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THE PLANAR DYNAMICS OF AIRSHIPS

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ABSTRACT: This paper will consider the forces and moments acting upon a LTA vehicle in order to develop parameters describing planar motion. Similar expressions for HTA vehicles will be given to emphasize the greater complexity of aerodynamic effects when buoyancy effects cannot be neglected. A brief summary is also given of the use of virtual mass coefficients to calculate loads on airships.

SYMBOLS

C_D	Drag coefficient
C_m	Pitching moment coefficient, $M_y/QS\ell$
C_{mq}	$\partial C_m/\partial (q\ell/2V)$
$C_{m\dot{q}}$	$\partial C_m/\partial (\dot{q}\ell^2/2V^2)$
$C_{m\alpha}$	$\partial C_m/\partial \alpha$
$C_{m\dot{\alpha}}$	$\partial C_m/\partial (\dot{\alpha}\ell/2V)$
C_z	Normal force coefficient, F_z/QS
C_{zq}	$\partial C_z/\partial (q\ell/2V)$
$C_{z\dot{q}}$	$\partial C_z/\partial (\dot{q}\ell^2/2V^2)$
$C_{z\alpha}$	$\partial C_z/\partial \alpha$
$C_{z\dot{\alpha}}$	$\partial C_z/\partial (\dot{\alpha}\ell/2V)$
D	Drag force
g	Gravitational acceleration

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$\bar{g}$	$g\,(1 - 1/S)$
Iy	Transverse moment of inertia
Ky	Transverse radius of gyration, $\sqrt{Iy/m\ell^2}$
L	Body length
ℓ	Reference length, body length
My	Pitching moment
m	Body mass
Q	Dynamic pressure, $1/2\rho Vo^2$
S	Reference area, $V^{2/3}$
s	Airship density to medium density, ρ_b/ρ
Vo	Airship speed
V	Airship volume
Xe,Ye,Ze	Inertial axes
X,Y,Z	Body axes
Z	Normal force
α	Angle of attack
θ	Angle of pitch
ϕ	Velocity potential
ρ	Density

INTRODUCTION

In studies of the dynamics of Heavier Than Air (HTA) vehicles, effects due to buoyancy are almost invariably neglected. Sustaining force is the result of relative motion existing between the HTA vehicle (or at least some portion of the vehicle) and the surrounding air mass. In short, the lift or sustaining force associated with HTA craft is entirely dynamic.

A somewhat reverse situation exists in the case of Lighter Than Air (LTA) craft. The principal sustaining force comes from buoyancy, with perhaps a small additional force (about 10 percent) available under some conditions from dynamic lift. To put the comparison between LTA and HTA craft on at least a semiquantitative basis, it is convenient to define a relative density parameter, s, as

$$S = \frac{\rho_b}{\rho} \tag{1}$$

It may be seen that s is of O (1) for a LTA vehicle, while for a HTA s is no less than O (10^{+2}) and for most cases O (10^{+4}).

In addition to buoyancy playing an essential role in LTA dynamics, there are in addition dynamic effects which for convenience might be lumped in the terms virtual mass. Such dynamic effects are taken to mean forces and moments arising from (and hopefully linear with) angular rate or linear acceleration. These virtual mass effects are essentially reactive forces and moments caused by imparting an angular velocity and a linear and angular acceleration to the surrounding air. Like buoyancy these virtual effects are usually neglected for HTA craft; for LTA vehicles, however, such effects form an essential part of the loads acting on the craft. Thus such effects enter prominently into any considerations of stability.

No originality is claimed in the following development of either the mathematical model of planar dynamics or the subsequent load calculation methods. The equations of planar motion originated with ballisticians such as Murphy [1]. However, because of the negligible effect of buoyancy, great simplifications are possible in the aeroballistic formulation. As will be shown, the airship equations are far more complex. The load calculation techniques follow from Bryson [2] originally and have been presented by Nielson [3]. Again these methods are applied to LTA vehicles rather than the HTA missiles which were the original motivation for Bryson's work.

