OBSERVATION OF ACOUSTIC-GRAVITY WAVES IN THE UPPER ATMOSPHERE DURING SEVERE STORM ACTIVITY

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July 1975

Prepared for
NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812
**Title and Subtitle:**
Observation of Acoustic-Gravity Waves in the Upper Atmosphere During Severe Storm Activity

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**Abstract:**
A nine-element continuum wave spectrum, high-frequency, Doppler sounder array has been used to detect upper atmospheric wave-like disturbances during periods with severe weather activity, particularly severe thunderstorms and tornadoes. Five events of severe weather activity, including extreme tornado outbreak of April 3, 1974, were chosen for the present study. The analysis of Doppler records shows that both infrasonic waves and gravity waves were excited when severe storms appeared in the North Alabama area. Primarily, in the case of tornado activity, S-shaped Doppler fluctuations or Doppler "fold-backs" are observed, while quasi-sinusoidal fluctuations are more common in the case of thunderstorm activity. A criterion for the production of Doppler "fold-backs" is derived and compared with possible tornado conditions.
# TABLE OF CONTENTS

ACKNOWLEDGEMENT ................................................................. i
SUMMARY ....................................................................... ii

I  INTRODUCTION ............................................................ 1

II  ATMOSPHERIC WAVES .................................................. 3

III EXPERIMENTAL OPERATIONS ........................................ 12

IV  PROCEDURES OF DATA ANALYSIS ................................. 22

V  GENERATION OF ACOUSTIC GRAVITY WAVES BY
   SEVERE WEATHER ACTIVITY ......................................... 29
A. Thunderstorm Observation ........................................... 29
B. Tornado Observation .................................................... 35
   1. November 20, 1973 .................................................. 35
   2. November 27, 1973 .................................................. 40
   3. April 3, 1974 ........................................................ 43

VI  DISCUSSIONS AND CONCLUSIONS ................................. 60

REFERENCES

APPENDIX
I. INTRODUCTION

The investigation of the dynamics of coupling between lower and upper atmosphere through wave generation, wave propagation and wave dissipation is essential for the understanding of the motion of the atmosphere. In particular, accurate information about atmospheric disturbances and wave propagation is required for the development of a model of the perturbed neutral atmosphere for the use in orbital/environment mission performance assessment of the Space Shuttle vehicle.

Coupling between the lower atmosphere and the ionosphere can be detected from ionospheric wave-like disturbances which are frequently observed on ground-based ionospheric sounding records as perturbations in the electron densities. Recently, Georges (1968, 1972), Baker and Davies (1969), and Davies and Jones (1972) reported observations of ionospheric wave-like disturbances which showed some correlation with severe weather activity, and suggested that infrasonic waves could be generated by the storm systems. Even though severe weather activity as possible sources of gravity-acoustic waves has been suggested, additional and more confirmative experimental evidence to support such a hypothesis is urgently necessary.

To provide more experimental evidence on the dynamics of upper atmospheric disturbances during severe weather activity, a continuous wave-spectrum, high frequency, Doppler sounder
array has been employed. The purpose of the study is to investigate the coupling between the lower and upper atmosphere (ionosphere) during severe weather activity.

Experimental observations of ionospheric wave-like disturbances during severe weather activity are from the NASA/ Marshall Space Flight Center CW Doppler sounder array located in the North Alabama region. Since January 1973, we have continuously monitored the Doppler frequency fluctuations of nine field transmitters at three stations which cover an area of 150 Km².

In Chapter II, we describe the characteristics of atmospheric waves. In Chapter III, experimental operations of Doppler sounder array which have been used to detect ionospheric disturbances are discussed. The procedures of data analysis from the records of Doppler fluctuations are presented in Chapter IV. In Chapter V, five events of severe thunderstorms and tornadoes, including the extreme tornado outbreak of April 3, 1974, were analyzed. In Chapter VI, we have discussed the effect of Doppler shift due to eastward wind which may affect the wave-period of acoustic-gravity waves observed during severe weather activity. Furthermore, the criterion for the excitation of S-shaped record or Doppler "fold-backs," which is very often observed during tornado activity, is derived and is compared with possible tornado conditions.
II. ATMOSPHERIC WAVES

Typical wave motions in the neutral atmosphere are the propagation of sonic waves. The normal frequency range of the sonic spectrum of the human ear is 20 Hz to 20 KHz. Based on this frequency range, the sonic waves are generally divided into the following three groups:

1. **Audio-Sonic Waves**
   - The frequency range is within the spectrum of human ear.

2. **Ultrasonic Waves**
   - The frequency range is above audio-sonic waves.
   - The upper frequency limits for ultrasonic waves approach the relaxation frequencies of metals, around 10^6 MHz, beyond which materials can no longer respond to the input of mechanical wave energy.

3. **Infrasonic Waves**
   - The frequency range is below audio-sonic waves.
   - In the real atmosphere, the presence of gravitational force causes the density to be vertically nonuniformly distributed which is responsible for the buoyancy oscillations of the fluid parcel of the atmosphere. The buoyancy frequency, which is also termed **Brunt-Väisälä frequency**, is given by

\[ f = \sqrt{\frac{g}{\gamma}} \]
\[ \omega_B = \left[ \frac{g}{T} (\Gamma_d + \frac{a_T}{dz}) \right]^{1/2} \] (2-1)

where \( g \) is the gravitational acceleration; \( T \), the temperature, \( z \), the spatial coordinate along the vertical height; and \( \Gamma_d \), the dry adiabatic lapse rate which is defined

\[ \Gamma_d = \frac{g}{R} \left( \frac{\gamma - 1}{\gamma} \right). \] (2-2)

