FLOWFIELD ANALYSIS FOR SUCCESSIVE OBLIQUE SHOCK WAVE-TURBULENT BOUNDARY LAYER INTERACTIONS

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A computation procedure is described for predicting the flowfields which develop when successive interactions between oblique shock waves and a turbulent boundary layer occur. Such interactions may occur, for example, in engine inlets for supersonic aircraft. Computations have been carried out for axisymmetric internal flows at $M_w = 3.82$ and $2.82$. The effect of boundary layer bleed has been considered for the $M_w = 2.82$ flow. A control volume analysis is used to predict changes in the flow field across the interactions. Two bleed flow models have been considered. A Turbulent boundary layer program has been used to compute changes in the boundary layer between the interactions. The results given are for flows with two shock wave interactions and for bleed at the second interaction site. In principle the method described may be extended to account for additional interactions. The predicted results are compared with measured results and are shown to be in good agreement when the bleed flow rate is low (on the order of 3% of the boundary layer mass flow), or when there is no bleed. As the bleed flow rate is increased, differences between the predicted and measured results become larger. Shortcomings of the bleed flow models at higher bleed flow rates are discussed.
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BOUNDARY LAYER INTERACTIONS

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University of Washington

SUMMARY

A computation procedure is described for predicting the flowfields which develop when successive interactions between oblique shock waves and a turbulent boundary layer occur. Such interactions may occur, for example, in engine inlets for supersonic aircraft. Computations have been carried out for axisymmetric internal flows at $M_\infty = 3.82$ and $2.82$. The effect of boundary layer bleed has been considered for the $M_\infty = 2.82$ flow. A control volume analysis is used to predict changes in the flow field across the interactions. Two bleed flow models have been considered. A turbulent boundary layer program has been used to compute changes in the boundary layer between the interactions. The results given are for flows with two shock wave interactions and for bleed at the second interaction site. In principle the method described may be extended to account for additional interactions. The predicted results are compared with measured results and are shown to be in good agreement when the bleed flow rate is low (on the order of 3% of the boundary layer mass flow), or when there is no bleed. As the bleed flow rate is increased, differences between the predicted and measured results become larger. Shortcomings of the bleed flow models at higher bleed flow rates are discussed.

INTRODUCTION

The interaction of an oblique shock wave with a turbulent boundary layer is known to induce drastic changes in the boundary layer properties and to cause substantial deviation of the supersonic flow field from the predicted inviscid flow. This deviation may be of sufficient magnitude to adversely affect the performance of aerodynamic devices. Suitable methods for predicting the boundary layer and the freestream flow characteristics in the presence of such disturbance are required by engineers responsible for the design of aerodynamic configurations in which shock wave boundary layer interactions occur.

A control volume method developed by Seebaugh, Paynter and Childs [1], and improved upon by Mathews [2], has been used successfully in the prediction of the boundary layer characteristics downstream of the interaction with a single oblique shock wave. However, in some aerodynamic devices such as mixed compression supersonic diffusers, the turbulent boundary layer is subjected to interactions with more than one shock wave. In this report, a computation procedure is described for predicting the flow field which develops when successive interactions of two oblique shock waves with a turbulent boundary layer occur. In principle the method described may be extended to account for additional interactions.
Computations have been carried out for axisymmetric internal flows at $M_\infty = 3.82$ and 2.82. Experiments have been conducted at these same Mach numbers with the interactions under study occurring at the walls of circular wind tunnels. In the Mach 2.82 study the effect of boundary layer bleed at the second interaction site has been considered. The predicted results are compared with experimentally observed results. The experimental configurations for which the analysis has been carried out are discussed in the section which follows. This is followed by a section which gives the details of the computational method.

SYMBOLS

<table>
<thead>
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<td>$A$</td>
<td>$[((\gamma-1)/2)M_e^2/(T_w/T_e)]^{1/2}$</td>
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<td>$a$</td>
<td>a constant in the wall-wake profile</td>
</tr>
<tr>
<td>$B$</td>
<td>$[(1+(\gamma-1)/2)M_e^2/(T_w/T_e)]^{-1}$</td>
</tr>
<tr>
<td>$C$</td>
<td>a constant in the Law of the Wall (usually equals 5.1)</td>
</tr>
<tr>
<td>$C_f$</td>
<td>skin friction coefficient, $\tau_w/(1/2)\rho e u_e^2$</td>
</tr>
<tr>
<td>$F$</td>
<td>entrainment function, see Eq. (6-5)</td>
</tr>
<tr>
<td>$(K_{1/1})_K$</td>
<td>kinematic shape factor, see Eq. (B-6)</td>
</tr>
<tr>
<td>$I_{Bx}$</td>
<td>$x$-momentum of the bleed flow</td>
</tr>
<tr>
<td>$K$</td>
<td>a constant in the Law of the Wall (usually equals 0.4)</td>
</tr>
<tr>
<td>$L$</td>
<td>shock wave boundary layer interaction length</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
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<tr>
<td>$m_B$</td>
<td>boundary layer mass bleed rate</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure</td>
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<tr>
<td>$R$</td>
<td>radial coordinate from tunnel centerline</td>
</tr>
<tr>
<td>$R_B$</td>
<td>radial coordinate of dividing stream surface separating bleed flow from main flow, (see Fig. 3)</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Reynolds number</td>
</tr>
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<td>$u$</td>
<td>velocity in streamwise direction</td>
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<td>$u^*$</td>
<td>vanDriest's generalized velocity, $(u_e/A)\arcsin\left{[(2A^2u/u_e)^2 - B]/(B^2+4A^2)^{1/2}\right}$</td>
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\( u_T = \text{friction velocity, } (\tau_w/\rho_w)^{1/2} \)

\( x = \text{axial coordinate, measured from shock generator tip} \)

\( y = \text{coordinate normal to the tunnel wall} \)

\( \gamma = \text{ratio of specific heats} \)

\( \Delta = \text{mass flow thickness, see Eq. (B-1)} \)

\( \Delta_E = \text{a thickness of freestream flow to allow for boundary layer mass entrainment, (see Fig. 3)} \)

\( \delta = \text{boundary layer thickness} \)

\( \delta^* = \text{displacement thickness of the boundary layer} \)

\( \eta = y/\delta \)

\( \theta = \text{momentum thickness of the boundary layer} \)

\( \nu = \text{kinematic viscosity} \)

\( \Pi = \text{coefficient of the wake function} \)

\( \rho = \text{mass density} \)

\( \sigma = [(\gamma-1)/2]M_e^2/(1+[(\gamma-1)/2]M_e^2) \)

\( \tau = \text{shear stress} \)

Subscripts

\( e = \text{conditions at the edge of the boundary layer} \)

\( w = \text{conditions at the wall} \)

\( \infty = \text{freestream conditions ahead of the first interaction} \)

EXPERIMENTAL CONFIGURATION AND INSTRUMENTATION

The experimental configurations which were used to produce the successive shock waves are shown schematically in Figure 1. The Mach 3.82 tunnel had a radius of 2.58 cm and a boundary layer thickness ahead of the first interaction of 0.43 cm. The shock wave generator was installed on the centerline of the tunnel at zero angle of attack. The generator had a 10-degree half-angle conical tip which broke to 13 degrees 6.60 cm downstream of the tip. For the Mach 2.82 tunnel the tunnel radius was 2.60 cm and the boundary layer thickness...
ahead of the first interaction was 0.40 cm. The shock generator had a 10-
degree conical tip which broke to 13 degrees 4.94 cm behind the tip. Both
generators were designed to provide as large a region of freestream flow as
possible between the first reflected and second incident shock waves while at
the same time keeping the expansion wave off the downstream corner of the
second conical surface from interfering with the second interaction.

About 7.62 cm from the tip of the Mach 2.82 shock generator, or approxi-
mately at the point where the second incident shock wave reached the wall, two
rows of thirty-eight 0.132 cm diameter bleed holes were drilled around the
periphery of the tunnel. The bleed system was operated in a choked flow con-
dition. With one row of the holes open, the bleed mass flux was about 2.8
percent of the boundary layer mass flux just ahead of the second interaction.
With two rows of holes open, the bleed mass flux was 5.0 percent of the boun-
dary layer mass flux.

Both tunnels were operated with a steady supply of dry air at 300°K. The
freestream unit Reynolds number for the $M = 3.82$ tunnel was $18.5 \times 10^6$ per
meter, that for the $M = 2.82$ tunnel, $19.0 \times 10^6$ per meter.

Standard instrumentation was used to obtain tunnel wall static pressures
and boundary layer pitot pressures profiles. Wall static pressures were taken
at 0.127 cm intervals along the tunnels. Pitot profiles were taken
in radial increments of 0.0127 cm at eighteen axial stations upstream of, with-
in, and downstream of the interaction region. Miniature total head tubes,
flattened to a dimension of 0.024 cm high by 0.066 cm wide were used for the
pitot profiles. Velocity profiles upstream of the first incident shock wave,
between the first reflected and second incident shock waves, and downstream of
the second reflected shock wave were calculated from the pitot profiles assum-
ing isoenergetic flow. A calibrated venturi meter was used to measure the
bleed flow.

ANALYSIS

Figure 2 shows the flow model used in the analysis. $R$ is the radial dis-
tance from the tunnel centerline and $x$ is the distance downstream from the tip
of the shock wave generator. Conditions at station 1 are assumed to be known.
The object of the analysis, given the shock generator shape and initial condi-
tions, is to compute the locations of the reflected shock waves and the boun-
dary layer properties at successive stations along the wall. Reflected compres-
sion waves are treated in the analysis as a single shock wave.

The boundary layer can be divided into three subregions associated with
the three steps in which the computation is carried out.

1. Region I extends from Station 1 where the first incident shock wave reaches
the boundary layer edge to Station 3 where the reflected shock wave emerges
from the boundary layer. Surface 2 is the stream surface which passes through
the intersection of the reflected shock wave with the boundary layer edge. Note
that this stream surface intersects the incident shock wave outside the boundary layer edge (see Fig. 3). This choice of surface 2 allows for mass entrainment into the boundary layer through the interaction region. The location of this surface and the pressure distributions along it are obtained from an inviscid conical flow solution. Using this surface, the tunnel wall, and the planes normal to the wall at 1 and 3 to define a control volume, a control volume analysis of the region may be used to determine the length of the interaction, and the boundary layer thickness and shape at 3. The method used here is quite similar to the methods used by Seebaugh [11] and Mathews [2] except for the velocity profile representations for the boundary layer. The velocity profile representation used at Stations 1 and 3 and throughout the analysis is the improved wall-wake velocity profile developed by Sun and Childs [31] (see Appendix A). For turbulent isoenergetic compressible boundary layer flow, the modified profile may be expressed in the form,

\[ \frac{u}{u_e} = \frac{1}{\sigma^2} \sin \{\text{arc sin} \sigma^2 [1 + \frac{1}{K} \frac{u^*}{u_e} (1 - \frac{a}{1-a})] - \ln (1 + (1-a)^{3/2}) \} - \frac{1}{a} \ln (1 + (1-a)^{3/2}) - \frac{\pi}{K} \frac{u^*}{u_e} (1 + \cos \theta) \} \]  

where

\[ \frac{\pi}{K} = \frac{1}{2} \left\{ \frac{u^*}{u^*} - \frac{1}{K} \ln \left( \frac{\delta u}{u^*} \right) - 5.1 + \frac{0.614}{a K} \right\} \]  

and \( a = 1 \) is assumed.

One other variation on the earlier method has been incorporated into the present analysis. The flow direction in the boundary layer downstream of the interaction has been taken as the average of the value at the wall (i.e., zero) and that at \( y = \delta_3 \) as determined from the inviscid solution. In the analyses by Seebaugh [11] and Mathews [2] the flow direction downstream of an interaction was taken to be parallel to the wall. Figures 3a and 3b show the control volumes used in the present analysis. Although boundary layer bleed was not employed at the first interaction site in the experimental studies, the control volumes shown do allow for that possibility. For the control volumes shown, the continuity equation may be expressed in the form,

\[ \int_{R_w^{-\delta_1-\Delta E}}^{R_w} 2\pi u R dR = \int_{R_w^{-\delta_3}}^{R_w} 2\pi u R dR + \dot{m}_b \]  

while the x-momentum equation may be written as,
\[
\int_{R_w^{-\delta_1-\Delta_E}}^{R_w} 2\pi p_1 RdR - \int_{R_w^{-\delta_1-\Delta_E}}^{R_w} 2\pi p_2 RdR - \int_{R_w^{-\delta_3}}^{R_w} \overline{p_3} 2\pi RdR - 2\pi R_w L_T\]

\[
= \int_{R_w^{-\delta_3}}^{R_w} 2\pi u^2 RdR - \int_{R_w^{-\delta_1-\Delta_E}}^{R_w} 2\pi u^2 RdR + \dot{I}_{Bx} \tag{4}
\]

where

\[\tau_w = (\tau_{w_1} + \tau_{w_3})/2 \text{ and } \overline{p_3}\]

is the average static pressure over the boundary layer at 3. Comparable equations may be written for the second interaction site. Allowance for boundary layer mass entrainment in the interaction region is made by assuming an entrainment rate equal to that for the flow upstream of an interaction.

(2) Region II extends from Station 3 to Station 5. This is the region of boundary layer flow between the first reflected shock and the second incident shock. Since no shock interactions are present in this region, the axial pressure gradient is relatively low. Starting with conditions at 3 as determined by the control volume analysis of the first interaction, changes in the boundary layer properties to Station 5 are computed using a turbulent boundary layer program suggested by Paynter and Schuelle [4]. The program uses a wall-wake profile to represent the velocity profile and the entrainment function concept proposed by Green [5] to solve the boundary layer equations (see Appendix B). An inviscid flow solution is used to provide the wall static pressure distribution needed for the boundary layer solution in this region. The inviscid solution, however, is obtained in a manner which allows for the effects of the first shock interaction. The method employed was to use an artificial wall position in the interaction region which would cause the reflected inviscid shock wave position to match that determined by control volume analysis of region I. The wall static pressure distribution \(p(x)\) as determined using the artificial wall position is then assumed to exist at the actual wall and is used in the boundary layer calculations.

(3) Region III extends from Station 5 to Station 7. This region covers the interaction of the second incident shock wave with the boundary layer. The method here is similar to that for Region I, except that the flow at control surface 6 is no longer conical. Conditions along this surface were obtained from a method of characteristics solution for flow past the double-cone centerbody. In the characteristics solution the interaction of the first reflected shock with the second incident shock must be considered. The location in the flow field of the reflected shock wave was determined using the artificial wall position.
In the control volume equations (3) and (4) the mass bleed rate in \( \dot{m}_B \) is assumed to be known. The \( x \)-momentum of the bleed flow depends on the manner in which the bleed flow is accomplished. In the analyses by Seebaugh [1] and Mathews [2] computations were made for three bleed models: porous wall suction, slot suction and scoop suction. Figure 3a shows the porous wall model. With this model, the \( x \)-momentum of the bleed flow, \( I_{Bx} \), was assumed to be zero. Figure 3b shows the slot suction model. With slot suction, \( I_{Bx} \) was assumed to have the same value as that possessed by the bleed mass as it entered the control volume, i.e.,

\[
I_{Bx} = \int_{R_B}^{W} 2\pi \rho u^2 R dR
\]

(5)

where \( R_B \) is determined from,

\[
\dot{m}_B = \int_{R_B}^{W} 2\pi \rho u R dR
\]

(6)

For the Mach 2.82 flow with bleed, the bleed holes were drilled normal to the wind tunnel wall. At first thought, then, the porous wall model might appear to provide a better representation of the bleed flow. However, the bleed hole diameter of 0.132 cm was on the order of one-half the boundary thickness at Station 3. Thus, there should be \( x \)-momentum associated with the bleed flow and the bleed flow behavior might then be expected to be somewhere between that for porous-wall suction and slot suction. As will be discussed in the section on results, this appears to have been the case.

**DESCRIPTION OF COMPUTATIONAL PROCEDURE**

The computation of the flow at \( M_{\infty} = 2.82 \) with 2.8 percent bleed rate will be used here to illustrate the computational procedure. The computation is carried out in the following steps:

(1) The boundary layer thickness and velocity profile shape at Station 1 are known from measurements. For the example under consideration the first incident shock wave is at an angle of 22.75 degrees to the axis of the tunnel and reaches the boundary layer edge at a station approximately 5.2 cm downstream of the tip of the shock wave generator. Measurements were not taken exactly at Station 1. However, measurements were taken just upstream at \( x = 5.08 \) cm. We then assume that the boundary layer properties at Station 1 can be approximated by those obtained at \( x = 5.08 \) cm. The best representation of the boundary layer velocity distribution by the wall-wake profile, Eq. (1), is obtained by using the computer program, LEAST, listed in TABLE 1-A.

In program LEAST, the boundary layer thickness \( \delta \) and the coefficient of wall friction \( C_f \) are regarded as two parameters and a double-loop-iteration scheme is used to solve for a least squares fit of the wall-wake profile to the
experimental velocity distribution. This program has been written for either isoenergetic or non-isoenergetic flow. The velocity distribution can be input in the form of pitot tube pressure or Mach number distribution. The input data listed in TABLE 1-B are pitot pressure profile data. Isoenergetic flow has been assumed. The description of the input is detailed in the comment cards listed in the program LEAST. For the case under consideration, the following boundary layer properties were obtained:

\[ \delta = 0.406 \text{ cm}, \ C_f = 0.002, \ u_t / u_e^* = 0.0445 \]

and the freestream Reynolds number \( Re = 19.0 \times 10^6 \) per meter.

(2) The control surface 2 is defined to coincide with the stream surface which extends upstream from the point of intersection of the reflected shock wave with the boundary layer edge. In the inviscid flow field behind a conical shock wave, the characteristic surfaces are conical surfaces originated from the tip of the shock generator. Therefore, the characteristics of the flow field are functions only of the half angles of the conical surfaces. Gootzait [6] developed a computer program, program \( \text{CONE} \), which is used to compute the flow field characteristics. The program \( \text{CONE} \) and the input for it are listed in TABLES 2-A and 2-B. There is only one input card for the program. The input \( \text{FCRMAT} \) is 7F10.6 and the input data are, in order, the specific heat ratio of the gas, the freestream Mach number, the half angle of the cone and the distance, in inches, behind the tip of the cone where the output is desired.

The conical stream surface can be described in non-dimensional form as,

\[ \frac{r}{r_0} = 1 + \int^{x/x_0}_1 \tan \alpha \frac{\tan \phi_s}{\tan \phi_s} d(x/x_0) \]

the reference point \((x_a, r_0)\) can be any point on the shock wave as shown in Figure 4a. \( \alpha \) is the flow deflection angle at the stream surface as measured with respect to the tunnel axis and \( \phi_s \) is the incident shock wave angle. (Note: the final reference radius is obtained by an iterative process which allows for boundary layer mass entrainment. This is taken care of automatically in the program). Figure 4b shows the choice of reference radius \( r_0 \) and the conversion of the stream surface to control surface 2. In the numerical computation a finite number of points is used to represent the control surface and the flow conditions such as Mach number, pressure and flow angle on each point are obtained from program \( \text{CONE} \) as functions of characteristic angle \( \phi \).

(3) The computer program ANAL for the control volume, originally developed by Seebaugh [7], modified by Mathews [2], and then improved upon by the present authors is used to compute boundary layer properties at Station 3. The program ANAL is listed in TABLE 3-A, the inputs in TABLE 3-B. The input values at Station 1 for \( u_t / u_e^* \), \( \delta \) and the Reynolds number based on boundary layer thickness, \( \mu \delta_l / \nu e^* \) are obtained in step 1 and the conditions on the control surface 2 are obtained from step 2. The flow is assumed to be isoenergetic;
therefore the recovery factor is assumed to be 1. There is no boundary layer suction in this region. The shear stress at the wall between stations 1 and 3 is assumed to be the average of the value at stations 1 and 3. The flow direction at station 3 is taken as the average of the values at the wall and at \( y = \delta_3 \) as determined from the inviscid solution.

In this computation, we use the shock wave angle \( \phi_s \), instead of the flow deflection angle \( \phi_2 \) across the first incident shock. We also choose to input conical flow conditions behind the first incident shock wave. The computed results include the interaction length and the boundary layer properties at station 3. The predicted boundary layer properties at station 3 include the boundary layer thickness, the coefficient of skin friction, the average static pressure across the boundary layer and the velocity distribution.

(4) The program ANAL, which is based on the control volume method, predicts only the properties of the boundary layer at the downstream and of the interaction region. For computing the boundary layer downstream of the interaction and for computations of the next interaction, the flow field outside the boundary layer must be known. This is obtained by a method of characteristics computer program. The program used here is a modified version of a program originally prepared by Cavalleri [8]. The program designates the region before the incident shock wave as region 2, the region between the incident reflected shock waves as region 3 and the region downstream of the reflected shock wave as region 4. The flow in region 2 is uniform and parallel to the centerline and the flow in region 3 is conical flow. Figure 5 shows the designated regions and input points at initial station \( x = 1.835 \) inches (4.65 cm) as used in the example under consideration. In computing the external flow field allowance must be made for the effect of the interaction. Otherwise the location of the reflected shock wave will be downstream of the actual reflected wave and the external flow field needed for downstream boundary layer calculations will be in error. In order to avoid this problem, an artificial wall location which would cause the reflected shock location to match that predicted by the control volume analysis was used. Determination of the appropriate artificial wall location is a trial and error process and for the computations which have been carried out to date has required three runs of the characteristics program. In this computation it was found that a tunnel radius of 0.99 inches (2.51 cm) caused the reflected shock wave position to match that determined by the control volume analysis. The inputs to the program are listed in Table 4. The reader should refer to reference 8 for the details of the inputs.

(5) From station 3 to station 5 we use a turbulent boundary layer program BLGRN (see Appendix B) which is a modified version of one developed earlier by Paynter and Schuelle [4] based on Green's [5] entrainment function concept. The control volume analysis in step 3 provides the initial conditions at station 3. For the boundary conditions on surface 4, we assume that the Mach number and pressure distributions are equal to those obtained along the artificial wall in the inviscid solution of step 4. We should also point out here that the computation is a two-dimensional approximation. However, in view of the ratio of boundary layer thickness to tunnel radius the results should be
applicable for the axisymmetric flow under consideration. The program, BLGRN, is listed in TABLE 5-A, the inputs in TABLE 5-B. The inputs are described in the comment cards at the beginning of the program listing.

(6) At 1.942 inches (4.94 cm) behind the tip, the shock generator broke from a 10 degree cone to 13 degrees and generated a second shock wave. The position of the second incident shock wave and the flow field characteristics behind the second shock wave are computed by using the same method of characteristics program which is used in step 4. Given the geometry of the centerbody and the inviscid solution (based on the artificial wall position) from step 4, we have the inputs for the computer program as listed in TABLE 6 and illustrated in Figure 6. As in the computation of step 4, the program designates the region before the second incident shock wave as region 2, the region between the second incident and reflected shock waves as region 3, and the region downstream of the second reflected shock wave as region 4. In region 2, unlike the situation in step 4 where the external flow is uniform, there exists a reflected shock wave. The program is not capable of solving for the intersection of two shock waves. In preparing for the input, the first reflected shock wave is replaced by a smoothed compression wave band. In the calculation described here a linear compression spaced over six input points was used. The flow between the artificial wall radius of 0.99 inch (2.51 cm) and the actual wall radius of 1.02 inches (2.60 cm) was assumed to be uniform and parallel to the wall. The result shows that the second incident shock wave has an angle of approximately 33 degrees and that it reaches the boundary layer edge at x = 2.85 inches (7.22 cm) where the boundary layer thickness is 0.125 (0.312 cm) as computed in step 5.

(7) The control surface 6 is also defined to lie along a stream surface behind the second incident shock wave, but the flow field in this region is quite complicated. Because of the nature of the characteristics program, it was necessary to run the program three separate times to determine flow properties at the three stations. The pressure and Mach number distributions at each station are obtained from the result of the previous step. At x = 2.85 inches (7.22 cm) where the second incident shock wave reaches the edge of boundary layer the cone radius is 0.552 inch (1.40 cm) while the distance from the centerline to the edge of the boundary layer is 0.895 inch (2.28 cm). The mass flux through the area between the radii is 0.412 lbm/sec (0.187 Kg/sec). At x = 2.988 and 3.205 inches (7.60 and 8.15 cm) for the mass flux to be equal to the 0.412 lbm/sec (0.187 Kg/sec) the radii are 0.907 and 0.922 inch (2.30 and 2.34 cm) respectively. Ideally, we can compute at more stations to determine the stream surface. But the preparation of pressure and Mach number profiles is very tedious. We determine the stream surface by graphically tracing a smooth curve through the above three points. The computer program MFLX is used here to compute mass flux from the profile of pressure and Mach numbers. Program MFLX and the input to the program are listed in TABLE 7.

(8) The computer program ANAL listed in TABLE 3-A is used for computation from station 5 to station 7. The boundary layer properties at station 5 are obtained from step 5 and the location of the control surface 6 is determined.
by step 7. After determining the control surface 6, the flow properties along
the surface are obtained from the output of step 6. The x-coordinate should
be transformed into the distance to the axial station at which the extension
of the second shock wave intersects with the tunnel centerline as shown in
Figure 8. For the points on the control surface the input to the program,
i.e., the x-coordinates, should be normalized by distance $x_0$ and the r-coordi-
nates should be normalized by radius $R_0$. The distance $x_0$ is the distance from
station 5 to the intersection point of the second shock wave with the center-
line and the radius $R_0$ is the distance from the tunnel centerline to the edge
of the boundary layer at station 5. For the flow considered in the sample cal-
culations boundary layer bleed takes place at the second interaction site. The
bleed mass flux is about 2.8 percent of the boundary layer mass flux at station
5. In this computation the slot suction model has been assumed for the bleed
flow. The input to the program ANAL is listed in TABLE 8. The details of the
input are described in the comment cards at the beginning of program ANAL list-
ed in TABLE 3-A.

