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SPACECRAFT SELF-CONTAMINATION DUE TO BACK-SCATTERING OF OUTGAS PRODUCTS

INTERIM REPORT

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FOREWORD

This document is an interim report describing results of the first ten months of effort on Contract NAS8-31440, "Space Shuttle Contamination Due to Backflow from Control Motor Exhaust." The contract was initiated on 10 April 1975 as a 10-month effort to develop analytical techniques for predicting the back-scattering of outgas contamination from an orbiting spacecraft.

The Scope of Work of the contract has been modified to include additional tasks and the period of performance extended for an additional nine months of effort. The final report is being deferred until the end of the extended period of performance.

This study was performed by personnel of the Lockheed Missiles & Space Company, Inc., Huntsville Research & Engineering Center, for the Space Sciences Laboratory of NASA-Marshall Space Flight Center. The NASA technical monitor for the study is Dr. R. J. Naumann. Dr. Naumann made significant contributions to this study through his own research efforts and through enlightening discussions with the author. His interest and assistance are greatly appreciated.

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Section 1
INTRODUCTION AND SUMMARY

It has long been recognized that the quality and resolution of optical astronomical observations are limited by the disturbing effects induced during passage of light through the earth's atmosphere. For this reason, measurements have been planned to be performed on board orbiting spacecraft at altitudes where the disturbing influence of the earth's atmosphere is negligible. Unfortunately, even in this near-vacuum environment, optical measurements can be degraded by the atmosphere induced by the spacecraft and instrumentation packages. A primary concern is the condensation and adsorption of outgas products on the various optical surfaces (lenses, mirrors, etc.) such that the optical properties of these surfaces are altered. Extremely small amounts of contamination can significantly affect the transmissivity and reflectivity of these optical surfaces, thus seriously degrading the intended astronomical measurements.

Outgas products can reach sensitive optical surfaces by: (1) direct flow from sources of outgas contamination; (2) scattering off spacecraft surfaces; or (3) back-scattering due to intermolecular collisions. The first two modes of transport are rather easily analyzed to obtain contamination flow estimates. The third mode is somewhat more difficult to analyze because of the complicated collision mechanics.

This study is addressed to the analysis of back-scattering of outgas contamination due to intermolecular collisions. The purpose is to develop analytical tools for making reasonably accurate quantitative estimates of outgas contamination return flux, given a knowledge of the pertinent spacecraft and orbit conditions. Two basic collision mechanisms were considered: (1) collisions involving only outgas molecules (self-scattering); and (2) collisions between outgas molecules and molecules in the ambient atmosphere (ambient-scattering). For simplicity, the geometry was idealized to a uniformly outgassing sphere and to a disk oriented normal to the freestream. The method of solution involved an integration of an approximation of the Boltzmann kinetic equation known as the BGK (or Krook) model equation (Ref. 1). Results were obtained in the form of simple equations relating outgas return flux to spacecraft and orbit parameters. Results are compared with previous analyses based on more simplistic models of the collision processes.

Section 2
MODEL DEVELOPMENT

The two basic mechanisms involved in back-scattering of outgas contamination are illustrated in Fig. 2-1. One mechanism involves collisions between the outward flowing outgas molecules such that some of the molecules are deflected back into the outgassing surface. The other mechanism involves collisions between the outgas molecules and molecules in the ambient atmosphere. The atmospheric molecules are moving at orbital velocities relative to the spacecraft and are thus highly energetic compared to the outgas molecules.

Molecular flow processes of this nature are described by the Boltzmann kinetic equation which describes the time and spatial rate of change in the velocity distribution functions as a result of intermolecular collisions. The full Boltzmann equation is given in Eq.(2.1) for one of the species in a binary (two species) gas:

$$\frac{\partial f_1}{\partial t} + \vec{v} \cdot \frac{\partial f_1}{\partial \vec{r}} = \iint \varphi_{11} v_{r_{11}} (F_1 F_1' - f_1 f_1') \, d^3 \vec{v}_1' \, d\omega + \iint \varphi_{12} v_{r_{12}} (F_1 F_2 - f_1 f_2) \, d^3 \vec{v}_2 \, d\omega \tag{2.1}$$

where the f's are velocity distribution functions for the separate species, the φ's are differential collision cross sections, and

$$v_{r_{11}} = \left| \vec{v}_1 - \vec{v}_1' \right| , \quad v_{r_{12}} = \left| \vec{v}_1 - \vec{v}_2 \right|$$

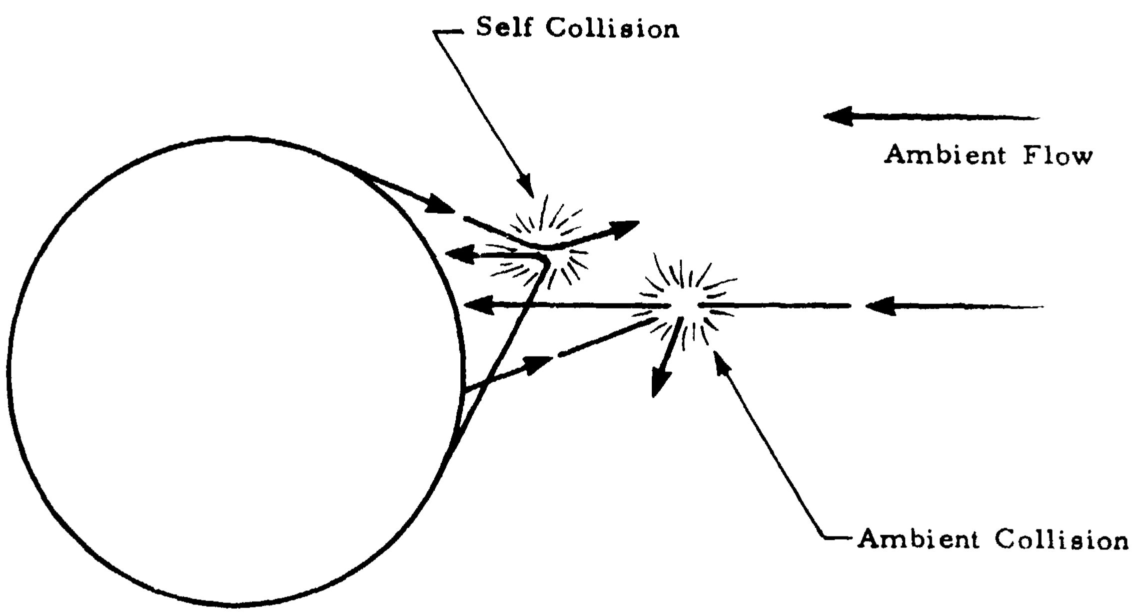


Fig. 2-1 - Return Flux Mechanism for Outgas Contamination

$$f_1 = f_1(\vec{v}_1), \quad f_1' = f_1(\vec{v}_1'), \quad F_1 = f_1(\vec{V}_1), \quad F_1' = f_1(\vec{V}_1')$$

$$f_2 = f_2(\vec{v}_2), \quad F_2 = f_2(\vec{V}_2)$$

The single subscripts "1" and "2" denote the individual species in the binary mixture. The double subscripts refer to collision parameters with the "11" subscripts referring to self-collisions and the "12" subscripts referring to cross collisions. The parameters in the equation include the molecular velocity vector $\vec{v}$, position vector $\vec{r}$ and time t. The primes refer to the "other" molecule in self-collisions. The capital letters refer to velocities after the collision. The relative velocities $v_{r_{11}}$ and $v_{r_{12}}$ refer to relative velocities between the colliding molecules prior to the collision. The solid angle element $d\omega$ contains the line of centers between the colliding molecules at the time of impact.

The left-hand side of Eq. (2.1) describes the rate of change in the distribution function, and the right-hand side contains the collision integrals which account for the effects of intermolecular collisions. When the right-hand side is zero, the flow is free molecular, and the left-hand side can be solved rather easily for a variety of problems. When the right-hand side is not zero, however, a rigorous solution would require an evaluation of the collision integrals at all points in the flow field, a formidable task which generally requires an enormous amount of computational effort. The only known exact solution of the Boltzmann equation with collisions is the Maxwellian distribution function toward which f relaxes at equilibrium.

In order to allow practical solutions of molecular flow processes with collisions, the BGK model equation (Ref. 1) was developed in which the collision integrals in the full Boltzmann equation were approximated by simple relaxation terms which simulate the relaxation of the full Boltzmann equation to the

Maxwellian equilibrium distribution. The BGK kinetic equations for a binary mixture are given as follows (Morse, Ref. 2):

$$\frac{\partial f_1}{\partial t} + \vec{v} \cdot \frac{\partial f_1}{\partial \vec{r}} = \nu_{11}(M_{11} - f_1) + \nu_{12}(M_{12} - f_1) \tag{2.2a}$$

$$\frac{\partial f_2}{\partial t} + \vec{v} \cdot \frac{\partial f_2}{\partial \vec{r}} = \nu_{22}(M_{22} - f_2) + \nu_{21}(M_{21} - f_2) \tag{2.2b}$$

where the ii subscripts indicate parameters for self-collisions, and the ij subscripts indicate cross-collisions. The symbol ν represents collision rates and M represents equilibrium Maxwellian distributions defined by

$$M_{ij} = \left(\frac{m_i}{2\pi k \Theta_{ij}}\right)^{3/2} n_i \exp\left[-\frac{m_i}{2k\Theta_{ij}}(\vec{v} - \vec{u}_{ij})^2\right] \tag{2.3}$$

