NASA TECHNICAL MEMORANDUM

NASA TM X-3312

NASA TM X-3312

CASE FILE COPY

THEORETICAL MONOCHROMATIC-WAVE-INDUCED CURRENTS IN INTERMEDIATE WATER WITH VISCOSITY AND NONZERO MASS TRANSPORT

Theodore A. Talay

Langley Research Center
Hampton, Va. 23665

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • DECEMBER 1975

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM X-3312		

4. Title and Subtitle	5. Report Date
THEORETICAL MONOCHROMATIC-WAVE-INDUCED CURRENTS IN INTERMEDIATE WATER WITH VISCOSITY AND NONZERO MASS TRANSPORT	December 1975
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Theodore A. Talay	L-10368
9. Performing Organization Name and Address	10. Work Unit No.
NASA Langley Research Center Hampton, Va. 23665	176-24-31-03
	11. Contract or Grant No.
12. Sponsoring Agency Name and Address	13. Type of Report and Period Covered
National Aeronautics and Space Administration Washington, D.C. 20546	Technical Memorandum
	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

This study reviews some wave-induced mass-transport current theories with both zero and nonzero net mass (or volume) transport of the water column. A relationship based on the Longuet-Higgens theory is derived for wave-induced, nonzero mass-transport currents in intermediate water depths for a viscous fluid. The relationship is in a form useful for experimental applications; therefore, some design criteria for experimental wave-tank tests are also presented. Sample parametric cases for typical wave-tank conditions and a typical ocean swell were assessed by using the relation in conjunction with an equation developed by Ünlüata and Mei for the maximum wave-induced volume transport. Calculations indicate that substantial changes in the wave-induced mass-transport current profiles may exist dependent upon the assumed net volume transport. A maximum volume transport, corresponding to an infinite channel or idealized ocean condition, produces the largest wave-induced mass-transport currents. These calculations suggest that wave-induced mass-transport currents may have considerable effects on pollution and suspended-sediments transport as well as buoy drift, the surface and midlayer water-column currents caused by waves increasing with increasing net volume transports. Some of these effects are discussed.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Wave-induced currents Stokes' drift Ocean currents Ocean waves Oceanography Mass transport	Unclassified – Unlimited Subject Category 48

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	27	$3.75

*For sale by the National Technical Information Service, Springfield, Virginia 22161

THEORETICAL MONOCHROMATIC-WAVE-INDUCED CURRENTS IN INTERMEDIATE WATER WITH VISCOSITY AND NONZERO MASS TRANSPORT

Theodore A. Talay
Langley Research Center

SUMMARY

This study reviews some wave-induced mass-transport current theories with both zero and nonzero net mass (or volume) transport of the water column. A relationship based on the Longuet-Higgens theory is derived for wave-induced, nonzero mass-transport currents in intermediate water depths for a viscous fluid. The relationship is in a form useful for experimental applications; therefore, some design criteria for experimental wave-tank tests are also presented. Sample parametric cases for typical wave-tank conditions and a typical ocean swell were assessed by using the relation in conjunction with an equation developed by Ünlüata and Mei for the maximum wave-induced volume transport. Calculations indicate that substantial changes in the wave-induced mass-transport current profiles may exist dependent upon the assumed net volume transport. A maximum volume transport, corresponding to an infinite channel or idealized ocean condition, produces the largest wave-induced mass-transport currents. These calculations suggest that wave-induced mass-transport currents may have considerable effects on pollution and suspended-sediments transport as well as buoy drift, the surface and midlayer water-column currents caused by waves increasing with increasing net volume transports. Some of these effects are discussed.

INTRODUCTION

The waters over continental margins are assuming a greater importance to many nations as considerations of economy, pollution, and transportation pose new requirements on our general knowledge of these waters. One major factor in the study of these waters is the oceanic circulation pattern, both global and local. Often, the local ocean current pattern must be investigated to determine the drift of a particular oil spill, ocean spoils dump, buoy, or school of fish. This local ocean circulation pattern is complicated by many interacting physical processes such as the ocean density field, sea surface topography, wind pattern and sea state, bathymetry, tides, zones of convergence and divergence of oceanic waters, and proximities of coasts.

A recent study (ref. 1) examined some of these conditions in a region of the mid-Atlantic continental shelf. One of the conclusions of this study was that currents caused by waves are often of the same order of magnitude as other currents in the region. Waves in passage are known to impose a nonclosed orbital motion on the fluid particles beneath the passing waves (ref. 2). Alternatively, this condition may be described as a closed orbital motion with a drift associated with the progression of the orbits. In shallow waters the orbital motion becomes flattened and resembles a back and forth "scrubbing" velocity which may lift sediment and wastes off the bottom and place them in suspension. The wave-induced drift may then act to transport these sediments or wastes. At or near the surface this wave-induced drift may influence the drift of surface pollutants and buoys, and thus represent a major component of the surface drift. A number of theories have been advanced to explain and predict this wave-induced drift, each theory limited by its own particular set of assumptions. Experimental wave-tank data in the form of drift velocity profiles have both confirmed the presence of a wave-induced drift and provided comparisons of the various theories.

Stokes' theory (ref. 2), assuming an inviscid fluid, presents equations for the wave-induced mass-transport current profile for cases of zero and nonzero net wave-induced mass transport of the water column. From experimental data for the zero net mass-transport cases, Stokes' theory appears to work well in deep waters. However, this theory does not explain observed velocities, especially a strong forward flow at the bottom in intermediate waters which are typical of continental shelf areas. The theory of Longuet-Higgens (ref. 3) follows an Eulerian approach to the equations of motion and allows for a nonzero fluid viscosity. For cases of zero net wave-induced mass transport of the water column, the Longuet-Higgens equation for the wave-induced mass-transport current profile qualitatively explains many of the effects observed in wave-tank experiments, even when the equation is applied beyond its original limitations. Recently, Ünlüata and Mei (ref. 4) examined the wave-induced velocity profiles from a Lagrangian approach to the equations of motion. Equations for the wave-induced mass-transport current and the corresponding net volume transport of the water column are presented. These equations involve wave and depth conditions as well as a constant pressure gradient in the flow direction. For a case of zero net mass (or volume) transport, the current profile was shown to reduce to that obtained by Longuet-Higgens. A maximum net mass transport of the water column was stated as that corresponding to a zero pressure gradient.

Kinsman noted that the assumption of a zero net horizontal mass transport caused by waves is appropriate for wave experiments carried out in a closed wave tank (ref. 5). In the ocean, however, it is unlikely that the net horizontal mass transport caused by waves will be zero.

This report discusses wave-induced current profiles associated with nonzero wave-induced mass transports. It reviews some previously published mass-transport current theories with a zero and a nonzero net mass transport of the water column. The appendix contains a derivation following closely the Eulerian approach of Longuet-Higgens (ref. 3), which results in an expression for the wave-induced mass-transport current profiles for intermediate water with viscous flow allowing a net mass transport of the water column. This expression is shown to be the same as that proposed by Longuet-Higgens although he did not explicitly state it in the form presented here. For experiments conducted in wave tanks where only current profiles may be generally measured, the derived expression is shown to be in a form directly applicable for comparing theoretical wave-induced currents with the measured data. Necessary wave-tank design criteria for such tests are discussed. In the absence of such experimental data, two parametric evaluations were conducted, one for wave and depth conditions from a typical wave-tank test and the other for a typical swell condition on the mid-Atlantic continental shelf. A theoretical expression for the maximum nonzero mass transport, given by Ünlüata and Mei in their Lagrangian analysis (ref. 4), was used in the analysis which follows to determine the upper limit for the net wave-induced mass (or volume) transport. A range of net mass (or volume) transport from zero to the theoretical maximum was used.

