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Inviscid Transonic Flow Computations with Shock Fitting

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Summary

First- and second-order numerical procedures for calculating two-dimensional transonic flows that treat shock waves as discontinuities are discussed. This short communication illustrates their application to a simple but non-trivial problem for which there are limited theoretical results.

Introduction

Some of the numerical difficulties that arise in inviscid transonic flow computations occur because the solution that is being sought is discontinuous and the numerical procedures often employed approximate the discontinuities by steep gradients; thus relatively small mesh sizes are required if dissipative and dispersive truncation errors are to be acceptable. Further, the difference equations used, as Murman has pointed out [1], must be chosen with care if entities that are conserved by the basic equations across discontinuities are to be conserved by the finite difference scheme.

We believe there are important advantages to computing such flows with numerical procedures that treat discontinuities as such, provided this can be done without major programming complexity. Moretti [2] has been the most constant advocate of doing precisely this, and he has computed a number of steady and unsteady flows using procedures that are essentially tailored to hyperbolic problems. Salas [3] has recently computed supersonic duct flows using somewhat analogous techniques, and Yu and Seebass [4] examined a transonic flow problem where the discontinuity could be traced from a boundary where the flow was hyperbolic.

The use of similar procedures for mixed flows, with a minimum of additional computational complexity has been pursued by Hafez and

Cheng [5], and by us. Some of our results are the subject of this brief communication.

Transonic Flow Past a Wedge

We have applied various numerical procedures to calculate flows past simple two-dimensional lifting airfoils; we illustrate one of them here for a very simple, but non-trivial, problem that models slightly subsonic flow past a wedge. The mathematical problem associated with the small perturbation approximation has been solved analytically for a sonic free stream, and we know the first derivative of the drag with Mach number evaluated at sonic conditons (Guderley and Yoshihara [6]; Liepmann and Bryson [7]). With the use of modern computers to evaluate the hypergeometric function, it would seem that precise results could be obtained for a range of Mach numbers.

The correct hodograph formulation of this problem was tackled numerically by Yoshihara [8] nearly two decades ago. While these results may not be definitive enough to validate the computation of the supersonic domain, they are accurate in their predictions of the pressure coefficient on the nose of the wedge, and consequently, the drag.

The mathematical formulation of this problem is simple and well-known. We need to solve

$$(\kappa + \phi_\xi)\phi_{\xi\xi} - \phi_{\eta\eta} = 0 \qquad \text{subject to}$$

$$\phi_\eta(\xi,0) = 0 \quad \text{for } \xi \notin [0,1], \quad \phi_\eta(\xi,0) = 1 \quad \text{for } \xi \in [0,1]. \tag{1}$$

The solution must behave as

$$\phi \sim \frac{\xi}{2\pi\sqrt{\kappa}(\xi^2 - \kappa\eta^2)} \left[1 + \frac{1}{2} \iint_{-\infty}^{\infty} \phi_\xi^{\,2}(\xi,\eta)\,d\xi\,d\eta \right]$$

for $\xi^2+\eta^2 \to \infty$. This expression is useful for supplying boundary data on a finite domain used in numerical computation. For small values of κ, i.e., for near sonic flow, a less severe asymptotic representation must be used. As $\kappa \to 0$ we can use the results of Reference 5. Here ϕ is proportional to the perturbation velocity

potential, κ is the usual transonic similarity parameter $(M_\infty^2 - 1) / [(\gamma+1)\,\delta]^{2/3}$, δ the wedge half angle and ξ and η the usual non-dimensional coordinates. The jump relations that follow directly from (1) and the irrotationality condition, are

$$[\kappa + \tfrac{1}{2}(\phi_\xi + \hat{\phi}_\xi)](\phi_\xi - \hat{\phi}_\xi)^2 = (\phi_\eta - \hat{\phi}_\eta)^2,$$

$$d\eta/d\xi = \pm\,[\kappa + \tfrac{1}{2}(\phi_\xi + \hat{\phi}_\xi)]^{-\frac{1}{2}}. \qquad (2)$$