Here \( R \) is the universal gas constant and \( \gamma = \frac{C_p}{C_v} \) where \( C_p \) is the specific heat at constant pressure, and \( C_v \) is the specific heat. The height variations of molecular temperature \( T \), sound speed \( a \), ratio of specific heat \( \gamma \), and Brunt-Väisälä wave period \( \tau_B (\equiv 2\pi/\omega_B) \) are given in Figures 1, 2, 3 and 4, respectively. It can be seen from Figure 4 that the Brunt-Väisälä frequency is practically less than 0.05 Hz in the atmosphere and it is therefore more convenient to speak of the period of the wave. This characteristic frequency caused by the buoyancy force is so small, as the period is several minutes to more than ten minutes in real atmosphere. Thus, only comparable low frequency waves would be affected by the buoyancy force. It is for this reason that the waves modified by the buoyancy force are limited to infrasonic waves in the atmosphere. In fact, the modification is so drastic that the resulting waves are called acoustic-gravity waves.

The characteristics of acoustic-gravity waves can be shown in simple calculations. To do so, the atmosphere is
Figure 1. The height variation of molecular temperature in real atmosphere.
Figure 2. The height variation of sound speed in real atmosphere.
\[ \gamma = \frac{C_p}{C_v} \]

Figure 3. The height variation of ratio of specific heat \(\gamma\) in real atmosphere.
Figure 4. The height variation of buoyancy or Brunt-Väisälä wave-period in real atmosphere.
assumed to be nonrotating (for a wave period shorter than two hours) with no large-scale neutral wind. The equations are the equation of continuity, the equation of motion, and the equation of energy which are replaced by the adiabatic equation in this case, namely,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \tag{2-3}
\]

\[
\rho \frac{D\mathbf{v}}{Dt} = -\mathbf{v} \rho + \rho g \tag{2-4}
\]

\[
\frac{D\rho}{Dt} = a^2 \frac{D\rho}{Dt} \tag{2-5}
\]

where the convective derivative is \(D/Dt = \partial/\partial t + \mathbf{v} \cdot \nabla\). Here \(\rho\) denotes mass density; \(\mathbf{v}\), velocity; \(P\), pressure; \(a = (\partial P/\partial \rho)^{1/2}\) sonic velocity; and \(g = (0, 0, -g)\), gravitational acceleration. For a static, motionless, isothermal atmosphere the mass density must be distributed exponentially as a function of height

\[
<\rho> = \rho_o e^{-z/H} \tag{2-6}
\]

where \(\rho_o\) is the density at \(z = 0\), and \(H = a^2/\gamma g\) (\(\gamma\) is the ratio of constant pressure specific heat to constant volume specific heat). The corresponding pressure distribution is

\[
<p> = P_o e^{-z/H} \tag{2-7}
\]

Let the isothermal atmosphere be perturbed according to the following scheme:
\[
\begin{align*}
\begin{cases}
\rho(\mathbf{r}, t) = \langle \rho(z) \rangle + \delta\rho(\mathbf{r}, t) \\
P(\mathbf{r}, t) = \langle P(z) \rangle + \delta P(\mathbf{r}, t) \\
\mathbf{V}(\mathbf{r}, t) = \delta\mathbf{V}(\mathbf{r}, t)
\end{cases}
\end{align*}
\tag{2-8}
\]

where \(< >\) denotes the equilibrium quantities and \(\delta\) implies the perturbations. After linearizing the equations, and assuming that the perturbation is proportional to
\[
\exp(i(\omega t - k \cdot \mathbf{r})
\]
we have the following dispersion relation
\[
\frac{k_x^2}{(1 - \omega_0^2/\omega^2)/(1 - \omega_a^2/\omega^2)} + \frac{k_z^2}{1 - \omega_a^2/\omega^2} = k_0^2
\tag{2-9}
\]

where \(k_0 = \omega/a; \omega_a = \gamma g/2a = a/2H = \text{acoustic cutoff frequency};\)
and \(\omega_b = (\gamma-1)^{1/2} \gamma g/a = \text{buoyancy frequency at isothermal atmosphere},\)

\[
\omega_b = \omega_B \left| \frac{dT}{dz} \right| \rightarrow 0
\tag{2-10}
\]

Let us discuss the properties of the dispersion relation (2-9) in the following three different ranges:

1. \(\omega < \omega_b\)
   This is gravity wave. The phase velocity of gravity waves is always less than the velocity of sound waves.

2. \(\omega_b < \omega < \omega_a\)
   This is cutoff region where no acoustic-gravity
waves exist under the condition of isothermal atmosphere.

(3) $\omega > \omega_a$

This is acoustic wave. The phase velocity of acoustic waves is equal or greater than the velocity of sound waves.

In the real atmosphere, the assumption of isothermal atmosphere is no longer valid. This implies that the cutoff region of the form discussed for isothermal atmosphere no longer exists. Therefore, the overlap between the regions of acoustic waves and gravity waves will be expected.
III. EXPERIMENTAL OPERATIONS

The continuous wave-spectra Doppler sounder array was developed by Davies (1962). Davies and Baker (1966) gave a
description of the theory and an interpretation of the data.

NASA/Marshall Space Flight Center CW Doppler sounder array
consists of three sites with nine field transmitters operating
at 4.0125, 4.759 and 5.734 MHz with receivers located at
NASA/Marshall Space Flight Center (NASA/MSFC), Alabama (see
Figure 5 for map description). The geographic locations of
the field transmitters are shown in Table 1. The received
signals from all nine transmitters are mixed with stable
oscillators of the same nominal frequencies, but are offset
several Hz higher. The beat frequencies are then recorded
on a slowly moving (one inch per minute) magnetic tape. For
data analysis, the tapes are played back at higher speeds
into an audio spectrum analyzer. The output of the spectrum
analyzer is connected to a facsimile recorder which gives a
strip chart record of the Doppler shifted frequency components
against time.