DYNAMICS OF PLANAR MOTION

Consider an airship undergoing planar motion as illustrated in Figure (1) below

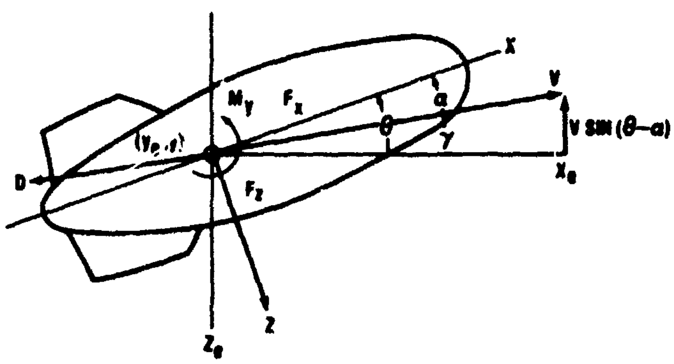


FIG. 1 FORCES AND MOMENTS ACTING ON AIRSHIP

The axes Xe,Ye,Ze are the inertial axes, while X,Y,Z are body-fixed axes. The equations of planar motion are the forces along axes Xe,Ze and the moment about axis Ye. Note that because of the definition of planarity axis Ye is identical to axis Y.

The moment and two force equations may be written as

$$m V \mathrm{Cos}(\theta-\alpha) = F_x \mathrm{Cos}\theta \cancel{x} - D \qquad (2a)$$

$$m\ddot{z}_e = F_z \cos\theta - F_x \sin\theta + mg\left(1 - \frac{1}{s}\right) \qquad (2b)$$

$$I\ddot{\theta} = M_y \qquad (2c)$$

where s is the relative density parameter of equation (1). In addition to the three load equations above, figure (1) also provides the following kinematic relationship:

$$\dot{z}_e = -V\sin(\theta - \alpha) \qquad (3a)$$

which gives upon differentiation

$$\ddot{z}_e = -\dot{V}\sin(\theta-\alpha) - V\cos(\theta-\alpha)[\dot{\theta} - \dot{\alpha}] \qquad (3b)$$

Under the assumption that the X axis does not greatly vary from the horizontal, Xe, it is possible to restrict θ and α to small angles. Subject to such small angle restrictions equations (2) and (3) become:

$$m\dot{V} = -D = F_x \qquad (4a)$$

$$m\ddot{z}_e = F_z - F_x\theta + mg(1 - 1/s) \qquad (4b)$$

$$I_y\ddot{\theta} = I_y\dot{q} = M_y \qquad (4c)$$

$$\dot{z}_e = -V(\theta - \alpha) \qquad (4d)$$

$$\ddot{z}_e = -\dot{V}(\theta - \alpha) - V(\dot{\theta} - \dot{\alpha}) \qquad (4e)$$

A first step might be the substitution of equation (4e) into equation (4b) to give:

$$-m[\dot{V}(\theta-\alpha) + V(\dot{\theta} - \dot{\alpha}) = F_z - F_x\theta + mg\left(1 - \frac{1}{s}\right) \qquad (5)$$

Equation (4a) may now be used to eliminate $\dot{V}$ in the above expression resulting in:

$$D(\theta - \alpha) - mV(\dot{\theta} - \dot{\alpha}) = F_z + D\theta + mg\left(1 - \frac{1}{s}\right) \qquad (6)$$

The above expression may be altered by introducing the following non-dimensional force coefficients

$$C_D = D\left(\tfrac{1}{2}\rho V_0^2 S\right)^{-1} \qquad C_z = F_z\left(\tfrac{1}{2}\rho V_0^2 S\right)^{-1} \qquad (7)$$

The coefficient C_z may be expanded in a Taylor series as

$$C_z = C_{z_0} + C_{z_\alpha}\alpha + C_{z_{\dot{\alpha}}}\left(\frac{\dot{\alpha} l}{2V}\right) + C_{z_q}\left(\frac{q l}{2V}\right) + C_{z_{\dot{q}}}\left(\frac{\dot{q} l^2}{2V^2}\right) \qquad (8)$$