SYMBOLS

A_1,A_2,A_3	coefficients in equation for $\overline{U}_{LH}(z)$ (defined by eqs. (7))
a	wave amplitude, meters
B_1,B_2,B_3	coefficients in equation for $\overline{U}_{LH,MT}(z)$ (defined by eqs. (A20a) and (A20b))
C	coefficient used in Stokes' mass-transport current (defined by eq. (3)), meters/second
H	wave height, trough to crest, 2a, meters
h	mean water-column depth, meters
k	wave number, $2\pi/L$, meters^{-1}
L	wavelength, meters
ℓ	wave-tank length, meters

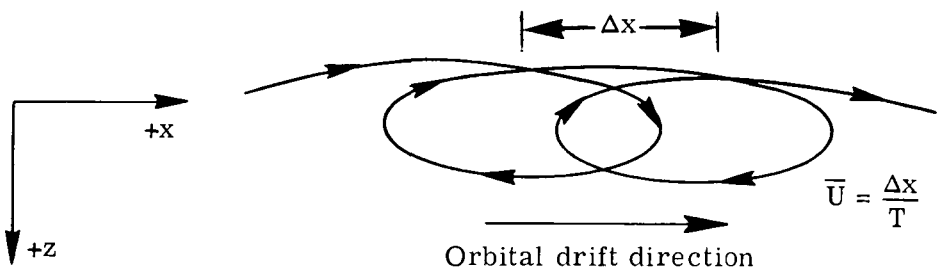
$O(\ )$	order of
$\overline{P}_{2a}$	second-order, time-averaged pressure gradient, newtons/meter^3
$R(z)$	function (defined by eqs. (A17) and (A18))
T	wave period, seconds
$\overline{U}(z)$	wave-induced mass-transport current, meters/second
V_x	net horizontal wave-induced volume transport per unit length of crest, meters^2/second
x	spatial coordinate measured positive in direction of wave advance, meters
z	vertical coordinate measured positive downwards from mean water level, meters
α_1	coefficient of $R(z)$, meters^{-3}
α_2	coefficient of $R(z)$, meters^{-2}
α_3	coefficient of $R(z)$, meters^{-1}
α_4	coefficient of $R(z)$
δ	boundary-layer thickness, meters
ϵ	small dimensionless quantity used in series expansion, $O(a/L)$
μ	dynamic viscosity of water, kilograms/meter-second
ν	kinematic viscosity of water, meters^2/second
ρ	density of water, kilograms/meter^3
σ	wave frequency, $2\pi/T$, seconds^{-1}
ψ	stream function for interior of fluid, meter·meter/second

Subscripts:

B	bottom
DP	in deep water
EXP	experiment
LH	Longuet-Higgens, under conditions of zero net mass transport
MAX	maximum
MT	conditions of nonzero net mass transport
S	surface
UM	Ünlüata and Mei, under conditions of zero net mass transport

REVIEW OF SOME WAVE-INDUCED CURRENT THEORIES

With the passage of a water wave, the fluid particles beneath the wave surface theoretically exhibit a nonclosed orbital motion in the vertical plane as shown in the following sketch:



Sketch (a)

This condition may be described, alternatively, as a closed orbital motion with a drift associated with the progression of the orbital motion. This steady, second-order drift velocity (also known as the mass-transport current) has been confirmed by several experimenters (ref. 3, p. 577; refs. 6 and 7). The following discussion reviews several

wave-induced current theories and also experimental wave-tank data to determine the applicability of such theories.

This discussion is limited to wave-induced currents only; thus, a number of simplifying assumptions are made. Monochromatic, two-dimensional, progressive surface waves are considered to be moving across the water column. The bottom is postulated to be flat and impermeable. Coriolis and surface tension forces are neglected, and the fluid is considered incompressible, inviscid, and irrotational with a constant, uniform surface pressure. A constant mean water-column depth is assumed. Wave-tank tests may approximate many of these conditions.

Given the above considerations, classical wave theory may be used to describe theoretically the phenomenon of a wave-induced current. Classical wave theory represents a solution of Newton's second law of motion (conservation of momentum) and the continuity equation (conservation of mass) subject to boundary conditions prescribed at the surface and bottom of the water. Solutions to these equations are found to be periodic, taking the form of classical waves. The first-order solution (Airy or linear theory) for small ratios of amplitude to wavelength of the waves prescribes a simple, sinusoidal waveform in which the fluid particles beneath the surface trace closed orbits with the wave passage. Each fluid particle returns to its original position after each wave cycle. In infinitely deep water, the orbits are circular; in water of finite depth, however, the orbits become ellipses. Higher-order wave theories describe a more complex situation.

Stokes' Wave-Induced Mass-Transport Current

For waves whose amplitudes are not small compared to the wavelength, the equation of the free surface is nonlinear. Stokes (ref. 2) found an approximate solution under this condition by using a free-surface equation consisting of a series of harmonic functions. To a second-order approximation, the Stokes solution exhibits the property that fluid particles beneath a passing wave trace out orbits which are not closed. After each wave cycle, the fluid particles are displaced a distance in the direction of wave advance. This nonperiodic drift of the fluid particles (and orbits) has been variously termed the mass-transport current, Stokes' drift, and wave-induced current. To a second order, Stokes' expression for the mass-transport current profile $\overline{U}_{ST}(z)$ is

$$\overline{U}_{ST}(z) = \frac{a^2 \sigma k \cosh\left[2k(z - h)\right]}{2 \sinh^2 kh} + C \tag{1}$$

where h is the mean water-column depth and C is an arbitrary coefficient. If it is assumed that the net mass transport for the water column is zero (assuming constant density),

$$\rho \int_0^h \overline{U}(z)\, dz = 0 \tag{2}$$

then

$$C = \frac{-a^2\sigma \sinh 2kh}{4h \sinh^2 kh} \tag{3}$$

These equations represent the conditions that exist in a closed wave tank where a constant return flow, given by C, is generated after sufficient time. For deep water ($kh > \pi$), $\sinh kh$ may be replaced by $e^{kh}/2$. When combined, equations (1) and (3) become

$$\overline{U}_{ST,DP}(z) = a^2\sigma k\left(e^{-2kz} - \frac{1}{2kh}\right) \tag{4}$$

Experiments by Mitchum and Mason (ref. 8, p. 32) have confirmed equation (4). Also, experiments in reference 6 for $3.2 < kh < 5.5$ show reasonable agreement with equation (4) in the upper half of the fluid.

To describe wave-tank experimental current profiles where the wave generator has not been running long enough to establish a return flow or to describe idealized ocean conditions where a return flow may not exist, the coefficient C is often allowed to be zero, thus allowing for a net mass transport. For intermediate waters ($kh < \pi$), this assumption yields

$$\overline{U}_{ST}(z)\Big|_{C=0} = \frac{a^2\sigma k \cosh\left[2k(z - h)\right]}{2 \sinh^2 kh} \tag{5}$$

and for deep waters ($kh > \pi$)

$$\overline{U}_{ST,DP}(z)\Big|_{C=0} = a^2\sigma k e^{-2kz} \tag{6}$$

for the Stokes mass-transport current profiles. Little data are available to confirm these results for finite net mass transport. However, Alofs and Reisbig (ref. 9) have found in the movement of lenses of oil on water, under conditions so that $5.50 < kh < 11.0$, mean surface velocities 35 percent to 150 percent higher than those predicted by Stokes' theory

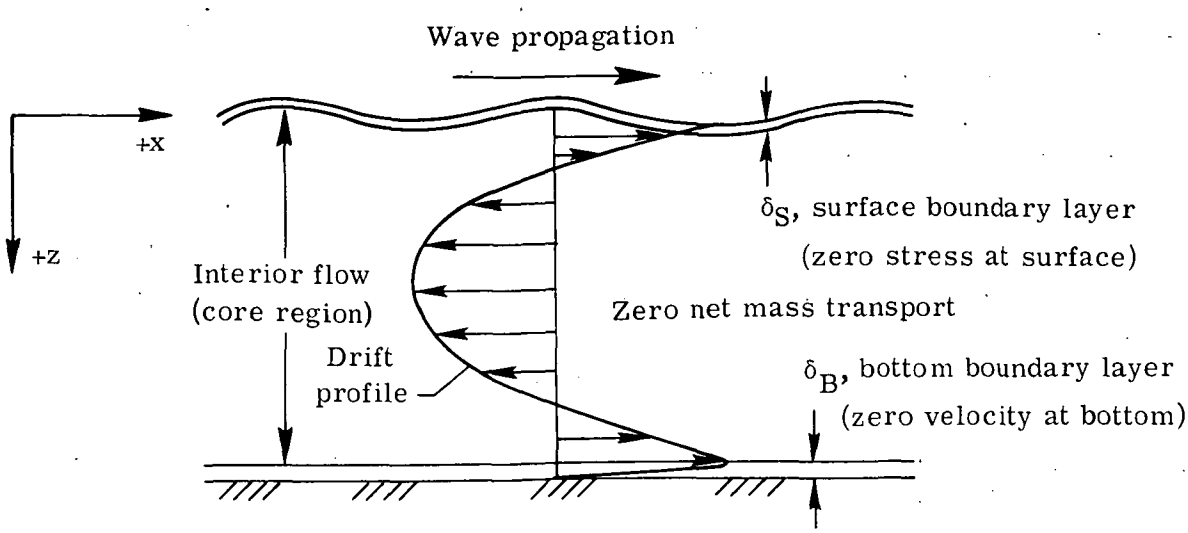
(eq. (6) at the surface ($z = 0$)). Care was taken to obtain wave-tank data before a return flow was set up.

