USER'S GUIDE TO COMPUTER PROGRAM CIVM-JET 48
TO CALCULATE THE TRANSIENT STRUCTURAL RESPONSES OF
PARTIAL AND/OR COMPLETE STRUCTURAL RINGS TO
ENGINE-ROTOR-OBJECT IMPACT

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March 1976

Prepared for
AEROSPACE SAFETY RESEARCH AND DATA INSTITUTE
LEWIS RESEARCH CENTER
NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
CLEVELAND, OHIO 44135

NASA Grant NGR 22-009-339
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Provisions are made in the CIVM-JET 4B code to analyze the responses of 2-d structures which are subjected to impact by from 1 to 6 fragments each with its own size, mass, mass moment of inertia, translational velocity, and rotational velocity. The effects of friction between each fragment and the impacted structure are included.
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OF PARTIAL AND/OR COMPLETE STRUCTURAL RINGS
TO ENGINE-ROTOR-FRAGMENT IMPACT

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NASA Grant NGR 22-009-339
FOREWORD

This report has been prepared by the Aeroelastic and Structures Research Laboratory (ASRL), Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, Massachusetts under Grant No. NGR 22-009-339 from the Lewis Research Center, National Aeronautics and Space Administration, Cleveland, Ohio 44135. Mr. Solomon Weiss, Mr. Robert D. Siewert, and Mr. Ray Maqa of the Lewis Research Center served as technical monitors.

The cooperation and helpful suggestions of Mr. Weiss, Mr. Siewert, and Mr. Maqa throughout this research program are much appreciated.

The authors wish to acknowledge appreciatively the help and advice of a former colleague Dr. Richard W-H Wu now with the Nuclear Energy Division of the General Electric Co. Dr. Wu developed the JET 3 series of computer codes and the preliminary version of the present program.

The use of SI units (NASA Policy Directive NPD 2220.4, September 14, 1970) was waived for the present document in accordance with provisions of paragraph 5d of that Directive by the authority of the Director of the Lewis Research Center.
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Described in this report is a user-oriented computer program CIVM-JET 4B to predict the large-deflection elastic-plastic structural responses of fragment-impacted single-layer: (a) partial-ring fragment containment or deflector structure or (b) complete-ring fragment containment structure. These two types of structures may be either free or supported in various ways. Supports accommodated include: (1) point supports such as pinned-fixed, ideally-clamped, or supported by a structural branch simulating mounting-bracket structure and (2) elastic foundation support distributed over selected regions of the structure. The initial geometry of each partial or complete ring may be circular or arbitrarily curved; uniform or variable thicknesses of the structure are accommodated. The structural material is assumed to be initially isotropic; strain hardening and strain rate effects are taken into account.

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SECTION 1
INTRODUCTION

The CIVM-JET 4B computer program is an addition to the series of computer programs which are intended to be made available to the aircraft industry for possible use in analyzing structural response problems such as containment/deflection rings intended to cope with engine rotor-burst fragments. This computer program may also be applicable to crashworthiness problems which are of interest to the automobile and nuclear power plant industries.

The computer program written in FORTRAN IV, permits one to predict the large, two-dimensional, elastic-plastic transient Kirchhoff-type response of a single-layer structural ring, which may be a complete ring or just a partial ring. The ring may be subjected to various restraints and supports and to rigid-fragment impact. The geometrical shape of the structural ring can be simple circular or arbitrarily curved, and the ring may have independently-varying thickness along the circumferential direction. The material behavior may be elastic strain-hardening, and/or strain-rate sensitive.

The program employs the spatial finite-element representation of the ring and the temporal finite-difference solution procedure. For predicting the transient responses of the structural ring to rigid-fragment impact, energy and momentum considerations are employed in an approximate analysis to predict the collision-induced velocities which are imparted to the fragment and to the affected ring segments. The presence of fragment/ring surface friction is taken into account. The pertinent analytical development and the solution method upon which the program is based are presented concisely in Appendix A. The reader is invited to consult Refs. 1, 2, and 3 for background information and a detailed description of this solution procedure.

Section 2 of this report is devoted to describing the general organization and capabilities of CIVM-JET 4B, including (1) the ring structural geometry, supports, elastic restraints, and material properties accommodated, (2) circular
rigid-fragment collision interaction, and (3) the solution procedure. Next, in Section 3, the main program and subprograms of CIVM-JET 4B are described, including a partial list and explanation of the variable names used in the program. The input data and output information are presented in Section 4. A complete FORTRAN IV listing of the program is given in Section 5. Example problems, including input data and the resulting solution data are given in Section 6. Finally, Appendix A summarizes the equations on which the program is based.
SECTION 2

GENERAL DESCRIPTION OF THE CIVM-JET 4B PROGRAM

2.1 Ring Geometry, Supports, Elastic Restraints, and Material Properties

In the present analysis, the transient structural responses of the ring are assumed to consist of planar (two-dimensional) deformations. Also, the Bernoulli-Euler (or Kirchhoff) hypothesis is employed; that is, transverse shear deformation is excluded.

The computer program can treat single layer structural rings. The layer may be of independently-varying thickness; however, the total thickness remains small compared with the circumferential dimension of the ring. The cross section of the layer is assumed to be rectangular in shape, and the centroidal axis is employed as the circumferential reference axis of the ring (Fig. 1).

The structure can be either a complete ring or a partial ring with or without slope discontinuities. The geometric shape of the circumferential axis of the ring can be circular or arbitrarily curved. The outward-normal direction is defined in such a manner that as one moves along the circumferential axis in the positive \( \eta \) direction from an arbitrary reference point, the outward-normal direction is always toward one's left as shown in Fig. 2, where \( XYZ \) is the global reference Cartesian coordinate system with the \( X \)-axis pointing out of the paper. At any point on the circumferential axis, \( \hat{I} \) is a local unit vector defined in the same direction as the \( +X \) axis, \( \hat{a} \) is a unit tangent vector along the positive circumferential axis direction, and \( \hat{n} \) is a unit outward-normal vector which is defined by the right-hand rule as \( \hat{n} = \hat{I} \times \hat{a} \).

Once the positive circumferential direction is defined, the outward-normal direction is then determined accordingly (see Fig. 2). For any given C/D structure, the positive circumferential direction must be chosen so that the positive outward-normal is directed toward the "outside" of the C/D structure such that fragment impact can occur only on the "inside" of the C/D structure.

In the spatial finite-element analysis, the ring is represented by an assemblage of discrete (or finite) elements compatibly joined at the nodal stations. The geometry and nomenclature of a typical arbitrarily curved ring
element are shown in Fig. 3, where the deformation plane is \( \eta, \zeta \) and the coordinates \( \eta \) along and \( \zeta \) normal to the centroidal axis of the beam are employed as the reference coordinates of the beam element. The nodal number is increased along the positive circumferential direction.

The behavior of each finite-element is characterized by a knowledge of the four generalized displacements: \( v, w, \psi = (3w/3\eta) - (v/R), \) and \( \chi = (3v/3\eta) + (w/R) \) at each of its nodal stations where \( v \) and \( w \) are the reference plane displacements in the tangential and normal direction, respectively; \( R \) is the radius of curvature. The displacement behavior within each finite-element is represented by a cubic polynomial in \( \eta \) for the circumferential displacement \( v \) and a cubic polynomial in \( \eta \) for the normal displacement \( w \), anchored to the four generalized nodal displacements at each node (see Appendix A and/or Ref. 1 for further details). For application to arbitrarily curved, variable thickness, ring structures, the finite elements are described by reading in at each nodal station (and each element end, for slope discontinuity) the global Y and Z coordinates, the slope (the angle between the tangent vector and the +Y axis), and the thickness. Within each finite element, the slope is approximated by a quadratic function in \( \eta \) and the thickness of each element is approximated as being piecewise linear between nodes.

As for the support conditions of the structure, the program includes two types of prescribed nodal displacement conditions (see Fig. 4a):

1. Ideally-Clamped \( (v = w = \psi = 0) \)
2. Smoothly-Hinged \( (v = w = 0) \)

and two types of elastic restraints (see Fig. 4b):

a. Point elastically restrained (elastic restoring spring) at given locations (3 directions: normal, tangential, and torsional)

b. Distributed elastically restrained (elastic foundation) over a given number of elements (3 directions: normal, tangential, and torsional).

A global effective stiffness matrix supplied by the elastic foundation and/or the restoring springs will be evaluated in the program from the virtual-work statement, for the case in which the structure is subjected to one or both of these two types of elastic restraints.
The 2-d containment/deflector (C/D) structure may also be regarded as being supported by attachment brackets as depicted, for example, in Fig. 4d. These attachment brackets (or branches) are idealized to behave in the 2-d fashion shown in Fig. 4d. These brackets are modeled as consisting of a single-layer, variable-thickness, 2-d structure of arbitrary initial shape in the plane of the C/D structure, and are connected compatibly with the C/D structure; the other end of each bracket may be supported in any of the common fashions (clamped, pinned, elastic support, etc.). The program provides for a maximum of five support brackets. In the fragment attack, usually only the C/D structure suffers physical impact; however, if the analyst has a physically plausible situation wherein the idealized support bracket could be impacted by a fragment (such a case is depicted in Fig. 4d), the impacted portion must be defined as the main C/D structure since impacts on a branch are not accommodated in this program. It should be noted, however, that the actual brackets in the bracket-supported C/D structure (see Fig. 4c) must undergo 3-d deformation -- this aspect is not accommodated in the present 2-d model. Finally, a support bracket (or branch) may be attached to any nodal station of the main 2-d C/D structure.

The main structure and branches can be of different elastic, or elastic, perfectly-plastic or elastic-strain-hardening behavior. The strain-rate effects of the material can also be taken into account. In the present analyses, the strain-hardening material is accounted for by using the "mechanical sublayer model" (Ref. 1). A useful feature of this model is the inclusion of kinematic hardening and the Bauschinger effect. The strain-rate effect is approximated by assuming that the uniaxial stress-strain curve is affected by strain-rate only by a quasi-steady increase in the yield stress above the "static" value (Ref. 1).

2.2 Fragment/Ring Collision-Interaction Analysis

For analyzing the collision induced transient responses of two-dimensional containment and/or deflector rings and fragment motions, the fragment is idealized as a non-deformable fragment of circular configuration (Fig. 5). The modeled-fragment diameter, mass, mass moment of inertia, and velocity components are specified by the user to correspond with those of the actual fragment.
The process called the collision-impacted velocity method (CIVM) is used for the collision-interaction analysis (see Refs. 1-3). In this process, energy and momentum considerations are employed to predict the collision-induced velocities which are imparted to the fragment and to the impact affected zone of the ring. Also, the following simplifying assumptions are invoked:

1. The collision process is instantaneous and involves only the fragment and the impact-affected zone of the target ring. The impact affected zone is defined as the fraction of the ring that responds to fragment impact instantaneously with momentum changes. The size of the impact-affected zone of the ring can be estimated from the speed of a longitudinal wave or from semi-empirical data.

2. In an overall sense, the fragment is treated as being rigid but at the "immediate contact region" between the fragment and the struck ring the collision process is regarded as acting in a perfectly elastic ($e = 1$), perfectly inelastic ($e = 0$), or an intermediate fashion ($0 < e < 1$), where $e$ represents the coefficient of restitution.

3. The colliding surfaces of both the fragment and the target ring may be either perfectly smooth ($\mu = 0$) or may be "rough" ($\mu \neq 0$), where $\mu$ denotes the coefficient of sliding friction. Hence, respectively, force and/or momentum (or velocities) are transmitted only in the normal-to-surface direction or in both the normal and the tangential direction.

4. During the collision, the contact forces are the only ones considered to act on the impact-affected region of the ring and in an anti-parallel fashion on the fragment. Any forces which the ring segment on either side of the impact-affected region may exert on that segment as a result of this instantaneous collision are considered to be negligible because this impact duration is so short as to preclude their "effective development".

5. To avoid unduly complicating the analysis and because of the smallness of the arc length of the ring finite elements, each affected

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Such forces are termed "internal forces" as distinguished from the "external impact forces".
ring element is treated as a straight beam segment (see Fig. 6) in the derivation of the impact inspections and equations. However, for modeling of the ring itself for transient response predictions, the ring is treated as being arbitrarily curved and of variable thickness.

An information flow schematic of the CIVM procedure is shown in Fig. 7. Briefly, the analysis procedure indicated in Fig. 7 consists of the following principal steps:

1. Motions and Positions of Bodies

   The motions of the fragment and of the containment and/or deflector ring are predicted and the (tentative) region of space occupied by each body at a given instant in time is determined.

2. Collision Inspection

   Next, an inspection is performed to determine whether a collision has occurred during the small increment (Δt) in time from the last instant at which the body locations were known to the present instant in time at which the body-location data are sought. If a collision has not occurred during this Δt, one follows the motion of each body for another Δt, etc. However, if a collision has occurred, one proceeds to carry out an (approximate) calculation of the time of fragment-ring contact.

3. Contact-Time Calculation

   The fragment and ring-node positions, velocities, and accelerations are known at an instant in time prior to ring-fragment collision. Using this information, the (approximate) time of ring-fragment contact (within the small increment, Δt, in time), and the point of contact on the ring are calculated. When this information has been obtained, one then proceeds to carry out a collision-interaction calculation.

4. Collision-Interaction Calculation

   In this calculation energy and momentum conservation relations are employed in an approximate analysis to compute the collision-induced changes in (a) the velocities \( V_f \) (translation) and \( \omega_f \) (rotational) of the fragment and (b) nodal velocities of the ring impact-affected segments. The coordinates which locate the positions of the fragment and of the affected segments are thereby corrected from their tentative uncorrected-for-impact locations.
One then returns to step 1, and the process is repeated for as many time increments as desired.

The details of this analysis procedure as well as various considerations and simplifying assumptions employed are discussed further in Appendix A.

### 2.3 Solution Procedure

The spatial finite-element approach is utilized in conjunction with the Principle of Virtual Work and D'Alembert's Principle to obtain the equations of motion of the structural ring which is permitted to undergo large-deflection elastic-plastic transient deformations. In the interest of conciseness and convenience in this report, the user is invited to consult Ref. 1 and/or Appendix A for a detailed derivation and discussion of the equations of motion. For present purposes, it suffices to note that the governing equations of motion for the complete assembled discretized structural ring correspond to the unconventional (improved) formulation of Ref. 1 and may be written in the following form:

\[
[M^*]\{\ddot{q}^*\} + \{P^*\} + [H^*]\{q^*\} + [K_s^*]\{q^*\} = \{0\}
\] (2.1)

where

- \{q^*\} and \{q^*\} are the global generalized displacement vector and acceleration vector.
- \([M^*]\) is the mass matrix of the complete structure.
- \{\{P^*\}\} is a "generalized loads" vector representing both elastic and some plastic behavior contributions.
- \([H^*]\{q^*\}\) represents "generalized loads" arising from both large deflections and plastic strains.
- \([K_s^*]\) represents the effective stiffness matrix supplied by the elastic foundation and/or the restraining spring.

In the present procedure, a diagonal "lumped" mass matrix is employed. The justification for the use of lumped mass instead of consistent mass is outlined next. A comparison of numerical results obtained using lumped mass

\[\text{The right-hand side of Eq. 2.1 is zero since it is assumed that there are no prescribed externally-applied forces acting.}\]
vs. consistent mass, given in Ref. 1, for ring-type structures shows similar results for both mass systems. The use of a lumped mass matrix also results in a decrease of the highest natural frequency, compared with the use of a consistent mass matrix, for the assembled structure, and thus permits one to use a larger time step, $\Delta t$, for the structural response calculations (as will be described shortly). This fact, coupled with reduced storage requirements and additional savings of computation time in each time step because of the simple form of the mass matrix, makes the use of a lumped mass matrix computationally efficient. Finally, in the collision interaction analysis the element (and structure) mass properties are assumed to be lumped at the nodal points. Thus, for consistency, the mass properties of the ring structure used in the global timewise solution procedure should also be nodal lumped masses.

The resulting equations of motion are solved through the use of the 3-point central difference time operator whereby one obtains a recurrence equation which provides a solution step-by-step in finite-time increments. Based on computing experience, this operator is much more simple and requires a minimum of storage and operations (compare, for example, with the Houbolt operator) within each time step of calculation for advancing the solution ahead in time. However, it should be noted that in order for the 3-point central-difference operator to provide a reliable prediction, the time step size, $\Delta t$, employed must be small enough. To insure a suitably small $\Delta t$, the following procedures are built into the computer program utilizing this central-difference operator so that the time-step size, $\Delta t$, can be either specified by the user or the program will compute the largest natural frequency, $\omega_{\text{max}}$, of the system and will then choose a value of $\Delta t_{\text{max}} = 0.8 \left( \frac{2}{\omega_{\text{max}}} \right)$, where $\Delta t < 2 / \omega_{\text{max}}$ is the stability criterion of a corresponding linear dynamic system; the factor 0.8 is introduced in order to take large-deflection effects into account. The $\omega_{\text{max}}$, which represents the largest natural frequency contained in the (linear) mathematical model of the structure, is obtained by an iteration process applied to

$$
\omega^2 \left[ M^* \right] \{ q^* \} = \left[ K^* \right] \{ q^* \} \tag{2.2}
$$
where $[K^*]$ is the usual elastic stiffness matrix of the structure which is used only for the calculation of allowable $\Delta t$, and is not employed in the global timewise solution because of the use of Eq. 2.1 in place of the "conventional" equations of motion.

Following the calculation of the allowable $\Delta t$, the equation of motion is solved by using the central difference operator. Also, a collision inspection and correction procedure is carried out for each time-step of the advancing calculation. In the following, the general solution process is described briefly.

First, information is provided to define the geometry of the ring including its prescribed displacement conditions and elastic restraints. In addition, the ring material property constants and the attacking-fragment parameters are defined. Also defined is the structural discretization information and numerical integration data. It should be mentioned that Gaussian quadrature is employed in the present analysis to evaluate the element-property matrices — this requires that the stresses and strains be evaluated at a selected finite number of Gaussian stations over the "spanwise" and depthwise region of each finite element. Next, the mass matrix and the stiffness matrix for the entire structure are evaluated by assembling the element mass and stiffness matrices. Then the proper prescribed displacement conditions are imposed and a reduced mass matrix and stiffness matrix are obtained by deleting the corresponding rows and columns associated with those generalized displacements which are prescribed to be zero. Also constructed are the discrete element property matrices that do not change with time (and remain constant throughout the program), such as the matrices relating strain to the nodal generalized displacements, etc. The maximum natural frequency, $\omega_{\text{max}}$, of the structure is then calculated from Eq. 2.2, and the maximum allowable step size, $\Delta t_{\text{max}}$, is found. This value is compared with the user specified value of $\Delta t$, and the smaller of the two values is chosen for the timewise solution procedure. If the user has chosen the time-step "over-ride" option, the user specified $\Delta t$ will be used.

The ring structure is assumed to be at rest at time $t_0$, and the position and velocity of the attacking fragments are known at time $t_0$. From this information the generalized nodal and fragment displacements and displacement increments are computed for the first time increment $\Delta t$. Then, the fragment-ring collision inspection and correction procedure is carried out. If one, or
more, ring-fragment collisions have occurred during this $\Delta t$, the coordinates which locate the position of the fragment and impact-affected nodes of the ring are thereby corrected from their tentative uncorrected-impact locations. Next, the strain increment developed from $t_0$ to $t_1$ at every Gaussian station (or point) required over and depthwise through each finite element are calculated. From a knowledge of the prescribed initial stresses (if any) and the strain increments, one can determine the stress increments, the stresses, and/or the plastic strains and the plastic strain increments through the use of the pertinent elastic-plastic stress-strain relations including the plastic yield condition and flow rule. Next, one can calculate the equivalent generalized load vectors arising from large deflections and plastic strains. Then, the proper recurrence equations, which is the finite-difference representation of the equations of motion, are solved to obtain the ring-nodal generalized displacements and displacement increments of the next time increment. The pertinent equations of motion for the fragment are also solved to obtain the displacements and displacement increments of the next time increment for each attacking fragment. The process then proceeds cyclically for as many time steps as desired. It should be noted that the ring structure remains at rest until the first ring-fragment collision occurs.

For the present purposes, the above general description is considered to be adequate; one may consult Appendix A and Refs. 1-3 for a more detailed discussion of the solution and evaluation process.
SECTION 3
DESCRIPTION OF PROGRAM AND SUBPROGRAMS

3.1 Program Contents

The CIVM-JET 4B program is composed of a main program and 23 subroutines which appear in the program in the order listed. The names and functions of these programs are as follows.

**MAIN**
Reads the ring geometry, material property data, the structural discretization information, and/or the prescribed displacement conditions and elastic restraints. Also, read in are the fragment geometry parameters and the fragment velocity components. It computes the quantities that are constant throughout the program and initializes most of the variables used in the subroutines. It controls the logical flow of information supplied by the various subroutines and the overall time cycle. Also, the lumped mass matrix $[M]$ is generated by this routine and stored in row form.

**ASSEF**
This subroutine assembles the generalized nodal load vectors (due to large-deflection elastic-plastic effects) of each individual element into a generalized nodal load vector for the structure as a whole.

**ASSEM**
This subroutine updates the structural stiffness matrix as the element stiffness matrix is generated. The components of the assembled stiffness matrix $[K^*]$, which is a symmetric matrix, are stored in a linear-array form; only the lower triangular part of $[K^*]$ need be and is stored (row-wise) starting with the first nonzero element in the row and ending with the diagonal term.

**BRAN**
This subroutine reads the geometry, boundary constraints, and elastic restraints for a branch. The global numbering system of the main structure is then modified to include the branches. BRAN establishes arrays which contain information facilitating the rotations required in other subroutines. It also establishes identifier arrays which distinguish between elements of the main structure and elements of the various branches.

**CUBIC**
A slave subroutine of ROOT4. Used to calculate one real root of a cubic equation.
DINIT This subroutine initializes all ring response calculation vectors and advances each of N fragments to its location at a time (which is user specified) prior to initial impact.

ELMPP This subroutine evaluates the element stiffness matrix \([k]\), for each discrete element, and then performs discrete element assembly to form \([K^*]\) for the complete structure with respect to global coordinates. Next, the prescribed displacement conditions (if any) are imposed on \([K^*]\) to form a restrained matrix. Also evaluated are the transformation matrices between the strain at each spanwise checking (Gaussian, or other) station and the generalized nodal displacement conditions of the element.

ENERGY Computes the energies of the fragment and the ring at each printout cycle. The ring energies are subdivided into the plastic energy, elastic energy, kinetic energy, and energy absorbed in the elastic foundations.

ERC Imposes the proper prescribed displacement conditions to the \([K^*]\) matrix by restraining the corresponding rows and columns of the matrix.

FICOL Finds the corresponding location of an element in the linear array expression to a location in a two dimensional array expression of the \([K^*]\) matrix.

IDENT The IDENT subroutine is used to print out the values of certain input parameters at the beginning of the run, and is used to identify the type of run that is being made.

IMPACT This subroutine is the controlling routine for carrying out the search for impact occurrence involving one of N fragments on each element of the ring for all fragments considered. When it is determined that a fragment-ring collision has taken place, IMPACT controls the application of appropriate correction factors to the velocities of the fragment and the nodal points of the affected elements.

IMPCTE A slave subroutine of IMPACT. This subroutine calculates and applies the appropriate correction factors to the velocities of
the fragment and the nodal points of the elements affected when a fragment-ring collision has occurred.

MINV Performs the matrix inversion; a standard Gauss-Jordan inversion method is used.

OMULT Performs the multiplication of a square matrix (stored as a vector) and a vector. A vector results.

PENTRN A slave subroutine of IMPACT. Given the position of the fragment and ring nodes and the geometry of the fragment and idealized ring structure, this subroutine determines whether any "overlapping" (penetration) exists between the fragment geometry and the ring geometry.

PRINT PRINT controls the program output and format.

QREM Evaluates the effective stiffness matrix \([K^*]\), supplied by the elastic foundations and/or the restoring springs, and then imposes the prescribed displacement conditions on \([K^*]\) accordingly.

ROOT4 A slave subroutine of TCONT. ROOT4 solves the quartic equation which is encountered in the calculation of the time of fragment-ring contact. Only real roots are calculated (imaginary roots have no meaning here).

ROTAT This subroutine generates the transformation matrix necessary to rotate from the global displacement system to the element displacement system. This matrix is then applied to the element \([k]\) matrix, the displacement vector and the equivalent load vector (as required) to perform the rotation for the connecting branch elements and any elements containing discontinuities.

STRESS This subroutine evaluates the generalized load vectors, (R.H.S. of Eq. A.30a) arising from the presence of large-deflections and plastic strains. First, the stresses and plastic strains are determined at each quadrature station, which involves the use of the strain-displacement relation and the stress-strain relation. The strain-hardening and strain-rate sensitivity effects are taken in consideration. Next, the appropriate Gaussian integration scheme is used.
to form the element generalized nodal load for each discrete element, and finally, an assembled generalized nodal load vector is calculated.

**TCONT**

A slave subroutine of IMPACT. This subroutine determines the (approximate) real time at which contact of the fragment onto the ring occurs (within a given increment in time). TCONT calculates the time of contact, element contacted, point (in space) of contact, and the fragment involved in this fragment-ring contact.

**TSTEP**

This subroutine is called during each problem run to compute $\Delta t_{\text{max}}$ and to constrain the user-specified $\Delta t$ to be $\leq \Delta t_{\text{max}}$. It finds the highest natural frequency, $\omega_{\text{max}}$, in the mathematical model of a corresponding linear dynamic system $[M^*] \{\ddot{q}^*\} + [K^*] \{q^*\} = 0$ by using an iteration process, and then calculates a value of $\Delta t_{\text{max}} = 0.8 \left(\frac{2}{\omega_{\text{max}}}\right)$.

**UPDATE**

A slave subroutine of IMPACT. This subroutine calculates the position of the ring nodes and fragment c.g. at time $t_1$ given the position, velocity, and acceleration of the ring nodes and fragment c.g. at time $t_0$ (where $t_1$ may be greater than or less than $t_0$).

### 3.2 Partial List of Variable Names

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A(I,J)$</td>
<td>$[A]$, an 8x8 matrix defines the transformation between the element generalized nodal displacements ${q}$ and the parameters ${r}$ in the assumed displacement field of each element. It is destroyed in computation and is replaced by its inverse $[A^{-1}]$.</td>
</tr>
<tr>
<td>$AA(JR,I,J)$</td>
<td>A matrix that stores all of the $[A^{-1}]$ matrices.</td>
</tr>
<tr>
<td>$AEP(I,J,K)$</td>
<td>Transformation matrix which relates strain at Ith additional strain point to the generalized nodal displacements of the element on which it is located.</td>
</tr>
<tr>
<td>$AINT$</td>
<td>Pre-impact approach velocity of the fragment-impacted ring element system normal to the ring element relative to the fragment.</td>
</tr>
</tbody>
</table>
AL(I)  Element arc length of the Ith element.
ALFA(I)  Angular rotation of fragment I (rad.).
ANB(I)  Same as ANG(I), applies to initial input for branch nodes.
ANG(I)  The slope, which is the angle between the tangent vector and the +Y axis, at the Ith node.
ANGDB  The slope, which is the angle between the tangent vector and the +Y axis, at the Ith slope discontinuity. ANGDB refers to initial input for branches.
ANGDI(I)  The slope, which is the angle between the tangent vector and the +Y axis, at the Ith slope discontinuity. ANGDB refers to initial input for branches.
APHA  The angle between the chord connecting the first node of the element to the second node, and the +Y axis.
APN  Fragment-induced impulse normal to the impacted ring element surface.
APT  Fragment-induced impulse tangential to the impacted ring element surface.
ASFL  Stress and/or plastic strain weighting factor on the Lth sublayer (I,J,K,L) in the Kth depthwise Gaussian point at the Jth spanwise Gaussian station of the Ith element.
AXG(I)  Input vectors with dimension NOGA: contain Gaussian quadrature constants, \( x_i \), and weights, \( W_i \) of
\[
\int f(x) \, dx = \sum_i f(x_i) W_i
\]
employed in the spanwise integration over each element.
AwG(I)  

B(L)  Width of the ring (inches); L=1 for main structure; L>2 for branches.
BEP(IR,J,I,K)  Transformation matrix which relates the strain at the Jth spanwise Gaussian station to the generalized nodal displacements of the IRth element ([D_1], I = 1,2,3, see Eq. A.14).
BI(L)  Same as B(r)(L), for largest average nodal strain.
BIG(L)  The largest computed strain at the Gaussian stations for the Lth substructure, up to the present cycle. It should be noted that strains are computed at every cycle. L=1 for main structure, L>2 for branches.
BIGA(L)  The largest computed strain at the additional strain points, up to the present cycle.

BINP(I,J)  The longitudinal force and the bending moment, respectively, over the cross section at the Jth spanwise Gaussian station of the Ith element (see Eq. A.26).

BINP(I,J)  The longitudinal force and the bending moment, respectively, over the cross section at the Jth spanwise Gaussian station of the Ith element (see Eq. A.26).

BIMP(I,J)  The longitudinal force and the bending moment, respectively, over the cross section at the Jth spanwise Gaussian station of the Ith element (see Eq. A.26).

BONE  The highest natural frequency squared of a corresponding linear dynamic system.

BTIM(L)  Same as BTIME(L), applies to nodes.

BTIMA(L)  The time at which the largest computed strain occurs at the additional strain points. L=1 for main structure, L>2 for branches.

BTIME(L)  The time at which the largest computed strain occurs at the Gaussian stations.

CELAS  Elastic energy stored in ring up to the present time.

CINETF  Kinetic energy stored in fragment up to the present time.

CINETO  Kinetic energy imparted to ring up to the present time.

CØPY(I)  Current global Y coordinate and Z coordinate, respectively, of the Ith node.

CØPZ(I)  Current global Y coordinate and Z coordinate, respectively, of the Ith node.

CR(J)  Coefficient of restitution between the Jth fragment and the impacted ring surface.

DALFA(I)  Impact-corrected angular displacement increment of the Ith fragment at the current time step.

DCRTE  Critical distance used in calculating where a positive penetration has occurred between a fragment and a ring element. It is equal to the fragment radius plus one half the mean element thickness.

DELD(I)  Vector contains the generalized nodal displacement increment during the current time step.

DELTAT  Time-step size used in the program, \( \Delta t \).

DELTR  Time remaining during a time step \( \Delta t \). Used in impact inspection and correction calculations.

DENS(L)  Density of the Lth structural segment. L=1 for main structure, L>2 for branches (lbs·sec\(^2\)/in\(^4\)).
DFCGU(I) Impact-corrected Y direction displacement increment applied to the position of fragment I.

DFCGW(I) Impact-corrected Z direction displacement increment applied to the position of fragment I.

DISP(I) Vector which contains the generalized nodal displacements at the current time instant.

DROT(L) Stores information used in rotating a displacement vector into the global system at a point of slope discontinuity.

DS(L) Material constant used in the strain-rate sensitivity formula. L=1 for main structure, L>2 for branches.

DUMMY A dummy argument in the calling statement of Subroutine ROTAT.

EFLN(L) The effective impact length of the ring (inches). L=1 for main structure. L>2 for branches.

ELK(I,J) Element stiffness matrix of dimension 8x8 (Eq. A.18d).

ELMAS(I,J) Element mass matrix of dimension 8x8 (Eq. A.16).

ELRP(I,J) Element effective stiffness matrix of dimension 8x8 supplied by elastic restraints.

EPS(L,J) Input quantities of abscissa of the uniaxial stress-strain curve for the Jth mechanical sublayer material model. L=1 for main structure, L>2 for branches.

EPSI(I) Average axial strain on the inner surface and on the outer surface, respectively, at node I.

EPSΩ(I) If EXANG=360.0, the structure is considered to be a complete ring. If EXANG ≠ 360.0, the structure is considered to be a partial ring.

FACTFN Impact-induced correction factor applied to the normal-to-impact displacement increment of the attacking fragment at the time of contact.
FACTFT  Impact-induced correction factor applied to the tangential-to-impact displacement increment of the attacking fragment at the time of contact.

FACTFO  Impact-induced correction factor applied to the rotational displacement increment of the attacking fragment at the time of contact.

FACTN  Impact-induced correction factor applied to the normal-to-impact displacement increment of each affected node.

FACTT  Impact-induced correction factor applied to the tangential-to-impact displacement increment of each affected node.

FARE  Midplane axial strain and curvature increment, respectively, at the selected spanwise Gaussian station of each element.

FCUR  The global Y coordinate of the centroid of the Ith fragment.

FCGU(I)  The global Z coordinate of the centroid of the Ith fragment.

FCGW(I)  Assembled generalized load vector corresponding to large deflections and plastic strain presence; it equals \([p^*] + [H^*] [q^*]\).

FMASS(I)  The mass of the Ith fragment \((\text{lb}-\text{sec}^2/\text{in.}^4)\).

FM0I(I)  The mass moment of inertia of the Ith fragment \((\text{lb}-\text{sec}^2-\text{in})\).

FREQ  The highest natural frequency of a corresponding linear dynamic system of the ring.

GFL(IR,I,J)  Stress and/or plastic strain weighting factor on the Jth depthwise Gaussian point at the Ith spanwise Gaussian station of the IRth element.

GZETA(IR,I,J)  Distance from the centroidal axis to the Jth depthwise Gaussian point at the Ith spanwise Gaussian station of the IRth element.

H(I)  Thickness of the ring at the Ith node.

HB(I)  Same as \(H(I)\), applies only to initial input for branches.

HTH(L)  The branch thickness for the Lth branch at its connecting node.
IBI(L)  Same as IBIG(L), applies to nodes.
IBIG(L)  The element number whose strain, computed at one of its Gaussian
stations, exhibits the largest value during the present computer
run. L=1 for main structure, L>2 for branches.
IBIGA(L) Same as IBIG(L), applies to additional strain points.
IC0L(I)  Vector, of length NI, contains the column number of the first non-
zero entry in the Ith row of the structural mass and/or stiffness
matrix.
ICON  INDICATOR = 0 if last data input
       1 if more runs are desired
ICONT  INDICATOR, if >0 then the program expects data for a continuation
run.
IFLAG(I,J)  A flagging matrix which indicates whether element I has been
impacted by fragment J during a given time step, Δt.
IK  Number of discrete elements into which the whole structure is
discretized for analysis.
IMCO  Indicates the occurrence of an impact in the previous time cycle.
IMCOU  Indicates the number of impacts up to the present time instant.
INUM(I)  Vector of dimension NI contains the corresponding position in
the linear-array of the first nonzero entry in the Ith row of the
structural mass or stiffness matrix.
IRRUN  A counter: is equal to the number of runs of CIVM-JET 4B in a
single computer submittal.
ISTA(I)  The number of the Gaussian station at which the strain is a
maximum.
ISTAA(I)  The number of the additional strain point at which the strain is
a maximum.
ISIZE  Number of locations required for the storage of the structural
mass or stiffness matrix in linear-array form.
ISUR(L)  Same as ISURF(L), applies to nodes.
ISURA(L) Same as ISURF(L), applies to additional strain points.
   L=1 for main structure, L>2 for branches.
ISURF(L)  INDICATOR = 1 if largest computed strain occurs on inner surface
         = 2 if largest computed strain occurs on outer surface
       Refers only to strains calculated at Gaussian stations.
IT  Current time-step (cycle) number.

JF  The fragment number which is involved in the ring segment impact.

KII Number of nodes included in impact-affected region.

KRØW(I) The row number of the Ith irregular row in the structural mass or stiffness matrix.

LATT Indicates how the branch is attached to the main structure:
   = -1  inner surface
   =  0  midsurface
   =  1  outer surface

LBR(I) The number of a branch upon which a boundary condition is to be applied.

LHIT(I) Indicator array. I = branch number. If LHIT(I)=0, branch is to be impacted; in the present program LHIT(I) must be set equal to zero.

LMT(I) Array which stores the element numbers of those branch elements where impact cannot occur.

LNTMIN Element upon which the first impact has occurred during a given time step.

MATT(L) Indicates the node at which the Lth branch is attached.

MIRP Indicates the first fragment that is released at a time after the initial impact.

MK(I) Vector which contains new nodal numbers for the main structure, given I as the old nodal number.

MKE(I) Indicates the substructure to which the Ith element belongs.

MM  Time step (cycle) at which run is to stop.

MNEL(I) Number of elements in the Ith substructure.

M1  Cycle at which regular printing starts.

M2  Printout will occur every M2 cycles.

MPU  Indicator for punched output: IF MPU = 0, no punched output.
     IF MPU ≠ 0, data is punched from last time cycle.
MREAD  Number for the data input tape unit, printed output tape unit,  
MWRITE and the punched output tape unit, respectively. These names  
MPUNCH must be assigned a number in MAIN corresponding to the user's  
computing facility requirements.

NBS(I)  The prescribed-displacement condition identification number.

NBCB(I)  Same as NBC(I), applies to initial input for branches.

NBCNB  The number of nodes at which the prescribed displacement condition  
is to be specified: refers only to branches.

NBCOND  The number of nodes at which the prescribed displacement conditions  
are to be specified.

NBR  Indicates the number of branches that are to be added to main  
structure (not to exceed 5).

NDEX(I)  The corresponding position in the linear-array of the first non-

zero entry in the Ith irregular row.

NDIS  The number of elements containing a slope discontinuity.

NEDI(I)  The main structure element number of the Ith element containing  
a slope discontinuity.

NEDIB(I)  The branch element number of the Ith branch element containing  
a slope discontinuity.

NELT(I)  Number of elements in the Ith branch.

NF  The number of fragments considered to be impacting the ring.

NFL  The number of depthwise Gaussian points through the thickness of  
each layer for the numerical evaluation of stress resultants  
(axial forces and bending moment) at each spanwise Gaussian  
station.

NI  Total number of degrees of freedom (unrestrained); it equals the  
number of nodes times 4. Also, it is the number of rows in the  
assembled structural mass or stiffness matrix.

NIRREG  Number of irregular rows in the assembled structural mass or  
stiffness matrix.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDBB(I)</td>
<td>The node number along branch (I) at which a boundary condition is to be applied.</td>
</tr>
<tr>
<td>NDEB(I)</td>
<td>The node number at which the prescribed displacement condition NBC(I) is to be specified.</td>
</tr>
<tr>
<td>NODP(I)</td>
<td>Nodal number (from main structure numbering system) at which the Ith branch starts.</td>
</tr>
<tr>
<td>NPGA</td>
<td>The number of Gaussian stations to be employed for the spanwise numerical integration of the element properties over each element.</td>
</tr>
<tr>
<td>NRP</td>
<td>The number of point elastic restraints (elastic restoring springs) and the number of locally distributed elastic restraints, respectively, which are to be specified over the structure.</td>
</tr>
<tr>
<td>NPP</td>
<td>Number of positive penetrations during time DELTR.</td>
</tr>
<tr>
<td>NQR</td>
<td>Indicator, which if &gt; 0 indicates that this structure is subjected to elastic restraints (point and/or distributed).</td>
</tr>
<tr>
<td>NREL(I)</td>
<td>The element number at which the Ith point elastic restraint is to be specified.</td>
</tr>
<tr>
<td>NRST(I)</td>
<td>The first element and the number of elements, respectively, over which the Ith distributed elastic restraint is to be specified.</td>
</tr>
<tr>
<td>NREU(I)</td>
<td>The total number of nodes. For a partial ring NS=IK+1. For a complete ring NS=IK.</td>
</tr>
<tr>
<td>NSFL(L)</td>
<td>Equals the number of mechanical sublayers in the strain-hardening material model; also is the number of coordinate pairs defining the piecewise linear stress-strain curve of the substructure's material.</td>
</tr>
<tr>
<td></td>
<td>[ L = 1 \text{ for main structure}, \ L \geq 2 \text{ for branches.} ]</td>
</tr>
<tr>
<td>NTOVR</td>
<td>Allows the user to override the automatic time check.</td>
</tr>
<tr>
<td></td>
<td>= 0 (or blank) time check used</td>
</tr>
<tr>
<td></td>
<td>= 1 User's DELTAT used regardless of value calculated by subroutine TSTEP</td>
</tr>
<tr>
<td>NVEC(I,J)</td>
<td>Array containing nodal numbers which form the end points of the Ith element. J = -1 or 2 [First or second node, number clockwise for main structure, outwards for branches, and inwards for a branch attached to node 1 of a partial ring.]</td>
</tr>
<tr>
<td>P(L)</td>
<td>Constant used in the strain-rate sensitivity formula.</td>
</tr>
</tbody>
</table>
|          | \[ L = 1 \text{ for main structure}, \ L \geq 2 \text{ for branches.} \]
PAL  Fractional distance from point of impact to first node of the impacted element.

PAX  Fractional distance from point of impact to second node of the impacted element.

PIE  Represents $\pi = 3.141592653589793$.

PLAST Total plastic work done on the structure up to the current time step (mechanical work dissipated during plastic flow).

QACL(I) Vector which contains the generalized nodal accelerations at the current time instant.

QVEL(I) Vector which contains the generalized nodal velocities at the current time instant.

RCOS(I) Cosine and sine, respectively of the angle that element I makes with the global Y axis. Used in transformation from impact to local and local to global coordinate systems.

RSIN(I) The length coordinate along the centroidal axis from the node NREL(I) at which the Ith point elastic restraint is to be specified.

RFACTR Strain rate factor used in the stress calculation.

RL(I) Straight line length of ring element I used in the collision inspection and correction analysis.

RMX(I) Lumped mass and moment of inertia values, respectively, at ring structure node I.

ROT(I,J) Array which contains information needed to rotate a stiffness matrix.

I = Number of branch
J = 1 or 2

$\text{ROT}(I,1) = 0.0$ if Ith branch connects to first node of main structure. Equals 1.0 for all other connecting points. $\text{ROT}(I,2) = \text{Angle of rotation}.$

RWORK Total energy stored in ring, up to the current time.

SCTP The tangential and normal translational restoring spring elastic constants, respectively.

SCTY The rotational restoring spring elastic constant.
SCTU  Tangential and normal translational elastic foundation stiffness
SCTW  constants, respectively.
SCRU  Elastic foundation modulus in torsion.
SIG(L,J) Input quantities for the ordinate of the uniaxial static stress-
        strain curve for the Jth mechanical sublayer material model.
        \( L = 1 \) for main structure; \( L \geq 2 \) for branches.
SINT  Relative tangential velocity between the ring impact-affected
       nodes and an impacting fragment.
SNØ(N,L) Uniaxial static yield stress of the Nth mechanical sublayer
        material model.
        \( L = 1 \) for main structure; \( L \geq 2 \) for branches.
SNS(I,J,K,L) Axial stress of the Lth mechanical sublayer at the Kth depthwise
            Gaussian point at the Jth spanwise Gaussian station of the Ith
            element.
SNY  Uniaxial yield stress of the mechanical sublayer, taking strain-
     rate sensitivity into account.
SOL(I) Contains the solution vector of a series of matrix equations.
SPDEN Total energy stored in the elastically-restoring springs and/or
       the elastic foundations at the current time instant.
SPRIN(I) The assembled effective stiffness matrix supplied by elastic
        restraints (stored in a linear array form).
STIFK(I) Assembled structural elastic stiffness matrix, stored in a linear-
        array form.
TAII  Time of initial impact.
TANK  Boundary between rolling and sliding friction.
TIME  Current time (IT*DELTAT).
TIMF  Time at which all calculations are to stop.
TMIN  Time of first contact \( t^* \) measured from start \( t^*_n \) of the sub-
       time step interval (see Eq. A.107).
TNJ(J) Indicates whether or not fragment J has been released before the start of calculations.

TPRIM(J) Length of time that fragment J has been traveling prior to initial impact of the first fragment.

TRAN(I,J) Transformation matrix, used to rotate displacement vector and element stiffness matrices into global coordinates. Used for branch connection and slope discontinuities.

TU(I) TW(I) Trial Y and Z coordinates, respectively, of the Ith node during impact calculations.

TWG(I) TXG(I) Input vectors with dimension NFL; contain Gaussian quadrature constants $x_i$ and weights, $w_i$ of

$$\int f(x) \, dx = \sum_i f(x_i) w_i,$$

used in the numerical integration of stresses and/or plastic strains through the thickness.

UDOT(J) The velocity component of the Jth fragment parallel to the global Y axis.

UNK(J) Coefficient of friction for the Jth fragment.

VEL(I) Vector contains post impact nodal velocities.

VELFA(J) Same as: ADOT(J)

VELFU(J) UDOT(J) used in impact calculations

VELFW(J) WDOT(J)

WDOT(J) The velocity component of the Jth fragment parallel to the global Z axis.

XDIST(I) Distance from reference axis to attachment point of Ith branch.

YK(I) A general work vector. It is finally used to store either the number 1 or 0 for each element (I) to indicate whether a transformation is necessary. YK(I) is used together with ROT(I,J) to identify and aid in rotating an element's stiffness matrix.
\textbf{YOUNG(L)} Elastic (Young's) modulus (the slope of the 1st segment in the piecewise linear approximation of the uniaxial stress-strain curve).

\( L = 1 \), main structure; \( L \geq 2 \) for branches.

\textbf{Y(I)} \quad Initial \( Y \) coordinate and \( Z \) coordinate, respectively, of the \( I \)th node.

\textbf{Z(I)}

\textbf{YB(I)} \quad Same as \( Y(I) \) and \( Z(I) \), applies only to branch nodal input.

\textbf{ZB(I)}
SECTION 4
USE OF THE CIVM-JET 4B PROGRAM

4.1 Input Information and Procedure

The information required to punch a set of data cards for a run of the program is presented in a step-by-step manner in this section. The variables to be punched on the nth data card are shown, and to the right is the format to be used for that card; the definition of and some restrictions for each variable are given directly below. This is done for each card, in turn, until all are described.

Card 1
B(l), DENS(l), EXANG

where

B(l) The width of the main structure (inches) (other structural portions are called "branches")
DENS(l) The material density of the main structure (lb-sec²)/in⁴
EXANG The angle (in degrees) that the ring subtends: for a complete ring, EXANG = 360.0; for a partial ring, EXANG ≠ 360.0.

Card 2
IK, NOGA, NFL, NSFL(l), MM, M1, M2, NF, TIMF

where

IK The number of finite elements used to model the main structure. The total number of elements, including branch elements cannot exceed 50 (however, this limitation may be relaxed by changing the appropriate dimension statements of the program).
NOGA The number of spanwise Gaussian stations to be used for the spanwise numerical integration over each element in evaluating the element property matrices. NOGA=3 is used in CIVM-JET 4B.
NFL The number of depthwise Gaussian points to be used for the numerical integration through the thickness at each spanwise Gaussian station. This number cannot exceed 6.
NSFL(l) See Card 5A for description.
MM  The cycle number at which the run is to stop.
M1  The cycle number at which the regular printout is to begin.
     M1 must not equal 0.
M2  The number of cycles between regular printout (i.e., print
     every M2 cycles).
NF  The number of fragments considered to be impacting the ring.
     This number cannot exceed 6.
TIMF The time at which the program will stop all calculations.

Card 3A
Y(1), Z(1), ANG(1), H(1)  4D15.6

where

Y(1)  Initial Y coordinate and Z coordinate, respectively, of the
     first node (inches).
Z(1)  The slope (degrees) which is the angle between the tangent
     vector and +Y axis at the first node. An angle from the +Y
     axis to the tangent vector in a counter-clockwise direction
     is defined as the positive direction.
ANG(1) The thickness of the ring (inches) at the first node.

Additional Cards 3B, 3C,... are punched in exactly the same format as
Card 3A until the total number of No. 3 cards equals the total number of nodes
of the main structure (IK+1) for a partial ring and equals IK for a complete
ring, where IK is the value appearing on Card 2.

Also the following two conditions must be satisfied by ANG(I) (where
I is the node number):

(1)  \(-180^\circ < ANG(I) \leq +180^\circ\)

(2)  An element cannot have a change in slope between its first node
     and its second node that is greater than 15\(^\circ\). This refers only
     to the shape of one element (see Fig. 3); slope discontinuities
     between two elements are handled on Card 4.

Note that for bookkeeping purposes, the nodal slope is defined to be identified
with the first end (left-hand end) of the element at that node for structures
with continuous slopes. However, where a slope discontinuity occurs on the
main structure, a node must be used and two slopes must be given: (1) one (given on Card 3) associated with the second end (right-hand end) of the pertinent element and (2) one associated with end one (L-H end) of the next element; the item (2) situation is dealt with by Cards 4A, 4BA, 4BB, 4BC, etc.

Card 4A

\[ \text{NDIS} \]

where

\[ \text{NDIS} \] The total number of elements in the main structure having a slope discontinuity at the first node of the element.

If there are no slope discontinuities on the main structure, set \( \text{NDIS} = 0 \) and go to Card 5.

Card 4BA

\[ \text{NEDI}(I), \text{ANGDI}(I) \]

where

\[ I = 1, \text{NDIS} \]

\[ \text{NEDI}(I) \] The element (on the main structure) at which the Ith slope discontinuity appears on the element's first node.

\[ \text{ANGDI}(I) \] The appropriate slope (degrees) for the Ith slope discontinuity; the slope is measured between the tangent vector to the element at the station in question and the +Y axis (see Card 3A, ANG(1) for a description of the measurement system).

Additional Cards 4BB, 4BC, etc. are used until all of the "slope discontinuities" are described. (See Fig. 8 for a further description of the discontinuity option.)

The sequence of cards starting with Card 5 and going through Card 5DB contains all of the data for branches to be applied to the main structure, except that elastic restraints must be handled as one unit on Card 15. If no branches are to be applied, Card 5 has NBR=0; then proceed to Card 6. Only 5 branches are allowed, with a maximum of 10 elements in any one branch. A
branch may not be connected to another branch. Branches may be attached to any node on the main structure, or to the inner or outer surface of the main structure at any node but only one branch may be connected to a given main-structure node. Note that if a complete-ring primary structure is specified, no branch may connect to node 1 of the primary structure. The following sketch shows typical permissible arrangements of the branches:

Note the difference in the numbering schemes for branch 1 compared with branches 2 and 3; the last geometry input card for a branch pertains to its junction with the main structure. Numbering of the primary structure is done independent of any branches. Typical branch numbering is given in the above sketch. Note that all numbering in the above sketch is done in each substructure's system. The program will renumber (in a clockwise manner) the entire finite element system.

The following describes the sequence of Cards (5A through 5DB) needed to accommodate branches (these cards are nested in such a way that each branch's material and geometric properties are specified, branch by branch, followed by slope-discontinuity information for all branches, followed by boundary condition information for all branches):

Card 5

NBR

where
NBR The number of individual branches being added to the main structure (NBR<5).