RESULTS

The results of the analysis for three different flow cases, one at $M_w = 3.82$
with no bleed, two at $M_w = 2.82$ with bleed at the second interaction site, are shown
in Figures 9 through 16. Results from the sample calculation described herein are
given in Figures 12 and 13. Comparisons are made between predicted and measured
results. The data for $M = 2.82$ flow are from an investigation by Teeter [9].

Figure 9 shows comparisons of the experimental and predicted shock wave
patterns and boundary layer thickness for the Mach 2.82 flow. Since the pre-
dicted shock wave locations are determined by the inviscid analysis used in
combination with the artificial wall position, no induced shock wave is pre-
dicted. Also shown is the pressure distribution at the tunnel side wall as a
function of the distance aft of the cone tip. The triangular points shown for
the analysis in the static pressure plot were determined by using the artifi-
cial wall and the inviscid flow solution. The predicted and observed values
are seen to be in good agreement along the entire length of the double shock
interaction.

Figure 10 shows $\delta_s$, $\theta$ and $C_f$ at several stations along the tunnel side
wall. Here also, predicted and observed results are in good agreement. The
experimental values shown for $C_f$ have been determined by a least-squares fit
of the modified wall-wake velocity profile to the experimentally determined
velocity profiles.

Figure 11 shows Mach number profiles for the boundary layer at the down-
stream end of the second interaction. The analysis predicts the end of the
second interaction to be at $x = 3.75$ inches. Since profiles were not taken at
this specific station, profiles taken just upstream at $x = 3.70$ inches (9.5 cm)
and just downstream at $x = 3.80$ inches (9.65 cm) are shown for comparison. It
is apparent that the analysis leads to a profile which provides a good repre-
sentation of the experimentally determined profile near the interaction end.
Figure 12 shows boundary-layer thicknesses, shock wave patterns and wall static pressure distributions for the Mach 2.82 flow with 2.8 percent boundary-layer bleed at the second interaction site. Figure 13 shows $\delta^*$, $\theta$, and $C_f$ for this flow.

Two sets of predicted results are given, one for the porous-wall suction model, the other for slot suction. (For the sample calculation described in the report slot suction has been assumed). As is shown, the differences between the results for the two suction models are not large. Differences in predicted results with the two models are due solely to the differences in values assigned to the x-momentum of the bleed flux. Since the bleed rate is low, the x-momentum associated with the slot suction model is small and not too different from the zero values for porous suction. The predicted and measured results are in reasonably good agreement.

Figure 14 shows boundary-layer thickness, shock wave patterns and wall static pressure distributions for $M = 2.82$ with 5.0 percent bleed. Values for $\delta^*$, $\theta$, and $C_f$ are shown in Figure 15, while Mach number profiles downstream of the second interaction are shown in Figure 16. The Mach number profiles represented by the solid lines in Figure 16 are predicted profiles for the two bleed flow models. They are shown on the figure at the axial positions predicted for the end of the second interaction. Since experimental profiles were not taken at these precise locations, experimental profiles taken in the neighborhood ($x = 3.20, 3.30$ and $3.40$ inches or $8.13, 8.38$ and $8.64$ cm.) of the predicted locations have been shown for comparison.

The flow conditions up to the second interaction are the same as those for the flow with 2.8 percent bleed. With the higher bleed rate the difference between the results for porous wall and slot suction are much more pronounced than with 2.8 percent. The slot-suction model gives a reflected shock location which is in better agreement with the observed results. On the other hand, the values of $\delta^*$, $\theta$, and $C_f$ obtained with the porous wall model agree better with experimental values than do the slot suction results. As was pointed out in the section on analysis, the bleed hole diameter of 0.132 cm was on the order of one-half of the boundary-layer thickness so that the bleed flow behavior might be expected to lie between that for porous wall suction and slot suction. The x-momentum of the bleed value might then, in turn, be expected to lie between the values used with the two models. Indeed, the use of a bleed flow momentum flux between the two limits would lead to better overall agreement between predicted and measured values of $\delta^*$, $\theta$, and $C_f$. Even then, however, the predicted interaction length would be too long. It should be remarked that in estimating $\Delta x$ for the slot-suction bleed flow model, no allowance is made for the turbulent shear stress along the stream surface separating the bleed flow from the main body of the flow, nor for the wall shear stress. Nor is the pressure force along the separating stream surface considered. The effects of the pressure force and wall shear tend to cancel the effect of the turbulent shear on the separating stream surface, but the extent to which they do so is not known. It should be remarked further that no allowance is made for the roughness effect of the holes on the wall shear. Further study is needed on the details of the bleed flow behavior, including the roughness effect of the holes, before the effects of bleed configuration can be resolved.
The computation procedure reported here represents the results of a continuing effort to improve analytical methods of predicting flowfields in the inlet of supersonic aircraft. In a recent analysis of inlet flowfields by Reyhner and Hickcox [10] the effect of the shock wave interaction on the inviscid flow was taken into account by first obtaining a control volume solution for the boundary-layer properties downstream of the interaction. Then, using an effective surface defined by the boundary-layer displacement thickness upstream and downstream of the interaction, and using a patching technique across the interaction region to construct an effective displacement surface for that region, the inviscid flow solution was obtained for the effective surface. A comparable technique was tried in the work reported here but it was not as successful as the scheme of using the simple reflection off the artificial wall.

CONCLUSIONS

A control volume analysis method, employed in conjunction with a turbulent boundary-layer computation scheme, has been used to predict the flowfield downstream of successive shock wave boundary-layer interactions for flows at $M_w = 3.82$ and $2.82$. The computational procedure has been outlined in detail. The effects of boundary layer bleed at the second interaction site have been considered. For flow with low bleed rates or no bleed the predicted interaction lengths and wall static pressures, as well as the boundary-layer properties downstream of the interactions show good agreement with measured results. With low bleed flow the predicted results for the slot-suction and porous-wall models differ only slightly since the momentum of the bleed flow is small. As the bleed flow rate is increased, predicted and measured results are also in reasonably good agreement. Here, however, differences between predicted results for the two suction models are larger since the difference between the momentum fluxes of the bleed flows is larger. A value of bleed flow momentum between the values used for the models would improve the agreement between predicted and measured results.
APPENDIX A

Modified Wall-Wake Profile

A simple representation of the mean velocity distribution in a turbulent boundary layer is very useful in integral analyses of turbulent flow problems. After an extensive survey of mean velocity profile measurement, Coles [11] suggested that for incompressible turbulent boundary layer flow the velocity profile may be represented by a linear combination of two universal functions in the form,

\[ \frac{u}{u_e} = \frac{1}{K} \ln \left( \frac{y u_t}{\delta} \right) + C + \frac{\pi}{K} \frac{W(y/\delta)}{K} = f(y) + g(y) \]  (A-1)

where

\[ f(y) = \frac{1}{K} \ln \left( \frac{y u_t}{\delta} \right) + C \]  (A-2)

is the Law of the Wall and

\[ g(y) = \frac{\pi}{K} W(y/\delta) \]  (A-3)

is the Law of the Wake.

Setting \( u/u_e \) and \( W(y/\delta) = 2 \) at \( y/\delta = 1 \) in Eq. (A-1) and subtracting the resulting equation from Eq. (A-1) leads to an expression for the velocity of the form,

\[ \frac{u}{u_e} = 1 + \frac{1}{K} (u_t/u_e) \ln (y/\delta) - \left( \frac{\pi}{K} \right) (u_t/u_e) [(2-W(y/\delta))] \]  (A-4)

Mathews et al., [12] have developed a wall-wake representation of the velocity profile in a form applicable for isoenergetic compressible boundary layers. Their profile is expressed as,

\[ \frac{u}{u_e} = (1/\sigma^{\frac{1}{2}}) \sin \left\{ \arcsin \left( \sigma^{\frac{1}{2}} \right) \left[ 1 + \frac{1}{K} (u_t/u_e^*) \ln (y/\delta) \right] - \left( \frac{\pi}{K} \right) (u_t/u_e^*) (1 + \cos \left( \frac{\pi y}{\delta} \right)) \} \]  (A-5)

where

\[ (u_t/u_e^*) = \frac{[C_F/2]}{(1-\sigma)^\frac{1}{2}} / \arcsin \left( \sigma^{\frac{1}{2}} \right) \]  (A-6)

and

\[ \frac{\pi}{K} = \left( \frac{1}{2} \right) \left\{ [1/(u_t/u_e^*)] - \frac{1}{K} \ln \left[ Re_\delta \left( C_F/2 \right)^\frac{1}{2} (1-\sigma)^{1.26} \right] - C \} \]  (A-7)

and where \( 2-W \) has been replaced by \( 1 + \cos \left( \frac{\pi y}{\delta} \right) \) for mathematical convenience.
Equation (A-5) has been found to provide a good representation of the boundary layer velocity profile for a range of external Mach numbers and wall static pressure gradients. However, with both Eq. (A-4) and Eq. (A-5) the velocity gradient at the boundary layer edge is found to have a non-zero value. In the modified wall-wake which is to be developed here this shortcoming is avoided.

The law of the wall may be derived from Prandtl's mixing length theory and the assumption that the shear stress is constant across the boundary layer (Cf. Schlichting [13]). However, an expression for $\tau$ of the form,

$$\tau = \tau_w [1 - (y/\delta)^a] = \tau_w (1 - n^a)$$

(A-8)

Where $a$ is a real constant should provide a more realistic relationship for the shear stress.

Using Eq. (A-8) we may write,

$$\tau_w (1-n^a) = \rho K^2 y^2 \ (du/dy)^2$$

(A-9)

Integration of Eq. (A-9) gives an expression for $u/u_T$ of the form,

$$u/u_T = (1/K) \ln \eta + (2/aK) \left\{ (1-n^a)^{1/2} - \ln [1+(1-n^a)^{1/2}] \right\} + C_1$$

(A-10)

Replacing $f(y)$ in Eq. (A-1) by Eq. (A-10) we have,

$$u/u_T = (1/K) \ln \eta + (2/aK) \left\{ (1-n^a)^{1/2} - \ln [1+(1-n^a)^{1/2}] \right\} + C_1$$

$$+ (\pi/K) W(\eta)$$

(A-11)

At the boundary layer edge ($\eta+1$) we have,

$$u/e/u_T = C_1 + (\pi/K) W(1) = C_1 + 2 \pi/K$$

(A-12)

while near the wall (as $n+0^+$),

$$u/u_T = (1/K) \ln (yu_T/v) - (1/K) \ln (\delta u_T/v) + C_1 + 0.614/aK$$

(A-13)

Near the wall the expression for the law of the wall as given by Eq. (A-2) is also applicable. Equating the expression for $u/u_T$ we may evaluate $C_1$,

$$C_1 = 5.1 - (0.614/aK) + (1/K) \ln (\delta u_T/v)$$

(A-14)
while from Eq. (A-12)

\[ \frac{\pi}{K} = \frac{1}{2} \left[ \left( \frac{u_e}{u_\tau} \right) - \frac{(1/K) \ln (\delta u_\tau/\nu)}{\nu} + 5.1 + 0.614/aK \right] \quad (A-15) \]

Following procedures similar to those used by Van Driest [14], Maise and McDonald [15] or Mathews [12],

\[ \frac{u}{u_e} = \frac{(B^2+4A^2)^{1/2}}{2A^2} \sin \left( \arcsin \frac{2A^2-B}{(B^2+4A^2)^{1/2}} \right) \left[ 1 + \frac{1}{K} \frac{u_\tau}{u_e} (1 - \ln n) + \frac{2(1-n^a)^{1/2}}{a} - \frac{2}{a} \ln \left( 1 + (1-n^a)^{1/2} \right) - \frac{\pi}{K} \frac{u_\tau}{u_e^*} (2-W(\eta)) \right] + \frac{B}{2A^2} \]

where

\[ \frac{\pi}{K} = \frac{1}{2} \left[ \left( \frac{u^*_e}{u_\tau} \right) - \frac{(1/K) \ln (\delta u_\tau/\nu_\tau)}{\nu_\tau} - 5.1 + 0.614/aK \right] \quad (A-17) \]

and

\[ \frac{u^*_e}{u_\tau} = \left( \frac{u_e}{u_\tau} \right) \left( \frac{1}{A} \right) \arcsin \left( \frac{2A^2-B}{(B^2+4A^2)^{1/2}} \right) \quad (A-18) \]

For isoenergetic flow, Eqs. (A-16) and (A-18) become, respectively,

\[ \frac{u}{u_e} = \frac{1}{\sigma^{1/2}} \sin \left( \arcsin \sigma^{1/2} \right) \left[ 1 + \frac{1}{K} \frac{u_\tau}{u_e^*} (1 + \ln n) + \frac{2(1-n^a)^{1/2}}{a} - \frac{2}{a} \ln \left( 1 + (1-n^a)^{1/2} \right) \right] - \frac{\pi}{K} \frac{u_\tau}{u_e^*} \left( 1 + \cos \pi n \right) \]

and

\[ \frac{u_\tau}{u_e^*} = \left( \frac{C_f/2}{\sigma/(1 - \sigma)} \right)^{1/2}/\arcsin \sigma^{1/2} \quad (A-20) \]

where \( 2-W(\eta) \) has been replaced by \( 1 + \cos \pi n \) for mathematical convenience. As \( a \to \infty \) Eq. (A-11) reduces to Eq. (A-1) while Eq. (A-19) reduces to the profile proposed by Mathews et al., Eq. (A-5).

The remaining problem is the selection of the constant \( a \). Based on measurements reported by Klebanoff [16] and Horstman and Owen [17], it appears that \( a = 1 \) represents a reasonable assumption. This amounts to the assumption of a linear shear stress distribution across the boundary layer.

The method of least squares has been used to fit the wall-wake profile, Eq. (A-19), for both \( a = 1 \) and \( a \to \infty \) to a number of experimental velocity
profiles by Seebaugh [7], Teeter [9] and Rose [18]. The computations can be carried out by using program LEAST listed in TABLE 1-A. An example of the results is given in Figure 17 which shows two profiles from the study by Seebaugh of an interaction between a conical shock wave and the turbulent boundary layer at the wall of an axially symmetric $M = 2.82$ wind tunnel. The profiles are for stations just upstream and just downstream of the interaction region and the experimental velocities have been calculated from pitot pressure and the wall static pressures under the assumption of isoenergetic flow. The 10-degree half angle cone used in the study did not produce a shock wave of sufficient strength to cause boundary layer separation. The values of $C_f$ and $\delta$ determined by the curve fits are listed on the figure along with values for the displacement and momentum thickness $\delta^*$ and $\theta$.

As is shown in the figure, both the modified wall-wake profile (a=1) and the profile for $a \to \infty$ provide good representations of the experimental velocity distribution over the ranges from $y = 0$ to the values determined for $\delta$ by the curve fits. The values of $C_f$, $\delta^*$ and $\theta$ determined for the two profiles differ only slightly. However, the values of $\delta$ as determined with the modified profile show much better agreement with the values of $\delta$ based on $\frac{u}{u_e} = 0.995$. Furthermore, the velocity gradient for the modified profile goes to zero at $y = \delta$. In all of the data examined to date the modified velocity distribution than the earlier version of the compressible wall-wake profile.
APPENDIX B

Boundary Layer Computations

For turbulent boundary layers in compressible flow, Green [5] derived from Head's (Cf. Green [5]) work for incompressible flow a procedure for simultaneously calculating the development of the momentum thickness and a quantity referred to as mass flow thickness, defined as,

\[ \Delta = \int_0^\delta \frac{\rho u}{\rho_e u_e} \, dy = \delta - \delta^* \]  

(B-1)

In this procedure the momentum-integral equation,

\[ \frac{d\theta}{dx} = \frac{C_f}{2} - (H+2-M_e^2) \frac{\theta}{u_e} \frac{du_e}{dx} - j \frac{\theta}{r} \frac{dr}{dx} \]  

(B-2)

is integrated simultaneously with an auxiliary equation which accounts for the rate at which the boundary layer entrains fluid from the free stream.

\[ \frac{d}{dx} \left( \rho_e u_e \Delta \right) = \rho_e u_e F \]  

(B-3)

In equation (B-2) the value of \( j \) is set equal to zero for two-dimensional flow, to unity for axisymmetric flow.

Equation (B-3) can be rearranged in the form,

\[ \frac{d\Delta}{dx} = F + (M_e^2 - 1) \frac{\Delta}{u_e} \frac{du_e}{dx} \]  

(B-4)

Green [5] found semi-empirically that \( F \) has the following form,

\[ F = 0.0306 \left( (H_1)_K - 3.0 \right)^{-0.653} \]  

(B-5)

where

\[ (H_1)_K = \frac{\int_0^\delta \frac{u}{u_e} \, dy}{\int_0^\delta \frac{u}{u_e} (1 - \frac{u}{u_e}) \, dy} \]  

(B-6)
Paynter and Schuehle [4], using empirical expressions for \((H_1)_K\) and \(C_f\) suggested by Green, developed a computer program to solve equations (B-2), (B-4) and (B-5). However, as it is suggested by Sun and Childs [3] the velocity in compressible turbulent boundary layer flow can be represented by equation (1). Using equation (1), we can solve equations (B-2), (B-4) and (B-5) without dependence on empirical formulas for \((H_1)_K\) and \(C_f\).

From equation (B-1) we have,

\[
\frac{d\theta}{dx} = \frac{\partial \theta}{\partial C_f} \frac{dC_f}{dx} + \frac{\partial \theta}{\partial \delta} \frac{d\delta}{dx} + \frac{\partial \theta}{\partial M_e} \frac{dM_e}{dx} \tag{B-7}
\]

\[
\frac{d\Delta}{dx} = -\frac{\partial \delta^*}{\partial C_f} \frac{dC_f}{dx} - \left(\frac{\partial \delta^*}{\partial \delta} - 1\right) \frac{d\delta}{dx} + \frac{\partial \delta^*}{\partial M_e} \frac{dM_e}{dx} \tag{B-8}
\]

Solving equations (B-7) and (B-8) for \(\frac{dC_f}{dx}\) and \(\frac{d\delta}{dx}\) yields,

\[
\frac{dC_f}{dx} = \frac{(1 - \frac{\partial \delta^*}{\partial \delta}) (\frac{d\theta}{dx} - \frac{\partial \theta}{\partial M_e} \frac{dM_e}{dx}) - (\frac{\partial \theta}{\partial C_f} \frac{d\Delta}{dx} - \frac{\partial \delta^*}{\partial \delta} \frac{dM_e}{dx})}{(1 - \frac{\partial \delta^*}{\partial \delta}) \frac{\partial \theta}{\partial C_f} + \frac{\partial \delta^*}{\partial \delta} \frac{\partial \theta}{\partial M_e}} \tag{B-9}
\]

\[
\frac{d\delta}{dx} = \frac{\frac{\partial \theta}{\partial C_f} (\frac{d\Delta}{dx} - \frac{\partial \delta^*}{\partial \delta} \frac{dM_e}{dx}) + \frac{\partial \delta^*}{\partial C_f} \frac{d\theta}{dx} - \frac{\partial \theta}{\partial M_e} \frac{dM_e}{dx}}{(1 - \frac{\partial \delta^*}{\partial \delta}) \frac{\partial \theta}{\partial C_f} + \frac{\partial \delta^*}{\partial \delta} \frac{\partial \theta}{\partial M_e}} \tag{B-10}
\]

To evaluate the displacement thickness and momentum thickness and their partial derivatives with respect to \(\delta\), \(C_f\) and \(M_e\) we use the expression given by Van Driest [14] for the temperature distribution through the boundary layer,

\[
\frac{T}{T_e} = \frac{T_w}{T_e} + (1 + \frac{\gamma - 1}{2} M_e^2 - \frac{T_w}{T_e}) \frac{u}{u_e} - \frac{\gamma - 1}{2} M_e^2 \left(\frac{u}{u_e}\right)^2 \tag{B-11}
\]

Equation (B-11) was used in the derivation of equation (1).

The displacement thickness and its partial derivatives with respect to \(\delta\), \(C_f\) and \(M_e\) can be written as,

\[
\delta^* = \int_0^\delta \left(1 - \frac{\rho u}{\rho e u_e}\right) dy = \int_0^\delta \left(1 - \frac{T_w}{T_{ue}}\right) dy \tag{B-12}
\]
Similarly, we have the momentum thickness and its partial derivatives as,

\[ \frac{\partial \delta^*}{\partial \delta} = \left(1 - \frac{T}{T_{u_e}}\right) + \int_0^\delta \frac{\partial}{\partial \delta} \left(1 - \frac{T}{T_{u_e}}\right) \, dy \]

\[ = - \int_0^\delta \frac{\partial}{\partial \delta} \left(\frac{T}{T_{u_e}}\right) \, dy \quad (B-13) \]

\[ \frac{\partial \delta^*}{\partial C_f} = \int_0^\delta \frac{\partial}{\partial C_f} \left(1 - \frac{T}{T_{u_e}}\right) \, dy \]

\[ = \int_0^\delta \frac{\partial}{\partial C_f} \left(\frac{T}{T_{u_e}}\right) \, dy \quad (B-14) \]

\[ \frac{\partial \delta^*}{\partial M_e} = - \int_0^\delta \frac{\partial}{\partial M_e} \left(\frac{T}{T_{u_e}}\right) \, dy \quad (B-15) \]

If initial values of \( \delta, C_f \) and \( M_e \) are known an initial velocity profile can be determined by substituting values of \( \delta, C_f \) and \( M_e \) into equation (1) and values of \( \theta \) and \( \delta^\star \) and their partial derivatives can be determined. Values of \( (H_1)_K \) can be computed by equation (B-6). We then use the values obtained and equations (B-2), (B-4) and (B-5) to compute \( d /dx \) and \( d /dx \). Substituting the obtained values into equations (B-9) and (B-10) gives values for \( dC_f/dx \) and
\[
d\frac{C_f}{dx} \text{. The computations for } \delta \text{ and } C_f \text{ may be carried out in step-by-step fashion. Let } \Delta x \text{ be a small increment of } x; \text{ then values of } \delta \text{ and } C_f \text{ at } x + \Delta x \text{ may be expressed as,}
\]

\[
\delta (x + \Delta x) = \delta(x) + \frac{d\delta}{dx} \Delta x \tag{B-20}
\]

\[
C_f (x + \Delta x) = C_f(x) + \frac{dC_f}{dx} \Delta x \tag{B-21}
\]

The boundary conditions \( M_e \) and \( dM_e/dx \) are assumed to be known, or may be determined from the method of characteristics solution. By repeating the computation, the properties can be computed throughout the boundary layer flow. Program BLGRN, listed in TABLE 5-A, has been developed for the numerical computations. A sample of the results computed with this program is shown in Hirst [19]. As is shown, derivations between computed and experimental values of \( H \) and \( Re_\theta \) begin to show up as a condition of separation is approached. However, the computed values of \( C_f \) agree quite well with the data over the entire range of the computations.
REFERENCES


FIGURE 1. EXPERIMENTAL CONFIGURATION
FIGURE 2. FLOW MODEL USED IN ANALYSIS
FIGURE 3. CONTROL VOLUME USED IN ANALYSIS
FIGURE 4. DETERMINATION OF STREAM SURFACE 2
**Figure 5.** Input for computing external flow field by method of characteristics

**Figure 6.** Input for computing external flow field for second interaction
INPUT DATA POINTS TO PROGRAM MFLX

FIGURE 7. COMPUTATION OF MASS FLUX AS A FUNCTION OF RADIAL COORDINATE

INPUT DATA POINTS FOR BOUNDARY LAYER EDGE STREAM LINE

FIGURE 8. LOCATION OF STREAM SURFACES
FIGURE 9. SHOCK WAVE POSITIONS, BOUNDARY LAYER THICKNESS AND WALL STATIC PRESSURES.  
$M_\infty = 3.82$  NO BLEED
FIGURE 10. $\delta^*$, $\theta$ AND $C_f$, $M_\infty = 3.82$, NO BLEED
MACH NUMBER PROFILES

- EXPERIMENTAL
- PREDICTED PROFILE AT LOCATION PREDICTED FOR END OF SECOND
INTERACTION (x = 9.525 cm). MEASUREMENT NOT TAKEN AT
x = 9.525 cm.

