A SUPERFLUID HELIUM SYSTEM
FOR AN LST IR EXPERIMENT

prepared by
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GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND 20771
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**Abstract**

This report describes the results of a study program directed toward evaluating the problems associated with cooling an LST instrument to 2 K for a year by using superfluid helium as the cooling means. It contains the results of the parametric analysis of systems using helium only and systems using helium plus a shield cryogen. A baseline system, using helium only is described. The baseline system is sized for an instrument heat leak of 50 mw. It contains 71 Kg of superfluid helium and has a total, filled weight of 217 Kg. A brief assessment of the technical problems associated with a long life, spaceborne superfluid helium storage system is also made. It is concluded that a one year life, superfluid helium cooling system is feasible, pending experimental verification of a suitable low g vent system.

**Distribution Statement**

Unclassified
PREFACE

An infrared instrument is being considered as a candidate experiment for the Large Space Telescope (LST). In order for such an instrument to achieve background limited operation and to gather sufficient scientific data, it is necessary to cool its detectors to or below the lambda point of helium (2.2K) for a minimum of one year. As a superfluid helium system has been proposed for cooling the instrument, this brief study program was directed towards investigating the problems, and the technical and financial risks associated with such a system.

The study program consisted of: 1) conducting parametric analyses of several types of superfluid helium storage systems, 2) selecting and designing a baseline system from among those analyzed, and 3) assaying the technical problems and risks associated with using a superfluid helium storage system in space.

The design analyses show that the type of cooler capable of a one-year hold time (helium only or helium with a shield cryogen) which has the smallest mass depends upon the instrument heat leak (i.e., the direct heat leak to the helium from the detection assembly plus radiation entering the optical-entrance aperture). The size and mass of the cooler also depends upon the instrument heat leak. For the anticipated range of instrument heat leaks (25-50 mw), a helium-only system is of less mass than a system incorporating a shield cryogen.

The baseline cooler, which was selected as a result of the parametric analysis, is a helium-only system sized to absorb approximately 50 mw of instrument heat leak for one year. It incorporates three vapor-cooled shields. The system contains 71 kg of superfluid helium and has a mass of 217 kg. Even though a one year hold time is significantly longer than the demonstrated performance of any present ground based or spaceborne helium storage systems, the development of such a system appears feasible if advanced state-of-the-art technology is carefully applied to the thermal design of the cooler and if a suitable low-g vent system for superfluid helium can be developed and demonstrated. Experimental
investigation of the problems associated with storing and venting superfluid helium in a low-g environment, and experimental demonstration of a suitable system is recommended.
<table>
<thead>
<tr>
<th>Table of Contents</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II. Parametric Analysis</td>
<td>3</td>
</tr>
<tr>
<td>A. General</td>
<td>3</td>
</tr>
<tr>
<td>B. Systems Using Helium Only</td>
<td>5</td>
</tr>
<tr>
<td>C. System with a Shield Cryogen</td>
<td>11</td>
</tr>
<tr>
<td>1. General Analysis of Shield Cryogens</td>
<td>11</td>
</tr>
<tr>
<td>2. Characteristics of Several Shielded Cryogen Systems</td>
<td>16</td>
</tr>
<tr>
<td>D. System Comparisons and Selection of Baseline System</td>
<td>21</td>
</tr>
<tr>
<td>III. Baseline System</td>
<td>26</td>
</tr>
<tr>
<td>A. Overall Configuration</td>
<td>26</td>
</tr>
<tr>
<td>B. Aperture Design and Instrument Heat Leak</td>
<td>36</td>
</tr>
<tr>
<td>C. Auxiliary Ground Equipment</td>
<td>44</td>
</tr>
<tr>
<td>IV. Technical Problem Areas</td>
<td>46</td>
</tr>
<tr>
<td>V. Conclusions and Recommendations</td>
<td>51</td>
</tr>
<tr>
<td>VI. New Technology</td>
<td>52</td>
</tr>
<tr>
<td>APPENDIX A Multilayer Insulation Systems</td>
<td>A-1</td>
</tr>
<tr>
<td>APPENDIX B Derivation of a General Expression for the Weight of Solid Cryogen Systems</td>
<td>B-1</td>
</tr>
</tbody>
</table>
# List of Illustrations

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Effect of Number of Vapor Cooled Shields on Helium Mass</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>Effect of Instrument Heat Leak on Helium Mass</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Mass of Helium Only Systems</td>
<td>10</td>
</tr>
<tr>
<td>4</td>
<td>Effect of Shield Cryogen Temperature on Helium Mass</td>
<td>12</td>
</tr>
<tr>
<td>5</td>
<td>Relative Mass of Solid Cryogen Systems</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>Comparison of Shield Cryogens</td>
<td>15</td>
</tr>
<tr>
<td>7</td>
<td>Effect of Instrument Heat Leak on Helium Mass for Shielded Systems</td>
<td>18</td>
</tr>
<tr>
<td>8</td>
<td>Effect of Instrument Heat Leak on Helium Mass for Several Ammonia Shielded Systems</td>
<td>19</td>
</tr>
<tr>
<td>9</td>
<td>Mass of Shielded Systems</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>Comparison of Total System Masses</td>
<td>22</td>
</tr>
<tr>
<td>11</td>
<td>Comparison of System Volumes</td>
<td>23</td>
</tr>
<tr>
<td>12</td>
<td>Baseline System</td>
<td>27</td>
</tr>
<tr>
<td>13</td>
<td>Schematic Flowsheet of Baseline Cooler</td>
<td>31</td>
</tr>
<tr>
<td>14</td>
<td>Aperture Details</td>
<td>38</td>
</tr>
<tr>
<td>15</td>
<td>Heat Leak for Various Duty Cycles and Aperture Designs</td>
<td>43</td>
</tr>
<tr>
<td>Table No.</td>
<td>Title</td>
<td>Page No.</td>
</tr>
<tr>
<td>----------</td>
<td>--------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>1</td>
<td>Parameters for Baseline System</td>
<td>25</td>
</tr>
<tr>
<td>2</td>
<td>Baseline Cooler Design Summary</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>Baseline Cooler Mass Breakdown</td>
<td>34</td>
</tr>
<tr>
<td>4</td>
<td>Baseline Cooler Heat Leak Summary</td>
<td>35</td>
</tr>
<tr>
<td>5</td>
<td>Baseline Instrument Parameters</td>
<td>37</td>
</tr>
<tr>
<td>6</td>
<td>Radiation Heat Leak Into Aperture</td>
<td>40</td>
</tr>
<tr>
<td>7</td>
<td>Time Average Instrument Heat Leak</td>
<td>41</td>
</tr>
</tbody>
</table>
LIST OF ABBREVIATIONS AND SYMBOLS

A = Surface area of cryogen vessel or (in Appendix A) surface area of the MLI system.
D = Diameter of cryogen vessel.
DF = Degradation factor for an MLI system (see Equation A2 for definition).
IHL = Instrument heat leak, i.e., the heat leak due to the instrument and its entrance aperture.
L = Length of helium vessel.
LST = Large Space Telescope.
MLI = Multilayer Insulation.
N = Number of layers in an MLI system.
q = Heat flow per unit time.
Q = Total amount of heat absorbed by the cryogen.
t = Hold time of cryogen vessel.
Tc = Temperature of colder (or cryogen) surface.
Th = Temperature of hotter (or vacuum shell) surface.
V = Volume of cryogen vessel.
VCS = Vapor Cooled Shield.
W = Mass of cryogen.
\Delta h = Latent heat of vaporization (or sublimation) of cryogen--per unit mass.
\varepsilon = Emittance of one layer in an MLI system.
\varepsilon = Effective emittance of an entire MLI system (see Equations A1 and A2 for definition).
\pi = 3.12
\rho = Density of cryogen.
\sigma = Stefan-Boltzman constant.
I. INTRODUCTION

An infrared instrument is being considered for the Large Space Telescope (LST). The heart of this instrument is at a focal plane where are located several detectors sensitive to radiation of wavelength from one micron to one millimeter. For an infrared instrument to be a viable LST experiment, it is essential that the detectors achieve background limited performance. This imposes the constraint that they be cooled to or below the lambda point of helium (2.2K). To effect substantial scientific returns, it is also desirable that the instrument have a minimum operational lifetime of one-year. The candidate for cooling the instrument is a superfluid helium system. The brief study summarized in this report was undertaken with the objective of evaluating the problems associated with cooling an infrared instrument on the LST for one-year to a temperature of approximately 2 K, using superfluid helium. A further objective was to assay the technical and financial risks involved in constructing such a cooling system.

The program's objectives were achieved through a generalized parametric analysis of cooling systems. The analysis was carried to the point where realistic estimates of system size, weight, and performance characteristics could be made. The primary intention of the study was not the optimization of any design, but rather the evaluation of conceptual alternatives to provide information that could be used in evaluating specific designs. However, a particular design was developed to the point where a realistic determination of heat leaks could be made, and where the problem areas associated with the cooler design could be identified and analyzed in a quantitative manner.

