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VISCOS/POTENTIAL FLOW ABOUT MULTI-ELEMENT TWO-DIMENSIONAL AND INFINITY-SPAN SWEPT WINGS: THEORY AND EXPERIMENT

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Abstract

The viscous subsonic flow past two-dimensional and infinite-span swept multi-component airfoils is studied theoretically and experimentally. The computerized analysis is based on iteratively coupled boundary-layer and potential-flow analysis. The method, which is restricted to flows with only slight separation, gives surface pressure distribution, chordwise and spanwise boundary-layer characteristics, lift, drag, and pitching moment for airfoil configurations with up to four elements. Merging confluent boundary layers are treated. Theoretical predictions are compared with an exact theoretical potential flow solution and with experimental measures made in the Ames 40- by 80-Foot Wind Tunnel for both two-dimensional and infinite-span swept wing configurations. Section lift characteristics are accurately predicted for zero and moderate sweep angles where flow separation effects are negligible.

Notation

A = aerodynamic influence coefficient matrix
a = aerodynamic influence coefficient
b = wing span, m
C_d = section drag coefficient = section drag/(
(\frac{1}{2} \rho U^2 C_o)
)
C_d_1 = wing induced drag coefficient = wing drag/(
(\frac{1}{2} \rho U^2 C_o b)
)
C_f = local skin friction coefficient
C_L = wing lift coefficient = wing lift/(
(\frac{1}{2} \rho U^2 C_o b)
)
C_t = section lift coefficient on wing centerline = section lift/(
(\frac{1}{2} \rho U^2 C_o \cos^2 \beta)
)
C_o = reference chord, m
C_p = pressure coefficient = (p - p_o)/\left(\frac{1}{2} \rho U^2 \cos^2 \beta\right)
D = minimum distance between adjacent elements
H = shape factor = \delta / \delta^*
R_k = component of freestream velocity normal to panels of kth component
p = static pressure, kN/m²
q = source strength
r = viscous/potential flow iteration relaxation factor
R = Gauss-Seidel relaxation factor
Re = Reynolds number = U C_o / \nu
R_e = Reynolds number based on momentum thickness = U \delta / \nu
s = boundary-layer coordinate along surface in chordwise direction, m
u = chordwise velocity, m/s
v = spanwise velocity, m/s
x, y = chordwise and spanwise wing coordinates, m
z = boundary-layer coordinate normal to airfoil surface, m

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The objective of the present work is to begin the extension of multi-component airfoil analysis into three-dimensional flows by considering the subsonic flow about swept wings of infinite aspect ratio. The basic analytic method is outlined in Section II. The present analysis extends the theoretical method of Dvorak and Woodward and compares predicted results with experiment. The theoretical extension of the work of Ref. 6 includes: 1) introduction of advanced potential-flow solution techniques; 2) modification of the viscous/potential flow coupling when only slight separation is present; and, 3) use of under-relaxation of surface source distributions in the iterative coupling of the viscous and potential flow solutions. In the supporting experimental program, tests of a two-element configuration were conducted in the Ames 40- by 80-Foot Wind Tunnel. The basic objective of this test was to obtain experimental data on two-dimensional and infinite aspect ratio swept-wing configurations. The experimental program is described in Section III and the results are compared with the theoretical predictions in Section IV.

II. Theoretical Method

General

The basic theoretical method is to couple iteratively potential-flow and boundary-layer analysis. The analysis is limited to flows with negligible upper-surface separation on any element. Provided there is no strong viscous interaction, limited lower surface separation closed bubbles, such as often occur in the cove on the lower surface of a wing with a slotted flap, are permitted. In the case of the infinite-span swept wing it is assumed that all spanwise gradients in the potential and viscous flows are equal to zero. Thus, in planes normal to the leading edge, the potential flow equations reduce to the two-dimensional form. For the infinite-span swept wing the chordwise boundary-layer characteristics are of primary interest. For the laminar case Jones shows that the vorticity distribution is located along each panel center, provides n equations. The additional equation required to close the system is supplied by specifying that the upper- and lower-surface velocities have a common limit at the trailing edge (i.e., the Kutta condition). This implies that the upper- and lower-surface vorticities be equal and opposite. For a unit freestream velocity the resulting system of equations to be solved for an airfoil with n panels is then

\[
\begin{align*}
\begin{pmatrix}
0 & \cdots & a_{1n} & \sin(a - \delta_1)
\end{pmatrix}
\begin{pmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_n
\end{pmatrix}
& =
\begin{pmatrix}
\sin(a - \delta_1) \\
\sin(a - \delta_2) \\
\vdots \\
\sin(a - \delta_n)
\end{pmatrix}
\end{align*}
\]

or in vector form

\[
[A][\gamma] = [\delta]
\]

where the nth column represents the combined influence coefficients for the upper- and lower-surface trailing edge vorticity panels.

