Preface

The work described in this report was performed by the Guidance and Control Division of the Jet Propulsion Laboratory.
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Abstract

This report describes three computer subroutines designed to solve the vector-dyadic differential equations of rotational motion for systems that may be idealized as a collection of hinge-connected rigid bodies assembled in a tree topology, with an optional flexible appendage attached to each body. Deformations of the appendages are mathematically represented by modal coordinates and are assumed small. Within these constraints, the subroutines provide equation solutions for (1) the most general case of unrestricted hinge rotations, with appendage base bodies nominally rotating at a constant speed, (2) the case of unrestricted hinge rotations between rigid bodies, with the restriction that those rigid bodies carrying appendages are nominally nonspinning, and (3) the case of small hinge rotations and nominally nonrotating appendages, i.e., the linearized version of case 2. Sample problems and their solutions are presented to illustrate the utility of the computer programs. Complete listings and user instructions are included for these routines (written in Fortran), which are intended as general-purpose tools in the analysis and simulation of spacecraft and other complex electromechanical systems.
Attitude Dynamics Simulation Subroutines for Systems of Hinge-Connected Rigid Bodies With Nonrigid Appendages

I. Introduction

Equations of motion which characterize the small, time-varying deformations of an elastic appendage attached to a rigid body experiencing arbitrary motions have been derived in detail for distributed-mass finite element models in Ref. 1, and for discrete mass models in Ref. 2. With the general structure of the appendage deformation equations established in these references, coordinate transformations are developed in Refs. 1 and 3 in order to allow representation of the elastic appendage in terms of a set of truncated modal coordinates far fewer in number than the original set. In Ref. 4, additional equations of motion are derived to describe the rotations of typical bodies in a system of hinge-connected rigid bodies arranged as a topological tree, with optional arbitrary nonrigid appendages attached to each rigid body in the system. In this respect, the results of Hooker in Ref. 5 and those of Ref. 4 are parallel.

It is the purpose of this report first to draw together the appendage equations and the equations describing rigid body motions of the tree system, assuming that some or all of the rigid bodies carry nonrigid appendages, and to derive a consistent and detailed set of system dynamical equations suitable for digital computer solution. Secondly, it is the purpose here to present general-purpose computer subroutines capable of solving the resulting system equations of rotational motion, and to demonstrate their utility and applicability to a wide class of spacecraft.
In generating the equations of motion for the hinge-connected tree of rigid bodies with nonrigid appendages, two specific formulations are obtained. The first formally constrains\(^1\) appendage base motion to small deviations from a nominal constant angular velocity in inertial space, thus allowing appendage rotation but with only small deviations from a constant rate of spin. The second formulation formally permits no spin and constrains appendage base motion to small deviations from a nominally zero angular velocity (and acceleration) in the inertial frame. However, both formulations permit otherwise unrestricted motions of the system rigid bodies consistent with the fundamental assumption of small appendage deformations from some nominal state. Computer subroutines (written in Fortran) are described which solve the equations produced by each of these approaches. In addition, a third subroutine is presented which solves the completely linearized equations for the nonrotating case, under the assumption that all rigid body rotations and their derivatives are small.

The computer programs are direct descendants of those described in Refs. 6 and 7, which are applicable to the hinge-connected rigid body tree without nonrigid appendages. All of the programs are designed to calculate the angular accelerations for every rigid and nonrigid body in the system but do not perform numerical integration. Thus, the routines are intended as general-purpose tools, to be called into action by the user's own particular simulation language, whether this be CSSL, CSMP, MIMIC, or some "homemade" variety. Each of the routines allows the user to prescribe the motion of any rigid body in the system rather than allow it to be calculated, a feature often useful for eliminating unwanted dynamics or for "rigidizing" certain joints in sensitivity studies.

II. Unrestricted Systems

A. Mathematical Model

Any problem of dynamic analysis must begin with the adoption of a mathematical model representing the physical system of interest. In what follows, it is assumed that the model consists of \(n + 1\) rigid bodies (labeled \(\ell_0, \ldots, \ell_n\)) interconnected by \(n\) line hinges (implying no closed loops and, hence, tree topology), with each body containing no more than three orthogonal rigid rotors, each with an axis of symmetry fixed in the housing body, and moreover with the possibility of attaching to each of the \(n + 1\) bodies a nonrigid appendage, with appendage \(\alpha_k\) attached to body \(\ell_k\).

If the actual connection between two massive portions of the physical system admits two (or three) degrees of freedom in rotation, then the analyst simply introduces one (or two) massless and dimensionless imaginary bodies into his model (as though they were massless gimbals). Since the number of equations to be derived here matches the number of degrees of freedom of the system, no price is paid in problem dimension by the introduction of imaginary bodies.

Each combination of a rigid body and its internal rotors and attached flexible appendage comprises a basic building block, referred to here as a substructure;

\(^1\) Deviations from nominal appendage base motion are treated as small in the sense that their products with appendage deformations are ignored, but nonlinear terms in these base motion deviations alone are retained. Thus, there is a formal limitation to small base motion deviations from nominal, but in practical applications, substantial deviations are accommodated quite satisfactorily.
thus, there are \( n + 1 \) substructures in the total system, so labeled that \( \sigma_k \) encompasses \( \delta_k, \alpha_k, \) and any rotors in \( \delta_k \).

### Definitions and Notations

Definitions and notational conventions are as follows (see Fig. 1):

**Def. 1.** Let \( n \) be the number of hinges interconnecting a set of \( n + 1 \) substructures.

**Def. 2.** Define the integer set \( \mathcal{B} \equiv \{0, 1, \ldots, n\} \).

**Def. 3.** Define the integer set \( \mathcal{P} \equiv \{1, \ldots, n\} \).

**Def. 4.** Let \( \delta_0 \) be a label assigned to one rigid body chosen arbitrarily as a reference body, and let \( \delta_0, \delta_1, \ldots, \delta_n \) be labels assigned to the rest of the rigid bodies in such a way that if \( \delta_j \) is located between \( \delta_0 \) and \( \delta_k \), then \( 0 < j < k \).

**Def. 5.** Define dextral, orthogonal sets of unit vectors \( b_1^k, b_2^k, b_3^k \) so as to be imbedded in \( \delta_k \) for \( k \in \mathcal{B} \), and such that in some arbitrarily selected nominal configuration of the total system, \( b_a^k = b_a^\prime \) for \( a = 1, 2, 3 \) and \( k, j \in \mathcal{B} \).

---

**Fig. 1.** Definitions for the \( k \)th substructure, with \( j < k \)
Def. 6. Define
\[ \{b^*\} = \begin{pmatrix} b_1^k \\ b_2^k \\ b_3^k \end{pmatrix}, \quad k \in \mathcal{B} \]

Def. 7. Define \( \{i\} \) as a column array of inertially fixed, dextral, orthogonal unit vectors \( i_1, i_2, i_3 \).

Def. 8. Let \( C \) be the direction cosine matrix defined by
\[ \{b^0\} = C \{i\} \]

Def. 9. Let \( \omega^0 \equiv (b^0)^T \omega \) be the inertial angular velocity vector of \( \mathcal{B}_\infty \) so that \( \omega^0 \) is the corresponding \( 3 \times 1 \) matrix in basis \( \{b^0\} \).

Def. 10. Let \( c_k \) be the mass center of the \( k \)th substructure, \( k \in \mathcal{B} \).

Def. 11. Let \( p_j \) be a point on the hinge axis common to \( \mathcal{B}_k \) and \( \mathcal{B}_j \) for \( j < k \) and
\[ k \in \mathcal{B} \]

Def. 12. Let \( p_j^0 \) be the position vector of the hinge point connecting \( \mathcal{B}_j \) and \( \mathcal{B}_k \) from the point \( o_k \) occupied by \( c_k \) when the \( k \)th substructure is in its nominal state.

Def. 13. Let \( c^k \) be the position vector from \( c_k \) to \( o_k \).

Def. 14. Let \( p^k \) be the position vector to \( c_k \) from the system mass center \( CM \).

Def. 15. Let \( X \) be the position vector to \( CM \) from an inertially fixed point \( \mathfrak{S} \), and let \( X = X \cdot \{i\} \).

Def. 16. Let \( \mathfrak{M}_k \) be the mass of the \( k \)th substructure, for \( k \in \mathcal{B} \).

Def. 17. Let \( \{p\}^k \) be a generic position vector from \( o_k \) to any point in the \( k \)th substructure.

Def. 18. Let \( Q_k \) be a point common to rigid body \( \mathcal{B}_k \) and flexible appendage \( \mathcal{A}_k \).

Def. 19. Let \( R^k = (b^k)^T R \) be the position vector fixed in \( \mathcal{B}_k \) locating \( Q_k \) with respect to \( o_k \).

Def. 20. Let \( (r)^k = (b^k)^T (r)^k \) be a generic symbol such that \( R^k + (r)^k \) locates a typical field point in \( \mathcal{A}_k \) with respect to \( o_k \) when the flexible appendage is in some nominal state (perhaps undeformed). For a discretized appendage \( \mathcal{A}_k \), let \( (r)^j = (b^k)^T (r)^j \) locate the \( j \)th node in the nominal state.

Def. 21. Define the generic deformation vector \( (u)^k \) in such a way that
\[ (p)^k \equiv R^k + (r)^k + (u)^k \]

and
\[ (p)^k = R^k + (r)^k + (u)^k \]

\(^2\) Superscripts on generic symbols such as \( p, r, \) and \( u \) will be omitted when obvious, as when the symbol appears within an integrand of a definite integral.
For a discretized appendage \( \alpha_k \), let \( (u^r)^k = (b^k)^T (u^r)^k \) be the deformation vector for node \( s \).

**Def. 22.** Let \( g^k \equiv (b^k)^T g^k \) be a unit vector parallel to the hinge axis through \( \alpha_k \).

**Def. 23.** For \( k \in \mathfrak{B} \), let \( \gamma_k \) be the angle of a \( g^k \) rotation of \( \delta_k \) with respect to the body attached at \( \alpha_k \). Let \( \gamma_k \) be zero when \( b^k = b^0(\alpha = 1, 2, 3; j, k \in \mathfrak{B}) \).

**Def. 24.** Let \( J^k \equiv (b^k)^T J^k (b^k) \) be the inertia dyadic of the \( k \)th substructure for \( o_k \), so that \( J^k \) is time-variable by virtue of deformations.

**Def. 25.** Let \( F^k \equiv (b^k)^T F^k \) be the resultant vector of all forces applied to the \( k \)th substructure except for those due to interbody forces transmitted at hinge connections.

**Def. 26.** Let \( T^k \equiv (b^k)^T T^k \) be the resultant moment vector with respect to \( c_k \) of all forces applied to the \( k \)th substructure except for those due to interbody forces transmitted at hinge connections.

**Def. 27.** Let \( \tau_k \) be the scalar magnitude of the torque component applied to \( \delta_k \) in the direction of \( g^k \) by the body attached at \( \alpha_k \).

**Def. 28.** Let \( F \equiv \sum_{k \in \mathfrak{B}} F^k = (b^0)^T F \) be the external force resultant for the total system.

**Def. 29.** Define the scalar \( \epsilon_{sk} \) such that for \( k \in \mathfrak{B} \) and \( s \in \mathfrak{B} \)

\[
\epsilon_{sk} \equiv \begin{cases} 
1 & \text{if } s \text{ lies between } \delta_0 \text{ and } \delta_k \\
0 & \text{otherwise} 
\end{cases}
\]

(The \( n(n + 1) \) scalars \( \epsilon_{sk} \) are called path elements.)

**Def. 30.** Define \( \mathfrak{M} \equiv \sum_{k \in \mathfrak{B}} \mathfrak{M}_k \), the total system mass.

**Def. 31.** Let \( C^j \) be the direction cosine matrix defined by \( (b^j) = C^j (b^r) \), \( r, j \in \mathfrak{B} \). (Note that in the nominal state, \( C^j = U \), the unit matrix.)

**Def. 32.** Let \( N_{kr} \) denote the index of the body attached to \( \delta_k \) and on the path leading to \( \delta_r \), and let \( N_{kk} \equiv k \). (These are the network elements.) For notational simplicity, use \( N_k \) for \( N_{ko} \).

**Def. 33.** For \( r \neq k \), let \( L^k \equiv p^k b^r \), and let \( L^k \equiv 0 \).

**Def. 34.** Define \( D^{jk} \equiv \sum_{j \in \mathfrak{B}} L^j \mathfrak{M}_j / \mathfrak{M} \) for \( k \in \mathfrak{B} \).

**Def. 35.** Let \( b_k \) be a point fixed in \( \delta_k \) such that \( D^{jk} \) is the position vector of \( o_k \) with respect to \( b_k \). (This point \( b_k \) is called the barycenter of the \( k \)th substructure in the nominal state.)

**Def. 36.** Define \( (b^k)^T D^k = b^k \equiv D^k + L^j \) for \( k, j \in \mathfrak{B} \).

**Def. 37.** Define the dyadic

\[
K^k \equiv \sum_{r \in \mathfrak{B}} \mathfrak{M}_r (D^{kr} \cdot D^{kr} U - D^{kr} D^{kr})
\]

---

3 For notational brevity, the set \( \mathfrak{B} - (k) \) is designated \( \mathfrak{B} - k \).
where \( \mathbf{U} \) is the unit dyadic, and define the corresponding matrix \( K^k \equiv (b^k) \cdot K^k \cdot (b^k)^T \).

**Def. 38.** Define

\[
\Phi^{kk} \equiv K^k + J^k \quad \text{and} \quad \Phi^{ik} \equiv (b^i) \cdot \Phi^{kk} \cdot (b^i)^T
\]

**Def. 39.** Define

\[
\Phi^{ij} \equiv - \mathbf{M} (D^j, \cdot \cdot \cdot D^j \cdot \mathbf{U} - \cdot \cdot \cdot D^j \cdot \mathbf{D}^j)
\]

with

\[
(b^i) \cdot \Phi^{ij} \cdot (b^j)^T = - \mathbf{M} (C^i, \cdot \cdot \cdot D^j \cdot \mathbf{C}^j \cdot \mathbf{D}^j - \cdot \cdot \cdot D^j \cdot \mathbf{D}^j)
\]

**Def. 40.** Let \( \omega^k = (b^k)^T \omega^k \) be the inertial angular velocity of \( \delta_k \).

**Def. 41.** Let \( h^k \) be the contribution of rotors in \( \delta_k \) to the angular momentum of the \( k \)th substructure relative to \( \delta_k \) with respect to \( \alpha_k \) and let \( h^k \equiv h^k \cdot (b^k) \).

**Def. 42.** Let \( B_r \) be the \( r \)th neighbor set for \( r \in B \), such that \( k \in B_r \) if \( \delta_k \) is attached to \( \delta_r \).

**Def. 43.** Let \( B_{jk} \) be the branch set of integers \( r \) such that \( r \in B_{jk} \) if \( k = N_r \). Thus, \( B_{jk} \) consists of the indices of those bodies attached to \( \delta_j \) on a branch which begins with \( \delta_k \).

**Def. 44.** Let the tilde symbol ("\( \sim \)"") signify, in application to a 3 by 1 matrix \( V \) with elements \( V_\theta \) (\( \theta = 1, 2, 3 \)), transformation to a skew-symmetric 3 by 3 matrix \( \tilde{V} \) given by

\[
\tilde{V} = \begin{bmatrix}
0 & -V_3 & V_2 \\
V_3 & 0 & -V_1 \\
-V_2 & V_1 & 0
\end{bmatrix}
\]

**B. The Equations**

The objective of this section is to begin with the general vector-dyadic equations derived in Ref. 4 and to proceed by sacrificing some of their generality in favor of a particular appendage model. Explicit results, in the form of both vector and matrix equations suitable for computer programming, will thereby be obtained.

In what follows, attention is confined to a special case of the finite element appendage model of Ref. 1, for which, as in Ref. 2, all mass of appendage \( k \) is concentrated in the \( n_k \) discrete nodal bodies of the appendage (with no distributed mass for the internodal elastic elements). All deformations from a nominal appendage state are assumed arbitrarily small, so that terms above the first degree in these deformations (and corresponding rates) can be neglected. Further, any rigid body \( \delta_k \) will be assumed to carry rotors, and, they will consist of an orthogonal triad whose axes parallel \( b^1_k, b^2_k, \) and \( b^3_k \).
The starting point for this development is the set of vector-dyadic equations of vehicle translation and substructure rotation as derived in Ref. 4 (Eqs. 9, 31-35):

\[ F = \mathcal{M} \ddot{X} \]  
(1)

\[ \sum_{k \in \mathcal{B}} W^k = 0 \]  
(2)

\[ \tau_z + g^z \cdot \sum_{k \in \mathcal{R}} \epsilon_{zk} W^k = 0 \quad (z \in \mathcal{R}) \]  
(3)

where

\[ W^k \equiv T^k + \sum_{r \in \mathcal{B}} D^{kr} \times F' + \epsilon^k \times \left( \frac{\mathcal{M}_k}{\mathcal{M}} F - F^k \right) \]

\[ + \sum_{r \in \mathcal{B}} \mathcal{M}_r D^{kr} \times [\dot{c}' + 2\omega' \times \dot{c}' + \dot{\omega}' \times c' + \omega' \times (\omega' \times c')] \]

\[ + \mathcal{M}_k \epsilon^k \times \sum_{r \in \mathcal{B}} [\dot{\omega}' \times D^{kr} + \omega' \times (\omega' \times D^{kr})] \]

\[ - \Phi^{kk} \cdot \omega^k - \sum_{r \in \mathcal{B} - k} \Phi^{kr} \cdot \dot{\omega}^r + \mathcal{M}_k \sum_{r \in \mathcal{B} - k} D^{kr} \times [\dot{\omega}' \times (\omega' \times D^{kr})] \]

\[ -\omega^k \times \Phi^{kk} \cdot \omega^k - \mathbf{h}^k - \omega^k \times \mathbf{h}^k - \Phi^{kk} \cdot \omega^k \]

\[ -\int_{a_k} p \times \dot{p} \, dm - \omega^k \times \int_{a_k} (p \times \dot{p}) \, dm \]  
(4)

and

\[ \omega^k = \omega^0 + \sum_{r \in \mathcal{R}} \epsilon_{kr} \gamma^r \]  
(5)

\[ \dot{\omega}^k = \dot{\omega}^0 + \sum_{r \in \mathcal{R}} \epsilon_{kr} [\gamma^r c' + \omega' \times \gamma^r] \]  
(6)

The adoption of a nodal body appendage model leads (as in Ref. 2, Eq. 58) to the following useful relation:

\[ \epsilon^k = -\sum_{i=1}^{n_k} \left( \frac{m_i}{\mathcal{M}_k} \right) u^i \]  
(7)

where appendage \( a_k \) has been idealized as \( n_k \) nodal bodies interconnected by massless elastic structure, with \( m_j \) the mass of nodal body \( s \), and \( u^i \) the displacement of the body \( s \) relative to \( b_k \) from the position occupied in the nominal state.

It will also be necessary to develop an expression for \( \Phi^{kk} \) in terms of appendage variables. From Def. 38, we know that
\[ \Phi_{kk} = K^{*} + J^{*} \]  

where \( K^{*} \), the "augmented" inertia dyadic, is a constant. \( J^{*} \), the inertia dyadic of the \( k \)th substructure for \( \alpha_k \), is time-variable due to appendage deformations and may be obtained from

\[ J^{k} = \int (p \cdot pU - pp) \, dm \]  

where \( U \) is the unit dyadic.

For the small-deformation appendage model adopted here, \( J^{k} \) may be evaluated (see Ref. 2, Eq. 126) as

\[
J^{k} = \bar{J}^{k} + [b^{*}]^{T} \left[ \sum_{k=1}^{a_k} \{ m_{i} [2(R^{k} + r^{i})u^{i}U - (R^{k} + r^{i})u^{i}r^{i}] - u^{i}(R^{k} + r^{i})^{T} + \ddot{\beta}^{i}I - \dot{\beta}^{i}I^{i} \} \} (b^{k}) \right]
\]

where \( \bar{J}^{k} \) is the nominal (constant) value of \( J^{k} \), and \( I^{i} \) is the constant inertia matrix of the \( i \)th nodal body for its own mass center and in its own body-fixed vector basis \( (n^{i})^{*} \), where in the nominal state, \( (n^{i})^{k} = (b^{k}) \).

Combining (8) and (10), we have

\[
\Phi_{kk}^{i} = [b^{*}]^{T} \left[ \sum_{i=1}^{a_k} \{ m_{i} [2(R^{k} + r^{i})u^{i}U - (R^{k} + r^{i})u^{i}r^{i}] - u^{i}(R^{k} + r^{i})^{T} + \ddot{\beta}^{i}I - \dot{\beta}^{i}I^{i} \} \} (b^{k}) \right]
\]

Finally, Eq. (4) requires more explicit expressions for the integrals over the appendage \( \alpha_k \). The appropriate expressions in this case may be found in Eq. (114) of Ref. 2, which simplifies to

\[
- \frac{d}{dt} \int_{a_k} p \times \dot{p} \, dm = - \int_{a_k} p \times \ddot{p} \, dm - \omega^{k} \times \int_{a_k} p \times \dot{p} \, dm
\]

or

\[
- \frac{d}{dt} \int_{a_k} p \times \dot{p} \, dm = - \sum_{i=1}^{a_k} (R^{k} + r^{i}) \times m_{i} \ddot{u}^{i} - \omega^{k} \times \sum_{i=1}^{a_k} (R^{k} + r^{i}) \times m_{i} \dddot{u}^{i}
\]

\[
- \sum_{i=1}^{a_k} \{ V \cdot \ddot{\beta}^{i} + \omega^{k} \times V \cdot \dot{\beta}^{i} \}
\]
Note that in Eqs. (7), (10), (11), and (12), the superscript \( k \) has been dropped from nodal body variables in the \( k \)th appendage (such as \( u^i \), which replaces \( u^i(k) \)).

Turning now to the appendage equations, we will make use of the nodal body finite element model case described by Eq. (95) of Ref. 2 (correcting the last algebraic sign within the braces on the right side of Eq. 95 by changing \(-\) to \(+\), and subtracting all nominally non-zero terms from the right side so as to make \( q \) a measure of the deviation from a nominal state in which the appendage might be deformed). In matrix form, the equation for the \( k \)th appendage becomes

\[
M^k \left( U - \Sigma_{U0} \Sigma_{U0}^T \frac{M^k}{\Omega_{k}} \right) \dot{q}^k + 2M^k \left[ \left( \Sigma_{U0} \omega^k \right)^{-} - \Sigma_{U0} \omega^k \Sigma_{U0}^T \frac{M^k}{\Omega_{k}} \right] \dot{q}^k
+ M^k \left( \Sigma_{U0} \omega^k \right)^{-} + \left( \Sigma_{U0} \omega^k \right)^{-} M^k - \left( M^k \Sigma_{U0} \omega^k \right)^{-} \right] \dot{q}^k

+ \left( \Sigma_{U0} \omega^k \right)^{-} M^k \left( \Sigma_{U0} \omega^k \right)^{-} \left( \Sigma_{U0} \omega^k \right)^{-} \left( M^k \Sigma_{U0} \omega^k \right)^{-}

- \Sigma_{U0} \left( \omega^k + \omega^k \omega^k \right) \Sigma_{U0}^T \frac{M^k}{\Omega_{k}} + \left( \Sigma_{U0} \omega^k \right)^{-} \left( \Sigma_{U0} \omega^k \right)^{-} + K^k \right] q^k

= \left[ \Sigma_{U0} \omega^k + \Sigma_{U0} \left[ \Theta \dot{X} - \ddot{R}^k \omega^k + \ddot{\omega}^k \omega^k R^k - \dddot{\omega}^k \dot{\omega}^k R^k \right]

+ \left( \Sigma_{U0} \omega^k \right)^{-} \left( \Sigma_{U0} \omega^k \right)^{-} \dot{r}_k - \left( \Sigma_{U0} \Omega^k \right)^{-} \left( \Sigma_{U0} \Omega^k \right)^{-} \dot{r}_k

- \dot{r}_k \Sigma_{U0} \omega^k \left] - \left( \Sigma_{U0} \omega^k \right)^{-} M^k \left( \Sigma_{U0} \omega^k \right)^{-} + \left( \Sigma_{U0} \Omega^k \right)^{-} M^k \left( \Sigma_{U0} \Omega^k \right)^{-} + \lambda^k \right \} \right)

\text{(13)}

where the assumption has been made that the appendage structure contains no damping. The symbol \( \lambda^k \) is a column matrix containing any forces or torques applied to the \( n_k \) sub-bodies of the appendage other than the structural interaction forces induced by deformations. For example, gravity forces or attitude control jet thrust would contribute to \( \lambda^k \). Also,

\[
q^k \equiv [u_1 u_2 \beta_1^k \beta_2^k u_2^2 \cdots \beta_3^k]^T
\]

a \( 6n_k \) by 1 matrix which fully characterizes the appendage deformations relative to some nominal state of deformation induced by the nominal constant value \( \Omega^k \) of \( \omega^k \).
\[
M^k \equiv \begin{bmatrix}
m^1 & 0 & 0 & 0 & \cdots & 0 \\
0 & I^1 & 0 & 0 & \cdots & 0 \\
0 & 0 & m^2 & 0 & \cdots & 0 \\
0 & 0 & 0 & I^2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & I^n
\end{bmatrix}
\]

a constant, symmetric \(6n_k\) by \(6n_k\) matrix defined in terms of the 3 by 3 partitioned matrices \(m^s, I^s\).

\[
m^s \equiv \begin{bmatrix}
m_s & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_s
\end{bmatrix}, \quad I^s = \begin{bmatrix}
I_{11}^s & I_{12}^s & I_{13}^s \\
I_{21}^s & I_{22}^s & I_{23}^s \\
I_{31}^s & I_{32}^s & I_{33}^s
\end{bmatrix} \quad (s = 1, \ldots, n_k)
\]

\[
\Sigma_{U0} = [U \ 0 \ U \ 0 \ \cdots \ U \ 0]^T \\
\Sigma_{0U} = [0 \ U \ 0 \ U \ \cdots \ 0 \ U]^T
\]

\(6n_k\) by 3 Boolean operator matrices, where \(U\) and 0 are the 3 by 3 unit and null matrices, respectively.

\[
r_k \equiv [r_1^T \ 0 \ r_2^T \ 0 \ \cdots \ r_n^T \ 0]^T
\]

\[
\tilde{r}_k \equiv \begin{bmatrix}
\tilde{r}^1 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
0 & 0 & \tilde{r}^2 & 0 & \cdots & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \cdots & \tilde{r}^n \\
0 & 0 & 0 & 0 & \cdots & 0
\end{bmatrix}
\]

a constant \(6n_k\) by \(6n_k\) matrix.

\[
K^k \equiv \text{the stiffness matrix that determines the structural interaction forces and torques induced by deformation of the \(k\)th appendage from its nominal state (a constant, symmetric \(6n_k\) by \(6n_k\) matrix).}
\]

It should now be recognized that the term \(\Theta \dot{X}\) in Eq. (13) must be replaced by the inertial acceleration of the mass center of the corresponding substructure in the local vector basis, which is assumed for each \(k\) to be zero in the "nominal" state. For substructure \(s_k\), this term is given by (see Eq. 54, Ref. 4)
\[ \Theta \ddot{X} = C^{k0}C \ddot{X} - (\ddot{\omega}^k c^k + 2\dot{\omega}^k \dot{c}^k + \omega^k \ddot{c}^k) + \sum_{\tau \in \Theta} \Theta^{-1} \left[ (\ddot{\omega}^\tau + \dot{\omega}^\tau \dot{c}^\tau) \left( D^{\tau0} + \frac{\partial}{\partial \tau} c^\tau \right) + \frac{\partial}{\partial \tau} (\ddot{c}^\tau + 2\dot{c}^\tau \dot{\omega}^\tau) \right] \tag{14} \]

and

\[ \dot{C} \ddot{X} = \frac{F}{\mathcal{M}} \tag{15} \]

treated as zero in the nominal state.

Equations (1)–(15) provide a rather complete system description (although the contribution of rigid rotors, i.e., \( k^* \), will be developed in more detail later). Since the number of nodes \( n_k \) in a single finite-element model of an elastic appendage is typically rather large, it is to be understood that the nodal body vibration equations, (Eqs. 13–15), will provide the basis for a transformation to distributed or modal coordinates for appendage deformations and that most of these will be deleted from consideration by truncating the matrix of deformation variables. Thus, the variables labeled \( \beta \) and \( \tau \) above will be replaced by appropriate combinations of new modal deformation variables.

The equations actually to be programmed for digital computer solution will therefore be the transformed and truncated versions of Eqs. (1)–(15). These will be described in the following sections as the system motions are confined to two particular cases of interest: (1) the case in which all appendage base-body angular rates \( \omega^k \) experience only slight deviations from some constant nonzero value (i.e., \( \omega^k \approx \Omega^k \), \( \dot{\omega}^k \approx 0 \)), or (2) the case in which \( \Omega^k \approx 0 \) (i.e., \( \omega^k \approx 0 \), \( \dot{\omega}^k \approx 0 \)) for all appendage base bodies.

In the first case, i.e., where \( \omega^k \approx \Omega^k \) and \( \dot{\omega}^k \approx 0 \), the approach taken in developing the system equations of motion, including linearization, coordinate transformation, and truncation, may be described as follows:

1. For the purposes of constructing a coordinate transformation for the appendages, assume that \( \omega^k \) experiences only small deviations from a constant \( \Omega^k \), and write the homogeneous form of the appendage equations.
2. Construct a coordinate transformation from these linear, constant-coefficient equations, and select the truncation level.
3. Return to the unrestricted \( \omega^k \) assumption, and substitute the transformations from (2) into all equations of motion.
4. In the homogeneous part of the appendage vibration equations only, ignore products of deformation variables and deviations of \( \omega^k \) from \( \Omega^k \). This step is not formally correct, since mathematically we cannot justify treating the deviation of \( \omega^k \) from \( \Omega^k \) as small only when it is multiplied by a deformation variable. On the basis of engineering judgment, however, the authors feel that it is probably justifiable and would be a less significant source of error than either modeling or truncation. The resulting equations contain all terms formally required for the analysis of a system with appendage base bodies experiencing small deviations from their nominal motions, but in applying these equations to systems with large deviations of base bodies from their
nominal motion one is suppressing products of these deviations with deformation variables. In fact, a very large change in base-body spin rate would change the effective structural stiffness of the appendage, and invalidate the modal analysis on which the appendage modal coordinate selection is based. In this respect, the equations would be tainted by truncation even if the suppressed terms were retained, and, since these terms would substantially complicate the analysis by coupling all variables into each vibration equation, they have been rejected here.

III. Systems With Rotating Appendages

A. Equations

Inspection of the appendage equations (Eqs. 13-15) reveals that the coefficients of \( q^k \) and \( \dot{q}^k \) depend upon \( \omega^k \), which characterizes the rotational motion of the appendage base. In general, \( \omega^k \) is an unknown function of time, to be determined only after the appendage equations are augmented by other equations of dynamics and control for the total vehicle and solved. Only if \( \omega^k \) can be assumed to experience, in a given time interval, small excursions about a constant nominal value \( \Omega^k \) is there any possibility of transforming Eq. (13) to a new set of uncoupled appendage coordinates. Any methods involving modal coordinates (see Ref. 1, Sect. 1) depend formally upon this assumption.