The dewar design requirements used in the study are as follows:

- Primary Cryogen: superfluid helium
- Instrument Temperature: 2 K
- Operating Life: one year
- Temperature of Vacuum Shell: 300 K
- Maximum Diameter of Vacuum Shell: one meter
- Temperature of the Environment: 300 K
- Instrument Related Heat Leaks to Helium: 0-50 mw
- Instrument Size: a cube 20 cm on a side
- Entrance Aperture: 2-8 cm diameter
- Duty Cycle: 0-100%
- Cold Filter Blockage During Operation: 0-97%

The desired values of the last three items are a 6 cm diameter entrance aperture, a 20% duty cycle, and 0% cold filter blockage during data taking periods.

The study program stressed the parametric analysis of two general types of superfluid helium storage systems: systems using helium only, and systems using superfluid helium for cooling the instrument plus a shield cryogen for reducing heat leak to the helium. The results of the parametric analysis are summarized in Section II.

At the end of the parametric analysis, a baseline design was selected for further evaluation. A preliminary design of the baseline cooler was produced, and its size, weight, and performance characteristics were determined. This design is described in Section III.

There are several general technical problem areas associated with storing superfluid helium in space for a one year period. Those which are particularly critical are described in Section IV. The conclusions and recommendations resulting from the study are presented in Section V, and a new technology statement is contained in Section VI.
II. PARAMETRIC ANALYSIS

A. GENERAL

The major emphasis in the study program was devoted to the parametric analysis, as the resulting general conclusions are useful in evaluating specific cooler designs. The parametric analysis was conducted as a screening process in which some components (for instance, multilayer insulation systems) were characterized by general performance parameters rather than being treated in detail. As the heat leak associated with the instrument is presently uncertain, it was treated as a variable.

The parametric analysis considered systems using only helium, as well as systems using helium plus a shield cryogen. In evaluating the shielded systems, a general appraisal of shield cryogens was made to provide a rationale for the selecting the shield cryogen; and then two shielded cryogen systems were examined. Finally, system comparisons were made and a baseline cooler was selected for further analysis.

The parametric analysis was conducted on the following basis:

- A system hold time of one year.
- A vacuum shell temperature of 300 K.
- A vacuum shell maximum diameter of approximately one meter.
- A variable instrument heat leak, with values ranging from zero milliwatts to approximately 50 milliwatts.
- A fifty-layer multilayer insulation system.
- A variable number of vapor cooled shields, with values ranging from 0-4 shields.

The performance of the multilayer insulation (MLI) system is critical to the thermal performance of the cooler. A general discussion of MLI systems is contained in Appendix A. The MLI system used in the parametric analysis was considered to consist of 50 layers of double aluminized mylar (with an emittance of .025 on each surface), with each
layer separated by two mesh spacers of either silk or dacron. Vapor cooled shields (or shields cooled by a second cryogen) were placed in this MLI system. The effect of the distribution of the layers of the MLI system (or alternatively, the number of layers of MLI on each shield) on the heat leak to the helium was analyzed, and an attempt was made to place the layers in the optimum location throughout the dewar assembly. The results presented are for a distribution of layers that produces a near minimum for the heat leaks to the helium. The chosen distribution may not be optimal, but is near enough to the optimum for purposes of this study.

Our practice in designing MLI insulated cryogenic vessels is to determine the heat leak by calculating the heat flow through the entire MLI system area as if it were a perfect, undisturbed system (i.e., a system which has no seams or penetrations). Then, to account for degradation encountered in practice, a degradation factor (DF) is applied to the "perfect" results, and to this are added the separately determined values for radiation heat leaks into the radiation traps at seams and penetrations plus conductive heat leaks via supports, piping, etc. In designing and fabricating the MLI system, we do everything possible to eliminate conductive paths through the undisturbed portions of the MLI system. In production we achieve this goal by carefully fabricating each layer of the system (spacers, as well as radiation shields) on a precisely dimensioned mandrel. Further, we minimize the radiation at penetrations by carefully designing the penetration, by controlling clearances to reduce gaps, and by installing low emittance edge guards and other design details where appropriate. The parametric analysis is based on such an MLI system. With systems designed and built according to this practice, we have been able to achieve heat leaks through the undisturbed portions of the MLI system which are within 25% of theoretical predictions (that is they have degradation factors, DF, of 1.25). To be conservative in the parametric analysis, we have used a degradation factor of 2 of the undisturbed portions of the MLI system. In order to facilitate the calculations, in the parametric analysis, we have accounted for other sources of heat leak (e.g., heat...
flux into radiation traps at penetrations and conductive heat flows) by degrading the performance of the MLI system by another factor of 2. That is, in the parametric analysis all heat leaks, except the instrument heat leak to the helium, are considered as heat flow through the MLI system, and are accounted for by applying a degradation factor of four to the expected thermal performance of an ideal MLI system.

The selection of 50 layers of MLI was somewhat arbitrary. A fifty-layer MLI system composed of layers each with an emittance of .025 has a theoretical effective emittance for the system of .00025. To our knowledge, no one has ever fabricated an MLI system which has demonstrated an effective emittance this low. However, in the systems which were analyzed in the parametric analysis, the 50 layers in the MLI systems were distributed over a number of shields — with 30 layers being the most used on a single shield. The theoretical effective emittance for a 30 layer MLI system is .00041. Considering the undisturbed portion of this system to be degraded by a factor of 2 results in an effective emittance of .00082. As we have built an 11 layer system which had a projected effective emittance of .0011 and a measured performance value of .0015, we feel that we have not extrapolated to an unwarranted degree beyond previously achieved thermal performance for MLI systems.

B. SYSTEMS USING HELIUM ONLY

For the helium-only systems a specific computer program was written to perform the heat and mass balances on the helium vessel and the vapor-cooled shields. Inputs to the program are as follows:

- fluid properties,
- temperatures of the helium vessel and the vacuum shell,
- hold time,
- number of vapor-cooled shields,
- effective emittance of the layers of multilayer insulation system,
• total number of layers of insulation in the MLI system,
• distribution of these layers on the helium vessel and the vapor-cooled shields,
• degradation factor to be used for the MLI system,
• maximum permissible diameter for the helium vessel,
• heat flux to the helium per unit area of the helium vessel.

Given these inputs, the computer program performs the heat and the mass balances on the helium vessel and each vapor-cooled shield and determines the size of the system, the mass of helium required and the allowable instrument heat leak. The program was used for analyzing the helium-only system, and the helium portions of systems employing shield cryogens.

The program determines the size of the helium vessel on the basis that it is a right circular cylinder. The length of this cylinder is set equal to its diameter for cases where the required vessel diameter does not exceed the specified maximum. However, if the needed volume of helium is so large that the diameter of the helium vessel would exceed the specified maximum, then the diameter of the vessel is set equal to the maximum, and its length is increased until its volume is sufficient to accommodate the required volume of helium.

The first item examined with the computer program was the effect of the number of vapor-cooled shields on the mass of helium required for a one year mission. The results of these calculations are shown in Figure 1. The data shown are for the case of no instrument heat leak to any portion of the system, i.e. no instrument heat leak to the helium or any of the vapor cooled shields. We see that the required helium mass decreases as the number of shields is increased, and that the incremental benefit decreases with the addition of each shield. Based on this data, we selected three vapor-cooled shields for the helium-only system, as the complication attendant to more shields did not seem justified by the savings in helium mass.
$t = 1$ YEAR
$L = D$
$N = 50$ LAYERS
$DF = 4$
$IHL = 0$ MW

FIGURE 1 EFFECT OF NUMBER OF VAPOR COOLED SHIELDS ON HELIUM MASS
Figure 2 shows, for a three vapor-cooled shield system, the required helium mass as a function of instrument heat leak at several values of the MLI degradation factor. A line indicating the amount of helium required to absorb only the instrument heat leak is also plotted on this figure. The total mass of helium required can be considered to consist of two components: a mass required to absorb the instrument heat leak and an additional mass required to absorb the parasitic heat leak from the dewar into the helium. For instance, as shown in Figure 2, for a system with a degradation factor of 4 and an instrument heat leak of 20 milliwatts, a total of 54 kilograms of helium are required: 27 kilograms to absorb the instrument heat leak and 27 kilograms to absorb parasitic heat leaks. At low values of instrument heat leak, the required mass of helium is determined primarily by the parasitic heat leak, and at high values of instrument heat leak the required mass of helium is determined primarily by the instrument heat leak. The curves show that, for each value of degradation factor, there is a region where the required amount of helium is determined by the instrument heat leak.

