A TECHNIQUE USING A NONLINEAR
HELICOPTER MODEL FOR DETERMINING
TRIMS AND DERIVATIVES

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SUMMARY

This paper describes a technique for determining the trims and quasi-static derivatives of a flight vehicle for use in a linear perturbation model; both the coupled and uncoupled forms of the linear perturbation model are included. Since this technique requires a nonlinear vehicle model, detailed equations with constants and nonlinear functions for the CH-47B tandem rotor helicopter are presented. Tables of trims and derivatives are included for airspeeds between -40 and 160 knots and rates of descent between ±10.16 m/sec (±2000 ft/min). As a verification, the calculated and referenced values of comparable trims, derivatives, and linear model poles are shown to have acceptable agreement.

INTRODUCTION

Avionics research for helicopters is in progress at the Langley Research Center as part of the VTOL approach and landing technology (VALT) program (ref. 1). An NASA/Army/Boeing Vertol CH-47B helicopter will be used as a tool to evaluate advanced research concepts relating to navigation, guidance, control, and displays. In order to assist this effort, a mathematical model of the CH-47B is required. Although a nonlinear model is available from the TAGS program (ref. 2), the complications become immense when the development of a feedback controller for the vehicle is considered a prime goal. Furthermore, the computational time during simulation can become significant, especially if the model contains a large number of nonlinear functions.

An alternate approach is to represent the vehicle by a linear perturbation model consisting of trims and stability and control derivatives. Since data for the complete CH-47 flight regime are unavailable in the literature, a method to calculate the trims and derivatives is needed. This report, therefore, describes a general procedure, using a nonlinear model, that automatically determines the trim conditions and the stability and control derivatives for any vehicle. The data generated in this paper are specifically for the CH-47B and are compared with existing data (ref. 3) where available.

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This report is divided into three main sections. The first section describes the approach for calculating the trims and the stability and control derivatives. An iterative solution to the six steady-state equations of motion is used to balance the forces and moments in a body-axis reference frame. Six trim variables are calculated for various flight regimes; these are the pitch and roll attitudes relative to a level-heading frame and the four control stick positions. Quasi-static derivatives (ref. 4) are determined by calculating the changes in the forces and moments due to a small perturbation in each of the state and control variables. The second section describes the twin-engine, tandem-rotor helicopter model used to calculate the forces and moments. This model is essentially the large maneuver model used in reference 2, with modifications to give a better representation of the CH-47B. The equations are divided into major modules representing specific areas or major functions. A brief description of each module is presented in this section and the actual equations are given in an appendix. Trims and derivatives obtained by the methods described in this paper are discussed in the third section and are compared with data from reference 3; this section includes plots of selected derivatives and trims for zero rate of climb and a velocity range of -40 to 160 knots. The data are presented generally in both SI Units and U.S. Customary Units. Calculations were made in U.S. Customary Units. Five appendixes include a discussion of the sign convention and major coordinate frames used (appendix A), equations for the helicopter nonlinear mathematical model (appendix B), constants for the CH-47B (appendix C), a linear perturbation model (appendix D), and a set of tables (appendix E) containing the trims and derivatives for velocities ranging from -40 to 160 knots and rates of descent over a range of ±10.16 m/sec (±2000 ft/min).

SYMBOLS

[A] stability derivative matrix

[B] control derivative matrix

[C] gradient matrix of $\bar{Y}$ with respect to $\delta_T$, $\theta$, and $\phi$

$\bar{C}_j$ jth column vector of [C]

$C_T$ rotor thrust coefficient

d$_{FR}$ interference factor of front rotor on rear rotor (see eqs. (B38) and (B41))

d$_{RF}$ interference factor of rear rotor on front rotor (see eqs. (B39) and (B40))
\( \vec{F}_T, \vec{M}_T \) vector of total force on vehicle, N, and vector of total moments on vehicle, N-m (see fig. D1)

\( F_X, F_Y, F_Z \) summation of thrust and aerodynamic forces along body x-axis, y-axis, and z-axis, respectively, N (see fig. A1)

\( g \) acceleration due to gravity, m/sec\(^2\)

\( H_f, H_r \) drag force of front or rear rotor perpendicular to shaft in downwind direction, N (see fig. A2)

\( h_f, h_r \) distance from center of gravity of helicopter to front or rear rotor hub, measured parallel to helicopter z-axis, m (see fig. A2)

\( I_{XX}, I_{YY}, I_{ZZ} \) helicopter moments of inertia about X-axis, Y-axis, and Z-axis, respectively, kg-m\(^2\)

\( I_{XZ} \) helicopter product of inertia in XZ-plane, kg-m\(^2\)

\( i \) angle of incidence of rotor shaft, rad (see fig. A2)

\( i, j \) dummy variables for counting

\( L, M, N \) total rolling, pitching, and yawing moments about center of gravity of helicopter, N-m (see fig. A1)

\( l_f, l_r \) distance from center of gravity of helicopter to projection of front or rear rotor hub on x-axis, m (see fig. A2)

\( m \) helicopter mass, kg

\( P, Q, R \) vehicle angular velocity about body x-axis, y-axis, and z-axis, respectively, rad/sec (see fig. A1)

\([S]\) 6 \( \times \) 10 stability derivative matrix defined by equation (15)

\( \vec{S}_j \) jth column vector of \([S]\)

\( T_f, T_r \) thrust force of front or rear rotor parallel to shaft, N (see fig. A2)
$U, V, W$ velocity of vehicle along body $x$-axis, $y$-axis, and $z$-axis, respectively, m/sec (see fig. A3)

$\vec{V}_B, \vec{V}_L$ vector of vehicle velocities expressed in a body frame or a local level frame, m/sec

$V_\infty$ free-stream velocity, m/sec

$X, Y, Z$ orthogonal coordinate system with $X$-axis parallel to Earth in direction of forward flight, $Z$-axis down toward center of Earth, and $Y$-axis completing a right-hand coordinate frame (see fig. A1)

$x, y, z$ orthogonal body-axis coordinate system with $x$-axis toward front of vehicle, $z$-axis pointing downward, and $y$-axis completing a right-hand coordinate frame (see fig. A1)

$x', y', z'$ rotor-hub axes, parallel to body axes but centered at rotor hub (see fig. A2)

$x_S, y_S, z_S$ rotor-shaft axes, centered at rotor hub with $z_S$-axis down along rotor shaft, $y_S$-axis coincident with $y'$-axis, and $x_S$-axis forward to complete a right-hand frame (see fig. A2)

$x_W, y_W, z_W$ rotor wind axes, related to rotor-shaft axes by rotor angle of sideslip (see fig. A2 and eq. (A3))

$\dot{X}$ vehicle horizontal airspeed, knots (see figs. 9 to 15)

$X, Y, Z$ summation in equation (6) of aerodynamic, thrust, and gravitational forces along body $x$-axis, $y$-axis, and $z$-axis, respectively, N

$X_{fus}, Y_{fus}, Z_{fus}$ fuselage aerodynamic force along body $x$-axis, $y$-axis, and $z$-axis, respectively, N

$\bar{X}$ general state vector (see appendix D)

$\bar{X}_N$ nominal state vector

$\dot{\bar{X}}_N$ derivative of nominal state vector
\( \bar{X}_p \)  perturbation state vector

\( \dot{\bar{X}}_p \)  derivative of perturbation state vector

\( \bar{X}_T \)  total state vector of vehicle

\( \dot{\bar{X}}_T \)  derivative of total state vector

\( \bar{Y} \)  output vector from HELICOP defined by equation (6)

\( Y_f, Y_r \)  side force of front or rear rotor perpendicular to shaft and directed 90° from downwind in direction of rotor rotation, N (see fig. A2)

\( \dot{Z} \)  vehicle vertical velocity, positive down toward Earth center, m/sec

\( \alpha_{\text{fus}} \)  fuselage angle of attack, deg (see fig. A3)

\( \beta_{\text{fus}} \)  fuselage angle of sideslip, deg (see fig. A3)

\( \beta' \)  sideslip angle of rotor, rad (see fig. A2)

\( \bar{\delta} \)  general control vector (see appendix D)

\( \delta_j \)  jth component of \( \bar{\delta}_T \)

\( \Delta \bar{\delta} \)  perturbation vector

\( \Delta p_j \)  jth component of \( \Delta \bar{\delta} \)

\( \Delta \bar{\delta}_T, \Delta \theta, \Delta \phi \)  correction terms to \( \bar{\delta}_T, \theta, \) and \( \phi \) for trim calculations (see fig. 2)

\( \Delta \bar{\xi} \)  perturbation vector in \( \bar{\xi} \) (see fig. 4)

\( \delta_B, \delta_C, \delta_S, \delta_R \)  longitudinal, collective, lateral, and directional controls, cm

\( \delta_N, \delta_P, \delta_T \)  nominal, perturbation, and total control vectors, m (see fig. D1)
\( \bar{\varepsilon} \) convergence tolerance vector for trim calculations (see fig. 2)

\( \theta \) pitch attitude of vehicle, deg (see fig. A1)

\( \lambda \) rotor inflow ratio

\( \lambda' \) component of inflow ratio due to free-stream velocity

\( \mu \) rotor advance ratio

\( \nu \) variable step parameter (see eq. (8))

\( \bar{\xi} \) 10 × 1 parameter vector defined by equation (14)

\( \rho \) air density, kg/m\(^3\)

\( \phi \) roll attitude angle of vehicle, deg

\( \Omega \) rotor rotational speed, rad/sec

\( \tilde{\Omega}_B \) vector of vehicle angular velocity expressed in a body frame, rad/sec

Subscripts:

f front

r rear

N nominal

P perturbation

Stability derivative notation:

\[ B_A = \left. \frac{\partial B}{\partial A} \right|_{\text{trim}} \]

where \( B \) can be \( X, Y, Z, L, M, N \) and \( A \) can be \( U, V, W, P, Q, R, \delta_B, \delta_C, \delta_S, \delta_R \)
CALCULATION OF TRIMS AND STABILITY DERIVATIVES

Background

The motion of a vehicle can be described by the general nonlinear vector differential equation

\[ \dot{X}_T = f(X_T, \delta_T) \]  \hspace{1cm} (1)

where \( X_T \) is the total state vector and \( \delta_T \) is the total control vector. The vehicle motion can also be represented as the sum of a nominal trim motion \((X_N, \delta_N)\) and a perturbation \((X_P, \delta_P)\) about the nominal. With this approach, both the state vector and control input vector are given by

\[ X_T = X_N + X_P \]  \hspace{1cm} (2)

\[ \delta_T = \delta_N + \delta_P \]  \hspace{1cm} (3)

The nominal trim variables \( X_N \) and \( \delta_N \) are then chosen to satisfy

\[ \dot{X}_N = f(X_N, \delta_N) = 0 \]  \hspace{1cm} (4)

Expanding equation (1) about the nominal trim values and retaining only first-order terms in \( X_P \) and \( \delta_P \) gives the linear vector differential equation

\[ \dot{X}_P = [A]X_P + [B]\delta_P \]  \hspace{1cm} (5)

Equation (5) is a good representation of the motion of the vehicle near the trim \( X_N, \delta_N \), i.e., for small \( X_P \) and \( \delta_P \). The matrices \([A]\) and \([B]\) are the stability and control derivatives of the vehicle. For convenience, matrices \([A]\) and \([B]\) are referred to as the stability derivative matrix. A detailed representation of equation (5) is given in appendix D.

Trims

The calculation of static trim conditions and associated stability derivatives makes repeated use of a large maneuver model of the CH-47B. The nonlinear model, discussed in a later section, is referred to as HELICOP. As shown in figure 1, inputs to HELICOP include constants such as air density \( \rho \), distances of rotor hubs from center of gravity
\( \dot{V}_L \), \( \Omega_B \), \( \phi \), pitch angle \( \theta \), and four control stick deflections \( \delta_T \).

\[ \dot{V}_L, \dot{\Omega}_B, \theta, \phi, \delta_T \]

Nonlinear model

\[ \dot{Y} = \begin{bmatrix} F_X - mg \sin \theta \\ F_Y + mg \cos \theta \sin \phi \\ F_Z + mg \cos \theta \cos \phi \\ L \\ M \\ N \end{bmatrix} \]

Figure 1.- Input-output definition for HELICOP.

The output from HELICOP is the \( 6 \times 1 \) vector of forces and moments \( \dot{Y} \). The first three components of \( \dot{Y} \) (i.e., \( X, Y, Z \)) are the sum of the body-axes aerodynamic and thrust forces \( F_X, F_Y, F_Z \) and gravitational forces. The second three components of \( \dot{Y} \) are the total moments \( L, M, N \) about the vehicle center of gravity expressed in body axes. Positive directions for the body-axes forces and moments are shown in appendix A, figure A1. The vector \( \dot{Y} \) is given by
\[
\vec{Y} = \begin{bmatrix}
F_X - mg \sin \theta \\
F_Y + mg \cos \theta \sin \phi \\
F_Z + mg \cos \theta \cos \phi \\
L \\
M \\
N
\end{bmatrix} = \begin{bmatrix}
X \\
Y \\
Z \\
L \\
M \\
N
\end{bmatrix}
\] (6)

where \( g \) is the acceleration due to gravity.

The trim condition implies a special set of the vehicle velocities, stick positions, and attitude angles that produce static equilibrium; i.e., the sum of the forces and moments equals zero \( (\vec{Y} = 0) \). The trim conditions defined herein are calculated and tabulated for fixed values of \( \rho, m, l_f, l_r, h_f, h_r, \) and \( \Omega_B \) and discrete values of \( \vec{V}_L \). For each \( \vec{V}_L \), the program iterates on \( \delta_T, \theta, \) and \( \phi \).

As shown in figure 2, initial guesses are made for the stick positions \( \delta_{T(0)} \) and attitudes \( \theta(0) \) and \( \phi(0) \). (The superscripts refer to the iteration number.) If \( \vec{Y}(i) \) is not within a predetermined convergence tolerance \( \epsilon \), then \( \delta_T, \theta, \) and \( \phi \) are updated by increments \( \Delta\delta_T, \Delta\theta, \) and \( \Delta\phi \). These updates are calculated using the negative of the gradient of \( \vec{Y} \) with respect to \( \delta_T, \theta, \) and \( \phi \) (ref. 5). The approach for calculating the gradient matrix \( [C] \) is described later in this section. A gradient step that would reduce \( \vec{Y}(i) \) to zero if the relation were linear is

\[
\begin{bmatrix}
\Delta\delta_T \\
\Delta\theta \\
\Delta\phi
\end{bmatrix} = -[C(i)]^{-1}\vec{Y}(i)
\] (7)

Since \( \vec{Y} \) is a nonlinear function of \( \delta_T, \theta, \) and \( \phi \), the step given by equation (7) may be too large (linearity could be violated); consequently, a parameter \( \nu \) is introduced so that the size of the gradient steps can be reduced to:

\[
\begin{bmatrix}
\Delta\delta_T \\
\Delta\theta \\
\Delta\phi
\end{bmatrix} = -\nu[C(i)]^{-1}\vec{Y}(i)
\] (8)
Calculate gradient matrix \([C]\)

\[
\begin{bmatrix}
\Delta \delta_T \\
\Delta \theta \\
\Delta \phi
\end{bmatrix} = -[C]^{-1} \tilde{\gamma}(i)
\]

Figure 2. - Flow chart for trim calculations.

