SENSOR DATA

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By intrack orbital error is meant a constant time adjustment, ΔT, that is applied to a set of ephemeris data which is otherwise correct. The ephemeris data may be in the form of an orbit tape or in the form of orbital elements with an associated orbit generator. The ΔT is simply added to the time before the ephemeris routine is accessed. It is implicit here that ΔT is a constant throughout the pass of data that we are considering, where the pass of data is typically a fraction of one orbit.

In terms of Keplerian orbital elements, we are applying an adjustment to one of the six orbital elements, and the other five are assumed to be correct. Why would we want to assume that five of the six orbital elements are correct? Those who are familiar with orbit determination problems understand that there are cases, particularly with a predicted orbit, where the predominant source of error will be an intrack error. We have seen numerical examples from real, predicted orbit tapes compared with later definitive and more accurate orbit tapes, which show that as much as 99.9 percent of the orbit error in a predicted tape can be removed, simply by applying a constant time adjustment throughout an orbit. Table 1 shows an example of intrack orbit error for the Small Scientific Satellite-A (SSS-A), which has an apogee height of 26,500 km, a perigee height of 220 km, and a period of 7 hours and 20 minutes. The error in the predicted orbit tape is determined by comparison with the definitive tape. The predicted tape is accessed at a time about 2 weeks beyond the available data used in the predicted tape.

Table 1
Example of Intrack Orbit Error

	Error in Predicted Tape (km)	Optimum Time Adjustment (s)	Error After Time Adjustment (km)
Near Apogee	123.	59.13	1.83
Near Perigee	586.	59.24	0.50

The primary motivation for determining this intrack adjustment is to improve the accuracy of our attitude determinations, particularly in cases where we are forced to determine an attitude in near real time within an hour of the time the data are received. In these cases, we have to use a predicted orbit tape; we do not have time to wait for a definitive orbit tape to be generated. Potentially, this technique has the capability of improving orbit determination for other users as well or of attaining the same orbit accuracy that we have now, but using less orbit data. That has not yet been done, but we are working on combining the orbit and attitude problems, that is, processing orbit tracking data with earth and sun sensor data in one system and thereby improving both the orbit accuracy and the attitude accuracy with the available data.

Figure 1 explains these attitude sensors, which have been used on at least four different missions–the Radio Astronomy Explorer-2 (RAE-2), the Interplanetary Monitoring Platform (IMP), the Small Scientific Satellite (S^3), and the Atmospheric Explorer (AE)–and which are planned to be used on the Synchronous Meteorological Satellite (SMS) and the Communications Technology Satellite (CTS). What all these missions have in common is an earth sensor telescope of some type, mounted at an angle to the spin axis so that the earth sensor scans a cone; if this cone intersects the earth, then the earth sensor will be triggered. The telescope may be sensitive to either infrared or visible light. In addition, there is a sun sensor on the spacecraft with a slit parallel to the spin axis; when the plane of that slit crosses the sun, the sensor triggers and also measures the angle between the spin axis and the sun direction.

The raw telemetry includes the angle between the spin axis and the sun, (β); the time that the sun sensor slit plane crossed the sun; the times that the earth sensor triggered on and off; and the inertial spin period, as defined by the time between two successive sun sightings.

As shown by figure 2, it is easier to visualize the information if it is considered in terms of the geometric parameters which it defines. It happens that all the information in a single frame of data defines only three angles in space. One of them, of course, is the sun angle, the angle between the spin axis and the sun, because it is measured directly. The other two geometric parameters are the dihedral angles labeled A_{in} and A_{out} in figure 2: A_{in} is the dihedral angle from the plane of the spin axis and the sun to the plane defined by the spin axis and the horizon vector, that is, the vector from the spacecraft to the horizon at the earth-in triggering; A_{out} is the same thing for the earth-out triggering. It is worth noting that these two horizon vectors are unknown quantities. Even if the vector from the spacecraft to the earth is known, the vector from the spacecraft to the horizon crossing points would not be known. The data define two dihedral angles measured with respect to unknown vectors.

We will now examine the resolution of these devices for the missions considered. The data are of course digitized before we receive them, so the value of the least significant bit is a lower limit on the resolution of the sensor. This is not to be confused with the accuracy of the sensor, because there may be systematic errors much larger than the resolution.