Geometry Definition

Up to four elements can be analyzed, with each element represented by as many as 60 pairs of surface coordinates. Each individual slat or flap segment location is related to the main wing or reference coordinate system by pivot point coordinates prescribed in both the main element coordinate system and the individual slat or flap segment coordinate system. The selection of a rotation angle measured relative to the main component completes the specification of the element position. It is generally convenient to pivot a leading-edge device about its trailing edge and to pivot a flap about its leading edge although the hinge point of a flap could, for example, provide a ready reference point. Also, as part of the geometry definition, the flap upper-surface longitudinal radius of curvature is determined using cubic splines for later use in the finite-difference boundary-layer calculations.

Potential Flow Analysis

The potential flow analysis is performed in a plane normal to the wing leading edge. The analysis is a method of singularities where each element is represented by a closed polygon of planer panels connecting the input coordinate pairs. A linear vorticity distribution is located along each panel with the requirement that the vorticity distribution be continuous across the panel corner points. Thus, if there are n panels there are n + 1 unknowns to be determined. The boundary condition of no flow through the surface, applied at each of the panel centers, provides n equations. The additional equation required to close the system is supplied by specifying that the upper- and lower-surface velocities have a common limit at the trailing edge (i.e., the Kutta condition). This implies that the upper- and lower-surface vorticities be equal and opposite. For a unit freestream velocity the resulting system of equations to be solved for an airfoil with n panels is then

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\sin(a - \delta_2) \\
\vdots \\
\sin(a - \delta_n)
\end{pmatrix}
\]

or in vector form

\[
[A][\gamma] = [\delta]
\]

where the nth column represents the combined influence coefficients for the upper- and lower-surface trailing edge vorticity panels.

If the trailing edge closes the vorticity must go to zero because the trailing edge then becomes a stagnation point. Although this solution is automatically given by Eq. (1) the geometry is often such that the influence coefficients in the nth column are quite small with the result that the matrix is poorly conditioned. In this case an alternate Kutta condition is used which specifies that the upper and lower trailing-edge vorticity strengths are equal to zero at the trailing edge. An additional unknown is supplied by introducing a constant strength source distribution on the surface of the airfoil. The resulting system of equations is the same as Eq. (1) except for the nth column of vorticity influence coefficients. That column is replaced by the constant source-distribution influence coefficients and \(\gamma_n\) is the unknown source distribution strength.

For a multi-element configuration with j components Eq. (2) can be written

\[
\begin{pmatrix}
A_{11} & \cdots & A_{1j} & \gamma_1 \\
A_{21} & \cdots & A_{2j} & \gamma_2 \\
\vdots & \ddots & \vdots & \vdots \\
A_{n_1} & \cdots & A_{n_j} & \gamma_j
\end{pmatrix}
= \begin{pmatrix}
n_1 \\
n_2 \\
\vdots \\
n_j
\end{pmatrix}
\]

The analysis is limited to flows with negligible upper-surface separation on any element. Provided there is no strong viscous interaction, limited lower surface separation closed bubbles, such as often occur in the cove on the lower surface of a wing with a slotted flap, are permitted. In the case of the infinite-span swept wing it is assumed that all spanwise gradients in the potential and viscous flows are equal to zero. Thus, in planes normal to the leading edge, the potential flow equations reduce to the two-dimensional form. For the infinite-span swept wing the chordwise boundary-layer characteristics are of primary interest. For the laminar case Jones shows that the vorticity distribution is located along each panel center, provides \(n\) equations. The additional equation required to close the system is supplied by specifying that the upper- and lower-surface velocities have a common limit at the trailing edge (i.e., the Kutta condition). This implies that the upper- and lower-surface vorticities be equal and opposite. For a unit freestream velocity the resulting system of equations to be solved for an airfoil with \(n\) panels is then

