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A THEORETICAL STUDY OF THE APPLICATION OF JET FLAP CIRCULATION CONTROL FOR REDUCTION OF ROTOR VIBRATORY FORCES - ADDENDUM

by

Andrew R. Trenka

October 1975

Prepared Under Contract No. NAS2-7307

For

U. S. Army Air Mobility Research and Development Laboratory
Ames Directorate
and
National Aeronautics and Space Administration
Ames Research Center
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FOREWORD

This Addendum presents additional results of a study to investigate the theoretical potential of a jet-flap control system for reducing the vertical and horizontal transmitted helicopter rotor blade root shears. The main body of results have been presented in NASA CR-137515. In the main effort and this extension a computer simulation was used to examine the reduction of each harmonic of the transmitted shears as a function of the jet parameters, the rotor operating conditions, and rotor configuration.

The research program was conducted by VIZEX, INC. under the joint sponsorship of the Ames Directorate-U.S. Army Air Mobility Research and Development Laboratory and the National Aeronautics and Space Administration. The contract No. was NAS2-7307. The effort reported in this Addendum commenced on October, 1974 and was completed in October, 1975.

The Technical Monitor was Mr. John McCloud of National Aeronautics and Space Administration. His direction, helpful technical comments, and discussions were of considerable value to the conduct of this effort.

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SUMMARY

Presented herein are the results of a supplimental study to investigate the theoretical potential of a jet-flap control system for reducing the vertical and horizontal non-cancelling helicopter rotor blade root shears. The techniques and results of that study (reported in NASA CR-137515) were also employed for this effort.

The conclusions reached in the main study were generally supported. One major exception was the conclusion that the dominant contributor to the rotor power requirements was the requirement to maintain moment trim as well as force trim. It was found in this supplimental study that the requirement to maintain moment trim did not entail a power penalty.

1.0 INTRODUCTION

VIZEX, INC. recently completed a study (Reference 1) to investigate the theoretical potential of a jet-flap control system for reducing the vertical and horizontal transmitted helicopter rotor blade root shears. A computer simulation was used to examine the reduction of each harmonic of the transmitted shears as a function of the jet parameters, the rotor operating conditions, and rotor configurations.

The general overall conclusion of Reference 1 was that the rotor jet-flap control system appears to be (theoretically) a practical means of achieving efficient higher harmonic control which could be used for many applications.

The results indicated that a jet-flap control system has the potential of reducing all of the vertical and inplane transmitted (non-cancelling) blade root shears simultaneously with a single jet control mode. Furthermore, the results indicated that the control angle schedule and additional power required were within practical limits (indicated in the subsequent discussion).

It was found that the blade torsional response can be an essential, beneficial element of the jet-flap control system. That is a jet-flap "torsionally controlled" rotor may be much more efficient than a "pure" (torsionally stiff) jet-flap rotor where the aerodynamic control is primarily from the jet angle rather than the blade angle.

The results of Reference 1 have been augmented by the study reported in this Addendum. Extensive use of the results of Reference 1 will be made by referal to the appropriate sections therein and will not, in general, be reproduced here. The scope of this effort reported herein is defined in the Section 2.0 in light of the results obtained in Reference 1. The results, conclusions and recommendations follow in Sections 3.0 and 4.0.

LIST OF SYMBOLS

A_n	Glauert coefficients, m/s
b	blade semichord, m
C_{D_p}	profile drag coefficient
$C_L, C_{L_{\alpha_0}}, C_{L_{\hat{t}}}$	section lift coefficient and lift-curve slope for α & $\hat{t}$ jet-off
c_j	generalized coordinate for j^{TH} control mode
C	matrix of aerodynamic mass, spring and damping coefficients
C_J	local jet momentum coefficient, $C_{J_{T_0}}\left(f_{cJ}/(\bar{r}+\mu\sin\psi)^2\right)$
$\bar{C}_{L_\alpha}, \bar{C}_{L_{\hat{t}}}, \bar{C}_{L_\tau}$	section lift-curve slope for $\alpha, \hat{t}$ & τ, jet-on
$C_{M_{\alpha_0}}, C_{M_{\hat{t}_0}}$	section moment-curve slope for α & $\hat{t}$, jet-off
$\bar{C}_{M_\alpha}, \bar{C}_{M_{\hat{t}}}, \bar{C}_{M_\tau}$	section moment-curve slope for $\alpha, \hat{t}$ & τ, jet-on
$C_{J_{T_0}}$	tip jet momentum coefficient based on rotor tip speed, $= \left(\frac{d\dot{m}_J}{dr}\right)V_J \Big/ b\rho(R\Omega)^2$
$\mathfrak{D}$	blade section drag per unit span, N/m
$\mathfrak{D}_{cJ}$	blade section jet dependent drag per unit span, N/m
D	matrix of mass, spring and damping coefficients
$\frac{d\dot{m}_J}{dr}$	jet mass flow rate per unit span, kg/s/m
e_0	distance of blade pitch axis forward of midchord, m
e	distance of blade elastic axis forward of midchord, m

E	matrix approximating $[C]$
$f_{g_i}(r)$	normalized deflection in the g^{TH} mode
f_{c_J}	nondimensional spanwise distribution of jet mass flow rate, unity at blade tip.
$\hat{F}$	matrix representing the matrix sum $[D + C]$
F_{TAIL}	tail rotor side force, N
$F_{X_{ROTOR}}$	rotor x-force in shaft plane, N
$F_{Y_{ROTOR}}$	rotor y-force in shaft plane, N
$F_{Z_{ROTOR}}$	rotor z-force in shaft plane, N
f_{τ_j}	nondimensional spanwise mode shapes for τ_j , $j = 1, 2$
F	$[D + E]$
$\overline{GF}$	$[\vec{Y}_0 - C\vec{X}_i]$
h	blade section plunging velocity relative to fixed axes, m/s
h_i	generalized coordinate for i^{TH} vertical deflection mode, m
H_i	generalized coordinate for i^{TH} inplane deflection mode, m
$h_{J_{TIP}}$	jet slot height at blade tip, m
I_K	quasi-steady part of Γ_K , m^2/s
k_0	jet velocity/maximum relative air velocity at tip
ℓ_{TAIL}	distance from shaft axis to tail rotor center
$\mathcal{L}_M$	blade section stalled lift per unit span, N / m
$\mathcal{L}$	blade section lift per unit span, N / m
m	blade mass per unit span, kg/m

m_M	blade section stalled pitching moment about midchord per unit span, m·N/m
m	blade section pitching moment about midchord per unit span, m·N/m
M_T	rotor tip mach number
M_N	local mach number
$\dot{M}_{J_T}$	total jet mass flow rate required for all blades, kg/s
N	number of rotor blades
NA	number of azimuth positions used in the computation
NR	number of blade radial segments used in the computation
NRA	total number of collocation points in rotor disc; NA•NR
p_{ATM}	freestream static pressure at infinity, N/m^2
P_T	total power required, rotor + compressor, W
PM_{ROTOR}	rotor pitching moment, m•N
P_R	power required by rotor, W
$\bar{p}_o$	compressor output pressure, N/m^2
P_c	compressor power required, W
q_i	generalized coordinate for i^{TH} vertical deflection, inplane deflection, torsional or control mode
RM_{ROTOR}	rotor rolling moment, m•N
R	total blade radius, m
r	radius to a blade section, m
$\bar{r}$	radius to a blade section, nondimensional (r/R)

r_J	inboard most radial position of jet, m
$\bar{R}$	gas constant
S_n	coefficients giving induced velocities due to mesh of vortex filaments in wake
$\bar{t}$	maximum chord displacement for parabolic camber, m
$\hat{t}_e$	equivalent parabolic camber, $b\dot{\alpha}_g/4V_1$
$\hat{t}_o$	geometric parabolic camber, $\bar{t}/2b$
T_{ROTOR}	rotor torque, m·N
t	time, s
τ	total jet angle, rad
τ_{nj}^c, τ_{nj}^s	cosine, sine components of the j^{TH} jet control mode at the n^{TH} rotor harmonic, rad
T_R	jet thrust recovery factor
T_{ATM}	freestream ambient temperature, K
v_k	velocity relative to the surface of the airfoil due to plunging motion at the k^{TH} collocation position, m/s
V_f	rotor translational (forward) velocity, m/s
V_1	component of total velocity of blade section perpendicular to the shaft and to the blade axes, m/s
V_J	jet velocity, m/s
w_k	normal induced velocity distribution at the k^{TH} collocation position, m/s
x	chordwise coordinate; distance aft of midchord, m
$\bar{x}_1$	distance of the CG aft of EA, m
$\vec{x}_o$	vector of unconstrained variables

$\vec{Y}_o$	vector of driving forces (forcing functions)
$\bar{\alpha}$	$= (1.25)\,\alpha_M$
α_i	induced angle, arc tan (V_2/V_1), rad
α_e	effective angle of attack of blade section relative to V_1, rad
α_g	geometric angle of attack of blade section relative to V_1, rad
$\dot{\alpha}_g$	time rate of change of geometric angle of attack, rad/s
α_M	stall angle for airfoil section $\lvert\alpha_M\rvert \le \pi/2$, rad
$\bar{\alpha}_M$	stall angle for airfoil section $\pi/2 < \lvert\alpha_M\rvert \le \pi$, rad
α_s	shaft angle relative to plane perpendicular to rotor translational velocity, rad
β	first harmonic flapping relative to a plane perpendicular to the shaft; $\beta = \beta_{1c}\cos\psi + \beta_{1s}\sin\psi$, rad
β_c	preconing angle, rad
Γ_K	total bound vorticity of blade section of K^{TH} segment, m^2/s
Γ_M	total stalled bound vorticity of blade section, m^2/s
γ_K	chordwise bound vorticity distribution at the K^{TH} collocation position, m/s
γ	ratio of specific heats (1.4 for air)
η_c	compressor efficiency
$\tilde{\theta}$	angular coordinate used to specify chordwise position; $x = -b\cos\tilde{\theta}$, rad
θ_B	built-in twist, rad
θ_i	generalized coordinate for i^{TH} torsion mode, rad
μ	advance ratio

ρ	air density, kg/m^3
σ	induced velocity coefficients of Γ-equations
ψ	azimuth, angle, rad
Ω	rotor angular velocity, rad/s
ω_{q_i}	natural frequency of q_i^{TH} mode, rad/s

Although all of the above symbols do not appear in this Addendum, they do all appear in the main report (Reference 1) and hence are reproduced here for completeness.

2.0 OVERALL EFFORT

As noted previously, the objective of this study was to investigate the theoretical potential of a jet-flap control system for reducing the helicopter vertical and horizontal noncancelling blade root shears (VBRS and HBRS respectively). The results and conclusions of the main study (Reference 1), while generally establishing the theoretical potential of the jet-flap control system, left several areas which required further study. These areas are the subject of this Addendum and are discussed below in Section 2.2 in light of the conclusions of Reference 1. For continuity, the conclusions of Reference 1 are presented in their entirety in Section 2.1

It is noted that all of the conclusions of Reference 1 were further supported in this continuation with the exception of conclusion (5). Conclusion (5) was not substantiated during this study; the pertinent discussion is presented in Section 3.7, p. 24.

2.1 Conclusions of Reference 1 (Section 6.0 of Reference 1)

The general overall conclusion of this study is that the rotor jet-flap control system appears to be (theoretically) a practical means of achieving efficient higher harmonic control which could be used for many applications.

The results indicate that a jet flap control system has the potential of reducing all of the vertical and inplane transmitted (non-cancelling) blade root shears simultaneously with a single jet control mode. Furthermore the results indicate that the control angle schedule and additional power required are within practical limits (indicated below).

It was found that the blade torsional response can be an essential, beneficial element of the jet-flap control system. That is a jet-flap "torsionally controlled" rotor may be much more efficient than a "pure" (torsionally stiff) jet-flap rotor where the aerodynamic control is primarily from the jet angle rather than the blade angle.

Because the "mechanics" of implementing the jet-flap control systems were not considered, the conclusions of this study are independent of such practical considerations.

The specific conclusions of this study are summarized as follows:

1) The jet-flap control system can suppress all the transmitted blade root shears to zero

2) Only one independent jet-flap control mode is required to suppress all the transmitted shears.

3) The jet deflection angles and additional power required are within practical limits.

   For example, the total jet deflection angle required (exclusive of the re-trim requirement) to suppress all the transmitted vertical and horizontal blade root shears never exceeded approximately 30°. Similarily the additional power required for the two-blade rotor was 117 HP, exclusive of that required to re-trim (301 HP).

4) It was possible to suppress all the transmitted shears with the $C_{j_{T_0}}$ as low as 0.005 and the jet deflection angles were still practical (at μ = 0.20, less than 30° including requirement for re-trim).

5) The jet-on trim requirement adopted for this study may be resulting in an unnecessary power penalty for trim.

The power required to maintain trim was greater than that required to suppress all the transmitted shears. The trim requirement adopted was that the rotor moments (and thus the shaft angles and fuselage attitudes) in addition to the forces shall be the same with the jet-on as with the jet-off. Accepting the small attitude changes may require significantly less power. The penalty may be due to the fact that the flapping rotor (with 5% hinge off-set) is an inefficient moment generator. Thus for a semi-rigid rotor this trim requirement may be a reasonable one.

6) Generally the blade dynamic bending response increased at all harmonics except that nearest the mode resonant frequency--there it generally decreased! However, the net result was that peak to peak bending stresses did not increase significantly.

7) Generally the more shear suppression required the greater the power required.

8) Interharmonic aerodynamic coupling due to the jet may be quite pronounced.

9) Suppression of one harmonic of shear affects the magnitude of all remaining shears--generally the shears at harmonics immediately above and below the suppressed harmonic are most affected.

10) For the rotor studi[illegible] 'lst torsion at (ω_{θ_1}/Ω= 5.3) torsion is an [illegible]s[illegible]ntial and beneficial element in the jet-flap control system.

2.2.1 Effect of Torsional Stiffness

In view of the behavior of the jet-flap rotor as a torsionally controlled rotor (conclusion 10) further studies to evaluate this possible mode of operation were undertaken. To suppliment the cases run in Reference 1, additional cases were run at $\mu = 0.2$ for $C_{J_{T_0}}$ 0.005 and 0.01 and 0. 02. Full trim requirements were maintained. These cases in conjunction with the case ($\mu = 0.2$ and $C_{J_{T_0}} = 0.03$) of Reference 1 indicate the behavior of a torsionally stiff blade with $C_{J_{T_0}}$ at a given advance ratio.

2.2.2 Trade-off Between Jet Momentum and Jet Angle

Because only one calculation was made at $C_{J_{T_0}} = 0.005$ ($\mu = 0.2$) in Reference 1, and because the jet deflection angles required to suppress shear were not excessive, (conclusion 4), further calculations were made to determine the practical lower bound on the jet momentum coefficient, $C_{J_{T_0}}$, and the jet deflection angle, τ_2, as a function of advance ratio.

To define these limits four cases were run at $\mu = 0.2$ and $C_{J_{T_0}} = 0.004$, 0.003, 0.002 and 0.001.

2.2.3 Effect of Removing Moment Trim Restraint

Because one of the ground rules of the study of Reference 1 required that the "jet-on" rotor be trimmed (both forces and moments) to the corresponding values of trim jet-off, very large control angles and hence power were required just to maintain trim. It was suspected that this requirement may have unduly penalized the jet-flap control rotor (conclusion 5). Hence the effect of removing the moment trim constraint was investigated in this Addendum study.

The blade is behaving similar to the Kaman CTR with the jet acting as the control flap. For the shaft driven rotor where the jet is being used primarily for control (i.e., low C_{JT_0}), the blade angle-of-attack is very much more effective in controlling the aerodynamic forces than is the jet deflection angle.

11) The power required to suppress shears and trim was significantly reduced by reducing the span of the jet by two-thirds and maintaining the same blowing coefficient.

2.2 Scope of Investigation Covered by This Addendum

The rotor configuration and blade properties employed in this Addendum study were those of the "basic rotor configuration" of Reference 1, Section 2.0. Two exceptions are noted:

(i) the study of the four bladed rigid rotor

(ii) the study of the four bladed Kaman-like "Controllable Twist Rotor"

Definition of the flight conditions and jet control parameters are also given in Section 2.0 of Reference 1.

The rotor systems studied in Reference 1 and this Addendum were assumed to be shaft powered rotor's, i.e. the jet was to be used primarily to control shears and not to power the rotor. Conventional collective and cyclic pitch was employed to provide rotor control in both the jet-off and jet-on flight conditions. In addition to the conventional pitch controls, one jet-flap control at the first harmonic was provided to allow maintaince of both force and moment trim conditions. That is the forces and moments obtained for the rotor in a given flight condition, jet-off were maintained when the jet was turned on. The additional jet-flap control was required because the rotor trim (primarily rolling and pitching moments) was upset by the jet effects.

Thus, only the force trim between jet-on and jet-off case was maintained while allowing moments to come out as they may. It could be argued that these moments would result in small, acceptable attitude changes of the fuselage, or that other trimming devices on the ship could be employed to maintain the same attitude.

One case at $\mu = 0.2$ and $C_{J_{T_0}} = 0.005$ was investigated to determine the power penalty assignable to maintaining force <u>and</u> moment trim.

2.2.4 Selected Shear Suppression

Because of some concern for the practical aspects of controlling the jet angle at harmonics much above 5Ω (Ω = rotor speed) and the questionable need to suppress shears much above the 5Ω, two cases were analyzed in which only non-cancelling shears up to 5Ω were suppressed. The two cases were at $\mu = 0.2$ with $C_{J_{T_0}} = 0.005$ and 0.01

2.2.5 Thrust Recovery Factor Variation

Because the calculated power required to suppress shears is influenced by the value of the thrust recovery factor, T_R, calculations were made at additional values of T_R. A value of 0.5 was used in Reference 1. For this study calculations with $T_R = 0.0$ and 1.0 were made; thus establishing bounds on its influence.

2.2.6 Advance Ratio Variation

No results on shear suppression were obtained above $\mu = 0.20$, in Reference 1. In this study, further efforts were expended to obtain results at a higher advance ratio.

Some justification existed for the supposition that results at $\mu = 0.3$ were not obtained in Reference 1 because of the blade loading requirements (C_T/σ) at this advance ratio were too high. Hence the study conducted under this effort was made at $\mu = 0.3$ for a 7500# rotor rather than the 10,000# rotor of Reference 1, all other properties were maintained the same as the "basic rotor configuration" (BRC). A total of three cases were run at $\mu = 0.3$ and $C_{J_{T_0}} = 0, 0.005$ and 0.01.

2.2.7 Four Bladed Rigid Rotor Configuration

The investigation of the potential of the jet-flap for shear suppression was extended to include the rigid rotor configuration. This provided information relative to the question of the possible benefits of this type of rotor configuration (with its greater control power) over the articulated rotor.