Input:
- \(\rho, m, l_r, l_y, h_r, h_y\)
- \(\hat{V}_L, \hat{h}_B, \theta(0), \phi(0), \delta_T(0)\)

\(i = 0\)

\(\tilde{\gamma}(i) \leq \tilde{\varepsilon}\)

Yes

Record trims:
- \(\rho, m, l_r, l_y, h_r, h_y, \hat{V}_L, \hat{h}_B,\)
- \(\delta_N = \delta_T, \theta_N = \theta(1), \phi_N = \phi(1)\)
where $0 \leq \nu \leq 1$. A value of 0.1 for $\nu$ was found to give rapid convergence for all trim cases. The control sticks and attitude angles are updated by

$$
\begin{bmatrix}
\delta_T(i+1) \\
\theta(i+1) \\
\phi(i+1)
\end{bmatrix} =
\begin{bmatrix}
\delta_T(i) \\
\theta(i) \\
\phi(i)
\end{bmatrix} - \nu \mathbb{C}(i)^{-1} \mathbf{Y}(i)
$$

This process is iterated until all components of $\mathbf{Y}$ are sufficiently close to zero. Convergence tolerances of 0.04448 N (0.01 lb) for the force components and 0.00136 N-m (0.001 ft-lb) for the moment components were used. When convergence is reached, the trim condition defined by the constants $\overline{V}_L$, $\overline{\delta}_B$, $m$, $\rho$, $l_f$, $l_r$, $h_f$, $h_r$, and the final values of the variables $\delta_T$, $\theta$, and $\phi$ is recorded. The trim values of $\delta_T$, $\theta$, and $\phi$ are designated $\delta_{TN}$, $\theta_N$, and $\phi_N$.

A flow chart illustrating the technique for calculating the gradient matrix $\mathbb{C}$ is shown in figure 3. Inputs include the final updated values $\delta_T(i)$, $\theta(i)$, $\phi(i)$, and $\mathbf{Y}(i)$ and a control-stick perturbation vector $\Delta \mathbf{p}$. Also required are the other basic inputs to HELICOP ($\rho$, $m$, $l_f$, $l_r$, $h_f$, $h_r$, $\overline{V}_L$, and $\overline{\Omega}_B$). The approach is to estimate the first four columns of $\mathbb{C}$, relating to the four control channels, by using a perturbation approximation to the derivative and to estimate the last two columns, relating to the attitudes $\theta$ and $\phi$, by using an analytic expression for the derivative. A new set of forces and moments $\overline{Y}_j$ are generated by separately applying a perturbation to each of the four control channels. Each of the first four columns of $\mathbb{C}$ are then approximated by calculating the change in $\mathbf{Y}$ due to the perturbation in control stick $\Delta \mathbf{p}_j$, as shown in

$$
\frac{\partial \mathbf{Y}}{\partial \delta_j} = \frac{\mathbf{Y}(i) - \overline{Y}_j}{\Delta \mathbf{p}_j} = \mathbf{C}_j \\
(j = 1, \ldots, 4)
$$

As an approximation to save computer time, the gradient of $\mathbf{Y}$ with respect to the attitude angles $\theta$ and $\phi$ is generated by performing the partial derivative operations on equation (6) and by assuming that $F_X$, $F_Y$, $F_Z$, $L$, $M$, and $N$ are not functions of $\theta$ and $\phi$. The last two columns of $\mathbb{C}$ are given by
Figure 3.- Flow chart for calculation of gradient matrix \([C]\).
The fifth and sixth columns of $[C]$ are then $\frac{\partial \vec{Y}}{\partial \theta}$ and $\frac{\partial \vec{Y}}{\partial \phi}$ evaluated at $\theta = \theta^{(i)}$ and $\phi = \phi^{(i)}$, i.e., the present estimates of the trim attitude angles.

The gradient matrix $[C]$ is

$$
[C^{(i)}] = 
\begin{bmatrix}
\vec{C}_1 & \vec{C}_2 & \vec{C}_3 & \vec{C}_4 & \frac{\partial \vec{Y}}{\partial \theta} & \frac{\partial \vec{Y}}{\partial \phi} \\
\end{bmatrix}_{\theta=\theta^{(i)}}
$$

Stability Derivatives

The stability derivatives are used to approximate the behavior of the vehicle about a trim condition when subjected to small perturbations in $\vec{V}_B$, $\vec{\Omega}_B$, and $\vec{\phi}_T$. For convenience, define the $10 \times 1$ parameter vector $\vec{\xi}$ as

$$
\vec{\xi} = 
\begin{bmatrix}
\vec{V}_B \\
\vec{\Omega}_B \\
\vec{\phi}_T 
\end{bmatrix}
$$
The 6 × 10 stability derivative matrix \([S]\) is defined as the gradient of \(\bar{Y}\) with respect to \(\bar{\xi}\), evaluated at trim conditions, as shown in the following equation:

\[
[S] = \begin{bmatrix}
\frac{\partial \bar{Y}_1}{\partial \bar{\xi}_1} \\
\frac{\partial \bar{Y}_2}{\partial \bar{\xi}_2} \\
\vdots \\
\frac{\partial \bar{Y}_N}{\partial \bar{\xi}_N}
\end{bmatrix}
\begin{bmatrix}
\bar{X}_{T=\bar{X}_N} \\
\bar{\delta}_{T=\bar{\delta}_N}
\end{bmatrix} = \frac{\partial \bar{Y}}{\partial \bar{\xi}}
\]  

(15)

The stability derivatives, shown in figure 4, are calculated by changing one component of the parameter vector \(\bar{\xi}_j\) by a small amount \((\Delta\bar{\xi}_j)\) and by leaving all other components at their trim values. (The subscript \(j\) refers to the element in \(\bar{\xi}\).) This new set of inputs is then put into HELICOP, resulting in a changed output \(\bar{Y}\). In order to get a reasonably good fit, both a positive and a negative perturbation are used to produce \(\bar{Y}^{(1)}\) and \(\bar{Y}^{(2)}\), respectively. The jth column of \([S]\) is then approximated as

\[
\bar{S}_j = \frac{\bar{Y}^{(1)} - \bar{Y}^{(2)}}{2 \Delta \bar{\xi}_j}
\]  

(16)

The size of \(\Delta \bar{\xi}_j\) was kept small to get a good approximation; 1 percent of the typical range of that variable was used. The perturbations are tabulated as follows:

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta U), m/sec</td>
<td>0.792</td>
</tr>
<tr>
<td>(\Delta V), m/sec</td>
<td>0.152</td>
</tr>
<tr>
<td>(\Delta W), m/sec</td>
<td>0.152</td>
</tr>
<tr>
<td>(\Delta P), rad/sec</td>
<td>0.005</td>
</tr>
<tr>
<td>(\Delta Q), rad/sec</td>
<td>0.005</td>
</tr>
<tr>
<td>(\Delta R), rad/sec</td>
<td>0.005</td>
</tr>
<tr>
<td>(\Delta \delta_B), cm</td>
<td>0.330</td>
</tr>
<tr>
<td>(\Delta \delta_C), cm</td>
<td>0.229</td>
</tr>
<tr>
<td>(\Delta \delta_S), cm</td>
<td>0.216</td>
</tr>
<tr>
<td>(\Delta \delta_R), cm</td>
<td>0.190</td>
</tr>
</tbody>
</table>
Figure 4.- Flow chart for stability derivatives.
HELICOPTER NONLINEAR MODEL

The CH-47 (Chinook) is a twin-engine tandem rotor helicopter (see fig. 5 taken from ref. 6) designed for all-weather, medium-sized transport type operations. The three bladed rotors are driven in opposite directions through interconnecting shafts which enable both rotors to be driven by either engine. The rotor heads are fully articulated, with pitch, flapping, and drag hinges.

Figure 5.- Three-view drawing of the CH-47C helicopter taken from reference 6 (1 ft = 0.3048 m; 1 in. = 0.0254 m).

The equations describing the helicopter are taken from reference 2, with a few modifications to give a better representation of the CH-47B, and are shown in appendix B for completeness. Only steady-state equations are included since the desired purpose of this research is to calculate trims and quasi-static derivatives. The only two areas affected by this approach are control mixing and rotor inflow ratio. Since the helicopter model (HELICOP) has been developed in modular form for computer programing, the transient response dynamics can be easily added if desired.
A flow chart for HELICOP is shown in figure 6. Each box represents a major module that is described in the following sections in the order shown on the flow chart.

Although HELICOP has been specialized for the CH-47B, only minor variations would be required to represent other tandem rotor helicopters.

Figure 6. - Flow chart for HELICOP.
Governor

For the purpose of this program, the rotor rotational speed $\Omega$ is a constant. The model can be expanded in the future to include engine dynamics and to investigate the sensitivity of the trims and derivatives to changes in angular velocity.

Transformation Parameters and Aerodynamic Input

Coordinate Transformation

The linear velocities of each rotor referenced to their respective shaft incidence angles and the angular velocities referenced to the rotor wind axes are calculated (see appendix B, eqs. (B2) to (B9) for the front rotor and eqs. (B12) to (B19) for the rear rotor). The relationship of the rotor wind axes to the body axes is shown in figure A2 of appendix A. Normalizing the horizontal and vertical velocities of each rotor by the rotor tip speed allows calculation of the rotor advance ratio $\mu'$ and the component of inflow ratio due to free-stream velocity $\lambda'$ (eqs. (B10), (B11), (B20), and (B21)).

Control Mixing and Coordinate Transformation

Equations for first and second stage mixing are combined and shown in simplified form (eqs. (B22), (B23), (B25), and (B26)). This simplification is possible since only quasi-static derivatives are desired. For each trim condition, the final stick trim positions were checked to verify that physical limits are not exceeded.

The CH-47B and CH-47C models have actuators at both front and rear rotors to automatically introduce forward longitudinal cyclic pitch as a function of airspeed. These trim schedules (fig. B1) affect the trim values at higher airspeeds. Finally, the lateral and longitudinal cyclic pitch angles are transformed from the body axis coordinate frame to the rotor wind axis frame (eqs. (B24) and (B27)).

Rotor Inflow Ratios, Thrust Coefficients, and Interference Parameters

There are four nonlinear equations that contain terms for the rotor inflow ratios $\lambda_f$, $\lambda_r$ and the thrust coefficients $C_{T,f}$, $C_{T,r}$. The problem is to find values that satisfy all equations. The approach used is to initially guess at two of the variables and then iterate until all four variables converge within a predefined tolerance. Figure 7 is a flow chart showing the major steps in calculating the four variables. Since trims are being calculated for flight conditions which differ by small steps in velocity, the past values of $\lambda_f$ and $\lambda_r$ are used as the initial guess for the new flight condition. By means of equations (B28) to (B35) and the initial guess, $C_{T,f}$ and $C_{T,r}$ are then calculated. A nonlinear function simulating rotor lift stall is included in the thrust coefficient calculations and is shown in figure B2.
Initial guess at $\lambda_f$, $\lambda_r$

Calculate $C_{T,f}$, $C_{T,r}$

Modify $\lambda_f$, $\lambda_r$

Calculate $dF_{FR}$, $dF_{RF}$, $\lambda_f$, $\lambda_r$

Calculate updated values of $C_{T,f}$, $C_{T,r}$

$\left(\Delta C_T\right)_{f,r} \leq 10^{-4}$ \%

Steady-state values $C_{T,f}$, $C_{T,r}$, $\lambda_f$, $\lambda_r$

Figure 7.- Convergence loop for calculating rotor inflow ratios and thrust coefficients.
Interference parameters $d_{FR}$, $d_{RF}$ between the two rotors are functions of the inflow ratio, advance ratio, and rotor sideslip angle (eqs. (B36) to (B41)). For backward flight, the interference parameters are interchanged; i.e., equations (B40) and (B41) are used in place of equations (B38) and (B39). New values for $\lambda_f$ and $\lambda_r$ are then calculated (eqs. (B41) and (B43)) by using the values obtained for the thrust coefficients, interference parameters, and components of inflow ratio due to the free-stream velocity.

The new values of $\lambda_f$ and $\lambda_r$ are then used to calculate new values for $C_{T, f}$ and $C_{T, r}$, as described previously. If $C_{T, f}$ and $C_{T, r}$ agree with the previous values within 0.0001 percent, the iteration is assumed complete. If the convergence tolerance is not met, $\lambda_f$ and $\lambda_r$ are modified by adding a fraction of the difference between the old and new inflow ratios to the previous values. The process is then repeated for another iteration. A number that works reasonably well for this fraction is 0.25.

**Rotor Forces and Moments**

Inputs to this module include rotor inflow and advance ratios, longitudinal and lateral cyclic pitch angles, collective pitch angles, blade twist, rotor angular velocities, and rotor parameters. These inputs are used to calculate thrust, coning angle, longitudinal and lateral flapping angles, horizontal and side forces, required torque, and both pitching and rolling hub moments. A correction factor to simulate stall at high airspeed is included by increasing the average rotor drag coefficient ($\delta_{FH}$ and $\delta_{RH}$ of appendix B). The force and moment calculations for the front and rear rotors are shown in equations (B44) to (B74).

Assumptions used in development of these equations include: (1) constant induced velocity through the rotor disk, (2) small flapping angle and inflow angle of attack, (3) negligible second and higher order harmonics for flapping, (4) negligible effect of the reverse flow region, and (5) infinitely rigid blades in all directions. The air density is assumed constant in all of these calculations. Provisions can be made to change air density as a function of altitude.

**Fuselage Forces and Moments**

Equations (B75) to (B90) show the approach for calculating the fuselage parasitic drag forces and moments. Analytical expressions relating the fuselage forces and moments to angle of attack $\alpha_{fus}$ and sideslip angle $\beta_{fus}$ are taken from reference 2. Figure A3 of appendix A illustrates the sign convention for $\alpha_{fus}$ and $\beta_{fus}$. A different approach was used to calculate the fuselage aerodynamic force along the body x-axis $X_{fus}$ since a numerical value for the flat-plate drag area was not available. A figure plotting the quotient of the drag to dynamic pressure as a function of $\alpha_{fus}$ and $\beta_{fus}$
can be found in reference 3 and is shown in figure B3. Interpolation is then used to obtain the flat-plate drag area.

**Total System Forces and Moments**

Equations (B91) to (B96) show the summation of the total aerodynamic forces and moments acting on the helicopter. The rotor forces and moments are transformed from the rotor wind axes to the aircraft body axes and are then summed with the fuselage forces and moments. The rotor forces also contribute aerodynamic moments about the helicopter center of gravity. As shown in figure A2, the positive directions for the front and rear rotor side forces are opposite each other.

**RESULTS AND DISCUSSION**

Using the techniques and the nonlinear model (HELICOP) described earlier, a set of trims and stability derivatives for the entire flight envelope was generated for the CH-47B. These data are presented in appendix E and are sufficient to allow the CH-47B to be represented by either the linear coupled model or the linear uncoupled model discussed in appendix D.

A major factor that impacts the validity of these trims and derivatives is the completeness of HELICOP. In order to establish confidence in HELICOP, a comparison is made with referenced data. The most complete set of data available in the literature is for level flight ($Z = 0$) at horizontal velocities of $V = 0, 40, 80, 120, 140, \text{ and } 160$ knots (ref. 3). These data include 2 angle trims, 4 control-stick trims, and 30 stability derivatives required in the uncoupled linear model. Since both the nonlinear model and the technique used to generate the referenced data are not documented, the cause of any difference between referenced and calculated data cannot always be determined. As shown in figure 8, the linear-model analysis consists of comparing the calculated and referenced values of trims, stability derivatives, and poles of the linear model.