Equation (6) may now be written in terms of C_z and C_D as

$$\left(\frac{\rho S l}{2m}\right)C_D(\theta - \alpha) - \left(\frac{q l}{2V}\right) + \left(\frac{\dot{\alpha} l}{2V}\right) = \left[C_{z_0} + C_{z_\alpha}\alpha + C_{z_{\dot{\alpha}}}\left(\frac{\dot{\alpha} l}{2V}\right)\right]\frac{\rho S l}{2m}$$
$$\frac{\rho S l}{2m}\left[+C_{z_q}\left(\frac{q l}{2V}\right) + C_{z_{\dot{q}}}\left(\frac{\dot{q} l}{2V}\right) + C_D\theta\right] + g\left(1 - \frac{1}{s}\right)\frac{l}{V^2} \qquad (9)$$

It is now possible to simplify equation (9) somewhat by the following redefinitions:

$$C_z^* = C_z\left(\frac{\rho S l}{2m}\right); \quad C_D^* = C_D\left(\frac{\rho S l}{2m}\right); \quad \tilde{g} = g\left(1 - \frac{1}{s}\right) \qquad (10)$$

Equation (10) now allows equation (9) to be rewritten and then rearranged as

$$\left(\frac{\dot{q}\ell^2}{2V^2}\right)C^*_{z_{\dot{q}}} + \left(\frac{q\ell}{2V}\right)[1+C^*_{z_q}] = \left(\frac{\dot{\alpha}\ell}{2V}\right)[1-C^*_{z_{\dot{\alpha}}}] - \alpha(C^*_D + C^*_{z_\alpha}) - C^*_{z_0} - \bar{g}\,\ell V^{-2} \quad (11)$$

Equation (4c), the moment equation, may be written as:

$$\dot{q} = \left(\frac{\rho S\ell V^2}{2I_y}\right)C_m = \left(\frac{V}{\ell}\right)^2\left(\frac{\rho S\ell}{2m}\right)\left(\frac{m\ell^2}{I_y}\right)C_m \quad (12)$$

where M_y has been replaced by C_m $(\rho s\ell V^2/2)$. Again replacing C_m by a Taylor series in α, $\dot{\alpha}$, q, and $\dot{q}$ and using the starred quantities gives for equation (12):

$$\left(\frac{\dot{q}\ell^2}{2V^2}\right)[1-K_y^{-2}C^*_{m_{\dot{q}}}] - \left(\frac{q\ell}{2V}\right)K_y^{-2}C^*_{m_q} - \left(\frac{\dot{\alpha}\ell}{2V}\right)K_y^{-2}C^*_{m_{\dot{\alpha}}} - \alpha K_y^{-2}C^*_{m_\alpha} = K_y^{-2}C^*_{m_0} \quad (13)$$

Equations (11) and (13) are now the basic equations of planar motion. The final goal remains to eliminate one of the variables between these simultaneous equations. For the present purposes the variable q will be eliminated and a single differential equation of motion in α will be written. As might be expected, this single equation is quite complicated. Before presenting this dynamic equation in a tractable form, an outline of the procedure will be given. A fairly straightforward approach is to eliminate $\dot{q}$ between equations (11) and (13). The resulting equation containing q, α, and $\dot{\alpha}$ is then differentiated to give an expression in $\dot{q}$, α, $\dot{\alpha}$, and $\ddot{\alpha}$. Returning to equations (11) and (13), eliminating this time q between them now provides a second expression $\dot{q}$, α, $\dot{\alpha}$, and $\ddot{\alpha}$. Elimination of $\dot{q}$ between these equations gives the single dynamic equation in α, $\dot{\alpha}$, and $\ddot{\alpha}$. In carrying out the above manipulation it is necessary to perform the differentiation of (ℓ/V). This operation may be written as,

$$\frac{d}{dt}\left(\frac{\ell}{V}\right) = \ell\frac{d}{dt}V^{-1} = -\frac{\ell}{V^2}\frac{dV}{dt} = -\frac{\ell}{V^2}\left(-\frac{C_D\rho S V^2}{2m}\right) = C^*_D \quad (14)$$