A block diagram of the equipment for one transmitting
station is shown in Figure 6. Three ovenized crystal
oscillators are built into one chassis, as shown in the
block diagram of Figure 7. Since each of the three transmitters
must be slightly different so they can be distinguished from
Figure 5. Locations of three sites of field transmitters and receivers of NASA/ Marshall Space Flight Center Continuous wave spectrum, high frequency Doppler sounder array system.
<table>
<thead>
<tr>
<th>Station</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASA/MSFC, Alabama</td>
<td>34° 39'N</td>
<td>86° 40'W</td>
</tr>
<tr>
<td>Ft. McClellan, Alabama</td>
<td>33° 44'N</td>
<td>85° 48'W</td>
</tr>
<tr>
<td>TVA Muscle Shoals, Alabama</td>
<td>34° 46'N</td>
<td>85° 38'W</td>
</tr>
<tr>
<td>TVA Nickajack Dam, Tennessee</td>
<td>35° 01'N</td>
<td>87° 38'W</td>
</tr>
</tbody>
</table>
Figure 6. Block diagram of transmitting system.
Figure 7. Block diagram of Oscillator chassis.
each other at the receiver. The actual frequencies for the field transmitters have been set as in Table 2. At the receiving station, the oscillators have been set exactly at the nominal frequencies so that in the absence of Doppler variations, the offsets of the transmitter oscillators are equal to the beat frequencies.

The transmitters are modified versions of the Heathkit DK 60B. The transmitting antennas are halfwave dipoles. In order to maximize vertical radiation and minimize horizontal radiation, the height of the dipoles above ground should be between one-eighth to one-fourth wavelength. Since all three antenna at a site are in one array, a height of approximately 30 feet has proven to be practical. To null the horizontal radiation in the direction of the receiver, the transmitting antenna at the sites are oriented so that the array is broadside to the receiving site and the ends of the antenna point to the receiver site (NASA/MSFC). Ground waves are not expected to be a problem since the sites are farther than 40 Km from the receivers. If a ground wave is received, it can be identified by its stable beat frequency on the Doppler records.

A block diagram of the receiving and recording system is shown in Figure 8. The reflected ionospheric echoes from the nine field transmitters are received by three Hammarlund SP-600-JX-17 communication receivers at three different nominal frequencies. These receivers use single conversion for channel
### Table 2

Nominal and Actual Operating Frequencies of Three Transmitters of NASA/ Marshall Space Flight Center Doppler Sounder Array System

<table>
<thead>
<tr>
<th>Nominal Frequency</th>
<th>Actual Frequency 1</th>
<th>Actual Frequency 2</th>
<th>Actual Frequency 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0125 MHz</td>
<td>4.0125 MHz + 10 Hz</td>
<td>4.0125 MHz + 20 Hz</td>
<td>4.0125 MHz + 30 Hz</td>
</tr>
<tr>
<td>4.759 MHz</td>
<td>4.759 MHz + 10 Hz</td>
<td>4.759 MHz + 20 Hz</td>
<td>4.759 MHz + 30 Hz</td>
</tr>
<tr>
<td>5.734 MHz</td>
<td>5.734 MHz + 10 Hz</td>
<td>5.734 MHz + 20 Hz</td>
<td>5.734 MHz + 30 Hz</td>
</tr>
</tbody>
</table>

**ORIGINAL PAGE IS OF POOR QUALITY**
Figure 8. Block diagram of receiving system.
frequencies below 7.4 MHz, and double conversion above 7.4 MHz. In either case, the internal crystal HF oscillator is used instead of the variable frequency oscillator. It is not necessary that the HF oscillator in the signal receiver has the stability of the reference receiver to be mixed with the signal from the reference oscillator. Thus, any variations in frequency disappear when the signal intermediate frequency (IF) is mixed with the reference IF.

In order to obtain the accurate timing of ionospheric disturbances, a time code is recorded on channel 4 of the data tape. When the tape is played back for spectrum analysis, the channel 4 output is mixed with whichever data channel is being played back, so that data and timing appear simultaneously on the records. In other words, channels 1, 2 and 3 are used for data, with a timing signal recorded on channel 4. The data tapes are played back on a four channel tape recorder at a speed of 15 inches per second. Each data tape is played back three times, with each playback, one data channel is mixed with the timing channel. The channels are recorded as follows:

Channel 1: 4.0125 MHz, low frequency
Channel 2: 4.759 MHz, middle frequency
Channel 3: 5.734 MHz high frequency
Channel 4: Timing signal.

The output from the tape recorder is spectrum analyzed using a Federal Scientific UA-6B, Ubiquitous audio spectrum analyzer. The output from the spectrum analyzer is fed to a
Gift GPR facsimile recorder which gives a record of frequency against time.

In the present study, we are interested in the analysis of atmospheric waves generated by severe weather activity. A few examples during severe weather activity, particularly thunderstorms and tornadoes, have been chosen for data analysis. The procedures of data analysis will be discussed in the following sections.
IV. PROCEDURES OF DATA ANALYSIS

The visible record of the Doppler data is obtained by frequency analysis of the signal. The frequency analysis is carried out by using a UA6B Ubiquitous audio spectrum analyzer. The output from the spectrum analyzer is displaced on a facsimile chart. In the present study, we are particularly interested in the investigation of the specific atmospheric-ionospheric coupling during thunderstorm and tornado activity which can be identified on the Doppler record. After visual identification of the events due to severe storm activity, the segments of the data tape containing the events are played back at a slower speed for better time resolution. By playing back the tapes at a slower speed and recording the data, one can achieve a time resolution of about five seconds, depending on the time-scale of interest, to study the fine structure in the Doppler variation.