Here ^ denotes values downstream of the shock wave. As Oswatitsch [9] noted many years ago, the wave drag of a body is directly related to the entropy produced by these waves. For this problem, the normalized drag coefficient may be determined, to lowest order, by the alternate expressions

$$\tilde{C}_D = \phi(1,0) - \phi(0,0) = (a_*/6U)\int(\phi_\xi - \hat{\phi}_\xi)^3 d\sigma$$

where a_* is the critical speed, U the free stream velocity, and $d\sigma$ an element of shock surface. For $\kappa \to 0$, $C_D \to 1.75 + 2\kappa$.

Numerical Procedures

For these calculations we have used a first-order accurate conservative scheme to calculate the solution to Equation (1). In conjunction with this scheme we have also used shock fitting. Because we anticipated treating the shock as a discontinuity and satisfying the jump relations across it, we were not initially concerned with using conservative differencing which has been shown to be essential across shock waves[1]. Our results indicated, however, that our shock-fitting procedure will not converge to the correct result if it is introduced into a converged non-conservative calculation.

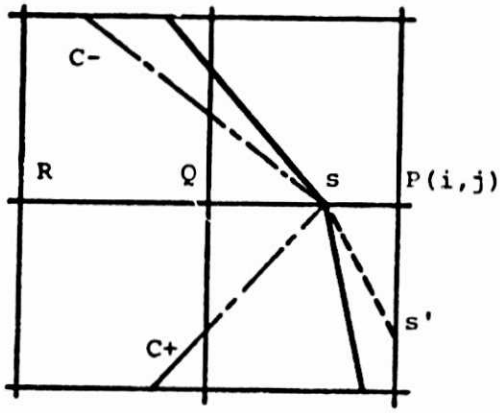
Equation (1) is of mixed type, and it is either hyperbolic or elliptic depending on whether $\kappa + \phi_\xi$ is positive or negative. Type dependent difference schemes, first introduced by Murman and Cole [10], have proved effective in solving such equations. To determine the type of the equation $\kappa + \phi_\xi$ is computed both by the central and by backward difference approximations. These results

are then used to select proper difference approximations for the derivatives. The only difference between the present scheme and the fully conservative scheme [1] is in the choice of the sonic point approximation. The fully conservative scheme failed because of the poor approximation of the differential equation at the sonic point caused by insisting upon flux conservation. Along the first characteristic of the expansion fan the first term in the differential equation (1) must precisely balance the second term as $\kappa + \phi_\xi \to 0$. Consequently, we have found it necessary to adopt a non-conservative approximation of the sonic point. It is obvious, of course, that at the shock point the flux conservative form must be used if the shock is not treated as a discontinuity. There the differential equation is of little consequence, but flux conservation is.

Shock Fitting

When shock waves are embedded in sharp gradients, fairly refined mesh spacing may be required if we want accurate results from a flux conservative difference approximation. An alternative, as we have mentioned, is to treat the shock as a discontinuity in the later stages of such calculations. For some problems this may prove to be essential; for others, it may be desirable, and for many simply superfluous. It may prove particularly beneficial for shock waves that originate with an elliptic behavior downstream and then undergo a transition to hyperbolic behavior there. Shock fitting also provides an easy mechanism for calculating the lowest order wave drag by integrating the entropy rise across the shock. For flows in which the flow behind the shock is elliptic, flux conservative calculations provide a reasonable definition of the shock and also can be used to calculate the entropy rise. For these calculations we have not introduced our shock-fitting procedures until we have obtained a reasonably well-converged first-order result. The initial shock position is determined by interpolating the sonic position in the supersonic to subsonic transitions, and the shock point (s) are then treated as regular computational points (see sketch). The flow properties ahead of the shock can be