By taking moments on Eqs. (2.2) for the conservation of mass, momentum and energy, the following relations are obtained for the model parameters:

$$n_i = \int f_i \, d^3 \vec{v} \tag{2.4a}$$

$$m_1 \vec{u}_{12} + m_2 \vec{u}_{21} = m_1 \vec{u}_1 + m_2 \vec{u}_2 \tag{2.4b}$$

$$n_1 \vec{u}_1 = n_1 \vec{u}_{11} = \int \vec{v} f_1 \, d^3 \vec{v} \tag{2.4c}$$

$$n_2 \vec{u}_2 = n_2 \vec{u}_{22} = \int \vec{u} f_2 \, d^3 \vec{v} \tag{2.4d}$$

$$\frac{3}{2} k \left(\Theta_{12} + \Theta_{21}\right) + \frac{m_1}{2} u_{12}^2 + \frac{m_2}{2} u_{21}^2$$

$$= \frac{3}{2} k \left(\Theta_1 + \Theta_2\right) + \frac{m_1}{2} u_1^2 + \frac{m_2}{2} u_2^2 \tag{2.4e}$$

$$\frac{3}{2}\ k n_1\, \Theta_1 = \frac{3}{2} k n_1 \Theta_{11} = \frac{1}{2} m_1 \int \left(v - u_1\right)^2 f_1\, d^3 v \qquad (2.4f)$$

$$\frac{3}{2}\ k n_2\, \Theta_2 = \frac{3}{2} k n_2 \Theta_{22} = \frac{1}{2} m_2 \int \left(\vec{v} - \vec{u}_2\right)^2 f_2\, d^3 \vec{v} \qquad (2.4g)$$

These conservation equations alone suffice to determine the self-collision parameters u_{ii} and Θ_{ii} in addition to the densities n_i. Additional constraints, however, are required to determine the cross-collision parameters u_{ij} and Θ_{ij}. Morse (Ref. 2) and Hamel (Ref. 3) required that the relaxation of momentum and temperature differences between the two species be the same as that derived from the full Boltzmann collision integral for Maxwell molecules. This leads to

$$\vec{u}_{12} = \vec{u}_{21} = (m_1\, \vec{u}_1 + m_2\, \vec{u}_2)/(m_1 + m_2) \qquad (2.5a)$$

$$\Theta_{12} = \Theta_1 + \frac{2m_1\, m_2}{(m_1 + m_2)^2} \left[(\Theta_2 - \Theta_1) + \frac{m_2}{6k} (\vec{u}_2 - \vec{u}_1)^2\right] \qquad (2.5b)$$

$$\Theta_{21} = \Theta_2 + \frac{2m_1\, m_2}{(m_1 + m_2)^2} \left[(\Theta_1 - \Theta_2) + \frac{m_1}{6k} (\vec{u}_2 - \vec{u}_1)^2\right] \qquad (2.5c)$$

It should be noted at this point that the developers of this kinetic model made use of the model for near equilibrium conditions, whereas the present application is for a decidedly non-equilibrium case. Willis (Ref. 4) cautioned about the use of the Krook model in highly non-equilibrium conditions such as these with his example of the calculation of sphere drag in hyperthermal, near free molecular flow. Before proceeding further in the use of the model and the model parameters defined in Eqs. (2.5), a check should be made of its validity in the present application. This was done by calculating directly the mean flow and kinetic temperature parameters for hyperthermal scattering of hard spheres and comparing with the relations in Eqs. (2.5). For hyperthermal flow, the outgas molecules (subscript 1) may be considered stationary and the ambient molecules (subscript 2) moving at the single velocity $\vec{u}_2$. Equations (2.5) then become

$$u_{12} = u_{21} = \frac{m_2}{m_1 + m_2} u_2 \tag{2.6a}$$

$$\Theta_{12} = \frac{1}{3k} \frac{m_1 m_2^2}{(m_1 + m_2)^2} u_2^2 \tag{2.6b}$$

$$\Theta_{21} = \frac{1}{3k} \frac{m_2 m_1^2}{(m_1 + m_2)^2} u_2^2 \tag{2.6c}$$

The mean flow velocities and kinetic temperatures for the hyperthermal scattering of hard spheres can be shown to agree exactly with the relations in Eqs. (2.6). These model parameters, therefore, can be used with great confidence in the present application.

The remaining parameters to be determined are the collision frequencies ν_{ij}. The appropriate values of these parameters depend on the type of collision process under consideration. For the present problem, they depend on whether the collision process is self-collisions or ambient-collisions. This determination will be made later as we consider each process separately. As a starting point, however, we can bear in mind that, for near-equilibrium flows, in which the distribution functions are small perturbations from the Maxwellian, taking moments on the single species BGK equation yields the Navier-Stokes equations with the correct viscosity-shear stress relation when (Ref. 5):

$$\nu = kTn/\mu \tag{2.7}$$

where T and n are the local temperature and number density, respectively, and μ is the Chapman-Enskog viscosity. This value has been used by a number of investigators for a wide variety of problems, even for highly non-equilibrium cases.

In our analysis of the return flow problem, we assume nearby free molecular flow conditions. The distribution function for the outgassing species is therefore a small perturbation from the free molecular solution:

$$f_1 = f_{1,0} + \hat{f}_1 \tag{2.8}$$

where $f_{1,0}$ is the free molecular solution and $\hat{f}_1$ is the perturbation due to collisions. For velocities directed away from the outgassing surface, $\hat{f}_1 << f_{1,0}$. Since return flow is due only to collision processes, (we consider only convex or flat surfaces), the free molecular solution $f_{1,0}$ is zero for velocities directed toward the outgassing surface. Applying these relations to the BGK equation for the outgassing species (Eq. (2.2a)) yields:

$$\frac{\partial \hat{f}_1}{\partial t} + \vec{v} \cdot \frac{\partial \hat{f}_1}{\partial \vec{r}} = \nu_{11}(M_{11} - \hat{f}_1) + \nu_{12}(M_{12} - \hat{f}_1) \tag{2.9}$$

for molecules flowing toward the surface. Also based on the small perturbation assumption, the parameters involved in M_{11} and M_{12} are obtained by using the free molecular distribution functions $f_{1,0}$ and $f_{2,0}$ in Eqs. (2.4).

Note that Eq. (2.9) contains contributions to the return flow distribution function due to both self-collisions and ambient collisions, and that the separate components cannot be decoupled in the form that Eq. (2.9) represents the collision process. Decoupling can be justified, however, on the basis of the following argument. An examination of the terms on the right-hand side of Eq. (2.9) shows that the terms containing the M's are production terms in which scattered molecules are produced at a rate equal to the product of the number density n_1 and the collision frequency ν_{11} or ν_{12}, depending on whether the collision process is self-collisions or ambient collisions. These terms are independent of $\hat{f}_1$ and, thus, are decoupling. The terms containing $\hat{f}_1$ are attenuation terms in which the scattered molecules are attenuated as a result of subsequent collisions. Since the scattered molecules returning to the surface are traveling in the same direction as the ambient flow and in opposition to the outgassing flow, attenuation should occur primarily as a result of collisions with the outward flowing outgas molecules. Based on this argument, we drop the attenuation terms involving ν_{12} and separate Eq. (2.9) into two parts containing contributions to the return flow due to self-collisions and ambient collisions:

$$\frac{\partial \hat{f}_{11}}{\partial t} + \vec{v} \cdot \frac{\partial \hat{f}_{11}}{\partial \vec{r}} = \nu_{11}(M_{11} - \hat{f}_{11}) \tag{2.10a}$$

$$\frac{\partial \hat{f}_{12}}{\partial t} + \vec{v} \cdot \frac{\partial \hat{f}_{12}}{\partial \vec{r}} = \nu_{12} M_{12} - \nu_{11} \hat{f}_{12} \tag{2.10b}$$

where $\hat{f}_{11}$ is the contribution due to self-collisions and $\hat{f}_{12}$ is the contribution due to ambient collisions. For nearly free molecular flow, the attenuation terms should be small and, hence, any error due to their consideration should be small.

Equations (2.10) are hyperbolic partial differential equations which can be integrated along characteristic lines in the direction of the velocity vector, $\vec{v}$. These integrations will be performed separately in the following paragraphs for the contributions due to self-collisions and ambient collisions.

Self-Collisions

Integrating Eq. (2.10a) from a point at infinity, $\vec{r}_\infty$, to a point on the outgassing surface, $\vec{r}_w$, yields the distribution function of return flow molecules on the outgassing surface:

$$\hat{f}_{11}(\vec{v},\vec{r}_w) = \hat{f}_{11}(\vec{v},\vec{r}_\infty)\exp\left[-\frac{1}{|\vec{v}|}\int_{\vec{r}_\infty}^{\vec{r}_w} \nu_{11}(\vec{r}')\,|d\vec{r}'|\right]$$

$$+\frac{1}{|\vec{v}|}\int_{\vec{r}_\infty}^{\vec{r}_w} \nu_{11}(r')M_{11}(\vec{v},\vec{r}')\exp\left[-\frac{1}{|\vec{v}|}\int_{\vec{r}'}^{\vec{r}_w}\nu_{11}(\vec{r}'')\,|d\vec{r}''|\right]|d\vec{r}'| \qquad (2.11)$$

At infinity the return flow distribution function is zero, thus eliminating the first term in Eq. (2.11). The return flow rate $q_{b_{11}}$ at the surface is now obtained by integrating the return flow distribution function over all velocities directed back into the surface.