Stokes' solutions have been obtained to third order (ref. 8, p. 33) and fifth order (ref. 10). Other nonlinear theories include cnoidal wave theory (ref. 8, p. 40) and stream function theory (ref. 11). These theories represent an effort to match a mathematical waveform to the actual one as the wave steepness grows and kh becomes small. All these theories, however, neglect the effects of viscosity by assuming an inviscid, irrotational flow. In intermediate waters ($kh < \pi$), phenomena occur that are not explained by the irrotational theories. Figure 1 shows the experimental mass-transport current profile for a wave-tank test conducted by Russell and Osorio (ref. 7). Intermediate water conditions existed ($kh = 1.25$), and the data were taken after the wave generator had been in operation sufficiently long to establish a return flow and a case of zero net mass transport for the water column. The wave period T was 1.5 seconds, the wave height H was 11.7 cm, and the water-column depth h was 50.8 cm. The Stokes wave-induced mass-transport current profile, as computed using equations (1) and (3), is shown as the dotted line in figure 1. The theory fails to predict the observed drift characteristics, especially at and near the bottom where a strong positive velocity is present.

Longuet-Higgens' Zero Mass-Transport Wave-Induced Current

Longuet-Higgens (ref. 3) examined Stokes' assumptions regarding the irrotationality of the flow. For a perfect fluid, the flow may have a tangential velocity relative to a solid boundary. For a viscous fluid, however, the particles of fluid directly in contact with the boundary must possess the boundary velocity, a condition of no slip; at the bottom of a wave tank or the ocean this boundary velocity is zero. At a short distance (the boundary-layer thickness) above the bottom there exist the oscillatory orbital velocities of the fluid particles caused by the wave passage. A strong velocity gradient within the bottom boundary exists; hence, strong vorticity. The vorticity may influence the rest of the fluid outside the boundary layer by conduction or convection, especially in intermediate waters; thus, the flow is rotational. Even if the boundary layer were infinitely thin, the vorticity present would be finite.

In Longuet-Higgens' description, the flow field consisted of a surface boundary layer of a thickness δ_S and a bottom boundary layer of a thickness δ_B in addition to an interior (core region) fluid flow (ref. 3). Boundary conditions included zero stress at the surface, zero velocity at the bottom, and the imposition of a zero net mass transport for the entire water column, as shown in sketch (b).



Sketch (b)

Longuet-Higgens proceeded with his analysis from an Eulerian point of view. Using a perturbation technique and matching the solution for the interior flow to the solution at the edges of the boundary layers, Longuet-Higgens showed (ref. 3), for a progressive wave where vorticity is assumed to spread only by conduction, that

$$\left.\begin{aligned}
\overline{U}_{LH}(z) &= \frac{a^2\sigma k}{4\sinh^2 kh}\left[2\cosh 2k(z-h) + A_1 + A_2\left(\frac{z}{h}\right) + A_3\left(\frac{z}{h}\right)^2\right] \\
A_1 &= kh\sinh 2kh - \frac{3}{2}\frac{\sinh 2kh}{kh} - \frac{3}{2} \\
A_2 &= -4kh\sinh 2kh \\
A_3 &= 3kh\sinh 2kh + \frac{3}{2}\frac{\sinh 2kh}{kh} + \frac{9}{2}
\end{aligned}\right\} \tag{7}$$

This equation applies to the interior of the fluid. Separate equations (ref. 3) exist for the mass-transport current profiles within the thin boundary layers. However, equations (7) describe the core region drift flow usually measured in wave-tank experiments. Shown in figure 1 is the Longuet-Higgens solution for the mass-transport current profile based on the previously described wave conditions. It is evident that a better qualitative fit to the experimental data curve is obtained. This observation is especially true at and near the bottom (outside the bottom boundary layer) where Stokes predicts a negative velocity, whereas the Longuet-Higgens theory predicts the strong forward flow

actually observed. It should be noted that within the bottom boundary layer (in this case 0.7 mm thick) the actual velocity would be zero at the bottom wall as is predicted in the bottom boundary-layer solution of Longuet-Higgens (ref. 3). Stokes' theory, which assumes inviscid flow, predicts a slip flow.

The conduction solution of Longuet-Higgens (eqs. (7)) is strictly valid only for wave amplitudes very much smaller than the boundary-layer thicknesses (which themselves are small), that is, $a/\delta_S \approx a/\delta_B << 1$. However, wave-tank tests by Russell and Osorio (ref. 7) show the equation to be of use for waves of considerable amplitude. For the particular example of figure 1, a is 0.058 meter and $\delta_B \approx \delta_S \approx 0.7$ mm yielding $a/\delta_S \approx a/\delta_B \approx 83$. Generally, Longuet-Higgens' theory, based on the experimental evidence of Russell and Osorio (ref. 7) where $a/\delta_S \approx a/\delta_B \approx O(100)$, appears to predict favorably wave-induced drift profiles in waters where $0.7 < kh < 1.5$. For $kh < 0.7$ the waves begin to exhibit an instability invalidating the Longuet-Higgens perturbation assumption. In deeper waters ($kh > 1.5$), the assumed surface condition leads the Longuet-Higgens solution to an unbounded condition near the surface (ref. 12).

Longuet-Higgens' analysis describes the wave-induced mass-transport current profile for a viscous, intermediate water condition of zero net mass transport of the water column. Closed wave-tank experiments have confirmed essential features of this theory for zero net mass transport. However, for cases of a nonzero mass transport, as in a long wave tank where the wave generator has not been running long enough to establish a return flow, or in the ocean, it is unlikely that a zero net mass transport caused by waves exists. Therefore, the use of a zero mass transport, Stokes' equation (eq. (1) or eq. (4)) or Longuet-Higgens' equations (eqs. (7)), would not be valid. Instead, Stokes' nonzero mass-transport theory (in deeper waters where viscous effects may not be important for most of the flow profile, eq. (5) or (6)) or an intermediate- and shallow-water viscous theory with nonzero mass transport should be used. To be of most value, any such theory would require confirmation by experiment.

Ünlüata and Mei's Nonzero Mass-Transport Theory

In a paper by Ünlüata and Mei (ref. 4), boundary-layer arguments were incorporated into the Lagrangian form of the equations of motion where the free surface may be treated as a fixed and known quantity (unlike the Eulerian Longuet-Higgens theory where complicated coordinate transformations of the free surface are required). Again, the fluid is divided into surface and bottom boundary layers with a core (interior) region. By perturbation analysis the second-order terms yield the following solution for the wave-induced current

$$\overline{U}_{UM,MT}(z) = \frac{\overline{P}_{2a}}{2\mu}\left(z^2 - h^2\right) + \frac{\sigma k a^2}{\sinh^2 kh}\left[\frac{3}{4} + \left(1 - \frac{z}{h}\right) kh \sinh 2kh + \frac{1}{2}\cosh 2k(z - h)\right] \tag{8}$$

and for the wave-induced net volume transport V_x

$$V_x = \frac{-\overline{P}_{2a}}{3\mu} h^3 + \frac{\sigma a^2 kh}{\sinh^2 kh}\left(\frac{3}{4} + \frac{kh}{2}\sinh 2kh + \frac{1}{4kh}\sinh 2kh\right) \tag{9}$$

In these equations, $\overline{P}_{2a}$ is a time-averaged, constant pressure gradient associated with the wave-induced current in the coordinate system of the fluid particles. According to Ünlüata and Mei, two limiting cases are possible: (1) when $V_x = 0$, the case of zero net volume transport induced by waves, equations (8) and (9) confirm the Eulerian approach of Longuet-Higgens (eqs. (7)); (2) in an infinite wave tank or idealized ocean where steady state has been reached $\overline{P}_{2a} = 0$, and a maximum net volume transport of the water column caused by waves occurs. Also, according to Ünlüata and Mei, the equations are most appropriate for $0.7 < kh < 1.5$ when compared to the Russell and Osorio experimental results. No discussion of possible intermediate cases of net volume transport is given in reference 4.

