USE OF STRAINRANGE PARTITIONING TO PREDICT HIGH-TEMPERATURE LOW-CYCLE FATIGUE LIFE

Marvin H. Hirschberg and Gary R. Halford
Lewis Research Center
Cleveland, Ohio 44135

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The fundamental concepts of the strain-range partitioning approach to high-temperature, low-cycle fatigue are reviewed. Procedures are presented by which the partitioned strain-range versus life relationships for any material can be generated. Laboratory tests are suggested for further verifying the ability of the method of strain-range partitioning to predict life.
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SUMMARY

The strainrange partitioning approach for predicting high-temperature, low-cycle fatigue life has undergone development at the NASA Lewis Research Center since its introduction in 1971 by Manson, Halford, and Hirschberg. The concept has been developed to the point where considerable confidence has been gained from its ability to encompass the behavioral patterns of alloys to which it has been applied. The approach holds a great deal of promise for materials specialists and designers.

The basic concepts are reviewed, and procedures are outlined for applying the method of strainrange partitioning to various laboratory high-temperature, low-cycle loading conditions. These procedures involve the determination of the four basic life relationships (for PP, CC, PC, and CP type inelastic strainranges) for any material and their use in conjunction with the interaction damage rule to predict the cyclic lives of laboratory specimens subjected to various combinations of the basic inelastic strainrange components. The degree of insensitivity of the four basic life relationships to test temperature as well as their utility in representing bounds on life are also discussed.

INTRODUCTION

This report is concerned with the strainrange partitioning approach for dealing with high-temperature, low-cycle fatigue life of metallic materials. Since the early 1950's, a variety of materials characterization and life prediction techniques have
been developed to fulfill the immediate needs of designers and builders of high-performance, energy-conversion equipment that must operate reliably for long periods of time at high temperatures. Although most of the approaches offered some unique advantage, they all also had deficiencies of one sort or another. Also, since the early 1950's a large quantity of high-temperature, low-cycle fatigue data has been generated on a host of alloys. Many of the data were determined in an attempt to answer a single specific question relative to a particular design requirement, and little regard was given to the generation of data that would have general applicability to future problems. As a consequence, the designer is faced with many approaches and a considerable quantity of diverse, sometimes contradictory, and often inappropriate low-cycle fatigue test data.

It was out of this environment that the strainrange partitioning approach was formulated in 1971 (ref. 1). In the intervening years we have developed considerable insight into its capabilities and recognize the technological impact the approach offers in the design of components that must resist failure due to the interaction of high-temperature creep and fatigue. The primary advantage of strainrange partitioning rests on its high degree of generality. Relatively few test data are necessary to characterize a material in terms of strainrange partitioning, and the results are directly applicable to complex strain cycles. Not only is the approach a general and fundamental one, but it also appears to offer significantly better accuracy in life prediction than has been possible heretofore based on our investigations with a fairly wide variety of materials (ref. 2). The various advantages of strainrange partitioning are reviewed in this text.

The main purposes of this report are to describe the method of strainrange partitioning, to provide guidance for generating the necessary data for determining the partitioned strainrange versus life relationships, and to suggest laboratory experiments that permit further verification of the ability of the method to deal with a variety of test conditions. It is hoped that, through the experience gained by laboratories in both generating strainrange partitioning data and further verifying the method with simple specimens, a greater degree of confidence in the soundness of the approach will result. Once this is accomplished, designers should feel more secure in applying the method to the life prediction of actual structural components.
REVIEW OF STRAINRANGE PARTITIONING CONCEPTS

The Lewis Research Center has long been involved in the development of theories and approaches for dealing with low-cycle fatigue behavior of materials. A most important transition from a stress approach to a strain approach occurred with the formulation of the Manson-Coffin law (ref. 3). We then went to the total strain approach with the inclusion of the elastic portion of the total strainrange (ref. 4). The Manson-Hirschberg method of universal slopes was then developed (ref. 5), which made it possible to estimate total strainrange versus life behavior from conventional short-time tensile properties. It was, of course, recognized that these approaches were all limited to temperatures below the creep limit. Our first attempts to account for creep effects were the 10 percent rule (ref. 6) and the creep-modified 10 percent rule (ref. 7). These approaches represented most of the available high-temperature test data rather well, but they were not sufficiently conservative for cases involving long hold times or where large creep strains could be accumulated. An attempt to remedy this was made with the life fraction approach (ref. 8). Unfortunately, that method involves extensive analytical procedures and was unable to explain some of the experimental observations regarding the damaging effects of compressive stress. The next step was the formulation of the method of strainrange partitioning which holds promise for overcoming deficiencies in previous methods.