Using the data of Figure 2, estimates were made, at each value of instrument heat leak, of the total cooler weight for the helium-only system. These weights include an estimated weight of structure in addition to the weight of the helium. Structure masses were estimated by assuming a certain thickness of metal for the vessel walls, vapor cooled shields, etc., and multiplying the mass per unit area of this metal times the area of the helium vessel. Estimates of structure mass were per unit area of this metal times the area of the helium vessel. Estimates of structure mass were made on two bases, a low limit estimate based on .635 cm of aluminum per unit of envelope surface area, and a high limit estimate based on 1.27 cm of aluminum per unit of envelope surface area. The resulting mass estimates are shown in Figure 3, where the total helium mass and the total system mass are shown as functions of instrument heat leak. The total system mass curve has the same general characteristics as the helium mass curves; i.e., the mass increases gradually with increasing heat leaks at low values of instrument heat leaks, but increases more rapidly as the instrument heat leak approaches 50 milliwatts.
FIGURE 2  EFFECT OF INSTRUMENT HEAT LEAK ON HELIUM MASS

- t = 1 YEAR
- N = 50 LAYERS
- DF = 4

3 VAPOR COOLED SHIELDS

DF = 4
DF = 3
DF = 2
t = 1 YEAR
N = 50 LAYERS
DF = 4

FIGURE 3  MASS OF HELIUM ONLY SYSTEMS
C. SYSTEMS WITH A SHIELD CRYOGEN

1. General Analysis of Shield Cryogens

The first step in analyzing systems employing a shield cryogen was to determine the effect of the shield cryogen temperature on the required amount of helium. Figure 4 shows the mass of helium required as a function of shield cryogen temperature. The curve is based on a 20 layer MLI system (but no vapor-cooled shield) being placed between the helium vessel and the shield cryogen, and no instrument heat leak. From this curve, we conclude that there is no incentive to use a shield cryogen of temperature less than 80-80 K, because for temperatures below this range the parasitic heat leak from the shield to the helium vessel is so low that the helium mass is determined essentially by the instrument heat leak. Further, we conclude that systems with higher shield-cryogen temperatures would be attractive only if vapor-cooled shields are used between the helium vessel and the cryogen shield.

As the parasitic heat leak to the helium in a shielded system can be reduced to the point where the mass of helium is determined by the instrument heat leak, the thermal control problem shifts from the helium vessel to the shield vessel. Therefore, the way to minimize the mass of such a system is to minimize the mass of the shield cryogen. Consequently, the problem reduces to one of selecting a shield cryogen of minimum mass. We screened candidate shield cryogens by analyzing a shield cryogen system by itself, disregarding the fact that is would enclose a helium vessel in the final system. The basis for using this screening procedure is the premise that the shield cryogen which is the most attractive by itself will be the most attractive when used as a shield for the helium vessel.

The analysis utilizes an idealized model in which the cryogen is stored in a spherical vessel which is insulated by an MLI system. The model also assumes that the only form of heat leak to the vessel is through the MLI system. For this situation, it can be shown (see Appendix B) that the mass of cryogen required is given by the expression:
FIGURE 4  EFFECT OF SHIELD CRYOGEN TEMPERATURE ON HELIUM MASS

t = 1 YEAR
N = 20 LAYERS
DF = 4
IHL = 0 MW
No Vapor Cooled Shield
Between Helium Vessel and Shield
The terminology is defined in the list of abbreviations and symbols. 

This expression indicates that the mass of cryogen is determined only by mission parameters, insulation properties, and fluid properties. Holding mission parameters and insulation properties constant, the above expression was used to generate the data needed to evaluate and rank different cryogens, by substituting the fluid properties for various candidate cryogens into the expression. Seven candidate solid cryogens were evaluated in this way, and the results are shown in Figure 5. The figure shows the relative mass of stored cryogen as a function of the temperature range over which each is useful as a solid. The upper temperature for each cryogen is its triple point, and the lower temperature corresponds to a minimum practical vapor pressure. The relative mass is the mass of each cryogen relative to the mass of ammonia for a given mission duration, shell temperature, and effective emittance of the insulation system. The relative mass figure is simply the quantity

\[ \frac{\left(\frac{1}{\rho^2 \Delta h^3}\right)_{\text{candidate cryogen}}}{\left(\frac{1}{\rho^2 \Delta h^3}\right)_{\text{ammonia}}} \]

From Figure 5 it is evident that the mass of any of the lower temperature shield cryogens is many times that of the higher temperature cryogens, and that ammonia is the most attractive shield cryogen.

In order to demonstrate this conclusion, we determined the mass of two pure solid-cryogen systems in which the cryogen was stored in a cylindrical vessel with its length equal to its diameter. The vessel was insulated with a 30 layer MLI system with a degradation factor of 4. The mass of two cryogens (carbon dioxide and ammonia) are plotted in Figure 6 as a function of that portion of the instrument heat leak which is absorbed on the shield cryogen. The range of heat leaks on the abscissa generally covers the expected range of instrument heat leak which would
ITEMS HELD CONSTANT:
MISSION DURATION
SHELL TEMPERATURE
EFFECTIVE EMMITANCE OF INSULATION

FIGURE 5  RELATIVE MASS OF SOLID CRYOGEN SYSTEMS
$t = 1 \text{ YEAR}$
$L = D$
$N = 30 \text{ LAYERS}$
$DF = 4$

**FIGURE 6** COMPARISON OF SHIELD CRYOGENS
be absorbed by a cryogen shield. The selection of a 30 layer MLI system on the cryogen vessel was based on the idea that in a full system, helium and shield, there would be an MLI system of 50 layers, with 20 layers being used between the helium vessel and the shield cryogen and the remaining 30 layers on the shield cryogen.

In interpreting the mass figures in Figure 6, it should be borne in mind that the area used in determining the parasitic heat leak (that is, the heat leak through the MLI system) is that of the shield cryogen vessel only—there is no allowance for the increased area due to the fact that the shield cryogen would enclose a helium vessel in the final system. From Figure 6 we see that for all values of heat leak the carbon dioxide system is significantly more massive than the ammonia system, demonstrating the validity of the general conclusions drawn from the results shown in Figure 5. We therefore conclude that ammonia is the most attractive cryogen to use in the shield. The helium system using an ammonia shield was analyzed further and the results are presented in the following section. A carbon dioxide shielded system was also analyzed somewhat further, because we believed that a somewhat lower heat flux to the helium might result from the use of a lower temperature shield and that the resulting savings in mass might produce a lower overall system mass.

2. Characteristics of Several Shielded-Cryogen Systems

The previous results show that a preferred shield cryogen would be one whose temperature lay between 80 to 90 K. However, no candidate cryogen of that temperature is acceptable, as a prohibitive mass of it would be required. The two potential shield cryogens are ammonia and carbon dioxide, which have substantially higher operating temperature. Thus, a complete dewar system with these as shield cryogens is by necessity a configuration of helium, vapor-cooled shields, MLI, and shield cryogen. For convenience, but without loss of generality, we can simulate real complete shielded systems by imposing, as an outer boundary condition on the helium vessel, the operational temperature of the chosen shield cryogen. On this basis, the helium mass needed to satisfy the
mission constraints was determined as a function of instrument heat leak for two cases: 1) an ammonia-shielded system with two vapor-cooled shields between the helium vessel and the ammonia, and 2) a carbon dioxide-shielded system with one vapor-cooled shield between the helium vessel and the carbon dioxide. Both systems employ a 20 layer MLI system between the helium vessel and the shield cryogen. For the ammonia system, 1 MLI layer is on the helium vessel, 4 layers are on the innermost vapor-cooled shield, and 15 layers are on the outermost vapor-cooled shield. For the carbon dioxide system, 4 MLI layers are on the helium vessel and 16 layers are on the vapor-cooled shield. While the MLI distributions are not proved optima, we expect little improvement could be effected by other choices. For both systems, as shown Figure 7, the required mass of helium is very low, and that mass is determined by the instrument heat leak and not by the heat flow from the shield.

The ammonia-shielded system was analyzed further to determine the effect of varying the number of vapor-cooled shields and the number of layers of MLI between the helium vessel and the shield. The results are shown in Figure 8. It can be seen that significantly higher helium mass is required if the number of vapor-cooled shields is reduced to 1, or if the number of layers of MLI between the helium vessel and the shield cryogen is reduced to 10. Thus, a design using two vapor-cooled shields and 20 layers of MLI was used for determining the total system mass of the shielded cryogen system.

The total mass of an ammonia-shielded cryogen system was determined in the same manner as for the helium-only system. Here the system was sized so the helium and ammonia hold times matched. The dewar contained a helium vessel, two vapor-cooled shields, and an ammonia cooled shield. The MLI layers were 50 in number, with 20 layers between the helium and the ammonia and 30 layers on the ammonia container. The results are shown in Figure 9, which shows masses as a function of instrument heat leak to the helium. The mass of carbon dioxide for a carbon dioxide-shielded system sized for 10 milliwatts instrument heat leak is also shown on this figure. As the mass of carbon dioxide is significantly higher than the mass of ammonia in the ammonia system, this strengthens
FIGURE 7  EFFECT OF INSTRUMENT HEAT LEAK ON HELIUM MASS FOR SHIELDED SYSTEMS
FIGURE 8  EFFECT OF INSTRUMENT HEAT LEAK ON HELIUM MASS FOR SEVERAL AMMONIA SHIELDED SYSTEMS
t = 1 YEAR
N = 50 LAYERS
DF = 4

FIGURE 9  MASS OF SHIELDED SYSTEMS
the conclusion that ammonia is the preferred shield cryogen.

D. SYSTEM COMPARISONS AND SELECTION OF BASELINE SYSTEM

The total system masses for the helium-only system and the preferable ammonia-shielded system as given in Figure 3 and 9 are replotted on Figure 10. It is clear that for low values of instrument heat leak the mass of the ammonia-shielded system is the lesser, but for instrument heat leaks greater than approximately 25 milliwatts the mass of the helium-only system is the lesser. The system volumes as functions of the instrument heat leak are shown in Figure 11. The trends are the same as for the masses, with the helium-only system being larger for lower values of instrument heat leak but smaller for higher values of the instrument heat leak.