The Doppler fluctuations printed in facsimile chart by playing back the tapes are then subjected to numerical digitizing. The digitized data will be used to determine time displacement and wave frequency of the fluctuations. In other words, for a given sample of the Doppler sounder record, the time displacements are determined by computing the cross-correlation functions. Consider two time series functions given by  and . The cross-correlation function is given by
\[ C_{\alpha\beta}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_\alpha(t) x_\beta(t + \tau) \, dt \] (4.1)

where \( \tau \) is the time lag, or the time displacement between the two functions being correlated. In a special case when \( x_\alpha = x_\beta \), Equation (4.1) becomes the auto-correlation function

\[ C_{\alpha\alpha}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x_\alpha(t) x_\alpha(t + \tau) \, dt \] (4.2)

In general, the correlation matrix \([C]\) can be expressed by

\[
[C] = \begin{bmatrix}
C_{11}(\tau) & C_{12}(\tau) & C_{13}(\tau) \\
C_{21}(\tau) & C_{22}(\tau) & C_{23}(\tau) \\
C_{31}(\tau) & C_{32}(\tau) & C_{33}(\tau)
\end{bmatrix}
\] (4.3)

In our case, if M, N and F refer to the ionospheric reflection points to the transmitters located at Muscle Shoals, Alabama; Nickajack Dam, Tennessee; and Fort McClellan, Alabama to the receivers at NASA/MSFC, respectively, the correlation matrix becomes

\[
[C] = \begin{bmatrix}
C_{MN}(\tau) & C_{MN}(\tau) & C_{MF}(\tau) \\
C_{NM}(\tau) & C_{NN}(\tau) & C_{NF}(\tau) \\
C_{FM}(\tau) & C_{FN}(\tau) & C_{FF}(\tau)
\end{bmatrix}
\] (4.4)

The power spectral density and cross power spectral density are obtained by Fourier transforming the auto and cross correlation functions, respectively, i.e.,
\[
P_{\alpha\alpha}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{\alpha\alpha}(\tau) e^{-i\omega \tau} d\tau \quad (4.5)
\]

and
\[
P_{\alpha\beta}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C_{\alpha\beta}(\tau) e^{-i\omega \tau} d\tau \quad (4.6)
\]

where \( \omega = 2\pi f \) is the angular frequency in radian/sec and \( f \) is frequency in Hz. In general, the cross-power spectral density is complex,

\[
P_{\alpha\beta}(\omega) = p^r_{\alpha\beta}(\omega) + i p^i_{\alpha\beta}(\omega) \quad (4.7)
\]

where \( p^r_{\alpha\beta}(\omega) \) is the real part and \( p^i_{\alpha\beta}(\omega) \) is the imaginary part of the cross spectrum function. The phase density spectrum can be obtained from

\[
\theta(\omega) = \theta_\beta(\omega) - \theta_\alpha(\omega)
\]

\[
= \tan^{-1} \left( \frac{p^i_{\alpha\beta}(\omega)}{p^r_{\alpha\beta}(\omega)} \right) \quad (4.8)
\]

where \( \theta_\alpha(\omega) \) is phase of time series \( x_\alpha(t) \), and \( \theta_\beta(\omega) \) is phase of time series \( x_\beta(\omega) \). The magnitude of the complex continuous spectrum, called the amplitude density spectrum, is

\[
|P_{\alpha\beta}(\omega)| = \left\{ [p^r_{\alpha\beta}(\omega)]^2 + [p^i_{\alpha\beta}(\omega)]^2 \right\}^{1/2} \quad (4.9)
\]

The power can be calculated from the integration of power spectral density.
The coherence estimates are given by

$$\gamma(\omega) = \frac{P_{\alpha\beta}(\omega)}{[P_{\alpha\alpha}(\omega) P_{\beta\beta}(\omega)]^{1/2}}$$

(4.11)

where $\gamma(\omega)$ is the complex coherence. The amplitude of the cross-spectrum at each frequency is the product of the corresponding amplitudes in the two time series. If a particular spectral component is absent from either time series, then it will be absent in the cross-spectrum and the coherence will be minimum at that frequency.

In our case the frequency of the ionospheric disturbances caused by severe weather activity can be computed from both power spectral density and cross-spectral density. The time delay between the reflection points can be obtained from cross-correlation function. Let $\tau_{MN}$ and $\tau_{MF}$ be the time delays between the reflection points M-N and M-F, respectively; and $D_{MN}$ and $D_{MF}$ be the distance between the similar points, respectively. The apparent trace velocities $V_{MN}$ and $V_{MF}$ are given by the following relations (See Figure 9):

$$V_{MN} = \frac{D_{MN}}{\tau_{MN}} \quad \text{and} \quad V_{MF} = \frac{D_{MF}}{\tau_{MF}}$$

(4.12)