Assuming then that \( \omega^k = \Omega^k \) and \( \dot{\omega}^k \approx 0 \), Eqs. (13)-(15) can be combined to provide the following appendage equation:

\[
M^k \left( U - \Sigma_{\omega^k} \dot{\Sigma}_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} \right) \dot{q}^k + \left( 2M^k \left[ (\Sigma_{\omega^k} \Omega^k)^{-} - \Sigma_{\omega^k} \dot{\omega}^k \Sigma_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} \right] + M^k (\Sigma_{\omega^k} \Omega^k)^{-} + (\Sigma_{\omega^k} \Omega^k)^{-} M^k - (M^k \Sigma_{\omega^k} \Omega^k)^{-} \right) \dot{q}^k \\
+ \left\{ -(\Sigma_{\omega^k} \Omega^k)^{-} (M^k \Sigma_{\omega^k} \Omega^k)^{-} + (\Sigma_{\omega^k} \Omega^k)^{-} M^k (\Sigma_{\omega^k} \Omega^k)^{-} \right\} q^k \\
+ M^k \left[ -\Sigma_{\omega^k} (\Sigma_{\omega^k} \Omega^k)^{-} \Sigma_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} + (\Sigma_{\omega^k} \Omega^k)^{-} (\Sigma_{\omega^k} \Omega^k)^{-} \right] + K^k \right\} q^k \\
= (-M^k \Sigma_{\omega^k} + M^k \Sigma_{\omega^k} \dot{\omega}^k + M^k \dot{\omega}^k \Sigma_{\omega^k} \omega^k - M^k \Sigma_{\omega^k} \sum_{r \in \bar{B}} C^{\bar{B}} (\ddot{\omega}^r + \dot{\omega}^r \dot{\omega}^r) D^{rk} \\
- M^k \left[ \Sigma_{\omega^k} C^{\bar{B}} \frac{F}{\Sigma_{\omega^k}} + \Sigma_{\omega^k} \dot{\omega}^k \omega^k \dot{R}^k + (\Sigma_{\omega^k} \omega^k)^{-} (\Sigma_{\omega^k} \omega^k)^{-} r_k \right] \\
- (\Sigma_{\omega^k} \omega^k)^{-} M^k (\Sigma_{\omega^k} \omega^k)^{-} \lambda^k + M^k \left[ \Sigma_{\omega^k} \dot{\omega}^k \dot{\omega}^k \dot{R}^k + (\Sigma_{\omega^k} \omega^k)^{-} (\Sigma_{\omega^k} \omega^k)^{-} r_k \right] \\
+ (\Sigma_{\omega^k} \omega^k)^{-} M^k (\Sigma_{\omega^k} \omega^k)^{-} r_k \\
- M^k \Sigma_{\omega^k} \sum_{r \in \bar{B}} C^{\bar{B}} \left[ -\Sigma_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} \dot{\omega}^r + 2\dot{\omega}^r \Sigma_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} \dot{c}^r + \dot{\omega}^r \dot{\omega}^r \Sigma_{\omega^k} \frac{M^k}{\Sigma_{\omega^k}} \dot{c}^r \right] \right] 
\]

\[ (16) \]
Equation (16) consists of $6n_k$ second-order scalar equations and can be written as a matrix equation with the following structure:

$$M_k'\ddot{q}^k + D_k'q^k + G_kq^k + K_kq^k + A_kq^k = L_k'$$

(17)

where

$$M_k' = M_k\left(U - \Sigma_{UO}^T \frac{\partial M_k}{\partial T}\right)$$

$$D_k' = 0$$

$$G_k' = 2M_k\left[(\Sigma_{UO}^a)^- - \Sigma_{UO}^\tilde{a} \Sigma_{UO}^T \frac{M_k}{\partial T}\right] + M_k(\Sigma_{UO}^a)^-$$

$$+ (\Sigma_{UO}^a)^- M_k - (M_k \Sigma_{UO}^a)^-$$

$$A_k' = -(\Sigma_{UO}^a)^- (M \Sigma_{UO}^a)^-$$

$$K_k' = (\Sigma_{UO}^a)^- M_k(\Sigma_{UO}^a)^- + K_k$$

$$+ M_k\left[ -\Sigma_{UO}(\tilde{\Omega}^k \tilde{\Omega}^k) \Sigma_{UO}^T \frac{M_k}{\partial T} + (\Sigma_{UO}^a)^- (\Sigma_{UO}^a)^- \right]$$

and

$$L_k' = -M_k[\Sigma_{UO} - \Sigma_{UO}^T (\tilde{R}^k + \tilde{D}^k) - \tilde{R}^k \Sigma_{UO}] \dot{\omega}^k - M_k \Sigma_{UO} \frac{C_{\omega}}{\partial T} F + \lambda^k$$

$$- M_k \Sigma_{UO} \sum_{\alpha \beta} C_{k\omega} (\tilde{\omega}^\alpha + \tilde{\omega}^\beta) D^k + N_k^\omega - N_k^c$$

$$- M_k \Sigma_{UO} \sum_{\alpha \beta} C_{k\omega} \left[ -\Sigma_{UO} \frac{M'_\omega}{\partial T} \ddot{q}^\alpha + 2\tilde{\omega}^\alpha \frac{M'_\omega}{\partial T} \dot{c}^\alpha + \tilde{\omega}^\alpha \frac{M'_\omega}{\partial T} \dot{c}^\alpha \right]$$

with

$$N_k^\omega = -M_k[\Sigma_{UO} \tilde{\omega}^k \tilde{\omega}^k (R^k + D^{kk})$$

$$+ (\Sigma_{UO}^a)^- (\Sigma_{UO}^a)^- r_k] - (\Sigma_{UO}^a)^- M_k(\Sigma_{UO}^a)^-$$

and

$$N_k^c = -M_k[\Sigma_{UO}(\tilde{\Omega}^k \tilde{\Omega}^k)(R^k + D^{kk}) + (\Sigma_{UO}^a)^- (\Sigma_{UO}^a)^- r_k] - (\Sigma_{UO}^a)^- M_k(\Sigma_{UO}^a)^-$$

Matrices $M_k'$, $D_k'$, and $K_k'$ are constant symmetric matrices, while $G_k'$ is a constant skew-symmetric matrix, and $A_k'$ has both symmetric and skew-symmetric parts. $N_k^\omega$, $N_k^c$
contains the nonlinear terms in $\omega^k$ due to centripetal accelerations of the appendage due to $\omega^k$, and $N'_{k\alpha}$ represents the nominal steady-state value of $N'_{k\alpha}$.

Notice that the form of Eq. (17) is identical to that of Eq. (140) in Ref. 2 (or Eq. 64, Ref. 1), with the exception of the additional right-hand-side terms

$$-M^k \sum_{r \in B - k} C^{kr} \left[ -\sum_{r=0}^{\infty} \frac{d^n}{dn} \frac{\partial}{\partial \omega_r} \tilde{\sigma}' + 2\omega' \frac{\partial}{\partial \omega_r} \tilde{c}' + (\tilde{\sigma}' + \tilde{c}')D' \right]$$

which describe the coupling of appendage $a_k$ to other rigid bodies and appendages of the system. Also, in comparing Eq. (17) to Eq. (140) of Ref. 2, note that $R$ has been replaced by $(R^k + D^{kk})$, a vector from the mass center (barycenter) of the undeformed augmented substructure to the point $Q_k$ (see Fig. 1 and Def. 35).

At this point in the development of the appendage equations, it is appropriate to elaborate upon what is meant by “nominal appendage state,” and what relationship this idea has to Eq. (17). We have already indicated that the approach to be taken is that of Ref. 1 (see pp. 1–3), namely that appendages are ideally considered as linearly elastic and that $u$ and $\beta$ are “small,” oscillatory appendage deformations, i.e., variational deformations. It is quite possible that these small oscillatory deformations will be superimposed on relatively large steady-state deformations, due to spin, for example.

In order to derive a suitable appendage equation, applicable for a “variational deformation” $q$, the substitution of an expansion for the total deformation $q'$ such as

$$q' = q + q_{ss}$$

has been made in Eq. (17), where $q_{ss}$ (= constant) is understood to be the steady-state appendage deformation due to spin. The steady-state deformation is given by

$$(K' + A')_{ss} = N'_{ss}$$

where

$$N'_{ss} = -M^k \left[ \sum_{u=0}^{\infty} \tilde{\sigma}^k \tilde{c}^k (R^k + D^{kk}) + (\sum_{u=0}^{\infty} \tilde{\sigma}^k) (\sum_{u=0}^{\infty} \tilde{c}^k)^{-1} r_k \right]$$

In effect then, in Eq. (17), we have linearized about the steady-state deformation induced by centrifugal forces due to spin of the $k$th substructure, with the mass center of this substructure inertially fixed. It should also be remembered that the original definitions of $\tilde{\sigma}_k$, $c_k$, and the vectors $(\eta)^k$, $(R)^k$, etc., remain intact but that the term “nominal state” is more clearly specified as the “steady state” of deformation due to the nominal (constant) spin of the $k$th substructure, with the mass center of that substructure inertially fixed. Also, the value of $K^k$ should include whatever increment to the elastic stiffness of the appendage is attributable to structural preload due to this spin; that is, $K^k$ includes the so-called “geometric stiffness matrix” of the structure.
The matrix $D'$, which in the general case would accommodate any viscous damping that may be introduced to represent energy dissipation due to structural vibrations, is zero here since such terms have been omitted. But they can still be inserted if one accepts the practice common among structural dynamicists of incorporating the equivalent of a term $D'\dot{q}^k$ into equations of vibration only after derivation of equations of motion and transformation of coordinates.

The nature of terms contributing to $G'_k$, $K'_k$, and $A'_k$ is discussed in some detail in Ref. 1. In particular, the matrix $A'_k$ is shown in Ref. 1 to disappear for the case of small base excursions about a nonzero constant spin only if the nodal bodies are particles or spheres, or if in the steady state of deformation, all nodal bodies have principal axes of inertia aligned with the nominal value of the angular velocity $\omega_k$ (i.e., $\omega_k^0 \approx \langle \omega_k^0 \rangle \Omega^k$). The latter restriction will henceforth be adopted in this report since it greatly reduces the computational task in transforming the homogeneous form of Eq. (17) to a set of completely uncoupled differential equations.

In order to transform Eq. (17) to a set of uncoupled equations, it is first necessary to rewrite it in first-order form, such as

$$\partial u_k \dot{Q}^k + \nu_k Q^k = \mathcal{E}_k$$

(18)

where

$$Q^k \equiv \begin{bmatrix} \dot{q}^k \\ \ddot{q}^k \end{bmatrix}, \quad \mathcal{E}_k \equiv \begin{bmatrix} 0 \\ L'_k \end{bmatrix}$$

$$\partial u_k \equiv \begin{bmatrix} K'_k & 0 & 0 \\ 0 & M'_k \end{bmatrix}, \quad \nu_k \equiv \begin{bmatrix} 0 & -K'_k \\ -K'_k & G'_k \end{bmatrix}$$

Now let $\Phi$ be a $(12n_k \times 12n_k)$ matrix of (complex) eigenvectors of the differential operator in Eq. (18), and let $\Phi'$ be a $(12n_k \times 12n_k)$ matrix of (complex) eigenvectors of the homogeneous adjoint equation

$$\partial u_k^T \dot{Q}^* + \nu_k^T Q^* = 0$$

(19)

so that $\Phi_k$ and $\Phi'_k$ are related by

$$\Phi_k^{-1} = i\Phi_k'^T$$

with $i$ a $(12n_k \times 12n_k)$ diagonal matrix which depends upon the normalization of $\Phi_k$ and $\Phi'_k$. Substitution into Eq. (18) of the transformation

$$Q^k = \Phi_k Y^k$$

and premultiplication by $\Phi'_k^T$ furnishes

$$(\Phi'_k^T \partial u_k \Phi_k) \dot{Y}^k + (\Phi'_k^T \nu_k \Phi_k) Y^k = \Phi'_k^T \mathcal{E}_k$$
The two coefficient matrices enclosed in parentheses are diagonal. If \( \Lambda_k \) is the 
\((12n_k \times 12n_k)\) matrix of the (complex) eigenvalues of the differential operator in 
Eq. (18), then upon premultiplication by \((\Phi_k^T \mathcal{Q}_k \Phi_k)^{-1}\), one obtains
\[
\dot{\bar{Y}}^k = \Lambda_k \bar{Y}^k + (\Phi_k^T \mathcal{Q}_k \Phi_k)^{-1} \Phi_k^T \bar{E}_k
\] (20)

Note that the matrix inversion in Eq. (20) consists simply of calculating the 
reciprocals of the diagonal elements of \(\Phi_k^T \mathcal{Q}_k \Phi_k\).

In practice, one may expect that physical interpretation of the new (complex) 
state variables \(Y_1^k, \ldots, Y_{12n_k}^k\) will permit truncation to a reduced set of variables
\[
\bar{Y}^k \equiv [Y_1^k \ldots Y_{N_k}^k Y_{N_k+1}^k \ldots Y_{2N_k}^k]^T
\] (21)
where \(N_k\) is the number of modes to be preserved in the simulation. The transfor-
mation matrix \(\Phi_k\) is accordingly truncated to the \((12n_k \times 2N_k)\) matrix \(\bar{\Phi}_k\), where
\[
\bar{\Phi}_k \equiv [\Phi_k^\prime \ldots \Phi_k^\prime \Phi_k^* \ldots \Phi_k^\prime \Phi_k^*] \]

The equation of motion of the appendage now becomes
\[
\bar{\Phi}_k \dot{\bar{Y}}^k + (\bar{\Phi}_k^T \mathcal{Q}_k \bar{\Phi}_k)^{-1} \bar{\Phi}_k^T \bar{E}_k = 0
\] (22)

Since, in the particular case studied here, the matrices \(\mathcal{Q}_k\) and \(\mathcal{V}_k\) in Eq. (18) 
are, respectively, symmetric and skew-symmetric, so that Eq. (19) becomes
\[
\mathcal{Q}_k \dot{Q}^k - \mathcal{V}_k Q^k = 0
\] (23)
the adjoint eigenvector matrix is available as the complex conjugate
\[
\Phi_k^\prime = \Phi_k^*
\] (24)
The final equations, after truncation of Eq. (24) and substitution into (22), are 
therefore obtained without the necessity of actually computing the eigenvectors 
constituting \(\Phi^k\). Thus, Eq. (22) becomes
\[
\bar{\Phi}_k \dot{\bar{Y}}^k = \Lambda_k \bar{Y}^k + (\bar{\Phi}_k^T \mathcal{Q}_k \bar{\Phi}_k)^{-1} \bar{\Phi}_k^T \bar{E}_k
\] (25)

Since the appendage modeling process thus far has assumed that the structure 
contains no damping, the diagonal matrix, \(\Lambda_k\), will contain only eigenvalues that
are purely imaginary, e.g., $\lambda_m = \pm i\sigma_m$. Conventional practice in structural dynamics, if some energy dissipation in the model is desired, is to rather arbitrarily add what amounts to a viscous damping term $D'kq^k$ to the appendage equation after completing the modal analysis, assuming that the structure of $D'_k$ is such that eigenvectors $\Phi'_k, \ldots, \Phi'^{12n_k}$ are undisturbed by this addition. Specifically, one substitutes $\lambda_m = -\xi_m\sigma_m \pm i\sigma_m$ into Eq. (22) or (25), where $\xi_m$ is the “percent of critical damping” and is chosen based on experience (including tests) with similar structures. (See Appendix A for a discussion of some ramifications of adding damping after transforming the appendage equations to modal coordinates.)

An apparent disadvantage of Eq. (25) is the fact that the quantities $\bar{Y}_k$, $\bar{\Phi}_k$, and $\bar{F}_k$ are complex. However, Eq. (25) can be written in terms of its real and imaginary parts and the resulting equations greatly simplified by the use of certain orthogonality relationships. The detailed development of the equations is shown in Ref. 3, and only the results are presented here.

Realizing that $\Phi'_k$ must have the form

$$\Phi'_k = \begin{bmatrix} \phi'_k \\ -\phi'_k \sigma_m \end{bmatrix}$$

where $\phi'_k = \psi'_k + i\Gamma'_k$, $(6n_k \times 1)$, and letting $Y^k = \delta^k + in_k$, $Y'^k = \delta^k - in_k$, $(\alpha = 1, \ldots, 6n_k)$, one can see from Ref. 3 that the real $N_k \times 1$ (truncated) matrices, $\delta^k$ and $\eta^k$, are the solutions to the equations

$$\delta^k = -\delta^k \eta^k - \delta^k \bar{\Phi}_k L_k \bar{\delta}^k - \bar{\xi}^k \bar{\sigma}^k \bar{\delta}^k$$

(26a)

and

$$\eta^k = \delta^k \bar{\eta}^k - \delta^k \bar{\psi}_k L_k \bar{\delta}^k - \bar{\xi}^k \bar{\sigma}^k \bar{\eta}^k$$

(26b)

As a result, the relationships between the real quantities $q^k$, $\dot{q}^k$, $\delta^k$, and $\eta^k$, in matrix terms, are as follows:

$$q^k = 2(\bar{\psi}_k \delta^k - \bar{\Gamma}_k \eta^k)$$

(27a)

and

$$\dot{q}^k = -2(\bar{\Gamma}_k \bar{\delta}^k + \bar{\psi}_k \bar{\eta}^k)$$

(27b)

so that

$$\ddot{q}^k = -2(\bar{\Gamma}_k \bar{\delta}^k + \bar{\psi}_k \bar{\eta}^k)$$

(27c)

In order to complete the set of model equations, particularly in the form suitable for computer solution, it is necessary to return to the vehicle equations, substituting the relations developed in Eqs. (7), (11), (12), etc., into Eq. (4), to obtain
\[ W^k = T^k + \sum_{r \in \mathfrak{g}} D^{kr} \times F' + c^k \times \left( \frac{\partial r_k}{\partial \mathfrak{g}} \mathfrak{g} - F \right) \]

\[ + \sum_{r \in \mathfrak{g}} \mathfrak{r}_r D^{kr} \times \left[ -\sum_{i=1}^{n_r} -m_i \mathfrak{r}_r \mathfrak{f}_i + 2\mathfrak{w}' \times \mathfrak{e}' + \mathfrak{w}' \times \mathfrak{c}' + \mathfrak{w}' \times (\mathfrak{w}' \times \mathfrak{c}') \right] \]

\[ + \mathfrak{r}_r c^k \times \sum_{r \in \mathfrak{r}} [\mathfrak{w}' \times D^{kr} + \mathfrak{w}' \times (\mathfrak{w}' \times D^{kr})] \]

\[ - \sum_{r \in \mathfrak{g}} \Phi^{kr} \cdot \mathfrak{w}^k - \mathfrak{h}^k - \mathfrak{w}^k \times \mathfrak{h}^k \]

\[ - (b^T)^T \sum_{j=1}^{n_r} \left[ m_j \left( 2(R^k + r^j)^T \hat{u}' \mathfrak{g} - (R^k + r^j) \hat{u}' \mathfrak{g} - \hat{u}' (R^k + r^j)^T \right) \right] \]

\[ + \beta \hat{I}' \mathfrak{f} - I' \beta \mathfrak{f} \right] (b^T) \cdot \mathfrak{w}^k \]

\[ \sum_{j=1}^{n_r} (R^k + r^j) \times m_j \mathfrak{u}' - \mathfrak{w}^k \times \sum_{j=1}^{n_r} (R^k + r^j) \times m_j \mathfrak{u}' \]

\[ - \sum_{j=1}^{n_r} (1 \cdot \beta \mathfrak{f} + \mathfrak{w}^k \times 1 \cdot \beta \mathfrak{f}) \quad (28) \]

Eliminating the use of \( \mathfrak{R}^k \) for simplicity (noting that this is an arbitrary vector fixed in \( \mathfrak{R}_k \) and it can always be chosen as zero) and substituting \( q^k \) and \( q^r \) where appropriate, the matrix form of Eq. (28) becomes

\[ W^k = T^k + \sum_{r \in \mathfrak{g}} \tilde{D}^{kr} C^{kr} F' + \left[ \tilde{F}^k - \left( C^k \mathfrak{r}_k \mathfrak{f} \right) \right] c^k \]

\[ + \sum_{r \in \mathfrak{g}} \mathfrak{r}_r \tilde{D}^{kr} C^{kr} \left[ -\Sigma_{\mathfrak{U}0} \frac{M^r}{\mathfrak{r}_r} \mathfrak{q}' + 2\mathfrak{q}' - \mathfrak{q}' - \mathfrak{q}' \mathfrak{q}' \right] \]

\[ + \mathfrak{r}_r \mathfrak{c}^k \sum_{r \in \mathfrak{g}} C^{kr} \left[ -\tilde{D}^{kr} \mathfrak{w}' + \tilde{D}^{kr} \mathfrak{w}' \mathfrak{D}^{kr} \right] \]

\[ - \sum_{r \in \mathfrak{g}} \Phi^{kr} C^{kr} \mathfrak{w}' + \mathfrak{r}_r \sum_{r \in \mathfrak{g} - k} \tilde{D}^{kr} C^{kr} \mathfrak{w} \mathfrak{D}^{kr} - \mathfrak{R}^k \Phi^{kr} \mathfrak{w}^k \]

\[ - \tilde{h}^k - \tilde{\mathfrak{h}}^k \mathfrak{h}^k - \left[ 2(M^k \mathfrak{r}_k) \right] ^T \mathfrak{q}^k \mathfrak{U} - \mathfrak{r}_k (M^k \mathfrak{q}^k)^T - (M^k \mathfrak{q}^k)^T \mathfrak{r}_k ^T \]

\[ + \Sigma_{\mathfrak{U}0} (\tilde{q}^k M^k - M^k \tilde{q}^k) \Sigma_{\mathfrak{U}0} \mathfrak{w}^k - \Sigma_{\mathfrak{U}0} \mathfrak{r}_k M^k \mathfrak{q}^k - \tilde{\mathfrak{h}}^k \Sigma_{\mathfrak{U}0} \mathfrak{r}_k M^k \mathfrak{q}^k \]

\[ - \Sigma_{\mathfrak{U}0} M^k \tilde{q}^k - \tilde{\mathfrak{h}}^k \Sigma_{\mathfrak{U}0} M^k \tilde{q}^k \quad (29) \]
where the operator \( \dagger \) reassembles the 3 by 1 submatrices of a column matrix into a three-row matrix, as illustrated by

\[
\begin{bmatrix}
    r_1^k \\
    r_2^k \\
    \vdots \\
    r_n^k \\
\end{bmatrix}
\equiv [r^1 \ 0 \ r^2 \ 0 \ \cdots \ r^n \ 0]
\]

Using the identity

\[
(M^{q^k})^T r_k \equiv (M^k r_k)^T q^k
\]

and regrouping some of the terms in (29), we have

\[
W^k = - \sum_{r \in \mathfrak{M}} \left[ \Phi^{kr} C^{kr} + \mathfrak{M}_k \delta_k C^{kr} D^r + \mathfrak{M}_k D^{kr} C^{kr} \right] \omega^r
\]

\[
- \left[ \sum_{r \in \mathfrak{M}} \sum_{k \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \bar{M}^r \right] M^{q^k} - \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \sum_{k \in \mathfrak{M}} \bar{M}^r M^{q^k} - h^k
\]

\[
+ T^k + \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} F^r + \left[ \tilde{F}^k - \left( C^{kr} \frac{\mathfrak{M}_k}{\mathfrak{M}_r} F^r \right) \right] c^k
\]

\[
+ \sum_{r \in \mathfrak{M}} \mathfrak{M}_r \bar{D}^{kr} C^{kr} \bar{D}^r \omega D^r - \bar{D}^{kr} \omega D^r - \bar{D}^{kr} \omega D^r
\]

\[
+ \mathfrak{M}_r \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \bar{D}^r \omega D^r - \bar{D}^{kr} \omega D^r - \bar{D}^{kr} \omega D^r
\]

\[
- [2 (M^{q^k})^T r_k U - r_k^T (M^{q^k})^T r_k - (M^{q^k})^T r_k]^T
\]

\[
+ \left[ \mathfrak{M}_r \bar{D}^{kr} C^{kr} \bar{M}^r \omega D^r - \bar{D}^{kr} \omega D^r - \bar{D}^{kr} \omega D^r \right]
\]

\[
+ \left[ \mathfrak{M}_r \bar{D}^{kr} C^{kr} \bar{M}^r \omega D^r - \bar{D}^{kr} \omega D^r - \bar{D}^{kr} \omega D^r \right]
\]

\[
- [2 (M^{q^k})^T r_k U - r_k^T (M^{q^k})^T r_k - (M^{q^k})^T r_k]^T
\]

\[
+ \sum_{r \in \mathfrak{M}} \sum_{k \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \bar{M}^r M^{q^k}
\]

The truncated modal coordinates, \( \delta^k \) and \( \bar{\eta}^k \), may now be introduced into the \( k \)th substructure equation by way of Eq. (27), as follows:

\[
W^k = - \sum_{r \in \mathfrak{M}} \left[ \Phi^{kr} C^{kr} + \mathfrak{M}_k \delta_k C^{kr} D^r + \mathfrak{M}_k D^{kr} C^{kr} \right] \omega^r
\]

\[
- \bar{A}_k^T \delta^k - \bar{A}_l^T \bar{\eta}^k + \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \left[ G_s \delta^r + P_s \bar{\eta}^r \right]
\]

\[
- \bar{\delta}^k + T^k + \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} F^r + \left[ \tilde{F}^k - \left( C^{kr} \frac{\mathfrak{M}_k}{\mathfrak{M}_r} F^r \right) \right] c^k
\]

\[
+ \sum_{r \in \mathfrak{M}} \left[ \mathfrak{M}_r \bar{D}^{kr} C^{kr} (2 \bar{D}^r \delta^r + \bar{D}^r \bar{\eta}^r) \right] + \mathfrak{M}_k \bar{D}^{kr} C^{kr} \omega D^r
\]

\[
+ \mathfrak{M}_r \sum_{r \in \mathfrak{M}} \bar{D}^{kr} C^{kr} \omega D^r - \bar{D}^{kr} \omega D^r - \bar{D}^{kr} \omega D^r
\]

\[
- j^k \bar{\omega}^k - \bar{\omega}^k (\bar{A}_k^T \delta^k + \bar{A}_l^T \bar{\eta}^k)
\]

(31)
where

\[ \Delta^k_R = -2 \bar{\theta}^k R^T_k M^k [\Sigma_{0 \Sigma} - \bar{\tau}_k \Sigma_{U0}] \]

\[ \Delta^k_f = -2 \bar{\theta}^k \bar{\theta}^T_k M^k [\Sigma_{0 \Sigma} - \bar{\tau}_k \Sigma_{U0}] \]

\[ \bar{F}_k = 2 \Sigma_{U0} M^k \bar{w}_k \]

\[ \bar{G}_k = 2 \Sigma_{U0} M^k \bar{r}_k \]

\[ j^k = 2 (M^k \dot{q}^k)^T r_k - r_k^T (M^k \dot{q}^k)^T - (M^k \dot{q}^k)^T r_k^T + \Sigma_{0 \Sigma} (\dot{q}^k M^k - M^k \dot{q}^k) \Sigma_{U0} \]

\[ \Phi^k = K^k + J^k \]

\[ \Phi^k = J^k \]

Using the relation in Eq. (6), the vehicle equations, (2) and (3), become (in matrix form)

\[ A^{00} \dot{\omega}_o + \sum_{j \in \vartheta} A^{0j} \dot{\omega}_j + \sum_{m \in \vartheta} A^{0m} \dot{\delta}_m + \sum_{m \in \vartheta} A^{0m} \dot{\eta}_m = \sum_{k \in \mathbb{B}} C^{0k} E^k \]  

(32)

and for \( s \in \vartheta \),

\[ A^{s0} \dot{\omega}_o + \sum_{j \in \vartheta} A^{sj} \dot{\omega}_j + \sum_{m \in \vartheta} A^{sm} \dot{\delta}_m + \sum_{m \in \vartheta} A^{sm} \dot{\eta}_m = g^{sT} \sum_{k \in \vartheta} \epsilon_{sk} C^{rk} E^k + \tau_s \]  

(33)

where

\[ A^{00} = \sum_{k \in \mathbb{B}} \sum_{r \in \mathbb{B}} C^{0k} \left( \Phi^{kr} C^{kr} + \Omega_k \tilde{c}^k C^{kr} \tilde{D}^{rk} + \Omega_p, \tilde{D}^{kr} C^{kr} \tilde{c}^r \right) C^{0r}, \]

3 by 3  

(34)

\[ A^{0j} = \sum_{k \in \mathbb{B}} \sum_{r \in \mathbb{B}} C^{0k} \left( \Phi^{kr} C^{kr} + \Omega_k \tilde{c}^k C^{kr} \tilde{D}^{rk} + \Omega_p, \tilde{D}^{kr} C^{kr} \tilde{c}^r \right) C^{0j}, \]

3 by 1  

(35)

\[ A^{s0} = g^{sT} \sum_{k \in \vartheta} \sum_{r \in \mathbb{B}} \epsilon_{sk} C^{ak} \left( \Phi^{kr} C^{kr} + \Omega_k \tilde{c}^k C^{kr} \tilde{D}^{rk} + \Omega_p, \tilde{D}^{kr} C^{kr} \tilde{c}^r \right) C^{0r}. \]

1 by 3  

(36)

\[ A^{sj} = g^{sT} \sum_{k \in \vartheta} \sum_{r \in \mathbb{B}} \epsilon_{sk} C^{ak} \left( \Phi^{kr} C^{kr} + \Omega_k \tilde{c}^k C^{kr} \tilde{D}^{rk} + \Omega_p, \tilde{D}^{kr} C^{kr} \tilde{c}^r \right) C^{0j}. \]

1 by 1  

(37)

\[ A^{0m}_R = C^{0m} \Delta^m_R - \sum_{r \in \mathbb{B}} C^{0r} \tilde{D}_m C^{0m} G_{m} \tilde{G}_{m}, \]

3 by \( N_m \)  

(38)

\[ A^{0m}_I = C^{0m} \Delta^m_I - \sum_{r \in \mathbb{B}} C^{0r} \tilde{D}_m C^{0m} \tilde{P}_m \tilde{G}_{m}, \]

3 by \( N_m \)  

(39)
\[ A_{km}^k = g^{*}(\epsilon_{sm} C^m \Delta_{km} - \sum_{r \in \mathcal{F}} \epsilon_{sr} C^r \tilde{D}^{rm} C^m \tilde{G}_{km} \tilde{\sigma}^m), \quad 1 \text{ by } N_m \] (40)

\[ A_{km} = g^{*}(\epsilon_{sm} C^m \Delta_{km} - \sum_{r \in \mathcal{F}} \epsilon_{sr} C^r \tilde{D}^{rm} C^m \tilde{P}_{km} \tilde{\sigma}^m), \quad 1 \text{ by } N_m \] (41)

\[ \mathcal{F} \equiv \text{the integer set containing the labels of only those rigid bodies of the system that possess a nonrigid appendage.} \]

\[ E^k = T^k - \tau^k - \tilde{\mathcal{K}}^k g^k (\ddot{\omega}^k + \dot{\psi}^k) + \sum_{r \in \mathcal{F}} \tilde{D}^{kr} C^{kr} F^r \]

\[ + \left[ \tilde{F}^k - \left( C^{k0} \mathcal{M}_k \mathcal{F} \right)^{-1} \right] c^k + \mathcal{M}_k \sum_{r \in \mathcal{F}} \tilde{D}^{kr} C^{kr} \omega^r \dot{D}^{rk} \]

\[ - \tilde{\mathcal{K}}^k \Phi^k \dot{\omega}^k - j^k \omega^k - \tilde{\mathcal{K}}^k (\Delta^k g^k + \Delta^k \eta^k) \]

\[ - \sum_{r \in \mathcal{F}} \left( \Phi^{kr} C^{kr} \mathcal{M}_k \tilde{D}^{rk} + \mathcal{M}_k \tilde{D}^{kr} C^{kr} \tilde{\omega}^r \right) \sum_{j \in \mathcal{F}} \epsilon_{jr} C^{jr} g^{jr} \]

\[ - \mathcal{M}_k \tilde{D}^{kr} C^{kr} (2 \tilde{\omega}^r \dot{c}^r + \tilde{\omega}^r \dot{c}^r) - \mathcal{M}_k \tilde{D}^{kr} C^{kr} \tilde{\omega}^r \dot{D}^{rk} \right], \quad 3 \text{ by } 1 \] (42)

and substitutions have been made for \( h^k \) and \( h^k \) based on restriction to three orthogonal axisymmetric rotors in \( \mathcal{A}_k \), with spin axes aligned to the unit vectors \( (b^k) \), and the following equations:

\[ h^k = g^k \dot{\psi}_R^k \] (43)

\[ \tau^k = g^k (\dot{\psi}_R^k + \dot{\omega}^k) \] (44)

\[ \vdots \quad \dot{h}^k = \tau^k - g^k \dot{\omega}^k \] (45)

where

\[ \dot{\psi}_R^k \equiv \dot{\psi}_R^k \cdot (b^k) = 3 \text{ by } 1 \text{ matrix of components of spin rate relative to } \mathcal{A}_k \text{ for three orthogonal axisymmetric rotors in } \mathcal{A}_k. \]

\[ g^k \equiv \text{spin-axis inertia matrix (diagonal) for the three axisymmetric rotors in } \mathcal{A}_k. \]

\[ \tau^k \equiv \tau^k \cdot (b^k) = 3 \text{ by } 1 \text{ matrix of applied torque on the three axisymmetric rotors in } \mathcal{A}_k. \]

It is to be understood that when symmetric rotors are present in the \( k \)th substructure, the rotors' mass and moments of inertia are to be included in \( \mathcal{F} \), the undeformed substructure's inertia dyadic for \( \mathcal{A}_k \). Of course, the mass of the rotors is also to be included in the substructure mass and c.m.-location calculations.

Equation (44) then provides up to three scalar differential equations which are uncoupled in acceleration from the system's vehicle/appendage equations. They
may be integrated and, with \( \omega^k \) and \( \tau^k_R \) known, can be solved for \( \psi^k_R \), which is then supplied to Eq. (42).