For the sun angle, the least significant bit typically has a value ranging from 0.25° to 1°, making the sun sensor a relatively coarse sensor. The earth-in and earth-out measurements are somewhat more sensitive. For these, the resolution depends on the clock rate of the spacecraft in relation to the spin period, and the resolution ranges from 0.01° to 0.7°. We

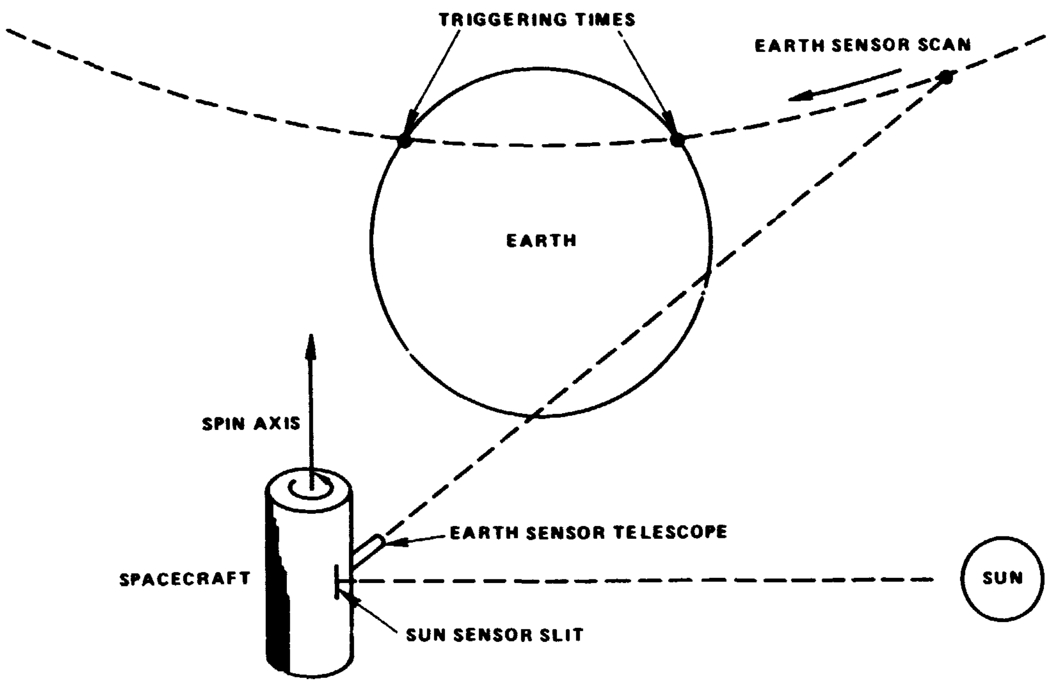

Figure 1. Earth and sun sensors.

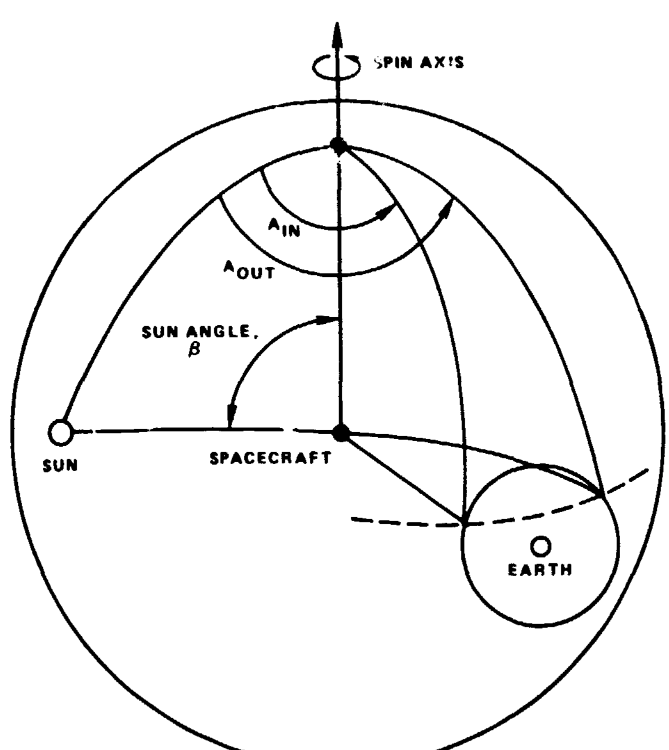

Figure 2. Geometrical observables from earth and sun sensors.

are mainly interested in missions where the resolution is closer to 0.01° because, in that case, the earth sensor is potentially a rather sensitive device, assuming that we can remove systematic errors, which may be as large as 1° or more, and which will be discussed later.