The calculated and referenced values of the stick and attitude trims are shown in figure 9. Additional calculated values are included to better define the shape of the trims. Both the shapes and magnitudes of the two sets of trim data agree very closely for all compared velocities. The maximum error in $\theta_N$ is at 80 knots and is less than 0.7$^\circ$; the maximum error in $\phi_N$ is less than 0.4$^\circ$ at 160 knots. The stick trims show excellent agreement with a slight divergence between the calculated and referenced values in high-speed flight. The maximum errors occur at 160 knots and are 1.09 cm (0.43 in.) for $\delta_B,N$, 1.01 cm (0.4 in.) for $\delta_C,N$, 0.23 cm (0.09 in.) for $\delta_S,N$, and 0.94 cm (0.37 in.) for $\delta_R,N$. 

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Plots of the referenced and calculated stability derivatives are presented in figures 10 to 15. Approximately two-thirds of the 30 stability derivatives show excellent agreement for all velocities. The remainder of the derivatives have some differences in magnitude, although the shapes generally agree quite well. An exception occurs at hover ($\dot{X} = 0$). For this velocity the calculated values of several derivatives, e.g., $Y_V$ and $N_V$, were found to be larger in order of magnitude than the referenced values. This would imply a radical change in the behavior of the vehicle in the vicinity of hover. Such behavior is not observed in the actual vehicle; it is concluded therefore that HELICOP and the calculated derivatives do not adequately describe the CH-47B at $\dot{X} = 0$. The nonlinear model representation (HELICOP) at hover is the suspected source of this anomaly. So as to generate reasonable values of calculated derivatives at $\dot{X} = 0$, a third-order curve was fitted through the calculated values at $\dot{X} = -40$, -20, 20, and 40 knots. This function was evaluated at $\dot{X} = 0$ to generate the values shown in figures 10 to 15 and in the tables of appendix E.

Approximately one-third of the stability derivatives exhibit some differences in magnitude between the two sets of data. A comparison of the linear-model poles for both referenced and calculated data (table I) shows the stability-derivative magnitude differences to be acceptable. Zeros of the models have also been calculated but are not included in table I since there is no simple and meaningful criterion for evaluating the effects of the zeros in a multi-input multi-output system.
Figure 9. - Control-stick and attitude-angle trims of vehicle.
Figure 9. - Concluded.
Figure 10.- Longitudinal force stability and control derivatives.
Figure 10. - Concluded.
Figure 11. - Normal-force stability and control derivatives.
Figure 11. Concluded.
Figure 12.- Pitching-moment stability and control derivatives.
Figure 12. - Concluded.
Figure 13. - Lateral-force stability and control derivatives.
Figure 13. - Concluded.
Figure 14.- Rolling-moment stability and control derivatives.
Figure 14. - Concluded.
Figure 15.- Yawing-moment stability and control derivatives.
Figure 15 - Concluded.
<table>
<thead>
<tr>
<th>$\dot{X}$, knots</th>
<th>Calculated poles</th>
<th>Referenced poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.2977</td>
<td>-0.272</td>
</tr>
<tr>
<td></td>
<td>-1.404</td>
<td>-1.040</td>
</tr>
<tr>
<td></td>
<td>0.078 ± 0.459j</td>
<td>0.094 ± 0.440j</td>
</tr>
<tr>
<td>40</td>
<td>0.388</td>
<td>0.427</td>
</tr>
<tr>
<td></td>
<td>-0.091 ± 0.286j</td>
<td>-0.064 ± 0.268j</td>
</tr>
<tr>
<td></td>
<td>-2.23</td>
<td>-1.90</td>
</tr>
<tr>
<td>80</td>
<td>0.555</td>
<td>0.645</td>
</tr>
<tr>
<td></td>
<td>-0.093 ± 0.242j</td>
<td>-0.067 ± 0.209j</td>
</tr>
<tr>
<td></td>
<td>-2.77</td>
<td>-2.55</td>
</tr>
<tr>
<td>120</td>
<td>0.666</td>
<td>-0.803</td>
</tr>
<tr>
<td></td>
<td>-0.053 ± 0.173j</td>
<td>-0.043 ± 0.142j</td>
</tr>
<tr>
<td></td>
<td>-3.14</td>
<td>-2.88</td>
</tr>
<tr>
<td>140</td>
<td>1.056</td>
<td>0.911</td>
</tr>
<tr>
<td></td>
<td>-0.050</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.002</td>
<td>-0.035 ± 0.096j</td>
</tr>
<tr>
<td></td>
<td>-3.31</td>
<td>-3.02</td>
</tr>
<tr>
<td>160</td>
<td>0.978</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>-0.020</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>-0.053</td>
<td>-0.052</td>
</tr>
<tr>
<td></td>
<td>-3.20</td>
<td>-3.06</td>
</tr>
</tbody>
</table>

(a) Longitudinal

<table>
<thead>
<tr>
<th>$\dot{X}$, knots</th>
<th>Calculated poles</th>
<th>Referenced poles</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.987</td>
<td>-0.942</td>
</tr>
<tr>
<td></td>
<td>0.064 ± 0.459j</td>
<td>0.123 ± 0.469j</td>
</tr>
<tr>
<td></td>
<td>-0.042</td>
<td>-0.082</td>
</tr>
<tr>
<td>40</td>
<td>-1.02</td>
<td>-1.03</td>
</tr>
<tr>
<td></td>
<td>0.065 ± 0.440j</td>
<td>0.089 ± 0.469j</td>
</tr>
<tr>
<td></td>
<td>-0.035</td>
<td>-0.062</td>
</tr>
<tr>
<td>80</td>
<td>-1.11</td>
<td>-1.11</td>
</tr>
<tr>
<td></td>
<td>0.077 ± 0.386j</td>
<td>0.096 ± 0.515j</td>
</tr>
<tr>
<td></td>
<td>-0.039</td>
<td>-0.049</td>
</tr>
<tr>
<td>120</td>
<td>-1.18</td>
<td>-1.13</td>
</tr>
<tr>
<td></td>
<td>0.127 ± 0.350j</td>
<td>0.131 ± 0.600j</td>
</tr>
<tr>
<td></td>
<td>-0.046</td>
<td>-0.053</td>
</tr>
<tr>
<td>140</td>
<td>-1.24</td>
<td>-1.17</td>
</tr>
<tr>
<td></td>
<td>0.177 ± 0.365j</td>
<td>0.156 ± 0.640j</td>
</tr>
<tr>
<td></td>
<td>-0.047</td>
<td>-0.063</td>
</tr>
<tr>
<td>160</td>
<td>-1.37</td>
<td>-1.21</td>
</tr>
<tr>
<td></td>
<td>0.260 ± 0.402j</td>
<td>0.204 ± 0.623j</td>
</tr>
<tr>
<td></td>
<td>-0.046</td>
<td>-0.098</td>
</tr>
</tbody>
</table>

(b) Lateral
The form of the referenced and calculated poles shown in table I agree at all velocities except for two longitudinal poles at 140 knots. This exception occurs at a transition point in the velocity sweep where a pair of complex roots at lower velocities become real roots at higher velocities.

Comparing poles near the origin (real parts less than 0.1 rad/sec) for both longitudinal and lateral cases shows that the real parts of all roots differ by less than 0.03 rad/sec, except for a pair of lateral roots at $\dot{X} = 0$ and a single lateral root at $\dot{X} = 160$ knots. For roots with real parts larger than 0.1 rad/sec, only a single longitudinal root at $\dot{X} = 0$ and a pair of lateral roots at $\dot{X} = 160$ knots have differences in their real parts greater than 20 percent. The majority of the imaginary part of the roots also exhibit similar close agreement.

The excellent agreement of the control-stick and attitude-angle trims, the shape and magnitude agreement of the majority of the stability derivatives, and the agreement of the pole location indicate that HELICOP and the referenced nonlinear model agree (except at $\dot{X} = 0$) with regard to the stability character and fundamental modes of response of the CH-47B. Exceptions do exist which produce differences in some zeros that could lead to different time responses.

CONCLUSIONS

This report contains a complete set of materials needed to mathematically describe the CH-47B helicopter. The material includes both a nonlinear model (HELICOP) and a linear coupled model with its 6 trims and 60 quasi-static stability derivatives covering the entire flight regime. The results from comparison with reference data show that the linear model and HELICOP are a good representation of the CH-47B, the exception being that HELICOP is questionable at zero airspeed. A comparison with data available in the literature was made for the trims, stability derivatives, and poles of the uncoupled linear model for positive velocities and zero rate of descent.

The two attitude-angle trims and four control-stick trims show excellent agreement at all velocities. The 30 stability derivatives comprising the uncoupled model show very good agreement regarding shapes, and the majority agree in magnitude. All but three of the poles of the linear model near the origin (real parts less than 0.1 rad/sec) agree within 0.03 rad/sec. Only three roots which have real parts greater than 0.1 rad/sec show differences in real parts greater than 20 percent. The majority of the imaginary parts also exhibit close agreement.
The technique used to generate the trims and stability derivatives is general and could be applied to other vehicles. All that is required is a nonlinear model of the vehicle, equivalent to HELICOP, which generates forces and moments from attitude angles and control-stick inputs.

Langley Research Center
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Hampton, Va. 23665
March 25, 1976
APPENDIX A

SIGN CONVENTION AND COORDINATE FRAMES

This appendix defines the sign conventions, angles, and coordinate frames used in this paper. Figure A1 illustrates the relationship between the local level axes (X, Y, Z) and the body axes (x, y, z). The local level frame has its origin at the vehicle center of gravity. The Z-axis is down and coincident with the Earth vertical; the X-axis is the projection of the vehicle's longitudinal body axis along the Earth with positive direction defined in forward flight; and the Y-axis completes a right-hand coordinate frame.

Figure A1.- Angular velocities, aerodynamic forces and moments, and relationships of local level axes and body axes with positive directions of vehicles.

The body frame has its origin at the vehicle center of gravity. The x-axis is along the reference longitudinal direction of the vehicle with the positive direction pointing forward; the z-axis is in the plane of symmetry of the fuselage with the positive direction through the floor; and the y-axis is positive to the right side, completing a right-hand coordinate frame. The transformation from the local level frame to the body frame is:
APPENDIX A

\[
\begin{align*}
\begin{bmatrix}
    x \\
    y \\
    z
\end{bmatrix} &=
\begin{bmatrix}
    \cos \theta & 0 & -\sin \theta \\
    \sin \theta \sin \phi & \cos \phi & \cos \theta \sin \phi \\
    \sin \theta \cos \phi & -\sin \phi & \cos \theta \cos \phi
\end{bmatrix}
\begin{bmatrix}
    X \\
    Y \\
    Z
\end{bmatrix} \\
\text{(A1)}
\end{align*}
\]

where \( \theta \) and \( \phi \) are the pitch and roll attitude angles. Positive directions for the body forces and moments \((F_X, F_Y, F_Z, L, M, N)\) and the body angular velocities \((P, Q, R)\) are shown in figure A1.

Figure A2. - Definition of rotor-shaft axes, wind axes, and rotor forces for front rotor and rear rotor.

Figure A2 defines the relationship of the body axes, rotor-shaft axes, and wind axes as used in the nonlinear model. (The subscripts \( f \) and \( r \) refer to the front rotor and the rear rotor.) Two rotor-hub axes \((x', y', z')\) are defined parallel to the body axes, one centered at the front rotor hub and the second at the rear rotor hub. The locations of the two rotor hubs relative to the vehicle center of gravity are given by \( l_f, l_r, h_f, \) and \( h_r \). The rotor-shaft axes \((x_S, y_S, z_S)\) are related to the body frame by
where \( i \) is the shaft incidence angle.

The wind axes for each rotor is related to the respective shaft axes by the rotor sideslip angles \( \beta_i \) and \( \beta_r \) as follows:

\[
\begin{bmatrix}
\mathbf{x}_{W,f} \\
\mathbf{y}_{W,f} \\
\mathbf{z}_{W,f}
\end{bmatrix} =
\begin{bmatrix}
\cos \beta_i & \sin \beta_i & 0 \\
-sin \beta_i & \cos \beta_i & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_{S,f} \\
\mathbf{y}_{S,f} \\
\mathbf{z}_{S,f}
\end{bmatrix}
\tag{A3a}
\]

\[
\begin{bmatrix}
\mathbf{x}_{W,r} \\
\mathbf{y}_{W,r} \\
\mathbf{z}_{W,r}
\end{bmatrix} =
\begin{bmatrix}
-cos \beta_r & -sin \beta_r & 0 \\
-sin \beta_r & cos \beta_r & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_{S,r} \\
\mathbf{y}_{S,r} \\
\mathbf{z}_{S,r}
\end{bmatrix}
\tag{A3b}
\]

Finally, the transformation from wind axes to rotor forces is:

\[
\begin{bmatrix}
\mathbf{H}_f \\
\mathbf{Y}_f \\
\mathbf{T}_f
\end{bmatrix} =
\begin{bmatrix}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_{W,f} \\
\mathbf{y}_{W,f} \\
\mathbf{z}_{W,f}
\end{bmatrix}
\tag{A4a}
\]

\[
\begin{bmatrix}
\mathbf{H}_r \\
\mathbf{Y}_r \\
\mathbf{T}_r
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & -1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_{W,r} \\
\mathbf{y}_{W,r} \\
\mathbf{z}_{W,r}
\end{bmatrix}
\tag{A4b}
\]

where the rear rotor forces constitute a left-hand coordinate frame. The positive directions for all rotor forces are shown in figure A2.
APPENDIX A

The definition of fuselage angle of attack ($\alpha_{\text{fus}}$) and angle of sideslip ($\beta_{\text{fus}}$) is shown in figure A3. The three linear velocities $U$, $V$, and $W$ are defined along the $x$, $y$, and $z$ body axes, respectively.

Figure A3. - Definition of fuselage angle of attack $\alpha_{\text{fus}}$ and angle of sideslip $\beta_{\text{fus}}$. 
APPENDIX B

EQUATIONS FOR HELICOPTER NONLINEAR MODEL

The 10 inputs to the helicopter nonlinear model (HELICOP) are the three body-axes velocities \( U, V, \) and \( W \), the three angular body rates \( P, Q, \) and \( R \), and the four control stick positions \( \delta_B, \delta_C, \delta_S, \) and \( \delta_R \). The six outputs are the three forces \( F_x, F_y, \) and \( F_z \) and the three moments \( L, M, \) and \( N \). Constants for the model are presented in appendix C. A complete list of equations for each module (found mainly in ref. 2) are presented in this appendix. (Subscripts \( F \) and \( R \) refer to the front and rear rotors, respectively.) The equations in this appendix are essentially in modular form for use in a computer program.