$$q_{b_{11}} = \int_{\vec{v}\cdot\vec{e}_n \geq 0} \vec{v}\cdot\vec{e}_n\,\hat{f}_{11}(\vec{v},\vec{r}_w)\,d^3\vec{v} \qquad (2.12)$$

where $\vec{e}_n$ is an inward directed surface normal. Expressing the velocities in terms of absolute velocities, v, directions and the solid angle, ω, enclosing the directions yields:

$$q_{b_{11}} = \int_{2\pi}\int_0^\infty v^3\cos\phi\,\hat{f}_{11}(\vec{v},\vec{r}_w)\,dv\,d\omega \qquad (2.13)$$

where ϕ is the angle between the surface normal $\vec{e}_n$ and the velocity vector $\vec{v}$. The solid angle integration is over the 2π steradians subtending the half-space outward from the surface. Combining Eqs. (2.11) and (2.13), along with the definitions of the Maxwellian M_{11} in Eq. (2.3), yields:

$$q_{b_{11}} = \int_{2\pi}\int_0^{\infty}\int_0^{\infty} \nu_{11} \cos\phi \left(\frac{m_1}{2\pi k \Theta_{11}}\right)^{3/2} n_1 \exp\left[-\frac{m_1}{2k\Theta_{11}}(\vec{v} - \vec{u}_{11})^2\right]$$

$$\exp\left[-\frac{1}{v}\int_0^{r'} \nu_{11}\, dr''\right] v^2\, dv\, dr'\, d\omega \qquad (2.14)$$

where the r's are distances along the integral path. As noted earlier, attenuation is small and, hence, the attenuation factor may be approximated by:

$$\exp\left[-\frac{1}{v}\int_0^{r'} \nu_{11}\, dr''\right] = 1 - \frac{1}{v}\int_0^{r'} \nu_{11}\, dr'' \qquad (2.15)$$

Equation (2.14) now becomes:

$$q_{b_{11}} = \int_{2\pi}\int_0^{\infty} \nu_{11} \cos\phi \left(\frac{m_1}{2\pi k \Theta_{11}}\right)^{3/2} n_1 \exp\left[-\frac{m_1}{2k\Theta_{11}} u_{11}^2 \sin^2\alpha_{11}\right]$$

$$\cdot\left\{\int_0^{\infty} v^2 \exp\left[-\frac{m_1}{2k\Theta_{11}}(v - u_{11}\cos\alpha_{11})^2\right] dv\right.$$

$$\left. - \int_0^{r'} \nu_{11}\, dr'' \int_0^{\infty} v \exp\left[-\frac{m_1}{2k\Theta_{11}}(v - u_{11}\cos\alpha_{11})^2\right] dv\right\} dr'\, d\omega \qquad (2.16)$$

where α_{11} is the angle between the mean flow velocity vector $\vec{u}_{11}$ and the velocity vector $\vec{v}$.

Integrating over the velocity yields:

$$q_{b_{11}} = \int_{2\pi}\int_{0}^{\infty} \nu_{11} \cos\phi \, n_1 (f_{11} - g_{11}) \, dr' \, d\omega \tag{2.17}$$

where f_{11} is the directional distribution function (scattering pattern) of the scattered molecules, and g_{11} is an attenuation term. These parameters are defined by:

$$f_{11} = \frac{1}{2\pi^{3/2}} e^{-\tilde{u}_{11}^2 \sin^2\alpha_{11}} \left\{ \tilde{u}_{11} \cos\alpha_{11} \, e^{-\tilde{u}_{11}^2 \cos^2\alpha_{11}} + \frac{\sqrt{\pi}}{2} (1 + 2\tilde{u}_{11}^2 \cos^2\alpha_{11}) \left[1 + \mathrm{erf}(\tilde{u}_{11} \cos\alpha_{11}) \right] \right\} \tag{2.18}$$

and

$$g_{11} = \frac{1}{2\pi^{3/2}} \frac{\int_0^{r'} \nu_{11} \, dr''}{\sqrt{2k\Theta_{11}/m_1}} e^{-\tilde{u}_{11}^2 \sin^2\alpha_{11}} \left\{ e^{-\tilde{u}_{11}^2 \cos^2\alpha_{11}} + \sqrt{\pi} \, \tilde{u}_{11} \cos\alpha_{11} \left[1 + \mathrm{erf}(\tilde{u}_{11} \cos\alpha_{11}) \right] \right\} \tag{2.19}$$

where

$$\tilde{u}_{11} = \sqrt{\frac{m_1}{2k\Theta_{11}}} \, u_{11} \tag{2.20}$$

The remaining integrations over r' and ω in Eq. (2.17) consist of simple volumetric integrals over the half-space outward from the outgassing surface (or the surface receiving the return flow).

The parameters u_{11} and Θ_{11} depend on the local density and distribution function of the outgassing molecules (Eqs. (2.4)), and, so, will in general depend on the shape of the outgassing body and will vary with position in the flow field. The collision frequency ν_{11} is assumed to be given by:

$$\nu_{11} = kT_w n_1/\mu(T_w) \tag{2.21}$$

where T_w is the outgassing surface temperature. In terms of the collision cross section σ_{11} and mean thermal velocity $\bar{v}_1$, this is equivalent to:

$$\nu_{11} = 1.111\ \bar{v}_1 \sigma_{11} n_1 \tag{2.22}$$

which compares with $1.414\ \bar{v}_1 \sigma_{11} n_1$ for a Maxwellian gas (a gas in local thermal equilibrium) and $0.806\ \bar{v}_1 \sigma_{11} n_1$ for a molecular beam. Our assumed collision frequency is thus seen to be a reasonable value, since the molecular flow would be expected to be much like a Maxwellian gas near the surface and a molecular beam far away from the surface.

Ambient Collisions

Equation (2.10b) is integrated in the same manner as in the preceding discussion to yield:

$$q_{b_{12}} = \int_{2\pi} \int_0^\infty \nu_{12} \cos\phi\, n_1 (f_{12} - g_{12})\, dr'\, d\omega \tag{2.23}$$

where

$$f_{12} = \frac{1}{2\pi^{3/2}} e^{-\tilde{u}_{12}^2 \sin^2\alpha_{12}} \Bigg\{ \tilde{u}_{12} \cos\alpha_{12}\, e^{-\tilde{u}_{12}^2 \cos^2\alpha_{12}} + \frac{\sqrt{\pi}}{2} (1 + 2\tilde{u}_{12}^2 \cos^2\alpha_{12}) \left[1 + \mathrm{erf}(\tilde{u}_{12} \cos\alpha_{12})\right] \Bigg\} \tag{2.24}$$

and

$$g_{12} = \frac{1}{2\pi^{3/2}} \frac{\int_0^{r'} \nu_{11}\, dr'}{\sqrt{2k\Theta_{12}/m_1}} e^{-\tilde{u}_{12}^2 \sin^2\alpha_{12}} \Bigg\{ e^{-\tilde{u}_{12}^2 \cos^2{}_{12}} + \sqrt{\pi}\, \tilde{u}_{12} \cos\alpha_{12} \left[1 + \mathrm{erf}(\tilde{u}_{12} \cos\alpha_{12})\right] \Bigg\} \tag{2.25}$$

These equations are nearly identical to those derived previously for self-collisions. Note that the double subscripts "11" are replaced by "12" everywhere except in the collision rate ν_{11} in Eq. (2.25).

According to Eqs. (2.4), the parameters u_{12} and Θ_{12} depend on both the outgassing flow properties, which are a function of position, and the ambient flow properties, which are essentially constant with position. As noted earlier, the relations reduce to the simple form given by Eq. (2.5) for the case of hyperthermal flow. Since the ambient atmospheric molecules are traveling at very high velocities compared to the outgassing molecules for the orbiting spacecraft we are considering, the use of these simpler relations in our analysis appears to be justified.

The collision frequency ν_{12} will again be assumed to take on the form assumed earlier in BGK model solutions:

$$\nu_{12} = k\hat{T}_{12} n_2/\hat{\mu} \tag{2.26}$$

In this case, however, the temperature and viscosity must be characteristic of the collision process occurring between ambient and outgas molecules. We will define the temperature $\hat{T}_{12}$ as a kinetic temperature characteristic of the energy of the flow (again neglecting the motion of the outgas molecules):

$$\hat{T}_{12} = \frac{\overline{m}_{12} u_2^2}{3k} \tag{2.27}$$

where $\overline{m}_{12}$ is a composite molecular mass for the two species. The viscosity μ is then defined in terms of this kinetic temperature and the collision cross section σ_{12} for collisions between the two species:

$$\hat{\mu} = \frac{\sqrt{\overline{m}_{12} k \hat{T}/\pi}}{\sigma_{12}} \tag{2.28}$$

The collision frequency ν_{12} then becomes

$$\nu_{12} = \sqrt{\frac{\pi}{3}} \; \sigma_{12} u_2 n_2$$

$$= 1.023 \; \sigma_{12} u_2 n_2 \tag{2.29}$$

Note that the composite mass $\overline{m}_{12}$ drops out and has no bearing on the collision rate. The resulting collision frequency compares very closely with $\nu_{12} = \sigma_{12} u_2 n_2$, the relation for hyperthermal collisions between hard spheres. The BGK model approach is therefore seen to lead to physically reasonable results.

Section 3
APPLICATION TO SPHERICAL SPACECRAFT

The outgas contamination return flow model developed in Section 2 is applied in this section to a spherical spacecraft. The contributions due to self-collisions and ambient collisions are treated separately in the following paragraphs.