In selecting a baseline system, all parametric analysis results were considered along with estimates of instrument heat leak during several meetings with staff members of NASA Goddard and members of the Infrared Instrument Definition Team. Although the details of the instrument and the optical-entrance aperture into the helium vessel were only tentatively fixed, the consensus was that the instrument heat leak would be toward the high end of the range considered (i.e., in the range of 25-50 milliwatts). As the accurate calculation of heat leaks is difficult, and as real systems often have heat leaks higher than predicted, a design based on a high instrument heat leak would contain a desired measure of conservatism, so a design sized for approximately 50 mw of instrument heat was selected for the baseline. For this condition, the helium-only system is the preferred system from a mass and possibly from a volume point of view. Other advantages of a helium-only system are as follows:

- The helium-only system is less complex than a shielded system since only one cryogen-containing vessel and one piping system are required. The structural support system also tends to be simpler for the helium-only system, as the cryogen mass is concentrated in one vessel.

- As only one cryogen is handled, the auxiliary ground equipment and the filling sequence are less complex. An ammonia-shielded
FIGURE 10 COMPARISON OF TOTAL SYSTEM MASSES
HELIUM WITH THREE VCS.

HELIUM WITH AN AMMONIA SHIELD AND TWO VCS.

FIGURE 11  COMPARISON OF SYSTEM VOLUMES
system requires not only the handling of ammonia, but also a cooling fluid (usually liquid nitrogen) for freezing the ammonia.

- A helium-only system provides simply the desired operation temperature—2 K. There is no need to match the hold time of the helium vessel to the hold time of the ammonia-shield vessel.

- The use of pure helium circumvents the possible detrimental effects of ammonia vent vapors on the measurements of the IR instrument or of any other experiments on the LST.

Based on foregoing, the helium-only system was selected for the baseline design. A system sized for approximately 50 milliwatts of instrument heat leak was selected for the baseline design, even though it was felt that the instrument heat leak might be somewhat less than this amount. This selection was made to provide an upper limit on the estimate of cooler size and to provide a degree of conservatism in the cooler sizing.

The desired characteristics of a baseline cooler correspond closely to one of the cases (Case No. 81) analyzed in the parametric analysis. The data shown in Table 1 was taken from this case and were used as the foundation for laying out and analyzing the baseline cooler.

Several design parameters were not completely fixed at this point, but were kept as variables and subjected to further trade-offs. These were the instrument associated variables of entrance aperture diameter, spacing of the vapor cooled shields in the vicinity of the entrance aperture, the duty cycle, and the blockage of a (possible) filter placed in the signal beam at the outermost vapor cooled shield.

As will be seen in the following section, some of the parameters listed in Table 1 were changed as the design of the baseline system progressed. The parasitic heat leak to the helium vessel was increased to 11 mw to provide more design margin in the thermal design, and the length of the helium vessel was increased to 102 cm to compensate for the fact that dished heads were used, to provide room for the instrument, and to provide some ullage volume.
### TABLE 1

PARAMETERS FOR BASELINE SYSTEM\(^{(1)}\)

<table>
<thead>
<tr>
<th>Surface</th>
<th>Temperature</th>
<th>Parasitic Heat Leak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium Vessel</td>
<td>2K</td>
<td>2 mw</td>
</tr>
<tr>
<td>VCS-1</td>
<td>23K</td>
<td>252 mw</td>
</tr>
<tr>
<td>VCS-2</td>
<td>102K</td>
<td>1,186 mw</td>
</tr>
<tr>
<td>VCS-3</td>
<td>204K</td>
<td>2,400 mw</td>
</tr>
<tr>
<td>Vacuum Shell</td>
<td>300K</td>
<td>-</td>
</tr>
</tbody>
</table>

System Type: Helium only  
Instrument Temperature: 2K  
Operating Life: One year  
Temperature of Vacuum Shell: 300K  
Total Time Average Heat Leak to Helium: 53 mw  
Layers of MLI: 50  
Distribution of Layers and $\bar{\varepsilon}$\(^{(2)}\) of System:  
- Helium Vessel: 1 layer ($\bar{\varepsilon} = 0.0063$)  
- VCS-1: 4 layers ($\bar{\varepsilon} = 0.0025$)  
- VCS-2: 15 layers ($\bar{\varepsilon} = 0.00079$)  
- VCS-3: 30 layers ($\bar{\varepsilon} = 0.00041$)  
Helium Mass: 71.3 kg  
Helium Vessel Size: 85.4 cm dia. x 85.4 cm long  
Target Thermal Performance:

---

\(^{(1)}\)From Computer Run Number 81.  
\(^{(2)}\)Theoretical, undegraded $\bar{\varepsilon}$ with $\varepsilon = 0.025$.  

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III. BASELINE SYSTEM

A. OVERALL CONFIGURATION

The baseline system selected was examined in moderate detail. A layout drawing of it was made, and its characteristics were determined. A point emphasized in the design effort was the thermal and mechanical design of the optical-entrance aperture to the helium vessel, because it is the major source of heat leak into the helium and the vapor-cooled shields. The baseline system design is based on the desired values of instrument related parameters, viz. a 6 cm diameter entrance aperture, a 20% duty cycle, and no cold filter blockage during data taking periods.

The baseline dewar is shown in Figure 12, and its overall characteristics are listed in Table 2. The system consists of a cylindrical helium vessel with flanged and dished heads, three vapor cooled shields, and a vacuum shell. The helium vessel contains a reentrant cavity in one end, into which the instrument is mounted. All heat leak into the instrument is conducted across the flanged joint on the instrument into the wall of the helium vessel. Access to the instrument is provided by a flange in the vacuum shell and circular joints in the three vapor-cooled shields. Drive motors for filter wheels in the instrument are mounted on the vacuum side of the access flange, and fiberglass drive shafts transmit motions to the instrument. The aperture cover and the cover actuation mechanism are mounted on the outside of the access flange. The aperture design is shown in more detail in a subsequent section.

The helium vessel is 86 cm in diameter by 102 cm long, and has an internal volume of 517 liters. This volume is sufficient to hold the required 488 liters of helium with a 6% ullage after initial filling of the vessel. The helium vessel is made of stainless steel. Stainless steel was selected because it will enable good quality vacuum joints to be made in the vessel itself, and will permit welding the stainless steel
### TABLE 2

**BASELINE COOLER DESIGN SUMMARY**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hold Time</td>
<td>1 Year</td>
</tr>
<tr>
<td>Cryogen</td>
<td>Superfluid Helium Only</td>
</tr>
<tr>
<td>Number of Vapor Cooled Shields (VCS)</td>
<td>3</td>
</tr>
<tr>
<td>MLI Distribution</td>
<td></td>
</tr>
<tr>
<td>Helium vessel</td>
<td>1 Layer</td>
</tr>
<tr>
<td>VCS - 1</td>
<td>4 Layers</td>
</tr>
<tr>
<td>VCS - 2</td>
<td>15 Layers</td>
</tr>
<tr>
<td>VCS - 3</td>
<td>30 Layers</td>
</tr>
<tr>
<td>Overall Diameter</td>
<td>105 CM</td>
</tr>
<tr>
<td>Overall Length</td>
<td>162 CM</td>
</tr>
<tr>
<td>Helium Vessel Diameter</td>
<td>86 CM</td>
</tr>
<tr>
<td>Helium Vessel Length</td>
<td>102 CM</td>
</tr>
<tr>
<td>Dry Mass</td>
<td>146 Kg</td>
</tr>
<tr>
<td>Cryogen Mass</td>
<td>71 Kg</td>
</tr>
<tr>
<td>Total Mass</td>
<td>217 Kg</td>
</tr>
</tbody>
</table>
fill line and vent line to the vessel without requiring metallurgical transition joints. The pressure vessel has been designed for 40 psig internal pressure and has a wall thickness of .076 cm. There is an aluminum extension of the cylindrical section of the helium vessel capped off with a flat circular aluminum plate to "square off" the ends of the helium vessel. This structure supports the MLI system of the helium tank and is used to provide a regular form on which to install the MLI system (it is easier to install an MLI system on a flat-headed vessel rather than on the dished head of the pressure vessel). The space between the aluminum structure and the dished head is used for fill and vent piping and for structural supports.

The three vapor-cooled shields are made of .076 cm thick flat ended aluminum cylinders. These shields also serve as a support for the MLI system on each shield. They are tied (thermally and structurally) to the structural supports on the helium vessel and are also used to heat station the fill and vent piping, the drive shafts to the instrument, and the electrical wiring to the instrument.

The MLI system is composed of double aluminized mylar, with two silk (or dacron) mesh spacers between each layer. Thermal performance of the MLI system has been based on the same number of layers as are used in the parametric analysis; viz., one layer on the helium tank, four layers on the innermost vacuum cooled shield, fifteen layers on the middle vacuum cooled shield, and 30 layers on the outermost vacuum cooler shield. A degradation factor of two has been used for the undisturbed portions of this MLI system, with heat leak into radiation traps at the various penetrations accounted for separately. The spacing of the vapor-cooled shields has been based on each layer of the MLI system being .076 cm thick.