If one now operates on the appendage equations, Eqs. (26), in a similar way, they may be expressed as

\[ m \in \mathcal{F} : \]

\[ \mathcal{Q}^m_0 + \sum_{j \in \mathcal{F}} \mathcal{Q}^m_j + \sum_{n \in \mathcal{F}} \mathcal{Q}^m_n + \sum_{n \in \mathcal{F}} \mathcal{Q}^m_{n^*} \mathcal{r}^n = \mathcal{Q}^m_0 \]  

\[ \mathcal{Q}^m_i + \sum_{j \in \mathcal{F}} \mathcal{Q}^m_j + \sum_{n \in \mathcal{F}} \mathcal{Q}^m_n + \sum_{n \in \mathcal{F}} \mathcal{Q}^m_{n^*} \mathcal{r}^n = \mathcal{Q}^m_i \]  

where

\[ \mathcal{Q}^m_0 = \frac{1}{2} \left[ \bar{D}^m \mathcal{C}^m_0 + \bar{m} \bar{e}^m \mathcal{C}^m_0 \sum_{r \in \mathcal{F}} \mathcal{C}^m \bar{d}^m \mathcal{C}^m_0 \right] \text{, } N_m \text{ by 3} \]  

\[ \mathcal{Q}^m_j = \frac{1}{2} \left[ \bar{D}^m \mathcal{C}^m_j + \bar{m} \bar{e}^m \mathcal{C}^m_j \sum_{r \in \mathcal{F}} \mathcal{C}^m \bar{d}^m \mathcal{C}^m_j \right] \mathcal{g}^j, \text{ } N_m \text{ by 1} \]  

\[ \mathcal{Q}^m_n = -\frac{1}{2} \bar{m} \bar{e}^m \mathcal{C}^m_n \bar{G}^n \mathcal{G}^n, \text{ } (m \neq n); \text{ } N_m \text{ by } N_n \]  

\[ \mathcal{Q}^m_R = U, \text{ } (m = n); \text{ } N_m \text{ by } N_m \]  

\[ \mathcal{Q}^m_I = -\frac{1}{2} \bar{m} \bar{e}^m \mathcal{C}^m I \mathcal{G}^n \mathcal{G}^n, \text{ } (m \neq n); \text{ } N_m \text{ by } N_n \]  

\[ \mathcal{Q}^m_I = 0, \text{ } (m = n); \text{ } N_m \text{ by } N_m \]  

\[ \mathcal{Q}^m_0 = \frac{1}{2} \left[ \bar{D}^m \mathcal{C}^m_0 + \bar{m} \bar{e}^m \mathcal{C}^m_0 \sum_{r \in \mathcal{F}} \mathcal{C}^m \bar{d}^m \mathcal{C}^m_0 \right] \text{, } N_m \text{ by 3} \]  

\[ \mathcal{Q}^m_j = \frac{1}{2} \left[ \bar{D}^m \mathcal{C}^m_j + \bar{m} \bar{e}^m \mathcal{C}^m_j \sum_{r \in \mathcal{F}} \mathcal{C}^m \bar{d}^m \mathcal{C}^m_j \right] \mathcal{g}^j, \text{ } N_m \text{ by 1} \]  

\[ \mathcal{Q}^m_R = -\frac{1}{2} \bar{m} \bar{e}^m \mathcal{C}^m R \bar{G}^n \mathcal{G}^n, \text{ } (m \neq n); \text{ } N_m \text{ by } N_n \]  

\[ \mathcal{Q}^m_R = 0, \text{ } (m = n); \text{ } N_m \text{ by } N_m \]  

\[ \mathcal{Q}^m_I = -\frac{1}{2} \bar{m} \bar{e}^m \mathcal{C}^m I \mathcal{G}^n \mathcal{G}^n, \text{ } (m \neq n); \text{ } N_m \text{ by } N_n \]  

\[ \mathcal{Q}^m_I = U, \text{ } (m = n); \text{ } N_m \text{ by } N_m \]
Recapping, the system equations (minus the rotor equations) are as follows:

\[ O_R^m = + \tilde{\sigma}^m \left[ - \tilde{\eta}^m - \tilde{\xi}^m \tilde{\delta}^m + \frac{1}{2} \tilde{\sigma}^m T \tilde{\eta}^m - \tilde{\sigma}^m X \right] - Z_R^m, \quad N_m \text{ by } 1 \quad (58) \]

\[ O_I^m = + \tilde{\sigma}^m \left[ \tilde{\delta}^m - \tilde{\xi}^m \tilde{\eta}^m + \frac{1}{2} \tilde{\sigma}^m T \tilde{\eta}^m - \tilde{\sigma}^m X \right] - Z_I^m, \quad N_m \text{ by } 1 \quad (59) \]

\[ V_m = C^m F + \sum_{r \in \mathcal{R}} C^m \tilde{\omega} \tilde{\omega} D^m \]

\[ + \sum_{r \in \mathcal{R} - m} C^m \frac{\partial}{\partial \tilde{\omega}} \left( \tilde{\omega} \tilde{\omega} C^r \right) - \tilde{\omega}^m \tilde{\omega}^m D^{mm}, \quad 3 \text{ by } 1 \quad (60a) \]

\[ X_m = \lambda^m - M^m (\Sigma_0 \omega^m)^{-} (\Sigma_0 \omega^m)^{-} r_m - (\Sigma_0 \omega^m)^{-} M^m (\Sigma_0 \omega^m) \]

\[ + M^m (\Sigma_0 \Omega^m)^{-} (\Sigma_0 \Omega^m)^{-} r_m + (\Sigma_0 \Omega^m)^{-} M^m (\Sigma_0 \Omega^m), \quad n_m \text{ by } 1 \quad (60b) \]

\[ Z_R^m = \frac{1}{2} \sum_{j \in \mathcal{R}} \left( \tilde{\Delta}^m_{ij} C_{ij}^m + \tilde{\sigma}^m \tilde{G}_{ij}^m \sum_{r \in \mathcal{R}} C^m \tilde{H}^m_{ij} C^r \right) \tilde{\omega}^m \tilde{\omega}^m, \quad N_m \text{ by } 1 \quad (61a) \]

\[ Z_I^m = \frac{1}{2} \sum_{j \in \mathcal{R}} \left( \tilde{\Delta}^m_{ij} C_{ij}^m + \tilde{\sigma}^m \tilde{G}_{ij}^m \sum_{r \in \mathcal{R}} C^m \tilde{H}^m_{ij} C^r \right) \tilde{\omega}^m \tilde{\omega}^m, \quad N_m \text{ by } 1 \quad (61b) \]

Recapping, the system equations (minus the rotor equations) are as follows:

\[ A^{00 \omega} + \sum_{j \in \mathcal{R}} A^{0j \dot{\omega}^j} + \sum_{m \in \mathcal{R}} A^{0m \dot{\delta}^m} + \sum_{m \in \mathcal{R}} A^{0m \dot{\eta}^m} = \sum_{k \in \mathcal{R}} C^{0k} E^k \quad (62a) \]

\[ s \in \mathcal{R}: \]

\[ A^{00 \dot{\omega}^s} + \sum_{j \in \mathcal{R}} A^{0j \dot{\omega}^s} + \sum_{m \in \mathcal{R}} A^{0m \dot{\delta}^m} + \sum_{m \in \mathcal{R}} A^{0m \dot{\eta}^m} = \tilde{g}^s \sum_{k \in \mathcal{R}} \epsilon_{sk} C^{ik} E^k + \tau, \quad (62b) \]

\[ m \in \mathcal{R}: \]

\[ \tilde{g}^{00 \omega^m} + \sum_{j \in \mathcal{R}} \tilde{g}^{0j \dot{\omega}^m} + \sum_{n \in \mathcal{R}} \tilde{g}^{0n \dot{\delta}^m} + \sum_{n \in \mathcal{R}} \tilde{g}^{0n \dot{\eta}^m} = Q_R^m \quad (62c) \]

\[ m \in \mathcal{R}: \]

\[ \tilde{g}^{00 \omega^m} + \sum_{j \in \mathcal{R}} \tilde{g}^{0j \dot{\omega}^m} + \sum_{n \in \mathcal{R}} \tilde{g}^{0n \dot{\delta}^m} + \sum_{n \in \mathcal{R}} \tilde{g}^{0n \dot{\eta}^m} = Q_I^m \quad (62d) \]

and these may be combined into the single matrix equation of the form \( A \dot{x} = B \), as shown in Eq. (63).
Except for $a^m_m$, $a^m_t$, $a^m_R$, and $a^m_R$ when $m = n$, the elements of system matrix $A$ are, in general, time-variable. Note also that, if the appendage equations are multiplied through by the factor 2, matrix $A$ becomes symmetric.

**B. Subroutine MBDYFR**

Equation (63) provides a complete set of rotational dynamics equations which lend themselves to solution by means of a generic computer program or subroutine for the rotating appendage case. When augmented by the rotor equations, control equations, and kinematical equations, they are fully descriptive of the system behavior.

The kinematical variables adopted in the preceding sections are as follows: $\gamma_k$ for $k \in G$ (Def. 23); $C^r$, $C^t$, and $C^R$ for $r, t \in B$ (Def. 31); and $\omega^0 = \{b^0\} \cdot \omega^0$ (Def. 9). Although the equations of motion have been expressed in terms of these quantities, the latter are not all independent. Relationships among kinematical variables developed in this section must therefore either be considered in conjunction with the dynamical equations or be substituted into them to remove redundant variables whenever a solution is sought.
The direction cosine matrix $C^r_j$ (Def. 31) relates sets of orthogonal unit vectors fixed in $r$, and $j$, and hence depends upon those angles $\gamma$ for which $\theta_0$ lies between $r$, and $j$, and also upon the corresponding unit vectors $g^e$ defining the intervening hinge axes. For the special case in which $r$ and $j$ are contiguous and $j < r$, it is always possible to express $C^r_j$ (and $C^j_r$) in terms of the single angle $\gamma$, and the single matrix $g^r$, as follows:

$$C^r_j = U \cos \gamma - \tilde{g}^r \sin \gamma + \tilde{g}^r g^r (1 - \cos \gamma),$$

and

$$C^j_r = U \cos \gamma + \tilde{g}^r \sin \gamma + \tilde{g}^r g^r (1 - \cos \gamma) = (C^r_j)^T.$$

It is only required that $C^r_j$ be determined where $r$ and $j$ are contiguous and, since $C^r_0 = C^0_j C^j_r$, to then derive matrices $C^0_r$ for $r \in \mathcal{R}$. An algorithm for accomplishing this task is described in Ref. 6, Appendix A.

The Fortran V subroutine, called MBDYFR, which provides the solution to Eq. (63), has been designed in much the same form as those subroutines described in Refs. 6 and 7. The routine may be exercised by means of either of two call statements. An initializing call statement supplies the routine with data that will remain constant throughout the dynamic simulation run.

The description which follows of the subroutine initialization statement includes some new variables which will now be defined. The use of these new variables is necessitated by the desire to make the subroutine MBDYFR more efficient. Therefore, the convention (described in Defs. 1-4) of labeling the system's rigid bodies from 0 to $n$, where each connection between bodies is a line hinge, will be modified. Rather than introduce imaginary massless bodies at connections with 2 or 3 degrees of rotational freedom, these types of connections will be handled directly by the routine and no new bodies will be introduced.

**Def. 45.** Let $n_c$ be the number of connections joining a set of $n_c + 1$ substructures. A connection is a 1-, 2-, or 3-degree-of-freedom joint at which all the rotational axes share a common point. The axes need not be mutually orthogonal.

**Def. 46.** Define the integer set $\mathcal{R} = \{0, 1, \ldots, n_c\}$.

**Def. 47.** Define the integer set $\mathcal{R}' = \{1, 2, \ldots, n_c\}$.

**Def. 48.** Let $\mathcal{B}_j$ be the $j$th neighbor set for $j \in \mathcal{R}'$, such that $k \in \mathcal{B}_j$ if $\delta_k$ is attached to $\delta_j$.

The rigid body labeling process is to be carried out precisely as prescribed in Def. 4, except that the last label will be $\delta_{n_c}$ rather than $\delta_n$. Note, however, that the connecting joint degrees of freedom are still labeled from 1 to $n$, so that one still has $\gamma_1, \gamma_2, \ldots, \gamma_n$ and $\mathbf{g}', \mathbf{g}^2, \ldots, \mathbf{g}'$ (The joints must be in the sequence corresponding to the body label sequence, as shown in Fig. 2). All references to the "$k$th substructure," when applying the MBDYFR subroutine, imply that $k \in \mathcal{B}'$.

**Def. 49.** For $k \in \mathcal{R}'$, let $\mathcal{A}_k$ denote the index label of the body attached to $\delta_k$ and on the path leading to $\delta_k$. The scalars $\mathcal{A}_k$ will be termed "connection elements." Thus, it is always true that $\mathcal{A}_1 = 0$. 

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Def. 50. Let $d_k$, $k \in \mathcal{Q}$, denote the number of degrees of freedom at the $k$th connection.

It is also necessary, when applying the subroutine, to relabel each of the nonrigid appendages $\sigma_k$ in the same sequence from 1 to $n_f$ (see Fig. 2) so that the labels become $\xi_1, \ldots, \xi_{n_f}$.

Def. 51. Let $n_f$ be the number of nonrigid appendages in the system (no more than one per substructure).

The first column of the input array, $F$, contains the index labels of those rigid bodies to which nonrigid appendages $\xi_i$ ($i = 1, \ldots, n_f$) are attached.

**Initializing Call Statement**

```fortran
CALL MBDYFR(NC, H, MB, MS, PB, PS, G, PI,
       NF, F, ER, EI, SR, MF, RF, WF, ZF)
```

where

- $NC =$ the integer $n_c =$ number of system connections (see Def. 45).
- $H(k, m) =$ array containing the connection elements $A_k$, $k \in \mathcal{Q}$, and the number of degrees of freedom, $d_k$, at the connection; $m = 1, 2$, $H(1, 1) = A_1$, $H(2, 1) = A_2$, $\ldots$, $H(n_c, 1) = A_{n_c}$, $H(1, 2) = d_1$, $H(2, 2) = d_2$, $\ldots$, $H(n_c, 2) = d_{n_c}$.
- $MB(j) =$ array of undeformed reference substructure ($\xi_0$) inertial constants $j = 1, \ldots, 7$. (Specifically: $MB(1) = J_{11}$, $MB(2) = J_{22}$, $MB(3) = J_{33}$, $MB(4) = -J_{12}$, $MB(5) = -J_{13}$, $MB(6) = -J_{23}$, $MB(7) = 0$.)
\( MS(i, j) \) = array of remaining substructure body (undeformed) inertial constants; \( i \in \mathcal{I}, j = 1, \ldots, 7 \). (Thus: \( MS(i, 1) = J^{T}_I \), \( MS(i, 2) = J^{T}_{22} \), \ldots, \( MS(i, 7) = \gamma^T_i \).

\( PB(i, j) \) = array containing elements of \( p^b_i \); \( i \in \mathcal{I}, j = 1, 2, 3 \).

\( PS(i, j, k) \) = array containing elements of \( p^l_i \); \( i \in \mathcal{I}, j \in \mathcal{I}, k = 1, 2, 3 \). (Exception!! If \( j < i \), set \( PS(i, i, k) = p^l_i \). Example: \( PS(3, 3, 1) = p^l_3 \). All \( PS(i, j, k) \), where \( j < i \), will be ignored.)

\( G(i, j) \) = array containing elements of \( g_i \); \( i \in \mathcal{I}, j = 1, 2, 3 \).

\( PI(i) \) = array of indicators; \( i = 1, 2, \ldots, n + 1 \). (If \( \gamma_i \) is a prescribed variable, \( PI(i) = 1 \). Otherwise, \( PI(i) = 0 \). Also, if \( PI(n + 1) = 1 \), system angular momentum \( HM \) will be calculated; otherwise, \( HM \) is set to zero.

\( NF \) = the integer \( n_f \), number of substructures with nonrigid appendages = number of nonrigid appendages.

\( F(n, m) \) = array containing the index labels of those rigid bodies with nonrigid appendages, the number of nodal bodies in each appendage’s finite element model, and the number of modes to be used in each appendage’s modal model; \( n = 1, 2, \ldots, n_f, m = 1, 2, 3 \). (Thus:

\( F(1, 1) \) = index label of rigid body carrying appendage \( a_1 \)

\( F(1, 2) \) = number of nodal bodies in appendage \( a_1 \)

\( F(1, 3) \) = number of modes representing appendage \( a_1 \)

\( F(2, 1) \) = index label of rigid body carrying appendage \( a_2 \)

\( \vdots \)

\( F(n_f, 3) \) = number of modes representing appendage \( a_{n_f} \).

\( ER(n, i, j) \) = array of elements of \( \vec{\gamma}_k \); \( n = 1, 2, \ldots, n_f; i = 1, 2, \ldots, 6n_k; k = F(n, 1); j = 1, 2, \ldots, N_k \).

\( EJ(n, i, j) \) = array of elements of \( \vec{\lambda}_k \); \( n = 1, 2, \ldots, n_f; i = 1, 2, \ldots, 6n_k; k = F(n, 1); j = 1, 2, \ldots, N_k \).

\( SR(n, j) \) = array of substructure nominal spin rates, \( \Omega^k \); \( k = F(n, 1); n = 1, 2, \ldots, n_f; j = 1, 2, 3 \).

\( MF(n, i, j) \) = array of nodal body inertial properties, \( M^k \), for each nonrigid appendage; \( n = 1, 2, \ldots, n_f; i = 1, 2, \ldots, n_k; k = F(n, 1); j = 1, 2, \ldots, 7 \). (Example: \( MF(2, 3, 1) = I^{T}_{11}, MF(2, 3, 2) = I^{T}_{22}, MF(2, 3, 3) = I^{T}_{33}, MF(2, 3, 4) = -I^{T}_{13}, \ldots, MF(2, 3, 7) = m_3, \) all for nonrigid appendage \( a_2 \), third nodal body.)
\[ RF(n, i, j) = \text{array of elements of } r_k, \quad k = F(n, 1), \text{ for each nonrigid appendage}; \quad n = 1, 2, \ldots, n, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, 3. \] (Example: 
\[ RF(1, 5, 1) = r^1_1, \quad RF(1, 5, 2) = r^2_1, \quad RF(1, 5, 3) = r^3_1, \quad \text{all for appendage } \mathcal{A}_1.\]

\[ WF(n, j) = \text{array of modal frequencies, } \bar{\sigma}^k; \quad k = F(n, 1), \text{ for each nonrigid appendage}; \quad n = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, N_k.\]

\[ ZF(n, j) = \text{array of modal damping factors, } \bar{\zeta}^k; \quad k = F(n, 1), \text{ for each nonrigid appendage}; \quad n = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, N_k.\]

The statement CALL MBDYFR (NC, H, \ldots) need only be executed once prior to a simulation run. However, as the simulation proceeds, the routine must be entered at every numerical integration step to compute the angular accelerations \( \bar{\omega}^0, \gamma^1, \ldots, \gamma^n \) and the modal coordinate acceleration vectors \( \bar{\delta}^k \) and \( \bar{\eta}^k (k \in \mathcal{A}). \) This is accomplished by executing the "dynamic" call statement.

**Dynamic Call Statement**

CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD, GMDD, 
\[ \text{DT, ET, WO, WDOT, DTD, ETD, HM} \])

where

- **NC** = the integer \( n_c \) = number of system connections.
- **TH(i)** = array containing the hinge torques, \( \tau_i; \quad i \in \mathcal{A}. \)
- **TB(j)** = array containing the elements of \( T^0; \quad j = 1, 2, 3. \)
- **TS(i, j)** = array containing the elements of \( T^i; \quad i \in \mathcal{A}', j = 1, 2, 3. \)
- **FB(j)** = array containing the elements of \( F^0; \quad j = 1, 2, 3. \)
- **FS(i, j)** = array containing the elements of \( F^i; \quad i \in \mathcal{A}', j = 1, 2, 3. \)
- **TF(n, i, j)** = array containing the torque elements of \( \lambda^k; \quad n = 1, \ldots, n, \quad k = F(n, 1), i = 1, \ldots, n, j = 1, 2, 3. \)
- **FF(n, i, j)** = array containing the force elements of \( \lambda^k; \quad n = 1, \ldots, n, \quad k = F(n, 1), i = 1, \ldots, n, j = 1, 2, 3. \)
- **GM(i)** = array of angles, \( \gamma_i; \quad i \in \mathcal{A}. \)
- **GMD(i)** = array of the angular velocities, \( \dot{\gamma}_i; \quad i \in \mathcal{A}. \)
- **GMDD(i)** = array of the prescribed angular accelerations, \( \ddot{\gamma}_i; \quad i \in \mathcal{A}. \)
- **DT(n, i)** = array of appendage modal coordinates, \( \bar{\delta}^k; \quad n = 1, \ldots, n, \quad k = F(n, 1), i = 1, \ldots, N_k. \)
- **ET(n, i)** = array of appendage modal coordinates, \( \bar{\eta}^k; \quad n = 1, \ldots, n, \quad k = F(n, 1), i = 1, \ldots, N_k. \)
- **WO(j)** = array containing the components of \( \omega^0; \quad j = 1, 2, 3. \)
- **WDOT(j)** = solution vector containing the elements of \( \dot{\omega}^0, \gamma^1, \ldots, \gamma^n; \quad j = 1, \ldots, n + 3. \) (WDOT(1) = \( \omega^0_1, \) WDOT (2) = \( \omega^0_2, \) WDOT (3) = \( \omega^0_3, \) WDOT (4) = \( \gamma_1, \ldots, \) WDOT (n + 3) = \( \gamma_n. \))
- **DTD(n, i)** = solution matrix for \( \dot{\delta}^k; \quad n = 1, \ldots, n, \quad k = F(n, 1), i = 1, \ldots, N_k. \)
ETD(n, i) = solution matrix for \( \dot{\eta}^k \); \( n = 1, \ldots, n_p, k = F(n, 1), i = 1, \ldots, N_k \).

HM = magnitude of the system angular momentum vector (see Appendix B for the momentum equations).

In summary, the call to MRATE supplies the subroutine with current instantaneous values for hinge torques and externally applied torques and forces on both rigid bodies and nonrigid appendages. Explicit expressions for computing these forcing functions, which may depend on \( \gamma_i, \dot{\gamma}_i \), and other system or control variables, are located in the main calling program (see sample problem that follows). Current values of \( \omega^k, \gamma_i, \dot{\gamma}_i \), and \( \tilde{\eta}^k \) are continuously produced by the main program's numerical integration operators and are therefore always available for input to MBDYFR.

It should be noted here that MBDYFR does not incorporate the terms in Eq. (42) that describe symmetric rotor torques on body \( \alpha_k \). As a result, the user is required, if rotors are present, to supply these terms as part of a "new" \( T^k \), i.e.,

\[
T^{\text{sk}} = T^k - \tau_R^k - \omega^k \dot{\omega}^k (\ddot{\omega}^k + \dot{\psi}^k)
\]

Thus, these terms must be formed in the main program along with Eq. (44), and \( T^{\text{sk}} \) is supplied to the subroutine as TB (if \( k = 0 \)) or TS in the MRATE call statement.

Note also that, if any of the \( \gamma \) are to be prescribed, the appropriate values of \( \gamma_i, \dot{\gamma}_i \), and \( \ddot{\gamma}_i \) must be supplied to the subroutine by way of GM, GMD, GMDD, respectively, in the MRATE call statement. An example of this is shown in Section IVC.

When the MBDYFR subroutine is used, the main calling program must contain Fortran V (or IV) statements which specify "type" and allocate storage for the variables and arrays being used. The mandatory specification statements are listed here.

**Required Specification Statements**

| INTEGER | NC, NF, H(n_r, 2), F(n_p, 3), PI(n + 1) |
| REAL | MB(7), MS(n_r, 7), PB(n_r, 3), PS(n_r, n_r, 3), G(n, 3), TH(n), TB(3), TS(n_r, 3), FB(3), FS(n_r, 3), GM(n), GMD(n), GMDD(n), ER(n_p, 6n_r, N_k), EI(n_p, 6n_r, N_k), MF(n_p, n_k, 7), RF(n_p, n_k, 3), WF(n_p, N_k), ZF(n_p, N_k), TF(n_p, n_k, 3), FF(n_p, n_k, 3), DT(n_p, N_k), ET(n_p, N_k), WO(3), SR(n_p, 3) |
| DOUBLE PRECISION | WDOT(n + 3), DTD(n_p, N_k), ETU(n_p, N_k) |

Also, in order that storage allocation for arrays internal to MBDYFR be minimized, the following statement must appear in the subroutine:

\[
\text{PARAMETER QH = n, QC = n_r, QF = n_p, NK = n_k, NKT = N_k}
\]
The proper placement of this statement in MBDYFR is shown in the listing (Appendix C).

C. A Sample Problem Simulation

To illustrate the use of subroutine MBDYFR, the dynamical system shown in Fig. 3 will be simulated. It consists of a rigid central body, $\delta_0$, to which is connected a rigid platform, $\delta_2$, with 2 degrees of rotational freedom relative to $\delta_0$. A spinning rotor, $\delta_1$, is also connected to $\delta_0$. The platform and the rotor each carry an elastic appendage, which will be modeled as a simple point mass supported by a massless elastic member.

For this test vehicle, the platform will be nominally nonrotating, while the rotor will have a nominal spin rate of $\omega_1$ about the spin axis fixed in $\delta_0$. The appendage modal models must now be derived from the appropriate discrete coordinate equations.

**Rotor Appendage Equations**

The general appendage equation is Eq. (17), where the matrices $M^i$, $G^i$, and $K^i$ for the rotor substructure are as follows:

\[
M^1 = \begin{bmatrix}
 m_1 & 0 & 0 & 1 \\
 0 & m_1 & 0 & 0 \\
 0 & 0 & m_1 & 1 \\
 0 & 0 & 0 & 0
\end{bmatrix}
\]  

\[\text{(6 x 6)}\]

\[
M_i = M^1 \left( U - \Sigma U_0 \Sigma^T U_0 \frac{M^1}{\mathcal{M}_1} \right) = \begin{bmatrix}
 \mu_1 & 0 & 0 & 1 \\
 0 & \mu_1 & 0 & 0 \\
 0 & 0 & \mu_1 & 1 \\
 0 & 0 & 0 & 0
\end{bmatrix}
\]  

\[\text{(6 x 6)}\]

where

\[
\mu_1 = m_1 - \frac{m_2}{\mathcal{M}_1}
\]

\[
\mathcal{M} = \mathcal{M}_0 + \mathcal{M}_1 + \mathcal{M}_2
\]

The rotor spin rate = $\Omega' = [0 \ 0 \ \omega_1]^T$.

\[
G_1' = 2 \begin{bmatrix}
 0 & -\omega_1 \mu_1 & 0 & 1 \\
 \omega_1 \mu_1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0
\end{bmatrix}
\]  

\[\text{(6 x 6)}\]
Fig. 3. MBDYFR simulation test vehicle

We will assume a symmetric stiffness matrix, \( K^1 \), of the form

\[
K^1 = \begin{bmatrix}
    k_1 & 0 & 0 \\
    0 & k_2 & 0 \\
    0 & 0 & k_3 \\
\end{bmatrix}
\]

(6 x 6)

where \( k_1, k_2, \) and \( k_3 \) are the respective stiffness coefficients which restrain linear motion in the \( b_1, b_2, \) and \( b_3 \) directions. Thus,

\[
K'_1 = \begin{bmatrix}
    k_1 - \omega^2 \mu_1 & 0 & 0 \\
    0 & k_2 - \omega^2 \mu_1 & 0 \\
    0 & 0 & k_3 \\
\end{bmatrix}
\]
The homogeneous rotor appendage equation may therefore be written as

\[
\begin{bmatrix}
\mu_1 & 0 & 0 \\
0 & \mu_1 & 0 \\
0 & 0 & \mu_1
\end{bmatrix}
\ddot{q}^1 + 2
\begin{bmatrix}
0 & -\omega_1\mu_1 & 0 \\
\omega_1\mu_1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
q^1 \\
+ \begin{bmatrix}
k_1 - \mu_1\omega_1^2 & 0 & 0 \\
0 & k_2 - \mu_1\omega_2^2 & 0 \\
0 & 0 & k_3
\end{bmatrix}
q^1 = 0
\]

where \( q^1 = [u^1, u^2, u^3]^T \) (realizing that \( \beta^1_1 = \beta^1_2 = \beta^1_3 = 0 \), since \( m_1 \) is a point mass).

If the equation is rewritten in first-order form, as in Eq. (18), it becomes

\[ q_{u1} \dot{Q}^1 + \gamma_1 Q^1 = 0 \]

where

\[
q_{u1} = \begin{bmatrix}
k_1 - \omega_1^2\mu_1 & 0 & 0 & | & 0 & 0 \\
0 & k_2 - \omega_2^2\mu_1 & 0 & | & 0 & 0 \\
0 & 0 & k_3 & | & \mu_1 & 0 \\
0 & 0 & 0 & | & \mu_1 & 0 \\
0 & 0 & 0 & | & 0 & \mu_1
\end{bmatrix}
\]

\[
\gamma_1 = \begin{bmatrix}
-\omega_1^2\mu_1 & 0 & 0 & | & 0 & 0 \\
0 & -\omega_2^2\mu_1 & 0 & | & 0 & 0 \\
0 & 0 & -\omega_3^2\mu_1 & | & 0 & 0 \\
0 & 0 & 0 & | & -\omega_1\mu_1 & 0 \\
0 & 0 & 0 & | & 0 & -\omega_2\mu_1 \\
0 & 0 & 0 & | & 0 & 0 \\
0 & 0 & 0 & | & 0 & -\omega_3\mu_1 \\
0 & 0 & 0 & | & 0 & 0 \\
0 & 0 & 0 & | & 0 & 0
\end{bmatrix}
\]

and

\[ Q^1 = [q^1, \dot{q}^1]^T \]

The rotor appendage equation eigenvalues, \( \lambda_j \), and corresponding eigenvectors, \( \Phi_j \), may then be found from

\[ [q_{u1}\lambda_j + \gamma_1]\Phi_j = 0 \]
From the characteristic equation, one finds that

$$\lambda_j = \pm i \left[ \frac{k}{\mu_1} + \omega_y^2 \mp 2\omega_y \sqrt{\frac{k}{\mu_1}} \right]^{\frac{1}{2}}$$

and

$$\lambda_j = \pm i \left[ \frac{k_j}{\mu_1} \right]^{\frac{1}{2}}$$

where $k = k_1 = k_2$.

If we now arbitrarily let $\sqrt{k/\mu_1} = 2\omega_y$ and $\sqrt{k_3/\mu_1} = 5\omega_y$, the eigenvalues become

$$\lambda_1 = i\omega_y,$$
$$\lambda_2 = i3\omega_y,$$
$$\lambda_3 = i5\omega_y,$$
$$\lambda_4 = -i\omega_y,$$
$$\lambda_5 = -i3\omega_y,$$
$$\lambda_6 = -i5\omega_y.$$ 

Note that the eigenvalues are imaginary as predicted and that they have been deliberately ordered to correspond to the form of Eq. (22), with conjugates in the lower half of $\Lambda_1$.

The eigenvectors corresponding to these eigenvalues may then be determined as

$$\Phi_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ i & -i & 0 & -i & i & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ -i\omega_y & i3\omega_y & 0 & -i\omega_y & -i3\omega_y & 0 \\ 0 & 0 & i5\omega_y & 0 & 0 & -i5\omega_y \end{bmatrix} = \begin{bmatrix} \Phi_1^T \\ \Phi_1\lambda_j \end{bmatrix}$$

Also,

$$\Phi_1^T \Phi_1 \Phi_1^T = \begin{bmatrix} 8\mu_1\omega_y^2 & 24\mu_1\omega_y^2 & 0 \\ 24\mu_1\omega_y^2 & 50\mu_1\omega_y^2 & 8\mu_1\omega_y^2 \\ 0 & 8\mu_1\omega_y^2 & 24\mu_1\omega_y^2 \\ 0 & 50\mu_1\omega_y^2 & 50\mu_1\omega_y^2 \end{bmatrix}$$
The final form of the appendage modal coordinate equations, shown in Eq. (26), can be obtained only if the eigenvectors are normalized so that $\Phi_1^T \Phi_1 = U$, the diagonal unit matrix (see Ref. 3). Thus, succeeding columns in $\Phi_1$ should be multiplied by $(8\mu_1\omega_2)^{-\frac{1}{2}}, (24\mu_1\omega_2)^{-\frac{1}{2}}, (50\mu_1\omega_2)^{-\frac{1}{2}}$, etc., for proper normalization in this case.