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\begin{pmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_n
\end{pmatrix}
= \begin{pmatrix}
\sin(a - \delta_1) \\
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\begin{pmatrix}
A_{11} & \cdots & A_{1j} & \gamma_1 \\
A_{21} & \cdots & A_{2j} & \gamma_2 \\
\vdots & \ddots & \vdots & \vdots \\
A_{n_1} & \cdots & A_{n_j} & \gamma_j
\end{pmatrix}
= \begin{pmatrix}
n_1 \\
n_2 \\
\vdots \\
n_j
\end{pmatrix}
\]
Gauss-Seidel method was selected as the best iterative method for the fixed relaxation factor. For example, consider an airfoil with a single slotted flap. If the flap is moved off to infinity, the two elements are essentially uncoupled. The off-diagonal block matrices of Eq. (3) which model the component interactions, are zero and two iterations of Eq. (6) give convergence (first iteration gets the solution and the second is required only to check the first). As \(|R-1|\) increases, the number of iterations required also increases.

If the elements are located in close proximity, as is typically the case, the interaction between components become important. Again, consider the single slotted flap. Assuming the initial solution \(\gamma_1^{(1)}, k = 1,2,\ldots\) is a null vector, the final coupled solution is obtained from Eq. (6) for \(R=1\) as follows: 1) \(\gamma_1^{(1)}\) is the solution vector for the main component in free air (because the upwash field from the flap is not present, the lift level for this component will be less than that for the coupled system); 2) \(\gamma_2^{(1)}\) is the solution for the flap in the downwash field of a wing that is carrying reduced lift, relative to the coupled system (this reduced downwash field causes the flap to carry excess lift); and 3) on the next iteration the wind lift will be too high because of the excess upwash from the flap. Similarly, the flap lift on the second iteration will be below that for the correctly coupled system.

Equations (5) and (6) are solved using the Jacobi iteration, and block-Gauss-Seidel iteration with relaxation. The solution is found by starting with some assumed initial solution vector,

\[
\begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_j
\end{bmatrix}^{(0)}
\]

The \(v\)th iteration is then expressed in one of the following ways:

1. Block Jacobi iteration:

\[
\begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_j
\end{bmatrix}^{(v)} = A^{-1} k \begin{bmatrix}
\gamma_k - \sum_{i=1}^{j} A_{ki} \gamma_i^{(v-1)}
\end{bmatrix}^{(v-1)}
\]

2. Block-Gauss-Seidel iteration with relaxation:

\[
\begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\vdots \\
\gamma_j
\end{bmatrix}^{(v)} = A^{-1} k \begin{bmatrix}
\gamma_k - \sum_{i=1}^{j} A_{ki} \gamma_i^{(v-1)}
\end{bmatrix}^{(v-1)}
\]

As one might expect the optimum relaxation factor for the Gauss-Seidel method is dependent on how closely the various lifting elements are coupled. For example, consider an airfoil with a single slotted flap. If the flap is moved off to infinity, the two elements are essentially uncoupled. The off-diagonal block matrices of Eq. (3), which model the component interactions, are zero and two iterations of Eq. (6) give convergence (first iteration gets the solution and the second is required only to check the first). As \(|R-1|\) increases, the number of iterations required also increases.

The oscillatory behavior indicates that under-relaxation should accelerate the rate of convergence and the amount of under-relaxation required should increase as the elements are moved closer together. An "optimum" relaxation factor given by

\[
R = 1 - \exp(-10 \min(D_i))
\]

experiments with two and three element configurations. \(D_i\) is the minimum distance in fraction of chord between the trailing edge of the ith component and the upper surface of the following component. With this relaxation factor the number of iterations required for convergence was generally reduced by a factor of 3 below that required with \(R=1\).

Computational times required for the direct method of solution have been compared to the computational times required for block-Gauss-Seidel iteration with relaxation method. The test cases were closely coupled two-component configurations. The total number of panels used for these comparisons was approximately 100. The iterative method obtained the converged solution in approximately one-half the time required by the direct method. Increasing the number of elements, and thus the total number of equations to be solved, had no significant impact on the number of iterations required by the iterative method. Thus the advantage of the iterative method over the direct will increase as the number of elements is increased.