The single equation in α that will represent dynamic planar motion will be written as:

$$\ddot{\alpha} + H_1\left(\frac{V}{\ell}\right)\dot{\alpha} - M_1\left(\frac{V}{\ell}\right)^2\alpha = A_1\left(\frac{V}{\ell}\right)^2 + G_1 \quad (15)$$

where

$$H_1 = \left[\frac{[1-K_y^{-2}C^*_{m_{\dot{q}}}][C^*_D + C^*_{z_\alpha}] + (1+C^*_{z_q})K_y^{-2}C^*_{m_{\dot{\alpha}}} + (1-C^*_{z_{\dot{\alpha}}})K_y^{-2}C^*_{m_q} + K_y^{-2}C^*_{z_{\dot{q}}}C^*_{m_\alpha}}{K_y^{-2}C^*_{z_{\dot{q}}}C^*_{m_{\dot{\alpha}}} - (1-C^*_{z_{\dot{\alpha}}})[1-K_y^{-2}C^*_{m_{\dot{q}}}]}\right] \quad (16a)$$

(16b)

$$M_1 = \left[\frac{\{[1-K_y^{-2}C^*_{m_{\dot{q}}}][C^*_D + C^*_{z_\alpha}] + K_y^{-2}C^*_{m_\alpha}C^*_{z_{\dot{q}}}\}C^*_D + (C^*_D + C^*_{z_\alpha})K_y^{-2}C^*_{m_q} - (1+C^*_{z_q})K_y^{-2}C^*_{m_\alpha}}{K_y^{-2}C^*_{z_{\dot{q}}}C^*_{m_{\dot{\alpha}}} - (1-C^*_{z_{\dot{\alpha}}})[1-K_y^{-2}C^*_{m_{\dot{q}}}]}\right] \quad (16c)$$

$$A_1 = \left[\frac{\{K_y^{-2}C^*_{z_{\dot{q}}}C^*_{m_0} + C^*_{z_0}[1-K_y^{-2}C^*_{m_{\dot{q}}}]\}C^*_D + K_y^{-2}C^*_{m_q}C^*_{z_0} - (1+C^*_{z_q})K_y^{-2}C^*_{m_0}}{K_y^{-2}C^*_{z_{\dot{q}}}C^*_{m_{\dot{\alpha}}} - (1-C^*_{z_{\dot{\alpha}}})[1-K_y^{-2}C^*_{m_{\dot{q}}}]}\right]$$

$$G_1 = \left[\frac{\bar{g}}{\ell} \left[\frac{K_y^{-2} C^*_{m_q} - C_D^* [1 - K_y^{-2} C^*_{m_{\dot q}}]}{K_y^{-2} C^*_{m_{\dot\alpha}} C^*_{m_q} - [1 - K_y^{-2} C^*_{m_{\dot q}}][1 - C^*_{z_{\dot\alpha}}]} \right] \right] \tag{16d}$$

Before attempting to simplify equations (16) it is necessary to reduce equation (15) to an equation with constant coefficients as the presence of (V/ℓ) introduces time into the coefficients. This may be accomplished by writing

$$\dot\alpha = \frac{d\alpha}{dt} = \frac{d\alpha}{dx}\frac{dx}{dt} = \frac{d\alpha}{d(x/\ell)}\frac{1}{\ell}\frac{dx}{dt} = \alpha' \left(\frac{V}{\ell}\right) \tag{17a}$$

where differentiation has been changed from time, t, to a non-dimensional arc length, (X/ℓ). In a similar fashion $\ddot\alpha$ may be written in terms of (X/ℓ) as

$$\ddot\alpha = \left(\frac{V}{\ell}\right)^2 \alpha'' - \left(\frac{V}{\ell}\right)^2 C_D^* \alpha' \tag{17b}$$

By replacing time derivatives by arc-length derivatives, equation (15) now becomes a second order constant coefficient equation:

$$\alpha'' + (H_1 - C_D^*)\alpha' - M_1\alpha = A_1 + G_1 \left(\frac{\ell}{V_0}\right)^2 \tag{18}$$

Admittedly, equations (16) are quite complex; for certain applications such as aeroballistics, great simplifications may be made.

