DOUBLE ION PRODUCTION IN MERCURY THRUSTERS

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### Abstract

Significant densities of doubly charged ions exist in the discharge chambers of electron bombardment ion thrusters. These ions are undesirable because they are a major plasma constituent effecting the sputtering damage which limits thruster lifetime. It would be desirable to reduce their density while maintaining good thruster performance. The development of a model which predicts the doubly charged ion density is discussed. The accuracy of the model is shown to be good for two different thruster sizes and a total of 11 different cases. The model indicates that in most cases more than 80% of the doubly charged ions are produced from singly charged ions. This result can be used to develop a much simpler model which, along with correlations of the average plasma properties, can be used to determine the doubly charged ion density in ion thrusters with acceptable accuracy. Two different techniques which can be used to reduce the doubly charged ion density, while maintaining good thruster operation, are identified as a result of an examination of the simple model. First, the electron density can be reduced and the thruster size then increased to maintain the same propellant utilization. Second, at a fixed thruster size, the plasma density, temperature and energy can be reduced and then to maintain a constant propellant utilization the open area of the grids to neutral propellant loss can be reduced through the use of a small hole accelerator grid. The reduction in the values of the plasma properties causes a decrease in the doubly charged ion density.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>1</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td>ii</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>iii</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>iii</td>
</tr>
<tr>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>THRUSTER OPERATION</td>
<td>2</td>
</tr>
<tr>
<td>THEORETICAL MODEL</td>
<td>6</td>
</tr>
<tr>
<td>Introduction</td>
<td>6</td>
</tr>
<tr>
<td>Electron Bombardment Reactions</td>
<td>10</td>
</tr>
<tr>
<td>Migration Losses</td>
<td>16</td>
</tr>
<tr>
<td>Photon Diffusion Losses</td>
<td>19</td>
</tr>
<tr>
<td>Determination of Specie Densities</td>
<td>21</td>
</tr>
<tr>
<td>EXPERIMENTAL PROCEDURES AND RESULTS</td>
<td>24</td>
</tr>
<tr>
<td>RESULTS AND DISCUSSION</td>
<td>32</td>
</tr>
<tr>
<td>SIMPLIFIED MODEL</td>
<td>39</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>55</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>56</td>
</tr>
<tr>
<td>APPENDIX A: Listing of the Computer Program HG</td>
<td>58</td>
</tr>
<tr>
<td>APPENDIX B: Listing of the Computer Program PROP</td>
<td>73</td>
</tr>
</tbody>
</table>
LIST OF TABLES

Table Number | Page
---|---
I Thruster Sizes, Configurations and Conditions | 26
II Experimental Results | 29
III Predicted Densities and Reaction Rates | 36
IV Determination of the Double Ion Density Using the Simplified Model | 51

LIST OF FIGURES

Figure Number | Page
---|---
1 Electron Bombardment Ion Thruster Schematic | 3
2 Discharge Chamber Reaction Schematic | 7
3 Mercury Cross Sections | 12
4 Plasma Property Profiles, 15 cm Thruster - SERT II Grids - 37 V Anode Voltage | 28
5 Double-to-Single Ion Density Ratio in a 15 cm Diameter Thruster | 33
6 Double-to-Single Ion Density Ratio in a 30 cm Diameter Thruster | 34
7 Rate Factors for Hg^+ → Hg^{++} | 41
8 Maxwellian Electron Temperature Correlation | 43
9 Primary Electron Energy Correlation | 45
10 Electron Density Correlation | 47
11 Primary Electron Density Correlation | 48
12 Uniformity Factor Correlation | 49
INTRODUCTION

Electron bombardment ion thrusters are presently being considered for use in deep space probes and for satellite stationkeeping functions. These devices which have the advantage of very high specific impulses also have the attendant disadvantage of low thrust densities. This low thrust characteristic necessitates thruster operation for long periods of time in order to accomplish typical missions (of the order of 10,000 hours). Thruster lifetimes can be limited by the erosion of ion chamber component parts with most of this erosion (or sputtering damage) being caused by doubly charged ions. Long thruster lifetimes therefore require control of doubly charged ion densities.

Many experiments could be performed to determine how the thruster should be operated so that thruster performance would be good and the doubly charged ion density would be reduced to an acceptable level. However, these experiments would have the disadvantage of being time consuming and costly and the results might be applicable to one size and type of thruster only. Instead a theoretical model could be developed which would accurately predict the doubly charged ion density over a wide range of conditions and thruster sizes. This model could be applied at low cost to determine the factors affecting the doubly charged ion density and how they should be adjusted to reduce the double ion density. It should also indicate what effects these changes would have on thruster performance. Such a model has been developed for electron bombardment ion thrusters and has been verified experimentally for thrusters which use mercury propellant. A discussion of the model's development and verification is presented in this paper along with some results and conclusions based upon the model.
THRUSTER OPERATION

Many of the assumptions and approximations used in the development of the model are based upon a knowledge of thruster operation. This section will briefly discuss thruster operation so that the development of the model in the next section will be more easily understood. An ion thruster has two basic tasks to perform:

1) Ionization of the neutral propellant atoms.

2) Acceleration of the ions to high velocities producing thrust.

These two topics will form the basis for the discussion of thruster operation.

Figure 1 shows a schematic for a typical electron bombardment ion thruster. The specific type shown has a strongly divergent magnetic field which is presently the most common type. However, the operation of all types of electron bombardment ion thrusters is very similar (1). Electrons are emitted from the cathode and are drawn toward the anode which is biased 30-40V positive with respect to the cathode. These electrons (called primary electrons) are injected into the primary electron region with an energy slightly less than that associated with the 30-40V anode voltage. Electrons in this region are kept from going immediately to the anode by a magnetic field set up between the cathode pole piece and the anode pole piece but collisions eventually facilitate electron diffusion across these magnetic field lines so that they can be collected by the anode. As a result of the magnetic field containment the electron density is much higher (\(10^{11}\) cm\(^{-3}\)) within this region than it is outside of it. The primary electron region's boundary is defined by the surface of revolution of the critical (magnetic) field line and the screen grid. Because the strength of the magnetic field
Figure 1 Electron Bombardment Ion Thruster Schematic
is fairly low within it, a fairly uniform electron density exists throughout the entire region.

Neutral propellant (e.g. mercury) is injected into the upstream end of the discharge chamber at low pressure \((10^{-4} \text{ torr})\). Most of the interactions between electrons and neutral propellant atoms take place in the primary electron region because higher electron densities and energies exist there. Electrons bombard the neutral atoms occasionally knocking an outer shell electron loose from the atom forming a singly charged ion. The production of a mercury single ion requires more than 10 eV of energy from the incident electron. This electron and the ejected electron then share the remainder of the energy which the incident electron carried originally. This reaction results in the replacement of one high energy electron with two lower energy electrons which rapidly randomize with similarly generated electrons to form a Maxwellian electron group. Ions are extracted from the plasma through holes in the screen and accelerator grids as a result of the large potential difference applied across these two grids. The rate of ion loss through the grids times the ionic charge is called the beam current.

Electron bombardment of atoms and ions also produces doubly charged ions. Many of these ions are extracted from the discharge chamber by the grids, however, some of them go to the walls. As ions near the walls (the cathode pole piece, screen grid, etc.) they are accelerated to high velocities by an electric field that exists at the plasma boundary. When these high velocity ions strike the walls they can knock atoms loose (sputter atoms) from the walls of the discharge chamber. The energy that doubly charged ions possess upon striking the
walls is twice that of singly charged ions, therefore the sputtering damage caused by a double ion is much greater than that caused by a single ion. Double ions are thought to cause most of the sputtering damage even though their density is typically an order of magnitude less than that of the single ions.
THEORETICAL MODEL

Introduction

In order to develop a simple model for determining the double ion density in the discharge chamber only those ionic and atomic species which were considered significant in determining the double ion density were included. The significant species were selected as those which have substantial electron impact cross sections of formation over the electron energy range of interest so that large numbers of these excited atoms or ions will be produced. These states also have sufficiently long effective lifetimes so that they can participate in production processes before they decay. Only those reactions which lead directly or indirectly to the production of double ions were included.

Figure 2 is a discharge chamber reaction schematic showing these dominant species and the reactions in which each species can participate. The model has been developed for thrusters using mercury propellant but the general procedure is valid for thrusters using other propellants.

The symbols used in Figure 2 represent the following species:

- $\text{Hg}^0$ - neutral ground state mercury
- $\text{Hg}^m$ - metastable neutral mercury ($6^3\text{P}_0$ and $6^3\text{P}_2$ states)
- $\text{Hg}^r$ - resonance state neutral mercury ($6^3\text{P}_1$ and $6^1\text{P}_1$ states)
- $\text{Hg}^+$ - singly ionized ground state mercury
- $\text{Hg}^{m+}$ - singly ionized metastable mercury ($6^1\text{D}_{5/2}$ and $6^3\text{D}_{5/2}$ states)
- $\text{Hg}^{++}$ - doubly ionized ground state mercury
Figure 2 Discharge Chamber Reaction Schematic
The arrows in Figure 2 indicate the various interaction routes considered in this analysis. Three different types of reactions are indicated in this figure. The first type of reaction occurs when an electron interacts with an atom or ion producing a more highly excited specie. This reaction is indicated in Figure 2 by an arrow going from one specie to another more highly excited specie (e.g. the production of double ions from single ions). The production of a highly excited specie also represents a loss mechanism for the less excited specie. The reverse reaction in which, for example, an ion captures an electron is improbable because the reaction requires three bodies to simultaneously collide.

The second type of process is that of an atom or ion going to a plasma boundary. Such a boundary could be either the discharge chamber wall on which the atom or ion would be de-excited or it could be a grid aperture in which case the atom or ion would be extracted from the discharge region. In either case this represents a loss rate for any of the excited states. These losses to the boundary are indicated in Figure 2 by the dotted lines to the wall of the chamber. The large arrow back to the neutral ground state represents the resupply of neutral ground state atoms either from the walls or from the propellant supply system.