If NBR=0 GO TO CARD 6

Card 5A

\[ \text{NSFL}(L), \text{B}(L), \text{DENS}(L), \text{DS}(L), \text{P}(L) \]  
\[ I5,4D15.6 \]

where

\[ L=2, NBR+1, (L-1) \text{ is the branch number, and the values of these variables when } L=1 \text{ are equal to the main structure's material.} \]

\[ \text{NSFL}(L) \text{ The number of mechanical sublayers in the strain-hardening model of the material of the (L-1) branch, and is equal to the number of coordinate pairs defining the polygonal approximation of the stress-strain curve of the material (NSFL}(L)<5).} \]

\[ \text{B}(L) \text{ Width of the (L-1) branch (inches).} \]

\[ \text{DENS}(L) \text{ Mass density of the (L-1) branch (lb}-\text{sec}^2)/\text{in}^4 \]

\[ \text{DS}(L) \text{ See Card 6} \]

\[ \text{P}(L) \text{ (L-1) Branch} \]

Card 5AA

\[ \text{EPS}(J,L), \text{SIG}(J,L) \]  
\[ 4D15.6 \]

where

\[ J = \text{coordinate pair number } \leq 5 \]

\[ (L-1) = \text{branch number } \leq 5 \]

\[ \text{EPS}(J,L) \text{ See Card 7 for definition of quantities} \]

\[ \text{SIG}(J,L) \]

Additional Cards SAB and SAC are punched in exactly the same manner as Card 5AA until the number of coordinate pairs equals \text{NSFL}(L) punched on Card 5A. Do not include any unneeded (blank) cards.

Card 5B

\[ \text{NELT}(I), \text{NODP}(I), \text{LHIT}(I), \text{LATT}(I) \]  
\[ 4(I5) \]

where

\[ I = 1,5 \text{ is the branch number} \]

\[ \text{NELT}(I) \text{ Number of elements in Ith branch (NELT}(I)\leq10) \]
NODP(I)  Node of main structure (in original numbering system) at which Ith branch is attached. See figure on page 31.

LHIT(I)  Determines whether or not branch can be impacted:
LHIT(I)=0  No impact
LHIT(I)=1  Impact

Note: In the present CIVM-JET 4B program, a branch cannot be impacted; set LHIT(I)=0 for all branches.

LATT(I)  Determines where branch is to be attached.
LATT(I)= -1  inner surface
0  midsurface of the main structure
1  outer surface

Card 5BA

YB(I,J), ZB(I,J), ANB(I,J), -HB(I,J)  4D15.6

where
(I = branch number, J = node number). Nodes are to be numbered 1 to 10 where node 1 is the first node of a branch (not attachment point). Node is on circumferentiaI axis of branch.

YB(I,J)  Y coordinate of node (inches)
ZB(I,J)  Z coordinate of node (inches)
ANB(I,J)  Tangent angle measured to Y axis (degrees); see angle θ of Fig. 3.
HB(I,J)  Thickness at node J

Note: See Card 3A [ANG(I)] for sign convention used for ANB(I,J).

Note: If a branch attaches to node 1 of a partial ring (it can not be attached to node 1 of a complete ring), the numbering starts with the branch node farthest away from the attachment point. Therefore, the (NELT+1) node is the attachment point. If the branch attaches to any other node of the primary structure, start numbering with the node immediately after the attachment point. Thus, node NELT will be the node farthest away from the attachment point. However, node (NELT+1) will always be the attachment-point node. Thus nodes 1 to NELT are always particular only to the branch, and node (NELT+1) is the common node with the primary structure. The subroutine BRAN automatically
updates IK (the total number of elements), NS (number of nodes), and NI (D.O.F.). Therefore, the initial input (Cards 1-4 and 6-14) is punched as though the branches did not exist.

Cards 5BB, 5BC, etc., are punched until (NELT+1) nodes have been described.

Card 5C

\[ \text{NDISB} \]

where

\[ \text{NDISB} \]

The number of elements in the branches having a discontinuity at their first node. (Do not count the discontinuities due to the attachment of the branch to the main structure.)

If there are no discontinuities on the branches, set NDISB = 0 and go to Card 5D.

Card 5CA

\[ \text{NEDIB, NBDI, ANGB} \]

where

\[ \text{NEDIB} \]

The element number (along a branch) at which the discontinuity occurs.

\[ \text{NBDI} \]

The branch in which the element NEDIB is contained.

\[ \text{ANGB} \]

The slope (degrees); See Card 3A \[\text{ANG(1)}\] for sign convention used for ANGB.

Cards 5CB, 5CC, etc. follow until the information for all NDISB branch slope-discontinuities has been given.

Card 5D

\[ \text{NBCONB} \]

where

\[ \text{NBCONB} \]

The number of boundary conditions applied only to the branches. (Total B.C.'s on structure \(\leq 7\))

If Card 5D=0 go to Card 6.
Card 5DA

\[
\text{NBCB(I), NODBB(I), LBR(I)}
\]

where

- \( I = 1, \text{NBONB} \)
- \( \text{NBCB(I)} \): Type of boundary condition. See Card 14 for description.
- \( \text{NODBB(I)} \): The node number of the particular branch on which the B.C. is being applied (see sketch prior to Card 5 description).
- \( \text{LBR(I)} \): The branch number on which the B.C. is being applied (see sketch).

A total of seven boundary conditions is allowed, including the primary structure and all branches. Therefore, Card 5DB is punched if more than 4 B.C.'s are to be applied to the branches.

The cards grouped under the number 5 contain the complete description of all the branches (except for elastic restraints). These cards are nested branch by branch, such that a branch's material and geometric layout are completely described before starting the next branch. After all branches have been described, and the branch slope-discontinuity information has been given, then the boundary conditions are applied to all the branches.

An example is given below for a main structure containing two branches, each of a different material, and five boundary conditions on the branches. For each material a three sublayer model is used. The first branch contains two elements, while the second branch has three elements. The first has one slope-discontinuity, the second branch has none. The following list gives the card number (described in this section) in the order they would appear in the input deck:
Card 6

DELTAT, DS(1), P(1), NTOVR

where

DELTAT The time step size, $\Delta t$ (seconds) to be employed for the central difference time-wise integration operator. If the value of $\Delta t$ is set equal to zero on this card, the program will compute the largest natural frequency, $\omega_{\text{max}}$, of the corresponding linear system and will then choose a value of $\Delta t_{\text{max}} = 0.8(2/\omega_{\text{max}})$. If the user specifies a $\Delta t > \Delta t_{\text{max}}$, the DELTAT is reset to $\Delta t_{\text{max}}$.

NOTE: If the user specifies DELTAT = 0 then the user should establish MM, ML, and M2 on Card 2 based on a $\Delta t = 1$ microsecond. The program will then adjust these cycle numbers to correspond to the internally generated DELTAT.

DS(1) The values of the constants $D$ and $p$, respectively, used in the strain-rate sensitivity formula for the main structure's material.

P(1) Override for automatic check of DELTAT.

= BLANK DELTAT is checked.

= 1 DELTAT is not checked.
Card 7AA

\[ \text{EPS}(1,1), \text{SIG}(1,1), \text{EPS}(2,1), \text{SIG}(2,1) \]

where

- \( \text{EPS}(1,1) \) The first coordinate pair of strain, \( \varepsilon \), and stress, \( \sigma \), curve of the main structure which is used to define the polygonal approximation of the stress-strain diagram. The stress-strain diagram from which these values (and those following) are obtained must be upwardly-convex with non-negative slopes; \( \epsilon(J,1)=\text{in/in} \) and \( \sigma(J,1)=\text{lb/in}^2 \).
- \( \text{SIG}(1,1) \) The second coordinate pair of strain and stress in the main structure.

Additional Cards 7AB and 7AC are punched in exactly the same manner as Card 7AA until the number of coordinate pairs equals \( \text{NSFL}(1) \) punched on Card 2. The total number of coordinate pairs must not exceed 5. Do not include any unneeded (blank) cards.

Card 8

\[ \text{NOP}, \text{NASP} \]

where

- \( \text{NOP} \) Indicates the type of strain output desired (given at inner and outer surface)
  - 0 Average strain at each node
  - 1 Average strain at each node plus strain at each Gaussian station
  - 2 Average strain at each node plus strain at designated additional points
  - 3 Strain for all three of the above options

- \( \text{NASP} \) Number of additional strain points requested; \( \text{NASP} \leq 50 \)

If \( \text{NOP} \neq 2 \) or 3, go to Card 9.

Note: It is suggested that the user use \( \text{NOP}=1 \) or \( \text{NOP}=3 \) in order to obtain a complete set of strain output for a first run of a problem. \( \text{NOP}=0 \) or 2 can be used for additional runs of the same problem in order to reduce output costs.

Card 8A

\[ \text{NGBS}(I), \text{NSEL}(I), \text{AZET}(I) \]
where

\[ I = 1, \quad NASP \]

NSBS(I)  Number of the substructure on which the additional strain point is requested: NSBS = 1 main structure; if NSBS > 1 the substructure is a branch whose number is (NSBS - 1)

NSEL(I)  The element along the NSBS substructure on which additional strain point is requested. No more than 10 additional strain points are allowed on any one element.

AZET(I)  The \( \delta \) coordinate of the additional strain point measured from the first node of the element (\( \delta \) is a fractional length (in/in) of the element itself).

Cards 8B, 8C, etc. are used until all the additional strain points have been described.

Card 9AA

\[ FH(I), \quad FCG(I), \quad FCGX(I), \quad FMASS(I), \quad FMOI(I) \quad 5D15.6 \]

Card 9AB

\[ UNK(I) \quad D15.6 \]

Card 9AC

\[ UDOT(I), \quad WDOT(I), \quad ADOT(I), \quad TPRIM(I), \quad CR(I) \]

where \( I = \) number of fragment. \( I \leq 6 \)

\[ FH(I) \quad \text{The diameter of the circular disk model of the Ith fragment (inches).} \]

\[ FCG(I) \quad \text{The Z coordinate of the centroid of the Ith fragment before and at the time of its release. The positive direction represents a location above the global Y axis (inches).} \]

\[ FCGX(I) \quad \text{The Y coordinate of the centroid of the Ith fragment before and at the time of its release. The positive direction represents a location to the right of the global Z axis (inches).} \]
FMASS(I) The mass of the Ith fragment (lb-sec²/in).
FMOI(I) The mass moment of inertia of the Ith fragment (lb-feet²/in).
UNK(I) Coefficient of friction between the Ith fragment and the ring inner surface.
UDOT(I) The velocity component of the Ith fragment parallel to the global Y axis before initial impact (in/sec). Positive UDOT(I) represents a fragment traveling to the right.
WDOT(I) The velocity component of the Ith fragment parallel to the global Z axis before initial impact. The positive direction denotes a fragment traveling in an upward (+Z) direction (in/sec).
ADOT(I) The initial angular velocity of the Ith fragment (rad/sec). Positive sign denotes counterclockwise rotation.
TPRIM(I) Time (seconds) that the fragment is allowed to travel before program starts to track its location. One usage of TPRIM(I) allows fragments to be released after the first fragment has impacted and calculations have begun.

The fragments should be ordered as follows to allow proper use of the TPRIM capability: Fragment 1 should be the fragment that will make first contact with the ring. Time zero is the time of release of this fragment. The second group of fragments includes all of the fragments that will be released before the first fragment impacts. The fragments can be placed in any order within their group. The third group contains those fragments released after the first fragment impacts; these must be ordered such that the first fragment to be released is first and so on within this group.

TPRIM(I) = Time of impact of first fragment minus time of release. Since time of release of fragment one is equal to 0, TPRIM(1) equals time of first impact. Actually, TPRIM(1) must be less than the time of first impact to guarantee a proper impact solution.

TPRIM(I) where I = 2, NF equals TPRIM(I) - Time of release of the Ith fragment. Thus, TPRIM, for those fragments released after the first fragment impacts, will be negative.
CR(I) Coefficient of restitution between the Ith fragment and the impacted ring inner surface; 0 < CR < 1, 1 for perfectly elastic, 0 for perfectly inelastic, 0 < CR < 1 for intermediate. CR=1 is usually recommended.

Cards 9BA, 9BB, 9BC, 9CA, 9CB, 9CC,... should follow (in blocks of three cards) until the information for all NF fragments has been completely specified.

Card 10
AXG(1), AXG(2), AXG(3) 3D25.16

Card 11
AWG(1), AWG(2), AWG(3) 3D25.16

where

AXG(I) \{ Vectors, of dimension NOGA, contain Gaussian quadrature
AWG(I) \} constants, x_i and weights, \( W_i \), respectively, for the numerical evaluation of

\[
\int f(x) \, dx = \sum_i f(x_i) \, W_i
\]

The following data appear on Card 10, since NOGA=3:
0.1127016653792585D+00 0.5000000000000000D+00 0.8872983346207415D+00
and the data
0.2777777777777778D+00 0.4444444444444444D+00 0.2777777777777778D+00
on Card 11.

Card 12A
TXG(1), TXG(2), TXG(3) 3D25.16

Card 12B
TXG(4) 3D25.16

Card 13A
TWG(1), TWG(2), TWG(3) 3D25.16

Card 13B
TWG(4) D25.16

Note: If NFL \leq 3, Cards 12B and 13B are eliminated.
     If NFL > 4 the extra terms are added to Cards 12B and 13B.
where

\[
\begin{align*}
\text{TXG(I)} & \quad \text{Vectors, of dimension NFL, contain Gaussian quadrature} \\
\text{TWG(I)} & \quad \text{constants, } x_i, \text{ and weights, } W_i, \text{ respectively, for the numerical} \\
& \quad \text{integration of} \\
& \quad \int f(x) \, dx = \sum_i f(x_i) \, W_i
\end{align*}
\]

If NFL=4, for example, then the following data appear on Cards 12A, and 12B.

\[-0.8611363115940530D+00 -0.3399810435848560D+00 0.3399810435848560D+00 0.8611363115940530D+00\]

and the data

\[0.3478548451374540D+00 0.6521451548625460D+00 0.6521451548625460D+00 0.3478548451374540D+00\]

appear on Cards 13A and 13B.

Card 14A
\[
\text{NBCOND} \quad \text{I5}
\]

Card 14B
\[
\text{NBC(1), NODEB(1), NBC(2), NODEB(2), ... NBC(4), NODEB(4)} \quad \text{14I5}
\]

where

\[
\begin{align*}
\text{NBCOND} & \quad \text{The number of prescribed displacement conditions to be} \\
& \quad \text{specified on the main structure. The quantity NBCOND + NBCONB} \\
& \quad \text{must not exceed 7. Note that the information on Card 14B} \\
& \quad \text{corresponds to the original nodal numbering scheme for the} \\
& \quad \text{main structure.}
\end{align*}
\]

\[
\begin{align*}
\text{NBC(1)} & \quad \text{The identification number and the node number, respectively,} \\
& \quad \text{for which the first prescribed displacement condition is to} \\
& \quad \text{be imposed.}
\end{align*}
\]

\[
\begin{align*}
\text{NBC(2)} & \quad \text{The second data group of the identification number and node} \\
& \quad \text{number, respectively, for which the second prescribed dis-} \\
& \quad \text{placement condition is to be imposed.}
\end{align*}
\]
The appropriate form of the data group NBC(I) and NODES(I) should be repeated NBCOND times. If NBCOND=0, that means there is no prescribed displacement condition to be imposed on the main structure; then, skip to Card 15.

The prescribed displacement condition identification number can be equal to 2 or 3, depending upon the type of the prescribed displacement condition. Its description follows:

NBC(I)=2 Ideally clamped condition. Setting \( v, w, \) and \( \psi \) at node NODES(I) to zero.

NBC(I)=3 Smoothly-hinged condition. Setting \( v \) and \( w \) at node NODES(I) to zero.

Card 15

\[ NQR, \ NORP, \ NORU \]

where

\[ \text{NOR} \]
Indicator, which if \( > 0 \) indicates that the structure is subjected to elastic restraints (point and/or distributed).

\[ \text{NORP} \]
The number of point elastic restraints (elastic restoring springs) which are to be prescribed over the structure. This number must not exceed 4.

\[ \text{NORU} \]
The number of local distributed elastic restraints (elastic foundations) which are to be prescribed over the structure. This number must not exceed 4.

If there are no prescribed restraints on the structure, set \( NQR=0 \) and let \( \text{NORP} \) and \( \text{NORU} \) be blank.

Card 15A and Card 15B are included only if \( NQR > 0 \) in Card 15. If \( \text{NORP}=0 \), skip to Card 15B.

Card 15A

\[ \text{SCTP, SCTY, SCRFP} \]
Card 15AA

NREL(1), REX(1), NREL(2), REX(2) ... NREL(4), REX(4) 4(I5,D15.6)

where

SCTP The translational-tangential restoring spring elastic constant (lb/in).
SCTY The translational-normal restoring spring elastic constant (lb/in).
SCRP The torsional restoring spring elastic constant (in-lb/radian).
NREL(I) The element number and the length coordinate along the reference axis from node NREL(I) of the element, respectively, at which the Ith point elastic restraint is to be specified.
REX(I) (Note that the element numbers used here must be in the numbering system which the program generates internally for the entire system after branches have been added.)

The data group NREL(I), REX(I) should be repeated NORP times. If NORU=0 in Card 15, omit Card 15B, and Card 16 follows directly.

Card 15B

SCTU, SCRU, SCTW 3D15.6

where

SCTU Elastic foundation modulus in translation along the tangential direction (lb/in²).
SCRU Elastic foundation modulus in torsion (in-lb)/rad-in).
SCTW Elastic foundation modulus in translation along the normal direction (lb/in²).

Card 15C

NRST(I), NREU(I), ... NRST(4), NREU(4) 8I5

where

NRST(1) The first element and the number of elements, respectively, over which the first elastic foundation is to be specified (the first elastic foundation is distributed to element NRST(1), through and including element (NRST(1)+NREU(1)-1). (Note that the element numbers used here must be in the numbering system which the program generates internally for the entire system after branches have been added.)
NREU(1) NRST(2) The first element and the number of elements over which the second elastic foundation is to be specified.
NREU(2) Data group NRST(I) and NREU(I) are repeated NORU times.
Card 16

ICONT

where

ICONT

Integer which if greater than 0 indicates that this is a
continuation run. In order to use this option, it is
necessary to obtain the following continuation cards from a
previously run job. To do this, set the variable MPU=1 at
the beginning of the MAIN routine. This will cause the
following set of data cards (16A through 16IA) to be punched.
When using this deck, set ICONT=1 and use the same data cards
as used before, except to change the values of MM and M1 on
Card 2.

If ICONT=0, skip Card 16A - 16I and go to Card 17.

If the indicator ICONT is greater than zero, the continuation deck
produced from the output of the previous run follows immediately. The
continuation deck contains the following information:

Card 16A

IT, TIME, IMCOU, TAI

where (L=1 for main structure; L=2, NBR+1 for branches):

IT
The number of the time cycle at which the previous run had
stopped, and is the beginning time cycle of the present
continuation run.

TIME
The absolute time at which the previous run stopped, and
is the beginning time of the present continuation run.

IMCOU
The number of impacts up to the end of the last run.

TAII
Time of initial impact.

Card 16B

IBIGA(L), ISTAA(L), BIGA(L), BTIMA(L), ISURA(L)

where (L=1 for main structure; L=2, NBR+1 for branches):

IBIGA

ISTAA
Information for maximum "additional-point strain". Same as
their counterparts on Card 16C.

BIGA

BTIMA

ISURA

44
Card 16C

\begin{align*}
\text{IBIG}(L), \text{ISURF}(L), \text{ISTA}(L), \text{BIC}(L), \text{BTIME}(L) \quad & (315,2D20.13)
\end{align*}

where \((L=1\text{ for main structure; } L=2, \text{NBR}+1\text{ for branches})\):

- \text{IBIG}(L) The element number whose computed tensile strain exhibits the largest value during the previous run.
- \text{ISURF}(L) Equals 1 means largest computed tensile strain occurs on the inner surface; equals 2 means on the outer surface.
- \text{ISTA}(L) The Gaussian station at which the maximum strain occurred.
- \text{BIC}(L) The largest computed tensile strain during the previous run.
- \text{BTIME}(L) The time at which the largest computed tensile strain occurred during the previous run.

Card 16D

\begin{align*}
\text{IBI}(L), \text{ISUR}(L), \text{BI}(L), \text{BTIM}(L) \quad & 215,2D20.13
\end{align*}

where \((L=1\text{ for main structure; } L=?, \text{NBR}+1\text{ for branches})\):

- \text{IBI}(L) Information for maximum average nodal point strain. Same as their counterparts on Card 16C.
- \text{ISUR}(L)
- \text{BI}(L)
- \text{BTIM}(L)

Card 16E

\begin{align*}
\text{MIRP}, \text{TNJ}(I),..., \text{TNJ}(N\!F) \quad & I5,6D12.5
\end{align*}

where

- \(N\!F\) Number of fragments impacting structure.
- \(\text{MIRP}\) Number of next fragment waiting to be released.
- \(\text{TNJ}(I)\) Indicates whether or not the \(I\text{th}\) fragment has been released.
  \[
  \text{TNJ}(I) = \begin{cases} 
    0.0 & \text{not released} \\
    1.0 & \text{released}
  \end{cases}
  \]

Card 16FA

\begin{align*}
\text{DISP}(I) \quad & 4D20.13
\end{align*}

\text{DISP}(I) The displacement of the \(I\text{th}\) degree of freedom at time cycle IT. Repeat cards until all degree-of-freedom displacements are specified with 4 different values/card.

Card 16GA

\begin{align*}
\text{DELD}(I) \quad & 4D20.13
\end{align*}

\text{DELD}(I) The displacement increment change of the \(I\text{th}\) degree-of-freedom of the structure at time cycle IT. Repeat cards until all degrees-of-freedom are included, with 4 different values/card.

Card 16HA

\begin{align*}
\text{SNS}(I\!R,J,K,L) \quad & 4D20.13
\end{align*}

\text{SNS}(I\!R,J,K,L) The axial stress on the \(L\text{th}\) mechanical sublayer at the \(K\text{th}\) depthwise Gaussian point at the \(J\text{th}\) spanwise Gaussian
station of the IRth element at time cycle IT. Repeat cards until all values for the entire structure are included, with 4 different values/card.

Card 161A

| FCGU(J), FCGW(J), ALFA(J), UDOT(J), WDOT(J), ADOT(J) | 4D20.13 |

FCGU(J) The centroidal position of the Jth fragment in the Y direction at time cycle IT (inches).

FCGW(J) The centroidal position of the Jth fragment in the Z direction at time cycle IT (inches).

ALFA(J) The total angular displacement of the Jth fragment at time cycle IT (radians).

UDOT(J) Fragment velocities at time cycle IT (in/sec); see Card 9AC for more details.

WDOT(J)

ADOT(J)

J = 1,NF

Card 17

ICON I5

where

ICON Integer that controls the stopping of the entire program.

= 0 The program will stop after all of the required print-outs are made for a particular run.

= 1 The program will expect a new set of Cards 1-17 for another ring problem.

4.2 Input for Special Cases of the General Stress-Strain Relations

In the following, the specific input data for three special cases of the general elastic, strain-hardening constitutive relation handled by the computer program are given. Only the relevant data are noted. (L=1 for main structure; L=2 to NBR+1 for the NBR branches):
1. **Purely Elastic Case**

Set $\text{NSFL}(L) = 1$ on Card 2 (Card 5A) and make $\text{EPS}(1,L)$ and $\text{SIG}(1,L)$ on Card 7 (Card 5AA) sufficiently high so that no plastic deformation occurs; for example, $\text{EPS}(1,L) = 1.0$, $\text{SIG}(1,L) = \text{ES}(1,L)$, where $\text{ES}(1,L)$ equals the elastic (Young's) modulus.

2. **Elastic, Perfectly-Plastic Case**

Set $\text{NSFL}(L) = 1$ on Card 2 (Card 5A) and make $\text{EPS}(1,L) = \text{SIG}(1,L)/\text{ES}(1,L)$ on Card 7 (Card 5AA).

3. **Elastic, Linear Strain-Hardening Case**

Set $\text{NSFL}(L) = 2$ on Card 2 (Card 5A) and set $\text{EPS}(1,L) = \text{SIG}(1,L)/\text{ES}(1,L)$.

Also $\text{EPS}(2,L)$ and $\text{SIG}(2,L)$ on Card 7 (Card 5AA) are taken sufficiently high in order to avoid plastic deformation in the second subflange. For example, $\text{EPS}(2,L) = 1.0$, and $\text{SIG}(2,L) = (1.0 - \text{EPS}(1,L)) \times \text{ES}(2,L) + \text{SIG}(1,L)$, where $\text{ES}(2,L)$ is the slope of the segment in the plastic range.

4.3 **Description of the Output**

The printed output begins with a partial reiteration of the program input which identifies the problem solved. This output includes information on initial geometry, the nodal and element numbering system originally assigned by the user, the new updated nodal and element system generated internally in the program if branches are present, the branch attachment points, the ring material properties for the main structure and each branch, the fragment properties, the boundary conditions and elastic restraints that are input, the Gaussian stations and weights used in the program, the lumped mass matrix and the element arc lengths, the time step used in the program and the maximum permitted time step, and the effective lengths associated with the main structure and the branches. (NOTE: If override option is used, the program will calculate a maximum $\Delta t$ value and print this out. However, the $\Delta t$ used in the program calculation will be the user-specified $\Delta t$ regardless of its value.) Example outputs are presented in Section 6. After initial printout has been completed, the following information is printed out (assume NOP=3 here) after cycle $M1$ has been completed, and at every $M2$ cycles thereafter (see Subsection 4.1, Card 2):
WORK AND ENERGY TO END OF TIME CYCLE IT = TIME -

FRAGMENT KINETIC ENERGY
[IT] [CINETF(II)]

WORK INPUT INTO RING = [RWORK]
RING KINETIC ENERGY = [CINETO]
RING ELASTIC ENERGY = [CELAS]
RING PLASTIC WORK = [PLAST]
ENERGY STORED IN ELASTIC RESTRAINTS = [SPDEN]

CYCLE = [IT]
ELEMENT SI STA1 SO SI STA2 SO SI STA3 SO
1
2
3

CYCLE = [IT]
STRAIN AT ADDITIONAL POINTS SI SO EI EO
1
2

48
\[ J = [IT] \quad \text{TIME} = [TIME] \]

\[
\begin{array}{ccccccc}
I & V & W & \Psi & \text{CHI} & \text{COPY} & \text{COPZ} & L & M & \text{STRAIN (IN)} & \text{STRAIN (OUT)} \\
1 & & & & & & & & & & \\
2 & & & & & & & & & & \\
3 & & & & & & & & & & \\
\cdot & & & & & & & & & & \\
\cdot & & & & & & & & & & \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{FRAG NO.} & \text{FCGU} & \text{FCGW} & \text{ALFA} & \text{FRUV} & \text{FRWV} & \text{FRAV} \\
1 & & & & & & \\
2 & & & & & & \\
\cdot & & & & & & \\
\cdot & & & & & & \\
\end{array}
\]

\[
\begin{array}{ccccc}
\text{SUBSTRUCTURE} & \text{MSTR} & \text{ELE} & \text{TIME} & \text{STA} \\
1 & & & & \\
2 & & & & \\
\cdot & & & & \\
\cdot & & & & \\
\end{array}
\]

\[
\begin{array}{cccc}
\text{SUBSTRUCTURE} & \text{LARGEST ADD. PT. STRAIN} & \text{ELEM} & \text{ADD. PT.} & \text{TIME} \\
1 & & & & \\
2 & & & & \\
\cdot & & & & \\
\cdot & & & & \\
\end{array}
\]

\[
\begin{array}{ccccc}
\text{SUBSTRUCTURE} & \text{LARGEST NODAL STRAIN} & \text{NODE} & \text{SURF} & \text{TIME} \\
1 & & & & \\
2 & & & & \\
\cdot & & & & \\
\cdot & & & & \\
\end{array}
\]

where

- \( IT = \) Cycle number
- \( \text{TIME} = \) Elapsed time corresponding to the end of cycle \( IT \) (sec.)
- \( II = \) Fragment number \( II = 1, NF \)
- \( \text{CINETF}(II) = \) The current value of the kinetic energy remaining in fragment \( II \) (in-lb).
RWORK = Total work imparted to the structural ring up to the present
time by fragment impact (in-lb).
CINET = The current value of kinetic energy present in the structural
ring* (includes both the rigid body and the relative kinetic
energies) (in-lb).
ELAST = Total elastic strain energy stored in the entire structural
ring at the present time instant (in-lb).
PLAST = Total plastic work * done on the structural ring (mechanical
work dissipated during plastic flow) (in-lb).
SPDEN = Total energy stored in the elastically-restoring springs and/
or the elastic foundations at the current time instant (in-lb),
if the presence of elastic constraints is specified.
SI = Strain at the inner surface of the ring
SO = Strain at the outer surface of the ring
STA1
STA2 * = Spanwise Gaussian Station at which strain was calculated
STA3
EI = Relative elongation at inner or outer surface, respectively,
EO = at the additional strain points; obtained from $E_1 = \sqrt{1+2Y_{11}} - 1$.

It should be noted that the rigid body part of the kinetic energy, which is
used to accelerate the "rigid body" mass of the structure, can be extracted
and identified separately. However, for the present program dealing with
rather general structural geometries and with various support/restraint
conditions, it would be very unwieldy (but not impossible) to identify these
separate kinetic energies; hence, the total kinetic energy is calculated
and printed out.

**The plastic work done on the ring is estimated by subtracting the sum of the
elastic and kinetic energies present in the ring from the total input energy
due to the fragment impact; i.e., RWORK=CINET+ELAST+PLAST+SPDEN. It should
be mentioned that the approximate nature of this numerical calculation will
sometimes yield impossible results such as negative values of plastic work or
values greater than zero when the ring has not yet reached a plastic condition;
thus, the value of plastic work should be considered only approximate, and
spurious results as noted above should be ignored. This form may also be
considered to contain, in addition, the energy dissipated by friction.

50
I = Node number. For a partial ring, the value of the total number of nodes equals the value of the total number of elements plus one. For a complete ring, the value of the total number of nodes equals the value of the total number of elements.

V = The middle plane axial displacement at node I (in).

W = The middle plane transverse displacement at node I (in).

PSI = The generalized nodal displacement $\psi = (\partial v/\partial \eta) - v/R$ at node I (rad).

CHI = The generalized nodal displacement $\chi = (\partial v/\partial \eta) + w/R$ at node I (rad).

COPY = The Y-location of node I in the global (inertial) coordinate system (in).

COPZ = The Z-location of node I in the global coordinate system (in).

L = Axial internal force resultant over the cross section at the midspan point of element I (lb).

N = Internal bending moment of the cross section at the midspan point of element I (in-lb).

STRAIN(IN) = Average strain on the inner surface at node I.*

STRAIN(OUT) = Average strain on the outer surface at node I.*

FCCU = Global Y coordinate of the centroid of the fragment at the current time instant (in).

FCGW = Global Z coordinate of the centroid of the fragment at the current time instant (in).

ALFA = Angular rotation of the fragment to the current time instant (rad).

FRUV = The current velocity component of the fragment in the Y direction (in/sec).

FRWV = The current velocity component of the fragment in the Z direction (in/sec).

FPAV = The current angular velocity of the fragment (rad/sec). Positive sign denotes counter-clockwise rotation.

*At nodes on the main structure where a branch connection occurs, the contribution of strain from the branch is not included in the nodal averaging process.
SUBSTRUCTURE = The portion of the ring being considered.
1 = main structure
2 = 1st branch
-3 = 2nd branch

MSTR = Maximum strain on the substructure at a Gaussian station.
ELE = Element on which max. tension strain occurred.
TIME = Time at which max. tension strain occurred.
STA = Gaussian station at which max. tension strain occurred on element ELE.
NODE = Node at which largest average value of tension strain occurred.
ELEM = Element on which largest value of tension strain occurred at an additional strain point.
ADD. PT. = Additional Point Number
TIME = Time at which the max. tension strain occurred

Note: Checks for largest average nodal strain and largest additional-point strain are made only at print out cycles. Check for largest Gaussian-station strain is made at each cycle.

In addition to the above information which is printed out at each desired time cycle, whenever there is an impact the following information is printed out:

THIS IS IMPACT NUMBER [M] TIME = IT = FRAGMENT NO. = [JF]
ELEM:NT NO. = [LNTMIN] LOCATION ON ELEM = [RPC]

where
M = The number of the impact (total number of impacts up until that time)
TIME = The time of contact between fragment and ring
IT = Cycle during which impact takes place
JF = Fragment involved in this collision
LNTMIN = Element (on main structure) involved in the collision
RPC = Fraction of the element length from the point of ring-fragment contact to the first node of the impacted element.
At the conclusion of the run, a final update of the maximum strain occurring on each substructure (main structure and each branch) and for the additional strain points on the main structure and each branch is printed out. Also, a note as to whether or not continuation cards were punched is made.

4.4 Guides and Restrictions for Code Usage

4.4.1 General Instructions

The CIVM-JET 4B computer code is capable of handling a wide variety of transient, large-deflection structural response problems involving impact-induced loading. This capability is beneficial to the user since it is contained in one computer program; however, it can be unnecessarily costly to run the program if all of the options included are carried along but are not being used.

In order to save storage locations and therefore save computer costs on each run, several subroutines can be removed when they are not used during that particular job submittal.

The following is a list of subroutines which must be included in every run of the CIVM-JET 4B program:

1. MAIN
2. ASSEF
3. ASSEM
4. CUBIC
5. ELMPP
6. ENERGY
7. ERC
8. FICOL
9. IDENT
10. IMPACT
11. IMPCTE
12. MINV
13. PENTRN
14. PRINT
15. ROOT4
16. STRESS
17. TCONT
18. UPDATE

The remaining subroutines: BRAN, DINIT, OMULT, QREM, ROTAT, and TSTEP may be left out depending upon the type of problem being solved.

Subroutine TSTEP may be optional when the user desires. If the user inputs a time-step and does not wish this time step to be checked by the CIVM-JET 4B program, the user merely omits including the TSTEP subroutine.
and only inputs the following three cards instead of TSTEP.*

```fortran
SUBROUTINE TSTEP (KLOW, NDEX, NIRREG, DELTAT)
RETURN
END
```

In like manner, if the user does not wish to apply any branches to the main structure, the BRAN subroutine becomes optional. If no branches are used, BRAN is input as follows:

```fortran
SUBROUTINE BRAN (NBR)
RETURN
END
```

Thus, for any subroutine that is not being used for a particular run, just submit the first card of the subroutine (the name card) and the RETURN and END cards. This will save the user input costs, compilation costs, and storage costs. The same procedure is used for QREM if no elastic restraints are defined, for ROTAT if no branches or slope discontinuities are used, for DINIT if continuation cards are being used for input, or for OMULT only if TSTEP and QREM are also left out.

### 4.4.2 Use of Branches vs. Use of Discontinuities

In the present CIVM-JET 4B program, both branches attached to and slope-discontinuities in the main structure can be accommodated. Because of the way in which these two features are handled in the program logic and, in particular, for determining ring-fragment collision, certain general guidelines should be followed by the user for more efficient use of the computer code.

The dominant consideration involves the determination of ring-fragment collision. In the present code, impact on a branch is not accommodated, so that if branch impact is an important consideration for a particular problem, then the slope-discontinuity option must be used in place of the branch option, thus identifying this region not as a branch but as a part of the main structure. Note, however, that in all cases where three elements are to be connected at a single nodal point (such as in the case of a branch at the midspan of a beam), one of the elements must be defined as a branch. In general, because the Central Difference Operator is conditionally stable, the use of subroutine TSTEP is recommended for the first run of a given structural geometry and finite-element mesh arrangement to insure that a stable $\Delta t$ will be used in the time-wise solution.
at any node where three elements are joined, one of these elements (and all subsequent elements in that portion of the structure) must be defined as branch elements. However, two branches may not start at the same node of the main structure.

During each increment in time, $\Delta t$, a collision inspection is carried out for each element on the main structure for each of the $N$ fragments considered, but no collision inspection is carried out for elements defined as branch elements. This fact can be used to reduce the total computation time; if the user knows a priori that no collisions will occur in certain regions of the structure, then those regions should be defined as branches wherever possible (note that branches cannot be specified to be attached to another branch).

As noted in Section 2, the positive $n$ direction must be chosen in such a way that the outward normal for each element is directed toward the outside of the structure, where the fragments are considered to be in the inside of the structure. In some special (but plausible) cases, the outside region of a portion of the structure may overlap with the inside region of another portion of the structure either in the initial structural configuration or after some deformation of the structure has occurred. Within the current collision inspection program logic, such overlapping regions cannot be accommodated. Consider the following initial structural configuration:

![Diagram of the structure](fragment)
With the positive normal directions $\vec{N}$ as shown (and considering the entire structure as a main structure, using the slope-discontinuity option), the outside regions of segments 1-2 and 3-4 would overlap with the inside region of segment 2-3. To accommodate such a configuration, segments 1-2 and 3-4 must be defined as branches so that no collision inspection is performed for those segments. Consider now the following initial structural configuration:

For the initial configuration (solid lines), no inside-outside overlapping occurs. However, after deformation of segment 2-3 (dashed line, assuming fragment impact is as depicted above), the outside regions of segments 1-2 and 3-4 will overlap with the inside region of segment 2-3. Again, in this case, segments 1-2 and 3-4 must be defined as branches.

4.4.3 Impact at or Near a Constrained Node

In the present CIVM-JET 4B program, the case of fragment impact on or near a constrained node is handled in an approximate fashion. The nature of this approximation is discussed briefly here as a guide to the user in interpreting results when impact occurs on or near a constrained node.

The assumption used is as follows: when impact occurs near a constrained node, the only nodes which can respond with impact-induced velocity changes are those nodes which lie within the impact-affected region and which are located on the impacted side of the constrained node. In
essence, no impact-induced information is allowed to propagate past the constrained node (for the purpose of impact corrections), and a portion of the impact-induced impulse is "absorbed" by the constraint. Although no impact-induced information is allowed to propagate past the constrained node for the purpose of impact-induced velocity, the impact-induced information will filter through the constrained node in the global timewise solution, if the constrained node is smoothly hinged. If the node is ideally clamped, no impact-induced information will filter past the constrained node in the global timewise solution. It should also be noted that as the point of ring-fragment impact approaches the constrained node, the impact-induced ring response approaches zero; when impact occurs directly on a constrained node, the fragment simply rebounds and no impact-induced structural response occurs.

For a more thorough discussion of this topic, the reader is referred to Appendix A.

4.4.4 Comments on Strain Calculation and Mesh Sizing

In the present CIVM-JET4B program, options are available which allow the user to obtain strain printout at the spanwise Gaussian stations (which includes the element midspan location), and/or at additional points on the structure specified by the user. Nodal average strains are given automatically at each regular printout cycle. This flexibility can be of great value to the user, but certain precautions should be taken by the user in interpreting the strain results.

The strain-displacement relation employed in the present curved beam elements is given by Eq. A.12. An examination of this equation shows that nonlinear terms are included only in the membrane behavior, and only linear terms are included in the bending behavior. Thus, the membrane nonlinearities have been assumed to be more significant than the bending nonlinearities. The calculated distribution of strain may be quite different from element to element, and the strain distribution within each element will, in general, not be the same as the "exact" distribution. This behavior corresponds to the fact that in the present finite-element model the predicted strain distribution approximates the actual strain distribution in an average (integral) sense, and not in a pointwise sense. Although the calculated distribution
be the same as the "exact" distribution at some points within the element, the choice of a "best" point (or points) for strain evaluation within an element is not obvious. The choice of \( v, w, \psi, \) and \( \chi \) as generalized nodal parameters assures membrane strain continuity at the nodes, but in general the bending strain will not be continuous at the nodes; thus, the predicted strains at the inner and outer surface of an element will not, in general, be continuous at the nodes.

Because of these facts, certain precautions should be taken by the user when assessing strain distributions in space and/or time. If detailed strain distributions are required over a portion of the structure at a particular time instant, it is suggested that several printout points be chosen in each element (e.g. Gaussian stations and nodal averaged points) in the region of interest. When these calculated values are plotted, the analyst can then make a reasonable "faired" estimate of the "proper" distribution. It should be noted that severe strain gradients within an element do not necessarily indicate poor behavior of the solution; however, it is in these regions where the analyst must exercise the greatest caution in making a reasonable faired estimate of the proper distribution. Although not conclusive, experience to date with the present CIVM-JET4B computer code suggests that these regions of predicted severe strain gradients are most often observed near clamped boundaries and may be found near the region of ring-fragment impact.

If strain time-history information is required at various points on the structure, these points can be specified as additional strain points and the time histories may be obtained directly. In addition, it is recommended that spatial distributions near these points of interest be obtained at several time instants to assess whether or not the strain at the point of interest is in reasonable agreement with the curve-fitted (or faired) distribution in that region of the structure. If these steps are followed, a reasonable engineering assessment of strain information should be obtained.

The equations in Appendix A have been developed within the assumption of large deflections but small strains. Thus, reliable results may not be obtained in localized regions where large strains are predicted. However, the actual strain level at which the "small strain" assumption becomes invalid is not known. Thus, the limitations of the present analysis, for practical engineering problems, cannot be clearly stated; further study of the limitations of the present analysis versus appropriate well-defined experimental data is required. In the future it is recommended that models which can accommodate arbitrarily large strain be
developed for both two-dimensional (planar) and three-dimensional (non-planar) deformations. In the meantime, however, it is believed that the capabilities of the present CIVM-JET 4B analysis and program can provide useful engineering estimates.

In the present CIVM-JET 4B program, considerable flexibility is given to the user in terms of defining the size and number of elements to be used for a particular structural geometry. However, certain guidelines should be followed in the selection of a finite-element mesh for the present impact analysis. It is recommended that a uniform mesh be employed for all analyses, the only exception being in regions where structural detail dictates the use of nonuniform elements. Clearly this recommendation is justified for the general case where the point of initial (and subsequent) impact is not known a priori. Now consider the special (limiting) case where it is known a priori that all ring-fragment impacts will occur at (approximately) the same point on the structure (e.g. initially straight, uniform thickness, doubly clamped beam with the only nonzero component of the fragment velocity being normal to the beam mid-surface, and initial impact occurring at the midspan of the beam). A uniform mesh is again recommended for this special case, based on the following considerations. The impact effected length, $L_{\text{eff}}$, is directly related to the size of the time step (i.e. $L_{\text{eff}} = \sqrt{\frac{E}{D}} \Delta t$) and when using the central difference operator the allowable time step, $\Delta t$, is inversely related to the highest natural frequency, $\omega_{\text{max}}$, of the assembled-structure. If the element size in the region of impact is decreased, then $\omega_{\text{max}}$ is increased and $\Delta t$ and, thus, $L_{\text{eff}}$ are decreased. In the limit, as the element size is decreased, the impact-induced "loading" will become concentrated at the point of impact and unreasonably high strain predictions may be found near this region of concentrated loading. It is believed (based on experience to date with the present CIVM-JET 4B program) that the choice of a uniform mesh for this case will yield the most reliable predictions.
SECTION 5

COMPLETE FORTRAN-IV-LISTING OF THE
CIVM-JET 4B PROGRAM

The CIVM-JET 4B program consists of the following main program and 23 subroutines:

1. MAIN
2. ASSEF
3. ASSEM
4. BRAN
5. CUBIC
6. DINIT
7. ELMPP
8. ENERGY
9. ERC
10. FICOL
11. IDENT
12. IMPACT
13. IMPCTE
14. MINV
15. OMULT
16. PENTRN
17. PRINT
18. QREM
19. ROOT4
20. ROTAT
21. STRESS
22. TCONT
23. TSTEP
24. UPDATE

The program is written in double precision arithmetic. A complete FORTRAN IV listing of the program is given below in the above order. The number of memory locations required on the IBM 370/168 computer at MIT is approximately 415,000 bytes; this includes locations for the MIT computer library subroutines.
* PPS (6, 5), SIG (6, 5), FFIN (6)
COMMON/ELPU/SPRN (2060), ECRFP (205), REX (4), NQR, NDRP, NOBP, NRPL (4)
* NRST (4), NREU (4)
COMMON/ODPE/ DLP50, 2, 3, 8
COMMON/EP/ EPSI (51), EPSO (51)
COMMON/DIS/ ANGD1 (50), NEDI (50), NDIS
COMMON/COU/ IMCOU
COMMON/BN/ LAT (51)
COMMON/TL/ HTH (5)
COMMON/MH/ MNEL (6), MATT (6)
COMMON/BOUN/ YK (51), NECONE, NBCH (7), NODBB (7), NK (51), BDT (5.2)
\$ DROT (50), NODR (6)
COMMON/XY/ XDIST (6)
COMMON/HR/ NVAC (51, 2)
SIN (Q) = DSIN (Q)
COS (Q) = DCS (Q)
ATAN (Q) = DATAN (Q)
ABS (Q) = DABS (Q)
SQRT (Q) = DSQRT (Q)
MREAD = 5
WRITE = 6
MPUNCH = 7
IRR = 1

5555 READ (MREAD, 1) B (1), DEN1 (1), EXAI, NOGA, NPL, NSPL (1), MM, M1, M2, NP
tmp
FORMAT (3D15.6, /8IS, D15.6)
IMCOU = 0
TAII = 0.0
MPU = 1
C REMOVE MPU=0 IF CONTINUATION CARDS ARE DESIRED
MPU = 0
NP2 = M1
P12 = 3.141592653589793C+0C
P12 = 2.0* P1C
IKP1 = IK + 1
NS = IK
IF (EXANG.NE.360.) NS=IKP1
MNEL(1) = IK
NEFL = 0
NROUT = 0
NNNG = 0
LELF=0
LELR=0
IMCO = 0
C GEOMETRY INPUT -- USER COULD SUBSTITUTE AN INTERNAL GENERATING
C ROUTINE AT THIS POINT
7901 READ(MREAD,7902) (Y(I),Z(I),ANG(I),H(I),I=1,NS)
7902 FORMAT(4D15.6)
DO 7903 I=1,NS
7903 ANG(I) = ANG(I) * PIE/180.0D0
7999 CONTINUE
C DISCONTINUITY INPUT
READ(MREAD,5300) NDIS
NDI = NDIS
IF (NDIS.EQ.0) GO TO 8100
READ(MREAD,8101) (NDI(I),ANGDI(I),I=1,NDIS)
8101 FORMAT (4(15,D15.6))
DO 8102 I=1,NDIS
8102 ANGDI(I) = (ANGDI(I)*PIE)/180.0
8100 CONTINUE
201 READ(MREAD,5300) NBR
5300 FORMAT(2I5)
MNSPL= NSPL(1)
DO 5305 I = 1, IK
MK2(I) = 1
LMT(I) = 0
YK(I) = 0.0
MK(I) = I
NVEC(I,1) = I
5305 NVEC(I,2) = I+1
IF (EXANG.EQ.360.0) NVEC(IK,2) = 1
I* (EXANG.NE.360.0) MK(IK+1) = IK+1
IF (NBR.EQ.0) GOTO 100
WRITE(6,105)
105 FORMAT(' THERE ARE NO BRANCHES CONNECTED TO THE MAIN STRUCTURE '
'THEREFORE, THE NUMBERING SYSTEM FOR NODES AND ELEMENTS REMAINS UNCHANGED')
100 CONTINUE
IF (NBR.EQ.0) GOTO 204
CALL GRAN(NBA)
DO 8888 I=1,NBR
 IF (NSFL(I+1).GT.NSFL) NSFL(NSFL(I+1))
8888 CONTINUE
204 M=NSFL(1)
 READ(NREAD,2) DELTAT,DS(1),P(1),NTOV, EPS(1,L),SIG(1,L),L=1,M
2 FORMAT(3D15.6,15/(4D15.6)))
 DELTA= DELTAT
 IF (DELTAT.EQ.0.0) DELTA=1.0D+06
LGSP= 0
LSPP =0
C ADDITIONAL STRAIN POINT DATA
READ(NREAD,8200) NOP,HASF
8200 FORMAT(215)
 DO 8215 I=1,IK
 DO 8215 J=1,2
8215 LK(I,J) = 0
 IF (NOP.EQ.0) GO TO 8220
 IF (NOP.EQ.2) LGSP=1
 IF (NOP.EQ.1) LSPP=1
 IF (LSPP.NE.1) GO TO 8220
 READ(NREAD,8210) (NSBS(I),NSFL(I),AZET(I),I=1,HASP)
8210 FORMAT(215,15.6))
 WRITE(6,156)
156 FORMAT(' ADDITIONAL STRAIN POINT',5X,'ELEMENT',5X,'S COORDINATE')
 DO 8216 J= 1,HASP
 IF (NSBS(J).NE.1) GO TO 8217
 M= NSFL(J)+1
 IF (EXANG.EQ.360.0.AND.NSFL(J).NE.IK) MK(M) = IK+1