Experimental Data

Few experiments have been performed to study wave-induced drift profiles under the conditions of a nonzero volume transport. The experiments of Alofs and Reisbig (ref. 9) were conducted in a closed wave tank under deep water conditions ($kh > 5.5$) and controlled conditions of a nonzero volume transport. Only surface drift velocities caused by waves were measured, but these velocities proved greater by 35 percent to 150 percent than predicted by the Stokes nonzero mass-transport theory. Oil lenses were used as surface drifters which may move at a different velocity than the water beneath them and influence the surface boundary-layer conditions also. Thus, it is difficult to assess these results when they are compared to a clean water condition. Some experiments, reported in reference 6, were conducted in intermediate waters ($kh < \pi$) before a return flow was set up, thus producing conditions of a possible nonzero volume transport caused by waves; but the results show a considerable scatter in the data. It is suspected, as reported in reference 6, that the scatter may be a result of the failure to achieve a stable waveform at the data station when the data were taken. Since the wave-induced mass-transport current is a secondary effect, the error introduced by an unstable waveform may be considerable. Thus, little usable information is available in cases of a nonzero volume transport caused by waves.

In wave-tank experiments, the usual measured data include (1) the wave and tank properties, and (2) the surface or bottom drift velocities, or the entire observed wave-induced drift profile. Small floats, neutrally buoyant particles, or dye streaks are the usual method of tracking water movements. The Ünlüata and Mei theory (eqs. (8) and (9)) suggests that the second-order, time-averaged pressure gradient $\overline{P}_{2a}$ is a necessary property for a comparison of the predicted and measured drift profiles. In practice, $\overline{P}_{2a}$ may be a difficult parameter to measure with sufficient accuracy. It is suggested, therefore, that an alternate form of the equation for the wave-induced mass-transport current profile may be more suitable for experimental purposes. A reexamination of the Longuet-Higgens analysis, explained in the next section, shows that such an equation may be determined.

LONGUET-HIGGENS' NONZERO MASS-TRANSPORT EQUATION FOR EXPERIMENTAL APPLICATIONS

Longuet-Higgens, in his paper on mass transport caused by waves (ref. 3), alluded to the possibility of a nonzero mass transport for the water column as a whole, and even suggested the form any additional terms, added to equations (7), might exhibit. However, he did not explicitly derive and state such an equation. An equation of this form would help describe those experiments conducted in wave tanks under nonzero mass-transport conditions. Such an equation would require only those wave parameters usually measured in these experiments including wave height, period, wavelength, the wave-induced current profile, and water depth.

To develop such an equation, Longuet-Higgens' original derivation was followed to the point where he assumed a case of zero mass transport (or zero volume transport for a constant density fluid) for the entire water column. At that point, an arbitrary nonzero total volume transport V_x was assumed to exist. The derivation of the final form of the theoretical equation is presented in the appendix. Equations (7) are modified to

$$\overline{U}_{LH,MT}(z) = \overline{U}_{LH}(z) + \frac{3}{2}\frac{V_x}{h}\left[1 - \left(\frac{z}{h}\right)^2\right] \tag{10}$$

for cases of nonzero volume transport of the water column caused by waves. It is also noted, from material presented in the appendix, that

$$V_x = \int_0^h \overline{U}_{LH,MT}(z)\, dz \tag{11}$$

The previously stated assumptions regarding the Longuet-Higgens theory are assumed to apply, with equation (10) most applicable in the range $0.7 < kh < 1.5$.

Equations (10) and (11) represent an arbitrary perturbation on the Longuet-Higgens zero mass-transport profile to account for a nonzero mass transport. Experimental data, as yet unavailable, must be used to validate this new approach.

A number of general design criteria exist in the planning of an experiment in a wave tank to study conditions of nonzero mass-transport wave-drift profiles. Wave amplitudes must be kept small so that the quantities $a/\delta_S \approx a/\delta_B$ are small although the Longuet-Higgens theory was shown by Russell and Osorio to be useful even when $a/\delta_S \approx a/\delta_B = O(100)$. Surface contamination of the water must be avoided. Huang (ref. 12) has shown such contamination to have a pronounced effect on the surface boundary condition and the resultant wave-drift profiles. Alofs and Reisbig (ref. 9) discuss a method whereby such effects of surface contamination are minimized. The method involves using a fan at the generating end of the wave tank to move the surface contaminants to the other end of the tank where they may be trapped or removed.

The time required for a backflow current to develop after wave generation has started in a wave tank is of the order of $\ell/\overline{U}(0)$ where ℓ is the length of the wave tank and $\overline{U}(0)$ is the surface wave-induced drift velocity (ref. 4). Longuet-Higgens (ref. 3) indicates that the time required to establish a steady state with regard to the conduction of vorticity is of the order of h^2/ν. To produce a condition of maximum wave-induced volume transport $V_{x,MAX}$, the wave generator should have been running long enough to establish a stable waveform and a steady state with regard to the conduction of vorticity. The wave generator should not, however, run so long that a return flow can be felt at the data measuring stations. The above discussion indicates that maximizing the wave-tank length ℓ and minimizing the wave-tank water depth h would produce the desired effects. Then, as time passes and a return flow develops, the net volume transport would decrease from its maximum value $V_{x,MAX}$ until, for a fully developed backflow, the net volume transport is zero ($V_x = 0$). At the far end of the wave tank, a sloping beach with wave-absorbing devices and materials should be used to prevent wave reflections and standing waves which would not satisfy the purely progressive wave assumption made in this analysis.

Under the experimental procedures outlined, an experimenter might obtain wave-induced current profiles $\overline{U}(z)$ (for a particular set of wave conditions) at successive times corresponding to varying backflow current conditions. Just after wave generation has begun (zero backflow current) a maximum net volume transport $V_{x,MAX}$ exists with the corresponding wave-induced drift profile $\overline{U}_{EXP,MT}(z)$. With time, a backflow current develops, and the wave-induced current profile proceeds through a series of stages corresponding to successively smaller net volume transports V_x. Finally, at a

sufficiently long time after the beginning of wave generation, a fully developed backflow current exists corresponding to zero net volume transport ($V_x = 0$) and to the wave-induced current profile $\overline{U}_{EXP}(z)$. Equations (10) and (11) may be rewritten as

$$\overline{U}_{LH,MT}(z) = \overline{U}_{EXP}(z) + \frac{3}{2}\frac{V_{x,MAX}}{h}\left[1 - \left(\frac{z}{h}\right)^2\right] \tag{12}$$

and

$$V_{x,MAX} = \int_0^h \overline{U}_{EXP,MT}(z)\, dz \tag{13}$$

where the experimental values of wave-induced current profiles at maximum net volume transport $\overline{U}_{EXP,MT}(z)$ and zero net volume transport $\overline{U}_{EXP}(z)$ are used in the calculations. Confirmation of the perturbation on the Longuet-Higgens zero mass-transport profile would exist if

$$\overline{U}_{LH,MT}(z) = \overline{U}_{EXP,MT}(z) \tag{14}$$

is obtained for the case of maximum net volume transport. Additional checks may be obtained by using the intermediate stage wave-induced current profiles corresponding to net volume transports $0 \leqq V_x \leqq V_{x,MAX}$ in equations (12) and (13).

For equation (10) to be a predictive technique, however, the net volume transport V_x must be parameterized. This process would, of necessity, involve a series of wave-tank tests to find the dependence of V_x on the wave and wave-tank test conditions. Thus, in general,

$$V_x = V_x(a, L, T, h, \ell, \nu, \ldots) \tag{15}$$

By dimensional-analysis and data-analysis techniques it should be possible to reduce the number of independent variables and to find a relation for V_x based on the wave-tank test data. The theoretical relation for $V_{x,MAX}$ by Ünlüata and Mei (ref. 4) for the limited range of applicability ($0.7 < kh < 1.5$) would be helpful in this analysis.