However, before considering this aspect of the problem, the conditions for stability of motion will be examined.

STABILITY CONSIDERATIONS

Equation (18) may be rewritten as

$$\alpha'' + 2\lambda\alpha' + \omega_n^2 \alpha' = \omega_n^2 \alpha_0^A + \omega_n^2 \alpha_0^g \tag{19}$$

where

$$\lambda = \frac{H_1 - C_D^*}{2} \tag{20a}$$

$$\omega_n^2 = -M_1 \tag{20b}$$

$$\alpha_0^A = -\frac{A_1}{M} \tag{20c}$$

$$\alpha_0^g = -\frac{G_1}{M_1}\left(\frac{\ell}{V_0}\right)^2 \tag{20d}$$

The term, λ, is the damping factor of the airship, the term, ω_n, is the undamped natural frequency. Only for small values of λ does the body oscillate at this frequency; in the presence of a significant amount of damping the planar oscillatory frequency, ω_d, is less than the undamped frequency, ω_n. The damped planar frequency, ω_d, can be expressed in terms of λ and ω_n as

$$\omega_d = \sqrt{\omega_n^2 - \lambda^2} = \omega_n \sqrt{(1 - \lambda^2/\omega_n^2)} \tag{21}$$

The term α_0^A in equation (20c) is the trim angle of attack due to aerodynamic asymmetries, while α_0^g in equation (20d) is the trim angle of attack due to gravitational path curvature. With regard to the term α_0^g, it might be of interest to note that if the airship is neutrally buoyant, i.e. s = 1, then from equation (10) $\dot g$ = o and hence from equation (20d), $\alpha_o^{\prime g}$ must be zero.

There are two conditions for oscillatory motion. These are:

$$\lambda < \omega_n \tag{22a}$$

and

$$\omega_n^2 > 0 \tag{22b}$$

The former condition allows us to write for the equality $\lambda = \lambda_c = \omega_n$,

$$\omega_d = \omega_n \sqrt{[1 - (\lambda/\lambda_c)]} \tag{23a}$$

where (λ/λ_c) is often called the damping ratio.

The condition for oscillatory motion given in equation (22b) is that ω_n is real which, in turn, from equation (20b) requires that

$$M_1 < 0 \tag{23b}$$

Under the condition where equations (22b) and (23b) are satisfied, stability (subsident motion) requires that

$$2\lambda \equiv [H_1 - C_D^*] > 0 \tag{24a}$$

$$\omega_n^2 = -M_1 > 0 \tag{24b}$$

Thus to assess dynamic and static stability (equations (24a,b) respectively) it is necessary to assign numerical values to the derivatives contained in equations (16a) and (16b). Numerical values for these terms are contained in Table I. While these values may vary with airship dimensions, they have been computed for the airship shown in Figure (2) below. An outline of the computational technique is given subsequently.

$$2\lambda = [H_1 - C_D^*]$$

$$\frac{[1 - K_y^{-2} C_{m_{\dot\gamma}}^*][C_D^* + C_{z_\alpha}^*] + (1 + C_{z_\gamma}^*) K_y^{-2} C_{m_{\dot\alpha}}^* + (1 - C_{z_{\dot\alpha}}^*) K_y^{-2} C_{m_\gamma}^* + K_y^{-2} C_{z_{\dot\gamma}}^* C_{m_\alpha}^*}{K_y^{-2} C_{z_{\dot\gamma}}^* C_{m_{\dot\alpha}}^* - (1 - C_{z_{\dot\alpha}}^*)[1 - K_y^{-2} C_{m_{\dot\gamma}}^*]}$$

$$- C_D^*$$

Inserting values from Table I gives,

$$\frac{[1+2.98][.044+.049]+(1+0)(-2.31)+(1+2.30)(2.31)+(22.36)(-.119)}{(-2.31)(-.119)-(1+2.297)(1+2.98)}$$

$$-.0437$$

$$-.234-.0437=-.278$$

Quite obviously the inequality of equation (24a) is not met so the airship is not dynamically stable.

