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COMAND — A FORTRAN PROGRAM FOR
SIMPLIFIED COMPOSITE ANALYSIS AND
DESIGN

Garret N. Vanderplaats

Ames Research Center
Moffett Field, Calif. 94035

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A FORTRAN program is presented for preliminary analysis and design of multilayered composite panels subjected to in-plane loads. All plies are of the same material. The composite is assumed symmetric about the midplane, but need not be balanced. Failure criterion include limit ply strains and lower bounds on composite in-plane stiffnesses. Multiple load conditions are considered.

The required input data is defined and examples are provided to aid the user in making the program operational. Average panel design times are two seconds on an IBM 360/67 computer. Results are compared with published literature. A complete FORTRAN listing of program COMAND is provided. In addition, the optimization program MIN is required for design.
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CONAND - A FORTRAN PROGRAM FOR SIMPLIFIED COMPOSITE ANALYSIS AND DESIGN

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INTRODUCTION

Early evaluation of composite materials in aerospace structures requires an efficient means of structural sizing for a given application. It is seldom possible to provide simple stress limits as is customary when designing with conventional isotropic materials, since failure of composites is dependent not only on the properties and orientations of the individual plys, but on the nature of the loading as well. Furthermore, by taking advantage of the ply orthotropy, the designer is free (within certain limits) to actually design the structural material through the proper choice of ply thicknesses and orientations.

COMAND is one of several programs being developed in the Advanced Vehicle Concepts Branch of Ames Research Center to provide a general and consistent approach to structural analysis and design. This program is for the analysis and design of a multilayered composite subject to inplane loads. The principal method of analysis and the failure criterion considered here are those used by Schmit and Farshi (Ref. 1). The optimization algorithm is the method of feasible directions using program CONMIN, which is described in Reference 2. COMAND is intended to provide first level design information for membrane structural behavior. Another program under development includes more general analysis, loading conditions, and failure criterion.*

*Program COMPOS by J. Mullen, Advanced Vehicle Concepts Branch, Ames Research Center.
The analysis and design capabilities and the basic assumptions of the program are presented in Section I. Section II describes the required input to the program and several examples of the results are presented in Section III. Possible future efforts in composite analysis and design are identified in Section IV. The principle equations used in the analysis are presented in Appendix A. Appendix B is a complete program listing.

SECTION I

ASSUMPTIONS AND RESTRICTIONS

Program COMAND can be used to analyze a given composite panel in which the ply thicknesses are prescribed, or to design the ply thicknesses to satisfy strain and stiffness limitations. Ply orientation angles are prescribed, and are not design variables. Typical loading conditions and ply orientations are shown in Figures 1 and 2, respectively.

The composite analysis and design is based on the following assumptions and restrictions.

1. The panel is subjected to in-plane loads NX, NY and NYX only. Bending and out-of-plane shear loads are not considered. Multiple loading conditions are considered and up to 10 independent loading conditions are allowed.

2. The composite is said to fail when the longitudinal, transverse or shear strain in any single ply exceeds a specified limit in the longitudinal, transverse or shear direction, respectively.

3. The composite is said to fail if the stiffness in the structural X, Y or XY direction is less than a specified lower limit.
4. The individual ply thicknesses are designed to give minimum total panel thickness. Ply thicknesses are treated as continuous variables and several pllys may be required to be of equal thickness.

5. All pllys are of the same material with the same elastic properties and strain limitations. Ply elastic properties (and therefore, those of the composite) are assumed to be the same in tension and compression.

6. Ply properties are required as program input. Micromechanics analysis is not performed in the program.

7. The composite is assumed to be symmetric about the midplane so that no bending-membrane coupling exists.

8. The composite need not be balanced. That is, a ply with +45 degrees fiber orientation need not be balanced with another ply of -45 degrees orientation. Up to 18 different ply orientations are permitted, allowing for design of composites with ply angles at 10 degree intervals. Ply fiber orientation angles are prescribed and are not design variables.

9. Temperature effects and temperature loading are not considered, except that the material properties and strain limits must be consistent with the design temperature.

SECTION II
PROGRAM INPUT

All program input is listed here. The variables and their definitions are presented first, followed by data organization. No units are provided
for the variables. It is required that all units be consistent. That is (for example), if loads are in newtons and thicknesses in meters, moduli must be given in newtons per square meter, strains in meters/meter and stiffness in newtons/meter.

Variables:

**TITLE(15)** Anything may be given as a title.

**NCALC** Calculation control. If NCALC=0, total composite thickness (weight) is minimized. If NCALC.NE.0, the given composite is analyzed only.