In the absence of experimental data obtained under conditions of a nonzero volume transport caused by waves, it is informative first to analyze parametrically some typical wave-tank and ocean-swell cases to examine: (1) the variation in wave-induced current profiles between extremes of zero net volume transport and maximum net volume transport and (2) typical magnitude ranges for the wave-induced currents. In this manner, the results of the next section may shed some light on what might be expected in an experimental program.

RESULTS AND DISCUSSION

The example of figure 1 has been examined for a range of possible volume transport values. Again, the particular wave conditions for this experiment were:

Water depth, h, cm	50.8
Wave period, T, sec	1.5
Wavelength, L, cm	256
Wave height, H, cm	11.7
kh	1.25
$a/\delta_S \approx a/\delta_B$	83

The experimental data presented in figure 1 represent a condition of zero net volume transport. Russell and Osorio (ref. 7) took this type of wave-tank data after the wave generator had been in operation for hours, when necessary, to achieve steady backflow conditions. The Ünlüata and Mei equation (9) was used to obtain a maximum theoretical volume transport. According to Ünlüata and Mei, this maximum occurs when $\overline{P}_{2a} = 0$ corresponding to an infinite wave tank. Equation (9) can be written for this case as

$$V_{x,MAX} = \frac{\sigma k a^2 h}{\sinh^2 kh}\left(\frac{3}{4} + \frac{kh}{2}\sinh 2kh + \frac{1}{4kh}\sinh 2kh\right) \tag{16}$$

The range of possible volume-transport values chosen for this parametric analysis included: (1) a solid wall wave-tank end ($V_x = 0$), (2) various degrees of wave-tank end "porosity" $\left(V_x = 0.25V_{x,MAX},\ V_x = 0.50V_{x,MAX},\ \text{and}\ V_x = 0.75V_{x,MAX}\right)$, and (3) an open-ended, infinite length channel $\left(V_{x,MAX}\right)$. With these values and the previously stated wave conditions, the Longuet-Higgens nonzero mass-transport equation (10) was used $\left(\text{with}\ \overline{U}_{LH}(z)\ \text{calculated from eqs. (7)}\right)$. The results are plotted in figure 2.

In figure 2 the case of $V_x = 0$ corresponds to the Longuet-Higgens zero volume- (or mass-) transport case originally presented in figure 1. However, as more net volume transport caused by waves is assumed (i.e., a smaller backflow), the effect is to shift the entire velocity profile forward to more positive values, thus rotating the curve about the point of interior flow near-bottom velocity. This represents a parabolic velocity distribution added to the original Longuet-Higgens equations (eqs. (7)) that vanishes at the bottom boundary layer and has a zero velocity gradient at the free-surface boundary layer as Longuet-Higgens had suggested (ref. 3). Finally, there is the extreme theoretical case of the infinite or open-ended channel where the volume transport is a maximum. A substantial wave-induced current profile, everywhere positive in the direction of wave advance, is present.

A case was also examined of a typical ocean swell, originally considered in reference 1. The particular wave conditions for the swell were:

Water depth, h, m	18.3
Swell period, T, sec	11.0
Swell wavelength, L, m	132.31
Swell height, H, m	1.83
kh	0.87

The value of kh in this example (0.87) is within the intermediate water limits best applicable for the use of the Longuet-Higgens nonzero mass-transport equation (10) ($0.7 < kh < 1.5$). Examination of the ratio a/δ is important to check the validity of using the Longuet-Higgens theory. To be strictly valid, $a/\delta_S \approx a/\delta_B$ should be much less than one, but Russell and Osorio showed the theory to predict reasonable results when $a/\delta_S \approx a/\delta_B$ was on the order of 100. The boundary-layer thickness δ is defined as

$$\delta = \left(\frac{2\nu}{\sigma}\right)^{1/2} \tag{17}$$

For water the kinematic viscosity ν is about 0.01 cm^2/sec and for this swell wave $\sigma = 2\pi/T = 0.571\ sec^{-1}$. This yields a boundary-layer thickness $\delta_S \approx \delta_B$ of 0.187 cm. The ratios a/δ_S and a/δ_B are calculated to be 489. These values are almost six times the value calculated for the wave-tank tests of Russell and Osorio. The question posed is whether or not the Longuet-Higgens theory may be applicable for these large values of a/δ. For this reason, extreme caution must be exercised in the interpretation of the results of any calculations made for this swell condition. Therefore, until experimental data are in hand, the following discussion is largely conjectual.

The swell height chosen (1.83 m) for this case is a winter statistical value for the Chesapeake Bight region so that 55 percent of the time the swell was at or below this value and 45 percent of the time above this value. Thus, the swell height may be considered typical. The Stokes mass-transport current calculated from equation (5) is shown as the dashed line in figure 3. The solid curves represent Longuet-Higgens' mass-transport current profiles using equation (10), with volume-transport values of $V_x = 0$, $V_x = 0.25V_{x,MAX}$, $V_x = 0.75V_{x,MAX}$, and $V_x = V_{x,MAX}$ where $V_{x,MAX}$ was again determined from Ünlüata and Mei's theory (eq. (16)). As figure 3 shows, a considerable range of current profiles is possible depending upon the volume-transport conditions. In all cases, the velocity near the bottom predicted by the nonzero volume-transport Longuet-Higgens theory is greater than that determined from Stokes' theory which neglects viscosity. For zero volume transport, a negative midlayer drift exists; whereas, for most positive values of volume transport V_x the drift profiles are everywhere positive. The

surface drift increases with increasing volume transport. In the cases of small net volume transport, the wave-induced surface current is on the order of 1 to 2 cm/sec, an amount considerably less than the static circulation of 5 to 20 cm/sec generally found along the Atlantic continental shelf (ref. 1, p. 24). At the higher values of net volume transport, however, the wave-induced surface current is of the order of 10 cm/sec or the same order as the static circulation on the continental shelf.

If these predictions are correct, the surface and midlayer drifts of pollutants, sediments, and buoys may be affected by the wave-induced current. In the case of buoys, large drogue plates or parachutes are set for a predetermined depth for the purpose of measuring the current at that depth. When the drogue plate or parachute drag area is much greater than that of the surface float and interconnecting cables, then the buoy drifts essentially with the current at the drogue plate or parachute depth. The surface float marks the position for quick determination of drift. Examination of figure 3 shows, for example, that for a particular depth on the continental shelf where the drogue is at a depth of 0.3 of the water-column depth, for zero volume transport the drift caused only by the swell-induced current is opposite to the direction of swell propagation (-1 cm/sec); however, for a case of maximum volume transport, the buoy drifts quickly in the direction of swell propagation (8 cm/sec). Thus, in the presence only of swell-induced currents, by this theory completely opposite buoy drifts are predicted dependent upon the net volume transport that exists. Further, in the interpretation of a known buoy drift where it is desired to subtract out, in some analytical manner, the influences of mass-transport currents (as well as tidal currents, wind stress currents, etc.) to obtain a residual "mean current" (ref. 8, p. 336), it is important to determine properly the total volume transport caused by waves to obtain the best estimate of the wave-induced mass-transport current profile.

Detailed, open-water continental shelf experiments which can be used to test the wave-induced mass-transport current calculations using equations (10) and (11) are not yet available. Wave-tank tests, however, appear to be a reasonable alternative for testing the Longuet-Higgens nonzero mass-transport theory based on the design criteria outlined previously. These tests would also aid in parameterizing the net volume transport V_x so that equation (10) may be used eventually as a predictive equation for future studies.

CONCLUDING REMARKS

This paper presents a review of some wave-induced mass-transport current theories both with and without a net mass (or volume) transport of the water column. A theoretical relationship based on the Longuet-Higgens theory was derived for the

wave-induced mass-transport current profiles in intermediate waters with viscous flow for cases of nonzero volume transport. The theoretical relationship is presented in a form useful for experimental applications. Some design criteria for experimental wave-tank tests using this relation are also presented. In the absence of such experimental data, sample parametric cases for monochromatic waves for a typical wave-tank test and a typical ocean swell were assessed using the derived relation. A maximum wave-induced volume transport was determined from an equation of Ünlüata and Mei as an upper limit for the parametric cases.