In considering the oscillatory frequency relationship for static stability (equation (24b)) we may write:

$$M_1=\frac{\{(1-K_y^{-2}C^*_{m\dot{q}})(C^*_D+C^*_{z\dot{\alpha}})+K_y^{-2}C^*_{z\dot{q}}C^*_{m\dot{\alpha}}\}C^*_D+(C^*_D+C^*_{z\alpha})K_y^{-2}C^*_{mq}-(1+C^*_{zq})K_y^{-2}C_{m\alpha}}{K_y^{-2}C^*_{m\dot{\alpha}}C^*_{z\dot{q}}-(1-C^*_{z\dot{\alpha}})[1-K_y^{-2}C^*_{m\dot{q}}]}$$

It can readily be shown that since the term in the braces is multiplied by C^*_D it is rather small. This allows the above expression to be numerically evaluated as

$$M_1 = 1.73$$

Obviously the second condition of equation (24b) is not met.

It might be expected that numerical values of the stability derivatives would vary from airship to airship. However, it would appear that no general simplifications may be made in the H_1 and M_1 coefficients except to omit terms multiplied by C^*_D. The results seem to indicate that for a satisfactory description of planar dynamics it is necessary to calculate the eight stability derivatives of Table I. Drag, as we have noted, is relatively unimportant for estimating the planar dynamics.

In passing it might be of interest to examine the equivalent expressions for H_1, M_1, A_1, and G_1, which are satisfactory for an HTA vehicle. If quantities such as $C^*_{m\dot{q}}$, $C^*_{z\dot{q}}$, C^*_{zq}, and $C^*_{z\dot{\alpha}}$ are ignored along with the product of starred quantities one, has

$$H_1=-[C^*_{z\alpha}+K_y^{-2}(C^*_{m\dot{\alpha}}+C^*_{mq})] \tag{25a}$$

$$M_1 = K_y^{-2}C^*_{m\alpha} \tag{25b}$$

$$A_1 = K_y^{-2}C^*_{m_D} \tag{25c}$$

$$G_1 = -[(g/\ell)K_y^{-2}C^*_{mq}] \tag{25d}$$

Quite clearly the criteria of equations (24) are met when $C^*_{mq}+C^*_{m\dot{\alpha}}$ and $C^*_{m\alpha}$ are negative for the HTA vehicle. An examination of equations (16a) and (16b) quickly show that dynamic stability cannot depend upon such simple criteria in the case of an LTA vehicle: stability considerations are far more complex for the LTA vehicle.

For a typical airship we have seen that the motion consists of one exponentially undamped mode and one exponentially damped mode since from equation (19)

$$\lambda_{1,2} = -\lambda \pm \sqrt{\lambda^2 - \omega_n^2} = -\lambda \pm \sqrt{\lambda^2 + M_1} \tag{26}$$

and using $\lambda = -.139$ and $M_1 = 1.73$ we obtain

$$\lambda_1 = .479 \qquad \lambda_2 = -1.18$$

As is well known, the fixed-wing HTA vehicle usually evidences two damped oscillatory modes.

CALCULATION OF AERODYNAMIC LOADS

A fairly straightforward method of calculating static and dynamic loads on an airship is the method of virtual mass. While this technique has its origin in the work of nineteenth-century hydrodynamicists, it has been applied with some success by Bryson[2] to HTA vehicles. Since space limitations do not permit even an outline of the derivation, reference should be made to either Bryson's work[2] or the more readable treatment of Nielson[3].