A four bladed rigid rotor, called configuration 3, with a gross weight of 88964N (20,000 lb.) was analyzed. This rotor blade configuration is directly comparable to the second rotor configuration of Reference 1.

The rigid blade flatwise mode shapes are presented in Figure 1. The edgewise and torsional mode shapes and frequencies are same as the "BRC". The corresponding flatwise frequencies at the rotor operating speed (300 RPM) are:

$$\omega_{h_1} = 6.2 \text{ HZ} \quad (\omega_{h_1}/\Omega = 1.24)$$

$$\omega_{h_2} = 18.0 \text{ HZ} \quad (\omega_{h_2}/\Omega = 3.60)$$

The cases investigated were for $\mu = 0.2$; $C_{J_{T_0}} = 0$ and $C_{J_{T_0}} = 0.005$.

2.2.8 Four Bladed Fully Articulated Rotor Configuration

A fully articulated blade, i.e. one having both flapping and lagging hinges, was also studied. The blade selected was a four bladed rotor having the physical properties of the Kaman "Controlable Twist Rotor", CTR, (See Reference 2). The control flap of the CTR was replaced by a jet-flap control.

The properties are given in Figures 2 through 7 inclusive. The blade frequencies and frequency ratios are given in Table 1.

The cases which were to have been run were $\mu = 0.33$; $C_{J_{T_0}} = 0$, 0.005 and 0.01.

2.2.9 Synopsis of Cases Run

A total of 18 cases involving variations of the above parameters are presented herein. The identification number sequence of Reference 1 was specified to keep track of the various cases run: e.g.

Case No. 1. 20. 03. 03

- 03 (last) — run number within the set
- 03 — designates tip jet momentum,
- 20 — designates advance ratio,
- 1 — designates rotor configuration
  - (1) 2 bladed, 10,000 lb. rotor
  - (2) 4 bladed, 20,000 lb. rotor

Table 2 is a synopsis of all cases discussed herein as well as those discussed in Reference 1. Those denoted by * were those investigated in this Addendum.

3.0 RESULTS

Presented in this section are the synopsized results of the investigation conducted as an extension to Contract NAS2-7307. The results reported herein are intended to augment those of Reference 1. The results are presented, as far as possible, in a format compatible with Reference 1.

The information is presented in the form of tables of amplitude and phase at each harmonic for all (transmitted and non-transmitted) shears and jet control angles (Tables 3, 4 and 5). Also presented are plots of the azimuthal variations of the total jet control angle required (Figures 8 through 19).

The total power required is presented for both jet-on and jet-off cases. The jet-on power includes estimates for the required jet-flap control compressor power. Jet-off results are discussed first, followed by the comparable jet-on results. Special jet-on cases are then discussed.

Additional results in the form of blade responses, bending moments, lift load distributions, etc. are introduced only as needed to help clarify results or presented as representative indicators of what happened in general.

In all cases in which shear suppression was required, the shears were suppressed to zero.

3.1 Effect of Torsional Stiffness

The cases JFSR-1.20.005.52C, 1.20.01.52C and 1.20.02.51 were cases run to further define the effect of increased torsional stiffness on the requirements for τ_2 to suppress all non-cancelling blade root shears as $C_{J_{T_0}}$ was varied. Figure 8, presents the azimuthal variation of τ_2 and is comparable to Figure 21 of NACA CR-137515. Figure 23 of Reference 1 presents a similar comparison for the $C_{J_{T_0}}$ = 0.03 case. The τ_2's required to achieve shear suppression for the torsionally stiff blade are substantially higher for all $C_{J_{T_0}}$'s investigated than for the less torsionally stiff blade. Note also that for the torsionally stiff blade, the overall magnitude of T_2 variation required increases as $C_{J_{T_0}}$ decreases. Just the opposite effect is observed for the flexible blade.

A substantial increase in the 3P, 6P and 9P harmonics of T_2 are observed (see Table 5a) as the blowing is decreased from 0.02 to 0.005 for the stiff blade. The above observations tend to substantiate the Conclusion 10 of Reference 1 (Also see Section 5.2.1 and 5.2.2, Reference 1).

It should also be noted that the overall effect of increasing higher harmonic T_2 as $C_{J_{T_0}}$ decreased would have been even more pronounced on the total T_2 required if it were not for the compensating effect of $1P\text{-}T_2$. Table 5a indicates a decreasing $1P\text{-}T_2$ required to maintain trim as $C_{J_{T_0}}$ decreases just as observed for the torsionally less stiff blade (See Figure 11 herein) up to $C_{J_{T_0}} = 0.005$. The discussion for this behavior is given in Section 5.2.5 of Reference 1

Figure 9 presents the total power required (including jet compressor power) vs. jet tip momentum for the Basic Rotor Configuration (BRC) and for the torsionally stiff blade. For low blowing coefficients ($C_{J_{T_0}} < 0.01$) the torsionally stiff blade requires only slightly less power than the more flexible (torsionally) blade, however, the requirements on T_2 increase substantially. Thus it appears that for low blowing coefficients, by paying a small power penalty and going to a torsionally flexible blade, substantial reduction in the T_2 required to suppress all non-cancelling blade root shears may be realized.

3.2 Trade-off Between Jet Momentun and Jet Angle

The trade-off between jet momentum and jet angle was investigated in cases:

JFSR	1.20.004.01
JFSR	1.20.003.01
JFSR	1.20.002.01
JFSR	1.20.001.01

In these cases $C_{J_{T_0}}$ was reduced from 0.004 to 0.001. The resulting τ_2 azimuthal variation required for each case is presented in Figure 10 . Figure 16 presents, for all $C_{J_{T_0}}$ examined, a synopsis of the harmonics of τ_2 required to suppress the appropriate non-cancelling shears. Also presented in Figure 12 is a plot of the compressor power, P_c , total mass flow, $\dot{m}_{J_T}$, and compressor pressure ratio $\bar{P}_0/P_{ATM}$ versus advance ratio, μ , for all $C_{J_{T_0}}$ run (for a discussion of these curves see Section 3.4 and Appendix III of Reference 1). Figure 9 presents the corresponding total power requirements

$$P_T = P_R + P_c$$

where P_R = rotor power required

P_c = jet compressor supply power required

From Figure 10, it is observed that even though the required τ_2 to suppress all non-cancelling shears substantially increases as $C_{J_{T_0}}$ is decreased to 0.001, the total angles required during a rotor revolution never excede 50°.

Presented in Figure 11 is a summary plot of the harmonics of jet angle amplitude required to suppress all non-cancelling blade root shears versus the jet tip momentum co-efficient at μ = 0.08 and 0.20.

A pronounced increase in τ_2 angle required at all harmonics EXCEPT 3, 4 and 7 is observed as $C_{J_{T_0}}$ is decreased. At the 3rd, 4th and 7th harmonics no clear pattern is evident. The increased angle required as $C_{J_{T_0}}$ is decreased, is believed due to the decreased effectiveness of the jet in suppressing the non-cancelling blade root shears. It is surprising that the variation of the harmonics of τ_2 are not as well behaved with $C_{J_{T_0}}$ as one might expect. Note in particular the plateaus in the τ_2 versus $C_{J_{T_0}}$ curves for the 5P through 11P curves between $0.003 \leq C_{J_{T_0}} \leq 0.005$. Note also the sharp increase in required $1P\text{-}\tau_2$ component which occurs between $0.001 \leq C_{J_{T_0}} \leq 0.002$. Recall that the $1P\text{-}\tau_2$ component is required to maintain jet off trim. It would appear that for values of $C_{J_{T_0}} < 0.002$ the upset in trim due to simply turning the jet on is greater than the corresponding effectiveness of the jet angle variation to maintain the original trim, i.e.

it no longer is true that "since lower values of C_{JT_0} upset jet-off tri conditions less, smaller values of $1P \cdot \tau_1$ are required to return to the initial jet-off trim".

The increased values of the harmonics of τ_1 required to suppress shears begin to be reflected in the total power required as shown in Figure 9. It appears that a minimum in the total power required does, in fact, occur at

$$C_{JT_0} \approx 0.002$$

Furthermore, a discontinuity in the total power required versus jet tip momentum curves, may arise as $C_{JT_0} \to 0$ due to the requirement to maintain trim to jet-off conditions. This is a mathematical difficulty associated with the requirement to maintain jet-off trim conditions.

3.3 Thrust Recovery Factor Variation

The variation of T_R (Thrust Recovery Factor) resulted in virtually no change in the required jet angle variation as can be seen in Figure 13. The largest change in the τ_1 required occured at 1P; changes at other harmonics were observable but small (1). The total power requirements decreased as T_R increases as expected. (See Table 6)

3.4 Advance Ratio Variation

In Reference 1, converged solutions were obtained only for the jet-off condition (Case 1.30.00.04) at high advance ratio ($\mu = 0.30$). Converged solutions could not be obtained with the jet on and the requirement that all shears be suppressed. It was determined that the problem lay with the suppression of the 1P harmonic of the inplane shear (See Section 5.1.2.1.1 of Reference 1). Large regions of stall were also noted.

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(1) Compare cases 1.20.005.03 and 1.20.005.04 in Table 5-c of this Addendum and case 1.20.005.01, in Table 6-a of Reference 1

In this Addendum, the same rotor configuration was run but at a lower blade loading, i.e. a reduction of 25% in C_T/σ. Again a converged solution was obtained for μ = 0. This solution was significantly different from that obtained at the higher blade loading. Some of the observed differences were:

(i) regions of stall were greatly reduced for the reduced C_T/σ case

(ii) rotor trim forces and moments were altered; in particular, the pitching moment (PM) increased in magnitude by a factor of 16.5 and changed sign:

	(C_T/σ) HIGH	(C_T/σ) LOW
PITCHING MOMENT	-87 ft.lb.	1431 ft.lb.
ROLLING MOMENT	4519 ft.lb.	5093 ft.lb.
TORQUE	1609 HP	1274 HP

It is noted that the 1431 ft.lb. pitching moment corresponds to a center of gravity offset of only 0.19 ft. from the effective rotor thrust force vector.

Also note that the reduced C_T/σ is reflected in the reduced power requirements.

(iii) The most significant changes in the harmonics of the blade root shears occurred in:

1P Vertical - increased by approximately 1.4

7P Horizontal - increased by approximately 5.0

(Compare Cases 1.30.00.04 and 1.30.00.73 in Tables 3d and 4d herein with 2 and 3 of Reference 1)

Initial runs to suppress all harmonics of non-cancelling vertical and horizontal blade root shears using $C_{J_{T_0}}$ = 0.005 and 0.01 resulted in τ_2's which exceeded 90°. Two major contributing harmonics were the 2P and 7P τ_2 components required to suppress the 2P vertical and 7P horizontal shear respectively. The larger of the two was the 2P component by far. It was decided to eliminate the 2P shear suppression requirement. The converged results (τ_2 azimuthal variation required) for this condition, i.e. suppress all non-cancelling blade root shears to zero except 2P, are presented in Figure 14. The very large 7P component of τ_2 is quite evident for both $C_{J_{T_0}}$'s; a significant increase in 7P-τ_2 occurs as $C_{J_{T_0}}$ is increased from 0.005 to 0.01.

It should be noted from Tables 3-d and 4-d that significant changes in the cancelling blade root shears have occurred as a result of suppressing the non-cancelling shears. In particular note the very large increase in the 7P vertical shear as $C_{J_{T_0}}$ is increased.

Thus while, the fuselage does not "see" these large loads, the flapping pin does.

For this case it was found that to suppress a 365 N (82.13 lb) horizontal blade root shear at 7P, the 7P vertical shear load increased from 41.8 N (9.391 lb), jet-off, to 756.6 N (170.1 lb) for $C_{J_{T_0}}$ = 0.005 to 3036 N (682.6 lb) for $C_{J_{T_0}}$ = 0.01.

Hence while it is possible to suppress the 7P horizontal non-cancelling shear with a "reasonable" jet flap amplitude at relatively low blowing coefficients, the penalty paid in terms of the increased flapping pin loads makes the desirability of suppression less attractive.

Recalling the discussion of the mechanism of shear suppression given in Section 5.2.3 of Reference 1, a brief study of the components involved in achieving suppression at μ = 0.3 was conducted. The implication of Reference 1 (sensitivity of total shear to amplitude and phase of τ_i) were further reinforced. Whether the sensitivity increases with advance ratio could not be definitively determined. The power requirements for these cases are given in Table 7.

3.5 Four Bladed Rigid Rotor Configuration

The results for the four bladed rigid rotor configuration are presented in Figure 15 and Tables 3-e, 4-e and 5-e for cases 3.20.00.05 and 3.20.005.01.

A comparison of the jet-off blade root shears for the four bladed rigid and flapping rotor (1) shows large increases in the 1P, 2P, 4P and 8P through 11P harmonics of the vertical blade root shear. The 3P and 5P through 7P vertical blade root shears decrease. The increases observed are substantially greater than the decreases. Notice particularly the very large 8P through 11P harmonics generated by the rigid rotor.

(1) See Case 2.20.00.02 Table 2 and 3 of Reference 1.

The horizontal shears do not change (compared to the flapping rotor) substantially with the exception of decreases at 3P and 7P and large increase at 5P.

The comparison of the rotor performance quantities are given in Table 8. The expected large rolling and pitching moments are obtained for the rigid rotor. Virtually no power change is observed.

Turning the jet on, maintain force and moment trim and suppressing all non-cancelling vertical and horizontal blade root shears required 2425 HP (1.8083×10^6 watts). Thus for a 11% increase in power all shears were suppressed to zero. However very large τ_2 's are required as evidenced by Figure 15. Dominating the τ_2 harmonics are the 1P, 2P and 4P values. The smallest harmonic amplitude of τ_2 required was 3°; all others exceeded this value. Thus it is conjectured that suppressing all higher harmonic shears for the rigid rotor will require substantially greater values of jet angle than for the flapping rotor.

3.6 Selected Shear Suppression

The effect of requiring shear suppression of only the 2nd through 5th harmonics of the non-cancelling blade root shears was investigated at $\mu = 0.2$ for three values of C_{JT} (i.e. 0.005, 0.01 and 0.03). The results obtained are typified by Figure 16 where the comparison of τ_2 azimuthal variation required for suppressing all and only 2P to 5P harmonics is presented. Elimination of the requirement for suppression of non-cancelling shears above the 5th can result in changes (in the required τ_2 at lower harmonics) of as much as 15%. Generally the changes are much less.

The effect of elimination of the higher harmonic shear suppression on the power is very small (see Table 9) until C_{JT} approaches 0.03. In all cases the power required is reduced when fewer harmonics of shear suppression are required.

At the lowest blowing coefficient for which these comparisons were made, $C_{JT} = 0.005$, we find a net increase (compared to no suppression) in power required of only 4% to suppress ALL non-cancelling blade root shears. To suppress only 2nd to 5th harmonics, a net increase in power required was 3%.

3.7 Effect of Removing Moment Trim Restraint

Figure 17 presents the azimuthal variation of τ_2 required to suppress all shears and require force and moment trim compared to the azimuthal variation of τ_2 required to suppress all shear but only require force trim. (Cases 1.20.005.01 and 1.20.005.02 respectively)

A similar comparison is made in Figure 18 for the case in which only partial shear suppression (2P to 5P) is required. (Cases 1.20.005.05 and 1.20.005.06 respectively) The corresponding total power required for these cases is presented in Table 10.

In both the full shear suppression and partial shear suppression cases, elimination of the requirement to maintain moment trim (i.e. $1P-\tau_2 = 0$) results in substantial changes the higher harmonic τ_2 required to suppress shear. Despite these substantial changes in τ_2 required almost no change in the total power is observed. These results tend to refute the arguments developed in Sections 5.1.3 and 5.2.5 of Reference 1 where it was argued that large $1P-\tau_2$ requirements contributed substantially to the jet power requirements.

An examination of the control settings and 1st torsional responses for these cases was made. No differences in the control setting were found. Differences in the torsional responses were found, particularly at 1P where the response was halved by elimination of the $1P-\tau_2$; the differences in torsional response were, as expected, directly proportional to the changes in τ_2 required. Thus the net effect of removing the requirement to maintain moment trim was one primarily of changing the jet angle harmonic content required to suppress shears. No power benifit was realized nor was any overall increase or decrease in the total τ_2 variation observed.

While no substantial power changes were observed for these cases, substantial changes did occur in the rolling and pitching moments generated by the rotor with and without full trim required. In particular, for the full shear suppression case, elimination of the moment trim requirement resulted in a decrease in the rolling moment. (1883 ft-lbs to 1537 ft-lbs) and a decrease in the pitching moment (415.8 ft-lbs to 241.7 ft-lbs). Accompaning such a change in pitching moment, in particular, would have been changes in fuselage attitude. These changes would have resulted in changes in the fuselage drag forces which would have altered the force trim requirements on the rotor. For example, for the assumption made, the fuselage angle change could be as much as two degrees. Conversely these changes could be interpreted as center of gravity shifts relative to rotor resultant force of less than 0.2 inches (for pitching moment) and 0.4 inches (for rolling moment). The effect of such changes on the rotor power were not examined.

3.8 Four Bladed Fully Articulated Rotor Configuration

Initial results obtained at $\mu = 0.33$ for jet-on / jet-off cases were suspect because of very low power requirements. A subsequent recheck of the mass-elastic integrals revealed two errors. One error involved the contribution of the lag degree of freedom to the edgewise shear; the other major error involved the mass (centrifugal) coupling term between flapping-pitch degrees of freedom. When the "corrected" values were introduced into the analysis converged solutions could not be obtained. The computations indicated diverging motions at 1P in flapping-lagging degrees of freedom with iteration. All attempts to obtain solutions, using techniques previously found to aid convergence, were unsuccessful. While it was suspected, and continues to be suspected, that an error exists in the computation of the mass-elastic integrals, none could be found and further efforts had to be abandoned.

4.0 CONCLUSIONS

With one exception the conclusions reached in Reference 1 and reproduced in Section 2.1 herein were generally supported in the study reported in this Addendum. The exception was that of Conclusion 5. The results of this Addendum did not support the conclusion that a power penalty may have resulted from the requirement to maintain moment trim as well as force trim. In fact no significant power change was observed between the cases requiring full trim and those requiring only force trim.

Some additional conclusions of this Addendum study are:

(1) For low blowing coefficients, $C_{J_{T_c}} < 0.01$, by paying a small power penalty and going to a torsionally flexible blade, substantial reductions in the jet angles required to suppress all non-cancelling blade root shears may be realized.

(2) At a given advance ratio, continued reduction in blowing coefficient will result in large increases in amplitude of the harmonics of τ_L required to suppress shears. A power minimum may also be observed.