A third type of reaction shown in Figure 2 is relevant only to the two resonance states. The resonance states differ from metastable states in that they have a very short lifetime before they de-excite spontaneously by emitting a photon of light. However, the energy of this photon is such that it is readily absorbed by a nearby neutral ground state atom producing another resonance state atom. Since the
transport time of the photon is small compared to the excited state lifetime the excited state can be considered to exist continuously. Eventually the photon can diffuse to a boundary where it will be lost; this is equivalent to the loss of a resonance state atom. This loss mechanism is represented in Figure 2 by a dotted line conveying a photon to the wall and a branching line going from the resonance atom to the neutral ground state atom.

Figure 2 also shows the dominant routes for the production and loss of all of the excited atoms and ions considered. For example, ground state single ions can be produced as a result of electron bombardment of neutral–ground–state, resonance state, and metastable state atoms and these single ions can be lost as a result of single ion migration to the plasma boundary and the production of metastable single ions and double ions by electron bombardment.

When equilibrium conditions exist in the discharge chamber the rate of production of each specie must equal its loss rate. If, for example, the production rate of single ions increases, the single ion density must also increase to keep the loss rate (which is directly proportional to the single ion density) equal to the higher production rate. This example illustrates the fact that the equilibrium density of any specie is determined by the associated production and loss rates. If equations determining the production and loss rates could be derived, these equations could then be solved for the equilibrium density of any specie under consideration. The remainder of this section is concerned with deriving equations for the production and loss mechanisms indicated in Figure 2 and then solving these equations for the equilibrium densities of the various states.
Electron Bombardment Reactions

The first reaction to be considered is the one which produces excited atoms or ions by electron bombardment from less excited atoms or ions. The total rate of production of any specie $\gamma$ from specie $\alpha$ (and hence the loss rate of $\alpha$ due to this reaction) is given by:

$$R^\gamma_\alpha = \int_{\text{Plasma Volume}} \int_{E=0}^{E=\infty} n_\alpha \sigma^\gamma_\alpha(E) v_e(E) \, dn_e d\psi$$

where $n_\alpha$ is the density of specie $\alpha$ at some point $\hat{r}$ in the plasma, $\sigma^\gamma_\alpha(E)$ is the cross section for the production of $\gamma$ from $\alpha$ at the electron energy $E$, $v_e$ is the electron velocity at energy $E$, $dn_e$ is the density of electrons with energies between $E$ and $E + dE$ at $\hat{r}$, and $d\psi$ is the infinitesimal volume element. The distribution of electrons over the energy spectrum of an ion thruster was assumed to be composed of a Maxwellian electron group which is described by a temperature ($T_{mx} \text{ -- eV}$) and a density ($n_{mx} \text{ -- cm}^{-3}$) and a monoenergetic group (primary electrons) which is described by an energy ($E_{pr} \text{ -- eV}$) and a density ($n_{pr} \text{ -- cm}^{-3}$). This type of electron distribution is generally accepted as appropriate for electron bombardment thruster plasmas.$^{(1,7)}$

Substituting this electron distribution into Equation (1) and combining terms to form new functions results in the following equation.

$$R^\gamma_\alpha = \int_{\text{Volume}} n [n_{pr} p^\gamma_{pr} (E_{pr}) + n_{mx} Q^\gamma_{mx} (T_{mx})] \, d\psi$$

where

$$p^\gamma_{pr} (E_{pr}) = v_e(E_{pr}) \sigma^\gamma_{pr} (E_{pr})$$

(3)
and
\[ Q^Y_{\alpha}(T_{\text{mx}}) = \int_{E=0}^{E=\infty} c^Y_{\alpha}(E) v_e(E) \frac{dn_{\text{mx}}(E)}{n_{\text{mx}}}. \] (4)

"\([dn_{\text{mx}}(E)/n_{\text{mx}}]\)" is the Maxwellian distribution function and the other terms are as defined previously.

Where possible the cross sections \(c^Y_{\alpha}\) required for Equations (3) and (4) were selected from experimental data. If experimental data were not available, theoretical cross sections were either obtained from the literature or calculated using the Gryzinski approximation. The Gryzinski approximation was modified for the cases of the metastable single ion production cross sections to reflect the significant value of the cross sections near the threshold. The cross sections used are presented in Figure 3 along with references indicating their origin.

Using integral equations like Equation (2) in the model would be inconvenient because it would then be very difficult to solve for the density of specie \(n_\alpha\) since \(n_\alpha\) appears within the integral. For this reason it would be desirable to convert Equation (2) into a simple algebraic equation. Fortunately the plasma is fairly uniform in the primary electron region which is where most of the reactions take place. This suggests using average properties in Equation (2) to obtain the following result.

\[ R^Y_\alpha = n^*_\alpha [n^*_\text{pr} P^Y_{\alpha\text{pr}} + n^*_\text{mx} Q^Y_{\alpha}(T^*_\text{mx})] V \] (5)

The asterisks indicate volume averaged quantities and \(V\) is the volume of the primary electron region.
Figure 3 Mercury Cross Sections
Figure 3 Mercury Cross Sections (continued)
In order to use Equation (5) to evaluate all of the production rates the volume averaged plasma properties \((n_{pr}^*, n_{mx}^*, \varepsilon_{pr}^*, T_{mx}^*)\) must be determined. Comparing Equations (2) and (5) the following definitions of the volume averaged properties must apply.

\[
    n_{pr}^* P^Y(\varepsilon_{pr}) = \int_{\text{Volume}} n_{pr} P^Y(\varepsilon_{pr}) \, d\Psi \tag{6}
\]

\[
    n_{mx}^* Q^Y(T_{mx}) = \int_{\text{Volume}} n_{mx} Q^Y(T_{mx}) \, d\Psi \tag{7}
\]

These two equations show that the volume averaged plasma properties will be weighted in some manner. In order to evaluate the integrals, species \(\alpha\) and \(\gamma\) must be chosen. The only specie density \((n_\alpha)\) that can be determined readily is the single ion density because plasma neutrality requires it to be approximately equal to the electron density. Specie \(\gamma\) must also be chosen in order to determine what \(P^\gamma\) and \(Q^\gamma\) to use. Figure 2 shows only two choices are possible—the singly ionized metastable states and the doubly ionized ground state. Since the whole purpose of the model is to determine the double ion density, specie \(\gamma\) was chosen as the doubly ionized ground state. Using these choices for species \(\alpha\) and \(\gamma\), Equations (6) and (7) were rewritten in the following form where the electron density \((n_e)\) has been used to approximate the single ion density.

\[
    n_e n_{pr} P^{++}(\varepsilon_{pr}) = \int_{\text{Volume}} n_e n_{pr} P^{++}(\varepsilon_{pr}) \, d\Psi \tag{8}
\]

\[
    n_e n_{mx} Q^{++}(T_{mx}) = \int_{\text{Volume}} n_e n_{mx} Q^{++}(T_{mx}) \, d\Psi \tag{9}
\]
The volume averaged values of the primary electron energy \( (\varepsilon_{pr}^*) \) and the Maxwellian electron temperature \( (T_{mx}^*) \) were defined as shown in Equations (10) and (11). These definitions were chosen because they give reasonable values for the properties involved (i.e. these volume averaged values can't be greater than the peak values, which was possible with some of the other definitions).

\[
P_+^{++}(\varepsilon_{pr}^*) = \frac{\int \text{Volume} n_e n_{pr} \, dV}{\int \text{Volume} \, dV} \tag{10}
\]

\[
Q_+^{++}(T_{mx}^*) = \frac{\int \text{Volume} n_e n_{mx} \, dV}{\int \text{Volume} \, dV} \tag{11}
\]

Equations (8) - (11) along with Equation (12), which says the volume averaged electron density is the sum of the volume averaged primary and Maxwellian electron densities, can be combined to obtain the following definitions of the remaining volume averaged plasma properties.

\[
n_e^* = n_{pr}^* + n_{mx}^* \tag{12}
\]

\[
n_e^* = \left( \frac{\int \text{Volume} n_e^2 \, dV}{\int \text{Volume} \, dV} \right)^{1/2} \tag{13}
\]

\[
n_{pr}^* = \left( \frac{\int \text{Volume} n_e n_{pr} \, dV}{\int \text{Volume} \, dV} \right)^{1/2} \tag{14}
\]

\[
n_{mx}^* = \left( \frac{\int \text{Volume} n_e n_{mx} \, dV}{\int \text{Volume} \, dV} \right)^{1/2} \tag{15}
\]
This concludes the mathematical development for electron bombardment reactions.

The volume averaged plasma properties must be evaluated in order to use Equation (5) to calculate the production rates. The plasma properties \( (n_{pr}, n_{mx}, c_{pr}, T_{mx}) \) are measured at many points inside the discharge chamber by a Langmuir probe. This data is then used to evaluate the integrals in Equations (10), (11) and (13)-(15) numerically yielding the needed volume averaged plasma properties.

Migration Losses

The second type of process to be considered is that of an excited atom or ion going to the plasma boundary. The equation for the plasma boundary loss rate of a specie \( \alpha \) is given by:

\[
R_{\alpha} = \int_{\text{plasma boundary}} n_{\alpha} v_{\alpha} \, dA
\]  

(16)

where \( n_{\alpha} \) is the density of specie \( \alpha \) at the boundary, \( v_{\alpha} \) is its average velocity toward the plasma boundary, and \( dA \) is the infinitesimal area. For neutral particles assumed to have a temperature equal to the discharge chamber wall temperature and having a mass \( m_0 \) the average velocity toward the boundary \( (v_0) \) is equal to one-fourth the average thermal speed

\[
v_0 = \frac{1}{4} \sqrt{\frac{8k T_{\text{wall}}}{m_0}}
\]

(17)

where \( k \) is the Boltzmann constant.
For ions this velocity is determined by the Bohm criterion (8,9) and is given by

$$v_q = \sqrt{\frac{T_{mx} q}{m_i} \left(1 + \frac{n_{pr}}{n_{mx}}\right)}$$

(18)

where $q$ is the ion charge (coul) and $m_i$ is the ion mass (kg), and $T_{mx}$, $n_{pr}$ and $n_{mx}$ are as defined previously.