The basic premise for strainrange partitioning is that in any hysteresis loop there are combinations of just two directions of straining and two types of inelastic strain. The two directions are, of course, tension (associated with a positive inelastic strain rate) and compression (associated with a negative inelastic strain rate); the two types of inelastic strain are time dependent (creep) and time independent (plastic). By combining the two directions with the two types of strain, we

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Since the preparation of this report, a pertinent paper (ref. 9) has been written describing techniques for separating inelastic strains. It was concluded that transient creep strain should be considered as plastic strain and only the steady-state component be considered as creep strain. The implications are significant, and anyone using strainrange partitioning should become familiar with this reference.
arrive at four possible kinds of strainranges that may be used as basic building blocks for any conceivable hysteresis loop. These define the manner in which a tensile component of strain is balanced by a compressive component to close a hysteresis loop and are described as follows:

1. Tensile plasticity reversed by compressive plasticity is designated a PP strainrange and represented by $\Delta \epsilon_{PP}$.

2. Tensile creep reversed by compressive plasticity is designated a CP strainrange and represented by $\Delta \epsilon_{CP}$.

3. Tensile plasticity reversed by compressive creep is designated a PC strainrange and represented by $\Delta \epsilon_{PC}$.

4. Tensile creep reversed by compressive creep is designated a CC strainrange and represented by $\Delta \epsilon_{CC}$.

The notation for the subscripts for the strainranges uses the type of tensile strain first, followed by the type of compressive strain. The name strainrange partitioning was chosen because it represented our premise that, in order to handle a complex high-temperature, low-cycle fatigue problem, the inelastic strainrange must first be partitioned into its components.

A schematic representation of the four component strainrange versus life relationships is shown in figure 1. What we have proposed then, is that the Manson-Coffin relationship (which represents the inelastic strainrange versus life behavior for materials below the creep limit) be expanded to four relationships for dealing with materials above the creep limit. It should be noted that the PP relationship is analogous to the original Manson-Coffin relationship since no creep strain is present.

Before we can use these four life relationships, we must first produce them experimentally. Figure 2 shows four types of idealized hysteresis loops that could be used to generate the strainrange versus life relationships. The PP type hysteresis loop (fig. 2(a)) can be generated in a conventional manner, but the other three (in figs. 2(b), (c), and (d)) require some less-conventional testing procedures. Detailed procedures for generating the required data for utilizing the method of strainrange partitioning will be described in the appendix.

Once the curves (schematically represented in fig. 1) have been generated for a material, there remains the question of how to apply these curves to a more complex loading problem in order to predict life. This is accomplished in two parts.
First, the hysteresis loop for the cycle being analyzed must be partitioned into its inelastic components. Second, a damage rule must be applied in order to predict the life associated with the combination of applied strainranges.

Figure 3 is a schematic representation of a complex hysteresis loop. This loop is made up of the following loading sequences: Starting at point 1, load is applied rapidly to point 2 and then a constant stress is held until point 3 is reached. We then unload elastically to point 4 and continue on rapidly to point 5. The compressive stress is then held constant until point 6 is reached, at which time the strain is held constant and the stress is relaxed to point 7. The cycle is completed by unloading elastically to point 1 where the cycle started. In the loop being described the inelastic strainrange is defined by the width of the loop AC. In going from A to C in the tension direction, we accumulated a tensile plastic strain AB and a tensile creep strain BC. In reversing the cycle, we accumulated a compressive plastic strain CD and a compressive creep strain DA. In this example the tensile plastic strain AB was reversed by a portion of the compressive plastic strain CD and, likewise, the entire compressive creep strain DA was reversed by only a portion of the tensile creep strain BC. We therefore had a PP strainrange of magnitude AB and a CC strainrange of magnitude DA. From the excess tensile creep strain and excess compressive plastic strain, we have a CP strainrange of magnitude BC-DA or CD-AB (since CD-AB = BC-DA). It should be noted that, in any hysteresis loop, it is possible to have a maximum of only three of the four types of strainranges. It is not possible for the PC and CP type strainranges to be components of the same hysteresis loop. Procedures for partitioning complex hysteresis loops will be referred to in the appendix.

For the model just described, we have concluded that the inelastic strainrange, $\Delta \epsilon_{\text{IN}} = AC$, is made up of the three components $\Delta \epsilon_{\text{PP}} = AB$, $\Delta \epsilon_{\text{CC}} = DA$, and $\Delta \epsilon_{\text{CP}} = (BC-DA)$. In equation form, $\Delta \epsilon_{\text{IN}} = \Delta \epsilon_{\text{PP}} + \Delta \epsilon_{\text{CC}} + \Delta \epsilon_{\text{CP}}$.