Additional symbols applicable to this appendix are defined as follows:

\[ A_{OF}, A_{OR} \] front/rear rotor coning angle, rad

\[ A_{IF}, A_{IR} \] front/rear longitudinal flapping angle, rad

\[ A_{ICF}, A_{ICR} \] front/rear lateral cyclic pitch angle in rotor-shaft axes, rad

\[ A_{ICFDR}, A_{ICRDR} \] rate of change of front/rear lateral cyclic with direction control, rad/m

\[ A_{ICFDS}, A_{ICRDS} \] rate of change of front/rear lateral cyclic with lateral control, rad/m

\[ A_{ICFR}, A_{ICRR} \] front/rear lateral cyclic pitch angle in wind axes, rad

\( a_s \) slope of rotor-blade lift curve per radian

\[ B_{IF}, B_{IR} \] front/rear lateral flapping angle, rad

\[ B_{ICF}, B_{ICR} \] front/rear longitudinal cyclic pitch angle in rotor-shaft axes, rad or deg

\[ B_{ICFR}, B_{ICRR} \] front/rear longitudinal cyclic pitch angle in wind axes, rad

\( b_S \) number of blades per rotor

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APPENDIX B

\( C_{FE} \) equivalent flat-plate drag area of fuselage, \( m^2 \)

\( C_{F1}, \ldots, C_{F4} \) dummy variables in equations (B91a) to (B91d)

\( C_{L\alpha} \) rate of change of fuselage lift force with \( \alpha \) normalized by dynamic pressure, \( m^2/\text{rad} \)

\( C_{L\beta} \) rate of change of fuselage rolling moment with \( \beta \) normalized by dynamic pressure, \( m^3/\text{rad} \)

\( C_{M\alpha} \) rate of change of fuselage pitching moment with \( \alpha \) normalized by dynamic pressure, \( m^3/\text{rad} \)

\( C_{N\beta} \) rate of change of fuselage yawing moment with \( \beta \) normalized by dynamic pressure, \( m^3/\text{rad} \)

\( C_{R1}, \ldots, C_{R4} \) dummy variables in equations (B91e) to (B91h)

\( C_{Y\beta} \) rate of change of fuselage side force with \( \beta \) normalized by dynamic pressure, \( m^2/\text{rad} \)

\( D_{FUS1} \) body-axes velocity component in longitudinal plane, m/sec

\( D_{FUS2} \) body-axes velocity component in lateral plane, m/sec

\( e_S \) rotor flapping hinge offset, m

\( F_H \) dummy variable, kg-m (see eq. (B44))

\( H_{CF}, H_{CR} \) front/rear rotor drag-force coefficient

\( L_{FUS} \) aerodynamic rolling moment about helicopter center of gravity due to fuselage, N-m

\( L_{HF}, L_{HR} \) front/rear lateral hub moment, N-m
APPENDIX B

$L_P$ rolling moment about helicopter center of gravity due to rotor drag and side forces, N-m

$M_D$ Mach number above which a nonlinear compressibility term is added

$M_{FUS}$ aerodynamic pitching moment about helicopter center of gravity due to fuselage, N-m

$M_{HF,H_R}$ front/rear longitudinal rotor hub moment, N-m

$M_P$ pitching moment about helicopter center of gravity due to rotor thrust, drag, and side forces, N-m

$M_{T90F,M90R}$ front/rear Mach number at rotor blade azimuth of 90°

$M_W$ mass moment of blade about flapping hinge, N-m

$N_{FUS}$ aerodynamic yawing moment about helicopter center of gravity due to fuselage, N-m

$N_P$ yawing moment about helicopter center of gravity due to rotor drag and side forces, N-m

$P_{F,P_R}$ front/rear components of vehicle rolling rate about rotor wind axes, rad/sec

$Q_{AEROF,QAEROR}$ front/rear torque required, N-m

$Q_{CF,QR}$ front/rear rotor torque coefficient

$Q_{DPR5}$ dynamic pressure, N/m²

$Q_{F,Q_R}$ front/rear components of vehicle pitching rate about rotor wind axes, rad/sec

$R_B$ rotor blade radius, m

$R_{F,R_R}$ front/rear components of vehicle yawing rate about rotor wind axes, rad/sec

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APPENDIX B

$R_{IF,IR}$  
front/rear rotor wake angle, rad

$T_C$  
dummy variable related to rotor thrust coefficients (see eqs. (B28) to (B35))

$(T_C)_{break}$  
break point above which a nonlinear function simulating rotor lift stall is included

tc  
ratio of thickness to blade section chord

$U_{F,R}$  
front/rear rotor relative wind velocity in rotor wind axes, m/sec

$U_{F1,UR1}$  
longitudinal velocity of front/rear rotor along x-axis of body, m/sec

$U_{F2,UR2}$  
longitudinal velocity of front/rear rotor in rotor-shaft axes, m/sec

$V_{F1,VR1}$  
lateral velocity of front/rear rotor along y-axis of body, m/sec

$W_{FUS}$  
vertical body axes velocity due to fuselage, m/sec

$W_{F1,WR1}$  
vertical velocity of front/rear rotor along z-axis of body, m/sec

$W_{F2,WR2}$  
vertical velocity of front/rear rotor in rotor wind axes, m/sec

$Y_{CF,CR}$  
front/rear rotor side-force coefficient

γ  
rotor blade lock number

$\Delta T_{CF,CR}$  
thrust limit nonlinear gain from reference 2 (see fig. B2)

$\delta_{FH,RH}$  
front/rear drag coefficient for rotor drag force and torque

$\delta_o$  
nominal average rotor drag coefficient without correction term

$\theta_{FDB,RDB}$  
rate of change of front/rear collective pitch with longitudinal stick, rad/m
APPENDIX B

\( \theta_{\text{FDC}}, \theta_{\text{RDC}} \) rate of change of front/rear collective pitch with collective stick, rad/m

\( \theta_{\text{OF}}, \theta_{\text{OR}} \) front/rear root collective pitch, rad

\( \theta_T \) rotor blade twist, rad

\( \theta_{\text{TF}}, \theta_{\text{TR}} \) root collective pitch at full \( \delta_C \), rad

\( \sigma \) rotor solidity ratio

\( \Omega_0 \) nominal rotor rotational speed, rad/sec

I Governor

\[
\Omega = \Omega_0
\]  

(B1)

II Transformation Parameters and Aerodynamic Coordinate Transformation

Front Rotor

Rotor velocities in body axes

\[
U_{F1} = U - h_f Q
\]

(B2)

\[
V_{F1} = V + \ell_f R + h_f P
\]

(B3)

\[
W_{F1} = W - \ell_f Q
\]

(B4)

Rotor velocities in rotor-shaft axes

\[
\begin{pmatrix}
U_{F2} \\
W_{F2}
\end{pmatrix} = \begin{bmatrix}
\cos i_F & \sin i_F \\
-sin i_F & \cos i_F
\end{bmatrix} \begin{pmatrix}
U_{F1} \\
W_{F1}
\end{pmatrix}
\]  

(B5)

Relative wind velocity in rotor wind axes

\[
U_F = \left[ (U_{F2})^2 + (V_{F1})^2 \right]^{1/2}
\]

(B6)
APPENDIX B

Rotor sideslip angle

\[
\sin \beta_F = \begin{cases} \frac{V_{F1}}{U_F} & (U_F \neq 0) \\ 0 & (U_F = 0) \end{cases} \tag{B7}
\]

\[
\cos \beta_F' = \begin{cases} \frac{U_{F2}}{U_F} & (U_F \neq 0) \\ 1 & (U_F = 0) \end{cases} \tag{B8}
\]

Angular velocities in rotor wind axes

\[
\begin{bmatrix} P_F \\ Q_F \\ R_F \end{bmatrix} = \begin{bmatrix} \cos i_F \cos \beta_F' & \sin \beta_F' & \sin i_F \cos \beta_F' \\ -\cos i_F \sin \beta_F' & \cos \beta_F' & -\sin i_F \sin \beta_F' \\ -\sin i_F & 0 & \cos i_F \end{bmatrix} \begin{bmatrix} P \\ Q \\ R \end{bmatrix} \tag{B9}
\]

Avance ratio

\[
\mu_F = \frac{U_F}{\Omega R_B} \tag{B10}
\]

Component of inflow ratio

\[
\lambda_F' = \frac{W_{F2}}{\Omega R_B} \tag{B11}
\]

Rear Rotor

Rotor velocities in body axes

\[
U_{R1} = U - h_r Q \tag{B12}
\]

\[
V_{R1} = V - l_r R + h_r P \tag{B13}
\]
APPENDIX B

\[ W_{R1} = W + \ell Q \]  

Rotor velocities in rotor-shaft axes

\[
\begin{bmatrix}
U_{R2} \\
W_{R2}
\end{bmatrix} =
\begin{bmatrix}
\cos i_R & \sin i_R \\
-sin i_R & \cos i_R
\end{bmatrix}
\begin{bmatrix}
U_{R1} \\
W_{R1}
\end{bmatrix}
\]  

Relative wind velocity in rotor wind axes

\[ U_R = \left( U_{R2}^2 + V_{R1}^2 \right)^{1/2} \]  

Rotor sideslip angle

\[
\sin \beta_R' = \begin{cases} 
\frac{V_{R1}}{U_R} & (U_R \neq 0) \\
0 & (U_R = 0)
\end{cases}
\]  

\[
\cos \beta_R' = \begin{cases} 
\frac{U_{R2}}{U_R} & (U_R \neq 0) \\
1 & (U_R = 0)
\end{cases}
\]  

Angular velocities in rotor wind axes

\[
\begin{bmatrix}
P_R \\
Q_R \\
R_R
\end{bmatrix} =
\begin{bmatrix}
\cos i_R \cos \beta_R' & -\sin \beta_R' & -\sin i_R \cos \beta_R' \\
-\cos i_R \sin \beta_R' & \cos \beta_R' & -\sin i_R \sin \beta_R' \\
\sin i_R & 0 & -\cos i_R
\end{bmatrix}
\begin{bmatrix}
P \\
Q \\
R
\end{bmatrix}
\]  

Advance ratio

\[ \mu_R = \frac{U_R}{\Omega_{RB}} \]
APPENDIX B

Component of inflow ratio

\[ \lambda_R^2 = \frac{W_{R2}}{\Omega_{RB}} \]  

(B21)

III Control Mixing and Coordinate Transformation

Front Rotor

\[ \theta_{OF} = \theta_{TF} + \theta_{FDB} \delta_B + \theta_{FDC} \delta_C \]  

(B22)

\[ A_{ICF} = A_{ICFDS} \delta_S + A_{ICFDR} \delta_R \]  

(B23)

\[ \left[ \begin{array}{c} A_{ICFR} \\ B_{ICFR} \end{array} \right] = \left[ \begin{array}{cc} \cos \beta_F & - \sin \beta_F \\ \sin \beta_F & \cos \beta_F \end{array} \right] \left[ \begin{array}{c} A_{ICF} \\ B_{ICF} \end{array} \right] \]  

(B24)

Rear Rotor

\[ \theta_{OR} = \theta_{TR} + \theta_{RDB} \delta_B + \theta_{RDC} \delta_C \]  

(B25)

\[ A_{ICR} = A_{ICRDS} \delta_S + A_{ICRDR} \delta_R \]  

(B26)

\[ \left[ \begin{array}{c} A_{ICRR} \\ B_{ICRR} \end{array} \right] = \left[ \begin{array}{cc} \cos \beta_R & \sin \beta_R \\ - \sin \beta_R & \cos \beta_R \end{array} \right] \left[ \begin{array}{c} A_{ICR} \\ B_{ICR} \end{array} \right] \]  

(B27)

The longitudinal cyclic trim terms \((B_{ICF}, B_{ICR})\) automatically introduce forward longitudinal cyclic pitch into both rotor systems as a function of airspeed, as shown in figure B1.

IV Thrust Coefficients

Front Rotor

\[ T_{CF} = \frac{\lambda_F}{2} + \frac{\theta_{OF}}{3} + \frac{\theta_T}{4} + \mu_F \left[ \frac{\theta_{OF}}{2} + \frac{\theta_T}{4} - \frac{B_{ICFR}}{2} \right] \]  

(B28)
APPENDIX B

\[(T_{CF})_{\text{break}} = \frac{0.288 - 0.48\mu_F}{a_S}\]  

\[T_{CF} = \begin{cases} 
T_{CF} & (T_{CF} \leq (T_{CF})_{\text{break}}) \\
(T_{CF})_{\text{break}} + \Delta T_{CF} & (T_{CF} > (T_{CF})_{\text{break}})
\end{cases}\]

where \(\Delta T_{CF}\) is computed from figure B2.

\[C_{TF} = \frac{a_S \sigma}{2} T_{CF}\]  

Figure B1.- Trim schedules of front and rear longitudinal cyclic pitch.
Figure B2.- Thrust limit nonlinear gain for both front and rear rotor.

Rear Rotor

\[
T_{CR} = \frac{\lambda_R}{2} + \frac{\theta_{OR}}{3} + \frac{\theta_T}{4} + \mu_R \left[ \frac{\theta_{OR}}{2} + \frac{\theta_T}{4} - \frac{B_{ICRR}}{2} \right]
\]  \hspace{1cm} (B32)

\[
(T_{CR})_{\text{break}} = \frac{0.288 - 0.48\mu_R}{a_S}
\]  \hspace{1cm} (B33)
APPENDIX B

\[ T_{CR} = \begin{cases} T_{CR} & (T_{CR} \leq (T_{CR})_{break}) \\ (T_{CR})_{break} + \Delta T_{CR} & (T_{CR} > (T_{CR})_{break}) \end{cases} \]  \hspace{1cm} (B34)

where \( \Delta T_{CR} \) is computed from figure B2.

\[ C_{TR} = \frac{a_{sg}}{2} T_{CR} \]  \hspace{1cm} (B35)

V Rotor Inflow Ratios and Interference Parameters

\[ R_{IF} = \tan^{-1}\left| \frac{\mu_F}{\lambda_F} \right| \]  \hspace{1cm} (B36)

\[ R_{IR} = \tan^{-1}\left| \frac{\mu_R}{\lambda_R} \right| \]  \hspace{1cm} (B37)

For \( U_X \geq 0 \):

\[ dF_{FR} = \left[ 0.356 + 0.321R_{IF} - 0.368(R_{IF})^2 + 0.392(R_{IF})^3 \right] \left(1 - |\sin \beta_F|\right) + \left[ 0.356 + 0.0131R_{IF} \right. \\
- 0.0764(R_{IF})^2 - 0.0085(R_{IF})^3 \left| \sin \beta_F \right| \]  \hspace{1cm} (B38)

\[ dF_{RF} = \left[ 0.356 - 0.151R_{IR} - 0.314(R_{IR})^2 + 0.164(R_{IR})^3 \right] \left(1 - |\sin \beta_R|\right) + \left[ 0.356 + 0.0131R_{IR} \right. \\
- 0.0764(R_{IR})^2 - 0.0085(R_{IR})^3 \left| \sin \beta_R \right| \]  \hspace{1cm} (B39)

For \( U_X < 0 \):

\[ dF_{RF} = \left[ 0.356 + 0.321R_{IR} - 0.368(R_{IR})^2 + 0.392(R_{IR})^3 \right] \left(1 - |\sin \beta_R|\right) + \left[ 0.356 + 0.0131R_{IR} \right. \\
- 0.0764(R_{IR})^2 - 0.0085(R_{IR})^3 \left| \sin \beta_R \right| \]  \hspace{1cm} (B40)