If we also arbitrarily truncate this modal transformation to just the first two modes, the resulting real and imaginary parts of $\Phi_1$ become

$$\tilde{\Phi}_1 = \begin{bmatrix} 0 & 0 \\ \frac{1}{2\omega_1\sqrt{2\mu_1}} & \frac{1}{2\omega_2\sqrt{6\mu_1}} \end{bmatrix}, \quad \tilde{\Gamma}_1 = \begin{bmatrix} \frac{1}{2\omega_1\sqrt{2\mu_1}} & -\frac{1}{2\omega_2\sqrt{6\mu_1}} \\ 0 & 0 \end{bmatrix}$$

Likewise,

$$\bar{\sigma}^1 = \begin{bmatrix} \omega_1 & 0 \\ 0 & 3\omega_2 \end{bmatrix}, \quad \bar{\xi}^1 = \begin{bmatrix} \xi_1^1 & 0 \\ 0 & \xi_2^1 \end{bmatrix}$$

**Platform Appendage Equations**

If the same process is applied to the nominally nonspinning platform appendage, its homogeneous equation of motion becomes

$$\begin{bmatrix} \mu_2 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_2 \end{bmatrix} \ddot{q}^2 + \begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \dddot{q} = 0$$

Using the first-order equations again,

$$\mathcal{U}_2 \ddot{Q}^2 + \mathcal{V}_2 Q^2 = 0$$

where

$$\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \mu_2 \\ 0 & \mu_2 & 0 \\ \mu_2 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & \mu_2 \\ 0 & \mu_2 & 0 \\ \mu_2 & 0 & 0 \end{bmatrix}$$
one can easily determine that the eigenvalues are

$$\lambda_j = \pm i \sqrt{\frac{k_1}{\mu_2}} , \pm i \sqrt{\frac{k_2}{\mu_2}} , \pm i \sqrt{\frac{k_3}{\mu_2}}$$

If we let $k = k_1 = k_2 = k_3$, and $\sqrt{k/\mu_2} = \sigma_2$, then

$$\Lambda_2 = \begin{bmatrix} \sigma_2 & i & 0 \\ \sigma_2 & i & \sigma_2 \\ 0 & -\sigma_2 & -\sigma_2 \end{bmatrix}$$

and

$$\Phi_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ \sigma_2 & 0 & 0 & -\sigma_2 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & -\sigma_2 & 0 \\ 0 & 0 & \sigma_2 & 0 & 0 & -\sigma_2 \end{bmatrix} = \begin{bmatrix} \Phi_2 \\ \Phi_2 \lambda_j \end{bmatrix}$$
The appropriate normalization factor for each $\phi_i$ is $(2\mu_i\sigma_i^2)^{-\frac{1}{2}}$. Thus, if the platform appendage modal model is truncated to the first two (transverse bending) modes, the needed quantities are

$$\bar{\Psi}_2 = \begin{bmatrix} \frac{1}{\sigma_2\sqrt{2}\mu_2} & 0 \\ 0 & \frac{1}{\sigma_2\sqrt{2}\mu_2} \end{bmatrix}, \quad \bar{\Gamma}_2 = 0$$

$$\bar{\varphi}^2 = \begin{bmatrix} \sigma_2 & 0 \\ 0 & \sigma_2 \end{bmatrix}, \quad \bar{\xi}^2 = \begin{bmatrix} \xi_1^2 & 0 \\ 0 & \xi_2^2 \end{bmatrix}$$

**Test Vehicle Constants**

To complete the specification of the test configuration shown in Fig. 3, numerical values can now be assigned to its various mass properties and other physical constants. First, let

- $\mu_0 = 399.9$ kg
- $\mu_1 = 50.1$ kg
- $\mu_2 = 50.0$ kg
- $m_1 = 1.0$ kg
- $m_2 = 5.0$ kg

$$\bar{J}_0 = \begin{bmatrix} 250. & 0. & 0. \\ 0. & 275. & 0. \\ 0. & 0. & 350. \end{bmatrix}, \text{kg-m}^2$$

$$\bar{J}_1 = \begin{bmatrix} 10. & 0. & 0. \\ 0. & 10. & 0. \\ 0. & 0. & 20. \end{bmatrix}, \text{kg-m}^2$$

$\therefore \quad \mu = \mu_0 + \mu_1 + \mu_2 = 500.0$ kg

$\therefore \quad \mu_1 = .998$ kg

$\therefore \quad \mu_2 = 4.95$ kg
\[ J^2 = \begin{bmatrix} 6.0 & 0 & 0 \\ 0 & 3.0 & 0 \\ 0 & 0 & 8.0 \end{bmatrix}, \text{kg} \cdot \text{m}^2 \]

Also, let

\[ \omega_1 = 10. \text{ rad/s}, \quad \xi_1^1 = \xi_1^2 = 0.01 \]

\[ \omega_2 = 9. \text{ rad/s}, \quad \xi_2^1 = \xi_2^2 = 0.01 \]

\[ \bar{\psi}_1 = \begin{bmatrix} 0 & 0 \\ 0.035391 & 0.020433 \end{bmatrix}, \quad \bar{\Gamma}_1 = \begin{bmatrix} 0.035391 & -0.020433 \\ 0 & 0 \end{bmatrix} \]

\[ \bar{\psi}_2 = \begin{bmatrix} 0.035313 & 0 \\ 0 & 0.035313 \end{bmatrix}, \quad \bar{\Gamma}_2 = 0 \]

The locations of the two point masses (see Figs. 4 and 5) relative to their substructure's mass center when they are in the nominal deformed state will be assumed as

\[ r_1 = [0.33 \quad 0 \quad -0.493]^T \text{ meters} \]

\[ r_2 = [0 \quad 0 \quad 0.56]^T \text{ meters} \]

Fig. 4. Substructure \( \alpha_1 \)
Fig. 5. Substructure \( z_2 \)

Locations for the interbody connections, relative to substructure mass centers, are

\[
p^{01} = [0, 0, -2]^T \text{ meters} \\
p^{02} = [0, 1, 0]^T \text{ meters} \\
p^{10} = [0, 0, 0]^T \text{ meters} \\
p^{20} = [0, -3, 0]^T \text{ meters}
\]

The three hinge directions are given by the direction cosines

rotor: \( g^1 = [0, 0, 1]^T \)

platform: \( g^2 = [0, 0, 1]^T \)

platform: \( g^3 = [1, 0, 0]^T \)

Also,

\( n_e = 2, \ n_f = 2, \ n_1 = 1, \ n_2 = 1, \ N_1 = 2, \ N_2 = 2 \)

\( h_1 = 0, \ h_2 = 0, \ d_1 = 1, \ d_2 = 2, \ n = 3 \)

As a result of these choices, the initializing call statement arguments become

\[
NC = 2
\]

\[
H = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix}
\]
\[ MB = \begin{bmatrix} 250. & 275. & 350. & 0. & 0. & 0. & 399.9 \end{bmatrix} \]

\[ MS = \begin{bmatrix} 10. & 10. & 20. & 0. & 0. & 0. & 50.1 \\ -6. & -3. & -8. & 0. & 0. & 0. & -50.0 \end{bmatrix} \]

\[ PB = \begin{bmatrix} 0. & 0. & -2. \\ 0. & 1. & 1. \end{bmatrix} \]

\[ PS(2, 2, j) = \begin{bmatrix} 0. & -3 & 0. \end{bmatrix} \text{ (all other PS elements are zero)} \]

\[ G = \begin{bmatrix} 0. & 0. & 1. \\ 0. & 0. & 1. \\ 1. & 0. & 0. \end{bmatrix} \]

\[ PI = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \text{ (assuming no prescribed hinge motions)} \]

\[ NF = 2 \]

\[ F = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 1 & 2 \end{bmatrix} \]

\[ ER(1, i, j) = \begin{bmatrix} 0. & 0. \\ .035391 & .020433 \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \end{bmatrix} \]

\[ EI(1, i, j) = \begin{bmatrix} .035391 & -0.020433 \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \\ 0. & 0. \end{bmatrix} \]
\[ ER(2, i, j) = \begin{bmatrix}
0.035313 & 0. \\
0. & 0.035313 \\
0. & 0. \\
0. & 0. \\
0. & 0. \\
0. & 0.
\end{bmatrix} \]

\[ EI(2, i, j) = 0. \]

\[ SR = \begin{bmatrix}
0. & 0. & 10. \\
0. & 0. & 0.
\end{bmatrix} \]

\[ MF(1, 1, j) = [0. \ 0. \ 0. \ 0. \ 0. \ 1.0] \]

\[ MF(2, 1, j) = [0. \ 0. \ 0. \ 0. \ 0. \ 5.0] \]

\[ RF(1, 1, j) = [.3333 \ 0. \ -.4930] \]

\[ RF(2, 1, j) = [0. \ 0. \ .56] \]

\[ WF = \begin{bmatrix}
10. & 30. \\
\end{bmatrix} \]

\[ ZF = \begin{bmatrix}
.01 & .01 \\
.01 & .01
\end{bmatrix} \]

**Test Vehicle Dynamics**

Before simulating a specific dynamic case for the test vehicle of Fig. 3, the characteristics of the interbody connections must be defined. The connection between \( \delta_0 \) and rotor \( \delta_1 \) will be assumed a frictionless bearing so that

\[ \tau_1 = 0 \]

The platform hinge connections will be assumed to be of the linear spring-damper type, i.e.,

\[ \tau_2 = -K_2(y_2 - y_{2c}) - B_2\dot{y}_2 \]

\[ \tau_3 = -K_3(y_3 - y_{3c}) - B_3\dot{y}_3, \]
where \( \gamma_{2c} \) and \( \gamma_{3c} \) are platform angular position commands. The values of the constants \( K_2, K_3, B_2, B_3 \) are arbitrarily chosen as

\[
K_2 = 250. \text{ n-m/rad}, \quad B_2 = 50. \text{ n-m-s/rad}
\]

\[
K_3 = 300. \text{ n-m/rad}, \quad B_3 = 50. \text{ n-m-s/rad}
\]

The dynamic response to be simulated here will be that due to a high-rate platform slew sequence. Slew commands \( \gamma_{2c} \) and \( \gamma_{3c} \) will be generated by integrating the time functions shown in Fig. 6. This will result in a 10-deg rotation about \( g^2 \) and a 10-deg rotation about \( g^3 \).

Initially, the rotor is spinning at 10 rad/s relative to \( \mathbf{e}_0 \) and the rotor appendage is at rest relative to the rotor but deflected radially outward in its steady-state deformed position. (One can show from Eq. (17), with the assumption \( k/\mu_1 = 4\omega_0^2 \), that the radial deformation (in the \( b_1 \) direction) due to spin is \( r_s/3 \), where \( r_s \) is the distance from the rotor spin axis to the appendage attachment point.) The platform, \( \mathbf{e}_1 \), as well as the base body, \( \mathbf{e}_0 \), are initially at rest. At \( t = 1 \) s, the command is issued to rotate the platform about \( g^2 \) at a rate of 10 deg/s until \( t = 2 \) s; again at \( t = 3 \) s, a command to rotate about the \( g^2 \) axis at 10 deg/s appears and ends at \( t = 4 \) s. The computer simulation program, employing MBDYFR, for this dynamic maneuver is shown in Fig. 7.

Notice that the necessary dimension specifications for each variable are stated in the JPL CSSL III simulation language as: ARRAY MB(7), MS(2, 7), . . . , etc.

An auxiliary routine, called HCK, is used in the simulation to keep track of the rotations of the reference body (\( \mathbf{e}_0 \)) relative to an inertially fixed frame. HCK uses Euler parameters to do this, and it is initialized using Euler angles. The variable, THET, is calculated in the program by means of HCK and represents the angular deviation of the \( b_0 \) axis from its initial, inertially fixed position, i.e., the reference body "nutation" angle.

The CSSL III function, "STEP," provides the unit step function when the independent variable, TIME, is greater than the specified constant. "INTEG(\( a_1, a_2 \)) signifies the integration of \( a_1 \) with respect to TIME, where \( a_2 \) is the initial condition.

![Fig. 6. Commanded slew rates](image)

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**START**

**PROGRAM** 3-BODY VEHICLE WITH SPINNING ROTOR AND 2 FLEXIBLE APPENDAGES

*SC*4020

**BLOG/198, BOX/601, CAMERA/149, FRAMES/50

ARRAY MB(7), MS(2,7), PB(2,1), PS(2,2,3), G(2,3), TH(2,3), TS(2,3)

ARRAY FB(3), FS(2,3), GM(3), GM2(3), GM3(3), ER(2,6,2), EI(2,6,2)

ARRAY HF(2,1,7), RF(2,1,3), WF(2,2,1), ZF(2,2,1), TF(2,1,3), FF(2,1,3)

ARRAY SM(2,3), DT(2,2), ET(2,2), WD(3), UD(2,1,3), UD(2,1,3)

**DOUBLE PRECISION** WD(6), DT(2,2), ET(2,2), EC(14)

**INTEGER** NC, NF, PB1(2,2), F(2,2), PI(4)

**DATA**

H/0,0,1,2/

MB/250.,275.,350.,10.,10.,10.,999.9/

MS(l,2)/10./MS(l,3)/20./MS(l,7)/50./

PB1(2,2)/PB1(1,2)/PB1(2,1)/PB1(3,1)/

PS(2,2,1)/PS(2,2,2)/PS(2,2,3)/PS(2,2,4)/

G(l,1)/G(l,2)/G(l,3)/G(l,1)/G(l,1)/G(l,1)/

F(l,1)/F(l,2)/F(l,3)/F(l,1)/F(l,2)/F(l,3)/

ER(l,2,1)/0.3539g75/ER(l,2,2)/0.2043284/

PI(1,1,1)/0.3539g75/PI(1,1,2)/0.2043284/

PI(1,1,1)/0.3539g75/PI(1,1,2)/0.2043284/

K=250., B2=50., K3=300., B3=50./

TFINAL=10., CLKTIM=900., PIE=3.14159265/

PHIZ=0., THETZ=0., PSIZ=0./

GM=0., GM2=0., GM3=0.,

**INITIAL**

NC=2, NF=2

CALL MBDYFR(NC, MB, MS, PB, PS, G, PI, NF, ER, EI, SR, RF, WF, ZF)

END

**DERIVATIVEAL**

IF (TIME.GE.TFINAL) GO TO 51

**OUTPUT**

W0(1)=W1, W0(2)=W2, W0(3)=W3

GM(1)=GM1, GM(2)=GM2, GM(3)=GM3

*Fig. 7. Simulation program for test vehicle dynamics using MBDYFR*
Fig. 7 (contd)
All arithmetic statements are in Fortran, although CSSL III allows several statements in a single line if separated by a "$". Variables to be plotted at every communication interval, CI, are listed in the PREPAR statement. Printed variables are listed in the OUTPUT statement.

The statement "CALL MBDYFR(NC, H, ...)" is located in the INITIAL section and is therefore executed only once, i.e., prior to the dynamic calculations. However, "CALL MRATE(NC, ... )" is in the DERIVATIVE section and is thus executed at every integration step. Note that two additional output variables have been added to the MRATE call statement argument list. They are U and UD, containing the appendage deformations $u_1, u_2, u_3, u_4, \ldots$ etc. and the deformation rates $\dot{u}_1, \dot{u}_2, \ldots$, respectively. These variables are always available internal to MBDYFR using the relations of Eq. (27) and are outputted here only to more clearly illustrate the dynamic response of the system. ($\beta_1, \beta_2, \ldots, \beta_1, \beta_2, \ldots$ etc. could also be obtained from the subroutine in those cases where the appendage nodal bodies have inertia.)

Results of the dynamic simulation are shown in the computer plots of Fig. 8, and the sample printout is presented in Fig. 9.

The solutions show, as expected, that all three components of the reference body angular velocity, $\omega_0$, are strongly perturbed by the platform as it accelerates or decelerates. Further, induced vibrations of the platform appendage are also in evidence on the reference body rates. Rotor spin rate, $\gamma_1$, relative to $\omega_0$ remains very close to its initial and nominal value of 10 rad/s, although the effect of slewing the platform about an axis parallel to rotor spin is quite evident as are the subsequent vibrations due to platform appendage motion. Platform hinge rates, $\gamma_2$ and $\gamma_3$, also show some appendage vibration, although it is very small compared to the slewing rate transients.

The components of rotor appendage deformation $u_1, u_2$ exhibit both modal frequencies, $\omega_k$ and $3\omega_k$, but are relatively small in amplitude compared to the platform appendage deflections $u_1, u_2$. An "X-Y" plot of the platform appendage's deflections relative to its locally fixed coordinate frame is also shown.

System angular momentum magnitude in this test simulation should remain constant since no external forces or torques are being applied. The plot of $HM$ shows this to be true very closely. Small deviations from a perfectly constant angular momentum in the simulations are to be expected due to the presence of modal damping (see Appendix A), numerical integration error, and round-off error.

IV. Systems With Nonrotating Appendages

A. Equations

In Part III, dynamical equations were developed for the substructure tree on the basis of (1) arbitrarily small flexible appendage deformations (and rates) from some nominal state and (2) arbitrarily small deviations of the angular rate of any rigid appendage base from a constant nonzero spin rate, $\Omega^k$. In this section, the assumption will be made that $\Omega^k = 0$ ($k \in K$), i.e., that the appendage bases are nonrotating.
Fig. 8. Test vehicle (with spinning rotor) simulation results using MBDYFR
Fig. 8 (contd)
Fig. 8 (contd)
Fig. 8 (contd)
Fig. 9. Simulation printout for test vehicle with spinning rotor
Equation (29) may now be simplified by the assumptions (for \( k \in \mathbb{F} \)) \( \omega^k \approx 0, \ q^k \approx 0, \ q^k \approx 0, \ c^k \approx 0, \ c^k \approx 0, \) to obtain
\[
(k \in \mathbb{F}) \quad W^k = T^k + \sum_{r \in \mathbb{R}} \tilde{D}^{kr} C^{kr} F^r + \left[ \tilde{F}^k - \left( C^{ko} \frac{\ddot{\mathcal{M}}_k}{\mathcal{M}} F^r \right)^r \right] c^k
\]
\[
- \sum_{r \in \mathbb{R}} \tilde{D}^{kr} C^{kr} \Sigma_{U0}^T M' q^r - \sum_{r \in \mathbb{R}} \Phi^{kr} C^{kr} \omega^r - \dot{h}^k
\]
\[
- \omega^k h^k - \Sigma_{U0}^T \tilde{r}_k M' \dot{q}^k - \Sigma_{\tilde{U}0}^T M' \dot{q}^k
\]
\[
+ \mathcal{M}_k \tilde{c}_k \sum_{r \in \mathbb{R} - \mathbb{F}} C^{kr} \omega^r \tilde{D}^{rk}
\]
\[
+ \mathcal{M}_k \sum_{r \in \mathbb{R} - \mathbb{F}} \tilde{D}^{kr} C^{kr} \omega^r \tilde{D}^{rk} - \omega^k \Phi^{kk} \omega^k
\]
\[
- \sum_{r \in \mathbb{R}} \mathcal{M}_k \tilde{c}^k \sum_{r \in \mathbb{R} - \mathbb{F}} C^{kr} \tilde{D}^{rk} \omega^r
\]  
(64)

The appendage equation (Eq. 16) may be simplified as well (letting \( R^k = 0 \)) to obtain
\[
(k \in \mathbb{F}) \quad M^k \left( U - \Sigma_{U0} \Sigma_{U0}^T \frac{M^k}{\mathcal{M}} \right) \ddot{q}_k + K^k \dot{q}_k
\]
\[
= -M^k (\Sigma_{U0} - \tilde{r}_k \Sigma_{U0}) \omega_k - M^k \Sigma_{U0} \sum_{r \in \mathbb{R}} C^{kr} \tilde{\omega}_r \tilde{D}^{rk}
\]
\[
- M^k \Sigma_{U0} C^{k0} \frac{F}{\mathcal{M}} + \lambda^k + M^k \Sigma_{U0} \sum_{r \in \mathbb{R} - \mathbb{F}} C^{kr} \Sigma_{U0}^T \frac{M'}{\mathcal{M}} \ddot{q}_r
\]
\[
- M^k \Sigma_{U0} \sum_{r \in \mathbb{R} - \mathbb{F}} C^{kr} \tilde{\omega}_r \tilde{D}^{rk}
\]  
(65)

This appendage equation is analogous to that in Eq. (207) of Ref. 2, whose homogeneous solution has the form
\[
q^k = \sum_{j=1}^{6n_k} a_j e^{\lambda_j t} \phi^k_j
\]
where \( \lambda_j \) and \( \phi^k_j \) are, respectively, eigenvalues and eigenvectors available from
\[
(M' \lambda_j^2 + K') \phi^k_j = 0
\]
and
\[
M' = M^k \left( U - \Sigma_{U0} \Sigma_{U0}^T \frac{M^k}{\mathcal{M}} \right)
\]
\[
K' = K^k
\]
If $\phi_k$ is the $6n_k$ by $6n_k$ matrix
\[
\phi_k \equiv [\phi_k^1 \phi_k^2 \ldots \phi_k^{6n_k}]
\]
the transformation
\[
q^k = \phi_k \eta^k
\]
may be used to transform Eq. (65) into
\[
\ddot{\eta}^k + \sigma_k^2 \eta^k = \phi_k^T L_k'
\]
where
\[
L_k = -M^k (\Sigma_{00} - \Sigma_{0U}) \omega^k - M^k \Sigma_{U0} \sum_{r \in H} C^{kr} \omega^r D^r
\]
\[
- M^k \Sigma_{U0} C^{k0} \frac{F}{3\Omega} + \lambda^k + M^k \Sigma_{U0} \sum_{r \in H - k} C^{kr} \Sigma_{0r} M^r \phi_r \eta^r
\]
\[
- M^k \Sigma_{U0} \sum_{r \in H - k} C^{kr} \omega^r D^r
\]
If the modal coordinates $\eta_1^k, \eta_2^k, \ldots, \eta_{6n_k}^k$ are now truncated to the set $\eta_1^k, \ldots, \eta_{6n_k}^k$ (as symbolized by the overbar) and modal damping is also incor-porated, Eq. (67) becomes
\[
\ddot{\eta}^k + 2\zeta_k \dot{\eta}^k + \omega_n^2 \eta^k = \phi_k^T L_k'
\]
Returning to the vehicle substructure equation, Eq. (64), the truncated modal transformation, $q^k \approx \phi_k \eta^k$, may be substituted and the result combined with Eqs. (2), (3), (5), and (6) to give
\[
A^{00} \omega^0 + \sum_{j \in \mathcal{S}} A^{0j} \dot{\gamma}_j + \sum_{k \in \mathcal{S}} A^{0k} \dot{\eta}^k = \sum_{k \in \mathcal{S}} C^{0k} E^k
\]
\[
(i \in \mathcal{S}) \quad A^{i0} \omega^0 + \sum_{j \in \mathcal{S}} A^{ij} \dot{\gamma}_j + \sum_{k \in \mathcal{S}} A^{ik} \dot{\eta}^k = g^T \sum_{k \in \mathcal{S}} e_{ik} C^{ik} E^k + \tau_i
\]
where
\[
A^{00} = \sum_{k \in \mathcal{S}} \sum_{r \in \mathcal{S}} C^{0k} \phi^{kr} C^{r0}, \quad 3 \text{ by } 3
\]
\[
A^{0j} = \sum_{k \in \mathcal{S}} \sum_{r \in \mathcal{S}} C^{0k} \phi^{kr} C^{rj} g^j, \quad 3 \text{ by } 1
\]
\[
A^{0k} = C^{0k} \left( \hat{\Delta}^{kT} + \sum_{r \in \mathcal{S}} C^{kr} \hat{D}^{rk} C^{rk} \right), \quad 3 \text{ by } N_k
\]
\[
A^{i0} = g^T \sum_{k \in \mathcal{S}} \sum_{r \in \mathcal{S}} C^{ik} e_{ik} \phi^{kr} C^{r0}, \quad 1 \text{ by } 3
\]
\[
A^{ij} = g^T \sum_{k \in \mathcal{S}} \sum_{r \in \mathcal{S}} C^{ik} e_{ik} \phi^{kr} C^{rj}, \quad 1 \text{ by } 1
\[ A^k = g^T \left( \epsilon_{kq} C^{k q} \Delta^k + \sum_{r \in S} \epsilon_{q r} C^{k r} \bar{D}^{k r} C^{k r} \bar{F}^k \right), \quad \text{by } N_k \]

\[ E^k = T^k - r^k - \bar{g}(k \bar{\omega}^k + \bar{\psi}^k) + \sum_{r \in S} \bar{D}^{k r} C^{k r} F^r \]

\[ + \left[ \bar{F}^k - \left( C^{k q} \frac{\bar{\omega}^k}{\bar{\omega}^q} \right) \bar{F}^k + \frac{\bar{\omega}^k}{\bar{\omega}^q} \sum_{r \in S \setminus q} \bar{D}^{k r} C^{k r} \bar{\omega}^r D^{k r} \right] \]

\[- \bar{\omega}^k \Phi^k \omega^k - \sum_{r \in S \setminus q} \Phi^k \sum_{j \in \mathcal{G}} C^{k r} \bar{\omega}^r \bar{\omega}^j D^{k r}, \quad \text{by } 3 \]

\[ \Phi^k = \bar{\Phi}^k C^{k r} + \bar{\Pi}_k \bar{\omega}^k C^{k r} + \bar{\Pi}_r \bar{D}^{k r} \bar{\omega}^r, \quad \text{by } 3 \]

\[ \bar{\Delta}^k = \bar{\Phi}^k M^k (\Sigma_{U0} - \tilde{r}_k \Sigma_{U0}), \quad N_k \text{ by } 3 \]

\[ \bar{F}^k = \Sigma_{U0} M^k \bar{\Phi}^k, \quad \text{by } N_k \]

(\( \Phi^k \) does not include the effects of appendage deformation.)

As in Eqs. (32) and (33), substitutions have been made for \( \bar{h}^k \) and \( \bar{h}^r \) based on restriction to three orthogonal axisymmetric rotors in \( S_k \), with spin axes aligned to the unit vectors \( (b^k) \), and the relations in Eqs. (43)-(45). Again, it is to be understood that any rotor's moments of inertia are to be included in \( J^k \), the undeformed substructure's inertia dyadic for \( \sigma_k \), and its mass is included in the substructure mass, \( \Pi_k \).

Operating on the appendage equation, Eq. (68), in a similar way provides

\[ \begin{align*}
(k \in \mathcal{G}) \quad A^{k 0} \omega^0 + \sum_{j \in \mathcal{G}} A^{k j} \gamma^j + \sum_{r \in \mathcal{G}} A^{k r} \bar{\gamma}^r &= Q^k
\end{align*} \]

where

\[ A^{k 0} = \bar{\Delta}^k C^{k 0} - \bar{F}^k T \sum_{r \in S \setminus q} C^{k r} \bar{D}^{k r} C^{r 0}, \quad N_k \text{ by } 3 \]

\[ A^{k j} = \left( \bar{\Delta}^k C^{k j} \epsilon^j_k - \bar{F}^k T \sum_{r \in S \setminus q} C^{k r} \bar{D}^{k r} C^{r j} \gamma^j \right) \gamma^j, \quad N_k \text{ by } 1 \]

\[ A^{k r} = -\bar{F}^k C^{k r} \bar{\Phi}^r \frac{\bar{\Pi}}{\bar{\Pi}}, \quad (r \neq k); \quad N_k \text{ by } N_r \]

\[ A^{k k} = U, \quad (r = k); \quad N_k \text{ by } N_k \]
\[ Q^k = -2\ddot{\xi}_k \dot{\eta}^k - \dddot{\eta}^k - \dddot{\eta}^k C \dot{\eta}_0 - \frac{F}{\partial \eta} \_\eta + \dddot{\eta}^k \lambda^k \]

\[ - \sum_{j \in \Omega} \left( \dot{\xi}^k C \dot{\eta}_j \_\eta^k - \dddot{\eta}^k \sum_{r \in \Omega} C^{kr} \dot{\eta}_r C \dot{\eta}_j \_\eta^r \right) \dot{\omega}^k \dot{\omega}_j \_\eta^j \]

\[ - \dddot{\eta}^k \sum_{r \in \Omega - \Omega} C^{kr} \dot{\omega}^r \dot{\omega}^r D^k, \quad N_k \text{ by } 1 \]

where modal damping, \( \dddot{\xi}_k \), has been added (see discussion in Section IIIA).

The substructure and appendage equations may now be combined into a single matrix equation of the form \( A \dot{x} = B \),

\[
\begin{bmatrix}
A^{00} & A^{0j} & A^{0k} \\
A^{j0} & A^{jj} & A^{jk} \\
A^{k0} & A^{kj} & A^{kk}
\end{bmatrix}
\begin{bmatrix}
\dot{\omega}^0 \\
\dot{\eta}_j \\
\dot{\eta}^k
\end{bmatrix}
= \begin{bmatrix}
\sum_{k \in \Omega} C^{0k} \dot{E}^k \\
\dddot{\eta}_j \sum_{r \in \Omega} \epsilon_{kr} C^{ik} \dot{E}^k + \dddot{\eta}_k \\
Q^k
\end{bmatrix}
\]

(71)

Again the elements of \( A \) are, in general, time-variable because of substructure relative motion. \( A \) is also symmetric.

Very often, one can justify making the assumption that all the variables, i.e., \( \omega^0 \), \( \gamma_j \), \( \eta^k \), and their derivatives are in some sense "small" and a complete linearization of Eq. (71) may be carried out. The computational benefits of a total linearization are quite substantial since the coefficient matrix, \( A \), then becomes formally constant, allowing its inverse to be computed only once, in advance of numerical integration.

If each symbol in Eq. (71) is expanded into three parts, the first being free of the variables \( \omega^0 \), \( \gamma_j \), \( \eta^k \), and their derivatives (indicated by overbar), the second being linear in these variables (indicated by overcaret), and the third containing terms above the first degree in the variables (indicated by three dots), and if one then determines explicit expressions for the new barred and caret symbols from their definitions, the linearized form of Eq. (71) becomes

\[ \overline{A}^{00} \overline{\omega}^0 + \sum_{j \in \Omega} \overline{A}^{0j} \overline{\gamma}_j + \sum_{k \in \Omega} \overline{A}^{0k} \overline{\eta}^k = \sum_{k \in \Omega} \left[ \overline{C}^{0k} (\overline{E}^k + \hat{E}^k) + \hat{C}^{0k} \overline{E}^k \right] \] (72a)

\[ \overline{A}^{i0} \overline{\omega}^0 + \sum_{j \in \Omega} \overline{A}^{ij} \overline{\gamma}_j + \sum_{k \in \Omega} \overline{A}^{ik} \overline{\eta}^k \]

\[ = g^{i r} \sum_{k \in \Omega} \epsilon_{kr} \left[ \overline{C}^{ik} (\overline{E}^k + \hat{E}^k) + \hat{C}^{ik} \overline{E}^k \right] + \dddot{\eta}_i + \dddot{\eta}_i \] (72b)

\[ \overline{A}^{k0} \overline{\omega}^0 + \sum_{j \in \Omega} \overline{A}^{kj} \overline{\gamma}_j + \sum_{r \in \Omega} \overline{A}^{kr} \overline{\eta}^r = \overline{Q}^k + \hat{Q}^k \] (72c)
where

\[ \hat{C}^i = \hat{C}^{i0} + \hat{C}^i + \cdots \]

\[ \tau_i = \hat{\tau}_i + \tilde{\tau}_i + \cdots \]

\[ A^i = \tilde{A}^i + \hat{A}^i + \cdots \]

\[ E^k = \tilde{E}^k + \hat{E}^k + \cdots \]

etc.,

and

\[ \hat{C}^j = \hat{C}^{jr} = U = 3 \text{ by 3 identity matrix} \]

\[ \hat{C}^j = -\gamma_r \bar{g}_r, \quad (r > j) \]

\[ \hat{C}^{jr} = \gamma_r \bar{g}_r = (\hat{C}^j)^T \]

Specifically,

\[ \hat{A}^{i0} = \sum_{k \in \mathbb{R}} \sum_{r \in \mathbb{R}} \tilde{A}^{kr} \]

\[ \hat{A}^{ij} = \sum_{k \in \mathbb{R}} \sum_{r \in \mathbb{R}} \tilde{A}^{kr} e_j e_i \]

\[ \hat{A}^{ik} = \tilde{A}^{ik} + \sum_{r \in \mathbb{R}} \tilde{D}^{ik} \tilde{F}^r \]

\[ \hat{A}^{i0} = \tilde{A}^{i0} + \sum_{r \in \mathbb{R}} e_i \tilde{A}^{kr} \]

\[ \hat{A}^{ij} = \tilde{A}^{ij} + \sum_{r \in \mathbb{R}} e_i e_j \tilde{A}^{kr} \]

\[ \hat{A}^{ik} = \tilde{A}^{ik} + \sum_{r \in \mathbb{R}} e_i \tilde{D}^{ik} \tilde{F}^r \]

\[ \hat{A}^{i0} = \tilde{A}^{i0} + \sum_{r \in \mathbb{R}} \tilde{D}^{ik} \tilde{F}^r \]

\[ \hat{A}^{ik} = \tilde{A}^{ik} + \sum_{r \in \mathbb{R}} \tilde{D}^{ik} \tilde{F}^r \]

\[ \hat{E}^k = \tilde{F}^k - \hat{\tau}_k^k + \sum_{r \in \mathbb{R}} \tilde{D}^{kr} \tilde{F}^r + \sum_{r \in \mathbb{R}} \hat{\tau}_k^r \tilde{F}^r + \left[ \tilde{F}^k - \left( \frac{\partial^2}{\partial \tau_k^i} \tilde{F} \right) \right] c_k \]

\[ \tilde{A}^{k0} = \tilde{A}^{k0} + \sum_{r \in \mathbb{R}} \tilde{D}^{kr} \]

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\[
\bar{A}^j = \left( \Delta^k_{jk} - \bar{F}^k - \sum_{r \in \mathcal{S}} \bar{D}^r_{k} \bar{G}_r \right) e^j
\]

\[
\bar{A}^r = -\bar{F}^r \frac{\bar{P}^r}{\bar{Q}} \quad (r \neq k)
\]

\[
\bar{A}^k = U_r \quad (r = k)
\]

\[
\bar{Q}^k = -\bar{F}^k \frac{\bar{F}}{\bar{Q}} + \bar{G}_k \bar{X}_k
\]

\[
\bar{Q}^k = -\bar{F}^k \frac{\bar{F}}{\bar{Q}} + \bar{G}_k \bar{X}_k
\]

It would remain then to determine \( \bar{T}^k, \bar{\tilde{T}}^k, \bar{\tilde{F}}^k, \bar{\tilde{E}}^k, \bar{\tilde{F}}, \bar{\tilde{E}}, \bar{\tilde{X}}^k, \bar{\tilde{X}}^k, \bar{\tilde{\tau}}, \bar{\tilde{\tau}}, \bar{\tilde{\tau}}, \) etc., for the particular system under study and to carry out the computations in Eq. (72). However, in constructing a subroutine to perform these computations, it was found to be more efficient to directly manipulate the combined form

\[
\bar{A}^{00} \omega^0 + \sum_{j \in \mathcal{S}} \bar{A}^j \bar{\omega}_j + \sum_{k \in \mathcal{S}} \bar{A}^{ik} \bar{\eta}^k = \sum_{k \in \mathcal{S}} \bar{C}^{ik} \bar{E}_k = \bar{Q}^k \quad (i \in \mathcal{S})
\]

\[
\bar{A}^{00} \omega^0 + \sum_{j \in \mathcal{S}} \bar{A}^j \bar{\omega}_j + \sum_{k \in \mathcal{S}} \bar{A}^{ik} \bar{\eta}^k = \bar{G}^r \sum_{k \in \mathcal{S}} \bar{e}_k \bar{E}_k + \bar{\tau}_i \quad (k \in \mathcal{S})
\]

where

\[
\bar{E}_k = \bar{E}_k + \hat{E}_k
\]

\[
\bar{C}^{ik} = \bar{C}^{ik} + \hat{C}^{ik}
\]

\[
\bar{\tau}_i = \bar{\tau}_i + \hat{\tau}_i
\]

etc.

By avoiding the separation into the parts \( \bar{E}_k, \hat{E}_k, \) etc., the computation becomes more efficient even though some second-order terms in the linearized variables are retained.

**B. Subroutines MBDYFN, MBDYFL**

The Fortran V subroutines MBDYFN and MBDYFL were written to provide the solutions to Eqs. (71) and (73), respectively. As in the case of MBDYFR, these routines are also exercised by either of two call statements, the first of which initializes the program with the system constants.
Initializing Call Statements

CALL MBDYFN(NC, H, MB, MS, PB, PS, G, PI,
NF, F, EIG, REC, RF, WF, ZF)

or

CALL MBDYFL(NC, H, MB, MS, PB, PS, G, PI,
NF, F, EIG, REC, RF, WF, ZF)

All the arguments in these call statements are defined exactly as given in IIIB, with the exception of the two new arguments, EIG and REC. Notice that the MBDYFR inputs ER, EI, SR, and MF no longer are used in these routines. The input arrays RF and EIG are used by the subroutine only if there are nonzero external forces and torques \( \lambda^k \) applied to an appendage.

\[
\text{EIG}(n, i, j) = \text{array of elements of } \phi_k^i; \quad n = 1, 2, \ldots, n_f; \quad i = 1, 2, \ldots, 6n_k; \quad k = F(n, 1); \quad j = 1, 2, \ldots, N_k. \quad \text{(Note! This array is not used by the routine if } \lambda^k, \text{ for all } k \in \mathcal{F}, \text{ is zero.)}
\]

\[
\text{REC}(n, i, j) = \text{array containing the "rigid-elastic coupling coefficients," } \tilde{A}^k \text{ and } \tilde{P}^k; \quad n = 1, 2, \ldots, n_f; \quad i = 1, 2, \ldots, 6; \quad k = F(n, 1); \quad j = 1, 2, \ldots, N_k. \quad \text{(For } i = 1, 2, 3, \text{ the elements of REC are those of } \tilde{P}^k; \text{ for } i = 4, 5, 6, \text{ the elements are those of } \tilde{A}^k.\)
\]

In order to compute the angular accelerations \( \dot{\omega}, \ddot{\gamma}, \ldots, \dddot{\gamma}, \) and the modal coordinate acceleration vectors \( \ddot{\eta}^k (k \in \mathcal{F}) \) at every numerical integration step, the simulation must repeatedly enter the subroutine using the dynamic call statement.

Dynamic Call Statement

CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM,
GMD, GMDD, ET, ETD, WO, WDOT, ETDD, HM)

where

\[
\text{ET}(n, i) = \text{array of appendage modal coordinates, } \eta^k; \quad n = 1, \ldots, n_f; \quad k = F(n, 1), \quad i = 1, \ldots, N_k.
\]

\[
\text{ETD}(n, i) = \text{array of modal coordinate rates, } \dot{\eta}^k; \quad n = 1, \ldots, n_f; \quad k = F(n, 1), \quad i = 1, \ldots, N_k.
\]

\[
\text{ETDD}(n, i) = \text{solution array for modal coordinate accelerations, } \ddot{\eta}^k; \quad n = 1, \ldots, n_f; \quad k = F(n, 1), \quad i = 1, \ldots, N_k.
\]

and all other arguments are defined exactly as in IIIB.