Let us now consider (with the aid of fig. 3) a numerical example with an inelastic strainrange $AC = 0.0050$, a tensile plastic strain $AB = 0.0015$, a tensile creep strain $BC = 0.0035$, a compressive plastic strain $CD = 0.0030$, and a compressive creep strain $DA = 0.0020$. The partitioned inelastic strainranges can now be determined from the following simple rules:

1. $\Delta \epsilon_{\text{PP}}$ is equal to the smaller of the plastic strains in the two directions. For the example, $AB = 0.0015$ and $CD = 0.0030$. Hence, $\Delta \epsilon_{\text{PP}} = 0.0015$. 
(2) \( \Delta \varepsilon_{cc} \) is equal to the smaller of the creep strains in the two directions. For the example, BC = 0.0035 and DA = 0.0020. Hence, \( \Delta \varepsilon_{cc} = 0.0020 \).

(3) \( \Delta \varepsilon_{cp} \) or \( \Delta \varepsilon_{pc} \) is equal to the remainder of the inelastic strainrange not assigned to \( \Delta \varepsilon_{pp} \) and \( \Delta \varepsilon_{cc} \). For the example, all but 0.0015 of the strainrange was assigned to \( \Delta \varepsilon_{pp} \) and \( \Delta \varepsilon_{cc} \). Hence, 0.0015 is the remainder, and it is a \( \Delta \varepsilon_{cp} \) strainrange since the excess of creep of 0.0015 is in tension and the excess of plasticity of 0.0015 is in compression.

There remains the final task of predicting the cyclic life resulting from this combination of partitioned strainranges. We have proposed the interaction damage rule (ref. 10) for this purpose. An example of how this rule is applied may be seen with the aid of figure 4. We must first have obtained the individual partitioned strainrange versus life relationships as shown in figure 4. We must also have determined, as above, the magnitude of the individual partitioned strainranges for the particular hysteresis loop being analyzed. We then perform the following steps:

(1) For the inelastic strainrange of interest (in this case \( \Delta \varepsilon_{IN} = AC \)), read the values of cyclic lives \( N_{pp}, N_{cc}, \) and \( N_{cp} \). It is not necessary to obtain a value of \( N_{pc} \) for this example since we have already determined that no such strainrange component exists (\( \Delta \varepsilon_{pc} = 0 \)) in our example hysteresis loop (fig. 3).

(2) Calculate the fractions for each of the partitioned strainranges as shown in figure 4. For this example, \( F_{pp} = \Delta \varepsilon_{pp}/\Delta \varepsilon_{IN} = AB/AC \), \( F_{cc} = \Delta \varepsilon_{cc}/\Delta \varepsilon_{IN} = DA/AC \), and \( F_{cp} = \Delta \varepsilon_{cp}/\Delta \varepsilon_{IN} = (BC-DA)/AC \). Note that \( F_{pp} + F_{cc} + F_{cp} = 1 \).

(3) The damage per cycle due to each of the components can be represented by \( F_{pp}/N_{pp}, F_{cc}/N_{cc}, \) and \( F_{cp}/N_{cp} \). The total damage per cycle is \( 1/N_{pred} \) and is equal to the sum of the individual damage contributions. Hence,

\[
1/N_{pred} = F_{pp}/N_{pp} + F_{cc}/N_{cc} + F_{cp}/N_{cp}
\]

(4) The predicted life \( N_{pred} \) for the hysteresis loop in question is then calculated using the above equation.

It is this total process of generating the individual failure life relationships, partitioning a hysteresis loop into its component strainranges, and combining the effects of these components to determine life, that we have called the method of strainrange partitioning.
VERSATILITY OF THE METHOD OF STRAINRANGE PARTITIONING

The following sections describe a number of the aspects of strainrange partitioning that we have used in the past to demonstrate the method's versatility in characterizing, correlating, and predicting the high-temperature, low-cycle fatigue behavior of alloys. In the appendix these are again mentioned along with detailed suggestions for conducting laboratory tests to further verify the various aspects of strainrange partitioning.

Material Characterization

To date, we have characterized several materials by the four independent partitioned strainrange versus life relationships. Examples for six materials are shown in figure 5. In all cases each relationship can be described adequately by a power law, that is, linear plots of strainrange versus life on log-log coordinates. It should be noted that the ordering of these partitioned strainrange versus life relationships is not the same for all the materials shown. For example, in some cases the CP and in others the PC type strainrange is the most damaging. Verification of the power law relationships will be of great practical value in that it will minimize the amount of data required by future investigators to characterize any one of the lines. In most cases five or six tests are required to adequately define any one of the lines.