3.1 SELF-COLLISIONS

Consider the geometry depicted in Fig. 3-1. The free molecular distribution function $f_{1,0}$ for outgas molecules at the point P is the Maxwellian distribution function corresponding to the sphere surface temperature T_w for velocity vectors enclosed by the solid angle Ω subtended by the sphere. Outside of this solid angle the distribution function is zero. The magnitude of the distribution function is determined by the outgas rate q_w such that

$$q_w = \left| \int_{2\pi} \vec{v} \, f_{1,0} \, d^3\vec{v} \right| \tag{3.1}$$

on the sphere surface. The Maxwellian distribution function satisfying these requirements is

$$\begin{aligned} f_{1,0} &= \frac{2}{\pi} q_w \left(\frac{m_1}{2kT_w}\right)^2 e^{-\frac{m_1}{2kT_w} v^2} && \vec{v} \text{ in } \Omega \\ &= 0 && \vec{v} \text{ not in } \Omega \end{aligned} \tag{3.2}$$

Using this distribution function, we obtain the following parameters:

$$n_1 = \int_{\Omega} f_{1,0} \, d^3\vec{v}$$

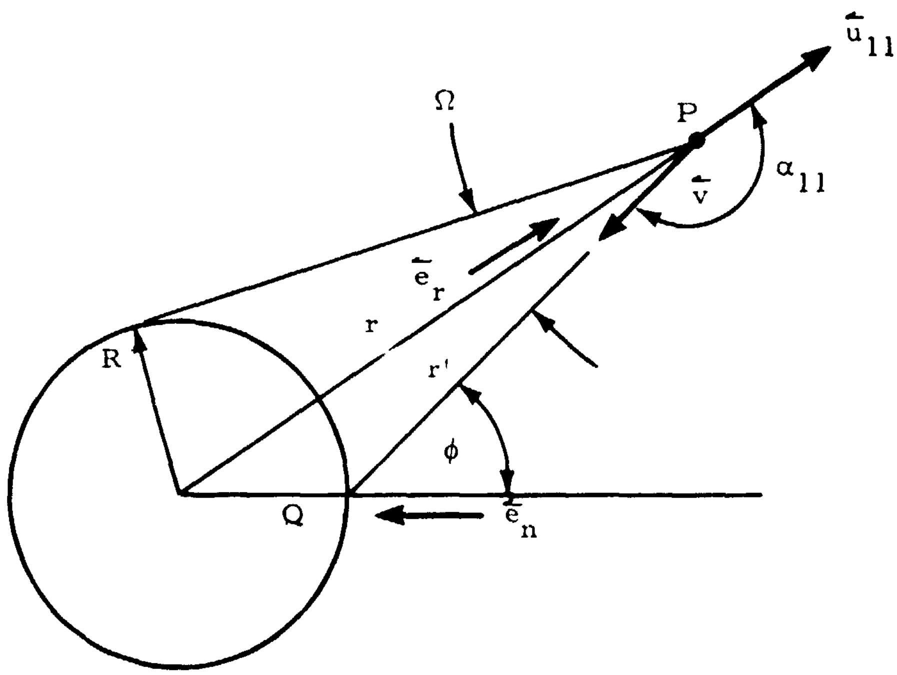


Fig. 3-1 - Geometry Involved in Back-Scattering Due to Self-Collisions

$$= \sqrt{\pi}\, q_w \left(\frac{m_1}{2\,k\,T_w}\right)^{1/2} \left(1 - \frac{\sqrt{\tilde{r}^2 - 1}}{\tilde{r}}\right) \tag{3.3}$$

$$\vec{u}_{11} = \frac{1}{n_1} \int_{\Omega} \vec{v}\, f_{1,0}\, d^3\vec{v}$$

$$= \left(\frac{2\,k\,T_w}{\pi\, m_1}\right)^{1/2} \frac{1}{\tilde{r}^2 - \tilde{r}\sqrt{\tilde{r}^2 - 1}}\; \vec{e}_r \tag{3.4}$$

$$\Theta_{11} = \frac{m_1}{3k} \int (\vec{v} - \vec{u}_{11})^2 f_{1,0}\, d^3\vec{v}$$

$$= T_w \left[1 - \frac{2}{3\pi(\tilde{r}^2 - \tilde{r}\sqrt{\tilde{r}^2 - 1})^2}\right]$$

$$= T_w \left(1 - \frac{2}{3}\, \tilde{\tilde{u}}_{11}^2\right) \tag{3.5}$$

where $\tilde{r} = r/R$, $\tilde{\tilde{u}}_{11} = \sqrt{\frac{m_1}{2\,k\,T_w}}\; u_{11}$ and $\vec{e}_r$ is a unit vector in the direction outward along r. These parameters are all seen to be functions of the radial distance, r. When they are included in the scattering pattern function f_{11} and the attenuation function g_{11}, Eq. (2.17) will yield the contribution to return flow due to self-scattering. Including these parameters in Eq. (2.17) and collecting into dimensionless groups yields:

$$\tilde{q}_{b_{11}} = 2\pi^2 \mathrm{Kn}_{11}^{-1} \int_0^{\pi/2} \sin\phi \cos\phi \int_0^{\infty} \tilde{n}_1' \, f_{11}\, d\tilde{r}'\, d\phi$$

$$- 2\pi^{3/2} Kn_{11}^{-2} \int_0^{\pi/2} \sin\phi \cos\phi \int_0^{\infty} \tilde{n}_1^2 \tilde{g}_{11} \, d\tilde{r}' \, d\phi \qquad (3.6)$$

where

$$\tilde{q}_{b_{11}} = q_{b_{11}}/q_w$$

$$Kn_{11} = \frac{2\mu}{m_1 R q_w}$$

$$\tilde{n}_1 = 1 - \sqrt{\tilde{r}^2 - 1}/\tilde{r}$$

$$\tilde{\Theta}_{11} = \Theta_{11}/T_w$$

$$\tilde{g}_{11} = \frac{1}{2\pi^{3/2}} \int_0^{\tilde{r}'} \tilde{n} \, d\tilde{r}'' \, \frac{e^{-\tilde{u}_{11} \sin^2\alpha_{11}}}{\sqrt{\tilde{\Theta}_{11}}} \left\{ e^{-\tilde{u}_{11}^2 \cos^2\alpha_{11}} \right.$$

$$\left. + \sqrt{\pi} \, \tilde{u}_{11} \cos\alpha_{11} \left[1 + \mathrm{erf}(\tilde{u}_{11} \cos\alpha_{11}) \right] \right\} \qquad (3.7)$$

The parameter Kn_{11} may be considered a Knudsen number characteristic of self-collisions. The parameter $\tilde{n}_1$ may be considered a dimensionless number density, and the integral

$$\tilde{N} = \int_0^{\tilde{r}'} \tilde{n}_1 \, d\tilde{r}' \qquad (3.8)$$

may be considered a dimensionless column density. This integral may be carried out in closed form by approximating $\tilde{n}_1$ by

$$\tilde{n}_1 = 1 - \sqrt{\tilde{r}^2 - 1}/\tilde{r} \cong \frac{1}{2}\left(\frac{1}{\tilde{r}^2} + \frac{1}{\tilde{r}^6}\right) \tag{3.9}$$

This approximation was made with good results in the study documented in Ref. 6. The resulting expression for $\tilde{N}$ is

$$\tilde{N} = \frac{1}{2\sin\phi}\left\langle \tan^{-1}\left(\frac{\tilde{r}' + \cos\phi}{\sin\phi}\right) + \phi - \pi/2 \right.$$
$$+ \frac{1}{4\sin^4\phi}\left\{\sin\phi\left\{\frac{\tilde{r}' + \cos\phi}{\tilde{r}^2}\left[\frac{\sin^2\phi}{\tilde{r}^2} + \frac{3}{2}\right] - \cos\phi\left(\sin^2\phi + \frac{3}{2}\right)\right\}\right.$$
$$\left.\left. + \frac{3}{2}\left\{\tan^{-1}\left(\frac{\tilde{r}' + \cos\phi}{\sin\phi}\right) + \phi - \pi/2\right\}\right\}\right\rangle \tag{3.10}$$

The radial distance $\tilde{r}$ can be expressed in terms of $\tilde{r}'$ by

$$\tilde{r} = \sqrt{1 + 2\cos\phi\;\tilde{r}' + \tilde{r}'^2} \tag{3.11}$$

The remaining integrals to be performed in Eq. (3.6) are simply a volumetric integral in axisymmetric spheric coordinates. These integrals, when evaluated numerically, yield the final expression for return flux due to self-collisions:

$$\tilde{q}_{b_{11}} = (0.01554 - 0.004058\,Kn_{11}^{-1})Kn_{11}^{-1} \tag{3.12}$$

This expression is plotted in Fig. 3-2 as a function of the inverse Knudsen number, Kn_{11}^{-1}, and compared with results obtained from a simple test particle