The vacuum shell is 102 cm in diameter by 155 cm long. It is made of 0.318 cm thick aluminum, and has three reinforcing rings on the cylindrical section.
Figure 12 indicates a structural support system consisting of fiberglass tubes and bands. There are six fiberglass tubes at the instrument end of the cooler. The tubes are arranged in three pairs which form, in effect, a truss. These tubes support all axial loads, all rotational loads, and half of the lateral loads. A lateral support system, composed of three fiberglass bands 120° apart, provides lateral support for the other end of the helium vessel. These tension bands do not support any axial load, but they do support half of the lateral load. All supports are heat stationed at each vapor cooled shield at a location where the temperature of the support equals the temperature of the shield. The supports have not been designed in detail in this study. They should receive careful attention during the final cooler design. Particular attention should be paid to their ability to maintain the alignment of the instrument within the specified dimensional tolerance under all expected static and dynamic loadings. The design of the point where the supports penetrate the MLI is of critical importance thermally, as heat leaks into radiation traps at these penetrations can be significant. This is particularly important on the outermost vapor cooled shield, where the MLI is the thickest and where the radiative heat flux from the warm vacuum shell is the greatest.

The piping and valving on the cooler is shown schematically in Figure 13. That portion of the piping which is part of the cooler itself consists of a vacuum-jacketed fill line, an insulated vent line, and an evacuation connection. The fill line contains a bayonet connection (which is capped off after filling), a shut-off valve and a burst disc (or relief valve) set at 37 PSIA. The fill line is vacuum-jacketed to minimize gas evolution during the filling process. The vent line contains pressure measuring instrumentation (a pressure transducer), a shut-off valve, and a back-pressure relief valve. The relief valve is set at approximately 23.9 mm of mercury (the vapor pressure of helium at 2K). This valve becomes operative only after launch. The precise setting of this valve can be determined only after
the pressure drop in the vent line has been determined. Alternatively, a pressure signal which is indicative of tank pressure (rather than the pressure at the valve) can be used to actuate the valve. This latter option, of course, requires another connection to the helium tank, with attendant problems of heat leak and thermal acoustic oscillations. The open end of the vent line downstream of the shut-off valve would also be capped prior to launch. A "pant-leg" is shown on the vent line at the point where it penetrates the vacuum shell, and an insulation system (comprised of, for instance, foam insulation) surrounds this line. These features are added to eliminate frosting of the vacuum shell and the line prior to launch. Since both the fill and vent lines are stainless steel and the cooler vacuum shell is aluminum, a transition joint between these two materials must be provided. These transition joints are at room-temperature locations - on the pant leg in the case of the vent line and at the bayonet joint and the warm end of the valve stem extension in the case of the fill line.

An evacuation line with pressure instrumentation, a burst disc, and a shut-off valve are included for evacuating the vacuum shell prior to launch. This line should be reopened after launch to provide a low conductance pumping path for the vacuum annulus in the vacuum shell - so that outgassing products can leave the vacuum space through this line rather than passing across any of the cold surfaces in the aperture. This will minimize the possibility of cryo deposits on either the optical surfaces or on the detectors in the instrument.

The piping and valving is not shown on the layout drawing, (Figure 12) but it would generally spiral out between the vapor cooled shields at the instrument end of the cooler. The vent line will be heat-stationed to the vapor-cooled shields by heat exchanger coils on the shields. The fill line will also be heat-stationed to the vapor cooled shields, but through "weak" thermal links which will heat-station the line during operation, but which will not constitute a major source of heat input to the line during the filling operation.
A mass breakdown of the baseline cooler is shown in Table 3. Some of the masses (e.g., supports and piping) are estimates, but we believe that they are representative of the masses of these items in a flight system.

A heat leak summary of the baseline cooler is shown in Table 4. Note that heat leaks to each of the vapor cooled shields, as well as heat leaks to the helium vessel have been tabulated. The instrument heat leak has been calculated in some detail because it is a significant source of heat leak. As previously mentioned, the radiation heat leaks through the undisturbed portions of the MLI have been calculated using a degradation factor of two. The allowable heat leak to each of the four stations has been taken from the heat balance in the parametric analysis. Subtracting the heat leak through the instrument and the heat leak through the MLI from the allowable heat leak at each temperature level yields an allowance for conductive heat leak down the supports and piping and for radiation into penetrations of the MLI. Heat leaks due to these latter causes have not been calculated in detail; but the allowances shown in Table 4 appear reasonable based on our experience, with the possible exception of the third (outermost) vapor cooled shield. If this allowance is not sufficient, and the heat input to the third vapor cooled shield due to supports and piping exceeds 605 milliwatts, then the entire shield system will run warmer than projected; and the helium mass flow will increase above that determined in the parametric analysis. Though some of the heat leaks shown in Table 4 have not been calculated in detail, we believe that all of the figures shown are reasonably close to what can be achieved by careful application of advanced, present-day technology. Thus, they should provide a reasonable basis for sizing the baseline cooler.
<table>
<thead>
<tr>
<th>Component</th>
<th>Mass (Kilograms)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium Tank</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>MLI System</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Three Vapor Cooled Shields</td>
<td>34</td>
<td></td>
</tr>
<tr>
<td>Vacuum Shell (1)</td>
<td>68</td>
<td>Including access flange at instrument, removable cover, and cover actuation mechanism.</td>
</tr>
<tr>
<td>Supports</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>Piping and Valving</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Instrument</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Total Dry Mass</td>
<td>146</td>
<td></td>
</tr>
<tr>
<td>Helium Mass</td>
<td>71</td>
<td></td>
</tr>
<tr>
<td>TOTAL SYSTEM MASS</td>
<td>217</td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4

BASELINE COOLER HEAT LEAK SUMMARY (Milliwatts)

HEAT LEAK INTO:

<table>
<thead>
<tr>
<th></th>
<th>Helium</th>
<th>VCS-1</th>
<th>VCS-2</th>
<th>VCS-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instrument Heat Leak for 20% Duty Cycle and 0% cold filter blockage during operation</td>
<td></td>
<td>66</td>
<td>265</td>
<td>158</td>
</tr>
<tr>
<td>Radiation through MLI (DF=2)</td>
<td>1</td>
<td>132</td>
<td>700</td>
<td>1,637</td>
</tr>
</tbody>
</table>
| Allowance for Support Conduction, Fill and Vent Line Conduction, and Radiation into MLI Penetra-
| tions                                    | 10     | 54    | 221   | 605   |
| Allowable Heat Leak (From Heat Balance in Parametric Analysis) | 53     | 252   | 1,186 | 2,400 |

(1) 28.7 mw from Heat Leak Calculations Plus 13.3 mw Contingency.

(2) See discussion accompanying Table 7.
B. APERTURE DESIGN AND INSTRUMENT HEAT LEAK

The heat leak due to the instrument arises from internal dissipation, radiation through the aperture, and conduction down lead wires and drive shafts. As the instrument design was not provided to us in sufficient detail to make a precise estimate of these heat leaks, the instrument heat leak was carried as a variable during the parametric analysis. During the course of the study, as results from the parametric analysis were generated and the sensitivity of the design to the magnitude of the instrument heat leak could be expressed in quantitative terms, the instrument parameters for the baseline design were developed. Table 5 lists the parameters which were selected for the baseline design during several meetings with the NASA technical monitor for the contract and the Infrared Instrument Definition Team.

The aperture design is shown on Figure 14. The aperture consists of a cylindrical tube 6 cm in diameter. The aperture tube, which is black on the inside, is not continuous, but rather consists of a number of distinct isothermal sections. Each section is heat stationed to its adjacent vapor cooled shield. The lengths of the sections have been selected to reduce the radiative heat leak to the colder portions of the system (particularly the helium vessel) to acceptably low levels. The spacing of the vapor cooled shields in the vicinity of the aperture has been set equal to the aperture section lengths (6 cm). The length of the detector lead wires and filter wheel drive shafts have also been set equal to the shield spacing. They are, of course, continuous, rather than being segmented like the aperture tube.