The pressure coefficient at the center of each panel is calculated from the surface velocity at that point. The lift and pitching-moment coefficients are obtained by integrating the pressures around the airfoil.

A comparison of the present method with the exact conformal mapping solution of Williams for...
a two element case is shown in Fig. 1. The angle of incidence is 0° and the flap deflection is 30°; agreement with the exact solution is excellent.

**Boundary-Layer Analysis**

As outlined in Fig. 2, a combination of integral and finite difference techniques are used. Integral methods are used for conventional boundary layers because of their computational efficiency whereas a finite-difference method is used for the more complex confluent boundary-layer analysis. The stagnation line, conventional boundary-layer, and confluent boundary-layer methods are discussed in the following sections.

**Stagnation Line Flow**

Cumpsty and Head\(^{10}\) experimentally investigated the stagnation-line flow characteristics of infinite-span swept wings. They found that the stagnation line boundary-layer integral parameters (H, δ, and C\(_f\)) and the state (laminar or turbulent) correlate with the parameter C\(_s\) = \(\frac{v_2}{\frac{du}{dz}}\). The stagnation line boundary-layer characteristics on each element are determined using these correlations.

**Conventional Boundary-Layer Methods**

Integral boundary-layer methods are used for all conventional boundary layers, such as the upper- and lower-surface boundary layers of all elements, and the upper-surface boundary layers of following elements up to the slot exists.

The two-dimensional equations of Curle\(^{11}\) are solved along external streamlines to determine the laminar boundary-layer characteristic. It is assumed that laminar cross-flow effects have a negligible influence on the overall calculation, at least for moderate sweep angles. Once the boundary-layer characteristic along the potential flow streamlines have been determined, the spanwise and chordwise parameters are determined.

These chordwise boundary-layer characteristics are used with the correlations of Smith\(^{12}\) to determine the point of laminar instability. These correlations relate R\(_g\) to a pressure gradient parameter. Knowing the point of instability, R\(_g\), and the pressure gradient; the location at which full transition has occurred is determined by using the correlations of Granville.\(^{13}\) The integral method of Cumpsty and Head\(^{14}\) for boundary layers on infinite-span swept wings is used for the turbulent calculation.

Should laminar separation occur prior to transition, the correlations of Gaster\(^{15}\) are used to determine whether turbulent reattachment occurs. If reattachment is predicted, the calculation continues as a turbulent flow; if catastrophic separation is predicted, the calculation is terminated.

**Confluent Boundary-Layer Method**

The finite-difference method used to solve the infinite-span swept-wing, confluent boundary-layer equations is described by Crank and Nicholson\(^{16}\) and by Dvorak and Head.\(^{17}\) The eddy viscosity model used here is a modification of the two-dimensional method for wall jets and turbulent boundary layers developed by Dvorak.\(^{18}\) These calculations include the effects of longitudinal surface curvature. The static pressure field, p(s,z), which is required for the solution of the boundary-layer equations, is determined directly from the potential flow solution.

The initial conditions required to start the finite-difference calculation at the slot exit are constructed from: 1) the integral boundary-layer solution at the slot exit on the upper surface of the component in question; 2) the laminar potential core as determined from the potential-flow solution; and 3) the upper- and lower-surface boundary-layer solutions at the trailing edge of the upstream element. If cove separation is present the boundary layer at the slot exit from the lower surface of the upstream element is assumed to follow the one-seventh power law with a thickness equal to one-third the slot-exit width. This assumed profile is representative of what is observed experimentally. With these initial conditions and with the static pressure field known, the boundary-layer equations are solved in a forward marching fashion to the trailing edge of the component.