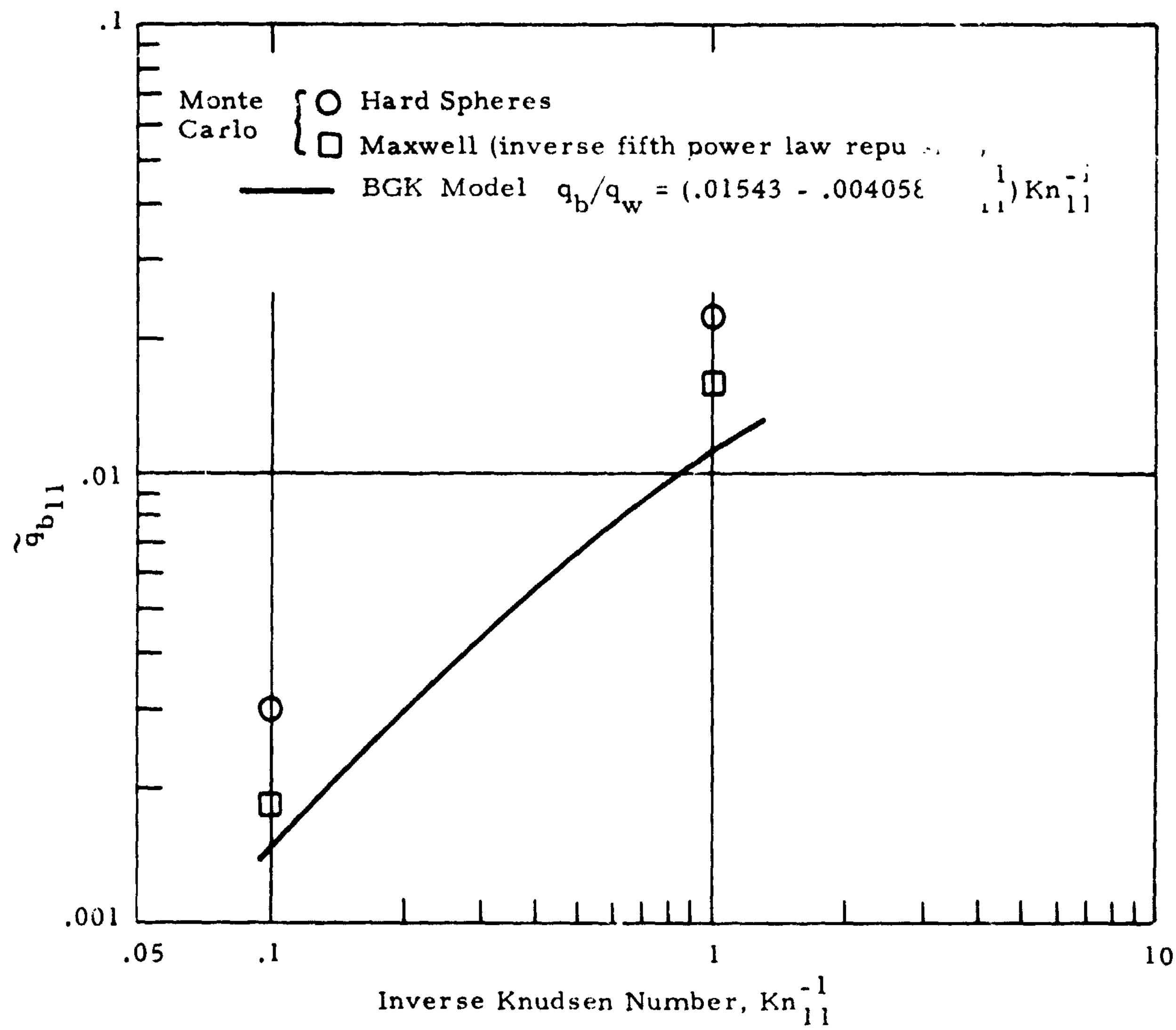


Fig. 3-2 - BGK Model Solution for Outgas Back-Scattering Due to Self-Collisions Compared with Monte Carlo Results

Monte Carlo program. This Monte Carlo program was developed solely to provide a source of comparison for the BGK model results. The BGK model results are seen to be somewhat less than the Monte Carlo results, but still well within the same order of magnitude.

3.2 AMBIENT COLLISIONS

The geometry involved in the integration of the BGK equation for ambient collisions is represented in Fig. 3-3. Spherical symmetry is no longer applicable in this case because of the directed flow of the ambient freestream. The return flux at a point Q on the sphere surface now depends on the angular displacement ϕ' away from the forward position (stagnation point). The collision rate ν_{12}, given by Eq. (2.29), and the local number density n_1, given by Eq. (3.3), may be included in Eq. (2.23) to yield the dimensionless form:

$$\tilde{q}_{b_{12}} = 1.023 \sqrt{\frac{\pi}{2}} \frac{S_b \sigma_{12}}{Kn_{22} \sigma_{22}} \left\{ \int_{2\pi} \cos\phi \, f_{12} \int_0^\infty \tilde{n}_1 \, d\tilde{r}' \, d\omega \right.$$

$$\left. - \sqrt{\pi} \, Kn_{11}^{-1} \int_{2\pi} \cos\phi \int_0^\infty \tilde{g}_{12} \tilde{n}_1 \, d\tilde{r}' \, d\omega \right\} \tag{3.13}$$

where

$$S_b = u_2 \sqrt{\frac{2kT_w}{m_1}} \tag{3.14}$$

$$Kn_{22} = \frac{1}{\sqrt{2} \, n_2 \sigma_{22} R} \tag{3.15}$$

and

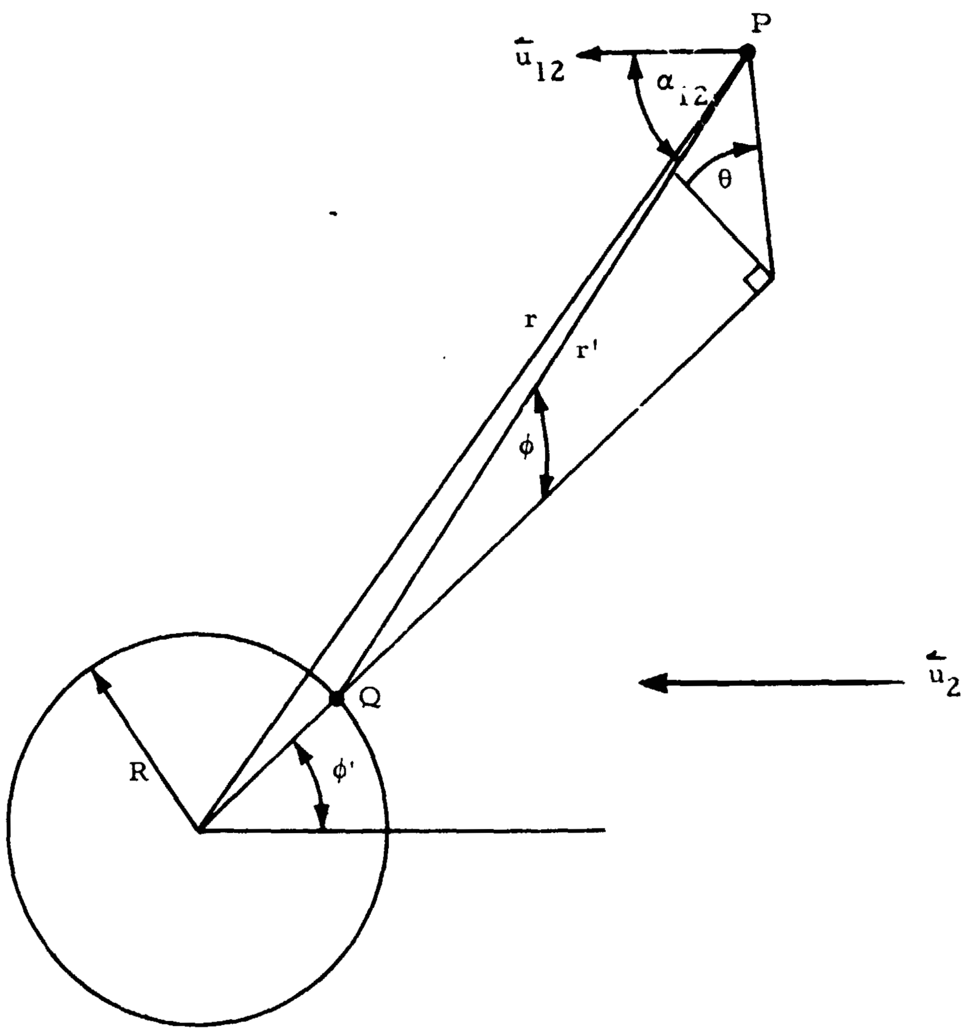


Fig. 3-3 - Geometrical Representation of Scattering by Ambient Atmospheric Molecules

$$\tilde{g}_{12} = \frac{1}{2\pi^{3/2}} \tilde{N} \frac{e^{-\tilde{u}_{12}^2 \sin^2\alpha_{12}}}{\sqrt{\tilde{\Theta}_{12}}} \left\{ e^{-\tilde{u}_{12}^2 \cos_{12}^2} + \sqrt{\pi}\, \tilde{u}_{12} \cos\alpha_{12} \left[1 + \mathrm{erf}\,(\tilde{u}_{12} \cos\alpha_{12})\right] \right\} \tag{3.16}$$

The parameter S_b is the speed ratio, and Kn_{22} is the freestream Knuden number. Using the values u_{12} and Θ_{12} given by Eqs. (2.6), the functions f_{12} and $\tilde{g}_{12}$ become:

$$f_{12} = \frac{1}{2\pi^{3/2}} e^{-\frac{3}{2}\sin^2\alpha_{12}} \left\{ \sqrt{\frac{3}{2}} \cos\alpha_{12}\, e^{-\frac{3}{2}\cos^2\alpha_{12}} + \frac{\sqrt{\pi}}{2} (1 + 3\cos^2\alpha_{12}) \left[1 + \mathrm{erf}\left(\sqrt{\frac{3}{2}} \cos\alpha_{12}\right)\right] \right\} \tag{3.17}$$

$$\tilde{g}_{12} = \sqrt{\frac{3}{2}} \frac{(1 + m_1/m_2)}{S_b} \tilde{N} h_{12} \tag{3.18}$$

where

$$h_{12} = \frac{1}{2\pi^{3/2}} e^{-\frac{3}{2}\sin^2\alpha_{12}} \left\{ e^{-\frac{3}{2}\cos^2\alpha_{12}} + \sqrt{\frac{3\pi}{2}} \cos\alpha_{12} \left[1 + \mathrm{erf}\,\left(\sqrt{\frac{3}{2}} \cos\alpha_{12}\right)\right] \right\} \tag{3.19}$$

After inclusion of the parameters defined by Eqs. (3.18) and (3.19), Eq. (3.13) becomes:

$$\tilde{q}_{b_{12}} = 1.023 \sqrt{\frac{\pi}{2}} \frac{\sigma_{12}}{\sigma_{22}} Kn_{22}^{-1} \left\{ S_b \int_{2\pi} \cos\phi f_{12} \int_0^{\infty} \tilde{n}_1 d\tilde{r}' d\omega \right.$$

$$\left. - \sqrt{\frac{3\pi}{2}} (1 + m_1/m_2) Kn_{11}^{-1} \int_{2\pi} \cos\phi h_{12} \int_0^{\infty} \tilde{N}\tilde{n}_1 d\tilde{r}' d\omega \right\} \tag{3.20}$$

The integration over the radial distance r' in the first term of Eq. (3.13) is just the dimensionless column density at infinity which from Eq. (3.10) is:

$$\int_0^{\infty} \tilde{n}_1 d\tilde{r}' = \tilde{N}_{\infty} = \frac{1}{2} \left(\frac{\phi}{\sin\phi} + \frac{3}{8} \frac{\phi}{\sin^5\phi} - \frac{1}{4} \frac{\cos\phi}{\sin^2\phi} - \frac{3}{8} \frac{\cos\phi}{\sin^4\phi} \right) \tag{3.21}$$