FIGURE 11. MACH NUMBER PROFILES DOWNSTREAM OF SECOND
INTERACTION, $M_\infty = 3.82$, NO BLEED.
FIGURE 12. SHOCK WAVE POSITION, BOUNDARY LAYER THICKNESS AND WALL STATIC PRESSURES, $M_\infty = 2.82$. 2.82 PER CENT BLEED.
FIGURE 13. $\delta^*, \theta, C_f, M_\infty = 2.82, 5.0$ PERCENT BLEED
FIGURE 14. SHOCK WAVE POSITIONS, BOUNDARY LAYER THICKNESS AND WALL STATIC PRESSURES. $M = 2.82, 5.0$ PERCENT BLEED.
BOUNDARY LAYER PROPERTIES

EXPERIMENTAL
PREDICTED (SLOT SUCTION MODEL)
PREDICTED (POROUS SUCTION MODEL)

AXIAL STATIONS
1. BEGINNING OF FIRST INTERACTION
2. END OF FIRST INTERACTION, PREDICTED
3. BEGINNING OF SECOND INTERACTION, PREDICTED
4. END OF SECOND INTERACTION, PREDICTED

FIGURE 15. \( \delta^*, \theta, C_f \). \( M_\infty = 2.82 \), 5.0 PERCENT BLEED.
FIGURE 16. MACH NUMBER PROFILES DOWNSTREAM OF SECOND INTERACTION $M_\infty = 2.82, 5.0$ PERCENT BLEED
FIGURE 17. VELOCITY PROFILES UPSTREAM AND DOWNSTREAM OF A SHOCK WAVE - BOUNDARY LAYER INTERACTION.
FIGURE 18. COMPARISON OF BOUNDARY LAYER PREDICTIONS OF BLGRN PROGRAM WITH DATA.
**TABLE 1-A: PROGRAM LEAST**

**PROGRAM LEAST** (INPUT, OUTPUT, TAPE 5=INPUT, TAPE 6=OUTPUT)

**PROGRAM FOR LEAST SQUARES FIT OF WALL WAKE PROFILE**

**INPUT FORMAT 7F10.6 EXCEPT CARD 1**

**CARD(s) COLUMNS**

1. TITLE, COLUMNS 1-72 HOLLERITH
2. 1-10 AT = 0, NO TOTAL TEMPERATURE DISTRIBUTION INPUT
   = 1, TOTAL TEMPERATURE DISTRIBUTION INPUT
3. 11-20 AIP = 0, CONSTANT PRESSURE DISTRIBUTION
   = 1, LINEAR PRESSURE DISTRIBUTION
4. 21-30 APT = 0, NO PITOT PRESSURE INPUT
   = 1, PITOT PRESSURE DISTRIBUTION INPUT
5. 31-40 AIM = 0, NO MACH NUMBER DISTRIBUTION INPUT
   = 1, MACH NUMBER DISTRIBUTION INPUT
6. 41-50 AIY = 0, EQUAL Y_INCREMENT
   = 1, POINTWISE Y INPUT
7. 51-60 AJB = 0, NO MORE JOB AFTER THIS INPUT
   = 1, MORE JOB AFTER THIS INPUT
8. 1-10 CA = 5.1
9. 11-20 BK = 0.4
10. 21-30 ATT = 1.
11. 31-40 AAK = 0.4
12. 41-50 UTFT = 0.4 FIRST ASSUMED VALUE FOR UT
13. 1-10 GAMMA = 1.4 GAS CONSTANT
14. 11-20 PTO1 TOTAL PRESSURE OUTSIDE B.L. IN PSIA
15. 21-30 TTO1 TOTAL TEMPERATURE OUTSIDE B.L. IN R
16. 1-10 AN NUMBER OF PROFILES INPUT
17. 1-10 PW STATIC PRESSURE AT WALL
18. 11-20 R LOCAL RADIUS OF TUNNEL IN INCH
19. 21-30 X STATION DISTANCE
20. 31-40 AN NUMBER OF POINT INPUT
21. 41-50 ANN APPROXIMATE NUMBER OF POINTS TO BL EDGE
22. 51-60 DY SIZE OF Y_INCREMENT IN INCH
23. 1-70 PT PITOT PRESSURE, AN VALUES IN PSIA (AP=1.)
24. 1-70 TT TOTAL TEMPERATURE AN VALUES IN R (AT=1.)
25. 1-70 EM MACH NUMBER AN VALUES (AIM=1.)
26. 1-70 YY Y INPUT IN I INCH, AN VALUE (AIY=1.)
27. 1-70 PP STATIC PRESSURE INPUT (AIP=2.)

**DIMENSION** PW(9), PEE(9), PTU(60, 9), PTUE(9), PT(60, 9), P(60, 9),
1. TT(60, 9), T(60, 9), TTE(9), EM(60, 9), RH0(60, 9),
2. R(9), X(9), ANN(9), Y(60, 9), ZZ(60)
**DIMENSION** Z(60), UZ(60), YZ(60)
**DIMENSION** TITLE(12), YY(60, 9)
DIMENSION ZP(60), ZM(60), ZT(60)
DIMENSION ZS(60), ZH(60)
COMMON AIC, BK, CA, GAMMA, GAM1, GAM2, GAM3, GAM4, GAM5, GAM6, EEM,
1 AN(9), CFF9, DEL9, RHOE(9), UE(9), TE(9),
2 UT(60,9), AAK, UTFT
90 READ(5,100) (TITLE(J), J=1,12)
READ(5,1000) AT, AIP, APT, AIM, AIV, AJB
READ(5,1000) GA, BK, AID, AAK, UTFT
READ(5,1000) GAMMA, PT01, TT01
GAM1=GA/MM-1.
GAM2=GA/MM+1.
GAM3=GA/MM/GAM1
GAM4=1./GAM3
GAM5=1./GAMMA
GAM6=GA/MM/2.
GAM7=1./GAM1
READ(5,1000) AM
M=AM
DO 8 J=1,N
1 READ(5,1000) PH(J), R(J), X(J), AN(J), ANN(J), DY
AN1=AN(J)
AN2=ANN(J)
N=AN1
N2=AN2
L=1
IF(AIP.EQ.1.) READ(5,1000) (PT(I,J), I=1,N)
IF(AIY.EQ.1.) READ(5,1000) (TT(I,J), I=1,N)
IF(AIM.EQ.1.) READ(5,1000) (EM(I,J), I=1,N)
IF(ATT.EQ.1.) GO TO 103
DO 102 I=1,N
TT(I,J)=TT01
102 CONTINUE
103 IF(AIV.EQ.1.) GO TO 104
READ(5,1000) (Y(I,J), I=1,N)
GO TO 223
104 Y(I,J)=0.
DO 2 I=2,N
Y(I,J)=Y(I-1,J)+DY
2 CONTINUE
223 IF(AIP.EQ.2.) READ(5,1000) (P(I,J), I=1,N)
DEL(J)=Y(N2,J)
11 DO 3 I=1,N
YY(I,J)=Y(I,J)/DEL(J)
ZII=YY(I,J)
ZZII=PT(I,J)
ZPII=PI(I,J)
ZTI=TT(I,J)
ZMI=EM(I,J)
3 CONTINUE
DLL=DEL(J)
IF(AIM.EQ.1.) GO TO 14
IF(AIP.EQ.2.) GO TO 15
C COMPUTE EME BY PT02/PT01
C IF(L.EQ.1.) PT02=PT(N2,J)
IF(L.GT.1) CALL INIP(Z, ZZ, DLL, PT02, N)
PT2PT1 = PT02 / PT01
EMOLD = 1.666 * PT2PT1 * PT2PT1 - 5.666 * PT2PT1 + 5.

4 AR1 = (GAM2 * EMOLD * EMOLD / (GAM1 * EMOLD * EMOLD + 2.)) ** GAMMA
AR2 = PT2PT1 ** GAMMA
AR3 = (AR1 * GAM2 / AR2 + GAM1) / (2. * GAMMA)
EMNEW = SQR(T(AR3))

IF(ABS(EMOLD - EMNEW) < 0.001) 10, 10, 20
20 EMOLD = EMNEW
GO TO 4
14 IF(L.EQ.1) EMNEW = EM(N2, J)
   IF(L.GT.1) CALL INTP(Z, ZM, DLL, EMNEW, N)
10 EM = EMNEW

C COMPUTE PE BY PT01 AND EM
C
AR4 = (1. + GAM1 * EM * EM / 2.) ** GAM3
PE = PT01 / AR4
PEE(J) = PE
IF(A1P.EQ.2.) GO TO 15
DO 5 I = 1, N
   P(I, J) = (PEE(J) - PW(J)) * YY(I, J) * A1P * PW(J)
5 CONTINUE
IF(A1P.EQ.0.) PE = PW(J)

15 IF(A1P.EQ.2.) CALL INTP(Z, ZP, DLL, PE, N)
   PEE(J) = PE
TS = TT01
IF(AT.EQ.1.) CALL INTP(Z, ZT, DLL, TS, N)
TTE(J) = TS

C COMPUTE EM BY PT2/P
C
IF(AIM.EQ.1.) GO TO 17
TEST = (0.5 * GAM2) ** GAM3
DO 6 I = 1, N
   PATIO = PT(I, J) / P(I, J)
IF(RATIO.LT.1.) GO TO 30
IF(RATIO - TEST) 40, 50, 60

C SUBSONIC
C
40 EM(I, J) = SQRT(2. * (RATIO ** GAM4 - 1.) / GAM1)
GO TO 6
C
PT2 LESS THAN P
C
30 EM(I, J) = 0.
GO TO 6
C SONIC
C
50 EM(I, J) = 1.
GO TO 6
C SUPERSONIC
C
60 EMOLD = 1.05
63 AR1 = ((2.0 * GAMMA * EMOLD * EMOLD - GAM1) / GAM2)**GAM5
AR2 = RATIO**GAM4
EMNEW = SQRT(2.0 * AR1 * AR2 / GAM2)
IF(ABS(EMOLD - EMNEW) - 0.001) 61, 61, 62
61 EM(I,J) = EMNEW
GO TO 6
62 EMOLD = EMNEW
GO TO 63
6 CONTINUE

COMPUTE UPSTREAM PROPERTIES

17 DO 7 I = 1, N
    PTU(I,J) = P(I,J) * (1.0 + GAM6 * EM(I,J) * EM(I,J))**GAM3
    T(I,J) = TT(I,J) / (1.0 + GAM6 * EM(I,J) * EM(I,J))
    RT = SQRT(T(I,J))
    UI(I,J) = EM(I,J) * 49.0 * RT
    RHO(I,J) = P(I,J) * 144.0 * 32.2 / (1716.0 * T(I,J))
7 CONTINUE
IF(APl.EQ.1.) GO TO 177
DO 117 I = 1, N
    PT(I,J) = PTU(I,J)
    IF(EM(I,J).GT.1.) PT(I,J) = P(I,J) * (0.5 * GAM2 * EM(I,J) * EM(I,J))
    1**GAM3 / (2.0 * GAMMA * EM(I,J) * EM(I,J) / GAM2 - GAM1 / GAM2)**GAM7
117 CONTINUE
177 IF(APl.EQ.1.) GO TO 18

COMPUTE EDGE CONDITION

TEST = (0.5 * GAM2)**GAM3
RATIO = PT2 / PEE(J)
IF(RATIO .LT. 1.) GO TO 39
IF(RATIO .GT. TEST) 49, 59, 69

SUBSONIC

49 EME = SQRT(2.0 * (RATIO**GAM4 - 1.0) / GAM1)
GO TO 96

PT2 LESS THAN P

39 EME = 0.
GO TO 96

SONIC

59 EME = 1.
GO TO 96

SUPERSONIC

69 EMOLD = 1.05
163 AR1 = ((2.0 * GAMMA * EMOLD * EMOLD - GAM1) / GAM2)**GAM5
AR2 = RATIO**GAM4
EMNEW = SQRT(2.0 * AR1 * AR2 / GAM2)
IF(ABS(EMOLD - EMNEW) - 0.001) 161, 161, 162
162 EMOLD = EMNEW
GO TO 163
161 EME=EMENew
96 CONTINUE

C COMPUTE UPSTREAM PROPERTIES
18 PTUE(J)=PBE(J)*1.4+GAM*EME*EME**GAM
    TE(J)=TT(E(J))/(1.+GAM*EME*EME)
    PT=SORT(TE(J))
    UE(J)=EME*9.4RT
    RHOE(J)=PBE(J)*144.*32.2/(1716.*TE(J))
    EME=EME
    TWT=T(T(1,J))/TTE(J)
    CALL LMLW(J,Z,SSN,NO.L,TWT)
    IF(ABS(SSN).LE.0.00000001) GO TO 21
    IF(L.GT.1) GO TO 12
L=L+1
    DEL=DEL(J)
    DEL(J)=0.9*DEL(J)
    SSN1=SSN
    GO TO 11
12 L=L+1
    SLAPE=SSN1/(SSN-SSN1)
    DE2=DEL(J)
    DEL(J)=DE1+SLAPE*(DE1-DE2)
    DE1=DE2
    SSN1=SSN
    GO TO 11
21 WRITE(6,210) (TITLE(I),I=1,12)
    WRITE(6,203) X(J),PT01,TT01
    WRITE(6,204)
    DO 210 I=1,N
    WRITE(6,205) I,Y(I,J),P(I,J),PT(I,J),EM(I,J),TT(I,J)
210 CONTINUE
    WRITE(6,210) (TITLE(I),I=1,12)
    WRITE(6,200) X(J),R(J),PW(J),DEL(J),CFF(J),UE(J),RHOE(J)
    WRITE(6,201)
    DO 211 I=1,N
    TREF=T(I,J)+198.5
    TRRF=T(I,J)**1.5
    VIS=2.77*TRRF/TREF/10.*8.
    U=U(I,J)*RHO(I,J)/32.2/VIS/10.*6.
    UR=U(I,J)/UE(J)
    PR=P(I,J)/PW(J)
    RR=RHO(I,J)/RHOE(J)
    PTR=PTU(I,J)/PT01
    TTR=T(T(I,J))/TT01
    UU=UR*UE(J)
    PO=RR*RHOE(J)
    RRO=RO/32.2
    ZS(I)=1.-UR*PR*(R(J)-Y(I,J))
    ZH(I)=1.-UR*PR*(R(J)-Y(I,J))
    WRITE(6,202) I,Y(I,J),YY(I,J),B,EM(I,J),UR,RR,PR,TTR,PTR
211 CONTINUE
    SUB=0.
    SUT=0.
    DO 212 I=2,N
    IS=I-1
44
SUD=SUD+0.5*(Y(I,J)-Y(IS,J))*(ZS(I)+ZS(IS))
SUT=SUT+0.5*(Y(I,J)-Y(IS,J))*(ZH(I)+ZH(IS))

CONTINUE
N0=N0+1
TREF=TE(J)+198.5
TRRF=TE(J)**1.5
VIS=2.27*TRRF/TREF/10.***8.
B=UE(J)*RHOE(J)/32.2/VIS/10.***6.
YZY=1.
UR=1.
PR=PE(J)/PM(J)
RR=RHOE(J)/RHOE(J)
TTR=TE(J)/TT01
PTR=PTUE(J)/PT01
WRITE(6,2002) N0,DLL,YZY,B,EEM,UR,RR,PR,TTR,PTR
SUD=SUD+0.5*(DLL-Y(N0,J))*(ZS(N0))
SUT=SUT+0.5*(DLL-Y(N0,J))*(ZH(N0))
SOQ=R(J)*R(J)-2.*SUD
SQJ=R(J)*R(J)-2.*SUT
SQD=SQD+SQD
SRT=SQRT(SQRT(SQD))
DELSTA=R(J)-SQD
THETA=R(J)-SRT
WRITE(6,2007) DELSTA,THETA
WRITE(6,2010) (TITLE(I),I=1,12)
WRITE(6,2000) X(J),R(J),PM(J),DEL(J),CFF(J),UE(J),RHOE(J)
WRITE(6,2006)
WRITE(6,2001)
DO 311 I=1,N
Z(I)=Y(I,J)
ZZ(I)=P(I,J)
CONTINUE
CALL PRPL(I,TTW0,UTZ,YZ,NO,UTSA)
DO 322 I=1,NO
T(I,J)=(TTW0+1.-TTW0)*UZ(I)=GAM6*EEM*EEM*UZ(I)*UZ(I)/(1.
+GAM6*EEM*EEM))**TTE(J)
TE=TE(J)/T(I,J)
TER=SQRT(TE)
EM(I,J)=EEM*UZ(I)**TER
Y(I,J)=DEL(J)*YZ(I)
TT(I,J)=T(I,J)**(1.+GAM6*EM(I,J)**EM(I,J))
YQ=Y(I,J)
CALL INTP(Z,ZZ,YQ,PQ,N)
P(I,J)=PQ
PTU(I,J)=P(I,J)**(1.+GAM6*EM(I,J)**EM(I,J))**GAM3
TREF=T(I,J)+198.5
TRRF=T(I,J)**1.5
VIS=2.27*TRRF/TREF/10.***8.
RHO(I,J)=P(I,J)**14.*32.2/(1716.*T(I,J))
B=UZ(I)*UE(J)*RHO(I,J)/32.2/VIS/10.***6.
PR=P(I,J)/PM(J)
RR=RHO(I,J)/RHOE(J)
PTR=PTUE(I,J)/PT01
TTR=TE(J)/TT01
ZS(I)=(1.-UZ(I)**RR)*(R(J)-Y(I,J))
ZH(I)=(1.-UZ(I)**RR*(R(J)-Y(I,J))
WRITE(6,2002) I,Y(I,J),Y(I,J),B,EEM(I,J),UZ(I),RR,PR,TTR,PTR
SUBROUTINE LWLW(J,Z,SN,NO,L,TWTO)
DIMENSION UHAT(100),Y(100),UT(100),PXA(100)
DIMENSION Z(60)
COMMON AID,BK,CA,GAMMA,GM1,GM2,GM3,GM4,GM5,GM6,EEM,
1 AN(9),CFF(9),DEL(9),RHOE(9),UE(9),TE(9),
2 U(60,9),AAK,UTFT
AN1=AN(J)
N=AN1
TRRF=TE(J)**1.5
TREF=TE(J)+198.5
VIS=2.27*TRRF/TREF/10**8
DEL=UE(J)*RHOE(J)*DEL(J)/12./32.2/VIS
SIGMA=GAME*EEM**2./(1.+GM6*EEM**2.)/TWTO
SIGMA1=1./TWTO-1.
SA=SQR(SIGMA)
SAB=SIGMA**2.+4.*SIGMA
ASB=SQR(SAB)
BAS=(4.*SIGMA-SIGMA)/ASB
DO 2 1=1,N
312 CONTINUE
SDU=0.*
SUT=0.*
DO 313 I=2,NO
IS=I-1
SDU=SDU+0.5*(Y(I,J)-Y(IS,J))*(ZS(I)+ZS(IS))
SUT=SUT+0.5*(Y(I,J)-Y(IS,J))*(ZH(I)+ZH(IS))
313 CONTINUE
SDQ=R(J)*R(J)-2.*SUD
SQ2=R(J)*R(J)-2.*SUT
SRD=SRD(SQ2)
SRT=SQRT(SQ2)
DEL=DEL(RJ)-SROT
WRITE(6,2008) DEL,THETA,UTSA
8 CONTINUE
IF(AUB.EQ.1.0) GO TO 90
1600 FORMAT(7F10.6)
1100 FORMAT(12A6)
2010 FORMAT(11H1,30X,12A6)
2020 FORMAT(/8X,10STATION X=,F10.6,3H IN,3X,2HR=,F10.3,3H IN,3X,
16HWHALL=,F10.6,5H PSPA,3X,4HDEL=,F10.6,5H IN,3X,3HCF=,F10.6,
1 2X,3HUE=,F7.2,2X,5HRHOD=,F9.6/)
2001 FORMAT(/1H *4,1H1,9X,1HY,9X,2HYY,8X,6HPR(FT),7X,2HEM,9X,
14HUE/UE,8X,8HRHO/RHOD,4X,4HP/PW,8X,6H4T/TE,6X,6HPNO/PN/)
2002 FORMAT(/2X,13X,9F12.6)
2003 FORMAT(/8X,10STATION X=,F10.6,3H IN ,3X,4HPTO=,F10.5,3X,
14HHTO=,F10.5/)
2004 FORMAT(/12X,11H1,12X,1HY,12X,1HP,12X,2HPTO,12X,1HM,12X,2HTO /)
2005 FORMAT(/10X,13X,2X,5F14.6)
2006 FORMAT(/4X,2H WALL-WAKE VELOCITY PROFILE /)
2007 FORMAT(/20X,7HDELTA=,F10.6,4H IN ,,4X,6HTHETA=,F10.6,4H IN ,)
2008 FORMAT(/20X,7HDELTA=,F10.6,4H IN ,,4X,6HTHETA=,F10.6,4H IN ,,
14X,7HHTO/UH=,F10.6)
END
IF(\(Z(I)-\text{DEL}(J)\)) \(2, 3, 4\)
2 CONTINUE
3 NO=I
   GO TO 30
4 NO=I-1
30 DO 1 I=1, NO
   URAT(I)=U(I,J)/UE(J)
   Y(I)=Z(I)/\text{DEL}(J)
1 CONTINUE
K=1
   CIAA=CA=0.614/(BK*AID)
   SIG2=2.*SIGMA/ASB
   SIG3=ASIN(BAS)
   FSIG=SIG3/SA
   UT(K)=UT(T)
   VWVE=(1./TWT-SIGMA)**1.76
   AAE=\text{REDEL}*FSIG*VWVE
6 IF(UT(K)) 5, 7, 5
7 PXA(K)=0.5
   SMM=0.
   SNN=0.
   GO TO 9
5 PXA(K)=0.5*(1.-UT(K))*((1./BK)*\text{ALOG}(*ABS(UT(K))*FSIG*VWVE)
1+CIAA))
   SMM=0.
   SNN=0.
9 DO 10 I=2, NO
   ACS=1.*COS(3.14*Y(I))
   YAA=SQRT(1.-Y(I))*AID)
   ALG=\text{ALOG}(Y(I))+2.\times(YAA-\text{ALOG}(1.+YAA))/AID
   ACT=1.+UT(K)*ALG/BK-PXA(K)*ACS
   AFF=\text{SIN}(SIG3*ACT)/SIG2+SIGMOB/2.*SIGMA
   ACD=(ALG+0.5*ACS*(\text{ALOG}(ABS(UT(K))))+\text{ALOG}(AAE)+BK*CIAA+1.))/BK
   SUN=URAT(I*AFF)*FSIG*COS(SIG3*ACT)*ACD
   SMM=SMM+SUN
   ADD=ACS*UT(K)*(2.*BK*\text{DEL}(J))-YAA*UT(K)/(BK*\text{DEL}(J))-PXA(K)*Y(I)*
13.14*SIG3*ACT)/\text{DEL}(J)
   SUU=(URAT(I)-AFF)*COS(SIG3*ACT)*FSIG*ADD
   SNN=SNN+SUU
10 CONTINUE
CFF(J)=2.*UT(K)*FSIG*FSIG*(1./TWT-SIGMA)*UT(K)
IF(ABS(SMM)*LE.0.0000001) GO TO 20
IF(K.GT.1.) GO TO 11
K=K+1
UT(K)=0.8*UT(K-1)
SMM=SMH
GO TO 6
11 K=K+1
SLOPE=SMM1/(SMM-SMM1)
UT(K)=UT(K-2)+SLOPE*(UT(K-2)-UT(K-1))
SMM=SMM
GO TO 6
20 RETURN
END
SUBROUTINE PRFL(J,TWTO,URT,Y,KAA,UT1)
DIMENSION URT(60),Y(60)
COMMON A10,BK,CA,GMMA,GAM1,GAM2,GAM3,GAM4,GAM5,GAM6,EEN,
1 AN(9),CFF(9),DEL(9),RHOE(9),UE(9),TE(9),
2 U(60,9),AAK,UTF
TRRF=TE(J)**1.5
TREF=TE(J)+198.5
VIS=2.77*TRRF/TREF/10.*8.
REDEL=UE(J)*RHOE(J)*DEL(J)/12./32.2/VIS
SIGMA=GAM6*EEM**2./(1.+GAM6*EEM**2.)/TWTO
SIGHB=1./TWTO-1.
SA=SQR(SIGMB)
SAB=SIGMB**2.+4.*SIGMA
ASB=SQR(SAB)
BAS=(2.*SIGMA-SIGMB)/ASB
CIAA=CA-0.614/8*EEN
SIG2=2.*SIGMA/ASB
SIG3=ASIN(BAS)
FSIG=SIG3/SA
UT2=CFF(J)/(2.*FSIG*FSIG*(1./TWTO-SIGMA))
UT1=SQR(ABS(UT2))
VVVE=(1./TWTO-SIGMA)**1.76
IF(UT1.GT.0.) GO TO 2
PX=0.5
GO TO 1
2 PX=0.5*(1.-UT1*(1./BK)*ALOG(REDEL*ABS(UT1)*FSIG*VVVE)+CIAA))
1 ANI=AAK+1.
KAA=ANI
Y(1)=0.
URT(1)=0.
DO 10 I=2,KAA
AI=I-1
Y(I)=AI/AAK
YAA=SQR(1.-Y(I)**AI)
ALG=ALOG(Y(I))+2.*(YAA-ALOG(1.+YAA))/AI
ACS=1.*COS(3.14*Y(I))
ACT=1.+UT1*ALOG/BK-PX*ACS
URT(I)=SIN(SIG3*ACT)/SIG2+SIGMB/2./SIGMA
10 CONTINUE
RETURN
END

SUBROUTINE INTP(X,Y,XOT,YOT,NO)
DIMENSION X(60),Y(60),V(60),VV(60)
DO 11 I=1,NO
IF(X(I)-XOT) 11,12,12
11 CONTINUE
12 NM=1
NU=NM+2
NL=NM+2
IF(NL.LT.1) NL=1
IF(NU.GT.NO) NU=NO
NW=NU-NL+1
DO 13 J=1,NW
L=NL+J-1
48
V(J)=X(L)
VV(J)=Y(L)
13 CONTINUE
CALL LAGRAN(V, VV, XOT, YOT, N)
RETURN
END

SUBROUTINE LAGRAN(X, Y, XOT, YOT, N)
DIMENSION X(60), Y(60)
YOT=O.
DO 1 I=I, N
AL=1.
DO 2 J=1, N
IF(J.EQ.I) GO TO 2
AL=AL*(XOT-X(I))/(X(I)-X(J))
2 CONTINUE
1 YOT=YOT+AL*Y(I)
RETURN
END

TABLE 1-B: INPUT TO PROGRAM LEAST

<p>| | | | | | |</p>
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<tr>
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<tbody>
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<td>XOT</td>
<td>YOT</td>
<td>N</td>
<td>N</td>
</tr>
<tr>
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<td>0.</td>
<td>1.</td>
<td>0.</td>
<td>0.</td>
<td>0.</td>
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<tr>
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<td>1.</td>
<td>50.</td>
<td>0.04</td>
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<td>1.4</td>
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<tr>
<td>1.</td>
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<td>1.02</td>
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<td>40.</td>
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<td>1.224</td>
<td>3.445</td>
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<td>4.679</td>
<td>5.122</td>
<td>5.542</td>
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</table>
TABLE 2-A: PROGRAM CONE