The profile drag for a streamwise section is determined by use of the method of Squire and Young\(^{19}\):

\[
C_d = 2\left(\frac{v_2}{u_2}\right) \frac{H_{le} + \frac{5}{2}}{H_{lee}}
\]

**III. Viscous/Inviscid Coupling**

The effect of boundary-layer displacement and mass entrainment on the potential flow is simulated by distributed sources on the panels used to describe the airfoil contour. The strengths of these source panels as determined directly from the boundary-layer solutions are q = \(\frac{3}{5} \left( u_2 \delta^* \right) \) where
is the chordwise potential-flow velocity at the edge of the boundary layer and $\delta^*$ is the boundary-layer displacement thickness given by

$$\delta^* = \int_0^\delta (1 - \frac{U}{U_e}) dz \quad (8)$$

A modified Kutta condition, which requires that the flow be tangential to the trailing edge panel at the trailing edge on the upper and lower surfaces, together with the boundary condition that the velocity normal to the surface be equal to the known source distribution, determines the potential flow. The resulting potential-flow pressure distribution is then used in subsequent boundary-layer calculations.

It has been observed (e.g., Brune, Rubbert, and Nark20) that conventional cyclic boundary-layer/inviscid flow matching usually results in a divergent or, at best, a slowly convergent iteration process, especially if the viscous interaction is relatively strong (as is often the case for high-lift configurations). In the present method the boundary-layer source distribution is under-relaxed according to the formula

$$q_I^{n+1} = q_I^n + r(q_I^n + q_I^f) \quad (9)$$

where $q_I^f$ is the source distribution strength used in the $ith$ potential flow solution and $q_I$ is equal to $dq_i \delta^*$ obtained from the $ith$ boundary-layer solution. A relaxation factor of 0.5 has been found to be sufficient for a large variety of configurations.

For the unswept case, separation is defined as the location where the skin friction goes to zero. For the swept case, separation is defined as the point where the wall skin-friction vector forms a $90^\circ$ angle with the local potential-flow velocity vector. The manner in which the viscous interaction is handled depends on what type of separation is being considered. If separation occurs at some point, $(x, y)_{sep}$, on the upper surface of a component, the boundary layer calculation is necessarily terminated at that point. Based on the experimental observation that the static pressure is nearly constant in the separated flow, $U_\delta$ is assumed constant and equal to its value at the separation point.

The displacement thickness in the separated zone is obtained by linear extrapolation from the point of separation. Thus, in the separated zone the blown boundary condition becomes $q = (U_\delta \delta^*)_{sep}$.

The method is not intended to model extensive flow separation; rather, the intent is to permit the calculation to continue if separation occurs, such as is often encountered after the first potential flow solution. Separation on the lower surface of the last element, which seldom occurs, is handled similarly. Lower-surface flow separation, such as might be encountered in a flap cove, is handled differently. Experimental observation indicates that even if cove separation occurs the boundary layer at the slot exit is generally thin and its displacement effect on the potential flow is expected to be minimal. Therefore, the source distribution in the cove-separation zone is assumed to be a linear interpolation made between its value at separation and zero at the trailing edge.

IV. Wind Tunnel Test Program

The rectangular planform finite wing, used in the experimental part of this study, is shown as it was mounted in the Ames 40- by 80-Foot Wind Tunnel in Fig. 3. This wing is equipped with a full-span, 40% chord, single slotted flap. The wing and flap were basically steel frameworks covered with wood; the wood in turn was coated with a glass reinforced plastic skin to give the desired contour. The wing span was 16 m with a reference chord of 1.7 m and a nominal extended chord of 2.1 m. The relatively high aspect ratio of 7.6 (based on the extended chord), combined with the uniform section is designed to provide a nearly constant span loading over the wing center section. In addition, the finite wing avoids the interference effects, caused by wind-tunnel side-wall junctions, that are usually encountered with models that span the test section. These interference effects become especially troublesome at high lift and when separation plays a significant role.

Fig. 3 Model mounted in 40- by 80-Foot Wind Tunnel

The flap brackets, which remain parallel to the chordwise direction when the wing is yawed, permit horizontal and vertical flap movement. The range of movement in the chordwise and normal-to-chord direction was 55 and 45% of $C_b$ respectively.

The basic airfoil section from which the wing-flap combinations were derived is an RAE 2815. The wing-flap combinations tested are shown in Fig. 4. A complete table of coordinates is listed by Foster et al. The 10° flap configuration had a faired cove whereas the 30° flap configuration had no cove fairing; the leading edge of the 30° flap configuration was dropped 10° with the pivot point...
on the lower surface at an $x_w/C_D$ of 0.15. The test Reynolds number, $Re$, was $3.8 \times 10^5$.