The major source of instrument heat leak to the helium and to the three vapor cooled shields is radiation from the 300K environment into the aperture. In order to reduce the time average value of this flux to an acceptable level, a rotatable wheel has been placed on the outermost vapor cooled shield (VCS-3). This wheel is heat stationed to VCS-3. During the 20% of the time when data is being taken, the wheel is rotated to bring an open hole in line with the aperture, i.e., the aperture is unobstructed during data taking periods. During the 80% of the time when
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1 Kg</td>
</tr>
<tr>
<td>Dimensions</td>
<td>Cube 20 cm on a Side</td>
</tr>
<tr>
<td>Lead Wires (1)</td>
<td>6 Wires, 36 ga. (.013 cm Dia)</td>
</tr>
<tr>
<td></td>
<td>Constantan. Length Equal to Shield Spacing</td>
</tr>
<tr>
<td>Drive Shafts for Filter Wheels Etc. (1)</td>
<td>6 Shafts, .63 cm Dia X .05 cm</td>
</tr>
<tr>
<td></td>
<td>Wall FRP Tubes. Length equal to Shield Spacing</td>
</tr>
<tr>
<td>Internal Dissipation (2)</td>
<td>9 MW</td>
</tr>
<tr>
<td>Spacing of Vapor Cooled Shields at Aperture</td>
<td>6 cm</td>
</tr>
<tr>
<td>Aperture Diameter</td>
<td>6 cm</td>
</tr>
<tr>
<td>Duty Cycle</td>
<td>On 20% of the Time, Standby 80% of the Time</td>
</tr>
<tr>
<td>Cold Filter Blockage During Data Taking</td>
<td>0%</td>
</tr>
</tbody>
</table>

(1) Lead Wires and Drive Shafts Heat Stationed at all Sheilds

(2) 3 Channels X 3 MW/Channel
the instrument is in the standby mode, a solid plate of polished aluminum (with an emittance of .025 on each surface) is rotated into the aperture. This plate blocks the 300K radiation from the helium vessel, VCS-1, and VCS-2. It also significantly reduces the amount of 300K radiation which is absorbed on VCS-3. The radiant heat leak into the aperture, for the conditions of standby and operation, is shown in Table 6. The time average radiant heat leak, based on a 20% duty cycle, is also shown in the table. As may be seen there is a significant difference in the radiant heat leak between the two conditions. Consequently the time average radiant heat leak is strongly dependent upon the duty cycle. The negative radiant heat flux from VCS-3 during standby results from the fact that less heat is absorbed from the 300K environment on the outside of the low emittance plate on VCS-3 than is rejected to colder surfaces in the aperture by the black cylindrical section of the aperture tube on VCS-3 and by the underside of the plate.

The time average, heat leak due to the total instrument is shown in Table 7. In this table, the time average radiation heat leak into the aperture is combined with other (constant) instrument heat leaks. As may be seen, radiation into the aperture is the major source of heat leak at all heat stations except VCS-3.

The heat leaks shown in Table 7 appear to be acceptably low for the baseline cooler to achieve its desired hold time. Nevertheless it would be desirable to reduce them even further either to provide a margin on the hold time or to permit reducing the amount of helium required to achieve the desired hold time. Within the constraints of a 6 cm diameter aperture, a 20% duty cycle, and an open aperture during data taking periods, there are two things that can be done to reduce the radiant heat leak into the aperture; increase the length of the isothermal aperture sections and/or rotate low emittance plates into the aperture at VCS-1 and VCS-2 during the standby period. The effect of increasing the length of the isothermal sections was not determined explicitly, but, based on the calculations for other aperture diameters, we estimate that the radiative heat leak from the aperture into the helium could be reduced.
### TABLE 6

**RADIATION HEAT LEAK INTO APERTURE**

(Milliwatts)

<table>
<thead>
<tr>
<th>Heat Leak Into:</th>
<th>Helium</th>
<th>VCS-1</th>
<th>VCS-2</th>
<th>VCS-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>During Standby</td>
<td>10.34</td>
<td>36.48</td>
<td>171.9</td>
<td>-193.3</td>
</tr>
<tr>
<td>During Operation</td>
<td>44.35</td>
<td>74.58</td>
<td>319.7</td>
<td>627.8</td>
</tr>
<tr>
<td>Time Average Heat Leak for a 20% Duty Cycle</td>
<td>17.14</td>
<td>44.10</td>
<td>201.5</td>
<td>-29.1</td>
</tr>
</tbody>
</table>

(1) Basis: 6 cm diameter aperture
6 cm shield spacing at aperture
0% cold filter blockage (i.e. open aperture) during data taking period
Low emittance plate (ε = .025 both sides) rotated into aperture at VCS-3 during standby.
TABLE 7

TIME AVERAGE INSTRUMENT HEAT LEAK\(^{(1)}\)

(Milliwatts)

<table>
<thead>
<tr>
<th>HEAT LEAK INTO:</th>
<th>Helium</th>
<th>VCS-1</th>
<th>VCS-2</th>
<th>VCS-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation into Aperture</td>
<td>17.1</td>
<td>44.1</td>
<td>202</td>
<td>-29</td>
</tr>
<tr>
<td>Radiation into MLI Penetration</td>
<td>--</td>
<td>5.7</td>
<td>33</td>
<td>152</td>
</tr>
<tr>
<td>Conduction Down Lead Wires</td>
<td>.1</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Conduction Down Filter Wheel</td>
<td>2.5</td>
<td>15.0</td>
<td>28</td>
<td>32</td>
</tr>
<tr>
<td>Drive Shafts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal Dissipation</td>
<td>9.0</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>TOTAL</td>
<td>28.7</td>
<td>66.3</td>
<td>265</td>
<td>158</td>
</tr>
</tbody>
</table>

\(^{(1)}\) Basis: 6 cm diameter aperture
6 cm shield spacing at aperture
20% duty cycle
0% cold filter blockage (i.e. open aperture) during
data taking period
Low emittance plate (\(e = .025\) both sides) rotated
into aperture at VCS-3 during standby.
by approximately a factor of two if the shield spacing were increased by 50% (to 9 cm), and could be reduced by approximately a factor of seven if the shield spacing were increased by a factor of 3 (to 18 cm). Conduction heat leaks would also be decreased by increasing the shield spacing; by a factor of 1.5 for the first case and by a factor of three for the second case.

The effect of three aperture variables on heat leak to the helium was calculated. The variables are the duty cycle of the instrument, the aperture diameter, and the percent of cold filter blockage during data taking periods. In these calculations it was assumed that a filter, located on, and heat stationed to, VCS-1 was rotated into the aperture during data taking periods. This filter would be carried on the same plate that is used to close off the aperture during standby periods, and would block a fraction of the 300K radiation. The range of variables considered was:

- Duty cycle 0-100%
- Aperture diameter 2, 4, 6, and 8 cm
- Cold filter blockage 0-97%

The length of the isothermal aperture sections was 6 cm in all cases. The results of the calculations are shown on Figure 15. Note that the ordinate on this figure is the time average total heat leak to the helium, i.e., the instrument heat leak plus 11MW of dewar heat leak (per Table 4). From the figure it can be seen that the time average total heat leak to the helium is essentially independent of duty cycle or cold filter blockage for the 2 cm and 4 cm apertures, but that these parameters become increasingly important as the aperture diameter increases.

The baseline cooler has been designed to have a one year hold time for a 53 MW time average total heat leak to the helium. Combinations of aperture parameters which result in heat leaks less than this will last longer than a year (i.e., they will have a design margin). Conversely, combinations of aperture parameters which result in heat leaks greater than 53 MW will last less than a year.
COLD FILTER BLOCKAGE, \%  
APERTURE DIAMETER, CM

6 CM SHIELD SPACING  
11 MW DEWAR HEAT LEAK 

\[ \text{DESIZED OPERATING POINT} \]

HEAT LEAK FOR ONE YEAR OPERATION 

\[ \text{TIME AVERAGE TOTAL HEAT LEAK TO HELIUM, MW} \]

DUTY CYCLE, PERCENT OBSERVING TIME

FIGURE 15  
HEAT LEAK FOR VARIOUS DUTY CYCLES AND APERTURE DESIGNS

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The foregoing discussion indicates the degree to which the thermal design of the instrument, and particularly its entrance aperture, influences the design of the helium storage system. The integration of the instrument with the cooler must receive very special attention during the design and development phases of the instrument/cooler. Very careful coordination of the efforts of the instrument designer and the cooler designer will be required in order that instrument heat leaks be held to an acceptable level.

C. AUXILIARY GROUND EQUIPMENT

The auxiliary ground equipment required for filling and operating the cooler prior to launch are shown schematically in Figure 13. This equipment will allow the cooler to be filled with helium at 2 K - a filling technique which will make maximum utilization of the volume of the helium vessel in the cooler. The fill circuit contains a precooling coil which is immersed in a tank containing vacuum pumped helium. The vent circuit contains a heat exchanger to heat the vent gas to room temperature and a vacuum pump to maintain tank pressure at or below 23.9 mm of mercury. The entire fill circuit is vacuum jacketed to reduce heat leaks and hence to keep gas evolution to a minimum. The high vacuum pumping system on the vacuum shells of the cooler and the precooling system are of conventional design.

A number of separate vacuum pumps have been shown in Figure 13. This would yield maximum flexibility and operation. However, it is probably possible to use only one helium pump and one high vacuum pumping system without sacrificing too much flexibility in operation. In addition to the equipment shown in Figure 13, temperature and pressure instrumentation would be required to monitor the filling process. The filling process would be generally as follows:

1. Evacuate both high vacuum systems to a pressure of least
   than \( 1 \times 10^{-5} \) mm of mercury.

2. Purge the helium system and fill it with room temperature,
   dry, gaseous helium.
3. Cool down the entire helium system by filling the precooling tank and the dewar with liquid helium at one atmosphere pressure.

4. Valve off and disconnect both high vacuum pumps.

5. Shut off the helium supply to the dewar.

6. Reduce the temperature of the entire helium system to 2 K by pumping on the dewar vent line and on the helium vessel in the precooling system. During this process, regulate the liquid helium flow to the helium vessel in the precooling system to maintain a liquid level above the precooling coil.