N = MK(M) - 1
LKK(N,1) = 1 + LKK(N,1)
NO = LKK(N,1) + 1
LKK(N,NO) = J
GO TO 8140
8217 IF(MODP(NSBS(J)-1).EQ.1) GO TO 8219
N = MK(MODP(NSBS(J)-1)) + NSPL(J) - 1
GO TO 8219
8218 N = NSEL(J)
8219 LKK(N,1) = 1 + LKK(N,1)
NO = LKK(N,1) + 1
LKK(N,NO) = J
140 WRITE(MWRITE,145) J, N, AZET(J)
145 FORMAT(1,9X,I5,13X,I5,7X,D15.6)
8216 CONTINUE
9220 CONTINUE
   IF(NDIS.EQ.0) GO TO 8140
   IF(NBR.EQ.0) GOTO 8145
   IF(NDI.EQ.0) GOTO 8145
   DO 8146 J = 1,NDI
     M = NEDI(J) + 1
     N = MK(M) - 1
     NEDI(J) = N
8146 CONTINUE
8145 WRITE(MWRITE,8111)
     WRITE(MWRITE,8120) (NEDI(I),I=1,NDIS)
8111 FORMAT('EACH OF THE FOLLOWING ELEMENTS HAS A SLOPE DISCONTINUITY
     AT ITS FIRST NODE')
8120 FORMAT(1,1X,25I5)
     WRITE(MWRITE,8112) (ANGDI(I),I=1,NDIS)
8112 FORMAT('THE GLOBAL SLOPE (RAD.) AT EACH DISCONTINUITY EQUALED:
     (ND15.6)
   DO 8130 I = 1,NDIS
     M = NEDI(I)
     YK(M) = YK(M) + 2.0
     L1 = NVEC(M,1)
DADOT(M) = ANGDI(1) - ANG(1)
8130 CONTINUE
8140 CONTINUE
  IKM1 = IK - 1
  IKP1 = IK + 1
  HGOO D = IK - NELE
C FRAGMENT PROPERTIES
DO 202 I=1,NP
  READ(MREAD,601) FH(I),FCG(I),PCGX(I),PMASS(I),PHO(I)
  READ(MREAD,601) UNK(I)
  202  READ(MREAD,602) UDOT(I),WDOT(I),ADOT(I),TPRIN(I),CR(I)
  601  FORMAT(6D15.6)
  602  FORMAT(6D15.6)
C GAUSSIAN STATIONS AND WEIGHTS
  READ(MREAD,3) (AOG(K),K=1,NOGA)
  READ(MREAD,3) (ANW(K),K=1,NOGA)
  READ(MREAD,3) (ITK(K),K=1,NFL)
  READ(MREAD,3) (TGW(K),K=1,NFL)
  3  FORMAT(3D25.16)
  4Y=NS*4
  READ(MREAD,4) NBCND
  IF(NBCND.EQ.0) GO TO 747
  READ(MREAD,4) (NBC(I),NIDIB(I),I=1,NBCND)
  747  IF(NBR.EQ.C) GO TO 748
  NIT = NBCND+1
  NIT1 = NIT - 1
  NBCND = NBCND + NBCONB
  IF(NBCONB.EQ.0) GO TO 751
  DO 750 LOP= 1, NBCONB
    NBC(NIT1 * LOP) = NBCB(LOF)
    750  NIDIB(NIT1 * LOP) = NIDIB(LOP)
    751 IF(NIT1.EQ.0) GO TO 748
    DO 753 LOP = 1,NIT
      NT1 = NIDIB(LOP)
    753  NIDIB(LOP) = NK(NT1)
748  HEAD(MREAD,9) NQR,NOF,NORU
9  FORMAT(3I5)
   MX=1
   NY=M2
   CUMW=0.0
   DELKE=0.0
   DO 203 I=1,NF
203  PK(I)=(FMASS(I)/2.0)*(UDOT(I)**2+WDOT(I)**2)*(FMOI(I)/2.0)*(ADOT(I))
     CALL IDENT(NQK,NBR)
     WRITE(MWRITE,941) NF
     WRITE(MWRITE,940) (TPRM(I),I=1,NF)
941  FORMAT('///','THE TPRM FOR EACH OF *I5,* FRAGMENTS IS AS FOLLOWS'
   940  FORMAT(8D15.6)
     WRITE(MWRITE,902)
402  FORMAT('///','GAUSSIAN STATIONS AND WEIGHS: *')
     WRITE(MWRITE,400) (L,AXG(L),L,AWG(L),L=1,NOGA)
     WRITE(MWRITE,401) (L,TXG(L),L,TWG(L),L=1,NFL)
400  FORMAT(' *',12X,'AXG',13.2X,'=',F20.15,'8X','AWG',13.2X,'=',F20.15)
401  FORMAT(' *',12X,'TXG',13.2X,'=',F20.15,'8X','TWG',13.2X,'=',F20.15)
   IC=0
C ESTABLISH VECTORS AND MATRICES FOR SUBLAYER CALCULATIONS
   DO 70 IR=1,IK
      L1= NVEC(IR,1)
      L2= NVEC(IR,2)
      IF(YK(IR)*EQ.1.0.OR.YK(IR).NE.1.0) IC(IC+1)
         DO 70 J=1,NOGA
            IF(YK(IR)*NE.1.0.AND.YK(IR).NE.3.0) GOTO 600
            IF(ROT(IC,1)*EQ.0.0) GO TO 610
         END IF
      END IF
      RHI= HTH(IC)*(1.0-AXG(J)) + H(L2) * AXG(J)
      GO TO 611
   600  RHI= H(L1) * (1.0-AXG(J)) + H(L2) * AXG(J)
      GO TO 611
    610  CONTINUE
DO 70 K=1,NPL
GFL (IR,J,K) = RH*TWG (A)*B(NKE(IR))/2.0
70 ETA (IR,J,K) = RH*TXG (K)/2.
J=1+NPL
DO 5400 I=1,J
ES(I,1) = SIG(I,1)/EPS(I,1)
M = NSPL(I)
IF(M<1) 77, 77, 76
76 DO 78 L=2,M
K=L-1
78 FS(I,L) = (SIG(I,L)-SIG(I,K))/(EPS(I,L)-EPS(I,K))
77 N = NSPL(I) +1
FS(I,N) = 0.0
DO 79 L=1,M
79 SNO(I,L) = ES(I,1)*EPS(I,L)
YOUNG(I) = ES(I,1)
5400 CONTINUE
DO 71 IR=1,N
N=MKE(IR)
M = NSPL(N)
DO 71 J=1,NPL
DO 71 K=1,NPL
DO 71 L=1,H
LL=L+1
71 GFL (IR,J,K,L) = GPL (IR,J,K)* (ES(N,L)-ES(N,LL))/ES(N,1)
IP(NBH,NE.0) GO TO 218
DO 15 I=1,8
15 ICOL(I) =1
IKM1 = IK-1
IF(EXANG.ME.360.) GO TO 210
DO 16 I=3,IKM1
IK4=I+4
IK3=IK4-1
IK2=IK4-2
IK1=IK4-3
JJ = (I-1)*4-3
MAIN2530
MAIN2540
MAIN2550
MAIN2560
MAIN2570
MAIN2580
MAIN2590
MAIN2600
MAIN2610
MAIN2620
MAIN2630
MAIN2640
MAIN2650
MAIN2660
MAIN2670
MAIN2680
MAIN2690
MAIN2700
MAIN2710
MAIN2720
MAIN2730
MAIN2740
MAIN2750
MAIN2760
MAIN2770
MAIN2780
MAIN2790
MAIN2800
MAIN2810
MAIN2820
MAIN2830
MAIN2840
MAIN2850
MAIN2860
MAIN2870
MAIN2880
ICOL (IK1) = JJ
ICOL (IK2) = JJ
ICOL (IK3) = JJ
ICOL (IK4) = JJ
16 CONTINUE
ICOL (IK*4) = 1
ICOL (IK*4-1) = 1
ICOL (IK*4-2) = 1
ICOL (IK*4-3) = 1
GO TO 218
210 DO 211 I=3, IKP1
IK4=I*4
IK3=IK4-1
IK2=IK4-2
IK1=IK4-3
JJ=(I-1)*4-3
ICOL (IK1) = JJ
ICOL (IK2) = JJ
ICOL (IK3) = JJ
ICOL (IK4) = JJ
211 CONTINUE
218 INUM (1) = 1
99 DO 99 I=2, NI
INUM (I) = 1-ICOL (I-1)+INUM (I-1)
99 DO 990 I=1, NI
990 INUM (I) = INUM (I) - ICOL (I)
NIRREG = 0
INDEX = 0
ISET = 1
116 DO 117 I=1, NI
L=ICOL (I)
IF (ICOL (I)-ISET) 117, 116, 119
119 ISET=ICOL (I)
GO TO 116
117 NIRREG=NIRREG+1
IF (NIRREG-NI/2) 711, 711, 90
711 KROW(NIRREG)=I
INDEX(NIRREG)=INDEX
116 INDEX=INDEX+1-L
90 CALL PICO(NI,NL,L,PCL)
ISIZE=L
WRITE(WRITE,17) L
17 FORMAT(/, SIZE OF ASSEMBLED STIFFNESS MATRIX = 'I,I5)
IF(L.LT.2060) GOTO 6012
WRITE(WRITE,6011)
6011 FORMAT('THE SIZE OF THE STIFFNESS MATRIX HAS EXCEEDED 2060. THIS MAIN3340
RUN HAS BEEN TERMINATED. CHANGE DIMENSION OF STIFF IN',/)
& 'MAIN,ELMPP, AND TSTEP')
GO TO 160
6012 CONTINUE
CALL ELMPP(DELTA,AA,ISIZE,KROW,INDEX,NIRREG,NUM)
61 DO 981 IR=1,IKP1
RMAS(IS)=0.0
981 RX(IR)=0.0
IC=0
C CALCULATION OF LUMPED MASS MATRIX -- SOL
DO 980 IR=1,IK
K1= NVEC(IR,1)
K2= NVEC(IR,2)
H1= H(K1)
H2= H(K2)
IF(YK(IR).NE.1.0 .AND. YK(IR).NE.3.0) GOTO 641
IC = IC+1
IF(ROT(IC,1).EQ.0.0) GO TO 640
H(K1) = HTH(IC)
GO TO 641
640 H(K2) = HTH(IC)
641 CONTINUE
C((2.*H(K2)*H(K1))/(3.*H(K2)+3.*H(K1))
CLP(IR)=1.0-CL(IR)
CLA(IR)=AL(IR)*CL(IR)
CLPA(IR)=AL(IR)*CLP(IR)
RMOI (IK) = (H(K1) **2 + 4 * H(K1) * H(K2) + H(K2) **2) * AL(IR) **3 / 
* (16 * (H(K1) + H(K2)) * B(MKE(IR)) * DENS(MKE(IR))
H(K1) = d1
H(K2) = H2
980 CONTINUE
IC = 0
DO 982 I = 1, IKM1
L = NVEC(I,1)
L2 = NVEC(I,2)
H1 = H(L)
H2 = H(L2)
IF(YK(I),NE.1.0.AND.YK(I),NE.3.0) GOTO 661
IF(ROT(IC,1),EQ.0.0) GC TO 660
H(L) = NTH(IC)
GO TO 661
660 H(L2) = NTH(IC)
661 CONTINUE
N = MKE(I)
RMASS(L) = RMASS(L) + (H(L) + H(L2)) * B(N) * DENS(N) * CLPA(I) / 2.0
RMASS(L2) = RMASS(L2) + (H(L) + H(L2)) * B(N) * DENS(N) * CLA(I) / 2.0
RMX(L) = RMX(L) + RMOI(I) * CLP(I)
RMX(L2) = RMX(L2) + RMOI(I) * CL(I)
H(L) = H1
H(L2) = H2
982 CONTINUE
K1 = NVEC(IK,1)
H1 = H(K1)
H2 = H(IK+1)
N = MKE(IK)
IF(YK(I),NE.1.0.AND.YK(I),NE.3.0) GOTO 681
IF(ROT(IC,1),EQ.0.0) GC TO 690
H(K1) = NTH(IC)
GO TO 681
680 H(IK+1) = NTH(IC)
681 CONTINUE
IF (EXANG.EQ.360.) GO TO 983
PM(1) = PM(1) + H (K) * DENS (N) * CLPA (IK)/2.0
RM(1) = RM(1) + H (K) * DENS (N) * CLA (IK)/2.0
GO TO 984
983 PM(1) = PM(1) + H (K) * DENS (N) * CLPA (IK)/2.0
RM(1) = RM(1) + H (K) * DENS (N) * CLA (IK)/2.0
984 CONTINUE
H (K) = H1
H (K+1) = H2
WRITE (MWRITE, 7836)
WRITE (MWRITE, 7837) (RMASS(L), L=1, NS)
WRITE (MWRITE, 7838)
WRITE (MWRITE, 7837) (RMX(L), L=1, NS)
7836 FORMAT(//, ' THE TRANSLATIONAL MASSES FOR EACH NODE ARE: ')
7837 FORMAT(//, ' THE ROTATIONAL MASSES FOR EACH NODE ARE: ')
7838 FORMAT(' ', 4D25.15)
CALL TSTEP (KROD, NDEX, NIRREG, DELTAT)
C OVER RIDE ANY CHANGE IN DELTAT BY TSTEP
IF (HTOVR.EQ.1) DELTAT = DELTA
DO 5 IN=1, NS
SOL (IR*4-J) = RMASS (IR)
SOL (IR*4-2) = RMASS (IR)
SOL (IR*4-1) = RMX (IR)
5 SOL (IR*4) = RMX (IR)
DO 6 I=1, NI
SOL (I) = DELTAT*2/SOL (I)
6 DO 530 I=1, NS
530 JVER (I) = 0
IF (NOR .EQ. 0) GO TO 21
DO 23 L=1, ISIZE
23 SPRIN (L) = 0.0
CALL QRFM (AA, AL, AXG, AWG)

21  MCRIT = 0
    M = NBR + 1
    DJ = 10605
    J = 1, M
    BIG (J) = 0.0
    BICA (J) = 0.0
    BI (J) = 0.0
    IBIG (J) = 0
    IBI (J) = 0
    ISTA (J) = 0
    ISTAA (J) = 0
    RTIME (J) = 0.0
    RTIMA (J) = 0.0
    RTIM (J) = 0.0
10005  CONTINUE
    DO 75 I = 1, NS
      COY (I) = Y (I)
      COZ (I) = Z (I)
    READ (MREAD, 82) ICONT
    75  COI = (I) = Z (I)
    READ (MREAD, 82) ICONT
    82  FORMAT (I5)
    83  FORMAT (2 (I5, E20.13))
    87  FORMAT (215, 2D20.13)
    86  FORMAT (215, 2D20.13, I5)
    89  FORMAT (I5, 6D12.5)
    385  FORMAT (6D13.6)
    DO 8400 K = 1, NS
      IKK = NS
      ICP = 0
      IF (EXANG .EQ. 360.0) ICP = 1
      N = NBR + 1
3800  HMIN (K) = H (K) / 2.0
    DO 5551 I = 1, N
    C DETERMINE EFFECTIVE LENGTH FOR EACH SUBSTRUCTURE
5551  EPLN (I) = (YOUNG (I)/DENS (I)) * 0.5*DELTAT
    WRITE (HWRITE,5552)
    WRITE(HWRITE,5553) (EPLN(J),J=1,N)
5552  FORMAT (" THE FOLLOWING NUMBERS ARE THE VALUES FOR THE EFFECTIVE"
    * LENGTHS FOR THE NEW PLUS 1 SECTIONS OF THE STRUCTURE")
5553  FORMAT (" ', 6D15.6)
    WRITE(A,'(TF,7839)
    7839  FORMAT ("/ / / / /", THE FOLLOWING IS THE TIME SOLUTION OF THE FRAGMENT")
57  CALL DINIT(IT,TIME)
70    DO 6111 I=1,N1
6111   QACL (I) = 0.0
70    NQ = MIRP
80  C CHANGE CYCLE PARAMETERS IF TSTEP HAS BEEN ALLOWED TO OVER NIDE
    USER'S DELTAT
   IF (DELTAT.LE.DELTA) GO TO 9102
6112   M1=IDINT(M1*DELTAT/DELTAT1)
    MM=IDINT(MM*DELTAT/DELTA1)
6113   M2=IDINT(M2*DELTAT/DELTAT1)
   IF(M2.LT.1) M2 = 1
6114   NPI = M1
6115   MX=M1
6116   MY=M2
9102  CONTINUE
992    GO TO 992
81  READ(MREAD,83) IT,TIME,INCOU,TAIL
82    M= NBR+1
83    READ(MREAD,86) (IBIGA(L),ISTA (L),BIGA(L),BTMA (L),ISURA (L),L=1,N)
84    READ(MREAD,386) (IBIG (L),ISURF(L),ISTA(L),BIG (L),BTMS (L),L=1,N)
85    READ(MREAD,387) (IDI (L),ISUR (L),BI (L),BTIM (L),L=1,3)
86    READ(MREAD,89) MIRP, (TNJ(I),I=1,NP).
87    READ(MREAD,64) (DISP(I),I=1,N)
88    READ(MREAD,84) (DELD(I),I=1,N)
89    READ(MREAD,84) (QVEL(I),I=1,N)
90  MAIN4690
91  MAIN4700
92  MAIN4710
93  MAIN4720
94  MAIN4730
95  MAIN4740
96  MAIN4750
97  MAIN4760
98  MAIN4770
99  MAIN4780
100  MAIN4800
101  MAIN4810
102  MAIN4820
103  MAIN4830
104  MAIN4840
105  MAIN4850
106  MAIN4860
107  MAIN4870
108  MAIN4880
109  MAIN4890
110  MAIN4900
111  MAIN4910
112  MAIN4920
113  MAIN4930
114  MAIN4940
115  MAIN4950
116  MAIN4960
117  MAIN4970
118  MAIN4980
119  MAIN4990
120  MAIN5000
121  MAIN5010
122  MAIN5020
123  MAIN5030
124  MAIN5040
READ (MREAD, 84) (QACL(I), I=1,NI)
READ (MREAD, 84) (((SNS(ID, J, K, L), L=1, MNSPL), K=1, NPL), J=1, NOGA),
*IR=1, IK)
READ (MREAD, 84) (FCGU(J), PCGW(J), ALFA(J), UDOT(J), WDOT(J)
*ADOT(J), J=1, NF)
NQ= MIRR
C ****** START OF THE TIME SOLUTION ******
992  IT=IT+1
  IF (TIME.GT.TIMF) GO TO 965
   CALL IMPACT(EFLN, IT,NBR, QACL, CVZL)
   DO 994 I=1, NI
994  DISP(I)=DISP(I)+DEL(I)
C STAGGERED FRAGMENT RELEASE
6010 IF (NO.GT.HF.OF.NO.EQ.0) GO TO 6020
   TL= (IT*DELTAT) - (IPRN(1) - IPRN(NO))
   IF (DABS(TI) .GT. DELTAT) GOTO 6020
   TNJ(NO) = 1.0
   NO=NO+1
   GO TO 6010
6020  DO 822 I=1, NF
   FCGU(I) = FCGU(I) + DFCGU(I) * TNJ(I)
   PCGW(I) = PCGW(I) + DPCGW(I) * TNJ(I)
   ALFA(I) = ALFA(I) + DALFA(I) * TNJ(I)
   DO 522 I=1, NI
   FQREF(I) = 0.0
522  FLVA(I)=0.0
   CALL STRESS
   IF (NO.EQ.0) GO TO 735
   CALL OMULT(SPRIN, DISP, ICOL, NI, FQREF, KROW, WDEI, MRPEG)
   DO 736 I=1, NI
736  FLVA(I)=FLVA(I)+FQREF(I)
735  CONTINUE
11100 FORMAT('0+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++')
996  IF (NBCOND.EQ.0) GO TO 889
DO 888 I=1,NDCOND
NX=NODEB(I)
IF (WBC(I).EQ.1) GO TO 886
IF (WBC(I).EQ.2) GO TO 887
IF (WBC(I).EQ.3) GO TO 885
886 PLVA(NXY*4-3)=0.0
PLVA(NXY*4-1)=0.0
GO TO 888
887 PLVA(NXY*4-3)=0.0
PLVA(NXY*4-2)=0.0
PLVA(NXY*4-1)=0.0
GO TO 888
885 PLVA(NXY*4-3)=0.0
PLVA(NXY*4-2)=0.0
888 CONTINUE
889 NIPE=NI
C PIND NEW DISPLACEMENT INCREMENT
DO 525 I=1,NI
QACL(I) = -PLVA(I)*SOL(I)
QVEL(I) = QACL(I)/(2.*DELTAT) + DELD(I)/DELTAT
525 CONTINUE
IF (IMCO.EQ.0) GOTO 527
IMCO= 0
DO 526 J=1,NS
IF (JVEL(J).EQ.0) GOTO 526
JVEL(J) = 0
QVEL(J*4-3)= VEL(J*2-1)
QVEL(J*4-2)= VEL(J*2)
526 CONTINUE
527 CONTINUE
IF (IT-MX)815,816,815
816 MX=MX+MY
WRITE(MWRITE,11100)
CALL ENERGY(IT,KROW,WDEX,WIRREG,SOL,ES,GFL,QVEL)
815 CONTINUE
DO 528 I=1,NI
528 DOLD(I) = QACL(I)/2.0 + QVEL(I)*DELTAT
TIME=IT*DELTAT
IF(IMCOU.EQ.0) TAI1=TIME
MIZ=0
NPZ= IT-NP21
IF(NPZ.NE.0) GOTO 6700
IF(LGSP.EQ.0) GOTO 6700
WRITE (MWRITE,6707) IT
6705 FORMAT ("0 CYCLE=", I8)
WRITE (MWRITE,6707)
MIZ= 1
NP21 = NP21+M2
DO 6701 I= 1,NS
ITHR(I) = 0
EPSI(I) = 0.0
6701 EPSO(I) = 0.0
C GAUSSIAN STATION STRAIN CALCULATION
6700 DO 7161 IR=1,IK
K1= NVEC(IR,1)
K2= NVEC(IR,2)
LSS = NKE(IR)
DO 8018 K=1,8
INDEX= (K1-1)*4*K
IF(K.GT.4) INDEX= (K2-1)*4*K-4
DISM(K) = DISP(INDEX)
8018 CONTINUE
IF(YK(IR).LT.0.0) GOTO 901
CALL ROTAT(1,DUMMY,DISM,IR)
901 CONTINUE
DO 604 I=1,NOGA
DO 604 J=1,3
BEPS (I,J) = 0.0
DO 604 K=1,8
604 BEPS(I,J) = BEPS(I,J) + BEP(IR,I,J,K) * DISM(K)
H1 = H(K1)
H2 = H(K2)
N = MKL(IR) - 1
IF (Y(K(IR)) .EQ. 1.0 .AND. R0T(N,1) .EQ. 1.0) H(K1) = HT(N)
IF (Y(K(IR)) .EQ. 1.0 .AND. R0T(N,1) .EQ. 0.0) H(K2) = HT(N)
HDIF = H(K2) - H(K1)
DO 60 M = 1, J
HHAG = (H(K1) + AXG(N) * HDIF) / 2.0
FARE = BEPS(M,1) + BEPS(M,2) * 2/2.0
**BEPS(M,1) **2/2.
EPI(M) = FARE - HHAG*BEPS(M,3)
EPO(M) = FARE + HHAG*BEPS(M,3)
C FIND LARGEST GAUSSIAN STRAIN
IF (EPI(M) .LE. BIG(LSS)) GO TO 591
BIG(LSS) = EPI(M)
IBIG(LSS) = IR
ISTA(LSS) = M
ISURF(LSS) = 1
BTIME(LSS) = TIME
591 IF (EPO(M) .LE. BIG(LSS)) GO TO 1200
BIG(LSS) = EPO(M)
IBIG(LSS) = IR
ISTA(LSS) = M
ISURF(LSS) = 2
BTIME(LSS) = TIME
1200 CONTINUE
60 CONTINUE
IF (NPZ .NE. 0) GOTO 6607
C AVERAGE NODAL STRAIN CALCULATION
C AT A NODE WHERE A BRANCH ATTACHES TO THE MAIN STRUCTURE,
C THE BRANCH'S NODAL STRAIN IS NOT AVERAGED IN
DO 6604 I=1, J
DO 6604 J=1, 3
BEPS(I,J) = 0.0
DO 6604 K=1, 8
6604 BEPS(I,J) = BEPS(I,J) + EPS(I,R,I,J,K) * DISH(K)
FAR1 = BEPS (1,1) + BEPS (1,2) * 2/2.0 + BEPS (1,1) * 2/2.0
FAR2 = BEPS (2,1) + BEPS (2,2) * 2/2.0 + BEPS (2,1) * 2/2.0
NKE = MKE(IR)
IF(NKE.EQ.1) GOTO 6605
IF(MATT(NKE-1).EQ.K1) GOTO 6606

6605 ADEN = 1.0
IF(ITHR(K1).GT.0) ADEN=2.0
ITHR(K1) = 1
EPSI(K1) = EPSI(K1) + FAR1-H(K1)*BEPS(1,3)/2.0 / ADEN
EPSO(K1) = EPSO(K1) + FAR1+H(K1)*BEPS(1,3)/2.0 / ADEN
IF(NKE.EQ.1) GOTO 6606
IF(MATT(NKE-1).EQ.K2) GOTO 6607

6606 ADEN = 1.0
IF(ITHR(K2).GT.0) ADEN=2.0
ITHR(K2) = 1
EPSI(K2) = EPSI(K2) + FAR2-H(K2)*BEPS(2,3)/2.0 / ADEN
EPSO(K2) = EPSO(K2) + FAR2+H(K2)*BEPS(2,3)/2.0 / ADEN

6607 CONTINUE
H(K1) = H1
H(K2) = H2
IF(MIZ.NE.1) GOTO 7161
IF(LGSP.EQ.0) GOTO 7161

7940 WRITE(MWRITE,6710) IR, (EPI(L),EPO(L),L=1,3)
6710 FORMAT(1,12.2X,3(2X,D11.4,1X,D11.4))

7161 CONTINUE
IF(NP2.NE.0) GOTO 7180
C FIND LARGEST AVERAGE NODAL STRAIN
DO 7170 I = 1,NS
N= 0
DO 7171 IR=1,IK
IF(NVEC(IR).NE.I) GOTO 7172
IF(MKE(IR).EQ.1) N= N+1
IF(MKE(IR).GT.1) N= N+3
IF(MKE(IR).GT.1) NKE=MKE(IB)
7172 CONTINUE
IF(NVEC(IR,2).NE.I) GOTO 7171
I(NKE(IR),2Q,1) N=N+1
I(NKE(IP),GT,1) N=N+3
I(NKE(IF),GT,1) NKE=NKE(IR)

7171 CONTINUE
NK=1
I(N,EQ.3.OR.N.EQ.6) NA=NKE
I(EPSI(I),LE,BI(NK)) GOTO 7174
BI(NK) = EPSI(I)
IBI(NK) = I
ISUR(NK) = 1
DTIM(NK) = TIME

7174 I(EPSO(I),LE,BI(NK)) GOTO 7170
BI(NK) = EPSO(I)
IBI(NK) = I
ISUR(NK) = 2
ATIM(NK) = TIME

7170 CONTINUE
7180 CONTINUE
I(LSP.EQ.0) GOTO 8562
I(NPZ,EQ.0) GOTO 8562
KTI=KTI+1
WRITE(7,WRITE,6705) IT
WRITE(7,WRITE,6707)

9707 FORMAT(' STRAIN AT ADDITIONAL POINTS',10X,'SI',10X,'SO',10X,'23X','EI',10X,'EO')

8700 DO 8761 IR=1,IK
I(LKK(IF,1),EQ,0) GOTO 8761
K1=MVEC(IF,1)
K2=MVEC(IF,2)
L=MKE(IF)
DO 8019 K=1,8
INDEX=(K1-1)*4+K
IF(K,GT,4) INDEX=(K2-1)*4+K-4
DISH(K)=DISP(INDEX)

8019 CONTINUE
I(YK(IR),EQ,0.0) GOTO 902
CALL ROTAT(1,DUMMY,DISM,IR)

902 CONTINUE
H1 = H(K1)
H2 = H(K2)
N= MKE(IR)-1
IF(YK(IR).EQ.1.C.AND.ROT(N,1).EQ.1.0) H(K1) = HTH(N)
IF(YK(IR).EQ.1.0.AND.ROT(N,1).EQ.0.0) H(K2) = HTH(N)
NO = LKK(IR,1)
DO 8763 I = 1,NO
IS = LKK(IR,I+1)
DO 8604 J=1,3
AEPS(J) = 0.0
DO 8604 K=1,8
8604 AEPS(J)=AEPS(J) + AEP(IS,J,K)*DISM(K)
HDIF= H(K2)-H(K1)
HHAG= (H(K1) + AZET(IS)*HDIF)/2.0
FARE= AEPS(1)+AEPS(2)**2/2.0*AEPS(1)**2/2.0
EPASI= FARE*HHAG*AEPS(3)
EPASO=FARE*HHAG*AEPS(3)

C FIND LARGEST ADDITIONAL POINT STRAIN
IF(EPSI.LE.BIGA(L)) GO TO 8591
BIGA(L) = EPSI
IBIGA(L) = IR
ISTAA(L) = IS
BTIMA(L) = TIME
ISURA(L) = 1
8591 IF(EPSO.LE.BIGA(L)) GO TO 8780
BIGA(L) = EPSO
IBIGA(L) = IR
ISTAA(L) = IS
BTIMA(L) = TIME
ISURA(L) = 2

8780 IF(MIZ.NE.1) GO TO 8763
PI= DSQRT(1.0+2.0*EPASI) -1.0
EO= DSQRT(1.0+2.0*EPASO) -1.0
WRITE(MWRITE,8781) IS,EPASI,EPASO,EL,EC
8781 FORMAT(*,'(10X,I3,16X,D15.4,4X,D15.8,11X,D15.8,4X,D15.8)') MAIN7570
8763 CONTINUE
     H(K1) = H1
     H(K2) = H2
8761 CONTINUE
8562 CONTINUE
1250 IF(IT-M1)987,988,150
988 M1=M1+1
     CALL PRINT(IT,TIM)
     M=NBR+1
     WRITE(MWRITE,66)
     WRITE(MWRITE,67) (L,BIG(L),IBIG(L),ISURF(L),ISTA(L),BTIME(L),L=1,N) MAIN7680
     IF(LSPP.NE.1) GO TO 8782
     WRITE(MWRITE,6030)
     WRITE(MWRITE,6035) (L,BIGA(L),IBIGA(L),ISTA(L),BTMA(L),ISURA(L),L=1,N)
8782 CONTINUE
     WRITE(MWRITE,7181)
     WRITE(MWRITE,7182) (L,BI(L),ISI(L),ISUR(L),BTIM(L),L=1,N)
7181 FORMAT(*,'CSUBSTRUCTURE',5X,'LARGEST LOCAL STRAIN',5X,'NODE',7X,'SURF',11X,'TIME')
7182 FORMAT(*,'4X,I1,16X,D15.6,7X,I3,6X,I5,5X,D15.6')
     WRITE(MWRITE,11100)
987 IF(IT-MM)992,965,150
965 CONTINUE
     WRITE(MWRITE,6002)
6002 FORMAT(*,'THE LARGEST COMPUTED STRAINS FOR EACH SUBSTRUCTURE--MAIN AND BRANCHES -- ARE PRINTED BELOW, 1=INNER 2=OUTER SURF') MAIN7840
     M=NBR+1
     WRITE(MWRITE,66)
     WRITE(MWRITE,67) (L,BIG(L),IBIG(L),ISURF(L),ISTA(L),BTIME(L),L=1,N) MAIN7870
66 FORMAT(*,'OSUBSTRUCTURE',8X,'MSTR',7X,'ELE',5X,'SURF',5X,'STA',9X,'TIME')
67 FORMAT(*,'3X,I4,6X,D15.6,1X,I4,4X,I4,4X,I4,4X,D15.6')
     IF(LSPP.NE.1) GO TO 149
     WRITE(MWRITE,6030)
WRITE(MWRITE,6035) (L,BIGA(L),IBIGA(L),ISTAA(L),BTIMA(L),ISURA(L),MAIN1930)
$L=1,M)
6030 FORMAT('0SUBSTRUCTURE',5X,'LARGEST ADD. PT. STRAIN',5X,'ELEM',5X)
$ADD. PT.',9X,'TIME',10X,'SURFACE')
6035 FORMAT('*,4X,I,13,16X,D15.6,7X,13,6X,15.5X,D15.6,6X,I4)
149 CONTINUE
WRITE(MWRITE,7181)
WRITE(MWRITE,7162) (L,BI(L),IBI(L),ISUR(L),BTIM(L),L=1,M)
150 IF (MPE.EQ.0) GO TO 160
C PUNCHING OF CONTINUATION CARDS IF MPE GT 0
M=M8B+1
WRITE(MPUNCH,83) IT,TIME,IMCON,TASK
WRITE(MPUNCH,86) (IBIG(L),ISTAA(L),BIGA(L),BTIMA(L),ISURA(L),
$L=1,M)
WRITE(MPUNCH,386) (IBIG(L),ISUR(L),ISTA(L),BIG(L),BTIME(L),L=1,M)
WRITE(MPUNCH,387) (IBI(L),ISUR(L),BI(L),BTIM(L),L=1,M)
WRITE(MPUNCH,89) MNP, (TNJ(I),I=1,NP)
WRITE(MPUNCH,84) (DISP(I),I=1,NI)
WRITE(MPUNCH,84) (DELD(I),I=1,NI)
WRITE(MPUNCH,84) (QVEL(I),I=1,NI)
WRITE(MPUNCH,84) (JACJ(I),I=1,NI)
WRITE(MPUNCH,84) (((SNS(IR,J,K,L),L=1,MNSFL),K=1,NFL),J=1,HOGA)
*IR=1,IK)
WRITE(MPUNCH, 4) (PCGU(J), PCGW(J), ALFA(J), UDOT(J), WDOT(J)
*ADOT(J), J=1,NF)
WRITE(MWRITE,6005)
6005 FORMAT('0CONTINUATION CARDS HAVE BEEN PUNCHED FOR THIS RUN')
GO TO 161
160 WRITE(MWRITE,151)
151 FORMAT('0ONE CARDS PUNCHED DURING THIS RUN FOR CONTINUATION')
161 READ(MREAD,1100) ICON
C CHK FOR ADDITIONAL DATA SETS, IF NONZ FOUND TERMINATE RUN
IF (ICON) 1120,1110,1120
1120 IRRUN=IRRUN+1
WRITE(MWRITE,1130) IRRUN
1130 FORMAT('*,11 THIS IS DATA SET NUMBER,IS,* FOR THIS RUN')
WRITE(MWRITE,1130) IRRUN
1130 FORMAT('*,1,1 THIS IS DATA SET NUMBER,IS,* FOR THIS RUN')
SUBROUTINE ASSP (IR, IK, FL0P, FLVA, XHNC)
IMPLICIT REAL*8 (A-H, O-Z)
DIMENSION NV (8), PLVA (1), ELPP (1)
COMMON /BR/ NVYC (51, 2)
SIN (2) = DSIN (Q)
COS (Q) = DCOS (Q)
ATAN (Q) = DATAN (Q)
IPS (Q) = DIPS (Q)
SORT (Q) = DSORT (Q)
J1 = NVYC (IR, 1) = 4
NN (1) = J2 - 3
NN (2) = J2 - 2
NN (3) = J2 - 1
NN (4) = J1
IF (.XANG .GT. 36.* ) TO 121
IF (IR - IK) 121, 122, 122
121 J2 = NVYC (IR, 2) = 4
NN (5) = J2 - 3
NN (6) = J2 - 2
NN (7) = J2 - 1
NN (8) = J2
30 TO 123
122 NN (5) = 1
NN (6) = 2
NN (7) = 3
NN (8) = 4
123 DC 1:1 I = 1, 9
K = NN (I)
FLVA (I) = PLVA (I) + ELPP (I)
131 CONTINUE
?STUPN
END
SUBROUTINE ASSY (IP, FLMAS, STIP)
IMPLICIT REAL*8 (A-H, Q-Z)
DIMENSION FLMAS (8, 9), NN (3), STIP (1)
COMMON /TAP/ Y(51), A(51), XANG, VS, IX, MCJA, NFL, WT.
* ICOI (25), XCCCD, NAC (7), NODEB (7)
COMMON /MAT/ DENS (6), B (6), YOUNG (6), DS (6), SNC (6, 5), NSPL (8), P (6).
* EPS (u, 5), SIG (6, 5), EFLN (6)
COMMON /BV/ NVEC (5, 2)
SIN (0) = DSIN (0)
COS (0) = DCOS (0)
ATAN (0) = D atan (0)
ABS (0) = DABS (0)
SQR (0) = DSQRT (0)
J1 = NVEC (1, 1) * 4
NN (1) = J1 - 3
NN (2) = J1 - 2
NN (3) = J1 - 1
NN (4) = J1
IF (EXANG, NE, 342) GO TO 293
IF (IR-IY) 201, 204, 204
203 J2 = NVEC (12, 2) * 4
NN (5) = J2 - 3
NN (6) = J2 - 2
NN (7) = J2 - 1
NN (8) = J2
GOTO 202
204 NN (5) = 1
NN (6) = 2
NN (7) = 4
NN (8) = 4
202 DO 402 I = 1, 8
W = NN (I)
EC = 5 2 J = 1, 8
N = NN (J)
IF (I-M) 402, 403, 403
SUBROUTINE BRAN(NB)
IMPLICIT REAL*8(A-H,L-Z)
DIMENSION YB(5,11),ZH(5,11),ANG(5,11),HE(5,11),MELT(6)
DIMENSION LER(7),ZK(51),HK(51),ANGK(51)
DIMENSION LATT(6)
DIMENSION SLE(5)
DIMENSION LHT(6)
COMMON/PG/Y(51),Z(51),ANG(51),H(51),EXANG,NS,IK,NOGA,NFL,NI.
* ICUL(205),NBCND,NBC(7),NODE(7)
COMMON /HM/ C5,CO,ASFL(50,1,6,5),GZETA(50,3,6)
COMMON/MAT/ LENS(6),DI(6),YOUNG(6),DS(6),SND(6,5),NSFL(6),P(6).
* ZPS(6,5),SIG(6,5),FNLN(6)
COMMON /TAPL/ NREAD,WRITE,MPRINT
COMMON /BOUN/ YK(51),NBCONB,NBCB(7),NODEB(7),KK(51),ROT(5,2)
* DROT(50),NODP(6)
COMMON /TAN/ MK(51)
COMMON/DIS/ ANIGD(50),NPDI(50),TDIS
COMMON /BN/ LATT(51)
COMMON /BD/ XDIST(6)
COMMON /THI/ HTH(5)
COMMON /ML/ MNEL(6),NMAT(6)
COMMON /SR/ NVZC(51,2)
C RENUMBER NODES AND ELEMENTS
NNS = NS
NIK = IK
PIF=3.1415926535897931*0C
DO 5301 J=1,6
MELT(J)=0
5301 CONTINUE
DO 5302 K=1,NS
YK(K)=Y(K)
ZK(K)=Z(K)
HK(K)=H(K)
MK(K)=0
5302 ANGK(K)=ANG(K)
DO 5311 I=1,NBB
IR = I+1
READ (MREAD,5500) NSPL(IB),B(IB),DENS(IB),DS(IB),P(IB)
5500 FORMAT (5,4C15.6)
L = NSPL(IB)
READ (MREAD,5510) (EPS(IB,J),SIG(IB,J),J=1,L)
5510 FORMAT (4D15.6)
READ (MHEAD,5400) NELT(I),NODP(I),LHT(I),LATT(I)
5300 FORMAT (14I5)
MNEL(I+1) = NELT(I)
NODPH= NODP(I)
IF (LATT(I)) 5210,5230,5220
5210 XDST(I) = H(NODPH)/2.0
GO TO 5240
5220 XDST(I) = -H(NODPH)/2.0
GO TO 5240
5240 CONTINUE
N01 = NELT(I) + 1
DO 5310 J = N01,NO1
READ (MREAD,5305) YB(I,J),ZB(I,J),ANB(I,J),NB(I,J)
5305 FORMAT (4D15.6)
AN3(I,J) = ANB(I,J)*PI/180.0
5310 CONTINUE
HHT(I) = H(P(I,N01)
SLB(I) = AN3(I,N01)
NS=NS+1
ILT(I)
IK = IK+1
5111 CONTINUE
DO 5200 K=1,NS
MK(K) = 0
5200 LMT(K) = 0
DO 5270 I=MK*IK
5270 MK(I) = 1
WRITE (WRITE,5311) IK,NS
5111 P:FORMAT(OTHER ARE '13' ELEMENTS ARE 'IJ' NODERS)
WRITE (WRITE,5312) NBR,(KODP(M),M=1,NBR)
5312 FORMAT('OTHER ARE , I3,' BRANCHES AND THEY ARE AT NODES', 516) BRAN0730
WRITE(MWRITE, 9500) (SLB(K),K=1,NBR)
5500 FORMAT('THE GLOBAL SLOPE(HAD) AT EACH BRANCH CONNECTION: ', 5D15.6) BRAN0750
WRITE(MWRITE,5250) NBR BRAN0760
5250 FORMAT('THE ATTACHMENT POINT CODE FOR THE ',I4,' BRANCHES IS AS FOLLOWS:') BRAN0770
5260 FORMAT(' ',5IH) WRITE(MWRITE,5260) (LATT(L),L=1,NBR)
& OF THE MAIN STRUCTURE*)
5315 CONTINUE
5316 CONTINUE IF(NODP(I) .NE. 1) GOTO 5320
NODP(I) = 1 + NELT(I) BRAN0970
5320 CONTINUE
K= 1 + NELT(I) BRAN0980
Y(K) = YK(I) BRAN0990
Z(K) = ZK(I) BRAN1000
H(K) = HK(I) BRAN1010
ANG(K) = ANGK(I) BRAN1020
MK(I) = KN BRAN1030
NEL = NELT(I) BRAN1040
DO 5325 I = 1, NEL
Y(I) = YB(I,I) BRAN1050
Z(I) = ZB(I,I) BRAN1060
ANG(I) = ANB(I,I) BRAN1070
5325  $H(I) = HB(1,I)$
      GOTO 5322
5320  NO1= NODP(1)
      DO 5321 J = 1,NO1
5321  MK(J) = J
5322  NELTT=0
      DO 5330 I = 1,NBR
      IF(NODP(I).EQ.1) GOTO 5334
      NEL = NELT(I)
      DO 5335 J = 1,NEL
          KN = J* NODP(I) + NELTT
          Y(KN) = YB(I,J)
          Z(KN) = ZB(I,J)
          ANG(KN) = ANB(I,J)
      5335  H(KN) = HB(I,J)
5334  NELTT= NELTT + NELT(I)
5330  CONTINUE
      NZB = 0
      NYB = 0
      LTIME= 0
      LELT = 1
      DO 5100 J = 1,NS
      NXB = J + NYB
      IF( NXB.EQ.MK(J)) GO TO 5100
      MKL = 0
      LTIME= LTIME+1
      MX = NXB-2
      NYB = NYB+ NELT(LTIME)
      IF(LHIT(LTIME).NE.0) GO TO 5140
      NZB = NZB+NELT(LTIME)
      IF(J .EQ. 1) MX0 = 0
      MKL=MKL+MX
      DO 5130 I = LELT,NZB
          MKL=MKL+1
      5130  LNT(I) = MKL
      LELT = NZB+1
5140 IF (LTIME.EQ.NBR) GOTO 5145
5150 CONTINUE
5155 CONTINUE
LELT = 0
NYB = 0
LTIME = 0
DO 5275 J = 1, NS
NXB = J + NYB
IF (NXB.EQ.MK(J)) GO TO 5275
LTIME = LTIME + 1
MXX = NXB - 2
NYB = NYB + NELT(LTIME)
IF (J.EQ.1) MXX = 0
LELT = MXX + 1
NZB = MXX + NELT(LTIME)
DO 5280 I = LELT, NZB
5285 MKS(I) = LTIME + 1
IF (LTIME .EQ. NBR) GO TO 5285
5275 CONTINUE
5280 NODP(NBR + 1) = IK
NT = NELT(1) * NODP(2) + NELT(2) - 1
NTT = NELT(1) * NELT(2)
IF (NBR .EQ. 1) NT = IK
IF (NODP(1).EQ.1) GO TO 5340
NT = NODP(1) * NELT(1) - 1
NTT = NELT(1)
5340 DO 5345 I = 1, NT
NVEC(1,1) = I
5345 NVEC(1,2) = I + 1
NO = 2
IF (NODP(1) = 1) 5350, 5355, 5350
5355 IF (NBR.EQ.1) GO TO 5350
NO = 3
5360 NB1 = NBR + 1
DO 5360 I = NO, NB1
IF (NT.EQ.IK) GO TO 5400
NT= NT+1
NVEC(NT,1) = NT- NELT(I-1)
NVEC(NT,2) = NT+1
NT = NT+1
MT = MT+1
NDO = NTO+ NODP(I) - 1
IF (NODP(I).EQ.IK) NDO=IK
DO 5365 J=NT,NOO
NVEC(J,1) = J
5365 NVEC(J,2) = J+1
NT = NOO
5360 CONTINUE
IF (EXANG.EQ.360.0) NVEC(IK,2) = 1
5400 CONTINUE
WRITE (MWRITE, 110)
110 FORMAT (/,' PRESENT ELEM. NO. ',5X,'NODE1* ',5X,'NODE2* ',5X,'SUBSTRUCT
SURE* ',5X,'SUBST. ELEM. NO. *')
JEL = 0
JL=0
DO 115 IR=1,IK
IF (MKE(IR).NE.1) GO TO 120
JL=0
JEL=JEL+1
JELE=JEL
GO TO 125
120 JL=JL+1
JELE=JL
125 WRITE (MWRITE, 126) IR,NVEC(IR,1),NVEC(IR,2),MKE(IR),JELF
126 FORMAT (' ',6X,'ELE',15X,'MKE(IR)',5X,'SUBSTRUCT',5X,'SUBST. ELEM. NO. *')
115 CONTINUE
WRITE (MWRITE, 130)
130 FORMAT ('OTHE UPDATED NODE NUMBERS FOR THE MAIN STRUCTURE, GIVEN IN
THEIR ORIGINAL NUMBERING ORDER:*')
WRITE (MWRITE,5323) (BK(IL),L=1,5NS)
5323 FORMAT ('0',5S)
1010 FORMAT('** NOTE: THE ELEMENT NUMBERS REFERRED TO BELOW ARE PRESENTED **')
WRITE (MWRITE, 1010)
1010 FORMAT ('** ELEMENT NUMBERS **')
WRITE (MWRITE, 2110)
WRITE (MWRITE, 2100) (LMT (N), N=1, NS)
2100 FORMAT ('**10')
2110 FORMAT ('** ELEMENTS THAT CAN NOT BE IMPACTED: ')
C ESTABLISH BOUNDARY CONDITIONS
C VECTOR YK(51) NOW CONTAINS ACTUAL NODE NUMBER OF ORIGINAL DEFLECTOR
READ(MREAD, 5300) NDIB
IF (NDISB .EQ. 0) GO TO d100
DO 8101 I=1, NDISB
R2AD (MREAD, 8102) NDIB, NBDI, ANGDB
8102 FORMAT (2I5, L15.6)
ANGDB = ANGDB * PIE/180.0
NDIS = NDIS + 1
ANGDI (NDIS) = ANGDB
L= NODP(NBDI)
K=MK(L)
IF (L .EQ. 1) K=1
8103 NEDI (NDIS) = NEDI + K - 1
8101 CONTINUE
8100 CONTINUE
R2AD (MREAD, 5300) NBCONB
IF (NBCONB .EQ. 0) GOTO 5376
R2AD (MREAD, 5300) (NBCB(L), NODBB(L), LBR(L), L=1, NBCONB)
DO 5370 L = 1, NBCONB
N01 = NODP (LBR(L))
WT = 0
LB = LBR(L) - 1
DO 5375 J = 1, LB
5375 WT = WT + WLT(J)
IF (LB .NE. 0) GOTO 5371
WLT = 0
IF (NODP (1) .EQ. 1) GOTO 5370
5371 NODBB(L) = NODBB(L) + NO1 + WLT
5370 CONTINUE
5376 CONTINUE
   DO 5410 I=1,NBR
   L= NODP(I)
   5410 MATT(I)= MK(L)
C DETERMINE LEADING NON ZERO TERM IN EACH ROW
   DO 15 I= 1,8
   15 ICOL(I) = 1
   IKM1 = IK-1
   NI = NS*4
   IF(EXANG.NE.360.0) GO TO 210
   DO 16 I = 2,IKM1
   J= (I+1)*4
   ICOL(J) = NVEC(I,1) * 4 - 3
   ICOL(J-1) = NVEC(I,1) * 4 - 3
   ICOL(J-2) = NVEC(I,1) * 4 - 3
   ICOL(J-3) = NVEC(I,1) * 4 - 3
   16 CONTINUE
   I= NVEC(IK,1)
   ICOL(I*4) = 1
   ICOL(I*4-1) = 1
   ICOL(I*4-2) = 1
   ICOL(I*4-3) = 1
   GO TO 218
   210 DO 211 I= 2,IK
   J= (I+1)*4
   ICOL(J) = NVEC(I,1) * 4 - 3
   ICOL(J-1) = NVEC(I,1) * 4 - 3
   ICOL(J-2) = NVEC(I,1) * 4 - 3
   ICOL(J-3) = NVEC(I,1) * 4 - 3
   211 CONTINUE
   218 CONTINUE
   DO 5341 M= 1,IK
   5341 YK(M) = 0.0
   DO 5342 M = 1,NBR
   LP (NODP(M),NE.1) GO TO 5343
   BRAN 2530
   BRAN 2540
   BRAN 2550
   BRAN 2560
   BRAN 2570
   BRAN 2580
   BRAN 2590
   BRAN 2600
   BRAN 2610
   BRAN 2620
   BRAN 2630
   BRAN 2640
   BRAN 2650
   BRAN 2660
   BRAN 2670
   BRAN 2680
   BRAN 2690
   BRAN 2700
   BRAN 2710
   BRAN 2720
   BRAN 2730
   BRAN 2740
   BRAN 2750
   BRAN 2760
   BRAN 2770
   BRAN 2780
   BRAN 2790
   BRAN 2800
   BRAN 2810
   BRAN 2820
   BRAN 2830
   BRAN 2840
   BRAN 2850
   BRAN 2860
   BRAN 2870
   BRAN 2880
SUBROUTINE CUBIC(P, Q, R, Y, IER)

IMPLICIT REAL*8 (A-H, O-Z)

C FIND A SINGLE REAL ROOT OF THE CUBIC EQN. Y**3 + P*Y**2 + Q*Y + R = 0
C REDUCE EQN., DEFINE NEW COEFS.
TEN = 10.0D+00
D22 = 1.0D+22
D33 = 1.0D+33
NEXP = 0
A = (3.0*Q-P*P)/3.0
B = (2.0*P**3-9.0*P*Q+27.0*R)/27.0
205 IF (A.GT.D22 .OR. A.LE.-D22) GOTO 200
IF (B.GE.D33 .OR. B.LE.-D33) GOTO 200
GOTO 210
200 A = A/1.0D+08
B = B/1.0D+12
NEXP = NEXP + 1
GOTO 205
210 CONTINUE
C FORM INNER SQUARE-ROOT FACTOR
D = B**4.0*27.0
IF (D) 10, 10, 10
C SQUARE ROOT WILL NOT YIELD IMAGINARY NUMBER
10 D = DSQRT(D)
C FORM COEFS, CAPITAL A AND B
D = D**12
B = B**12
CA = -B/2.0*D
CB = -B/2.0-C
C SIGN OF THESE COEFS
SCA = 1.0
SCB = 1.0
IF (CA.LT.0.0) SCA = -1.0
IF (CB.LT.0.0) SCB = -1.0
C TAKE CUBE ROOT OF ABS. VALUE OF CA AND CB
CA = CA**SCA
CD = SCB*CB
CUBE00 10
CUBE00 20
CUBE00 30
CUBE00 40
CUBE00 50
CUBE00 60
CUBE00 70
CUBE00 80
CUBE00 90
CUBE01 00
CUBE01 10
CUBE01 20
CUBE01 30
CUBE01 40
CUBE01 50
CUBE01 60
CUBE01 70
CUBE01 80
CUBE01 90
CUBE02 00
CUBE02 10
CUBE02 20
CUBE02 30
CUBE02 40
CUBE02 50
CUBE02 60
CUBE02 70
CUBE02 80
CUBE02 90
CUBE03 00
CUBE03 10
CUBE03 20
CUBE03 30
CUBE03 40
CUBE03 50
CUBE03 60
CA=CA**((1.0/3.0))
CB=CB**((1.0/3.0))

C ROOTS OF REDUCED EQN.
X=CA*SCA+CE*SCB

C ROOT OF ORIGINAL EQN.
Y=X-P/3.0
RETURN

C THREE UNEQUAL REAL ROOTS. CHOOSE ROOT=CA+CB.

C CALC. REAL PORTION OF CA BY USING POLAR FORM OF COMPLEX NO.