The integral over r' in the second term is:

$$\int_0^{\infty} \tilde{n}_1 \tilde{N} d\tilde{r}' = \frac{1}{2} \tilde{N}_{\infty}^2 \tag{3.22}$$

Equation (3.20) now becomes:

$$\tilde{q}_{b_{12}} = 1.023 \sqrt{\frac{\pi}{2}} \frac{\sigma_{12}}{\sigma_{22}} Kn_{22}^{-1} \left\{ S_b \int_{2\pi} \cos\phi f_{12} \tilde{N}_{\infty} d\omega \right.$$

$$\left. - \frac{1}{2} \sqrt{\frac{3\pi}{2}} (1 + m_1/m_2) Kn_{11}^{-1} \int_{2\pi} \cos\phi h_{12} \tilde{N}_{\infty}^2 d\omega \right\} \tag{3.23}$$

The angle α_{12} is given by (see Fig. 3-3):

$$\alpha_{12} = \cos^{-1}(\cos\phi' \cos\phi - \sin\phi' \sin\phi \cos\theta) \qquad (3.24)$$

At the forward station ($\phi' = 0$), $\alpha_{12} = \phi$ and the integral over the 2π steradians in Eq. (3.23) becomes a single integral over ϕ from 0 to $\pi/2$. At other stations, the integral requires a double integration of ϕ from 0 to $\pi/2$ and θ from 0 to 2π. In all cases, the integration must be performed numerically. The resulting form of the solution is:

$$\tilde{q}_{b_{12}} = \frac{\sigma_{12}}{\sigma_{22}} Kn_{22}^{-1} \left[S_b F(\phi') - (1 + m_1/m_2) Kn_{11}^{-1} G(\phi') \right] \qquad (3.25)$$

where the factors F and G depend on sphere station. The computed values of F and G are given in Table 3-1 compared to results obtained previously using a "first collision" model approach (Ref. 6). The "first collision" results did not consider an attenuation factor, G. The BGK and "first collision" model results are seen to compare fairly closely over the entire range of sphere stations with the exception of the rear stations. The first collision model does not allow upstream scattering, and, hence, the return flux approaches zero at the rear station (180 deg). The BGK model, on the other hand, implicitly includes a combination of first and multiple collisions which allow some upstream scattering. The BGK model thus results in a nonzero return flux at the rea station.

Table 3-1

COMPUTED PARAMETERS FOR BACKSCATTER TO SPHERE
DUE TO COLLISIONS WITH AMBIENT ATMOSPHERE

ϕ' (deg)	F (BCK)	G (BGK)	F ("First Collisions,") Ref. 6
0	.607	.304	.564
10	.606	.304	.556
20	.587	.298	.534
30	.554	.287	.501
40	.508	.268	.459
50	.451	.244	.410
60	.385	.216	.358
70	.315	.184	.303
80	.245	.150	.248
90	.181	.118	.196
100	.128	.0890	.150
110	.0877	.0659	.110
120	.0590	.0485	.0757
130	.0399	.0361	.0479
140	.0278	.0278	.0267
150	.0205	.0223	.0122
160	.0162	.0190	.0039
170	.0140	.0172	.0005
180	.0134	.0167	.0000

Section 4
APPLICATION TO FLAT PLATE DISK

The return flow model developed in Section 2 was also applied to a flat plate disk oriented normal to the direction of the ambient flow. In this application the return flow was calculated at points at various elevations above the surface of the disk. The method of application was similar to that described for the sphere, but some liberties were taken to simplify the analysis. Consider the disk geometry depicted in Fig. 4-1. The return flux is to be determined at the point Q located at an elevation, H, above the center of the disk surface. The free molecular distribution function, $f_{1,0}$, for the outgas molecules at the point P is the same as given for the sphere in Eq. (3.2) for velocity vectors enclosed by the solid angle subtended by the disk. Outside of this solid angle, $f_{1,0}$ is zero. The outgas flow parameters are easily obtained by integration of the distribution function along the line perpendicular to the disk center ($\phi' = 0$). For points off this line, however, the integrations become much more complicated and are probably not obtainable in closed form. To simplify matters it was assumed that, for points within a distance of one disk radius of the disk center ($r \leq R$), the local number density, n_1, is equal to the centerline density at the same distance from the disk center point. For points beyond one disk radius ($r > R$), the density was calculcated based on a far field approximation ($r >> R$). In the far field, the outgassing disk is approximated by a point area source emitting with a Lambertian (cosine) distribution. The mean flow velocity u_{11} and temperature Θ_{11} are not strongly dependent on position, and so are assumed to always take on centerline values. The resulting outgas flow parameters are:

$$n_1 = \sqrt{\pi}\; q_w \left(\frac{m_1}{2kT_w}\right)^{1/2} \left(1 - \frac{\sqrt{\tilde{r}^2 - 1}}{\tilde{r}}\right) \qquad r \leq R$$

$$= \sqrt{\pi}\; q_w \left(\frac{m_1}{2kT_w}\right)^{1/2} \frac{\cos\phi'}{2\tilde{r}^2} \qquad r > R \qquad (4.1)$$

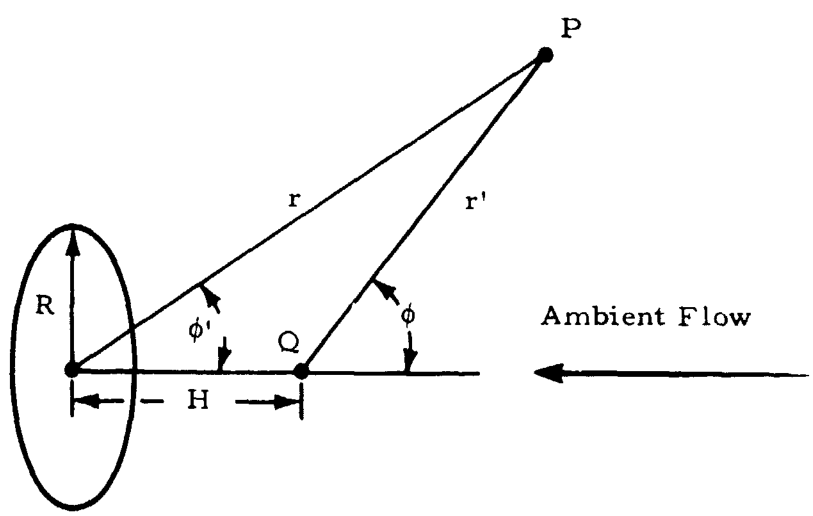


Fig. 4-1 - Geometry Involved in Back-Scattering to Points On and Above Flat Plate Disk

$$\vec{u}_{11} = \left(\frac{2kT_w}{\pi m_1}\right)^{1/2} \frac{1}{\tilde{r}^2 - \tilde{r}\sqrt{\tilde{r}^2 + 1} + 1} \vec{e}_r \tag{4.2}$$

$$\Theta_{11} = T_w \left[1 - \frac{2}{3\pi} \frac{1}{(\tilde{r}^2 - \tilde{r}\sqrt{\tilde{r}^2 + 1} + 1}\right]$$

$$= T_w (1 - \frac{2}{3} \tilde{\tilde{u}}_{11}^2 \tag{4.3}$$

where $\tilde{r} = r/R$, $\tilde{u}_{11} = \sqrt{\frac{m_1}{2kT_w}}\, u_{11}$ and $\vec{e}_r$ is a unit vector in the direction outward along r. The integration of the BGK model equation for the flat plate disk follows closely the process described in Section 3 for the sphere. The resulting expressions for the self-scattering and ambient scattering contributions to the outgas return flux are:

$$\tilde{q}_{b_{11}} = \left[F(H) - G(H)\,Kn_{11}^{-1}\right] Kn_{11}^{-1} \tag{4.4}$$

$$\tilde{q}_{b_{12}} = \frac{\sigma_{12}}{\sigma_{22}} Kn_{22}^{-1} \left[S_D F(H) - (1 + m_1/m_2)\,Kn_{11}^{-1}\, G(H)\right] \tag{4.5}$$

The parameters F and G are plotted for the case of self-collisions in Fig. 4-2 and for the case of ambient collisions in Fig. 4-3.