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APPENDIX B

\[
dF_{FR} = \left[ 0.356 - 0.151R_{IF} - 0.314(R_{IF})^2 + 0.164(R_{IF})^3 \right] \left( 1 - | \sin \beta_F^1 | \right) + \left[ 0.356 + 0.0131R_{IF} - 0.0764(R_{IF})^2 - 0.0085(R_{IF})^3 \right] | \sin \beta_F^1 |
\]

\[
\lambda_F = \lambda_F^1 - \frac{C_{TF}}{2 \left[ (\lambda_F)^2 + (\mu_F)^2 \right]^{1/2}} - \frac{dF_{RF}}{2 \left[ (\lambda_R)^2 + (\mu_R)^2 \right]^{1/2}} \frac{C_{TR}}{2 \left[ (\lambda_F)^2 + (\mu_F)^2 \right]^{1/2}}
\]

\[
\lambda_R = \lambda_R^1 - \frac{C_{TR}}{2 \left[ (\lambda_R)^2 + (\mu_R)^2 \right]^{1/2}} - \frac{dF_{RF}}{2 \left[ (\lambda_F)^2 + (\mu_F)^2 \right]^{1/2}} \frac{C_{TF}}{2 \left[ (\lambda_F)^2 + (\mu_F)^2 \right]^{1/2}}
\]

VI Rotor Forces and Moments

\[
F_H = \pi \rho (R_B)^4
\]

Front Rotor
Drag coefficient

For \( M_{T90F} - M_D \leq 0 \),

\[
\delta_{FH} = \delta_o + 2.07(T_{CF})^2
\]

For \( M_{T90F} - M_D > 0 \),

\[
\delta_{FH} = \delta_o + 2.07(T_{CF})^2 + 0.096(M_{T90F} - M_D) + 0.8(M_{T90F} - M_D)^3
\]

where

\[
M_D = 0.955 - 1.25tc
\]

and

\[
M_{T90F} = \frac{\Omega R_B}{331.6} \left( 1 + \sqrt{\left( \mu_F \right)^2 + \left( \lambda_F^1 \right)^2} \right)
\]
APPENDIX B

Thrust

\[ T_f = F_H C_T F \omega^2 \]  

(B48)

Coning angle

\[ A_{OF} = \frac{\gamma}{12} \left[ \frac{4}{3} T_{CF} + \frac{2}{3} \theta_{OF} + \frac{3}{4} \mu_F B_{ICFR} \right] - \frac{B_{ICFR}}{\gamma \Omega} - \frac{16}{\gamma \Omega} \left[ 1 + \frac{(\mu_F)^2}{2} \right] \]  

(B49)

Longitudinal flapping angle

\[ A_{IF} = \frac{4}{3} \left[ \frac{\mu_F}{1 - \frac{(\mu_F)^2}{2}} \left( \frac{\lambda_F}{2} + \frac{3}{4} \theta_{OF} + \frac{3}{8} \mu_F B_{ICFR} \right) - \frac{B_{ICFR}}{\gamma \Omega} \left[ 1 + \frac{(\mu_F)^2}{2} \right] \right] \]  

(B50)

Lateral flapping angle

\[ B_{IF} = \frac{4}{3} \left[ \frac{\mu_F}{1 + \frac{(\mu_F)^2}{2}} \right] A_{OF} + A_{ICFR} - \frac{16}{\gamma \Omega} \left[ 1 + \frac{(\mu_F)^2}{2} \right] \]  

(B51)

Side-force coefficient

\[ Y_{CF} = T_{CF} B_{IF} + \mu_F \left[ A_{IF} \left( \frac{B_{IF}}{4} - A_{ICFR} \right) - \frac{B_{ICFR}}{2} - \frac{3}{4} \theta_{OF} - \frac{3}{2} \lambda_F - \frac{\theta_T}{2} \right] \]  

(B52)

\[ + \lambda_F \left( \frac{B_{IF}}{4} - \frac{A_{ICFR}}{4} \right) + A_{OF} \left( \frac{B_{ICFR}}{6} + \frac{A_{IF}}{6} \right) \]

Side force

\[ Y_f = \frac{a_{SO}}{2} Y_{CF} F H \omega^2 \]  

(B53)
APPENDIX B

Horizontal-force coefficient

\[ H_{CF} = T_{CF} A_{IF} + \frac{\mu_F \delta_{FH}}{2a_S} \]  
(B54)

Horizontal force

\[ H_f = \frac{a_S \sigma}{2} H_{CF} F_H \Omega^2 \]  
(B55)

Torque coefficient

\[ Q_{CF} = \mu_F \left\{ \mu_F \left[ \frac{\delta_{FH}}{4a_S} + \frac{B_{ICFR} A_{IF}}{16} - \frac{3}{16} (A_{IF})^2 + \frac{A_{ICFR} B_{IF}}{16} - \frac{(B_{IF})^2}{16} - \frac{(A_{OF})^2}{4} \right] \right. \\
\left. + \lambda_F \left( \frac{B_{ICFR}}{4} - \frac{A_{IF}}{2} \right) - \frac{\lambda_{OF} A_{ICFR}}{6} + \frac{A_{OF} B_{IF}}{3} \right\} + \frac{\delta_H}{4a_S} - \frac{\theta_{OF} \lambda_F}{3} - \frac{\theta \lambda_F}{4} \]

\[ - \frac{B_{ICFR} A_{IF}}{8} + \frac{A_{ICFR} B_{IF}}{8} - \frac{(\lambda_F)^2}{2} - \frac{(A_{IF})^2}{8} - \frac{(B_{IF})^2}{8} \]  
(B56)

Torque required

\[ Q_{AEROF} = \frac{a_S \sigma}{2} Q_{CF} F_H \Omega^2 R_B \]  
(B57)

Longitudinal hub moment

\[ M_{HF} = \frac{e_S b_S}{2} M_W \Omega^2 A_{IF} \]  
(B58)

Lateral hub moment

\[ L_{HF} = \frac{e_S b_S}{2} M_W \Omega^2 B_{IF} \]  
(B59)
Rear Rotor

Drag coefficient

For $M_{T90R} - M_D \leq 0$,

$$\delta_{RH} = \delta_0 + 2.07(T_{CR})^2$$  \hspace{1cm} (B60)

For $M_{T90R} - M_D > 0$,

$$\delta_{RH} = \delta_0 + 2.07(T_{CR})^2 + 0.096(M_{T90R} - M_D) + 0.8(M_{T90R} - M_D)^3$$  \hspace{1cm} (B61)

where $M_D$ is defined by equation (B47a) and where

$$M_{T90R} = \frac{\Omega_{RB}}{331.6} \left(1 + \sqrt{\left(\mu_R\right)^2 + \left(\lambda_R\right)^2}\right)$$  \hspace{1cm} (B62)

Thrust

$$T_r = C_{TR}F_H\Omega^2$$  \hspace{1cm} (B63)

Coning angle

$$A_{OR} = \frac{\gamma}{12} \left[4T_{CR} + \frac{\theta_{OR}}{6} + \frac{\theta_T}{5} - \frac{(\mu_R)^2\theta_{OR}}{2}\right]$$  \hspace{1cm} (B64)

Longitudinal flapping angle

$$A_{IR} = \frac{4}{1 - \left(\frac{(\mu_R)}{2}\right)^2} \left[\mu_R \left(\frac{\lambda_R}{2} + \frac{2}{3}\theta_{OR} + \frac{\theta_T}{2} - \frac{3}{8} \mu_R B_{ICRR}\right) - \frac{B_{ICRR}}{4} \right] - \frac{16QR}{\gamma\Omega} \left[1 + \left(\frac{(\mu_R)}{2}\right)^2\right]$$  \hspace{1cm} (B65)

Lateral flapping angle

$$B_{IR} = \frac{4}{3} \left[\frac{\mu_R}{1 + \left(\frac{(\mu_R)}{2}\right)^2}\right] A_{OR} + A_{ICRR} - \frac{16PR}{\gamma\Omega} \left[1 - \left(\frac{(\mu_R)}{2}\right)^2\right]$$  \hspace{1cm} (B66)
Side-force coefficient

\[ Y_{CR} = T_{CR}B_{IR} + \mu_{R} \left[ A_{IR} \left( \frac{B_{IR}}{4} - \frac{A_{ICRR}}{4} - \mu_{RA_{OR}} \right) + A_{OR} \left( \frac{\mu_{RB_{ICRR}}}{2} - \frac{3}{4} \theta_{OR} - \frac{3}{2} \lambda_{R} - \frac{\theta_{T}}{2} \right) \right] \]

\[ + \lambda_{R} \left( \frac{B_{IR}}{4} - \frac{A_{ICRR}}{4} \right) + A_{OR} \left( \frac{B_{ICRR}}{6} + \frac{A_{IR}}{6} \right) \]

(B67)

Side force

\[ Y_{r} = \frac{a_{g}^{\sigma}}{2} Y_{CR} F_{H} \Omega^{2} \]  

(B68)

Horizontal-force coefficient

\[ H_{CR} = T_{CR}A_{IR} + \frac{\mu_{R} \delta_{R}}{2a_{S}} \]  

(B69)

Horizontal force

\[ H_{r} = \frac{a_{g}^{\sigma}}{2} H_{CR} F_{H} \Omega^{2} \]  

(B70)

Torque coefficient

\[ Q_{CR} = \mu_{R} \left\{ \mu_{R} \left[ \frac{\delta_{RH}}{4a_{S}} + \frac{B_{ICRR}A_{IR}}{16} - \frac{3}{16} \left( A_{IR} \right)^{2} + \frac{A_{ICRR}B_{IR}}{16} - \frac{\left( B_{IR} \right)^{2}}{16} - \frac{\left( A_{OR} \right)^{2}}{4} \right] \right\} \]

\[ + \lambda_{R} \left( \frac{B_{ICRR}}{4} - \frac{A_{IR}}{2} \right) - \frac{A_{OR}A_{ICRR}}{6} + \frac{A_{OR}B_{IR}}{3} \right\} \]

\[ + \frac{\delta_{H}}{4a_{S}} - \frac{\theta_{OR} \lambda_{R}}{3} - \frac{\theta_{T} \lambda_{R}}{4} - \frac{B_{ICRR}A_{IR}}{8} \]

\[ + \frac{A_{ICRR}B_{IR}}{8} - \frac{\left( \lambda_{R} \right)^{2}}{2} - \frac{\left( A_{IR} \right)^{2}}{8} - \frac{\left( B_{IR} \right)^{2}}{8} \]  

(B71)

Torque required

\[ Q_{AEROR} = \frac{a_{g}^{\sigma}}{2} Q_{CR} F_{H} \Omega^{2} R_{B} \]  

(B72)
APPENDIX B

Longitudinal hub moment

\[ M_{HR} = \frac{e_{bs} S}{2} M_{w} \Omega_{AIR}^2 \]  \hspace{1cm} (B73)

Lateral hub moment

\[ L_{HR} = \frac{e_{bs} S}{2} M_{w} \Omega_{BIR}^2 \]  \hspace{1cm} (B74)

VII Fuselage Forces and Moments

Vertical velocity with downwash

\[ W_{FUS} = W + \left[ (\lambda_F - \lambda_F^1) + (\lambda_R - \lambda_R^1) \right] \Omega_{RB} \]  \hspace{1cm} (B75)

\[ D_{FUS1} = \left[ U^2 + (W_{FUS})^2 \right]^{1/2} \]  \hspace{1cm} (B76)

\[ D_{FUS2} = (U^2 + V^2)^{1/2} \]  \hspace{1cm} (B77)

Fuselage angle of attack

\[ \sin \alpha_{fus} = \begin{cases} \frac{W_{FUS}}{D_{FUS1}} & (D_{FUS1} \neq 0) \\ 0 & (D_{FUS1} = 0) \end{cases} \]  \hspace{1cm} (B78)

\[ \cos \alpha_{fus} = \begin{cases} \frac{U}{D_{FUS1}} & (D_{FUS1} \neq 0) \\ 1 & (D_{FUS1} = 0) \end{cases} \]  \hspace{1cm} (B79)

\[ \alpha_{fus} = \begin{cases} \tan^{-1} \left( \frac{W_{FUS}}{U} \right) & (D_{FUS1} \neq 0) \\ 0 & (D_{FUS1} = 0) \end{cases} \]  \hspace{1cm} (B80)
APPENDIX B

Fuselage angle of sideslip

\[
\sin \beta_{\text{fus}} = \begin{cases} \frac{V}{D_{\text{FUS}2}} & (D_{\text{FUS}2} \neq 0) \\ 0 & (D_{\text{FUS}2} = 0) \end{cases}
\]

\[
\cos \beta_{\text{fus}} = \begin{cases} \frac{U}{D_{\text{FUS}2}} & (D_{\text{FUS}2} \neq 0) \\ 1 & (D_{\text{FUS}2} = 0) \end{cases}
\]

\[
\beta_{\text{fus}} = \begin{cases} \tan^{-1}\left(\frac{V}{U}\right) & (D_{\text{FUS}2} \neq 0) \\ 0 & (D_{\text{FUS}2} = 0) \end{cases}
\]

Dynamic pressure

\[
Q_{\text{DPRES}} = \frac{1}{2} \rho (U^2 + V^2 + W_F^2_{\text{FUS}})
\]

Fuselage forces

\[
X_{\text{fus}} = \begin{cases} -C_{\text{FE}} Q_{\text{DPRES}} & (U \geq 0) \\ C_{\text{FE}} Q_{\text{DPRES}} & (U < 0) \end{cases}
\]

where the flat-plate drag of the fuselage \(C_{\text{FE}}\) is found from figure B3 as a function of the angles of attack and sideslip.

\[
Y_{\text{fus}} = -C_Y \beta Q_{\text{DPRES}} \sin \beta_{\text{fus}}
\]

\[
Z_{\text{fus}} = -C_L \alpha Q_{\text{DPRES}} \sin \alpha_{\text{fus}}
\]

Fuselage moments

\[
L_{\text{FUS}} = -C_L \beta Q_{\text{DPRES}} \sin \beta_{\text{fus}} \cos \beta_{\text{fus}} \left(1 - |\sin \alpha_{\text{fus}}|\right)
\]
APPENDIX B

\[ M_{\text{FUS}} = C_{Ma} Q \cdot \text{D PRESS} \sin \alpha_{\text{fus}} \cos \alpha_{\text{fus}} \]  \hspace{1cm} (B89)

\[ N_{\text{FUS}} = -C_{N\beta} Q \cdot \text{D PRESS} \sin \beta_{\text{fus}} \cos \beta_{\text{fus}} (0.94 \sin \alpha_{\text{fus}} + 0.342 \cos \alpha_{\text{fus}}) \]  \hspace{1cm} (B90)

Figure B3.- Flat-plate drag area of fuselage as a function of angle of attack and sideslip angle.