Again, it should be noted that MBDYFN and MBDYFL do not incorporate the terms in \( E^k \) that describe rotor torques on \( \delta_k \). The user must include these terms, if rotors are present, in \( T^k \) (or \( \tilde{T}^k \)) as it is formed in the main program.
Also, if any of the $\gamma_i$ are to be prescribed, appropriate values of $\dot{\gamma}_i$, as well as $\gamma_i$ and $\dot{\gamma}_i$, must be supplied to the subroutine by way of the MRATE dummy arguments GMDD, GM, and GMD, respectively.

When either the MBDYFN or the MBDYFL subroutine is used, the main calling program must contain Fortran “type” and storage allocation statements. The mandatory statements are:

Required Specification Statements

```
INTEGER NC, NF, H(nc, 2), F(nf, 3), PI(n + 1)
REAL MB(7), MS(nc, 7), PB(nc, 3), PS(nc, nc, 3),
     G(n, 3), TH(n), TB(3), TS(nc, 3), FB(3), FS(nc, 3),
     GM(n), GMD(n), GMDD(n), EIG(n, 6nk, Nk), REC(n, 6, Nk),
     RF(n, nk, 3), WF(nk, NK), ZF(nk, NK),
     TF(n, nk, 3), FF(n, nk, 3), ET(nk, NK),
     ET(nk, NK), WO(3)
DOUBLE PRECISION WDOT(n + 3), ETDD(n, NK)
```

In order that storage allocation for arrays internal to MBDYFN and MBDYFL be minimized, the following statement must appear in the subroutine:

```
PARAMETER QH = n, QC = nc, QF = nf, NK = nk, NKT = Nk
```

The proper placement of this statement in MBDYFN and MBDYFL is shown in their listing (Appendices D and E).

C. Sample Problems

To illustrate the use of subroutines MBDYFN and MBDYFL, a sample problem suitable for computer simulation will be described. The test vehicle to be simulated has the configuration shown in Fig. 10—a rigid central body, $\delta_0$, a rigid platform, $\delta_1$, which is hinged to $\delta_0$ (2 degrees of freedom), and a flexible appendage, $\sigma$, also attached to $\delta_0$.

![Fig. 10. MBDYFN, MBDYFL simulation test vehicle](image-url)
For this example, the numbers used to describe the test vehicle's mass properties, including the appendage, were taken from an actual spacecraft design. The appendage model includes the characteristic vibration modes of four solar panels, a parabolic antenna, and several other structural members.

**Test Vehicle Constants**

The following numerical constants are required for initializing the subroutines:

$$m_0 = 79.0 \text{ kg}$$

$$m_1 = 1.93 \text{ kg}$$

$$\bar{J}^0 = \begin{bmatrix} 1230. & 16.29 & 43.45 \\ 1290. & -61.75 \\ \text{sym.} & \text{1650.} \end{bmatrix} \text{ kg-m}^2$$

$$\bar{J}^1 = \begin{bmatrix} 4.75 & 0. & 0. \\ 5.53 & 0. \\ \text{sym.} & 1.32 \end{bmatrix} \text{ kg-m}^2$$

Let the modal model for appendage $$\alpha_0 (Q_1)$$ be truncated to seven modes, i.e., $$N_0 = 7$$. Thus,

$$\bar{F}^0 = \sum_{0}^T M^0 \phi_0 =$$

$$\begin{bmatrix}
.0338 & .0106 & .0023 & .0032 & -.0655 & -.3050 & -.0276 \\
.0017 & .0011 & -.0182 & .0010 & -.5381 & 1.753 & 1.051 \\
-.8678 & -.00005 & 0. & 2.234 & 1.962 & .5585 & .3919
\end{bmatrix} \text{ kg-m}$$

$$\bar{\Delta}^0 = (\sum_{0}^T + \sum_{0}^T \tilde{\phi}) M^0 \phi_0 =$$

$$\begin{bmatrix}
.0814 & .4236 & 21.30 & -.4081 & 7.577 & -.4.320 & 2.032 \\
17.17 & 12.30 & -.2386 & 5.930 & .4020 & -.1589 & -.2.061 \\
0.0080 & 0.0019 & 0.0009 & -.0521 & 2.520 & -.9205 & .2761
\end{bmatrix} \text{ kg-m}^2$$

$$\bar{\phi}_0 = 2\pi [.5756 \quad .6134 \quad .6134 \quad .6307 \quad 2.723 \quad 2.963 \quad 3.047]^T \text{ rad/s}$$

$$\bar{\xi}_0 = [\ .20 \quad .20 \quad .20 \quad .20 \quad .05 \quad .05 \quad .01]^T$$
Also, let
\[ g^1 = [0, 0, 1]^T \]
\[ g^2 = [1, 0, 0]^T \]
\[ p^{01} = [0, 0, 0]^T, \quad p^{10} = [0, 0, 0]^T \]

Since no external forces or torques will be applied to the appendage, the eigenvector matrix \( \bar{\phi}_0 \) is not needed, nor is the matrix \( \bar{\tau}_0 \). Finally,
\[ n_x = 1, \quad n_f = 1, \quad n_0 = 1, \quad N_0 = 7 \]
\[ h_1 = 0, \quad d_i = 2, \quad n = 2 \]

The integer \( n_o \), which indicates the number of sub-bodies in the appendage model and is only required if external forces and torques are applied to appendage \( \alpha_o \), has been set to the smallest acceptable value that satisfies dimensioning requirements.

The initializing call statement arguments therefore become
\[ NC = 1 \]
\[ H = [0, 2] \]
\[ MB = [1230, 1290, 1650, -16.29, -43.45, 61.75, 79.0] \]
\[ MS = [4.75, 5.53, 1.32, 0, 0, 0, 1.93] \]
\[ PB = 0 \]
\[ PS = 0 \]
\[ G = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \]
\[ PI = [0, 0, 1] \] (assuming no prescribed hinge motions)
\[ NF = 1 \]
\[ F = [0, 1, 7] \]
\[ EIG = 0 \]
\[ REC = \]
\[
\begin{bmatrix}
0.0338 & 0.0106 & 0.0023 & 0.0032 & -0.6055 & -0.3050 & -0.0276 \\
0.0017 & 0.0111 & -0.0182 & 0.0010 & -0.5381 & 1.753 & 0.1051 \\
-0.8678 & -0.0005 & 0 & 2.234 & 1.962 & 0.5585 & 0.3919 \\
0.0814 & 0.4236 & 0.2130 & -0.4081 & 7.577 & -4.320 & 2.032 \\
17.17 & 12.30 & -0.2386 & 5.930 & 0.4020 & -0.1589 & -2.061 \\
0.0080 & 0.0019 & 0.0009 & -0.0521 & 2.520 & -0.9205 & 0.2761 \\
\end{bmatrix}
\]
Test Vehicle Dynamics

As before, the platform hinge connections will be defined as being of the linear spring and viscous damper type, but the position commands will be deleted, so that

\[
\tau_1 = -K_1 \gamma_1 - B_1 \dot{\gamma}_1
\]

\[
\tau_2 = -K_2 \gamma_2 - B_2 \dot{\gamma}_2
\]

where

\[
K_1 = 900. \text{ n-m/rad}
\]

\[
K_2 = 850. \text{ n-m/rad}
\]

\[
B_1 = 100. \text{ n-m-s/rad}
\]

\[
B_2 = 100. \text{ n-m-s/rad}
\]

The vehicle response to be simulated in this example will be that due to an arbitrary sequence of force and torque pulses applied to the reference body, \( \delta_0 \). A rectangular pulse of thrust will be applied in the \( b^0_2 \) direction with magnitude 300 n and a duration of 2 s, starting at \( t = .5 \) s. This will be followed by a 1-s torque pulse in the \( b^0_1 \) direction of magnitude 10. n-m, starting at \( t = 3.5 \) s. And the last disturbance will be a 1-s torque pulse in the \( b^0_1 \) direction of magnitude 10. n-m, starting at \( t = 6.5 \) s. The computer program for this dynamic simulation is given in Fig. 11.

Initially, the system is assumed to be completely at rest. Again, the CSSL III language function, “STEP,” is used to construct the applied pulses. Only the angular rates of \( \delta_0 \) are calculated in this example; its inertial angular position is not computed. Appendage modal coordinate rates and positions are both provided, although only the rates are plotted in the system responses of Fig. 12. A sample of the printed output is shown in Fig. 13.

Notice that by far the greatest disturbing effect to both platform and flexible appendage is due to the applied force. However, the changes in \( \omega_0 \) magnitude due to the torque disturbances are quite significant. It is not clear to what extent the platform vibrations are coupling with appendage vibrations and reference body motion, although the platform rotations are small in magnitude.

It is apparent that the applied force (fixed with respect to \( \delta_0 \)) caused some slight accumulation of system angular momentum as the system mass center moved in response to platform and appendage vibrations. This small amount (.17 n-m-s) was dwarfed, however, by the next pulse of torque, so that after 4.5 s, the angular momentum should have been approximately 10 n-m-s. The last torque pulse, applied orthogonally to the preceding one, would then raise the total angular momentum magnitude to slightly more than \( \sqrt{(10)^2 + (10)^2} = 14.14 \) n-m-s. The simulation printout shows a computed value of 14.25 n-m-s.
```
*** START

PROGRAM 2-BODY VEHICLE WITH FLEX. APPENDAGE
*SCRN020
BLO6/198,802/401,CAMERA/91N.FRAMES/50

COMMENT
ARRAY MB(7),HS(1,7),PS(1,1,3),G(2,3)
ARRAY EI(1,1,7),RF(1,1,3),REC(1,1,3),RF(1,1,7),ZF(1,7)
ARRAY TB(3),TS(1,3),FS(1,3),GM(2),GMD(2)
ARRAY TH(2),BD(3),TF(1,1,3),EF(1,1,3),ET(1,1,7),ETD(1,7)
DOUBLE PRECISION MMDT(5),ETD(1,7)
INTEGER NC,NF,H(1,2),F(I(3)),P(3)
DATA H(1,1)'/0'/H(1,2)'/7'/P(0)'/1'/
DATA P(1,2)'1'/F(1,3)'/7'/
DATA MB'/1230.0'/'1290.0'/'1550.0'/'1629.0'/'43.45'/'1.75'/'79.0/
DATA NS'/4.75'/'53.1'/'32.0'/'0.0'/'11.93/
DATA G(1,1)'/6.11'/11'/
DATA REC'/0337.0'/'0164.0'/'9478.0'/'0135.17'/'170795.0'/'
CONSTANT FINTIM-'IO'KCLKTM='I000 PIEl='3014159265'
CONSTANT k=' IO0 'k2=' 100 '

INITIAL
NC='1'NF='1'
DO 57 L=1,7
57 NR(1,1)='F(1,1)=2.0*PIE
CALL MBDFYN(INC,MB,HS,PS,G,P,NC,F,REC,RF,RF,F)

END

DYNAMIC
IF(TIMEGT+FINTIM)GO TO FIN
STPCLK CLKWITM
OUTPUT 10,W52,H3,NX,NY,FZ,ETA1,ETA2,ETA4,ETA5,ETAM1,ETAM2,ETA7,.....
ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,ANGR,ANGM=0,M3=0,30=0,31=O....
GM1=GM2=GM1=GM2
PREPAR W1,W2,W3,NX,NY,FZ,ETD1,ETD2,ETD3,ETD4,ETD5,ETD6,ETD7,.....
ANGM=ANGH=GM1=GM2=GM1=GM2
DERIVATIVE BODY2F
VARIABLE TIME=0.0 S CINTERVAL CI=0.1
XERROR =I=I.0 E6 S MERROR =I=I.0 E6

NOISORT
GM(1)=GM1 S GM(1)=GM1
GM(2)=GM2 S GM(2)=GM2
ET(1,1)=E1A1 S ET(1,1)=E1A1 S ET(1,1)=E1A1 S ET(1,1)=E1A1
ET(1,2)=ET12 S ET(1,2)=ET12 S ET(1,2)=ET12 S ET(1,2)=ET12
ETD(1,1)=ETD1 S ETD(1,1)=ETD1 S ETD(1,1)=ETD1 S ETD(1,1)=ETD1
ETD(1,2)=ETD2 S ETD(1,2)=ETD2 S ETD(1,2)=ETD2 S ETD(1,2)=ETD2
ETD(1,3)=ETD3 S ETD(1,3)=ETD3 S ETD(1,3)=ETD3 S ETD(1,3)=ETD3
ETD(1,4)=ETD4 S ETD(1,4)=ETD4 S ETD(1,4)=ETD4 S ETD(1,4)=ETD4
ETD(1,5)=ETD5 S ETD(1,5)=ETD5 S ETD(1,5)=ETD5 S ETD(1,5)=ETD5
W0(1)=1 W0(2)=2 W0(3)=3 S ANGM=MM

COMMENT

HINGE TORQUES

```

Fig. 11. Simulation program for test vehicle dynamics using MBDFYN
COMMENT
TH(1)==K1*GM1 = B1*GMID
TH(2)==K2*GM2 = B2*GMID
COMMENT**   FORCE EQUATION
COMMENT
FZ=(STEP(15,TIME)-STEP(12,TIME))*300.
FB(3)==FZ
COMMENT**   ENGINE TORQUE
COMMENT
NX=(STEP(3-5,TIME)-STEP(4-5,TIME))*10.
NY=(STEP(6+5,TIME)-STEP(7+5,TIME))*10.
TB(1)==NX  $  TB(2)==NY
COMMENT**   SOLUTION FOR SYSTEM ACCELERATIONS
COMMENT
CALL MRATEINCTH,TB,T5,FB,FS,TF,FF,GMID,GMDD,ET,ETD,NDOT....
ET0D,TH,
ND=NDOT(1)  $  ND2=NDOT(2)  $  ND3=NDOT(3)
COMMENT**   SYSTEM RATES AND POSITIONS
COMMENT
$1=INTEG(NDOT(1),0+)
$2=INTEG(NDOT(2),0+)
$3=INTEG(NDOT(3),0+)
ET01=INTEG(ET0D(1,1),0+)  $  ETA1=INTEG(ET01,0+)
ET02=INTEG(ET0D(1,2),0+)  $  ETA2=INTEG(ET02,0+)
ET03=INTEG(ET0D(1,3),0+)  $  ETA3=INTEG(ET03,0+)
ET04=INTEG(ET0D(1,4),0+)  $  ETA4=INTEG(ET04,0+)
ET05=INTEG(ET0D(1,5),0+)  $  ETA5=INTEG(ET05,0+)
ET06=INTEG(ET0D(1,6),0+)  $  ETA6=INTEG(ET06,0+)
ET07=INTEG(ET0D(1,7),0+)  $  ETA7=INTEG(ET07,0+)
GMID=INTEG(NDOT(4),0+)  $  GM1=INTEG(GMID,0+)
GM2D=INTEG(NDOT(5),0+)  $  GM2=INTEG(GM2D,0+)
END
END
END
TERMINAL
FIN... CONTINUE
END
END

Fig. 11 (contd)
Fig. 12. Test vehicle simulation results using MBDYFN
Fig. 12 (contd)
Fig. 12 (contd)
Fig. 12 (contd)
Exactly the same simulation can be made using the linearized subroutine version, MBDYFL. The only change necessary in the simulation program of Fig. 11 to allow the use of the linearized version is the change of "CALL MBDYFN(NC, . . . )" to "CALL MBDYFL(NC, . . . )" in the initialization section. This was done and resulted in solutions for the system response which are virtually indistinguishable from those plotted in Fig. 12. However, some slight deviations are detectable in the printed output shown in Fig. 14 when compared with the MBDYFN results of Fig. 13. The major difference between the two simulations in this case is reflected in the computer running time. A total of 2 min of accountable central processor time (Univac 1108) was required by the program using MBDFN as contrasted with only 1 min of central processor time used by the MBDYFL program. In addition, memory storage is considerably reduced by the use of MBDYFL, so that the overall cost of producing the desired solutions in this case is significantly reduced.

Another convenient method of reducing computation time and therefore cost under certain circumstances is to use these subroutines' prescribed variable option. By setting $P(i) = 1$, the hinge angle variables $\gamma_i$, $\dot{\gamma}_i$, and $\ddot{\gamma}_i$ may be prescribed, i.e., defined by the user in the main program rather than computed within the subroutine. When this is done, any expression in the main program defining the hinge torque $\tau_i(TH(i))$ is ignored by the subroutine. The equations normally solved by the subroutine to obtain $\ddot{\gamma}_i$ are then deleted from consideration, thus reducing the system order and speeding up calculations.

For an example of this approach, we can return to the program of Fig. 11, using MBDYFN, and change $P(1)$ so that $P(1) = 1$ and $P(2) = 1$ (leaving $P(3)$ = 1 unchanged so that the angular momentum calculation is still performed), as shown in Fig. 15. This means that the platform hinge rotations are to be prescribed. However, by not defining any function for GMDD(1) and GMDD(2), these variables remain zero, as will their integrals. Thus, the simulation will proceed as before but with $\ddot{\gamma}_i = \dot{\gamma}_i = \gamma_i = 0$ $(i = 1, 2)$; i.e., the platform will be "frozen" or rigidly connected to $\theta_p$.

The system response (with identical disturbances) in this configuration was simulated, and the plotted results were indistinguishable from those in Fig. 13. A sample of the simulation's printed output, shown in Fig. 16, indicates clearly that "freezing" the platform has had no significant effect on the dynamic response of the reference body or the appendage modal coordinates. However, some numerical differences are discernible in the printout.

Thus, prescribing the platform's "motion" in this case did not appreciably change the overall result and, as a matter of fact, took 15 s less computation time than the original run with no prescribed variables, a saving of $\frac{1}{6}$.

V. Summary and Conclusions

In this report, detailed mathematical models have been developed, suitable for describing the attitude dynamics of vehicles that may be idealized as systems of interconnected rigid bodies with possible terminal flexible appendages. The resulting mathematical formulations apply to two kinds of system behavior: (1) generally arbitrary rigid-body rotations with the restriction that appendage base body deviations from some nominal constant spin rate are small, and (2) unrestrained rigid-body rotations with the restriction that appendage base motion deviations
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Fig. 13. Simulation printout for program using MBDYFYN
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Fig. 14. Simulation printout for program using MBDYFL
Fig. 15. Simulation program for test vehicle with prescribed platform motion using MBDYFN
COMMENT
TH1) = K1*GM1 = B1*GM1D
TH2) = K2*GM2 = B2*GM2D

COMMENT
FORCE EQUATION

COMMENT
FZ = (STEP1,5,TIME)*STEP(2,5,TIME) + 300
FB13) = FZ

COMMENT
ENGINE TORQUE

COMMENT
NM = (STEP(3,5,TIME) - STEP(4,5,TIME))*10
NY = (STEP(6,5,TIME) - STEP(7,5,TIME))*10
TB1) = NM S TB2) = NY

COMMENT
SOLUTION FOR SYSTEM ACCELERATIONS

COMMENT
CALL MKATHECTh,TH,TB,TS,FB,FS,TF,FF,GM,GM1D,GM2D,ET,ETO,NO,ROOT,
END(ET)

COMMENT
SYSTEM RATES AND POSITIONS

COMMENT
W1 = INTEG(WDOT(1),0)
W2 = INTEG(WDOT(2),0)
W3 = INTEG(WDOT(3),0)
ETD1 = INTEG(EYDD(1,1),0) S ETA1 = INTEG(EYD1,0)
ETD2 = INTEG(EYDD(1,2),0) S ETA2 = INTEG(EYD2,0)
ETD3 = INTEG(EYDD(1,3),0) S ETA3 = INTEG(EYD3,0)
ETD4 = INTEG(EYDD(1,4),0) S ETA4 = INTEG(EYD4,0)
ETD5 = INTEG(EYDD(1,5),0) S ETA5 = INTEG(EYD5,0)
ETD6 = INTEG(EYDD(1,6),0) S ETA6 = INTEG(EYD6,0)
ETD7 = INTEG(EYDD(1,7),0) S ETA7 = INTEG(EYD7,0)
GMT1 = INTEG(GMDT(1),0) S GM1 = INTEG(GM1D,0)
GMT2 = INTEG(GMDT(2),0) S GM2 = INTEG(GM2D,0)

END
END
END
TERMINAL
FIN, CONTINUE
END
END

Fig. 15 (contd)
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<td>-1.007563×10^03</td>
<td>-1.250840×10^02</td>
<td>-1.171715</td>
</tr>
<tr>
<td>ETA5</td>
<td>2.355918×10^02</td>
<td>-6.491051×10^02</td>
<td>-1.191509×10^03</td>
</tr>
<tr>
<td>ETA6</td>
<td>-5.530666×10^03</td>
<td>4.777599×10^03</td>
<td>-1.191549×10^03</td>
</tr>
<tr>
<td>GM1</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>GM2D</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Fig. 18. Simulation printout for program using MBDYFN with prescribed platform motion.
from a nominally zero angular rate are small. The second approach was then further restricted to the often very useful assumption that all system rotations are small, permitting a formal linearization with respect to hinge and reference body rotations. Of course, appendage deformations are assumed small in every case.

Three FORTRAN subroutines were then described which solve the equations of motion for these three cases, namely, MBDYFR (for spinning appendages), MBDYFN (for nonspinning appendages), and MBDYFL (linearized for small rotations). Each of the routines has much the same functional appearance as those programs described in Ref. 6., i.e., an initializing entry and a dynamic entry point, with the only differences being the addition of appendage-related parameters, variables, and forcing functions. The routines also retain the option of user-prescribed rotations at selected hinge connections. However, an additional option provided in these programs is that of calculating angular momentum magnitude, which at times provides a valuable check on computational accuracy.

In applying MBDYFR, one can conclude that the mathematical difficulties introduced by spin have forced not only a first-order transformation to obtain uncoupled coordinates but, as a consequence, two coordinates per mode must be solved for in the subroutine. However, what appears to be a computational disadvantage in this case may well be softened by the necessity to consider fewer modes. Some other difficulties are also introduced by this particular modal transformation. The presence of both the modal coordinate position and rate in the expressions for appendage deformation and deformation rate can lead to significant error if modal damping is inserted (thus disturbing eigenvector orthogonality) and large steady-state appendage deformations are present. The user must ensure that any appendage deformations in the damped case remain essentially oscillatory about a nominally zero mean. MBDYFR, as it now stands, also forces the user, regardless of which appendages are spinning or not spinning, to formulate each appendage's modal description using only the first-order transformation, i.e., as if it were subject to spin. While it was much more convenient to program MBDYFR in this way, future requirements for improved computational efficiency may make a modification of MBDYFR desirable. Still, in spite of these particular characteristics, it is felt that MBDYFR can be successfully employed in a wide variety of applications because of its inherent generality and versatility. In addition to the prescribed variable and angular momentum calculation options, the user may also choose to use MBDYFR to directly calculate the steady-state deformations due to centrifugal forces. This is accomplished by setting SR, the nominal appendage spin rate, to zero even though, in the simulation, the appendage is spinning. Setting SR to zero restores the centrifugal force-terms to the equations, and appropriate deformations will appear in the solution. However, as indicated before, the greater the modal damping under these circumstances, the larger the numerical error will be in the steady-state deformations due to spin.

The routines MBDYFN and MBDYFL are of more immediate utility at JPL since current spacecraft designs here are three-axis-stabilized. They represent a generalization of the hybrid-mode concept, developed in Ref. 2, to the rigid-body-tree approach. As a result, it is no longer necessary to add special terms and re-derive equations of motion in order to accommodate discrete rigid-body rotations (or translations) in the system (as was done, for example, in Ref. 9 for the Viking Orbiter with flexible appendages and rigid propellant slosh masses). Even translational dampers can be reasonably well approximated within the hinge-connected tree system. Because of its speed advantages and because it usually
provides acceptable solution accuracy even when rotations are not strictly small, the completely linearized version, MBDYFL, will offer the greatest utility among the three programs at JPL for routine control design studies.

To make these subroutines more easily available to the aerospace industry, they have been submitted to COSMIC (Computer Software Management and Information Center), University of Georgia, Athens, Georgia, for evaluation and dissemination to interested agencies and institutions.

References


Appendix A

Effects of Damping on Rotating Appendage Equations

In Section IIIA, it was pointed out that the addition of viscous damping-like terms to the already transformed appendage equations, particularly for the case of a nominally rotating appendage/base, is mathematically not justified. However, the insertion of modal damping terms is usually thought to be justified on the practical basis that it reasonably and more conveniently represents the physical response of systems as determined from actual test data.

However, it may be useful to illustrate how and to what extent the mathematical inconsistencies so introduced may affect computational results. For example, one can show that the insertion of modal damping into Eq. (26) introduces errors in the steady-state values of \( \delta^k \), \( \eta^k \), and therefore the deformations \( q^k \) and \( \dot{q}^k \). This can be seen from the following. Repeating Eqs. (26) and (27), we have

\[
\dot{\delta}^k = -\dot{\sigma}^k \eta^k - \ddot{\sigma}^k \overline{T}^T_k \dot{L}_k' - \dot{\xi}^k \dot{\sigma}^k \delta^k \tag{A-1}
\]

\[
\dot{\eta}^k = \ddot{\sigma}^k \delta^k - \ddot{\sigma}^k \overline{\psi}^T_k \dot{L}_k' - \dot{\xi}^k \ddot{\sigma}^k \eta^k \tag{A-2}
\]

\[
q^k = 2(\overline{\psi}^k \delta^k - \overline{\pi}_k \eta^k) \tag{A-3}
\]

\[
\dot{q}^k = -2(\overline{\pi}_k \sigma^k \delta^k + \overline{\psi}^k \sigma^k \eta^k) \tag{A-4}
\]

\[
\ddot{q}^k = -2(\overline{\pi}_k \sigma^k \delta^k + \overline{\psi}^k \sigma^k \eta^k) \tag{A-5}
\]

If we now examine \( q^k \) and \( \dot{q}^k \) when \( \delta^k \) and \( \eta^k \) have reached a steady-state condition, i.e., when \( \dot{\delta}^k = \dot{\eta}^k = 0 \), we have, from (A-1),

\[
\dot{\sigma}^k \eta^k = -\dot{\sigma}^k \overline{T}^T_k \dot{L}_k' - \dot{\xi}^k \dot{\sigma}^k \delta^k
\]

and from (A-2),

\[
\ddot{\sigma}^k \delta^k = \ddot{\sigma}^k \overline{\psi}^T_k \dot{L}_k' + \dot{\xi}^k \ddot{\sigma}^k \eta^k
\]

Substituting from (A-2) into (A-1),

\[
\dot{\eta}^k = -\overline{T}^T_k \dot{L}_k' - \ddot{\sigma}^k \dddot{\xi}^k \dddot{\sigma}^k \left[ \overline{\psi}^T_k \dot{L}_k' + \ddot{\sigma}^k \dot{\xi}^k \dot{\sigma}^k \eta^k \right]
\]

or

\[
\dot{\eta}^k = -\overline{T}^T_k \dot{L}_k' - \dddot{\xi}^k \overline{\psi}^T_k \dot{L}_k' - \dot{\xi}^k \dot{\sigma}^k \eta^k
\]
or
\[ \bar{\eta}^k = \left( U + \bar{\xi}^k \bar{\xi}^k \right)^{-1}(\bar{\Gamma}^k_L - \bar{\xi}^k \bar{\psi}^k) L^k_{km} \]  \hspace{1cm} (A-6)

Substituting from (A-1) into (A-2),
\[ \bar{\delta} = \bar{\psi}^T L^k_{km} + \bar{\xi}^k \bar{\psi}^k \left[-\bar{\Gamma}^k L^k_{km} - \bar{\xi}^k \bar{\psi}^k \bar{\delta}^k \right] \]

or
\[ \bar{\delta} = \bar{\psi}^T L^k_{km} - \bar{\xi}^k \bar{\Gamma}^k L^k_{km} - \bar{\xi}^k \bar{\xi}^k \bar{\delta}^k \]

or
\[ \bar{\delta} = \left( U + \bar{\xi}^k \bar{\xi}^k \right)^{-1} \left( \bar{\psi}^T - \bar{\xi}^k \bar{\Gamma}^k L^k_{km} \right) \]  \hspace{1cm} (A-7)

From (A-3), (A-6), and (A-7),
\[ \bar{q}^k = 2 \bar{\psi}^T \left(U + \bar{\xi}^k \bar{\xi}^k \right)^{-1} \left( \bar{\psi}^T - \bar{\xi}^k \bar{\Gamma}^k L^k_{km} \right) - 2 \bar{\Gamma}^k \left(U + \bar{\xi}^k \bar{\xi}^k \right)^{-1} \left( \bar{\psi}^T - \bar{\xi}^k \bar{\Gamma}^k L^k_{km} \right) \]

From (A-4), (A-6), and (A-7),
\[ \bar{q}_{km} = 2 \left[ \bar{\psi}^T L^k_{km} \right] \]  \hspace{1cm} (A-8)

where
\[ U_t = \left( U + \bar{\xi}^k \bar{\xi}^k \right) \]

From (A-4), (A-6), and (A-7),
\[ \bar{q}_{km} = 2 \left[ \bar{\psi}^T L^k_{km} \right] \]  \hspace{1cm} (A-9)

Notice that from (A-9), \( \bar{q}_{km} \neq 0 \) in general! However, as \( \bar{\xi}^k \) becomes infinitesimally small, (A-8) and (A-9) approach
\[ \bar{q}^k = 2 \left[ \bar{\psi}^T \bar{\psi}^k L^k \right] \]

and
\[ \bar{q}_{km} = 2 \left[ \bar{\psi}^T \bar{\Gamma}^k \right] \]
due to orthogonality relations between \( \bar{\psi}^k \) and \( \bar{\Gamma}^k \).

The discovery above that, in general, \( \bar{q}_{km} \neq 0 \) when modal damping is introduced is rather disconcerting. It is further disturbing to realize that if the appendage deformation rates \( \bar{q}^k \) are not zero when the modal coordinates appear to indicate an appendage at rest, then the angular momentum calculations of the subroutines, based on \( \bar{q}^k \), will be in error as well.
Fortunately, we have assumed that the appendage deformations, $q_k$, and their derivatives are small and represent only the oscillatory component of the total possible deformation. This tends to imply that $L_k$ must be very small to begin with and that the steady-state levels of $q_k$ (or its derivatives) after damping are "small" compared to its transient oscillatory amplitudes. Therefore the errors introduced in (A-8) and (A-9) should be of relatively little significance. However, one should be aware of their existence and that they can add to other computational errors.
Appendix B

System Angular Momentum Computation

In Ref. 5, Hooker shows that for a dynamical system of the type considered here, namely, a topological tree of rigid bodies any one of which may carry a flexible appendage, the equations are of the general form

\[ A \dot{x} = B \]

where

\[
A = \begin{bmatrix}
    a_{00} & a_{0k} & b_0 \\
    a_{0k}^T & a & b \\
    b_0^T & b^T & c \\
\end{bmatrix}, \quad \text{and} \quad x = \begin{bmatrix}
    \omega_0 \\
    \gamma \\
    \dot{\eta} \\
\end{bmatrix}
\]

and Hooker proves that the angular momentum of this system about its mass center is the product of the first row of \( A \) with \( x \):

\[ H = a_{00}\omega_0 + a_{0k}\gamma + b_0\dot{\eta} \quad (B-1) \]

and that the 3 by 3 matrix \( a_{00} \) represents the instantaneous system inertia. The relation (B-1) is precisely that implemented in each of the subroutines MBDYFR, MBDYFN, and MBDYFL to calculate \( H \) (3 by 1). \( H \) is a 3 by 1 vector matrix whose elements are the components of the system angular momentum vector in the reference body frame. These three elements are available within the subroutine if the user wishes to extract them. He may also wish to transform them to an inertial reference frame in certain situations as a check on his simulation accuracy. However, the normal subroutine function as shown here in the examples and listings is to supply the user with only the magnitude of \( H \), i.e.,

\[ |H| = (h_1^2 + h_2^2 + h_3^2)^{\frac{1}{2}} \]

where

\[ H = [h_1, h_2, h_3]^T \]
Appendix C

Subroutine MBDYFR Listing and User Requirements

Subroutine Entry Statements

CALL MBDYFR(NC, H, MB, MS, PB, PS, G, PI, NF, F, ER, EI, SR, MF, RF, WF, ZF)
CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD, GMDD, DT, ET, WO, WDOT, DTD, ETD, HM)

Input/Output Variable Type and Storage Specifications

INTEGER NC, NF, H(n_c, 2), F(n_f, 3), PI(n + 1)

REAL MB(7), MS(n_c, 7), PB(n_c, 3), PS(n_c, n_c, 3), G(n, 3), TH(n), TB(3), TS(n_c, 3), FB(3), FS(n_c, 3), GM(n), GMD(n), GMDD(n), ER(n_f, 6 n_k, N_k), EI(n_f, 6 n_k, N_k), MF(n_f, n_k, 7), RF(n_f, n_k, 3), WF(n_f, N_k), ZF(n_f, N_k), TF(n_f, n_k, 3), FF(n_f, n_k, 3), DT(n_f, N_k), ET(n_f, N_k), WO(3), SR(n_f, 3)

DOUBLE PRECISION WDOT(n + 3), DTD(n_f, N_k), ETD(n_f, N_k)

External Subroutines Called

CHOLD—double precision subroutine for solving matrix equations of the form

\[ Ax = B \]

where \( A \) is a square, symmetric, positive-definite matrix (see statement 1291).

Subroutine Setup

Insert the Fortran statement

\[
\text{PARAMETER QC = n_c, QH = n, QF = n_f, NK = n_k, NKT = N_k}
\]

(If more than one appendage is present, use the largest \( n_k \) and \( N_k \) for the \text{PARAMETER} statement to provide sufficient storage.)
Data Restrictions

\[ n > 1, \eta > 1, \eta_n > 1, n_k > 1, N_k > 1 \]

Core Storage Required

Code: 6500 words

Data: \( \sim 500 \) words (minimum; increases with \( n, \eta, \eta_n, \) etc.)