7. Admit helium to the dewar through the precooling system while continuing to pump on both helium vessels. The helium flow to the dewar should be adjusted to make the temperature of the helium leaving the precooling coil equal to or less than 2 K. The flow of helium to the helium vessel in the precooling circuit should be adjusted to maintain liquid level in the tank.

8. When the dewar is full, shut off the fill-line valve on the dewar, and disconnect and cap the fill line. Continue to pump on the vent line from the dewar.

9. Just prior to launch, close the shut-off valve in the dewar vent line and disconnect and cap the line.

   During launch, the pressure of the helium in the dewar will increase as heat flows into the helium. However, the vessel will pump down after the back pressure valve in the vent line becomes operative upon achieving orbit. Variations on the above sequence may be used during ground hold and checkout operations, or when the tank is topped off.
IV. TECHNICAL PROBLEM AREAS

Technical problem areas associated with storing superfluid helium in space for one year are related to achieving the desired hold time, and hence are centered around the thermal design of the cooler and the development of design features which will be functionally adequate yet will result in an acceptably low heat leak. The following paragraphs cover those areas which appear to be most critical.

From the results of the parametric analysis, it is evident that the cooler size and weight depend very strongly on the instrument heat leak. In order to minimize this heat leak, it is imperative that detailed attention be directed to the thermal design of the instrument and its entrance aperture to insure that radiation and conductive heat fluxes are held to an absolute minimum. Since the radiative heat flux in the entrance aperture is a major source of heat leak, efforts should be directed toward reducing it either by reducing the aperture area or by increasing the aperture length.

The design and fabrication of an MLI system which has a sufficiently low effective emittance is a perennial problem in high performance, cryogen storage systems. The MLI is particularly important in the present case because the performance requirements for the MLI system on VCS-3 exceeds those of any existing system of which we are aware. We believe that the technology available today is adequate to produce a system with the required performance, but only if strict attention is directed to all details of the system (both during design and fabrication) to minimize solid conductive paths between layers of the MLI system and to minimize radiation traps at seams and penetrations.

The structural support system is another perennial problem area in liquid helium storage vessels, particularly in those which must withstand high static or dynamic loads. A number of design concepts have been used for low conductance, high strength structural support systems. They include the use of high strength tension members of, for instance, stainless steel, inconel, or fiberglass; tubular members which support
as a function of time during the venting process. The helium tank must, of course, be able to withstand the peak pressure. If it cannot, one must redesign the tank to withstand the peak pressure, or use another line size, length, or geometry which will result in the peak pressure remaining within the pressure holding capability of the tank, or add a non-vacuum insulation (e.g., foam) to reduce the heat flux into the tank to a level which is low enough that the pressure in the tank will not exceed the allowable pressure.

In order to determine whether the vent line would constitute a problem, we calculated the tank pressure as a function of time for two cases. In both cases we assumed that the tank was full and that there was a catastrophic loss of vacuum. The burst disc (or relief valve) on the tank was assumed to be set at 2.5 atmospheres (absolute). In the first case analyzed, the helium tank was insulated with only MLI, and the vent line was 1.2 meters long with a 2.54 cm inside diameter. For this case, the tank pressure rose to approximately 8 atmospheres pressure and then decayed to the relief valve setting (or ambient pressure if a burst disc were used). The contents of the tank vented completely within several minutes. The helium vessel in the baseline design is designed for 2.72 atmospheres internal pressure with a vacuum outside the vessel. It weighs 24 kg. In order that the vessel be able to withstand 8 atmospheres pressure the wall thickness of the vessel would have to be increased. The resulting vessel would weigh 71 kg, or 47 kg more than the baseline vessel.

In the second case analyzed we assumed that the helium vessel was covered with 0.63 cm of closed cell urethane foam. The weight of the foam insulation is 3 kg, and its purpose is to reduce the heat flux into the helium, even after a loss of vacuum. The vent line was assumed to be 2.54 cm in diameter (the same as the first case) but its length was increased to 31 meters. Even for this length of vent line, the pressure in the helium vessel will not rise above the relief setting during the entire venting process.
In comparing the two cases, it is clear that the addition of a relatively thin layer of insulation on the helium vessel will prevent tank pressures from rising to high levels if there is a loss of vacuum, even for relatively long vent lines. Thus, if foam insulation is used, lighter weight tankage may be used and longer vent lines (with lower heat leak) may be used.

Thermal acoustic oscillations may occur spontaneously in tubes connecting a liquid helium reservoir to ambient temperatures. These oscillations cause a heat pumping effect which may increase the heat leak into the helium reservoir up to 1,000 times that of normal conduction down the tube wall. Physically, the oscillation is a traveling wave phenomenon which transports energy from the warm end of the tube to the cold end and which ejects discreet pulses of warmed vapor from the cold end of the tube. Some of the most significant characteristics of the phenomenon are as follows:

- Freely vented tubes do not oscillate. Tubes with very large volumes at the warm end approximate a freely vented line and will generally not oscillate.

- The heat pumping rate increases with increasing pressure amplitude of oscillation and increasing frequency of oscillation. The pressure amplitude of the oscillation in turn is an increasing function of the length-to-diameter ratio of the tube.

- Oscillations can occur in tubes which cover a wide range of diameter and lengths.

- For liquid containing systems (that is, systems operating below the critical pressure) the oscillations are more severe when the cold end of the tube terminates in the gas space than when it terminates below the liquid surface.

- Tubes with closed cold ends do not oscillate.

- The oscillation intensity can be increased by heating the tube along a portion of its length, and it can be reduced by cooling a portion of the length of the tube. This result points to the value of thermally shorting vent lines to radiation shields.
There is not sufficient quantitative understanding of thermal-acoustic oscillations to be able to predict with confidence whether a given tube will or will not have self-sustaining oscillations. Consequently, many systems which contain liquid helium have thermal oscillations when initially constructed. A number of techniques for suppressing or eliminating thermal acoustic oscillations have been conceived. Some of these techniques are:

- Close off the cold end of the oscillating tube.
- Restrict the flow, particularly in the cold end of the tube, with foam, sintered metal, fibers, beads, knotted string, etc. The restriction will reduce oscillations but also restrict the flow.
- Place expanded sections of tubing or accumulators in the ambient temperature piping to dampen the oscillation energy. These accumulators can take the form of Helmholtz resonators.
- Install a large volume at the warm end of the oscillating tube.

Since thermal/acoustic oscillations frequently occur in helium systems, they should be expected, and some thought should be given to a means for suppressing them while the system is in the design stages. The conservative approach is to expect oscillations in all lines leading from ambient temperature to the helium vessel, to provide space to add dampers to suppress them if they occur, and to identify any lines which do oscillate early enough in the cooler development program that modifications which suppress them can be incorporated in the system.

The most uncertain problem associated with storing superfluid helium in the LST is the behavior of the superfluid helium itself in near zero-g. There is no precedent for storing or venting superfluid helium in this environment, and little is known about the low-g behavior of superfluid helium. Three problem areas are perceived: (1)

The effect of low-g on the distribution of superfluid helium inside the storage vessel and the thermal transport resulting from this distribution. Does superfluid helium behave in such a way that the helium will not adequately cool the load?

The effect of low-g on the phase separation and vent system. Does superfluid helium behave in such a way that liquid is lost through the vent system or that heat fluxes in the vent system are excessive?

The effect of low-g on fluid dynamics. Does superfluid helium behave in such a way that serious thermo-mechanical oscillations build up inside the helium vessel, thus upsetting the satellite attitude control system or adversely effecting thermal transport or venting?

Some ground-based work on superfluid helium vent systems employing a superfluid plug has been performed.\(^1\) This work has, we understand, demonstrated the adequacy of such a vent system in a one-g environment. However, the consequences of any of the postulated problems are so serious that further understanding of its behavior is necessary before undertaking the development of a spaceborne, superfluid helium storage system. We believe that adequate understanding can be developed only in an experimental program which results in the demonstration of a suitable system.

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V. CONCLUSIONS AND RECOMMENDATIONS

The conclusions resulting from the study are:

- A cooler with a one year life is feasible if no major problems are encountered with the vent system.

- A helium only system is preferred for the probable range of instrument heat leak.

- The major technical risk is operating lifetime.

- Advanced technology must be applied to minimize heat leak.

- The zero-g vent system for superfluid helium is the major undemonstrated feature of the cooler.

Based on these conclusions, we make the following recommendations:

- Experimentally demonstrate a zero-g vent system for superfluid helium.

- Structure the cooler development program for close coordination between dewar contractor and instrument contractor.

- Devote detailed attention to the thermal design of the experiment/cooler subsystem.
VI. NEW TECHNOLOGY

No new technology has been developed under this contract.
APPENDIX A

MULTILAYER INSULATION SYSTEMS

This appendix discusses multilayer insulation system performance in general and presents test data on a number of systems.

The performance of an actual multilayer insulation system may be related to the performance of an ideal system, composed of completely free-floating shields, by the relationship:

\[
\dot{q} = \sigma A \varepsilon (T_h^4 - T_c^4)
\]

\[(A1)\]

(The terminology is defined in the list of abbreviations and symbols.)