Calculations indicate that substantial changes in the wave-induced mass-transport current profiles exist dependent upon the assumed net volume transport. The maximum volume transport, corresponding to an infinite channel or idealized ocean condition, produces the largest wave-induced mass-transport currents. The surface and midlayer water-column currents caused by waves increase with increasing net volume transport. These results suggest that wave-induced mass-transport currents may have a considerable effect upon pollution and suspended sediment transport and buoy drift. Also, the parametric analysis indicates that a more detailed examination of wave-induced mass-transport currents based on extensive wave-tank testing is necessary to substantiate the Longuet-Higgens nonzero mass-transport theory and to parameterize the wave-induced net volume transport.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
November 3, 1975

APPENDIX

AN EQUATION FOR WAVE-INDUCED MASS-TRANSPORT CURRENT PROFILES FOR INTERMEDIATE WATER, VISCOUS FLOW WITH NONZERO VOLUME TRANSPORT

Longuet-Higgens (ref. 3) examined the problem of wave-induced currents for a fluid of small viscosity. His analysis included solving for the stream function of the interior flow (between the edges of the surface and bottom boundary layers), but the analysis used the conditions at the interface of these layers as necessary boundary conditions. By assuming a zero net mass transport of the water column, Longuet-Higgens was able to obtain a single solution for the stream function. Longuet-Higgens alluded to the possibility of a nonzero net mass transport caused by waves and suggested the form any additional terms added to the stream function would exhibit. This appendix is an explicit statement of his inferences. An equation is derived for the wave-induced velocity profile in a form that would be useful for experimenters studying wave-induced currents. It is shown that the term added to the original Longuet-Higgens zero mass-transport equation is, in fact, of the form suggested by Longuet-Higgens.

This appendix closely follows the Longuet-Higgens derivation for a zero net mass transport, and the reader may consult reference 3 for additional details. Only monochromatic, two-dimensional, progressive waves are assumed to be moving through the water column in a manner similar to waves in a wave tank or swell in the ocean. A constant density fluid is assumed. The net mass transport, then, is simply the product of the constant density and the net volume transport. The boundary layers at the surface and bottom of the fluid δ_S and δ_B are very small compared to the depth of the interior flow. Thus the distances from the surface to the top and bottom interfaces may be written

$$\left.\begin{aligned} z &= \delta_S \approx 0 && \text{(Top interface)} \\ z &= h - \delta_B \approx h && \text{(Bottom interface)} \end{aligned}\right\} \tag{A1}$$

In this instance equations (291), (286), (287), (288), and (289) of reference 3 may be rewritten, using the notation of this paper, as

$$\nabla^4\psi(z) = \nabla^4 \frac{\sigma a^2 \sinh 2k(z - h)}{4 \sinh^2 kh} \tag{A2}$$

$$\epsilon^2\left(\frac{\partial\psi}{\partial z}\right)_{z=h} = \frac{5\sigma k a^2}{4\sinh^2 kh} \tag{A3}$$

$$\epsilon^2\left(\frac{\partial^2\psi}{\partial z^2}\right)_{z=0} = \frac{-2\sigma k^2 a^2 \sinh 2kh}{\sinh^2 kh} \tag{A4}$$

$$\epsilon^2(\psi)_{z=0} = 0 \tag{A5}$$

$$\epsilon^2(\psi)_{z=h} = \text{Constant} \tag{A6}$$

Equation (A2) relates the stream function for the interior of the fluid to the wave and mean water-column depth conditions and is a function of z. Equation (A3) is a boundary condition at the edge of the bottom boundary layer. The equation represents, in fact, the horizontal wave-induced drift velocity at that point. The quantity ϵ is a small dimensionless quantity of the order of a/L. Equation (A4) represents the gradient of the wave-induced drift velocity at the surface boundary layer. The stream function ψ is constant at the boundaries. At the surface, this constant may be chosen to make ψ vanish, yielding equation (A5). The stream function at the bottom boundary layer is also a constant. Longuet-Higgens examines this and, for the case of zero net wave-induced mass transport, sets equation (A6) equal to zero. As Longuet-Higgens indicates, however, this condition may be relaxed to allow a nonzero net mass transport. As $\epsilon^2\psi(z)$ is the stream function for the mass transport velocity $\overline{U}_{LH,MT}(z)$ (ref. 3), then

$$\epsilon^2 \frac{\partial\psi}{\partial z}(z) = \overline{U}_{LH,MT}(z) \tag{A7}$$

Integrating equation (A7) over the water column and considering equation (A5) results in

$$\epsilon^2(\psi)_{z=h} = \int_0^h \overline{U}_{LH,MT}(z)\, dz \tag{A8}$$

The value of $\epsilon^2(\psi)_{z=h}$ is the wave-induced mass-transport velocity profile integrated over the water column. This value is equivalent to the net volume transport (per unit crest width of the wave). If an arbitrary value for the integral (e.g., V_x) is prescribed instead, then equation (A6) becomes

$$\epsilon^2(\psi)_{z=h} = V_x \tag{A9}$$

where V_x is the volume transport per unit length of the wave crest.

APPENDIX

To solve equations (A2) to (A5) and (A9), a solution of the form

$$\epsilon^2\psi(z) = \frac{\sigma a^2}{4 \sinh^2 kh}\left[\sinh 2k(z - h) + R(z)\right] \tag{A10}$$

is assumed as proposed by Longuet-Higgens for the case of zero net volume (mass) transport. It is then noted that

$$\overline{U}_{LH,MT}(z) = \epsilon^2 \frac{\partial\psi}{\partial z}(z) = \frac{\sigma a^2 k}{4 \sinh^2 kh}\left[2 \cosh 2k(z - h) + \frac{1}{k}\frac{\partial R(z)}{\partial z}\right] \tag{A11}$$

where $\overline{U}_{LH,MT}(z)$ indicates the expanded Longuet-Higgens equation to account for a nonzero volume (or mass) transport.

Equation (A10) may be differentiated as required, and the boundary conditions at $z = 0$ or $z = h$ substituted as needed, using equations (A2) to (A5) and equation (A9). After the necessary algebraic operations,

$$\frac{d^4R(z)}{dz^4} = 0 \tag{A12}$$

$$\left[\frac{dR(z)}{dz}\right]_{z=h} = 3k \tag{A13}$$

$$\left[\frac{d^2R(z)}{dz^2}\right]_{z=0} = -4k^2 \sinh 2kh \tag{A14}$$

$$\left[R(z)\right]_{z=0} = \sinh 2kh \tag{A15}$$

and

$$\left[R(z)\right]_{z=h} = \frac{4 \sinh^2 kh}{a^2\sigma} V_x \tag{A16}$$

are obtained. Again, equations (A12) to (A15) are the same as presented by Longuet-Higgens, but equation (A16) is different from zero, thus the effects of a net volume transport are indicated.

From equation (A12), $R(z)$ must have the form

$$R(z) = \alpha_1 z^3 + \alpha_2 z^2 + \alpha_3 z + \alpha_4 \tag{A17}$$

where α_1, α_2, α_3, and α_4 are functions only of the wave and water-column depth conditions and the assumed value of volume transport. Using equation (A17) in equations (A13) to (A16), four equations result in four unknowns: α_1, α_2, α_3, and α_4. Solution of these equations yields

$$\left.\begin{aligned}
\alpha_1 &= \frac{-2\sinh^2 kh}{h^3 a^2 \sigma} V_x + \frac{3k}{2h^2} + \frac{k^2}{h}\sinh 2kh + \frac{1}{2h^3}\sinh 2kh \\
\alpha_2 &= -2k^2 \sinh 2kh \\
\alpha_3 &= \frac{-3k}{2} + \left(k^2 h - \frac{3}{2h}\right)\sinh 2kh + \frac{6\sinh^2 kh}{a^2 \sigma h} V_x \\
\alpha_4 &= \sinh 2kh
\end{aligned}\right\} \tag{A18}$$