determined either by extrapolation from the upstream conditions at Q and R, or by using the characteristic relations along C ±. For these calculations the simple extrapolation method was used. Behind the shock $\hat{\phi}_\eta$ is extrapolated by the usual difference method using the old value at P and $\hat{\phi}_\xi$ is obtained from the jump relation (2). Should the shock slope extrapolation provide a shock intersection at s', as indicated by the dotted line, then the value of ϕ is fixed at s' by using the values of $\phi = \hat{\phi}$, $\hat{\phi}_\xi$ and $\hat{\phi}_\eta$ at s. At each stage of the calculation we correct the position of the shock by the simple procedure



$$\xi_s^{n+1} = \xi_s^n + k\Delta\xi\,(\phi_\xi^{n+1} - \phi_\xi^n)_s. \qquad (3)$$

Various values of k have been tried; values near one seem to be the most satisfactory. The rationale for (3) is simple: if the flow ahead of the shock has increased in speed, i.e., $\phi_\xi^{n+1} - \phi_\xi^n > 0$, (3) allows the shock to be swept downstream and flow properties are re-calculated using the new shock position. Such corrections are repeated with each iteration until the changes in ϕ are judged sufficiently small. We examined a simple one-dimensional problem (discussed in Bauer et al. [11]) using the procedures described above. We found that the shock approaches the exact location independent of the initial guess for its location. The computation time for this simple model was comparable to that of an equivalent shock point operator scheme, and the accuracy slightly better.

Hafez and Cheng [8] have pursued somewhat similar calculations. They calculated the transonic flow over a parabolic-arc airfoil using a shock-fitting scheme. In their scheme the shock is located

by a numerical procedure that detects a jump in ϕ_ξ, and the difference approximation at the point following the shock is constructed using the jump relation. Such a scheme may be inherently less accurate than ours; it is also less complex and may be less sensitive to the initial data used. The results they report are in good agreement with the fully conservative results of Murman [1] and with our results. If we start with a converged flux conservative calculation and introduce shock fitting we find minor changes in the shock shape and the local shock slope. For flow past curved profiles we find a behavior that closely approximates the Oswatitsch-Zierep [12] singularity expected behind the shock.

Results and Discussion

The supersonic portion of the flow field calculated using a first-order shock-fitting scheme is depicted in Figure 1 for $\kappa = -0.5$ and a mesh spacing that corresponds to twenty points on the wedge nose. The total computational region was $-2 \leq \xi \leq 4$ and $0 \leq \eta < 6$. A weak shock seems to originate near the wedge corner but it has been smeared out by truncation errors. There is further evidence of such a shock in the second-order calculations which we have carried out. Figure 2 depicts the pressure coefficient $C_p^* = -2(\kappa + \phi_\xi)$ for three values of η. Figure 3 depicts the shock image in the $(q^*, \bar{\theta})$-plane were $q^* = \kappa + \phi_\xi$ and $\bar{\theta}$ is the normalized local flow deflection angle θ/δ. Near the tip of the shock, the shock is of the supersonic-supersonic type.

The normalized drag coefficient $\tilde{C}_D$ for $\kappa = -0.5$ has been calculated using various schemes. The shock-fitting algorithm gives a normalized drag coefficient of 0.656 when evaluated by pressure integral, and 0.561 when evaluated by entropy production. The values of $\tilde{C}_D$ for non-conservative first- and second-order approximations are 0.677 and 0.723 respectively. Until we are able to carry out calculations with more refined mesh spacings and at smaller values of κ it makes little sense to compare them to the result $\tilde{C}_D \sim 1.75 + 2\kappa$ for $\kappa \to 0$. Nevertheless, it is probably reasonable to conclude that this theoretical value underestimates the drag (see [8]), and hence so do the calculations we have reported here.

As noted earlier, we have applied the algorithm outlined here to flows past airfoils. The small perturbation velocity distribution for a flux conservative calculation is compared in Figure 4 to that obtained using shock fitting for the same computational grid. Aside from the better definition of the singular behavior of the pressure gradient behind the shock and minor changes in shock shape (not shown), the results are comparable.