Note that \(\varepsilon\) expresses the fraction of the black-body radiant heat flow between two surfaces. Some authors prefer to use a shielding factor, \(\mu\) (which is the reciprocal of \(\varepsilon\)), to express the efficacy of the insulation system; others prefer to define the performance of an MLI system in terms of an effective conductivity, and then treat the radiation heat transfer as conductive heat exchange. We do not recommend the latter approach, for it tends to obscure important factors in the performance of the insulation system.

The effective emittance, \(\overline{\varepsilon}\), for parallel plates (and for other geometries in which the view factor is unity), is given by:

\[
\overline{\varepsilon} = \frac{\dot{q}/A}{\sigma (T_h^4 - T_c^4)} = \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 \left[ \frac{1}{N + 1} \right] \]

\[(A2)\]

The term containing \(\varepsilon\) indicates the degree to which black-body radiation is attenuated by the emissivity of the facing surfaces of the insulation system, and the term containing \(N\) indicates the degree to which black-body radiation is attenuated by the number of layers of insulation. The degradation factor, \(DF\), relates the performance of an actual insulation system to the performance of an ideal system (that is, one in which there
is no conduction between layers and no edge effects at penetrations, etc). DF is defined as the heat transferred through an actual insulation system divided by the calculated heat leak for the same system as determined by Equation A1 using a degradation factor of unity.

Since the actual heat leak will always be greater than the theoretical heat leak, the degradation factor is always greater than 1. It has been demonstrated experimentally that the degradation factor can approach unity for systems installed in guarded flat-plate calorimeters or for systems which have been very carefully applied to relatively large tanks. In general, however, the degradation factor is significantly greater than unity—especially for small tanks to which many layers have been applied.

Figure A-1 shows a plot of the calculated value of effective emittance, \( \bar{\varepsilon} \), of an aluminized Mylar multilayer insulation system as a function of the number of layers in the system and degradation factor. The values are based on an emissivity of 0.025 for the aluminum surfaces. Also shown in Figure A-1 is the thickness of the insulation system, based on 0.076 cm of thickness per layer in insulation, a value which we have found to be optimum for systems that employ aluminized Mylar shields and either silk or nylon tule spacers.\(^1\)

Table A-1 shows performance data for a number of multilayer insulation systems. This summary has been compiled from experimental data reported in the literature—some of it taken at Arthur D. Little, Inc. Table A-2 lists the references from which the data was taken. Note (Table A-1) that the effective emittance of systems incorporating 5 and 10 layers is equal to or less than the effective emittance for systems which incorporate many times this number of layers. This is also indicated by the fact that the effective emittance per shield for the systems incorporating few layers is significantly less than the effective emittance per shield for the many-layered systems. We believe that this

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\(^1\) References for the labeled points in Figure A1 are indicated in Table A1.
CURVES ARE BASED ON $\varepsilon_1 = \varepsilon_2 = 0.025$

FIGURE A-1  EFFECTIVE EMITTANCE AND THICKNESS OF AN ALUMINIZED MYLAR MULTILAYER INSULATION SYSTEM
<table>
<thead>
<tr>
<th>SYSTEM CHARACTERISTICS</th>
<th>INSTALLATION DATA</th>
<th>THERMAL PERFORMANCE</th>
<th>REFERENCE FOR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO. SHIELDS</td>
<td>SYSTEM WEIGHT</td>
<td>SYSTEM SURFACE AREA</td>
</tr>
<tr>
<td>1/4 mil polyester gold coated both sides</td>
<td>2 layers silk coating per shield</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2 layers silk coating per shield</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2 layers glass fabric per shield</td>
<td>No</td>
<td>5</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2 layers glass fabric per shield</td>
<td>Yes</td>
<td>5</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>20 mil open cell polyurethane foam</td>
<td>No</td>
<td>20</td>
</tr>
<tr>
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<td>25 mil polyurethanes foam</td>
<td>No</td>
<td>25</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2.8 mil Reflective paper</td>
<td>No</td>
<td>20</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>1/3 mil polyester biaxial Al. coated both sides</td>
<td>No</td>
<td>36</td>
</tr>
<tr>
<td>1/4 mil Al.</td>
<td>Multi-layer composite 0.1 mis Al. foil 5 mil fiberglass 1/4 mil polyester</td>
<td>No</td>
<td>20</td>
</tr>
<tr>
<td>1/4 mil Al. covered both sides</td>
<td>No</td>
<td>Yes</td>
<td>42</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2 layers silk per shield</td>
<td>Yes</td>
<td>10</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>2 layers silk per shield</td>
<td>No</td>
<td>10</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>&quot;Superfloe&quot; (Durres mesh)</td>
<td>No</td>
<td>120</td>
</tr>
<tr>
<td>1/4 mil Al.</td>
<td>Glass fiber paper</td>
<td>No</td>
<td>26</td>
</tr>
<tr>
<td>1/4 mil Al.</td>
<td>Rayon fabric</td>
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<td>36</td>
</tr>
<tr>
<td>1/4 mil Al.</td>
<td>Glass fiber web</td>
<td>No</td>
<td>21</td>
</tr>
<tr>
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<td>Glass fiber web</td>
<td>No</td>
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<tr>
<td>1/4 mil Al.</td>
<td>Glass fiber web</td>
<td>No</td>
<td>30</td>
</tr>
<tr>
<td>1/4 mil polyester Al. coated both sides</td>
<td>&quot;Superfloe&quot; (Durres mesh)</td>
<td>No</td>
<td>44</td>
</tr>
</tbody>
</table>

1Based on measured heat flux corrected to hot boundary temperatures of 90°F (32°C) and cold boundary temperatures of -32°F (7°C).
TABLE A-2

LIST OF REFERENCES FOR TABLE A-1


5. Linde Company Data - Personal Communication from Mr. N. Gibbon, 1971.


difference is due to the fact that the systems with the best performance (Systems A and B in Figure A1) were applied very carefully—with extreme care being taken to insure that there was no compression at corners, that edge effects at penetrations were reduced to a minimum, etc., whereas the many-layered systems were not applied with the same care.
APPENDIX B

DERIVATION OF A GENERAL EXPRESSION FOR THE WEIGHT
OF SOLID CRYOGEN SYSTEMS

This appendix contains a derivation of an expression for the weight of cryogen required in order that a stored cryogen system have a specified hold time. The expression is derived in terms of cryogen properties and specified boundary conditions using the following idealized basis:

1. The cryogen vessel is spherical.

2. The vessel is insulated with a MLI system which can be characterized by an effective emittance $\varepsilon$.

3. The only form of heat leak into the cryogen is through the MLI system.

The analytical model is shown in Figure B1, and the terminology is defined in the list of abbreviations and symbols.

The heat balance on the cryogen vessel is that the heat absorbed by the cryogen during the mission equals the product of the heat leak per unit time and the hold time. This also equals the mass of cryogen in the vessel times the latent heat. Expressed analytically:

$$ Q = qt = WAh $$

Solving for $W$ yields:

$$ W = \frac{qt}{Ah} \quad (B1) $$

The heat leak through the MLI system is:

$$ q = \sigma A \varepsilon (T_h^4 - T_c^4) \quad (B2) $$
FIGURE B-1  ANALYTICAL MODEL OF IDEALIZED CRYOGEN STORAGE VESSEL
The area and volume of the cryogen vessel are given by:

\[ A = \pi D^2 \quad (B3) \]

\[ V = \frac{\pi D^3}{6} \quad (B4) \]

The mass of cryogen is:

\[ W = \rho \quad (B5) \]

Equation (B5) may be rearranged:

\[ V = \frac{W}{\rho} \quad (B6) \]

Substituting this value of \( V \) into equation (B4) gives:

\[ \frac{W}{\rho} = \frac{\pi D^3}{6} \quad (B7) \]

Solving for \( D \) yields:

\[ D = \left( \frac{6W}{\pi \rho} \right)^{1/3} \quad (B8) \]

Substituting this value of \( D \) into equation (B3) yields:

\[ A = \pi \left( \frac{6W}{\pi \rho} \right)^{2/3} \quad (B9) \]

Substituting this value of \( A \) into equation (B2) gives:

\[ q = \sigma \left[ \pi \left( \frac{6W}{\pi \rho} \right)^{2/3} \right] \frac{1}{\varepsilon} \left( T_h - T_c \right) \quad (B10) \]

Substituting this into equation (B1) yields:

\[ W = \frac{\sigma \left[ \pi \left( \frac{6W}{\pi \rho} \right)^{2/3} \right] \frac{1}{\varepsilon} \left( T_h - T_c \right) t}{\Delta h} \quad (B11) \]
Solving for $W$ results in:

$$W = \frac{36\pi}{\rho^2} \left[ \sigma \varepsilon \left( \frac{T_h^4 - T_c^4}{\Delta h} \right) t \right]^3$$  \hspace{1cm} \text{(B12)}$$

Now, if $T_c^4 \ll T_h^4$, this reduces to:

$$\dot{W} = \frac{36\pi}{\rho^2} \left[ \sigma \varepsilon \frac{T_h^4}{\Delta h} t \right]^3$$  \hspace{1cm} \text{(B13)}$$