This is what the data look like in a single frame, and now I want to define the unknowns in this problem. The first is the attitude of the spacecraft, because that is what the sensor was put on board to determine. The attitude can be described by a two-element state vector, right ascension (α) and declination (δ), if we assume that the attitude is constant and there is no nutation. I am assuming throughout this presentation that any nutation in the spacecraft is negligible. Therefore, attitude is a two-element state vector. The second unknown is the intrack time adjustment, Δt, which is the primary topic of this paper. We are assuming that we have a source of orbital ephemeris, which is correct, with the possible exception of this intrack time adjustment.

So these are three primary state parameters: two for attitude and one for time adjustment. In the ideal case, those would be the only three unknowns in the problem, and the problem would be relatively simple. In practice, it has been found that, on all of the missions we have supported, there are significant systematic biases in the sensors that have to be removed when the data are processed in order to meet the attitude requirements of the mission, which may be, for example, ±1° for attitude.

Therefore, some additional parameters must be determined from the data. In general, the unknowns include the elevation of the earth sensor with respect to the spin axis, the aximuth of the earth sensor with respect to the sun sensor, and the elevation of the sun sensor with respect to the spin axis, that is, a bias in the measured sun angle. In addition, there is a possible earth sensor triggering threshold or sensitivity error. These earth sensors do not have a very narrow field of view; the field of view may be as wide as 3° in diameter. If the sensor threshold is not accurately known, there may be an uncertainty of several degrees as to where the sensor is pointing at the time it triggers. Finally, there is the possibility of a constant time delay on either the earth-in or the earth-out triggering due to electronic delay between the time the event occurs and the time it is recorded.

In principle, all of these quantities can be measured on the ground before the spacecraft is launched. In practice, they are subject to change. The alignments, of course, could change due to thermal distortion of the spacecraft. It is even more likely that the apparent alignments with respect to the spin axis would change because the spin axis shifts with respect to the geometric body axis, for example, due to uneven fuel usage between fuel tanks. Also, the electronic parameters can change, for example, if the temperature of the electronic components changes.

To model all these sources of error, it is necessary to introduce five additional angular parameters. These are biases with nominal values of zero. There is a bias on the earth sensor elevation and a bias on the earth-in or the earth-out rotation angle. This includes both the effect of an azimuth offset between the earth and sun sensors and a possible difference between the time delays for the earth-in and the earth-out triggerings. There is also a bias on sun angle and a bias on the apparent angular radius of the earth as seen from the spacecraft.

The bias on the angular radius of the earth is intended to correct for the earth sensor triggering threshold. Without discussing the details, it can be shown that, for an earth sensor with a circular field of view, any constant triggering threshold can be exactly compensated for by adding or subtracting a constant bias on the apparent radius of the earth.

Thus, the total number of unknowns includes five angular biases and three primary state parameters, for a total of eight parameters that have to be determined. However, a frame of data includes only three observables: sun angle and earth-in and earth-out angles. Clearly, on the basis of a single frame of data, we could determine, at most, three of these unknowns. To have any hope of determining all eight unknowns, we need more than one frame of data, and the frames have to be independent in some sense. This is where the real problem occurs.

Typically, we do have a large number of frames of data, but the sun angle may be constant throughout the entire block because, as mentioned previously, the sun sensor is a relatively coarse sensor, and the sun angle is changing very slowly. So it is not uncommon for the measured sun angle to be constant throughout the pass, in which case, regardless of the number of frames of data, there is only one actual observable for the sun angle.

The earth-in and earth-out angles are more useful, because they do vary with the spacecraft position. Still, two frames of data taken at nearby positions in the orbit will be redundant. Speaking qualitatively, in order to determine all eight of these parameters, it is clear that we need a significant fraction of an orbit of data in order to have independent observables and not just the three observables that occur in one frame.

We have developed a program called OABIAS, which processes data of the type described and determines a state vector, including the unknowns listed below:

- Attitude (α, δ)
- Earth sensor elevation bias
- Bias on earth-in angle, A_{in}
- Bias on earth-out angle, A_{out}
- Bias on angular radius of earth
- Bias on sun angle, β
- In-track orbit time adjustment, Δt

The program is a standard, weighted least-squares recursive estimator. Any number of the above listed parameters can be fixed at constant values and not determined.

The program works as a standard recursive estimator: the state vector is used to predict the observables, the residual is computed for each observable, and the partial derivatives of each observable with respect to each element in the state are computed. Then the residuals and partial derivatives are used to update the state vector. The partial derivatives can be computed analytically for every case except the case of interest here, the intrack orbital time adjustment.