**NPLY** Number of plies. Up to 18 plies are allowed.

**NDV** Number of design variables. This is the number of ply thicknesses which are allowed to change independently in the optimization process or the number of different thicknesses prescribed for analysis. 1.LE.ND.V.LE.NPLY

**NLC** Number of loading conditions. Up to 5 loading conditions are allowed.

**IPRINT** Print control for the optimization program, CONMIN. IPRINT = 0 gives no print during the optimization. IPRINT = 1 to IPRINT = 4 provide increasing degrees of output during optimization. IPRINT = 2 is usually desirable.

**LNK(NPLY)** Design variable linking. LNK(I) gives variable number (ply thickness) associated with the ITH ply. For example, in a four ply problem (NPLY = 4), LNK^T = (1, 2, 2, 3) will impose the requirement that plys 2 and 3 are of the same thickness. In this case NDV = 3.
X(NDV) Initial thickness of the design variables (i.e. $X^T = .05, .03, .04$). If NCALC.NE.0., the composite is analyzed for ply thicknesses defined in X and linked according to LNK. If J = LNK(I), the thickness of the ITH ply is stored in X(J).

VLB(NDV) Lower bounds on the design variables. VLB(I).GE.0, I = 1,NDV. It is usually desirable to set at least one VLB(I) = 1.0E-10 if lower bounds of zero are desired, in order to prevent the optimization program from attempting to analyze a panel of zero thickness. If NCALC.NE.0. VLB(I) = 0, I = 1, NDV may be input.

THN(NPLY) Ply orientations in degrees, referenced to the structural X-axis. THN(I) = Ply orientation of the ITH ply.

EL Ply longitudinal modulus.

ET Ply transverse modulus.

GLT Ply shear modulus.

PRLT Ply major Poisson's ratio (ply transverse Poisson's ratio, PRXL, is calculated internally).

EPLC Ply longitudinal compressive strain limit (negative number).

EPLT Ply longitudinal tensile strain limit (positive number).

EPTT Ply transverse tensile strain limit (positive number).

GMLT Ply maximum shear strain limit (positive number).

A11L Lower bound on composite stiffness in the structural X-DIRECTION.

A22L Lower bound on composite stiffness in the structural Y-DIRECTION.

A66L Lower bound on composite shear stiffness.

PN(3, NLC) Loads, column I corresponds to loading condition I, I = 1, NLC. Row J corresponds to load NX, NY and NXY for J = 1, 2 and 3, respectively of load condition I.
**Data Organization:**

<table>
<thead>
<tr>
<th>No. of Cards</th>
<th>Information</th>
<th>Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Title — Anything may be given here</td>
<td>15A4</td>
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<tr>
<td>1</td>
<td>NCALC, NPLY, NDV, NLC, IPRINT</td>
<td>515</td>
</tr>
<tr>
<td>1</td>
<td>LNK(I), I=1,NPLY</td>
<td>1515</td>
</tr>
<tr>
<td>1-3</td>
<td>X(I), I=1,NDV</td>
<td>8F10.2</td>
</tr>
<tr>
<td>1-3</td>
<td>VLB(I), I=1,NDV (Blank card(s) if NCALC.NE.0)</td>
<td>8F10.2</td>
</tr>
<tr>
<td>1-3</td>
<td>THN(I), I=1,NPLY</td>
<td>8F10.2</td>
</tr>
<tr>
<td>1</td>
<td>EL, ET, GLT, FRLT</td>
<td>4F10.2</td>
</tr>
<tr>
<td>1</td>
<td>EPLC, EPLT, EPTC, EPTT, GMLT, A11L, A22L, A66L</td>
<td>8F10.2</td>
</tr>
<tr>
<td>NLC</td>
<td>PN(J,I), J=1,3 (One card per loading condition)</td>
<td>3F10.2</td>
</tr>
</tbody>
</table>

Begin with next set of data. Program terminates if 2 blank cards are read here.

This information is duplicated in Table 1, along with a data form for convenient reference.

**SECTION III**

**EXAMPLES**

Several examples are presented here to aid the user in making the program operational and to provide some insight into design using composite materials. All examples are for a high strength graphite-epoxy composite.