PROGRAM CONE (INPUT, OUTPUT, PUNCH, TAPE 5 = INPUT, TAPE 6 = OUTPUT, TAPE 7 = PUNCH)

INPUT FORMAT 7F10.6

CARD(S) COLMNS
1 1-10 GAMMA = 1.4 SPECIFIC HEAT RATIO OF AIR
11-20 EMI FREE STREAM MACH NUMBER
21-30 PHC HALF ANGLE OF CONE TIP IN DEGREE
31-40 XL STATION OF OUTPUT DESIRED

THIS PROGRAM COMPUTES THE CONICAL FLOW PROPERTIES GIVEN THE SHOCK ANGLE AND THE SURFACE MACH NUMBER

DIMENSION EMINF(12), PHC(6)
DIMENSION PH25(12), PH50(12), PH75(12), PH100(12), PH125(12), PH150(12)
DIMENSION EM25(12), EM50(12), EM75(12), EM100(12), EM125(12), EM150(12)
DIMENSION PC(61, 0(6)
DIMENSION SAVE1(300), SAVE2(300), SAVE3(300), SAVE4(300), SAVE5(300)
SAVE6(300), SAVE7(300)
DIMENSION SAVE8(300)
TT = 2000.
R = 53.3

THE FOLLOWING CARDS READ IN THE DATA AS TAKEN FROM THE CONICAL FLOW TABLES BY SIMS

DATA EMINF/1.5, 1.75, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0, 8.0/
DATA PHC/0.43633, 0.67266, 1.13090, 1.17453, 2.18177, 2.61807/
DATA PH25/0.72980, 0.60832, 0.52370, 0.41169, 0.34011, 0.29017, 0.25379, 0.22495,
1.22529, 1.2973, 1.69367, 1.46238, 1.29317/
DATA PH50/0.73709, 0.60935, 0.52525, 0.41425, 0.34410, 0.29592, 0.26104,
1.23484, 2.14589, 1.89654, 1.66280, 1.5262/
DATA PH75/0.73490, 0.61445, 0.53140, 0.42358, 0.35701, 0.31239, 0.28078,
1.25747, 2.39731, 2.14813, 1.98465, 1.87127/
DATA PH100/0.74466, 0.62364, 0.54465, 0.44136, 0.37900, 0.33790, 0.30918,
1.28822, 2.72421, 2.5049, 2.36333, 2.2664/
DATA PH125/0.76158, 0.66399, 0.56518, 0.46631, 0.40760, 0.36939, 0.34296,
1.32933, 3.0953, 2.89907, 2.7740, 2.6895/
DATA PH150/0.78592, 0.6969, 0.59192, 0.49663, 0.44085, 0.40494, 0.38032,
1.36266, 0.34955, 0.33174, 0.32092, 0.31301/
DATA EM25/0.48669, 1.73403, 1.98086, 2.47295, 2.96276, 3.45099,
13.93477, 4.16624, 0.89553, 5.84407, 6.77969, 7.70188/
DATA EM50/0.54579, 1.70046, 1.94512, 2.41940, 2.89138, 3.35716,
13.81645, 4.26906, 4.71481, 5.38525, 6.42692, 7.23908/
DATA EM75/0.51973, 1.65580, 1.89123, 2.35286, 2.80480, 3.24675,
13.78394, 4.0946, 4.50971, 5.29688, 6.03849, 6.73371/
DATA EM100/0.37485, 1.60451, 1.83404, 2.27888, 2.7101, 3.12751,
13.58057, 3.18914, 2.92184, 3.92879, 5.63165, 6.21039/
DATA EM125/0.32490, 1.55131, 1.77219, 2.00162, 2.61039, 3.00266,
13.376145, 3.73078, 4.06627, 4.68017, 5.22455, 5.69227/
THE NEXT PART OF THE PROGRAM INTERPOLATES TO FIND THE SHOCK ANGLE AND THE SURFACE MACH NUMBER.

```
DATA EM150/1.27074,1.49255,1.70688,2.11794,2.50675,2.87312,
 13.21670,3.53743,3.83599,4.36681,4.81780,5.19815/
1 READ (5,10) GAMMA,EM1,PHIC,XL
WRITE(6,2000) GAMMA,EM1,PHIC,XL
PHIC=PHIC/57.2957795
KKK=1
GAMMA=1.0
GAMMA=1.0

CALL TAINT (EMINF,PH25,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,PH50,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,PH75,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,PH100,EM1,PC(5),12,2,NER,DMON)
CALL TAINT (EMINF,PH125,EM1,PC(6),12,2,NER,DMON)
AMON=0.0
CALL TAINT (PHC,PC,PHIC,PHIC,6,2,NER,AMON)
DMON=0.0
CALL TAINT (EMINF,EM25,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,EM50,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,EM75,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,EM100,EM1,EM2,12,2,NER,DMON)
CALL TAINT (EMINF,EM125,EM1,EM2,12,2,NER,DMON)
BMON=0.0
CALL TAINT (PHC,EMC,PHIC,6,2,NER,BMonden)

CALCULATE THE MAXIMUM VELOCITY

VH=SQRT(2.0/GAMMA)*SQRT(GAMMA*TT*32.2)
INITIALIZE THE VELOCITY IN THE ANGULAR DIRECTION

PSIO=0.0

CALCULATE THE VELOCITY ON THE CONE

VROU=(EMC**2)*(VM**2)*(GAMMA/2.0)
VROD=1.0+(EMC**2)*(GAMMA/2.0)
VRO=SQRT(VROU/VPOD)
PHIC=PHIC

CALCULATE THE ENTROPY FUNCTION

D1=ALOG((17.0)*(EM1**2)*(SIN(PHIW)**2)-1.0)/6.0
D2=(6.0)*(EM1**2)*(SIN(PHIW)**2)
D3=((EM1**2)*(SIN(PHIW)**2)+5.0)
DS1=(D1-1.4)*ALOG(D2/D3))/0.4
DP=EXP(-DS)
SAVE1(KKK)=XL
SAVE2(KKK)=XL*SIN(PHIC)/COS(PHIC)
SAVE3(KKK)=PHIC*57.2957795
SAVE4(KKK)=EMC
SAVE5(KKK)=DP
SAVE6(KKK)=PHIC*57.2957795
```
EMX = EMC
A1 = 1 + 0.5 * GAMMA * EM1 / EM1**2
A11 = ALOG(A1)
B1 = A11 * GAMMA / GAMM1
B11 = EXP(B1)
A2 = 1 + 0.5 * GAMMA * EM1 / EM1**2
A21 = ALOG(A2)
B2 = A21 * GAMMA / GAMM1
B21 = EXP(B2)
SAVE7(KKK) = DP * A11 / B21
KKK = 2

C SET THE INCREMENTAL VALUE OF THE ANGLE TO BE USED
50 DELPHI = (PHI - PHIC) / 117.0
WRITE (6, 2050)
60 DD 200 K = 1, 117
STEP = K
X = STEP * DELPHI
PHI = PHIC + X
FUZZ = STEP - 1.0
IF (FUZZ) GO TO 68

C CALCULATE THE VELOCITY IN THE ANGULAR DIRECTION
68 P3 = (GAMMA) * (RO) * ((VM**2) - (VRO**2))
D = -(GAMMA) * (RO) * ((VM**2) - (VRO**2))
PSIX = PSI0 + (DELPHI * P3) / D
GO TO 80
70 P1 = -(GAMMA) * (RO) * ((PSI0**2) * (COS(PHIC) / SIN(PHIC)))
P2 = -(GAMMA) * (RO) * ((VM**2) - (VRO**2)) * (PSI0) * (COS(PHIC) / SIN(PHIC))
P3 = (GAMMA) * (RO) * ((VM**2) - (VRO**2))
P4 = -(GAMMA) * (RO) * (PSI0**2)
D = (GAMMA) * (RO) * (PSI0**2) - (GAMMA) / 2.0 * ((VM**2) - (VRO**2))
PSIX = PSI0 + (DELPHI) * (P1 + P2 + P3 + P4) / D

C CALCULATE THE VELOCITY IN THE RADIAL DIRECTION
80 VRX = PSI0 * DELPHI + VRO

C CALCULATE THE FLOW ANGLE
100 THETAX = PHI + ATAN (PSIX / VRX)
110 VX = SQRT (VRX**2 + PSIX**2)
120 AX = SQRT ((GAMMA) / (VM**2 - VX**2))

C CALCULATE THE MACH NUMBER
130 EMX = VX / AX
EM1U = (EMX**2) * (GAMMA / 2.0)
EM1D = 1.0 + (EMX**2) * (GAMMA / 2.0)
EM1T = SQRT (EM1U / EM1D)

140 YL = XL * (SIN(PHI) / COS(PHI))
BURP = PHI * 57.2957795
WRITE (6, 2100) VRX, PSI0, BURP, AX
WRITE (6, 2110) XL, YL, THETAX, EM1T, DP
SAVE1(KKK) = XL
SAVE2(KKK) = YL
SAVE3(KKK) = THETAX * 57.2957795
SAVE4(KKK) = EMX
SAVE5(KKK) = DP
SAVE6(KKK) = BURP
A1 = 1 + 0.5 * GAMMA / GAMM1
A11 = ALOG(A1)
B1 = A11 * GAMMA / GAMM1
B11 = EXP(B1)
A2 = 1 + 0.5 * GAMMA / GAMM1
\[ A21 = \text{ALOG}(A2) \]
\[ B2 = A21 \times \text{GAMMA/GAMMA1} \]
\[ B2 = \text{EXP}(B2) \]
\[ \text{SAVE7}(KKK) = DP \times R11 / B2 \]
\[ \text{SAVE8}(KKK) = 1.02 \times \text{SAVE2}(KKK) \]
\[ \text{PSIX} = \text{PSI} \times \text{VRX} \]
\[ \text{PHI} = \text{PHI} \times \text{KKK} + 1 \]

200 EMCMEXM
WRITE(6,3001)
WRITE(7,3002)
WRITE(7,3003)
1

3000 FORMAT(1H1,12X,1HX,14X,1HY,14X,3HDEL,13X,1HM,15X,8HPT/PTINF,10X,
15HTHETA,12X,4HPR/P1)
3001 FORMAT(2X,7F15.6)
3002 FORMAT(4F10.6)
3003 FORMAT(5F10.6)
1000 FORMAT(2F10.3)
1010 FORMAT(4F10.6)
2000 FORMAT(1H1,7H4HMC =,F15.6,10X,5HPSI =,F15.6,10X,6HPHIC =,F15.6)
2050 FORMAT(1H1,4S5X,7HRESULTS)
2100 FORMAT(1HO,4HVR =,F15.6,10X,5HPSI =,F15.6,10X,
16PHI =,F15.6,10X,3HHA =,F15.6)
2110 FORMAT(1HO,3HX =,F10.5,8X,3HY =,F10.5,8X,8HTHETAX =,F10.5,8X,
16MSTR =,F10.5,8X,10HPT/PTINF =,F10.5)

END

SUBROUTINE TAINT(XTAB,FTAB,X,FX,N,K,NER,MON)
DIMENSION XTAB(1),FTAB(1),T(10),C(10)
CPS0400 TAINT SUBROUTINE- IN FORTRAN II.
IF (N - K) 1,1,2
1 NER = 2
RETURN
2 IF (K-9) 3,3,1
3 IF (MON) 4,4,5
5 IF (MON-2) 6,7,4
4 J = 0
11 NER = 3
12 NM1 = N - 1
DO 8 I = 1, NM1
10 J = J + 1
8 CONTINUE
11 CONTINUE
12 MON = 1
13 IF (J) 12, 6, 6
12 MON=2
7 DO 13 I=1,N
14 IF (X-XTAB(I)) 14,14,13
17 J=I
13 CONTINUE
GO TO 15
6 DO 16 I=1,N
16 IF (X-XTAB(I)) 16,17,17
19 J=1
20 M=J+K
18 J=(J-(K+1)/2
19 J=J
22 J=J-1
21 KPl=K+1
JSAVE=J
26 DO 23 L=1,KP1
C(L)=X-XTAB(J)
23 T(L)=FTAB(J)
26 DO 24 J=1,K
24 I=J+1
25 T(J)=(C(J)*T(I)-C(I)*T(J))/((C(J)-C(I))
23 I=I+1
END
TABLE 2-B: INPUT TO PROGRAM CONE

| 1.4  | 2.82  | 10.  | 1.035 |

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### TABLE 3-A: PROGRAM ANAL

Program ANAL (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT)

Two-dimensional and axially symmetric shock wave-boundary layer interaction

**Program Input**

Input Format 7F10.6 Except Cards 1 and 2

**Card(s)**

1 and 2 Titles, Columns 1-72, Hollerith

3

<table>
<thead>
<tr>
<th>COL. 1-10</th>
<th>ENE1</th>
<th>Upstream Mach No.</th>
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<tbody>
<tr>
<td>11-20</td>
<td>UTUE1</td>
<td>Upstream Dimensionless Friction Velocity</td>
</tr>
<tr>
<td>21-30</td>
<td>RED1</td>
<td>Upstream Reynolds No. (Delta)</td>
</tr>
<tr>
<td>31-40</td>
<td>AINPT</td>
<td>1.0 Input Theta1</td>
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<tr>
<td>41-50</td>
<td>RECFA1</td>
<td>Recovery Factor</td>
</tr>
<tr>
<td>51-60</td>
<td>AX1</td>
<td>0.0 or 1.0 2-D Case</td>
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<tr>
<td>61-70</td>
<td>SHORT</td>
<td>0.0 Long Output</td>
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4

<table>
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<th>COL. 1-10</th>
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<td>11-20</td>
<td>BLEED2</td>
<td>Entrainment Rate</td>
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<tr>
<td>21-30</td>
<td>AIB</td>
<td>0.0 No Bleed</td>
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<td>1.0 Porous Suction</td>
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<td>2.0 Slot Suction</td>
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<td></td>
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<td>3.0 Scoop Suction</td>
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<tr>
<td>31-40</td>
<td>BK</td>
<td>Constant in Wall-Wake Profiles (Usually 0.4)</td>
</tr>
<tr>
<td>41-50</td>
<td>C</td>
<td>Constant in Wall-Wake Profiles (Usually 5.1)</td>
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<tr>
<td>51-60</td>
<td>STC0EF</td>
<td>0.5 If No Shear Force Is Desired</td>
</tr>
<tr>
<td>61-70</td>
<td>AIDD</td>
<td>Parameter in (1-(Y/DELTA)**AIDD) (=1. is recommended at this time)</td>
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5

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<thead>
<tr>
<th>COL. 1-10</th>
<th>TH1D</th>
<th>Theta1 When AINPT=1.0</th>
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<tr>
<td></td>
<td>ALP1D</td>
<td>Alpha1 When AINPT=2.0</td>
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<tr>
<td>11-20</td>
<td>TH1Z</td>
<td>Theta1 When AINPT=2.0</td>
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FOLLOWING FOR AXIALLY SYMMETRIC CASES ONLY (AXI=2.0)

6

<table>
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<th>COL. 1-10</th>
<th>DEL1R</th>
<th>Delta1/R</th>
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<tr>
<td>11-20</td>
<td>AIE2</td>
<td>1.0 B.L. Edge Streamline Data Input</td>
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<tr>
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<td>2.0 Conical Flow Data Input</td>
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<tr>
<td></td>
<td></td>
<td>3.0 Internal Flow Field Input Data</td>
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<tr>
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<td></td>
<td>4.0 Constant Pressure Boundary</td>
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<tr>
<td>21-30</td>
<td>FLDIR3</td>
<td>Flow Direction Onstream of Axisym. Interactns</td>
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<th>COL. 1-10</th>
<th>AKK</th>
<th>No. Points of Input Data</th>
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<td>FOLLOWING FOR B.L. EDGE STREAMLINE INPUT(AIE2=1.0)</td>
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8

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<th>COL. 1-10</th>
<th>XYD</th>
<th>Mach Number</th>
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<td>11-20</td>
<td>RPM</td>
<td>R/R0</td>
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<tr>
<td>21-30</td>
<td>EME2S</td>
<td>Mach Number</td>
</tr>
<tr>
<td>31-40</td>
<td>PE2P1</td>
<td>P/P0</td>
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DIMENSION TTRAT(200), PTRAT(200), PTRNS(200), URAT(200), EM(200)
DIMENSION YY(200), UTUES(100), E(100), WR(200), PHIR(200)
DIMENSION REDEL(100)

COMMON TTRAT, PTRAT, PTRNS, URAT,
  EM, WR, PHIR,
  SHORT, BLEED1, BLEED2, BLEED,
  AIB, TWTTE, GAMMA, TH10,
  BLMN, BLMON, BLMI, BLMOI,
  DSO, DSSD, BK, EME1, SHFAC,
  SIG1, SIGMA1, SIGS1, FSI1, WMVE1,
  CFHW1, C, STCOEF

COMMON /UWU/AIDD
COMMON /WUW/AINPT, TH12Z

READ DATA CARDS 1 AND 2
1 CALL PRIM(ILINE,1)

READ DATA CARDS 3 AND 4

READ(5,1000) EME1, UTUES1, REDEL1, AINPT, RECFAC, AXI, SHORT
READ(5,1300) BLEED1, BLEED2, AIB, BK, C, STCOEF, AIDD
C=C-0.614/(BK*AIDD)
INPT=AINPT
BLEED=BLEED1*BLEED2
GAMA=1.4
G1=EME1*EME1
TWTTE=RECFAC*(1.-RECFAC)/(1.+(GAMA-1.)*G1/2.)
SIG1=0.2*EME1*EME1
SIGMA1=SIG1/(1.+SIG1)
SIGS1=SQRT(SIGMA1)
FSI1=ASIN(SIGS1))/SIGS1
WMVE1=(TWTTE*(1.+SIG1))**1.76
FUUE1=UTUES1*FSI1
CFHW1=2.*FUUE1*FUUE1/(TWTTE*(1.+SIG1))
TAU1P1=0.7*CFHW1*EME1*EME1
PX1=.5*(1.-UTUES1*(1./BK))*ALOG(REDEL1*ABS(UTUES1))
  1*FSI1/WMVE1*C)

VARIABLES WHICH MUST BE INITIALIZED
IZ=0
YS=0.
WRITE(6,1024)
GO TO (10,20), INPT
INPUT PARAMETER IS THETA1
READ DATA CARD 5

10 READ(5,1000) TH1D
   WRITE(6,1010) UTUES1, REDEL1, PX1, TH1D, EME1, RECFAC, TWTTE
   ILINE=ILINE+3
   TH1=TH1D*.01745329
   Z=SIN(TH1)**2.
   ANUM=2.*(G1*Z-1.)
   DNUM=(2.*G1*(GAMMA+1.-2.*Z))*TAN(TH1)
   ALP1=ATAN(ANUM/DNUM)
   ALP1D=ALP1D*.01745329

FIND PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM MACH NO.
   CALL SHOCK(2, ALP1D, TH1, EME1, EME2, P2P1)
   GO TO 30

INPUT PARAMETER IS ALPHA1
READ DATA CARD 5

20 READ(5,1000) ALP1D, TH1Z2
   WRITE(6,1011) UTUES1, REDEL1, PX1, ALP1D, EME1, RECFAC, TWTTE
   ILINE=ILINE+3
   ALP1=ALP1D*.01745329
   IF(ALP1D.EQ.0.) GO TO 310

FIND SHOCK WAVE ANGLE, PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM MACH NO.
   CALL SHOCK(1, ALP1, TH1, EME1, EME2, P2P1)
   TH1D=TH1D*.01745329
   ROOT=SIN(TH1)
   IF(ROOT.GT.1.) GO TO 1

ERROR EXIT IF SIN(THETA) GREATER THAN 1.0
   IF(ROOT.GT.1.) GO TO 1

EQUATE ALPHA1 AND ALPHA2

30 ALP2=ALP1
   ALP2D=ALP1D
   L=1
   WRITE(6,1025) L, BLEED1
   L=2
   WRITE(6,1025) L, BLEED2

FIND SHOCK WAVE ANGLE, PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM MACH NO.
   IF(ALP1D.EQ.0.) GO TO 310
   CALL SHOCK(1, ALP2, TH2, EME2, EME3, P3P2)
   ROOT=SIN(TH2)

ERROR EXIT IF SIN(THETA) GREATER THAN 1.0
IF(ROO.GT.1.0) GO TO 1
TH20=TH2/.01745329
310 IF(ALP10.GT.0.) GO TO 311

PARAMETERS FOR NO SHOCK

EME2=EME1
EME3=EME1
P2P1=1.
P3P2=1.
P3P1=1.
GO TO 312
311 P3P1=P3P2*P2P1
WRITE(6,1012) P2P1,P3P2,P3P1,EME1,EME2,EME3
ILINE=ILINE+6
WRITE(6,1013) ALP10,TH10,ALP20,TH20
ILINE=ILINE+6

DETERMINE UPSTREAM BOUNDARY LAYER PROFILE PROPERTIES

312 CALL PRFL(TUES1,PX1,EME1,1.0,2,YY)
IF(SHORT.EQ.0.) CALL PRIN(ILINE,3)
II=1
WRITE(6,1014) II
ILINE=ILINE+4
IF(SHORT.EQ.1.) GO TO 4000
WRITE(6,1028)
DO 40 I=1,101
WRITE(6,1015) I,YY(I),EM(I),URAT(I),PTRN(I),TTRAT(I)
ILINE=ILINE+1
CALL PRIN(ILINE,2)
IF(ILINE-2) 31,31,40
31 WRITE(6,1014) II
WRITE(6,1028)
ILINE=ILINE+4
40 CONTINUE
4000 DUM=0.