The magnitude horizontal trace velocity $V_H$ and the direction $\phi$ (measured east from north) can be determined from the following set of equations:
Figure 9. Graphical representation of wave front moving across the doppler sounder array.
\[
\cot \phi = \frac{V_{MN} \sin \psi_{MN} - V_{MF} \sin \psi_{MF}}{V_{MN} \cos \psi_{MN} - V_{MF} \cos \psi_{MF}}
\]  
\(4.13\)

and the magnitude of \(V_H\) is given by

\[
V_H^2 = \frac{V_{MN}^2 V_{MF}^2 \sin^2 (\psi_{MN} - \psi_{MF})}{V_{MN}^2 + V_{MF}^2 - 2 V_{MN} V_{MF} \cos (\psi_{MN} - \psi_{MF})}
\]  
\(4.14\)

where \(\psi_{MN}\) and \(\psi_{MF}\) and the angles between the geographic north and lines MN and MF, respectively. The true velocity may be determined by computing the vertical trace velocity \(V_v\). The vertical trace velocity \(V_v\) may be determined from the time displacements at various frequencies and the difference of reflection points for a given transmitter and receiver location, are assumed to lie on a vertical plane. The heights of reflections for various frequencies involved are determined from the ionogram analysis. From the values of horizontal phase velocity \(V_H\) and vertical trace velocity \(V_v\), the true phase velocity may be determined as shown in Figure 10.

Fortran listing of computer program for the computation of cross correlation functions, auto correlation functions, power spectral density, cross power spectral density, cross spectral amplitudes, cross spectral phases, cross spectral coherences, etc., are included in the appendix.
Figure 10. Graphical representation for computing phase velocity of the wave front.
V. GENERATION OF ACOUSTIC GRAVITY WAVES BY
SEVERE WEATHER ACTIVITY

In this section, we present the experimental evidence of wavelike disturbances in the ionosphere caused by severe weather activity, particularly due to thunderstorms and tornadoes. The atmospheric waves observed in the ionospheric height are in the category of acoustic-gravity waves. As described in Section III and Section IV, the data obtained from the Doppler sounder are then subjected to power spectrum and cross power spectrum analyses. The results of experimental evidence is shown as follows:

A. Thunderstorm Observation

Two thunderstorm events of May 26, 1973 and March 20, 1974, have been chosen and discussed here.

(1) May 26, 1973

The Doppler record for this event is shown in Figure 11 at an operating radio frequency of 4.0125 MHz. It is evident in this figure that the Doppler fluctuations are highly disturbed until 1100 CDT. Both power spectrum density and cross power spectrum density analyses indicates that the periodicity of the Doppler frequency fluctuations are in the 3 to 5 minute range. The magnitude of the horizontal phase velocities obtained from the cross power spectrum...
Figure 11. CW Doppler record at the operating frequency of 4.0125 MHz (event on May 26, 1973).
density analysis or cross-correlogram is shown to be
1000 m/sec propagating from the south and southwest.

(2) May 20, 1974

Figures 12, 13 and 14 are the CW Doppler records
at operating frequencies 4.0125, 4.759 and 5.734 MHz,
respectively, under thunderstorm conditions of March 20,
1974. No tornadoes were sighted in or around North
Alabama on this day; therefore, it can be classified
as a severe thunderstorm active day. The thunderstorm
associated disturbances are clearly shown on all the
records beginning at 1610 CDT and continuing through
the early morning hours. Wavelike fluctuations with
long periods of 4 to 5.5 minutes are clearly shown on
the 4.0125 MHz record beginning at 2200 CDT and lasting
through 0300 CDT. At higher operation frequencies,
namely 4.759 and 5.734 MHz, the ionospheric reflected
signal was lost after 2300 CDT. The CW Doppler records
of March 20, 1974, were subjected to cross correlation
analysis to compute the phase velocities. The horizontal
trace velocities showed a magnitude of 1000 m/sec
propagating from a direction southwest of the Doppler
sounder. Examination of the Doppler sounder records
also reveals that the frequency fluctuations have a
constant amplitude and that they persisted for 4 to 5
hours.
Figure 12. CW Doppler record at the operating frequency of 4.025 MHz (event on March 20, 1974).
Figure 13. CW Doppler record at the operating frequency of 4.759 MHz (event on March 20, 1974).
Figure 14. CW Doppler record at the operating frequency of 5.743 MHz (event on March 20, 1974).
B. Tornado Observation

The three tornado events of November 20, 1973, November 27, 1973 and the extreme tornado outbreak of April 3, 1974 have been chosen and discussed in this report.

(1) November 20, 1973

Figures 15 and 16 show a CW Doppler record for November 20, 1973, at the operating frequencies of 4.0125 MHz, and 4.759 and 5.734 MHz, respectively. The wave-like fluctuations are clearly seen beginning at 1230 CST and continuing through the evening. The periodicity of these wave-like disturbances based on the analyses from power spectrum and cross power spectrum is shown to be in the 6 to 8 minute range. The phase velocities of the disturbances, derived from a cross correlation analysis, are propagating from the south-west with a magnitude of 600-700 m/sec. The Doppler record of November 20, 1973, is expanded on the time axis to clearly show the time variations (Figure 17). It can be seen from Figure 17 that the Doppler fluctuations are not quasi-sinusoidal but are S-type waves with several Doppler fold backs. Radar weather summary charts for November 20, 1973 are shown in Figure 18. The weather map clearly shows the areas of radar echoes, lines of radar echoes and heights of echo tops. The lines of radar echoes are from the southwest to the north of the Doppler system. The
Figure 15. CW Doppler record at the operating frequency of 4.0125 MHz (event on November 20, 1973).
Figure 16. Cr. Depplet record at the operating frequency of 5.734 and 4.759 MHz (event on November 20, 1973).
Figure 17. The expansion of time axis of Doppler record on November 20, 1973 at the operating frequency 4.0125 MHz.
heights of echo tops are of the order of 40,000 to 50,000 feet (or about 12 to 15 Km). The radar summary charts indicate that the severe weather activity is located southwest of our Doppler array, which is the direction from which the wavelike disturbances seen on Doppler records are propagating.