Typical ply unidirectional properties are listed in Table 2 for a fiber volume fraction of 0.6. The table is reproduced directly from Reference 3. Note that the ultimate strain limits are not specified for longitudinal and transverse strain or for shear. However, reasonable values are readily
obtained by analyzing a single ply of unit thickness, subject to a set of loads which are equal to the ultimate stresses. For example, given a longitudinal load of 180,000 lb/in. the resulting longitudinal strain will be ultimate strain. Therefore, a single ply composite is analyzed for the following load conditions:

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>NX</th>
<th>NY</th>
<th>NXY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>180000.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>3</td>
<td>0.</td>
<td>-30000.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>0.</td>
<td>8000.</td>
<td>0.</td>
</tr>
<tr>
<td>4</td>
<td>0.</td>
<td>0.</td>
<td>12000.</td>
</tr>
</tbody>
</table>

Note that a negative NX load is not imposed because the ultimate longitudinal compressive stress is the same in magnitude as the tensile stress. Therefore, the ultimate strains are also equal in magnitude (but opposite in sign).

The program input variables are now:

- **TITLE:** Determination of strains - G/E composite.
- **NCALC = 1** Analysis
- **NPLY = 1** One ply.
- **NDV = 1** One thickness.
- **NLC = 4** Four load conditions.
- **IPRINT = 0** Not used for analysis.
- **LNX(1) = 1** Ply thickness = X(1).
- **X(1) = 1.0** Composite thickness.
- **VLB(1) = 0.** Not used for analysis.
- **THN(1) = 0.** Zero degree ply orientation.
EL = 21,000,000 Longitudinal modulus.
ET = 1,700,000 Transverse modulus
GLT = 650,000 Shear modulus.
PRLT = 0.21 Major Poisson's ratio.

EPLT = EPLC = EPTT = EPTC = GMLT = 0 - Strain limits set to zero since they are not known.
A11L = A22L = A66L = 0 Not meaningful here

PN(I,J) - Loads, given above.
The input data is listed in Table 3 with the corresponding output in Figure 3.
The ultimate strains are now the actual ply strains in the direction of the applied load for the corresponding loading condition. For example, since load condition 1 is the ultimate longitudinal stress, the longitudinal strain, EPL, under this load condition is also ultimate. That is:

EPLT = 0.00857 (table 2 gives 0.00870)

Similarly,
EPLC = -0.00857
EPTC = -0.0176
EPTT = 0.00471 (table 2 gives 0.00475)
GMLT = 0.0185

These are now the limit strains to be used in design.

Example 1 - Quasi-isotropic composite

In order to draw a comparison between graphite epoxy composites and the familiar aluminum materials, a simple case is first considered in which plys are oriented at 15 degree intervals (NPLY = 12) and subject to a single
unidirectional load, \(NX = 20,000 \text{ lb/in.} \) (NY=NXY=0). All plys are required to be of the same thickness so that NDV=1 and LNK(I)=1, I,NPLY. The total thickness is minimized. No minimum stiffness limits are imposed, so that \(A_{11L}=A_{22L}=A_{66L}=0\). Lower bounds on the thicknesses are arbitrarily set to 0.00001 in. Initial ply thickness is prescribed as 0.05 in. The input data is listed in Table 4, where the print control for the optimization program, CONMIN, is taken as IPRINT = 2. The program output is listed in Figure 4. The optimum composite thickness is 0.525 inches. The design is constrained by the transverse strain limit in the 90 degree direction (ply number 12). The average stress in the structural X-direction (direction of load) in the composite is 38,000 PSI. Note that this is significantly less than the ultimate stress of 60,000 PSI for a typical aluminum alloy. However, the density of the composite is 0.056 lb/in.\(^3\) as compared to 0.101 lb/in.\(^3\) for aluminum. Therefore, the relative weight of graphite epoxy as compared to aluminum for this example is \(0.056*60000/(0.101*38000) = 0.875\) giving a 12.5 percent weight savings.

Note that even though the 90 degree ply has failed, some additional load may be carried before all plys fail. Therefore, the failure stress predicted here may be considered analogous to the limit stress, with the ultimate stress being (usually) somewhat higher.