DETERMINE UPSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES

CALL FLUX(101,YY,1,EME1,DUM)
HII=(SHFAC-SIG1)/(TWIT*(1.+SIG1))
RETH=RETH1*DUSD
PRAND=.72
PR13=PRAND**((1.+3.)
TBART0=5.*TWIT+.22*PR13+.5-.22*PR13/(1.+SIG1)
CFLT=1.246*EXP(-1.561*HII)*(RETH**(-.268))
1((TBART0*1.+SIG1)**0.7963)
WRITE(6,1016) DSO,DSO,DLMN,BLM0N,SHFAC,HI1,CFWW1,CFLT1
DSD1=DSD
DSDD=DSD
ILINE =ILINE+6

CONSTANTS FOR MASS AND MOMENTUM BALANCES

H1=1.+(GAMMA-1.)/2.*EME3*EME3
H2=SQRT(H1)
\[ H_3 = 1.0 + \left( \frac{\text{GAHMA} - 1.0}{2.0} \right) \]
\[ H_4 = \sqrt{H_3} \]
\[ H_5 = 1.0 - \text{DSO} - \text{DSO} \]

IF \( \text{AIB} \leq 1.0 \) GO TO 410
IF \( \text{BLEED1} = 1.0 \) GO TO 410
WS = \|\text{BLEED1}\| * \text{BLMN}
CALL \text{INTRP} (\text{WR}, \text{YY}, \text{WS}, \text{YS}, 101)
CALL \text{INTRP} (\text{YY}, \text{PHIR}, \text{YS}, \text{PHIS}, 101)
WRITE (6, 1027) \text{WS}, \text{YS}, \text{PHIS}

C AIB = 1 POROUS PLATE SUCTION
C AIB = 2 SLOT SUCTION
C AIB = 3 SCOOP SUCTION

C ITERATION TO FIND SOLUTION TO CONTINUITY AND MOMENTUM EQUATIONS

410 IF \( \text{AIB} \leq 1.0 \) \( \text{BLMOM1} = 0.0 \)
   IF \( \text{AIB} > 1.0 \) \( \text{BLMOM1} = -\text{GAHMA} * \text{EME1} * \text{EME1} * \text{PHIS} \)
   KB = 1
   WRITE (6, 1026) KB, BLMOM1
   BLMOM2 = \|\text{BLEED2}\| * (1.0 - \|\text{DMO}\|) * \text{GAHMA} * \text{EME1} * \text{EME2} * \cos(\text{ALP1})
   I*H4/\sqrt{H_1 + (\text{GAHMA} - 1.0) / 2.0 * \text{EME2} * \text{EME2}}
   KB = 2
   WRITE (6, 1026) KB, BLMOM2
   BLMOMF = BLMOM1 + BLMOM2
   KB = 3
   WRITE (6, 1026) KB, BLMOMF

C FOR APPROXIMATE SOLUTION ASSUME THAT TAU/\text{P1} = 2.0 * STCOEF * TAUI/\text{P1}

C TAUP1 = 2.0 * STCOEF * TAUIP1
   H6 = P2P1 - 1.0 - \text{GAHMA} * G1 * H5 - BLMOMF * TAUP1 / \tan(\text{ALP1})
   IF \( \text{SHORT} = 0.0 \) WRITE (6, 1022) H1, H2, H3, H4, H5, H6, TAUP1
   A1 = P3P1 * \text{EME3} * \text{EME1} * H2 / (1.0 - \|\text{DMO}\|) / (1.0 * \|\text{BLEED}\|)
   A2 = (P2P1 - P3P1) / H6
   A3 = -\text{GAHMA} * P3P1 * \text{EME3} * \text{EME3} / H6
   A4 = TAUP1 / \tan(ALP1) / H6
   IF \( \text{SHORT} = 0.0 \) WRITE (6, 1017) A1, A2, A3, A4
   IF \( \text{SHORT} = 0.0 \) CALL \text{PRINT} (ILINE, 3)
   IF \( \text{SHORT} = 0.0 \) WRITE (6, 1018)
   ILINE = ILINE + 2

C APPROXIMATE 2-D SOLUTION (ASSUME REDEL3 = REDEL1 AND TAU = TAUI)

C M = 1
   REDEL3(1) = REDEL1
   SIG3 = 2.0 * \text{EME3} * \text{EME3}
   SIGMA3 = SIG3 / (1.0 + SIG3)
   SIG3 = \sqrt{SIG3}
   FSIG3 = (\text{ASIN} \text{SIG3}) / SIG3
   VMVE3 = (\text{THTTE} * (1.0 + SIG3)) ** 1.76
   UTUE3(1) = 1.5 * UTUES1
   IF \( \text{AIB} = 1.0 \) \( \text{UTUE3}(1) = 3.0 * \text{UTUES1} \)
   PX3 = 0.5 * (1.0 - UTUE3(1)) ** (1.0 / 8K) * \text{ALOG} \text{REDEL3(M)} * \|\text{ABS} \text{UTUE3}(1)\|
1*FSIG3/VWVE3)+C))
J=1
GO TO 43
41 J=J+1
IF(J.EQ.80) GO TO 50
UTUE3(J)=UTUE3(J-1)-01
PX3=.5*(1.-UTUE3(J)*((1./BK)*ALOG(REDEL3(M))*ABS(UTUE3(J))
1*FSIG3/VWVE3)+C))
43 CALL PRFL(UTUE3(J),PX3,EME3,P3P1,1,YY)
CALL FLUX(101,YY,1,EME3,DUM)
IF(SHORT.EQ.1.) GO TO 440
WRITE(6,1019) J,UTUE3(J),E(J)
ILINE=ILINE+3
CALL PRIN(ILINE,2)
IF(ILINE-2)=430,430,440
430 WRITE(6,1018)
ILINE=ILINE+2
440 TEST=ABS(E(J))
IF(TEST.LE.0.0001) GO TO 60
IF(E(J).LT.0.) GO TO 45
IF(J.JT.9) GO TO 41
45 SLOPE=(E(J-1)-E(J))/(UTUE3(J-1)-UTUE3(J))
UTUE3(J+1)=UTUE3(J)-E(J)/SLOPE
47 J=J+1
PX3=.5*(1.-UTUE3(J)*((1./BK)*ALOG(REDEL3(M))*ABS(UTUE3(J))
1*FSIG3/VWVE3)+C))
IF(J.EQ.80) GO TO 50
GO TO 43
50 WRITE(6,1020)
IZ=1
GO TO 80

SOLUTION OBTAINED (APPROXIMATE)

Determine downstream boundary layer profile properties

60 CALL PRFL(UTUE3(J),PX3,EME3,P3P1,2,YY)
IF(SHORT.EQ.0.) CALL PRIN(ILINE,3)
II=3
WRITE(6,1014) II
WRITE(6,1033) UTUE3(J), REDEL3(M), PX3
ILINE=ILINE+6
IF(SHORT.EQ.1.) GO TO 7000
WRITE(6,1028)
DO.70 I=1,101
WRITE(6,1015) I,YY(I),EM(I),URAT(I),PTRNS(I),TTRAT(I)
ILINE=ILINE+1
CALL PRIN(ILINE,2)
IF(ILINE-2)=61,61,70
61 WRITE(6,1014) II
WRITE(6,1028)
ILINE=ILINE+4
70 CONTINUE
C DETERMINE DOWNSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES
C
7000 CALL FLUX(101,YY,1,EME3,DUM)
FUTURE3=UTUE3(J)*FSIG3
CFW3=2.*FUTURE3*FUTURE3/(TWTTE*(1.+SIG3))
FM1=1.+SIG1
FM3=1.+SIG3
HI3=(SHFAC-SIG3)/(TWTTE*FM3)
RETH3=REDEL3(M)*DSD
TARTC=2.*TWTTE+2.*PR13+(.5+.22*PR13)/(1.+SIG3)
CFLT3=.246*EXP(1.561*HI3)*RETH3**(.267)
1(TARTC*(1.+SIG3)**.647963)
WRITE(6,016) DSD,DSDO,BLMN,BLON,SHFAC,H13,CFW3,CFLT3
D13=A1*(1.-DSD)
D31=1.-D13
D31=DSD*D31/DSD1
DOS31=DSD*D31/DSD1
IF(ALP10.EQ.0.) GO TO 79
C
C INTERACTION LENGTH
C
ELD1=1.-D31/TAN(ALP1)
IF(ALB.EQ.0.) ELD1=1.-D31/YS/TAN(ALP1)
79 IF(ALP10.EQ.0.) ELD1=0.
WRITE(6,1021) D31,US31,DDSD1,ELD1
C
C TWO-DIMENSIONAL EXACT SOLUTION (ONLY FOR 2-D CASES)
C
IF(M.GT.1.) GO TO 80
IF(AX1.EQ.2.) GO TO 80
IF(D31.LT.0.) WRITE (6,1034)
IF(D31.LT.0.) GO TO 1
CALL PRIN(ILINE,3)
WRITE(6,1029)
IF(SHORT.EQ.1.) GO TO 500
WRITE (6,1030)
WRITE (6,1031) M,REDEL3(M)
ILINE=ILINE+4
500 M=M+1
IF(M.EQ.11.) GO TO 530
REDEL3(M)=REDEL1*D31*P3P1*(EME3/EME1)*(FM3/FM1)**1.26
FUTURE3=UTUE3(J)*FSIG3
CFW3=2.*FUTURE3*FUTURE3/(TWTTE*(1.+SIG3))
TAUTP1=0.7*CFW3*EME3*EME3*P3P1
TAUTP1=SIGCOEF*(TAUTP1+TAUTP1)
IF(SHORT.EQ.1.) GO TO 650
WRITE (6,1031) M,REDEL3(M)
ILINE=ILINE+2
CALL PRIN(ILINE,2)
WRITE(6,1032)
650 IF(ILINE.EQ.2) WRITE(6,1032)
ILINE=ILINE+2
600 WRITE (6,1030)
ILINE=ILINE+2
TTEST=ABS(REDEL3(H-1)-REDEL3(M))/REDEL3(H-1)
IF(TIFEST.LE.0.01) GO TO 60
UTUE3(1)=1.5*UTUE1
IF(ALB.EQ.1.) UTUE3(1)=3.*UTUE1
PX3=.5*(1.-UTUE3(1)*(1./BK)+ALOG(REDEL3(H)*ABS(UTUE3(1)))
J=1
GO TO 503
501 J=J+1
IF(J.EQ.80) GO TO 50
UTUE3(J)=UTUE3(J-1)+.01
PX3=.5*F1(1.UTUE3(J)*(1.+BK)*ALOG(RED13*(M)*ABS(UTUE3(J))
*FSIG3/VWVE3)+C))
503 CALL PRL(UTUE3(J),PX3,EME3,P3P1,YY)
CALL Flux(101,YY,1,EME3,DUM)
H6=P2P1-I.-GAMA*G1*H5-GLMOMF+TAUP1/TAN(ALP1)
IF(TAUP.EQ.0.) WRITE(6,1022) H1,H2,H3,H4,H5,H6,TAUP1
A2=(P2P1-P3P1)/H6
A3=-GAMA*P3P1*EME3*EME3/H6
C A4 IS THE SHEAR TERM IN THE MOMENTUM EQUATION
C
A4=TAUP1/TAN(ALP1)/H6
IF(TAUP.EQ.0.) WRITE(6,1017) A1,A2,A3,A4
TEST=ABS(E(J))
IF(TEST.LE.0. GO001) GO TO 520
IF(J.LT.3) GO TO 501
505 SLOPE=(E(J-1)-E(J))/(UTUE3(J-1)-UTUE3(J))
UTUE3(J+1)=UTUE3(J)-E(J)/SLOPE
J=J+1
PX3=.5*F1(1.-UTUE3(J)*((1.+BK)*ALOG(RED13*(M)*ABS(UTUE3(J))
*FSIG3/VWVE3)+C))
510 J=J+1
IF(J.EQ.80) GO TO 50
GO TO 503
520 D13=A1*(1.-DSD)
D31=1./D13
GO TO 500
530 WRITE(6,1032)
IZ=1
GO TO 80
80 IF(ALP1.LT.2.) GO TO 1
IF(ALP1.GT.0.) CALL AXIS(ELD1,UTUES1,UTUE3(J),
REDE1,PIX1,ALP1,EME1,IZ)
IF(ALP1.EQ.0.) CALL ANOS(UTUES1,UTUE3(J),REDE1,PIX1,
EME1,IZ)
GO TO 1
1000 FORMAT(7F10.6)
1010 FORMAT(15H0, 9X,21HOPTION 1 INPUT THETA1/10X,9HUTUES1 = ,F10.6,
15X,24HREYNOLDS NO. (DELTA1) = ,E14.6,5X,6HPX1 = ,F10.6/5X,
25X,9HETETA1 = ,F10.6,8H DEGREES,5X,5HM1 = ,F10.6/
310X,18HRECOVERY FACTOR = ,F10.6,5X,9HTW/TTE = ,F10.6)
1011 FORMAT(15H0, 9X,21HOPTION 2 INPUT ALPHA1/10X,9HUTUES1 = ,F10.6,
15X,24HREYNOLDS NO. (DELTA1) = ,E14.6,5X,6HPX1 = ,F10.6/5X,
25X,9HALPHA1 = ,F10.6,8H DEGREES,5X,5HM1 = ,F10.6/
310X,18HRECOVERY FACTOR = ,F10.6,5X,9HTW/TTE = ,F10.6)
1012 FORMAT(15H0/10X,30HBOUNDARY LAYER EDGE CONDITIONS//
110X,8HP2/P1 = ,F10.6,5X,8HP3/P2 = ,F10.6,5X,8HP3/P1 = ,F10.6/
219X,5HM1 = ,F10.6,8X,5HM2 = ,F10.6,8X,5HM3 = ,F10.6)
1013 FORMAT(15H0/10X,32HSHOCK AND FLOW DEFLECTION ANGLES//10X,
115HINCIDENT SHOCK,5X,8HALPHA = ,F10.6,8H DEGREES,5X,
28HTHETA = ,F10.6,8H DEGREES/10X,15HREFLECTED SHOCK,5X,8HALPHA = ,
3F10.6,8H DEGREES,5X,8HTHETA = ,F10.6,8H DEGREES)
1014 FORMAT(1H4, 9X, 35HBOUNDARY LAYER PROFILE DATA STATION, I3)
1015 FORMAT(1H4, 6X, I3,6F14.6)
1016 FORMAT(1H4/10X,14HDELSTAR/DEL = ,F14.6,4X,10HMOM/DEL = ,
1F14.6/10X,25HNON-DIMENSIONAL MASS FLUX = ,F10.6/
210X,32HNON-DIMENSIONAL MOMENTUM FLUX = ,F10.6/10X,
31HSHAPE FACTOR = ,F10.6,5X,31HINCOMPRESSIBLE SHAPE FACTOR = ,
4 F10.6/10X,16HCF(WALL-WAKE) = F10.6,5X,22HCF(LUDWEBIG TILLMAN) = ,
5F10.6)
1017 FORMAT(1H4, 10X, 5HHA1 = ,F10.6,5X,5HHA2 = ,F10.6,5X,5HHA3 = ,F10.6,
1 5X,5HHA4 = ,F10.6)
1018 FORMAT(1H4, 10X,32HRESULTS OF ITERATIONS FOR UTUE*3/)
1019 FORMAT(1H4, 6X, 25HNO CONVERGENCE FOR UTUE*3)
1020 FORMAT(1H4, 10X,12HDEL3/DDEL1 = ,F10.6/10X,
12HDELSTAR3/DELSTAR1 = ,F10.6/10X,12HMOM3/MOM1 = ,F10.6/10X,
29H/DEL1 = ,F10.6)
1022 FORMAT(1H4, 10X,25HNO CONVERGENCE FOR REDEL3)
1024 FORMAT(1H4, 10X,54H TWO-DIMENSIONAL APPROXIMATE SOLUTION (REDEL3 = R)
1EDELI))
1025 FORMAT(1H4, 9X,8H(M)BLEED, I1,11H/(M)B.L. = ,F10.6,
120H (BLEED POSITIVE IN))
1026 FORMAT(1H4, 9X,40H MOMENTUM FLUX ASSOCIATED WITH B.L. BLEED, I1,
13H = ,F14.6,14H (POSITIVE IN))
1027 FORMAT(1H4, 10X,5HWS = ,F10.6,5X,5HYS = ,F10.6, 5X,7HPHIS = ,F10.6)
1028 FORMAT(1H4, 9X,1HY, 4X,1HM, 12X,4HWE/UE, 9X,6HPT/PT1, 6X,1GHT(NS)/PT1,
2 6X,6HT/TTL/1/
1029 FORMAT(1H4, 7X,30H TWO-DIMENSIONAL EXACT SOLUTION)
1030 FORMAT(1H4,10X,31HRESULTS OF ITERATION FOR REDEL3)
1031 FORMAT(1H4,10X,4HMD = ,I3,5X,22HREYNOLDS NO.(DELTA) = ,E14.6)
1032 FORMAT(1H4,6X,25HNO CONVERGENCE FOR REDEL3)
1033 FORMAT(1H4,10X,9HUTUE*3 = ,F10.6,5X,9HREDEL3 = ,E14.6,5X,
16HXP3 = ,F10.6)
1034 FORMAT(1H4,6X,28HDELS/DELI NEG. (NO SOLUTION))
END

SUBROUTINE AXIS(ELD1,UTUE1,UTUE3,REDEL1,PX1,ALP1,EME1,IZ)
C AXIALLY SYMMETRIC FLOW ANALYSIS
DIMENSION TRAT(200), PRTAT(200), PTRANS(200), UMAT(200), EM(200)
DIMENSION E1(100), E2(100), EDEL(100), UTUE1(100), YTUE3A(100), YY(200)
DIMENSION XXO(50), RHO(50), EME25(50), PE2P2(50)
DIMENSION PHI(50), ALPH(50), RR(50), DELS30(50)
DIMENSION E12(50), ELD(50), XR(50), WR(200), PHIR(200)
COMMON TRAT, PRTAT, PTRANS, UMAT, 1
1 EM, WR, PHIR, 1
2 SHORT, BLEED1, BLEED2, BLEED, 1
3 AIB, TWTI, GAMMA, T110, 1
4 BLMN, BLMON, BLMI, BLMOI, 1
5 DDS, DDSD, BK, EME1, SHFAC, 1
6 SIG1, SIGMA1, SIG1, FSIG1, WAVE1, 1
7 CFWM1, C, STCOEF
COMMON /HUW,AINPT,TH1ZZ

READ DATA CARD 6

READ(5,1000) DEL1R,AIE2,FLDIR3
CALL PRFL(UTUES1,PX1,HEME1,1,0,2,YY)
CALL FLUX(101,YY,2,HEME1,DEL1R)
SN1=(1-OSD)/DDSD
SN2=SN1-3.0
SN3=ALOG(SN2)
SN4=-0.6169*SN3
SN5=EXP(SN4)
FEN=0.0299*SN5
FENT=FEN*ELD1/BLMN
FETM=FENT*BLMI/BLMOI
R00=1.-DEL1R
R00=ROO+ROO-2.*BLMI*FENT
R01=SQR(R00)
IE2=AIE2
ALP10=ALP1/0.1745329
CALL PRIN(IINE,3)
WRITE(6,1011)
GO TO (10,20,30),IE2

BOUNDARY LAYER EDGE STREAMLINE DATA ARE INPUT

READ DATA CARD 7

10 READ(5,1000) AKK
   KK=AKK
   DO 11 I=1,KK

READ DATA CARDS 8

11 CONTINUE
   WRITE(6,1025)
   ILINE=ILINE+5
   DO 12 I=1,KK
   WRITE(6,1025) I,XXO(I),RRO(I),HEME2S(I),PE2P1(I),ALPHA(I)
   ILINE=ILINE+1
   CALL PRIN(ILINE,2)
   IF(ILINE=2) 13,13,12
   13 WRITE(6,1025)
   ILINE=ILINE+5
   12 CONTINUE
   GO TO 25

PROGRAM CALCULATES CONICAL FLOW STREAMLINE FROM INPUT DATA FROM
CONICAL FLOW PROGRAM
ARRAYS IN ORDER SHOCK TO CONE

20 READ(5,1000) AKK
   KK=AKK
   DO 21 I=1,KK

PHI IS IN DEGREES
C ALPHA IS IN DEGREES
C
READ(5,1000) PHI(I), ALPHA(I), EME2S(I), PE2P1(I)
21 CONTINUE
IF(SHORT.EQ.0.) WRITE(6,1027)
ILINE=ILINE+5
DO 822 I=1,KK
PHI(I)=PHI(I)*.01745329
ALPHA(I)=ALPHA(I)*.01745329
C PHI IS IN RADIANS
C ALPHA IS IN RADIANS
C
IF(SHORT.EQ.1.) GO TO 822
WRITE(6,1028) I,PHI(I),ALPHA(I),EME2S(I),PE2P1(I)
ILINE=ILINE+1
CALL PRIN(ILINE,2)
IF(ILINE.EQ.1) 821,821,822
821 WRITE(6,1027)
ILINE=ILINE+5
822 CONTINUE
C CONICAL FLOW STREAMLINE CALCULATION
C
TERM1=0.
TERM2=0.
XX0(I)=1.
RRO(I)=1.
TP1=TAN(PHI(I))
DO 22 K=E,KK
PHIBAR=(PHI(K)+PHI(K))/2.
ALPBAR=(ALPHA(K)+ALPHA(K))/2.
TPB=TAN(PHIBAR)
TAB=TAN(ALPBAR)
TERM1=TERM1+TPB*TPB*(PHI(K)-PHI(K-1))/(TAB-TPB)
XX0(K)=EXP(TERM1)
TERM2=TERM2+TAB*(XX0(K)-XX0(K-1))
RRO(K)=1.+TERM2/TP1
IF(XX0(K).GT.20.) GO TO 922
22 CONTINUE
922 IF(SHORT.EQ.0.) WRITE(6,1025)
ILINE=ILINE+5
DO 24 I=1,KK
ALPHA(I)=ALPHA(I)/.01745329
C ALPHA IS IN DEGREES
C
IF(SHORT.EQ.1.) GO TO 24
WRITE(6,1026) I, XX0(I), RRO(I), EME2S(I), PE2P1(I),ALPHA(I)
ILINE=ILINE+1
CALL PRIN(ILINE,2)
IF(ILINE.EQ.1) 23,23,24
23 WRITE(6,1025)
ILINE=ILINE+5
24 CONTINUE
C CONVERSION OF STREAMLINE COORDINATES TO RS/R AND L/DEL1
C
25 ROR=I.-DELI
ROR=ROR1
TH11=TH10*0.01745329
IF(AINPT.EQ.2.) TH11=TH12*0.31745329
TP1=TAN(TH11)
XOR=1.*ROR/TP1
DO 26 K=1, KK
RR(K)=RRO(K)*ROR
XR(K)=XXO(K)*ROR/TP1
ELD2(K)=(XR(K)-XOR)/DELI
DEL30R(K)=1.-RR(K)
26 CONTINUE
IF(SHORT.EQ.1.) GO TO 2800
WRITE(6,1029)
ILINE=ILINE+5
DO 28 I=1, KK
WRITE(6,1033) I, XXO(I), RRC(I), RR(I), ELD2(I), DEL3OR(I), XR(I)
ILINE=ILINE+1
CALL PRIN(Iライン,2)
IF(ILINE-2) 27,27,28
27 WRITE(6,1029)
ILINE=ILINE+5
28 CONTINUE
C
CALCULATION OF FORCE ALONG BOUNDARY OF CONTROL VOLUME ADJOINING
C REGION 2
C
2800 EI2(I)=0.
DO 280 K=2, KK
RRBAR=(RR(K-1)+RR(K))/2.
PE2PIB=(PE2P1(K-1)+PE2P1(K))/2.
EI2(K)=EI2(K-1)-PE2PIB*RRBAR*(RR(K-1)-RR(K))
280 CONTINUE
IF(SHORT.EQ.1.) GO TO 3000
WRITE(6,1030)
ILINE=ILINE+5
DO 30 I=1, KK
WRITE(6,1031) I, XXO(I), RRC(I), RR(I), ELD2(I), DEL3OR(I), EME2S(I)
1,PE2P1(I), EI2(I), ALPHA(I)
ILINE=ILINE+1
CALL PRIN(Iライン,2)
IF(ILINE-2) 29,29,30
29 WRITE(6,1030)
ILINE=ILINE+5
30 CONTINUE
3000 GO TO 37
C
C CONSTANT PRESSURE BOUNDARY
C FIND SHOCK WAVE ANGLE, PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM
C MACH NO.
C
35 CALL SHOCK1(ALP1,TH1,EME1,EME2,P2P1)
ALP10=ALP1/.01745329
ALP2=ALP1
ALP2D=ALP10
C
C FIND SHOCK WAVE ANGLE, PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM MACH NO.

CALL SHOCK(1, ALP2, TH2, EME2, EME3, P3P2)
TH2 = TH2 / 0.1745329
P3P2 = P3P2 * P2P1
WRITE (6, 1034)
WRITE (6, 1012) P2P1, P3P2, P3P1, EME1, EME2, EME3
ILINE = ILINE + 6
WRITE (6, 1013) ALP10, TH10, ALP20, TH20
ILINE = ILINE + 6

C DETERMINE UPSTREAM BOUNDARY LAYER PROFILE PROPERTIES

37 CALL PRFL(U1UES1, PX1, EME1, 1.0, 2, YY)
IF (SHORT.EQ.0.) CALL PRIN (ILINE, 3)
II = 1
WRITE (6, 1014) II
ILINE = ILINE + 6
IF (SHORT.EQ.1.) GO TO 4000
WRITE (6, 1039)
DO 40 I = 1, 101
WRITE (6, 1015) I, YY(I), EME(I), U0R8(I), PPRR5(I), TTRR5(I)
ILINE = ILINE + 1
CALL PRIN (ILINE, 2)
IF (ILINE - 2) 38, 38, 40
38 WRITE (6, 1014) II
ILINE = ILINE + 6
WRITE (6, 1039)
40 CONTINUE

C DETERMINE UPSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES

4000 CALL FLUX(101, YY, 2, EME1, DEL1R)
HH1 = (SHFAC - SIG1) / (TWTTE * (1. + SIG1))
RETH1 = REL1 * DODS
PRANDL = .72
PR13 = PRANL ** (1. / 3.)
TBART0 = 5. * TWTTE + 22 * PR13 + (5. - .22 * PR13) / (1. + SIG1)
CFLT1 = .246 * EXP(-1.561 * HI) * (RETH1 ** (-.268)) / 1((TBART0 * (1. + SIG1)) ** 0.7963)
WRITE (6, 1016) DSD, DDDS, BLMN, BLMON, SHFAC, HI1, CFMW1, CFLT1
ILINE = ILINE + 6
DSD1 = DSD
DSD1 = DODS
WRITE (6, 1017) DEL1R, BLMI, BLM01
ILINE = ILINE + 3
I = 1
U1UES3A(I) = UCUE3
FM1 = BLMI
FM01 = BLM01
FM01 = FM01 * (1. + FMT)
FM1 = FM1 * (1. + FMT)
J = 1
EL01A(J) = EL01
R3 = BLEED * 2. * FM1
L = 1

67
WRITE(6,1036) L,BLEED1
L=2
WRITE(6,1036) L,BLEED2
WRITE(6,1035) BLEED,R3
IF(I2.EQ.1) RETURN
IF(ELO1.LT.0.) RETURN
YRS=0.
IF(AIB.EQ.1.) GO TO 500
IF(BLEED1.EQ.0.) GO TO 500
WSA=A5(BLEED1)*2.*FM1
CALL INTRP(HR,YY,WSA,YYS,101)
CALL INTRP(YY,PHIR,YYS,PHISA,101)
YRS=YYS*DEL1R
WRITE(6,1037) WSA,YRS,PHISA
R3=1.
IF(AIB.EQ.3.) R3=1.-YRS
C
C AIB=1 POROUS PLATE SUCTION
C AIB=2 SLOT SUCTION
C AIB=3 SCOOP SUCTION
C
500 IF(AIB.LE.1.) BLHOM1=0.
IF(AIB.LE.3.) R3=1.
IF(AIB.GE.2.) BLHOM1=-PHISA
KB=1
WRITE(6,1038) KB,BLHOM1
IF(SHORT.EQ.1.) GO TO 50
CALL PRIN(ILINE,3)
WRITE(6,1018)
ILINE=ILINE+2
50 GO TO (51,51,51,55), IIE2
C
C ITERATION PROCEDURE FOR SOLUTION OF CONTINUITY EQUATION
C
51 CALL INTRP(EL02,EME2S,ELD1A(J),EME2,KB)
CALL INTRP(ELD2,P2P1,ELD1A(J),P2P1,KB)
CALL INTRP(ELD2,ALPHA,ELD1A(J),ALP2D,KB)
ALP2=ALP2D+.01745329
C
ALP2=ALP2-FLDIR3*.01745329
ALP2D=ALP2/.01745329
C
DETERMINE SHOCK WAVE ANGLE, PRESSURE RATIO ACROSS SHOCK AND
C DOWNSTREAM MACH NO.
C
CALL SHOCK(1,ALP2,TH2,EME2,EME3,P3P2)
P3P1=P3P2*P2P1
TH2D=TH2/.01745329
IF(SHORT.EQ.1.) GO TO 55
WRITE(6,1012) P2P1,P3P2,P3P1,EME1,EME2,EME3
ILINE=ILINE+6
WRITE(6,1013) P3P1,TH1D,ALP2D,TH2D
ILINE=ILINE+6
55 R1=2.
GAM1=4*GAMMA-1.)*2.
R1A=1.*GAE1*EHE3*EHE3
R1B=1.*GAE1*EHE1*EHE1
R2=-2.*P3P1*EHE3*SQRt(R1A/R1B)/EHE1
IF(AIB.EQ.3.) R2=R2*R3R*R3R
R3=BLEED+2.*F1
IF(IE2.EQ.3) D3P=1.*ELD1A(J)*TAN(ALP1)
IF(IE2.EQ.3 AND AIB.EQ.3) D3P=1.*YRS=ELD1A(J)*TAN(ALP1)
IF(IE2.LT.3) CALL INTRP(ELD2,DEL3R,ELD1A(J),DEL3RP,KK)
IF(IE2.LT.3 AND AIB.EQ.3) DEL3R=DEL3RP=YRS
IF(IE2.LT.3) D3P=DEL3RP/CHEL1R
REDEL3=REDEL1*P3P1*(EHE3/EHE1)*(R1A/R1B)**1.26
SIG3=GAE1*EHE3*EHE3
SIGMA3=SIG3/R1A
SIGS3=SQRT(SIGMA3)
FSIG3=(ASIN(SIGMA3))/SIGS3
VWVE3=(TWT*E1A)***1.76
IF(REDEL3.LE.0.) RETURN
PX3=-S(S1-UTUE3A(I))*((1./BK)*ALOG(REDEL3.*ABS(UTUE3A(I))
1*FSIG3/VWVE3+C))
60 CALL PRFL(UTUE3A(I),PX3,EHE3,P3P1,1,YY)
GO TO (51,61,62,61,IE2)
61 CALL INTRP(ELD2,E12,ELD1A(J),E12PR,KK)
CALL INTRP(ELD2,DEL3R,ELD1A(J),DEL3R,KK)
IF(SHORT.EQ.1.) GO TO 63
WRITE(6,1032) ELD1A(J),E12PR,DEL3R
ILINE=ILINE+1
CALL PRINT(IINE,2)
IF(IINE-2)610,610,620
610 WRITE(6,1018)
ILINE=ILINE+2
620 GO TO 63
62 D31=1.-ELD1A(J)*TAN(ALP1)
DEL3R=D31*DEL1R
63 CON=DEL3R
D3PR3=(DEL3R-YRS)/R3R
IF(AIB.EQ.3.) DEL3R=D3PR3
IF(SHORT.EQ.0.) WRITE(6,1040) R3R,D3PR3
ILINE=ILINE+2
CALL FLUX(101,YY,2,EHE3,DEL3R)
DEL3R=CON
FM3=BLHI
FM3=BLMO
E1(J)=R1*FM1+R2*FM3+R3
IF(SHORT.EQ.1.) GO TO 65
WRITE(6,1019) R1,R2,R3,FM1,FM01,FM3,FM03
WRITE(6,1020) I,UTUE3A(I),E2(I),J,ELD1A(J),E1(J)
ILINE=ILINE+6
CALL PRINT(IINE,2)
IF(IINE-2)64,64,65
64 WRITE(6,1018)
ILINE=ILINE+2
65 TEST=ABS(E1(J))
IF(TEST.LE.0.00001) GO TO 80
IF(J.GE.2) GO TO 70
J=2
ELD1A(J)=1.*ELD1A(I)
GO TO 50
STOP CASE NO CONVERGENCE ON J

WRITE(6,1021)
RETURN

STOP CASE L/D/E 1 LESS THAN 0.