Through the use of this virtual mass technique it may be shown that the derivatives used in the previous expressions for H_1 and M_1 are given as,

$$C_{z_\alpha} = 2BC_D^* - 2A_{11} \tag{27a}$$

$$C_{z_{\dot\alpha}} = -4B_{11} \tag{27b}$$

$$C_{z_q} \doteq 4\bar{A}_{11}(x/\ell)_b \tag{27c}$$

$$C_{z_{\dot q}} = 4C_{11} \tag{27d}$$

$$C_{m_\alpha} = -2C_{11}C_D^* + 2\left(\frac{x}{\ell}\right)_b \bar{A}_{11} + 2B_{11} \tag{27e}$$

$$C_{m_{\dot\alpha}} = 4C_{11} \tag{27f}$$

$$C_{m_q} = -4\left(\frac{x}{\ell}\right)_b \bar{A}_{11} - 4C_{11} \tag{27g}$$

$$C_{m_{\dot q}} = -4D_{11} \tag{27h}$$

where

$$A_{11} = \pi\, \overline{a(x)}^2 / S \tag{28a}$$

for body-alone and that

$$A_{11} = \pi\, \overline{s(x)}^2 \left[1 - \frac{\overline{a(x)}^2}{\overline{s(x)}^2} + \frac{\overline{a(x)}^4}{\overline{s(x)}^4}\right] \tag{28b}$$

for the body in the presence of fins. a(x) is the body radius as a function of body station and s(x) is fin span (center-line to tip) as a function of body station. In addition B_{11}, C_{11} and D_{11} are defined as

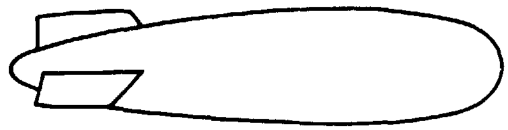
$$B_{11} = \int_{(x/\ell)_b}^{(x/\ell)_n} A_{11}\left(\frac{x}{\ell}\right) d\left(\frac{x}{\ell}\right) \tag{29a}$$

$$C_{11} = \int_{(x/\ell)_b}^{(x/\ell)_n} \left(\frac{x}{\ell}\right) A_{11}\, d\left(\frac{x}{\ell}\right) \tag{29b}$$

$$D_{11} = \int_{(x/\ell)_b}^{(x/\ell)_n} \left(\frac{x}{\ell}\right)^2 A_{11}\, d\left(\frac{x}{\ell}\right) \tag{29c}$$

when "n" and "b" refer to nose and base respectively.

The above integrals have been evaluated numerically for the airship shown in Figure (2) from tabular values of a(x) and s(x). The calculations of equations (27) were carried out to give the results shown in Table I.



LENGTH (FT)	DIAMETER (FT)	VOLUME (FT3)	MOM. OF INERTIA (SLUG-FT2)
517	120	$4.4 \times 10^{+6}$	$1.42 \times 10^{+8}$

FIG. 2 REPRESENTATIVE AIRSHIP

TABLE I

C_{z_α}	$C_{z_{\dot\alpha}}$	C_{z_q}	$C_{z_{\dot q}}$	C_{m_α}	$C_{m_{\dot\alpha}}$	C_{m_q}	$C_{m_{\dot q}}$
.03165	-1.458	10^{-9}	-.0756	.7305	-.0756	+.0756	-.09725

Also calculated in the program is the airship volume $V = 4.4158 \times 10^{+6}$. Assuming neutral buoyancy it can be shown that $(\rho SL/2m)$, equals 1.57, which together with values in Table I allows the starred derivatives (equation (10)) to be calculated.

CONCLUSION

This paper has taken a brief look at the hydrodynamic complexities of planar dynamics of airships. It has been shown that the equations of motion for a LTA vehicle are far more complex than the corresponding equations of a HTA vehicle. A method has been presented for calculating all loads (except drag) acting on a moving airship.

REFERENCES:

1. Murphy, C. H., "Free Flight Motion of Symmetric Missiles," BRL Report 1216, Ballistic Research Laboratories (July 1963).

2. Bryson, A. E., "Stability Derivatives for a Slender Missile with Application to Wing-Body-Vertical-Tail Configurations," Journal of the Aeronautical Sciences, Vol. 20, No. 5, pp. 297-308 (1953).

3. Nielson, J. N., Missile Aerodynamics, McGraw-Hill Publishing Co. (1960).