A survey of the literature dealing with high-temperature fatigue shows one of the other major advantages of the proposed approach. In the past it took a great variety of tests to completely characterize a material. Besides test temperature (which we shall deal with separately in a later section), specific design curves were necessary to account for a variety of testing frequencies, applied wave shapes, as well as for tensile and compressive hold times. All these requirements made it almost impossible to completely characterize a material so that the design data would be easily and generally applicable to all problems. It was therefore usually left to an investigator to match his own specific test program to his own application. The strainrange partitioning method, which involves only the four partitioned strainrange versus life relationships to characterize material behavior, should eliminate this dilemma.

Designers will still have to determine the type and magnitude of the strain being
applied to their structure, but the failure curves, as represented by the partitioned
strainrange versus life relationships, are general and can be applied to any specific
situation. This should reduce the number and kinds of tests previously necessary
to predict life.

Figure 6 was prepared to demonstrate the ability of the approach to predict lives
for a variety of alloys. Tests were conducted on 12 alloys, and the observed cyclic
lives are in agreement with the calculated lives within factors of two. These tests
were run at various frequencies and in many instances with tensile or compressive
hold times under either constant stress or strain conditions. These results demon-
strate the ability of the method of strainrange partitioning to characterize the high-
temperature, low-cycle fatigue behavior of these materials.

Bounds on Life

Early in the design process, it is likely that the inelastic strains at critical loca-
tions in a structure will be known with at least a limited degree of accuracy. The
method of strainrange partitioning allows us to take advantage of this preliminary
information and determine the expected upper and lower bounds on life without
having to perform the partitioning of a hysteresis loop.

As can be seen from the illustrative example of figure 4, if the inelastic strain-
range \( \Delta \epsilon_{IN} \) were entirely of the PP type, the most the life could possibly be is
\( N_{PP} \). On the other hand, if \( \Delta \epsilon_{IN} \) were made up entirely of the most damaging
type of strainrange (CP in this illustrative case), the life could be no less than
\( N_{CP} \). In other words, the upper and lower bounds on life for any given inelastic
strainrange can be obtained from the assumption that the actual life must lie between
the most conservative and least conservative of the partitioned strainrange versus
life relationships.

These bounds on life can be further narrowed if one wishes to make some addi-
tional assumptions regarding the types of strainrange components that might exist
in the structure being analyzed. For example, for certain thermal fatigue problems,
the designer might know that the CP type of cycle is not possible in the critical
location being analyzed. This information might eliminate the CP failure mode
from consideration and thereby alter the lower bound on life from \( N_{CP} \) to \( N_{CC'} \),
as illustrated in figure 4.
For some materials, two or more of the four partitioned strain range versus life relationships can be quite close to each other (fig. 5). This could result in upper and lower bounds that are also quite close. In such a case any further refinement of the calculations required to more accurately determine the types of strainranges present might not be justified. This could greatly reduce the effort required for life prediction without any appreciable sacrifice in accuracy.

Temperature Insensitivity

Experience indicates that low-cycle fatigue life may be sensitive to test temperature. In general, for the same applied loading, the higher the temperature, the lower the life. This trend can be rationalized by recognizing that the effects of temperature can be twofold: its effect on the flow behavior and on failure behavior. It is well known that the flow behavior of materials is highly temperature sensitive. The higher the temperature, the greater the likelihood for creep deformation. The failure behavior, which is represented by the life relationships, on the other hand, is not so sensitive to temperature. We have investigated (ref. 11) the influence of temperature on the failure relationships for two alloys, 316 stainless steel and 2\(\frac{1}{4}\) Cr-1Mo steel. A summary of these results is shown in figure 7 where it can be seen that temperature has a negligible effect on the failure relationships. Reasons for the behavior were documented in reference 11. Hence, the role of temperature in influencing fatigue life for these two alloys is to alter the partitioning of creep and plasticity in a cycle, but not to alter the failure relationships. This insensitivity is an important aspect that can enhance the utility of strainrange partitioning. Fewer tests are required to characterize the high-temperature fatigue behavior, and the analysis of cycles involving changing temperatures is greatly simplified.

Application to Complex Cycles

We have to date obtained only limited data indicating that the method of strain-range partitioning works well for predicting lives for complex cycles. We used the partitioned strainrange versus life relationships for 316 stainless steel generated at 705\(^\circ\) C (fig. 5) to predict lives for tests in which the temperature was cycled both
in-phase and out-of-phase with the applied strain. The results of these predictions are shown in figure 8.