2. November 27, 1973

Figure 19 shows the Doppler records at the operating frequency of 4.0125 MHz on November 27, 1973 under tornado conditions. This figure clearly shows the wavelike fluctuations beginning about 1100 CST and continuing through the evening hours. The period of the fluctuations obtained from both power spectrum and cross power spectrum analysis are indicated to be on the order of 6 to 8 minutes. The horizontal trace velocities of the disturbances, derived from a cross correlation analysis, are also propagating from the southwest with a magnitude of 600-700 m/sec. The radar weather summary charts for November 27, 1973, are shown in Figure 20. The weather map also clearly shows that the lines radar echoes are from the southwest to the north of the Doppler system. The heights of echo tops are of the order of 40,000 to 50,000 feet (or about 12 to 15 Km). Thus, the radar summary charts indicate that the severe weather activity is located
Figure 19. CW Doppler record at the operating frequency of 4.759 and 4.0125 MHz (event on November 27, 1973).
southwest of our Doppler array, which is the direction from which the wavelike disturbances seen on Doppler records are propagating. The oscillations of Doppler fluctuations in this case persist more than 6 hours.

(3) April 3, 1974

During the extreme tornado outbreak of April 3, 1974, we observed wave-like fluctuations other than the ordinary traveling ionospheric disturbances (TID). Figures 21, 22 and 23 show the CW Doppler records for this day at the operating frequencies of 4.0125, 4.759 and 5.734 MHz, respectively. The observed wave-like fluctuations show several S-shaped Doppler traces or Doppler frequency "fold backs". The periods of these atmospheric waves can be determined by auto-correlating the Doppler records. Figures 24, 25, 26 and 27 show the auto-spectral density of the wave periods observed during different time periods. In Figure 24 three peaks corresponding to waves with periods of 3 min., 6 min., and 11 min. are apparent in the data for the 0700 to 0800 CDT time period. As shown in Figure 4, at 180 Km altitude, the Brunt-Väisälä wave-period is about 10 min. Therefore, waves with 3 min. and 6 min. wave periods are acoustic waves and waves with 11 min. wave periods are gravity waves. The horizontal phase velocity of the disturbances, calculated from a cross-correlation analysis, shows they are propagating in a direction
Figure 21. CW Doppler record at the operating frequency 4.0125 MHz (event on April 3, 1974).
Figure 22. CW Doppler record at the operating frequency 4.759 MHz (event on April 3, 1974).
Figure 23. CW Doppler record at the operating frequency 5.734 MHz (event on April 3, 1974).
Figure 24. Auto spectrum density of the ionospheric disturbances on April 3, 1974 at 0700-0800 CDT.
Figure 25. Auto spectrum density of the ionospheric disturbances of April 3, 1974 at 1200-1300 CDT.
Figure 26. Auto spectrum density of the ionospheric disturbances of April 3, 1974 at 1300-1400 CDT.
4.759 MHz Doppler
Nickajack Dam, Tenn.
to
Huntsville, Ala.

1400-1500 CDT
April 3, 1974

Figure 27. Auto spectrum density of the ionospheric disturbances of April 3, 1974 at 1400-1500 CDT.
103° from geomagnetic north at a speed of 225 m/sec which is a typical phase velocity for gravity waves at F-region ionospheric heights. Gravity waves are predominating over infrasonic waves during 0700 - 0800 CDT time period. Possible sources of the disturbances will be discussed in later sections.

The horizontal phase velocity of the disturbances can be calculated from cross-correlograms. This is because the zero frequency crossings are essentially independent of the amplitude of the change of height of the reflecting surface and they can be used to determine the average period and average time displacement. Figure 28 shows cross-correlograms during the time interval 1400-1500 CDT on 3 April 1975. The average wave-period shown in this cross-correlogram is 13 min. This is in good agreement with the wave-periods during the same time interval obtained from auto-spectral analysis, as shown in Figure 27. The time displacements of the signals between two stations (height peaks from correlograms) are as follows:

\[
(\Delta T)_{F-M} = 650 \text{ sec} \\
(\Delta T)_{F-N} = 330 \text{ sec} \\
(\Delta T)_{N-M} = 320 \text{ sec}
\]

where the subscript F-M denotes the time delay between Ft. McClellan and Muscle Shoals; subscript F-N, time
Figure 28. Cross correlograms of the zero crossing from Nickajack Dam (N), Muscle Shoals (M) and Ft. McClellan (F).
delay between Ft. McClellan and Nickajack Dam; and subscript \( N-M \), time delay between Nickajack Dam and Muscle Shoals. Acceptable combinations of time displacements must satisfy the relationship

\[
(\Delta T)_{F-N} + (\Delta T)_{N-M} = (\Delta T)_{F-M} \tag{5-2}
\]

In our case, it can be shown that the highest peaks of the correlogram, as shown in Equation (5-1), satisfy Equation (5-2). The horizontal phase velocities (both magnitude and direction clockwise from geomagnetic north) during various time intervals on April 3, 1974 are shown in Table 3.

