ABSTRACT: This paper will deal with two computer aided design methods for the design and construction of strong, lightweight structures which require complex and precise geometric definition. The first, flexible structures, is a unique system of modeling folded plate structures and space frames. In the latter it is possible to continuously vary the geometry of a space frame to produce large, clear spans with curvature. The second method deals with developable surfaces where both folding and bending are explored with the observed constraint of available building materials and what minimal distortion would result in maximum design capability. We are developing alternative inexpensive fabrication techniques to achieve computer defined enclosures which are extremely lightweight and mathematically highly precise.

Folded Plate Systems

My discovery of kinematic folded plate systems, which I term folded mosaic structures, began some twelve years ago with a curiosity about the dynamic behavior of a crumpled wad of paper. An extended observation led me to develop an operational procedure for diagramming the bounding edges of what appeared to be the essential plates involved in the formation of an individual wrinkle. I have created diagrams of folded plate patterns which I subsequently integrated into continuous patterns by the use of symmetry operations.

Hundreds of these patterns have been made and investigated. The unique property of each is that, by allowing only folding of the sheet along the lines of the pattern, a flat sheet may be transformed into a variety of three dimensional shapes, Figure 1. These include domes, warped surfaces, and complex shells involving both. It is also possible to create structures which envelope a space by closing back on themselves. An additional feature of each pattern is that the entire system is composed of the repetition of a small number of non-identical plates. For example, the pattern shown has as few as two unique triangles generating the entire system, Figure 2.

* Associate Research Professor, Computer Science, University of Utah, Salt Lake City, Utah, U.S.A.
Although there has been some formal study in the area of kinematics -- namely, mechanics and the kinematics of machine -- there has rarely been an exploration of geometric systems of the type and to the extent of those I have conducted. This work is unique in its discovery, and in its development of new kinematic systems. My initial interest was only to examine what sort of total system behavior results when a specific configuration of geometry is brought together.

Early investigation was directed toward discovery of new patterns and observations of how each moved. This was done simply by folding up large sheets of paper on which a pattern had been scribed. The model was then moved by hand to change it from one shell form into another, Figure 3. The inherent beauty of these forms, and the facility with which one could directly change them at will, immediately gripped the imagination. The design potential for creating lightweight architectural shells, or other three dimensional enclosures, was more than apparent.

Larger and larger models were made, first of paper and then of cardboard, and scored and folded by hand. As study progressed I began to use computers, and to develop computer aided design techniques for observing the behavior of three dimensional structures, by creating simulated images, developing shading techniques, and investigating structural analysis.

We can now fold these patterns in such a way that almost any surface shape, that a designer can specify as an enclosure, can be constructed as a precise folded plate shell form. While this was always possible to demonstrate empirically, a precise calculation of the three dimensional geometry was not possible until 1971 when I collaborated with Professor Hank Christiansen, a structural analyst, who wrote a kinematic analysis program for this purpose. The computer aided design techniques achieve a series of versatile structural systems which are capable of producing an infinite variety of enclosure shapes.

**Computer Aided Structural Analysis**

Initial work in computer simulated structural analysis is complete on these systems. We are able to show, by computer simulated color photographs, the stress distribution throughout the structure. We have developed these versatile geometric systems by producing drawings, diagrams, and three dimensional models using computer assisted design techniques. Under computer simulation one can continuously change the plate geometry, Figure 6, make a selection of a specific arrangement of plates, and then continuously fold them for study and selection of some desired form of single curvature, Figure 7. The plates may also be folded to achieve an approximation to a doubly curved, or warped surface, Figure 8. An arrangement of a series of these folded structures would be suitable for creating the envelope of a rigid airship. As well as being both lightweight and strong, the modular foldings would lend themselves to economic mass production techniques.

**Curved, Plate Truss Structures**

The folded plate systems can also produce space frame structures. With these it is possible to continuously vary the geometry of the space frame to produce structures other than the usual flat, or occasional geodesic, types. Structures which require large, clear spans, such as airport hangars, are usually accommodated by the standard flat octet space truss, to which current methods of design and construction are limited.
There is an obvious need for clear span trusses which have some curvature. To achieve flexibility in the design and the construction of such structures, we have completed a working computer program which allows the specification of any surface of revolution. It will then construct a truss on top of that surface, the depth of which may also be specified, and it will output a control tape for the creation of all the plates of the given structure. Figure 9 is a photograph of actual models, showing the standard truss at the bottom, with two trusses of increasing curvature above.

The Developable Surface Program

The aerospace industry has brought a growing need for strong, lightweight structures which require complex and precise geometric definition. The usual solution has been by costly numeric controlled milling of solid blocks to achieve these required structures. Our work is attempting to develop alternative inexpensive fabrication techniques to achieve computer defined space forms.

It is well understood that to fold a metal plate along a straight line strengthens it; and that bending it to some radius of curvature will increase its structural stability. I have observed that one can combine these two structural properties by introducing a curved, folded edge to a plate. From this basic structural observation we have created a Developable Surface computer program to allow completely general design freedom. It was not at all apparent at the outset, however, that one could generalize a folded edge to any space curve. A thorough mathematical analysis revealed that such a generalization was possible. With this determined, we developed the program.

From this research we have developed generic systems and construction techniques which have the following potential applications, and are beginning to produce numeric controlled engraving and fabrication of folded metal prototypes for same:

1. Airship envelopes
2. Curved, clear span structures
3. Solar energy reflectors
4. Liners for liquid natural gas tankers
5. Lightweight gas tanks for airplane wings
6. Concrete formwork for space curve structures
7. Lightweight guideways for rapid transit monorails
8. Lightweight complex bridge interchanges

A controlled, curved surface, or pathway, can be achieved by declaring the space curve to be a folded edge defining two developable surfaces. This program makes the ordinarily difficult task of physical construction of a precise, complex space curve, relatively simple and direct, while using flat sheet materials and requiring limited joining. Figures 10 and 11 are photographs of an actual model of a complex structure defined by the Developable Surface program. Additional typical forms created with this program can be seen as a part of the film presentation of this paper as computer simulated color video pictures.
Ruled Surface Program, An Approximation to Warped Surfaces

This program constructs a triangular network in a zig-zag manner between the alternate points on two space curves. The curves are definable in the same ways as the curve in the developable surface program. The triangular network may be flattened out to form a flat network, and numeric controlled tapes and pictures of this are available, as well as plots and display pictures of the three dimensional objects. Several networks may also be found and displayed at the same time.

Hyperbolic paraboloids have been extensively used in architecture, for example, because they are both elegant and structurally efficient. They suffer, however, from demanding difficult and expensive formwork. The ruled surface program allows us to directly build any hyperbolic paraboloid by triangular approximation.

These are a few of a number of techniques we have developed for the definition and construction of extremely lightweight and mathematically controlled surfaces and enclosures.
FIG. 4

FIG. 5

REPRODUCIBILITY OF THE ORIGINAL PAGE IS POOR
Reproducibility of the original page is poor.