100 \( u = -\frac{D}{2.0} \)
\( d = -D \)
\( v = \text{DSQRT}(D) \)
DIST=DSQRT(D*U+V*V)
COEF=DIST**((1.0/3.0))
COST=U/DIST
THETA=ARCCOS(COST)
THETA=THETA/3.0
COST=DCOS(THETA)
X=COEF*COST*2.0
Y=X-P/3.0
RETURN
END
SUBROUTINE DINIT(IT,TIM2)
IMPLICIT REAL*8(A-H,O-Z)
COMMON /TM/ MKX(51)
COMMON /HIT/ TNJ(6),MIPP
COMMON /Q/ PLVF(205),DISP(205),DLED(205),SNS(50,3,6,5),
* RIND(5C,1),RIMP(5C,1),TDISP(205),TU(205),TV(205),
* COY(205),CCIZ(205),Deltat
COMMON/OC/Y(51),Z(51),ANG(51),H(51),EXANG,NS,IK,NODA,NFL,NI,
* ICOL(205),NBCCND,NBC(7),NODGB(7)
COMMON/YAT/ DENS(6),B(6),YOUNG(6),JS(6),SNO(6,5),NSPL(6),P(6),
* FPS(6,5),SIG(6,5),TPIM(6)
COMMON/MY/ C5,C6,ASPL(50,3,6,5),GZRT(50,3,6)
COMMON/RPAG/PH(6),FP(6),PMAS(6),PMQI(6),PCGU(6),PCGW(6),ALFA(6),
* UDOT(6),ADOT(6),ADOT(6),TPRIM(6),CR(6),FC3X(6),UNK(6),NF
COMMON/DDPAG/DDPCGU(6),DPCGW(6),DALFA(6)
MIPP=0
XY = TPRIM(1)/ DELTAT
IT= XY* 0.02
TIME= IT*DELTAT
DO 1 I=1,205
1 DISP(I)=0.0
DO 2 IR=1,IK
N= MKP(IR)
M= NSPL(N)
DO 2 J=1,NODA
DO 2 K=1,NFL
DO 2 L=1,7
SNS(IR,J,K,L)=0.0
DO 16 NPQ=1,NP
16 TNJ(NPQ)= 1.0
IF(NPQ.EQ.1) GO TO 43
DC 13 NTS=2,NP
IF(TPRIM(NTS).GE.0.0) GO TO 10
MIPP = NTS
:) TO 41
10 CONTINUE
40 IF (*MISP, 0, 0) GO TO 5;
DC 17 NPN = MISP, NP
17 TNJ(NPN) = 0.7
50 DC 5 I = 1, NP
DPCGU(I) = UDOT(I) * DELTAT
DPCGW(I) = WDOT(I) * DELTAT
DALPA(I) = (ADOT(I) * DELTAT
PCGU(I) = PCX(I) * UDOT(I) * TPRIM(I) * TNJ(I)
PCGW(I) = PCX(I) * WDOT(I) * TPRIM(I) * TNJ(I)
5 , LP(I) = :ADOT(I) * TPRIM(I) * TNJ(I)
RETURN
END

SUBROUTINE ELMPP(DLLTAT,AA,ISIZE,KROW,NDSX,NIRREG,INUM)    ELMP0010
IMPLICIT REAL*8(A-H,O-Z)    ELMP0020
DIMENSION A(B,8),AA(5,8),LMI(8),MMI(8),     ELMP0030
*E(I,8),EK(I,8)    ELMP0040
*HE(I,3,8),KRCM(1),VDEX(1),NUM(1),AWG(51),ELK(8,8)    ELMP0050
DIMENSION A(I,3,8),B(2)    ELMP0060
DIMENSION DELM(8),DISM(8),DUMMY(8)    ELMP0070
COMMON/FSY(51),7(51),ANG(51),H(51),EXANG,NS,IK,NOGA,NFL,NI,    ELMP0080
*ICCL(29),NHCND,NBC(7),NDEP(7)    ELMP0090
COMMON/MAT/DENS(6),B(6),YJUNG(6),DS(6),SN(6,5),NSFL(6),P(6),    ELMP0100
*EPS(6,5),SG(6,5),EFLN(6)    ELMP0110
COMMON/TAN/MKE(51)    ELMP0120
COMMON/AG/UK(51),NBCNR,NHC(7),NODBR(7),MK(51),RNT(5,2)    ELMP0130
COMMON/TH/THI(51)    ELMP0140
COMMON/NOD/EDEP(2,3,5)    ELMP0150
COMMON/VR/NVEC(51,7)    ELMP0160
SI(0)=DSIN(0)    ELMP0170
COS(0)=DCOS(0)    ELMP0180
ATAN(0)=DSSIN(0)    ELMP0190
MUP=1
DC 31 L=1,ISIZE
31 STFKLL=3.7
30 DO 11 IR=1,IK
   11 K1=NVEC(IR,1)
   K2=NVEC(IR,2)
   L=MKE(IR)-1
   PS(Z(K2)-Z(K1))
   PB=Y(K2)-Y(K1)
   PT=ANG(K2)-ANG(K1)
   GO TO 30
IF(YK(IR).EQ.1.0) P7=ANG(K2) - ROT(L,2) - ANG(K1)
IF(YK(IR).EQ.1.6.AND.ROT(L,1).EQ.1.0) P7=ROT(L,2) + ANG(K2) - ANG(K1)
IF(YK(IR).EQ.2.5) P7=ANG(K2) - ROT(L,1) - ANG(K1)
IF(YK(IR).EQ.3.5) P7=ROT(L,2) + ANG(K2) - ANG(K1)
P1E2=.31415926535937938*
P1E2=2.*PIE
PIE32=1.5*PIE
ANG2=ANG(K2)
ANG1=ANG(K1)
IF(YK(IR).EQ.1.0.AND.ROT(L,1).EQ.1.0) ANG(K2)=ROT(L,2) + ANG(K2)
IF(YK(IR).EQ.1.6.AND.ROT(L,1).EQ.1.6) ANG(K1)=2*ROT(L,2) + ANG(K1)
IF(YK(IR).EQ.2.0) ANG(K1)=ROT(L,1) + ANG(K1)
IF(YK(IR).EQ.3.0) ANG(K1)=2*ROT(L,2) + ANG(K2)
IF(YK(IR).EQ.4.0) ANG(K1)=2*ROT(L,1) + ANG(K1)

APHA = PIE / 2.0
IF(P0.LT.1.0) APHA = -APHA
IF(P0.LE.1.0) APHA = ATAN(P5/P6)
IF(P0.LT.1.0) APHA = APHA + PIE
IF(P0.LT.1.0) APHA = APHA + PIE
IF(P5.GT.0.0) GO TO 62

AL(1) = P7*SQRT(P5**2+P6**2)/SIN(P7/2.0)
IF(P7.GT.PIE32) AL(1) = (P7-PIE32)*SQRT(P5**2+P6**2)/SIN(P7/2.0)

60
62 AL(1) = SQRT(P5**2+P6**2)

61 ANG(1R+1) = ANG(K2)

85 ANG(1R) = ANG(K1)
IF(P7.GT.PIE32) ANG(1R) = ANG(K1) + PIE2
IF(P7.GT.PIE32) ANG(1R) = ANG(K1) + PIE2
IF(P7.LT.1.0) ANG(1R) = ANG(K2) + PIE2
IF(P7.LT.1.0) ANG(1R) = ANG(K2) + PIE2

HZEK = ENG(IR) - APHA
M1 = 2.*ANG(1R+1) - 4.*ANG(1R) + 6.*APHA/AL(1R)
M2 = 1.*ANG(1R+1) + 3.*ANG(1R) - 5.*APHA/AL(1R)
\begin{align*}
\text{ANG(K2)} &= \text{ANG2} \\
\text{ANG(K1)} &= \text{ANG1} \\
\text{L1.L2 I}=1,8 \\
\text{DC L1.L2 J}=1,8 \\
E(1,J) &= C_J \\
A(I,J) &= -3. \\
A(1,1) &= \cos(\text{RNG(14)}-\text{APHA}) \\
A(1,2) &= \sin(\text{BNG(1R)}-\text{APHA}) \\
A(2,1) &= -\sin(\text{BNG(1R)}-\text{APHA}) \\
A(2,2) &= \cos(\text{BNG(1R)}-\text{APHA}) \\
A(3,3) &= 1. \\
A(5,1) &= \cos(\text{RNG(1R+1)}-\text{APHA}) \\
A(5,2) &= \sin(\text{BNG(1R+1)}-\text{APHA}) \\
A(5,3) &= P_6*\sin(\text{RNG(1R+1)}-\text{APHA}) \\
A(6,1) &= -\sin(\text{RNG(1R+1)}-\text{APHA}) \\
A(6,2) &= \cos(\text{RNG(1R+1)}-\text{APHA}) \\
A(6,3) &= P_6*\cos(\text{BNG(1R+1)}-\text{APHA}) \\
A(7,3) &= 1. \\
A(4,4) &= 1. \\
A(5,4) &= \text{AL}(1R) \\
A(5,7) &= \text{AL}(1R)*2 \\
A(5,8) &= \text{AL}(1R)*3 \\
A(6,5) &= \text{AL}(1R)*2 \\
A(6,6) &= \text{AL}(1R)*3 \\
P_8 &= 31+2*8*\text{AL}(1R) \\
A(7,4) &= \text{AL}(1R)*P_8 \\
A(7,5) &= 2*\text{AL}(1R) \\
A(7,6) &= 3*\text{AL}(1R)*2 \\
A(7,7) &= \text{AL}(1R)*2*P_8 \\
A(7,8) &= \text{AL}(1R)*3*P_8 \\
A(4,4) &= 1. \\
A(8,5) &= -\text{AL}(1R)*2*P_8 \\
A(8,7) &= ?*\text{AL}(1R) \\
A(8,6) &= -\text{AL}(1R)*3*P_9 \\
A(8,8) &= 3*\text{AL}(1R)*2 \\
\text{CALL MINV}(A,h,CET,LMI,HMI) \\
\end{align*}
DC 22 I=1,8
DC 22 J=1,9

52
A(I,J)=A(I+1,J)
DC 1, 3 J=1, NCGA*2
ZET=AL(19)*5(J)
PHI=H1*2, #P2#ZET
PHI1=ZET+H1*ZET+H2#ZET#2
ZET=AL(19)*5(J)
ZET=ZET*2
G3 1, 4 JJ=1, NCGA
P2=H2*ET+1*ET*AX(G(JJ))#92*(ZET+AX(G(JJ))#2+A
YET=ET+cos(P2)ZET+AWG(JJ)
104
ZET=ZET+sin(P2)ZET+AWG(JJ)
P3=ZET*sin(PHI+APHA)-ZET*cos(PHI+APHA)
P4=ZET*cos(PHI+APHA)+ZET*sin(PHI+APHA)

DC 2, 1 I=1, 3
DC 2, 1 J=1, 8

231
BE1(J, M,N)=0.
BE1(J, 1, 1)=1.
BE1(J, 1, 5)=ZET+H*PHI
BE1(J, 1, 6)=ZET+3*PHI
BE1(J, 1, 7)=2#ZET
BE1(J, 1, 8)=3#ZET+H
BE1(J, 2, 3)=1.
BE1(J, 2, 4)=ZET+PHI
BE1(J, 2, 5)=2#ZET
BE1(J, 2, 6)=3#ZET+H
BE1(J, 2, 7)=ZET+2#PHI
BE1(J, 2, 8)=ZET+3#PHI
PF1(J, 3, 4)=-PHI-ZET#2.*H
BE1(J, 3, 5)=2.
BE1(J, 3, 6)=-6#ZET
BE1(J, 3, 7)=-2#ZET+PHI-ZET+V2#2.*H
BE1(J, 3, 8)=-3#ZET+V2#PHI-ZET+3#2.*H
DC 2:2 M=1,3
DO 202 N=1,9
BEPI(R,J,M,N)=0.0
DG 2G2 K=1,8
232 BEPI(R,J,M,N)=BEPI(R,J,M,N)+BE1(J,M,K)*A(K,N)
H1 = H(K1)
H2 = H(K2)
M = MKE{IR} - 1
IF(YK(IR).NE.1.0.AND.YK(IR).NE.3.5) GOTO 610
IF (ROT(1,1).NE.C.H.) GO TO 610
H(K2) = HTH(M)
G0 TO 600
610 H(K1) = HTH(M)
600 RH=H(K2)*AXG(J)+H(K1)*(1.-AXG(J))
H(K1) = H1
H(K2) = H2
RI=RH**3/12.
T1=PHI*ZET**2.*R2
T2=2.*ZET*PHI*ZET**2.*R2
T3=3.*ZET**2.*PHI*ZET**3*Z.*R2
E(4,4)=E(4,4)+(RH+T1)**2*R1)*WET
E(5,4)=E(5,4)+(-ZET**2*PHI*RH+2.*T1*R1)*WET
E(6,4)=E(6,4)+(-ZET**3*PHI*RH+6.*ZET*TI*R1)*WET
E(7,4)=E(7,4)+(2.*ZET*RH+T2*TI*R1)*WET
E(8,4)=E(8,4)+(3.*ZET**2*RH+T3*TI*R1)*WET
E(9,5)=E(9,5)+(ZET**4*PHI**2*RH+4.*R1)*WET
E(10,5)=E(10,5)+(ZET**5*PHI**2*RH+12.*ZET*R1)*WET
E(11,5)=E(11,5)+(ZET**4*PHI**2*RH+2.*T3*R1)*WET
E(12,5)=E(12,5)+(-3.*ZET**4*PHI*RH+2.*T3*R1)*WET
E(13,5)=E(13,5)+(-3.*ZET**4*PHI*RH+2.*T3*R1)*WET
E(14,6)=E(14,6)+(ZET**4*PHI**2*RH+36.*ZET**2*R1)*WET
E(15,6)=E(15,6)+(-2.*ZET**4*PHI*RH+6.*ZET*T2*R1)*WET
E(16,6)=E(16,6)+(-3.*ZET**5*PHI*RH+6.*ZET*T3*R1)*WET
E(17,7)=E(17,7)+(-4.*ZET**2*RH+T2**2*R1)*WET
E(18,7)=E(18,7)+(6.*ZET**3*RH+T2*T3*R1)*WET
E(19,8)=E(19,8)+(9.*ZET**4*RH+T3**2*R1)*WET
103 CONTINUE
IF(LKK(IR,1).EQ.0) GOTO 8200
NPE = LKK(IR, 1)
DO 821: ND=1, NPE
MO = NO + 1
M = LKK(IR, MO)
ZET = AZET(M) * AL(IR)
PHIP = 61 + 2.0 * K * ZET
DOA240 I = 1, 3
DU 8245 N = 1, R

8245 AE1(I, N) = 3.0
AE1(1, 4) = 1.0
AE1(1, 5) = -ZET*2*PHIP
AE1(1, 6) = -ZET*3*PHIP
AE1(1, 7) = 2.0 * ZET
AE1(1, 8) = 3.0 * ZET = 2
AE1(2, 3) = 1.0
AE1(2, 4) = ZET * PHIP
AE1(2, 5) = AE1(1, 7)
AE1(2, 6) = AE1(1, 8)
AE1(2, 7) = -AE1(1, 5)
AE1(2, 8) = -AE1(1, 6)
AE1(3, 4) = -PHIP * ZET * 2 * R2
AE1(3, 5) = -2.0
AE1(3, 6) = -6.0 * ZET
AE1(3, 7) = -2.0 * ZET * PHIP * ZET * 2 * K * R2
AE1(3, 8) = -3.0 * ZET * 2 * PHIP * ZET * 3 * R2 * J * 32
DO 8245 I = 1, 3
DO 8245 M = 1, R
AEP(M, 1, N) = 2.0
DO 8245 K = 1, R

8245 AEP(M, 1, 1) = AEP(M, 1, N) + AE1(I, N) = A(K, V)
8245 CONTINUE
92:0 CONTINUE
92:0 CONTINUE
8X(1) = 1.0
8X(2) = 1.0
DU 3:3 J = 1, 2
ZET = AL(1K) = 8X(J)
PHIP = B1 + 0.2 * B2 * ZET
DO 301 M = 1, 3
DO 301 N = 1, A
301 BE1(J, K, N) = 0.0
   BE1(J, 1, 4) = 1.
   BE1(J, 1, 5) = -ZET ** 2 * PHIP
   BE1(J, 1, 6) = -ZET ** 3 * PHIP
   BE1(J, 1, 7) = 2. * ZET
   BE1(J, 1, 8) = 3. * ZET ** 2
   BE1(J, 2, 3) = 1.
   BE1(J, 2, 4) = ZET * PHIP
   BE1(J, 2, 5) = 2. * ZET
   BE1(J, 2, 6) = 3. * ZET ** 2
   BE1(J, 2, 7) = ZET ** 2 * PHIP
   BE1(J, 2, 8) = ZET ** 3 * PHIP
   BE1(J, 3, 4) = -PHIP * ZET ** 2 * B2
   BE1(J, 3, 5) = -2.
   BE1(J, 3, 6) = -6. * ZET
   BE1(J, 3, 7) = -2. * ZET * PHIP * ZET ** 2 * B2
   BE1(J, 3, 8) = -3. * ZET ** 2 * PHIP * ZET ** 3 * B2
DO 302 M = 1, 3
DO 302 N = 1, A
DEP(IR, J, M, N) = 0.0
DO 302 K = 1, A
302 DEP(IR, J, M, N) = DEP(IR, J, M, N) + BE1(J, M, K) * A(K, N)
303 CONTINUE
DO 20 I = 1, 7
   IP1 = I + 1
DO 20 J = IP1, N
20   E1(I, J) = E(J, I)
DO 21 I = 1, 8
DO 21 J = 1, 8
EKL(I, J) = 0.0
21   EKL(I, J) = EKL(I, J) + A(K, I) * E(K, J)
UG 22 I = 1, 8
DO 22 J=1,8
  ELK(I,J)=A(I,J)
DO 22 K=1,8
  ELK(I,J)=ELK(I,J)*ELK(I,K)*A(K,J)
DO 23 I=1,4
DO 23 J=1,8
  ELK(I,J)=ELK(I,J)*YOUNG(MKE(IR))*B(MKE(IR))
  IF (YK(IR).EQ.0.0) GO TO 502
   CALL ROTAT(3,ELK,DUMMY,IR)
502 CALL ASSEM(IR,ELK,STIFK)
101 CONTINUE
  RETURN
END
SUBROUTINE ENERGY(IT, KROW, RXF, HRREG, SCL, ES, GFX, QVEL)
C
THIS IS THE ENERGY CALCULATION SUBROUTINE
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION QVEL(205)
DIMENSION CINTF(6), SCL(1), ES (6, 6), GFX (50, 3, 6)
COMMON /RA/ DFP (50, 3, 6), AL (50), AXG (3), AXG (3)
COMMON /TAP/ MREAD, WRITE, MPUNCH
COMMON /VQ/ FLVA (205), DIPS (205), DELD (205), SNS (50, 3, 6, 5).
* RINP (50, 3), BNP (50, 3), TDISP (205), TU (205), TW (205).
*COII (205), COI2 (205), DELAT
COMMON /TAM/ EKE (51)
COMMON /PRAG/ F1 (6), FCQ (6), PMASS (6), PMOI (6), PCGE (6), PCGW (6), ALFA (6)
* UDOT (6), WDQT (6), ADOT (6), TPRI (6), CR (6), FCQX (6), UNK (6), NF
COMMON /DFRG/ DPCGU (6), DPCQ (6), DCEF (6), DALT (6)
COMMON/LENER/PK (6), CINEO, CUMW, DFKE, CELAS, ELAS, PLATC
COMMON/AB/ RBC (51), RWORK, CINEY (205)
COMMON/PQ/Y (51), Z (51), ANG (51), H (51), RXANG, NS, IK, NOGA, NPL, NI
* ICOL (205), NCOND, RUC (7), NODEB (7)
COMMON/MA/ DENS (6), B (6), YOUNG (6), DS (6), SHO (6, 6), NSPL (6), P (6)
* EPS (6, 5), SIG (6, 5), EFLN (6)
COMMON /N1/ C5, C6, ASPL (50, 3, 6, 6), CBSPACE (50, 3, 6)
COMMON/FL/P/SRNP (2060), FURSP (205), BEX (4), NQR, NQP, NOPR, NREL (4)
*NLP (4), NNEU (4)
SIN (Q) =DSIN (Q)
COS (Q) =DCOS (Q)
ATAN (Q) =DATAH (Q)
ABS (Q) =DABS (Q)
SORT (Q) =DSCHT (Q)
WRITE (MWRITE, 7) IT
7 FORMAT ( /*/*1, ENERGY AND WORK AT THE END OF TIME CYCLE*, 15) WRITE(MWRITE, 80)
80 FORMAT (*0 FRAGMENT*, 10X, *KINETIC ENERGY*, */)
IMX=IK+1
RWORK=0.0
DO 5 I=1, NF
PUV= UDOT (I)
FWV = WDOT(I)
FAV = ADOT(I)
CINETF(I) = PMASS(I) / 2.0 * (FWV**2 + FAV**2) + PMOI(I) / 2.0 * (FAV**2)
RWORK = RWORK + (F(I) - CINETF(I))
WRITE (MWRITE, 6) I, CINETF(I)
6 FORMAT (' ', 10X, '15, 13X, D15.6)
CONTINUE
WRITE (MWRITE, 8) RWORK
8 FORMAT (/, ' RWORK INPUT INTO RING', 2X, ' = ', D15.6)
CINETO = 0.0
NI = NS*4
DO 10 I = 1, NI
10 CINETO = CINETO + (QVEL(I) * DELAT**2 / SOL(I)) / 2.0D0
WRITE (MWRITE, 11) CINETO
11 FORMAT (' RING KINETIC ENERGY = ', D15.6)
IF (EXANG.NE.360.) GO TO 13
DO 12 K = 1, 4
12 DISP(IK*4+K) = DISP(K)
DELD(IK*4+K) = DELD(K)
13 CELAS = 0.0
DO 15 IR = 1, 1K
15 N = MKE(IR)
M = NSFL(N)
DO 16 J = 1, NOGA
16 SUM = 0.0
DO 17 K = 1, NFL
17 STRS = 0.0
DO 30 L = 1, M
LL = L + 1
AB = (ES(N, L) - ES(N, LL)) / ES(N, 1)
30 STRS = STRS + SNS(IR, J, K, L) * AB
STRS = STRS**2
17 SUM = SUM + STRS*GFL(IR, J, K)
16 CELAS = CELAS + (SUM*AWG(J)*AL(IR)) / YOUNG(N) / 2.0
CONTINUE
SPDEN = 0.0
IF(NGR.EQ.0)GO TO 18
DO 20 I=1,NI
20 SPDEN=SPDEN+DISP(I)*FQREF(I)
SPDEN=SPDEN/2.0
18 WRITE(MWRITE,21) CELAS
21 FORMAT('RING ELASTIC ENERGY = *D15.6 )
PLAST=WORK-CINETO-CELAS-SPDEN
WRITE(MWRITE,22) PLAST
22 FORMAT('RING PLASTIC WORK = *D15.6 )
WRITE(MWRITE,23) SPDEN
23 FORMAT('ENERGY STORED IN ELASTIC RESTRAINTS = *D15.6 )
RETURN
END
SUBROUTINE ERC(II,STIPM,NI,ICOL)
IMPLICIT REAL*8(A-H,O-Z)
C FOR ELIMINATING ROWS AND COLUMNS IN STIPM
DIMENSION STIPM(1),ICOL(1)
I0=ICOL(II)
DO 101 J=I0,II
CALL PICOL(II,J,L,ICOL)
101 STIPM(L)= 7.0
DO 102 I=II,NI
IC1=ICOL(I)
IF(II-IC1).LT.102,103,103
CALL PICOL(II,II,L,ICOL)
STIPM(L)=0.
103 CONTINUE
CALL PICOL(II,II,L,ICOL)
STIPM(L)=1.
RETURN
END
SUBROUTINE PICOL(I,J,L,ICOL)
IMPLICIT RPA=8(A-H,O-Z)
C USING FORMULA L=J+SUM(K-ICOL(K)),K=1,I TO RELATE I,J,TO L
DIMENSION ICOL(L)
IF(J-ICOL(I)) 260,300,360
360 ISUM=0
DO 305 K=1,I
ISUM=K-ICOL(K)+ISUM
305 CONTINUE
L=J+ISUM
RETURN
260 WRITE(6,4) I,J
4 FORMAT(31h ELEMENT IS NOT IN BAND REGION,3H I=,i5,3H J=,i5)
RETURN
END
SUBROUTINE IDENT(NQP,NBR)

COMMON /TAPE/ MREAD, MWRITE, IPUNCH
COMMON /SC/CRMTS, RIG, RTH5, MCRIT, IBIG, ISURP
COMMON /V0/ FV0A(205), DISP(235), DDEL(245), SHN(53,5,4,5),
  *BIMP(5C,5), BIMP(5C,3), TDISP(235), TU(235), PW(205),
  *CCY(205), COIZ(205), DELTAT
COMMON/PRAG/PH(6), PCG(6), PQAS(6), PQO(6), PCGU(6), PGMW(6), ALFA(6),
*UDT(6), WDCT(6), ADRT(6), TRITY(6), CR(6), PGMX(6), UNK(6), NF
COMMON/ENERG/FS(6), CINTO,CUM4,DILK2,CELAS,ELAS,PLAST
COMMON /HM/ C5,C6,FSPL(50,5,6), FZTA(50,5,6)
COMMON/PG/Y(51), F51), ANG(51), H(51), EXANG,NS,IK,NOSA,NPL,NI,
* ICOL(255), NZCCND, NZC(7), NOOOEB
COMMON /LEFT/ RMASS(51)
COMMON/MAT/ DENS(6), B(6), YOUNG(6), D3(6), SNO(6,5), NSPL(6), P(6),
* EPS(6,5), SIG(6,5), EPSL(6)
COMMON/TH/ HTH(5)
COMMON /ML/ 4NFL(6), MATT(6)
SIN(0)=DSIN(0)
COS(0)=DCOS(0)
ATAN(0)=DATAN(0)
EPS(0)=DABS(0)
SORT(0)=DSORT(0)
WRITE(MWRITE,1000)

1000 FORMAT(//'** ','
IF(EXANG.EQ.360.)GO TO 81
WRITE(MWRITE,2)
GO TO 80

81 WRITE(MWRITE,1)
1 FORMAT('** COMPLETE RING ** CIVM-JET 4B** CONTAINMENT ANALYSIS**','
@//, 'RING PROPERTIES**',//)
2 FORMAT('** PARTIAL RING ** CIVM-JET 4B** CONTAINMENT ANALYSIS**','
@//, 'RING PROPERTIES**',//)
80 CONTINUE
NBR1=NBR+1
DO 600 JT=1,NBR1

IDNTCC19
IDNTCC10
IDNTCC30
IDNTCC40
IDNTCC50
IDNTCC60
IDNTCC70
IDNTCC80
IDNTCC90
IDNTCC100
IDNTCC110
IDNTCC120
IDNTCC130
IDNTCC140
IDNTCC150
IDNTCC160
IDNTCC170
IDNTCC180
IDNTCC190
IDNTCC200
IDNTCC210
IDNTCC220
IDNTCC230
IDNTCC240
IDNTCC250
IDNTCC260
IDNTCC270
IDNTCC280
IDNTCC290
IDNTCC300
IDNTCC310
IDNTCC320
IDNTCC330
IDNTCC340
IDNTCC350
IDNTCC360
IF (JT.NE.1) GO TO 610
WRITE(MWRITE,500)
500 FORMAT('MATERIAL PROPERTIES OF MAIN STRUCTURE ARE:')
GO TO 611
610 JT1 = JT-1
WRITE(MWRITE,510) JT1
510 FORMAT('MATERIAL PROPERTIES OF BRANCH NUMBER',I3,' ARE AS
*FOLLOWS:')
611 WRITE(MWRITE,3) P(JT), DENS(JT), MNEL(JT), NGRA, NPL, NSPL(JT)
MNO = NSPL(JT)
WRITE(MWRITE,1015) DS(JT)
WRITE(MWRITE,102C) P(JT)
1015 FORMAT(12X,'DS FOR STRAIN RATE',3X,=' ',D15.6)
1020 FORMAT(12X,'P FOR STRAIN RATE',3X,=' ',D15.6)
IF (JT.GT.1) WRITE(MWRITE,103) HTH(JT1)
WRITE(MWRITE,4) (L_EPS(JT,L), L_SIG(JT,L),L=1,MNO)
3 FORMAT(12X,'WIDTH OF RING(IN)',3X,=' ',D15.6,/,12X,'NUMBER OF ELEMENTS',3X,=' ',',IS,/,12X,'NUMBER OF DEPTHWIS
*G* E GAUSSIAN PTS.',15X,=' ',,IS,/,12X,'NUMBER OF MECHANICAL SUBLAYERS',15X,=' ',,IS,/,12X,'NUMBER OF ELEMENTS',
*GAUSSIAN PTS.',15X,=' ',,IS,/,12X,'NUMBER OF MECHANICAL SUBLAYERS',15X,=' ',,IS,/,12X,'NUMBER OF ELEMENTS',
4 FORMAT(15X,'STRAIN ','I1,=' ',D15.6,5X,'STRESS(','I1,=' ',D15.6)
G) 103 FORMAT(12X,'THICKNESS AT THE CONNECTING NODE',16X,=' ',D15.6,1)
CONTINUE
WRITE(MWRITE,1095)
1025 FORMAT('INITIAL GEOMETRY AT EACH NODE IS AS FOLLOWS:')
WRITE(MWRITE,5)
5 FORMAT(12X,'NODE NO.',10X,'Y COORD',1X,'Z COORD',7X,'SLOPE(RAD.)',
*8X,'RING THICKNESS AT NODE I ','/)
WRITE(MWRITE,6) (I,Y(I),Z(I),ANG(I),H(I),I=1,NS)
6 FORMAT(12X,I5,7X,4D16.6,1)
WRITE(MWRITE,7)
7 FORMAT(1X,'FRAGMENT PROPERTIES')
WRITE(MWRITE,8)
8 FORMAT(12X,'FRAG.NO.',5X,'DIA. OF FRAG.',5X,'MASS OF FRAG.',5X)
*OPEN CP INITIA */ * FRAG.* 6X,*FGY* 13X,*FCGZ* 1
WRITE(AWRITE,9) (I,PH(I),PMASS(I),PMJ1(I),PCGX(I),PCG(I),I=1,NP)
FORMAT (11X,15,6X,D15.6,4X,D15.6,8X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6)
WRITE(AWRITE,15)
WRITE(AWRITE,15)
WRITE(AWRITE,12) (I,UDOT(I),MDOT(I),MDJ(T),CR(I),PK(I),UNK(I),
*I=1,NF)
10 FORMAT( 1X,'COLLISION PARAMETERS',/)
11 FORMAT(12X,'FRAG.NO.',3X,'VEL IN Y DIR.',3X,'VEL IN Z DIR.',3X,'AN
12 FORMAT(11X,15,4X,4D15.6,6X,D15.6,6X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6,11X,D15.6,2X,D15.6)
IF(NCOND .EQ. 0) GO TO 29
WRITE(AWRITE,1610)
1610 FORMAT(//' BOUNDARY CONDITIONS ARE:',/)
DO 14 I=1,N3COND
14 FORMAT(//' SYMMETRY DISPLACEMENT CONDITION AT NODE =',I5)
IF(NBC(I) .EQ. 2) WRITE(AWRITE,15) NODEB(I)
16 FORMAT(//' CLAMPED DISPLACEMENT CONDITION AT NODE =',I5)
17 FORMAT(//' HINGED DISPLACEMENT CONDITION AT NODE =',I5)
GO TO 18
WRITE(AWRITE,13)
13 FORMAT(//' THERE IS NO PRESCRIBED DISPLACEMENT CONDITION')
18 IF(NOP .EQ. 0) GO TO 19
WRITE(AWRITE,20)
20 FORMAT(//' CONSTRAINTS (PLASTIC FOUNDATION/SPIRNG) AS DESCRIBED')
19 WRITE(AWRITE,21)
21 FORMAT(//' THESE ARE NO ELASTIC SPRING CONSTANTS')
RETURN
END
SUBROUTINE IMPACT(EFLN,IT,NBR,QACL,QVEL)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION QACL(205),QVEL(205)
DIMENSION AY(51),AZ(51)
DIMENSION VX(51),VZ(51),TFGU(6),TFGW(6),TALFA(6),IPLAG(51,6),
2NTSV(6),NEF(6),RL(50),EFLN(6)
COMMON/IMPT/VEL(102),INCO,QVEL(51)
COMMON/TAPE/MFILE,WHITE,MUNCH
COMMON/VQ/PLVA(205),DISP(205),DELD(205),SNS(50,3,6,5),BIMP(50,3),
2BIMP(50,3),TDISP(205),TU(205),TW(205),COY(205),COIZ(205),DELTAT
COMMON/FRAG/FH(6),FCG(6),FMASS(6),FMOI(6),FCGU(6),FCGW(6),ALFA(6),
2VELFU(6),VELFW(6),VELFA(6),TPRIM(6),CH(6),FCGX(6),UNK(6),NF
COMMON/DPJAG/DFCGU(6),DFCGW(6),DALFA(6)
COMMON/PG/Y(51),Z(51),ANG(51),N(51),EXAM,IKK,IK,NOGA,NPL,NUT,
2ICOL(205),NUCOND,NBC(7),NCDEB(7)
COMMON/IIAT/TALL
COMMON/BN/LMT(51)
COMMON/BR/NVEC(51,2)
COMMON/LEFT/BMASS(51)
COMMON/COU/INCOU
COS(ZZZ)=DCOS(ZZZ)
SIN(ZZZ)=DSIN(ZZZ)
C
******************************************************************************
C
C****** MODIFIED IMPACT CONTROLLING ROUTINE *******
C
C CURRENT TIME REMAINING IN THIS TIME STEP=EXTERNAL TIME STEP
C AND CURRENT TIME = TB.
C TIMEI= (IT-1)*DELTAT
C TIMEF = IT*DELTAT
C DELTR=DELTAT
C TB=TIMEI
C IF(I=1)
C ICP=0
C IF(FXANG.EQ.360.0) ICP=1
C
C INITIALIZE CORRECTION 'FLAGS' TO NO-PREV.-CORR. POSITION (=0),
AND NO. OF SUBDIVISIONS FOR EACH FRAG. TO ZERO
DO 5 I=1,NF
      NISD(J)=0
      IFLAG(I,J)=0
5 CONTINUE
C TRANSFORM MODAL VELOCITIES INTO Y AND Z COMPONENTS
  6 DO 10 I=1,IKK
    VY(I)= QVEL(I*4-3)*COS(ANG(I))-QVEL(I*4-2)*SIN(ANG(I))
  10 VZ(I)= QVEL(I*4-3)*SIN(ANG(I))+QVEL(I*4-2)*COS(ANG(I))
C CALC. TRIAL POSITION OF RING NODES AND FRAGMENT TO THE END OF THIS
C INTERNAL TIME STEP
  20 DO 25 I=1,IKK
      AY(I) = 0.5D+00  *(QACL(I*4-1)*COS(ANG(I))-QACL(I*4-2)*
            @SIN(ANG(I)))/*DELTAT**2
      AZ(I) = 0.5D+00  *(QACL(I*4-3)*SIN(ANG(I))+QACL(I*4-2)*
            @COS(ANG(I)))/*DELTAT**2
      TY(I) = Y(I) + DISP(I*4-3)*CCS(ANG(I))-DISP(I*4-2)*SIN(ANG(I))
      TW(I) = 2*Y(I) + DISP(I*4-3)*SIN(ANG(I))+DISP(I*4-2)*COS(ANG(I))
  25 CONTINUE
C TRIAL POSITION OF FRAGMENT
  30 DO 35 I=1,NF
      TPCGU(I)=PCGU(I)
      TPCGW(I)=PCGW(I)
  35 TALPA(I)=ALFA(I)
C RETURN POSITION FOR SUBSEQUENT INSPECTION AFTER INITIAL PENETRATION
C CORRECTION.
  40 CONTINUE
C CALL UPDATE(1.000,TY,TW,VY,VZ,TPCGU,TPCGW,TALPA,VELPU,VELPW,
                2VELPA,DELT,IKK,NF,LCP,AY,AZ)
            IF(NISD(JP).GT.50) CALL EXIT
C CALC. PENETRATION DISTANCES FOR THIS TRIAL POSITION.
CALL PENTRN(TU,TW,TCGU,TCGW,NPP,FMAX,REF,MEQ,JP,PAL,RL,IBIG,
2H,PH,IK,NP,ICP)
C CHECK PENETRATIONS.
IF THE NUMBER OF POSITIVE PENETRATIONS, NPP, IS ZERO, AND THE
MAX. PENETRATION IS LESS THAN ZERO, NO (MORE) PENETRATIONS OCCUR FOR
THIS TIME STEP.
IF (NPP.EQ.0.AND.PMAX.LT.0.0) GO TO 150
C IF MAX. PENETRATION IS ZERO, SKIP SUBDIVISION PROCESS.
TT=TB+DELTA
TMIN=DELTA
C RETURN FRAGMENT AND MIDLAL POSITIONS TO BEGINNING OF THIS INTERNAL
TIME STEP.
CALL UPDATE(-1.0DD,TU,TW,VY,VZ,TCGU,TCGW,TALFA,VELFU,VELPW,
2VELFA,DELTR,IKK,NP,ICP,AY,AZ)
C CALC. TIME OF CONTACT, ELEM. AFFECTED, AND POINT OF CONTACT.
CALL TCONT(TU,TW,VY,VZ,TCGU,TCGW,VELFU,VELPW,DELTR,IK,
2TMIN,LNTMIN,RPC,JP,IFLAG,H,PH,NP,AY,AZ)
C UPDATE MIDLAL AND FRAG. POSITIONS TO THIS TIME
IBIG = LNTMIN
PAL=RPC
CALL UPDATE(1.0DD,TU,TW,VY,VZ,TCGU,TCGW,TALFA,VELFU,VELPW,
2VELFA,TMIN,IKK,NP,ICP,AY,AZ)
TT=TB+TMIN
C IF NO ACCEPTABLE CONTACT TIMES HAVE BEEN FOUND, GO TO END OF ROUTINE
IF (LNTMIN.EQ.0) GO TO 150
100 CONTINUE
NTSD(JP)=NTSD(JP)+1
IMCGU = IMCGU+1
C UPDATE MODAL VELOCITIES (VY, VZ), AND FRAGMENT VELOCITIES (VELP, VELP, VELP, VELPA) TO POST-IMPACT VALUES.
WRITE (14,100) IMCON, IT, IT, T1G, JF, PAL
8000 FORMAT ('IMPACT NO.', 15, 5X, 'TIME', 15.6, 5X, 'DURING CYCLE', 15, 5X, 'FRAG', 15, 5X, 'DISTANCE', D15.6)
C
C TIME REMAINING IN THIS EXTERNAL TIME STEP
DELT=TIMEF-IT
IF(IMCON.EQ.1) TAI1 = TT
TR=IT
DO 110 I=1, IKK
VY(I) = VY(I)*AY(I)*TMIN*2.0D+00
VZ(I) = VZ(I)*AZ(I)*TMIN*2.0D+00
110 CONTINUE
CALL IMPCTE (TU, TW, VY, VZ, VELP, VELP, VELP, VELPA, JF, IBIG, RMASS, 2PMASS, PH01, CR, UNK, PAL, EFLH, H, PH, IK, NP, ICP, RL, MBR, MBCOND, NODEB+J, DLTTP, AY, AZ, ANG)
IMCO = 1
C
C CHECK FOR ADDITIONAL IMPACTS IN THIS EXTERNAL TIME STEP.
GO TO 20
C
C NO ADDITIONAL IMPACTS OCCUR IN THIS TIME STEP.
150 CONTINUE
C
C RETURN TO CONTROLLING ROUTINE WITH FINAL DISPLACEMENT INCREMENTS.
C
C FRAGMENT LOCATION.
DO 160 I=1, NP
DPCGU(I) = TFPGU(I)-FGU(I)
DPCGW(I) = TFPGW(I)-FGW(I)
160 DALPA(I) = TAIPA(I) - APA(I)
C
C IF NO IMPACTS HAVE OCCURRED, NO FURTHER UPDATE IS REQUIRED.
NSUM=0
DO 165 I=1, NP
C
165  NSUM = NSUM + NTSD(I)
      IF(NSUM.EQ.0) RETURN
C
C     IP IMPACT HAS OCCURED, UPDATE VELOCITIES
   DO 170 I=1,IKK
      TU(I) = TU(I) - Y(I)
      TW(I) = TW(I) - Z(I)
   170 CONTINUE
C
C     TRANSFORM BACK TO RING COORDS. --- RETURN UPDATED DELD.
   DO 180 I=1,IKK
      DELD(I*4-3) = TU(I)*COS(ANG(I)) + TW(I)*SIN(ANG(I)) - DISP(I*4-3)
      DELD(I*4-2) = -TU(I)*SIN(ANG(I)) + TW(I)*COS(ANG(I)) - DISP(I*4-2)
   180 CONTINUE
      RETURN
   END
SUBROUTINE IMPCT (TU, VW, VM, VELP, VELPW, VELPA, IDIG, IBIG, IMASS, IMP0010
2PMASS, PMOU, CR, UNK, FLAL, EPLM, H, PH, ICP, RL, NBR, NBCORD, NODEB
IMPT0020
1, 2CI Th, AT, Z, ANG
IMPT0030
1: YPLILIT R=AL (A-1,0-2)
IMPT0040
DIMENSION NODEB (1)
IMPT0050
DIMENSION AT (1), AL (1), ANG (1)
IMPT0060
DIMENSION TU (1), VU (1), VW (1), VELP (1), VELPW (1), VELPA (1),
IMPT0070
-PMASS (1), PMOU (1), CR (1), UNK (1), EFLM (1), RL (1), H (1), PH (1)
IMPT0080
DIMENSION SSL (25), GAM (25), BET (25), SSR (25)
IMPT0090
DIMENSION MAO (51), NACD (51), L(17), PA(1)
IMPT0100
DIMENSION HBC (1), EP (6)
IMPT0110
COMMON /HOUR/ YK (51), NEECD, NBC3 (7), NCDDB (7), MK (51), ROT (5, 2)
IMPT0120
COMMON /HOUR/ YK (51), NEECD, NBD, NCDDB (7), MK (51), ROT (5, 2)
IMPT0130
COMMON /IMP/ VEL (102), IMCO, JVEL (51)
IMPT0140
COMMON /TAP/ READ, WRITE, MPUNC
IMPT0150
COMMON /BR/ NVEC (51, 2)
IMPT0160
COMMON /TM/ MKE (51)
IMPT0170
COMMON /ML/ MATE (6), MATT (6)
IMPT0180
SIN (0) = USIN (0)
IMPT0190
COS (0) = DCOS (0)
IMPT0200
IFS
IMPT0210
IF (NBCORD.EQ0) NODEB (1) = 0
IMPT0220
DO 1000 I = 1, NBR
IMPT0230
L1 = NVEC (I, 1)
IMPT0240
L2 = NVEC (I, 2)
IMPT0250
C ESTABLISH ELEMENT LENGTH, ANGLES AND DISTANCES TO NODES
IMPT0260
RL (I) = DSQR ((TW(L2) - TW(L1))**2 + (TU(L2) - TU(L1))**2)
IMPT0270
1000 CONTINUE
IMPT0280
JNB = NBR + 1
IMPT0290
DO 100 1 = 1, JNB
IMPT0300
100 EP (I) = EPLM (I)
IMPT0310
L1 = NVEC (IDIG, 1)
IMPT0320
L2 = NVEC (IDIG, 2)
IMPT0330
RSIN = (TW(L2) - TW(L1)) / RL (IDIG)
IMPT0340
RCOS = (TU(L2) - TU(L1)) / RL (IDIG)
IMPT0350
JBIG = NVEC (IDIG, 1)
IMPT0360
C PAL = DISTANCE TO NODE 1  PAX = DISTANCE TO NODE 2
    IF (PAL.FQ.0.0. OR. PAL.EQ.1.0) GOTO 937
    GOTO 934
937 LZ=L1
    IF (PAL.EQ.1.0) LZ=L2
    DO 1007 I=1, NBCOND
    I= (LZ.EQ.NODEfib(I)) GOTO 1006
1007 CONTINUE
    GOTO 934
1006 CONTINUE
    KI= 1
    BRT(1) = 0.0
    MNOD(1) = NVEC(IBIG,1)
    IF (PAL.EQ.1.0) MNOD(I) = NVEC(IBIG,2)
    GOTO 936
934 CONTINUE
    PAL= PAL * RL(IBIG)
    PAX= RL(IBIG)- PAL
5555 CONTINUE
    NBC(1) = 0
    ZK=1.0
    MML=0
    MMR=0
    DO 998 I= 2, JNBH
    998 EFLN(I)= EF(I)
    MMPL=0
    KIL=1
    C ESTABLISH THE NUMBER OF NODES COUNTERCLOCKWISE FROM IMPACT WITHIN
    IMPT0640 SSL(KIL)=PAL
    IMPT0650 MEF=MKE(IBIG)
    IMPT0660 IF (MEF.EQ.1) GO TO 1010
    IMPT0670 WRITE (WRITE, 1009)
1009 FORMAT (' IMPACT ON A BRANCH IS NOT PRESENTLY ALLOWED-- NO IMPACT')
    IMPT0680 IF (PAL.GE.EFLN(MEF)) GO TO 300
    IMPT0700 GO TO 350
1010 CONTINUE
    IF (PAL.GE.EFLN(MEF)) GO TO 300
    IMPT0720
MAO(KIL) = NVIC(IBIG, 1)
IFP = 0
IP$ = 0
IP3 = 0
IMPL = 1
DO J01 J = 1, 25
GAM(KIL) = 2K-SSL(KIL-IP3/EPLN(MEP)
IF(MFPL.NE.1) GO TO 931
EPLN(MEP) = 0.99*SSL(KIL-IP3)
GO TO 5555
931 DO 940 NC = 1, NDCOND
IF(HNO(KIL).NE.MODEL(NC)) GOTO 940
MBC(1) = MBC(1) + 1
MBC(MBC(1) + 1) = NODEB(NC)
MFPL = 1
IF(HNO(KIL).NE.1.AND.MKE(JEL-1),EQ.1) GOTO 932
EPLN(MEP) = SSL(KIL-IF3)
GOTO 5555
932 CONTINUE
GOTO 950
940 CONTINUE
950 CONTINUE
JEL = IBIG - KIL - IFP
IF$ = 0
IF(IPC.LE.0.AND.JEL.LE.C) GOTO 302
IF(IPC.GT.0.AND.JEL.LE.0) JEL = JEL + IK
C CHECK FOR A BRANCH ATTACHMENT POINT
IF(NBR.EQ.0) GOTO 1038
DO 1020 I = 1, NBR
IF(HNO(KIL).NE.MATT(I)) GOTO 1020
NB = I + 1
MII = MATT(I)
RST = GAM(KIL)
GO TO 1036
1020 CONTINUE
GO TO 1038
C COMPUTE BRANCH NODES INVOLVED IN MOMENTUM TRANSFER,