For the in-phase tests the minimum applied strain occurred at the minimum test temperature (230° C) and both the strain and temperature were increased at a constant rate until the peak strain was reached at the same time the peak temperature was reached (750° C). The strain and temperature were cycled in this manner until failure occurred. For the out-of-phase test the temperature cycle was reversed so that the minimum strain occurred with the maximum temperature and the maximum strain with the minimum temperature.

Since almost all the creep strain was introduced into the hysteresis loop near the peak temperature, the in-phase cycle was a combination of PP and CP types of strain, and the out-of-phase cycle was a combination of PP and PC types of strain. Predicted lives for these tests (fig. 8) fell within the same factors of two of the observed lives as did the isothermal test results of figure 7.

CONCLUDING REMARKS

The method of strainrange partitioning and various aspects of its utility are described. Procedures are presented in the appendix for generating the required partitioned strainrange versus life relationships and for using them to predict high-temperature, low-cycle fatigue life. It is hoped that application of the strainrange partitioning method as suggested herein, will provide additional verification of its usefulness. The authors would appreciate being made aware of the results of the applications of this method for it is through the sharing of such results that an objective assessment of the strengths and weaknesses of the approach can be obtained.

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APPENDIX - STRAINRANGE PARTITIONING PROCEDURES

In order for potential users to gain experience with strainrange partitioning procedures, it is suggested that the following aspects be investigated for any material of interest. This will permit generation of the necessary material characterization data and will indicate the capability of this method for predicting fatigue life. The suggested approach involves the following:

1. Material characterization by determination of the basic life relationships for the PP, CC, PC, and CP type strainranges
2. Demonstration of the transition in life between two bounding curves as a result of systematically changing one of the testing parameters such as cyclic straining rate
3. Determination of the effect of test temperature on the life relationships established previously (items (1) and (2))
4. Analyses of tests with complex or "mixed" cycling conditions by partitioning strains, computing damage components, and predicting lives

This program, if conducted in its entirety, would require approximately 25 to 50 high-temperature, low-cycle fatigue tests. In addition, it is also desirable to conduct a few standard tensile and creep-rupture tests to document the more conventional properties.

It is suggested that a baseline elevated temperature be selected for conducting the majority of the tests. This temperature should be high enough that creep effects can be readily introduced within reasonable times. For example, a testing temperature of 650° C would be a good choice for stainless steels, but 900° C would be more appropriate for an advanced nickel-base superalloy. Baseline test temperatures should be chosen such that some tests could be conducted at temperatures of 100° C above and below the baseline temperature and still be able to introduce creep effects in a practical time span.

Material Characterization

The four basic partitioned strainrange versus life relationships shown schematically in figure 9 can be determined by conducting isothermal laboratory tests that feature the individual strainranges of interest. Six test points are suggested
for determining each of the curves. Examples of tests that feature the four components will now be presented.

**PP type of tests.** - This test consists of only plastic strain in both the tensile and the compressive halves of the cycle. No creep strain is permitted. The hysteresis loop for a PP type of cycle is shown in figure 10(a). To achieve this condition at high temperatures, it is necessary to cycle the strain at a frequency sufficiently high to preclude the introduction of time-dependent creep strain. Continuous strain cycling tests at frequencies on the order of 0.5 to 2.0 hertz have been used in previous research (ref. 1) and have been found to be acceptable. The minimum frequency required to exclude creep effects can be determined experimentally by conducting a series of tests at progressively higher and higher frequencies until a plateau in stress-strain response or in cyclic life is reached. All PP type of tests would then be conducted at or above this minimum frequency. The programmed strain-time wave shape should be either a sine wave or a triangular wave, with a preference for the second of these. Strain ranges for the six PP type of tests should be selected so as to give lives covering the range between approximately 50 to 50,000 cycles to failure. A plot (fig. 9) can then be constructed of \( \Delta \epsilon_{pp} \) versus the number of cycles to failure, \( N_{pp} \) on logarithmic axes. It is necessary that this relationship be established before the determination of the other three life relationships.

**CP type of tests.** - A number of testing techniques can be used to achieve a cycle that features the CP strain range. Of the three most common cycles, there is a preference for the one involving a constant tensile creep stress as shown in figure 10(b). This cycle is the most efficient in terms of obtaining the greatest amount of creep strain in the least amount of testing time. Furthermore, the magnitude of the creep strain is readily identifiable. We have conducted numerous tests of this type at Lewis and have found the technique to work well. The test can be executed under servo-load control with external strain limits superimposed. A tensile creep stress is programmed to act for as long as necessary to produce a predetermined amount of tensile strain. When this tensile strain limit is reached, the servo-controller calls for a compressive stress that is somewhat greater than the capacity of the material. To prevent the compressive loading from occurring too rapidly and from producing large compressive strains, we have purposely throttled down the oil supply to the hydraulic actuator with a small orifice needle valve. The compressive deformation rate is thereby controlled at a rate that has been previously determined to be high.
enough to preclude creep effects but not so high as to cause overshooting of the compressive strain limit (equal in magnitude to the tensile strain limit). When this limit is reached, the servo-controller calls for the previously applied tensile stress, and the cycle proceeds and repeats itself until failure occurs.