VIII Total System Forces and Moments

Define the following:

\[ C_{F1} = \sin \beta'_F \]  \hspace{1cm} (B91a)

\[ C_{F2} = \cos \beta'_F \]  \hspace{1cm} (B91b)
APPENDIX B

\[ C_{F3} = \sin i_F \]  
\[ C_{F4} = \cos i_F \]  
\[ C_{R1} = \sin \beta'_R \]  
\[ C_{R2} = \cos \beta'_R \]  
\[ C_{R3} = \sin i_R \]  
\[ C_{R4} = \cos i_R \]  

Total forces

\[
\begin{bmatrix}
  F_X \\
  F_Y \\
  F_Z
\end{bmatrix} = \begin{bmatrix}
  X_{fus} \\
  Y_{fus} \\
  Z_{fus}
\end{bmatrix} + \begin{bmatrix}
  -C_{F2}C_{F4} & -C_{F1}C_{F4} & C_{F3} \\
  -C_{F1} & C_{F2} & 0 \\
  -C_{F2}C_{F3} & -C_{F1}C_{F3} & -C_{F4}
\end{bmatrix} \begin{bmatrix}
  H_f \\
  Y_f \\
  T_f
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
  -C_{R2}C_{R4} & C_{R1}C_{R4} & C_{R3} \\
  -C_{R1} & -C_{R2} & 0 \\
  -C_{R2}C_{R3} & C_{R1}C_{R3} & -C_{R4}
\end{bmatrix} \begin{bmatrix}
  H_r \\
  Y_r \\
  T_r
\end{bmatrix}
\]  

Moments resulting from rotor forces

\[
L_P = h_f(-C_{F1}H_f + C_{F2}Y_f) - h_r(C_{R1}H_r + C_{R2}Y_r)
\]  

\[
M_P = h_f(C_{F2}C_{F4}h_f + C_{F2}C_{F3}l_f) + h_r(C_{R2}C_{R4}h_r - C_{R2}C_{R3}l_r) + Y_f(C_{F1}C_{F4}h_f + C_{F1}C_{F3}l_f)
\]

\[
+ Y_r(-C_{R1}C_{R4}h_r + C_{R1}C_{R3}l_r) + T_f(-C_{F3}h_f + C_{F4}l_f) + T_r(-C_{R3}h_r - C_{R4}l_r)
\]  

\[
N_P = l_f(-C_{F1}H_f + C_{F2}Y_f) + l_r(C_{R1}H_r + C_{R2}Y_r)
\]
APPENDIX B

Total moments

\[
\begin{bmatrix}
  L \\
  M \\
  N
\end{bmatrix} = \begin{bmatrix} L_{FUS} \\ M_{FUS} \\ N_{FUS} \end{bmatrix} + \begin{bmatrix} L_P \\ M_P \\ N_P \end{bmatrix} + \begin{bmatrix}
  C_{F2}C_{F4} & -C_{F1}C_{F4} & -C_{F3} \\
  C_{F1} & C_{F2} & 0 \\
  C_{F2}C_{F3} & -C_{F1}C_{F3} & C_{F4}
\end{bmatrix} \begin{bmatrix} L_{HF} \\ M_{HF} \\ Q_{AEROF} \end{bmatrix}
\]

\[
\begin{bmatrix}
  -C_{R2}C_{R4} & -C_{R1}C_{R4} & C_{R3} \\
  -C_{R1} & C_{R2} & 0 \\
  -C_{R2}C_{R3} & -C_{R1}C_{R3} & -C_{R4}
\end{bmatrix} \begin{bmatrix} L_{HR} \\ M_{HR} \\ Q_{AEROR} \end{bmatrix}
\]

(B96)
APPENDIX C

CONSTANTS FOR CH-47B

Constants used in this report are for the center of gravity of the vehicle located at fuselage station line 338 (0.1778 m) aft and at water line 18.7 (1.309 m). Numerical values of these constants are as follows:

\[ \text{AICFDR} = 125 \text{ deg/m (3.18 deg/in.)} \]
\[ \text{AICRDR} = 125 \text{ deg/m (3.18 deg/in.)} \]
\[ \text{AICFDS} = 75.2 \text{ deg/m (1.91 deg/in.)} \]
\[ \text{AICRDS} = -75.2 \text{ deg/m (-1.91 deg/in.)} \]
\[ a_S = 5.75 \]
\[ b_S = 3 \]
\[ C_{L\alpha} = 32.5 \text{ m}^2/\text{rad (350 ft}^2/\text{rad)}} \]
\[ C_{L\beta} = 6.57 \text{ m}^3/\text{rad (232 ft}^3/\text{rad)}} \]
\[ C_{M\alpha} = 142 \text{ m}^3/\text{rad (5000 ft}^3/\text{rad)}} \]
\[ C_{N\beta} = 51.5 \text{ m}^3/\text{rad (1820 ft}^3/\text{rad)}} \]
\[ C_{Y\beta} = 43.4 \text{ m}^2/\text{rad (467 ft}^2/\text{rad)}} \]
\[ e_S = 0.203 \text{ m (0.667 ft) } \]
\[ h_f = 2.093 \text{ m (6.867 ft)} \]
\[ h_r = 3.527 \text{ m (11.57 ft)} \]
APPENDIX C

\( I_{XX} = 50\,386.3\,\text{kg}\cdot\text{m}^2 \ (37\,163\ \text{slug}\cdot\text{ft}^2) \)

\( I_{YY} = 273\,536\,\text{kg}\cdot\text{m}^2 \ (201\,750\ \text{slug}\cdot\text{ft}^2) \)

\( I_{ZZ} = 257\,685\,\text{kg}\cdot\text{m}^2 \ (190\,059\ \text{slug}\cdot\text{ft}^2) \)

\( I_{XZ} = 19\,838.3\,\text{kg}\cdot\text{m}^2 \ (14\,632\ \text{slug}\cdot\text{ft}^2) \)

\( i_f = 9.0^\circ \)

\( i_r = 4.0^\circ \)

\( l_t = 6.425\ \text{m} \ (21.08\ \text{ft}) \)

\( l_r = 5.450\ \text{m} \ (17.88\ \text{ft}) \)

\( M_W = 510.2\ \text{kg}\cdot\text{m} \ (114.7\ \text{slug}\cdot\text{ft}) \)

\( m = 14\,968.6\ \text{kg} \ (1\,025.67\ \text{slug}) \)

\( R_B = 9.144\ \text{m} \ (30\ \text{ft}) \)

\( \gamma = 8.26 \)

\( \delta_0 = 0.0094 \)

\( \theta_{FDB} = 24.2\ \text{deg/m} \ (0.615\ \text{deg/in.}) \)

\( \theta_{RDB} = -24.2\ \text{deg/m} \ (-0.615\ \text{deg/in.}) \)

\( \theta_{FDC} = 73.4\ \text{deg/m} \ (1.864\ \text{deg/in.}) \)

\( \theta_{RDC} = 73.4\ \text{deg/m} \ (1.864\ \text{deg/in.}) \)
APPENDIX C

\( \theta_T = -9.14^\circ \)

\( \theta_{TF} = 7.85^\circ \)

\( \theta_{TR} = 7.85^\circ \)

\( \rho = 1.227 \text{ kg/m}^3 \ (0.00238 \text{ slug/ft}^3) \)

\( \sigma = 0.067 \)

\( \Omega_0 = 24 \text{ rad/sec} \)
APPENDIX D

LINEAR PERTURBATION MODEL

This appendix presents the development of the linear perturbation model in both the coupled and uncoupled forms. For completeness, details are included which show the relationship of the nonlinear model and the perturbation linear model, the method of handling a general trim condition (nonstatic trim conditions), and the assumption required to arrive at the coupled and uncoupled models used in this report.

A nonlinear model of a vehicle can be considered as a transformation of the vehicle state $\vec{x}_T$ and control $\vec{\delta}_T$ into forces $\vec{F}_T$ and moments $\vec{M}_T$ applied to the vehicle (fig. D1(a)); that is,

\[
\begin{align*}
\vec{F}_T &= f_1(\vec{x}_T, \vec{\delta}_T) \\
\vec{M}_T &= f_2(\vec{x}_T, \vec{\delta}_T)
\end{align*}
\]

An equivalent representation of the vehicle motions, which is more convenient for control system design, is to have as outputs the rate of change of the state variables (see fig. D1(b)). The general nonlinear representation is

\[
\dot{\vec{x}}_T = f(\vec{x}_T, \vec{\delta}_T)
\]

The transformation between equations (D1) and (D2) is given in reference 7. It is possible to represent equation (D2) by a series expanded about a nominal condition $(\vec{x}_N, \vec{\delta}_N)$ as

\[
\dot{\vec{x}}_T = \dot{\vec{x}}_N + \left. \frac{\partial f}{\partial \vec{x}_T} \right|_{\vec{x}_T = \vec{x}_N, \vec{\delta}_T = \vec{\delta}_N} (\vec{x}_T - \vec{x}_N) + \left. \frac{\partial f}{\partial \vec{\delta}_T} \right|_{\vec{x}_T = \vec{x}_N, \vec{\delta}_T = \vec{\delta}_N} (\vec{\delta}_T - \vec{\delta}_N) + \text{Higher order terms}
\]

If the perturbations (fig. D1(c))

\[
\vec{x}_P = \vec{x}_T - \vec{x}_N
\]

and

\[
\vec{\delta}_P = \vec{\delta}_T - \vec{\delta}_N
\]
APPENDIX D

(a) Force and moment output of nonlinear model.

(b) Angular and linear acceleration output of nonlinear model.

(c) Angular and linear acceleration output of linear model.

Figure D1.- Nonlinear and linear representation of vehicle dynamics.

are sufficiently small, equation (D3) can be represented by

\[ \dot{\vec{X}}_T = \dot{\vec{X}}_N + [A]\vec{X}_P + [B]\vec{\delta}_P \]  

(D6)

where

\[ [A] = \left. \frac{\partial f}{\partial \vec{X}_T} \right|_{\vec{X}_T = \vec{X}_N, \vec{\delta}_T = \vec{\delta}_N} \]  

(D7)

\[ [B] = \left. \frac{\partial f}{\partial \vec{\delta}_T} \right|_{\vec{X}_T = \vec{X}_N, \vec{\delta}_T = \vec{\delta}_N} \]  

(D8)
APPENDIX D

In this report the state vector is defined by

\[
\bar{X} = \begin{bmatrix}
\vec{V}_B \\
\vec{\omega}_B \\
\theta \\
\phi
\end{bmatrix}
\]  

(D9)

where \( \vec{V}_B = (U, V, W)^T \) is the vehicle linear velocity expressed in body coordinates, \( \vec{\omega}_B = (P, Q, R)^T \) is the vehicle body rate expressed in body coordinates, and \( \theta \) and \( \phi \) are the vehicle pitch and roll angles. The control vector \( \bar{\delta} \) is given by

\[
\bar{\delta} = \begin{bmatrix}
\delta_B \\
\delta_C \\
\delta_S \\
\delta_R
\end{bmatrix}
\]  

(D10)

where \( \delta_B \) is differential collective stick, \( \delta_C \) is collective stick, \( \delta_S \) is cyclic stick, and \( \delta_R \) is differential cyclic stick.

The nominal values \( \bar{X}_N, \bar{\delta}_N \) and the coefficient matrices \( [A], [B] \) are functions of the flight condition. This report considers nonaccelerating and nonrotating flight paths, i.e., \( \dot{\bar{X}}_N = 0 \), for which the nominal state is given by

\[
\bar{X}_N = \begin{bmatrix}
U_N \\
V_N \\
W_N \\
0 \\
0 \\
\theta_N \\
\phi_N
\end{bmatrix}
\]  

(D11)

For this condition, equation (D6) is given by
\[
\begin{bmatrix}
\frac{X_{\delta B}}{m} & \frac{X_{\delta C}}{m} & \frac{X_{\delta S}}{m} & \frac{X_{\delta R}}{m} \\
\frac{Y_{\delta B}}{m} & \frac{Y_{\delta C}}{m} & \frac{Y_{\delta S}}{m} & \frac{Y_{\delta R}}{m} \\
\frac{Z_{\delta B}}{m} & \frac{Z_{\delta C}}{m} & \frac{Z_{\delta S}}{m} & \frac{Z_{\delta R}}{m}
\end{bmatrix}
\]
\[
\begin{bmatrix}
\frac{M_{\delta B}}{I_{YY}} & \frac{M_{\delta C}}{I_{YY}} & \frac{M_{\delta S}}{I_{YY}} & \frac{M_{\delta R}}{I_{YY}}
\end{bmatrix}
\]
\[
\begin{bmatrix}
\frac{(I_1 L_{\delta B} + I_3 N_{\delta B})}{I_{XX}} & \frac{(I_1 L_{\delta C} + I_3 N_{\delta C})}{I_{XX}} & \frac{(I_1 L_{\delta S} + I_3 N_{\delta S})}{I_{XX}} & \frac{(I_1 L_{\delta R} + I_3 N_{\delta R})}{I_{XX}} \\
\frac{(I_2 L_{\delta B} + I_1 N_{\delta B})}{I_{ZZ}} & \frac{(I_2 L_{\delta C} + I_1 N_{\delta C})}{I_{ZZ}} & \frac{(I_2 L_{\delta S} + I_1 N_{\delta S})}{I_{ZZ}} & \frac{(I_2 L_{\delta R} + I_1 N_{\delta R})}{I_{ZZ}}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

where

\[
I_1 = \frac{I_{XX}I_{ZZ}}{I_{XX}I_{ZZ} - I_{XZ}^2} \quad I_2 = \frac{I_{XX}I_{XZ}}{I_{XX}I_{ZZ} - I_{XZ}^2} \quad I_3 = \frac{I_{ZZ}I_{XZ}}{I_{XX}I_{ZZ} - I_{XZ}^2}
\]
Equation (D12) represents the coupled motion of the longitudinal variables (U, W, Q, \( \theta \)) and the lateral variables (R, P, V, \( \phi \)). Coupling refers to the fact that perturbations in longitudinal variables affect the rate of change of lateral variables and vice versa. For many vehicles, this coupling is not strong enough to influence the motion of the vehicle. For these cases, the cross derivatives and \( \phi_N \) are assumed equal to zero, and the set of eight coupled equations can be written as two sets of four equations each. The longitudinal equations are

\[
\begin{bmatrix}
\dot{U}_P \\
\dot{W}_P \\
\dot{Q}_P \\
\dot{\theta}_P
\end{bmatrix} =
\begin{bmatrix}
\frac{X_U}{m} & \frac{X_W}{m} & \left(\frac{X_Q}{m} - W_N\right) & -g \cos \theta_N \\
\frac{Z_U}{m} & \frac{Z_W}{m} & \left(U_N + \frac{Z_Q}{m}\right) & -g \sin \theta_N \\
M_U & M_W & M_Q & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
U_P \\
W_P \\
Q_P \\
\theta_P
\end{bmatrix}
+ \begin{bmatrix}
\frac{X_\delta_B}{m} & \frac{X_\delta_C}{m} \\
\frac{Z_\delta_B}{m} & \frac{Z_\delta_C}{m} \\
M_\delta_B & M_\delta_C \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\delta_{B,P} \\
\delta_{C,P}
\end{bmatrix}
\tag{D13}
\]

and the lateral equations are

\[
\begin{bmatrix}
\dot{P}_P \\
\dot{\phi}_P \\
\dot{R}_P \\
\dot{V}_P
\end{bmatrix} =
\begin{bmatrix}
\frac{I_1 L_P}{I_{XX}} + \frac{I_3 N_P}{I_{ZZ}} & 0 & \frac{I_1 L_R}{I_{XX}} + \frac{I_3 N_R}{I_{ZZ}} & \frac{I_1 L_V}{I_{XX}} + \frac{I_2 N_V}{I_{ZZ}} \\
1 & 0 & \tan \theta_N & 0 \\
\frac{I_2 L_P}{I_{XX}} + \frac{I_1 N_P}{I_{ZZ}} & 0 & \frac{I_2 L_R}{I_{XX}} + \frac{I_1 N_R}{I_{ZZ}} & \frac{I_2 L_V}{I_{XX}} + \frac{I_1 N_V}{I_{ZZ}} \\
W_N + \frac{Y_P}{m} & g \cos \theta_N & \frac{Y_R}{m} - U_N & \frac{Y_V}{m}
\end{bmatrix}
\begin{bmatrix}
P_P \\
\phi_P \\
R_P \\
V_P
\end{bmatrix}
+ \begin{bmatrix}
\frac{I_1 L_\delta S}{I_{XX}} + \frac{I_3 N_\delta S}{I_{ZZ}} & \frac{I_1 L_\delta R}{I_{XX}} + \frac{I_3 N_\delta R}{I_{ZZ}} \\
0 & 0 \\
\frac{I_2 L_\delta S}{I_{XX}} + \frac{I_1 N_\delta S}{I_{ZZ}} & \frac{I_2 L_\delta R}{I_{XX}} + \frac{I_1 N_\delta R}{I_{ZZ}} \\
Y_\delta S & Y_\delta R
\end{bmatrix}
\begin{bmatrix}
\delta_{S,P} \\
\delta_{R,P}
\end{bmatrix}
\tag{D14}
\]
APPENDIX E