Example 2 - (0, +45, 90) composite design

Due to practical considerations, it is improbable that many different ply orientations will be used in most structures. In this example, the composite is required to be balanced so that the thicknesses of the +45 and -45 degree plys are the same. Then there are three independent design
variables (NDV = 3) and the ply thickness linking vector becomes $\text{LNK}^T = (1, 2, 2, 3)$. The ply orientation vector is $\text{THN}^T = (0., 45., -45., 90.)$. A minimum stiffness of 500,000 lb/in. is required in the structural X-direction. The composite is required to support the following four independent loading conditions:

<table>
<thead>
<tr>
<th>Load Condition</th>
<th>NX</th>
<th>NY</th>
<th>NXY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20000.</td>
<td>0.</td>
<td>0.</td>
</tr>
<tr>
<td>2</td>
<td>15000.</td>
<td>-15000.</td>
<td>5000.</td>
</tr>
<tr>
<td>3</td>
<td>-15000.</td>
<td>10000.</td>
<td>10000.</td>
</tr>
<tr>
<td>4</td>
<td>0.</td>
<td>0.</td>
<td>20000.</td>
</tr>
</tbody>
</table>

The input data is listed in Table 5 and the corresponding output in Fig. 5. The print control for CONMIN is set to IPRINT = 0 in this example and in example 3 for brevity. The optimum composite thickness is 0.578 inches. The active constraints are transverse strain limits and are identified by safety factors of unity in Fig. 5 (3 constraints are active).

Example 3 - (0. +30, +60, 90) composite design.

This composite is designed subject to the same constraints and loading conditions as example 2. The only difference is the number of plies and their orientations. The composite is again required to be balanced. In this case, NDV = 4, NPLY = 6, $\text{LNK}^T = (1, 2, 2, 3, 3, 4)$, and $\text{THN}^T = 0., 30., -30., 60., -60., 90.)$. The input data and output are listed in Table 6 and Fig. 6, respectively. The optimum composite thickness is 0.532 inches and there are six active strain limit constraints as seen from Fig. 6. Note that although the number of plies and their orientations are different from example 2, the total composite thickness is reduced by less than ten percent.
An additional exercise of interest is to eliminate plies which comprise a small percentage of the total thickness, and solve the optimization problem again. For example, a composite made up of ±30 and ±60 degree ply orientations results in an optimum thickness of 0.526 inches. It is instructive to design the 12 ply composite of example 1 subject to this same set of loads, but allowing for different ply thicknesses (require that the composite be balanced for consistency with examples 2 and 3). The resulting thickness is 0.588 inches. Solution of this case is left as an exercise.

Example 4 - Limit stress vs. ply thickness distribution

In order to assess the applicability of this program to preliminary composite design, results obtained using COMAND are compared here with design curves for a (0, ±45, 90) composite subjected to uniaxial tension, compression and shear loading (applied separately). Figures 7-10 are reproduced from Reference 3. A composite with various relative ply thicknesses was analyzed under these separate loading conditions. No stiffness constraints were imposed and the lowest factor of safety was found for all strain failure criterion. The calculated stress was then multiplied by this factor to give the failure (limit) stress. The results are plotted on Figures 7-10 for 25 and 50 percent zero degree plys. Figure 10 compares the extensional modulus, $E_x$.

The results indicate reasonable comparison for compressive stress, shear stress and extensional modulus. However, considerable discrepancy is found in comparing tensile stress limits. This is because the composite is constrained by transverse strain limits on the 90 degree plys. In Reference 3, one or more plies are allowed to fail without assuming composite failure.
When a single ply fails, this ply is assumed to carry no load. The composite is said to fail only when all plies fail individually. This again demonstrates the difference between the limit stress calculated here and the ultimate stress presented in Reference 3. The difference in results between these two assumptions is usually reduced when multiple sets of combined loadings (practical design situations) are considered.

SECTION IV
DISCUSSION

A short program has been presented by which first estimates are readily obtained for design requirements of composite structures. The program is easily used and requires minimal execution time. Because the failure criterion are extremely load dependent, some judgement is necessary in choosing permissible ply orientations, so that the existence of a given ply orientation does not prevent attainment of an optimum design. This problem is much less prevalent under multiple loading conditions. However, it does suggest that development of an optimization algorithm capable of completely eliminating plies may be fruitful.

For the results to be meaningful, it is important that this program be applied only to structures satisfying (at least approximately) the restrictions imposed in Section I. Of particular importance are the restrictions of in-plane loading and composite symmetry about the midplane.

Recognizing the complexities of composite analysis and design as well as the benefits to be gained through the use of these materials, future development work in this area appears warranted.**

**Several of the topics identified here are currently being included in the COMPOS program by J. Mullen at ARC.
These efforts should include more complex loading such as bending, out of plane shear, and temperature loads on nonsymmetric composites. This necessarily requires the inclusion of more sophisticated analysis techniques and failure criterion. Panel buckling under various force and displacement boundary conditions is also an area of interest because, with increased composite strengths, stiffness requirements become increasingly important, since the probability of failure in this mode is increased with reduced plate thicknesses. Additionally, analysis and design of composites made up of plies of differing elastic properties is a needed and straightforward extension. This will provide the capability of selective reinforcement of conventional isotropic materials as well as use of various combinations of advanced materials. Finally, these capabilities should be incorporated into a general finite element analysis and design program for application to large scale structures of practical interest.
APPENDIX A

COMPOSITE ANALYSIS AND DESIGN EQUATIONS

Analysis Equations

The equations used for analysis and design are presented here. These equations are consistent with the assumptions listed in Section I. Equation numbers beginning with the letter A are consistent with Reference 1.