WRITE(6,1042) J,ELD1A(J),I
RETURN

ITERATION PROCEDURE FOR SOLUTION OF MOMENTUM EQUATION

S1=1.
S2=-1.
S3=2.
S4=2.*GAMMA*EME1*EME1
S5=2.*GAMMA*EME3*EME3*P3P1*(.-1.)
S6=GAMMA*EME1*EME1*BLMOM1
S7=-1.
TAU1P1=0.7*CFHW1*EME1*EME1
FUTUE3=UTUE3A(I)*FSIG3
CFWH3=2.*FUTUE3*FLTUE3*(TWTUE*(1.*SIG3))
TAU3P1=0.7*CFHW3*EME3*EME3*P3P1
TAU1P1=STCCEF*(TAU1P1+TAU3P1)

S8 IS THE SHEAR TERM IN THE MOMENTUM EQUATION
S8=2.*TAU1P1*DEL1R*ELD1A(J)
R1C=1.*GAMMA*EME2*EME2

S9 IS THE ENTRAINMENT TERM IN THE MOMENTUM EQUATION
S9=2.*BLEED2*FM1*COS(ALP1)*EME1*EME2*GAMMA*SQRTR(R1B/R1C)
T1=2.*DEL1R-DEL1R*DEL1R
T2=2.*DEL3R-DEL3R*DEL3R
IF (IE2.EQ.3.) T1=2.*DEL1R+DEL1R*DEL1R
IF (IE2.EQ.3.) T2=2.*DEL3R+DEL3R*DEL3R
T3=P2P1*(T1-T2)/2.
T4=0.
IF(AIB.EQ.3.) T2=(2.*O3PR3-O3PR3*O3PR3)*R3R*R3R
IF(AIB.EQ.1.) T4=2.*YRS-YRS*YRS
IF(AIB.EQ.3.) S5=S5*R3R*R3R
IF(E2.EQ.4.) T3=E12PR
E21I=SL*T1+SL*T2+P3P1*S3*T3+S4*FM01+S5*FM03+S6*S7*T4-S8+S9
IF(SHORT.EQ.1.) GO TO 82
WRITE(6,1022) S4,S5,S8,T1,T2,T3,T4
WRITE(6,1020) I,UTUE3A(I),E2(I),J,ELD1A(J),E1(J)
ILINE=ILINE+6
CALL PRINT(ILINE,2)
IF (ILINE-2) 81, 81, 82
81 WRITE(6,1018)
ILINE=ILINE+2
82 TEST =ABS(E2(I))
IF (TEST .LE. 0.00001) GO TO 110
IF (I .GE. 2) GO TO 90
I=2
UTUE3A(I)=0.9*UTUE3
IF (EME1.LT.2.5 .AND. ALP10.LT.3.5) UTUE3A(I)=1.1*UTUE3
GO TO 100
90 SLOPE2=(E2(I-1)-E2(I))/(UTUE3A(I)-UTUE3A(I))
UTUE3A(I+1)=UTUE3A(I)-E2(I)/SLOPE2
I=I+1
IF (I.EQ.51) WRITE(6,1023)
IF (I.EQ.51) RETURN
100 ELDA1A(I)=ELDA1A(J)
J=1
GO TO 50

C SOLUTION TO CONTINUITY AND MOMENTUM EQUATIONS HAS BEEN DETERMINED
C
110 UTUE3=UTUE3A(I)
ELDA1=ELDA1A(J)

C DETERMINE DOWNSTREAM BOUNDARY LAYER PROFILE PROPERTIES
C
CALL FLFL(UTUE3,PX3,EME3,F3P1,2,YY)
IF (SHORT.EQ.0.) CALL PRINT(ILINE,3)
IF (SHORT.EQ.0.) WRITE(6,1018)
IF (ALP10.EQ.0.) GO TO 1100
WRITE(6,1012) P2P1,P3P2,P3P1,EME1,EME2,EME3
WRITE(6,1013) ALP10,TH10,ALP20,TH20
1100 IF (SHORT.EQ.0.) CALL PRINT(ILINE,3)
II=3
WRITE(6,1014) II
WRITE(6,1043) UTUE3,REDEL3,PX3
ILINE=ILINE+6
IF (SHORT.EQ.1.) GO TO 1200
WRITE(6,1039)
DO 120 I=1,101
WRITE(6,1015) I,YY(I),EM(I),URAT(I),PTRAT(I),PTRNS(I),TTRAT(I)
ILINE=ILINE+1
CALL PRINT(ILINE,2)
IF (ILINE-2) 111, 111, 120
111 WRITE(6,1014) II
ILINE=ILINE+4
WRITE(6,1039)
120 CONTINUE

C DETERMINE DOWNSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES
C
1200 IF (AIB.EQ.3.) DEL3R=D3P3
CALL FLUX(110,YY,P3,EME3,DEL3R)
FUTUE3=UTUE3*FSIG3
CFW3=2.*FUTUE3*FUTUE3/(TWTE*(1.+SIG3))
HI3=(SHFAC-SIG3)/(TWTE*(1.+SIG3))
RETH3=REDEL3+DOSD
TBARTO=2.5*TWTE+22*PR13*(5.22*PR13)/(l.*SIG3)
CFL3=.246*EXP(-1.561*HI3)*(RETH3**(-.268))/
1*(TBARTO**(1.+SIG3)**.7963)
WRITE(6,1016) DSD,DOSD,BLMN,BLMON,SHFAC,H13,CFWM3,CFL3
IF(A1B.EQ.3.) DEL3R=D3PR3*R3R
WRITE(6,1117) DEL3R,BLMI,BLM0I
ILINE=ILINE+3
D31=DEL3R/DEL1R
D31=D30*D31/DSD1
D31=0.5*D30*D31/DSD1
WRITE(6,1024) D31,D31,D30,DEL1F
IF(A1B.EQ.3.) WRITE(6,1041) YRS,D3PR3,R3R
RETURN
1000 FORMAT(7F10.6)
1011 FORMAT(1H6/10X,3SHSOLUTION FOR AXIALLY SYMMETRIC FLOW)
1012 FORMAT(1H6/10X,30HBOUNDARY LAYER EDGE CONDITIONS//
110X,8HP2/P1 = ,F10.6,5X,8HP3/P2 = ,F10.6,5X,8HP3/P1 = ,F10.6/
210X,5HM1 = ,F10.6,8X,5HM3 = ,F10.6,8X,5HM3 = ,F10.6)
1013 FORMAT(1H6/10X,32HSHOCK AND FLOW DEFORMATION ANGLES//10X,
115HINCIDENT SHOCK 5X,8HALPHA = ,F10.6,8H DEGREES/10X,15HREFLECTED SHOCK 5X,8HALPHA = ,
3F10.6,8H DEGREES/10X,8HTHETA = ,F10.6,8H DEGREES)
1014 FORMAT(1H6,9X,35HBOUNDARY LAYER PROFILE DATA STATION,13)
1015 FORMAT(1H6,5X,13,6F14.6)
1016 FORMAT(1H6/10X,14HDELSTAR/DEL = ,F14.6,4X,10HMON/DEL = ,
1F14.6/10X,29HNON-DIMENSIONAL MASS FLUX = ,F10.6/
219X,32HNON-DIMENSIONAL MOMENTUM FLUX = ,F10.6/10X,
315HSHAPE FACTOR = ,F10.6,5X,30HINCOMPRESSIBLE SHAPE FACTOR = ,
4 F10.6/10X,16HCF(WALL-WAKE) = ,F10.6,5X,22HCF(LUWEIG TILLMAN) = ,
5F10.6)
1017 FORMAT(1H6,9X,8HDEL/R = ,F10.6/10X,21HBL/L. MASS INTEGRAL = ,
1F10.6/10X,25HBL/L. MOMENTUM INTEGRAL = ,F10.6/
1018 FORMAT(1H6,9X,42HRESULTS OF ITERATION FOR UTUE+3 AND L/DEL1)
1019 FORMAT(1H6,9X,5HRI = ,F10.6,6X,5HR2 = ,F10.6,5X,5HR3 = ,F10.6/
110X,6HM1 = ,F10.6,5X,7HFM01 = ,F10.6/10X,6HM3 = ,F10.6,5X,
27HFM03 = ,F10.6)
1020 FORMAT(1H6,9X,4HI = ,I3,3X,9HUTUE*3 = ,F10.6,5X,5HE2 = ,F10.6/
110X,3HJ = ,I4,5X,7H/01 = ,F10.6,5X,5HE1 = ,F10.6)
1021 FORMAT(1H6,19HCONVERGENCE ON J)
1022 FORMAT(1H6,9X,5H5S4 = ,F10.6,5X,5H5S5 = ,F10.6,5X,5H5S8 = ,F10.6/
110X,5H5T = ,F10.6,5X,5H5T2 = ,F10.6,5X,5H5T3 = ,F10.6,5X,5H5T4 = ,
2 F10.6)
1023 FORMAT(1H6,6X,19HCONVERGENCE ON I)
1024 FORMAT(1H6/10X,12HDEL3/DEL1 = ,F10.6/10X,
129HDELSTAR3/DELSTAR1 = ,F10.6/10X,12HMON3/MOM1 = ,F10.6//10X,
29HDL/DEL1 = ,F10.6)
1025 FORMAT(1H6/10X,35HBOUNDARY LAYER EDGE STREAMLINE DATA//
1 5X,1HK, 8X,4HX/X0,10X,4HR/RO,11X,3HM2E,10X,5HPE2P1,
2 9X,5HALPHA//)
1026 FORMAT(1H6,6X,13,5F14.6)
1027 FORMAT(1H6+10X,34HCONICAL FLOW STREAMLINE INPUT DATA//
1 9X,1HK,9X,3PHHI,10X,5HALPHA, 9X,3HM2E, 8X,5HPE2P1//)
1028 FORMAT(1H6,6X,13,5F14.6)
1029 FORMAT(1H6/10X,25HCONVERTED STREAMLINE DATA//
1 9X,1HK,8X,4HX/X0,10X,4HR/RO,10X,4HRS/R, 9X,6HL/DEL1, 8X,6HDEL3/R,
2,10X,3HX/R//)}
1030 FORMAT(1HO/10X,31HSUMMARY OF EDGE STREAMLINE DATA/
  1 9X,1H,K,4X,4H/X/0,10X,4HR/R, 8X,4HRS/R, 7X,
  26HL/DEL1, 7X,6HDEL3/R, 8X,3HME2, 7X,5HPE2P1, 7X,
  35H(1E2, 7X,5HALPHA/)
1031 FORMAT(1H, 6X,13,9F12.6)
1032 FORMAT(1H, 10X,9HL/DEL1 = ,F14.6,5X,8H(I)E2 = ,F14.6,5X,
  18HDEL3R = ,F14.6)
1033 FORMAT(1H, 6X,13,6F14.6)
1034 FORMAT(1HO/10X,38HCONSTANT PRESSURE BOUNDARY IN REGION 2)
1035 FORMAT(1H, 9X,19H/M)BLEED/(M)B.L. = ,F10.6/
  110X,19H/(M)BLEED/(M)DUCT = ,F10.6)
1036 FORMAT(1H, 9X,8H(M) BLEED,(I1,11H/(M) B.L. = ,F10.6,
  120H (BLEED POSITIVE IN))
1037 FORMAT(1HO/10X,6HWSA = ,F10.6,5X,6HRS = ,F10.6,5X,8HPHISA = ,
  1F10.6)
1038 FORMAT(1HO, 9X,4HMOMENTUM FLUX ASSOCIATED WITH B.L. BLEED,(I1,
  13H = ,F14.6,14H (POSITIVE IN))
1039 FORMAT(1HO, 1 9X,1H,K,8X,1H,Y,14X,1HM,12X,4HU/UE, 9X,6HPT/PT1, 6X,10HPT(NS)/PT1,
  2 6X,6HT/TT1/)
1040 FORMAT(1HO, 10X,7HR3/R = ,F10.6,5X,12HDEL3PR/R3 = ,F10.6)
1041 FORMAT(1HO, 8X,26HSCOOP CASE,SCOOP HEIGHT/R = ,F10.6,5X,
  112HDEL3PR/R3 = ,F10.6,5X,7HR3/R = ,F10.6)
1042 FORMAT(1HO/ 6X,17HSTOP CASE L0DEL1,13,4H) = ,F10.6,5X,4HI = ,I3)
1043 FORMAT(1HO, 10X,9HUTUE*3 = ,F10.6,5X,9HREDEL3 = ,E14.6,5X,
  16HPX3 = ,F10.6)
END

SUBROUTINE ANOS(UTUES1,UTUE3,REDEL1,PX1,03120,EME,I2)
C AXIALLY SYMMETRIC FLOW ANALYSIS BLEED WITH NO SHOCK WAVE
DIMENSION TTRAT(200),PTRAT(200),PTRNS(200),URAT(200),EM(200)
DIMENSION E11100), E2(100),DEL3OR(100), UTUE3A(100), YY(200)
DIMENSION WR(200), PHIR(200)
COMMON TTRAT , PTRAT , PTRNS , URAT 
1 EM , WR , PHIR 
2 SHORT , BLEED1 , BLEED2 , BLEED 
3 AIB , TWITE , GAMMA , TH10 
4 BLMN , BLMON , BLMI , BLMOI 
5 DSD , DDSO , BK , EME1 , SHFAC 
6 SIG1 , SIGMA1 , SIGS1 , FSIG1 , WMVE1 
7 CFWM1 , C , STCOEF 
C READ DATA CARD 6
C READ(5,1000) DEL1R
CALL PRIN(ILINE,3)
WRITE(6,1011)
C DETERMINE UPSTREAM BOUNDARY LAYER PROFILE PROPERTIES
C 37 CALL PRFL(UTUES1,PX1,EME1,1.0,2,YY)
C PARAMETERS FCR NO SHOCK
C EME3=EME1

73
EME2=EME1
P2P1=1.
P3P2=1.
P3P1=1.
II=1
WRITE(6,1014) II
ILINE=ILINE+4
IF(SHORT.EQ.1.) GO TO 4000
WRITE(6,1039)
DO 40 I=1,101
WRITE(6,1015) I,YY(I),EM(I),URAT(I),PTRAT(I),PTNS(I),TTRAT(I)
ILINE=ILINE+1
CALL FRLN(ILINE,2)
ILINE=ILINE+3,38,40
38 WRITE(6,1014) II
ILINE=ILINE+4
WRITE(6,1039)
40 CONTINUE

C DETERMINE UPSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES
C
4000 CALL FLUX(101,YY,2,EME1,DEL1R)
HII=(SHFAC-SIG1)/(TWLTE*(1.+SIG1))
RETHI=REDEL1*DDSD
PRANDL=72
PR13=PRANDL**((1./3.)
TBARTO=.5*TWLTE+.22*PR13+(-.22*PR13)/(1.*SIG1)
CFLT1=.246*EXP(-1.561*HI1)**(RETHI**(-1.268))/
 discTBARTO(1.+SIG1))**0.7963)
WRITE(6,1016) DSD,DDSD,BLMN,BLMON,SHFAC,HII,CFWW1,CFLT1
ILINE =ILINE+6
DSD1=DSD
DDSD1=DDSD
WRITE(6,1017) DEL1R,BLMI,BLMOI
ILINE=ILINE+3
I=1
UTUE3A(I)=UTUE3
FM1=BLMI
FM01=BLMOI
J=1
DEL30R(J)=0312D*DEL1R
R3=BLEED*2.*FM1
L=1
WRITE(6,1036) L,BLEED1
L=2
WRITE(6,1036) L,BLEED2
WRITE(6,1039) BLEED,R3
IF(II.EQ.1.) RETURN
YRS=0.
IF(A1B.EQ.1.) GO TO 500
IF(BLEED1.EQ.0.) GO TO 500
WSA=AES(BLEED1)*2.*FM1
CALL INTRP(WR,YY,WSA,YYS,101)
CALL INTRP(YY,PHIR,YYS,PHISA,101)
YRS=YYS*DEL1R
WRITE(6,1037) WSA,YRS,PHISA
R3R=1.
IF(AIB.EQ.3.) R3R=1.-YRS

C
C AIB=1 POROUS PLATE SUCTION
C AIB=2 SLOT SUCTION
C AIB=3 SCOOP SUCTION
C
500 IF(AIB.LE.1.) BLHOM1=0.
   IF(AIB.GE.2.) BLHOM1=-PHISA
   KB=1
   WRITE(6,1038) KB,BLHOM1
   IF(SHORT.EQ.1.) GO TO 50
   CALL PRIN(ILINE,3)
   WRITE(6,1018)
   ILINE=ILINE+2
C
C ITERATION PROCEDURE FOR SOLUTION OF CONTINUITY EQUATION
C
50 R1=2.
   GAM1=(GAMMA-1.)/2. 
   R2=-2.
   IF(AIB.EQ.3.) R2=R2*R3R*R3R
   P3=BLEEO*Z.'FHI
   60 DEL3RP=DEL3R(J)
   IF(AIB.EQ.3.) DEL3RP=DEL3RP-YRS
   D3RP=DEL3RP/DEL1R
   REDEL3=RED1L*D31P
   SIG3=GAM1*EME3*EME3
   SIGMA3=SIG3/(1.+SIG3)
   SIGS3=SQRT(SIGMA3)
   FSIG3=(ASIN(SIGS3))/SIGS3
   VWVE3=(TWTE*SIG3)**1.76
   PX3=.5*(1.-UTUE3A(I)*((1./BK)*ALOG(REDEL3*ABS(UTUE3A(I))
                  1*FSIG3/VWVE3)+C))
   CALL PRFL(UTUE3A(I),PX3,EME3,P3P1,1,YY)
   DEL3R=DEL3R(J)
   CON=DEL3R
   D3PR3=(DEL3R-YRS)/R3R
   IF(AIB.EQ.3.) DEL3R=D3PR3
   IF(SHORT.EQ.0.) WRITE(6,1040) R3R,D3PR3
   ILINE=ILINE+2
   CALL FLUX(101,YY,2,EME3,DEL3R)
   DEL3R=CON
   FM3=BLMI
   FM03=BLHOM1
   E1(J)=R1*FM1+R2*FM3+R3
   IF(SHORT.EQ.1.) GO TO 65
   WRITE(6,1019) R1,P2,R3,FM1,FM01,FM3,FM03
   WRITE(6,1020) I,UTUE3A(I),E2(I),J,DEL3R(J),E1(J)
   ILINE=ILINE+6
   CALL PRIN(ILINE,2)
   IF(ILINE-2) 64,64,65
64 WRITE(6,1018)
   ILINE=ILINE+2
65 TEST=ABS(E1(J))
   IF(TEST.LE.0.00001) GO TO 80
   IF(J.GE.2) GO TO 70
   J=2

75
DEL3OR(J) = 0.9*DEL3OR(I)
GO TO 60
70 SLOPE1 = (E1(J-1) - E1(J)) / (DEL3OR(J-1) - DEL3OR(J))
DEL3OR(J+1) = DEL3OR(J) - E1(J) / SLOPE1
J = J + 1
IF(J .EQ. 51) GO TO 71
GO TO 60
C
C STOP CASE NO CONVERGENCE ON J
C
71 WRITE (6, 1021)
RETURN
C
C ITERATION PROCEDURE FOR SOLUTION OF MOMENTUM EQUATION
C
60 S1 = 0.
   S2 = 0.
   S3 = 0.
   S4 = 2.
   S5 = -2.
   S6 = BLMOMI
   S7 = 0.
   T1 = 0.
   T2 = 0.
   T3 = 0.
   T4 = 0.
IF(AIB .EQ. 3.) S5 = S5*R3*R3R
E2(I) = S1*T1 + S2*T2 + S3*T3 + S4*T4 + S5*FM01 + S6*FM03 + S7*T4
IF(SHORT .EQ. 1.) GO TO 82
WRITE (6, 1022) S4, S5, T1, T2, T3, T4
WRITE (6, 1020) I, UTUE3A(I), E2(I), J, DEL3OR(J), E1(J)
ILINE = ILINE + 6
CALL PRIN (ILINE, 2)
IF (ILINE - 2) 81, 81, 82
61 WRITE (6, 1018)
ILINE = ILINE + 2
82 TEST = ABS(E2(I))
IF (TEST .LE. 0.00001) GO TO 110
IF (I.GE. 2) GO TO 90
I = 2
UTUE3A(I) = 0.9*UTUE3
GO TO 100
90 SLOPE2 = (E2(I-1) - E2(I)) / (UTUE3A(I-1) - UTUE3A(I))
UTUE3A(I+1) = UTUE3A(I) - E2(I) / SLOPE2
I = I + 1
IF(I .EQ. 51) WRITE (6, 1023)
IF(I .EQ. 51) RETURN
100 DEL3OR(I) = DEL3OR(J)
J = 1
GO TO 60
C
C SOLUTION TO CONTINUITY AND MOMENTUM EQUATIONS HAS BEEN DETERMINED
C
C
110 UTUE3 = UTUE3A(I)
C
C DETERMINE DOWNSTREAM BOUNDARY LAYER PROFILE PROPERTIES
C

76
CALL PRFL(UTUE3, PX3, EME3, P3P1, 2, YY)
IF(SHORT.EQ.0.) CALL PRIN(ILINE, 3)
WRITE(6,1018)
1100 II = 3
WRITE(6,1014) II
WRITE(6,1013) UTUE3, REDEL3, PX3
ILINE = ILINE + 6
IF(SHORT.EQ.1.) GO TO 1200
WRITE(6,1039)
DO 120 I = 1, 101
WRITE(6,1015) I, YY(I), EM(I), URAT(I), PTRAT(I), PTRNS(I), TTRAT(I)
ILINE = ILINE + 6
CALL PRIN(ILINE, 2)
IF(ILINE-2) 111, 111, 120
111 WRITE(6,1014) II
ILINE = ILINE + 4
WRITE(6,1039)
120 CONTINUE

C DETERMINE DOWNSTREAM BOUNDARY LAYER INTEGRAL PROPERTIES

1200 IF(AIB.EQ.3.) DEL3R = D3PR3
CALL FLUX(101, YY, 2, EME3, DEL3R)
FUTURE3 = UTUE3 * FSIG3
CFHW3 = 2 * FUTURE3 * FUTURE3 / (TWTTE * (1 + SIG3))
HI3 = (SHFAC - SIG3) / (TWTTE * (1 + SIG3))
RETH3 = REDEL3 * DDSO
TBARTO = 5 * TWTTE * 22 * PR13 * (5 - 22 * PR13) / (1 + SIG3)
CFLT3 = 0.246 * EXP(-1.561 + HI3) * (RETH3 ** (-0.269))
WRITE(6,1017) DELZ, BLMN, BLM01, HIS, CFLT3
IF(AIB.EQ.3.) WRITE(6,1041) YRS, D3PR3, R3R
RETURN

1000 FORMAT(7F10.6)
1011 FORMAT(1HO/10X, 3HF10.6/10X, 1HO/10X, 3HF10.6/10X, 1HO/10X, 3HF10.6/10X)
1016 FORMAT(1HO, 9X, 3H10.6/10X, 1HO, 9X, 3H10.6/10X, 1HO, 9X, 3H10.6/10X)
1017 FORMAT(1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X)
1018 FORMAT(1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X)
1019 FORMAT(1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X)
1020 FORMAT(1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X, 1H, 9X, 3H10.6/10X)
SUBROUTINE SHOCK(ISH, ALP, TH, EMX, EMY, PRAT)

C SUBROUTINE TO CALCULATE CHANGES IN PROPERTIES ACROSS AN OBLIQUE SHOCK WAVE

DIMENSION TTRAT(200), PTRAT(200), PTRNS(200), UMAT(200), EMX(200), EMY(200)

COMMON TTRAT , PTRAT , PTRNS , UMAT ,
  EM , WR , PHIR ,
  SHORT , BLEED1 , BLEED2 , BLEED ,
  AIB , TWTE , GAMMA , THID ,
  BLMN , BLMON , BLMI , BLMOI ,
  OSD , DOSD , BK , EME1 , SHFAC ,
  SIG1 , SIGMA1 , SIGS1 , FSIG1 , VWVE1 ,
  C , STCOEF

G1=EMX*EMX
G2=G1*G1

IF(ISH=1) 10,10,20

C DETERMINE SHOCK WAVE ANGLE

10 G3=(G1**2.1)/G1
G4=(G2**1.1)/G2
G5=(GAMMA+1.)*(GAMMA+1.)/4.*((GAMMA-1.)/G1
SOS=SIN(ALP)**2.
AC(1)=(SOS-1.)/G2
AC(2)=G4+G5*SOS
AC(3)=-G3-GAMMA*SOS
AC(4)=1.