TRIM AND DERIVATIVE DATA

A complete set of trims and derivatives are shown in tables E1 and E2 for sea-level conditions and for the flight envelope \(-40 \leq \dot{X} \leq 160\) knots and \(-10.16 \leq \dot{Z} \leq 10.16\) m/sec (\(\pm 2000\) ft/min). The trims and derivatives are given in the International System of Units (SI) in table E1 and in the U.S. Customary Units in table E2. These data are based upon a center of gravity of 0.1778 m (7 in.) aft of the vehicle reference point. All of the force derivatives are normalized by \(m\), the rolling moments by \(I_{XX}\), the pitching moments by \(I_{YY}\), and the yawing moments by \(I_{ZZ}\). Values for \(m\), \(I_{XX}\), \(I_{YY}\), and \(I_{ZZ}\) are given in appendix C.
## APPENDIX E

### TABLE E1. - STABILITY DERIVATIVES AND TRIMS IN SI UNITS

(a) \( \dot{Z} = -10.16 \text{ m/sec} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit (a)</th>
<th>Value of parameter at X, knots, of</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>1/\text{sec}</td>
<td>( -10.16 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( l )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( m )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( n )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( o )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( p )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( q )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( r )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( s )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( t )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( u )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( v )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( w )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( x )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( y )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>( z )</td>
<td>1/knots</td>
<td>( -0.0000 \times 10^{-6} )</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

(a) Denotes \( \dot{Z} = -10.16 \text{ m/sec} \)
TABLE E1. - Continued

(b) $\dot{z} = -7.62 \text{ m/sec}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>-40</th>
<th>-20</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
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<td>$\xi / \text{m}$</td>
<td>1/SEC</td>
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<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
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<td>0.02592</td>
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</tr>
<tr>
<td>$\eta / \text{m}$</td>
<td>1/SEC</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
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<td>0.02592</td>
<td>0.02592</td>
</tr>
<tr>
<td>$\dot{\xi} / \text{m}$</td>
<td>1/SEC</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
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<tr>
<td>$\dot{\eta} / \text{m}$</td>
<td>1/SEC</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
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</tr>
<tr>
<td>$\xi$</td>
<td>1/SEC</td>
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<td>$\eta$</td>
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<td>1/SEC</td>
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<td>0.02592</td>
<td>0.02592</td>
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<tr>
<td>$\dot{\eta}$</td>
<td>1/SEC</td>
<td>0.02592</td>
<td>0.02592</td>
<td>0.02592</td>
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</tr>
<tr>
<td>$\xi$</td>
<td>1/SEC</td>
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<td>0.02592</td>
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<td>$\dot{\eta}$</td>
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</table>

* DENotes the fitting of a three range group (one near zero, $\pm 20$ and $\pm 40$) values.
### APPENDIX E

**TABLE E1.** - Continued

(c) \( \hat{z} = -5.08 \text{ m/sec} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at ( X ), knots, of</th>
<th>( 120 )</th>
<th>( 140 )</th>
<th>( 160 )</th>
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<tbody>
<tr>
<td>( U_{AV}, F )</td>
<td>( \text{m/s} )</td>
<td>( -0.0472 )</td>
<td>( -0.0527 )</td>
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<td>( -0.0232 )</td>
<td>( -0.0271 )</td>
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<td>( -0.0282 )</td>
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<td>( -0.0345 )</td>
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<td>( -0.0446 )</td>
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<td>( -0.0505 )</td>
<td>( -0.0545 )</td>
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<td>( -0.0946 )</td>
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<tr>
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<td>( -1.123 )</td>
<td>( -1.180 )</td>
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<td>( -1.084 )</td>
<td>( -1.150 )</td>
<td>( -1.221 )</td>
</tr>
</tbody>
</table>

* Derived by fitting a third degree curve through \( x_1, x_2, x_3 \) and \( x_4 \) values.
TABLE E1.- Continued

(d) \( \dot{Z} = -2.54 \text{ m/sec} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
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<th>-20</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
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<tbody>
<tr>
<td>( u_{k} )</td>
<td>/SEC</td>
<td>-2.69</td>
<td>-2.24</td>
<td>-1.79</td>
<td>-1.33</td>
<td>-0.88</td>
<td>-0.41</td>
<td>-0.05</td>
<td>0.45</td>
<td>0.90</td>
<td>1.35</td>
<td>1.80</td>
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<tr>
<td>( v_{k} )</td>
<td>/SEC</td>
<td>-2.69</td>
<td>-2.24</td>
<td>-1.79</td>
<td>-1.33</td>
<td>-0.88</td>
<td>-0.41</td>
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<td>0.90</td>
<td>1.35</td>
<td>1.80</td>
</tr>
<tr>
<td>( w_{k} )</td>
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<td>-2.69</td>
<td>-2.24</td>
<td>-1.79</td>
<td>-1.33</td>
<td>-0.88</td>
<td>-0.41</td>
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<td>0.90</td>
<td>1.35</td>
<td>1.80</td>
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<tr>
<td>( k_{x} )</td>
<td>/SEC</td>
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<td>-2.24</td>
<td>-1.79</td>
<td>-1.33</td>
<td>-0.88</td>
<td>-0.41</td>
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<td>-2.24</td>
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<td>-0.88</td>
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<td>0.90</td>
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<td>1.80</td>
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<tr>
<td>( \dot{Z} )</td>
<td>/SEC</td>
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<td>-2.24</td>
<td>-1.79</td>
<td>-1.33</td>
<td>-0.88</td>
<td>-0.41</td>
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<td>-0.41</td>
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<td>1.35</td>
<td>1.80</td>
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</table>

* Derived by fitting a 9-term power curve through 900, 750, 600 and 450 values.
APPENDIX E

TABLE E1.- Continued

(e) \( \dot{z} = 0 \) m/sec

<table>
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<th>Parameter</th>
<th>Unit</th>
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<th>120</th>
<th>140</th>
<th>160</th>
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<tbody>
<tr>
<td>( u_1 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
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<td>-0.0000</td>
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</tr>
<tr>
<td>( u_2 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
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<td>-0.0020</td>
<td>-0.0010</td>
<td>-0.0005</td>
<td>-0.0003</td>
<td>-0.0002</td>
<td>-0.0001</td>
<td>-0.0000</td>
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</tr>
<tr>
<td>( v_1 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
<td>-0.0010</td>
<td>-0.0005</td>
<td>-0.0003</td>
<td>-0.0001</td>
<td>-0.0000</td>
<td>-0.0000</td>
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<td>-0.0000</td>
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<tr>
<td>( v_2 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
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<td>-0.0002</td>
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<td>-0.0000</td>
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</tr>
<tr>
<td>( w_1 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
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<td>-0.0000</td>
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<tr>
<td>( w_2 )</td>
<td>( \frac{\text{m}}{\text{s}} )</td>
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<td>-0.0000</td>
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<tr>
<td>( \theta_1 )</td>
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</tbody>
</table>

Value of parameter at \( X \), knots, of

- \( u_1 \) = 0 m/sec
- \( u_2 \) = 0 m/sec
- \( v_1 \) = 0 m/sec
- \( v_2 \) = 0 m/sec
- \( w_1 \) = 0 m/sec
- \( w_2 \) = 0 m/sec
- \( \theta_1 \) = 0 deg
- \( \theta_2 \) = 0 deg
TABLE E1. - Continued

(f) \( \dot{z} = 2.54 \text{ m/sec} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
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<th>(-20)</th>
<th>(0)</th>
<th>Value of parameter at ( X ), knots, of</th>
</tr>
</thead>
<tbody>
<tr>
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<td>(0.0185)</td>
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<tr>
<td>( v/\text{m} )</td>
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<td>(-0.0123)</td>
<td>(-0.0127)</td>
<td>(0.0064)</td>
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</tr>
<tr>
<td>( w/\text{m} )</td>
<td>1/SEC</td>
<td>(-0.0294)</td>
<td>(0.0219)</td>
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<tr>
<td>( \gamma )</td>
<td>1/SEC</td>
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<td>(0.0219)</td>
<td>(0.0034)</td>
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<tr>
<td>( K_{\text{eff}} )</td>
<td>1/SEC</td>
<td>(-0.0294)</td>
<td>(0.0219)</td>
<td>(0.0034)</td>
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<tr>
<td>( K_{\text{eff}} )</td>
<td>1/SEC</td>
<td>(-0.0294)</td>
<td>(0.0219)</td>
<td>(0.0034)</td>
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</table>

* Obtained by fitting to the values given for \( x_{\text{eff}}, \gamma, K_{\text{eff}} \) and \( X \) and \( X \) values.

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### APPENDIX E

### TABLE E1.- Continued

(g) $\tilde{z} = 5.08 \text{ m/sec}$

<table>
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<th>(10^3 60)</th>
<th>(10^3 80)</th>
<th>(10^3 100)</th>
<th>(10^3 120)</th>
<th>(10^3 140)</th>
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</thead>
<tbody>
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<td>#/sec</td>
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<td>(3.119)</td>
<td>(3.119)</td>
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<td>(v/v)</td>
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<td>(v/v)</td>
<td>#/sec</td>
<td>(2.524)</td>
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<tr>
<td>(v/v)</td>
<td>#/sec</td>
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<td>(3.119)</td>
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<td>(3.119)</td>
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</tr>
<tr>
<td>(v/v)</td>
<td>#/sec</td>
<td>(2.524)</td>
<td>(3.119)</td>
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<td>(3.119)</td>
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<td>(v/v)</td>
<td>#/sec</td>
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<td>(3.119)</td>
<td>(3.119)</td>
<td>(3.119)</td>
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</tbody>
</table>

*Note: \(v/v\) values are the ratio of the flow velocity to the wind velocity.*

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APPENDIX E

TABLE E1. - Continued

(h) \( \dot{Z} = 7.62 \text{ m/sec} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
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<th>(0)</th>
<th>(10)</th>
<th>(20)</th>
<th>(40)</th>
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<th>(80)</th>
<th>(100)</th>
<th>(120)</th>
<th>(140)</th>
<th>(160)</th>
</tr>
</thead>
<tbody>
<tr>
<td>XU (n)</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.172)</td>
<td>(-0.162)</td>
<td>(-0.152)</td>
<td>(-0.143)</td>
<td>(-0.134)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
</tr>
<tr>
<td>XU (v)</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.161)</td>
<td>(-0.152)</td>
<td>(-0.143)</td>
<td>(-0.134)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
</tr>
<tr>
<td>XU (w)</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.151)</td>
<td>(-0.143)</td>
<td>(-0.134)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
<td>(-0.045)</td>
</tr>
<tr>
<td>(v_{x})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.151)</td>
<td>(-0.143)</td>
<td>(-0.134)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
<td>(-0.045)</td>
</tr>
<tr>
<td>(w_{y})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.141)</td>
<td>(-0.134)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
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<td>(-0.036)</td>
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<tr>
<td>(w_{z})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.131)</td>
<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
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<td>(-0.054)</td>
<td>(-0.045)</td>
<td>(-0.036)</td>
<td>(-0.027)</td>
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<tr>
<td>(v_{x})</td>
<td>(\text{ft} / \text{sec})</td>
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<td>(-0.125)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
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<tr>
<td>(w_{y})</td>
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<td>(-0.121)</td>
<td>(-0.116)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
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<td>(-0.036)</td>
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<tr>
<td>(w_{z})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.111)</td>
<td>(-0.107)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
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<tr>
<td>(v_{x})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.101)</td>
<td>(-0.098)</td>
<td>(-0.090)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
<td>(-0.045)</td>
<td>(-0.036)</td>
<td>(-0.027)</td>
<td>(-0.018)</td>
<td>(-0.009)</td>
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</tr>
<tr>
<td>(w_{y})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.091)</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
<td>(-0.045)</td>
<td>(-0.036)</td>
<td>(-0.027)</td>
<td>(-0.018)</td>
<td>(-0.009)</td>
<td>(-0.000)</td>
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<td>(-0.000)</td>
</tr>
<tr>
<td>(w_{z})</td>
<td>(\text{ft} / \text{sec})</td>
<td>(-0.081)</td>
<td>(-0.072)</td>
<td>(-0.063)</td>
<td>(-0.054)</td>
<td>(-0.045)</td>
<td>(-0.036)</td>
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<td>(-0.018)</td>
<td>(-0.009)</td>
<td>(-0.000)</td>
<td>(-0.000)</td>
<td>(-0.000)</td>
<td>(-0.000)</td>
</tr>
</tbody>
</table>

* Definitions: \( X \equiv \text{X-component of \( \dot{X} \)}, \( Y \equiv \text{Y-component of \( \dot{Y} \)}, \( Z \equiv \text{Z-component of \( \dot{Z} \)} *, etc., where \( \theta \), \( \phi \), \( \psi \), and \( \gamma \) are constant.
### APPENDIX E

#### TABLE E1 - Concluded

(i) $\dot{Z} = 10.16 \text{ m/sec}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at X, knots, of</th>
<th>Value of parameter at X, knots, of</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{TU}^1$</td>
<td>$\text{SEC}$</td>
<td>$-0.84243$</td>
<td>$-0.12805$</td>
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<tr>
<td>$\text{TU}^1$</td>
<td>$\text{SEC}$</td>
<td>$0.5745$</td>
<td>$0.12735$</td>
</tr>
<tr>
<td>$\text{V}^1$</td>
<td>$\text{SEC}$</td>
<td>$0.0817$</td>
<td>$0.07139$</td>
</tr>
<tr>
<td>$\text{SOC}^1$</td>
<td>$\text{SEC}$</td>
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<td>$0.0161$</td>
</tr>
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<td>$\text{SEC}$</td>
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</tr>
<tr>
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<td>$\text{SEC}$</td>
<td>$0.0006$</td>
<td>$0.0006$</td>
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<td>$0.0005$</td>
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<tr>
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<td>$\text{SEC}$</td>
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<tr>
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<td>$\text{SEC}$</td>
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<td>$\text{SEC}$</td>
<td>$0.0001$</td>
<td>$0.0001$</td>
</tr>
</tbody>
</table>