![Diagram of airfoil configurations tested.](image)

**Fig. 4 Airfoil configurations tested.**

**Wing Forces and Moments**

Overall wing forces and moments were obtained from the wind tunnel scale system and corrected for tunnel wall-interference effects. The total lift was used in conjunction with the vortex lattice theory of Hough to determine a section angle of attack at the center section of the wing according to the formula

$$\alpha = \alpha_w - \Delta \alpha$$  (10)

where $\Delta \alpha$ is the induced angle of attack at the center section due to finite wing effects. The induced angle of attack, or mean downwash at the wing center line, as given by the vortex lattice theory is

$$\Delta \alpha = \left[ \frac{1}{C_D} \left( \frac{C_L}{C_D} \right) \right] (C_L)_{c,t}$$  (11)

where it is assumed that the induced angle of attack scales with the measured total lift of the wing.

**Pressure Measurement:**

The center section of the wing was instrumented with three chordwise rows of static pressure orifices. One row was located on the model centerline, a second row at $y = 0.44 C_0$, and a third row at $y = -0.22 C_0$. No flap brackets were located between $y = -0.22 C_0$ and $y = 0.44 C_0$. At each of these three spanwise stations there were 64 orifices on the main element and 30 orifices on the flap. The method of measuring pressures was with scanivalves and transducers with ranges of $\pm 17 \text{kN} m^{-2} (\pm 2.51 \text{lb/in}^2)$; the scanivalves were automatically sequenced. The transducer output was digitized and punched onto data cards for subsequent data reduction on a digital computer.

Preliminary tests at sweep angles of $0^\circ$ and $25^\circ$ showed no significant spanwise gradients in the center section of the wing. All data presented in this paper were obtained on the model centerline.

**Hot-Wire Boundary-Layer Surveys**

Velocity profiles near the main element trailing edge and above the flap upper surface were obtained using a translating hot-wire probe. The drive motor was located inside the flap, thus minimizing the aerodynamic interference due to the survey device. Anemometer output was linearized to give a linear relationship between voltage and velocity and then recorded on a X-Y plotter as a function of probe position.

Hot-wire calibrations were made using a free jet powered by a variable speed blower. The anemometer bridge was temperature compensated to minimize the influence of wind-tunnel static-temperature variation on anemometer output.

**V. Comparison of Theoretical and Experimental Results**

**Section Lift**

Figures 5-7 show comparisons of the measured section lift characteristics, as functions of section angle of attack, with those determined theoretically. Figure 5 is for the $10^\circ$ flap configuration without sweep; Fig. 6 is for the $30^\circ$ flap configuration with $0^\circ$ sweep; and Fig. 7 is for the $30^\circ$ flap configuration with $25^\circ$ sweep. The theoretical calculations have been terminated at the angle of attack where upper-surface separation first occurred. This type of separation greatly inhibited or prevented convergence of the viscous-potential flow iteration process, and resulted in generally unreliable results. The fact that in all cases separation occurred at lift levels below, but within $10\%$ of, $C_L_{\text{max}}$ indicates that the existence of separation on either the main component or flap provides a reasonably accurate and conservative estimate of maximum lift.

![Graph showing section lift characteristics for plain leading edge.](image)

**Fig. 5 Section lift characteristics for plain leading edge: $\beta = 0^\circ$, $\delta_f = 10^\circ$.**

![Graph showing section lift characteristics for drooped leading edge.](image)

**Fig. 6 Section lift characteristics for drooped leading edge: $\beta = 0^\circ$, $\delta_f = 30^\circ$.**

The general character of the results for the three configurations shown in Figs. 5-7 are similar. Comparison of the experimental measurements and the potential-flow solutions show strong viscous interactions resulting in observed lift coefficients well below those predicted by pure potential-flow theory. The addition of viscous interaction effects brings the theoretical predictions into much better agreement with the experimental results with the largest difference occurring, somewhat surprisingly, at lower angles of attack. The reason why the lift
over-predicted at the lower angles of attack is shown by the section pressure distributions presented in Figs. 8 and 9.