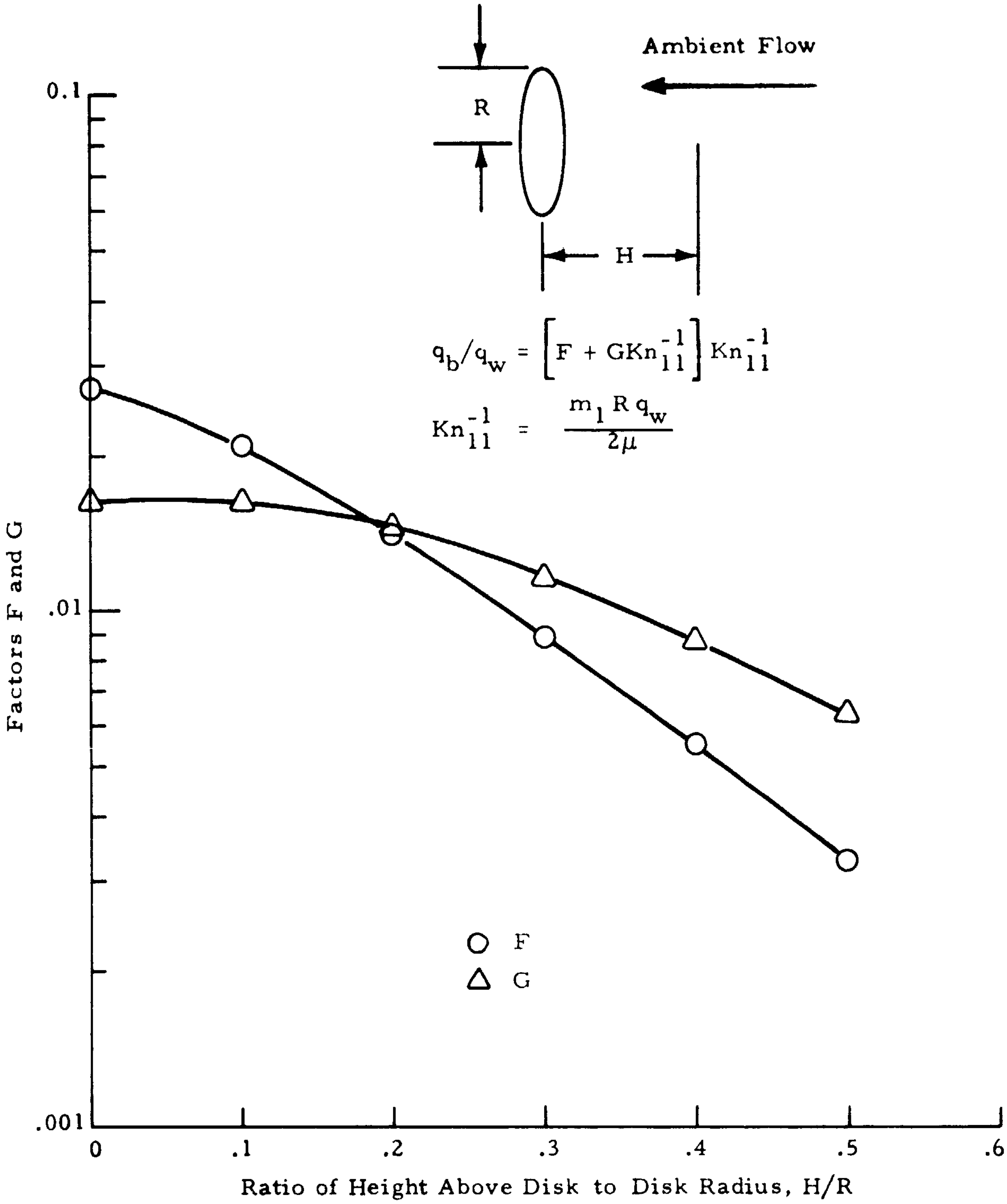


Fig. 4-2 - Return Flux to Points On and Above Flat Plate Disk Due to Self-Collisions

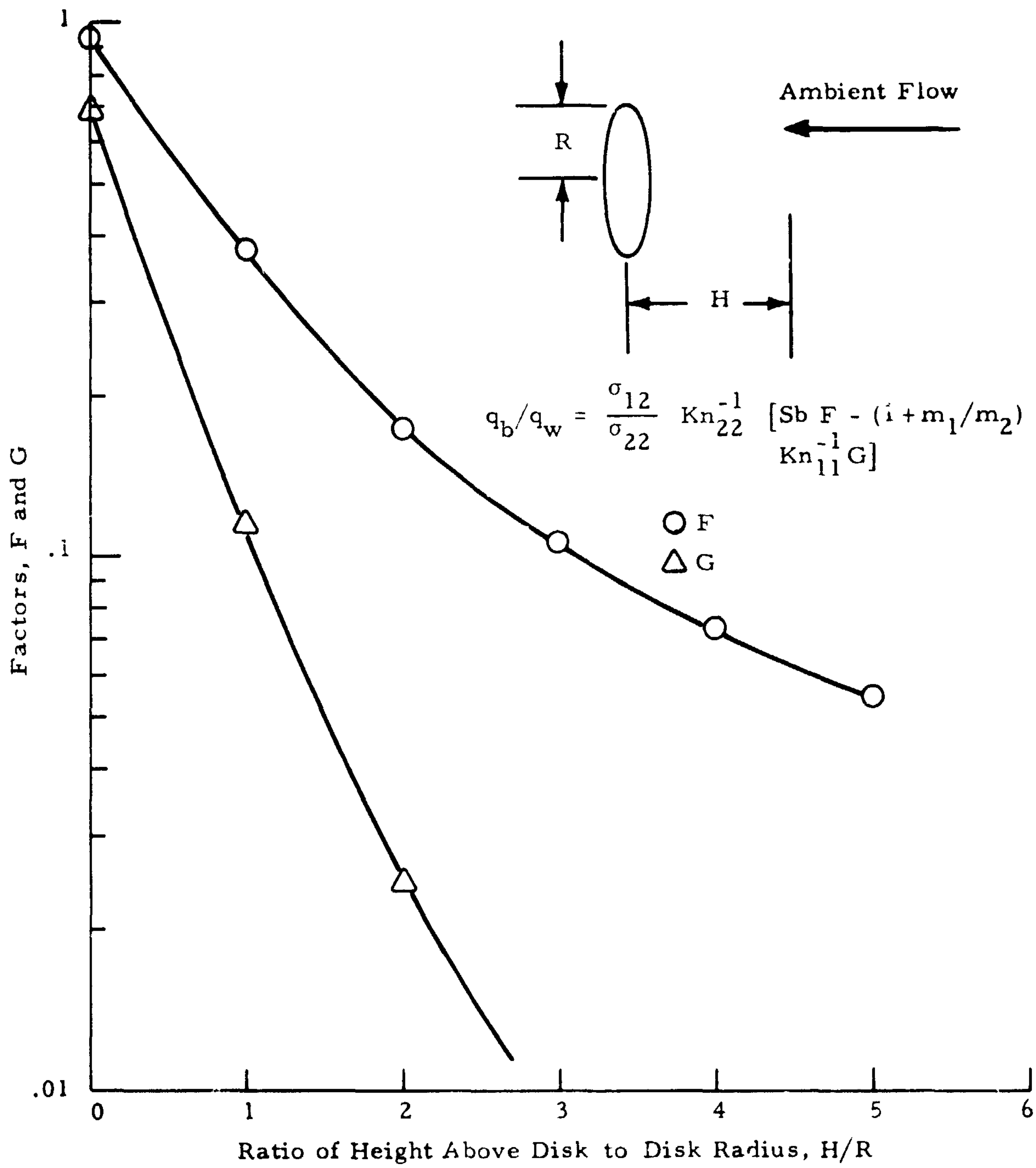


Fig. 4-3 - Return Flux to Points On and Above Flat Plate Disk Due to Ambient Collisions

Section 5
APPLICATION TO SPACE SHUTTLE

The equations developed in Section 3 for a sphere and Section 4 for a flat plate disk were used to estimate the backscatter of outgassing from the Space Shuttle vehicle. The Space Shuttle Orbiter is approximately 35 m long and 24 m wide at the base. We assumed an overall equivalent diameter of 20 m for use in the sphere and disk return flow equations. Calculations were made based on a circular orbit altitude of 400 km. A surface temperature of 300°K, an outgas rate q_w of 10^{-8} g/cm^2/sec, and an outgas product molecular weight of 100 were assumed. Atmospheric data were taken from Ref. 7 which provided a value of 41.9×10^{-16} cm^2 for the ambient cross section σ_{22}. Based on molecular data in Ref. 8, a value of 43.0×10^{-16} cm^2 was assumed for the collision cross section, σ_{11}, of the outgas molecules at 300°K. This value was also taken as the cross section σ^*_{12} for collisions between outgas and ambient molecules at energies corresponding to a reference temperature T^* of 300°K. The cross section σ_{12} corresponding to orbital interaction velocities was estimated based on the temperature variation of viscosity:

$$\frac{\sigma_{12}}{\sigma^*_{12}} = \left(\frac{\hat{T}_{12}}{T^*}\right)^{-\left(n - \frac{1}{2}\right)} \tag{5.1}$$

where $\hat{T}_{12}$ is the kinetic temperature defined by Eq. (2.27) and n is the exponent in the power law variation of viscosity with temperature. Assuming a composite molecular weight of 50 for the interacting species, and a value of 0.8 for the exponent n (Ref. 8), the interaction temperature $\hat{T}_{12}$ is found to be about 100,000°K, and the cross section ratio σ_{12}/σ_{22} is about 0.2.

The self-scattering return flux was calculated and found to be 2.25×10^{-12} g/cm^2/sec (1.35×10^{10} molecules/cm^2/sec) using the sphere formula and 3.80 x 10^{-12} g/cm^2/sec (2.29 x 10^{10} molecules/cm^2/sec) using the disc formula for the zero elevation. The self-scattering contribution is the same for all stations on the sphere. The ambient scattering return fluxes on the sphere were found to be 4.83 x 10^{-11} g/cm^2/sec (2.90 x 10^{11} molecules/cm^2/sec) at the forward station ($\phi' = 0$) and 1.07 x 10^{-12} g/cm^2/sec (6.40 x 10^9 molecules/cm^2/sec) at the rear station ($\phi' = 180$ deg). Note that the predicted ambient scattering return flux is about 20 times greater than the self-scattering return flux at the forward station and about half the self-scattering return flux at the rear station. The fact that the predicted ambient scattering return flux at the rear station is within the same order of magnitude as the self-scattering return flux is noteworthy, since previous studies based on first collision theory did not allow any upstream scattering in ambient collisions. The validity of these upstream scattering predictions will be discussed later in Section 6.

The ambient-scattering return flux on the disk was found to be 7.43 x 10^{-11} g/cm^2/sec (4.46 x 10^{11} molecules/cm^2/sec). The predicted return flux to the disk is seen to be about 50% greater than that to the sphere at the forward station.