Listing

```fortran
SUBROUTINE HBDYFR(NC,C,CH,HA,PB,PA,G,P1,NF,F,E1,SR,RF,ZF)
! ADJUSTABLE DIMENSIONS

INTEGER P1(1),(NC,2)
REAL R1(1,HA,NC,7),P1(1,NC,3),P2(1,NC,3)
PARAMETER QC=2,QH=3,QF=2,NK=1,NKT=2
PARAMETER NAK=6,NK,SR=1,V=QH+3,V4=9+V5=16+V6=QH,N=QH,NH=QH
PARAMETER ST=V*2,QF*NKT,SN=ST

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION AI0T(SI,ST),ST,ST),B"ASS<S",2,NC,?,PBINC.SI,PA="NC,NC,2>
INTEGER EPS(8tS),CPSI(UC,S?,M(Q),H1 <S» ,FI tsi ,F CNF, 3)
REAL A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>

ADDITIONAL DIMENSIONED VARIABLES

DOUBLE PRECISION A00(3,3),AB(3,3),AQF(OF,3,NKT),AKFR(NF,QM,NTK)

PARAMETER NAK="",NK,*C*I,V=qH*3,VH="V",S3="J",S,Q="QH",NH="QH"
PARAMETER J=«+2«QF/*NC,?,PBINC.SI,PA="NC,NC,2>
DO 1 1=1,NB
1 EPS(1,1)=CPS(1,1)

COMPUTE H(1)=C, WHERE I=HINGE LABEL AND C=CONNECTION LABEL

I=0

DO 6 J=2,NB
6 KK=C(J-1,2)

DO 8 K=1,KB
8 I=1+1

8 I=J=1

COMPUTE H(I)=J, WHERE I=BODY LABEL=1 AND J=NEAREST HINGE LABEL

DEFINE F(J)=K, WHERE J=BODY LABEL=1 AND K IS APPENDAGE LABEL

(If k=0, body has no flex. appendage)

DO 239 N=1,UB
239 F(IN)=N

DO 242 K=1,NF
242 JNF(K)=K

N=NF

NB=NA

DEFINE SUBSTRUCTURE MASSES

M58(I)=MB(I)

DO 248 N=2,UB
248 M58(N)=MA(N-1,7)

DEFINE TOTAL NUMBER OF FLEX. APPENDAGE MODES TO BE RETAINED

NTMO=0

DO 461 K=1,NF
461 NTMO=NTMO+F(K,3)

NTZ=2*NTMO

INITIAL CALCULATION OF BARYCENTER VECTORS W.R.T. BODY C.G.S.

AND HINGE POINTS

I=1

DO 35 J=2,MB
35 T=TBMASS(I)

DO 149 I=1,MB
122 11 = i - 1
123 DO 149 J = 1, NB
124 J = J + 1
125 IF (I,J,EQ,J) GO TO 163
126 IF (I,J,EQ,J) GO TO 70
127 IF (I,J,EQ,J) GO TO 80
128 IF (CPS(I,J), EQ, 1) GO TO 400
129 70 L X (1, J) = PA ( I (1, I, 1 )
130 L Y (1, J) = PA ( I (1, I, 2 )
131 L Z (1, J) = PA ( I (1, I, 3 )
132 GO TO 149
133 400 CONTINUE
134 DO 400 K = 1, J
135 IF (CPS(K,J),EQ,1) GO TO 500
136 400 CONTINUE
137 GO TO 149
138 500 L X ( 1, J ) = PA ( I ( K, 1 )
139 L Y (1, J) = PA ( I ( K, 2 )
140 L Z (1, J) = PA ( I ( K, 3 )
141 GO TO 149
142 80 DO 80 L = 1, 1
143 IF (CPS(L,J),EQ,1) GO TO 101
144 90 CONTINUE
145 GO TO 149
146 101 L X ( 1, J ) = PB ( L, 1 )
147 L Y (1, J) = PB ( L, 2 )
148 L Z (1, J) = PB ( L, 3 )
149 GO TO 149
150 163 L X (1, J) = O X
151 L Y (1, J) = O Y
152 L Z (1, J) = O Z
153 199 CONTINUE
154 DO 13 N = 1, NB
155 DO 13 J = 1, NB
156 DO (N,J) = L(N,J)
157 D Y (N, J ) = L T ( N , J )
158 D Z (N,J) = L Z ( N , J )
159 DO 13 K = 1, NB
160 D X (N,K) = D X ( N , J ) - (B M A S S ( K ) / T M ) * L X ( N , K )
161 D Y (N,J) = D Y ( N , J ) - (B M A S S ( K ) / T M ) * L Y ( N , K )
162 D Z (N,J) = D Z ( N , J ) - (B M A S S ( K ) / T M ) * L Z ( N , K )
163 C
164 NOMINAL SPIN RATE CENTRIFUGAL FORCES
165 C
166 DO 736 K = 1, NF
167 1 = F ( K , 1 )
168 R 1 = S R ( K , 1 )
169 R 2 = S R ( K , 2 )
170 R 3 = S R ( K , 3 )
171 D 1 = D X ( 1 , 1 )
172 D 2 = D Y ( 1 , 1 )
173 D 3 = D Z ( 1 , 1 )
174 W W D E ( K , 1 ) = -R 3 *( N 3 * O 1 - R 1 * D 1 ) + R 2 *( -R 2 * O 2 + R 1 * D 1 )
175 W W D E ( K , 2 ) = -R 3 *( R 3 * O 2 + R 2 * D 2 ) - R 1 *( R 1 * O 2 - R 2 * D 1 )
176 736 W W D E ( K , 3 ) = -R 2 *( -R 3 * O 2 - R 2 * D 3 ) + R 1 *( R 3 * D 1 + R 2 * D 3 )
177 C
178 CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY
179 C
180 DO 31 N = 1, NB
181 P H ( N , 1 , 1 ) = I X ( N )
182 P H ( N , 1 , 2 ) = I Y ( N )
183 P H ( N , 1 , 3 ) = I Z ( N )
184 P H ( N , 2 , 1 ) = I Y ( N )
185 P H ( N , 2 , 2 ) = I Z ( N )
186 C
187 DO 30 J = 1, NB
189 P H ( N , 1 , 2 ) = P H ( N , 1 , 2 ) * B M A S S ( J ) * D X ( N , J ) * O Y ( N , J )
190 P H ( N , 1 , 3 ) = P H ( N , 1 , 3 ) * B M A S S ( J ) * D X ( N , J ) * O Z ( N , J )
PH(N,2,2) = PH(N,2,2) + BMASS(J) * (DX(N,J) + 2 * DZ(N,J))
PH(N,2,3) = PH(N,2,3) - BMASS(J) * DT(N,J) * DZ(N,J)
PH(N,3,3) = PH(N,3,3) * BMASS(J) * (DX(N,J) + 2 * DT(N,J))
PH(N,3,1) = PH(N,3,1)
PH(N,3,2) = PH(N,3,2)

C computes PK and GK (3 x NKT arrays)

DO 201 K = 1, NFK
  LNK = F(K,2)
  JNT = F(K,3)
  DO 203 J = 1, JNT
    DO 202 L = 1, LNK
      PK(K,J,L) = PK(K,J,L) + MF(K,L,7) * ER(K,L,L)
    END DO 202
    GK(K,J,1) = GK(K,J,1) + MF(K,L,7) * EI(K,L,L)
  END DO 203
  CONTINUE

DO 204 DLRK(K,1,J,L) = 0
  L1 = L - 1
  L2 = L - 2
  L3 = L - 3
  L4 = L - 4
  L5 = 0
  L6 = 0
  DLRK(K,1,J,L) = DLRK(K,1,J,L) + MF(K,L,7) * EI(K,L,L)
  CONTINUE
  RETURN

ENTRY MRATEINC (T, B, TA, FB, FA, TF, FF, GM, GMDD, DT, ET, WC, OTD, ETD, HM, U, O00)

REAL TFIOF, NK, 3, FF, BF, NK, 3, CM, GM, GMDD, TH, 3, 0(3), NXO, YO, ZO

C computes DLR AND DLR = TRANSPOSE matrices (3 x NKT arrays)

DO 205 K = 1, NFK
  LNK = F(K,2)
  JNT = F(K,3)
  DO 202 J = 1, JNT
    DO 201 L = 1, LNK
      DLRK(K,1,J,L) = DLRK(K,1,J,L) + MF(K,L,7) * ER(K,L,L)
    END DO 201
      DLRK(K,2,J,L) = DLRK(K,2,J,L) + MF(K,L,7) * EI(K,L,L)
  END DO 202
  CONTINUE
  RETURN
DOUBLE PRECISION EC(ST), DTD(GF, NKT), ETD(GF, NKT), BOLT(V)

BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES

DO 335 J=1,NM
 MM=J-1
 N=M(J-1)
 S2=M*SGNH(GM(J-1))
 CGM=CSGNGM(J-1)
 CGM=1.-CGM
 G1=CGNH*G(J-1)
 G2=CGM*G(J,2)
 G3=CGM*G(J,3)
 SG1=SGM*G(J,1)
 SG2=SGM*G(J,2)
 SG3=SGM*G(J,3)
 G1S=G1*G(J,1)
 G2S=G2*G(J,2)
 G3S=G3*G(J,3)
 G2S=G2*G(J,2)
 G3S=G3*G(J,3)
 G1S=G1*G(J,1)
 G2S=G2*G(J,2)
 G3S=G3*G(J,3)
 G2S=G2*G(J,2)
 G3S=G3*G(J,3)
 IF(J.EQ.1) GO TO 3350
 DO 321 L=M,1
 IF(EPS(L,M).*EQ.1) Q() TO 322
 CONTINUE
 GO TO 3350

321 CONTINUE

GO TO 3350

322 K=L

DO 334 L=1,3
 DO 339 M=1,3
 T(J,L,M)=0.
 DO 334 L=1,3
 T(J,L,M)=T(J,L,M)+AB(L,J)*G(K,J,M)
 GO TO 335

CONTINUE

334 T(J,L,M)=T(J,L,M)+AB(L,J)*G(K,J,M)

335 CONTINUE

CONTINUE

CONTINUE

COORD. TRANSFORMATION OF 6 VECTORS (TO REF. BODY FRAME)

DO 362 J=1,NM
 DO 362 J=1,NM
 G0(1,J)=0.
 DO 362 J=1,NM
 AB(J,J)=G0(1,J)*T(I,K,J)*G(I,K)
 CONTINUE

ANG. VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)

DO 366 K=1,NM
 GG(K,1)=GM0(K)*G0(K,1)
 GG(K,2)=GM0(K)*G0(K,2)
 GG(K,3)=GM0(K)*G0(K,3)
 DO 361 J=1,NB
 KV=HI(J)
 WX0(J)=WX0(J)
 WYO(J)=WYO(J)
 ZW0(J)=ZW0(J)
DO 36 K=1,KV
330* IF(EPS(K,J),EQ.0) GO TO 36
331* NKO(J)=NKO(J)+GG(K,1)
332* WYD(J)=WYD(J)+GG(K,2)
333* NWZ(J)=NWZ(J)+GG(K,3)
334* 36 CONTINUE
335* 361 CONTINUE
336* C
337* C ANG. VELOCITY COMPONENTS AT EACH HINGE (IN REF. BODY FRAME)
338* C
339* DO 3666 M=1,NH
340* M1=M+1
341* MC=M(MC)
342* M1=M1(MC)
343* WHX=WHX(MC)
344* WHT=WH12(MC)
345* NWZ=NWZ(MC)
346* IF(M1.EQ.M) GO TO 3667
347* DO 3666 N=M1,NH
348* WHX=WHX=WHX(N,1)
349* WHT=WH12=WH12(N,2)
350* NWZ=WHZO=WHZO(N,3)
351* 3666 CONTINUE
352* 3667 CONTINUE
353* IF(WH11(N,1))=GG(N,1) & WHT=GG(N,2) & NWZ=WHZ0(N,3)
354* WWJ(N,1))=GG(N,1) & NWZ=WHZ0(N,3)
355* 3666 CONTINUE
356* C
357* C TRANSFORM PK AND G6 MATRICES TO REFERENCE BODY BASIS=MULT*BY FREQ.
358* C
359* DO 468 K=1,NF
360* KK=F(KK,1)
361* JNT=F(KJ,1)
362* IF(KK,EQ.1) GO TO 4720
363* M1=M(KK)
364* DO 472 J=1,M
365* DO 472 J=1,JNT
366* DLKRO(K,I,J)=-0.
367* DLK10(K,I,J)=-0.
368* PKOS(K,I,J)=-0.
369* GKOS(K,I,J)=-0.
370* DO 469 L=1,J
371* DLKRO(K,I,J)=-DLKRO(K,I,J) & DLK10(K,I,J) & DLK10(K,I,J) & DLK10(K,I,J)
372* PKOS(K,I,J)=-PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J)
373* GKOS(K,I,J)=-GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J)
374* 469 CONTINUE
375* PKOS(K,I,J)=-PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J)
376* GKOS(K,I,J)=-GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J)
377* 472 GO TO 468
378* 4720 CONTINUE
379* DO 4721 I=1,3
380* DO 4721 J=1,JNT
381* DLKRO(K,I,J)=-DLKRO(K,I,J) & DLK10(K,I,J) & DLK10(K,I,J) & DLK10(K,I,J)
382* PKOS(K,I,J)=-PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J) & PKOS(K,I,J)
383* GKOS(K,I,J)=-GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J) & GKOS(K,I,J)
384* 4721 CONTINUE
385* 468 CONTINUE
386* C
387* C COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.)
388* C
389* FEXO(I)=FB(1)
390* FEXO(I)=FB(2)
391* FEXO(I)=FB(3)
392* IF(F(I),EQ.0) GO TO 254
393* IL=F(I)
394* JN=F(L,J)
395* DO 253 J=1,JN
396* FEXO(I)=FEXO(I)+FF(I,J,1)
397* FEXO(I)=FEXO(I)+FF(I,J,2)

ORIGINAL PAGE IS OF POOR QUALITY.
398* 253 FEZ0(1)=FEZ0(1)*FF(1L,J,3)  
399* 254 CONTINUE.  
400* 255 FS(I,1)=FEZ0(1)  
401* 256 FS(1,2)=FE Z0(1)  
402* 257 FS(1,3)=FEZ0(1)  
403* 258 DO 246 N=2,48  
404* 259 K=N+1  
405* 260 DO 246 N=1,3  
406* 2460 FS(I,N)=FA(1K,L)  
407* IF(F(N)>ED,O) GO TO 246  
408* 2461 IWF(I,N)  
409* JN=FI(1L,2)  
410* 2462 DO 245 J=1,JN  
411* 2463 DO 245 I=1,3  
412* 2464 FS(I,N)=FS(IN,S)*FF(1L,J,1)  
413* 2465 CONTINUE  
414* 2466 C COMPUTE TRANSL. AND ROTAT. DISPLACEMENTS OF APPENDAGE SUB-BODIES  
415* 2467 C  
416* 2468 DO 232 K=1,NF  
417* 2469 JN=FI(1K,2)  
418* 2470 LK=FI(1K,3)  
419* 2471 DO 233 J=1,JN  
420* 2472 DO 233 I=1,3  
421* 2473 U(I,K,J)=0.  
422* 2474 B(I,K,J,1)=0.  
423* 2475 UO(I,K,J)=0.  
424* 2476 BD(I,K,J,1)=0.  
425* 2477 ID=FI(I,J,1)+1.  
426* 2478 IF(ID=3)  
427* 2479 DO 233 L=1,LK  
428* 2480 U(K,J,1)=U(K,J,1)+2.*ER(K,1L,D)*DT(K,L)=2.*E1(K,1L,D)*ET(K,L)  
429* 2481 B(K,J,1)=B(K,J,1)+2.*ER(K,1L,R)*DT(K,L)=2.*E1(K,1L,R)*ET(K,L)  
430* 2482 UO(K,J,1)=UO(K,J,1)+2.*ER(K,1L,L)*DT(K,L)=2.*E1(K,1L,L)*ET(K,L)  
431* 2483 BD(K,J,1)=BD(K,J,1)+2.*ER(K,1L,L)*DT(K,L)*MF(K,L)  
432* 2484 S =2*E1(K,1L,L)*DT(K,L)*MF(K,L)  
433* 2485 S =2*E1(K,1L,L)*DT(K,L)*MF(K,L)  
434* 2486 CONTINUE  
435* 2487 C COMPUTE C.M. PERTURBATION (FROM NON-UNDEFORMED LOCATION) ON EACH  
436* 2488 SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORDS.)  
437* 2489 C  
438* 2490 DO 262 K=1,NF  
439* 2491 I=N+1  
440* 2492 JN=FI(IK,2)  
441* 2493 DO 263 L=1,3  
442* 2494 MCK(I,K,1)=0.  
443* 2495 263 MCK(I,K,1)=0.  
444* 2496 DO 265 J=1,JN  
445* 2497 DO 265 L=1,3  
446* 2498 MCKD(I,K,1)=MCKD(I,K,1)*UD(K,J,1)*MF(K,J,1)  
447* 2499 265 MCKD(I,K,1)=MCKD(I,K,1)*UD(K,J,1)*MF(K,J,1)  
450* 2500 DO 266 L=1,3  
451* 2501 CKD(I,K,1)=CKD(I,K,1)*MSB(IK)  
452* 2502 C K(D(K,1)=CKD(I,K,1)*MSB(IK)  
453* 2503 CONTINUE  
454* 2504 C COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE w.r.t. ITS  
455* 2505 INSTANTANEOUS C.M. (IN LOCAL COORDS.)  
456* 2506 C  
457* 2507 DO 268 L=1,3  
458* 2508 TS(I,L)=TB(L)  
459* 266 DO 267 N=2,48  
460* 267 K=N+1  
461* 268 DO 267 L=1,3  
462* 267 TS(N,L)=TA(K,L)  
463* 268 DO 2670 N=1,48  
464* 267 IL=FI(N)  
465* 267 IL=FI(N)  
466* 2670 IF(IL=EO,D) GO TO 2670
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ADDITIONAL AUGMENTED INERTIA DYADICS (IN REF. BODY FRAME)

DO 37 1=1,NB
DO 37 J=1,NB
IF(J.GE.J) GO TO 37
DXZ=DXO(J,J)+DZ(J,J)
DYZ=DO(J,J)+DO(J,J)
DZ=DO(J,J)+DO(J,J)
PS(I,J,J,1)=TH(DYZ+DOZ)
PS(I,J,J,2)=TH(DXZ+DOX)
PS(I,J,J,3)=TH(DXZ+DOX)
PS(I,J,J,4)=TH(DXZ+DOX)
PS(I,J,J,5)=TH(DXZ+DOX)
PS(I,J,J,6)=TH(DXZ+DOX)
PS(I,J,J,7)=TH(DXZ+DOX)
PS(I,J,J,8)=TH(DXZ+DOX)
CONTINUE
DO 751 J=1,NB
DO 751 M=1,NB
DO 751 N=1,NB
CONTINUE
DO 236 K=1,NF
KK=NF(K,1)+1
NN=NF(K,2)
J=NF(K,3)
VJ(I,J)=0.
VJO(I,J)=0.
U1(U(K,J,1))
U2(U(K,J,2))
U3(U(K,J,3))
B1(B(K,J,1))
B2(B(K,J,2))
B3(B(K,J,3))
VJ(1,1)=VJ(1,1)+2.*MS*(R2+R3+U3)+12*B3*13*B2
VJ(1,2)=VJ(1,2)+2.*MS*(R2+R3+U3)+12*B3*13*B2
VJ(1,3)=VJ(1,3)+2.*MS*(R2+R3+U3)+12*B3*13*B2
CONVERT INERTIA MATRIX TO REF. BODY COORDS.

DO 1 366 K=1,3
   DO 356 L=1,JNT
      PS(K,K,K,L)=PS(K,K,K,L)*T(H,L,K)*AC(L,1)
   CONTINUE
   DO 357 J=1,JNT
      DO 355 I=1,KK
         PS(I,I,J,H)*PS(I,I,J,H)+T(M,L,J)*AB(L,K)
      CONTINUE
CONTINUE

TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME

DO 363 I=2,NB
   M=HH(I,1)
   DO 364 J=1,3
      DO 365 K=1,3
         AB(J,K)=0.
      CONTINUE
      DO 364 L=1,3
         AB(J,K)=AB(J,K)*PS(I,1,J,L)*T(M,L,K)
      CONTINUE
   CONTINUE

COMPUTE THE PGSO, GPSO, AND DOSO VECTORS FOR EACH FLEX. APPEND.

DO 208 K=1,NF
   K=KK(KK,K)
   M=HH(KK)
   JNT=JNT(KK)
   DO 207 I=1,3
      CV(I)=0.
   DO 207 J=1,JNT
      CV(I)=CV(I)+DLK*(K,K,J,O)*ET(K)*DLO(K,K,J)*ET(K,J)
   IF(KK*EQ=1) 60 TO 2090
   DO 207 J=1,3
      PGSO(K,K)=0.

VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING.

QUADRIC TERMS INVOLVING THE CONNECTING BODY ANGULAR VELOCITIES AND THE MUTUAL BARTCENTER-HINGE VECTORS:

DO 261 K=1,NF
   IF(K,NF)>1
      DUX=WSO(K,2)*PSO(K,3)-YSO(K,1)*PSO(K,2)
      DUY=WZO(K,3)*PSO(K,1)-XSO(K,1)*PSO(K,3)
      DUY=WZO(K,2)*PSO(K,1)-XSO(K,2)*PSO(K,3)
      DUY=WZO(K,1)*PSO(K,2)-XSO(K,3)*PSO(K,3)
   CONTINUE

DO 2301 L=1,NB
   IL=I(M)
   IF(IL<1) GO TO 2303
   DCPX=DCPX*YTO(L)-DZO(N,L)+DUZ(L)
   DCPY=DCPY*YTO(L)+VZ0(N,L)-DUZ(L)
   DCPZ=DCPZ*YTO(L)-VZ0(N,L)+DUZ(L)
   CONTINUE

IF(I,E0.3) GO TO 482
   CWD(N,1)=CWD(N,1)+WFDX
   CWD(N,2)=CWD(N,2)+WFDY
   CWD(N,3)=CWD(N,3)+WFDZ
   CONTINUE

CPFX=CPFX+WFDX
CPFY=CPFY+WFDY
CPFZ=CPFZ+WFDZ
CONTINUE
CONTINUE
DFX = 0.0
HY = 0.0
HZ = 0.0
PGF = 0.0
PGFY = 0.
PGFZ = 0.0
PMDZ = 0.0
PMDY = 0.0
PMDZ = 0.0
WDSO = 0.0
WDSY = 0.0
WDSZ = 0.0
CONTINUE
K = 3*(N+1)
E(K+1,1) = HX*WZO(N) - HZ*WYO(N) + TXO(N)*CPFX
DFX = 0.0
PGF = 0.0
PGFY = 0.
PGFZ = 0.0
PMDZ = 0.0
PMDY = 0.0
PMDZ = 0.0
WDSO = 0.0
WDSY = 0.0
WDSZ = 0.0
CONTINUE
DO 3001 I = 1, N
DO 3001 J = 1, N
3001 AOO(I,J) = 0.0
DO 3001 I = 1, N
3001 AOO(I,1) = AOO(I,1) + PS(I,J,3)
3001 AOO(I,2) = AOO(I,2) + PS(I,J,2)
3001 AOO(I,3) = AOO(I,3) + PS(I,J,1)
3001 CONTINUE
FLEX. APPEND CONTRIBUTION TO AOO MATRIX COMPUTATION (3X3)
DO 210 K=1,NB
   KK=I(K)
DO 210 L=1,NB
IF(K.EQ.0) GO TO 210
DO 2103 I=1,3
DO 2103 J=1,3
2103 PSF(K,L,I,J)=0.
LL=I(L)
IF(K.EQ.0) GO TO 2101
DP1=PSGO(KK,1)*DOO(L,K)
DP2=PSGO(KK,2)*DOO(L,K)
DP3=PSGO(KK,3)*DOO(L,K)
PSF(K,L+1,1)=DP2-DO3
PSF(K,L+2,2)=DP1-DO3
PSF(K,L+3,3)=DP1-DO2
PSF(K,L+1,2)=PSGO(KK,2)+DOO(L,K)
PSF(K,L+2,1)=PSGO(KK,1)+DOO(L,K)
PSF(K,L+3,1)=PSGO(KK,1)+DOO(L,K)
PSF(K,L+2,3)=PSGO(KK,3)+DOO(L,K)
PSF(K,L+3,2)=PSGO(KK,2)+DOO(L,K)
CONTINUE
IF(K.LE.L) GO TO 2101
CONTINUE
2101 CONTINUE
IF(L.LE.K+J) GO TO 210
DO 214 I=1,3
DO 214 J=1,3
AB(I,J)=PSF(K,L+1,J)
CONTINUE
214 CONTINUE
AF(I,J)=PSF(K,L+1,J)+AB(J,I)
CONTINUE
215 CONTINUE
210 CONTINUE
DO 2151 K=1,NA
DO 2151 L=1,NA
IF(K.LE.L) GO TO 2151
DO 2141 I=1,3
DO 2141 J=1,3
PSF(K,L+1,J)=PSF(L,K,J+I)
CONTINUE
2141 CONTINUE
DO 3004 K=1,NB
   KK=I(K)
DO 3004 L=1,NB
LL=I(L)
IF(K.EQ.0).AND.(LL.EQ.0) GO TO 3004
DO 3003 I=1,3
DO 3003 J=1,3
A00(I,J)=A00(I,J)+PSF(K,L+1,J)
3003 CONTINUE
3004 CONTINUE
C
C A0K VECTOR ELEMENT COMPUTATION (3X1)
C
C A0M SCALAR ELEMENT COMPUTATION
C
DO 14 M=1,NN
   Q=M(M)+1
AV(M,1) = 0
AV(M,2) = 0
AV(R,1) = 0
DO 7 J = 1, NM
DO 7 I = 1, NB
DO 10 L = 1, 13
IF(EPS(M,1), EQ, 0) GO TO 7
PSG(J,1,N) = 0
DO 10 L = 1, 13
PSG(J,1,N) = PSG(J,1,N) + (PS(J,1,N,L) * PSG(J,1,N,L)) * GO(M,1, L)
AV(M,N) = AV(M,N) + PSG(J,1,N)
7 CONTINUE
DO 14 K = 1, NM
IF(K > GT,M) GO TO 14
JQM(K) = 1
AIS(1) = 0
AIS(2) = 0
AIS(3) = 0
DO 15 J = 1, NB
DO 15 I = 1, 13
IF(EPS(K,J), EQ, 0), OR, (EPS(M,1), EQ, 0) GO TO 15
DO 18 N = 1, 3
AIS(N) = AIS(N) + PSG(J,1,N)
15 CONTINUE
18 AIS(N) = 0
CONTINUE
AIS(K,N) = GO(K,1) * AIS(1) + GO(K,2) * AIS(2) + GO(K,3) * AIS(3)
15 CONTINUE
DO 219 K = 1, NF
JK = F(K,3)
JQM = F(K,1) + 1
DO 222 L = 1, 13
DO 222 J = 1, 13
222 AB(I,J) = 0
DO 221 L = 1, NB
AB(I,2) = AB(I,2) - DZO(L,JQ)
AB(I,3) = AB(I,3) - DYO(L,JQ)
221 AB(I,2) = AB(I,2) - DZO(L,JQ)
AB(I,3) = AB(I,3) - DYO(L,JQ)
220 AB(I,1) = AB(I,1)
AB(I,2) = AB(I,2)
DO 220 L = 1, 13
220 AB(I,1) = AB(I,1)
AB(I,2) = AB(I,2)
AB(I,3) = AB(I,3)
219 CONTINUE
225 AOFR(K,1,J) = DLKRO(K,1,J)
226 AOFI(K,1,J) = DLKIR(K,1,J)
227 DO 220 L = 1, 13
220 AOFR(K,1,J) = AOFR(K,1,J) - AB(I,L) * KGOS(K,L,J)
229 AOFI(K,1,J) = AOFI(K,1,J) - AB(I,L) * PKOS(K,L,J)
219 CONTINUE
AKFR VECTOR COMPUTATION (1XKT) (FLEX COUPLING WITH RIGID SUBSTRUCTURE
AKVF VECTOR COMPUTATION (1XKT) (FLEX COUPLING WITH RIGID SUBSTRUCTURE
DO 224 K = 1, NF
JK = F(K,3)
JQM = F(K,1) + 1
DO 229 J = 1, JK
229 ZSH(K,J) = 0
DO 224 L = 1, NM
ZSH(K,J) = 0
224 ZSF(K,J) = 0
DO 223 L = 1, NM
ZSH(K,J) = 0
223 ZSF(K,J) = 0
DO 231 L = 1, 13
231 AB(I,J) = 0
DO 226 L = 1, NB
IF(EPS(M,L), EQ, 0) GO TO 226
AB(I,2) = AB(I,2) - DZO(L,JQ)
AB(I,3) = AB(I,3) - DYO(L,JQ)
950• 
951• 226 CONTINUE 
952• AB(2,3) = AB(2,3) - DXO(L,JQ) 
953• CONTINUE 
954• AB(2,4) = AB(2,4) 
955• DO 226 J = 1,3 
956• DO 226 J = 1,3 
957• DUR(I,J) = DLKRO(K,I,J) 
958• DUR(I,J) = DLK10(K,I,J) 
959• IF (EPS(M,K),EQ.0) DUR(I,J) = 0. 
960• IF (EPS(M,K),EQ.0) DUR(I,J) = 0. 
961• DO 228 L = 1,3 
962• DUR(I,J) = DUR(I,J) - AB(I,L)*DGOS(K,L,J) 
963• DUR(I,J) = DUR(I,J) - AB(I,L)*DGOS(K,L,J) 
964• DO 228 J = 1,3 
965• DO 228 J = 1,3 
966• ZSR(K,J) = ZSR(K,J) + DUR(I,J)*MGJ(M,J) 
967• ZSR(K,J) = ZSR(K,J) + DUR(I,J)*MGJ(M,J) 
968• DO 229 J = 1,3 
969• AKFR(K,M,J) = 0. 
970• AKFI(K,M,J) = 0. 
971• DO 229 J = 1,3 
972• AKFR(K,M,J) = AKFR(K,M,J) + GO(M,J) + DUR(I,J) 
973• AKFI(K,M,J) = AKFI(K,M,J) + GO(M,J) + DUR(I,J) 
974• CONTINUE 
975• C 
976• C COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR 
977• C 
978• DO 41 J = 2, NB 
979• JD = 2, NB 
980• C 
981• 111 CM(J,L) = 0. 
982• DO 42 K = 1, JK 
983• IF (EPS(K,J),EQ.0) GO TO 42 
984• CM(J,L) = CM(J,L) + MGJ(K,L) 
985• CM(J,L) = CM(J,L) + MGJ(K,L) 
986• CM(J,L) = CM(J,L) + MGJ(K,L) 
987• CONTINUE 
988• C 
989• C 
990• DO 40 I = 1, NA 
991• DO 40 I = 1, NA 
992• EA(1) = 0. 
993• DO 40 I = 1, NA 
994• DO 40 I = 1, NA 
995• DO 40 I = 1, NA 
996• DO 40 I = 1, NA 
997• EAM = EAM + (PS(1,J,M,L) - PSF(1,J,M,L)) + CM(J,L) 
998• CONTINUE 
999• C 
1000• K = 3*(I-J) 
1001• E(K+1,1) - E(K+1,1) = EA(I) 
1002• E(K+1,1) - E(K+1,1) = EA(I) 
1003• E(K+1,1) - E(K+1,1) = EA(I) 
1004• CONTINUE 
1005• DO 55 M = 1, 3 
1006• DO 55 M = 1, 3 
1007• DO 55 M = 1, 3 
1008• DO 55 M = 1, 3 
1009• E(M) = E(M) - E(K,1) 
1010• DO 55 M = 1, 3 
1011• JK = M(K) 
1012• IF (PI(K,M),EQ.0) GO TO 60 
1013• I = I + 1 
1014• CONTINUE 
1015• DO 60 M = 1, 3 
1016• E(M) = 0. 
1017• DO 60 M = 1, 3 
1018• IF (EPS(K,J),EQ.0) GO TO 61
DO 65 M=1,3
65 J=3*(J-1)+1
CE(M)=CE(M)+E(J,1)
61 CONTINUE
DO 66 L=1,3
66 E(J,3)=EC(J,3)*GO(K,L)+CE(L)
EC(J,3)=EC(J,3)+TM(K,1)
60 CONTINUE
DO 610 I=1,3
610 K=0
1=3
DO 612 I=1,NM
IF(P(I,J).NE.0) GO TO 610
EC(I)=EC(I)-AV(I,J)*GADD(J)
610 CONTINUE
CONTINUE
DO 612 CONTINUE
612 CONTINUE
C COMPUTE RT. HAND SIDE OF APPENDAGE EQUATIONS (IN APPEND. COORDS.)
DO 477 K=1,NF
477 CONTINUE
CDU(K,1)=0
DO 478 L=1,NF
IF(K.EQ.L) GO TO 478
CDU(K,2)=CDU(K,2)+DY0(L)
CDU(K,1)=CDU(K,1)+DX0(L)
CDU(K,3)=CDU(K,3)+DU0(L)
478 CONTINUE
DO 477 CONTINUE
477 CONTINUE
DO 483 K=1,NF
483 I=F(K,1)=1
MH(11)
483 CONTINUE
CQ(1)=FTXQ*CDU(K,1)/TM+CWD(1,1)
CQ(2)=FTXQ*CDU(K,2)/TM+CWD(1,2)
CQ(3)=FTXQ*CDU(K,3)/TM+CWD(1,3)
IF(1,1).EQ.1) GO TO 4840
484 VE(K,J)=WDE(K,J)
484 CONTINUE
DO 484 L=1,3
VE(K,J)=VE(K,J)+TM(J,L)*CQ(L)
484 CONTINUE
GO TO 483
483 CONTINUE
DO 487 J=1,3
487 VE(K,J)=CQ(J)+WDE(K,J)
487 CONTINUE
DO 485 K=1,NF
485 NL=F(K,2)
485 CONTINUE
F(K,1)=1
MH(11)
RJ=SR(K,1)
R2=SR(K,2)
R3=SR(K,3)
IF(1,1).EQ.1) GO TO 4870
487 MJ=J,1
WY(J)=TM(J,1)+MXO(1)+TM(J,2)*WYO(1)+TM(J,3)*WZ0(1)
487 CONTINUE
GO TO 4872
4872 CONTINUE
1158  DO 463 J=1,NL
1159  JL=NV*J
1160  JO=JL+NTMO
1161  A(I,JL)=0.
1162  A(I,J0)=0.
1163  A(I0,JL)=0.
1164  A(I0,J0)=0.
1165  IF(J.EQ.JL) A(I,JL)=2.*
1166  IF(J.EQ.J0) A(I0,J0)=2.*
1167  463 CONTINUE
1168  462 NV=NV+NL
1169  C   ENTER COEFF. WHICH COUPLE REF. BODY AND FLEX. APPENDAGES INTO A
1170  C   NV=1V
1171  C   DO 464 K=1,NF
1172  C   NL=F(K,3)
1173  C   DO 465 J=1,N
1174  C   IF(P(K,J).NE.0) GO TO 467
1175  C   J=J+1
1176  C   DO 466 I=1,NL
1177  C   IL=NV+I
1178  C   IO=IL+NTMO
1179  C   A(I,JL)=QFR(K,J,J)
1180  C   A(I0,JL)=QFR(K,J,J)
1181  C   A(I0,J0)=QFR(K,J,J)
1182  C   A(I0,J0)=A(I0,J0)
1183  C   465 CONTINUE
1184  C   464 NV=NV+NL
1185  C   ENTER COEFF. WHICH COUPLE SUBSTR. BODIES AND FLEX. APPEND. INTO A
1186  C   NV=1V
1187  C   DO 466 K=1,NF
1188  C   NL=F(K,3)
1189  C   J=0
1190  C   DO 467 J=1,N
1191  C   IF(P(K,J).NE.0) GO TO 467
1192  C   J=J+1
1193  C   DO 468 I=1,NL
1194  C   IL=NV+I
1195  C   IO=IL+NTMO
1196  C   A(I,JL)=0.
1197  C   A(I0,JL)=0.
1198  C   A(I0,J0)=0.
1199  C   A(I0,J0)=A(I0,J0)
1200  C   468 CONTINUE
1201  C   467 CONTINUE
1202  C   466 NV=NV+NL
1203  C   CALCULATE FLEX. BODY COUPLING COEFF. AND ENTER INTO A MATRIX
1204  C   NV=1V
1205  C   DO 473 L=1,NF
1206  C   NL=F(L,3)
1207  C   NR=1V
1208  C   DO 474 K=1,NF
1209  C   NR=F(K,3)
1210  C   IF(K.EQ.L) GO TO 474
1211  C   DO 475 J=1,NL
1212  C   JK=NC0+J
1213  C   JO=JK+NTMO
1214  C   A(I,K,J)=C0.
1215  C   A(I0,K,J)=C0.
1216  C   A(I0,K0)=C0.
1217  C   DO 475 J=1,NL
1218  C   JK=NC0+J
1219  C   JO=JK+NTMO
1220  C   A(I,K,J)=C0.
1221  C   A(I0,K,J)=C0.
1222  C   A(I0,K0)=C0.
1223  C   DO 475 J=1,NL
1224  C   JK=NC0+J
1225  C   A(I,K,J)=A(I,K,J)+KOS(K,N,J)*KOS(L,N,J)/TM
1226  C   A(I0,K,J)=A(I0,K,J)+KOS(K,N,J)*KOS(L,N,J)/TM
CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

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CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

ANGULAR MOMENTUM OF THE SYSTEM

SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANGULAR ACCELERATION AND RELATIVE ROTATIONAL ACCELERATIONS
GO TO 910
1297* 913 CONTINUE
1298* K=J-3
1299* IF(IPI(K,MG,O)) GO TO 911
1300* ECI(J)=EC(IKV)
1301* KV=KV-1
1302* GO TO 910
1303* 911 ECI(J)=GMDD(K)
1304* 910 CONTINUE
1305* DO 9003 I=1,V
1306* 9003 IOT(I)=EC(I)
1307* I=V
1308* DO 9001 K=1,MF
1309* NLF(K,J)
1310* DO 9002 N=1,ML
1311* I0=I+1
1312* IL=1O+NTMO
1313* DTD(K,N)=EC(I0)
1314* 9002 ETO(K,N)=EC(I1)
1315* 9001 I=E+NL
1316* 92 CONTINUE
1317* RETURN
1318* END

DIAGNOSTICS

ACTION TIME = 49.98 SUPS

CSSL=TRAN+CSSL
Appendix D

Subroutine MBDYFN Listing and User Requirements

Subroutine Entry Statements

CALL MBDYFN(NC, H, MB, MS, PB, PS, G, PI,
NF, F, EIG, REC, RF, WF, ZF)
CALL MRATE(NC, TH, TB, TS, FB, FS, TF, FF, GM, GMD,
GMDD, ET, ETD, WO, WDOT, ETD, HM)

Input /Output Variable Type and Storage Specifications

INTEGER NC, NF, H(nc, 2), F(nf, 3), PI(n + 1)
REAL MB(7), MS(nc, 7), PB(/i, 3), PS(nf, nc, 3),
G(n, 3), TH(n), TB(3), TS(nc, 3), FB(3), FS(nc, 3),
GM(n), GMD(n), GMDD(n), EIG(nf, 6nk, Nk),
REC(nf, 6, Nk), RF(nf, nk, 3), WF(nf, Nk),
ZF(nf, Nk), TF(nf, nk, 3), FF(nf, nk, 3),
ET(nf, Nk), ETD(nf, Nk), WO(3).

DOUBLE PRECISION WDOT(n + 3), ETDD(nf, Nk)

External Subroutines Called

CHOLI>— (see Appendix C and statement 1013)

Subroutine Setup

Insert the Fortran statement

PARAMETER QC = nc, QH = n, QF = nf, NK = nk, NKT = Nk

(If more than one appendage is present, use the largest nk and Nk for the
PARAMETER statement to provide sufficient storage.)

Data Restrictions

n > 1, nf > 1, nc > 1, nk > 1, Nk > 1
Core Storage Required

Code: 4500 words

Data: ~500 words (minimum; increases with \( n, \eta \)).

Listing