C FIND ROOTS OF CUBIC EQUATION IN (SIN(THETA))**2

END
CALL CUBIC(AC,Z)  
ROOT=SQRT(Z)  
IF(ROOT-1.) 12,12,11  
C  
ERROR EXIT IF SIN(THETA) GREATER THAN 2.0  
11 WRITE(6,1010) ROOT  
TH=ASIN(ROOT)  
RETURN  
12 TH=ASIN(ROOT)  
C  
DETERMINE PRESSURE RATIO ACROSS SHOCK AND DOWNSTREAM MACH NO.  
20 GAM1=Gamma+1.  
GAM2=Gamma-1.  
GAM3=GAM1*GAM1  
Z=SIN(TH)**2.  
PRAT=(GAMMA*G1+Z-GAM2)/GAM1  
ANUM=GAM3*G2*Z-4.*(G1-Z-1.)*(GAMMA*G1*Z+1.)  
DENO=(GAMMA*G1+Z-GAM2)*(GAM2*G1*Z+2.)  
EMY=SQRT(ANUM/DENO)  
RETURN  
1010 F0RMAT(1HG,10X,10HSIN(TH) = .F10.6)  
END  
SUBROUTINE FLUX(K,Y,INDC,EME,DELRR)  
C  
SUBROUTINE TO CALCULATE MASS AND MOMENTUM FLUX OF B.L.  
C  
ALSO CALCULATES DISPLACEMENT AND MHDENTUH THICKNESSES  
C  
INDC=1 TWO-DIMENSIONAL FLOW  
C  
=2 AXIALLY SYMMETRIC FLOW ON COWL  
C  
=3 AXIALLY SYMMETRIC FLOW ON CENTERBODY  
DIMENSION TRAT(200),PRTAT(200),PTRN(200),URAT(200),EM(200)  
DIMENSION Y(200),YY(200),BLHR(200),BLMOR(200),BLMIR(200)  
DIMENSION EMOR(200),YR(200),WR(200),PHIR(200)  
COMMON TRAT,PRTAT,PTRN,URAT,  
1 EM,WR,PHIR,  
2 SHORT,BLEED1,BLEED2,BLEED,  
3 A1B,TWIE,GAMMA,T10,  
4 BLMN,BLMON,BLMI,BLMOI,  
5 DSD,ODSD,BK,EME1,SHFAC,  
6 SIG1.SIGMA1.SIGS1.FSIG1,VMVE1,  
7 CFMW1,G,STCOEF  
DO 100 I = 1,K  
PRAT=1.  
TTO=1.+(GAMMA-1.)*EM/I*EM/I/2.  
TOTE=1.+(GAMMA-1.)*EM*EM/T2.  
TOTE=TOTE*TTRAT/I/TTOT  
RHRAT=PRAT/TOTE  
GO TO (10,20,30),INDC  
C  
TWO-DIMENSIONAL FLOW  
10 BLMR(I)=RHRAT*URAT(I)  
BLMOR(I)=BLMR(I)*URAT(I)  
IF(URAT(I).LE.0) BLMOR(I)=-BLMOR(I)
YY(I) = Y(I)
GO TO 100

C AXIALLY SYMMETRIC FLOW ON COWL

20 BLMR(I) = RHAT * URAT(I)
BLMOR(I) = BLMR(I) * URAT(I)
IF(URAT(I), LE, 0.) BLMOR(I) = -BLMOR(I)
YY(I) = Y(I)
YR(I) = YY(I) * DELR
BLNIR(I) = BLMR(I) * (1. - YR(I))
BMOIR(I) = BLMOR(I) * (1. - YR(I))
GO TO 100

C AXIALLY SYMMETRIC FLOW ON CENTERBODY

30 GO TO 100
100 CONTINUE
GO TO (110, 120, 130), INDIC

C TWO-DIMENSIONAL FLOW

110 DO 115 I = 1, K
CALL INTEG(I, YY, BLMR, AREA1)
CALL INTEG(I, YY, BLMOR, AREA2)
WR(I) = AREA1
PHIR(I) = AREA2
115 CONTINUE
BLMN = AREA1
BLMON = AREA2
DSD = 1. * BLMN
DSDS = BLMN - BLMON
SHFAC = DSD / DSDS
RETURN

C AXIALLY SYMMETRIC FLOW ON COWL

120 DO 125 I = 1, K
CALL INTEG(I, YR, BLMIR, AREA5)
CALL INTEG(I, YR, BMOIR, AREA6)
WR(I) = AREA5 * 2.
PHIR(I) = AREA6 * 2.
125 CONTINUE
BLMI = AREA5
BLMOI = AREA6
BLMN = 2. * BLMI / (2. * DELR - DELR * DELR)
BLMON = 2. * BLMOI / (2. * DELR - DELR * DELR)
DSD = (1. - SQRT(1. - 2. * DELR * DELR + 2. * BLMI)) / DELR
DSDS = (1. - SQRT(1. - 2. * BLMI + 2. * BLMOI)) / DELR
SHFAC = DSD / DSDS
130 RETURN
END

SUBROUTINE PRFL(UWEST, PX, EME, PKP1, IOPT, YY)
C SUBROUTINE TO CALCULATE DISTRIBUTIONS OF PROPERTIES
C FOR BOUNDARY LAYER WITH WALL-WAKE VELOCITY PROFILE

DIMENSION TTTRAT(200), PTRA T(200), PTRNS(200), URAT(200), EM(200)

DIMENSION YY(200), WR(200), PHIR(200)

COMMON TTTRAT, PTRAT, PTRNS, URAT,
1 EM, WR, PHIR,
2 SHORT, BLEED1, BLEED2, BLEED,
3 AIB, TWTTE, GAMMA, TH1D,
4 BLMN, BLMON, BLMI, BLMOI,
5 DSD, DDSO, BK, EME1, SHFAC,
6 SIG1, SIGMA1, SIGS1, FSIG1, WWVE1,
7 CFHW1, C, STCOEF

COMMON /UWU/AIDD

PI=3.1415927

EXP2=1./GAMMA-1.

EXP3=1./GAMMA-1.

GAM1=(GAMMA-1.)/2.*

GAM2=GAMMA+1.

GAM3=GAMMA-1.

G1=EME1*

SIGMA=1/(1.+GAM1*G1)

SIGG1=SQRT(SIGMA)

SIGG2=1./SIGG1

SIGG3=ASIN(SIGG1)

URAT(1)=0.

TTTRAT(1)=TWTTE

EM(1)=0.

YY(1)=0.

PTRAT(1)=(1./(1.+GAM1*EME1*EME1))**EXP2*PKP1

PTRNS(1)=PTRAT(1)

DO 100 I=2,101

AI=I-1

YY(I)=AI/100.

YAA=SQRT(1.-YY(I))**AIDD)

ALG=ALOG(YY(I))+2.*(YAA-ALOG(1.+YAA))/AIDD

URAT(I)=SIGG2*SIN(SIGG3*SIGG3*PX*(1.+COS(PI*YY(I))))*

11./BK)**UTEST*SIGG3*ALG

TTTRAT(I)=TWTTE+(1.-TWTTE)*ABS(URAT(I))

U2=URAT(I)*URAT(I)

EM(I)=SQR T(U2/((1.+G1+GAM1)*TTTRAT(I)-GAM1*U2))

IF(URAT(I).LE.0.) EM(I)=-1.*EM(I)

IF(IOPT-1) 100,100,20

C C CALCULATION OF TOTAL PRESSURE DOWNSTREAM OF NORMAL SHOCK

C

20 PTRAT(I)=((1.+GAM1*EM(I)*EM(I))/(1.+GAM1*EME1*EME1))**EXP2*PKP1

IF(EM(I)-1.) 40,40,50

40 PRINTS(GAMMA)=PTRAT(I)

GO TO 100

50 PTRNS(I)=(GAM2*EM(I)*EM(I)/2.)/(1.+GAM1*EME1*EME1))**EXP2*(GAM2/

1(2.*GAMMA*EM(I)*EM(I)-GAM3))**EXP3*PKP1

100 CONTINUE

RETURN

END

SUBROUTINE INTRP(X,Y,XX,YY,N)
C LINEAR INTERPOLATION
DIMENSION X(200), Y(200)
DC 30 I=1,N
IF (XX-X(I)) 10,20,30
10 K=I-1
YY=Y(K)+(Y(I)-Y(K))*(XX-X(K))/(X(I)-X(K))
RETURN
20 YY=Y(I)
RETURN
30 CONTINUE
RETURN
END

SUBROUTINE CUBIC(C,Z)
C SUBROUTINE TO DETERMINE ROOTS OF A CUBIC EQUATION
C NASA, AMES RESEARCH CENTER, MOFFETT FIELD, CALIF.
DIMENSION C(4), Y(Z)
ACOS(X)=ATAN(SQRT(1.0-X**2)/X)
P=-C(3)**2/3.0 + C(2)
Q=2.0*C(3)**3/27.0 - C(2)*C(3)/3.0 + C(1)
RSQ = -0.5*Q/ SQRT(-P**3/27.0)
IF (ABS(RSQ) GT 1.0) RSQ=SIGN(1.0,RSQ)
PHI=ACOS(RSQ)
TEM=2.0*SQRT(-P/3.0)
X1= TEM*COS(PHI/3.0)
X2= TEM*COS(PHI/3.0 + 2.09439510)
X3= TEM*COS(PHI/3.0 + 4.18879020)
IF (X2-X3) 150, 150, 160
150 Y1=AMAX1(Y1, X1, X2)
Y1=AMIN1(Y1, X3)
GO TO 175
160 Y1=AMIN1(Y1, X2)
Y1=AMAX1(Y1, X3)
175 Y1=Y1-C(3)/3.0
8 Z=Y1
RETURN
END

SUBROUTINE INTEG(K,Y,Z,AREA)
C INTEGRATION USING SIMPSON#S RULE
DIMENSION Y(230), Z(200)
IF (K.EQ.5) GO TO 1
IF (K.EQ.1) GO TO 21
IF (K.EQ.2) GO TO 22
IF (K.EQ.3) GO TO 23
IF (K.EQ.4) GO TO 24
1 AK=K
BK=AK/2.
KK=BK
CK=KK
IF (BK-CK) 4,2,4
C C K IS EVEN
```plaintext
C
2 N=K-1
   GO TO 5
C
4 N=K
5   ODD=0.
    EVEN = 0.
    J=N-3
   DO 10 I=2,J,2
    EVEN = EVEN + Z(I)
    ODD = ODD + Z(I+1)
   10 CONTINUE
    AREA = (Y(2)-Y(1))/3.*(Z(1)+Z(N)+4.*(EVEN+Z(N-1)))+2.*ODD)
    IF(K=CK) 14,12,14
C
K IS EVEN
12   AREA = AREA + (Y(K)-Y(K-1))*(Z(K)+Z(K-1))/2.
    RETURN
C
K IS ODD
14   RETURN
21   AREA=0.
    RETURN
22   AREA=(Y(2)-Y(1))*(Z(2)+Z(1))/2.
    RETURN
23   AREA=(Y(2)-Y(1))*(Z(3) +4.*Z(2)+Z(1))/3.
    RETURN
24   AREA=(Y(2)-Y(1))*((Z(4)+Z(3))/2.+(Z(3)+4.*Z(2)+Z(1))/3.)
    RETURN
END

SUBROUTINE PRIN(ILINE,IND)
C SUBROUTINE TO COUNT LINES OF OUTPUT ON PAGE, CHANGE PAGE WHEN NECESSARY AND WRITE TITLE ON EACH PAGE
C DIMENSION TITLE(12),TITL1(12)
   GO TO (10,20,30),IND
C
FIRST PAGE TITLE
10   READ(5,1000)(TITLE(J),J=1,12)
   READ(5,1000)(TITL1(J),J=1,12)
   ILINE=2
   WRITE(6,1010)(TITLE(J),J=1,12)
   WRITE(6,1012)(TITL1(J),J=1,12)
   IPAGE=1
   WRITE(6,1011)IPAGE
   RETURN
C
TEST FOR END OF PAGE
20   IF(ILINE=48)21,22,22
```
21 RETURN

C CHANGES PAGE
C
22 ILINE=2
IPAGE=IPAGE+1
C
PAGE TITLE
C
WRITE(6,1011) (TITLE(J),J=1,12)
WRITE(6,1012) (TITL1(J),J=1,12)
WRITE(6,1013) IPAGE
RETURN
C
PROGRAMMED PAGE CHANGE
C
30 IPAGE=IPAGE+1
WRITE(6,1011) (TITLE(J),J=1,12)
WRITE(6,1012) (TITL1(J),J=1,12)
WRITE(6,1013) IPAGE
ILINE=2
RETURN
1000 FORMAT(12A6)
1010 FORMAT(1H1,30X,12A6)
1011 FORMAT(1H*,100X,5HPAGE,I3)
1012 FORMAT(1H*,30X,12A6)
END

TABLE 3-B: INPUT TO PROGRAM ANAL

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<th>10 DEGREE CONE NC ALPHA=1.</th>
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### TABLE 4: INPUT TO METHOD OF CHARACTERISTICS PROGRAM

**MACH 2.82 10 DEGREE CONE**

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TABLE 5-A: PROGRAM BLGRN

PROGRAM BLGRN(INPUT,TAPE 5=INPUT,OUTPUT,TAPE6=OUTPUT)
C INPUT FORMAT 10F7.0 EXCEPT CARD 1
C CARD(S)
C 1 TITLE; COLUMNS 1-72, HOLLERITH
C 2 COL.1-7 AJ AJ=0, FOR 2-D FLOW
C 8-14 TO TOTAL TEMPERATURE (R)
C 15-21 TW WALL TEMPERATURE (R)
C 22-28 REYNOLDS FREE STREAM REYNOLDS NO. PER FT /10000.
C 29-35 SIGMA PRANDTL NUMBER (= .72)
C 36-42 CF COEFFICIENT OF SKIN FRICTION
C 43-49 DEL BOUNDARY LAYER THICKNESS (IN.)
C 3 COL. 1-7 X0 VALUE OF X FOR STARTING POINT (MAY EQUAL TO ZERO)
C 8-14 DXMN MINIMUM PERMISSIBLE STEP SIZE (IN.)
C 15-21 DXHX MAXIMUM PERMISSIBLE STEP SIZE (IN.)
C 22-28 ANP FREQUENCY OF PRINTOUT AS MULTIPLE OF STEP SIZE
C 29-35 ANXP NUMBER OF POINTS AT WHICH EXTRA PRINT IS REQUIRED
C 4 COL.1-7 XEND VALUE OF X FOR ENDING POINT
C 8-14 ASE A PARAMETER IN (1.-(Y/DEL)**ASEA) (=1, IS RECOMMENDED)
C 5 COL.1-7 ANME NUMBER OF VALUES GIVEN IN MACH NUMBER ARRAY(4, AND 200.)
C 8-14 ANSM NUMBER OF SMOOTHING PASSES ON MACH NO. DATA
C 6 COL.1-70 AME VALUES OF MACH NUMBER ARE INPUT, MORE THAN ONE CARD CAN BE USED IF NECESSARY
C 7 COL.1-70 X CENTERLINE DISTANCE(IN.) AT WHICH MACH NUMBERS ARE INPUT
C 8 COL.1-70 ANR NUMBER OF VALUES GIVEN IN RADIUS ARRAY (AT LEAST 4
C 8-14 ANSR NUMBER OF SMOOTHING PASSES ON INPUT DATA
C 9 COL.1-70 R VALUES OF R ARE INPUT. MORE THAN ONE CARD CAN BE USED IF NECESSARY
C 10 COL.1-70 X CENTERLINE DISTANCE (IN) AT WHICH R VALUE ARE INPUT
COMMON/XCHGE/SIGNI,P,FF, TORR
COMMON/HOME/H1,H2
COMMON/ANS/THTA,DELSTIA,GASCT,BMU,GAM21
COMMON/ARRR/KR,NR,XIR(200),AIR(200),DIP(200),XCR(200)
COMMON/BBDD/AJ,AIME(200),AAR(200)
COMMON/CONST/R,SG,PI,GW,TOW,B,E91,E92,H,AME,DEME,PT3,REY3,TE,ITYR
COMMON/INTEG/CF,DEL,DCF,DDL,DX,DXMN,DXMX,X,XEND,MEND
COMMON/RFIN/T0,TW,SIGMA,GAMMA,REYN,ASEA
COMMON/ARME/KME,NME,XIME(200),AIME(200),DIME(200),XME(200)
925 FORMAT(10F7.0)
904 FORMAT(1X,13F10.6)
903 FORMAT(/7X,1HX,7X,5HTHETA,5X,8HDISP,TH,3H H,9X,2HH2,8X,1HP,9X,1
1HF,7X,9HR*TH/1000,3X,2HME,8X,5HDELTA,5X,2HCF,6X,7HDDEL/DX,4X,6HDCF
2/DX/)
902 FORMAT(/17H INITIAL DELTA =,F9.6,7H INCHES,5X,13H INITIAL CF =,F
19.6)
901 FORMAT(* TO = *,F7.2/)
1  * TW = *F7.2/
2  * SIGMA = *F6.3/
3  * GAMMA = *F6.3/
4  * PSIA = *F7.3/
5  * REYNO = *F9.2/
6  * XO = *F7.2/
7  * DXMIN = *F7.4/
8  * DXMAX = *F7.4/
9  * XEND = *F7.2/
10  * NPRINT = *I3)

40 FORMAT(8A10)
41 FORMAT( //,8A10//)
49 FORMAT(1H1)
12344 FORMAT(16H PROGRAM RAN AT I2,1H.,I2,3H.,2A10///)

200 FORMAT(1H15X,100ITEM35 - A COMPUTER PROGRAM TO PREDICT THE DEVELOPMENT OF A COMPRESSIBLE TURBULENT BOUNDARY LAYER )

DIMENSION TITLE(8)
CALL CLOCK(IHR,MIN,ISEC)
CALL DATE
WRITE(6,200)
WRITE(6,12344)IHR,MIN,A7,B7
5 CONTINUE
C READ HEADING
READ(5,40)(TITLE(I),I=1,8)
IF(EOF95)6,7
6 CALL EXIT
7 CONTINUE
IRY=0
READ(59925)AJYTO,TWPREYNO,SIEMA,CFJCEL
TOLER=.0000005
GAMMA=1.4
REYNO=REYNO/12.
REYNO=REYNO*100000.
READ(5,925)XO,DXMN,DXMX,ANP,ANXP
NPRINT=ANP+0.1
C NXP IS NUMBER OF POINT AT WHICH EXTRA OUTPUT IS REQUIRED.
NXP=ANXP+0.1
READ(5,925)XEND,ASEA
NME = 0
WRITE(6,41)(TITLE(I),I=1,8)
C READ AND PRINT MACH NUMBER DISTRIBUTION
KME=NME+1
READ(5,925)ANME,ANSME
NME=KME+IFIX(ANME-0.9)
IF(ANME)230,225,229
225 NSME=ANSME+0.1
READ(5,925)AIME(I),I=KME,NME
READ (5,925)(XIME(I),I=KME,NME)
DO 3388 I=KME,NME
3388 AAME(I)=AIME(I)
C CHECK THAT X IS INCREASING
KP=KME+1
DO 228 I=KP,NME
IF (XIME(I)-XIME(I-1)) .226,226,228
226 WRITE (6,934) XIME(I-1),XIME(I)
IE=-1
228 CONTINUE
C READ AND PRINT CONTOUR
NR=0
READ (5,925) ANSR
NR=KR+IFIX(ANR-0.9)
IF(ANSR) 410,420,420
READ(5,925)AIR(I),I=KR,NR
DO 3399 I=KR,NR
3399 AAR(I)=AIR(I)
C K THAT X IS INCREASING
KP=KR+1
DO 422 I=KP,NR
IF (XIR(I)-XIR(I-1)) 421,421,422
IE=-1
421 WRITE (6,934) XIR(I-1),XIR(I)
CONTINUE
422 CONTINUE
C SMOOTH RADII AND PRINT SMOOTHED VALUES
CALL SMOOTH(NSR,KR,NR,XIR,AIR,DIR)
IF(KR.NE.1) GO TO 425
XCR(I)=XIR(I)
425 CONTINUE
C OBTAIN ME AS A FUNCTION OF ARC LENGTH
DO 440 I=KME,NMEXME(I)
CALL PI(KR,NR,XIR,XCR,XIME(I),XME(I))
440 CONTINUE
C CONSTANT
C SQRT(2.)=1.414214
R=53.35
G=32.174
S=0.5*GAMMA-0.5
PI=3.141592
GW=TW/TO
TOW=TO/TW
B=TOW-1.
E91 =1./0.915
E92=E91-1.
CV=4290.
T015=2.27*TO**1.5
BMU=T015*E-8/(TO+198.6)
GASQ3 = GAMMA/(2.*SW**CV**TO)
GASQT=SQRT(GASQ3)
GAM21=(GAMMA-2.)/(GAMMA-1.)
DX=DXMN
MEND=0
NPC=1
CALL DERIV
PT3=REYN0*12./(REY3*AME)
PTT=PT3/144.
RRRT = REYN0 * THTA / 1000.
WRITE(6, 901) TO, TW, SIGMA, GAMMA, PTT, REYN0, X0, DXMN, DXMX, XEND, NPRINT
WRITE(6, 902) DEL, CF
WRITE(6, 935) ANME, ANSME
935 FORMAT (* ONME = *, F7.2, * NSME = *, F7.2*)
WRITE(6, 936)
936 FORMAT (* ME VALUES*)
WRITE(6, 937) (AAME(I), I = KME, NME)
937 FORMAT (1X, 10F10.3)
WRITE(6, 938)
938 FORMAT (* CORRESPONDING X VALUES*)
WRITE(6, 939) (XIME(I), I = KME, NME)
934 FORMAT (* TWO SUCCESSIVE X FOUND WHICH ARE NOT INCREASING *, 1 F12.4)
C SMOOTH MACH NUMBER AND PRINT SMOOTH VALUE
230 WRITE (6, 939)
939 FORMAT (* SMOOTHED MACH NUMBERS*/
1 * X ME DME*)
WRITE (6, 940) (XIME(I), AIME(I), DIME(I)), I = KME, NME
940 FORMAT (1X, 3F10.3)
C
WRITE(6, 955) ANR, ANSR
955 FORMAT (* ONR = *, F7.2, * NSR = *, F7.2*)
WRITE(6, 956)
956 FORMAT (* R VALUES*)
WRITE (6, 937) (AAR(I), I = KR, NR)
WRITE(6, 938)
WRITE (6, 939) (XIR(I), I = KR, NR)
410 WRITE (6, 959)
959 FORMAT (* SMOOTHED RADII*/
1 * X R DR*)
WRITE (6, 940) (XIR(I), AIR(I), DIR(I)), I = KR, NR
WRITE(6, 903)
C
WRITE(6, 904) X, THTA, DELSTA, H, H2, P, FF, RRRT, AME, DEL, CF, DDL, DCF
IAY=2
42 CONTINUE
10 CALL MERSON
11 IF (MEND.EQ.1) GO TO 16
IF (NPC.EQ.NPRINT) GO TO 16
NPC = NPC+1
GO TO 10
C RESET COUNTER
16 NPC = 1
C
RRRT = REYN0 * THTA / 1000.
WRITE(6, 904) X, THTA, DELSTA, H, H2, P, FF, RRRT, AME, DEL, CF, DDL, DCF
IF (CF.GT.0.) AND. (CF.LT. TDLRER)) GO TO 43
IF (CF.LT.0.) AND. (DX.LT. CC0065)) GO TO 43
IF (MEND.EQ.0.) GO TO 10
GO TO 88
43 CF = CF - DXABS*2.*ABS(CF)/ABS(DCF)
X = X + DXABS
DEL = DEL + DDL*DXABS
TDLRER = ABS(CF)
CALL DERIV
SUBROUTINE MERSON
C MERSON INTEGRATION
C V IS THE VARIABLE ARRAY (BOTH INPUT AND OUTPUT), D IS THE DERIVATIVE
C ARRAY.
C DX IS CURRENT STEP LENGTH, DXMN AND DXMX MINIMUM AND MAXIMUM VALUES OF
C DX. X IS INDEPENDENT VARIABLE, AND XEND ITS UPPER LIMIT.
C
COMMON/CONST/R,S,G,PI,GW,TOW,BS91,ES92,H,AME,DME,PT3,TE,Y
COMMON /INTEG/V(Z),D(2),DX,DXMN,DXMX,X,XEND,MEND
DIMENSION U(Z),V(Z),DA(Z),DC(Z),DD(Z),DE(Z)
C BU, BL ARE UPPER AND LOWER BOUNDS ON TRUNCATION ERROR.
DATA BU, BL/0.0005, 0.00005/
C
7 MARK IS SET NON-ZERO TO PREVENT INCREASING THE STEP LENGTH IF EITHER
C XEND IS REACHED, OR THE STEP LENGTH HAS JUST BEEN DECREASED.
MARK=0
IF (X+DX-XEND) 5,7,7
C PREVENT OVERSHOOTING X END.
7 MARK=1
DX=XEND-X
MEND=1
C
SAVE POINT A.
5 DO 10 I=1,2
10 U(I)=V(I)
C
15 DX2=DX/2.
DX3=DX/3.
DX6=DX/6.
DX8=DX/8.
C FIND POINT B.
CALL DERIV
DO 20 I=1,2
DA(I)=D(I)
20 V(I)=U(I)+D(I)*DX
C FIND POINT C.
X=X+DX3
CALL DERIV
V(1)=(D(I)+DA(I))*DX6+U(I)
V(2)=D(2)+DA(2))*DX6+U(2)
C FIND POINT D.
CALL DERIV
X=X+DX6
40 DO I=1,2
DC(I)=D(I)
40 V(I)=U(I)+DX8*(DA(I)+3.*D(I))
C FIND POINT E.
CALL DERIV
X=X+DX\^2
DO 50 I=1,2
DD(I)=D(I)
V(I)=U(I)+DX\^2*(DA(I)-3.*DC(I)+4.*D(I))
50 VE(I)=V(I)
C FIND POINT F.
  CALL DERIV
  MINC=0
  MDEC=0
  DO 60 I=1,2
  V(I)=U(I)+DX\^2*(DA(I)+4.*DD(I)+D(I))
  IF (VE(I)) 51,60,51
C TRUNC IS A MEASURE OF THE TRUNCATION ERROR.
  51 TRUNC=ABS(1.-V(I)/VE(I))
  IF (TRUNC-BU) 54,54,52
  52 MDEC=MDEC+1
  54 IF (TRUNC-BL) 56,56,60
  56 MINC=MINC+1
  60 CONTINUE
  IF (MDEC.GT.0) GO TO 70
  IF (MINC.EQ.2.AND.MARK.EQ.0) GO TO 80
  65 RETURN
  70 HINC=1
  71 X=X-DX
C IF EITHER TRUNCATION ERROR IS ABOVE THE UPPER BOUND, DECREASE STEP LEN
  70 MARK=1
  IF (DX-DXMN) 65,65,71
  65 RETURN
C HALVE STEP LENGTH.
  DX=AMAX1(DX\^2,DXMN)
C IN CASE MEND HAS BEEN SET, RESET IT.
  MEND=0
  GO TO 15
C IF BOTH TRUNCATION ERRORS ARE BELOW THE LOWER BOUND, INCREASE STFP LEN
  80 IF (DX-DXMX) 81,65,65
  65 RETURN
C DOUBLE STEP LENGTH.
  DX=AMIN1(DX\^2,DXMX)
  GO TO 15
END