(3) The roll played by the thrust recovery factor in determining rotor power is negligibly small.

(4) The suppression of non-cancelling shears may result in unacceptably large cancelling blade root shears being generated, especially at high advance ratios. These loads may pose fatigue problems at the blade attachment points.

(5) For the case investigated the rigid rotor required larger values of jet angle to achieve shear suppression than a similar flapping rotor.

REFERENCES

1. Piziali, R.A. and Trenka, A.R. A Theoretical Study of the Application of Jet Flap Circulation Control for Reduction of Rotor Vibratory Forces May 1974 NASA CR 137515 (Vizex CR-74-1)

2. Lemnios, A.Z. and Smith, A.F. An Analytical Evaluation of the Controllable Twist Rotor Performance and Dynamic Behavior May 1972 USAAMRDL TR 72-16

3. Evans, W.T. and McCloud, J.L.,III Analytical Investigation of a Helicopter Rotor Driven and Controlled by a Jet Flap NASA TN D 3028 September 1965

MODE	FREQUENCY AT $\Omega = 3.48$ HZ	HARMONIC (ω/Ω)
RIGID FLAPPING	3.59 HZ	1.032
1st FLAPWISE BENDING	9.81 HZ	2.82
2nd FLAPWISE BENDING	15.80 HZ	4.54
1st EDGEWISE BENDING	.80 HZ	0.23
2nd EDGEWISE BENDING	14.20 HZ	4.08
1st TORSION	10.13 HZ	2.91

TABLE 1. BLADE FREQUENCIES AND FREQUENCY RATIOS FOR CONFIGURATION 4

TABLE 2. LIST OF ALL CASES PRESENTED

C_{T_0} \ μ	0.08	0.02	0.03	0.33
0	1.08.00.03	1.20.00.03 1.20.00.50 (5) 2.20.00.02 * 3.20.00.05	1.30.00.04 * 1.30.00.72(8)	4.33.00.07 **
0.001 0.002 0.003 0.004		* 1.20.001.01 * 1.20.002.01 * 1.20.003.01 * 1.20.004.01		
0.005		1.20.005.01 * 1.20.005.02 (6) * 1.20.005.03 } (7) * 1.20.005.04 } * 1.20.005.05(2) * 1.20.005.52C(5) * 1.20.005.06(9) * 3.20.005.01	* 1.30.005.79(10)	
0.01	1.08.01.01	1.20.01.01 * 1.20.01.03(2) * 1.20.01.52C(5)	1.30.01.01 ** * 1.30.01.77(10)	
0.02		1.20.02.01 * 1.20.02.51		
0.03	1.08.03.02	1.20.03.06 (2) 1.20.03.56 (3) 1.20.03.08 1.20.03.09 } (1) 1.20.03.10 } 1.20.03.20 (4) 1.20.03.58 (5) 2.20.03.02	1.30.03.01 **	

(SEE NEXT PAGE FOR LEGEND)

LEGEND FOR TABLE 2

NOTES:

(1) Only vertical shear suppressed at 2nd and 5th harmonic respectively

(2) Non-cancelling shears suppressed at 2nd through 6th (or 5th) harmonics only

(3) Short jet, only vertical shear suppressed at 5th harmonic

(4) No shears suppressed, only jet off rotor trim-forces and moments-required

(5) Increased torsional stiffness, jet off and jet on

(**) Convergence not obtained

(*) Cases analysed under Mod. 3 Extension

Case No. 1 20. 03. 03

- 03 — run number within the set
- 03. — designates tip jet momentum,
- 20. — designates advance ratio,
- 1 — designates rotor configuration

  (1) 2 bladed, 44,482 N (10,000 lb.) rotor flap only
  (2) 4 bladed, 88,964 N (20,000 lb.) rotor flap only
  (3) 4 bladed, 88,964 N (20,000 lb.) rotor-rigid
  (4) 4 bladed, 51,152 N (11,500 lb.) rotor-flap/lag

(6) Only force trim maintained; all non-cancelling shears suppressed

(7) Thrust recovery factor, Tr , varied from 0.5 (in 1.20.005.01) to 0 and 1.0 in -03 and -04 respectively.

(8) Configuration 1 with only change being 7500# wt. instead of 10,000#

(9) Only force trim maintained; non-cancelling shears suppressed at 2nd thru 5th harmonics only

(10) Trim and all non-cancelling blade root shears constrained except @ 2P

TABLE 3-a

VERTICAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.02.51		1.20.01.52C		1.20.005.52C			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	22299.	.0	22299.	.0	22299.	.0		
1	8794.	-80.88	6183.	-77.24	4862.	-76.28		
2	.0	.0	.0	.0	.0	.0		
3	1045.	-145.8	1821.	-108.3	1687.	-117.4		
4	.0	.0	.0	.0	.0	.0		
5	322.1	-68.49	270.	-165.1	426.	-137.4		
6	.0	.0	.0	.0	.0	.0		
7	87.2	-136.4	72.1	-133.4	59.6	-134.3		
8	.0	.0	.0	.0	.0	.0		
9	104.7	-165.8	104.	-122.0	170.	-106.7		
10	.0	.0	.0	.0	.0	.0		
11	93.6	-40.92	19.3	-71.72	32.6	-162.5		

TABLE 3-b

VERTICAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.004.01		1.20.003.R1		1.20.002.R1		1.20.001.C1	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	22299.	.0	22299.	.0	22299.	.0	22299.	.0
1	4937.5	-72.45	4559.4	-70.68	4240.9	-69.38	3830.8	-67.08
2	.0	.0	.0	.0	.0	.0	.0	.0
3	2184.5	154.1	2318.8	152.2	2313.5	150.0	2274.8	146.8
4	.0	.0	.0	.0	.0	.0	.0	.0
5	988.4	-90.62	996.4	-89.89	988.8	-89.56	978.6	88.80
6	.0	.0	.0	.0	.0	.0	.0	.0
7	82.91	-119.8	79.93	-112.8	88.52	-98.30	89.68	-90.41
8	.0	.0	.0	.0	.0	.0	.0	.0
9	62.99	-48.95	58.81	50.18	56.89	-57.83	56.49	67.26
10	.0	.0	.0	.0	.0	.0	.0	.0
11	25.82	88.15	17.29	78.56	16.29	66.13	16.42	58.45

TABLE 3-c

VERTICAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.005.06		1.20.005.02		1.20.005.04		1.20.005.03	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	22299.	.0	22299.	.0	22299.	.0	22299.	.0
1	2806.	-77.37	2814.4	-78.56	5262.	-73.65	5280.	-73.46
2	.0	.0	.0	.0	.0	.0	.0	.0
3	2441.	149.2	2384.7	149.0	2184.1	156.1	2154.3	156.0
4	.0	.0	.0	.0	.0	.0	.0	.0
5	1042.	-91.17	981.3	-90.37	991.5	-92.42	987.1	-92.46
6	208.4	-5.203	.0	.0	.0	.0	.0	.0
7	121.0	30.41	116.1	-133.8	68.10	-123.5	67.43	-135.9
8	77.0	-1.921	.0	.0	.0	.0	.0	.0
9	109.9	165.4	58.18	67.25	68.77	43.18	65.03	-40.89
10	93.6	96.71	.0	.0	.0	.0	.0	.0
11	83.2	-66.05	22.39	58.70	28.82	87.36	27.05	87.23

TABLE 3-d

VERTICAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.30.00.73		1.30.005.79		1.30.01.76			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	17041.	.0	17041.	.0	17041.	.0		
1	15902.	-67.86	17490.	-73.61	17877.	-72.99		
2	2973.	174.0	3494.	-173.5	3849.	-163.2		
3	1811.	169.2	1204.	154.7	817.	59.62		
4	227.	76.15	.0	.0	.0	.0		
5	165.	39.83	152.7	11.82	493.	87.18		
6	25.6	78.29	.0	.0	.0	.0		
7	41.8	59.24	756.6	109.0	3036.	.3569		
8	5.06	52.08	.0	.0	.0	.0		
9	7.26	-13.68	86.1	90.77	146.	62.74		
10	26.9	66.93	.0	.0	.0	.0		
11	29.2	158.2	22.2	-93.24	98.4	-75.59		

TABLE 3-e

VERTICAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	3.20.00.05		3.20.005.01		1.20.01.03		1.20.005.05	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	22299.	.0	22299.	.0	22299.	.0	22299.	.0
1	4434.	119.2	4568.	119.0	6574.	-75.27	5271.	-73.57
2	722.	34.84	.0	.0	.0	.0	.0	.0
3	458.	-139.5	1213.	-131.0	1898.	161.6	2212.5	155.3
4	871.	54.19	.0	.0	.0	.0	.0	.0
5	655.	-44.03	306.	-82.61	1043.5	-93.37	1048.0	-92.53
6	281.	121.4	.0	.0	209.1	13.46	216.6	12.51
7	47.8	-54.23	433.	-67.87	76.60	-31.80	75.04	-30.95
8	98.6	-100.4	.0	.0	68.99	-21.52	69.04	-24.35
9	119.	102.4	131.	18.48	87.36	166.3	85.89	165.7
10	97.4	55.99	.0	.0	100.35	94.63	102.71	94.18
11	179.	-93.02	81.3	-48 .7	75.71	-66.58	75.84	68.63

TABLE 4-a

HORIZONTAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.02.51		1.20.01.52C		1.20.005.52C			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	1031.	.0	1463.	.0	1697.	.0		
1	653.0	-16.58	1503.	-6.718	1952.	-4.364		
2	292.0	-72.77	253.	-60.44	184.	-54.41		
3	.0	.0	.0	.0	.0	.0		
4	16.9	137.8	9.39	-178.3	15.5	166.8		
5	.0	.0	.0	.0	.0	.0		
6	154.2	-99.24	138.	99.49	112.	-102.8		
7	.0	.0	.0	.0	.0	.0		
8	194.5	-145.2	149.	-161.9	146.	-172.1		
9	.0	.0	.0	.0	.0	.0		
10	22.2	-35.81	17.1	-20.25	17.8	-4.464		
11	.0	.0	.0	.0	.0	.0		

TABLE 4-b

HORIZONTAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.004.01		1.20.003.R1		1.20.002.R1		1.20.001.C1	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	1750.4	.0	1808.6	.0	1867.8	.0	1934.5	.0
1	2056.4	-5.189	2180.5	-5.024	2294.8	-4.775	2437.6	-4.487
2	415.1	3.121	451.05	-3.118	461.7	-9.194	470.6	-16.23
3	.0	.0	.0	.0	.0	.0	.0	.0
4	11.08	-145.8	14.43	-179.0	12.28	148.7	13.73	112.7
5	.0	.0	.0	.0	.0	.0	.0	.0
6	45.02	-1.826	48.22	-2.885	49.20	-2.108	50.44	-1.497
7	.0	.0	.0	.0	.0	.0	.0	.0
8	48.49	-177.0	44.66	-170.5	46.31	-166.3	47.73	160.9
9	.0	.0	.0	.0	.0	.0	.0	.0
10	3.24	-9.277	2.62	-34.59	3.71	-43.68	5.11	-46.28
11	.0	.0	.0	.0	.0	.0	.0	.0

TABLE 4-c

HORIZONTAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.20.005.06		1.20.005.02		1.20.005.04		1.20.005.03	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	1699.	.0	1700.1	.0	1690.8	.0	1705.0	.0
1	2189.	-2.804	2189.0	-2.644	1938.1	-5.295	1960.8	-5.258
2	489.3	-8.988	4906.6	-8.736	405.3	6.966	397.5	7.331
3	.0	.0	.0	.0	.0	.0	.0	.0
4	14.1	121.6	14.34	134.4	12.66	-127.6	13.52	-121.4
5	.0	.0	.0	.0	.0	.0	.0	.0
6	4.47	12.08	48.84	1.614	42.24	-3.847	41.74	-3.674
7	42.4	-149.2	.0	.0	.0	.0	.0	.0
8	75.0	179.6	43.14	-173.4	49.60	-176.4	46.26	177.7
9	40.5	-33.75	.0	.0	.0	.0	.0	.0
10	28.9	-77.86	4.78	-39.02	2.82	-2.957	2.21	-3.058
11	23.8	104.5	.0	.0	.0	.0	.0	.0

TABLE 4-d

HORIZONTAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	1.30.00.73		1.30.005.79		1.30.01.76			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	2697.	.0	2472.	.0	2329.	.0		
1	2839.	-9.741	2333.	-10.57	2152.	-15.30		
2	284.	-159.4	441.	-143.2	679.	-122.0		
3	82.2	-43.64	.0	.0	.0	.0		
4	150.	38.30	82.9	5.769	275.	-8.431		
5	158.	94.23	.0	.0	.0	.0		
6	161.	134.5	82.4	83.67	442.	70.24		
7	365.	-169.1	.0	.0	.0	.0		
8	195.	90.88	113.5	-46.84	311.	71.06		
9	27.9	149.6	.0	.0	.0	.0		
10	4.04	-170.6	11.6	94.48	43.3	-166.0		
11	9.18	-148.5	.0	.0	.0	.0		

TABLE 4-e

HORIZONTAL BLADE ROOT SHEARS - AMPLITUDE AND PHASE (N/DEGREES)

CASE	3.20.00.05		3.20.005.01		1.20.01.03		1.20.005.05	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	2372.	.0	2300.	.0	1469.7	.0	1698.	.0
1	3470.	1.810	3390.	1.236	1526.2	-6.488	1950.5	-5.278
2	184.	7.086	271.	-140.3	366.6	26.73	400.5	7.096
3	17.7	158.6	.0	.0	.0	.0	.0	.0
4	67.9	95.55	422.	29.03	31.21	-74.96	10.08	-100.6
5	111.	153.5	.0	.0	.0	.0	.0	.0
6	48.5	-95.87	58.4	69.89	22.90	-98.36	18.77	-96.20
7	92.4	-61.38	.0	.0	23.77	-79.90	30.27	-115.6
8	50.5	-46.16	20.7	17.98	73.84	-165.9	78.60	-178.7
9	42.1	28.37	.0	.0	50.40	-19.96	43.18	-31.32
10	20.3	-2.902	37.4	47.23	20.72	-81.83	27.94	-77.16
11	28.7	113.1	.0	.0	25.19	117.1	25.94	106.4

TABLE 5-a

HARMONICS OF TIP JET ANGLE REQUIRED TO SUPPRESS SHEARS - AMPLITUDE AND PHASE (DEGREES/DEGREES)

CASE	1.20.02.51		1.20.01.52C		1.20.005.52C			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	.0	.0	.0	.0	.0	.0		
1	24.09	10.11	15.98	18.17	9.958	25.24		
2	3.003	-172.5	4.788	166.7	7.621	162.7		
3	15.04	155.2	35.42	144.6	45.57	145.1		
4	3.356	-60.32	6.961	80.58	9.153	-75.8		
5	4.965	-68.63	7.306	-41.90	8.332	-29.52		
6	9.352	-123.5	13.27	-123.6	17.26	-126.8		
7	5.358	34.91	4.910	33.03	6.208	35.40		
8	.9364	50.88	1.765	51.33	1.935	25.59		
9	3.286	10.98	6.431	-45.93	11.48	-53.62		
10	1.820	152.6	1.710	154.4	1.876	159.2		
11	1.501	162.9	1.466	114.2	2.342	84.72		

TABLE 5-b

HARMONICS OF TIP JET ANGLE REQUIRED TO SUPPRESS SHEARS - AMPLITUDE AND PHASE (DEGREES/DEGREES)

CASE	1.20.004.01		1.20.003.R1		1.20.002.R1		1.20.001.C1	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0.	.0	.0	.0	.0	.0	.0	.0	.0
1	11.63	25.27	8.508	36.58	5.917	66.52	10.21	135.4
2	17.35	32.73	21.47	20.18	26.64	8.766	39.30	-3.459
3	1.947	151.6	2.489	136.1	2.705	130.9	3.750	129.7
4	1.692	5.944	1.205	-2.774	1.456	5.748	2.081	11.75
5	1.531	-28.43	2.271	40.27	2.840	-38.12	4.268	-35.59
6	2.915	170.9	2.892	165.0	3.580	165.7	5.221	165.01
7	1.715	-174.6	2.007	168.6	1.852	176.2	1.935	175.4
8	1.049	163.1	1.837	176.3	2.362	175.1	3.454	175.8
9	3.320	-26.88	4.327	34.22	5.222	-36.21	7.357	-38.82
10	2.084	-76.66	2.389	-54.10	2.978	-54.16	4.334	-54.18
11	3.111	115.1	4.384	115.7	5.269	114.5	7.431	113.1

TABLE 5-c

HARMONICS OF TIP JET ANGLE REQUIRED TO SUPPRESS SHEARS - AMPLITUDE AND PHASE (DEGREES/DEGREES)

CASE	1.20.005.06		1.20.005.02		1.20.005.04		1.20.005.03	
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	.0	.0	.0	.0	.0	.0	.0	.0
1	.0	.0	.0	.0	13.32	20.65	14.28	21.76
2	19.66	7.182	19.93	7.454	15.60	40.73	15.66	42.45
3	2.395	130.6	2.529	130.1	1.897	155.2	1.953	153.7
4	.5901	-24.67	.5303	-21.20	1.578	5.317	1.577	4.634
5	2.297	-31.95	2.250	-36.97	1.337	-30.78	1.336	-30.43
6	.0	.0	2.374	153.4	2.540	169.1	2.530	168.8
7	.0	.0	2.737	173.8	1.481	176.6	1.595	170.5
8	.0	.0	1.385	-165.1	.8751	163.7	.8840	165.5
9	.0	.0	3.474	-32.93	3.231	-25.86	3.140	-24.26
10	.0	.0	1.502	-57.80	1.740	-77.59	1.771	-76.69
11	.0	.0	3.104	116.6	2.795	117.7	2.761	-118.2

TABLE 5-d

HARMONICS OF TIP JET ANGLE REQUIRED TO SUPPRESS SHEARS - AMPLITUDE AND PHASE (DEGREES/DEGREES)

CASE	1.30.005.79		1.30.01.76					
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	.0	.0	.0	.0				
1	24.80	-25.05	22.16	-24.25				
2	.0	.0	.0	.0				
3	2.526	111.9	7.172	151.6				
4	2.693	109.3	3.175	109.1				
5	2.575	11.12	4.711	5.796				
6	1.848	27.33	6.988	-59.24				
7	11.52	96.31	39.97	10.65				
8	.9835	-84.39	4.414	125.4				
9	1.693	90.40	1.169	61.00				
10	.4003	-93.83	.9559	-123.2				
11	.8346	-49.97	2.180	-62.13				

TABLE 5-e

HARMONICS OF TIP JET ANGLE REQUIRED TO SUPPRESS SHEARS - AMPLITUDE AND PHASE (DEGREES/DEGREES)

CASE	3.20.005.01		1.20.01.03		1.20.005.05			
HARMONIC	AMP	PHASE	AMP	PHASE	AMP	PHASE	AMP	PHASE
0	.0	.0	.0	.0	.0	.0		
1	18.61	-71.51	20.03	19.96	13.73	21.04		
2	20.46	-139.3	13.76	68.81	15.53	41.71		
3	3.516	-31.79	2.403	154.1	1.901	156.5		
4	16.07	81.50	1.152	3.088	1.534	7.391		
5	6.657	77.29	1.049	-31.13	1.454	-27.82		
6	6.062	-78.19	.0	.0	.0	.0		
7	5.984	-107.9	.0	.0	.0	.0		
8	3.179	68.67	.0	.0	.0	.0		
9	4.440	-24.94	.0	.0	.0	.0		
10	3.009	-138.7	.0	.0	.0	.0		
11	5.039	112.9	.0	.0	.0	.0		

TABLE 6

TOTAL POWER REQUIRED TO SUPPRESS ALL SHEARS FOR SEVERAL VALUES OF THRUST RECOVERY FACTOR (T_R)

T_R	TOTAL POWER	
	HP	WATTS
1.0	920.5	$.6864 \times 10^6$
0.5	924.0	$.6890 \times 10^6$
0	927.4	$.6916 \times 10^6$

TABLE 7

TOTAL POWER REQUIRED TO SUPPRESS ALL SHEARS, EXCEPT 2P, AT ADVANCE RATIO = 0.30

C_{J_T}	TOTAL POWER	
	HP	WATTS
0	1274	$.9508 \times 10^6$
.005	1308.2	$.9755 \times 10^6$
.010	2551.4	1.9026×10^6

TABLE 8

PERFORMANCE VALUES FOR FOUR BLADED RIGID ROTOR AND FOUR BLADED FLAPPING ROTOR

	Rigid Rotor	Flapping Rotor
$F_{X\ ROTOR}$	0	0
$F_{Y\ ROTOR}$	6551.9 N (-1473 lbs)	6596.4 N (-1483 lbs)
$F_{Z\ ROTOR}$	89182.4 N (20,050 lbs)	89191.3 N (20,052 lbs)
RM_{ROTOR}	-62966.2 N-m (-48250 ft-lbs)	3411.3 N-m (2614 ft-lbs)
PM_{ROTOR}	-15931.4 N-m (-12208 ft-lbs)	1166.1 N-m (893.6 ft-lbs)
POWER	1.6316×10^6 watts (2188 HP)	1.6420×10^6 watts (2202 HP)
POWER (JET ON) (C_{JT_0} =.005)	1.8083×10^6 watts (2425 HP)	Not analysed for C_{JT_0} = 0.005

NOTE: (1) The rigid rotor CG/thrust offset corresponding to the PM rctor noted in 0.612 ft.