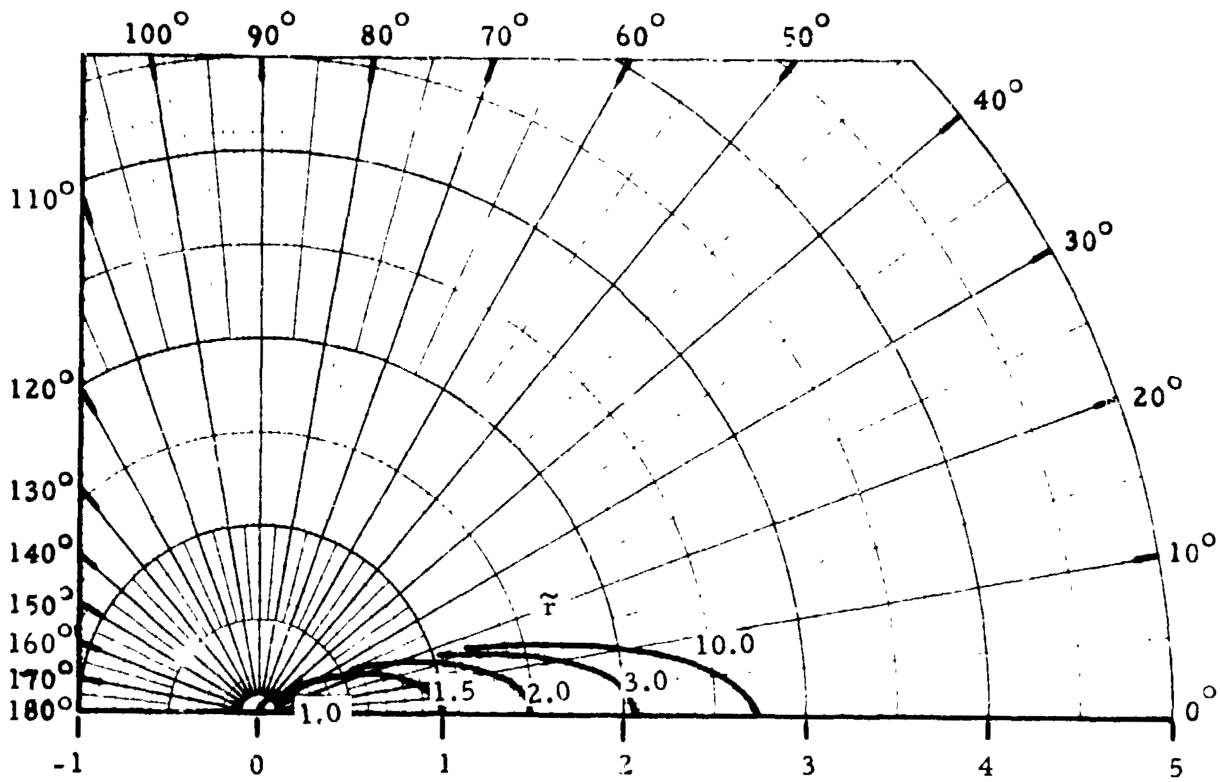
The contamination control criteria given in Ref. 9 specifies a return flux less than 10^{12} molecules/cm^2/sec. The predicted return fluxes for the 400 km orbit satisfy these criteria.

Section 6
PHYSICAL INTERPRETATION OF THE BGK MODEL SOLUTION

In light of the BGK model development described in Section 2, the physical interpretation of the BGK model solution can be stated as follows. Scattered molecules are produced locally at all points in the flow field at a rate equal to the local collision frequency. These molecules take on velocities and directions consistent with a gas in equilibrium at a hypothetical temperature and with a superimposed mean flow velocity. The hypothetical temperature and mean flow velocity are determined such that overall momentum and energy are conserved in the collision process. The BGK model scattering pattern, therefore, satisfies the basic requirements of mechanics, but it is not unique in doing so.

The scattering pattern for self-collisions for outgassing from a sphere is shown in Fig. 6-1, with distance away from the sphere center as a parameter. The scattering is seen to be highly directed in the direction of the mean flow away from the sphere, with the distributions becoming more directed as the distance increases. The scattering pattern approaches a constant distribution in the far field. Although the distributions are highly directed in the direction of the mean flow, there is in all cases a small, but nonzero, component of backscattering. Mathematically, this is brought about simply because the high velocity "tail" of the assumed Maxwellian distribution function contains some molecules with upstream components greater than the superimposed mean flow velocity. There is a good deal of physical basis for the predicted upstream scattering for the case of self-scattering from a sphere, since even "first" collisions of molecules emitted at high angles from the surface normal can cause deflections back into the surface.

More difficult to explain is the predicted upstream scattering for the case of ambient collisions. The predicted BGK scattering pattern for this case is shown in Fig. 6-2 compared to the cosine distribution characteristic



Fractional Scattering Distribution per Unit Solid Angle, f_{11}

Fig. 6-1 - BGK Model Scattering Pattern for Return Flux Due to Self-Scattering

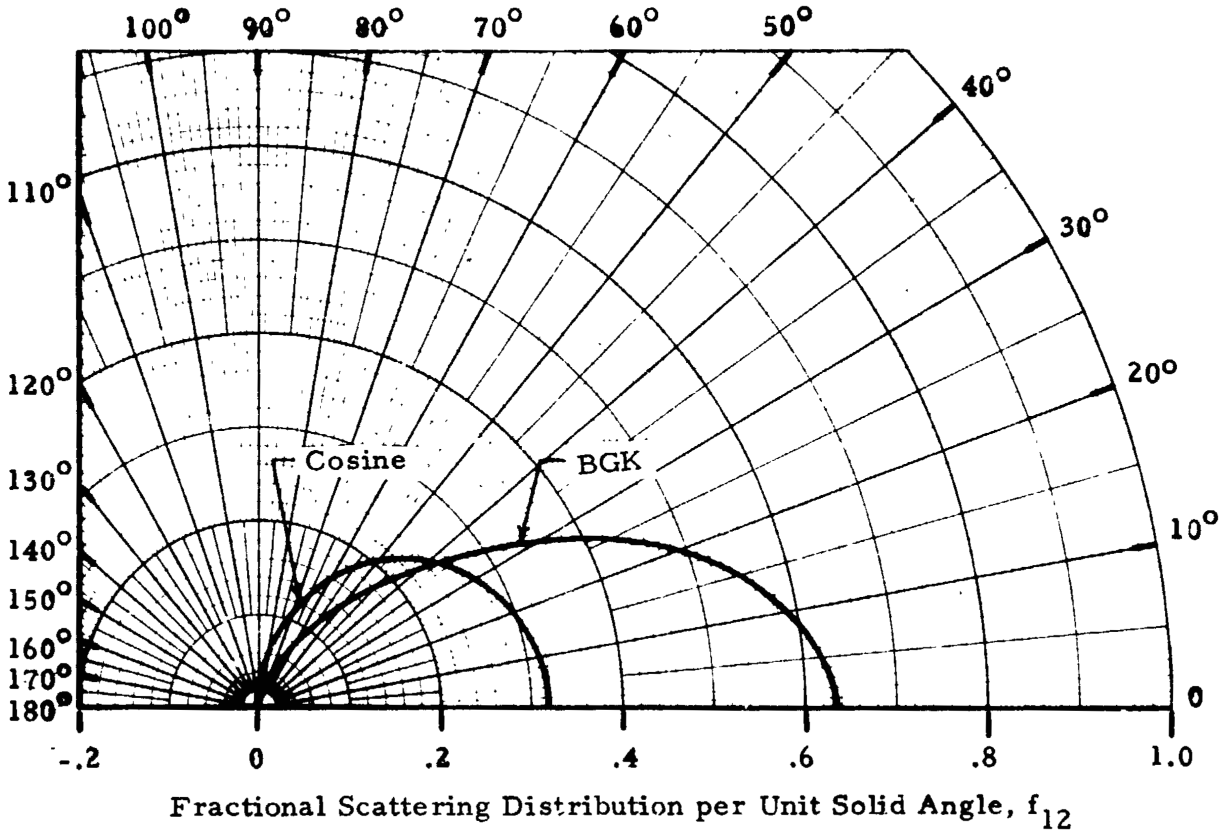


Fig. 6-2 - Scattering Patterns for Return Flux Due to Collisions with Ambient Atmosphere – BGK Model vs Cosine Law of "first" collisions in hyperthermal flow. Again, there is a small, but non-zero, component in the BGK predicted upstream scattering. Since "first" collisions between outgas molecules and freestream molecules do not permit upstream scattering, the physical interpretation of the upstream scattering components is that the overall collision process consists of a spectrum of collision events including not only "first" collisions, but also more complicated multiple collisions. Based on a cosine distribution, a small fraction of molecules will be directed in a near-lateral direction with respect to the ambient velocity vector after a "first" encounter. Some of these molecules will be directed in opposing directions such that "second" collisions can occur between these molecules to produce upstream as well as downstream deflections. Other more complicated collision events can also produce upstream deflections. Since the BGK model made no attempt to incorporate a rational combination of these collision events in the collision process, and since the upstream components form a relatively small portion of the overall scattering pattern, it is probably safe to say that there is a considerable margin of error in the BGK model prediction of upstream scattering for the case of ambient collisions. Probably the truth lies somewhere between the BGK and cosine distributions, which means that the actual upstream scattering for ambient collisions is less than the BGK model prediction.

Section 7
CONCLUDING REMARKS

The BGK model has been shown to be a useful device for estimating return flux for both self and ambient-scattering. The functional form of the return flow equations developed from the BGK model has the same form as equations developed previously by Scialdone (Ref. 10) and Robertson (Ref. 6) using the first collision model approach. The present approach, however, includes an attenuation term and provides for a small amount of upstream scattering in ambient collisions. As discussed in Section 6, the upstream scattering in ambient collisions is the primary area of doubt in the BGK approach. It was shown in Section 6 that a mechanism for upstream scattering exists, but at this point, it is impossible to quantify it. It is certainly likely that the BGK estimates provide an upper bounds. The return flux calculations in Section 3. therefore, may be considered reasonably accurate for self-collisions and for ambient collisions on the forward side of the sphere. The wake side calculations for ambient scattering, however, must be regarded as upper bounds only.

A significant point was noted by Naumann (Ref. 11) that the return flux of an outgassing species is dependent on the total molecular outflow. The flux q_w in the Knudsen number Kn_{11} for self-collisions, therefore, would be the total outflow rate. Thus, a contaminant species diluted in a large efflux of benign species would be returned at a higher rate.

Section 8

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