Since the integral equation used to define production rates has been reduced to an algebraic equation in terms of average properties Equation (16) should also be simplified in this manner. Because the migration loss is a surface phenomenon, however, it is necessary to use surface averaged densities and velocities based on surface average properties to obtain

$$R_{\beta\alpha} = n_{\alpha}^{S} v_{\alpha}^{S} A$$

(19)

where $A$ is the total surface area of the primary electron region and the superscript "s" designates values based on surface averaged properties. Equation (19) could be made more convenient for use in the model if it were based on volume averaged properties as Equation (5) is. Equation (19) was for this reason rewritten in terms of volume averaged densities as follows

$$R_{\beta\alpha} = n_{\alpha}^{*} v_{\alpha}^{*} A/F_{\alpha}$$

(20)

where $v_{\alpha}^{*}$ is $v_{\alpha}(T_{mx}, n_{pr}/n_{mx})$ and $F_{\alpha}$ is a plasma uniformity factor given by Equation (21) which relates the volume and surface averaged density-velocity product.
This concludes the mathematical development for the migration loss of excited species, but some additional discussion and quantification of the uniformity factor $F_\alpha$ is necessary before it can be used in Equation (20). The evaluation of $F_\alpha$ for neutral excited states is difficult because it is difficult to measure their densities. The migration of excited neutrals to the plasma boundary is however a minor loss mechanism compared to the losses due to the conversion of neutral excited atoms into single ions. Therefore $F_\alpha$ for neutral excited states can be set equal to unity without introducing a significant error into the total loss rate calculation. In the case of ions, however, migration to the boundary is a major loss mechanism and $F_\alpha$ must be evaluated in order to obtain accurate results. For the singly ionized ground state the approximation, $n_+ = n_e$, can again be used in order to evaluate $F_+$ using Equation (21). The uniformity factor for the singly ionized metastable states was set to unity since these states have a very minor effect on the double ion density. The determination of $F_{++}$ is based on the observation\(^{(10)}\) that the volume averaged double ion density $(n_{++}^*)$ is proportional to the volume averaged electron density squared $(n_e^*)^2$. It has been assumed that this proportionality holds locally and this results in the following definition of $F_{++}$.

\[
F_{++} = \frac{(n_{++}^*)^2}{\left[ \int n_e^* v_{++} \, dA \right] / A} \tag{22}
\]
Photon Diffusion Losses

The third type of process to be considered is the loss of resonance state atoms due to photon diffusion to the walls of the discharge chamber. From diffusion theory the rate of photon loss across any plasma boundary and hence the rate of resonance state atom loss by this mechanism is given by the equation

$$R_{r} = \int D \Delta n_p \, dA = DA[\Delta n_p] . \tag{23}$$

In this equation $n_p$ is the photon density and $D$ is the photon diffusion coefficient which is given by

$$D = \frac{1}{3\pi(n_0\sigma_c)^2} \tag{24}$$

"τ" in this equation is the average lifetime of the resonance state atom, $n_0^*$ is the neutral ground state atom density, and $\sigma_c$ is the cross section for absorption of the photons by neutral ground state atoms. (11) The second equality in Equation (23) reflects the fact that average properties are being used in this analysis. The photon density has been assumed constant up to a point one photon mean free path from the boundary. From this point the density is assumed to decay linearly to zero at the boundary. This assumption yields the following conservative estimate for the photon loss rate

$$R_{r} = DA \frac{n_p^*}{\lambda_f} \tag{25}$$

where $\lambda_f$ is the mean free path for photon absorption ($\frac{1}{n_0\sigma_c}$). This
approximation is valid when the photon mean free path is much less than
the characteristic dimension of the plasma, a condition that is readily
satisfied for this case where the photon mean free paths are very small
($\lambda_p = 1 \text{ cm}$).

Since the neutral density is assumed uniform over the discharge
region the photon density profile is similar to the resonance state
atom density profile and the following approximation between the photon
and resonance state atom density at any location in the plasma applies:

$$n_p = n_r \beta ~. \quad (26)$$

$\beta$ is a proportionality constant that can be thought of as the ratio of
the probability that a photon will be "free" in the plasma to the prob-
ability that it will be "bound" forming a resonance state atom. This
ratio of probabilities can also be expressed as the ratio of the
average lifetime of a free photon ($\frac{1}{\lambda_p}$) to the resonance state atom
lifetime ($\tau$). Therefore, $\beta$ is given by:

$$\beta = \frac{1}{\lambda_p n_r \sigma_c} \quad (27)$$

where $c$ is the speed of light and the other quantities have already
been defined. Combining Equations (25), (26 and 27) one obtains the
following equation for the loss rate of resonance state atoms due to
photon diffusion:

$$R_{\text{\gamma}} = \frac{n_r^*}{3c} A \left[ -\frac{1}{\lambda_p} - \frac{1}{\tau} \right] n_r \sigma_c \quad (28)$$
Determination of Specie Densities

The equations derived so far in this section can now be combined to determine the equilibrium density of each specie included in the model. Equations of the form of (5) and (20) -- and (28) for the case of resonance state atoms -- along with the values for the volume averaged plasma properties and the plasma uniformity factors can be used to determine the rates of production and loss for each specie in the plasma. The steady state density of these species can then be calculated by equating their total production rates to their total loss rates. For example, the $6^3P_0$ metastable atom density is determined by equating the production rate of this metastable atomic state from neutral ground state atoms to the sum of the associated loss rates due to 1) migration to the wall, 2) production of single ions, and 3) the production of double ions, that is

$$n^*_m \left[ n^*_{pr} p^m_{o}(r_{pr}) + n^*_{mx} Q^m_{o}(T^*_{mx}) \right] \Psi = \frac{n^*_m v^*_m A}{F_m}$$

$$+ \ n^*_m \left[ n^*_{pr} p^+_{m}(r_{pr}) + n^*_{mx} Q^+_{m}(T^*_{mx}) \right] \Psi + n^*_m \left[ n^*_{pr} p^{++}_{m}(r_{pr}) + n^*_{mx} Q^{++}_{m}(T^*_{mx}) \right] \Psi.$$

Solving this for the metastable atom density ratio one obtains

$$\frac{n^*_m}{n^*_o} = \left[ n^*_{pr} p^m_{o}(r_{pr}) + n^*_{mx} Q^m_{o}(T^*_{mx}) \right] \left\{ \frac{v^*_m}{\Psi/A F_m} + \left[ n^*_{pr} p^+_{m}(r_{pr}) \right. \right.$$

$$+ \ n^*_{mx} Q^+_{m}(T^*_{mx}) \left. \right] \left[ n^*_{pr} p^{++}_{m}(r_{pr}) + n^*_{mx} Q^{++}_{m}(T^*_{mx}) \right] \right\} \]$$

where $n^*_m$ is the volume average metastable state density and $v^*_m$ is the average velocity of metastable neutral atoms toward the boundary. A similar type of equation can be derived for each of the other excited states but they are all as complex or more complex than Equation (30).
For example, the equation for the doubly ionized ground state density has eight terms in the numerator. Each of these terms has the same form as the bracketed quantities in the numerator of Equation (30).

The quantity $\psi/A$ in the denominator of Equation (30) has an interesting physical interpretation. It is contained in a term which represents the loss rate per unit volume of metastable state atoms to the plasma boundary. This term shows the manner in which the size and shape of the primary electron region enters into the model. For a large thruster $\psi/A$ will be large and an ion or excited neutral must, on the average, travel great distances to reach the plasma boundary, and so the loss rate per unit volume of these species will be small. For a small thruster $\psi/A$ will be small and on the average the ions and excited species are near the boundary and can reach it readily resulting in a large loss rate of these species per unit volume.

At this point only the relative density of each excited specie $(n^*_i/n^*_o)$ can be calculated. However one additional fact can be added to the model, the requirement that the plasma be neutral (i.e., $n^*_e = n^*_+ + n^*_{m+} + 2n^*_{++}$). This requirement when added to the relative density equations of the ionized states implies unique single, metastable single and double ion densities. These in turn imply a unique neutral atom density and hence a unique density for each specie considered in the analysis. One must however iterate to arrive at these densities because a neutral ground state atom density must be assumed initially to determine photon loss rates from Equation (28). At the conclusion of the analysis then the calculated ground state atom density must agree with the assumed value.
A computer program has been written which calculates the densities of all the species considered in the model. The densities are calculated by using relative density equations similar to Equation (30) and the plasma neutrality condition. The input needed to make these calculations includes the volume averaged plasma properties, the plasma uniformity factors and the volume-to-surface area ratio of the primary electron region. A listing of this computer program entitled "HG" is included as Appendix A.
EXPERIMENTAL PROCEDURES AND RESULTS

The model developed in the previous section will predict the specie densities if the volume averaged plasma properties, uniformity factors and geometric quantities, which are collectively called the input parameters, are known. In order to verify the accuracy of the model, data must be gathered so that the model’s input parameters can be determined. These input parameters can be used by the model to predict the specie densities which can then be compared to the measured densities to determine the model’s accuracy. The model’s accuracy has been determined by comparing the measured and predicted double ion densities since the double ion density is the model’s main concern.

In order to test the accuracy of the model over a wide range of conditions data were used from different thrusters and operating conditions. The 15 cm diameter SERT II thruster was operated with two different grid sets and at three different power levels in each of these configurations. Data were collected at each condition allowing the accuracy of the model to be verified at six different points. Data for the 30 cm diameter thruster were also obtained from Hughes Research Laboratories (12) so that the model could be verified over a wider range of thruster sizes, configurations, and operating conditions. Both thrusters have strongly divergent magnetic fields. Their general configuration and manner of operation have been described in the "Thruster Operation" section. More detailed thruster specifications, etc. are available in the literature. (9, 13, 14)

The data gathering procedure for the 15 cm thruster will be used to illustrate the general manner in which the needed data were obtained.
Before the gathering of data could begin thruster operation and flow rates were kept stable for approximately thirty minutes. This insured that thruster conditions would change little in the twenty minute period during which the data were obtained. Table I lists the conditions and configurations at which the 15 cm thruster was operated at C.S.U. along with those for the 30 cm thruster as obtained from Hughes Research Laboratories. This table indicates the changes in configuration for both thrusters resulted from using different grid types. The SERT II grids, listed in Table I, are flat grids with hole diameters of \( \leq 0.4 \) cm and in operation are separated by a gap of \( 0.23 \) cm. The high perveance dished grids are dished slightly to prevent the grids from shorting during operation due to their thermal expansion. Their hole diameter is smaller \( (0.25 \text{ cm}) \) as is their separation gap \( (0.079) \). More detailed specifications for the two grid types can be found in Reference 9. The EM (Engineering Model) grids are similar to the high perveance dished grids described above. The SHAG (Small Hole Accelerator Grids) grids have an accelerator hole diameter that is \( \approx 70\% \) of the EM grids' accelerator hole diameter. This smaller hole size reduces the loss of neutral propellant. These two grid types are described in more detail in References 15 and 16. Table I shows, for example, that the 15 cm thruster with the SERT II grids was operated at one condition where the amount of current collected at the anode \( (I_{\text{arc}}) \) was 1.7 A, while the voltage difference between the anode and cathode \( (V_{\text{arc}}) \) was 37.2 V and the ion current through the grids \( (I_{\text{beam}}) \) was 0.258 A.