The analysis is based on the ply materials properties $E_L$, $E_T$, $G_{LT}$, $v_{LT}$ and $v_{TL}$, ply thicknesses, $t_i$, and orientations, $\theta_i$.

The force deformation equations for the $k$th load condition are;

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}_k = [A] \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k$$

where

$$\begin{aligned}
\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix}_k &= \begin{bmatrix} N_{xk} \\ N_{yk} \\ N_{xyk} \end{bmatrix} \\
\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}_k &= \begin{bmatrix} \varepsilon_{xk} \\ \varepsilon_{yk} \\ \gamma_{xyk} \end{bmatrix}
\end{aligned}$$

$[N]_k$ is the vector of applied in-plane loads referenced to the structural $x$-axis and $[\varepsilon]_k$ is the corresponding strain state. $u$ and $v$ are the displacements in the coordinate $x$ and $y$ directions, respectively.

$$A_{rs} = \sum_{i=1}^{NPLY} (C'_{rs}) t_i \quad r,s = 1,2,6$$

where $t_i$ is the thickness of the plys oriented at angle $\theta_i$ with respect to the structural $x$-axis. Coefficients $(C'_{rs})_i$ are defined in terms of $\theta_i$ and
and the ply elastic constants as

\[
(c'_{11})_i = (c_{11})_i \varepsilon_i^4 + 2(c_{12})_i \varepsilon_i^2 m_i^2 + (c_{22})_i m_i^4 + 4(c_{66})_i m_i^2 \varepsilon_i^2
\]

\[\text{[A3]}\]

\[
(c'_{12})_i = (c_{11})_i \varepsilon_i^2 m_i^2 + (c_{12})_i (\varepsilon_i^4 + m_i^2)
\]

\[\text{[A4]}\]

\[
(c'_{16})_i = (c_{11})_i \varepsilon_i^3 m_i + (c_{12})_i (m_i^3 \varepsilon_i - \varepsilon_i^3 m_i)
\]

\[\text{[A5]}\]

\[
(c'_{22})_i = (c_{11})_i m_i^4 + 2(c_{12})_i m_i^2 \varepsilon_i^2 + (c_{22})_i \varepsilon_i^4 + 4(c_{66})_i m_i^2 \varepsilon_i^2
\]

\[\text{[A6]}\]

\[
(c'_{26})_i = (c_{11})_i m_i^3 \varepsilon_i + (c_{12})_i (m_i^3 \varepsilon_i - m_i^3 \varepsilon_i)
\]

\[\text{[A7]}\]

\[
(c'_{66})_i = (c_{11})_i m_i^2 \varepsilon_i^2 - 2(c_{12})_i m_i^2 \varepsilon_i^2 + (c_{22})_i \varepsilon_i^2 + (c_{66})_i (\varepsilon_i^2 - m_i^2)^2
\]

\[\text{[A8]}\]
where

\[ \lambda_i = \cos \theta_i \quad m_i = \sin \theta_i \]  

\[ (c_{11})_i = \frac{E_{Li}}{(1-v_{LTi}v_{TLi})} \]  

\[ (c_{12})_i = \frac{v_{TLi} E_{Li}}{(1-v_{LTi}v_{TLi})} = \frac{v_{LTi} E_{Ti}}{(1-v_{LTi}v_{TLi})} \]  

\[ (c_{22})_i = \frac{E_{Ti}}{(1-v_{LTi}v_{TLi})} \]  

\[ (c_{66})_i = c_{LTi} \]

Note that the subscript \( i \) is not required on equations \([A10]-[A13]\) since the elastic properties are assumed the same for all plies. The subscript is retained here for consistency.