```fortran
SUBROUTINE MDOYFN(NC,CMB,MA,PB,PA,GP,RF,IFGE,REC,RF,RF,ZF)

ADJUSTABLE DIMENSIONS

INTEGER PI(NC),C(NC+2)
REAL MB(NC),MA(NC,7),PB(NC,3),PA(NC,NC,3)
PARAMETER QC(1,NC=2),UF=1,NIK=1,NKT=7
PARAMETER MB=6+NX+NC,C1=1,V=UH+3,V=QV+5,S3=3,S4=UH+NH=NM
PARAMETER ST=V+RF*NKT,S4=S+ST

ADDITIONAL DIMENSIONED VARIABLES

COUNT DOUBLE PRECISION A(3,NK),M(3),B(4,3),E(3)
REAL A0013,3),AB(3,3),AOF(3,3),AKT(3,3),AL1(3,3)
C ADDITIONAL DIMENSIONED VARIABLES

DO 86 K=1,NC
DO 86 J=1,NB
IF(K.EQ.(J-1)) CPS(K,J)=1
IF(K.LE.(J-1)) GO TO 86
GO TO 86
CONTINUE

CONTINUE

CONTINUE

L=Q
DO 1 J=1,NC
K=K(J,2)
DO 1 K=1,NC
L=L+1
DO 1 I=1,NB
EPSL(I,1)=CPS(J,1)
CONTINUE

COMPUTE H(I)=C, WHERE H=HINGE LABEL AND C=CONNECTION LABEL

END
```

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590  C  COMPUTE $H_{ij}(l_j, J_i)$, WHERE $i$ = BODY LABEL + 1 AND $j$ = NEAREST MINGE LABEL
600  C
610  C
620  C
630  C
640  C
650  C
660  C
670  C
680  C
690  C
700  C
710  C
720  C
730  C
740  C
750  C
760  C
770  C
780  C
790  C
800  C
810  C
820  C
830  C
840  C
850  C
860  C
870  C
880  C
890  C
900  C
910  C
920  C
930  C
940  C
950  C
960  C
970  C
980  C
990  C
1000  C
1010  C
1020  C
1030  C
1040  C
1050  C
1060  C
1070  C
1080  C
1090  C
1100  C
1110  C
1120  C
1130  C
1140  C
1150  C
1160  C
1170  C
1180  C
1190  C
1200  C
1210  C
1220  C
1230  C
1240  C
1250  C
1260  C
1270  C

COMPUTE $H_{ij}(l_j, J_i)$, WHERE $i$ = BODY LABEL + 1 AND $j$ = NEAREST MINGE LABEL

IF $i$ = $J_i$ GO TO 47
K1 = $H_{i}(1)$
K2 = $H_{i}(2)$
IF $k_1$ = $EQ_k_2$ GO TO 47
$H_{i}(2+1) = 1$
CONTINUE

DEFINE $F_{ij}(j) = k$, WHERE $i$ = BODY LABEL + 1 AND $k$ IS APPENDAGE LABEL
IF $k = 0$, BOUT HAS NO FLEX, APPENDAGE

DO 239 $H = 1, n_B$
IF 242 $k = 1, n_F$
$J = F_{i}(k, 1) + 1$
$F_{i}(J) = k$
$N_F = N_F$
$N_B = N_B$

DEFINE SUBSTRUCTURE MASSES

$M_S(1) = n_B(7)$
$M_S(n) = M_A + n_B(1, 7)$

TOTAL NUMBER OF FLEX, APPENDAGE NODES TO BE RETAINED

$N_T = 0$
$N_T = N_T + F_{i}(k, 3)$

INITIAL CALCULATION OF BARYCENTER VECTORS $B_0 + T_0$ BODY $C_0 + S$ AND MINGE POINTS

$1X(1) = n_B(1)$
$1Y(1) = n_B(2)$
$1Z(1) = n_B(3)$
$1X(1) = n_B(4)$
$1Z(1) = n_B(5)$
$1Y(1) = n_B(6)$
$B_M(1) = M_B(7)$
$T_M = B_M(1)$
$T_0 = T_M(1)$
$N_0 = 35$ $J = 2, n_B$
$1X(J) = N_A(J + 1, 1)$
$1Y(J) = N_A(J + 1, 2)$
$1Z(J) = N_A(J + 1, 3)$
$1X(J) = N_A(J + 1, 4)$
$1Z(J) = N_A(J + 1, 5)$
$1Y(J) = N_A(J + 1, 6)$
$B_M(1) = N_A(J + 1, 7)$
$T_M = T_M(1)$
$N_0 = 14$ $J = 1, n_B$
$1X(J) = 1$
$1Y(J) = 1$
$1Z(J) = 1$
$N_0 = 149$ $J = 1, n_B$

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DO 500 K=1, N

IF(CPS(K,J)+E=1) GO TO 500

CONTINUE

GO TO 199

LX(I,J)=PA(I,K,J)

LT(I,J)=PB(I,J)

LZ(I,J)=PB(I,J)

GO TO 199

DO 90 L=1,N

JF(CPS(L,J)+E=1) GO TO 101

CONTINUE

GO TO 199

LX(I,J)=PB(I,J)

LT(I,J)=PB(I,J)

LZ(I,J)=PB(I,J)

CONTINUE

GO TO 13 N=1,NB

GO TO 13 J=1,NB

DZ(N,J)=LX(N,J)

DY(N,J)=LY(N,J)

DT(N,J)=LZ(N,J)

DO 13 K=1,NB

DZ(N,J)=DZ(N,J)+(BMASS(K)/TM)*LX(N,K)

DY(N,J)=DY(N,J)+(BMASS(K)/TM)*LY(N,K)

DT(N,J)=DT(N,J)+(BMASS(K)/TM)*LZ(N,K)

13 CONTINUE

CALCULATION OF AUGMENTED INERTIA DYADICS FOR EACH BODY

DO 3 1=1,NB

PM(N,1,1)=IX(N)

PM(N,1,2)=IX(N)

PM(N,1,3)=IX(N)

PM(N,2,3)=IY(N)

PM(N,3,3)=IZ(N)

PM(N,3,3)=IZ(N)

DZ(N,J)=DZ(N,J)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,1,2)=PM(N,1,2)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,1,3)=PM(N,1,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,2,2)=PM(N,2,2)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

30 PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

31 PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

PM(N,3,3)=PM(N,3,3)+(BMASS(J))*DZ(N,J)*DZ(N,J)

C DEFINE PEK(3 X NKT ARRAY)

C DEFINE DLK=TRANSPOSE MATRIX (3 X NKT ARRAY)

DO 201 K=1,NF

JNT=F(K,3)

DO 201 J=1,N

PK(K,J)=REC(K,J)

DO 201 K=1,3

DLK(K,J)=REC(K,J)

RETURN

ENTRY HRATE(NC,TM,TB,TA,FB,PA,TF,FP,NH,NK,E0,ET,ETD,ED,EDT,EDT,EDT)

SD=N

REAL TF(3),FP(3),NH(3),NK(3),ET(3),ETD(3),ED(3)

100 S(1)=ET(1)

RETURN

DOUBLE PRECISION ECI(T),ETD(1)(NKT),EDOT(V)

C BODY-TO-BODY COORDINATE TRANSFORMATION MATRICES
DO 325 J=1,NN
MN=J-1
MN(J+1)=1
SGM=SGM(GM(J))
CMH=CMH(CM(J))
CMH=1+CMH
G1=CMH(G(J+1))
G2=CMH(G(J+2))
G3=CMH(G(J+3))
SG1=SGM(G(J+1))
SG2=SGM(G(J+2))
SG3=SGM(G(J+3))
G1=G1+G(J+1)
G2=G2+G(J+2)
G3=G3+G(J+3)
G1=G1+G(J+1)
G2=G2+G(J+2)
G3=G3+G(J+3)
AB(1,1)=CMH*G1
AB(1,2)=SGM*G2
AB(1,3)=SGM*G3
AB(2,1)=SGM*G2
AB(2,2)=CMH*G2
AB(2,3)=SGM*G3
AB(3,1)=SGM*G3
AB(3,2)=SGM*G3
AB(3,3)=CMH*G3
IF(J*EPS(J+1)) GO TO 3350
DO 321 L=M(N)
IF(EPS(L,N)>EPS(1)) GO TO 322
CONTINUE
GO TO 3350
K=L
DO 339 L=1,J
DO 339 M=1,J
T(L,L,M)=0
DO 339 L=1,J
DO 339 L=1,J
T(L,L,M)=AB(1,L,1)*T(L,L,M)
GO TO 335
CONTINUE
DO 335 L=1,J
DO 335 M=1,J
CONTINUE
T(L,L,M)=AB(1,L,1)
CONTINUE
COORD* TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
DO 362 I=1,N
DO 362 J=1,J
DO 362 K=1,K
GO(J,J)=GO(J,J)+T(I,I,J)*G(K,J)
CONTINUE
ANG* VELOCITY COMPONENTS OF EACH BODY (IN REF. BODY FRAME)
DO 366 K=1,N
GG(K,1)=GMD(K)*G(K,1)
GG(K,2)=GMD(K)*G(K,2)
GG(K,3)=GMD(K)*G(K,3)
DO 361 J=1,N
KV=M(J)
WXO(J)=WXO(J)
WXO(J)=WXO(J)+GG(K,1)
WXO(J)=WXO(J)+GG(K,2)
ANG. VELOCITY COMPONENTS AT EACH Hinge (in Ref. Body Frame)

DO 3444 NM = 1, NH
     M = M + 1
     MC = CMC(M) + 1
     WH = WH(M) + 1
     WHO = WHO(M) + 1
     WHZ = WHZ(M) + 1
     IF(M = NH) GO TO 3467
     DO 3466 NH = NH + 1
     WH = WH x WH = WH(M) + 1
     CMC = CMC(M) + 1
     CH = CMCM(C) + 1
     IF(N = NH) GO TO 3447
     CONTINUE
     NH = NH + 1
     CMCM = CMCM(C) + 1
     CONTINUE
865
     WCM(M, 1) = GM(M, 1) * WCH(M) + WH = GM(M, 2) * WHZ
     WCM(M, 2) = GM(M, 2) * WHZ + GM(M, 3) * WCH
     WCM(M, 3) = GM(M, 3) * GM(M, 2) * WHX + GM(M, 1) * WCH
     CONTINUE

C TRANSFORM PK AND DLK TO REF. BODY BASIS

DO 3444 K = 1, NF
     KK = F(I, 1) + 1
     JNT = F(I, 2)
     IF(KK = JNT) GO TO 4720
     NH = NH + 1
     DO 4721 J = 1, JNT
     PKO(K, 1, J) = 0
     DO 4721 J = 1, JNT
     CONTINUE
     CONTINUE

COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN Ref. Coord.)

DO 254 N = 1, NB
     IF(N = NM) GO TO 259
     JLM = F(1, 2)
     JLM = F(1, 2)
     JLM = F(1, 2)
     DO 253 J = 1, JLM
     CONTINUE
     CONTINUE
     CONTINUE
     CONTINUE

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3350  DO 245 J=1, JN
3360  DO 245 I=1, J
3370  245  FSI(N,I)*FSI(N,1)*FF(I,L,J)
3380  CONTINUE
3390  C
3400  C  COMPUTE TRANSLATIONAL AND ROTATIONAL DISPLACEMENTS OF ENCLOSED SUB-BODIES
3410  C
3420  DO 232 K=1, KF
3430  JNF(K,2)
3440  LNF(K,3)
3450  C  DO 233 J=1, JN
3460  DO 233 I=1, J
3470  U(K,J,I)=0,
3480  [D0(I,J)=0]
3490  DO 233 L=1, LK
3500  233  U(K,J,L)=[U(K,J,L)]=EIG(K,I,D,L)*ET(K,L)
3510  CONTINUE
3520  C
3530  C  COMPUTE C*M* PERTURBATION (FROM NON-UNDEFORMED LOCATION) ON EACH
3540  C  SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORDS)
3550  C
3560  DO 262 K=1, KF
3570  [KNF(K,1)*]
3580  JNF(K,3)
3590  C  DO 263 J=1, JN
3600  MCK(K,1)=0,
3610  DO 265 J=1, JN
3620  DO 265 I=1, J
3630  263  MCK(K,1)=MCK(K,1)-PK(K,1,J)*ET(K,J)
3640  DO 266 I=1, J
3650  266  MK(K,1)=MCK(K,1)/MSB(JK)
3660  CONTINUE
3670  C
3680  C  COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE WRT ITS
3690  C  INSTANTANEOUS C*M* (IN LOCAL COORDS)
3700  C
3710  DO 268 L=1, J
3720  268  TS(I,L)=TB(L)
3730  DO 267 N=2, NB
3740  K=N-1
3750  DO 267 L=1, J
3760  267  TS(N,L)=TA(K,L)
3770  DO 2670 N=1, NB
3780  LN=Fl(N)
3790  IF (LN.EQ.0) GO TO 2670
3800  JNF(L+2)
3810  DO 2671 J=1, JN
3820  DO 2671 L=1, J
3830  2671  TS(N,L)=TS(N,L)+TF(I,L,J)
3840  CONTINUE
3850  DO 269 N=1, NB
3860  K=N-1
3870  IF (K.EQ.0) GO TO 269
3880  TS(N,1)=TS(N,1)+CK(K,2)*FS(N,1)+CK(K,1)*FS(N,2)
3890  TS(N,2)=TS(N,2)+CK(K,2)*FS(N,1)+CK(K,1)*FS(N,3)
3900  TS(N,3)=TS(N,3)+CK(K,2)*FS(N,2)+CK(K,1)*FS(N,4)
3910  CONTINUE
3920  DO 271 N=1, NB
3930  K=N-1
3940  IF (K.EQ.0) GO TO 271
3950  JNF(K,2)
3960  DO 272 J=1, JN
3970  RUXRF(K,J)=U(K,J+1)
3980  RUYRF(K,J)=U(K,J+2)
3990  RUXRF(K,J+1)=U(K,J+3)
4000  TS(N,1)=TS(N,1)+RUXRF(K,J)+RUYRF(K,J)
4010  TS(N,2)=TS(N,2)+RUXRF(K,J)+RUYRF(K,J)
4020  TS(N,3)=TS(N,3)+RUXRF(K,J)+RUYRF(K,J)
4030  CONTINUE
4040  JPL TECHNICAL REPORT 32-1598
TRANSFORM VECTORS TO REF. BODY FRAME

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,H)*TS(H,J)

M=TS(H,J)*TS(H,J)+TS(H,J)*TS(H,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)

DO 17 J=1,NB

K=N

L*F(K,J)=F(K,J)+FS(K,J)*TS(J,J)

M=TS(J,J)*TS(J,J)+TS(J,J)*TS(J,J)
CONTINUE

DO 781 M=1,3

781 PS1(M,1) = PH1(M,1)

C TRANSFORM AUGMENTED BODY INERTIA DYADICS TO REF. BODY FRAME

DO 363 I=2,NB

M=M1(I)

DO 364 J=I,3

AB(J,K) = 0.

DO 364 L=I,3

AB(J,K) = AB(J,K) + PH1(I,J,L) * TIM(L,K)

CONTINUE

DO 365 J=1,3

DO 365 K=1,3

PS1(I,I,J,K) = 0.

DO 365 L=1,3

PS1(I,I,J,K) * TIM(L,J) * AB(L,K)

CONTINUE

CONTINUE

CONTINUE

DO 208 K=1,NF

KK = F(KK) = I

M=M1(KK)

JNT = F(JNT)

IF(KK.EQ.1) GO TO 2090

DO 209 I=1,3

PS0(I,I) = 0.

DO 209 K=1,3

PS0(K,I) * PS0(K,I) + TIM(M,J,L) = MCK(K,J)

GO TO 208

207

208 CONTINUE

CONTINUE

DO 2091 I=1,3

2091 PS0(K,I) = MCK(K,I)

CONTINUE

CONTINUE

VECTORS CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING

VECTORS SCALE FACTORS INVOLVING THE CONNECTING ROOT ANGULAR

VELOCITIES AND THE MUTUAL BART CENTER-HINGE VECTORS

CONTINUE

CONTINUE

DO 320 N=1,NB

I=F1(N)

CONTINUE

DO 476 J=1,3

C=M0(D(N,J)) = 0.

CPX = 0.

CPY = 0.

CPZ = 0.

CPFX = 0.

CPFY = 0.

CPFZ = 0.

DCPX = 0.

DCPY = 0.

DCPZ = 0.

CONTINUE

CONTINUE

DO 7301 L=1,NB

CONTINUE

IF(L.EQ.0) GO TO 7149

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

WDX = WY0(L) + DZ0(L,N) - WZ0(L) + DT0(L,N)

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE
CONTINUE

IF(N.EQ.0) GO TO 487

CMBD(N+1) = CMBD(N) + NMBDY

CMBD(N+2) = CMBD(N+1) + NMBDFZ

CMBD(N+3) = CMBD(N+2) + NMBFZ

487 CONTINUE

CPFX = CPFX + NMBDZ

CPFY = CPFY + NMBFY

CPFZ = CPFZ + NMBFDZ

CPFX = CPFX + NMBDX

CPFY = CPFY + NMBDY

CPFZ = CPFZ + NMBDZ

2301 CONTINUE

DFX = DTYO(N+1) + FEZ0(N) - DZ0(N+1) - FETYO(N)

DFY = DZ0(N+1) + FEYO(N) - DAXO(N+1) - FETO(N)

DFZ = DAXO(N+1) + FETO(N) - DTYO(N+1) - FEY0(N)

IF(1.NE.0) GO TO 7146

487 CONTINUE

HA = 0

HT = 0

HZ = 0

7146 CONTINUE

IF(1.EQ.0) GO TO 243

FACT = MSB(N)/TM

FTX = FTXO / FACT

FTY = FTYO / FACT

FZ = FTZO / FACT

PSFX = (PG50(1,2) * (FEZ0(N) - FTM) - PG50(1,3) * (FEYO(N) - FTM)) / MSB(N)

PSFY = (PG50(1,1) * (FEYO(N) - FTM) - PG50(1,2) * (FEZ0(N) - FTM)) / MSB(N)

PSFY = (PG50(1,1) * (FEYO(N) - FTM) - PG50(1,2) * (FEZ0(N) - FTM)) / MSB(N)

PMWDX = PG50(1,2) * CPFZ - PG50(1,3) * CPFY

PMWDX = PG50(1,1) * CPFY - PG50(1,2) * CPFZ

GO TO 244

243 CONTINUE

PSFX = 0

PSFY = 0

PSFZ = 0

PMWDX = 0

PMWDY = 0

PMWDFZ = 0

244 CONTINUE

K = 3*N(N+1)

E(K+1,1) = H*Z0(N) - H*Z0(N) - TAO(N) * CPX * OFX + PGFX = P*MWDX

E(K+2,1) = H*X0(N) + H*X0(N) + TAO(N) * CPY * OFY + P*MY 

E(K+3,1) = H*TO(N) - H*AO(N) + TAO(N) * CPZ * OFZ + PGFZ = P*MWDFZ

230 CONTINUE

AOD MATRIX ELEMENT COMPUTATION (3x3)

DO 3001 J = 1,3

DO 3001 J = 1,3

3001 AOD(1,1) = 0

DO 3 = 3 * N + 1, N + 8

D10 AOD(1,1) = AOD(1,1) + P*S(J)
ADD(1,2)*ADD(1,2)*PS(1,J+1,2)
ADD(1,3)*ADD(1,3)*PS(1,J+1,3)
ADD(2,2)*ADD(2,2)*PS(1,J+2,2)
ADD(2,3)*ADD(2,3)*PS(1,J+2,3)
ADD(3,3)*ADD(3,3)*PS(1,J+3,3)
CONTINUE
ADD(2,1)*ADD(1,2)
ADD(3,1)*ADD(1,3)
ADD(3,2)*ADD(2,3)
C
FLEX APPEND CONTRIBUTION TO ADD MATRIX COMPUTATION (3X3)
DO 210 K=1,NB
KK=FL(K)
DO 210 L=1,NB
IF(K.GT.L) GO TO 210
DO 210 J=1,3
DO 210 J=1,3
210 CONTINUE
LLL=FL(L)
IF(KK.EQ.K) GO TO 2101
DO1=PSO(K+1,K,1)*DXO(L,K)
DO2=PSO(K+1,K,2)*DXO(L,K)
DO3=PSO(K+1,K,3)*DXO(L,K)
DO 2101 PSF(K+1,L,1)=DO1+DO2+DO3
DO 2101 PSF(K+1,L,2)=DO1+DO2+DO3
DO 2101 PSF(K+1,L,3)=DO1+DO2+DO3
PSF(K+1,L,1)=PSF(K+1,L,1)*DXO(K,L)
PSF(K+1,L,2)=PSF(K+1,L,2)*DXO(K,L)
PSF(K+1,L,3)=PSF(K+1,L,3)*DXO(K,L)
PSF(K+1,L,2)=PSF(K+1,L,2)*DXO(K,L)
PSF(K+1,L,3)=PSF(K+1,L,3)*DXO(K,L)
PSF(K+1,L,2)=PSF(K+1,L,2)*DXO(K,L)
PSF(K+1,L,3)=PSF(K+1,L,3)*DXO(K,L)
CONTINUE
IF(LL.EQ.L) GO TO 210
DO 2102 J=1,3
PSF(K,L+1,J)=AB(1,J)*PSF(K,L+1,J)
DO 2102 J=1,3
DO 2102 J=1,3
210 CONTINUE
GO TO 210
DO 2103 J-1,3
PSF(L+1,J)=AB(1,J)*AB(J+1)
DO 2103 J-1,3
DO 2103 J-1,3
210 CONTINUE
DO 2104 J=1,3
IF(K.LE.L) GO TO 2151
DO 2104 J=1,3
DO 2104 J=1,3
2104 CONTINUE
DO 2105 J=1,3
PSF(K+1,L,J)=PSF(K+1,L,J+1)
2105 CONTINUE
DO 2106 J=1,3
DO 2106 J=1,3
2106 CONTINUE
DO 2107 J=1,3
DO 2107 J=1,3
2107 CONTINUE
DO 2108 J=1,3
DO 2108 J=1,3
2108 CONTINUE
IF(KK.EQ.0) AND (LLL.EQ.U) GO TO 3004
CONTINUE

AOK VECTOR ELEMENT COMPUTATION (3x1)

DO 14 M=1:NM
14 IQ=M(M)+1

AV(M,1)=Q0
AV(M,2)=Q0
AV(M,3)=Q0
DO 7 J=1:NB
7 IF(EPS(M,J)+EQ.0) GO TO 7

PSF(J+1,N)=Q0
DO 10 L=1:NM
10 PSF(J+1,N)=PSG(J+1,N)+(PS(J+1,N)+PSF(J+1,N))=GO(L,N)

AV(M,N)=AV(M,N)+PSG(J+1,N)
CONTINUE

DO 20 J=1:NB
20 AB(J,1)=0.

DO 22 L=1:NB
22 AB(L,1)=AB(1,2)+DZ0(L,J)

AB(1,3)=AB(1,3)+DZ0(L,J)

DO 22 K=1:3
22 AB(2,K)=AB(2,K)+DZ0(K,1)

DO 22 K=1:3
22 AB(3,K)=AB(3,K)+DZ0(K,1)

DO 22 K=1:3
22 AB(3,1)=AB(3,2)+DZ0(1,1)

CONTINUE

DO 22 K=1:3
22 AB(K,1)={0.0}

CONTINUE

CONTINUE

DO 22 K=1:3
22 AB(K,1)={0.0}

CONTINUE

CONTINUE

JPL TECHNICAL REPORT 32-1598
DO 231 J=1, J
231 AB(I,J)=0,
DO 226 L=1, NB
IF(EPS(K+L+EQ+G) GO TO 226
226 AB(I,J)=AB(I,J)+D20(L, J)
DO 210 L=1, J
DO 228 J=1, J
IF(EPS(K+J)+EQ+G) DUR(I, J)+=0
DO 226 L=1, J
DUR(I, J)=DUR(I, J)+AB(I,J)*PKO(K+L, J)
DO 221 J=1, J
DO 2241 J=1, J
DO 229 J=1, J
AKF(K, M, J)=0,
DO 229 J=1, J
AKF(K, M, J)=AKF(K, M, J)*GO(M, I)*DUR(I, J)
C COMPUTE CORRECTION ELEMENTS FOR (E) VECTOR
DO 41 J=2, NB
JK=M[1, J]
DO 411 M=1, J
411 CM(J, M)=0.
DO 42 K=1, JK
IF(EPS(K+J)+EQ+G) GO TO 42
CM(J, 1)=CM(J, 1)+MGJ(K, 1)
CM(J, 2)=CM(J, 2)+MGJ(K, 2)
CM(J, 3)=CM(J, 3)+MGJ(K, 3)
CONTINUE
CONTINUE
DO 49 J=1, NB
EA(1)=0.
EA(2)=0.
EA(3)=0.
DO 40 J=2, NB
DO 4507 M=1, J
DO 4507 L=1, J
507 EA(M)=EA(M)+EPS(I, J)+M+L)*PSF(I, J, M, L)+CM(I, J, L)
401 CONTINUE
K=J+1
E(K)+J=E(K)+J=E(A(I))
E(K)+J=E(K)+J=E(A(2))
E(K)+J=E(K)+J=E(A(3))
CONTINUE
DO 55 M=1, J
40 EC(M)=EC(M)+E(K, 1)
DO 52 J=2, NB
DO 52 M=1, J
K=M+1
EC(M)=EC(M)+E(K, 1)
I=0
DO 60 M=1, NB
JK=M[1, J]
IF(EPS(K)+EQ+G) GO TO 60
I=I+1
EC(I+3)=0.
DO 60 J=1, NB
EC(M)=0.
CONTINUE
DO 61 J=1, NB
IF(EPS(K)+EQ+G) GO TO 61
DO 65 K=1,N
   J=3*(J-1)+M
621* DO 65 CE(N)*CE(J)*CE(J,1)
622* CONTINUE
622* DO 46 L=1;V
624* DO 46 EC(I,J)*EC(J,1)*GE(R,L)*CE(L)
625* CONTINUE
626* DO 40 GO TO 610
627* IF(P(I,J)+EQ.0) GO TO 610
628* EC(I,J)*AV(J,J)*GM(J)
631* CONTINUE
632* K=0
633* IV=3
634* GO TO 612
635* IF(P(I,J)+NE.0) GO TO 612
636* K=1
637* IV=1
638* DO 611 J=1,NH
639* IF(P(I,J)+EQ.0) GO TO 611
640* AV(J,J)*AS(J,J)*AS(J,J)+GM(J)
641* CONTINUE
642* CONTINUE
643* CONTINUE
644* C COMPUTE RT, HAND SIDE OF APPENDAGE EQUATIONS (IN APPEND COORDS)
645* C
646* DO 483 K=1;NF
647* IM=K+1
648* MH=1
649* M=1
650* CQ(I,J)*FTXO/TH + CM6D(I,J)
651* CQ(2)*FTY0/TH + CM6D(I,J)
652* CQ(3)*FTZ0/TH + CM6D(I,J)
653* IF(I+EQ.1) GO TO 484
654* DO 484 J=1,NH
655* VE(K,J)+0
656* DO 484 L=1,NH
657* VE(K,J)*VE(K,J)*TM(J,L)*CM6D(L)
658* GO TO 483
659* CONTINUE
660* DO 484 J=1,NH
661* VE(K,J)*CM6D(J)
662* CONTINUE
663* DO 485 K=1;NF
664* NL=K+2
665* DO 486 N=1,HL
666* NB=N+1
667* DO 488 J=1,NH
668* JM=NH+J
669* JM=NH+J
670* VB(K,JM)=FF(K,N,J)
671* VB(K,JM)=FF(K,N,J)
672* CONTINUE
673* CONTINUE
674* NV=1
675* DO 491 K=1;NF
676* JM=K+3
677* NL=K+2
678* NL=K+2
679* DO 492 J=1;MN
680* I=NH+J
681* VF=VF(K,J)+Z*ZF(K,J)*TFD(K,J)+RF(K,J)+ETL(K,J)
682* DO 493 N=1,NL
683* VF=VF+G(K,N)+VF(K,N)
684* DO 494 N=1,3
685* VF=VF+G(K,N)+VF(K,N)
686* VF=VF+ZRF(K,N)
687* EC(I,J)=VF

ORIGINAL PAGE IS OF POOR QUALITY
DO 406 J=1,NL
   DO 407 K=1,ML
      NL=NV(J,K)
   CONTINUE
END

END
LOAD SYSTEM MATRIX (A) WITH AOQ+ADQ+AKM ELEMENTS

DO 23 J=1,3
DO 23 J=1,3
23 A(I,J)=AOQ(I,J)
DO 24 I=1,3
24 K=0
DO 29 J=1,NH
IF(I(I,J)+NEQ) GO TO 24
KW=1
AIK*3,1]=AV(J,1)
AI(I,K*3]=AV(J,1)
CONTINUE
29 K=0
DO 250 I=1,NH
IF(I(I,J)+NEQ) GO TO 250
KM=1
LQ=0
DO 25 J=1,NH
IF(I(I,J)+NEQ) GO TO 25
LJ=LJ+1
IF(K*3+L)=26
AIK*3,L]=AS(I,J)
GO TO 25
26 AIK*3+L]=AL(L+3,K+3)
25 CONTINUE
250 CONTINUE
ANGULAR MOMENTUM OF THE SYSTEM
IF(I(I,NH)+I)+NEQ) GO TO 8752
DO 5651 I=1,3
5651 HH(I)*Q=0
DO 5651 J=1,3
5651 HH(I)*HH(I)*AI(I,J)*BD(J)
DO 5652 I=1,3
5652 DO 5652 J=1,3
5652 HH(I)*HH(I)*AV(J,1)*BD(J)
DO 5653 I=1,3
DO 5653 K=1,4F
5654 NL=FK(I,3)
DO 5654 J=1,4F
5654 HH(I)*HH(I)*AV(I,J)*BD(K,J)
5653 CONTINUE
5653 CONTINUE
H=SQRT(HH(I)*HH(I)*HH(2)*HH(3))
8752 CONTINUE
SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANG. ACCELERATION AND HINGE
RELATIVE ROTATIONAL ACCELERATIONS

NT=V*NTMO
IT=J*NTMO
KV=IV
CALL CHOLD(B92,A,ST,IT,EC,G,1+0=7)
DO 10 J=NT+4-1
IF(J*LE*V) GO TO 913
JY=J*IV=IV
EC(J)*EC(JI)
GO TO 910
910 CONTINUE
913 CONTINUE
K=J-3
IF(I(I,K)+NEQ) GO TO 911
EC(J)*EC(KY)
KY=KV=1
GO TO 910
911 EC(J)*GMOD(K)
910 CONTINUE
1027*  DO 9003 I=1,N
1028*  9003  WRITE(I*I+1,E3)
1029*  T=S
1030*  DO 9001 K=1,NF
1031*     NL=F(K,3)
1032*  DO 9002 N=1,NL
1033*     I=I+M
1034*  9002  ET0D(K,N1)=EC(10)
1035*  9001  I=I+NL
1036*  92  CONTINUE
1037*  RETURN
1038*  END

DIAGNOSTICS

ATIUN TIME = 30.73 SUPS

CSSL=TRAN,CSSL

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Appendix E

Subroutine MBDYFL Listing and User Requirements

Subroutine Entry Statements

Same as MBDYFN (see Appendix D)

Input/Output Variable Type and Storage Specifications

Same as MBDYFN (see Appendix D)

External Subroutines Called

AINVD—double precision matrix inversion subroutine. Inverts any real square, nonsingular matrix, A, and leaves the result in A (see statement 419).

Subroutine Setup

Same as MBDYFN (see Appendix D)

Data Restrictions

Same as MBDYFN

Core Storage Required

Code: 3500 words

Data: ~500 words (minimum; varies with n, n_f)

Listing