If the partial derivative of any arbitrary observable is considered with respect to the time adjustment, Δt, it is equal to the partial derivative of the observation with respect to the spacecraft position vector, $\vec{R}$, multiplied by the partial derivative of the spacecraft position with respect to Δt:

$$\frac{\partial\,(\text{observation})}{\partial \Delta t} = \frac{\partial\,(\text{observation})}{\partial \vec{R}} \cdot \frac{\partial \vec{R}}{\partial \Delta t}$$

$$= \frac{\partial\,(\text{observation})}{\partial \vec{R}} \cdot \vec{V}$$

The derivative of the spacecraft position with respect to Δt is one that cannot be computed analytically, because we do not have an analytical expression for spacecraft position as a function of time. However, it is not necessary, because we can get the velocity ($\vec{V}$) of the spacecraft from the orbit tape.

In practice then, we analytically compute the derivation of each observation with respect to $\vec{R}$, then multiply that by the velocity vector obtained from an orbit tape. The important point here is that this method is not restricted to any particular type of orbit. We are not assuming, for example, a Keplerian orbit. Any orbit that can be described by an orbit tape can be handled correctly using this technique.

Figure 3 shows some results obtained using simulated data for the Communications Technology Satellite, which is scheduled to be launched next year. The data were simulated for transfer orbit, with a perigee of 190 km and an apogee of 36,000 km. The attitude in this case is pointing 49° below the plane of the orbit; the earth sensor is an IR sensor mounted at 85° from the spin axis. These facts together imply that the earth sensor will scan the earth only during the indicated portions of the orbit. For the rest of the orbit, the earth sensors will miss the earth; there will be no useful data from the earth sensor for those periods.

Both of these sections of the orbit were simulated and the data were combined. The data includes 100 frames evenly spaced over 240 minutes over both of those segments of the orbit–40 minutes near perigee and 200 minutes near apogee. Gaussian noise of 0.012° is applied to the earth rotation angles, the earth-in and earth-out angles. That corresponds to the clock rate expected for the spacecraft.

We were attempting to determine the complete eight-element state vector, so biases of 1° were applied to each of the angular parameters that describe the bias on the sun angle, the earth-in and earth-out azimuth angles, the earth sensor elevation, and the apparent angular radius of the earth. For the time adjustment, an error of 60 seconds was applied to the ephemeris data, which corresponds to a very large, 6° error in spacecraft position at perigee, which would make the perigee data virtually useless for attitude determination without correcting for it. But the time adjustment corresponds to an error of only 0.15° true anomaly at apogee.

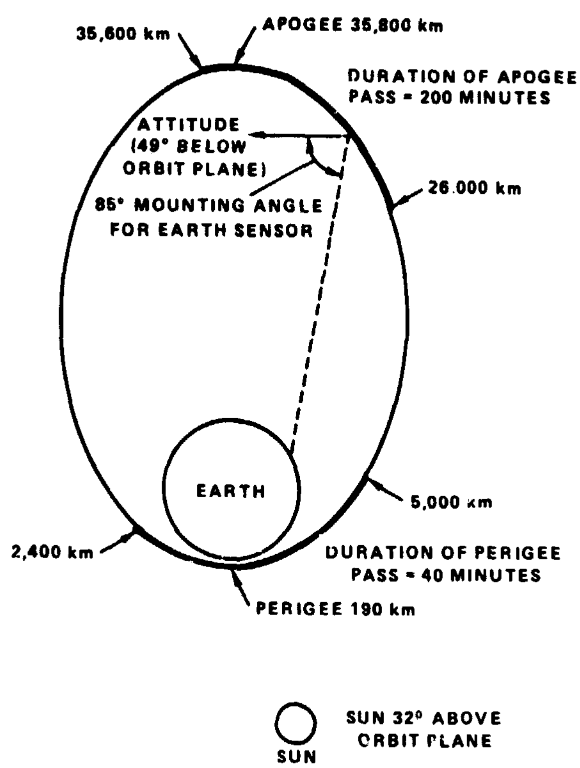

Figure 3. Example of position determination with simulated data for the Communications Technology Satellite.

The results for these simulated data are shown in table 2. The biases are initially estimated at zero, because they are unknown quantities; the initial attitude was obtained from a deterministic processing of the data, before correcting for biases. It can be seen that this initial attitude is more than 2° off, which would violate mission constraints for this mission, since we have a 1° attitude accuracy requirement. The final result from the program shows that all of the unknowns are determined to an accuracy of 0.05°, and the intrack time adjustment is determined to an accuracy of one-half second. This indicates that, first of all, the program works. Secondly, it means that the problem is feasible. That is, we really should be able to determine all eight of these state parameters, assuming that we have a sufficient amount of data and that there are no substantial systematic errors that have not been considered in the state vector.