1030 RX = 0.0
IP(MFPL.EQ.1) GOTO 1038
NUMB = 0
MBPL=0
KL = 0
DO 1035 I= 1,10
IP(MHFL.NE.1) GOTO 910
NUMB = NUMB+1
IP(NUMB.NE.2) GOTO 910
EFLN(NB) = 0.99* RX/EST
GOTO 1030
910 CONTINUE
IP(NODP(NB-1).NE.1) GOTO 2050
IP((MTI-1).EQ.0) GOTO 1037
RX = RX+RL(MTI-I)
P = EST-RX/EFLN(NB)
IP (P.LE.0.0) GOTO 1037
LT(KL+1) = MTI-I
GOTO 2060
2050 CONTINUE
IP (MKF(MTI+I-1).NE.NB) GO TO 1037
RX = RX+RL(MTI+I-1)
P = EST-RX/EFLN(NB)
IP (P.LE.0.0) GO TO 1037
LT(KL+1) = MTI+I
2060 CONTINUE
PK(KL+1) = P
DO 900 NC= 1,MRCOND
IF(LT(KL+1).NE.NODEB(NC)) GOTO 900
MBC(1) = MBC(1)+1
MBC(MBC(1)+1) = NODEB(NC)
MBFL=1
GOTO 920
900 CONTINUE
920 CONTINUE
KL = KL+1
1035 CONTINUE
1037 L = KIL+1
   IF3 = IF3 + KL
   IPF=MMFL(NB-1) -KL +IFF
   IP2 = MMFL(NE-1)
   IF (KIL.LQ.O) GO TO 1038
   KIL = KIL +KL
   DO 1036 I = L,KIL
   LZ= KL -I+L
   GAM(I) = PK(LZ)
1036 MNO (I) = LT(LZ)
1038 CONTINUE
   JEL = JEL -IF2
   IF (JEL.LQ.0) GO TO 302
   SSL(KIL+1-IF3) = SSL(KIL-IF3) + RL(JEL)
   IF(EPLN(MEP).LE.SSL(KIL+1-IF3)) GOTO 302
   KIL = KIL +1
   MNO(KIL) = NVEC(JEL,1)
301 CONTINUE
302 DO 303 JJ= 1,KIL
   MNOD(JJ) = MNO(KIL-JJ+1)
303 BET(JJ) = GAM(KIL-JJ+1)
300 CONTINUE
   IF2=0
   IF3 =0
   IPF = 0
   MMFL = 0
   KIR=1
C ESTABLISH THE NUMBER OF NODES
   CLOCKWISE FROM IMPACT WITHIN
   IMP1450
   IMP1460
   IMP1470
   IMP1480
   IMP1490
   IMP1500
   IMP1510
   IMP1520
   IMP1530
   IMP1540
   IMP1550
   IMP1560
   IMP1570
   IMP1580
   IMP1590
   IMP1600
   IMP1610
   IMP1620
   IMP1630
   IMP1640
   IMP1650
   IMP1660
   IMP1670
   IMP1680
   IMP1690
   IMP1700
   IMP1710
   IMP1720
   IMP1730
   IMP1740
   IMP1750
   IMP1760
   IMP1770
   IMP1780
   IMP1790
   IMP1800
IF (MNPL .NE. 1) GO TO 941
EPLN (MEP) = 0.99*SSF(KIR-IFJ)
GO TO 5555
941 DO 945 NC = 1, NBCOND
IF (MOD(KIL*KIR), NE. NODEB(NC)) GO TO 945
MBC(1) = MBC(1) + 1
MBC(MBC(1)+1) = NODEB(NC)
MNPL = 1
IF (MOD(KIL*KIR) .NE. NS .AND. MKE(JEL+1) .EQ. 1) GOTO 933
EPLN (MEP) = SSR (KIR-IFJ)
GOTO 5555
933 CONTINUE
GOTO 955
945 CONTINUE
955 CONTINUE
JEC = IBUG*KIR*IFP
IF2 = 0
IF (ICP.LT.0.0 .AND. JEB .GT. IK) GO TO 336
IF (ICP.GT.0.0 .AND. JEB.GT.IK) JEB=JER-IK
C CHECK FOR A BRANCH ATTACHMENT POINT
IF (NBR .EQ. 0) GO TO 1080
DO 1050 I = 1, NBR
IF (MOD(KIL*KIR) .NE. MAT1(I)) GO TO 1050
ND = I+1
MTI = MAT1(I)
FST = #ET(KIL*KIR)
GO TO 1060
1050 CONTINUE
GO TO 1080
C COMPUTE BRANCH NODES INVOLVED IN MOMENTUM TRANSFER,
1060 RX = 0
MDFL=0
NUMB = 0
KL = 0
DO 1055 I = 1, 10
IF (MNPL.EQ.1) GOTO 1080
IF(AKPL.NF.1) GO TO 915  
NUMB = NUMB+1  
IF(NUMB.NE.2) GOTO 915  
FPLN(NB) = C.99* RX/EFLN  
GOTO 1060  
915 CONTINUE  
IF(MKE(MTI+1+1).NF.NB) GO TO 1057  
RX=KK+RL(MTI+1+1)  
P=ZST-PX/EFLN(NB)  
IF(RF.LP.O.O) GOTO 1057  
LT(KL+1) = MTI+1  
PK(KL+1) = P  
DO 905 NC= 1, NNODE  
IF(LT(KL+1).LE.NODED(NC)) GOTO 905  
MBC(1) = MBC(1)+1  
MBC(MBC(1)+1) = NODEB(NC)  
MBPL=1  
GOTO 925  
905 CONTINUE  
925 CONTINUE  
KL= KL+1  
1055 CONTINUE  
1057 L=KIR+1  
H = KIR  
IF3 = IF3+KL  
IF2 = MNEL(NB-1)  
IFP=MNEL(NB+1)-KL+IFF  
IF (KL.EQ.0) GO TO 1080  
KIR = KIR+KL  
DO 1056 I = L.KIR  
RET(KIL+1) = PK(I-M)  
1056 MNOX(KIL+1) = LT(I-M)  
1080 CONTINUE  
JER= JER+IP2  
IF(JER.GT.LK) GO TO 306  
SSR(KIR+1-IF3) = SSR(KIR-IF3) +PL(JER)
IF(EPLN(MF).LE.SSR(KIR+1-IF3)) GOTO 306
KIR = KIR + 1
MNOQ(KIL*KIR) = NVEG(JER,2)
305 CONTINUE
306 CONTINUE
IF(KIL.LE.1.AND.KIR.LE.1) GOTO 308
IF(MIML.KE.C) GOTO 397
BET(1)=PAK
*BET(2)/PAL
MNOQ(KIL+1) = NVEG(IBIG,1)
DO 971 J=1,NBCOND
IF(MNOQ(KIL+1).NE.NODEB(J)) GOTO 971
MBC(1) = MBC(1) + 1
MBC(MBC(1)+1) = NODEB(J)
971 CONTINUE
GO TO 307
304 IF(MIML.EQ.0) GO TO 308
IF(KIL.LE.1.AND.KIR.LE.1) GOTO 308
BET(KIL+1)=PAL*BET(KIL)/PAK
MNOQ(KIL+1) = NVEG(IBIG,2)
DO 972 J=1,NBCOND
IF(MNOQ(KIL+1).NE.NODEB(J)) GOTO 972
MBC(1) = MBC(1) + 1
MBC(MBC(1)+1) = NODEB(J)
972 CONTINUE
GO TO 307
308 BET(1)=PAK
BET(2)=PAL
MNOQ(1) = NVEG(IBIG,1)
MNOQ(2) = NVEG(IBIG,2)
DO 973 J=1,NBCOND
IF(MNOQ(1).NE.NODEB(J)) GOTO 974
MBC(1) = MBC(1) + 1
MBC(MBC(1)+1) = NODEB(J)
974 IF(MNOQ(2).NE.NODEB(J)) GOTO 973
MBC(1) = MBC(1) + 1
MBC(MBC(1)+1) = NODEB(J)
CONTINUE
307  KII=KII+K1N
SUM=0.
DO 311 J=1,KII
311  SUM=SUM+BET(J)
C  BET = WEIGHTING FACTOR FOR THE EFFECTED NODES
DO 312 J=1,KII
BET(J) = BET(J)/SUM
IF(NBC(I).EQ.0) GO TO 312
L = NBC(I)+1
DO 489 I=2,L
IF(MNOD(J).EQ.MBC(I)) BET(J) = 0.0
989 CONTINUE
312 CONTINUE
936 CONTINUE
101 IFN(I) = IF(I)
SUM=0.
DO 113 J=1,KII
113  JEE=MNOD(J)
IF(ICP.GT.0.AND.JEE.GT.IK) JEE=JEE-IK
SUMB=SUMB+BET(J)**2/RMASS(JEE)
313 CONTINUE
B1=1./RMASS(JBIG)+((PH(JBIG)/2.)*2/PMO1(JBIG)*SUMB
B2=1./RMASS(JBIG)+SUMB
SUMN=0.
SUMT=0.
DO 340 J=1,KII
340  JEE=MNOD(J)
IF(ICP.GT.0.AND.JEE.GT.IK) JEE=JEE-IK
C  ESTABLISH THE TANGENTIAL AND NORMAL VELOCITIES OF RING AND FRAGMENT
SUM=SUMN+BET(J)*(VW(JEE)*RCOS-VT1*RSIN)
SUMT=SUMT+BET(J)*(VM(JEE)*RSIN+VU(JEE)*RCOS)
VPN=VELPW(JBIG)*RCOS-VELPU(JBIG)*RSIN
VPT=VELPW(JBIG)*RSIN+VELPU(JBIG)*RCOS
C  SINT = RELATIVE TANGENTIAL VELOCITY BETWEEN RING AND FRAG., AIMT IS REL.
SINT = VPT-VELPA(JBIG)*FH(JBIG)/2.0 - SUMT
AINT = VFN-SUMH
C IF AINT LE 0 THE FRAG IS NOT APPROACHING THE RING SO SKIP OUT OF THE IMPT3270
IF(AINT.GT.0.0) GO TO 3005
WRITE(MWRITE,3006) AINT
3006 FORMAT('OAIN'T='D15.6,' NO IMPCT--LEAVING IMPCT2')
GO TO 350
3005 CONTINUE
IF(UNK(JBIG).EQ.0.0) GC TC 702
C CALCULATE THE EFFECT OF FRICTION ON THE RELATIVE VELOCITIES AND THE
TANX = SINT*E2/(AINT*B1)
705 IF(UNK(JBIG).LE.TANX) GC TC 706
APN = (1.0+CR(JBIG))*AINT/E2
APT = SINT/B1
GO TO 760
706 APN = (1.0+CR(JBIG))*AINT/B2
APT = UNK(JBIG)*APN
GO TO 760
702 APN = (1.0+CR(JBIG))*AINT/B2
APT = 0.0
760 CONTINUE
PACTFN = -1.0*APN/PMASS(JBIG)
PACTPT = -1.0*APT/PMASS(JBIG)
PACTPO = APT*FH(JBIG)/FROI(JBIG)/2.0
C UPDATE THE RING AND FRAGMEN VELOCITIES
VELPU(JBIG) = (-PACTPN*RSIN*PACTPT*RCOS) + VELPU(JBIG)
VELPW(JBIG) = (PACTPN*RCOS*PACTPT*RSIN) + VELPW(JBIG)
VELPA(JBIG) = PACTPO + VELPA(JBIG)
DO 350 J=1,KII
JEE = MOD(J)
IF(ICP.GT.0.AND.JEE.GT.1K) JEF = JEE-1K
PACTN = ET(J)*APN/RMSS(JEE)
PACTT = ET(J)*APT/RMSS(JEE)
VU(JEE) = (-PACTN*RSIN*PACTT*RCOS) + VU(JEE)
VW(JEE) = (PACTN*RCOS*PACTT*RSIN) + VW(JEE)
A = VU(JEE)*AY(JEE)*DELTR*2.0D+00
IMPT3250
IMPT3260
IMPT3270
IMPT3280
IMPT3290
IMPT3300
IMPT3310
IMPT3320
IMPT3330
IMPT3340
IMPT3350
IMPT3360
IMPT3370
IMPT3380
IMPT3390
IMPT3400
IMPT3410
IMPT3420
IMPT3430
IMPT3440
IMPT3450
IMPT3460
IMPT3470
IMPT3480
IMPT3490
IMPT3500
IMPT3510
IMPT3520
IMPT3530
IMPT3540
IMPT3550
IMPT3560
IMPT3570
IMPT3580
IMPT3590
IMPT3600
\[ B = \sqrt{\text{JEE}^2 + \text{AZ}^2} \times \text{DELT} \times 2.0 \times 0.0 \]
\[ VEL(\text{JEE}^2 - 1) = A \times \cos(\text{ANG(\text{JEE})}) + B \times \sin(\text{ANG(\text{JEE})}) \]
\[ VEL(\text{JEE}^2) = -A \times \sin(\text{ANG(\text{JEE})}) + B \times \cos(\text{ANG(\text{JEE})}) \]
\[ JVEL(\text{JEE}) = 1 \]

350 CONTINUE
RETURN
END
SUBROUTINE MINV(\$, N, DET, L, M)
IMPLICIT REAL*8(A-H, O-Z)

SEARCH FOR THE LARGEST ELEMENT

DIMENSION A(1), L(1), M(1)
DET=1.0
NK=-N
DO 80 K=1, N
NK=NK+N
L(K)=K
M(K)=K
KK=NK+K
BIGA=A(KK)
DO 20 J=K, N
IZ=N* (J-1)
DO 20 I=K, N
IJ=IZ+I
10 IF (DABS(BIGA)-DABS(A(IJ))) 15, 20, 20
15 BIGA=A(IJ)
L(R)=I
M(K)=J
20 CONTINUE

INTERCHANGE ROWS

J=L(K)
IF (J-K) 35, 35, 25
25 KI=K-N
DO 30 I=1, N
KI=KI+N
HOLD=A(KI)
JI=KI-K+J
A(KI)=A(JI)
30 A(JI)=HOLD

INTERCHANGE COLUMNS
C

35 I=M(K)
   IF (I-K) 45, 45, 38
38 JP=N*(I-1)
   DO 40 J=1, N
   JK=NK+J
   JI=JP+J
   HOLD=-A(JK)
   A(JK)=A(JI)
40 A(JI)=HOLD

C C C

DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT ELEMENT IS
CONTAINED IN BIG)

C C C

45 IP(BIGA) 48, 46, 48
46 DET=0.
   RETURN
48 DO 55 I=1, N
   IP(I-K) 50, 55, 50
50 IK=NK+I
   A(IK)=A(IK)/(-BIGA)
55 CONTINUE

C C C

REDUCE MATRIX

DO 65 I=1, N
   IK=NK+I
   HOLD=A(IK)
   IJ=I-N
   DO 65 J=1, N
   IJ=IJ+N
   IP(I-K) 60, 65, 60
60 IP(J-K) 62, 65, 62
62 KJ=IJ-I+K
   A(IJ)=HOLD*A(KJ)+A(IJ)
65 CONTINUE

MINV0372
MINV0380
MINV0390
MINV0400
MINV0410
MINV0420
MINV0430
MINV0440
MINV0450
MINV0460
MINV0470
MINV0480
MINV0490
MINV0500
MINV0510
MINV0520
MINV0530
MINV0540
MINV0550
MINV0560
MINV0570
MINV0580
MINV0590
MINV0600
MINV0610
MINV0620
MINV0630
MINV0640
MINV0650
MINV0660
MINV0670
MINV0680
MINV0690
MINV0700
MINV0710
MINV0720
DIVIDE ROW BY PIVOT

KJ = K - N
DO 75 J = 1, N
KJ = KJ + N
IF (J - K) 76, 75, 70
70 A(KJ) = A(KJ) / BIGA
75 CONTINUE

PRODUCT OF PIVOTS

DET = DET * BIGA

REPLACE PIVOT BY RECIPROCAL

A(KK) = 1.0 / BIGA
80 CONTINUE

FINAL ROW AND COLUMN INTERCHANGE

K = N
100 K = (K - 1)
   IF (K) 150, 150, 105
105 I = L(K)
   IF (I - K) 120, 120, 108
108 JQ = N * (K - 1)
   JR = N * (I - 1)
   DC 110 J = 1, N
   JK = JQ + J
   HOLD = A(JK)
   JI = JB + J
   A(JK) = -A(JI)
110 A(JI) = HOLD
120 J = M(K)
   IF (J - K) 100, 100, 125
125  \( KI = K - N \)
    DO 130 I = 1, N
    KI = KI + N
    HOLD = A(KI)
    JI = KI - K + J
    A(KI) = -A(JI)
130  A(JI) = HOLD
    GO TO 100
150  RETURN
    END
SUBROUTINE OMULT(SQVCT, RWVCT, NCOL, NRROWS, ACC, KROW, INDEX, NIRREG)
IMPLICIT REAL*8(A-H,O-Z)
C TO FIND ACC OF (SQVCT)* (RWVCT) = (ACC)
DIMENSION SQVCT(1), RWVCT(1), NCOL(1), ACC(1), KROW(1), INDEX(1)
INDEX=0
NROWS=NROWS-1
IF(NIRREG .GT. 0) GO TO 200
C HIGH SPEED PRODUCT FOR REGULAR MATRICES
DO 100 NN=1, NROWS
SUM=C .0
IP1=NN+1
KST=NCOL(NN)
INDEX=INDEX+NN-KST
DO 101 KPL=KST, NN
IJ=INDEX+KPL
101 SUM=SUM+SQVCT(IJ)*RWVCT(KPL)
C NOW FOR THE COLUMN ELEMENTS
INDEX=IJ
DO 102 KPL=IP1, NRROWS
IF(NN.LT.NCOL(KPL)) GO TO 100
INDEX=INDEX+KPL-NCOL(KPL)
102 SUM=SUM+SQVCT(INDEX)*RWVCT(KPL)
100 IACC(NN)=ACC(INN)+SUM
C NOW FOR THE LAST ROW
104 KADD=NCOL(NROWS)
SUM=0.0
INDEX=INDEX+NRROWS-KADD
DO 103 KPL=KADD, NRROWS
IJ=INDEX+KPL
103 SUM=SUM+SQVCT(IJ)*RWVCT(KPL)
ACC(NROWS)=ACC(NROWS)+SUM
RETURN
C MEDIUM SPEED PRODUCT FOR NIRREG .LE. NRROWS/2
200 IF(NIRREG .GT. NRROWS/2) GO TO 201
DO 105 NN=1, NRROWS
IP1=NN+1
KST=NCOL(NN)
INDEX=INDEX+NN-KST
SUM=J.
DO 106 KPL=KST,NN
IJ=INDEX+KPL
106 SUM=SUM+SOVCT(IJ)*RWVCT(KPL)
NCK=0
INDEX=IJ
DO 107 KPL=IP1,NROWS
IP(NN,LT. NCOL(KPL))GOTO 109
INDEX=INDEX+KPL-NCOL(KPL)
108 SUM=SUM+SOVCT(JINDEX)*RWVCT(KPL)
GO TO 105
109 NCK=NCK+1
IF (NCK.GT. NROWS)GOTO 105
IP(KPL,GE. KROW(NCK))GOTO 109
IP1=KROW(NCK)
INDEX=INDEX(NCK)+NN
GO TO 107
105 ACC(NV)=ACC(NN)+SUM
GO TO 104
201 DO 503 NN=1,NROWS
IP1=NN+1
K=NCOL(NN)
INDEX=INDEX+NN-K
SUM=0.
DO 502 KRX=K,NN
IJ=INDEX+KRX
502 SUM=SUM+SOVCT(IJ)*RWVCT(KRX)
INDEX=IJ
DO 504 KRX=IP1,NROWS
K=NCOL(KRX)
INDEX=INDEX+KRX-K
IF (NN.LT. K)GOTO 504
SUM=SUM+SQVCT(JINDEX)*RWVCT(KRX)
504 CONTINUE
SUBROUTINE PENTRN(TU,TW,TPCGU,TPCGW,NPP,PMAX,NF,NEQ,JP,PAL,RL
2,IBIG,N,PH,IK,NP,ICP)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION PN(51,5,6),PAE(51,5,5)
DIMENSION TU(1),TW(1),TPCGU(1),TPCGW(1),NEP(1),RL(1),Q(1),PH(1)
COMMON /TAPE/ WRITE,READ,MPUNCH
COMMON/BN/ LMT(51)
C CHECK FOR NODAL IMPACT AND ELEMENT IMPACT
DC 11C IF = 1, NP

L=1
DO 107 IR=1,IK
PD(IR,IP) = -10.0
PND(IR,IP) = -10.
L1= NVEC(IR,1)
L2=NVEC(IR,2)
IF(IR,NE,LMT(L)) GO TO 90
L=L+1
GO TO 100
90 CONTINUE
C CALCULATE DIST FROM PRAG TO NODE1 -- DPN AND LENGTH OF ELEMENT DEL
DPN=D5OR( (TPCGU(IP)-TU(L1))**2 +(TPCGW(IP)-TW(L1))**2)
DEL=D5OR( (TU(L2)-TU(L1))**2+(TW(L2)-TW(L1))**2)
RL(IR) = DEL
C DCRIN IS THE CRITICAL DISTANCE = HALF ( PRAG DIA. + AVG ELEMENT THIC
DCRIN= (PH(IP) + (H(L1)+H(L2))/2.) /2.)
DCRTE=DCRIN

DU= TU(L1)-TPCGU(IP)
DW= TW(L1) - TPCGW(IP)
DEU= TU(L2) -TU(L1)
DEW= TW(L2) - TW(L1)
TCOS = DEU / DEL
TSIN = DEW / DEL
P1L = -(DU*TCOS+DW*TSIN)
PALE(IR,IP) = PAL
IF (PAL.LT.0.0) GO TO 100

PENT0010 PENT0020 PENT0030 PENT0040 PENT0055
PENT0060 PENT0070 PENT0080 PENT0090
PENT0100 PENT0110 PENT0120 PENT0130 PENT0140
PENT0150 PENT0160 PENT0170 PENT0180 PENT0190
PENT0200 PENT0210 PENT0220 PENT0230 PENT0240
PENT0250 PENT0260 PENT0270 PENT0280 PENT0290
PENT0300 PENT0310 PENT0320 PENT0330 PENT0340
PENT0350 PENT0360
IF (PAL.GT.DEL) GO TO 100
DFE = (-DW*COS* DU*T SIN)
PD(IR,IP) = DCRT3-DFP
100 CONTINUE
110 CONTINUE
C CALCULATE THE LARGEST PENETRATION DIST. AND NUMBER OF PENETRATIONS
K = 0
IF (ICP.GT.0) K=1
DO 290 J=1,NF
290 NEF(J) = 0
NEQ=0
PMA= -5.0
NPP = 0
PAL = -1.5
DC 300 IR=1,IK
L1= NVFC(IR,1)
L2= NVFC(IR,2)
DO 30C IF= 1, NF
335 IP (PD(IR,IF).GT.0.0) NPP=NPP+1
IP (PD(IR,IF)-PMA) 360 ,350,340
340 PMAX= PD(IR,IF)
PMA = PMA(IR,IF)/ RL(IR)
IRIG = IR
NEQ=0
IP=IF
DO 345 J=1,NF
345 NFP(J) =0
GO TO 360
350 NEP(IF) =1
NEQ= NEQ+1
360 CONTINUE
306 CONTINUE
NRETURN
LND
SUBROUTINE PRINT(IT, TIME)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION COPY(51), COP2(51), FAILI(51), FAILO(51)
COMMON /VL/ PLVA(205), DISP(205), DEDL(205), SBX(50, 3, 6, 5),
       *BRNP(50, 3), RBNP(50, 3), TNP(205), TB(205), TW(205).
       *COII (205), COIIZ(205), DETA
COMMON/FG/I(51), Z(51), ANG(51), H(51), EXANG, NS, IK, NOGA, NFL, NI,
       *ICOL(205), NBCOMD, NBC(7), NODEB(7)
COMMON/IIAT/ TAI
COMMON/NAT/ DENS(6), B(6), YOUNG(6), DS(6), SN0(6, 5), NSPL(6), P(6),
       *EPS(6, 5), SIG(6, 5), EFLN(6)
COMMON/HM/ C5, C6, C8L(50, 3, 6, 5), GZET(50, 3, 6)
COMMON/BA/ BEP(50, 3, 3, 8), AL(50), AMG(3), AMG(3)
COMMON/FRAZ/PH(6), PCG(6), FMAS(6), FMOI(6), PCGU(6), PCGW(6), ALFA(6),
       *UDOT(6), WDOT(6), ADOT(6), TFRM(6), CR(6), PCX(6), UNK(6), NP
COMMON/DPRD, DPCGU(6), DPCGW(6), DALFA(6)
COMMON/TAE/ NREAD, NWRITE, NPRINT
COMMON/EP/ EPSI(51), EPSO(51)
SIN(Q) = DSIN(Q)
COS(Q) = DCOS(Q)
ATAN(Q) = DATAN(Q)
ABS(Q) = DABS(Q)
SORT(Q) = DSORT(Q)

CALCULATING PRESENT POSITION OF EACH NODE
DO 11 I = 1, NS
   COPY(I) = Y(I) + DISP(I*4-3)*COS(ANG(I)) - DISP(I*4-2)*SIN(ANG(I))
   11 COPIZ(I) = Z(I) + DISP(I*4-3)*SIN(ANG(I)) + DISP(I*4-2)*COS(ANG(I))
TAIT = TIME - TAI
WRITE(NWRITE, 1)IT, TIME, TAIT
1 FORMAT(//, 'J=', J, ', ', 'TIME=', T, 'TIME AFTER INITIAL IPRNO000')
1 FORMAT(//, 'IACT=', , D15.6)
WRITE(NWRITE, 2)

WRITE(NWRITE, 22)I, DISP(I*4-3), DISP(I*4-2), DISP(I*4-1), DISP(I*4),
                  PRINN0030
                  PRINN0031
                  PRINN0032
                  PRINN0033
*COPY(I), COPZ(I), BIMP(I, 2), BIMP(I, 2), EPS1(I), EPSO(I)
IF (EXAMG. EQ. 360.) GO TO 189
22 FORMAT(I5, 9D12.4, 2X, D12.4)
IKP1 = IK + 1
WRITE (HWRITE, 23) IKP1, DISP(IKP1*4-3), DISP(IKP1*4-2), DISP(IKP1*4-1)
DO 36 I = 1, NF
36 WRITE (HWRITE, 37) I, PFGU(I), PFGW(I), ALFA(I), UDOT(I), WDOT(I), ADOT(I)
37 FORMAT(10X, I5, 3X, 6D15.6, 15X)
RETURN
END
SUBROUTINE GELM(AA, AL, AXGL, AAG)
IMPLICIT REAL*8(A-H, O-Z)

TO FIND EFFECTIVE STIFFNESS MATRIX DUE TO ELASTIC RESTRAINTS
DIMENSION AA(50,8,8), AL(1), AXGL(1), AAG(1), BNU(51)
* ELR(8,8), ELR(8,8), ELR(8,8)
DIMENSION DELA(4), DLSH(8), DUMMY(9)
COMMON/FSY(51), Z(51), ANG(51), H(51), EXANG, NS, IK, NGLA, NFL, NI,
* ICOL(205), NPGCD, WBC(7), NODEB(7)
COMMON/MAT, DENS(8), D(6), YOUNG(6), DS(6), SNO(6,5), NSPL(6), P(6),
* EPS(6,5), SIG(6,5), FELN(6)
COMMON/ELPU/SPIRIP(2060), EQRER(205), REX(4), NQR, NORP, NORU, NREL(4),
* NEST(4), NREL(4)
COMMOM/TAPE, MHEA, MWRITE, MPUNCH
COMMON/BB, NVEC(51,2)
COMMON/ENON, YK(51), NECN, BBCB(7), NODPB(7), MK(51), ROT(5,2)

IF(DROT(50), NODP(6))
COMMON/XD, XIST(6)
COMMON/TAM, MKE(51)
SIN(0) = DSIN(0)
COS(0) = DCOS(0)
ATAN(0) = DATAN(0)
ABS(0) = DAHS(0)
SORT(0) = DSCRT(0)
PIE = 3.141592653589797L+00
PIE2 = 2.0*PIE
PIE32 = 1.5*PIE

IF (NORP .EQ. 0) GO TO 1
READ(MHEA,2) SCTP, SCTV, SCRP, (NREL(I), REX(I), I=1, NORP)
2 FORMAT(3D15.6/(4(15, D15.6)))
WRITE(MWRITE,1100) NOZE
1100 FORMAT(/'THE CONSTANTS FOR',' I1', 'ELASTIC POINT CONSTRAINTS ARE')
WRITE(MWRITE,777) SCTP, SCTV, SCRP
WRITE(MWRITE,1140)
1140 FORMAT(/'I', 'ELEMENT', 'I', 'S COORDINATE')
WRITE(MWRITE,1145) (NREL(I), REX(I), I=1, NORP)

QREM0010
QREM0020
QREM0030
QREM0040
QREM0050
QREM0060
QREM0070
QREM0080
QREM0090
QREM0100
QREM0110
QREM0120
QREM0130
QREM0140
QREM0150
QREM0160
QREM0170
QREM0180
QREM0190
QREM0200
QREM0210
QREM0220
QREM0230
QREM0240
QREM0250
QREM0260
QREM0270
QREM0280
QREM0290
QREM0300
QREM0310
QREM0320
QREM0330
QREM0340
QREM0350
QREM0360
**FORMAT (I, 10X, I, 13X, D13.6)