The value of the tensile stress to be used in such cycles must be low enough that the creep strain is the dominant strain in the tensile portion of the cycle, yet large enough that the creep time per cycle is not prohibitively large. Should a material undergo significant cyclic hardening or softening, the time duration of the creep portion of the cycle can change drastically unless the creep stress level is periodically and appropriately increased or decreased. Since the important aspect of these tests is to impose and measure an amount of creep strain, there should be little concern for the magnitude of the stress responsible for producing this strain. This is analogous to permitting, and essentially ignoring, the stress level changes that are associated with cyclic hardening or softening during high frequency cycling to establish the PP type behavior.

When the desired inelastic strain ranges are too small for practical application of these cycles, the tensile strain, hold-time cycle (fig. 10(c)) is suggested. Here, the straining is done in exactly the same way as for the PP type of cycle except that a hold period is introduced at the peak tensile strain. During the hold period, stress relaxation occurs by a creep mechanism. The total amount of creep strain incurred per cycle is equal to the amount of elastic strain that is relaxed as a function of time. Only a small amount of creep strain can be obtained in this fashion, and since it is always desirable to feature the type of strain range of interest, this test cycle should only be used when the desired inelastic strain range is small. The magnitude of the tensile creep strain per cycle is equal to the CP strain range and the balance of the inelastic strain range in the cycle must therefore be equal to the PP strain range.

A third testing technique that features the CP strain range is a strain cycle that involves a very low straining rate during tensile deformation but a very high straining rate during the compressive deformation portion of the cycle (fig. 10(d)). Although this type of cycle is relatively simple to use, the interpretation of the results is not as straightforward as the two techniques just discussed. The major problem is in the interpretation of how much of the tensile strain is creep strain and how much is plastic strain. Since the stress and strain response are not of a form from
which the creep strain can be immediately identified, the tensile strain must be partitioned into its component strains by some independent means (ref. 9). This cycling technique is recommended only if the other two types of cycles cannot be achieved with available testing equipment.

Before the $\Delta \epsilon_{\text{CP}} - N_{\text{CP}}$ relationship can be plotted, the damage due to the presence of any PP type strainrange must be accounted for. Hence, it is necessary that the $\Delta \epsilon_{\text{PP}} - N_{\text{PP}}$ relationship be the first one established. In this case, only PP and CP strainrange components are present, and the fractions, $F_{\text{PP}}$ and $F_{\text{CP}}$ are known. The interaction damage rule (fig. 4) can now be used in a reverse procedure. The experimentally observed life is substituted for the predicted life, and the equation is solved for the unknown value of $N_{\text{CP}}$. This $N_{\text{CP}}$ life represents the life that would have been obtained had there been no contribution from PP type of damage. Hence, the inelastic strainrange of the cycle is plotted against this $N_{\text{CP}}$ life value. These results can then be plotted as shown in figure 9. Strainranges for the six CP type of tests should cover approximately the same regime as was used to establish the $\Delta \epsilon_{\text{PP}} - N_{\text{PP}}$ relationship.

**PC type of tests.** - Tests appropriate for the determination of the life relationship are almost identical to the CP tests. The only difference between the two is in the direction of the creep portion of the cycle as can be seen by comparing the CP and PC cycles in figures 10(b) to (g). Hence, by interchanging the tensile and compressive notations in the previous section, the techniques for producing the PC straining can be obtained.

**CC type of tests.** - There are also three basic testing procedures for generating strain cycles with completely reversed creep strains, that is, CC type strainranges. In fact, such cycles are, in principle, formed by using just the tensile portion from the CP test and the compressive portion from the PC test. The preferred cycle (fig. 10(h)) involves the use of the cyclic creep-rupture test (ref. 12) wherein a constant tensile stress is servo-controlled until a preset tensile strain limit is reached by creep. When the limit is reached, the direction of the stress is rapidly reversed and an equal-valued constant compressive stress is servo-controlled until a preset compressive strain limit is reached. Upon reaching this limit, the tensile stress is reapplied and the test is continued in the same fashion until failure occurs. Again, it may be necessary to periodically increase or decrease the magnitude of the creep stresses as the test progresses in order to maintain a desired creep strain-range or time per cycle. Our experience has been that under CC type straining
conditions, creep rates increase throughout the test when the stress amplitude is held constant.