The values of the volume averaged plasma properties and the uniformity factors must be known in order to calculate the theoretical double ion density. In order to determine the values of these average...
### Table I
Thruster Sizes, Configurations and Conditions

<table>
<thead>
<tr>
<th>Thruster Diameter (cm)</th>
<th>Grid Type</th>
<th>Anode Current ($I_{arc}$ --A)</th>
<th>Anode Voltage ($V_{arc}$ --V)</th>
<th>Ion Beam Current ($I_{beam}$ --A)</th>
<th>Mass Flow Rate (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.</td>
<td>SERT II</td>
<td>1.00</td>
<td>33.</td>
<td>0.180</td>
<td>0.310</td>
</tr>
<tr>
<td>15.</td>
<td>SERT II</td>
<td>1.70</td>
<td>37.2</td>
<td>0.258</td>
<td>0.307</td>
</tr>
<tr>
<td>15.</td>
<td>SERT II</td>
<td>2.05</td>
<td>42.6</td>
<td>0.272</td>
<td>0.308</td>
</tr>
<tr>
<td>15.</td>
<td>Dished</td>
<td>3.02</td>
<td>32.2</td>
<td>0.499</td>
<td>0.735</td>
</tr>
<tr>
<td>15.</td>
<td>Dished</td>
<td>4.06</td>
<td>37.5</td>
<td>0.654</td>
<td>0.725</td>
</tr>
<tr>
<td>15.</td>
<td>Dished</td>
<td>4.13</td>
<td>40.4</td>
<td>0.622</td>
<td>0.650</td>
</tr>
<tr>
<td>30.</td>
<td>EM</td>
<td>5.0</td>
<td>37.</td>
<td>1.0</td>
<td>1.25</td>
</tr>
<tr>
<td>30.</td>
<td>EM</td>
<td>7.5</td>
<td>37.</td>
<td>1.5</td>
<td>1.76</td>
</tr>
<tr>
<td>30.</td>
<td>EM</td>
<td>10.0</td>
<td>37.</td>
<td>2.0</td>
<td>2.29</td>
</tr>
<tr>
<td>30.</td>
<td>SHAG</td>
<td>9.5</td>
<td>30.</td>
<td>1.5</td>
<td>1.74</td>
</tr>
<tr>
<td>30.</td>
<td>SHAG</td>
<td>11.7</td>
<td>30.</td>
<td>2.0</td>
<td>2.30</td>
</tr>
</tbody>
</table>
plasma properties, the primary and Maxwellian electron densities and energies must be determined everywhere in the discharge chamber. The plasma properties at some point in the plasma can be measured using a Langmuir probe and analyzed using the procedure described in Reference 17. The plasma properties at sixteen different points in the plasma were measured, for each 15 cm thruster condition listed in Table I, using the movable Langmuir probe and associated circuitry described in Reference 18. The results of a typical survey (15 cm thruster - SERT II grids -37 V anode voltage) are plotted in Figure 4. This figure shows the spatial variation of the Maxwellian electron temperature, primary electron energy and the primary and Maxwellian electron densities in the discharge chamber. The Maxwellian electron temperature is seen to average approximately 9 eV over the primary electron region defined by the critical field line while the primary electron energy averages about 30 eV. The average Maxwellian electron density is about $10^{11} \text{cm}^{-3}$ while the average primary electron density is approximately $10^{10} \text{cm}^{-3}$ over the same region. The electron densities and energies are seen to be fairly uniform in the primary electron region but drop off rapidly outside this region. Using data similar to that plotted in Figure 4, Equations (10)-(15) and (21) were evaluated numerically by the computer program "PROP" (listed in Appendix B) to obtain the volume averaged properties and uniformity factors for each case. The results are listed in Table II along with the volume-to-surface area ratio of the primary electron region ($V/A$) and the thruster operating specifications which are reproduced from Table I.

The average values which resulted from an examination of Figure 4 ($T_{mx} = 9 \text{ eV}, \ n_{mx} = 10^{11} \text{cm}^{-3}$, $n_{pr} = 10^{10} \text{cm}^{-3}$) are seen to
Figure 4  Plasma Property Profiles, 15 cm Thruster - CRT II Grids - 37 V Anode Voltage
<table>
<thead>
<tr>
<th>Operating Variables</th>
<th>15.</th>
<th>15.</th>
<th>15.</th>
<th>15.</th>
<th>15.</th>
<th>30.</th>
<th>30.</th>
<th>30.</th>
<th>30.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thruster Diameter (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grid Type</td>
<td>SERT II</td>
<td>HIGH PERVEANCE DISHED EM EM EM SHAG SHAG</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Arc Current (I_{arc} - A)</td>
<td>1.0</td>
<td>1.7</td>
<td>2.05</td>
<td>3.02</td>
<td>4.06</td>
<td>4.13</td>
<td>5.0</td>
<td>7.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Arc Voltage (V_{arc} - V)</td>
<td>33.</td>
<td>37.2</td>
<td>42.6</td>
<td>32.2</td>
<td>37.5</td>
<td>40.4</td>
<td>37.</td>
<td>37.</td>
<td>37.</td>
</tr>
<tr>
<td>Beam Current (I_{beam} - A)</td>
<td>0.180</td>
<td>0.258</td>
<td>0.272</td>
<td>0.499</td>
<td>0.654</td>
<td>0.622</td>
<td>1.0</td>
<td>1.5</td>
<td>2.0</td>
</tr>
<tr>
<td>Mass Flow Rate (A)</td>
<td>0.310</td>
<td>0.307</td>
<td>0.308</td>
<td>0.735</td>
<td>0.725</td>
<td>0.650</td>
<td>1.25</td>
<td>1.76</td>
<td>2.29</td>
</tr>
<tr>
<td>Phase Volume to Surface Area Ratio (V/A - cm)</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>1.4</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Average Maxwellian Electron Temperature (T_{MAX} - eV)</td>
<td>4.2</td>
<td>9.1</td>
<td>12.2</td>
<td>4.3</td>
<td>7.1</td>
<td>10.2</td>
<td>3.3</td>
<td>3.6</td>
<td>3.8</td>
</tr>
<tr>
<td>Average Primary-to-Maxwellian Electron Density Ratio (n_{p}/n_{MAX})</td>
<td>0.034</td>
<td>0.033</td>
<td>0.00166</td>
<td>0.017</td>
<td>0.042</td>
<td>0.134</td>
<td>0.50</td>
<td>0.35</td>
<td>0.25</td>
</tr>
<tr>
<td>Average Primary Electron Energy (E_{p} - eV)</td>
<td>27.5</td>
<td>29.6</td>
<td>38.4</td>
<td>21.5</td>
<td>23.4</td>
<td>31.0</td>
<td>25.4</td>
<td>25.5</td>
<td>27.2</td>
</tr>
<tr>
<td>Average Electron Density (n_{e} x 10^{-11} cm^{-3})</td>
<td>9.80</td>
<td>9.10</td>
<td>8.07</td>
<td>36.0</td>
<td>24.3</td>
<td>18.2</td>
<td>7.51</td>
<td>8.97</td>
<td>16.4</td>
</tr>
<tr>
<td>Uniformity Factor F_1</td>
<td>2.3</td>
<td>2.1</td>
<td>2.3</td>
<td>2.0</td>
<td>1.9</td>
<td>1.8</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>Uniformity Factor F_2</td>
<td>3.1</td>
<td>2.5</td>
<td>2.6</td>
<td>2.5</td>
<td>2.1</td>
<td>2.0</td>
<td>1.8</td>
<td>1.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table II
Experimental Results

<table>
<thead>
<tr>
<th>Input Parameters</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>245</th>
<th>245</th>
<th>245</th>
<th>245</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured Double-to-Single Ion Current Ratio (I^+ / I^-)</td>
<td>0.024</td>
<td>0.073</td>
<td>0.12</td>
<td>0.036</td>
<td>0.081</td>
<td>0.18</td>
<td>0.080</td>
<td>0.125</td>
<td>0.167</td>
</tr>
</tbody>
</table>
agree well with the volume-averaged values listed in Table II

\( T_{\text{mx}} = 9.1 \text{ eV}, \quad \nu_{\text{pr}} = 29.6 \text{ eV}, \quad n_{\text{mx}} = 8.4 \times 10^{19} \text{cm}^{-3}, \quad n_{\text{pr}} = 7.0 \times 10^{19} \text{cm}^{-3} \). 

\( F_+ \) and \( F_{++} \), which are defined as the ratio of the volume average ion flux to the average flux at the surface of the primary electron region, are seen to have values of 2.1 and 2.5 respectively for the case being discussed. The average plasma properties listed in Table II are observed to cover a wide range in plasma conditions; a situation which is desirable for verification of the model.

The double ion density inside the discharge chamber must also be determined. This can be accomplished indirectly by determining the double-to-single ion density ratio \( n_{++}/n_+ \) in the discharge chamber and the single ion density. The single ion density can be determined with sufficient accuracy by equating the single ion density to the electron density. The value of the double-to-single ion density ratio can be determined from measurements of the ratio of the double ion current to the single ion current in the exhaust beam \( (I_{++}/I_+) \), and the equation

\[
n_{++}/n_+ = \frac{I_{++}}{I_+} \cdot \left( \frac{2}{2} \right) \cdot \left( \frac{2}{2} \right).
\]

The quantity \( \frac{2}{2} \) accounts for charge and Bohm criterion velocity differences between double and single ions.