(2) Remember when comparing the 4-bladed and 2-bladed results that not only was the total vehicle weight increased by a factor of 2, but also the effective fuselage flatplate drag was increased by a factor of 2 ($8.18\,m^2 \times 2 = 16.36\,m^2$).

TABLE 9

COMPARISON OF POWER REQUIREMENTS TO SUPPRESS ALL AND ONLY FIRST FIVE HARMONICS OF THE NON-CANCELLING BLADE ROOT SHEARS

CASE ID	HARMONICS SUPPRESSED	TOTAL POWER	
		H.P.	WATTS x 10^{-6}
1.20.00.09	None	892.	.6652
1.20.005.01	ALL	927.3	.6915
1.20.005.05	2 to 5	924.4	.6893
1.20.01.01	ALL	969.5	.7230
1.20.01.03	2 to 5	968.2	.7220
1.20.03.08	ALL	1310.3	.9771
1.20.03.06	2 to 6	1290.	.9620

TABLE 10

TOTAL POWER REQUIRED TO SUPPRESS SPECIAL CONSTRAINT CASES

CASE IDENTIFICATION	TOTAL POWER	
	HP	WATTS
1.20.005.01	927.3	$.6915 \times 10^6$
1.20.005.02	929.6	$.6932 \times 10^6$
1.20.005.05	924.4	$.6893 \times 10^6$
1.20.005.06	929.1	$.6928 \times 10^6$

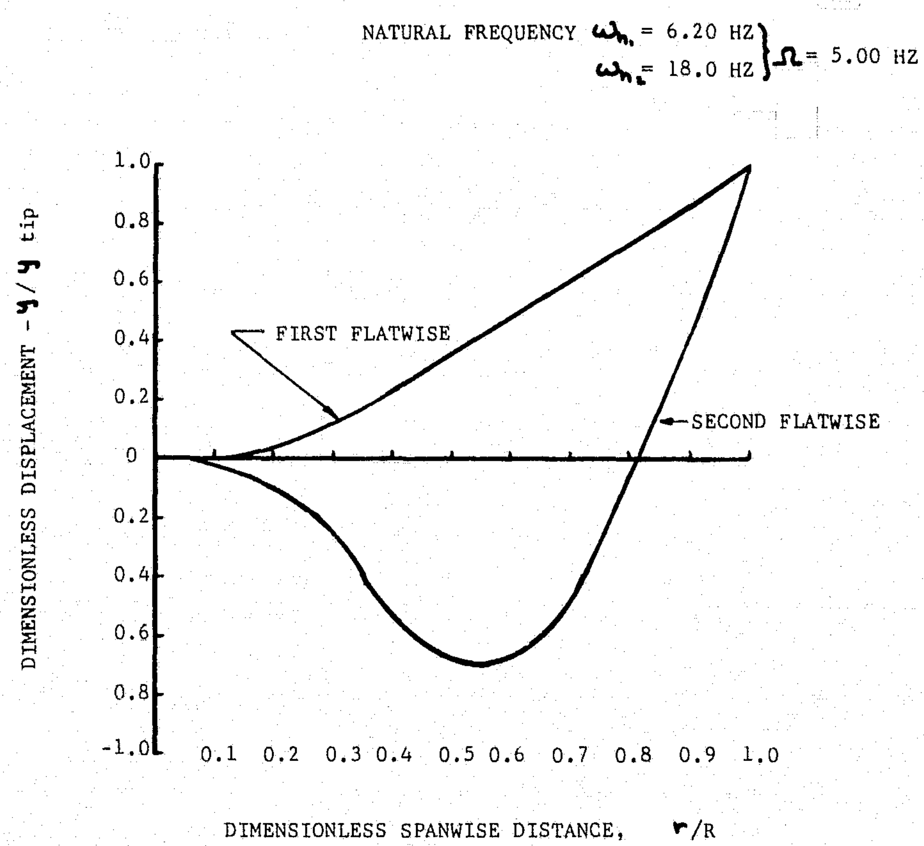


FIGURE 1. CANTILEVER FLATWISE BENDING MODE SHAPES FOR RIGID ROTOR (Configuration 3)