```
* SUBROUTINE MBDYFL(NC,CMB,MA,MB,PA,GB,PI,NF,F,ELG,REC,RF,NF,ZF)
* ADJUSTABLE DIMENSIONS
*
* INTEGER PI(1,NC,2)
* REAL MB(1,MA(NC,3),MB(NC,3),PA(NC,NC,3))
* PARAMETER QC=1,GH=2,GF=1,NF=1,NKT=7
* PARAMETER NFK=6*NF,NK=1,VQ=1,V*Q=V3,S=3,S=Q,W,NH=QH
* PARAMETER ST=Y*Q*ST,NF=ST
*
* ADDITIONAL DIMENSIONED VARIABLES
*
* DOUBLE PRECISION A(ST,ST),NRK(SH),BMASS(S)
* INTEGER EPSY(SI,SI),EPS(SI,SI),QH(SI,SI),FI(SI,SI),F(NF,3)
* REAL AQG(4,3),AB(4,3),ADTF(4,3,NKT),AKF(4,4,NKT),A5(4,4),AV(4,4)
* $,AS(3,CL(3),CF,3),CH(3),CL(3),QF(3),QF(3),QF(3),QF(3),QF(3)
* $,AX(3),AX(3),AX(3),AX(3),AX(3),AX(3),AX(3),AX(3),AX(3)
* $,AY(3),AY(3),AY(3),AY(3),AY(3),AY(3),AY(3),AY(3),AY(3)
* $,B(3),B(3),B(3),B(3),B(3),B(3),B(3),B(3),B(3)
* $,C(3),C(3),C(3),C(3),C(3),C(3),C(3),C(3),C(3)
* $,F(3),F(3),F(3),F(3),F(3),F(3),F(3),F(3),F(3)
* $,G(3),G(3),G(3),G(3),G(3),G(3),G(3),G(3),G(3)
* $,H(3),H(3),H(3),H(3),H(3),H(3),H(3),H(3),H(3)
* $,K(3),K(3),K(3),K(3),K(3),K(3),K(3),K(3),K(3)
* $,L(3),L(3),L(3),L(3),L(3),L(3),L(3),L(3),L(3)
* $,M(3),M(3),M(3),M(3),M(3),M(3),M(3),M(3),M(3)
* $,N(3),N(3),N(3),N(3),N(3),N(3),N(3),N(3),N(3)
* $,O(3),O(3),O(3),O(3),O(3),O(3),O(3),O(3),O(3)
* $,Q(3),Q(3),Q(3),Q(3),Q(3),Q(3),Q(3),Q(3),Q(3)
* $,S(3),S(3),S(3),S(3),S(3),S(3),S(3),S(3),S(3)
* $,U(3),U(3),U(3),U(3),U(3),U(3),U(3),U(3),U(3)
* $,W(3),W(3),W(3),W(3),W(3),W(3),W(3),W(3),W(3)
* $,X(3),X(3),X(3),X(3),X(3),X(3),X(3),X(3),X(3)
* ORIGTNAC PAGES
```
DEFINE EPS(K, J) USING C

C

DO 86 J = 2, NB

IF(K.EQ.(J-1)) CPS(K, J) = 1
IF(K.LT.(J-1)) GO TO 87

GO TO 86

CONTINUE

C

DO 89 J = 1, NC

J = J + 1

DO 89 L = J, J + 1

IF(K.EQ.(L-1)) GO TO 89
IF((CPS(K, L).EQ.1).AND.(C(J-1, L).EQ.(L-1))) CPS(K, J) = 1

CONTINUE

C

COMPUTE M(J) = C, WHERE J = MINGE LABEL AND C = CONNECTION LABEL

C

DO 98 J = 1, NB

K = C(J-1, 2)

DO 98 K = 1, KK

L = L + 1

DO 98 I = 1, NB

EPS(J-1) = CPS(J, I)

C

COMPUTE M(J) = J, WHERE J = BODY LABEL+1 AND J = NEAREST MINGE LABEL

C

M(J) = 1

M(NB) = NB

DO 97 J = 1, NB

IF(J.EQ.1) GO TO 97

K = M(J)

K2 = M(J+1)

IF(K.EQ.K2) GO TO 97

M(J+1) = J + 1

CONTINUE

C

DEFINE F(J) = K, WHERE J = BODY LABEL+1 AND K IS APPENDAGE LABEL

IF K = 0, BODY HAS NO FLEX APPENDAGE

C

DO 239 M = 1, NB

F(J) = 0

DO 239 M = 2, NB

J = K + 1, NF

F(J) = K

NF = NF

NB = NB

C

DEFINE SUBSTRUCTURE MASSES

C

MNB(1) = MB(7)

DO 248 M = 2, NB

MNB(M) = MB(M)

MNB = MNB

C

TOTAL NUMBER OF FLEX APPENDAGE NODES TO BE RETAINED

C

MNB = 0

DO 961 K = 1, NF
INITIAL CALCULATION OF BARYCENTER VECTORS AND HINGE POINTS

1. \( \mathbf{I} \times \mathbf{X}(1) = \mathbf{M}(1) \)
2. \( \mathbf{I} \times \mathbf{Y}(1) = \mathbf{M}(2) \)
3. \( \mathbf{I} \times \mathbf{Z}(1) = \mathbf{M}(3) \)
4. \( \mathbf{I} \times \mathbf{Y}(1) = \mathbf{M}(4) \)
5. \( \mathbf{I} \times \mathbf{Z}(1) = \mathbf{M}(5) \)
6. \( \mathbf{I} \times \mathbf{Z}(1) = \mathbf{M}(6) \)
7. \( \mathbf{BMASS}(1) = \mathbf{M}(7) \)
8. \( \mathbf{TM} = \mathbf{BMASS}(1) \)
9. \( \text{DO} \ 35 \ J = 1, N \)
10. \( \mathbf{I} \times \mathbf{M}(J) = \mathbf{M}(J+1) \)
11. \( \mathbf{I} \times \mathbf{Y}(J) = \mathbf{M}(J+1,2) \)
12. \( \mathbf{I} \times \mathbf{Z}(J) = \mathbf{M}(J+1,3) \)
13. \( \mathbf{I} \times \mathbf{Z}(J) = \mathbf{M}(J+1,4) \)
14. \( \mathbf{I} \times \mathbf{Z}(J) = \mathbf{M}(J+1,5) \)
15. \( \mathbf{I} \times \mathbf{Z}(J) = \mathbf{M}(J+1,6) \)
16. \( \mathbf{BMASS}(J) = \mathbf{M}(J+1,7) \)
17. \( \mathbf{TH} = \mathbf{TM} \times \mathbf{BMASS}(J) \)
18. \( \text{DO} \ 149 \ I = 1, N \)
19. \( I = I + 1 \)
20. \( \text{DO} \ 149 \ J = 1, N \)
21. \( J = J + 1 \)
22. \( \text{IF} \[ \mathbf{I} \times \mathbf{EQ} + J \] \text{GO TO} \ 143 \)
23. \( \text{IF} \[ \mathbf{I} \times \mathbf{EQ} + J \] \text{GO TO} \ 70 \)
24. \( \text{IF} \[ \mathbf{I} \times \mathbf{EQ} + J \] \text{GO TO} \ 90 \)
25. \( \mathbf{LX}(I,J) = \mathbf{PA}(I,J,1) \)
26. \( \mathbf{LY}(I,J) = \mathbf{PA}(I,J,2) \)
27. \( \mathbf{LZ}(I,J) = \mathbf{PA}(I,J,3) \)
28. \( \text{GO TO} \ 149 \)
29. \( \text{CONTINUE} \)
30. \( \text{DO} \ 500 \ K = 1, N \)
31. \( \text{IF} \[ \mathbf{CP}(I,J) \times \mathbf{EQ} + I \] \text{GO TO} \ 500 \)
32. \( \text{CONTINUE} \)
33. \( \text{GO TO} \ 149 \)
34. \( \text{CONTINUE} \)
35. \( \text{GO TO} \ 149 \)
36. \( \text{CONTINUE} \)
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160. \( \text{CONTINUE} \)

C CALCULATION OF AUGMENTED INERTIA STADICS FOR EACH BODY

DO 31 N = 1, NB

PH(N,1,1) = IX(N)
ADDITIONAL AUGMENTED INERTIA DYNAMICS (IN REF. BODY FRAME)

CONTINUE
DO 7 J=1,NB
   DO 7 I=1,NB
   7 CONTINUE
   IF(EPS(M,J)+EQ.0) GO TO 7
   PSG(J,J)+=0.
   DO 10 I=1,J
      PSG(J,J)+=PSG(J,I,N)+PSG(J,J,L)+G(I,L)
   10 CONTINUE
   IF(K+6+M) GO TO 14
   J=AH(K)+1
   AIS(1)=0
   AIS(2)=0
   AIS(3)=0
   AIS(N)=AIS(N)+PSG(J,J,N)
   CONTINUE
   AIS(K,N)=G(K,1)+AIS(1)+G(K,2)+AIS(2)+G(K,3)+AIS(3)
   CONTINUE
   C
   C
   Define PK(3 x NKT Array)
   DO 201 K=1,N
      J=EK(K,J)
   201 CONTINUE
   Define DLK=TRANSPOSE MATRIX (3 X NKT Array)
   DO 202 I=1,N
      J=EK(I,J)
   202 CONTINUE
   Define AKF MATRIX (3 x NKT) (Ref. Body/Flex Appendage Coupling)
   DO 219 K=1,N
      J=EK(K,J)
   219 CONTINUE
   Define AKF VECTOR (1 x NKT) (Flex Coupling with Rigid Substructures)
   DO 229 K=1,N
      J=EK(K,J)
   229 CONTINUE
2990   AB(2,3)=AB(2,3)-DLJ
3000   CONTINUE
3100   AB(2,3)=AB(2,3)+DLJ
3200   AB(3,2)=AB(3,2)-DLJ
3300   DO 228 L=1,3
3400   DT=228 J=1,3
3500   IF(ABS(K(J,M))EQ=0) DUR(1,J)=0,
3600   DO 228 L=1,3
3700   DUR(1,J)=DUR(1,J)-AB(1,L)*PR(K,L,J)
3800   DO 228 J=1,3
3900   AKF(K,M,J)=G,
4000   DO 229 J=1,3
4100   AKF(K,M,J)=AKF(K,M,J)+G(M,J)*DUR(1,J)
4200   CONTINUE
4300   C ENTER CONSTANTS INTO FLEX BODY PORTION OF COEFF. MATRIX A
4400   C
4500   IY=3
4601   DO 129 I=1,NH
4701   IF(F(I,J)NE=0) GO TO 129
4801   IY=IY+1
4901   CONTINUE
5000   NV=IY
5100   DO 422 K=1,NF
5200   NL=FL(K,J)
5300   DO 422 J=1,NL
5400   IL=NY+1
5500   DO 422 J=1,NL
5600   JL=NY+J
5700   A(L,JL)=A(K,L,J)
5800   IF(1*EQ=J) A(L,LJ)=1
5900   CONTINUE
6000   NV=NV+NL
6100   C ENTER COEFF. WHICH COUPLE REF. BODY AND FLEX. APPENDAGES INTO A
6200   NV=NV+NL
6300   C
6400   DO 469 K=1,NF
6500   NL=FL(K,J)
6600   DO 469 J=1,3
6700   DO 469 J=1,3
6800   IL=NY+J
6900   A(L+J)=A(K,J+J)
7000   A(L,J+J)=A(K,L,J)
7100   NV=NV+NL
7200   C ENTER COEFF. WHICH COUPLE SUBSTR BQUIES AND FLEX. APPEND. INTO A
7300   NV=NV+NL
7400   DO 499 K=1,NF
7500   NL=FL(K,J)
7600   J=0
7700   DO 497 J=1,NH
7800   IF(F(I,J)NE=0) GO TO 497
7900   J=J+1
8000   DO 497 J=1,3
8100   IL=NY+1
8200   A(L,J+3)=A(K,J+3)
8300   A(J+3,L)=A(L+J+3)
8400   CONTINUE
8500   CONTINUE
8600   NV=NV+NL
8700   C CALCULATE FLEX BODY COUPLING COEFF. AND ENTER INTO A MATRIX
8800   NV=NV+NL
8900   NV=NV+NL
9000   DO 473 L=1,NF
```
380 CONTINUE
381 A(JK,IK)=A(JK,IK)
382 CONTINUE
383 474 NRO=NRO+1
384 475 NC0=NC0+NL
385 C LOAD SYSTEM MATRIX A WITH AQG,AOK,AKM ELEMENTS
386 C
387 C DO 23 J=1,3
388 C DO 23 J=1,3
389 23 A(1,J)=AQG(1,J)
390 DO 24 J=1,3
391 DO 24 J=1,3
392 K=0
393 DO 24 J=1,NH
394 IF(P(I(J)+NEQ)) 24,25,K=1
395 A(K+3,J)=AY(J,J)
396 A(I,K+3)=AY(J,J)
397 24 CONTINUE
398 C K=0
399 25 CONTINUE
400 DO 26 J=1,NH
401 IF(P(I(J)+NEQ)) 26,27,K=1
402 K=1
403 L=0
404 DO 26 J=1,NH
405 IF(P(I(J)+NEQ)) 26,27,L=1
406 A(K+3,L)=AS(I,J)
407 A(K+3,L)=AS(I,J)
408 GO TO 25
409 C A(K+3,L)=AS(I,J)
410 26 CONTINUE
411 27 CONTINUE
412 25 CONTINUE
413 C SOLVE SYSTEM MATRIX FOR REFERENCE BODY ANGLE ACCELERATION AND HINGE
414 (RELATIVE) ROTATIONAL ACCELERATIONS
415 C
416 C
417 C
418 C NT=V+NT0
419 C IT=I+NT0
420 CALL ADNQ(A,ST,IT,51095,RRK)
421 CONTINUE
422 RETURN
423 ENTRY RATE(NC,TH,TB,TA,FB,FA,TF,FF,GM,GH,600,ET,ET,ET,ET,ET)
424 CONTINUE
425 DO 28,NH
426 REAL TF(QF,MK,31,FF,QF,MK,31,ETQF,NK),ETD,QF,NK),TB(3),TA(3),TAMC(3))
427 S1FB3(3),FAINC,3),GM11,GM011,GM0D11,TH11,RO13,ES(S311)
428 DOUBLE PRECISION EC(ST),ED(D(QF,NK),FDOT(V),EQ(ST)
429 C BODY TO BODY COORDINATE TRANSFORMATION MATRICES
430 C
431 C DO 335 J=1,NH
432 MM=J-1
433 AB(J,1)=GM(J)=GM(J)=GM(J)=GM(J)
434 AB(J,1)=GM(J)=GM(J)=GM(J)
435 AB(J,1)=GM(J)=GM(J)
436 AB(J,1)=GM(J)
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COORD TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)

COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.)

COORD TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)

COMPUTE TOTAL EXTERNAL FORCE ON EACH SUBSTRUCTURE (IN REF. COORD.)

COORD TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)

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COORD TRANSFORMATION OF G VECTORS (TO REF. BODY FRAME)
DO 233 L=1,14
507* \mathcal{U}(k,j_{1}l)\mathcal{V}(k,j_{1}l)\mathcal{E}(k,j_{1}l)\mathcal{F}(k,j_{1}l)
508* CONTINUE
509* C
510* C COMPUTE C*M* PERTURBATION (FROM MOM UNDEFORMED LOCATION) ON EACH
511* C SUBSTRUCTURE WITH AN APPENDAGE (LOCAL COORDS.)
512* C
513* DO 262 K=1,NF
514* IF(K,F1) GO TO 263
515* JN=K
516* DO 263 J=1,N
517* MCK(K,j)=0
518* DO 265 J=1,N
519* DO 265 J=1,N
520* MCK(K,j)=PK(K,j)*ET(K,j)
521* DO 266 J=1,N
522* C(K,j)=MCK(K,j)/MSB(JK)
523* CONTINUE
524* C
525* C COMPUTE TOTAL EXTERNAL TORQUE ON EACH SUBSTRUCTURE & R*T* ITS
526* C INSTANTANEOUS C*M* (IN LOCAL COORDS.)
527* C
528* DO 268 L=1,3
529* TS(1,L)=TB(L)
530* DO 267 N=2,NB
531* K=N
532* DO 267 L=1,3
533* TS(N,L)=TA(K,L)
534* DO 267 N=1,NB
535* IF(L,F1) GO TO 268
536* JN=K
537* DO 268 L=1,3
538* TS(N,L)=TS(N,L)*TF(L,JN)
539* CONTINUE
540* 2670 CONTINUE
541* 2670 C
542* DO 269 N=1,NB
543* IF(K,E9,0) GO TO 269
544* TS(N,1)=TS(N,1)*CK(K,2)*FS(N,3)*CK(K,3)*FS(N,2)
545* TS(N,2)=TS(N,2)*CK(K,3)*FS(N,1)*CK(K,1)*FS(N,3)
546* TS(N,3)=TS(N,3)*CK(K,1)*FS(N,2)*CK(K,2)*FS(N,1)
547* CONTINUE
548* 269 CONTINUE
549* DO 271 N=1,NB
550* K=F1(N)
551* IF(K,E9,0) GO TO 271
552* JN=K
553* DO 272 J=1,JN
554* RUX=RF(K,J,1)+U(K,J,1)
555* RUY=RF(K,J,2)+U(K,J,2)
556* RUX=RF(K,J,3)+U(K,J,3)
557* TS(N,1)=TS(N,1)*RUX*FF(K,J,3)-RUX*FF(K,J,2)
558* TS(N,2)=TS(N,2)*RUX*FF(K,J,1)+RUX*FF(K,J,3)
559* TS(N,3)=TS(N,3)*RUX*FF(K,J,2)-RUX*FF(K,J,1)
560* CONTINUE
561* C
562* C TRANSFORM VECTORS TO REF* BODY FRAME
563* C
564* TAO(I)=TS(I,1)
565* TYO(I)=TS(I,2)
566* TZO(I)=TS(I,3)
567* DO 17 I=2,NB
568* M=111
569* K=1
570* L=1(K,1)
571* FE0(I)=TM(I,1,1)*FS(I,1,1)+TM(M,2,1)*FS(I,2,1)+TM(M,3,1)*FS(I,3,1)
572* FE0(I)=TM(I,1,2)*FS(I,1,2)+TM(M,2,2)*FS(I,2,2)+TM(M,3,2)*FS(I,3,2)
573* FE0(I)=TM(I,1,3)*FS(I,1,3)+TM(M,2,3)*FS(I,2,3)+TM(M,3,3)*FS(I,3,3)
574* TXO(I)=TM(I,1,1)*TS(I,1,1)+TM(M,2,1)*TS(I,1,2)+TM(M,3,1)*TS(I,1,3)
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>575</td>
<td>TTO(I) *T(M,1,2)*TS(I,1,1)*T(M,2,2)*TS(I,1,2)*T(M,3,2)*TS(I,2,1,1)*T(M,2,2,1,1)*DZ(I,1,1)</td>
</tr>
<tr>
<td>576</td>
<td>TTO(I) *T(M,1,2)*TS(I,1,1)*T(M,2,2)*TS(I,1,2)*T(M,3,2)*TS(I,2,1,1)*T(M,2,2,1,1)*DZ(I,1,1)</td>
</tr>
<tr>
<td>577</td>
<td>DZ(I,1,1)*T(M,1,1)*DZ(I,1,1)*T(M,2,2)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>578</td>
<td>DZ(I,1,1)*T(M,1,2)*DZ(I,1,1)*T(M,2,2)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>579</td>
<td>DZ(I,1,1)*T(M,1,2)*DZ(I,1,1)*T(M,2,2)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>580</td>
<td>DZ(I,1,1)*T(M,1,2)*DZ(I,1,1)*T(M,2,2)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>581</td>
<td>DZ(I,1,1)*T(M,1,2)*DZ(I,1,1)*T(M,2,2)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>582</td>
<td>DZ(I,1,1)*T(M,1,3)*DZ(I,1,1)*T(M,2,3)*DY(I,1,1)*T(M,3,3)*DZ(I,1,1)</td>
</tr>
<tr>
<td>583</td>
<td>DO 17 J=1,NB</td>
</tr>
<tr>
<td>584</td>
<td>IF(I,EQ,J) GO TO 17</td>
</tr>
<tr>
<td>585</td>
<td>IF(CPS(K,J)+EQ,1) GO TO 177</td>
</tr>
<tr>
<td>586</td>
<td>IF(CPS(K,J)-EQ(*)J=1)) GO TO 17</td>
</tr>
<tr>
<td>587</td>
<td>DO 20(I,J)=DZ(I,1,1)</td>
</tr>
<tr>
<td>588</td>
<td>DZO(I,J)=DZ(I,1,1)</td>
</tr>
<tr>
<td>589</td>
<td>DZO(I,J)=DZ(I,1,1)</td>
</tr>
<tr>
<td>590</td>
<td>DO 17 CONTINUE</td>
</tr>
<tr>
<td>591</td>
<td>177</td>
</tr>
<tr>
<td>592</td>
<td>DZ(I,1,J)*T(M,1,1)*T(M,2,1)*DY(I,1,J)*T(M,3,1)*DZ(I,1,1)</td>
</tr>
<tr>
<td>593</td>
<td>DZ(I,1,J)*T(M,1,2)*T(M,2,2)*DY(I,1,J)*T(M,3,2)*DZ(I,1,1)</td>
</tr>
<tr>
<td>594</td>
<td>17 CONTINUE</td>
</tr>
<tr>
<td>595</td>
<td>DO 367 I=1,NB</td>
</tr>
<tr>
<td>596</td>
<td>DZ(I,1,1)=DZ(I,1,1)</td>
</tr>
<tr>
<td>597</td>
<td>DZO(I,1,J)=DZ(I,1,1)</td>
</tr>
<tr>
<td>598</td>
<td>DZO(I,1,J)=DZ(I,1,1)</td>
</tr>
<tr>
<td>599</td>
<td>DO 367 CONTINUE</td>
</tr>
<tr>
<td>600</td>
<td>C COMPUTE TOTAL EXTERNAL FORCE ON VEHICLE (IN REF. COORD.)</td>
</tr>
<tr>
<td>601</td>
<td>C</td>
</tr>
<tr>
<td>602</td>
<td>FT2O=0</td>
</tr>
<tr>
<td>603</td>
<td>FT2O=0</td>
</tr>
<tr>
<td>604</td>
<td>FT2O=0</td>
</tr>
<tr>
<td>605</td>
<td>DO 247 N=1,NB</td>
</tr>
<tr>
<td>606</td>
<td>FT2O=FT2O+FEQ1(N)</td>
</tr>
<tr>
<td>607</td>
<td>FT2O=FT2O+FEQ1(N)</td>
</tr>
<tr>
<td>608</td>
<td>FT2O=FT2O+FEQ1(N)</td>
</tr>
<tr>
<td>609</td>
<td>C COMPUTE THE PGSO VECTORS FOR EACH FLIGHT APPEARANCE</td>
</tr>
<tr>
<td>610</td>
<td>C</td>
</tr>
<tr>
<td>611</td>
<td>C</td>
</tr>
<tr>
<td>612</td>
<td>C DO 208 K=1,NF</td>
</tr>
<tr>
<td>613</td>
<td>KK=F(K,1)+1</td>
</tr>
<tr>
<td>614</td>
<td>M=KK/IKK</td>
</tr>
<tr>
<td>615</td>
<td>JNK=F(K,3)</td>
</tr>
<tr>
<td>616</td>
<td>IF(KK+EQ+1) GO TO 2096</td>
</tr>
<tr>
<td>617</td>
<td>DO 209 I=1,N</td>
</tr>
<tr>
<td>618</td>
<td>KGSO(K,1)=0</td>
</tr>
<tr>
<td>619</td>
<td>DO 208 J=1,N</td>
</tr>
<tr>
<td>620</td>
<td>PGSO(K,1)=KGSO1(N,FK)(J)=MCK(K,J)</td>
</tr>
<tr>
<td>621</td>
<td>GO TO 208</td>
</tr>
<tr>
<td>622</td>
<td>208 CONTINUE</td>
</tr>
<tr>
<td>623</td>
<td>C DO 209 I=1,N</td>
</tr>
<tr>
<td>624</td>
<td>C PGSO(K,1)=MCK(K,J)</td>
</tr>
<tr>
<td>625</td>
<td>C CONTINUE</td>
</tr>
<tr>
<td>626</td>
<td>C</td>
</tr>
<tr>
<td>627</td>
<td>C VECTOR CROSS PRODUCTS DESCRIBING SYSTEM ROTATIONAL COUPLING</td>
</tr>
<tr>
<td>628</td>
<td>C (QUADRATIC TERMS INVOLVING THE CONNECTING AND ANGULAR VELOCITIES AND THE MUTUAL HINGE VECTORS)</td>
</tr>
<tr>
<td>629</td>
<td>C</td>
</tr>
<tr>
<td>630</td>
<td>C</td>
</tr>
<tr>
<td>631</td>
<td>C</td>
</tr>
<tr>
<td>632</td>
<td>C</td>
</tr>
<tr>
<td>633</td>
<td>C</td>
</tr>
<tr>
<td>634</td>
<td>C</td>
</tr>
<tr>
<td>635</td>
<td>C</td>
</tr>
<tr>
<td>636</td>
<td>C DO 230 I=1,N</td>
</tr>
<tr>
<td>637</td>
<td>CPX=CPX*DT(I+1,N)+FEQ1(L)*DZ(I+1,L)*FEQ1(L)</td>
</tr>
<tr>
<td>638</td>
<td>CPX=CPX*DT(I+1,N)+FEQ1(L)*DZ(I+1,L)*FEQ1(L)</td>
</tr>
<tr>
<td>639</td>
<td>CPX=CPX*DT(I+1,N)+FEQ1(L)*DZ(I+1,L)*FEQ1(L)</td>
</tr>
<tr>
<td>640</td>
<td>C CONTINUE</td>
</tr>
<tr>
<td>641</td>
<td>C IF(I+EQ+1) GO TO 243</td>
</tr>
<tr>
<td>642</td>
<td>C FACT=MSB(N)/TH</td>
</tr>
<tr>
<td>643</td>
<td>C</td>
</tr>
</tbody>
</table>

**Note:** The code is written in FORTRAN, a programming language used for scientific and engineering computing. The code is for computing forces on a vehicle in a given reference frame.
DO 55 N=1,3
56 EC(M)=E(M+1,1)
DO 52 J=2,NB
DO 52 M=1,3
K=M-1(I)4M
52 EC(M)=EC(M)+E(K+1,1)
K=M
IF(K(N)=NEQ GO TO 60
I=1
EC(I+3)=Q
DO 401 M=1,3
401 CE(M)=Q
DO 41 J=K+1,NB
IF(EPS(K,J)*EQ.0 GO TO 61
DO 40 M=1,3
J=3(J-1)+M
40 CE(M)+CE(M)+E(I,J+1,1)
41 CONTINUE
DO 46 L=1,3
46 EC(I+3)=EC(I+3)+SO(K,L)+CE(L)
45 CONTINUE
DO 40 P=1,3
DO 410 M=1,3
410 EC(I)=EC(I)+AV(J,1)*GM0(J)
410 CONTINUE
K=Q
IV=3
DO 492 M=1,3
492 IF(P(I,J)*NEQ GO TO 612
K=K+1
492 IV=IV+1
DO 492 M=1,3
492 IF(P(I,J)*EQ.Q GO TO 611
700 IF(LST(J)=A(I,J))A(I,J)+A(J,I)
700 EC(K+3)=EC(K+3)+AS(I,J)+GM0(J)
702 CONTINUE
703 CONTINUE
704 CONTINUE
705 C COMPUTE RT, HAND SIDE OF APPENDAGE EQUATIONS IN APPEND. COORDS.
706 C DO 493 K=1,3
707 IFLK(J)+1
708 M=I(I)
710 CQ(I)+FTXO/TH
711 CQ(J)+FTYO/TH
712 CQ(J)+FTZO/TH
713 IF(I*EQ.1) GO TO 4849
DO 464 J=1,3
715 DO 464 K=1,3
716 VE(K,J)=D0
717 DO 464 L=1,3
718 GO TO 463
719 CONTINUE
720 DO 464 J=1,3
721 VE(K,J)=CV(J)
722 CONTINUE
723 DO 465 K=1,NF
724 NL=FL(K,2)
725 DO 467 M=1,NL
726 NH=NH(J+1)
727 DO 465 J=1,3
728 JNH=J
729 JNH=J+3
730 VB(K,JN)=FF(K,NH(J))
731 CONTINUE
732 VB(K,JN)=FF(K,NH(J))
733 CONTINUE
734 NV=IV
735 DO 491 K=1,NF
736 JNH=J(K,3)
737 NL=FL(K,2)
738 NL=NL(K)
739 DO 492 J=1,3
740 IL=LY(J)
741 W(V)=BF(K,J)+(2*ZF(K,J)*ELO(K,J)+VF(K,J)*EFT(K,J))
742 DO 493 N=1,NL
743 W(V)=W(V)+EFL(K,NH(J))
744 DO 499 N=1,3
745 W(V)=W(V)+PK(K,NH(J))
746 EC(IL)=W(V)
747 DO 4920 L=1,NH
748 IF(P(L)+EQ.0) GO TO 4920
749 EC(IL)=EC(IL)+AKFL(K,J)*GMDL(L)
750 CONTINUE
751 CONTINUE
752 CONTINUE
753 C ANGULAR MOMENTUM OF THE SYSTEM
754 C
755 IF(P(JH(J)+NE(J)+1) GO TO 8752
756 DO 5651 I=1,3
757 MM(I)=0
758 DO 5651 J=1,3
759 MM(I)=MM(I)+ADD(I,J)*B(I,J)
760 DO 5652 I=1,3
761 MM(I)=MM(I)+QH(I)
762 MM(I)=MM(I)+AV(I,J)*GMD(J)
763 DO 5653 I=1,3
764 MM(I)=MM(I)+FU(I,J)
765 DO 5653 K=1,NF
766 NL=FL(K,3)
767 DO 5654 J=1,NL
768 MH(I)*ADD(I,J)*BF(K,1,J)+ELO(K,J)
769 CONTINUE
770 MM=SQR(MM(I)**2+MM(2)**2+MM(3)**2)
771 CONTINUE
772 C SOLVE SYSTEM MATRIX FOR REF BODY ANGLE ACCEL., SUBSTRUCTURE
773 C MING ANGLE ACCEL. AND FLEX. BODY MODE ACCEL.
774 C
775 DO 671 I=1,1
776 EQ(I)=Q
777 DO 671 J=1,1
778 EQ(I)=EQ(I)+AV(I,J)*EC(I,J)
779 KV=EQ
780 DO 910 J=M+1
781 IF(J.LE.V) GO TO 913
782
DIAGNOSTICS

ACTION TIME = 24.46 SECS

CSSL,TRAN,CSSL