Radar summary charts provided by the National Weather Service, Figures 29, 30, 31 and 32 were used in comparing results calculated from both auto-spectral analysis (Figures 24 to 27) and cross-correlograms (Figure 28 and Table 3) show line echoes and cloud tops from weather radar summaries at different time intervals. During the time interval 0700 to 0800 CDT, Figure 24 and Table 3 show that gravity waves with wave periods of 11 min. and horizontal phase speeds of 225 m/sec propagating from the northwest (towards 103° from north). Figure 29 suggests that the disturbances may have originated in the severe storm cells in the lines of radar echoes with highest
<table>
<thead>
<tr>
<th>DATE</th>
<th>DOPPLER FREQUENCY (MHz)</th>
<th>OBSERVED TIME (CDT)</th>
<th>HORIZONTAL TRACCE VELOCITY (M/SEC)</th>
<th>DIRECTION (DEGREE FROM MAGNETIC-NORTH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4-3-1974</td>
<td>4.0125</td>
<td>0700-0800</td>
<td>225</td>
<td>103°</td>
</tr>
<tr>
<td></td>
<td>4.759</td>
<td>1200-1300</td>
<td>101</td>
<td>252</td>
</tr>
<tr>
<td></td>
<td>4.759</td>
<td>1300-1400</td>
<td>217</td>
<td>255</td>
</tr>
<tr>
<td></td>
<td>4.759</td>
<td>1400-1500</td>
<td>160</td>
<td>146</td>
</tr>
</tbody>
</table>
echo tops from 30,000 to 48,000 ft. (or 10 to 15 Km). During the time interval 1200 to 1300 CDT, Figure 25 and Table 3 indicate that gravity waves with wave periods of 12 min. and horizontal phase speeds of 101 m/sec were propagating from the northeast (toward 252° from north). Figure 30 indicates that the source of these disturbances may be in the severe storm cells which are shown in lines of radar echoes with highest echo tops from 47,000 to 50,000 ft. (or 14 to 16 Km). During the time interval 1300 to 1400 CDT, Figure 26 and Table 3 indicate that gravity waves with wave periods of 13 min. and horizontal phase speed of 217 m/sec were propagating from the northeast (toward 225° from north). Figure 31 shows that the source of these disturbances may be in the severe storm cells which are shown in lines of radar echoes with highest echo tops from 37,000 to 56,000 ft. (or 11 to 16 Km). During the time interval 1400 to 1500 CDT, Figure 27 and Table 3 show that gravity waves with wave periods of 12 min. and horizontal phase speeds of 160 m/sec were propagating from the northwest (toward 146° from north).

Again, Figure 32 indicates the source of these disturbances may be in the severe storm cells which are shown in lines of radar echoes with the highest echo tops from 50,000 to 62,000 ft. (or 15 to 19 Km).
VI. DISCUSSIONS AND CONCLUSIONS

Georges (1968; 1973), Baker and Davies (1969), and Davies and Jones (1972) reported observation of ionospheric disturbances during severe storms and suggested that infrasonic waves could be generated by the storm system. During the past two years, the experimental observations accomplished by our group also indicate that infrasonic waves with wave periods of 2 to 8 minutes and with horizontal phase velocity 600-1000 m/sec which is in the range of acoustic velocity at ionospheric height are generated by severe thunderstorm and tornado activity. Furthermore, during the tornado outbreak of April 3, 1974, we observed gravity waves with wave periods of 11 to 13 min. and horizontal phase speeds of 100 to 225 m/sec with several Doppler fold backs in the Doppler records. This result is slightly different from those of the Boulder group; in particular, the gravity waves in this case had wave periods of 11 to 13 min. as compared with the infrasonic waves with wave periods of 3 to 5 min. in the observations by the Boulder Group. However, recent observations made by a Canadian group at London, Ontario, also contained thunderstorm induced gravity waves with wave periods of 12 to 20 min. (Curry and Murry, 1974). Similarly, French and U.K. groups also reported observations of gravity waves with wave periods of the order
of 20 min. in the vicinity of a weather disturbance (Bertin, et al., 1975).

It is possible that waves with periods of 2 to 8 min. are transformed to waves with periods of 11 to 13 min. by the superimposition of Doppler shifts due to neutral winds by the following relation

\[ \omega' = \omega - \mathbf{k} \cdot \mathbf{U} \]  

(6-1)

where \( \omega \) is the wave frequency observed on the ground; \( \omega' \), the intrinsic frequency as seen by the observer moving with the wind; \( \mathbf{k} \), the wave vector of the disturbances; and \( \mathbf{U} \), the wind velocity. However, waves with periods of 2 to 8 min. at ionospheric heights belong to the category of acoustic waves while those with periods of 11 to 13 min. are in the category of gravity waves. This transformation of acoustic waves to gravity waves is possible only when the wave-wave interaction dominates the dynamics of upper atmosphere (Sadkeev and Galeev, 1969). The super-imposition of a Doppler shift due only to neutral winds cannot transform infrasonic waves into gravity waves. This seems to indicate that: (1) severe weather activity is responsible for the generation of both gravity waves and infrasonic waves; and (2) the dynamics of convective motion, wind shear and/or other unknown mechanisms might be responsible for the process of wave-wave interaction required to transform acoustic waves which were generated by severe weather activity to gravity waves.
The waveforms of the Doppler records shown in Figures 15, 16, 17, 19, 21, 22 and 23 are highly distorted in the S-shaped fashion. This S-shaped record has been explained in terms of a corrugated reflecting surface that moves horizontally with constant speed (Davies and Jones, 1972). We have assumed that the transmitter and receiver both move with speed V while the ionosphere is stationary. If the amplitude of the corrugated surface is \( A \), the mean height of the reflecting surface is \( H \), the path, \( S \), from the transmitter to the reflecting point is given by (Davies and Jones, 1972)

\[
S = (H + A \cos \theta) \left(1 + k^2 A^2 \sin^2 \theta\right)^{1/2}
\]  

(6-2)

where \( \theta = kx \) is the phase of the reflecting point on the corrugated surface, and \( x \) is the horizontal spatial axis which the transmitter moves along. The relation between phase, \( \theta \), and time, \( t \), is given by

\[
\frac{\theta}{k} - kA (H + A \cos \theta) \sin \theta = Vt
\]  

(6-3)

Thus, the instantaneous Doppler shift, \( \Delta f \), for a carrier frequency, \( f \), is:

\[
\frac{\Delta f}{\lambda} = 2kA \frac{V}{c} \frac{\sin \theta}{\left(1 + k^2 A^2 \sin^2 \theta\right)^{1/2}} x
\]

\[x [1 - k^2 HA \cos \theta - k^2 A^2 \cos 2\theta]
\]  

(6-4)
Davies and Jones (1972) dropped the second and third terms in the square bracket in Equation (6.4). It will be shown later that the second term should have been retained since it is on the order of unity which is essentially the same order as the first term.