DO 10 I=1,NRNP
SL=REX (1Q)
N2=NREL (1Q)
K1= NVEC (NQ, 1)
K2= NVEC (NE, 2)
MOP = MKE (NE) - 1
L=MOP
P5=Z (K2) - Z (K1)
P6=Y (K2) - Y (K1)
P7=ANG (K2) - ANG (K1)
IF (YK (NF) . EQ. 1.0) P7=ANG (K2) - ROT (L, 2) - ANG (K1)
IF (YK (NE) . EQ. 1.0. AND. ROT (L, 1) . EQ. 0.0) P7=ROT (L, 2) + ANG (K2) - ANG (K1)
IF (YK (NE) . EQ. 2.0) P7= ANG (K2) - DPOT (NF) - ANG (K1)
IF (YK (NE) . EQ. 3.0) P7=ROT (MOP, 2) * ANG (K2) - DROT (NE) - ANG (K1)
ANG2=ANG (K2)
ANG1=ANG (K1)
IF (YK (NE) . EQ. 1.0. AND. ROT (L, 1) . EQ. 0.0) ANG (K2) = ROT (L, 2) + ANG (K2)
IF (YK (NE) . EQ. 1.0. AND. ROT (L, 1) . EQ. 1.0) ANG (K1) = ROT (L, 2) + ANG (K1)
IF (YK (NE) . EQ. 2.0) ANG (K1) = DROT (NF) + ANG (K1)
IF (YK (NE) . EQ. 3.0) ANG (K2) = ROT (MOP, 2) + ANG (K2)
IF (YK (NE) . EQ. 3.0) ANG (K1) = DROT (NE) + ANG (K1)
APHA = PIE / 2.0
IF (P5.LT.0.0) APHA= -APHA
IF (P6.LT.0.0) APHA= ATAN (P5/P6)
IF (P6.LT.0.0 AND. P5.LT.0.0) APHA=APHA-PIE
IF (P6.LT.0.0 AND. P5.GE.0.0) APHA=APHA+PIE
BNG (NE+1)=ANG (K2)
BNG (NF)=ANG (K1)
IF (P7.GT. (PIE/32) AND. APHA.LT.0.0) BNG (NE+1)=ANG (K2) - PIE2
IF (P7.GT. (PIE/32) AND. APHA.GT.0.0) BNG (NE)=ANG (K1)+PIE2
IF (P7.LT.(-PIE/32) AND. APHA.GT.0.0) BNG (NE+1)=ANG (K2) + PIE2
IF (P7.LT.(-PIE/32) AND. APHA.LT.0.0) BNG (NE)=ANG (K1)-PIE2
HZER=BNG (NE)-APHA
P1= (-2. * BNG (NE+1) - 4. * BNG (NF) * 6. * APHA) / AL (NE)
B2= (3. * BNG (NF+1) + 3. * BNG (NE) - 5. * APHA) / AL (NE)**2**
ANG(K2) = ANG2
ANG(K1) = ANG1
PHI = WZER + B1*SL + B2*SL**2
PHIP = B1 + 2*B2*SL
YZET = 0.0
ZZET = 0.0
DO 104 JJ = 1, NOGA
  P2 = BZLR + B1*SL + AXG(JJ) + B2*(SL + AXG(JJ))***2*APHA
  YZET = YZET + COS(P2) + SL*APHA(JJ)
  ZZET = ZZET + SIN(P2) + SL*APHA(JJ)
  P3 = YZET*SIN(PHI) + ZZET*COS(PHI + APHA)
  P4 = YZET*COS(PHI) + ZZET*SIN(PHI + APHA)
  ELR (1, 1) = SCTP*COS(PHI)**2 + SCTY*SIN(PHI)**2
  ELR (1, 2) = (SCTP - SCTY)*COS(PHI)*SIN(PHI)
  ELR (3, 1) = P3*COS(PHI) + SCTP - P4*SIN(PHI)*SCTY
  ELR (4, 1) = SL*COS(PHI) + SCTP
  ELR (5, 1) = SL**2*SIN(PHI) + SCTP
  ELR (6, 1) = SL**3*SIN(PHI) + SCTP
  ELR (7, 1) = SL**2*COS(PHI) + SCTP
  ELR (8, 1) = SL**3*COS(PHI) + SCTP
  ELR (2, 2) = SCTP*SIN(PHI)**2 + SCTY*COS(PHI)**2
  ELR (3, 2) = P3*SIN(PHI) + SCTY + P4*COS(PHI) + SCTY
  ELR (4, 2) = SL*SIN(PHI) + SCTP
  ELR (5, 2) = SL**2*COS(PHI) + SCTP
  ELR (6, 2) = SL**3*COS(PHI) + SCTP
  ELR (7, 2) = SL**2*SIN(PHI) + SCTP
  ELR (8, 2) = SL**3*SIN(PHI) + SCTP
  ELR (3, 3) = P3**2*SCTP + P4**2*SCTY + SCRIP
  ELR (4, 3) = P3 + SCTY + SL*PHIP + SCRIP
  ELR (5, 3) = P4 + SL**2*SCTY + 2*SL*SCRIP
  ELR (6, 3) = P4 + SL**3*SCTY + 3*SL**2 + SCRIP
  ELR (7, 3) = (P3 + SCTP) + SL**2
  ELR (8, 3) = (P3 + PHIP + SCRIP) + SL**3
  ELR (4, 4) = (SCTP + PHIP + SCRIP) + SL**2
  ELR (5, 4) = 2*SL**2 + PHIP + SCRIP
  ELR (6, 4) = 3*SL**3 + PHIP + SCRIP
QREM0730
QREM0740
QREM0750
QREM0760
QREM0770
QREM0780
QREM0790
QREM0800
QREM0810
QREM0820
QREM0830
QREM0840
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QREM0870
QREM0880
QREM0890
QREM0900
QREM0910
QREM0920
QREM0930
QREM0940
QREM0950
QREM0960
QREM0970
QREM0980
QREM0990
QREM1000
QREM1010
QREM1020
QREM1030
QREM1040
QREM1050
QREM1060
QREM1070
QREM1080
ELR (7,4) = (SCTP+PHIP**2*SCR) *SL**3
ELR (8,4) = (SCTP+PHIP**2*SCR) *SL**4
ELR (5,5) = SL**4*SCTTY + SL**2*SCR
ELR (6,5) = SL**5*SCTY + SL**3*SCR
ELR (7,5) = 2.*SL**3*PHIP*SCR
ELR (8,5) = 2.*SL**4*PHIP*SCR
ELR (6,6) = SL**6*SCTY + SL**4*SCR
ELR (7,6) = 3.*SL**4*PHIP*SCR
ELR (8,6) = 3.*SL**5*PHIP*SCR
ELR (7,7) = (SCTP+PHIP**2*SCR) *SL**4
ELR (8,7) = (SCTP+PHIP**2*SCR) *SL**5
ELR (8,8) = (SCTP+PHIP**2*SCR) *SL**6
DO 12 I = 1,7
IP1 = I+1
DO 12 J = IP1,8
12 ELR (I,J) = ELR (J,I)
DO 13 I = 1,8
DO 13 J = 1,8
ELR (I,J) = 0.0
DO 13 K = 1,8
13 ELR (I,J) = ELR (I,J)*ELR (I,K)*AA (NE,K,J)
DO 14 I = 1,8
DO 14 J = 1,8
ELR (I,J) = 0.0
DO 14 K = 1,8
14 ELR (I,J) = ELR (I,J)*AA (NE,K,I)*ELR (K,J)
IF (YK(NE).EQ.0.0) GO TO 502
CALL ROTAT (3, ELRP, DUMMY, NE)
502 CONTINUE
CALL ASSEM (NE,ELRP,SPRIN)
10 CONTINUE
1 IF (NORU .EQ.0) GO TO 4
READ (MFAD,3) SCTU, SCRU, SCTW, NRST (I), NREU (I), I = 1, NORU
3 FORMAT (3D15.6/(815))
WHITE (WHITE, 1120) NORU
1120 FORMAT (///, ' THE CONSTANTS FOR*,I3,* ELASTIC FOUNDATIONS ARE:*')
WRITE(MWRITE,777)SCTU,SCTW,SCRU
777   FORMAT(/,'10X,'THE VALUE OF THE TANGENTIAL SPRING CONSTANT IS =',D15.6,/) QREM1460
     *5.6,/,10X,'THE VALUE OF THE NORMAL SPRING CONSTANT IS =',D15.6,/) QREM1460
     *10X,'THE VALUE OF THE TORSIONAL SPRING CONSTANT IS =',D15.6,/) QREM1460
WRITE(MWRITE,1150)
1150   FORMAT(/,'10Y,'FIRST ELEMENT',10X,'NUMBER OF ELEMENTS') QREM1500
WRITE(MWRITE,1155) (NAST(I),NRP(D),I=1,NORU)
1155   FORMAT(' ',13X,I3,24X,I3) QREM1520
DO 15 IQ=1,NORU
15 NSTAT=NSTAT(IQ)
NEND=NRPU(IQ)
DO 16 IR=1,NEND
16 NE=(NSTAT-1)*IR
IF(NE.GT.IK) NL=NE-IK
K1 = NVEC(NE,1)
K2 = NVEC(NE,2)
MOP = MKE(NE) - 1
L=MOP
P5=Z(K2) -2(K1)
P6=Y(K2) -Y(K1)
P7=ANG(K2) -ANG(K1)
IF(YK(NP),EQ.1.0) P7=ANG(K2) - ROT(L,2) -ANG(K1)
IF(YK(NP),EQ.1.0,AND.,ROT(L,1).EQ.0.0) P7=ROT(L,2) +ANG(K2) -ANG(K1)
IF(YK(NP),EQ.2.0) P7=ANG(K2) - DROT(NE) -ANG(K1)
IF(YK(NP),EQ.3.0) P7=ROT(MOP,2) +ANG(K2) -DROT(NE) -ANG(K1)
ANG2=ANG(K2)
ANG1=ANG(K1)
IF(YK(NP),EQ.1.0,AND.,ROT(L,1).EQ.0.0) ANG(K2)=ROT(L,2) +ANG(K2)
IF(YK(NP),EQ.1.0,AND.,ROT(L,1).EQ.1.0) ANG(K1)=ROT(L,2) +ANG(K1)
IF(YK(NP),EQ.2.0) ANG(K1)=DROT(NE) +ANG(K1)
IF(YK(NP),EQ.3.0) ANG(K2)=ROT(MOP,2) +ANG(K2)
IF(YK(NP),EQ.3.0) ANG(K1)=DROT(NE) +ANG(K1)
APHA = PIE/2.0
IF(P5.LT.0.0) APHA = -APHA
IF(P6.LE.0.0) APHA = ATAN(P5/P6)
IF(P6.LE.0.0,AND.,P5.LE.0.0) APHA = APHA - PIE QREM1800
```
IP (P6.LT.0.0 .AND. P5.GE.0.0) APHA = APHA + PI
BNG (NR+1) = ANG (K2)
BNG (NE) = ANG (K1)
IP (P7.GT.(PIE32) .AND.APHA.LT.0.0) BNG (NE+1) = ANG (K2) - PI
IP (P7.GT.(PIE32) .AND.APHA.GT.0.0) BNG (NE) = ANG (K1) + PI
IP (P7.LT.(-PIE32) .AND.APHA.GT.0.0) BNG (NE+1) = ANG (K2) + PI
IP (P7.LT.(-PIE32) .AND.APHA.LT.0.0) BNG (NE) = ANG (K1) - PI
BZER = BNG (NE) - APHA
B1 = (-2. * BNG (NE+1) - 4. * BNG (NE) + 6. * APHA) / AL (NE)
ANG (K2) = ANG2
ANG (K1) = ANG1
DO 102 I = 1, 8
DO 102 J = 1, 8
FLR (I, J) = 0.0
FLR (I, J) = 0.0
ZET = AL (NE) * AXG (J)
PHI = B1 + B2 * ZET
PHI = BZER + B1 * ZET + B2 * ZET**2
WET = AL (NE) * AWG (J)
YZET = ZET**2
ZZET = ZET**2
DO 105 JJ = 1, NOGA
P2 = BZFR + B1 * ZET**2 + B2 * (ZET**2 + AXG (JJ))
YZET = YZET + COS (P2) * ZET**2 + ANG (JJ)
ZET = ZET**2
YZET = ZET**2 + SIN (P2) * ZET**2 + AWG (JJ)
YZET = YZET**2
P3 = YZET + SIN (PHI + APHA) - ZET**2
P4 = YZET**2
ELR (1, 1) = ELR (1, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (2, 1) = ELR (2, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (3, 1) = ELR (3, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (4, 1) = ELR (4, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (5, 1) = ELR (5, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (6, 1) = ELR (6, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (7, 1) = ELR (7, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
ELR (8, 1) = ELR (8, 1) + (SCtu * COS (PHI)**2 + SCTW * SIN (PHI)**2) * WET
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QREM 1810
QREM 1820
QREM 1830
QREM 1840
QREM 1850
QREM 1860
QREM 1870
QREM 1880
QREM 1890
QREM 1900
QREM 1910
QREM 1920
QREM 1930
QREM 1940
QREM 1950
QREM 1960
QREM 1970
QREM 1980
QREM 1990
QREM 2000
QREM 2010
QREM 2020
QREM 2030
QREM 2040
QREM 2050
QREM 2060
QREM 2070
QREM 2080
QREM 2090
QREM 2100
QREM 2110
QREM 2120
QREM 2130
QREM 2140
QREM 2150
QREM 2160
ELR (6, 2) = ELR (6, 2) + (ZET**3 * SCTW*cos (PHI)) * WET
ELR (6, 3) = ELR (6, 3) + (P)**2 * SCTU + P**2 * SCT + SCR.U * WET
ELR (5, 3) = ELR (5, 3) + (P)**2 * SCTW + ZET**2 + 2.0 * SCR + ZET) * WET
ELR (6, 3) = ELR (6, 3) + (P)**2 * SCTW + ZET**3 + 3.0 * SCR + ZET**2 * WET
ELR (5, 5) = ELR (5, 5) + (ZET**4 * SCTW + 4.0 * ZET**2 + SCR) * WET
ELR (6, 5) = ELR (6, 5) + (ZET**5 * SCTW + 5.0 * ZET**3 + SCR) * WET
ELR (6, 6) = ELR (6, 6) + (ZET**6 * SCTW + 6.0 * ZET**4 + SCR) * WET

FLR (4, 1) = FLR (4, 1) + ZET**2 + COS (PHI) * SCTU * WET
ELR (7, 1) = ELR (7, 1) + ZET**2 + COS (PHI) * SCTU * WET
ELR (8, 1) = ELR (8, 1) + ZET**2 + COS (PHI) * SCTU * WET
ELR (4, 2) = ELR (4, 2) + ZET**2 + SIN (PHI) * SCTU * WET
ELR (7, 2) = ELR (7, 2) + ZET**2 + SIN (PHI) * SCTU * WET
ELR (8, 2) = ELR (8, 2) + ZET**2 + SIN (PHI) * SCTU * WET
ELR (4, 3) = ELR (4, 3) + (P)**3 + SCTU + PHIP* SCR) * ZET * WET
ELR (7, 3) = ELR (7, 3) + (P)**3 + SCTU + PHIP* SCR) * ZET * WET
ELR (8, 3) = ELR (8, 3) + (P)**3 + SCTU + PHIP* SCR) * ZET * WET
FLR (4, 4) = FLR (4, 4) + (SCTU + PHIP**2 + SCR) * ZET**2 + WET
ELR (5, 4) = ELR (5, 4) + (SCTU + PHIP**2 + SCR) * ZET**2 + WET
ELR (6, 4) = ELR (6, 4) + (SCTU + PHIP**2 + SCR) * ZET**2 + WET
ELR (7, 4) = ELR (7, 4) + (SCTU + PHIP**2 + SCR) * ZET**2 + WET
ELR (8, 4) = ELR (8, 4) + (SCTU + PHIP**2 + SCR) * ZET**2 + WET

10 3 CONTINUE
DO 5 I=1, 7
1P1=I+1
DO 5 J=IP1, 8
ELR (I, J) = ELR (J, I)
DO 6 I=1, 8
DO 6 J=1, 8
ELR (I, J) = 0.0
DO 6 K = 1, 8
6  ELLR(I, J) = ELLR(I, J) + ELLR(I, K) * AA(NE, K, J)
DO 7 I = 1, 8
DO 7 J = 1, 8
7  ELLP(I, J) = 0.0
DO 7 K = 1, 8
IF (YK(NF) .EQ. 0.0) GO TO 503
   CALL ROTAT(I, ELLP, DUMMY, NE)
503 CONTINUE
   CALL ASSEM(NF, ELLP, SPRIN)
16 CONTINUE
15 CONTINUE
4  IF (NBCND .EQ. 0) RETURN
DO 91 I = 1, NBCND
91  JT4 = NOD2B(I) * 4
JT4M3 = JT4 - 3
JT4M2 = JT4 - 2
JT4M1 = JT4 - 1
   CALL ERC(JT4M3, SPRIN, NI, ICOL)
   IF (NBC(I) .EQ. 1 .OR. NBC(I) .EQ. 2) CALL ERC(JT4M1, SPRIN, NI, ICOL)
   IF (NBC(I) .EQ. 2 .OR. NFC(I) .EQ. 3) CALL ERC(JT4M2, SPRIN, NI, ICOL)
91 CONTINUE
RETURN
END
SUBROUTINE ROOT4(A,B,C,D,X,IER,IMAGN)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION X(4),IMAGN(4)
C
C FIND REAL ROOTS OF QUARTIC EQN. X**4+A*X**3+B*X**2+C*X+D=0
C IF IMAGN(I).EQ.0, THEN I-TH ROOT IS REAL.
C IF IMAGN(I).EQ.1, THEN I-TH ROOT IS IMAGINARY AND IS SET EQUAL
C TO A REAL NUMBER OF ORDER 10**-25.
C
DO 5 I=1,4
5 IMAGN(I)=0
C DEPEN D COEFFS. OF RESOLVENT CUBIC
IER=0
P=-B
Q=A*C-4.0*D
R=-A*A*D+4.0*C**2-D*C
C FIND ROOT, Y, OF RESOLVENT CUBIC
CALL CUBIC(P,Q,R,Y,IER)
C DEPEN D R*H
R=A*A/4.0-B+Y
C IF R*H IS LESS THAN ZERO, CANNOT CONTINUE
IF(R.GE.0.0)GO TO 10
IER=1
WRITE(6,200)
200 FORMAT('0+',R*H LESS THAN 0.0-- NO ROOTS FOUND FOR QUARTIC')
RETURN
C DEPEN D H
10 R=DSQRT(R)
IF(R.LT.1.0D-30)R=0.0
C DEPEN D COEFFS. E AND P (SQUARED)
IF(R.LE.0.0)GO TO 20
C1=(4.0*A*B-8.0*C-A*A*A)/(4.0*R)
E=3.0*A*A/4.0-R*R-2.0*B
P=E-C1
E=E+C1
GO TO 10
C FOR R=0.0
20 C2=Y+Y-4.0*D
IF (C2.GE.0.0) GO TO 25
IEN=-1
WRITE(6,300)
300 FORMAT(10,'*Y*Y-4=D.LT.0.0--NO ROOTS FOUND FOR QUARTIC*)
25 C2=DSQRT(C2)
C2=2.0*C2
E=3.0*A*A/4.0-2.0*B
P=E-C2
E=P+C2
C IF EITHER E OR F (SQUARED) ARE NEGATIVE, IMAGINARY ROOTS WILL
C RESULT. SET THEM TO LARGE VALUES.
30 CONTINUE
IF (E.GE.0.0) GO TO 35
E=1.0D50
IMAGN(1)=1
IMAGN(2)=1
35 CONTINUE
IF (P.GE.0.0) GO TO 40
P=1.0D50
IMAGN(3)=1
IMAGN(4)=1
40 CONTINUE
C CALC. E AND F
E=DSQRT(E)
P=DSQRT(P)
C CALC. THE FOUR ROOTS.
X(1)=-A/4.0+B/2.0-E/2*C
X(2)=-A/4.0+B/2.0-E/2.0
X(3)=-A/4.0-B/2.0+F/2.0
X(4)=-A/4.0-B/2.0+F/2.0
RETURN
END
SUBROUTINE ROTAT(INC, ELK, YLW, IR)
C         THIS SUBROUTINE TRANSFORMS MATRICES FROM ELEMENT SYSTEMS TO
C         GLOBAL SYSTEM AND VICE-VERSA.  INC = 1 FOR GLOBAL VECTOR INTO
C         ELEMENT VECTOR, INC = 2 FOR ELEMENT VECTOR INTO GLOBAL VECTOR.  INC = 3
C         FOR ELEMENT MATRIX INTO GLOBAL SYSTEM.
C         ELK IS THE MATRIX TO BE TRANSFORMED, YLW IS THE VECTOR TO BE
C         TRANSFORMED, WHILE IR IS THE ELEMENT NUMBER YLW, ELK, AND DROT USED
C         BELOW, Indicate whether a branch or a discontinuity is being con-
C         sidered and what the angle of rotation is.
C         IMPLICIT REAL*8 (A-H, O-Z)
C         DIMENSION ELK (8, 9), YLW (9), TR (4, 4), TRAN (8, 8), WORK (8), ELM (8, 8)
C         COMMON /BOUN/, YK (51), NBCONB, NBCB (7), NODBB (7), MK (51), ROT (5, 2)
C         O, DROT (51), NODE (6)
C         COMMON /TAM/, MKF (51)
C         COMMON /XO/, XDIST (6)
C         COMMON /BR/, NVEC (51, 2)
C         COMMON /TAPE/, MREAD, MWRITE, MPUNCH
C         COMMON /TIME/, IT
C         SIN (Q) = DSIN (Q)
C         COS (Q) = DCOS (Q)
C         ATAN (Q) = DATAN (Q)
C         ABS (Q) = DABS (2)
C         SQRT (Q) = DSQRT (Q)
C         DC 1100  I = 1, 4
C         DO 1100  J = 1, 4
  1100  TP (I, J) = 0.0
C         TR (3, 3) = 1.0
C         TR (4, 4) = 1.0
C         MOP = MKF (IR) - 1
C         DC 1110  J = 1, 8
C         DC 1110  K = 1, 8
C         1110  TRAN (J, K) = 0.0
C         DC 1115  J = 1, 8
C         1115  TRAN (J, I) = 1.0
C         ANGK = DROT (IR)
C         IF (YK (IR) .LT. 1.0 .OR. YK (IR) .EQ. 3.0) ANGK = ROT (MOP, 2)
TR (1, 1) = DCOS(ANGK)
TF (1, 2) = DSIN(ANGK)
TP (2, 1) = -DSIN(ANGK)
TP (2, 2) = DCOS(ANGK)
IF (YK(TR), EQ. 2.0) GO TO 1120
IF (ROT(MOP, 1), NE. 0.0) GO TO 1120
DO 1130 J = 5, 8
DO 1130 K = 5, 8
1130 TRAN (J, K) = TR (J-4, K-4)
TRAN (5, 7) = XDIST(MOP) * TR (1, 1)
TRAN (6, 7) = XDIST(MOP) * TR (2, 1)
GOTO 1150
1120 DO 1140 J = 1, 4
DO 1140 K = 1, 4
1140 TRAN (J, K) = TR (J, K)
IF (YK(TR), EQ. 2.0) GO TO 1190
TRAN (1, 3) = XDIST(MOP) * TR (1, 1)
TRAN (2, 3) = XDIST(MOP) * TR (2, 1)
1150 CONTINUE
IF (YK(TR), NE. 3.0) GO TO 3000
ANGZ = DROT(IR)
TRAN (1, 1) = DCOS(ANGZ)
TRAN (1, 2) = DSIN(ANGZ)
TRAN (2, 1) = -DSIN(ANGZ)
TRAN (2, 2) = DCOS(ANGZ)
3000 CONTINUE
IF (IND.EQ.3) GOTO 800
IF (IND.EQ.1) GOTO 130
DO 210 I = 1, 8
DO 210 J = 1, 8
210 ELRR (I, J) = TRAN (J, I)
DO 215 I = 1, 8
DO 215 J = 1, 9
215 TRAN (I, J) = ELRR (I, J)
100 CONTINUE
110 CONTINUE
DC 116C I = 1, 8
WORK(I) = 0.0
DO 116C J = 1, 8
116C WORK(I) = WORK(I) + TRAN(I, J) + ELM(J)
DC 117C I = 1, 8
117C ELM(I) = WORK(I)
RETURN
800 CONTINUE
DO 216C I = 1, 8
DO 216C J = 1, 8
FLRR(I, J) = 0.0
DO 216C K = 1, 8
216C FLRR(I, J) = FLRR(I, J) + ELM(I, K) + TRAN(K, J)
DO 217C I = 1, 8
DO 217C J = 1, 8
ELM(I, J) = 0.0
DO 217C K = 1, 8
217C ELM(I, J) = ELM(I, J) + TRAN(K, I) + FLRR(K, J)
RETURN
END
SUBROUTINE STRESS
IMPLICIT REAL*8(A-H,O-Z)
C TO EVALUATE GENERALIZED NODAL LOAD VECTOR DUE TO LARGE
AND PLASTIC-PLASTIC STRAIN
C
DIMENSION ELPP(8), EPS(3), CEPS(3,3), DEMPW(3), BIMPW(3), NWB(3,3)
*PM(8), ZINL(8)
DIMENSION DDEL(8), DVM(8), DDMY(8,8)
COMMON /YCN/ YK(51), NSCONB, NSCB(7), NNODEB(7), MK(51), ROT(5,2)
 Strand 5, NODE(6)
 COMMON /TV/ FLV(205), DISP(225), DZLD(225), SNS(50,3,6,5)
*EMD(52,53), EMIP(50,3), TDISP(205), TV(225), TW(225)
*CIILY(205), CIIZ(205), DLTAT
 COMMON /TM/ MEK(51)
 COMMON/FG/Y(51), Z(51), ANG(51), H(51), EXAXG, NS, Y, NCGA, NFL, NF
* ICL(205), N[](7), NNODEB(7)
 COMMON /MN/ DZNS(6), Y(6), YOUNG(6), D5(6), SN(R,6,5), NSPL(6), P(6)
* EPS(6,5), SIT(6,5), FPLN(6)
 COMMON /NH/ CS, C6, ASPF(50,3,6,5), GZIT(50,3,6)
 COMMON /FA/ BFP(50,3,3,4), AL(50), AXG(3), AWM(3)
 COMMON /XD/ XDIST(6)
 COMMON /VF/ NVFC(51,2)
 COMMON /TAPE/ MREAD, MWRITE, MPUNCH
SIN(0)=DSIN(Q)
COS(0)=DCOS(Q)
ATAN(0)=DATAN(Q)
ABS(0)=DABS(2)
SORT(0)=DSORT(0)
MOP=0
DO 502 IR=1,IK
K1= NVFC(IR,1)
K2= NVFC(IR,2)
DO 8000 K=1,3
INDEX=(K1-1)*4+K
IF(K.GT.4) INDEX=(K2-1)*4+K-4
DELM(K)=DDEL(INDEX)
DISM(K)=DISP(INDEX)

STSSC010
STSSC020
STSSC030
STSSC040
STSSC050
STSSC060
STSSC070
STSSC080
STSSC090
STSSC100
STSSC110
STSSC120
STSSC130
STSSC140
STSSC150
STSSC160
STSSC170
STSSC180
STSSC190
STSSC200
STSSC210
STSSC220
STSSC230
STSSC240
STSSC250
STSSC260
STSSC270
STSSC280
STSSC290
STSSC300
STSSC310
STSSC320
STSSC330
STSSC340
STSSC350
STSSC360
9000 CONTINUE
   IF(YK(IR),EQ,0,0) GOTO 901
   CALL ROTAT(1, DUMMY, DELM, IR)
   CALL ROTAT(1, DUMMY, DISM, IR)
901 CONTINUE
   M = MKF(IR)
   DO 503 J=1, N2GA
      RIMP(IR,J) = J.
      FIMP(IR,J) = J.
   503 CONTINUE
   DC 402 I=1,3
   LFPS(I) = 3.
   DO 402 K=1, 8
   402 FEPS(I) = EPS(1) + EPS(IFIR, I, K) * DELM(K)
   CEPS(J, 1) = (?.)
   CEPS(J, 2) = (?.)
   DO 403 K=1, 8
      CFPS(J, 1) = EPS(J, 1) + EPS(IFIR, J, 1, K) * DISM(K)
      CFPS(J, 2) = EPS(J, 2) + EPS(IFIR, J, 2, K) * DISM(K)
   403 CONTINUE
   205 PAR = EPS(1) + EPS(2) - EPS(2) * EPS(1) == 2/2.
   * + EPS(J, 1) * EPS(1) - EPS(1) == 2/2.
      PCUR = EPS(3)
   D) 151 K=1, NPL
      5KNP = 0.
      EPSX = PAR + GZETA(1P, J, K) * PCUR
      IF(DS(M), EQ, 0, 0) GO TO 5100
      C5 = 1.0 / P(M)
      C6 = 1.0 / DS(M) / DELTAT
      RFACTR = 1.0 * (C6 * ABS(EPSX)) ** C5
5100 N = NSFL(M)
   DO 15 L=1, N
      SNS(IR, J, K, L) = SNS(IR, J, K, L) + YOUNG(M) * EPSX
      IF(DS(M), EQ, 0, 0) GO TO 255
      IF (SNS(IP, J, K, L) - SNS(M, L)) > 30, 31, 11
      91 SNY = SNC(M, L) * FACTR
      IF (SNS(IR, J, K, L) - SNY) > 30, 31, 22
   20 SNS(IR, J, K, L) = SNY
GO TO 301
30 IF (SN5(IR,J,K,L)+SNO(M,L)) 92,311,3C1
25 SN5 = SNO(M,L)+RPACIR
25 IF (SN5(IR,J,K,L)+SN5) 40,301,331
40 SNS(IR,J,K,L)=-SN5
GO TO 301
255 IF (SN5(IR,J,K,L)-SNO(M,L)) 19,311,17
17 SNS(IR,J,K,L) = SNO(M,L)
GO TO 301
18 IF (SN5(IR,J,K,L)+SNO(M,L)) 19,301,301
19 SNS(IR,J,K,L) = -SNO(M,L)
3;1 IFNP=BNP+SN5(IR,J,K,L)=ASPL(IR,J,K,L)
35 CONTINUE
EINP(IR,J)=BINP(IR,J)+BNP
BNP(IR,J)=BINP(IR,J)+BNP*GZETA(IR,J,K)
51 CONTINUE
503 CONTINUE
107 DO 101 J=1,4
WB(J,1)=CEPS(J,1)*AWG(J)*BINP(IR,J)*AL(IR)
WB(J,2)=CEPS(J,2)*AWG(J)*BINP(IR,J)*AL(IR)
LTNPW(J)=EINP(IR,J)*AWG(J)*AL(IJ)
BITPW(J)=BINP(IR,J)*AWG(J)*AL(IJ)
101 CONTINUE
GO 102 I=1,4
PN(I)=....
P(I)=... HNL(I)=J
DO 102 J=1,4
PN(I)=PN(I)+BEP(IR,J,1,I)*BINP(J)
P(I)=PN(I)+BEP(IR,J,1,I)*BINP(J)
132 HNL(I)=HNL(I)+BEP(IR,J,2,I)*HWS(J,2)
**BEP(IR,J,1,I)*HWS(J,1)
207 DO 105 I=1,4
105 TLFP(I)=PN(I)+PN(I)+HNL(I)
IF(YK(IR),FQ,1,0) GC TO 502
CALL RCTAT(2,DUMMY,ELFP,IR)
SUBROUTINE TCO: : (TY, T2, VY, VZ, FCGR, FCGR, VELPS, VELPW, DELTP. NET, THMM, TCON0010
2LMTMIN, RFC, NFT:: IFLAG, H, PH, MP, AP, AZ)
IMPLICIT REAL*: (-H, 0. -2)
DIMENSION Y(4), IMAGN (4)
DIMENSION AY (51), AZ (51)
DIMENSION TY (1), T2 (1), VY (1), VZ (1), FCGR (1), FCGR (1), VELPS (1),
VELPW (1), IFLAG (51, 6), H (1), PH (1)
COMMON /TAPE/, MREAD, MWRITE, MPUNCH
COMMON /HIT/, TNJ (6), MNP
COMMON/RN/LMT (51)
COMMON/BR/NVEC (51, 2)
C
C ROUTINE TO CALCULATE THE TIME OF CONTACT OF FRAGMENT ON RING.
C
C INITIALIZE MIN., TIME AND ELEM. NO..,
MIN = DELTR * 1.1
LMTMIN = 0
EPS = DELTR * 1.0D-03
EPSN = - EPS
TEN = 10.0D+00
NUM = -6
C
C LOOP OVER ALL ELEMS. (ASSUME CLOCKWIE NUMBERING OF NODES)
DO 100 IF = 1, NF
VELPS = VELPS (IP) * TNJ (IF)
VELPW = VELPW (IF) * TNJ (IF)
L = 1
DO 100 LNUM = 1, NET
C
C IF THIS ELEMENT IS A BRANCH ELM. FOR WHICH NO IMPACT
C CAN OCCUR, SKIP TO NEXT ELEMENT.
   IF (LNUM .NE. LMT (L)) GO TO 20
   L = L + 1
   GO TO 100
C
C DEFINE CA, CB,.....
20  L1 = NVEC (LNUM, 1)
```
I2= NVEC (LNUM, 2)
TTT= (H (I1) + H (I2)) / 2.0
DC= (TTT + H (IF)) / 2.0
CA=AZ (I1) * (AY (I1) +AY (I2)) -AY (I1) * (AZ (I2) +AZ (I1))
CB= (VY (I1) - VEFUT) * (AZ (I1) -AZ (I2)) + (VZ (I1) - VZ (I2)) *AY (I1) +
CB= (EFUT - VZ (I1)) * (AY (I1) -AY (I2)) + (VY (I2) - VY (I1)) *AZ (I1)
CC= (FCGU (IF) - TY (I1)) * (AZ (I2) -AZ (I1)) + (VEPUT - VY (I1)) * (VZ (I2) -
CVZ (I1)) + (TZ (I1) - T2 (I2)) *AY (I1) + (TZ (I1) - FCGW (IF)) * (AY (I2) -AY (I1)) +
CD= (FCGU (IF) - TY (I1)) * (VZ (I2) - VZ (I1)) + (VEPUT - VY (I1)) * (TZ (I2) -
CTZ (I1)) + (TZ (I1) - FCGW (IF)) * (VY (I2) - VY (I1)) + (VZ (I1) - VEFUT) *
CT= (TY (I2) - TY (I1))
C= (TZ (I1) - TZ (I2)) * (TY (I1) - FCGU (IF)) - (TY (I1) - TY (I2)) *
C= (TZ (I1) - FCGW (IF))
CP= (AY (I2) - AY (I1)) **2 + (AZ (I2) -AZ (I1)) **2
CG= ((AY (I2) - AY (I1)) * (VY (I2) - VY (I1)) + (AZ (I2) -AZ (I1)) *
CH= (VZ (I2) - VZ (I1)) **2 + (VZ (I2) - VZ (I1)) **2 + ((AY (I2) - AY (I1)) *
CT= (TY (I2) - TY (I1)) + (AZ (I2) -AZ (I1)) * (TZ (I2) - TZ (I1)) ) **2
CI= 2.0 * ( (TY (I2) - TY (I1)) * (VY (I2) - VY (I1)) + (TZ (I2) - TZ (I1)) *
C= (VZ (I2) - VZ (I1))
CJ= (TY (I2) - TY (I1)) **2 + (TZ (I2) - TZ (I1)) **2
CJ= DSORT (CJ)
A=CA-DC * (3.0 / 16.0) * CI**2 * CH/CJ**5 - 5.0 * CI**4 / (128.0 * CJ**7)
B= (CH**2 + 6.0 * CI * CG) / (8.0 * CJ**3) + CP / (2.0 * CJ)
C=CB-DC * (CI**3 / (16.0 * CJ**5) - CI*CH / (4.0 * CJ**3) + CG / (2.0 * CJ)
D=CD-DC * CI / (2.0 * CJ)
F= CE-DC * CJ
TM = 0.0
IF (E.LT.0.0).AND.(IFLAG (LNUM, IF)) .EQ. 2) E = 0.0
IF ( E.EQ.0.0) GOTO 30
AP= D/E
BP= C/E
CP= B/E
DP= A/E
```
RTLED = 1.0/(DELTR*1.1)
RT2=RTLED*RTLED
RT3=RT2*RTLED
RT4=RT3*RTLED
DB=RT4*AP*RT3+BP*RT2+CP*RTLED*DP
IF(DB.LT.0.0) GOTO 25
CB = 4.0*RT3+3.0*RT2+AP+2.0*RTLED*BP+CP
IF(CB.LT.0.0) GOTO 25
BB = 6.0*RT2+3.0*RTLED*AP+EP
IF(BB.LT.0.0) GOTO 25
AB = 4.0*RTLED*AP
IF(AB.LT.0.0) GOTO 25
B1 = BB-CB/AB
IF(B1.LE.0.0) GOTO 25
C1 = CB-AB*CB/B1
IF(C1.LE.0.0) GOTO 25
GOTO 100
25 CONTINUE
AP=AP* TEN**NUM
BP= BP* TEN**((NUM*2)
CP= CP* TEN**((NUM*3)
DP= DP* TEN**((NUM*4)
CALL ROOT4(AP,BP,CP,DP,X,IER,IMAGN)
TM = 1000.0
IF(IER.NE.0) GOTO 100
DO 40 I= 1,4
IF(IMAGN(I).EQ.1) GOTO 40
IF(X(I).EQ.0.0) GOTO 40
T= TEN**NUM/X(I)
IF(T.GT.EPSN.AND.T.LT.0.0) T=0.0
IF(T.LT.0.0) T= 1.0D+25
IF(T.LT.PFS.AND.IFLAG(LNUM,IF).EQ.1) GOTO 40
IF(T.LT.TM) TM=T
40 CONTINUE
XY=1.1* DELTR
IF(TM.LE.XY) IFLAG(LNUM,IF) = 2
IF(TM.GT.TMIN) GOTO 100
30 CONTINUE
IF(TM.LT.EXS.AND.IFLAG(LNUM,IF).FP.1) GOTO 100
IFLAG(LNUM,IF) = 2

C CHECK FOR CONTACT ON ELEMENT.
C UPDATE POSITIONS (FOR THIS ELEM.) TO TM
Y1 = TY(I1) + TM*VY(I1) + AY(I1)*TM**2
Y2 = TY(I2) + TM*VY(I2) + AY(I2)*TM**2
Z1 = T2(I1) + TM*VZ(I1) + A2(I1)*TM**2
Z2 = T2(I2) + TM*VZ(I2) + A2(I2)*TM**2
YP = PCGU(IP) + TM*VFPMT
ZP = PCGW(IP) + TM*VFPWT

C MAGNITUDE OF VECTORS P1 AND P2
P1 = DSQRT((Y2 - Y1)**2 + (Z2 - Z1)**2)
P2 = DSQRT((YF - Y1)**2 + (ZF - Z1)**2)

C DEFN P2*COS(THETA)
P2CT = ((Y2 - Y1)*(YF - Y1) + (Z2 - Z1)*(ZP - Z1))/P1

C IF P2*COS(THETA) IS LESS THAN ZERO OR GREATER THAN P1, CONTACT
C HAS OCCURRED OUTSIDE OF ELEMENT LIMITS. GO TO NEXT ELEM.
IF(P2CT.LT.0.0 OR P2CT.GT.P1) GO TO 100

C OTHERWISE, CONTACT HAS OCCURRED. UPDATE TMIN, LNUM, AND
C RPC (RELATIVE POSITION OF CONTACT---FRACTION OF ELEM LENGTH FROM
C NODE 1)
TMIN = TM
LNUM = LNUM
NPTMIN = IF
IF(TMIN.GT.DELTR) TMIN = DELTR
RPC = P2CT/P1

C LOOP TO NEXT ELEMENT
100 CONTINUE
XY = 0.999*D*CO = DELTR
IF (TMIN.GT.XY) TMIN = DELTR

C SET FLAG FOR THIS CONTACT TO 1
IFLAG(LNTMIN,NPTMIN) = 1

C IF MODAL IMPACT, SET FLAG FOR ADJACENT ELEMENT
IF (RPC.GT.0.01 .AND. RPC.LT.0.99) RETURN
IF (RPC.LE.0.01) NN = NVSC(LNTMIN,1)
IF (RPC.GE.0.99) NN = NVSC(LNTMIN,2)
DO 120 I = 1,2
DO 120 J = 1,NPT
IF (NVSC(J,I).EQ.NN) IFLAG(J,NPTMIN) = 1
120 CONTINUE
RETURN
END
SUBROUTINE TSTEP(      
KROW, NDEK, NDEKG, DELTA) TSTP031C
      
C       JST 3C       
C 
C TO FIND DELTA IF IT IS NOT SPECIFIED 
C       EXPLICIT PEAT=9 (A-H, J-2) 
C       DIMENSION ANK(205), TRIAL(205), VYULT(205), VECTA(205), 
*       XPCW (1), NDEK (1) 
CMON/ABC/PMX (51), NWCRK, SIVY(215) 
CMON/PG/Y(51), Z(51), ANG (51), H (51), VXLNG, NS, IK, NPGA, NFL, NI, 
*       ICOL (215), NSCOND, NAC (7), NGDEB (7) 
CMON/MA/ DENS (6), B (6), YOUNG (6), DS (6), SIG (5, 5), NSPL (6), P (6), 
*       EPS (5, 5), SIG (6, 5), EPLN (6) 
CMON /LEFT/ RMASS (51) 
CMON /ST/ SIFK (206) 
CMON /TAPE/ MREAD, MWRITE, MPUNCH 
SIN (Q) = DSIN (Q) 
COS (Q) = DCO S (Q) 
ATAN (Q) = DATAN (Q) 
ABS (Q) = DABS (Q) 
SORT (Q) = DSORT (Q) 
INT (Q) = IDINT (Q) 
MREAD = 5 
MWRITE = 5 
IMX = IK + 1 
IF (EXANG .NE. 360.) GO TO 1 
DO 5 T = 1, IK 
AMK3 (T*4-3) = RMASS (I) 
AMKF (T*4-2) = RMASS (I) 
AMKF (T*4-1) = RMX (I) 
5 AMKE (T*4) = PMX (I) 
AMKF (IMX*4-3) = RMASS (1) 
AMKE (IMX*4-2) = RMASS (1) 
AMKF (IMX*4-1) = PMX (1) 
AMKF (IMX*4) = RMX (1) 
GO TO 75C 
1 DO 20 I = 1, IMX 
AMKE (T*4-3) = RMASS (I) 
20 CONTINUE
AMKE(I*4-2)=RMASS(I)
AMKE(I*4-1)=RMX(I)
2C AMKE(I*4)=RMX(I)
730 DC 3 F=1,NI
3 TRYAL(K)=1,0
IF(NBCCYD .EQ. 0) GO TO 90
D0 91 Y=1,NBCCYD
JT4=NCDEB(I) *4
JT4+3=JT4-3
JT4+2=JT4-2
JT4+1=JT4-1
CALL PROC(JT4+3,STIFK,NI,ICOL)
TRYAL(JT4+3)=0.0
IF(NBC(I) .EQ. 1 .OR. NBC(I) .EQ. 2) CALL ERC(JT4+1,STIFK,NI,ICOL)
IF(NBC(I) .EQ. 1 .OR. NBC(I) .EQ. 3) CALL ERC(JT4+2,STIFK,NI,ICOL)
IF(NBC(I) .EQ. 1 .OR. NBC(I) .EQ. 2) TRYAL(JT4+1)=0.0
IF(NBC(I) .EQ. 1 .OR. NBC(I) .EQ. 3) TRYAL(JT4+2)=0.0
91 CONTINUE
90 MRANK=NI
BCNE=NI
LPSLN=1,JE=07
2 BCLD=1,0
DC 14 ITK=1,4
DC 12 ILL=1,5C
DO 4 I=1,MRANK
4 VMULT(I)=0.0
CALL OMULT(STIFK,TRYAL,ICOL,NI,VMULT,KRCW,NDEX,NI,REG)
310 DO 320 JT= 1,NI
320 VECTR(JT) = VMULT(JT)/AMKE(JT)
BNW=1.0
DO 6 K=1,MRANK
IF (BNW=ABS(VECTR(K))) 60,60,6
60 CONTINUE=ABS(VECTR(K))
6 CONTINUE
DC 7 K=1,MRANK
IF (BNW=ABS(VECTR(K))) 7,3,7
CONTINUE
MP=K
BNEW=VECTR(K)
DO 9 K=1,MRANK
9 TIAL.(K)=VECTR(K)/BNEW
I"( ABS(BNEW/BOLD-1.C)-EPSLN) 15,15,13
C ITERATION
10 BTH=BOLD
LCLD=BNEW
12 CONTINUE
EPSLN=EPSLN-1.
14 CONTINUE
C NOT CONVERGING AFTER IL•IK ITERATIONS
EPSLN=1.C
LCLD=BNEW
GO TO 32
C EIGEN VALUE FOUND
15 FCVE=3NF4
32 WRITE(WRITE,24) (TIAL(J),J=1,NI)
24 FORMAT(/*," EIGEN VECTOR OF HIGHEST MODE",/.*,14X,*,14X,*,14X,*,13XTSIP0923)
*," PSI",12X,*,CHI",/,(11X,4E15.6)
*FREQ= SORT(BOND)
F:CTCL=.A
DELTAN=FACTCL*2./FREQ
WRITE(WRITE,25) FREQ
25 FORMAT(/*," HIGHEST NATURAL FREQUENCY (RAD/SEC) =",D25.16)
WRITE(WRITE,31) DELTAN
31 FORMAT(* THE COMPLETE VALUE OF THE MAX DELTAT =",D25.16)
MP=0
3C DELTAN=DELTAN*10.)
MP=MP+1
IF(DELTAN.LT.10.C) GO TO 3
DELTAN= INT(DELTAN) *10.C**(-MP)
IF(DELTAN.GT.DELTAY) DELTAY = DELTAN
IF(DELTAY.EQ.9.C) DELTAY = DELTAY
WRITE(WRITE,28) DELTAY
28 FORMAT( 'DELTAT SHOULD EQUAL: ', 5X, D13.6)
WRITE( MYPRINT, 33 ) DELTAT
33 FORMAT( 'THE VALUE OF DELTAT USED IN THE PROGRAM IS: ', D15.6)
RETURN
END
SUBROUTINE UPDATE(SIGN, TU, TW, VV, VZ, TPCGU, TPCGW, TALFA, VELFU, VELFW, UPDA0010
2VELFA, DELTR, IKK, NP, ICP, AY, AZ)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION AY(51), AZ(51)
DIMENSION TU(1), TW(1), VV(1), VZ(1), TPCGU(1), TPCGW(1), TALFA(1),
2VELFU(1), VELFW(1), VELFA(1)
COMMON /HIT/ TNJ(6), NLP
C UPDATE NODAL POSITIONS
DO 10 I=1, IKK
TU(I)= TU(I) + SIGN*(VV(I)*DELTR+AY(I)*DELTR**2)
TW(I)= TW(I) + SIGN*(VZ(I)*DELTR+AZ(I)*DELTR**2)
10 CONTINUE
IF (ICP.LE.0) GO TO 20
TU(IKK+1)=TU(I)
TW(IKK+1)=TW(I)
C UPDATE FRAGMENT POSITION
20 DO 30 I=1, NP
TPCGU(I)= TPCGU(I) + VELFU(I) *DELTR*SIGN*TNJ(I)
TPCGW(I)= TPCGW(I) + VELFW(I) *DELTR*SIGN*TNJ(I)
30 TALFA(I)= TALFA(I) + VELFA(I) *DELTR*SIGN*TNJ(I)
RETURN
END
UPDA0020
UPDA0030
UPDA0040
UPDA0050
UPDA0060
UPDA0070
UPDA0080
UPDA0090
UPDA1000
UPDA1010
UPDA1020
UPDA1030
UPDA1040
UPDA1050
UPDA1060
UPDA1070
UPDA1080
UPDA1090
UPDA1100
UPDA1110
UPDA1120
UPDA1130
UPDA1140
UPDA1150
UPDA1160
UPDA1170
UPDA1180
UPDA1190
UPDA1200
UPDA1210
UPDA1220
ILLUSTRATIVE EXAMPLES

The following two examples are presented to assist the user in checking the adaptation of CIVM-JET 4B to his computer facility.

6.1 A Variable-Thickness, Partial Ring -- Includes Branches, Slope Discontinuities, and an Elastic Foundation

6.1.1 Problem Description

The geometry of the main structure, as shown in Fig. 9, is a partial ring composed of an initially-straight portion and a circular portion. The straight section is 10.0 in long, 1.5 in wide, and varies linearly in thickness from 0.3 in at its pinned end to 0.1 in where it joins the circular portion. The circular section has a 5.0 in mean radius, a 1.5 in width, a 0.1 in uniform thickness, and consists of a 60° arc. The partial ring is supported by a pinned joint at its left-hand end, a branch connected at the straight-circular junction, and an elastic foundation located as depicted in Fig. 9a. This foundation consists of arbitrarily chosen normal $k_N$ and tangential $k_T$ stiffness equal to 1500 psi and 3000 psi, respectively.

The "main structure" of the partial ring is called substructure one (1) and is assumed to consist of aluminum material with a yield stress of 46,000 psi, an elastic modulus of $10^7$ psi, and is represented by a two-mechanical-sublayer model defined by the following stress-strain $(\sigma, \varepsilon)$ pairs: $\sigma_1, \varepsilon_1 = 46,000$ psi, 0.0046 and $\sigma_2, \varepsilon_2 = 58,000$ psi, 0.18000. The strain-rate constants were chosen to be $D = 6500 \text{ sec}^{-1}$ and $P = 4$. The mass density is $0.25 \times 10^{-3} \text{ (lb-sec}^2)/\text{in}^4$.

The branch (termed substructure 2) is a steel structure 1.0 in wide, 2.23607 in long, and has a constant thickness of 0.4 in. The branch has a slope discontinuity between its two equal-length elements. The branch attaches to the outer surface of the main structure at the eleventh node of the ring and is clamped at its other end. The branch material is represented by a three-mechanical-sublayer model defined by $\sigma_1, \varepsilon_1 = 80,950$ psi, 0.00279; $\sigma_2, \varepsilon_2 = 105,300$ psi, 0.02250; and $\sigma_3, \varepsilon_3 = 121,000$ psi, 0.20000 (with an elastic
modulus of 29 x 10^6 psi and a yield stress of 80,950 psi). The strain rate
constants for the branch are D = 40.4 sec^{-1} and P = 5, with a mass density of
0.733085 x 10^{-3} (lb-sec^2)/in^4.

The variable-thickness straight portion of the structure is
modeled by 10 equal-length finite elements; 6 equal-arc finite elements repre-
sent the constant thickness curved section; and 2 equal-length elements repre-
sent the constant thickness branch. This makes a total of 18 finite elements
used for the entire structure.

The elements of the main structure are initially numbered consecu-
tively from 1 to 16, and the branch elements are initially numbered from 1 to
2; this is depicted in Fig. 9b. The program will then renumber the elements
from left to right to include the branch elements in the global system; the
resulting renumbering is shown in Fig. 9c.

The attacking fragment has the following parameters (see Fig. 9a):
- radius r_f = 0.5 in, mass m_f = .385610 x 10^{-3} (lb-sec^2/in); mass moment of
  inertial I_f = 0.482014 x 10^{-4} (lb-sec^2-in); initial translational velocity
  components: \( \dot{Y}_f = 2507.96 \) in/sec, \( \dot{Z}_f = 1482.75 \) in/sec; initial rotational
  velocity \( \dot{\beta}_f = 0.0 \); initial C.G. position \( Y_{CG} = 6.0 \) in, \( Z_{CG} = -2.0 \) in. The
  value of the coefficient of restitution, e, is set at 1.0 to represent a
  perfectly-elastic impact reaction, and the coefficient of friction is set to
  0.0.

The strain is to be calculated at each of the three spanwise
Gaussian stations and each node of the main structure and the branch.
Also, 3 additional points at which strain predictions are desired are
requested. Two of these are on main structural elements 9 and 11; the
point on element 9 is located near the point of first impact and the point
on element 11 is located near the branch connection (\( s \) coordinates 0.53
and 0.05, respectively). The additional strain point on the branch is located
at \( s = 0.50 \) of the first element. This corresponds to the same location as
the second Gaussian station on this element. The strains should be exactly
the same at this point since both the Gaussian station at \( s \) and the additional
point are at the same physical location.

The CIVM-JET 4B program will be used to calculate the structural
response of the ring and the motion of the fragment, using a time step of 1
microsecond. Printout of structural responses and fragment position data are
desired at intervals of every 40 cycles until 600 cycles have been completed.

6.1.2 Input Data

The values to be punched on the data cards are as follows:

Card 1

<table>
<thead>
<tr>
<th>Field</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E(1)</td>
<td>0.150000D+01</td>
</tr>
<tr>
<td>DENS(1)</td>
<td>0.250000D-03</td>
</tr>
<tr>
<td>EXANG</td>
<td>0.360000D+02 (partial ring)</td>
</tr>
</tbody>
</table>

(arbitrary value; EXANG ≠ 360.0 for partial ring)

Card 2

<table>
<thead>
<tr>
<th>Field</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>IK</td>
<td>16</td>
</tr>
<tr>
<td>NOGA</td>
<td>3</td>
</tr>
<tr>
<td>NFL</td>
<td>4</td>
</tr>
<tr>
<td>NSFL(1)</td>
<td>2</td>
</tr>
<tr>
<td>MM</td>
<td>1580</td>
</tr>
<tr>
<td>M1</td>
<td>780</td>
</tr>
<tr>
<td>M2</td>
<td>40</td>
</tr>
<tr>
<td>NF</td>
<td>1</td>
</tr>
<tr>
<td>TIMF</td>
<td>0.158000D-02</td>
</tr>
</tbody>
</table>

Card 3A

<table>
<thead>
<tr>
<th>Field</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y(1)</td>
<td>0.0</td>
</tr>
<tr>
<td>Z(1)</td>
<td>0.0</td>
</tr>
<tr>
<td>ANG(1)</td>
<td>0.0</td>
</tr>
<tr>
<td>H(1)</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Additional cards are provided until all 17 nodal stations of the
main structure are described.
There are no slope discontinuities on the main structure; skip to Card 5.

Card 5

NBR = 1

Card 5A

NSFL(2) = 3
B(2) = 0.10000D+01
DENS(2) = 0.733085D-03
DS(2) = 0.404000D+02
P(2) = 0.500000D+01

Card 5AA

EPS(1,2) = 0.279000D-02
SIG(1,2) = 0.809500D+05
EPS(2,2) = 0.225000D-01
SIG(2,2) = 0.105300D+06
EPS(3,2) = 0.200000D+00
SIG(3,2) = 0.121000D+06

Card 5B

NELT(1) = 2
NODP(1) = 11
LHIT(1) = 0
LATT(1) = 1
Card 5BA

\[
\begin{align*}
YB(1,1) &= 0.1050000D+02 \\
ZB(1,1) &= 0.1000000D+01 \\
ANB(1,1) &= 0.634349D+02 \\
HB(1,1) &= 0.4000000D+00 \\
\end{align*}
\]

Additional cards 5BB are provided until all branch nodes of this branch are described.

Card 5BC below contains information about the branch attachment point.

\[
\begin{align*}
YBB(1,3) &= 0.1000000D+02 \\
ZB(1,3) &= 0.0 \\
ANB(1,3) &= 0.634349D+02 \\
HB(1,3) &= 0.4000000D+00 \\
\end{align*}
\]

Card 5C

\[
\begin{align*}
NDISB &= 1 \\
\end{align*}
\]

Card 5CA

\[
\begin{align*}
NEDIB &= 2 \\
NBDI &= 1 \\
ANGB &= 0.265651D+02 \\
\end{align*}
\]

Card 5D

\[
\begin{align*}
NBCOND &= 1 \\
\end{align*}
\]

Card 5DA

\[
\begin{align*}
NBCB(1) &= 2 \\
NODBB(1) &= 2 \\
LBR(1) &= 1 \\
\end{align*}
\]

Card 6

\[
\begin{align*}
DELTAT &= 0.1000000D-05 \\
DS(1) &= 0.6500000D+04 \\
P(1) &= 0.4000000D+01 \\
NTOVR &= BLANK \\
\end{align*}
\]
Card 7AA

\[ \begin{align*}
\text{EPS}(1,1) &= 0.460000 \times 10^{-02} \\
\text{SIG}(1,1) &= 0.460000 \times 10^{05} \\
\text{EPS}(2,1) &= 0.180000 \times 10^{00} \\
\text{SIG}(2,1) &= 0.580000 \times 10^{05}
\end{align*} \]  

Card 8

\[ \begin{align*}
\text{NOP} &= 3 \\
\text{NASP} &= 3
\end{align*} \]

Card 8A

\[ \begin{align*}
\text{NSBS}(1) &= 1 \\
\text{NSEL}(1) &= 9 \\
\text{AZET}(1) &= 0.530000 \times 10^{00}
\end{align*} \]

... Additional cards are punched until all 3 additional strain points are described.

\[ \begin{align*}
\text{NSBS}(3) &= 1 \\
\text{NSEL}(3) &= 11 \\
\text{AZET}(3) &= 0.500000 \times 10^{-01}
\end{align*} \]

Card 9AA

\[ \begin{align*}
\text{FH}(1) &= 0.100000 \times 10^{01} \\
\text{FCG}(1) &= -0.200000 \times 10^{01} \\
\text{FCGX}(1) &= 0.600000 \times 10^{01} \\
\text{FMASS}(1) &= 0.385610 \times 10^{01} \\
\text{FMOI}(1) &= 0.482014 \times 10^{-04}
\end{align*} \]

Card 9AB

\[ \begin{align*}
\text{UNK}(1) &= 0.0
\end{align*} \]

Card 9AC

\[ \begin{align*}
\text{UDOT}(1) &= 0.260796 \times 10^{04} \\
\text{WDOT}(1) &= 0.148275 \times 10^{04} \\
\text{ADOT}(1) &= 0.0
\end{align*} \]
TPRIM(1) = 0.960000D-03
CR(1) = 0.100000D+01

Card 10
AXG(1) = 0.1127016653792585D+00
AXG(2) = 0.5000000000000000D+00
AXG(3) = 0.8872983346207415D+00

Card 11
AWG(1) = 2.777777777777778D+00
AWG(2) = 0.4444444444444440D+00
AWG(3) = 0.777777777777778D+00

Card 12A
TXG(1) = -0.8611363115940539D+00
TXG(2) = -0.3399810435848560D+00
TXG(3) = 0.3399810435848560D+00

Card 12B
TXG(4) = 0.8611363115940530D+00

Card 13A
TWG(1) = 0.3478548451374540D+00
TWG(2) = 0.6521451548625460D+00
TWG(3) = 0.6521451548625460D+00

Card 13B
TWG(4) = 0.3478548451374540D+00

Card 14A
NBCOND = 1

Card 14B
NBC(1) = 3
NODEB(1) = 1
Card 15

NQR = 2
NORP = 0
NORU = 2

Ship Card 15A and go to Card 15B

Card 15B

SCTU = 0.300000D+04
SCTU = 0.0
SCTU = 0.150000D+04

Card 15C

NRST(1) = 9
NREU(1) = 2
NRST(2) = 13
NREU(2) = 3

Card 16

ICON = 0

Skip to Card 17

Card 17

ICON = 1

Note: Setting ICON = 1 causes the program to search for another complete set (Cards 1-17) of data cards. In this case the data cards for example number 2 (described in Section 6.2.1) followed immediately after the data cards for example number 1 and both problems were run during the same computer submittal. If ICON = 0 the job will terminate.
<table>
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<th>900</th>
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<th>20</th>
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<td>33.0</td>
<td></td>
<td>0.0</td>
<td>0.02</td>
<td>0.32500000+02</td>
<td>0.50000000+03</td>
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<td></td>
<td></td>
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<td>C.</td>
<td>1111</td>
<td>100</td>
<td>0.0</td>
<td>0.02</td>
<td>0.29000000+02</td>
<td>0.25000000+03</td>
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<td></td>
<td></td>
<td></td>
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<td>100</td>
<td>0.0</td>
<td>0.02</td>
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<td>0.24000000+03</td>
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<td></td>
<td></td>
<td></td>
</tr>
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<td>0.0</td>
<td>0.02</td>
<td>0.22000000+03</td>
<td>0.21000000+03</td>
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<td></td>
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<td>C.</td>
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<td></td>
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</tr>
<tr>
<td>C.</td>
<td>5555</td>
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<td>0.0</td>
<td>0.02</td>
<td>0.17000000+03</td>
<td>0.15000000+03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>0.0</td>
<td>0.02</td>
<td>0.14000000+03</td>
<td>0.12000000+03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.</td>
<td>3333</td>
<td>100</td>
<td>0.0</td>
<td>0.02</td>
<td>0.10000000+03</td>
<td>0.09000000+03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.</td>
<td>2222</td>
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<td>0.0</td>
<td>0.02</td>
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<td>0.05000000+03</td>
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<td></td>
<td></td>
<td></td>
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<tr>
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<td>0.02</td>
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<td>0.01000000+03</td>
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<td></td>
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<td>0.02</td>
<td>0.00000000+03</td>
<td>0.00000000+03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**THIS IS THE INPUT DECK FOR EXAMPLE 6.1**

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<th>40000000</th>
<th>10000000</th>
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</tr>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
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<td>C.</td>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
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</tr>
<tr>
<td>C.</td>
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<td>0.000000</td>
<td>0.000000</td>
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<td>0.000000</td>
<td>0.000000</td>
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<td>C.</td>
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<td>C.</td>
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</tr>
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**ORIGINAL PAGE OF POOR QUALITY**
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<table>
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<tr>
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<th></th>
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<th></th>
<th></th>
</tr>
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<tr>
<td>1</td>
<td>9</td>
<td>0.5100000D+06</td>
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<td></td>
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</tr>
</tbody>
</table>
6.1.3 Solution Output Data

The following is the output obtained as a result of the CIVM-JET 4B analysis of this partial ring example.

The numbering system for the nodes and elements is listed as well as an identification of the branch attachment point and the slopes at the branch connection and at the slope discontinuity. The partial ring initial geometry, boundary conditions, and elastic foundations are defined as well as all the necessary data pertaining to the impacting fragment. A "maximum allowable" time step is computed and the user generated time step is checked against this.

Each impact is recorded (there are 7 impacts during this run) and the essential data concerning element number, fragment number, time, and location are output. For each printout cycle, an update of each nodal position, the fragment position, the strains at each Gaussian point, each node, and each additional strain point is given.

Initial impact occurs on element 9 at 967.796 microsecond after fragment release. During this computer run the maximum strain reaches 5.79% on the main structure and only 1.28% on the branch.

Note that for conciseness only a portion of the requested output is presented here. Included are: all initial problem data, printout at time cycles 980, 1020, 1060, 1100, 1140, 1180, ... skip to 1540, 1580 (last); a record of all impacts occurring up to time cycle 1580 is retained.
There are 19 elements and 19 nodes.
There are 1 branches and they are at nodes 11.
The global slope (rad) at each branch correction: 7.11075E+01
The attachment point code for the 1 branches 11 as follows:

<table>
<thead>
<tr>
<th>Element No.</th>
<th>Node1</th>
<th>Node2</th>
<th>Substructure</th>
<th>Subst. Element No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
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<tr>
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<td>2</td>
</tr>
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<td>18</td>
<td>19</td>
<td>1</td>
<td>16</td>
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The updated nodal numbers for the final structure, given in their original numbering order:
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

Note: The element numbers appended to below are physical element numbers.

Elements that can not be injected:
11 12 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Additional strain point element in element:
1 9 0.50E-03 0.10E-03 0.13E-03 0.34E+00
2 11 0.50E-03 0.10E-03 0.13E-03 0.34E+00
3 13 0.50E-03 0.10E-03 0.13E-03 0.34E+00

Each of the following elements has a slope discontinuity at its first node:
12

The global slope (rad) at each discontinuity equals:
0.536492E+00

182
PENTAL RIV. CIVIL-JET NOX CONTENT ANALYSIS

RING PROPERTIES

MATERIAL PROPERTIES OF MAIN STRUCTURE ARE:

- Material of Main (E)
- Density of Ring
- Width of Elements
- Angle of Stress Gaussian
- Mass of Stiffness Gaussian
- Number of Mechanical Subelements

DS FOR STRAIN RATE

P FOR STRAIN RATE

<table>
<thead>
<tr>
<th>Strain (1)</th>
<th>Stress (1)</th>
<th>Stress (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.460000E+02</td>
<td>0.400000E+05</td>
<td>0.590000E+05</td>
</tr>
</tbody>
</table>

MATERIAL PROPERTIES OF SUBRING NUMBER 1 ARE AS FOLLOWS:

- Material of Subring (E)
- Density of Ring
- Width of Elements
- Angle of Stress Gaussian
- Mass of Stiffness Gaussian
- Number of Mechanical Subelements

DS FOR STRAIN RATE

P FOR STRAIN RATE

<table>
<thead>
<tr>
<th>Strain (1)</th>
<th>Stress (1)</th>
<th>Stress (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.400000E+02</td>
<td>0.500000E+01</td>
<td>0.400000E+00</td>
</tr>
</tbody>
</table>

THICKNESS AT THE CONNECTING NODE

<table>
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<th>Strain (1)</th>
<th>Stress (1)</th>
<th>Stress (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.400000E+02</td>
<td>0.400000E+00</td>
<td>0.400000E+00</td>
</tr>
</tbody>
</table>

INITIAL GEOMETRY AT ZACL NODE IS AS FOLLOWS:

<table>
<thead>
<tr>
<th>NODE NO.</th>
<th>X COORD</th>
<th>Y COORD</th>
<th>SLOPE (RAD.)</th>
<th>RING THICKNESS AT NODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.300000E+00</td>
</tr>
<tr>
<td>2</td>
<td>0.100000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.250000E+00</td>
</tr>
<tr>
<td>3</td>
<td>0.200000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.250000E+00</td>
</tr>
<tr>
<td>4</td>
<td>0.300000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.240000E+00</td>
</tr>
<tr>
<td>5</td>
<td>0.400000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.220000E+00</td>
</tr>
<tr>
<td>6</td>
<td>0.500000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.200000E+00</td>
</tr>
<tr>
<td>7</td>
<td>0.600000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.180000E+00</td>
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<td>8</td>
<td>0.700000E+01</td>
<td>0.0</td>
<td>0.0</td>
<td>0.160000E+00</td>
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ORIGINAL PAGE IS OF POOR QUALITY
<table>
<thead>
<tr>
<th>FRAG NO.</th>
<th>VELOCITY 1</th>
<th>VELOCITY 2</th>
<th>COLLISION ENERGY 1</th>
<th>COLLISION ENERGY 2</th>
<th>CONTACT ENERGY 1</th>
<th>CONTACT ENERGY 2</th>
<th>IMPACT ENERGY 1</th>
<th>IMPACT ENERGY 2</th>
<th>CONTACT IMPACT ENERGY 1</th>
<th>CONTACT IMPACT ENERGY 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2074914+04</td>
<td>0.1421231+04</td>
<td>0.0</td>
<td>0.1000000+01</td>
<td>0.1735235+04</td>
<td>0.0</td>
<td>0.1000000+01</td>
<td>0.1735235+04</td>
<td>0.1000000+01</td>
<td>0.1735235+04</td>
</tr>
</tbody>
</table>

**COLUMNS:**
- VELOCITY 1
- VELOCITY 2
- COLLISION ENERGY 1
- COLLISION ENERGY 2
- CONTACT ENERGY 1
- CONTACT ENERGY 2
- IMPACT ENERGY 1
- IMPACT ENERGY 2
- CONTACT IMPACT ENERGY 1
- CONTACT IMPACT ENERGY 2

**NOTES:**
- These parameters are used to simulate the forces and energies involved in the collision between two bodies.
- They are crucial for understanding the dynamics of the event.

**SIMULATION SETTINGS:**
- **Initial Conditions:**
  - VELOCITY 1: 0.2074914+04
  - VELOCITY 2: 0.1421231+04
  - CONTACT ENERGY 1: 0.1000000+01
  - CONTACT ENERGY 2: 0.1735235+04
- **Contact Impacts:**
  - CONTACT IMPACT ENERGY 1: 0.1000000+01
  - CONTACT IMPACT ENERGY 2: 0.1735235+04

**CONCLUSIONS:**
- The simulation shows how the forces interact during a collision, influencing the outcome of the event.
- Further analysis can help in predicting and controlling such events in real-world applications.
SHELL OF REVOLVING STIFFNESS RATIO α = 510

THE TRANSITIONAL AXES FOR EACH ELEMENT

THE INITIAL AXES FOR EACH ELEMENT

RELATIVE VALUES OF SIGNIFICANT FIGURES

HIGHEST NATURAL FREQUENCY (RAD/S) = \( \frac{\pi}{10} \times 27532.653272 \times 10^6 \)

THE COMPLETE VALUES FOR THE GXG DELTA = 1.123274\times10^7 \times 1.000000000000000

DIVERGENCE SHOULD BE: 0.3747060-25

THE VALUE OF DELTA USED IN THE FIGURE IS 0.1000000-25

THE CONSTANTS FOR FLEXURAL FOUNDATIONS ARE:

THE VALUE OF THE TRANSFORMAL ELASTIC CONSTANT IS = 0.3000000-04

THE VALUE OF THE K. (RELATIVE LENGTH) CONSTANT IS = 0.1500000-04

THE VALUE OF THE TOL. DENUMERATIVE CONSTANT IS = 0.0.5

FIRST ELEMENT NUMBER OF ELEMENTS = 13

THE FOLLOWING SHOWS THE VALUES FOR THE EFFECTIVE LENGTHS FOR THE BAND PLUS 1 SECTIONS OF THE STRUCTURE

ORIGINAL PAGE IN POOR QUALITY
## CIRCUMFERENCE AND WORK AT THE END OF EACH CYCLE 1020

### FRACTIONAL ELASTIC ENERGY (\(C_{\text{eff}}\))

<table>
<thead>
<tr>
<th>Cycle</th>
<th>(R/W)</th>
<th>(R)</th>
<th>(W)</th>
<th>(E)</th>
<th>(\lambda)</th>
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</thead>
<tbody>
<tr>
<td>0.100</td>
<td>1.050</td>
<td>0.110</td>
<td>0.280</td>
<td>0.390</td>
<td>0.230</td>
</tr>
<tr>
<td>0.100</td>
<td>1.050</td>
<td>0.110</td>
<td>0.280</td>
<td>0.390</td>
<td>0.230</td>
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<tr>
<td>0.100</td>
<td>1.050</td>
<td>0.110</td>
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<td>1.050</td>
<td>0.110</td>
<td>0.280</td>
<td>0.390</td>
<td>0.230</td>
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### CYCLE 1020

#### Strain at Additional Points

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Tensile Stress (MPa)</th>
<th>Tensile Strain (%)</th>
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</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.330</td>
<td>0.280</td>
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<td>1.00</td>
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#### Original Page

**Original Page is of Poor Quality**
<table>
<thead>
<tr>
<th>INPUT NO.</th>
<th>TIME</th>
<th>DISTANCE</th>
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**Input and Data at the End of Time Cycle 1540**

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**Matrix Element**

<table>
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<tr>
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<th>Value</th>
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### Cycle 1500

<table>
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<th>V</th>
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<th>ES</th>
<th>COF</th>
<th>COPE</th>
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<tr>
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<td>0.773070-02</td>
<td>0.693900-02</td>
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### Cycle 1500 Stress at Additional Points

<table>
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<th>V</th>
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<th>ES</th>
<th>COF</th>
<th>COPE</th>
<th>L</th>
<th>N</th>
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<tr>
<td>0.6</td>
<td>0.0</td>
<td>0.197520-02</td>
<td>0.136700-02</td>
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<td>0.187530-02</td>
<td>0.243212-03</td>
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### Data

- **Cycle:** 1500
- **Time:** 0.150000-02
- **Time After Initial Impact:** 0.612290-03

### Substructure

<table>
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<th>Sum</th>
<th>Time</th>
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</thead>
<tbody>
<tr>
<td>0.759122-21</td>
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<tr>
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### Substructure Largest Local Strain (L) Local Strain Time

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<th>Time</th>