Given the loads \([N]_k\), the membrane strains are obtained from equation \([A1]\) as

\[ \{\varepsilon\}_k = [A]^{-1} \{N\}_k \]

Finally the strains in the \( i \)th ply (\( k \)th load condition) are determined from

\[ \varepsilon_{lik} = \lambda_i^2 \varepsilon_{xk} + m_i^2 \varepsilon_{yk} + m_i \lambda_i \gamma_{xyk} \]

\[ \varepsilon_{zik} = m_i^2 \varepsilon_{xk} + \lambda_i^2 \varepsilon_{yk} - m_i \lambda_i \gamma_{xyk} \]  

\[ \gamma_{12ik} = -2m_i \lambda_i \varepsilon_{xk} + zm_i^2 \varepsilon_{yk} + (\lambda_i^2 - m_i^2) \gamma_{xyk} \]  

\[ \varepsilon_{11} = \cos \theta \quad m = \sin \theta \]  

\[ (c_{11}) = \frac{E}{(1-v^2)} \]  

\[ (c_{12}) = \frac{v E}{(1-v^2)} \]  

\[ (c_{22}) = \frac{E}{(1-v^2)} \]  

\[ (c_{66}) = c \]
If the stresses in the ith ply are required, these may be obtained from the orthotropic elastic stress-strain relationships to be

\[\sigma_{1ik} = (c_{11})_i \varepsilon_{1ik} + (c_{12})_i \varepsilon_{2ik}\]

\[\sigma_{2ik} = (c_{22})_i \varepsilon_{1ik} + (c_{22})_i \varepsilon_{2ik}\]

\[\tau_{12ik} = (c_{66})_i \gamma_{12ik}\]

**Design Equations**

The design objective is to minimize the total composite thickness (and therefore weight);

\[\text{Minimize } W = \sum_{i=1}^{N\text{PLY}} t_i\]

Constraints on the design include limit ply strains and lower bounds on stiffness.

The limit strains imposed on the individual plies are expressed as constraint functions as follows:

\[G_{11k} = \frac{\varepsilon_{11k}}{EPLC} - 1. \leq 0 \quad i = 1, N\text{PLY}, k = 1, N\text{LC}\]

\[G_{21k} = \frac{\varepsilon_{11k}}{EPLT} - 1. \leq 0 \quad i = 1, N\text{PLY}, k = 1, N\text{LC}\]

\[G_{31k} = \frac{\varepsilon_{21k}}{EPTC} - 1. \leq 0 \quad i = 1, N\text{PLY}, k = 1, N\text{LC}\]

\[G_{41k} = \frac{\varepsilon_{21k}}{EPTT} - 1. \leq 0 \quad i = 1, N\text{PLY}, k = 1, N\text{LC}\]

\[G_{51k} = \frac{\gamma_{12ik}}{GMLT} - 1. \leq 0 \quad i = 1, N\text{PLY}, k = 1, N\text{LC}\]
where subscript \(i\) denotes ply number and subscript \(k\) denotes load condition.

Lower bounds on stiffness are expressed as constraint functions:

\[
\bar{c}_1 = 1. - \frac{A(1,1)}{A_{11L}} \leq 0.
\]

\[
\bar{c}_2 = 1. - \frac{A(2,2)}{A_{22L}} \leq 0.
\]

\[
\bar{c}_3 = 1. - \frac{A(3,3)}{A_{33L}} \leq 0.
\]

Constraints on strains are nonlinear functions of the design variables, \(t_i\). The values of these constraints are stored in vector \(G\), (five values per ply, one ply after another) for each load condition in sequence.

Constraints \(\bar{c}_1\), \(\bar{c}_2\) and \(\bar{c}_3\) on stiffness are linear functions of the design variables. The values of these constraints are stored after constraints on strains in vector \(G\).

There are \(5 \times NPLY \times NLC\) nonlinear constraints and three linear constraints on the optimization problem. Program "CONMIN" defines a nonlinear constraint as "active" if its value is greater than or equal to a specified value \(CT\) (a small negative number). Linear constraints are "active" if their value equals or exceeds a value of \(CTL\). If a given constraint is active the analytic gradient of this constraint with respect to the independent design variables, \(t_i\), must be supplied. This information is obtained by direct differentiation of the constraint functions and is readily calculated using the equations of analysis.
A complete FORTRAN listing of program "COMAND" is given here. In addition, program "CONMIN" is required and this program is described in reference 2. The general program organization is shown in block diagram form in figure 11.
COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMMAND
JULY, 1974

DO 10 I=1,NPLT
10 GNLT=GNLTS
WHITE (10,276) 1,TNL(1),TML(1),TME(1)
WHITE (10,265) 1,PNL(1),PNLT(1),PNLTS(1)
WHITE (10,270) 1,PML(1),PMLS(1),PMLTS(1)
WHITE (10,280) DO 10 I=1,NPLT