There will always be a PP component of strainrange in the CC type of test. Furthermore, there may also be an unbalanced component of strainrange such as CP or PC, depending on whether the unbalance of creep strain is in the tensile or compressive half of the cycle. Before the undesirable damaging effects due to the presence of any PP and of CP or PC type of strainrange can be factored out of the test results, it is necessary to have conducted the PP, CP, and PC tests and to have already established these three life relationships.

When the desired inelastic strainrange is small, these CC types of cycle may be difficult to control. In that event, an alternative test (fig. 10(i)) is a rapid strain cycling test with hold periods superimposed at both the peak tensile and compressive strains. During these peak strain hold periods, the stresses relax and elastic strain is in effect converted into creep strain. If the amount of stress relaxation is the same in both tension and compression, there is no component of unbalanced strainrange present in the cycle. Hence, the CC strainrange is equal to the amount of creep strain relaxed during either the tensile or compressive halves of the cycle. The balance of the inelastic strainrange is the PP strainrange.

The third means of producing balanced creep strain in a cycle (fig. 10(j)) is simply to perform a completely reversed strain cycle at a low enough frequency that creep can occur. As discussed earlier with respect to CP type of testing, this procedure introduces another difficulty since the creep strain that is incurred is not readily identifiable. Hence, a supplementary partitioning test would be required to partition the inelastic strainrange into its CC and PP components. An unbalanced strainrange component (PC or CP) would not be expected in this type of test provided the straining rates in tension and compression were equal.

**General comments.** In selecting testing techniques to be used, there are some practical limitations that must be taken into consideration. For example, when a CC type of test is being conducted to establish the $\Delta \varepsilon_{CC} - N_{CC}$ relationship, the dominant damage in the cycle must be due to the CC strainrange component. In reference 1, we suggested using the criterion that at least one half of the damage $\varphi$ in a test with an observed life $N_{OBS}$ must be due to the strainrange component of interest (e.g., $\varphi = F_{CC} \left( N_{OBS}/N_{CC} \right) > 0.5$ for a CC cycle). Only if this criterion is met can a test point be used in the establishment of the life relationship.
We have also found from experience, that it is desirable to impose still another restriction on whether a test point should be used in the establishment of one of the basic life relationships. Although not a mandatory requirement, the strainrange component of interest should be the dominant component; that is, the strainrange fraction $F$ should be greater than 0.5 (e.g., $F_{CC} = (\Delta \epsilon_{CC}/\Delta \epsilon_{IN}) > 0.5$ for a CC cycle).

**Bounds on Life**

Having once determined the four partitioned strainrange versus life relationships in accordance with the tests described in the previous section, the premise that strainrange partitioning can be used to represent bounds on cyclic life can now be evaluated. As an illustration, consider a material whose life relationships are positioned as shown in figure 11. For this material no cycle will produce lives greater than that indicated by the PP line, and lives cannot be achieved that are lower than indicated by the three curves involving creep strain in the three different extreme modes. A balanced strain cycle, for example, would have its lowest life bounded by $N_{CC}$ and the highest life by $N_{PP}$. Or, considering an unbalanced cycle wherein compressive creep can be ruled out, the lowest possible life would be $N_{CP}$ and the highest possible life would be $N_{PP}$. A check on this premise of strainrange partitioning could be obtained with approximately six specimens by performing a series of tests in which a single controlled variable (such as frequency, hold time, tensile creep stress, or whatever variable is convenient) is systematically changed from one test to the next. The simplest example would be to select a balanced strain cycle, and, at a given test temperature and strainrange, vary the test frequency. At the highest frequency the strainrange would be all PP. As frequency is decreased, CC type of strain would gradually displace the PP type of strain, and the cyclic life would drop from a maximum of $N_{PP}$ to a minimum of $N_{CC}$. The results might take on the appearance as indicated schematically in figure 11.

**Temperature Insensitivity**

A potentially beneficial aspect of characterizing the high-temperature, low-cycle fatigue behavior of materials in terms of the four partitioned strainrange versus life
relationships is that for a number of materials, these life relationships should not be expected to vary appreciably as the testing temperature is either increased or decreased. Although temperature has a profound influence on constitutive relationships (i.e., flow behavior, or stress–strain–time response), the influence of temperature on the cyclic fracture process is not particularly a strong one. In fact, it can be neglected for some engineering materials. The degree of temperature insensitivity can be assessed in the following way.