The flap pressure distribution for the unswept 30° flap configuration of Fig. 8 is predicted quite accurately, whereas the load level for the main element is over-predicted. This discrepancy is attributed to extensive flow separation in the cove on the lower surface of the main component. This separation, although predicted by the boundary-layer analysis, results in a strong viscous interaction which is not modeled accurately in the theoretical analysis.

As a result, the contribution to lift of the pressure distribution in the cove is over-predicted. In addition, the cove separation appears to have reduced the net circulation on the main element, thereby influencing the upper surface pressure distribution as well. At higher angles of attack, such as the 7.5° shown in Fig. 9, the orientation of the cove relative to the freestream is such that the influence of cove separation (although still present) is suppressed; the result is a much improved agreement between theory and experiment. Increasing the sweep angle to 25° did not significantly alter this behavior for either the 10° or 30° flap configurations.

Figure 10 compares the effect of angle of attack on computed integral boundary-layer characteristics with its effect on experimentally measured characteristics for the unswept 10° flap configuration. The comparisons are made for a point located at the trailing edge of the main component upper surface. The shape factor, \( H = \frac{\delta}{\delta_v} \), is predicted accurately throughout the angle of attack range. The tendency to slightly over-predict the displacement thickness is caused, at least in part, by the increased wing loading due to the previously mentioned cove separation effects.

Velocity profiles on the flap upper surface for the 30° flap configuration are shown in Figs. 11 and 12 for an angle of attack of 9°. The mean velocity profile in Fig. 11 is for a station 0.5 downstream of the main component trailing edge. The dominate feature is the wake from the main component. The hot-wire probe in this case did not get close enough to the flap upper surface to get into the very thin flap upper-surface boundary layer. Agreement between theory and experiment is reasonably good. Inspection of the mean velocity profile indicates, as assumed in the theoretical analysis, that a laminar core exits in the slot efflux. Also shown in Fig. 11 is the turbulence level, \( \sqrt{u'^2/u_e} \), which provides a qualitative measure of the potential for turbulent transport. The turbulence level, significant throughout the slot efflux, indicates that a true laminar core is not present. This turbulence is undoubtedly comprised of remnants of the turbulent mixing process associated with the cove separation. The predicted and measured velocity profiles at the flap trailing edge are compared in Fig. 11. Although the total boundary-layer thickness and the point of minimum velocity associated with the wing wake are predicted accurately, the experimental measurements show that the wing wake and flap boundary layer have merged to a greater extent than that predicted theoretically implying that the turbulence in the so-called laminar core has a significant influence on the confluent boundary-layer development.
Concluding Remarks

A theoretical method for analyzing the viscous/potential flow around two-dimensional and infinite-span multi-component airfoils has been described. The analysis is based on iteratively coupled boundary-layer and potential-flow calculations. The viscous-flow analysis considers the confluent boundary layer, where appropriate, and computes both spanwise and chordwise boundary-layer characteristics. In support of the theoretical program an experimental study was conducted in the Ames 40- by 80-Foot Wind Tunnel. Comparisons of the present theoretical method with other exact solutions and with the experimental data resulted in the following conclusions:

1. The present theoretical method accurately predicts section lift characteristics of multi-component configurations through moderate sweep angles where flow separation effects are negligible.

2. For the configurations tested, the angle of attack for the first occurrence of upper surface separation defines a reasonably accurate and conservative estimate of $C_{\text{max}}$ at sweep angles of $0^\circ$ and $25^\circ$.

3. The present method accurately predicts the optimum flap gap for maximum lift at a fixed angle of attack.

4. In addition to the study of two-dimensional and infinite-span swept multi-component airfoils, this method will be useful in the analysis of high-lift characteristics of relatively high aspect ratio, moderately swept, finite wings.

5. The influence of cove separation on the state (laminar or turbulent) of the slot efflux can have a strong influence on the development of the confluent boundary layer over the flap upper surface.

6. The method of Squire and Young for computing drag does not (in its original form) accurately predict the drag of multi-component configurations.

7. Potential-flow matrix computer solution times for multi-component configurations are significantly reduced, from those required for direct triangular decomposition solution, by using block-Gauss-Seidel iteration with relaxation.

8. The present potential-flow method, which utilizes linear vorticity, gives excellent agreement with exact solutions for the inviscid flow around multi-component configurations.
References