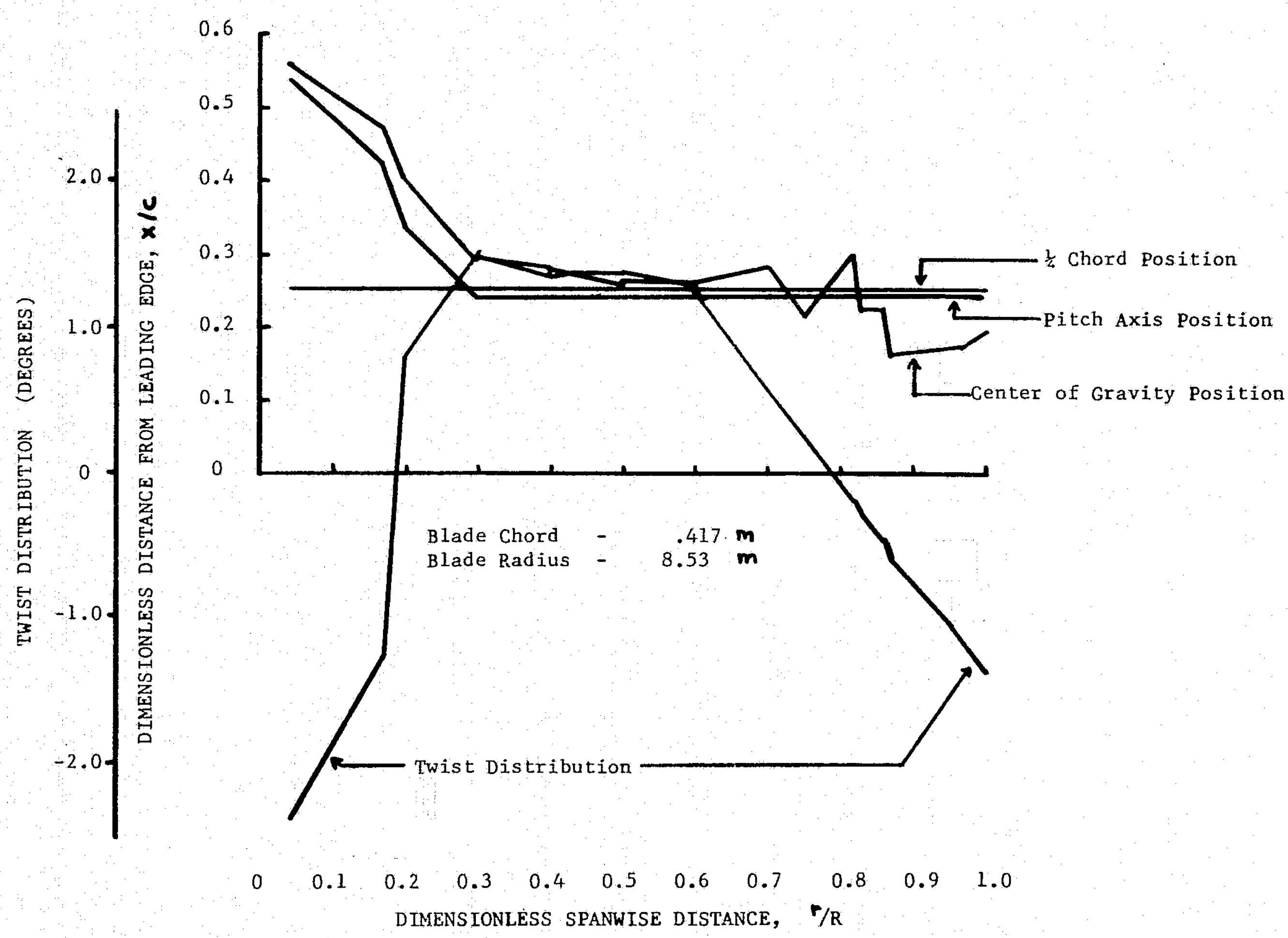


FIGURE 2. GEOMETRIC PROPERTIES FOR CONFIGURATION 4

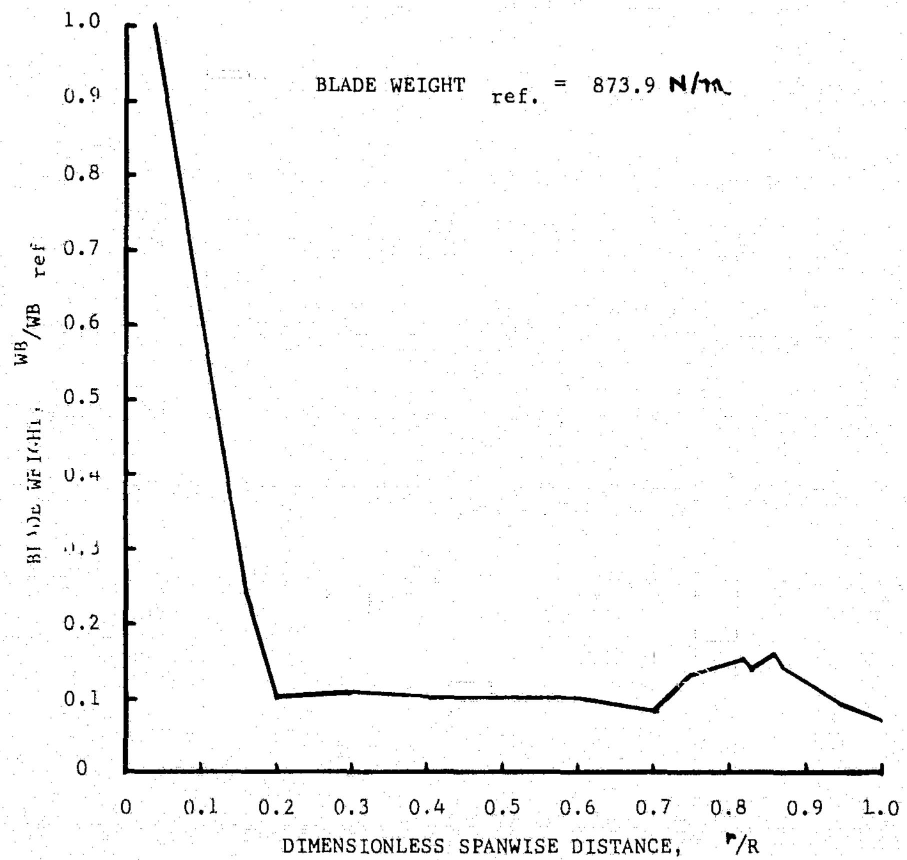


FIGURE 3. BLADE WEIGHT DISTRIBUTION FOR CONFIGURATION 4

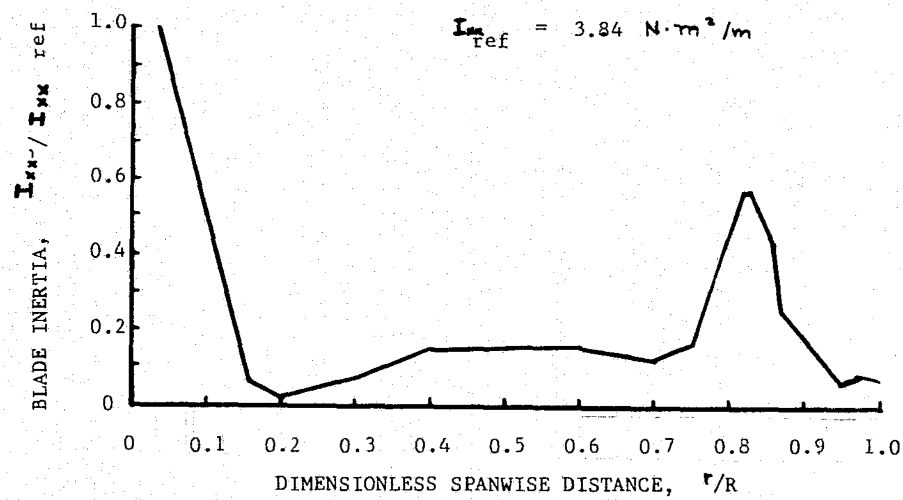


FIGURE 4. BLADE PITCH INERTA DISTRIBUTION FOR CONFIGURATION 4

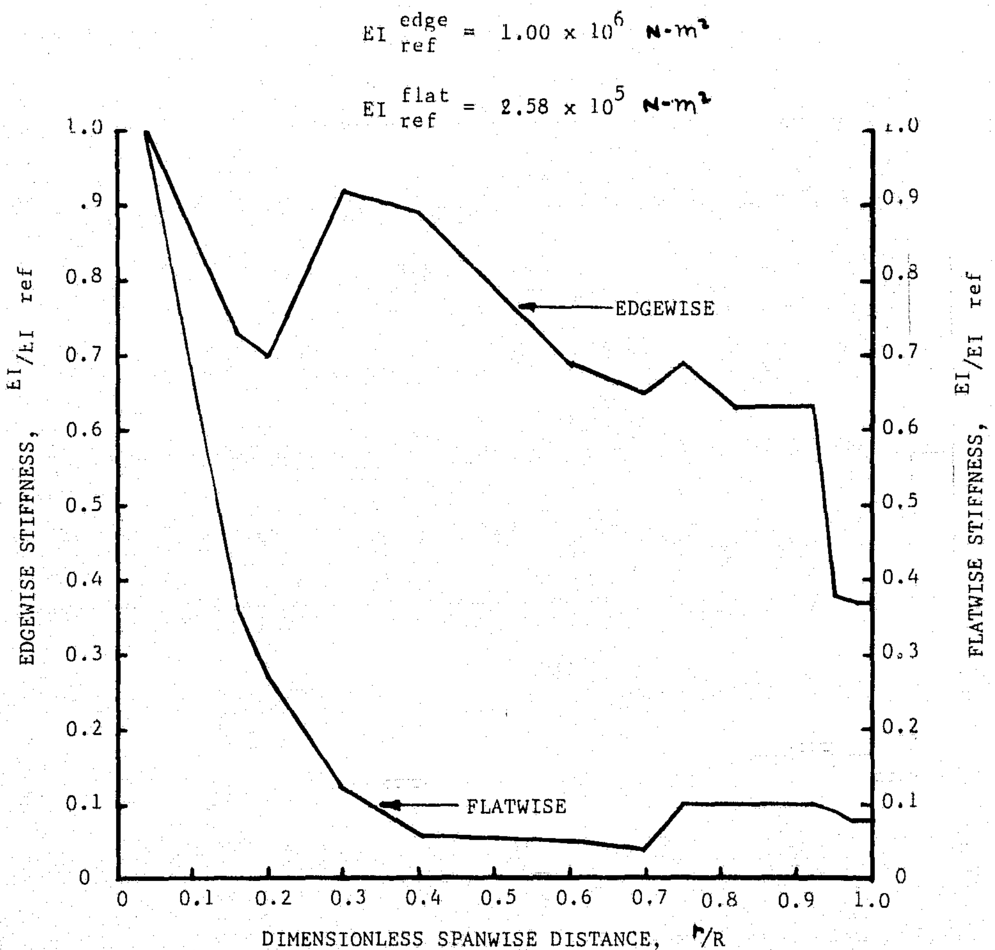


FIGURE 5. BLADE BENDING STIFFNESS DISTRIBUTION FOR CONFIGURATION 4

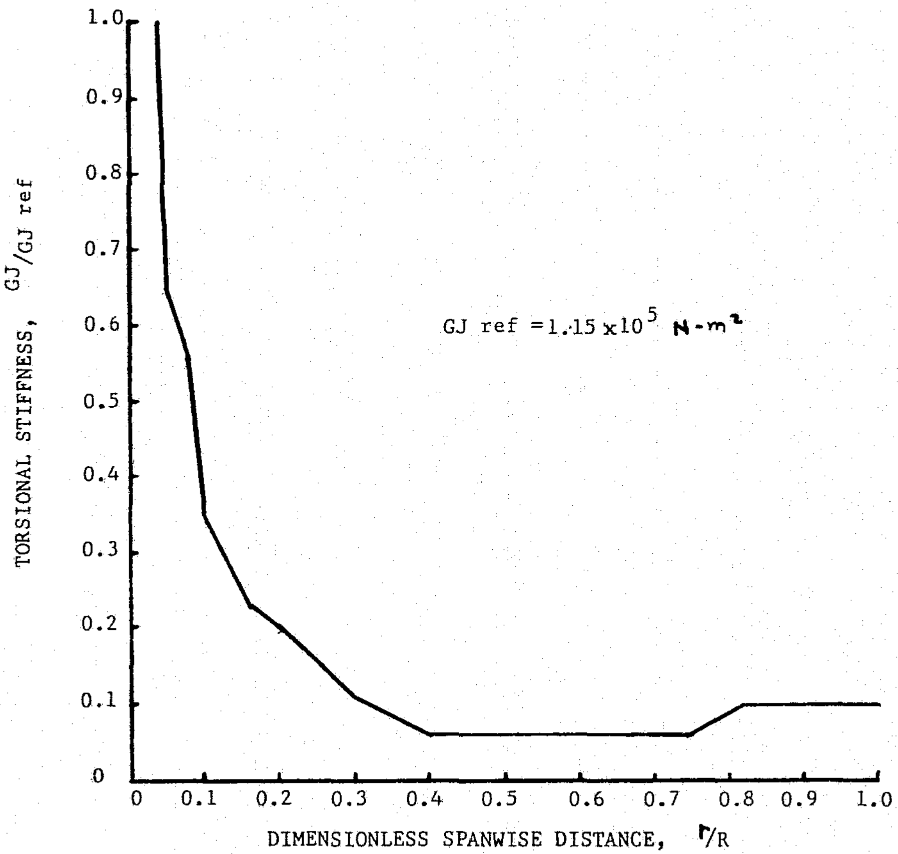


FIGURE 6. BLADE TORSIONAL STIFFNESS DISTRIBUTION FOR CONFIGURATION 4

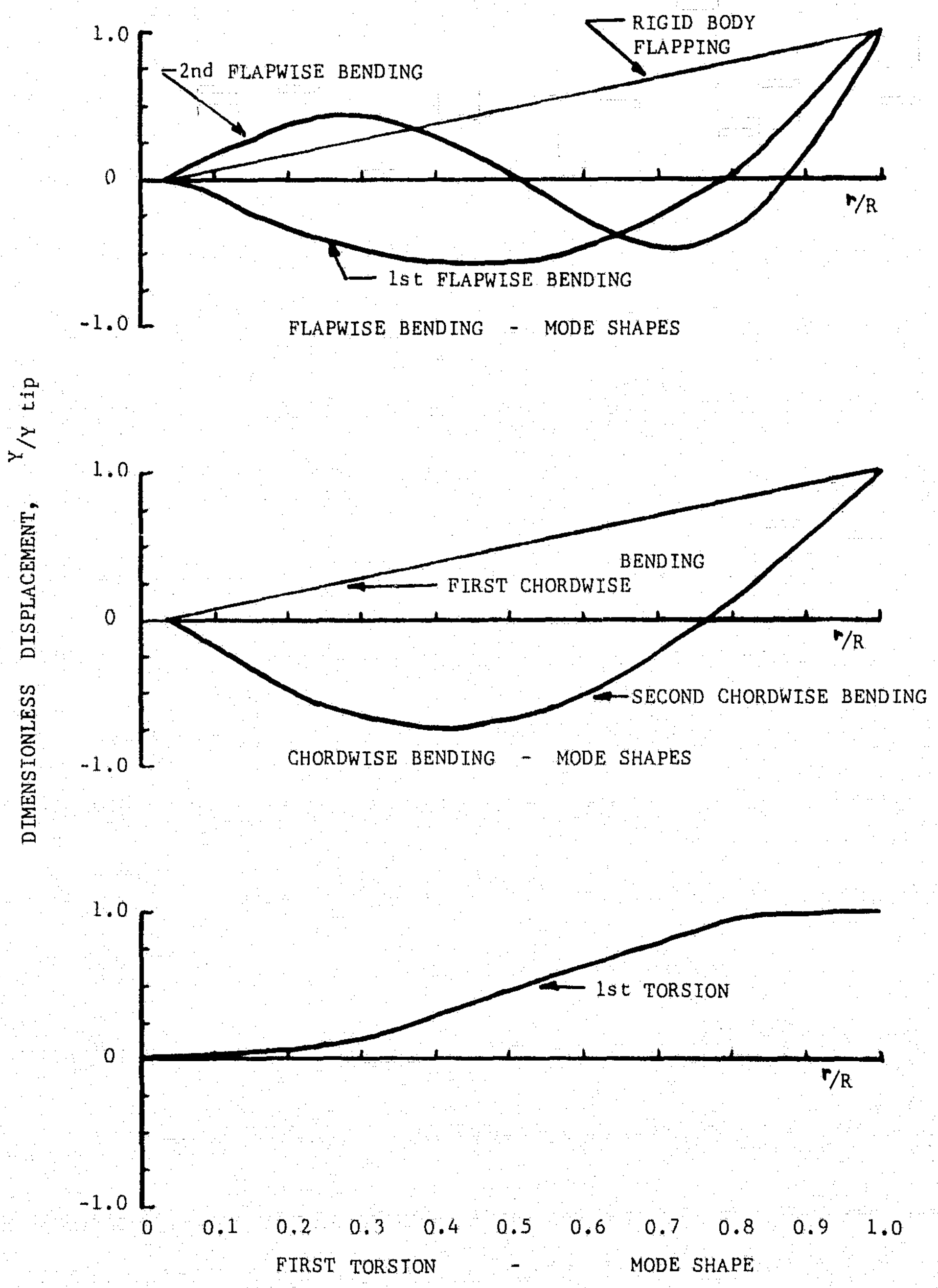


FIGURE 7. MODE SHAPES FOR CONFIGURATION 4

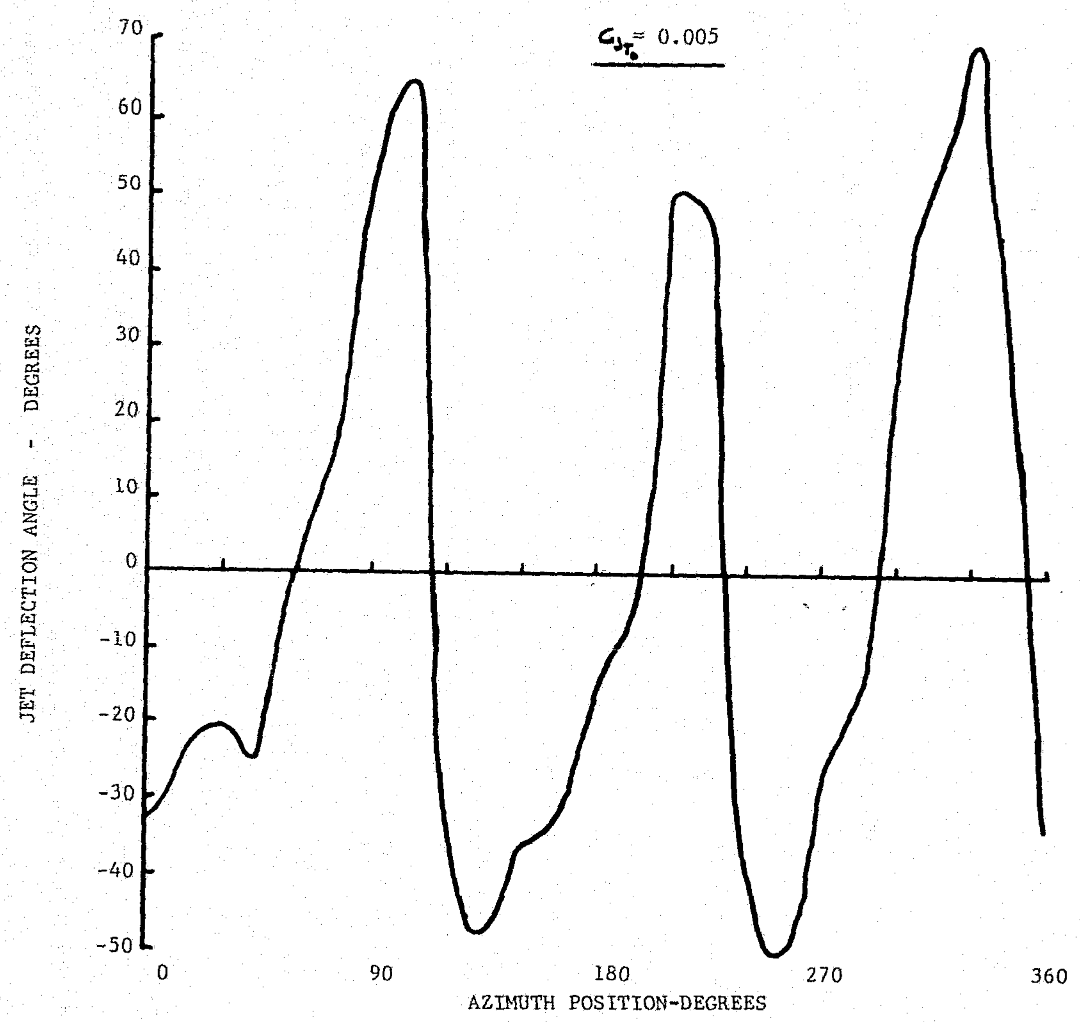


FIGURE 8a AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL TRANSMITTED SHEARS TO ZERO FOR TORSIONALLY STIFF BLADE AT $\mu = 0.20$ AND VARIOUS $C_{J_{T_0}}$.

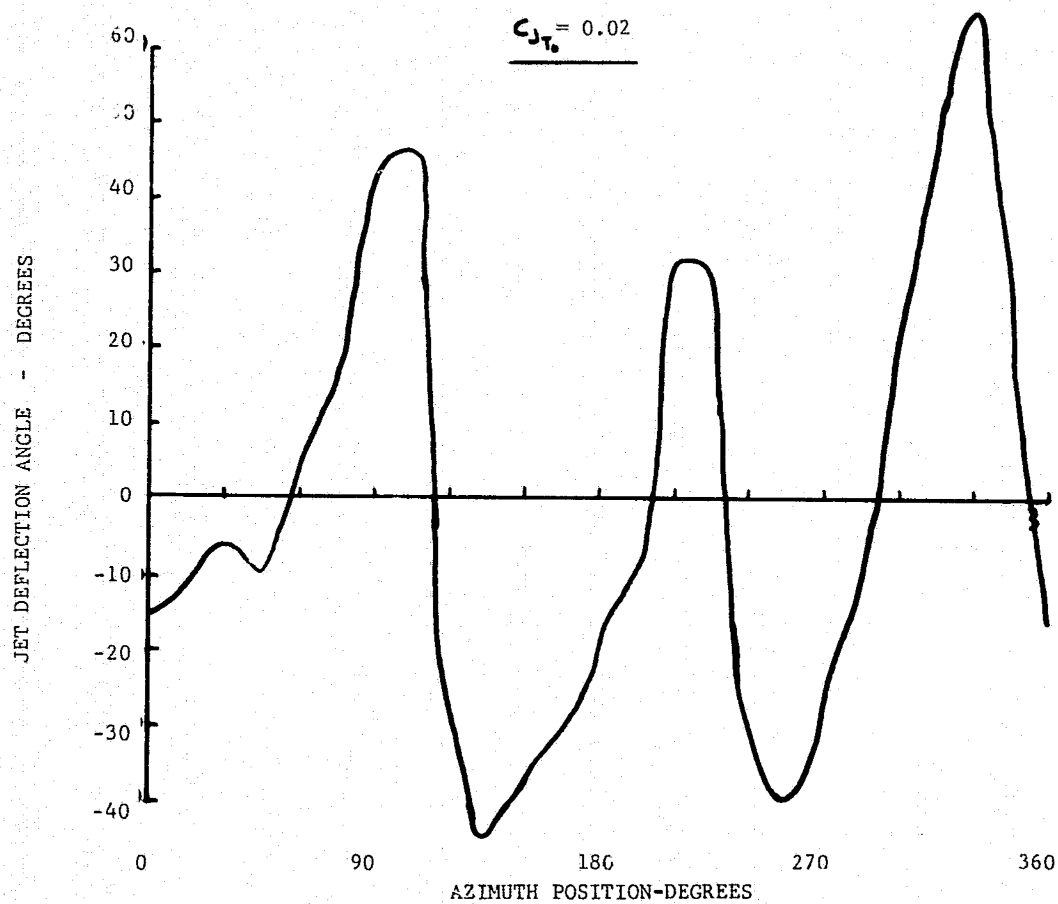


FIGURE 8b

CONTINUED

$C_{J_{T_0}} = 0.01$

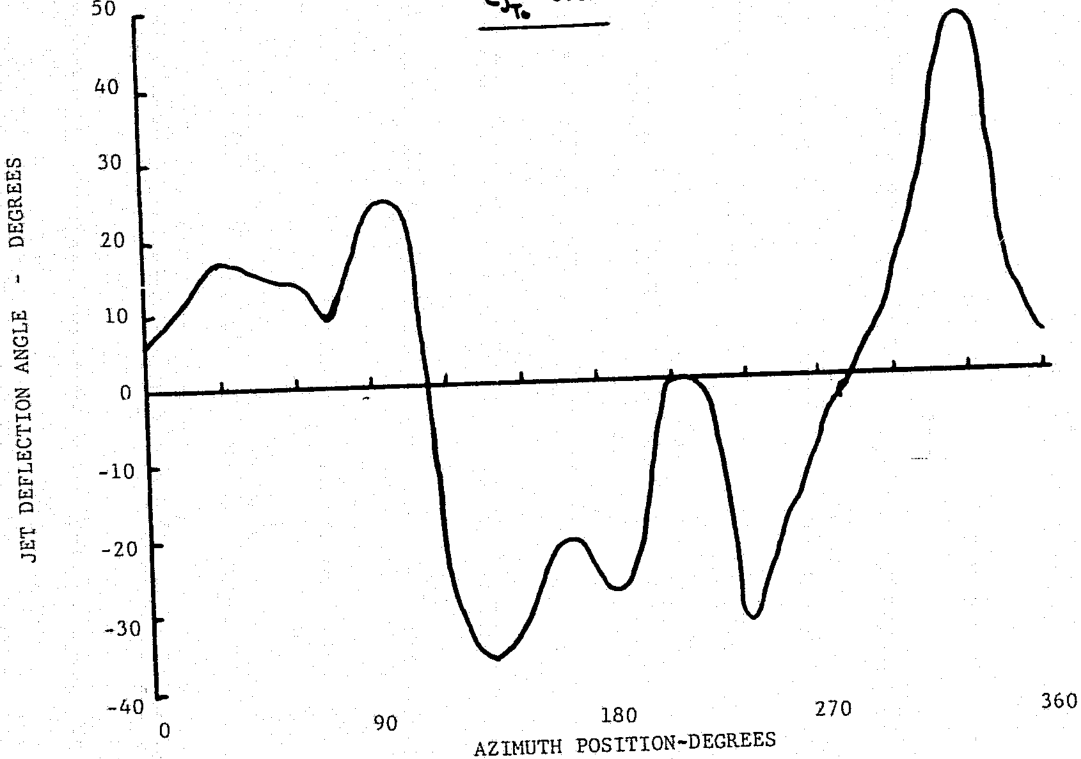


FIGURE 8c CONCLUDED

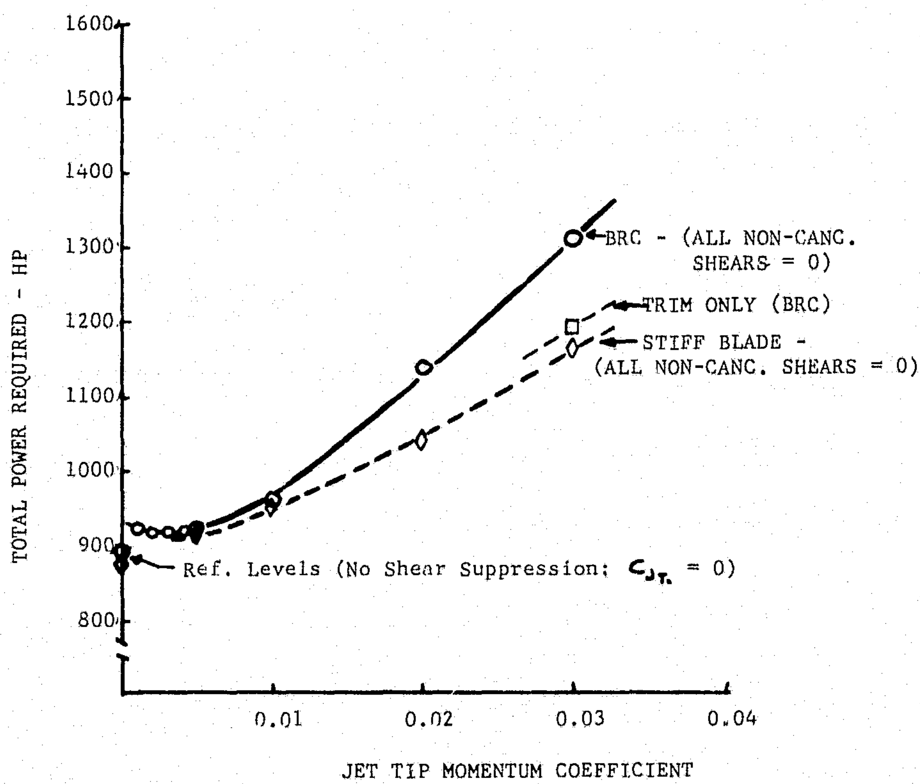


FIGURE 9. TOTAL POWER REQUIRED vs JET TIP MOMENTUM COEFFICIENT AT μ = 0.20

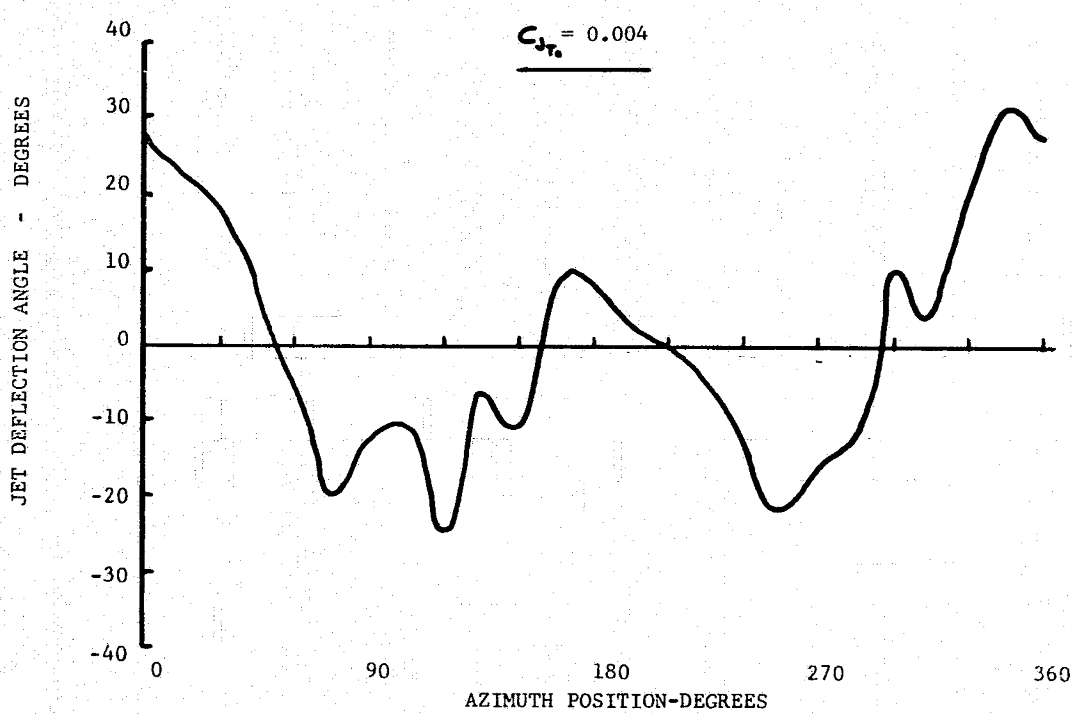


FIGURE 10a AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL TRANSMITTED SHEARS TO ZERO FOR μ = 0.20, AND VARIOUS $C_{J_{T_0}}$ FOR BASIC ROTOR CONFIGURATION.