The quantity \( \frac{I_{++}}{I_+} \) was measured using a mass spectrometer. The methods used for data acquisition and analysis using such a device are described in Reference 20 for the 15 cm thruster data and in Reference 19 for the 30 cm thruster. The results obtained are listed in the last row of Table II. They show, for example, a double-to-single ion current ratio of 1.3 for the 30 cm thruster, which is higher than that measured for the 15 cm thruster.
anode voltage case. The general trend observed from these data is that an increase in power input \((I_{\text{arc}} \times V_{\text{arc}})\) for a certain thruster configuration results in an increase in the ratio \(I^{++}/I^+\).
RESULTS AND DISCUSSION

The values of the average plasma properties, listed in Table II, are observed to vary over large ranges. For example, the average Maxwellian electron temperature ranges from a low value of 3.3 eV to a high value of 12.2. Similarly, the average primary-to-Maxwellian electron density ratio varies from 0.02 to 0.50. This large variation is considered sufficient to allow a general decision to be made about the accuracy of the model. Comparisons of the experimental and theoretical values of the double-to-single ion density ratio have been used to verify the model's accuracy because this quantity \( \frac{n_{++}}{n_+} \) was determined experimentally. The theoretical and experimental values of the double-to-single ion density ratio are plotted as a function of propellant utilization in Figures 5 and 6. The curves labeled "THEORETICAL" result from predictions made by the model using the "Input Parameters" listed in Table II. The curves labeled "EXPERIMENTAL" result from measurements of the ratio \( \frac{i^{++}}{i^+} \) made using the mass spectrometer. The trends exhibited by the THEORETICAL and EXPERIMENTAL curves are very similar, and the agreement between the THEORETICAL and EXPERIMENTAL values of the double-to-single ion density ratio is good for plasma physics work with the average error being 3%. The maximum error of 40% is observed at low double-to-single ion density ratios in the 30 cm thruster. These error values indicate the model is accurate over a wide range of plasma conditions and thruster configurations.

Since the model has been shown to be accurate in its predictions of the double-to-single ion density ratio over a wide range of conditions, there is a distinct possibility that the specie densities and
Figure 5  Double-to-Single Ion Density Ratio in a 15 cm Diameter Thruster
Figure 6 Double-to-Single Ion Density Ratio in a 30 cm Diameter Thruster
reaction rates used to predict the double-to-single ion density ratio are also accurate. The remainder of this section will examine the model's predictions of these specie densities and reaction rates. These quantities are listed in Table III along with the thruster operating variables and the model's input parameters which were reproduced from Table II.

The section in Table III titled "Calculated Normalized Densities" lists the model's predictions of the normalized densities of the states considered in the model where the normalized density of some specie is defined as the specie density divided by the total heavy particle density. The sum of the normalized densities for any thruster condition should therefore equal unity. Table III shows, for example, that the 15 cm diameter thruster operating with SERT II grids at 37 V anode voltage would be predicted to have 68% neutral ground state atoms, 19% neutral resonance state atoms, 6.9% singly charged ground state ions and .2% doubly charged ground state ions. The normalized density of the single ions agrees fairly well in all cases with the 10% value quoted as typical in the literature. As expected the neutral ground state atoms are the most numerous.

These normalized density trends can be explained in terms of variations of plasma properties. For example, the normalized single ion density increases with increasing power input \((I_{\text{arc}} \times V_{\text{arc}})\) to the thruster in all cases. This occurs because an increase in the values of the volume averaged plasma properties causes the ratio of the production rate of single ions to the total neutral density to increase. The increase in the ratio indicates a smaller total neutral density i.
## Table III

### Predicted Density and Reactor Wetted Area

<table>
<thead>
<tr>
<th>Reactor Diameter (in)</th>
<th>Wetted Area (ft²)</th>
<th>Reactor Volume (ft³)</th>
<th>Reactor Wetted Area (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>25</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>35</td>
<td>30</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>40</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>45</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>50</td>
<td>45</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>

Note: The values in parentheses are the diameter of the reactor wetted area with a factor of safety of 1.5.

### Measured Value

- Reactor Volume (ft³)
  - Measured Value: 30

### Calculated Value

- Reactor Wetted Area (ft²)
  - Calculated Value: 25
needed to maintain a specified single ion density and so the normalized single ion density increases as previously observed.

The last section in Table III shows the calculated production rates for singly and doubly charged ions through the various intermediate states. These production rates have been normalized by the total production rate of the specie indicated. The fraction of the associated interactions effected by the primary electrons is indicated in parenthesis. For example, at the 15 cm thruster's 37 V, SERT II grid operating point, 59% of the single ions are produced as a result of electron interaction with neutral ground state atoms and 28% resulted from electron bombardment of neutral resonance state atoms. The neutral ground state-to-single ionic state interactions were induced by primary electrons 23% of the time and by Maxwellian electrons the remainder (77%) of the time.

Thruster performance is determined primarily by the mechanism for the production and loss of single ions. The production of these ions is, according to this model, quite dependent on the neutral metastable and neutral resonance states which are ignored in most other analyses. The manner in which single ions are produced however differs a great deal between the two thrusters. In the 15 cm thruster most of the single ions are produced as a result of Maxwellian electron bombardment while primary electrons are unimportant because of their low densities. This indicates that for 15 cm thruster operation the primary electron region is the important reaction region because it is the region where high densities of high energy Maxwellian electrons occur. In the 30 cm thruster, however, relatively high primary electron densities exist and since the Maxwellian electron temperature is low most of the single ion
production results from primary electron bombardment. So for the 30 cm thruster the primary electron region is the important reaction region because it contains high densities of high energy primary electrons.

Table III indicates in all cases a large percentage of the double ions are produced from single ions. This is as one would expect because the minimum energy required to produce a double ion from a single ion is 18.7 eV while 29 eV is required to produce a double ion from a neutral ground state atom. As the power input to the thruster increases the number of electrons with energies greater than 29 eV increases causing the relative importance of the neutral-to-double transition to increase. The least energy is required for the production of double ions via singly ionized metastable states, but the densities of these states are so low that this production mechanism is unimportant.
SIMPLIFIED MODEL

In the previous section it has been shown that most double ions are produced as a result of electron bombardment of single ions. In order to simplify the analysis of the "Theoretical Model" section the other intermediate states for double ion production can therefore be ignored with no significant loss in the accuracy of the double ion density calculations. In the simplified model presented here the approximation is made that the total rate of production of double ions equals the rate of production of double ions from single ions. This production rate is given by:

\[ R_{p}^{++} = R_{p+}^{++} = n_{+}^{*} \left[ n_{pr}^{*} p_{+}^{++}(r_{pr}) + n_{mx}^{*} q_{+}^{++}(T_{mx}) \right] \Psi . \]  

(32)

The total loss rate of double ions is given by the equation

\[ R_{L}^{++} = \frac{n_{++}^{*} v_{++}^{*} A}{F_{++}} . \]  

(33)

Equating the loss and production rates and then solving the resultant equation for the double ion density one obtains

\[ n_{++}^{*} = n_{+}^{*} \frac{\left[ n_{pr}^{*} p_{+}^{++}(r_{pr}) + n_{mx}^{*} q_{+}^{++}(T_{mx}) \right] \Psi}{v_{++}^{*} A / F_{++}} . \]  

(34)

The approximation \( n_{e}^{*} = n_{+}^{*} \) can now be used and Equation (18) can be substituted for the double ion velocity to obtain the following equation.
The double ion density can now be determined for a given thruster operating condition using this equation and the plots of $P^{++}(\xi_{pr})$ and $Q^{++}(T_{mx})$ found in Figure 7 if the volume averaged plasma properties and the uniformity factor $F_{++}$ are known. This equation will consistently predict lower double ion densities than the complete model since it ignores the production of double ions from neutral states and the singly ionized metastable states, but this error should generally be small. The error will be greatest for plasmas with high energy electrons which can produce double ions directly from neutral states.

The last section of Table III can be used to determine the magnitude of this error for the 11 cases considered in this study. Since the simplified model considers only the single-to-double transition the error associated with this approximation can be determined from the listed value of the percentage of double ions produced from single ions. For example, for the 15 cm-SERT II grid - 37 V anode voltage case the percentage of double ions produced from single ions is 78%. This means that the value of the double ion density predicted by the simplified model would be 78% of that predicted by the complete model. Examination of Table III indicates the double ion densities calculated using the simplified model will agree well with the complete model's predictions for all the 30 cm thruster conditions because in these cases the percentage of double ions produced from single ions is greater than 97%. 

$$n_{++} = n^*_e \frac{F_{++}}{\left[ T_{mx} q \left( 1 + \frac{n^*_e}{n^*_e} / \left( \frac{m_1}{m_1} \right) \right) \right]}$$

(35)
Figure 7 Rate Factors for Hg$^+$ + Hg$^{++}$
In each of these cases few electrons have energies in excess of 29 eV (the minimum energy required for the neutral-to-double transition). The simplified model will however, according to Table III, yield results which are generally low for the 15 cm thruster data (e.g. 30% low for the SERT II grid - 42 V anode voltage condition) because in these cases sufficiently high Maxwellian electron temperatures exist to cause a relatively large percentage of the electrons to have energies in excess of 29 eV.

The most accurate way to determine the values of the average plasma properties required in Equation (35) would be to conduct a Langmuir probe survey of the discharge chamber under consideration to determine the plasma properties at many different points and to then use this information in Equations (10) to (15) and (21) to determine average plasma properties. The collection of the plasma property data is however costly and time consuming. For this reason average plasma property correlations were developed. The correlating parameters used are composed of thruster operating parameters (e.g. $I_{arc}$) and geometric properties (e.g. $V/A$). Using the Maxwellian electron temperature data listed in Table III, for example, one obtains the correlation presented in Figure 8. The terms used in the correlating parameter are defined in Table III. The correlating parameter used in Figure 8 was determined by trial and error. The shape of a curve through the resultant data points was picked to match the trends observed in the data points. For example, the slope of the curve in the neighborhood of the low Maxwellian electron temperature points is seen to decrease. This agrees with the trend observed in the data and also agrees with a prediction, based on inelastic collision cross section data, which says a lower
Figure 8 Maxwellian Electron Temperature Correlation
bound on the Maxwellian electron temperature should exist roughly in
the neighborhood of 5 eV (1).