This brings us to the case of the real data. We have processed real data from four spacecraft—IMP, S^3, RAE-2, and AE—but the results cannot be presented in this form because the true state is not known for any of these spacecraft. In fact, since the attitude changes, or is changed, from one day to the next, we do not generally have more than one pass of data to examine to define a given state vector.

Table 2
OABIAS Results*

Parameter	True State	Initial Estimate	Final Result
Attitude α	330.00	329.50	330.02
Attitude δ	-21.75	-23.88	-21.80
Earth Sensor Elevation Bias	-1.00	0.0	-1.01
Earth-in Azimuth Bias	-1.00	0.0	-0.99
Earth-out Azimuth Bias	-1.00	0.0	-0.96
Bias on Angular Radius of Earth	1.00	0.0	1.01
Sun Angle Bias	-1.00	0.0	-0.99
Intrack Time Adjustment	-60.00	0.0	-59.59

* All values are in degrees, except intrack time adjustment, which is in seconds.

Thus, only three things can be said about the real data: either it can or cannot be fitted, or we can find an infinite number of state vectors which would fit the data equally well to within the noise level. It happens that this third cause is not uncommon for the missions that we examined. The reason is that, when trying to determine an eight-element state vector on a small section of an orbit, it is to be expected that the entire state vector will not be determinable.

However, we have generally found that the results are consistent with what we would expect, based on simulation. That is, where there are enough data to find the state vector, it can be determined uniquely. Where there is not enough information, we can determine any number of state vectors that would fit the data. We have not, as a rule, encountered the other problem, that of not being able to find any state vector that fits the data; this would indicate the presence of some systematic error that we have not modeled.

It should be pointed out that, if the misalignment parameters and biases could be eliminated and the state vector thereby reduced from eight elements to three elements, it would obviously be a simpler problem. It would take much less data to determine the state. In fact, since there are three observables in a single frame of data, and as there would be only three unknowns in that case, the time adjustment and the attitude could be determined, based on just a single frame of data. But, based on our experience, it is not feasible to ignore all of these systematic bias parameters.

DISCUSSION

VOICE: Is this technique being used operationally?

SHEAR: It has been applied operationally to all four spacecraft mentioned; currently, the operational work is concentrated on AE-C. It has improved the attitude determination accuracy for those missions. It is not used to process every pass of data. It is used on the initial passes of data in the mission to try to determine the biases that will be used throughout the mission.

VOICE: Are the simulation results that you've shown for the same static attitude estimator? When you generated data, did you use the model you have in your estimator?

SHEAR: That's right. It's the same model. There are no systematic errors that aren't accounted for.

VOICE. There are no attitude dynamics?

SHEAR: Correct, there are no attitude dynamics.

VOICE: In the case of the real data, the accuracy of your intrack time adjustment could be checked against orbit tracking data, couldn't it?

SHEAR: It is possible to get an independent confirmation of the intrack orbit error. I think our problem has been more often that we can't determine all eight state parameters including the time adjustment; consequently, if the time adjustment does not agree with the orbit data, we don't know whether we've got the right answer for one of those biases or not.

I think that it is feasible to determine the intrack time adjustment on the S^3 spacecraft. The S^3 has an orbit that is fairly similar to the orbit I just showed you for the simulated example. In that case, I think it's feasible, but we haven't processed S^3 data extensively yet.

VOICE: I have worked on some of the problems of bias determination and the biases are usually so highly correlated with what you're trying to measure that they can't be separated. Is your simulation realistic?

SHEAR: Well, there are different geometry cases to be considered. I could point out, for example, a circular orbit. For any circular orbit, we cannot determine this complete eight-element state vector, regardless of the attitude of the spacecraft, the sensor mounting angles, or the amount of data. It can be proven that there is a perfect correlation between the in-track error and the sensor misalignments; we can find an infinite number of state vectors that will fit the data.

Referring back to figure 3, if we were to delete either one of these passes of data, I don't think it would be feasible to do the problem; the complete eight-element state vector could not be determined with either pass alone. Of course, if we could reduce the state vector to fewer elements, it could be done. But we were not able to solve for all eight elements, even on simulated data, with just one of those sections.

VOICE: As a practical problem, can you get track-in data when you're at perigee?

SHEAR: Probably not directly at perigee, for such a low perigee. But it should be possible to get most of these data. The S^3 spacecraft, which has the same orbit except for a slightly lower altitude, frequently obtains a pass of data which covers most of what is indicated here as the perigee pass.

VOICE: For your simulated data, where did the sun lie with respect to the orbit flight?

SHEAR: The sun is shown in figure 3, but I didn't mention it. The sun is 32° above the orbit plane, and it's roughly in the direction shown in the figure.