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### Substructure

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### Substructure

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<tr>
<th>Rens</th>
<th>Elel</th>
<th>Sum</th>
<th>Time</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

### Original Page is of Poor Quality
6.2 A Uniform-Thickness, Unsupported Complete Circular Ring Subjected to a T-58 Rotor Tri-Hub Burst

6.2.1 Problem Description

The geometry of the free-ring containment structure, as shown in Fig. 10, is a free circular ring, 0.4 in thick, 2.5 in wide, with a mean radius of 7.7 in. The ring is subjected to a tri-hub burst (consisting of three perfectly symmetric fragments which are idealized as being circular and non-deformable) with each fragment being released at different times. Forty uniform finite elements are used to model the complete ring.

The 4130 cast steel ring material is represented by a three-mechanical-sublayer model defined by $\sigma_1, \epsilon_1 = 80,950 \text{ psi}, 0.00279; \sigma_2, \epsilon_2 = 105,300 \text{ psi}, 0.0225; \text{ and } \sigma_3, \epsilon_3 = 121,000 \text{ psi}, 0.2000$ with an elastic modulus of $29 \times 10^6$ psi and a yield stress of 80,950 psi. The strain rate constants are $D = 40.4 \text{ sec}^{-1}$ and $P = 5$, and the mass density is taken to be $0.733085 \times 10^3 \text{ (lb-sec}^2/\text{in}^4)$. The attacking fragments (Fig. 10) have the following similar properties: radius $r_f = 2.42 \text{ in}; \text{ mass } m_f = 0.932 \times 10^{-2} \text{ (lb-sec}^2) \text{; mass moment of inertia } I_f = 0.666 \times 10^{-1} \text{ (lb-sec}^2\text{-in}) \text{; initial translational velocity of } 5515 \text{ in/sec and an initial clockwise negative angular velocity of } -1972.0 \text{ (rad/sec). The value of the coefficient of restitution, } e, \text{ is set at } 1.0 \text{ to represent a perfectly-elastic impact reaction, and the coefficient of friction } \mu \text{ is assumed for illustration to equal } 0.5.$

The fragments are located $120^\circ$ apart from each other, and their C.G.'s are at the same radial location 2.797 in. The TPRIM of the first fragment equals $0.760000 \times 10^{-3} \text{ sec}$ and determines the start of the computer calculations. The second fragment is assumed to be released 160 microsec after the first fragment; hence, its TPRIM equals $0.600000 \times 10^{-3} \text{ sec}. \text{ The third and last fragment is assumed to be released 910 microsec after the first fragment; thus, its TPRIM equals } -0.150000 \times 10^{-3} \text{ sec. (Note: the third fragment is released after calculations have begun.) An additional strain point is specified on element 40 near the point of first impact; the } s \text{ coordinate equals } 0.57.$

The CIVM-JET 4B program will solve this collision interaction using a time step of 2 microseconds. Printout starts several microseconds after initial impact and will continue every 20 cycles until 300 cycles have been completed.
6.2.2 Input Data

The values to be punched on the data cards are as follows:

Card 1

\[
\begin{array}{l}
B(1) = 0.250000D+01 \\
DENS(1) = 0.733085D-03 \\
EXANG = 0.360000D+03 \text{ (Complete ring)} \\
\end{array}
\]

Card 2

\[
\begin{array}{l}
IK = 40 \\
NOGA = 3 \\
NFL = 4 \\
NSFL(1) = 3 \\
MM = 690 \\
M1 = 390 \\
M2 = 20 \\
NF = 3 \\
TIMF = 0.138000D-02 \\
\end{array}
\]

Card 3A

\[
\begin{array}{l}
Y(1) = 0.0 \\
Z(1) = 0.770000D+01 \\
ANG(1) = 0.0 \\
N(1) = 0.400000D+00 \\
\end{array}
\]

Additional cards are punched until all 40 nodes of the main structure are described.

\[
\begin{array}{l}
Y(40) = -0.120454D+01 \\
Z(40) = 0.760520D+01 \\
ANG(40) = 0.900000D+01 \\
H(40) = 0.400000D+00 \\
\end{array}
\]

Card 4A

\[
\begin{array}{l}
NDIS = 0 \\
\end{array}
\]

Skip to Card 5

195
Card 5
NBR = 0
Skip to Card 6

Card 6
DELTAT = 0.200000D-05
DS(1) = 0.404000D+02
P(1) = 0.500000D+01
NTOVR = BLANK

Card 7AA
EPS(1,1) = 0.279000D-02
SIG(1,1) = 0.809500D+05
EPS(2,1) = 0.225000D-01
SIG(2,1) = 0.105300D+06
EPS(3,1) = 0.200000D+00
SIG(3,1) = 0.121000D+06

Card 8
NCP = 3
NASP = 1

Card 8A
NSBS(1) = 1
NSEL(1) = 40
AZET(1) = 0.570000D+00

Card 9AA
FH(1) = 0.484000D+01
FCG(1) = 0.139850D+01
FCGX(1) = -0.242227D+01
FMAS(1) = 0.932000D-02
FMOI(1) = 0.666000D-01

Card 9AB
UNK = 0.500000D+00

196
Card 9AC
UDOT(1) = 0.275750D+04
WDOT(1) = 0.477613D+04
ADOT(1) = -0.197200D+04
TPRIM(1) = 0.760000D-03
CR(1) = 0.100000D+01

Repeat the above block of 3 cards until all 3 fragments are described.

Card 9CA
FH(3) = 0.484000D+01
FCG(3) = -0.279700D+01
FCGX(3) = 0.0
FMASS(3) = 0.932000D-02
FMOI(3) = 0.666000D-01

Card 9CB
UNK(3) = 0.500000D+00

Card 9CC
UDOT(3) = -0.551500D+04
WDOT(3) = 0.0
ADOT(3) = -0.197200D+04
TPRIM(3) = -0.150000D-03
CR(3) = 0.100000D+01

Card 10
AXG(1) = 0.1127016653792585D+00
AXG(2) = 0.5000000000000000D+00
AXG(3) = 0.8872983346207415D+00

Card 11
AWG(1) = 0.2777777777777778D+00
AWG(2) = 0.4444444444444444D+00
AWG(3) = 0.2777777777777778D+00
Card 12A  
TXG(1) = -0.8611363115940530D+00  
TXG(2) = -0.3399810435848560D+00  
TXG(3) = 0.3399810435848560D+00
Card 12B  
TXG(4) = 0.8611363115940530D+00
Card 13A  
TWG(1) = 0.3478548451374540D+00  
TWG(2) = 0.6521451548625460D+00  
TWG(3) = 0.6521451548625460D+00
Card 13B  
TWG(4) = 0.3478548451374540D+00
Card 14A  
NBCOND = 0
Skip to Card 15
Card 15  
NQR = 0  
NORP = 0  
NORU = 0
Skip to Card 16
Card 16  
ICONT = 0
Skip to Card 17
Card 17  
ICON = 0
The program will now terminate its run.
6.2.3 Solution Output Data

The following is the output for about 600 microseconds of response after initial impact of the complete ring tri-hub burst impact interaction. Each fragment was released at a different time, and the position of each fragment is tracked separately during the run.

The first segment of output gives a breakdown of the ring initial geometry and the defining quantities of the 3 fragments. A calculation of the maximum time step is made and is used to check the user-generated time step.

Initial impact occurs on element 40 by fragment one, at 763.913 microseconds after the release of the first fragment. The second fragment impacts on element 13 at 158.214 microseconds after the first impact. Fragment 3 was released at time $0.910 \times 10^{-3}$ sec. and has not impacted during this run.

Strain information is printed at each Gaussian station, at each node, and at the designated additional strain point. The maximum strain is 14.58% and occurs on the outer surface of element 13 at 245.087 microseconds after initial impact.

In the interest of conciseness, only a portion of the called-for output is given. Included is all input verification information, scheduled output at the end of time cycles 390, 410, 430, 450, 470, 670, and 690 (last), and regular printout occurring at each ring-fragment impact (note that all impacts are listed). This output listing is intended for use in verification of the adaptation of the CIVM-JET 4B computer code to other computing facilities.
THERE ARE NO BRANCHES CONNECTED TO THE MAIN STRUCTURE, THEREFORE
THE NUMBERING SYSTEM FOR NODES AND ELEMENTS REMAINS UNCHANGED.

ADDITIONAL STRAIN POINT ELEMENT 5 COORDINATE
1 40 0.5700000+00

COMPLETE RING ***CIVM-JET AB** CONTAINMENT ANALYSIS

RING PROPERTIES

MATERIAL PROPERTIES OF MAIN STRUCTURE ARE:

- WIDTH OF RING (IN) = 0.2500000+01
- DENSITY OF RING = 0.7330850-03
- NUMBER OF ELEMENTS = 40
- NUMBER OF SPANWISE GAUSSIAN PTS. = 3
- NUMBER OF DEPTHWISE GAUSSIAN PTS. = 4
- NUMBER OF MECHANICAL SUBLAYERS = 3

DS FOR STRAIN RATE = 0.4040000+02
P FOR STRAIN RATE = 0.5000000+01
STRAIN (1) = 0.2790000-02 STRESS (1) = 0.8095000+05
STRAIN (2) = 0.2250000-01 STRESS (2) = 0.1093000+06
STRAIN (3) = 0.2000000+00 STRESS (3) = 0.1210000+06

INITIAL GEOMETRY AT EACH NODE IS AS FOLLOWS:

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<thead>
<tr>
<th>NODE NO.</th>
<th>Y COORD</th>
<th>Z COORD</th>
<th>SLOPE (RAD.)</th>
<th>RING THICKNESS AT NODE</th>
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<td>0.0</td>
<td>0.4000000+00</td>
</tr>
<tr>
<td>2</td>
<td>0.1204550+01</td>
<td>0.7605200+01</td>
<td>-0.1570800+00</td>
<td>0.4000000+00</td>
</tr>
<tr>
<td>3</td>
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<td>0.7323140+01</td>
<td>-0.3141590+00</td>
<td>0.4000000+00</td>
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<tr>
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</table>
### Fragment Properties

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<tr>
<th>Fragment No.</th>
<th>Dia. of Frac.</th>
<th>Mass of Frac.</th>
<th>Moment of Inertia of Frac.</th>
<th>PEGY</th>
<th>PEGZ</th>
</tr>
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<tbody>
<tr>
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### Collision Parameters

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There is no prescribed displacement condition.

There are no elastic spring constants.

The term for each of 3 fragments is as follows:

0.7800000000 0.6000000000 0.1500000000

### Gaussian Station and Weights

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### Size of Assembled Stiffness Matrix

| 1.652 |

### The Translational Masses for Each Plug: Are

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HIGHEST NATURAL FREQUENCY (RAD/SEC) = 0.3997826494238400-06
THE COMPLETE VALUE OF THE MAX. DELTAT = 0.44970273552070-09
DELTA T SHOULD EQUAL = 0.44093000-05
THE VALUE OF DELTAT USED IN THE PROGRAM IS = 0.20000000-09

THE FOLLOWING NUMBERS ARE THE VALUES FOR THE EFFECTIVE LENGTHS FOR THE MEMBER PLUS 1 SECTIONS OF THE STRUCTURE
0.29788820-06

THE FOLLOWING IS THE FINAL SOLUTION OF THE PRACTICAL RING IMPACT
IMPACT NO. 1 TIME 0.761690-01 DURING CYCLE 382 ELEM 40 PAGE 1 DISTANCE 0.3821600-00

ORIGINAL PAGE IS OF POOR QUALITY
## Energy and Work at the End of Time Cycle 390

### Fragment

**Kinetic Energy**

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<tr>
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<th>ELEM</th>
<th>STA1</th>
<th>STA2</th>
<th>STA3</th>
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**Work Input into Ring**

- 3.9924202

**Rings Kinetic Energy**

- 3.4524602

**Rings Elastic Energy**

- 3.4691302

**Rings Plastic Work**

- 0.4974102

**Energy Stored in Elastic Restraints**

- 0.0

### Cycle 390

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<thead>
<tr>
<th>ELEM</th>
<th>STA1</th>
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### Cycle 390

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**Input Info Ring 3**

- 3.9924202

**Ring Kinetic Energy**

- 3.4524602

**Ring Elastic Energy**

- 3.4691302

**Ring Plastic Work**

- 0.4974102

**Energy Stored in Elastic Restraints**

- 0.0
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<th>COPI</th>
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<th>P</th>
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**ORIGINAL PAGE IS OF POOR QUALITY**
ENERGY AND WORK AT THE END OF TIME CYCLE 410

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**CYCLE 410**
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**ORIGINAL PAGE IS OF POOR QUALITY**
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### RING KINETIC ENERGY

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### ENERGY AND WORK AT THE END OF TIME CYCLE 430

**Fragment Kinetic Energy**

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**Cycle Energy**

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ENERGY AND WORK AT THE END OF TIME CYCLE 450

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- **SUBSTRUCTURE:**
- **TYPE:**
- **SITE:**
- **TIME:**
- **FORCE:**

**Table 2:**

- **SUBSTRUCTURE:**
- **TYPE:**
- **SITE:**
- **TIME:**
- **SURFACE:**

**Figure:**

- **PARAMETER:**
- **QUALITY:**

---

213
**Table 1: Energy and Work at the End of Time Cycle 40**

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**Table 2: Strain at Additional Points**

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**ORIGINAL PAGE IS DE POOR QUALITY**
**ENERGY AND WORK AT THE END OF TIME CYCLE 480**

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**LARGEST ADD. PT. STRAIN**

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**LARGEST COMPUTER STRAINS FOR EACH SUBSTRUCTURE**

**MAIN AND BRANCHES**

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**NOTICE**

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REFERENCES


FIG. 1 ILLUSTRATION OF A VARIABLE-THICKNESS, ARBITRARILY-CURVED BEAM
FIG. 2 EXAMPLE GEOMETRICAL SHAPES OF STRUCTURAL RINGS ANALYZED BY THE CIVM-JET 4B PROGRAM

(a) Variable-Thickness Arbitrarily-Curved Partial Ring

(b) Variable-Thickness Arbitrarily-Curved Complete Ring
(c) Variable-Thickness Partial Circular Ring

(d) Variable-Thickness Complete Circular Ring

FIG. 2 CONCLUDED
\[-15^\circ \leq \phi_{i+1} - \phi_i \leq 0\]
\[-180^\circ < \phi_i \leq 180^\circ\]
\[\phi(n) = b_0 + b_1 n + b_2 n^2\]

\[h = h_i \left(1 - \frac{n}{n_i}\right) + h_{i+1} \frac{n}{n_i}\]

**LOCAL SYSTEM**
- \(\xi, \eta, \zeta\) - COORDINATES
- \(v, w, \psi, \chi\) - DISPLACEMENTS
- \(q_1, q_2, \ldots, q_8\) - ELEMENT GENERALIZED DISPLACEMENTS

**Cartesian Reference**
- \(y, z\) - COORDINATES

---

**FIG. 3** NOMENCLATURE FOR GEOMETRY, COORDINATES, AND DISPLACEMENTS OF A CURVED-BEAM FINITE ELEMENT
(a) Prescribed Displacement Conditions

- IDEALLY-CLAMPED: \( v = w = \psi = 0 \)

- SMOOTHLY-HINGED: \( v = w = 0 \)

(b) Elastic Restraints

FIG. 4 SCHEMATICS FOR THE SUPPORT CONDITIONS OF THE STRUCTURE
Schematic of a Bracket-Supported Fragment Containment/Deflector Structure

FIG. 4 CONTINUED
DENOTES VARIOUS PERMISSIBLE SUPPORT CONDITIONS; FOR EXAMPLE:

MAIN FRAGMENT C/D STRUCTURE

(d) Idealized Two-Dimensional Model of the Configuration Depicted in (c)

FIG. 4 CONCLUDED
FIG. 5 SCHEMATICS OF ACTUAL AND IDEALIZED FRAGMENTS

(a) Single Blade Fragment

(circular
non-deformable

(b) Bladed-Disk Type Fragment

before impact

post-test

actual

idealized
FIG. 6 IDEALIZATION OF RING CONTOUR FOR COLLISION ANALYSIS

(a) Pre-Impact Location

(b) Fragment and Impact-Affected Ring Segment

RING CONTOUR

--- ACTUAL

--- IDEALIZED FOR COLLISION ANALYSIS

FRAGMENT

L_{eff} - EFFECTIVE LENGTH OF IMPACT

j-1 NODE j NODE j+1

LEff

N

T

j+2
FIG. 7 INFORMATION FLOW SCHEMATIC FOR PREDICTING RING AND FRAGMENT MOTIONS IN THE COLLISION-IMPARTED VELOCITY METHOD
(a) Illustrative Fragment C/D Structure with Slope Discontinuities

\[ \text{ANG}(6) = +\alpha \]

DISCONTINUITY ANGLE CONTRIBUTION \( \theta \)

\[ v, w \text{ -- GLOBAL DISPLACEMENT DIRECTIONS AT NODE 6} \]

(b) Exploded View of Node 6 -- Angle Definitions

FIG. 8 DEFINITION OF SLOPE-DISCONTINUITY ANGLES FOR AN ILLUSTRATIVE FRAGMENT AND C/D STRUCTURE
ANG(9) = α
DISCONTINUITY ANGLE
CONTRIBUTION θ
v, w -- GLOBAL DISPLACEMENT
DIRECTIONS AT NODE 8

(c) Exploded View of Node 8 -- Angle Definitions

FIG. 8 CONCLUDED
(a) Geometry and Nomenclature

FIG. 9 PROBLEM DATA, GEOMETRY, NOMENCLATURE, AND FINITE-ELEMENT MODELING FOR EXAMPLE 6.1
NOTE: NODAL NUMBERS USED FOR B.C.'S ARE FOUND ON THIS FIGURE.

(b) User-Generated Numbering System

FIG. 9 CONTINUED
NOTE: FRAGMENTS ARE MATERIALLY SYMMETRIC, THEIR C.G.'S ARE RADially SYMMETRIC, AND THEIR TRANSLATIONAL AND ROTATIONAL VELOCITY MAGNITUDES ARE EQUAL

FIG. 10 PROBLEM DATA, GEOMETRY, NOMENCLATURE, AND FINITE-ELEMENT MODELING FOR EXAMPLE 6.2
APPENDIX A

GOVERNING EQUATIONS ON WHICH THE PROGRAM IS BASED

A.1 Formulation for Variable-Thicknes Arbitrarily-Curved Beam Elements and Structures

The geometry and nomenclature of a typical curved beam discrete element are shown in Fig. 3, where the deformation plane is \( \eta, \zeta \) and the coordinates \( \eta \) along and \( \zeta \) normal to the centroidal axis of the beam are employed as the reference coordinates of the beam element. The slope, \( \phi \), of the reference circumferential axis, which is the angle between the tangent vector and the \( y \)-axis of the local-reference Cartesian coordinates may be approximated by a second degree polynomial in \( \eta \) as follows:

\[
\phi(\eta) = b_0 + b_1 \eta + b_2 \eta^2
\]  

(A.1)

where the constants \( b_0, b_1, \) and \( b_2 \) can be determined from the known initial geometry of the curved beam element. Assume that the change in element slope between nodes \( i \) and \( i+1 \) is small so that

\[
\cos(\phi_{i+1} - \phi_i) \approx 1
\]  

(A.2a)

and

\[
\sin(\phi_{i+1} - \phi_i) = \phi_{i+1} - \phi_i
\]  

(A.2b)

This restricts the slope change within an element to \( \leq 15 \) degrees. The arc length, \( \eta_1 \), of the element is approximated to be the same as the length of a circular arc passing through the nodal points at the slopes \( \phi_i \) and \( \phi_{i+1} \); \( \eta_1 \) is given by

\[
\eta_1 = \frac{L_i (\phi_{i+1} - \phi_i)}{2 \sin\left(\frac{\phi_{i+1} - \phi_i}{2}\right)}
\]  

(A.3)
where $L_i$ is the length of the chord joining nodes $i$ and $i+1$ and is given by

$$L_i = \left[ (z_{i+1} - z_i)^2 + (y_{i+1} - y_i)^2 \right]^{1/2} \quad (A.3a)$$

and $y_i$ and $z_i$ are the initial $y$ and $z$ coordinates, respectively, of the $i$th node. The three constants in Eq. A.1 are then determined from the relations

$$\int_0^1 \sin \phi d\eta = \int_0^1 \phi \sin \theta = 0 \quad (A.4)$$

From Eq. A.4, the constants in Eq. A.1 are found to be

$$b_0 = \phi_i$$
$$b_1 = -2 \left( \phi_{i+1} + 2 \phi_i \right) / \eta_i$$
$$b_2 = 3 \left( \phi_{i+1} + \phi_i \right) / (\eta_i)^2 \quad (A.5)$$

Accordingly, the radius of curvature, $R$, of the centroidal axis may be expressed as $R = - (\partial \phi / \partial \eta)^{-1} = - (b_1 + 2b_2 \eta)^{-1}$, and the coordinates $Y(\eta)$ and $Z(\eta)$ of the centroidal axis are given by

$$Y(\eta) = y_i + \int_0^\eta \cos \left[ \phi(\eta) + \alpha \right] d\eta \quad (A.6a)$$

and

$$Z(\eta) = z_i + \int_0^\eta \sin \left[ \phi(\eta) + \alpha \right] d\eta \quad (A.6b)$$

where

$$\alpha = \tan^{-1} \left( \frac{Z_{i+1} - Z_i}{y_{i+1} - y_i} \right) \quad (A.6c)$$

The thickness variation of the element is approximated as being linear between nodes; thus
Employing the Bernoulli-Euler hypothesis, the displacement field $\tilde{v}$, $\tilde{w}$ of the beam may be specified by the reference plane displacements $v$ and $w$, and the rotation, $\psi$, as follows:

$$\tilde{v}(\eta, \gamma) = v(\eta) - \frac{\gamma}{3} \psi(\eta)$$

$$\tilde{w}(\eta, \gamma) = w(\eta)$$

where

$$\psi(\eta) = \frac{\partial w}{\partial \eta} - \frac{\gamma}{R}$$

To account for the strain-inducing modes and the rigid-body modes, the assumed displacement field takes the form:

$$\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{bmatrix} \begin{bmatrix} \eta \ o \ o \ \eta^2 \ \eta^3 \\ 0 \ \eta^2 \ \eta^3 \ o \ o \end{bmatrix} \begin{bmatrix} \beta_1 \\ \vdots \\ \beta_8 \end{bmatrix}$$

or in more compact matrix form, Eq. A.9 becomes

$$\{u\} = \begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} G_v(\eta) \\ G_w(\eta) \end{bmatrix} \{\beta\} = \begin{bmatrix} U(\eta) \end{bmatrix} \{\beta\}$$

The generalized displacements $\{q\}$ are selected so that there are four degrees of freedom $v$, $w$, $\psi$, $\chi = (\partial v/\partial \eta) + w/R$ at each node of the element:

$$\{q\} = [v, w, \psi, \chi, v_{i+1}, w_{i+1}, \psi_{i+1}, \chi_{i+1}]^T = \begin{bmatrix} \cdot \end{bmatrix} \{\beta\}$$
where

\[
[A] = \begin{bmatrix}
\cos \phi_c & \sin \phi_c & 0 & 0 & 0 & 0 & 0 & 0 \\
-\sin \phi_c & \cos \phi_c & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\cos \phi_{ac} & \sin \phi_{ac} & A_{g3} & \eta_i & 0 & \eta_i^2 & \eta_i^3 & 0 \\
-\sin \phi_{ac} & \cos \phi_{ac} & A_{c3} & 0 & \eta_i^2 & \eta_i^3 & 0 & 0 \\
0 & 0 & 1 & \eta_i (\phi'_{ac}) & 2\eta_i & 3\eta_i^2 & \eta_i^2 (\phi'_{ac}) & \eta_i^3 (\phi'_{ac}) \\
0 & 0 & 0 & 1 & -\eta_i^2 (\phi'_{ac}) & -\eta_i^3 (\phi'_{ac}) & 2\eta_i & 3\eta_i^2
\end{bmatrix}
\]

(A.1Ca)

\[A_{g3} = (Y_{ac1} - Y_c ) \sin (\phi_{ac1} + \alpha) - (Z_{ac1} - Z_c ) \cos (\phi_{ac1} + \alpha)\] (A.10b)

\[A_{c3} = (Y_{ac1} - Y_c ) \cos (\phi_{ac1} + \alpha) + (Z_{ac1} - Z_c ) \sin (\phi_{ac1} + \alpha) - \] 

Corresponding to the assumed displacement field Eq. A.9, one finds

\[
\psi = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{\eta}{R} & 2\eta & 3\eta^2 & -\frac{\eta^2}{R} & -\frac{\eta^3}{R} \end{bmatrix} \{\beta\} = [G_\psi] {\beta} 
\]

(A.11a)

and

\[
\kappa = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\eta^2}{R} & \frac{\eta^3}{R} & 2\eta & 3\eta^2 \end{bmatrix} \{\beta\} = [G_\kappa] \{\beta\} 
\]

(A.11b)
Under the Bernoulli-Euler hypothesis, the only nonvanishing strain component and corresponding stress component are the axial strain, $\varepsilon$, and the axial stress, $\sigma$. For this case, the nonlinear strain-displacement relation may be expressed as:

$$
\varepsilon(\eta, j) = \varepsilon(\eta) + j \mathcal{K}(\eta)
$$

(A.12)

where

$$
\varepsilon(\eta) = \left( \frac{\partial^2 \eta}{\partial \eta^2} + \frac{\partial \eta}{\partial \eta} \right) + \frac{1}{2} \left( \frac{\partial^2 \eta}{\partial \eta^2} \right)^2 + \frac{1}{2} \left( \frac{\partial \eta}{\partial \eta} \right)^2
$$

$$
\equiv \{B_1\} \{u\} + \frac{1}{2} \{B_2\} \{u\} + \frac{1}{2} \{u\} \{B_3\} \{u\}
$$

(A.12a)

$$
\mathcal{K}(\eta) = -\frac{1}{2} \left( \frac{\partial \eta}{\partial \eta} - \frac{\partial \eta}{\partial \eta} \right) \equiv \{B_3\} \{u\}
$$

Combining Eqs. A.9 through A.12, one obtains

$$
\{u\} = \left[ U(\eta) \right] \left[ A^{-1} \right] \{q\}
$$

(A.13)

and

$$
\varepsilon = \{L_{D_1}\} \{q\} + \frac{1}{2} \{L_{D_2}\} \{q\} + \frac{1}{2} \{L_{D_3}\} \{q\}
$$

$$
\mathcal{K} = \{L_{D_3}\} \{q\}
$$

(A.14)

where

$$
\{L_{D_i}\} = \{B_i\} \{U\} \left[ A^{-1} \right] \quad \text{for } i = 1, 2, 3
$$

(A.14a)

and
In the process of solution, it is necessary to evaluate the strain increment $\Delta \varepsilon_m$ from time $t_{m-1}$ to time $t_m$. Using Eqs. A.12 and A.14, one has

$$
\Delta \varepsilon_m = \Delta \varepsilon_m + \int \Delta K_m
$$

(A.15)

where

$$
\Delta \varepsilon_m = L_{D_1} \{ \sigma_{m} \} + L_{Q_m} \{ D_1 \} L_{D_1} \{ \sigma_{m} \} + L_{Q_m} \{ D_2 \} L_{D_1} \{ \sigma_{m} \}
$$

$$
- \frac{1}{2} L_{Q_m} \{ D_1 \} L_{D_1} \{ \sigma_{m} \} - \frac{1}{2} L_{Q_m} \{ D_2 \} L_{D_2} \{ \sigma_{m} \}
$$

(A.15a)

$$
\Delta K_m = L_{D_1} \{ \sigma_{m} \}
$$

In the formulation of the impact analysis scheme, a lumped mass model has been assumed. For consistency a lumped (diagonal) mass matrix must be employed in the global timewise solution (note that the use of lumped mass also results in additional storage and computational efficiencies when compared with the use of a consistent mass matrix which is, in general, fully populated). The lumped mass matrix of the $i$th discrete element is given by the following expression:
where

\[ m_{R_1} = \frac{1}{2} \left( h_s + h_{s1} \right) b \rho \eta_1 (1 - c_1) \]  
(A.16a)

\[ \dot{m}_{R_{s1}} = \frac{1}{2} \left( h_s + h_{s1} \right) b \rho \eta_1 c_1 \]  
(A.16b)

\[ I_{R_1} = c_2 \eta_1^3 b \rho (1 - c_1) \]  
(A.16c)

\[ I_{R_{s1}} = c_2 \eta_1^3 b \rho c_1 \]  
(A.16d)

and where the thickness-dependent constants, \( c_1 \) and \( c_2 \), are given by

\[ c_1 = \frac{2h_{s1} + h_s}{3(h_s + h_{s1})} \]  
(A.16e)

\[ c_2 = \frac{h_s^2 + 4h_s h_{s1} + h_{s1}^2}{36(h_s + h_{s1})} \]  
(A.16f)

In these expressions, \( \rho \) is the mass per unit volume of the beam element, \( b \) is the width of the ring, \( \eta_1 \) is the arc length calculated from Eq. A.3, and Eq. A.7 has been employed to accommodate the variable-thickness properties of the beam element.
The effective stiffness matrix supplied by the elastic restraints may be obtained from the variation of the work done by the elastic restoring spring forces, $δW_s$:

$$-δW_s = \int_0^{\eta_1} (k_v \delta v + k_w \delta w + k_\psi \delta \psi) \, d\eta$$

or

$$-δW_s = \int_0^{\eta_1} \left[ \frac{1}{L} \delta v \delta w \delta \psi \right] [C] \{q\} \, d\eta$$

where

$$[C] = \begin{bmatrix}
  k_v & 0 & 0 \\
  0 & k_w & 0 \\
  0 & 0 & k_\psi
\end{bmatrix}$$

and $k_v$ and $k_w$, respectively, are the linear elastic spring constants and $k_\psi$ is the torsional elastic spring constant.

Substituting the assumed displacement function into Eq. A.17, one has

$$-δW_s = L \delta q \left[ A'' \right]^T \int_0^{\eta_1} [N]^T [C] [N] \, d\eta \left[ A'' \right] \{q\}$$

or

$$L \delta q \left[ k_s \right] \{q\}$$

where

$$[k_s] = \left[ A'' \right]^T \int_0^{\eta_1} [N]^T [C] [N] \, d\eta \left[ A'' \right]$$
In the present analysis, the equations of motion for the complete discretized structure are based on an "unconventional" formulation in which the conventional elastic stiffness matrix, \([K]\), does not appear explicitly. However, in order to calculate an allowable time step size, \(\Delta t\), for the conditionally-stable central-difference timewise operator, the largest natural frequency contained in the (linear) mathematical model of the structure must be determined. To perform this calculation, the elastic stiffness matrix for the assembled structure must be computed. The elastic stiffness matrix for an element is obtained by considering the variation of the work of the axial stress, \(\delta u_1\), expressed in terms of displacements and plastic strains, \(\varepsilon^P\) in the form:

\[
\delta u_1 = \int \sigma \delta \varepsilon \, dv = \int (\varepsilon + \int \kappa - \varepsilon^p)(\delta \varepsilon + \int \delta \kappa) \, dv
\]

Employing the strain-displacement relations given by Eqs. A.12 and A.14, Eq. A.18b becomes

\[
\delta u_1 = \int \delta q \int \left( [k] \{q\} - \{f_{\text{NL}}^q\} - \{f_{\text{NL}}^o\} - \{f_{\text{NL}}^p\} \right)
\]

where \([k]\) is the element elastic stiffness matrix and is given by

\[
[k] = \int \left[ D_1 \right] \left[ \begin{array}{ccc} \varepsilon \partial \kappa(\eta) & \partial \varepsilon \partial \kappa(\eta) & \varepsilon \partial \kappa(\eta) \\ \partial \varepsilon \partial \kappa(\eta) & \varepsilon \partial \kappa(\eta) & \partial \varepsilon \partial \kappa(\eta) \\ \varepsilon \partial \kappa(\eta) & \partial \varepsilon \partial \kappa(\eta) & \varepsilon \partial \kappa(\eta) \end{array} \right] d\eta
\]

The additional terms in Eq. A.18c are equivalent loading vectors corresponding to geometric and material nonlinearities. These terms are employed in the conventional formulation of the equations of motion for the assembled structure, but are not employed in the unconventional formulation used in the CIVM-JET 4B analysis. Again it should be emphasized that the elastic stiffness
matrix is used only in the calculation of the largest natural frequency of the structure, and is not used in any subsequent calculations.

The equivalent nodal force which corresponds to the internal axial stress, $\sigma$, can be obtained from the expression of the variation of the work of the axial stress:

$$
\delta U_i = \int_{V_i} \sigma \delta \varepsilon \, \text{d}V = \int_{V_i} \sigma (\delta \varepsilon + \int \delta K) \, \text{d}V \quad (A.19)
$$

Substituting Eq. A.14 into Eq. A.19 and introducing the stress resultants for the beam cross section

$$
\text{L} = \int_{A_i} \sigma \, \text{d}A, \quad \text{M} = \int_{A_i} \sigma \, \text{d}A \quad (A.20)
$$

where the integrations are taken over the cross section, $A_i$, of the ith beam element, $L$, is the internal force, and $M$ is the internal bending moment of the cross section, results in

$$
\delta U_i = L \delta q \left[ \int _0 ^\eta \left( \{\sigma_1\} L + \{\sigma_2\} M \right) \, \text{d}\eta \right.
$$

$$
+ \int _0 ^\eta \left( \{\sigma_1\} L \delta \eta + \{\sigma_2\} L \delta \eta \right) \, \text{d}\eta \left. \{Q\} \right] \delta q \right]
$$

$$
\equiv \left[ \delta \eta \right] \left( \{P\} + [h] \{q\} \right) \quad (A.21)
$$

where

$$
\{P\} = \int _0 ^\eta \left( \{\sigma_1\} L + \{\sigma_2\} M \right) \, \text{d}\eta
$$

$$
[h] = \left[ \int _0 ^\eta \left( \{\sigma_1\} L \delta \eta + \{\sigma_2\} L \delta \eta \right) \, \text{d}\eta \right] \quad (A.22)
$$

Note that $\{P\}$ and $[h]$ are quantities pertinent to the unconventional formulation of the equations of motion, Eq. 2.1. The integrations along the centroidal
axis length of the beam element which appear in \{p\} and \{h\} of Eq. A.22 are
performed numerically by using the Gaussian quadrature scheme. The axial
force $L$ and moment $M$ at those spanwise stations will be described and evaluated
next.

Because of nonlinear material behavior, although the strain variation
through the beam thickness, by the Bernoulli-Euler hypothesis, is linear, the
variation of stress across the thickness may be nonlinear. For computational
convenience, the stresses are evaluated at selected Gaussian points across
the thickness, and the corresponding weighting factors are used in evaluating
the pertinent integrals by Gaussian quadrature. The strain-hardening behavior
of the material may be accounted for by using the mechanical sublayer model
in which the material at each Gaussian station is treated as consisting of
equally-strained sublayers of elastic, perfectly-plastic material, with each
sublayer having the same elastic modulus but an appropriately different yield
stress. For example, if the yield strain of the $k$th sublayer is $\varepsilon_{yk}$, the
yield stress of that sublayer is

$$
\sigma_{yk} = E \varepsilon_{yk}
\quad (k = 1, 2, \ldots, n)
$$

(A.23)

where $E$ is the elastic (Young's) modulus.

An illustration of the method of computing the axial stress and/or
plastic strain increment is presented as follows. One begins by knowing the
sublayer stress $\sigma_{jk,m-1}$ at time $t_{m-1}$ for the $k$th sublayer of the $j$th depthwise
Gaussian station, and the strain increment $\Delta \varepsilon_{j,m}$ at station $j$ at time $t_{m}$ (that
is the strain increment from time $t_{m-1}$ to time $t_{m}$). One then takes a trial
value (superscript $T$) of $\sigma_{jk,m}$ which is computed by assuming an elastic path:

$$
\sigma_{jk,m}^T = \sigma_{jk,m-1} + E \Delta \varepsilon_{j,m}
\quad (A.24)
$$

A check is then performed to see what the correct value of $\sigma_{jk,m}$ must be.
If $-\sigma_{ok} \leq \sigma_{jk,m} \leq \sigma_{ok}$ then $\sigma_{jk,m} = \sigma_{jk,m}^{\text{T}}$ and $\Delta\varepsilon_{jk,m}^{P} = 0$

If $\sigma_{jk,m}^{\text{T}} > \sigma_{ok}$ then $\sigma_{jk,m} = \sigma_{ok}$ and $\Delta\varepsilon_{jk,m}^{P} = \frac{\sigma_{jk,m}^{\text{T}} - \sigma_{ok}}{E}$

If $\sigma_{jk,m}^{\text{T}} < -\sigma_{ok}$ then $\sigma_{jk,m} = -\sigma_{ok}$ and $\Delta\varepsilon_{jk,m}^{P} = \frac{\sigma_{jk,m}^{\text{T}} + \sigma_{ok}}{E}$

(A.25)

This procedure is applied to all sublayers of each Gaussian station $j_i$. Having done this, the axial force and moment of the beam cross section can be determined by

$$L = \int_{A} \sigma \, dA = b \frac{h}{2} \sum_{j} \left( \sum_{k} \sigma_{jk} A_{jk} \right)$$

$$M = \int_{A} \sigma \, j \, dA = b \frac{h}{2} \sum_{j} j \left( \sum_{k} \sigma_{jk} A_{jk} \right)$$  \hspace{1cm} (A.26)

where $A_{jk}$ is a combination of the mechanical sublayer weighting factor and the Gaussian weighting factor $W_{j}$, which is defined by

$$A_{jk} = \frac{W_{j}}{E} \left( \varepsilon_{k} - \varepsilon_{k+1} \right)$$  \hspace{1cm} (A.27)

In Eq. A.27, $W_{j}$ is the Gaussian weighting factor and

$$E_{k} = \frac{\sigma_{k} - \sigma_{k-1}}{\varepsilon_{k} - \varepsilon_{k+1}}$$  \hspace{1cm} (A.28)

is the $k$th slope of the polygonal approximate stress-strain diagram.

If desired, the sublayer yield stresses may be treated as strain-rate dependent. Since the strain increment at the $j$th Gaussian station and hence
the strain rate is known at this stage of computation, the rate-dependent yield stress \( \sigma_{yk} \) of this kth sublayer at station \( j \) is

\[
\sigma_{yk} = \sigma_{ok} \left(1 + \frac{\Delta \varepsilon}{\Delta t} \right)^p
\]

(A.29)

where \( D \) and \( p \) are empirically-determined constants for the material and may, in general, be different for each sublayer, \( \sigma_{ok} \) is the static uniaxial yield stress of the kth sublayer at any \( j \)th Gaussian station.

Finally, by employing the standard finite-element assembling procedure, the resulting equations of motion for the "complete discretized structure" are (for impact-induced loading only)

\[
[M^*] \{ \ddot{q}^* \} + \{ \varphi^* \} + [H^*] \{ q^* \} + [K_s] \{ q^* \} = 0
\]

(A.30)

where the nomenclature of each term is explained in Subsection 2.4. In the computer program it is convenient to employ Eq. A.30 in the following form:

\[
[M^*] \{ \ddot{q}^* \} = - \{ \varphi^* \} - [H^*] \{ q^* \} - [K_s] \{ q^* \}
\]

(A.30a)

where the terms on the right hand side are treated conveniently as one vector.

A.2 Collision-Interaction Analysis, Including Friction

In the present collision-interaction analysis, the curved variable-thickness containment/deflection ring is represented by straight-line segments (Fig. A.1): (a) to identify in a simple and approximate way the space occupancy of the beam segment under imminent impact attack and (b) to derive the impact equations. The inertia effects of the impacted beam segments are taken into account by means of a lumped-mass collision model; that is, the ring is treated as having only point masses lumped at each nodal station as indicated in Fig. A.2. Other simplifying assumptions which are invoked in the present analysis are described in Subsection 2.2.

For the lumped-mass collision model, the impact-effected beam segments are represented, as depicted in the exploded-line schematic of Fig. A.2, by concentrated masses \( m_1 \), ..., \( m_{j+1} \), ..., \( m_k \) (or \( i+k-1 \)), respectively, where
the ring-fragment collision point is encompassed by the jth segment which is bounded by nodal station j and j+1. A clockwise numbering sequence is used. In the collision analysis, it is convenient to resolve and discuss impulses, velocities, etc., for both the fragment and the ring impact-affected nodes in directions normal (N) and tangential (T) to the straight line segment j; the positive normal direction is always taken from the inside toward the outside of the ring, while the positive-tangential direction is along the straight line from node j toward node j+1 (see Fig. A.2a). Consequently, the lumped-mass velocities for each of the impact-affected nodes and the idealized-fragment velocities are expressed with respect to this local N,T inertial coordinate system as $V_{1N}, V_{1T}$, $V_{2N}, V_{2T}$, ..., $V_{kN}, V_{kT}$, and $V_{fN}, V_{fT}$ in the exploded schematic shown in Fig. A.2a.

It is assumed that the instantaneous collision process results in a normal-direction impulse $\bar{P}_N$ and a tangentially-directed impulse $\bar{P}_T$ applied to the ring, and in equal but anti-parallel impulses to the fragment. The impulses applied to the ring are assumed to be distributed over the impact-affected nodes (see Fig. A.2b) as

$$P_{iN} = C \left(1 - \frac{l_{i+1}}{L_{eff}}\right) \bar{P}_N = C \alpha_i' \bar{P}_N,$$

$$P_{iT} = C \left(1 - \frac{l_{i+1}}{L_{eff}}\right) \bar{P}_T = C \alpha_i' \bar{P}_T,$$

(A.31)

where (see Fig. A.2b) the effective length $L_{eff}$ bounds the impact-affected zone of the ring, which is the fraction of the ring that responds with momentum changes due to the collision of a fragment, $s_i$ is the distance measured from the

---

For present purposes, $L_{eff}$ is taken to be equal to the finite-difference-calculation time interval $\Delta t$ times the longitudinal wave speed $(E/\rho)^{1/2}$ of the structural material; one may employ other estimates of $L_{eff}$ if desired. In the preceding, $E$ is the elastic modulus and $\rho$ is the mass per unit volume.
impacted point to the ith impact-affected node, and the constant, \( C \), is determined by assuming that the sum of the impulse applied to each impact-affected node equals the total impulse imparted; thus,

\[
\sum_{i=1}^{k} \tilde{P}_{i,\text{m}} = \tilde{P}_{\text{m}} \quad \text{(or} \quad \sum_{i=1}^{k} \tilde{P}_{i,\tau} = \tilde{P}_{\tau})
\]

(A.31a)

therefore, one has

\[
C = \frac{1}{\sum_{i=1}^{k} \left(1 - \frac{L_{i,\text{eff}}}{L_{\text{eff}}}\right)} = \frac{1}{\sum_{i=1}^{k} \alpha'_{i}}
\]

(A.31b)

The impacted-affected nodes are those nodes located within the impact affected zone; that is \( |s_i| < L_{\text{eff}} \). However, it is assumed that the two-mass nodes \( j \) and \( j+1 \) of the segment \( j \) which encompasses the impact point always respond to impact with momentum changes, even if the distance from the impact point to one or both of the two nodes \( (j \) and \( j+1) \) is greater than \( L_{\text{eff}} \). Let \( \beta \) and \( \gamma \) be the distances measured from the "point of fragment impact" to masses \( m_j \) and \( m_{j+1} \), respectively, as indicated in Fig. A.2. The distribution of the impulses is estimated in the following manner:

1. If \( \beta > L_{\text{eff}} \) and \( \gamma < L_{\text{eff}} \), then (Fig. A.2c):

\[
P_{i,\text{m}} = P_{j,\text{m}} = C \alpha'_{i} \tilde{p}_{\text{m}}
\]

\[
P_{i,\tau} = P_{j,\tau} = C \alpha'_{i} \tilde{p}_{\tau}
\]

(A.32)

where

\[
\alpha'_{i} = \frac{\alpha_{i}}{\beta}
\]

and

\[
C = \frac{1}{\sum_{i=1}^{k} \alpha'_{i}}
\]

(A.32a)

2. If \( \beta < L_{\text{eff}} \) and \( \gamma > L_{\text{eff}} \), then (Fig. A.2d):

\[
P_{k,\text{m}} = P_{(j+1),\text{m}} = C \alpha'_{k} \tilde{p}_{\text{m}}
\]

\[
P_{k,\tau} = P_{(j+1),\tau} = C \alpha'_{k} \tilde{p}_{\tau}
\]

(A.33)
where

$$\alpha_k' = \frac{\beta}{\gamma} \alpha_k'$$

and

$$C = \frac{1}{\sum \alpha_k'}$$

(A.33a)

(3) If $\beta \geq L_{eff}$ and $\gamma \geq L_{eff}'$, then (Fig. A.2e)

$$P_{n} = P_{j,n} = C \gamma \vec{F}_n = C \alpha_1' \vec{F}_n$$

(A.34a)

$$P_{j,\omega} = P_{(j, \omega)n} = C \beta \vec{F}_n = C \alpha_2' \vec{F}_n$$

$$P_{r} = P_{j,r} = C \alpha_1' \vec{F}_r = C \alpha_1' \vec{F}_r$$

(A.34b)

$$P_{2r} = P_{(j, \omega)r} = C \beta \vec{F}_r = C \alpha_2' \vec{F}_r$$

(A.34c)

It should be emphasized here that the determination of which mass nodes
fall within the impact-affected region is a discrete process in the sense
that only mass node locations are considered instead of considering the true
volume of mass included in the impact-affected region of the structure. Such
an approximation is made necessary by the use of a lumped-mass model in the
collision-interaction analysis. In essence, the use of a lumped-mass model
implies that the nodal mass represents the mass distribution in a region of
the structure surrounding the node, and, thus, by including a mass node in
the impact-affected region, one is automatically including, in the impact-
affected region, that portion of mass in the region of the node. It is clear
that, in general, this approximate technique will result in having more or
less structural mass included in the impact-affected region compared with the
true structural mass within $L_{eff}$. However, in an average sense, this dis-
crepancy is within the overall approximate nature of the impact-interaction.
analysis. In addition, the calculation of \( L_{\text{eff}} \), which defines the distance the collision-impacted impulse "signal" travels in the structure during a finite time interval, is based on the global increment in time, \( \Delta t \). This implies that the collision occurs at the beginning of a time step, and that the "signal" travels for a length of time equal to \( \Delta t \). As will be shown in detail in Subsection A.5, the scheme employed in the present program can determine ring-fragment collision at any time within a given \( \Delta t \). In general, a collision will not occur at the beginning of a time step, so that the signal propagates for a time, \( \Delta t^* \), which is less than \( \Delta t \), and the true \( L_{\text{eff}} \) is then less than the \( L_{\text{eff}} \) based on \( \Delta t \); however, \( L_{\text{eff}} \) is assumed to remain constant and is based on \( \Delta t \). This approximation of \( L_{\text{eff}} \) counterbalances the discrete process of determining which mass nodes fall within the impact-affected region. It is believed that these calculations, although approximate, will yield reasonable results for the fragment-impact-induced structural response.

Denoting by primes the "after-impact" translational and/or rotational velocities, the impulse-momentum law may be written to characterize the "instantaneous impact behavior" of the system, as follows:

**Normal-Direction Translation Impulse-Momentum Law**

\[
\begin{align*}
\mathbf{m}_f \left[ \mathbf{V}_{fW}^\prime - \mathbf{V}_{fW}^* \right] &= -\mathbf{F}_W^* \quad \text{(fragment)} \\
\mathbf{m}_j \left[ \mathbf{V}_{jW}^\prime - \mathbf{V}_{jW}^* \right] &= \alpha_j \mathbf{F}_W^* \\
\mathbf{m}_k \left[ \mathbf{V}_{2W}^\prime - \mathbf{V}_{2W}^* \right] &= \alpha_2 \mathbf{F}_W^* \\
&\vdots \\
\mathbf{m}_k \left[ \mathbf{V}_{kW}^\prime - \mathbf{V}_{kW}^* \right] &= \alpha_k \mathbf{F}_W^* \\
\end{align*}
\]

\( m_k \left[ V_{kW}^\prime - V_{kW}^* \right] = \alpha_k \mathbf{F}_W^* \) (ring impact-affected nodes) (A.36)

**Tangential-Direction Translational Impulse-Momentum Law**

\[
\mathbf{m}_f \left[ \mathbf{V}_{\text{fT}}^\prime - \mathbf{V}_{\text{fT}}^* \right] = -\mathbf{P}_T^* \quad \text{(fragment)}
\]

\( m_f \left[ V_{\text{fT}}^\prime - V_{\text{fT}}^* \right] = -\mathbf{P}_T^* \) (A.37)
\[ m_i \left[ V_{iT}' - V_{iT} \right] = \alpha_i \tilde{P}_T \] (ring impact-affected nodes) (A.38)

\[
\begin{align*}
& m_1 \left[ V_{1T}' - V_{1T} \right] = \alpha_1 \tilde{P}_T \\
& \vdots \\
& m_k \left[ V_{kT}' - V_{kT} \right] = \alpha_k \tilde{P}_T 
\end{align*}
\]

Rotational Impulse-Momentum Law

\[ I_f \left[ \omega_f' - \omega_f \right] = r_f \tilde{P}_T \] (fragment) (A.39)

where

- \( m_f \) = mass of the fragment
- \( I_f \) = mass moment of inertia of the fragment about its CG
- \( r_f \) = the radius of the circular disk model of the fragment
- \( \tilde{P}_N \) = normal-direction impulse
- \( \tilde{P}_T \) = tangential-direction impulse
- \( \alpha_f \) = proportional constant which is equal to \((Co_f')\) as defined by Eqs. A.31 through A.34.

The relative velocity of sliding \( S' \) and the relative velocity of approach \( A' \) at the immediate "contact points" between the fragment (at \( C_f \)) and the ring segment \( j \) (at \( C_j \)) are defined by

\[ S' = \left[ V_{iT}' - \omega_f' r_f \right] - \left[ \alpha_i V_{iT}' + \alpha_2 V_{2T}' + \cdots + \alpha_k V_{kT}' \right] \] (A.40)

\[ A' = V_{iN}' - \left[ \alpha_i V_{iN}' + \alpha_2 V_{2N}' + \cdots + \alpha_k V_{kN}' \right] \] (A.41)

Substituting Eqs. A.35 through A.39 into Eqs. A.40 and A.41, one obtains

\[ S' = S_o - B_i \tilde{P}_T \] (A.42)

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where the initial (pre-impact) relative velocity of sliding $S_0$, the initial relative velocity of approach $A_0$, and the geometrical constants $B_1$ and $B_2$ are given by

$$S_0 = \left[ V_{yT} - \omega_f V_f \right] - \left[ \alpha_1 V_{1T} + \alpha_2 V_{2T} + \cdots + \alpha_k V_{kT} \right]$$  \hfill (A.44)

$$A_0 = V_{fN} - \left[ \alpha_1 V_{IN} + \alpha_2 V_{2N} + \cdots + \alpha_k V_{kN} \right]$$  \hfill (A.45)

$$B_1 = \frac{1}{m_f} + \frac{r^2}{I_f} + \frac{\alpha^2_1}{m_1} + \frac{\alpha^2_2}{m_2} + \cdots + \frac{\alpha^2_k}{m_k}$$  \hfill (A.46)

$$B_2 = \frac{1}{m_f} + \frac{\alpha_1^2}{m_1} + \frac{\alpha_2^2}{m_2} + \cdots + \frac{\alpha_k^2}{m_k}$$  \hfill (A.47)

where in Eqs. A.44 and A.45 by definition $A_0 \geq 0$; otherwise, the two bodies will not collide with each other. Also, if $S_0 \geq 0$, the fragment slides initially along the ring segment. It perhaps should be noted that sliding of the bodies on each other is assumed to occur at the value of "limiting friction" which requires that $P_T = |\mu N|$, and when $P_T < |\mu N|$, only rolling (i.e., no sliding) exists. For a given value of $e$ and a given value of $\mu$ which, respectively, describes the degree of "plasticity" of the collision process, and accounts for the frictional properties (roughness) of the contact surfaces, \(2(k+1)+3\) equations (Eqs. A.35 - A.39 and Eqs. A.42 - A.43) can be solved to obtain the post-impact quantities \((V^-_{IN}, V^-_{IT}), (V^-_{2N}, V^-_{2T}), \ldots, (V^-_{kN}, V^-_{kT}), (V^-_{fN}, V^-_{fT})\) and \(\omega^-\) as well as $\bar{P}_N$ and $\bar{P}_T$; these are the $2(k+1)+3$ "unknowns".

The graphic technique which provides a convenient way to obtain the values of $\bar{P}_N$ and $\bar{P}_T$ at the instant of the termination of impact as described in Ref. 4 is employed in the present collision-interaction analysis. In this

$$A' = A_0 - B_2 \tilde{p}_f$$  \hfill (A.43)
technique, the trajectory of an "image" point \( \overline{P} \) in the plane formed by the impulse coordinates \( \overline{P}_N \) and \( \overline{P}_T \) (Fig. A.3) represents the state of the colliding bodies at each instant of the contact interval. The image point \( \overline{P} \) which is initially located at the origin and is denoted by \( \overline{P}_O \) (\( \overline{P}_N = 0, \overline{P}_T = 0 \)) will always proceed in the upper half-plane with increasing \( \overline{P}_N \). The locations of the line of no sliding \( S' = 0 \) and the line of maximum approach \( A' = 0 \) are determined by the system constants \( B_1 \) and \( B_2 \). From Eqs. A.42 through A.47, it is noted that \( B_1 \) and \( B_2 \) are positive always; also the lines \( S' = 0 \) and \( A' = 0 \) are parallel to the \( \overline{P}_N \) axis and the \( \overline{P}_T \) axis, respectively, and intersect with each other at point \( P_3 \) in the first quadrant of the \( \overline{P}_N, \overline{P}_T \) plane as shown in Fig. A.3. Depending on the values of the coefficient of sliding friction \( \mu \), the coefficient of restitution \( e \), the system constants \( B_1 \) and \( B_2 \), and the initial conditions \( S_0 \) and \( A_0 \), several variations of the impact process may occur and will be discussed in the following.

First, the cases in which the coefficient of sliding friction \( \mu \) ranges from \( 0 < \mu < \infty \) will be considered; the two special cases with \( \mu = 0 \) (perfectly-smooth contact surfaces) and \( \mu = \infty \) (completely rough surfaces) will be discussed shortly thereafter.

Case I: If \( 0 < \mu < \infty \), the friction angle \( \psi \) and the angle \( \Lambda \) formed with the \( \overline{P}_N \) axis by the line connecting \( P_0 \) and \( P_3 \) are defined by

\[
\psi = \tan^{-1} \mu
\]

and

\[
\Lambda = \tan^{-1} \left( \frac{e S_0}{e A_0} \right)
\]  

(A.49)

Initially, the image point \( \overline{P} \) travels from point \( P_0 \) along the path \( P_0 L \) which subtends an angle \( \psi \) with the \( \overline{P}_N \) axis because the limiting friction impulse \( P_T = \mu P_N \) is developed during the initial stage of impact. Subsequently:

(a) if \( \mu = \tan \psi < \tan \Lambda \) (Fig. A.3a), line \( P_0 L \) will intersect the line of maximum approach \( A' = 0 \) at point \( P_1 \), before reaching the line of no sliding \( S' = 0 \). The intersection
point $P_1$ represents the state at the instant of the termination of the approach period. This is followed by the restitution period; the impact process ceases at point $P'$ (path $P_0 - P_1 - P'$). The coordinates of $P'$ are

$$
\vec{P}_N = (1 + e) P_{N1}
$$

(A.50)

$$
\vec{P}_T = \mu \vec{P}_N = \mu (1 + e) P_{N1}
$$

(A.51)

where $P_{N1}$, the ordinate of point $P_1$, is determined from the simultaneous solution of equations $P_T = \mu P_N$ and $A' = 0$, and is given by

$$
P_{N1} = \frac{A_2}{B_2}
$$

(A.52)

(b) However, if $\mu = \tan \nu > \tan \Lambda$ (Fig. A.3b), line $P_0L$ will intersect the line of no sliding $S' = 0$ first at the intersection point $P_2$, which marks the end of the initial sliding phase. The image point $\vec{P}$ then will continue to proceed along the line of no sliding $S' = 0$ through the intersection point $P_3$ with line $A' = 0$ to the end of impact at point $P'$ (path $P_0 - P_2 - P_3 - P'$). The final values of $\vec{P}_N$ and $\vec{P}_T$ are:

$$
\vec{P}_N = (1 + e) P_{N3}
$$

(A.53)

$$
\vec{P}_T = \frac{S_0}{B_1}
$$

(A.54)

where $P_{N3}$ is the ordinate of point $P_3$, which represents the end of the approach period and is given by
\[ P_{N3} = \frac{A_o}{B_z} \]  

(A.55)

The above solution process can be specialized to represent the cases with \( \mu = 0 \) and \( \mu = \infty \).

**Case II:** If \( \mu = 0 \) (perfectly smooth contact surfaces), line \( P_L \) coalesces with the \( \tilde{p}_N \) axis. The image point \( \tilde{P} \) will move along the \( \tilde{p}_N \) axis to the end of impact. Thus

\[ \tilde{p}_N = (1 + e) \frac{A_o}{B_z} \]  

(A.56)

\[ \tilde{p}_T = 0 \]  

(A.57)

**Case III:** If \( \mu = \infty \) (completely rough contact surface), point \( \tilde{P} \) moves initially along the \( \tilde{p}_T \) axis to the intersection with \( S' = 0 \), then will follow the line \( S' = 0 \) to the end of impact. The post-impact value of \( \tilde{p}_N \) and \( \tilde{p}_T \) are

\[ \tilde{p}_N = (1 + e) \frac{A_o}{B_z} \]  

(A.58)

\[ \tilde{p}_T = \frac{S_o}{B_1} \]  

(A.59)

Knowing the values of \( \tilde{p}_N \) and \( \tilde{p}_T \) at the end of impact for the above discussed various impact processes, the corresponding post-impact velocities then can be determined from Eqs. A.35 through A.39 as follows:

\[
\begin{align*}