The correlation for the primary electron energy is shown in
Figure 9. The correlating parameter contains the quantity \( n_c \) which
is the corrected propellant utilization. The corrected utilization
was used in the correlating parameter, instead of the measured pro-
pellant utilization, because a better fit of the data points resulted
from its use. The propellant utilization (\( n \)) of an ion thruster
depends upon the plasma properties, the effective open area for the
loss of neutral atoms through the grids (\( A_0 \)) and the effective open
area for the loss of ions through the grids (\( A_+ \)). The propellant util-
ization is defined by the equation

\[
n = \frac{n_+ v_+ A_+}{n_+ v_+ A_+ + n_{ot} v_0 A_0} = 1 - \frac{n_{ot} v_0 A_0}{n_+ v_+ A_+}
\]  

(36)

where \( n_{ot} \) is the total neutral atom density. The primary energy
(and other average plasma properties) of a given thruster correlate
with the propellant utilization as defined above, but correlation be-
tween grid sets having different values of the ratio \( A_0/A_+ \) is poor.
The problem caused by the utilization's dependence upon grid sets can
be corrected by eliminating the ratio \( A_0/A_+ \) from Equation (36) and then
substituting in its place the value of the ratio \( A_0/A_+ \) for some stand-
ard grid set. The resultant quantity is the corrected utilization and
is defined by the equation

\[
c = 1 - \frac{n_{ot} v_0}{n_+ v_+} \left[ \frac{A_0}{A_+} \right]_{\text{standard}}
\]  

(37)
Figure 9: Primary Electron Energy Correlation
The open area for the loss of ions \( A_+ \) from a thruster is proportional to the open area fraction of the screen grid \( (\phi_s)_1 \). Equilibrium flow theory \( (1) \) can be used to determine that the open area for the loss of neutral atoms \( A_o \) is proportional to the quantity \( (\phi_s \phi_a)/(\phi_s + \phi_a) \) where \( \phi_a \) is the open area fraction of the accelerator grid. These two approximations can be used to define the ratio \( A_o/A_+ \) as follows

\[
\frac{A_o}{A_+} = \left( \frac{\phi_s \phi_a}{\phi_s + \phi_a} \right) / \phi_s = \frac{\phi_a}{\phi_s + \phi_a}.
\]

If Equations (36) - (38) are combined the following result is obtained,

\[
\eta_c = 1 - .5(1 - n) \frac{\phi_s + \phi_a}{\phi_a},
\]

where the constant ".5" defines \( \phi_a/(\phi_s + \phi_a) \) for the standard grid set.

Figures 10-12 show correlations for the remaining input parameters. These correlations were developed by trial and error in a manner similar to that used to obtain those in Figures 8 and 9. It should be noted that the correlation in Figure 11 is for the quantity \( n_{pr}[4/A]^{-1.5} \) not the primary electron density \( (n_{pr}) \).

It should be understood that the correlations of Figures 8-12 are based on data obtained from strongly divergent magnetic field thrusters. The average plasma properties predicted using these figures may be inaccurate for other types of thrusters (e.g. multipole or radial field thrusters). Therefore Langmuir probe surveys should be made for these other types in order to obtain good estimates of the average plasma properties and hence accurate predictions of the double ion density.
Figure 10 Electron Density Correlation
Figure 11 Primary Electron Density Correlation
Application of the simplified model can be best demonstrated through an example. Consider a 15 cm thruster operating at the conditions defined by the first section of Table IV. The corrected utilization ($\eta_c$) is first calculated using Equation (39) and a value of 68% is obtained. Next the correlating parameters are calculated. For example, the value of the correlating parameter

$$
\left( \frac{I_{arc}}{V_{arc}} \right)^2 \left( \frac{I_{beam}}{V_{arc}} \right)^{1/3} (\psi/A)^{-1/4}
$$

used in Figure 8 is 8.3 volts cm$^{-1/4}$ m$^{-1/4}$. This value indicates the average Maxwellian electron temperature would be 4.6 eV. The remainder of the average plasma properties were determined in a similar manner. The results obtained are listed in the second section of Table IV. Using the values of the primary electron energy and the Maxwellian electron temperature one can enter Figure 7 and determine $P^{++}(22 \text{ eV})$ and $Q^{++}(4.6 \text{ eV})$. These quantities, together with the average densities, the uniformity factor and the volume-to-surface area ratio for this thruster are then substituted into Equation (35) to obtain the double ion density as shown in the last section of Table IV. The double ion density calculated using the simplified model is $6.2 \times 10^6 \text{ cm}^{-3}$ while the value calculated using the complete model is $5.2 \times 10^6 \text{ cm}^{-3}$. The major reason for the discrepancy is that the electron temperature in Table IV (4.6 eV) is larger than the value used by the complete model (4.3 eV). This higher electron temperature causes $Q^{++}(T_{\text{mx}}^*)$ to be too large and results in the over-estimate of the double ion density.
Table IV.
Determination of the Double Ion Density Using the Simplified Model

Measured Thruster Variables

(15 cm Thruster)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{arc}$</td>
<td>3. amps</td>
</tr>
<tr>
<td>$V_{arc}$</td>
<td>32.2 volt</td>
</tr>
<tr>
<td>$I_{beam}$</td>
<td>0.499 amps</td>
</tr>
<tr>
<td>$n$</td>
<td>0.68</td>
</tr>
<tr>
<td>$n_c$</td>
<td>0.68</td>
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</tbody>
</table>

Approximate Plasma Properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{mx}$</td>
<td>4.6 eV</td>
</tr>
<tr>
<td>$\xi_{pr}$</td>
<td>22. eV</td>
</tr>
<tr>
<td>$F_{++}$</td>
<td>2.55</td>
</tr>
</tbody>
</table>

$\phi_s = 0.67$
$\phi_a = 0.67$
$\psi/A = 1.4 \text{ cm}$

Calculation of the Double Ion Density

$$n_{++}^* = \frac{(3.51 \times 10^{11} \text{ cm}^{-3})^2 (1.4 \text{ cm}) (2.55)}{[9.6 \times 10^9 \frac{\text{cm}^2}{\text{sec}^2 \cdot \text{eV}} (4.6 \text{ eV}) (1.013)]^{1/3}}$$

$$\times [0.013 (0.55 \times 10^{-7} \frac{\text{cm}^3}{\text{sec}}) + 0.987 (0.23 \times 10^{-8} \frac{\text{cm}^3}{\text{sec}})]$$

$$= 6.2 \times 10^3 \text{ cm}^{-3}$$
An examination of Equation (35) will indicate some general trends which should be considered in the design and operation of electron bombardment thrusters. For example, the double ion density varies linearly with the volume-to-surface area ratio. Therefore if two thrusters have the same average plasma properties the larger thruster will have a higher double ion density. Equation (35) suggests it would be desirable to reduce the electron density since the double ion density is proportional to the square of the electron density. However, making arbitrary adjustments in the plasma properties to reduce the double ion density may have an adverse effect on other aspects of thruster performance which must also be considered. An examination of the effect of electron density on propellant utilization will indicate one of the effects such an adjustment would have. The propellant utilization previously defined in Equation (36), is reproduced below.

\[ n = 1 - \frac{n_{ot}^* v_o A_o/A_+}{n_+ v_+} \]  

(40)

The single ion density \(n_+^*\) can be approximated by the electron density \(n_e^*\). The total neutral density \(n_{ot}^*\) is the sum of the densities of all the neutral species and can be calculated using the equation

\[ n_{ot}^* = n_o^* (1 + n_{int}^*/n_o^* + n_{rt}^*/n_o^*) \]  

(41)

where \(n_{int}^*\) and \(n_{rt}^*\) are the total metastable and resonance states densities. The values of the ratios in Equation (41) can be calculated using equations similar in form to Equation (30). The neutral ground state density can be calculated using the equation
where the ratio \( \left( \frac{n^*_o}{n^*_+} \right) \) again takes a form similar to that of Equation (30). Combining these results into Equation (40) a result of the following form is obtained. "f" is a function of the Maxwellian electron temperature, primary electron energy, primary-to-Maxwellian electron density ratio and the uniformity factor \( F_+ \). The dependence of the propellant utilization on the electron density and thruster parameters is explicitly shown.

\[
n = 1 - \frac{A^*_o/A^*_+}{n^*_e} \frac{V/A}{f(T^*_m, T^*_p, n^*_p/n^*_m, F_+)} \tag{43}
\]

One can see that a reduction in the electron density to reduce the double ion density will also have the undesirable effect of reducing the propellant utilization. However, if some changes in thruster design are made along with a reduction in the electron density the propellant utilization can be held constant while the double ion density is reduced. For example, if a new thruster were being designed one might double the volume-to-surface area ratio by making the thruster larger than its predecessor. It could then be operated at one-half the electron density of the predecessor allowing the propellant utilization to remain constant while exhibiting half the double ion density in accordance with Equation (35).

It might also be desirable to reduce the double ion density of a certain size thruster while maintaining the same propellant utilization. The propellant utilization could be held constant by reducing both the ratio \( A^*_o/A^*_+ \) (which reduces the relative escape rate of neutrals) and
the electron density in a manner that keeps the ratio \( \frac{A_o}{A_i} / n_e^* \) constant. According to Equation (35) this would result in a large reduction in the double ion density which varies as the square of the electron density. The data in Table III for the two 30 cm thruster configurations at 1.5 and 2.0 amps beam current can be used to determine if theory and experiment agree for this method of double ion density reduction. The only difference in these two thruster configurations is the open area fraction of the accelerator grid. The EM accelerator grid has an open area fraction \( \phi_a \) of 45% while the open area fraction for the SHAG accelerator grid is 23%. Both sets have a 69% open area fraction for the screen grid. The value of the ratio \( A_o/A_i \) can be calculated for both grid sets using Equation (38). For the EM grids the ratio \( A_o/A_i \) has a value of .39 while for the SHAG grids the value of the ratio is .25. The change from EM grids to SHAG grids then allowed operation at a given propellant utilization to occur at a lower arc voltage and hence a lower electron densities and energies and as a result lower double ion densities. In this particular case the double-to-single ion density ratios dropped from 4.4% and 6.0% to 2.2% and 2.8% respectively at two different utilizations when the SHAG grids were used. The theoretical model predicted essentially the same quantitative changes.
CONCLUSIONS

A discharge chamber model for an electron bombardment ion thruster has been developed which considers metastable, resonance and ground state atomic and ionic production and loss mechanisms. The model can be used to predict doubly charged ion densities from plasma property information. These calculated double ion densities agree with measured values to within 40% for low values of the double-to-single ion density ratio \( n_{++}/n_+ < 2\% \) and to within 20% for the rest of the data. Correlations, which relate average plasma properties to thruster operating variables such as anode current, can be used to estimate the average plasma properties in strongly divergent magnetic field thrusters when the properties themselves are not available. Singly charged ions are produced, according to this analysis, in significant numbers in two step processes through intermediate metastable and resonance states in addition to direct ionization from the neutral ground state. Doubly charged ions are produced predominantly via the singly ionized ground state with direct ground state neutral-to-double ion production becoming more significant in plasmas with high Maxwellian electron temperatures and primary electron energies. A simplified model which considers only the singly ionized ground state in double ion production can be used to predict double ion densities that agree with the complete model's predictions to within 5% when primary electron energies and Maxwellian electron temperatures are less than 29 eV and 5 eV respectively. The recent experimental observation\(^{(12)}\) that the use of small hole accelerator grids in conjunction with lower anode voltages provides a means for reducing double ion densities in thrusters, without degrading performance, is supported by the model.
REFERENCES


APPENDIX A

The computer program "HG", which can be used to predict the densities of excited atomic and ion states considered in the complete model, is listed below. The input parameters needed by this program can be approximated using the correlations in Figures 8-12. More accurate input parameters can be determined using the computer program "PROP", listed in Appendix B, and data obtained from a Langmuir probe survey of the discharge chamber. The computer program "HG" uses the equations developed in the "Theoretical Model" section and carries out the calculations in the manner suggested at the end of that section. Comment cards are included in the listing to indicate what calculations, etc. are to be carried in each section.