*Note: Values are approximate and may be subject to variation.*
### APPENDIX E

**TABLE E2: STABILITY DERIVATIVES AND TRIMS IN U.S. CUSTOMARY UNITS**

(a) $\dot{Z} = -2000$ ft/min

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>-40</th>
<th>-20</th>
<th>0</th>
<th>20</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{Y}$</td>
<td>l/deg</td>
<td>0.5118</td>
<td>0.5040</td>
<td>0.5086</td>
<td>0.5140</td>
<td>0.5344</td>
<td>0.5380</td>
<td>0.5465</td>
<td>0.5421</td>
<td>0.5075</td>
<td>0.5022</td>
</tr>
<tr>
<td>$K_{X}$</td>
<td>l/deg</td>
<td>0.3985</td>
<td>0.3949</td>
<td>0.3994</td>
<td>0.4048</td>
<td>0.4070</td>
<td>0.4091</td>
<td>0.4075</td>
<td>0.4070</td>
<td>0.4075</td>
<td>0.4070</td>
</tr>
<tr>
<td>$K_{Y}$</td>
<td>l/deg</td>
<td>0.3985</td>
<td>0.3949</td>
<td>0.3994</td>
<td>0.4048</td>
<td>0.4070</td>
<td>0.4091</td>
<td>0.4075</td>
<td>0.4070</td>
<td>0.4075</td>
<td>0.4070</td>
</tr>
<tr>
<td>$K_{X}$</td>
<td>l/deg</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0005</td>
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</tr>
<tr>
<td>$P_{Y}$</td>
<td>deg</td>
<td>0.0419</td>
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<td>deg</td>
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<td>0.6977</td>
<td>0.6977</td>
<td>0.6977</td>
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<td>0.6977</td>
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<td>0.6977</td>
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</tr>
<tr>
<td>$P_{K}$</td>
<td>deg</td>
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<td>0.0054</td>
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<td>$P_{K}$</td>
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<td>0.0194</td>
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<td>$P_{K}$</td>
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<tr>
<td>$P_{K}$</td>
<td>deg</td>
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<td>0.0294</td>
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<td>0.0294</td>
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</tr>
<tr>
<td>$P_{K}$</td>
<td>deg</td>
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<td>0.0294</td>
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<td>$P_{K}$</td>
<td>deg</td>
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</table>

*Value of parameter at $\dot{Z}$, knots, of*
### APPENDIX E

**TABLE E2. - Continued**

(b) \( \dot{Z} = -1500 \text{ ft/min} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at ( \dot{X} ), knots, of ( \dot{Z} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F_{U1} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.5631)</td>
</tr>
<tr>
<td>( V_{U1} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0356)</td>
</tr>
<tr>
<td>( X_{U1} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0204)</td>
</tr>
<tr>
<td>( Y_{U1} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0114)</td>
</tr>
<tr>
<td>( Z_{U1} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0059)</td>
</tr>
<tr>
<td>( \Delta F_{U2} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0093)</td>
</tr>
<tr>
<td>( \Delta V_{U2} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0054)</td>
</tr>
<tr>
<td>( \Delta X_{U2} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0059)</td>
</tr>
<tr>
<td>( \Delta Y_{U2} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0059)</td>
</tr>
<tr>
<td>( \Delta Z_{U2} )</td>
<td>( \text{ft/sec} )</td>
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<tr>
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<td>( \text{ft/sec} )</td>
<td>(-0.0070)</td>
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<td>( \text{ft/sec} )</td>
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<tr>
<td>( \Delta \dot{Y}_{U3} )</td>
<td>( \text{ft/sec} )</td>
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<td>( \Delta \dot{Z}_{U3} )</td>
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<tr>
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<tr>
<td>( \Delta Y_{U4} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0079)</td>
</tr>
<tr>
<td>( \Delta Z_{U4} )</td>
<td>( \text{ft/sec} )</td>
<td>(-0.0079)</td>
</tr>
</tbody>
</table>

\* Derived by fitting a third order curve through \( \dot{X} =, \dot{Z} =, \dot{X} = 0 \) and \( \dot{X} = \).
### APPENDIX E

**TABLE E2. - Continued**

(c) \( \dot{Z} = -1000 \text{ ft/min} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value of parameter at ( \dot{X} ), knots, of</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
<td>Unit</td>
</tr>
<tr>
<td>( X/U )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/V )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/L )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\ddot{X} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^2 )</td>
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</tr>
<tr>
<td>( X/\dot{X}^3 )</td>
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</tr>
<tr>
<td>( X/\dot{X}^4 )</td>
<td>I/SEC</td>
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<td>( X/\dot{X}^5 )</td>
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<tr>
<td>( X/\dot{X}^6 )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^7 )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^8 )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^9 )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^{10} )</td>
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</tr>
<tr>
<td>( X/\dot{X}^{11} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^{12} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^{13} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^{14} )</td>
<td>I/SEC</td>
</tr>
<tr>
<td>( X/\dot{X}^{15} )</td>
<td>I/SEC</td>
</tr>
</tbody>
</table>

* DERIVED BY FITTING A THIRD ORDER CURVE TO ROUGHLY 40, 80, 120 AND 160 VALUES.*
### APPENDIX E

#### TABLE E2. - Continued

(d) $\dot{Z} = -500$ ft/min

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at $X$, knots, of</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1/SEC</td>
<td>-0.047</td>
</tr>
<tr>
<td>V</td>
<td>1/SEC</td>
<td>-0.0016</td>
</tr>
<tr>
<td>WV</td>
<td>1/SEC</td>
<td>-0.0007</td>
</tr>
<tr>
<td>XV</td>
<td>1/SEC</td>
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<tr>
<td>XV</td>
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<td>-0.0158</td>
</tr>
<tr>
<td>XY</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>XZ</td>
<td>1/SEC</td>
<td>-0.0057</td>
</tr>
<tr>
<td>YZ</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>VX</td>
<td>1/SEC</td>
<td>-0.008</td>
</tr>
<tr>
<td>VV</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>XV</td>
<td>1/SEC</td>
<td>-0.004</td>
</tr>
<tr>
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<td>-0.002</td>
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<tr>
<td>XZ</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>YZ</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>VX</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>VV</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>XV</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>XY</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>XZ</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
<tr>
<td>YZ</td>
<td>1/SEC</td>
<td>-0.002</td>
</tr>
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</table>

* Denoted by fitting a third order curve through 900, 200, 400 and 600 values.
## APPENDIX E

### TABLE E2. - Continued

(e) $\dot{Z} = 0$ ft/min

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at $\dot{X}$, knots of</th>
</tr>
</thead>
<tbody>
<tr>
<td>X/M</td>
<td>1/SEC</td>
<td>0.02400, 0.02049, 0.02114, 0.02529, 0.02264, 0.02592, 0.02901, 0.02362, 0.02427, 0.02773</td>
</tr>
<tr>
<td>Y/M</td>
<td>1/SEC</td>
<td>0.07032, 0.05075, 0.00003, 0.00022, 0.00162, 0.00206, 0.00201, 0.00003, 0.00020, 0.00001</td>
</tr>
<tr>
<td>Y/R/M</td>
<td>1/SEC</td>
<td>0.03747, 0.02811, 0.02329, 0.02507, 0.02695, 0.02354, 0.02352, 0.02002, 0.02015, 0.02015</td>
</tr>
<tr>
<td>Y/R/M</td>
<td>F/T/SEC2</td>
<td>0.17130, 0.13262, 0.11406, 0.10849, 0.10866, 0.12686, 0.14817, 0.16818, 0.18612, 0.15894</td>
</tr>
<tr>
<td>Y/R/M</td>
<td>F/T/SEC3</td>
<td>0.00167, 0.00608, 0.01959, 0.02017, 0.02023, 0.02082, 0.02062, 0.02020, 0.02018, 0.02015</td>
</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.00012, 0.00068, 0.00011, 0.00011, 0.00011, 0.00011, 0.00011, 0.00011, 0.00011, 0.00011</td>
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<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.11863, 0.10930, 0.09883, 0.08821, 0.07759, 0.06700, 0.05639, 0.04578, 0.03516, 0.02454</td>
</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.82746, 0.81458, 0.75521, 0.69553, 0.63586, 0.57619, 0.51642, 0.45665, 0.39688, 0.33711</td>
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<tr>
<td>X/Y</td>
<td>1/SEC</td>
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</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
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</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
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</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.00003, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019</td>
</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.00003, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019</td>
</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.00003, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019</td>
</tr>
<tr>
<td>X/Y</td>
<td>1/SEC</td>
<td>0.00003, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019, 0.00019</td>
</tr>
</tbody>
</table>

* Derived by fitting a third order curve through -20, -20, 20 and 400 values.
## TABLE E2. - Continued

(f)  \( \dot{z} = 500 \text{ ft/min} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at ( X ), knots, of</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>( \text{ft/sec} )</td>
<td>( \theta )</td>
</tr>
<tr>
<td>( \dot{z} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{z} )</td>
</tr>
<tr>
<td>( \dot{X} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{X} )</td>
</tr>
<tr>
<td>( \dot{Y} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{Y} )</td>
</tr>
<tr>
<td>( \dot{W} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{W} )</td>
</tr>
<tr>
<td>( \dot{I} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{I} )</td>
</tr>
<tr>
<td>( \dot{J} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{J} )</td>
</tr>
<tr>
<td>( \dot{K} )</td>
<td>( \text{ft/sec} )</td>
<td>( \dot{K} )</td>
</tr>
</tbody>
</table>

* DERIVED BY FITTING A THIRD ORDER CURVE THROUGH \( \text{X}, \( \dot{X} \), \( \Phi \) AND \( \Phi \) VALUES.
### APPENDIX E

**TABLE E2.** Continued

(g) $\dot{Z} = 1000 \text{ ft/min}$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value of parameter at $X$, knots, of</th>
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</thead>
<tbody>
<tr>
<td>$K_{X/}$</td>
<td>SEC</td>
<td>-40</td>
</tr>
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<td>$K_{Y/}$</td>
<td>SEC</td>
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<td>-0.0159</td>
</tr>
<tr>
<td>$Y_{x/}$</td>
<td>FT/SEC</td>
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</tr>
<tr>
<td>$Y_{y/}$</td>
<td>FT/SEC</td>
<td>-0.7007</td>
</tr>
<tr>
<td>$Y_{z/}$</td>
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</tr>
<tr>
<td>$Z_{y/}$</td>
<td>FT/SEC</td>
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<td>FT/SEC</td>
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</tr>
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<td>SEC</td>
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<tr>
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<td>SEC</td>
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<td>$X_{z/}$</td>
<td>SEC</td>
<td>-2.3959</td>
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<tr>
<td>$X_{y/}$</td>
<td>SEC</td>
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<td>-2.3959</td>
</tr>
<tr>
<td>$X_{y/}$</td>
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<td>-2.3959</td>
</tr>
<tr>
<td>$X_{z/}$</td>
<td>SEC</td>
<td>-2.3959</td>
</tr>
<tr>
<td>$X_{y/}$</td>
<td>SEC</td>
<td>-2.3959</td>
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<tr>
<td>$X_{z/}$</td>
<td>SEC</td>
<td>-2.3959</td>
</tr>
</tbody>
</table>

* DERIVED BY FITTING A THIRD ORDER CURVE THROUGH -40, -20, 0 AND 40 ft/min VALUES.

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TABLE E2. - Continued

(h) \( \bar{Z} = 1500 \text{ ft/min} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>-40</th>
<th>-20</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X' / X )</td>
<td>1 SEC</td>
<td>0.9741</td>
<td>0.9466</td>
<td>0.9322</td>
<td>0.9087</td>
<td>0.8417</td>
<td>0.4175</td>
<td>0.2172</td>
<td>0.2554</td>
<td>0.5259</td>
<td>0.3583</td>
<td>0.4012</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/2 SEC</td>
<td>0.9711</td>
<td>0.9372</td>
<td>0.9233</td>
<td>0.8993</td>
<td>0.8361</td>
<td>0.4824</td>
<td>0.2428</td>
<td>0.2650</td>
<td>0.4520</td>
<td>0.4336</td>
<td>0.5205</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/4 SEC</td>
<td>0.9703</td>
<td>0.9363</td>
<td>0.9224</td>
<td>0.8985</td>
<td>0.8352</td>
<td>0.4815</td>
<td>0.2421</td>
<td>0.2646</td>
<td>0.4514</td>
<td>0.4332</td>
<td>0.5203</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/8 SEC</td>
<td>0.9664</td>
<td>0.9316</td>
<td>0.9178</td>
<td>0.8938</td>
<td>0.8314</td>
<td>0.4796</td>
<td>0.2406</td>
<td>0.2632</td>
<td>0.4500</td>
<td>0.4329</td>
<td>0.5190</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/16 SEC</td>
<td>0.9664</td>
<td>0.9316</td>
<td>0.9178</td>
<td>0.8938</td>
<td>0.8314</td>
<td>0.4796</td>
<td>0.2406</td>
<td>0.2632</td>
<td>0.4500</td>
<td>0.4329</td>
<td>0.5190</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/32 SEC</td>
<td>0.9664</td>
<td>0.9316</td>
<td>0.9178</td>
<td>0.8938</td>
<td>0.8314</td>
<td>0.4796</td>
<td>0.2406</td>
<td>0.2632</td>
<td>0.4500</td>
<td>0.4329</td>
<td>0.5190</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/64 SEC</td>
<td>0.9664</td>
<td>0.9316</td>
<td>0.9178</td>
<td>0.8938</td>
<td>0.8314</td>
<td>0.4796</td>
<td>0.2406</td>
<td>0.2632</td>
<td>0.4500</td>
<td>0.4329</td>
<td>0.5190</td>
</tr>
<tr>
<td>( X' / X )</td>
<td>1/128 SEC</td>
<td>0.9664</td>
<td>0.9316</td>
<td>0.9178</td>
<td>0.8938</td>
<td>0.8314</td>
<td>0.4796</td>
<td>0.2406</td>
<td>0.2632</td>
<td>0.4500</td>
<td>0.4329</td>
<td>0.5190</td>
</tr>
</tbody>
</table>

* Derived by fitting a third order curve through -40, -20, 0, 20 and 40 values.
### TABLE E2. - Concluded

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>-40</th>
<th>-20</th>
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<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>120</th>
<th>140</th>
<th>160</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
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<tr>
<td>$y$</td>
<td>ft/min</td>
<td><strong>200</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
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<tr>
<td>$z$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$w$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
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<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$v$</td>
<td>ft/min</td>
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<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$u$</td>
<td>ft/min</td>
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<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$t$</td>
<td>ft/min</td>
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<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$s$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$r$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
<td><strong>0.030</strong></td>
<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$q$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
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<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
<tr>
<td>$p$</td>
<td>ft/min</td>
<td><strong>30</strong></td>
<td><strong>0.084</strong></td>
<td><strong>0.066</strong></td>
<td><strong>0.054</strong></td>
<td><strong>0.045</strong></td>
<td><strong>0.037</strong></td>
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<td><strong>0.025</strong></td>
<td><strong>0.020</strong></td>
<td><strong>0.016</strong></td>
<td><strong>0.012</strong></td>
</tr>
</tbody>
</table>

The values in the table are derived by fitting a third-order curve through -40, -20, 0, 20, 40, 60, 80, 100, 120, 140, and 160 values.
REFERENCES


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