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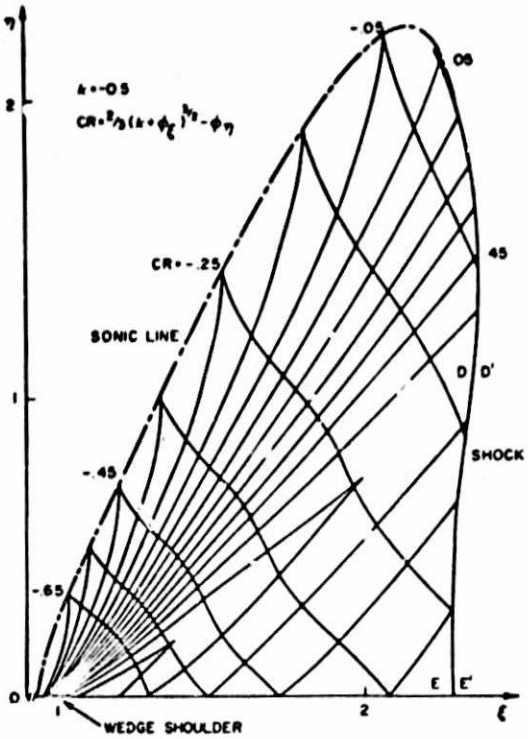

Fig. 1 Sonic region and characteristics.

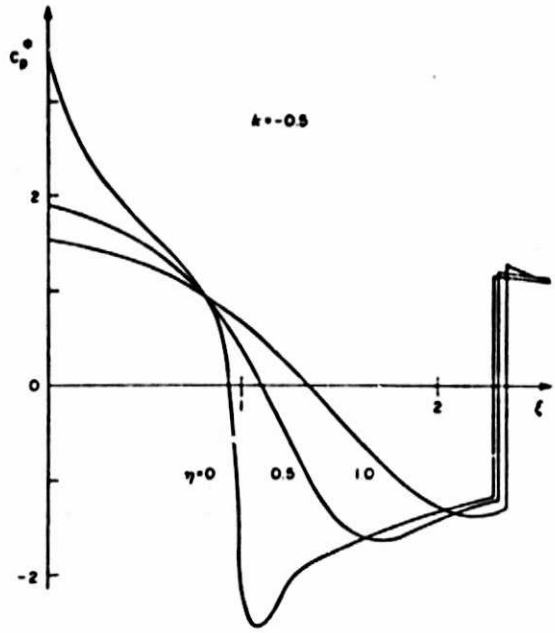

Fig. 2. Pressure coefficient versus ξ for three values of η.

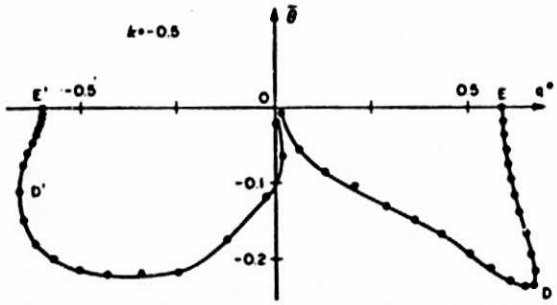

Fig. 3. Shock polar showing supersonic conditions behind tip of the shock.

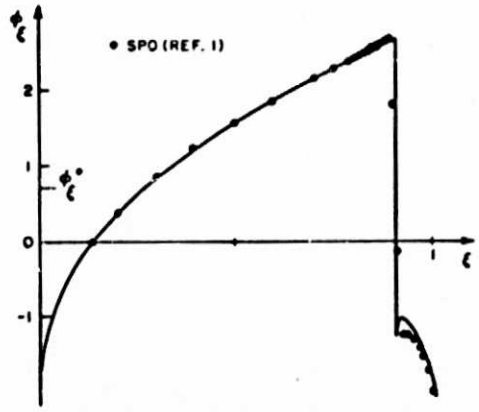

Fig. 4. Velocity distribution along the chord of a 6% thick parabolic arc for $M_\infty = 0.909$ with and without shock fitting.