For each of the life relationships already determined, additional tests could be conducted featuring each of the four basic strainranges. For each of the four basic types of cycle, a minimum of two tests could be conducted at temperatures of 50° and 100° C above and below the baseline temperature used in generating the basic life relationships. The ensuing test results could then be used to establish new points for direct comparison with the original life relationships. A suggested method of comparison of these new test results with the baseline results is indicated in figure 12.

Application to Complex Cycles

Another advantageous aspect of strainrange partitioning is its inherent ability to handle any generalized strain cycle (for example, fig. 13(a)) regardless of its complexity. Any inelastic strain can be separated into time-dependent and time-independent components. We have proposed both analytical and experimental techniques (ref. 9 and reply to Husslage's and Kreipl's discussions of ref. 13) for partitioning strains within a mixed cycle, that is, a cycle with combinations of PP, CP or PC, and CC type strainranges.

Once partitioned, and once the life relationships have been determined as in figure 13(b), the process of predicting the life \( N_{\text{PRED}} \) associated with a mixed cycle is reduced to a simple algebraic exercise of solving the following expression.

\[
\frac{F_{PP}}{N_{PP}} + \frac{F_{CC}}{N_{CC}} + \frac{F_{CP}}{N_{CP}} + \frac{F_{PC}}{N_{PC}} = \frac{1}{N_{\text{PRED}}}
\]

As a demonstration of the potential of strainrange partitioning in predicting cyclic lives of mixed cycles, it is suggested that a few strain cycling tests with some
complicated wave shape or pattern be performed, the inelastic strains partitioned into creep and plastic components, and the cyclic lives predicted and compared with experimentally observed lives. Approximately six test specimens could be devoted to such an evaluation, and the results can be displayed as shown schematically in figure 13(c).
REFERENCES


Figure 1. - Typical partitioned strainrange-life relationships used to characterize material behavior in the creep-fatigue range.

Figure 2. - Idealized hysteresis loops used in defining individual partitioned strainrange-life relationships.
Figure 3. - Defining partitioned strainrange components of complex hysteresis loop.

Figure 4. - Definition of terms for interaction damage rule.
Figure 5. Partitioned strain-range-life relationships for six alloys.

(a) Alloy, 316 stainless steel; temperature, 705°C.
(b) Alloy, 2.25 Cr-1 Mo steel; temperature, 595°C.
(c) Alloy, A-286 (vacuum); temperature, 595°C.
(d) Alloy, H-13 steel; temperature, 595°C.
(e) Alloy, IN-100; temperature, 925°C.
(f) Alloy, T-111 (vacuum); temperature, 1150°C.
Figure 6. - Ability of strainrange partitioning to characterize material behavior in creep-fatigue range. (Data for 12 alloys: Ni, Co, Ta, Fe, and Cu-base.)

Figure 7. - Comparison of observed and predicted life at different temperatures using strainrange partitioning approach.
Figure 8. - Ability of strainrange partitioning to predict lives for in-phase and out-of-phase tests from isothermal data. 316 Stainless steel, temperature, 230°C ≈ 750°C.

Figure 9. - Procedure for generating isothermal partitioned strainrange-life relationships.
Figure 10. Examples of isothermal test cycles to determine partitioned strain-range-life relationships.
Figure 11. - Use of partitioned strainrange-life relationships to obtain bounds on life.

Figure 12. - Use of partitioned strainrange-life relationships obtained at one temperature $T_o$ to predict behavior at other temperatures.
(a) Determination of $\Delta \varepsilon_{IN}$, $F_{PP}$, $F_{CC}$, $F_{PC}$, and $F_{CP}$.

\[
\begin{align*}
F_{PP} &= \frac{\Delta \varepsilon_{PP}}{\Delta \varepsilon_{IN}} \\
F_{CC} &= \frac{\Delta \varepsilon_{CC}}{\Delta \varepsilon_{IN}} \\
F_{PC} &= \frac{\Delta \varepsilon_{PC}}{\Delta \varepsilon_{IN}} \\
F_{CP} &= \frac{\Delta \varepsilon_{CP}}{\Delta \varepsilon_{IN}}
\end{align*}
\]

(b) Prediction of life.

\[
\frac{1}{N_{PRED}} = \frac{F_{PP}}{N_{PP}} + \frac{F_{CC}}{N_{CC}} + \frac{F_{PC}}{N_{PC}} + \frac{F_{CP}}{N_{CP}}
\]

(c) Plot of predicted versus observed lives.

Figure 13. Use of partitioned strainrange-life relationships and interaction damage rule to predict lives for complex cycles.
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—National Aeronautics and Space Act of 1958

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