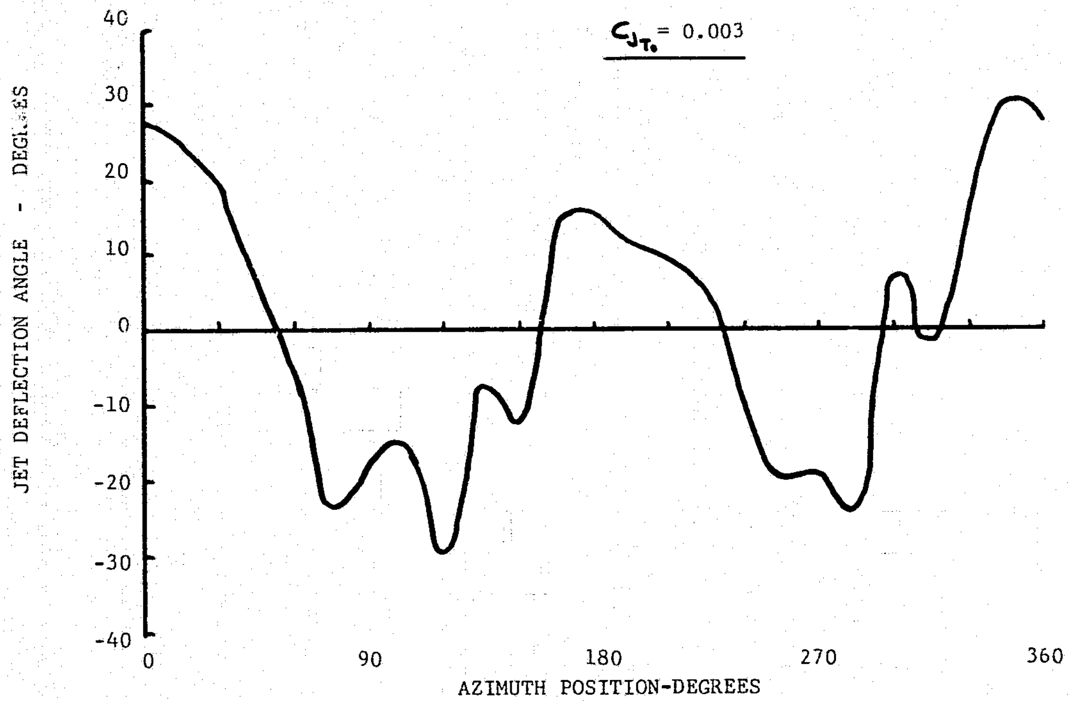


FIGURE 10b

CONTINUED

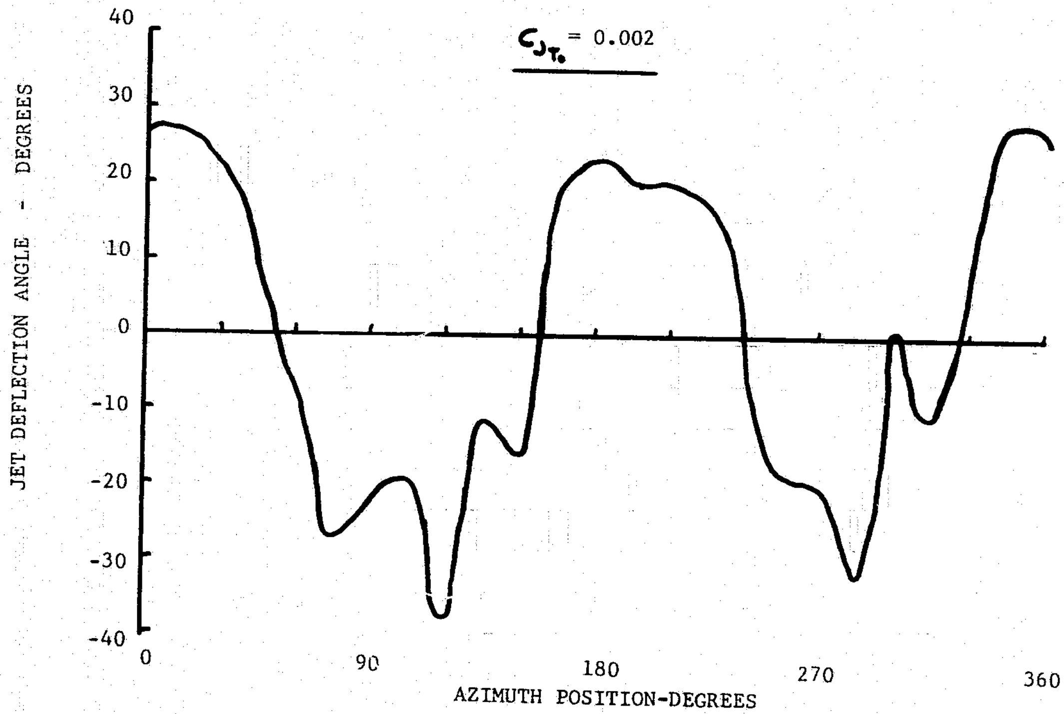


FIGURE 10c

CONTINUED

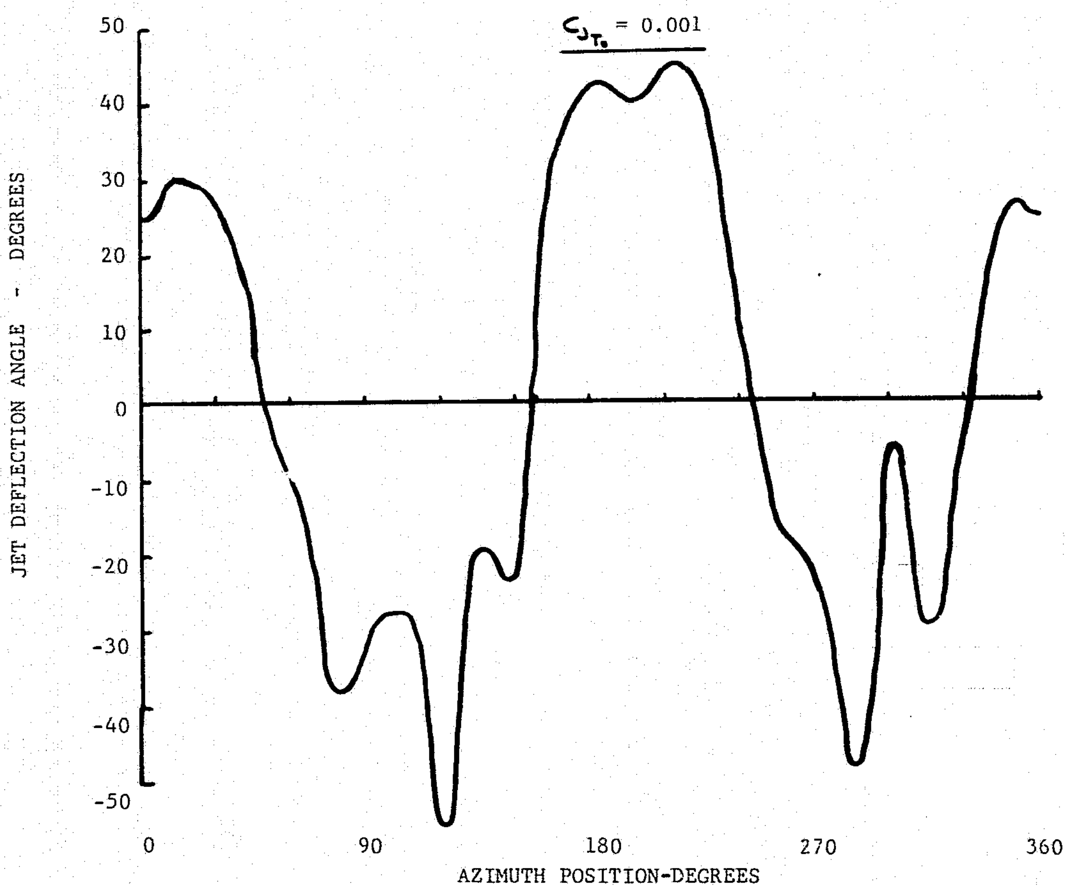


FIGURE 10d CONCLUDED

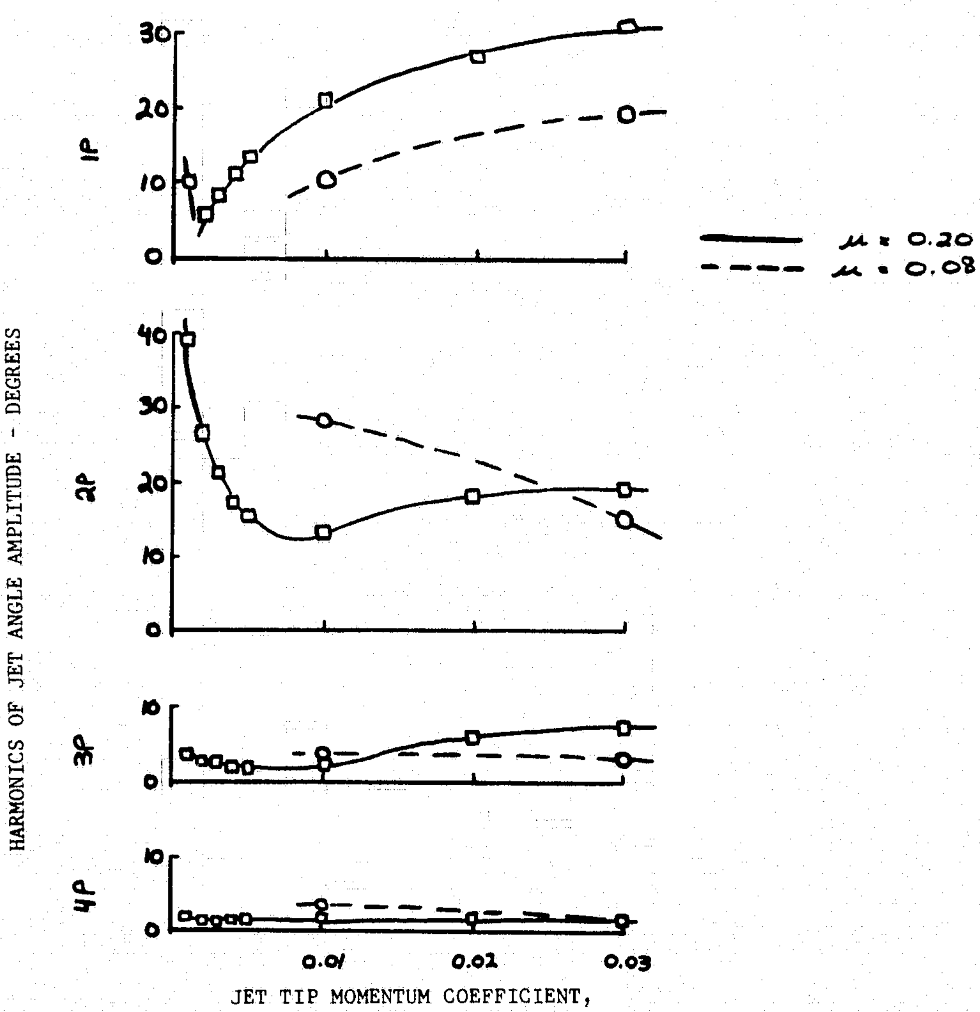


FIGURE 11a HARMONICS OF JET ANGLE AMPLITUDE REQUIRED TO SUPPRESS ALL TRANSMITTED SHEARS vs JET TIP MOMENTUM COEFFICIENT AT μ = 0.08, 0.20.