DO (10,310) 1,EPH(1) ,I=1,J=1
C INITIALIZE COMMON PARAMETERS TO DEFAULT VALUES.
ITEMS=30
KSLC=1
ICNH=40
NSCL=4
EPCLC=4
CTC=1
CTM=100
CTL=100
CPI=5
THEIC=1
THEIC=1
THEL=1210
TML=1220
TMS=1230
THEM=1240
LISTME=1250
IPRF=0
C CONNECT PLY ANGLES TO RADIAN.
DO 70 J=1,MPT
70 ITEM=ITEM+10
70 chute variables are currently set at 100.
VPL=1+100.
F L Y S T I F F H S C O E F C I M T S ,
CALL (EMP) INL(1),PLT(1),PLTS(1),PLT(1),PLTS(1)
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COMPOSITE ANALYSIS AND DESIGN PROGRAM - CONAND

**CONTINU**

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**WRITE** (6,130)
SUBROUTINE CLP3 (INFO,OBJ,X,OF,G,IC,NC,VA,VS,JS,NS,CC,NCV,AC,NCV,VC,NCV,OC,NC

DIMENSION X(50),INFO(50)

CALL CLNP4 (INFO,OBJ,X,OF,G,IC,NC,VA,VS,JS,NS,CC,NCV,AC,NCV,VC,NCV,OC,NCV)

RETURN
END

COMPUTER ANALYSIS AND DESIGN PROGRAM — CUMAND — COMPA JULY, 1974

NASA-RES RESEARCH CENTER, VERNETT FIELD, CALIF.

CALL CLMP (INFO,OBJ,X,OF,G,IC,NC,VA,VS,JS,NS,CC,NCV,AC,NCV,VC,NCV,OC,NCV)

RETURN
END

CALL CLMP (INFO,OBJ,X,OF,G,IC,NC,VA,VS,JS,NS,CC,NCV,AC,NCV,VC,NCV,OC,NCV)

RETURN
END
**COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COM4**  
**JULY, 1974**

<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>106</td>
<td>IF</td>
<td>IF (GADIENT - ACTIVE) LONGITUDINAL TENSILE STRAIN CONSTRAINT.</td>
</tr>
<tr>
<td>107</td>
<td>THEN</td>
<td>THEN</td>
</tr>
<tr>
<td>108</td>
<td>MAC, NAC1</td>
<td>MAC, NAC1</td>
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<tr>
<td>109</td>
<td>RETURN</td>
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<td>110</td>
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<tr>
<td>111</td>
<td>MAC, NAC1</td>
<td>MAC, NAC1</td>
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<td>113</td>
<td>ENDIF</td>
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**JULY, 1974**

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<tbody>
<tr>
<td>114</td>
<td>IF</td>
<td>IF (GADIENT - ACTIVE) TRANSVERSE COMPRESSION STRAIN CONSTRAINT.</td>
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<td>115</td>
<td>THEN</td>
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<td>130</td>
<td>IF</td>
<td>IF (GADIENT - ACTIVE) SHEAR STRAIN CONSTRAINT.</td>
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<td>138</td>
<td>IF</td>
<td>IF (GADIENT - ACTIVE) TRANSVERSE TENSILE STRAIN CONSTRAINT.</td>
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<td>IF (GADIENT - ACTIVE) SHEAR TENSILE STRAIN CONSTRAINT.</td>
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**JULY, 1974**

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**COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND - COM4**  
**JULY, 1974**

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**JULY, 1974**

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**JULY, 1974**

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**Composite Analysis and Design Program - COMAND - COMP4**

**July 1974**
References


**COMAND DATA ORGANIZATION:**

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**Begin with next set of data - Program terminates if 2 blank cards are read here.**

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TABLE 1 - DATA ORGANIZATION - CONCLUDED
# Table 1.2.1-III. Key Unidirectional Properties

**High-Strength Graphite/Epoxy [-0]**

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*Typical Design Allowable, reference section 1.2.0

References: 1.2-15, -19, -21

1.2.1 Table 2.- Material properties.
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**COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND**

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**TABLE 3 - DETERMINATION OF LIMIT STRAINS - G/E COMPOSITE**
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COMPOSITE ANALYSIS AND DESIGN PROGRAM - COMAND

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| F         | 0.  | 15. | -15. | 30. | -30. | 45. | -45. | 60. |
|           | -60.| 75. | -75. | 90. |

| G         | 21000000. | 17000000. | 650000. | .21 |
|           | -.00857    | .00857     | -.0176   | .00471 |
|           | .0184      | 0.         | 0.       | 0.   |

| H         | 20000.     |

| I         |            |

TABLE 4 - QUASI-ISOTROPIC COMPOSITE UNDER UNIAXIAL LOAD - EXAMPLE 1
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TABLE 5 - (0, ±45, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 2
TABLE 6 - (6, ±30, ±60, 90) GRAPHITE EPOXY COMPOSITE - EXAMPLE 3
Inplane loads $N_x$, $N_y$, $N_{xy}$
Load condition $k$.