Values of the functions $P_{\alpha}(\xi_{pr})$ and $Q_{\alpha}(T_{mx})$ are listed immediately after the computer program listing. The particular initial state ($\alpha$) and final state ($\gamma$) are indicated in the last twenty columns. For example, the label "HGM-HG+ 3PO" indicates the initial state for the reaction is the $6^3P_o$ metastable state and the final state is the singly ionized ground state. The first seven cards listed with a particular identifying label contain the values for $P_{\alpha}(r_{pr})$ while the second seven list values for $Q_{\alpha}(T_{mx})$. 
C ELECTRON DENSITIES FOR ITERATION ON MASS FLOW RATE
C
READ (4,140) (TMP(L),L=1,NS)
GO TO 109
107 ELNFS=TEMP(JJ)
GO TO 109
108 WRITE (6,139)
109 CONTINUE
XNF(JJ)=ELNFS
EFLD=ELNFS*5/1,F11
FMCONF (44,144,11) VA*ELD*TEMP*DEN
DO 128 II=1,NS
SF=0.
WHITE (4,145) VA*ELDENS,TEMP,PH,PM3,DEN
WRITE (4,146) VOL,OPN,OPC,F1,F2
TOTA=1.,1./DEN
NP=ELDENS,1/0T
NM=ELDENS/181,OPNR
PFM=P0,TEM
CALL YINTE (5,02,5,02,1.,SUMM,1,PSUM)
C
C CALCULATION OF METASTABLE/NEUTRAL GROUND STATE RATIO
C
DO 112 J=1,2
DO 110 I=1,2
DO 110 K=1,2
SF(I,K)=5*X00M(J,I,K)
CALL TINTE (SF,SSF,1.,SUMM,PSUM)
DO 111 I=1,2
DO 111 K=1,2
SF(I,K)=5*X00M(J,I,K)
CALL TINTE (SF,SSF,1.,SUMM,PSUM)
X00M=SSF,1.,V0,4.*WA=V0,4.*X00M
X00M=SUM,(V0,4.*SUM,SSF,1.,SUMM)
WHITE (6,147) TN(J),PAH01(J)
WHITE (6,148)
WHITE (6,149)
WHITE (6,149)
WHITE (6,149)
WHITE (6,149)
WHITE (6,149)
X00M=SSF,1.,SUM,PSUM
X00M=SSF,1.,SUM,PSUM
PAX=LOOS,SSF,1.,SUM,PSUM
PAX=LOOS,SSF,1.,SUM,PSUM
XX=SUM,1.,SUM,PSUM
XX=SUM,1.,SUM,PSUM
WHIT (6,149) XX=SUM,1.,SUM,PSUM
112 CONTINUE
X00M=PAH01(J),PAH01(J),100.
C
C CALCULATION AND ITERATION FOR RESONANCE/NEUTRAL, SINGLE/NEUTRAL, C NEUTRAL DENSITY
C
JNL=1
IF JNL=0
T2=1+2
K=11-1
IF (II,I,T2) XX=(2*PSU(44,II,II)-SSU(II,J,J)*100.
XNF=1.*XX*XX
XX=II-10.
C
C GUESS NEUTRAL DENSITY
C
113 XNF=1.DEN/X0/1000000.
CALCULATE RESONANCE/NEUTRAL RATIO

NO 117 J=1+1
    IF (FLAG,FN+1) GO TO 116
    NO 114 I=1+Z
    NO 114 X=1+2
    SX(I,J,X,J,K)
    SK(I,K)=SK(K,J,J,K)
    CALL YINTEG (SK,J,J,K,XSM(J),XPSM(J))
    NO 115 I=1+2
    NO 115 X=1+2
    SX(I,J,X,J,K)
    SK(I,K)=SK(K,J,J,K)
    CALL YINTEG (SK,J,J,K,XSM(J),XPSM(J))
    CONTINUE
    NO 116 X=3+3*J/J
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J),SUM)
    IF (FLAG,FU+1) GO TO 118
    CALL SUMIT (SX,J,J,K,XSM(J),XPSM(J),SUM,DEC)
    X(2)=0,
    G(2)=0,
    CALL SUMIT (SX,J,J,K,XSM(J),XPSM(J),SUM,DEC)
    IF (FLAG,FU+1) GO TO 119

C CALCULATION OF SINGLE/NEUTRAL RATIO

VP=SQRT(TEP**4,KNJ**2*KNP*KNM)**1.0
    VV=V/V
    CALL SUMIT (SX,J,J,K,XSM(J),XPSM(J),SUM,DEC)
    CALL YINTEG (SK,J,J,K,XSM(J),XPSM(J),SUM,DEC)
    RN=(SUM+SUM2+TSJ)/KLOS**2
    RN=SUM+SUM2+TSJ
    RN=KLOS**2

C CHECK OF FUNCTION IN GUESS OF NEUTRAL DENSITY

WNP=GM**4*(1.0+KNP)*WNP0
    KJ=KJ
    XJ=KJ
    J=J
    JK=J
    IF (FLAG,G) GO TO 113

C NEUTRAL GROUND STATE ATOM DENSITY

X=KJ*SM**2/XXJ
    X=KJ
    XXJ=XXJ
    XJ=KJ
    J=J
    JK=J
    IF (FLAG,G) GO TO 113

C PRINT OUT RESONANCE ATOM DENSITY RATIO

NO 118 J=1+1
    X=1+1
    X=1+1
    X=1+1
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))
    CALL YINTEG (SX,J,J,K,XSM(J),XPSM(J))

```
WRITE (6,157) VALL*PHLOSS(J)*ASMM(J)*XPSMN(J)*SUMM*PRSHG
1     2640
(1M) 6.250
TT=VALL*PHLOSS(J)*ASMM(J)*SUMM
AA=VALL/TT
AH=ASMM(J)/TT
AC=SUMM/TT
WRITE (6,151) AA+AD+AH+AC
CONTINUE

C PRINT OUT SINGLE ION DENSITY RATIO

WRITE (6,154) MHD0
SIN(J+II)+RNI0*NO
PRIF=FR(H,C)
WRITE (6,161)
PRIF=FR(H,C)
PRIF=FR(H,E)
WRITE (6,167) SUM1*PRSUM1*TSUM1*PRIF*TSS*PRIG
SE=SFV*SUM1*HPSUM1*TSUM1*PRIF*TSS*PRIG
TT=SUM1*TSUM1*TSS
AA=SUM1/TT
AE=TSUM1/TT
AC=TSUM1/TT
WRITE (6,154) AA+AH+AC
WRITE (6,162)
WRITE (6,167) WLOSS*SUM2*PRSUM2*TSUM2*PRIM
TT=WLOSS/TT
AE=WLOSS/TT
AC=TSUM2/TT
WRITE (6,154) AA+AH+AC

C CALCULATION OF SINGLE PLTA/KEUTR KeR A R A T I O

C

121 123 J=1,2

C

122

C

123

C CONTINUE

C
```

```
WRITE (6,113) XX
WRITE (6,114) X(1),M(1)
DO 106 I=1,25
106 WRITE (6,115) X(1),M(1)
WRITE (6,116) (PH[1],I1=1,10)
WRITE (6,117) (G(I1),PH[1],I1=1,10)

C PLOT OF UTILIZATION VERSUS DISCHARGE POWER
C
IF (1,6,6,0,.E0,13) GO TO 107
GO TO 111
107 IF (1,6,6,0,.E0,13) GO TO 104
DO 109 L=1,25
108 UTIL(I)=Y(I)
GO TO 111
109 DO 110 L=1,25
110 CONTINUE

CALL MAPM (2,XX,YPY,1),X1M,M1L,M1V,YM1,M1X,M1Z)
112 CALL MAPM (2,XX,YPY,1),X1M,M1L,M1V,YM1,M1X,M1Z)
113 CALL MAPM (2,XX,YPY,1),X1M,M1L,M1V,YM1,M1X,M1Z)
114 RETURN
C
CEND
SUBROUTINE XI_INTERP (X,Y,X,YIN,NINT,INTERP)

INT 10
INT 20
INT 30
INT 40
INT 50
INT 60
INT 70
INT 80
INT 90
INT 100
INT 110
INT 120
INT 130
INT 140
INT 150
INT 160
INT 170

SUBROUTINE YINTEGRATE (SIGMA,SIGN,PH,PUSH,SUMP)

NTG 10
NTG 20
NTG 30
NTG 40
NTG 50
NTG 60
NTG 70
NTG 80
NTG 90
NTG 100
NTG 110
NTG 120
NTG 130
NTG 140
NTG 150
NTG 160
NTG 170
C
CEND
```
MAXWELLIANS ELECTRONS

DO 102 I=1,Z1
   SX(I)=SIGMA(S2I)
   CALL AITKEN (SF, SIG, SX, I, T, TH)
   TSUM=PROP+TR
   SUM=TSUM+PSUM
   PSUM=PSUM/SUM
   RETURN