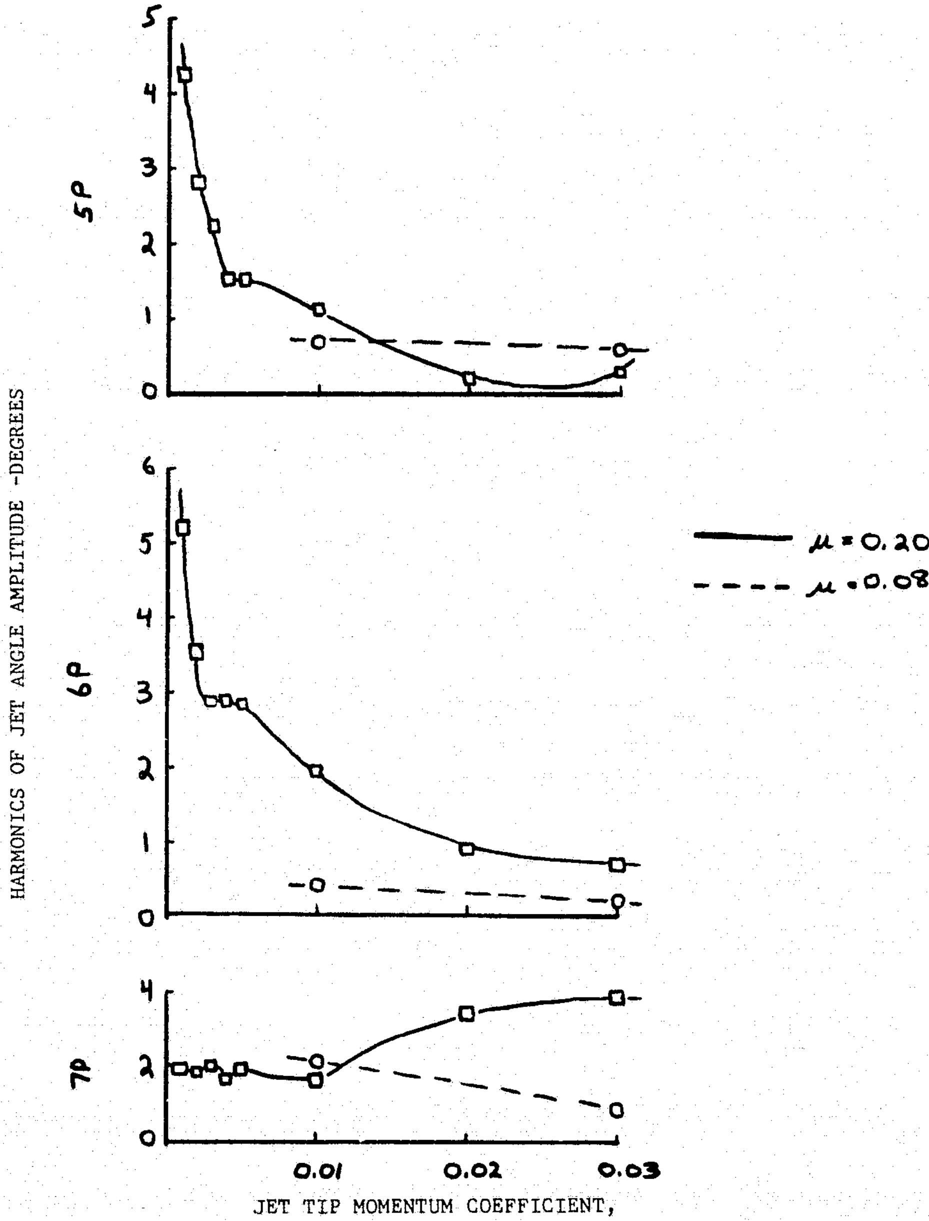


FIGURE 11b CONTINUED

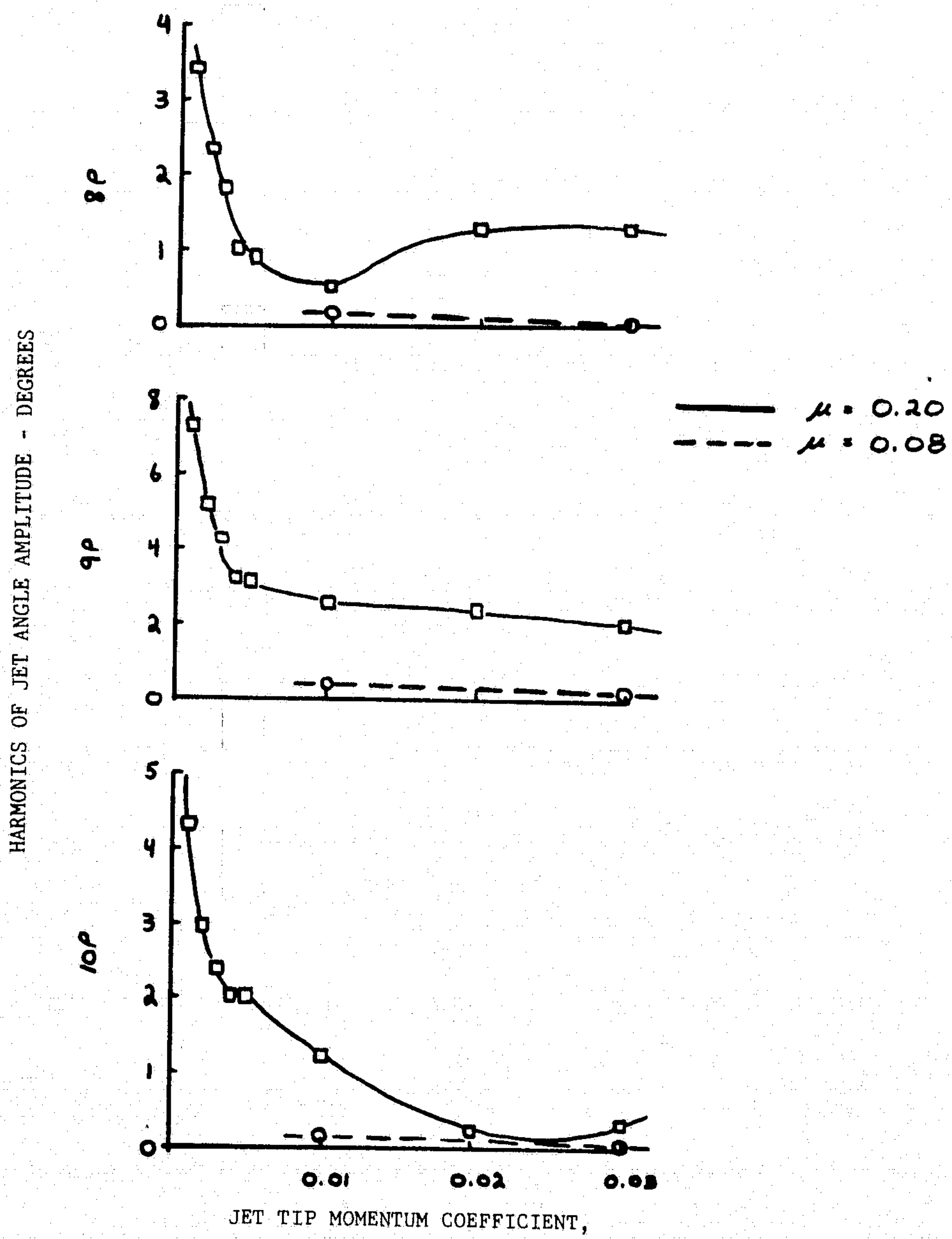
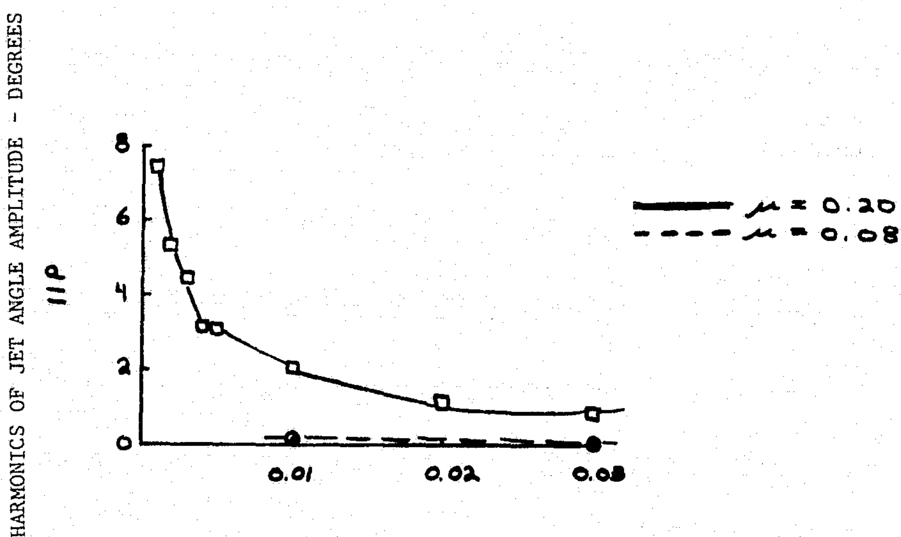


FIGURE 11c CONTINUED



JET TIP MOMENTUM COEFFICIENT,

FIGURE 11d CONCLUDED

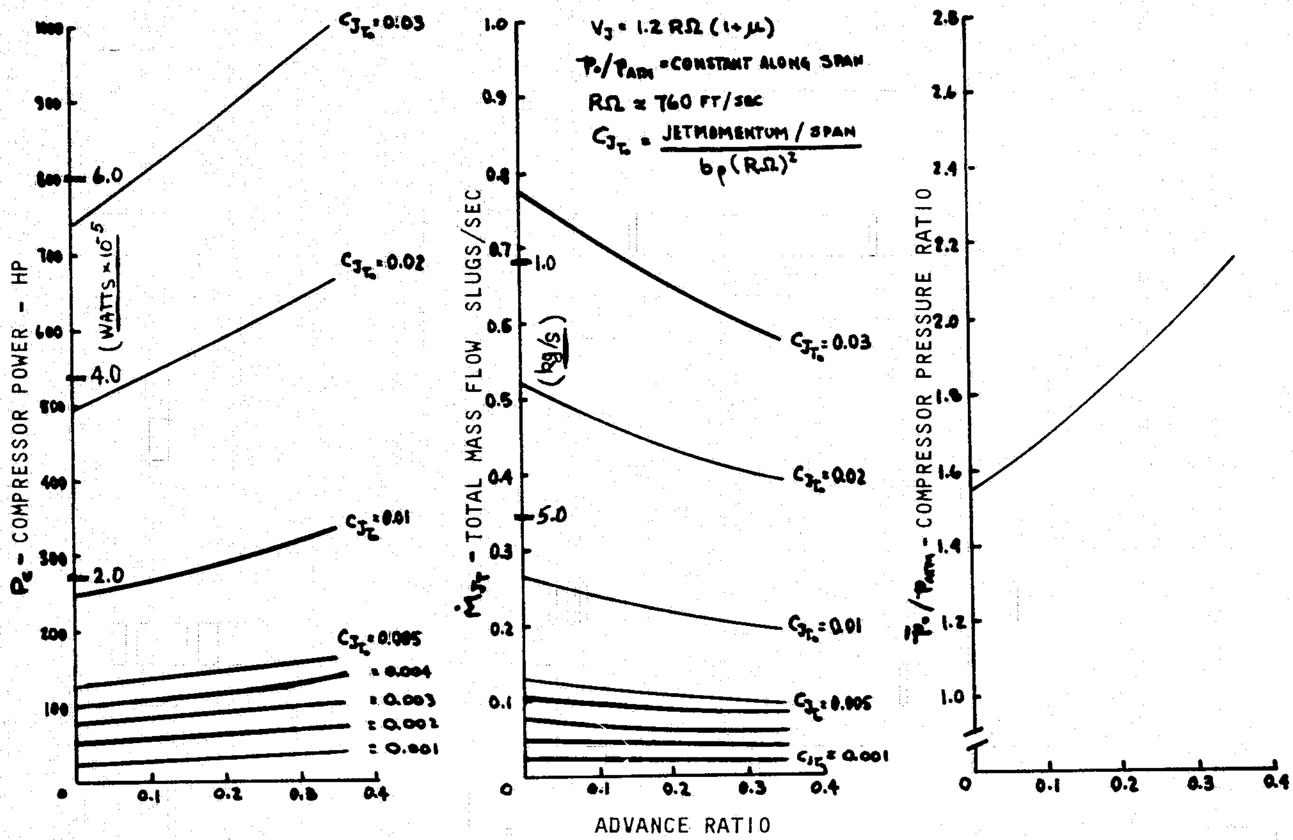


Figure 12 . JET PRESSURE RATIO, VELOCITY AND COMPRESSOR POWER PLOTS.

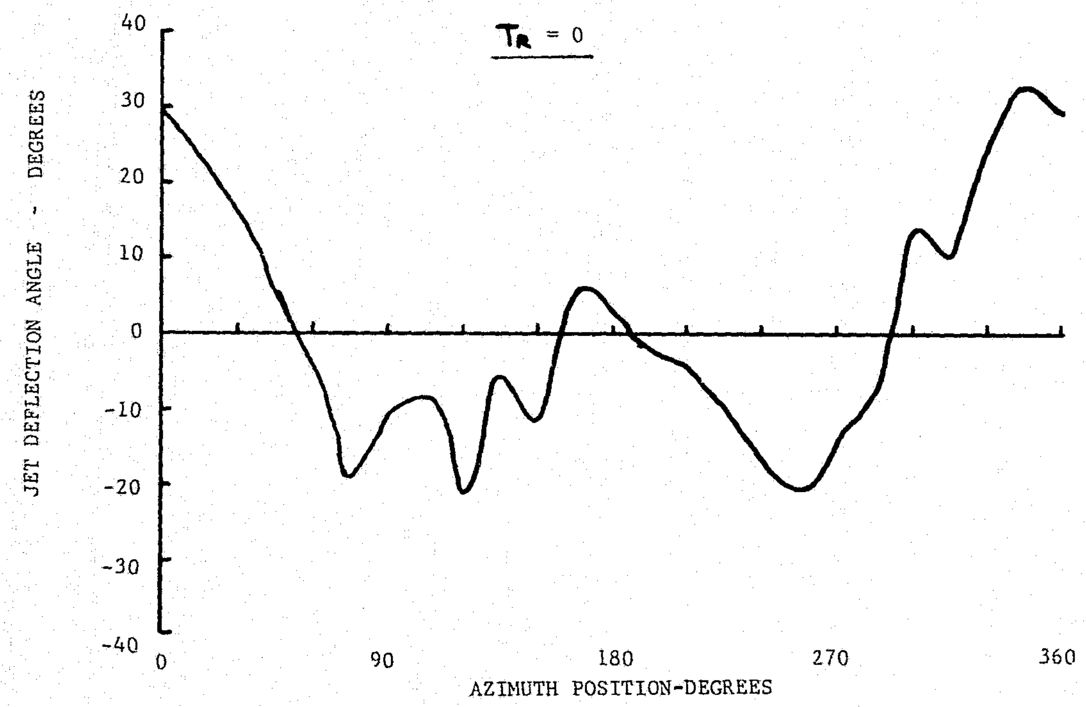


FIGURE 13a AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL TRANSMITTED SHEARS TO ZERO FOR μ = 0.20, $C_{J_{T_0}}$ = 0.005 FOR VARIOUS THRUST RECOVERY FACTORS

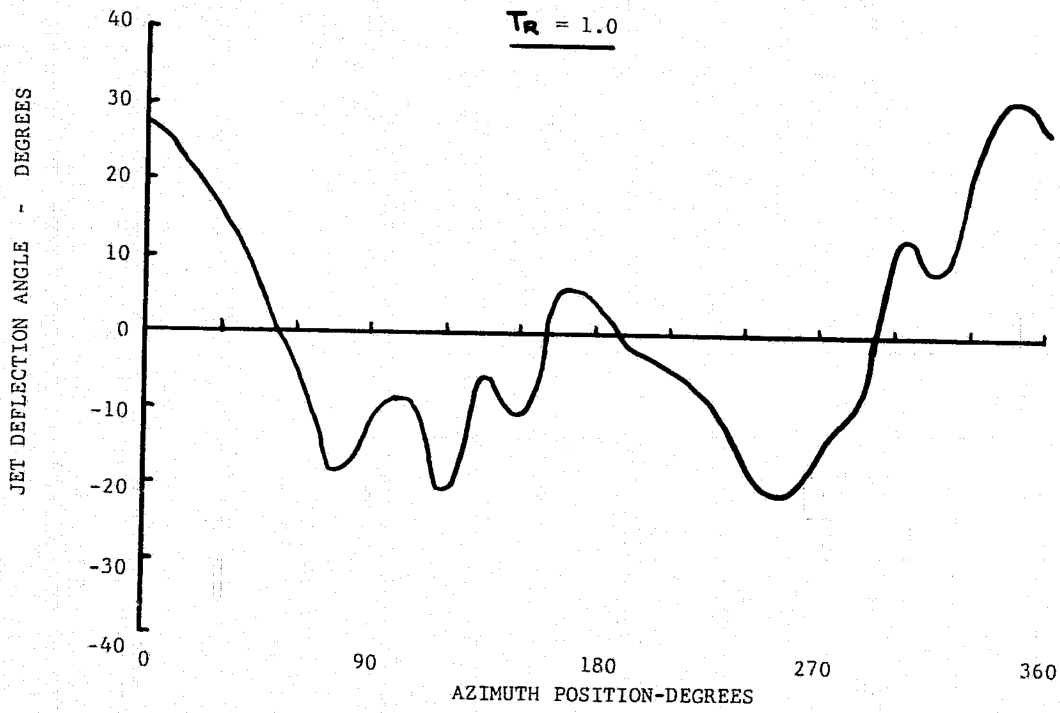


FIGURE 13b

CONTINUED

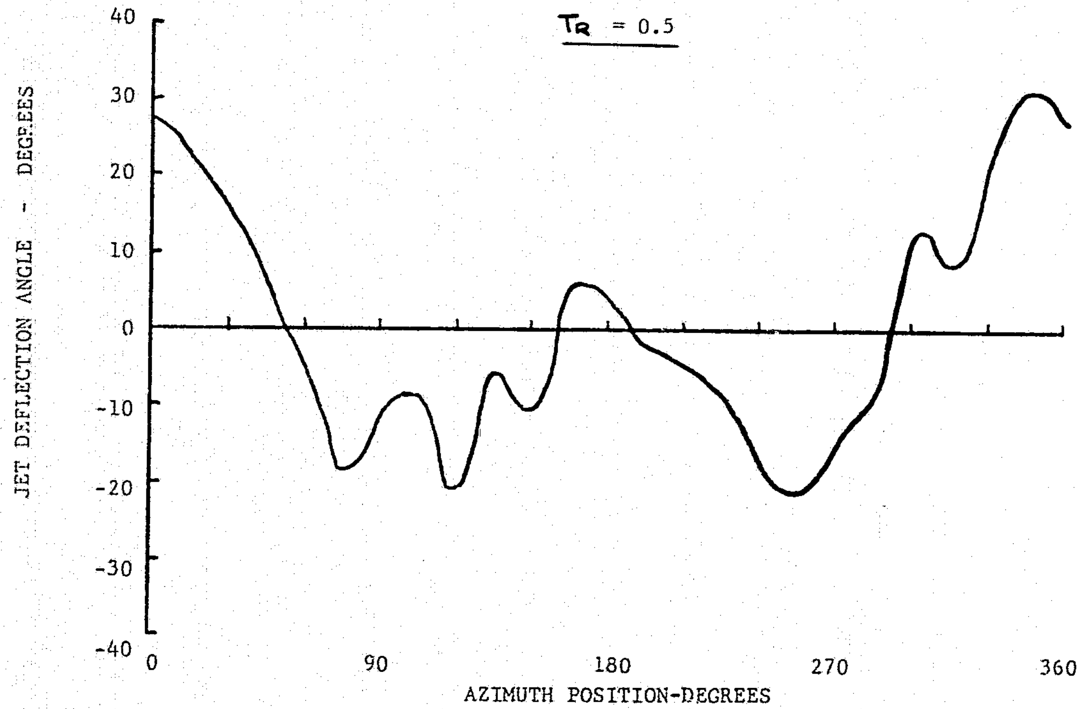


FIGURE 13c

CONCLUDED

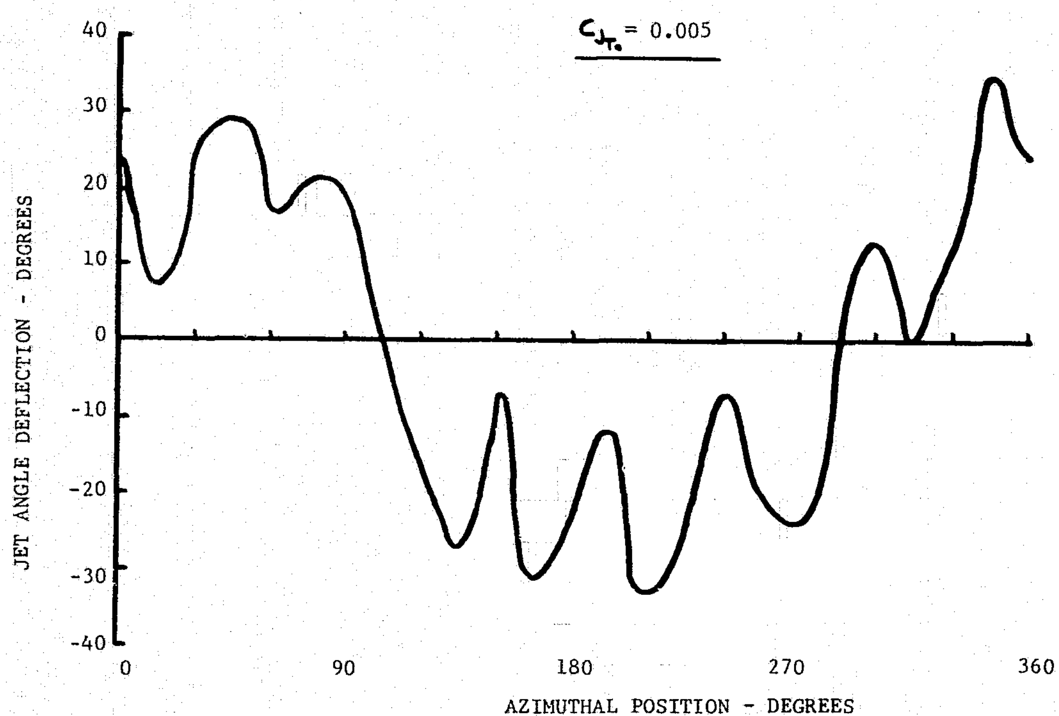


FIGURE 14a AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL TRANSMITTED SHEARS, EXCEPT 2P, TO ZERO FOR μ = 0.30, AND FOR VARIOUS $C_{J_{T_0}}$