Figure 1.- Typical composite loading.
Symmetric composite layup

Figure 2.- Typical ply orientation.
**Analysis of Synthetic Composite Panel**

**Title:** Determination of Limit Strains - G/E Composite

### Ply Properties
- All Ply Identical
- Longitudinal Modulus: 1.23X10^6 psi
- Transverse Modulus: 1.7X10^5 psi
- Shear Modulus: 1.9X10^4 psi
- Poisson's Ratio: 0.31
- Ply Thickness: 0.004 in.

### Ply Thicknesses, Directions, and Design Variables

<table>
<thead>
<tr>
<th>Ply No.</th>
<th>Thickness</th>
<th>Theta (deg)</th>
<th>Var. No.</th>
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### Ply Strain Limits

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<th>Transverse Strain</th>
<th>Shear Strain</th>
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<tbody>
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<td>Limit</td>
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### Stiffness Limits

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### Composite Stresses Reference to Structural Axes

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</table>

### Figure 3: Determination of limit strains - G/E composite.
Figure 3.— Concluded.

Figure 4.— Quasi-isotropic G/E composite under uniaxial load — Example 1.
Figure 4. - Continued.
Figure 4. - Continued.

Final optimization iteration

<table>
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<tr>
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<th>Constraint Value (Function)</th>
<th>Constraint Value (Gradient)</th>
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Legend:
- **N** indicates a non-binding constraint.
- **B** indicates a binding constraint.
- **V** indicates a violated constraint.
- **T** indicates a terminated constraint.

Notes:
- Maximum number of iterations: 10
- Maximum number of function evaluations: 50
- Maximum number of gradient evaluations: 20
- Maximum number of constraint function evaluations: 15
- Maximum number of constraint gradient evaluations: 10
### Ply Information

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### Ply Strains

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### Composite Strain Reference to Structural Axes

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### Design

**Title:** Symmetric Composite Panel

**Parameters:**
- No. of Ply
- No. of Load Conditions

**Ply Properties:** All Ply Identical
- Longitudinal Modulus (L) = $170000000$
- Transverse Modulus (T) = $650000000$
- Poisson's Ratio (L,T) = $0.10$
- Poisson's Ratio (L,0) = $0.10$

**Ply Thicknesses, Orientations, and Design Parameters**

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**Figure 5:** (0, ±45, 90) Graphite epoxy composite - Example 2.
Figure 5.- Concluded.
Figure 6. - (0, ±30, ±60, 90) Graphite epoxy composite — Example 3.
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**Figure 6.** Concluded.
Figure 7.- Ultimate tensile strength $F_{x}^{tu}$ for high-strength graphite/epoxy - $[0_{l}^{2}/\pm 45_{l}/90_{k}]_{x}$ family.
Figure 8.- Ultimate compressive strength $F_{x}^{cu}$ high-strength graphite/epoxy - [0/±45°/90°] family.
Figure 9. - Ultimate shear strength $F_{xy}^{u}$ high-strength graphite/epoxy - $[0_1/\pm 45/90_k]$ family.
Figure 10. - Extensional modulus $E_x$ high-strength graphite/epoxy - $[0_{1}/±45_{3}/90_{k}]$ family.
'COMAND'
MAIN PROGRAM TO READ AND ORGANIZE DATA AND PRINT RESULTS

COMP2
PLY STIFFNESS COEFFICIENTS (CALLED ONLY ONCE)

COMP3
COMPOSITE MEMBRANE STIFFNESSES AND FLEXABILITIES

COMP4
CALCULATE OBJECTIVE FUNCTION VALUE, OBJ
CALCULATE CONSTRAINT VALUES, G
CALCULATE GRADIENT OF OBJECTIVE, DF
CALCULATE GRADIENTS, ∇G, OF ACTIVE CONSTRAINTS

CONMIN
CONSTRAINED FUNCTION MINIMIZATION

BUFFER BETWEEN CONMIN AND COMP4

FIGURE 11.- 'COMAND' BLOCK DIAGRAM.