END

SUBROUTINE SUMIT (SIG, SIG+, PROP+, TSUM, SUM, PSUM+)
   TSUM=0.
   DO 102 I=1,N
      DO 101 J=1,N
      DO 101 K=1,P
         SX(J,K)=SIG(I,J,K)
      101 SX(J,K)=SIG(I,J+K)
      CALL YNTFG (SX(I), STR, PRSUM(I), SUM(I), PSUM(I))
   102 TSUM=TSUM+SUM(I)
   RETURN

C
C SUBROUTINE AITKEN (X,Y,N,K,XH,YH)
C
C AITKEN INTERPOLATION SUBROUTINE
C
C CALLING SFUNCF***
C CALL AITKEN(X,Y, N,K, XH, YH)
C X IS A ONE DIMENSIONAL ARRAY OF INDEPENDENT
C VARIABLE (INCREASING OR DECREASING)
C Y IS A ONE DIMENSIONAL ARRAY OF DEPENDENT
C VARIABLE
C N IS NO. OF X,Y PAIRS
C K IS degree OF INTERPOLATING POLYNOMIAL (MAX = 10)
C XH & YH are INDEPENDENT VARIABLE ARGUMENT
C YH IS INTERPOLATED RESULT
C
C **********************************************************************
C
C TYPE, DIMENSION AND LABELLED COMMON STATEMENTS
C
C DIMENSION X(N), Y(N), XX(11), YY(11)
C X(1)=1.
C IF (X(N)-X(1)) 110,101,101
C 101 IF (Y(1)-Y(1)) 102,102,103
C 102 L=0
C GO TO 119
C
C 103 IF (X(N)-XH) 104,104,105
C 104 L=L+1
C GO TO 119
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<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
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APPENDIX B

The computer program "PROP" is listed below. It can be used to determine the values of the volume averaged plasma properties and the uniformity factors needed by the computer program "HG." The data needed to determine these quantities is obtained from a Langmuir probe survey of the discharge chamber in which the plasma properties are determined at many different locations within the chamber. This data is used to numerically evaluate Equations (10) to (15) and (21) yielding the volume averaged plasma properties and the uniformity factors. Comment cards are included in the computer program to indicate the purpose of each section. A CDC 6400 computer will use approximately thirty seconds of Central Processor time to evaluate five sets of data obtained from five Langmuir probe surveys.
PROGRAM PROG (INPUT,OUTPUT,TAPS=INPUT,TAPE6=OUTPUT)

THIS PROGRAM CALCULATES THE AVERAGE PROPERTIES
PLS=RADIAL POSITION OF LAMINAR PROPE POINTS (1-CENTERLINE)
POS=POS. OF OBSERVED DATA POINTS IF IFLAG=4
POST-AXIAL POSITION OF LAMINAR PROPE POINTS (1-UPSTREAM PNT.)
N=NO. OF TESTED PSI SPT
IFLAG=NO. OF RADIAL POINTS
IFLAG=1 IF ONE WANTS TO PRODUCE A SET OF POINTS UPSTREAM
IFLAG=2 POINTS PRODUCED AT THE RANUS
DISTANCE FROM SCREEN TO POINT WHERE THE GENERATED SET IS TO
AF PLAYED
KAP-LINE OF PRIMARY ELECTRON HEATON AT CENTERLINE
DIMENSION XND(70), YND(70), Z(70), V(70), Z1(21), W(21), F(70)
DIMENSION AI(70), AQ(70), AN(70), ANM(70), AI(11), C(11), X(11), Y(11), Z(11), POS(10), A(10)
DATA POS(0), X(1), Y(1), Z(1), XH(1), YH(1), ZH(1)
DATA PLS /0.,1.,2.,3.,4. /
DATA PLS /0.,1.,2.,3.,4. /
DO 103 I=1,NT
103 READ (5,126) XH(I),YH(I),ZH(I),XH(I),YH(I),ZH(I)
IF (IFLAG,.NE.0) NN=2*NN+1
READ (5,127) POSZ(I), I=1,NN
DO 104 J=1,NN
104 YH(I), ZH(I)
DO 105 K=1,NN
105 YH(I), ZH(I)
CALL ATKN (PLS(I),XH(I),YH(I),ZH(I),POSZ(I),AI(I))
CALL ATKN (PLS(I),XH(I),YH(I),ZH(I),POSE(I),AI(I))
CALL ATKN (PLS(I),XH(I),YH(I),ZH(I),POSE(I),AI(I))
DC 107 J=1*NT
  T(J)=AT(J)
  Z(J)=AZ(J)
  XNP(J)=ANP(J)
  107 XNW(J)=XNP(J)
  WHTE (I=130) (I=IN)XNP(IN)XNW(IN)N=1*NT
  109 IF (IFLAG,F=0.0) GO TO 114

C     CALCULATION OF THE EXTRA SET OF POINTS
C
N=1
DO 110 I=1,4
  N=I
  X=0
  IF 109 J=1*NT*4
  L=1
  Y(J)=T(J)
  YP(J)=XNP(J)
  YW(J)=XNW(J)

  109 YZ(L)=Z(J)
  K=NT+1
  CALL AITKEN (POZ+Y+L+U*NT+AT(K))
  K=NT
  CALL AITKEN (POZ+Y+L+U*NT+AT(K))
  CALL AITKEN (POZ+Y+L+U*NT+AT(K))
  CALL AITKEN (POZ+Y+L+U*NT+AT(K))
  CALL AITKEN (POZ+Y+L+U*NT+AT(K))
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0
  IF (AT(K),LT,0.0) AT(K)=0.0

110 CONTINUE

C     RESHUFFLING OF POINTS
C
N=N-1
  M=M-1
  IF 109 J=1*NT*4

111 POSZ(J)=POSZ(J-1)

END1=1
  PCSP(1)=FIN
  NTAT=4
  NDA=NT/4

DC 112 I=X=NT+1
  K=NT+4-1
  J=4
  X=5
  Z(J)=J(J)
  7(J)=7(J)
  XNP(J)=XNP(J)

XNP(K)=XNP(K)
  ATW=NT

DC 113 I=NT+4
  I=NT+4
  T(J)=AFF(J)
  2(J)=AZ(J)
  XNP(J)=ANP(J)

114 XNW(J)=XNP(J)

C     START OF THE CALCULATION
C
DC 111 J=1*50
  ++[111]+1
  ++[111]+1
  ++[111]+1
THIS PRODUCES AN ARRAY OF FFT AT CONSTANT Z POINTS

DIM 111, J=1, K

M=IP(T)(M)M

CALL AIJ+K (I, K, Z1 + 2, Z1 + 2)

CALL AIJ+K (I, K, Z1 + 2, Z1 + 2)

IF (M=1) GO TO 115

IF (M=1) GO TO 116

115

VIF (J) = .25 IF (J) + 2, R3 IF (J)

A(J) = W(J) IF (J)

B(J) = W(J)

116

VIF (M) = 0

117

FF (M) = 0

INTEGRATION OF F (EVALUATING G11)

M=4

N=1


A(J) = 1

IF (J=1.5) GO TO 119

IF (J=1.5) CALL ITAY (M*3, 3, W, W, W, W, W, W)

118


IF (J=1.5) GO TO 119

IF (J=1.5) CALL ITAY (M*3, 3, W, W, W, W, W, W)

119


C(J) = 1

IF (J=1.5) GO TO 120

IF (J=1.5) CALL ITAY (M*3, 3, W, W, W, W, W, W)

120


C(J) = 1

IF (J=1.5) GO TO 121

IF (J=1.5) CALL ITAY (M*3, 3, W, W, W, W, W, W)

121


C(J) = 1

IF (J=1.5) GO TO 122

IF (J=1.5) CALL ITAY (M*3, 3, W, W, W, W, W, W)

122


VIF (J) = .25 IF (J) + 2, R3 IF (J)

A(J) = W(J) IF (J)

B(J) = W(J)

CONTINUE
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
OF=CH=HC  
CALL OTGI (POSZ*C*V+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)  
CALL AIYKEN (POSZ*V+NO*+*0+1+HC)  
QCA=QC=HC  
CALL OTGI (POSZ*V+4+HC)
CALL CHECK (V + 4 * NFLAG)
Q(1) = N + 2 * N + 1 + V
Q(2) = N + 2 * N + 1 + V
Q(3) = N + 2 * N + 1 + V
Q(4) = N + 2 * N + 1 + V
CALL OTFG (NTH + 2 * N + 1 + V)
CALL AITKEN (NTH + 2 * N + 1 + V)
DO 105 IJ = 1, 4
Q(IJ) = Q(IJ) * P(IJ)
CALL QTGF (Q(IJ) + P(IJ))
CALL AITKEN (Q(IJ) + P(IJ))
A = A * 2 * T
A = A * 2 * T
IF (IFLAG .NE. 1) GO TO 10A

CATHODE POLE PIECE SECTION

CALL AITKEN (DTH * XI + N + N + CZU + P(1))
CALL AITKEN (DTH * XI + N + N + CZU + P(2))
CALL AITKEN (DTH * XI + N + N + CZU + P(3))
CALL AITKEN (DTH * XI + N + N + CZU + P(4))
CALL CHECK (P + 4 * NFLAG)
P(5) = P(4) / 4,
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL AITKEN (DTH * V3 + V1 + V2 + V3 + V4)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL AITKEN (DTH * V5 + V6 + V7 + V8 + V9)
CALL CHECK (V + 4 * NFLAG)
V(5) = V(4) / 4,
107 FORMAT (215+4+10,4)
108 FORMAT (2X, -91, (CV-1)=x11.4)
109 FORMAT (2X, 40!NEGATIVE VELOCITY EXTRAPOLATION OCCURRED!5*7H)
110 FORMAT (10X,A(?10,3,2A))
111 FORMAT (PF10.5)
112 FORMAT (3F10.5 )
113 FORMAT (3F10.5 )
114 FORMAT (3F10.5 )
115 FORMAT (2X, 3HF1=+I0.4+2X, 3HF2=+I0.4+2X, 13H+I+ FACTOR=+F10.4)
116 FORMAT (2X, 39!NEGATIVE RENSITY EXTRAPOLATION OCCURRED!5*7H)
C
END