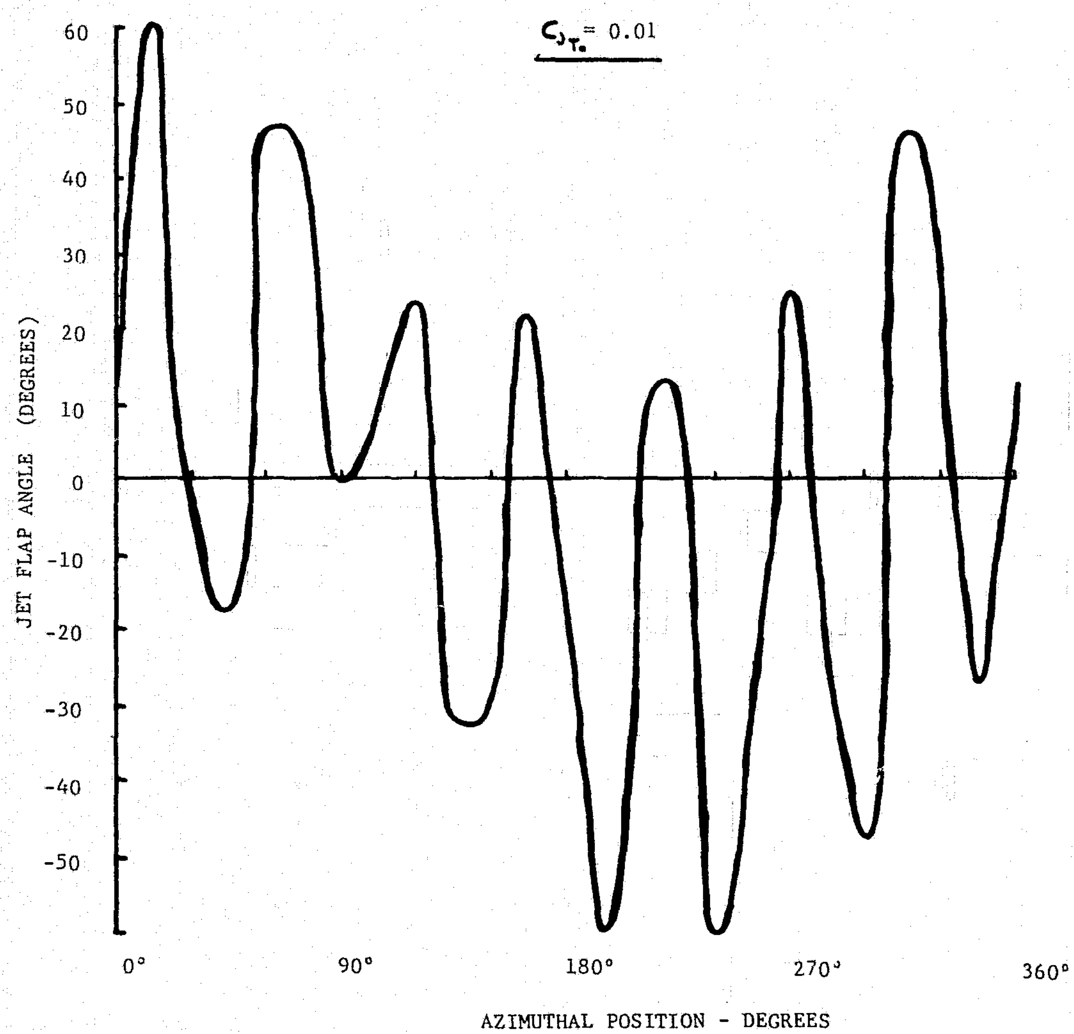


FIGURE 14b

CONCLUDED

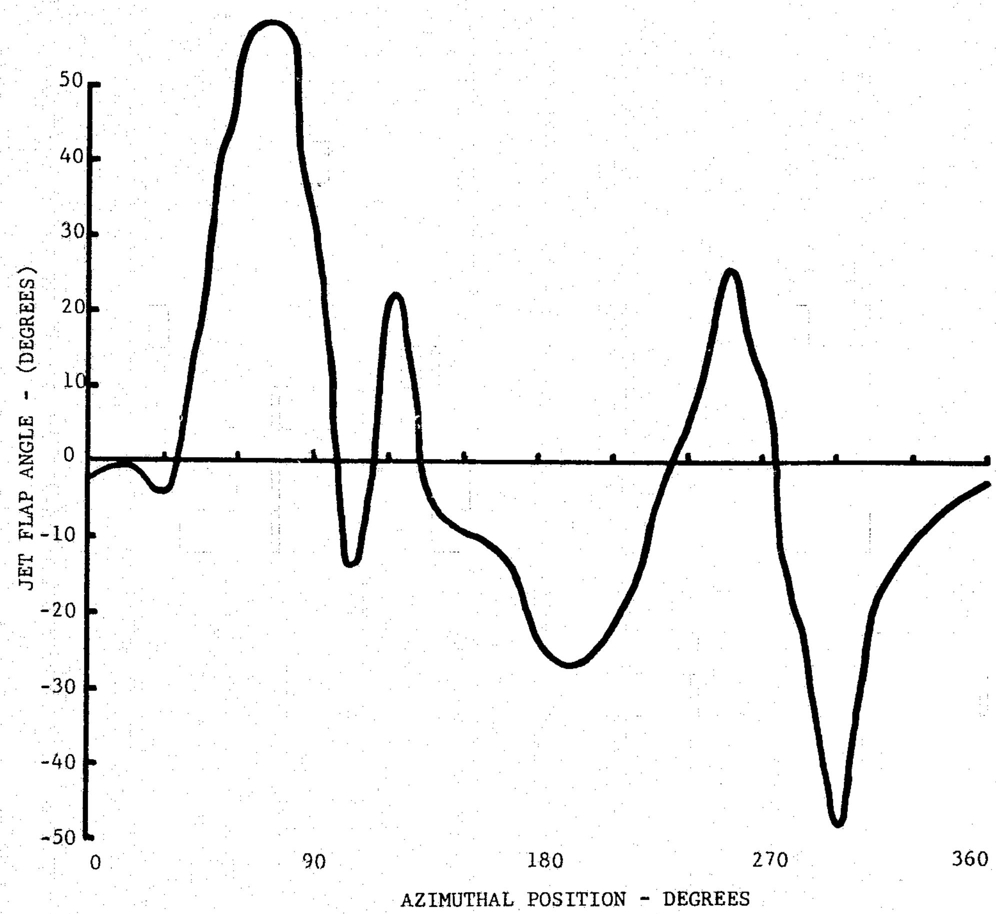


FIGURE 15 AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL TRANSMITTED SHEARS TO ZERO FOR A 4-BLADED RIGID ROTOR AT μ = 0.20, $C_{J_{T_0}} = 0.005$

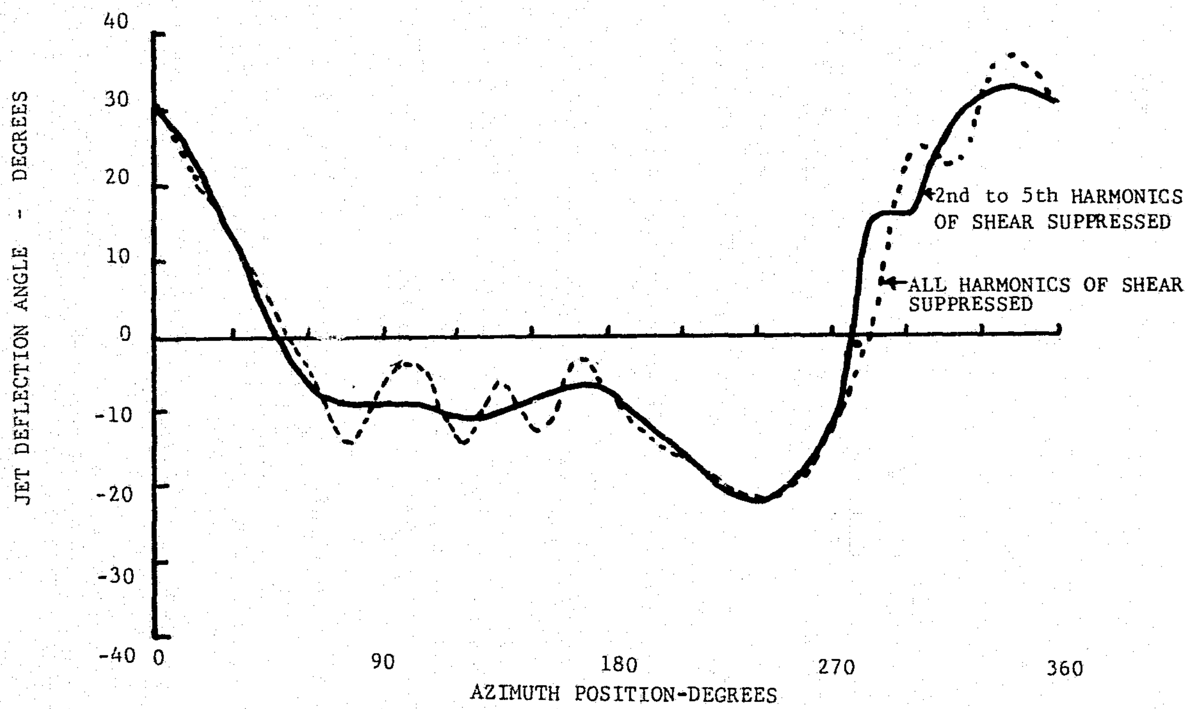


FIGURE 16 AZIMUTHAL VARIATION OF REQUIRED JET ANGLE FOR ALL AND PARTIAL SHEAR SUPPRESSION AT $\mu = 0.2$, $C_{J_{T_0}} = 0.01$

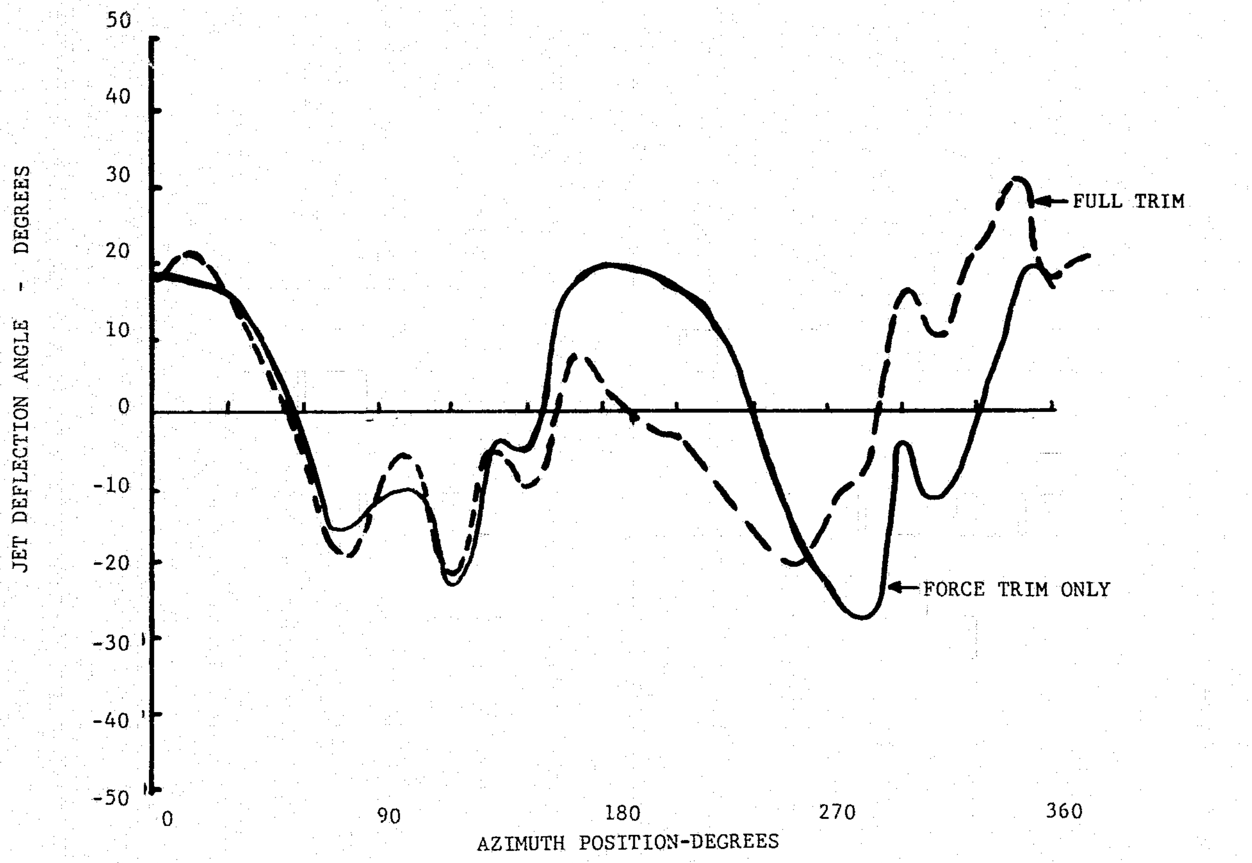


FIGURE 17. AZIMUTHAL VARIATION OF REQUIRED JET ANGLE TO SUPPRESS ALL SHEARS; COMPARISON OF FULL TRIM AND FORCE TRIM ONLY REQUIRED AT $\mu = 0.20$, $C_{J_{T_0}} = 0.005$

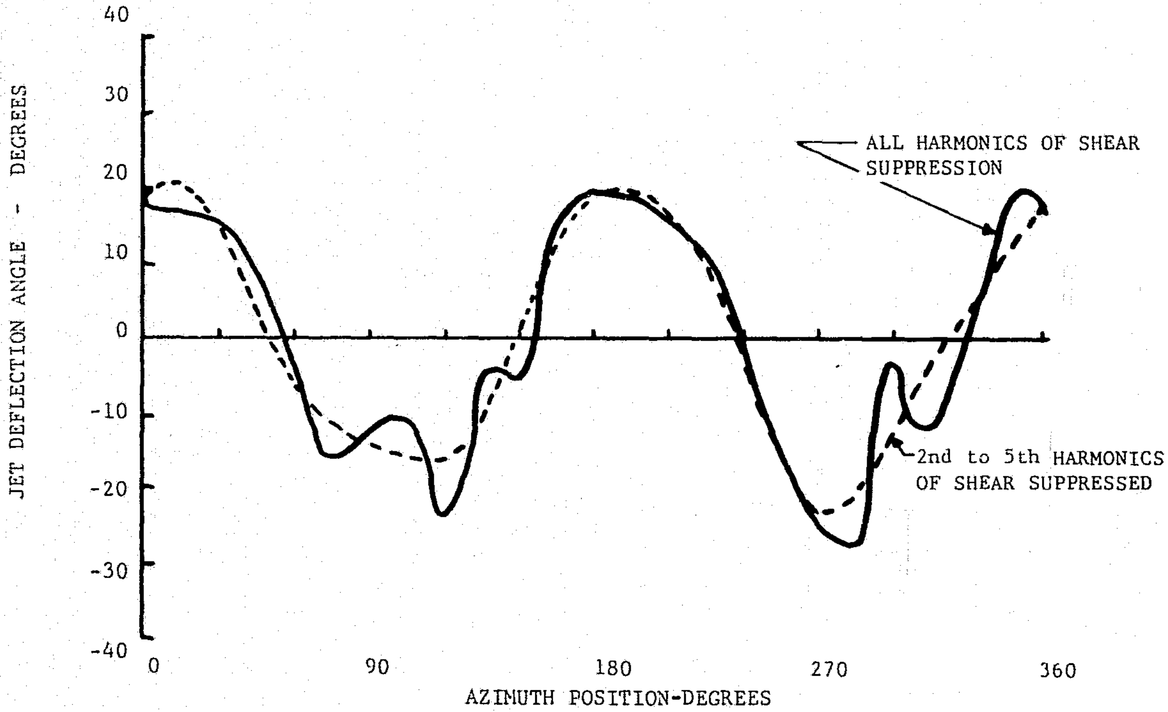


FIGURE 18. AZIMUTHAL VARIATION OF REQUIRED JET ANGLE WITH ONLY FORCE TRIM MAINTAINED: COMPARISON OF PARTIAL AND FULL SHEAR SUPPRESSION AT $\mu = 0.20$, $C_{J_{T_0}} = 0.005$

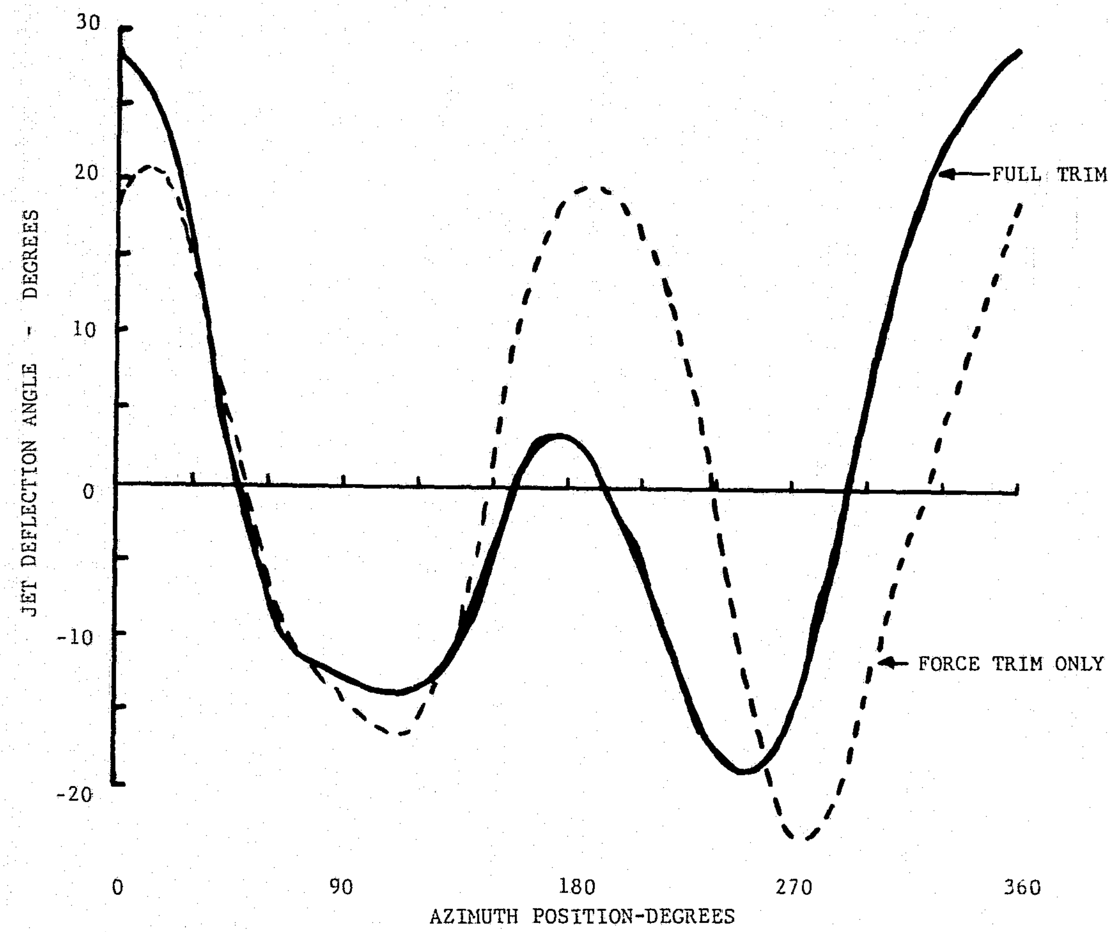


FIGURE 19. AZIMUTHAL VARIATION OF REQUIRED JET ANGLE FOR PARTIAL SHEAR SUPPRESSION; COMPARISON OF FULL TRIM WITH FORCE TRIM ONLY MAINTAINED AT $\mu = 0.20$, $C_{J_{T_0}} = 0.005$