Differentiating equation (A17) with respect to depth yields

$$\frac{dR(z)}{dz} = 3\alpha_1 z^2 + 2\alpha_2 z + \alpha_3 \tag{A19}$$

Equations (A11), (A18), and (A19) may all be combined to obtain

$$\overline{U}_{LH,MT}(z) = \frac{a^2 \sigma k}{4\sinh^2 kh}\left[2\cosh 2k(z - h) + B_1 + B_2\left(\frac{z}{h}\right) + B_3\left(\frac{z}{h}\right)^2\right] \tag{A20a}$$

where

$$\left.\begin{aligned}
B_1 &= kh\sinh 2kh - \frac{3}{2}\frac{\sinh 2kh}{kh} - \frac{3}{2} + \frac{6\sinh^2 kh}{a^2 \sigma kh} V_x \\
B_2 &= -4kh\sinh 2kh \\
B_3 &= 3kh\sinh 2kh + \frac{3}{2}\frac{\sinh 2kh}{kh} + \frac{9}{2} - \frac{6\sinh^2 kh}{a^2 \sigma kh} V_x
\end{aligned}\right\} \tag{A20b}$$

The basic difference between this expanded form of the Longuet-Higgens current profile equation for nonzero volume (or mass) transport and the original Longuet-Higgens equation (ref. 3) is the additional term $\frac{6\sinh^2 kh}{a^2 \sigma kh} V_x$ which appears in both the B_1 and

B_3 coefficients. This term represents changes as a result of a net volume transport caused by the waves. Finally, equations (A20a) and (A20b) may be rewritten as

$$\overline{U}_{LH,MT}(z) = \overline{U}_{LH}(z) + \frac{3}{2}\frac{V_x}{h}\left[1 - \left(\frac{z}{h}\right)^2\right] \tag{A21}$$

where $\overline{U}_{LH}(z)$ is the Longuet-Higgens wave-induced current profile equation for zero net volume transport (ref. 3). Longuet-Higgens indicated that a term representing a parabolic velocity distribution, vanishing at the bottom and with zero velocity gradient at the free-surface boundary layer, would be added to the current profile equation for $\overline{U}_{LH}(z)$. The term containing V_x in equation (A21) is of this form. The net volume transport V_x is an unknown quantity which must be parameterized, if not theoretically, then by experiments, to obtain unique solutions of equation (A21).

REFERENCES

1. Whitlock, Charles H.; and Talay, Theodore A.: The Influence of Surface Waves on Water Circulation in a Mid-Atlantic Continental-Shelf Region. NASA TN D-7771, 1974.
2. Stokes, George Gabriel: Mathematical and Physical Papers. Vol. 1. Cambridge Univ. Press, 1880.
3. Longuet-Higgens, M. S.: Mass Transport in Water Waves. Phil. Trans. Roy. Soc. London, ser. A, vol. 245, no. 903, Mar. 31, 1953, pp. 535-581.
4. Ünlüata, Ümit; and Mei, Chiang C.: Mass Transport in Water Waves. J. Geophys. Res., vol. 75, no. 36, Dec. 1970, pp. 7611-7618.
5. Kinsman, Blair: Wind Waves – Their Generation and Propagation on the Ocean Surface. Prentice-Hall, Inc., c.1965.
6. Beach Erosion Board: A Study of Progressive Oscillatory Waves in Water. Tech. Rep. No. 1, U.S. Army Corps of Engineers, 1941.
7. Russell, R. C. H.; and Osorio, J. D. C.: An Experimental Investigation of Drift Profiles in a Closed Channel. Proc. Sixth Conf. Coastal Engineering, Engr. Foundation Res. on Wave Research, University of California, 1958, pp. 171-193.
8. Wiegel, Robert L.: Oceanographical Engineering. Prentice-Hall, Inc., c.1964.
9. Alofs, D. J.; and Reisbig, R. L.: An Experimental Evaluation of Oil Slick Movement Caused by Waves. J. Phys. Oceanography, vol. 2, no. 4, Oct. 1972, pp. 439-443.
10. De, S. C.: Contributions to the Theory of Stokes Waves. Proceedings Cambridge Philosophical Soc., vol. 51, pt. 4, Cambridge Univ. Press, Oct. 1955, pp. 713-736.
11. Dean, R. G.: Stream Function Representation of Nonlinear Ocean Waves. J. Geophys. Res., vol. 70, no. 18, Sept. 1965, pp. 4561-4572.
12. Huang, Norden E.: Mass Transport Induced by Wave Motion. J. Mar. Res., vol. 28, no. 1, Jan. 15, 1970, pp. 35-50.

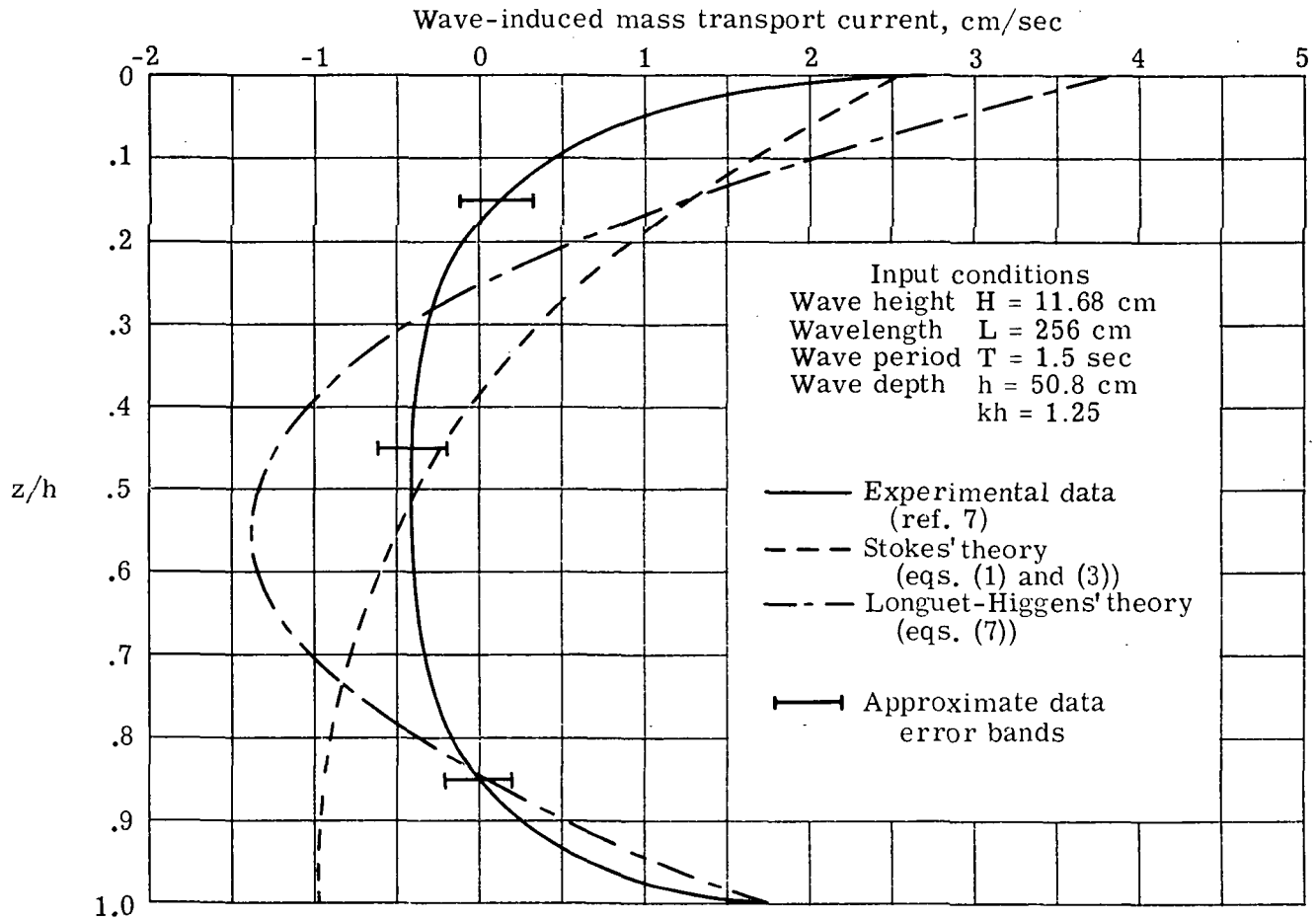


Figure 1.- Wave-induced mass-transport current plotted against nondimensional depth for zero net volume transport $\left(V_x = 0\right)$. Surface and bottom boundary layers are not shown.