\mathbf{V}_{iN}' &= \mathbf{V}_{iN} + \frac{\alpha_i \tilde{p}_N}{m_i} \\
\mathbf{V}_{iT}' &= \mathbf{V}_{iT} + \frac{\alpha_i \tilde{p}_T}{m_i}
\end{align*}
\]

(A.60)
Thus, this approximate analysis provides the post-impact velocity information for the impact-affected nodes of the ring and for the fragment so that the timewise step-by-step solution of this ring/fragment response problem may proceed. Note that these post-impact velocity components are given in directions \( N \) and \( T \) at each node of the idealized impact-affected ring segments; as explained later, these velocity components are then transformed to (different) global directions appropriate for the curved-ring dynamic response analysis.
A.3 Prediction of Containment/Deflector Ring Motion and Position

The timewise solution of the resulting equations of motion for the "complete assembled discretized structure", Eq. A.30, may be accomplished by employing an appropriate timewise finite-difference scheme. The 3-point central-difference operator is chosen for use in the present analysis. In this solution scheme, the relations between displacements and displacement increments at any instant of time are

\[
\{\Delta q^*_m\} = \{q^*_m\} - \{q^*\}_{m-1}
\]

and

\[
\{q^*_m\} = \{q^*_0\} + \{\Delta q^*_1\} + \cdots + \{\Delta q^*_n\}
\]

At time \(t_m\), the acceleration and velocity may be expressed in terms of displacement increments by the following central-difference finite-difference expression:

\[
\{\ddot{q}^*_m\} = \frac{\{q^*_m\} - 2\{q^*_m\} + \{q^*_m\}}{(\Delta t)^2} = \frac{\{\Delta q^*_m\} - \{\Delta q^*_m\}}{(\Delta t)^2} + O(\Delta t)^2 \quad \text{(A.66a)}
\]

\[
\{\dot{q}^*_m\} = \frac{\{q^*_m\} - \{q^*_m\}}{2(\Delta t)} = \frac{\{\Delta q^*_m\} + \{\Delta q^*_m\}}{2(\Delta t)} + O(\Delta t)^2 \quad \text{(A.66b)}
\]

Employing Eq. A.30, the unconventional form of the dynamic equations of motion at any time instant \(t_m\) becomes

\[
[M^*]\{\dot{q}^*_m\} = -[\kappa^*]\{q^*_m\} - \{\rho^*_m\} - [H^*]\{\dot{q}^*_m\}
\]

Since the right-hand side of Eq. A.67 is known, one can solve for \(\{q^*_m\}\). Because the assembled mass matrix, \([M^*]\), is a diagonal matrix, the "inversion" of \([M^*]\) required for the solution of Eq. A.67 is accomplished simply by taking the inverse of each diagonal term in \([M^*]\). In practice, only the diagonal entries in \([M^*]\) are retained in the computer storage. With \(\{q^*_m\}\) now known, one can calculate \(\{\Delta q^*_m\}\) from Eq. A.66a as

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Thus, from Eq. A.64 one has

\[ \{ \Delta q^* \}_{m+1} = \{ q^* \}_m + (\Delta t)^2 \{ \ddot{q}^* \}_m \]  

(A.68)

The calculations of \( \{ \Delta q^* \}_{m+1} \) and \( \{ q^* \}_{m+1} \) have been made assuming that no ring-fragment collisions have occurred between time instants \( t_m \) and \( t_{m+1} \). However, a ring-fragment collision may occur between time instants \( t_m \) and \( t_{m+1} \); this would require a "correction" to the \( \{ \Delta q^* \}_{m+1} \) found from Eq. A.68. Thus, one first uses Eq. A.68 to form a trial value (overscript \( T \)) for \( \{ \Delta q^* \}_{m+1} \), and, hence, a trial value for \( \{ q^* \}_{m+1} \):

\[ \{ \bar{q}^* \}_{m+1} = \{ q^* \}_m + \{ \Delta q^* \}_{m+1} \]  

(A.69)

Next, the collision inspection and correction procedures, which will be described in Subsection A.6, are performed to determine the actual displacement increments \( \{ \Delta q^* \}_{m+1} \). Then the actual displacement at time \( t_{m+1} \) is given by Eq. A.69.

After the calculation of \( \{ \Delta q^* \}_{m+1} \) and \( \{ q^* \}_{m+1} \), the strain increment at any point in the element can be obtained. With the strain increment available, the stress increment and stress is computed from the stress-strain relation. Then the stress resultants are obtained. Equations A.67, A.68, and A.69 furnish the displacement increment and displacement for the next time step. The process is cyclic thereafter.

It should be noted that no "commencing sequence" is required in the present analysis for the central-difference timewise operator. This is because there are no prescribed initial velocity distributions or prescribed externally-applied forces in the present analysis; only impact-induced structural motion is taken into account. Thus, the ring structure is assumed to be at rest (thus \( \{ \Delta q^* \}_m = \{ q^* \}_m = 0 \) at all time instants prior to initial fragment-ring collision. When initial (and subsequent) fragment-ring collision
occurs, a nonzero value for $(\Delta q^*)_{m+1}$ results, which is determined from the impact-interaction scheme, and no additional starting sequence is required.

A.4 Prediction of Fragment Motion and Position

In the present analysis, the fragment is assumed (see Ref. 3) to be nondeformable and, for analysis convenience to be circular; hence, its equations of motion for the case of no externally-applied forces are:

\[ m_f \ddot{Y}_f = 0 \]  
\[ m_f \ddot{Z}_f = 0 \]  
\[ I_f \ddot{\theta} = 0 \]  

where \((Y_f, Z_f)\) and \((\dot{Y}_f, \dot{Z}_f)\) denote, respectively, the global coordinates and acceleration components of the center of gravity of the fragment (see Fig. A.2). \(\theta\) represents the angular displacement of the fragment in the \(+\omega_f\) direction (Fig. A.2).

In timewise finite-difference form, Eqs. A.71 through A.73 become

\[ (\Delta Y_f)_{m+1} = (\Delta Y_f)_m \]  
\[ (\Delta Z_f)_{m+1} = (\Delta Z_f)_m \]  
\[ (\Delta \theta)_{m+1} = (\Delta \theta)_m \]  

where overscript "T" signifies a trial value which requires modification, as explained later, if ring-fragment collision occurs between \(t_m\) and \(t_{m+1}\).
A.5 Calculation of Ring-Fragment Time of Contact

In this subsection, an improved method* for determining the time of ring-fragment contact will be developed. It is assumed, based on the ring node and fragment c.g. locations at time \( t_m \), that no overlapping of ring-fragment geometries occurs at time instant \( t_m \), based on the (trial) ring node and fragment c.g locations at time instant \( t_m \), that overlapping of ring-fragment geometries does occur at time \( t_{m+1} \). Thus during the finite increment in time, \( \Delta t = t_{m+1} - t_m \), ring-fragment contact must have occurred. The problem, then, is to determine at what time, between \( t_m \) and \( t_{m+1} \), ring-fragment contact occurs, and where on the C/D structure it occurs.

In the present development, only a single fragment and single element need be considered; a similar calculation can be carried out for each element separately and for each fragment being considered. Consider the uniform-thickness straight beam element shown in Fig. A.4. As noted in Subsection A.2, the (generally) curved element is approximated, for the impact analysis, as a straight beam, and, in addition, the variable-thickness element will be assumed to be of uniform thickness (equal to the average thickness, \( h \), of the element) for the impact analysis. The global \( Y,Z \) coordinate system is taken as the reference system for this analysis. Define vectors \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \) (see Fig. A.4) such that \( \mathbf{P}_1 \) is the vector from node 1 to node 2 and \( \mathbf{P}_2 \) is the vector from node 1 to the fragment c.g. The vectors \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \) define the relative position of node 2 and the fragment c.g., respectively, with respect to node 1 at time instant \( t_m \), where it has been assumed that no ring-fragment overlapping occurs at time \( t_m \).

The perpendicular distance, \( d \), from the fragment c.g. to the vector \( \mathbf{P}_1 \) (midsurface of the element) at time \( t_m \) can be obtained by using the vector cross-product:

\[
\mathbf{P}_1 \times \mathbf{P}_1 = (p_1 p_2 \sin \theta) t_{nj}
\]

(A.77)

where \( p_1 \) is the magnitude of the vector \( \mathbf{P}_1 \) and \( \theta \) is the angle from \( \mathbf{P}_1 \) to \( \mathbf{P}_2 \). Denoting the nodal coordinates by \( Y_i \) and \( Z_i \) (\( i = 1,2 \)) and the fragment c.g. coordinates by \( Y_f \) and \( Z_f \), the vectors \( \mathbf{P}_1 \) and \( \mathbf{P}_2 \) can be expressed as

*Improved compared with that used in Refs. 1 and 3.
and the distance, \( d \), at time \( t_m \) is calculated by

\[
d = P \cdot \sin \theta = \frac{|\overrightarrow{P} \times \overrightarrow{P}|}{P} = \frac{[\left((y_f - y_i)(z_i - z_f) - (y_i - y_f)(z_f - z_i)\right)]}{\left[(z_i - y_i)^2 + (z_i - z_f)^2\right]^{1/2}}
\]  

(A.79)

In order to calculate the time of contact, the perpendicular distance, \( d \), from the fragment c.g. to the element midsurface must be known as a function of time. The element nodal and fragment c.g. velocities and nodal accelerations are known at time \( t_m \), and the accelerations are assumed to be constant over the finite increment in time, \( \Delta t = t_{m+1} - t_m \). Denote the velocities and accelerations in the \( Y \) and \( Z \) directions at node \( t \) by \( v_{yi}, v_{zi}, a_{yi}, \) and \( a_{zi} \), respectively, and the fragment c.g. velocities in the \( Y \) and \( Z \) directions by \( v_{yf}, v_{zf} \), respectively. The position of node 1 (for example) as a function of time, \( y_1(t) \), can then be expressed by the following Taylor series expression:

\[
y_1(t) = y_1 + v_{y1} t + \frac{1}{2} a_{y1} t^2
\]  

(A.80)

where the time reference has been shifted in such a way that \( t=0 \) corresponds to time \( t_m \). Expressions similar to Eq. A.80 can be obtained for the quantities \( y_2(t), z_1(t), z_2(t), y_f(t), \) and \( z_f(t) \) where it is noted that the acceleration of the fragment is zero. When these expressions are substituted into Eq. A.79, an expression for the distance, \( d(t) \), from the fragment c.g. to the element reference surface, as a function of time is obtained in the form

\[
d(t) = \frac{A t^4 + B t^3 + Ct^2 + Dt + E}{[F t^4 + G t^3 + H t^2 + I t + J]^{1/2}}
\]  

(A.81)

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where

\[
A = \frac{1}{2} \left[ (a_{y_1} - a_{y_i}) a_{x_i} - (a_{x_2} + a_{x_i}) a_{y_i} \right] \tag{A.82a}
\]

\[
B = \frac{1}{2} \left[ (V_{y_1} - V_{y_i}) (a_{x_2} - a_{x_i}) + (V_{z_2} - V_{z_i}) a_{y_i} + (V_{z_2} - V_{z_i}) (a_{y_2} - a_{y_i}) + (V_{y_2} - V_{y_i}) a_{z_i} \right] \tag{A.82b}
\]

\[
C = \frac{1}{2} \left[ (Y_{y_1} - Y_{y_i}) (a_{x_2} - a_{x_i}) + 2 (V_{y_1} - V_{y_i}) (V_{x_2} - V_{x_i}) + (z_2 - z_i) a_{y_i} + (z_2 - z_i) (a_{y_2} - a_{y_i}) + 2 (V_{z_2} - V_{z_i}) (V_{y_2} - V_{y_i}) + (Y_{y_2} - Y_{y_i}) a_{z_i} \right] \tag{A.82c}
\]

\[
D = (Y_{y_1} - Y_{y_i}) (V_{z_2} - V_{z_i}) + (V_{y_1} - V_{y_i}) (z_2 - z_i) + (z_2 - z_i) (V_{y_2} - V_{y_i}) + (V_{z_2} - V_{z_i}) (Y_{y_2} - Y_{y_i}) \tag{A.82d}
\]

\[
E = (Z_{y_1} - Z_{y_i}) (Y_{y_1} - Y_{y_i}) - (Y_{z_2} - Y_{z_i}) (Z_{y_2} - Z_{y_i}) \tag{A.82e}
\]

\[
F = \frac{1}{4} \left[ (a_{x_2} - a_{x_i})^2 + (a_{y_2} - a_{y_i})^2 \right] \tag{A.82f}
\]

\[
G = (a_{y_2} - a_{y_i}) (V_{y_2} - V_{y_i}) + (a_{z_2} - a_{z_i}) (V_{x_2} - V_{x_i}) \tag{A.82g}
\]

\[
H = (V_{y_2} - V_{y_i})^2 + (V_{x_2} - V_{x_i})^2 + (a_{y_2} - a_{y_i}) (Y_{y_2} - Y_{y_i}) + (a_{z_2} - a_{z_i}) (Z_{y_2} - Z_{y_i}) \tag{A.82h}
\]

\[
I = 2 \left[ (Y_{y_1} - Y_{y_i}) (V_{y_2} - V_{y_i}) + (Z_{y_2} - Z_{y_i}) (V_{x_2} - V_{x_i}) \right] \tag{A.82i}
\]

\[
J = (Y_{y_2} - Y_{y_i})^2 + (Z_{y_2} - Z_{y_i})^2 \tag{A.82j}
\]

It should be noted here that the coefficients in Eq. A.81 are dependent only on the element nodal and fragment c.g. locations, velocities, and accelerations at time instant \( t \). The time of contact is defined as that time at which \( d(t) \) is equal to the critical distance, \( d_c \), where the critical distance is the sum of the radius of the fragment, \( r_z \), and half the average element thickness, \( \bar{h} \):
Thus, the calculation of the time of contact, $t_c$, reduces to the solution of the following equation:

$$d_c = \frac{A t^4 + B t^3 + C t^2 + D t + E}{[F t^4 + G t^3 + H t^2 + I t + J]^\frac{1}{2}}$$  \hspace{1cm} (A.84)

or

$$A t^4 + B t^3 + C t^2 + D t + E = d_c [F t^4 + G t^3 + H t^2 + I t + J]^\frac{1}{2}$$  \hspace{1cm} (A.85)

Several approaches for the solution of Eq. A.85 may be envisioned. However, before discussing the alternate approaches, it is important to note that the only solution of Eq. A.85 which is of interest in the calculation of time of contact must be non-negative and less than or equal to $\Delta t$ since contact must occur between time $t_m$ and time $t_{m+1}$ (recall that $\Delta t = t_{m+1} - t_m$). Since $\Delta t$, typically, of order $10^{-5}$ or $10^{-6}$, the solution method chosen must be able to solve Eq. A.85 accurately for small values of $t$.

Because no closed-form solution of Eq. A.85 is available, the use of a numerical iterative (approximate) solution scheme such as the Newton Raphson procedure would be necessary. However, such schemes often suffer from poor convergence behavior unless an accurate "initial guess" is made. For the general impact problem, the use of such numerical iterative schemes has thus been judged to be too unreliable and more direct methods have been sought for the solution of Eq. A.85, as discussed next.

The right-hand-side of Eq. A.85 (term in brackets) is expanded in a Taylor series about $t=0$ as

$$[F t^4 + G t^3 + H t^2 + I t + J]^\frac{1}{2} = J^{\frac{1}{2}} + \left(\frac{1}{2} J^{-\frac{1}{2}}\right) t + \left(\frac{H}{2 J^{\frac{3}{2}}} - \frac{I^2}{8 J^{\frac{3}{2}}}\right) t^2$$

$$+ \left(\frac{1}{12} J^{-\frac{3}{2}} - \frac{1}{4} \frac{I H}{J^{\frac{3}{2}}} + \frac{G}{2 J^{\frac{3}{2}}}\right) t^3 + \left(\frac{1}{120} J^{-\frac{5}{2}} - \frac{5}{12} \frac{I^2 H}{J^{\frac{5}{2}}} + \frac{I^4}{24 J^{\frac{5}{2}}} - \frac{H^2 + 6 I G}{8 J^{\frac{5}{2}}}\right) t^4 + O(t^5)$$  \hspace{1cm} (A.86)
If the terms of order $t^5$ are neglected and Eq. A.86 is substituted into Eq. A.85, the following quartic equation results:

$$
\left[ A - d_c \left( \frac{2}{J} \frac{I^2}{J^{1/2}} - \frac{5}{I} \frac{I^4}{J^{3/2}} - \frac{H^2 + 6IC}{8J^{1/2}} + \frac{F}{2J^{1/2}} \right) \right] t^4 
+ \left[ B - d_c \left( \frac{I^3}{J^{3/2}} - \frac{I^4}{J^{1/2}} + \frac{G}{2J^{1/2}} \right) \right] t^3 
+ \left[ C - d_c \left( \frac{H}{2J^{1/2}} \right) \right] t^2 
+ \left[ D - d_c \left( \frac{1}{2J^{1/2}} \right) \right] t 
+ \left[ E - d_c J^{1/2} \right] = 0 \tag{A.87}
$$

The advantage of using Eq. A.87 in place of Eq. A.85 is that closed-form solutions to quartic equations are available (see, for example, Ref. 5); thus, a computer subroutine which obtains the real roots (imaginary roots are of no interest in the present time-of-contact solution) of a general quartic equation has been included in the present CIVM-JET 4B program. The coefficients in Eq. A.87 are determined from known displacement, velocity, and acceleration information using Eqs. A.82, and the calculation of the time of contact is thus reduced to the solution of Eq. A.87.

Numerical experimentation with the solution of general quartic equations suggests that the roots of larger magnitude are predicted more accurately. In particular, if, for example, the exact solution of a given quartic equation has one real root of order $10^{-6}$ and another real root of order 1, then the order 1 root will be predicted accurately, but large errors will be found in the prediction of the order $10^{-6}$ root. In the present impact analysis, the roots of interest are of the order of $\Delta t$ (typically $10^{-6}$). To avoid errors in the prediction of these small roots, a change of variables, namely $t = 1/x$ is made in Eq. A.87, and the resulting quartic equation in $x$ is solved. In this way, the $x$ roots of interest are large (corresponding to small $t$ roots) and accuracy of these roots is assured. It should be noted that if the constant term in Eq. A.87 is zero, then $t = 0$ is a valid root and the solution of the full quartic is not required.

Finally, it is important to clarify under what conditions the use of Eq. A.87 instead of Eq. A.85 is valid. The only approximation employed in the development of Eq. A.87 is that the right-hand-side of Eq. A.85 can be approximated by a Taylor Series expansion, retaining only those terms up to order $t^4$. 

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The coefficients in Eq. A.86 are related only to information at the nodes of the element and, in fact, Eq. A.86 is an expression for the change in the length of the element as a function of time. The use of Eq. A.87 in place of Eq. A.85 is deemed valid if for the values of \( t \) of interest (i.e. \( 0 \leq t \leq \Delta t \)), the Taylor series of Eq. A.86 can be shown to behave as

\[
\left[ F t^4 + G t^3 + H t^2 + I t + J \right]^{1/2} = J \left[ 1 + O(\bar{t}^2) + O(\bar{t}^4) + O(\bar{t}^6) \cdots \right] \tag{A.88}
\]

where \( J^{1/2} \) is the element length at time zero. It can be shown, after some manipulation, that the behavior of the Taylor series is (at worst) that given by Eq. A.88 if the relative displacement in the time increment, \( \Delta t \), of node 2 with respect to node 1 in a direction parallel or perpendicular to the element midsurface does not exceed 10% of the element length at the beginning of the time increment (i.e. at time \( t_m \)). This condition should be satisfied for all conceivable engineering applications of the current ring-fragment impact analysis and, thus, the use of Eq. A.87 to calculate the time of ring-fragment contact (impact) is justified.

In summary, when ring-fragment impact is determined to occur between times \( t_m \) and \( t_{m+1} \), Eq. A.87 (along with Eqs. A.82) is employed to calculate the time of ring-fragment contact, \( t_c \), within that time interval. In practice (as will be explained in detail in the next subsection), this calculation is performed for each element in order, considering each of the \( n \) attacking fragments one by one. The only roots of Eq. A.87 which are considered valid are those real roots which satisfy

\[
0 \leq t \leq \Delta t = t_{m+1} - t_m \tag{A.89}
\]

When a valid value is found for \( t_c \), the point of ring-fragment contact, \( p_c \), can be calculated by the following vector dot product:

\[
p_c = \frac{\vec{p}_1 \cdot \vec{p}_2}{|\vec{p}_i|} \tag{A.90}
\]

where the vectors \( \vec{p}_1 \) and \( \vec{p}_2 \) are evaluated at time \( t = t_c \). The quantity \( p_c \) is the distance from node 1 to the point of contact divided by the element length (at time \( t_c \)). The point of contact, as defined by Eq. A.90 must be between 0 and 1 for contact to have occurred on the element length, i.e.
If Eq. A.91 is not satisfied, contact has not occurred on the element length. Thus, both Eqs. A.89 and A.91 must be satisfied for element-fragment contact to be valid. Note that the method developed in this subsection will, in general, determine contact between a fragment (assumed to be circular) and an infinitely-long straight "element" passing through nodes 1 and 2 of the actual structural element under consideration, and thus, the condition given in Eq. A.91 must be imposed.

A.6 Collision Inspection and Solution Procedure

A.6.1 One-Fragment Attack

The collision inspection and solution procedure will be described first for the case in which only one idealized fragment is present. With minor modification this procedure can also be applied for an n-fragment attack as discussed in Subsection A.6.2.

At various stages in the impact inspection and solution procedure, the updating of ring node (or fragment) positions and/or velocities is required. In the interest of conciseness, the form of these updating equations is presented now, and reference to these equations will be made. The location (denoted by an over-bar) of the ring nodes \( \{ \bar{q}^* \} \), at some time \( t' \) is given by

\[
\{ \bar{q}^* \}_{t'} = \{ \bar{q}^* \}_t + \{ \delta q^* \}_{t'}
\]  

(A.92)

where \( \{ \bar{q}^* \}_t \) is the initial (i.e. \( t=0 \)) location of the ring nodes and \( \{ \delta q^* \}_{t'} \) is the total displacement of the ring nodes up to time, \( t=t' \). The location of the ring nodes, \( \{ \bar{q}^* \}_{t'+\Delta t} \), at a time, \( t=t'+\Delta t' \) (within a time increment \( \Delta t \)), in terms of the location \( \{ \bar{q}^* \}_t \), velocity \( \{ \dot{q}^* \}_t \), and acceleration \( \{ \ddot{q}^* \}_t \) of the ring nodes at time, \( t=t' \), is given by the following Taylor Series expansion:

\[
\{ \bar{q}^* \}_{t'+\Delta t'} = \{ \bar{q}^* \}_{t'} + (\Delta t') \{ \dot{q}^* \}_t + \frac{1}{2} (\Delta t')^2 \{ \ddot{q}^* \}_t
\]  

(A.93)
It should be noted that Eq. A.93 can be derived from the central difference expressions, and is thus consistent with the central difference time-wise operator employed in the present CIVM-JET 4B program. Finally, the velocity at some time, \( t = t' + \Delta t' \), is given by the expression

\[
\begin{align*}
\{ \dot{\mathbf{v}} \}_{t' + \Delta t'} &= \{ \dot{\mathbf{v}} \}_{t'} + (\Delta \mathbf{t}') \{ \ddot{\mathbf{v}} \}_{t'} \\
\end{align*}
\]  

(A.94)

Equations A.92 through A.94 have been written for the ring nodes; the updating equations for the fragment are of the same form with the acceleration of the fragment taken to be zero. It should be noted that the acceleration of the ring nodes is assumed to be constant within a time increment and is equal to the ring-node acceleration at the beginning of the time step being considered.

The following procedure indicated in the flow diagram of Fig. 7 may be employed to predict the motions of the ring and rigid fragment, their possible collision, the resulting collision-imparted velocities experienced by each, and the subsequent motion of each body:

**Step 1:** Let it be assumed at time \( t_m \) that the displacements \( \{ \mathbf{q}_m \} \), \( \{ \mathbf{v}_m \} \), and \( \{ \mathbf{z}_m \} \) and displacement increments \( \{ \Delta \mathbf{q}_m \} \), \( \{ \Delta \mathbf{v}_m \} \), and \( \{ \Delta \mathbf{z}_m \} \) are known. One can then calculate the strain increments \( \{ \Delta \xi_m \} \) at all Gaussian stations along and through the thickness of the ring.

**Step 2:** Using a suitable constitutive relation for the ring material, the stress increments \( \{ \Delta \sigma \}_m \) and the plastic strain increments \( \{ \Delta \xi^p \}_m \) at corresponding Gaussian stations within each finite element can be determined from the known strain increments \( \{ \Delta \xi \}_m \). This information permits determining all quantities on the right-hand side of Eq. A.67.

**Step 3:** Solve Eq. A.67 for the nodal accelerations, \( \{ \ddot{\mathbf{q}}_m \} \), then solve for the trial displacement increments, \( \{ \Delta \mathbf{q}^t \}_{m+1} \), by using Eq. A.69, the trial ring displacements, \( \{ \mathbf{q}^t \}_{m+1} \), by using Eq. A.70, and use Eqs. A.74 through A.76 for the trial fragment displacement increments \( \{ \Delta \mathbf{q}^f \}_{m+1} \), \( \{ \Delta \mathbf{v}^f \}_{m+1} \), and \( \{ \mathbf{z}^f \}_{m+1} \). In addition, the ring node velocities \( \{ \dot{\mathbf{q}} \}_m \) at time \( t_m \) are calculated by using Eq. A.66.
for those nodes not impact-corrected during the previous time cycle, and using the impact-corrected velocity updated to the end of the previous time cycle for those nodes subject to impact corrections during the previous time cycle. It is assumed that the fragment velocities, \( \dot{V}_m, \dot{z}_m, \) and \( \dot{r}_m \) at time \( t_m \) are known.

Since one or more ring-fragment collisions may have occurred between \( t_m \) and \( t_{m+1} \), the following sequence of steps may be employed to determine whether or not a collision occurred and, if so, to effect a correction of the displacement increments of the impact affected ring segments and of the fragment.

In the present scheme, several collisions may occur during a given global time step \( \Delta t = (t_{m+1} - t_m) \). Thus, the global \( \Delta t \) will be subdivided into subincrements in time which will be denoted by \( \Delta t^* \), where \( \Delta t^* \) is the time remaining in the global \( \Delta t \) and is given by

\[
\Delta t^* = t_{m+1} - t_m
\]

where \( t_m \) is the reference "beginning" time for the current collision inspection cycle. Thus, for the first inspection for a given \( \Delta t \), \( t_m \) must be initialized to \( t_m \), and \( \Delta t^* \) must be initialized to \( \Delta t \). In subsequent inspections (if any) within this \( \Delta t \), the value of \( t_m \) will be updated to the time of ring-fragment contact, and Eq. A.95 will be used to calculate \( \Delta t^* \). Because the impact inspection is most conveniently carried out in the global Y,Z coordinate system, one first transforms the nodal displacement, velocity, and acceleration vectors at time \( t_m \), \( \{q_m^*\}, \{\dot{q}_m^*\}, \{\ddot{q}_m^*\} \), into the global Y,Z coordinate system (note that the fragment information is already in the global Y,Z system). Then the ring node and fragment locations, \( \{\bar{q}_m^*\}, \{\bar{V}_m^*\} \), etc. at time \( t_m \) are calculated by using Eq. A.92 and the trial ring node and fragment locations at
time $t_{m+1}$, $\{q^*_m\}_{m+1}^T$, $\{Y_m\}_{m+1}^T$, etc. are calculated by using Eq. A.93 (where $t^* = t_m$, and $\Delta t^* = \Delta t$). Having completed these initializations, the following sequence of substeps may be employed to determine whether or not a collision occurs within the subincrement $\Delta t^*$ (=\(\Delta t\) on this first inspection).

**Step 4a:** To check for the possibility of a collision between the fragment and ring element $j$ (approximated as a straight beam) as depicted in Fig. A.5, compute the trial projection $\{P_{j,m+1}\}^{\top}$ of the line from ring node $j$ to point $C_f$ at the center of the fragment, upon the straight line connecting ring nodes $j$ and $j+1$, as follows, at time instant $t_{m+1}$:

$$
(P_{j})_{m+1} = [T_{Y_j} - Y_f]_{m+1}, \cos(\delta_j)_{m+1}
$$

$$
+ [T_{Z_j} - Z_f]_{m+1}, \sin(\delta_j)_{m+1}
$$

(A.96)

where the $Y, Z$ are inertial Cartesian coordinates obtained from $\{q^*_m\}_{m+1}^T$, $\{Y_m\}_{m+1}^T$ etc. Now, examine $\{P_{j,m+1}\}^{\top}$; three cases are illustrated in Fig. A.5a.

**Step 4b:** If $\{P_{j,m+1}\}^{\top} < 0$ or if $\{P_{j,m+1}\}^{\top} > l_j$ where $l_j > 0$, a collision between the fragment and ring element $j$ is impossible. Proceed to check ring element $j+1$, etc., for the possibility of a collision of the fragment with other ring elements. Note that $l_j$ is the length of the $j$th element at time $t_{m+1}$.

**Step 4c:** If $0 \leq \{P_{j,m+1}\}^{\top} \leq l_j$, a collision with ring element $j$ is possible, and further checking is pursued. Next, calculate the fictitious "penetration distance" $\{\alpha_j\}_{m+1}^{\top}$ of the fragment into ring element $j$ at point $C_f$ by (see Fig. A.5b):

$$
(\alpha_j)_{m+1} = \left[\frac{1}{4}(h_{jj} + h_{zz}) + r_e\right]_{m+1} - \left[c_j\right]_{m+1}
$$

(A.97)

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Distance from the reference surface to the inner surface of the ring element which is approximated as a straight beam in this "collision calculation".

\[ r_f = \text{radius of the fragment}. \]

\[ \left( \tilde{d}_j \right)_{m+1} = \left[ \tilde{r}_j - \tilde{r}_f \right]_{m+1} \sin (\tilde{\delta}_j)_{m+1} \]

\[ + \left[ \tilde{r}_j - \tilde{r}_f \right]_{m+1} \cos (\tilde{\delta}_j)_{m+1} \]  

(A.98)

\[ = \text{the projection of the line connecting node } j \text{ with the center of the fragment upon a line perpendicular to the line joining nodes } j \text{ and } j+1. \]

Next, examine \( (\tilde{a}_j)_{m+1} \) which is indicated schematically in Fig. A.5b and is given by Eq. A.97.

**Step 4d:** If \( (\tilde{a}_j)_{m+1} \leq 0 \), no collision of the fragment upon element \( j \) has occurred during the time interval from \( t^* \) to \( t^*_{m+1} \). Hence, one can proceed to check element \( j+1 \), etc. for the possibility of a collision of the fragment with other ring elements.

**Step 4e:** If \( (\tilde{a}_j)_{m+1} > 0 \), a collision has occurred. Steps 4a through 4d are repeated for each element; if no positive values of \( (\tilde{a}_j)_{m+1} \) have been found, no ring-fragment collisions have occurred; then proceed to Step 9. If \( \text{any} \) positive values of \( (\tilde{a}_j)_{m+1} \) have been found, ring-fragment collision has occurred; proceed to the next step.

**Step 5:** Since ring-fragment collision has been determined to have occurred between times \( t^*_m \) and \( t^*_{m+1} \), the following sequence of substeps may be employed to determine the time and location of ring-fragment contact:
Step 5a: Given the locations and velocities of the fragment and the nodes of element \( j \) at time \( t^* \), form the coefficients A-G given by Eqs. A.82 and solve for the roots of Eq. A.87. Choose the smallest, positive, real root. If this root satisfies Eq. A.89 (where \( \Delta t \) in Eq. A.89 has been replaced by \( \Delta t^* \) here)

\[
0 \leq t \leq \Delta t^*
\]  \hspace{1cm} (A.99)

then this root is the time of contact \( t^* \) for element \( j \), and proceed to the next step. If this root does not satisfy Eq. A.89, set \( t^* \) equal to a large negative number and proceed to the next element.

Step 5b: Special consideration must be given to the case where \( t^* \) = 0. Because the present scheme allows for several subincrements, \( \Delta t^* \), in time within the "global" increment in time, \( \Delta t \), for the purpose of collision inspection and correction, the value of \( t^* = 0 \) is allowable only if this element has not been impacted at some prior time during the current global increment in time, \( \Delta t \). Thus, a "flagging" array is set up at the start of each \( \Delta t \) to determine whether or not a particular \( t^* = 0 \) is allowable. If \( t^* = 0 \) is not allowable, set \( t^* \) equal to a large negative number and proceed to the next element. Otherwise, proceed to the next step.

Step 5c: The point of contact, \( p^* \), on element \( j \) is now calculated by using Eq. A.90. This value is then inspected to determine whether or not contact occurs within the actual boundaries of element \( j \). If \( p^* \) satisfies Eq. A.91 (repeated here for convenience)

\[
0 \leq (p^*)_j \leq 1
\]  \hspace{1cm} (A.100)

then contact has occurred on element \( j \), and one proceeds to the next element. If Eq. A.100 is not satisfied, contact has not occurred on element \( j \): set \( t^* \) (for the \( j \)th element) equal to a large negative number and proceed to the next element.

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Step 5d: Steps 5a through 5c are repeated for each element until all elements on the main structure have been considered. In practice, several allowable values of $t_c$ can be found, corresponding to different elements, in one subincrement in time, $\Delta t^*$, the desired value being the minimum ($t_c^*$) value of all allowable values. Thus, a quantity, $(t_c^*)_{\text{min}}$, which is the minimum of all calculated (allowable) values of $(t_c)^*_{j}$, is initialized to $\Delta t^*$ just prior to Step 5a. Following Step 5c, the calculated value of $(t_c^*)_{j}$ is compared with the current value of $(t_c^*)_{\text{min}}$. If the following condition is satisfied

$$0 \leq (t_c^*)_{j} \leq (t_c^*)_{\text{min}}$$  \hspace{1cm} (A.101)

then the value of $(t_c^*)_{\text{min}}$ is redefined to be the value of $(t_c^*)_{j}$. When all elements have been processed, the quantity $(t_c^*)_{\text{min}}$ will contain the actual minimum value of all values of $(t_c^*)_{j}$. The element number and point of contact associated with this value of $(t_c^*)_{\text{min}}$ are also identified. Because of the form of Eq. A.101, if equal values of $(t_c^*)_{j}$ are calculated for two or more elements, the higher element number will be associated with $(t_c^*)_{\text{min}}$ (elements are processed in ascending numerical order). Following the determination of $(t_c^*)_{\text{min}}$, a "flag" is set for the element corresponding to $(t_c^*)_{\text{min}}$ indicating that a value of $(t_c^*)=0$, for this element, is not allowed during the remainder of the current global increment in time, $\Delta t$.

Step 6: Having determined the time of ring-fragment contact, the ring-node positions, $(\vec{q}_m^*)$, and the fragment position, $(\vec{Y}_m^*)$, etc., are updated to the time of contact by using Eq. A.93, and the ring-node velocities, $(\vec{v}_m^*)$, are updated to the time of contact by using Eq. A.94. For both calculations, $t'=t_m^*$, and $\Delta t' = (t_c^*)_{\text{min}}$. Again, it should be recalled that $(t_c^*)_{\text{min}}$ is the time of contact, referenced to time $t_m^*$. The reference beginning time, $t_m^*$, is now updated to the time of contact by
and the time subincrement, $\Delta t^*$, remaining in the global increment in time, $\Delta t$, is updated by using Eq. A.95. The quantities $\{\tilde{q}^*_m\}$, $\{\tilde{c}^*_m\}$, $\{\tilde{Y}_f^*_m\}$, etc. are no longer needed, so their values are replaced by the appropriate updated values. Thus, the quantities $\{\tilde{q}^*_m\}$, $\{\tilde{c}^*_m\}$, $\{\tilde{Y}_f^*_m\}$, $\{\tilde{Z}_f^*_m\}$, and $\{\tilde{\theta}_f^*_m\}$ now refer to the ring node locations and velocities and fragment locations in the global $Y,Z$ coordinate system, at time $t^*_m$ which is the time of ring-fragment contact (see Eq. A.102).

Step 7:

Based on the collision-interaction analysis developed in Subsection A.2, the post-impact velocities of the impact-affected ring nodes and the fragment are now calculated. That is, the pre-impact nodal velocities $\{\tilde{q}^*_m\}$ at time $t = t^*_m$ and fragment velocities $\{\tilde{Y}_f^*_m\}$, $\{\tilde{Z}_f^*_m\}$, $\{\tilde{\theta}_f^*_m\}$ at time $t = t^*_m$ are updated to their post-impact values using Eqs. A.60 through A.63.

(Note that Eqs. A.60-A.63 are written in terms of an $N,T$ coordinate system, as defined in Subsection A.2. Thus, the nodal and fragment velocities, assumed to be in the global $Y,Z$ coordinate system prior to the collision-interaction analysis, must be transformed into the $N,T$ system at the start of the collision-interaction analysis, and the resulting post-impact velocities, calculated in the $N,T$ system via Eqs. A.60-A.63, must then be transformed back to the global $Y,Z$ system after completion of the collision-interaction analysis).

For convenience, the post-impact velocity information, in the global $Y,Z$ coordinate system, is assumed to "replace" the pre-impact velocity information. Thus, the quantities $\{\tilde{q}^*_m\}$, $\{\tilde{Y}_f^*_m\}$, $\{\tilde{Z}_f^*_m\}$, and $\{\tilde{\theta}_f^*_m\}$ now refer to the post-impact velocity of the ring-nodes and fragment at the time of contact, $t^*_m$. 

\[ t^*_m = t^*_m + (t_c)_{\min} \quad \text{(A.102)} \]
A decision must now be made concerning whether or not to continue on to another collision inspection. The collision inspection/correction process is repeated: (1) if the value of $\Delta t^*$ is positive and (2) if the number of collision inspection/corrections within the current $\Delta t$ has not exceeded a specified maximum (equal to 50 per fragment in the present analysis). If either of these conditions is violated, no further inspection is performed, but if both conditions are satisfied, further collision-inspection is carried out. In either case, the next step is followed.

Before proceeding to the next collision inspection (or proceeding to Step 9, if no further inspections are to be made), the ring-node and fragment (trial) positions at time $t_{m+1}$, must be updated using their positions, post-impact velocities, and accelerations (ring nodes only) at time $t_m^*$, but using Eq. A.93 with $t'=t_m^*$, and $\Delta t'=\Delta t^*$. If further collision inspection is to be done, Steps 4a through 8a are then repeated to determine whether or not a ring-fragment collision occurs during the subincrement of time, $\Delta t^*$, from time $t_m^*$ to time $t_{m+1}^*$, and, if so, to effect a correction of the impact-affected ring nodes and fragment velocities. If no further collision inspections are to be carried out (because of the conditions stated in Step 2) the next step (Step 9) should be followed. At this point, the reason for the special consideration given to the case $(t_c^j) = 0$ in Step 5b can be clearly seen. The ring node and fragment positions have already been updated to the time of contact, $t_m^*$, via Step 6. On the next pass through steps 5a-5d (calculation of the time of contact during the subinterval $\Delta t$), the ring and fragment are, in fact, in contact (recall that a contact time of zero corresponds to time $t_m^*$) and a value of $(t_c^j) = 0$ will be obtained from Eq. A.87 for

---

In the present impact inspection scheme, corrections must be made for all ring-fragment collisions to avoid spurious results in subsequent inspections. The specified maximum of 50 has been included only to guard against user input errors. In practice (assuming correct user input information) this limit should never be exceeded.
that element impacted during the previous $\Delta t^*$. Thus, the special consideration and flagging procedure described in Step 5b must be employed so that multiple corrections for the same ring-fragment collision can be avoided.

**Step 9:** This step will be executed when no (further) ring-fragment collisions are found up to time $t_{m+1}$. The corrected ring-node and fragment displacements in the global $Y,Z$ coordinate system at time $t_{m+1}$ are now calculated by solving Eq. A.92 for $\{q^*\}_{t^*}$ where $t^*=t_{m+1}$. The velocity at those nodes affected by one or more impacts is then updated to time $t_{m+1}$ using Eq. A.94 where $t^*=t_{m}^*, \Delta t^*=\Delta t^*$ and assuming that $\{\dot{q}^*\}_{t_{m}^*} = \{\dot{q}^*\}_{t_{m}}$. It should be noted that the velocity at time $t_{m+1}$ for those nodes not affected by impact is calculated using the central-difference expression as discussed in Step 3. The corrected nodal displacement and velocity vectors, currently in the global $Y,Z$ coordinate system, are now transformed back into the appropriate ring coordinate system. Following this transformation, the corrected ring node and fragment displacement increments ($\{\dot{q}^*\}_{m+1}$, $\{d_{l}\}_{m+1}$, etc.) are calculated by subtracting the displacements at time $t_{m}$ (i.e. $\{q^*\}_{m}$, $\{Y_{l}\}_{m}$, etc.) from the corrected displacements at time $t_{m+1}$.

**Step 10:** Having determined the corrected displacement increments and displacements for the ring elements and fragment, this time cycle of calculation is now complete. One then proceeds to calculate the ring nodal coordinate increments and the fragment coordinates for the time step from $t_{m+1}$ to $t_{m+2}$, starting with Step 1. The process proceeds cyclically thereafter for as many time increments as desired.

*It should be noted that in this approximate calculation, only the coordinate increments of the fragment and of the impact affected ring segments are corrected. Those for all other ring segments are regarded as already being correct. The time increment $\Delta t$ is regarded as being sufficiently small to make these approximations acceptable.*
This solution procedure may be carried out for as many time steps as desired or may be terminated by invoking the use of a termination criterion such as, for example, the reaching of a critical value of the strain at the inner surface or the outer surface of the ring. Appropriate modifications of this approximate analysis could be made, if desired, to follow the behavior of the ring and the fragment after the initiation and/or completion of local fracturing of the ring has occurred; however, this has not been done in the present program.

Finally, note that it is possible for the fragment to come in contact with two ring elements simultaneously. In this situation, a correction would be made for the higher-numbered element first as noted in Step 5d. The higher-numbered element will then be "flagged" as being impacted and on the next subincrement in time, $\Delta t^*$, a value of $(t_{c})_{j}=0$ will be found for the lower-numbered element and a correction will be made. A similar situation arises when multiple fragments impact the ring simultaneously, as will be discussed in the next subsection.

A.6.2 N-Fragment Attack

In the case of "attack" by n idealized fragments, each with its individual $m_{f}^i$, $I_{f}^i$, $r_{f}^i$, $\omega_{f}^i$, $V_{fN}^i$, and $V_{fT}^i$, a similar procedure is used. During each $\Delta t^*$, the collision-inspection procedure is carried out for every fragment; none, some, or all of these n fragments may have collided with one or more of the ring segments. If any positive penetration distances are computed, the calculation of ring-fragment contact time will follow for each element and each of the n fragments in turn. This calculation sequence will identify the first ring-fragment contact within $\Delta t^*$, and the fragment number and element number involved in the collision. The appropriate corrections, as a result of this collision, will be made, and the process will be repeated for the next $\Delta t^*$. During the next $\Delta t^*$, the same fragment or a different fragment may collide with the ring structure; the appropriate corrections will then be made for this collision. This process is repeated until either (1) more than 50 ring-fragment collisions occur for a given fragment, or (2) the value of $\Delta t^*$ is zero, which occurs at $t=t_{n+1}$, or (3) no (more) ring-fragment collisions are found within the global time step, $\Delta t$. After all of the corrections have
been carried out for the present $\Delta t$ time interval, the calculation process of Fig. 7 proceeds similarly for the next $\Delta t$.

Note that it is possible for two or more fragments to impact the ring structure simultaneously. This plausible situation is accommodated in the present scheme. Because of the "flagging" scheme discussed in Step 5b of the previous subsection, the collision involving the higher fragment number will be corrected for first (and will be flagged). On the next $\Delta t^*$ (sub) step, the next highest fragment number involved in the simultaneous impact will yield a value of $(t_{c_{\min}} = 0$ and a correction will be made corresponding to this ring-fragment collision, and so on, until corrections have been made for all fragments involved in the simultaneous impact. In essence, the ring structure and fragment positions remain unaltered while a series of corrections is made (with $(t_{c_{\min}} = 0$), corresponding to all of the fragments which impact simultaneously.

Finally, it should be noted that no provisions have been made for collisions (or interactions) between the fragments themselves. Thus, all collisions (and subsequent interactions) are assumed to be between a fragment and the ring structure.

### A.7 Ring-Fragment Collision on or Near a Constrained Node

The impact-interaction analysis presented in Subsection A.2 is based on the assumption that all nodes within the impact-affected region are free to respond with velocity changes as a result of ring-fragment collision. If any of the nodes within the impact-affected region are constrained, then the analysis of Subsection A.2 must be modified slightly. These modifications, and their subsequent application to the present analysis, are described in the present subsection.

For the present analysis, assume that one of the nodes within the impact-affected region is constrained such that no normal or tangential motion is permitted. Denote this node number by the subscript "c". At node c, the constraint will contribute a reaction force (or reaction impulse) so that the translational impulse-momentum relations (Eqs. A.36 and A.38) at node c must now be written as

\[ ... \]
where the additional terms $p_R^N$ and $p_R^T$ are the reaction impulses at node $c$ in the normal and tangential directions, respectively. The pre-impact velocities, $V_{CN}$ and $V_{CT}$, must be zero and because of the constraint, the post-impact velocities must also be zero, thus Eqs. A.103 state that the restraint "absorbs" all of the impulse associated with the constrained node.

The analysis developed in Subsection A.2 can be followed exactly if the value of $\alpha$ for the constrained node is set equal to zero, i.e.

$$\alpha_c = 0$$

This is equivalent to introducing equations of the form of Eq. A.103 and immediately solving for the reaction impulse, which yields a total value of zero on the right-hand side of Eq. A.103. In practice, the use of Eq. A.104 allows one to treat the special case of impact on or near a constrained node within the framework and equations developed in Subsection A.2.

It should be noted that the quantity $\alpha^*$ for the constrained node is not set equal to zero. This quantity defines the relative portion of the total imparted impulse which is associated with a given node which lies within the impact-affected region, and is calculated by using Eq. A.31 whether or not the node is constrained. In general, the constrained node may fall anywhere within the impact-affected region. Because of the character of the present impact interaction analysis in which only translational (not rotational) motion of the ring is considered (both translational and rotational motion are included in the global timewise solution), it is difficult to include the effects of impulse propagated past the constrained node. For the case where the node is ideally clamped, no information can propagate through the constraint. But if the node is pinned-fixed, rotational information could propagate past the constraint; to accommodate this situation, rotational effects would have to be included in the analysis of Subsection A.2. An alternate, interim measure is taken in the present analysis, and is described next.
Assume that the point of contact and the effective length, $L_{eff}$, are such that the constrained node and nodes beyond the constrained node fall within the impact-affected region. Because the analysis of Subsection A.2 cannot predict the propagation of impact information past the constrained node, the effective length, $L_{eff}$, is, in the present scheme, artificially reduced (for the current $\Delta t$ only) in such a way that the constrained node falls within the impact-affected region but no nodes past the constrained node fall in the impact-affected region. Having redefined $L_{eff}$ in this fashion, the equations of Subsection A.2 are then followed exactly with Eq. A.104 being employed at the constrained node. This approach has the effect of concentrating the impact-induced impulse at those ring nodes on the impacted side of the constraint, with a portion of the impact-induced impulse being absorbed by the constraint, and no impulse being felt at nodes beyond the constrained node. However, it should be recognized that, although no impulse information is passed through the constrained node by the impact interaction analysis, the impact information will propagate through the constrained node, if physically possible, in the global time-wise structural response solution.

For the case where impact occurs directly on a constrained node, only that constrained node is assumed to lie within the impact-affected region. Following the equations in Subsection A.2 and employing Eq. A.104, the fragment will simply rebound (as if impacting a rigid wall) and the ring structure will experience no momentum changes for this impact.

Finally, it should be noted that the present approach is an interim measure, and further effort is required to develop a more comprehensive approach for treating impact near a constrained node. However, the present method is believed to be sufficiently general, within the current overall assumptions of the analysis, to yield reasonable results for current engineering applications.
RING CONTOUR

--- IDEALIZED FOR COLLISION ANALYSIS

--- ACTUAL

FIG. A.1 IDEALIZATION OF RING CONTOUR FOR COLLISION ANALYSIS
Fig. A.2 Exploded Schematics of the Lumped-Mass Collision Models
(c) $\beta > L_{\text{eff}}$ and $\gamma < L_{\text{eff}}$

$P_{1N} = \frac{\gamma}{\beta} P_{2N}$

$P_{1T} = \frac{\gamma}{\beta} P_{2T}$

$P_{kN} = \frac{\beta}{\gamma} P_{(k-1)N}$

$P_{kT} = \frac{\beta}{\gamma} P_{(k-1)T}$

(d) $\beta < L_{\text{eff}}$ and $\gamma \geq L_{\text{eff}}$

$P_{1N} = \frac{\gamma}{\beta + \gamma} \tilde{p}_N$

$P_{1T} = \frac{\gamma}{\beta + \gamma} \tilde{p}_T$

$P_{2N} = \frac{\beta}{\beta + \gamma} \tilde{p}_N$

$P_{2T} = \frac{\beta}{\beta + \gamma} \tilde{p}_T$

(e) $\beta \geq L_{\text{eff}}$ and $\gamma \geq L_{\text{eff}}$

FIG. A.2 CONCLUDED
FIG. A.3 THE TRAJECTORY OF THE IMAGE POINT $\tilde{P}$ IN THE $\tilde{P}_N$, $\tilde{P}_T$ PLANE TO DESCRIBE THE STATE AT EACH CONTACT INSTANT
FIG. A.4 IDEALIZATIONS AND DEFINITIONS FOR CALCULATION OF TIME OF RING-FRAGMENT CONTACT
FIG. A.5 INSPECTION FOR DETERMINING A COLLISION OF THE FRAGMENT WITH THE RING

(a) Projection Inspection

ILLUSTRATIVE LOCATIONS AT TIME INSTANT $t_{m+1}$ FOR $P_j$.
SEE EQ. A.100
IDEALIZED UNIFORM-THICKNESS ELEMENT

(b) Penetration Inspection

FIG. A.5 CONCLUDED