SUBROUTINE PI(K,N,X,V,X1,V1)
C INTERPOLATES BY POLYNOMIAL FITTING.
C GIVEN V(K),V(K+1),...,V(N) AT X(K),X(K+1),...,X(N), FITS A SECOND
C ORDER POLYNOMIAL TO THE THREE POINTS NEAREST X1 AND RETURNS A VALUE V1
C AT X1. FAILS IF ANY TWO OF THE X ARE EQUAL. IF N-K EQUALS ZERO OR ONE
C THE SUBROUTINE ALSO RETURNS A RESULT.
C
DIMENSION X(1),V(1)
C CHECK IF ONLY ONE OR TWO POINTS IN ARRAY.
  IF (N-K-1) 2,4,6
  C RETURN CONSTANT VALUE IF N=K
  2 V1=V(K)
  GO TO 100
C INTERPOLATE LINEARLY IF N=K+1
4 \text{vl} = v(k) + (v(n) - v(k)) / (x(n) - x(k)) \times (x_1 - x(k)) \\
\text{go to 100}

\text{c}

\text{lagrangian interpolation.}

\text{c}

\text{find nearest three points.}

6 \text{do 10 } j = k, n \\
15 \text{i} = j \\
10 \text{continue}

\text{i} = n \\
17 \text{if } (i. \text{gt. } k + 1) \text{ go to 20}

\text{c use first three points.}

\text{i} = k - 1

20 \text{if } (i. \text{ne. } n) \text{ go to 22}

\text{c use last three points.}

\text{i} = n - 3

25 \text{if } (i. \text{eq. } n) \text{ go to 29}

\text{c interpolate using lagranges formula.}

22 \text{if } (x(i - 2) + x(i + 1) - 2 \times x_1) \text{ go to 25}

23 \text{i} = i - 2

24 \text{go to 25}

\text{c subroutine smooth(k, l, m, x, y, dy)}

\text{input y(i) as a function of x(i) for i = l, l + 1, \ldots, m.}

\text{smooth in k passes.}

\text{output y(i) smoothed and the derivatives dy(i).}

\text{routine fails if any two x equal}

\text{dimension x(1), y(1), dy(1)}

\text{n is the number of points to be smoothed.}

n = m - l + 1

\text{do not smooth if less than four points.}

\text{if } (k. \text{eq. } 0 \text{ or } n. \text{lt. } 4) \text{ go to 50}

\text{c}

\text{smooth}

\text{c}

m3 = m - 3

40 \text{do } j = 1, k

\text{smooth successive sets of four points.}

40 \text{do } i = l, m3
A = (Y(I+2) - Y(I)) / (X(I+2) - X(I))
B = Y(I) - A * X(I)
C = (Y(I+3) - Y(I+1)) / (X(I+3) - X(I+1))
D = Y(I+1) - C * X(I+1)

C DO NOT SMOOTH IF THE FOUR POINTS ARE COLINEAR.
   IF (A - C) 10, 12, 10
   12 IF (B - D) 14, 40, 14

C XINT IS INTERSECTION OF LINES JOINING ALTERNATE POINTS
10 XINT = (B - D) / (C - A)
   IF (XINT.GE.X(I+1).AND.XINT.LE.X(I+2)) GO TO 40
   14 Y(I+1) = 0.5 * (Y(I+1) + A * X(I+1) + B)
   Y(I+2) = 0.5 * (Y(I+2) + C * X(I+2) + D)
   40 CONTINUE

C C CALCULATE DERIVATIVES
C 50 IF (N.EQ.1) GO TO 70
   IF (N.EQ.2) GO TO 80

C AT FIRST POINT.
D1 = X(I+1) - X(I)
D2 = X(I+2) - X(I)
S1 = (Y(I+1) - Y(I)) / D1
S2 = (Y(I+2) - Y(I)) / D2
DY(L) = (D2 * S1 - D1 * S2) / (D2 - D1)

C AT INTERMEDIATE POINTS.
L1 = L + 1
M1 = M - 1
60 DO I = L1, M1
   D1 = X(I) - X(I-1)
   D2 = X(I+1) - X(I)
   S1 = (Y(I) - Y(I-1)) / D1
   S2 = (Y(I+1) - Y(I)) / D2
   60 DY(I) = (D1 * S2 + D2 * S1) / (D1 + D2)

C AT LAST POINT.
D1 = X(M) - X(M-1)
D2 = X(M) - X(M-2)
S1 = (Y(M) - Y(M-1)) / D1
S2 = (Y(M) - Y(M-2)) / D2
DY(M) = (D2 * S1 - D1 * S2) / (D2 - D1)

66 RETURN

C C ONLY ONE POINT GIVEN.
70 DY(L) = 0.
   GO TO 66

C ONLY TWO POINTS GIVEN.
80 DY(L) = (Y(M) - Y(L)) / (X(M) - X(L))
   DY(M) = DY(L)
   GO TO 66
END

SUBROUTINE DERIV
COMMON/XCHGE/SIGN1,P,FF,TOSSR
COMMON/HONE/H1,H2
COMMON/ARRR/ARRK,ARR,ARRR/XIR(200),ARR(200),ARR(200),XCR(200)
COMMON/BBDD/AJ,AMM(200),ARY(200)
COMPonn/ARMe/KMe,NMe,xMe(200),AIME(200),DIME(200),xMc(200)
COMMON/ANS/THTA,DELS/8,GASOT,6MU,GAMA21
COMMON/REIN/TO,TW,SIGMA,GAMMA,REyNO,ASEA
COMMON/CONST/R,S,G,PI,GW,TOW,B,E91,E92,H,AME,DME,PT3,REy3,TE,IRy
COMMON/INTEG/CF,DEl,DCF,DLx,DX,DXMN,DXMX,XXEND,MEND
CALL PI(KMc,NMc,xMc,AIME,x,AME)
CALL PI(KR,NR,XCR,AIR,x,RR)
CALL PI(KR,NR,XCR,DIR,x,DR)
SIGN1=1.
IF(CF.GT.0.) SIGN1=1.
C51=5.1-0.614/(0.4*ASEA)
C52=5.1
G1K=0.
G2K=0.
HG1K=0.
HG2K=0.
G1=0.
G2=0.
HG1=0.
HG2=0.
DG1CF=0.
DG2CF=0.
DG1DL=0.
DG2DL=0.
DG1ME=0.
DG2ME=0.
HDG1CF=0.
HDG2CF=0.
HDG1DL=0.
HDG2DL=0.
HDG1ME=0.
HDG2ME=0.
AME=AME*AME
S1=1.*S*AME2
C1=GW*S1
C22=S1-S1*GW
C33=-S*AHE2
TE=TE/Sl
TOTT=(TO/TE)**GAM21
TE198=(TE+198.6)/(TO+198.6)
REy3=CASTOT*TOTT*TE198/BMU
IF(IRy.LT.1) GO TO 40
REyNO=PT3*AHE*PEy3/12*
CONTINUE
ASC=S*AME2*TOW/S1
ASSORT(ASC)
ASB=2.*ASQ-B
XYZ1=2.*S*AME
STWE=SORT(0.5*TW/TE)
B24=B*B+4.*ASQ
SQBA=SORT(B24)
SARC=ASIN(ASB/SQBA)
USTT=A/SARC
USTA=STWE*USTT
CF3=ABS(CF)
CF2=SGRT(CF3)
IF(CF3.LT.0.) SIGN=-1.
SCF=SIGN*CF2
R1=SCF*USTA
Q1=0.5*SQBA/ASQ
Q2=0.5*B/ASQ
TETW=TE/TW
TAT=REYN*STWE*TETW**1.76
TATD=TAT*CF2
TATDC=TATD*DEL
IF(CF3.LT.TORR) GO TO 83
R2=2.5*ALOG(TATDC)+C51
P=0.5-0.5*R1*R2
GO TO 84
83 CONTINUE
P=0.5
84 CONTINUE
UU=SQRT((GAMMA*R*TE)*AME**12.*SQRT(G)
SIG3=5*AME2*SIGMA**0.333333+1.
TR=TO*SIG3/S1
GAM1=(GAMMA-1.)*AME
C DIFFERENTIATE WITH RESPECT TO ME
DTEME=-T0*2.5*AME/(S1*S1)
DAME=SQRT(TOW*S/S1)*(1.-5*AME2/S1)
DASQME=2.*AME*DAME
DT17ME=1.76*(TETW)**0.76+DTEME/TW
DBAME=-0.5*B*DASQME/(ASQ*ASQ)
DSQBME=DASQME/(ASQ*SQBA)-0.5*SCBA*DASQME/(ASQ*ASQ)
DA2BME=(2.*ASQ-B)*Z.*DASQME/(ASQ*ASQ)*SQBA
TIN=1.-ASB*ASB/S24)
TINSQ=SQRT(TIN)
DARCME=DA2BME/TINSQ
DSQTM=-0.5*STWE*DTEME/TE
DAARM=(DA2BME/ASQ+SARCE/SARC
IF(CF3.LT.TORR) GO TO 85
DR1ME=SCF*(DSQTM*USTT+STWE*DAARM)
DR2ME=2.5*DT17ME/TETW**1.76+2.5*DSQTM/STWE
DPME=-0.5*R2*DR1ME-0.5*R1*DR2ME
DRICF=0.5*R1/CF3
DR2CF=1.25/CF3
DPCF=-0.5*DR1CF*R2-0.5*DR2CF*R1
DR2DL=2.5/DEL
DPDL=-0.5*2.5*R1/DEL
GO TO 86
85 CONTINUE
DPME=0.
DPCF=0.
DPDL=0.
86 CONTINUE
E605=EXP((10.-C52)/2.5)
DEL10=DEL/10.
Y1=E605/TATD
IF(Y1.GT.DEL10) Y1=DEL10
DY=(DEL-Y1)/50.
M=1
N=1
77 DO I=M,N
\[ Y = Y_1 + (5I - I)_Y^D Y \]
\[ YD = Y/Y_D \]
\[ PYD = PI/YD \]
\[ YDDC = YD**ASEA \]
\[ YD1 = 1-YDDC \]
\[ YD1 = YD2+1\]
\[ ALGY = ALOG(YD) + 2 \times (YD2 - ALOG(YD1))/ASEA \]
\[ UEST = 1 + 2.5 \times R1*ALGY \]
\[ UUE = Q1 * SIN(UEST * SACR) + Q2 \]
\[ UUE2 = UUE * UUE \]
\[ Z = C11 + C22 * UUE + C33 * UUE2 \]
\[ 
\]
\[ \text{CONTINUE} \]
HGD2CF=HGD2CF+DG2CF
HGD1DL=HGD1DL+DG1DLP
HGD2DL=HGD2DL+DG2DLP
HGD1ME=HGD1ME+DG1MEP
HGD2ME=HGD2ME+DG2MEP
IF(M.NE.1) GO TO 11
M=2
N=51
GO TO 77
11 CONTINUE
G1I=(G1-0.5*HG1)*DY+0.5*Y1*G1P
G2I=(G2-0.5*HG2)*DY+0.5*Y1*G2P
G1KI=(G1K-0.5*HG1K)*DY+0.5*Y1*G1KP
G2KI=(G2K-0.5*HG2K)*DY+0.5*Y1*G2KP
DELSTA=DEL-G1I
DG1CFI=(DG1CF-0.5*HGD1CF)*DY+0.5*Y1*DG1CFP
DG2CFI=(DG2CF-0.5*HGD2CF)*DY+0.5*Y1*DG2CFP
DG1DLI=(DG1DL-0.5*HGD1DL)*DY+0.5*Y1*DG1DLP
DG2DLI=(DG2DL-0.5*HGD2DL)*DY+0.5*Y1*DG2DLP
DG1MEI=(DG1ME-0.5*HGD1ME)*DY+0.5*Y1*DG1MEP
DG2MEI=(DG2ME-0.5*HGD2ME)*DY+0.5*Y1*DG2MEP
DTHCF=DG1CFI-DG2CFI
DTHDL=DG1DLI-DG2DLI
DTHME=DG1MEI-DG2MEI
DDSCF=-DG1CFI
DDSEM=-DG1MEI
DDSDL=-DG1DLI
H=DELSTA/THTA
A8=((H+1.-TR/TE)*TETY-1.)/1.12
IF(A8.LT.0.) WRITE(6,700) A8,H
700 FORMAT(5X,3HA8,=F10.6,12X,2HH,=F10.6)
IF(A8.LT.0.) WRITE(6,701) G1I,G2I,DEL,DELSTA,CF,DX
701 FORMAT(5X,4HG1I,=F10.6,5X,4HG2I,=F10.6,5X,4HDHL,=F10.6,5X,4HDME,=F10.6,5X,
17HDELSTA,=F10.6,3X,3HCF,=F10.6,3X,3HDX,=F10.6)
AA=A8**E91+2.*
A1=E91*A8**E92 /1.12
DTRDX=DME*GAM1*TO*(SIGMA**.33333-TR/TO)/S1
FRAC=(4.-2.*AA)/(4.-2.*AA+2.*AA)
H1=(1.-AA*AA)*(4.-2.*AA)/(4.-2.*AA)
H2=DEL/THTA-H
H2K=G1KI/G1KI-G2KI
IF(CF.GT.0.) GO TO 50
C
FF=(H2+4.2)*.0067
GO TO 51
50 CONTINUE
C
FF=.0306*(H2K-3.0)**-0.653
51 CONTINUE
DUX1=12.*SQRT(GAMMA*R)
TSE2=TO*AM2*(GAMMA-1.)/(2.*SQRT(TE)*S1*S1)
DUX=DUX1*(SQRT(TE)-TSE2)*DME
DUDX = THTA * DUDX / UU

DH2DX = (FF - H2 * CF / 2.) / THTA + H2 * (H + I) * DUDX / UU
DTHDX = CF / 2. - (H + 2. - AME2) * DUDX
IF (AJ .NE. 0.) DTHDX = DTHDX - THTA * DR / RI
DTEDX = -TG * 2. * S * AME * DME / (S1 * S1)
T15 = (DTRDX - DTEDX) * THTA / TE
DDSD1 = DDSDFL - 1.
DETERM = DTHCF * DDSDL - DTHDL * DDSDF
THME = DTHDX - DME * DTHME
DDSDC = -THTA * DH2DX - H2 * DTHDX
DSME = DDSDL - DME * DDSME
DCF = (DDSDL * THME - DTHDL * DSME) / DETERM
DDL = (DTHCF * DSME - DDSDF * THME) / DETERM
DDSDX = DDL + DDSDL

30 RETURN
END

---

TABLE 5-B: INPUT TO PROGRAM BLGRN

<table>
<thead>
<tr>
<th>2.82</th>
<th>10 + 13 TEETER</th>
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<tr>
<td>0.</td>
<td>536.6</td>
</tr>
<tr>
<td>2.65</td>
<td>0.001</td>
</tr>
<tr>
<td>3.</td>
<td>1.</td>
</tr>
<tr>
<td>7.</td>
<td>4.</td>
</tr>
<tr>
<td>2.4514</td>
<td>2.4277</td>
</tr>
<tr>
<td>2.65</td>
<td>2.708</td>
</tr>
<tr>
<td>7.</td>
<td>4.</td>
</tr>
<tr>
<td>2.65</td>
<td>2.708</td>
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**TABLE 6: INPUT TO METHOD OF CHARACTERISTICS PROGRAM**

10+13 FROM 2.4

<p>| | | | | | |</p>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 10+13 | 2.4
| 2.65  | 2 | 0 | 47 | 1 | 0 |
| 2.62  | 1.4| 24. | 25.31 | 4.1| 1.0 |
| 0.00061| 0.001| 0.0001 | 0.001 | 0.001 |
| 3.00  | 0.001| 0.001 | 0.001 | 5.0|
| 4.0   | 3.0 | 3.235 | 3.25 | 5.0|
| 0.342375 | 13.0 | -10.0 | -10.0 |
| 2.02  | 3.02 | 5.0 |
| 1.02  | 0.0 |
| 6.335 | 0.999 | 2.584 |
| 2     | 3   | 2  | 1  | 2 | 26 |
| 2.40  | 1.02 | 0.0 | 2.53637 | 0.99835 |
| 2.40  | 1.01 | 0.0 | 2.53637 | 0.99835 |
| 2.40  | 1.00 | 0.0 | 2.53637 | 0.99835 |
| 2.40  | 0.99 | 0.0 | 2.53637 | 0.99835 |
| 2.40  | 0.9797 | 1.0 | 2.53637 | 0.99835 |
| 2.40  | 0.97827 | 0.0 | 2.538211 | 0.99835 |
| 2.40  | 0.9617 | 2.4 | 2.66272 | 0.999 |
| 2.40  | 0.9516 | 3.0 | 2.6588 | 0.999 |
| 2.40  | 0.9416 | 3.631 | 2.655 | 0.999 |
| 2.40  | 0.9315 | 3.7305 | 2.6515 | 0.999 |
| 2.40  | 0.921477 | 3.8277 | 2.6482 | 0.999 |
| 2.40  | 0.91144 | 3.923 | 2.645 | 0.999 |
| 2.40  | 0.9014 | 4.0164 | 2.6418 | 0.999 |
| 2.40  | 0.881519 | 4.19945 | 2.6359 | 0.999 |
| 2.40  | 0.86174 | 4.3785 | 2.63035 | 0.999 |
| 2.40  | 0.84213 | 4.555 | 2.62511 | 0.999 |
| 2.40  | 0.8227 | 4.7292 | 2.620 | 0.999 |
| 2.40  | 0.80348 | 4.904 | 2.6154 | 0.999 |
| 2.40  | 0.78445 | 5.078366 | 2.6109 | 0.999 |
| 2.40  | 0.765465 | 5.25314 | 2.60662 | 0.999 |
| 2.40  | 0.7471 | 5.4289 | 2.6025 | 0.999 |
| 2.40  | 0.728777 | 5.6062 | 2.598 | 0.999 |
| 2.40  | 0.7107 | 5.7853 | 2.5948 | 0.999 |
| 2.40  | 0.69297 | 5.9648 | 2.5912 | 0.999 |
| 2.40  | 0.67553 | 6.15 | 2.58775 | 0.999 |
| 2.40  | 0.6584 | 6.3367 | 2.58447 | 0.999 |
| 2.40  | 0.44812 | 13.0 | 2.42914 | 0.998 |
| 2.40  | 0.46611 | 12.33 | 2.4298 | 0.998 |
| 2.40  | 0.50812 | 11.2254 | 2.43212 | 0.998 |
| 2.40  | 0.51993 | 10.9846 | 2.43446 | 0.998 |
| 2.40  | 0.581397 | 9.8785 | 2.447921 | 0.998 |
| 2.40  | 0.60356 | 9.5647 | 2.451 | 0.998 |
| 2.40  | 0.616255 | 9.44656 | 2.4505 | 0.998 |
| 2.40  | 0.524544 | 9.375146 | 2.4500 | 0.998 |
| 2.4012 | 0.63984 | 9.24714 | 2.4495 | 0.998 |
PROGRAM MFLX

PROGRAM FOR COMPUTATION OF MASS AND MOMENTUM FLUX

INPUT FORMAT 7F10.6 EXCEPT CARD 1

CARD(S) COLUMNS
1 TITLE, COLUMNS 1-72 HOLLERITH
2 1-100 TO TOTAL TEMPERATURE DEGREE R
11-20 CR GAS CONSTANT (=53.3)
21-30 AN NO. OF POINTS INPUT
31-40 CR AVERAGE INCREMENT (INCH)
41-50 RC RADIUS OF LOCAL CONE SURFACE (INCH)
51-60 AIN = 0. NO MORE JOB AFTER THIS INPUT
61-70 AIN = 1. MORE JOB AFTER THIS INPUT
7-70 CR RADIUS AN VALUES (INCH)
8-70 CR MACH NUMBER AN VALUES
9-70 CR STATIC PRESSURE AN VALUES (PSIA)

DIMENSION EH(200), R(200), ROU(200), ROUU(200)

DIMENSION TITLE(J), I READ(5,1000) (TITLE(J), J=1,12)
WRITE(6,2000) (TITLE(J), J=1,12)
READ(5,100) TO, CR, AN, DY, RC, AIN

WRITE(6,400) TO, RC

READ(5,100) (R(I), I=1,N)
READ(5,100) (EM(I), I=1,N)
READ(5,100) (P(I), I=1,N)
WRITE(6,300)
SUM=0.
BU=0.
BUU=0.

DO 2 I=1,N
AI=I
A=SQRT(TOT)
TCT=1.+0.2*EM(I)*EM(I)
A3=SQRT(TCT)
Rou(I)=49.*144.*P(I)*EM(I)*AB/(CR*A)
U=49.*EM(I)*A/AB
ROUU(I)=ROU(I)*U
IF(I.EQ.1) GO TO 20
RIV=0.5*(ROU(I-1)+ROU(I))
RIVU=0.5*(ROU(I-1)+ROUU(I))
ARE=3.1415*(R(I)*R(I)-R(I-1)*R(I-1))/144.
BU=RIV*ARE
BUU=RIVU*ARE

102
<table>
<thead>
<tr>
<th>X = 2.988</th>
<th>MASS CHECK</th>
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<tbody>
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<tr>
<td>0.615</td>
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<tr>
<td>1.770</td>
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</tr>
<tr>
<td>1.750</td>
<td>1.748</td>
</tr>
<tr>
<td>1.725</td>
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</tr>
<tr>
<td>2.150</td>
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<table>
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</table>
### TABLE 8: INPUT TO PROGRAM ANAL

<table>
<thead>
<tr>
<th>SECOND SHOCK M=2.82</th>
<th>10+13</th>
<th>SLOT BLEED=0.028</th>
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NASA-Langley, 1976