For small amplitude waves, \( k^2A^2 \ll 1 \), and Equation (6.2) becomes

\[
\cos^2 \theta + \left( \frac{H}{A} \right) \cos^2 \theta - \frac{2}{k^2A^2} \cos \theta + \frac{2}{k^2A^2} (S - H) = 0
\]  

(6-5)

which is a third order algebraic equation of \( \cos \theta \). In general, Equation (6-5) has three roots which means that three echoes at different phases can be observed at the same time. However, for a longer wavelength wave, \( k \) is shorter. Thus, \( 2/k^2A^2 >> 1 \), and Equation (6-5) reduces to a first order algebraic equation since \( |\cos \theta| \ll 1 \). From Equation (6-5) the condition for the existence of multiple-valued echoes is

\[
k^2AH \geq 1
\]  

(6-6)

When this condition exists, the possibility of observing S-shaped Doppler traces is solely dependent on decreasing the wavelength and/or increasing the amplitude of the disturbances.

Equations (6-2), (6-3) and (6-4) have been solved numerically. Figure 33 shows phase path vs. time to reach
Wavelength = 200 km
Height = 180 km
Radio Frequency = 5 MHz
Phase Velocity = 800 m/sec

Figure 33. Variation of phase path with time (A = amplitude, \( \lambda \) = wavelength).
the receiver. It is clear that the phase path is sinusoidal when the ratio of the amplitude of the wavelength is less than 0.03. When the ratio of the amplitude to the wavelength is greater than 0.03, it is possible to observe a three-valued phase path. Figure 34 shows the Doppler phase shift, $\Delta f$, vs. time to reach the receiver. When the ratio of the amplitude to the wavelength is less than 0.03, the Doppler phase shift is practically quasi-sinusoidal. Doppler fold-backs or S-shaped Doppler traces are formed when the ratio of the amplitude to the wavelength is greater than 0.03 for this case.

In conclusion, the analysis of the Doppler records during the severe weather activity reveals the following results: (1) both infrasonic waves with wave-periods of 2 to 8 min. and with horizontal phase velocity of 600-1000 m/sec, and gravity-waves with wave-periods of 11 to 13 min. and horizontal phase velocity of 100 to 225 m/sec are observed during severe thunderstorm activity and tornado activity; (2) several Doppler fold-backs are observed during the tornado-active day and quasi-sinusoidal wave-profiles are observed during the severe-thunderstorm-active day; (3) the Doppler fold-backs phenomenon is due to radio wave reflection from disturbances where the amplitude to wavelength ratio is high; (4) the eastward wind between the source and the observed fluctuations might be responsible for the change of the wave-period from infrasonic waves to gravity-waves.
Wavelength = 200 Km
Height = 180 Km
Radio Frequency = 5 MHz
Phase Velocity = 800 m/sec

Figure 34. Variation of doppler shift (Δf) with time (A = amplitude, λ = wavelength).
REFERENCES


APPENDIX
DO 0410 710
12 FORMAT (46HCROSS SPECTRAL COHOMENCES RELATIVE TO SERIES 618)
0411 720 WRITE (6,15)
0412 730 DO 39 K=1,NW
0413 740 39 WRITE (6,4) TITLE
0414 750 WRITE (6,4) TITLE
0415 760 DO 40 J=1,NSENS
0416 770 WRITE (6,13) J, (J,K=1,NSENS)
0417 780 END FORMAT (3HCROSS SPECTRAL GAINS OF SERIES J,JM RELATIVE TO SERIES 618)
0418 790 WRITE (6,15)
0419 800 DO 40 K=1,NW
0420 810 40 WRITE (6,8) K, (J=1,JM) J, NSENS)
0421 820 WRITE (6,4) TITLE
0422 830 WRITE (6,4) TITLE
0423 840 DO 41 J=1,NSENS
0424 850 WRITE (6,13) J, (K=1,NSENS)
0425 860 14 FORMAT (5I10) STANDARD DEVIATION OF THE AUTO SPECTRAL DENSITIES /
0426 870 INJ SERIES 618)
0427 880 WRITE (6,15)
0428 890 DO 41 K=1,NW
0429 900 41 WRITE (6,6) J, (I,J=1,JM) J, NSENS)
0430 910 C PLOT CURVE
0431 920 DIMENSION X(I),Z(I),X5(I),Y5(I)
0432 930 2 X(I)=FLOAT(I) 0433 940 X(I)=80.0
0434 950 Z(I)=2.0
0435 960 Z(I)=0.0
0436 970 DO 34 J=1,NSENS
0437 980 DO 32 J=1,NSENS
0438 990 32 Z(J)*I=ABS(I)
0500 1000 CALL PRTPLT(AZ,A5,Y5,I)
0501 1010 33 FORMAT ("AUTO SPECTRAL DENSITY WITH RESPECT TO SERIES J,JM"
0502 1020 GU TO 20
0503 1030 END

END OF COMPILED; NO DIAGNOSTICS.