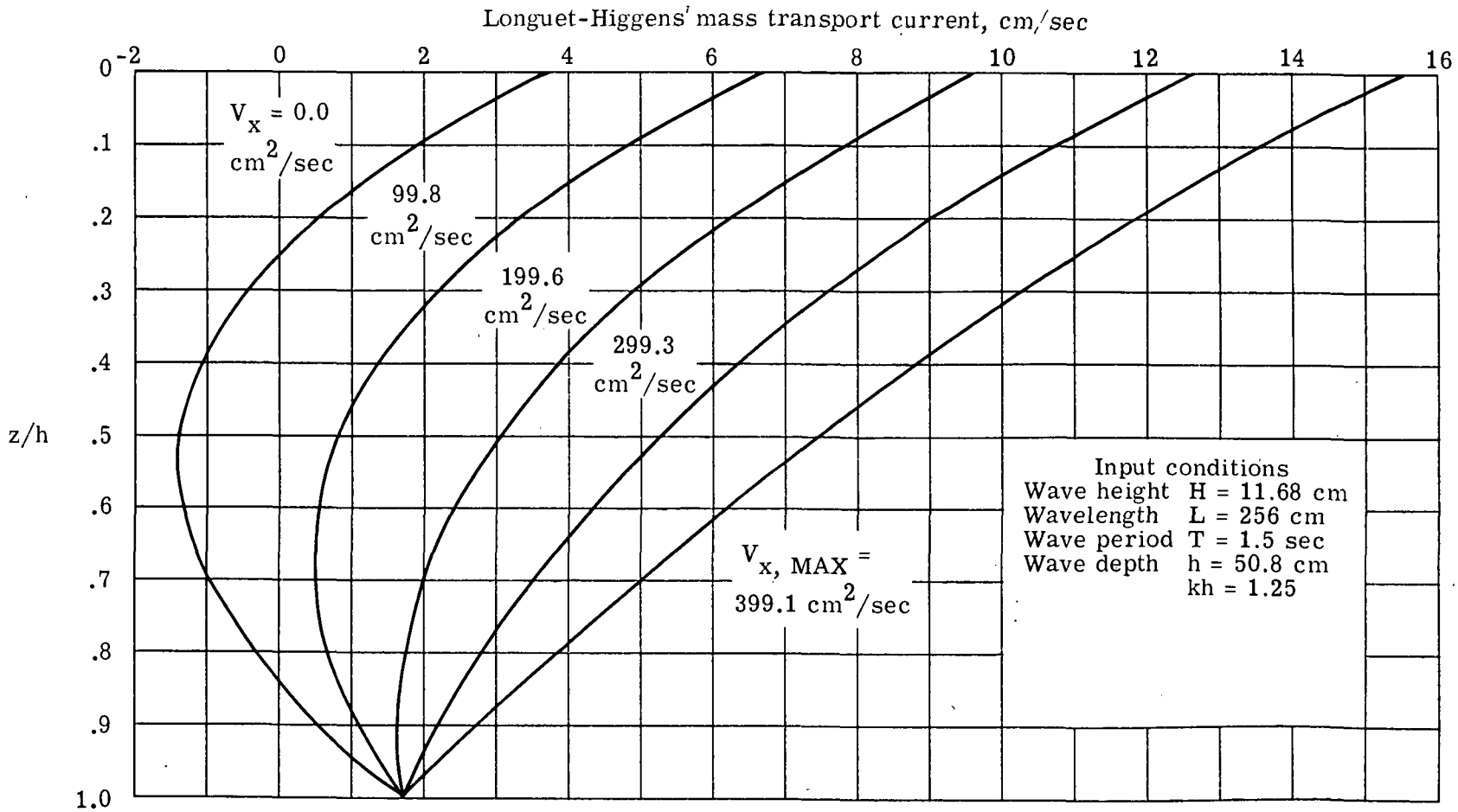


Figure 2.- Longuet-Higgens' mass-transport current plotted against nondimensional depth (with and without net volume transport V_x of water column) using equation (10). Surface and bottom boundary layers are not shown.

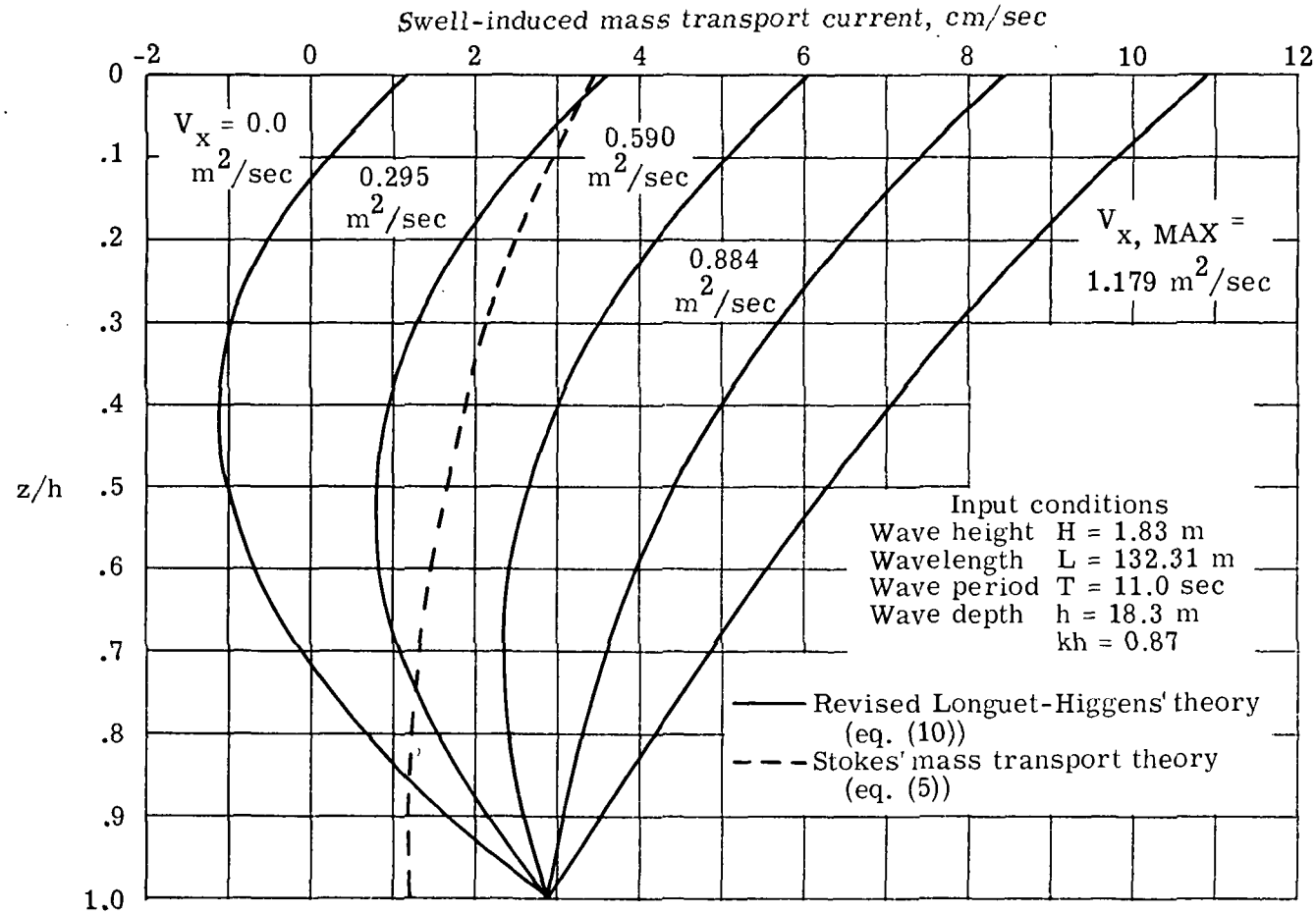


Figure 3.- Swell-induced mass-transport current plotted against nondimensional depth $\left(V_x \text{ variable}\right)$. Surface and bottom boundary layers are not shown.

NASA-Langley, 1975 L-10368

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546

OFFICIAL BUSINESS
PENALTY FOR PRIVATE USE $300

SPECIAL FOURTH-CLASS RATE
BOOK

POSTAGE AND FEES PAID
NATIONAL AERONAUTICS AND
SPACE ADMINISTRATION
451

U.S.MAIL

POSTMASTER: If Undeliverable (Section 158 Postal Manual) Do Not Return

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546