NASA TECHNICAL NOTE

NASA TN D-8246

NASA TN D-8246

TRANSFORMATION OF APPARENT OCEAN WAVE SPECTRA OBSERVED FROM AN AIRCRAFT SENSOR PLATFORM

Lamont R. Poole

Langley Research Center
Hampton, Va. 23665

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JUNE 1976

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TN D-8246		

4. Title and Subtitle	5. Report Date
TRANSFORMATION OF APPARENT OCEAN WAVE SPECTRA OBSERVED FROM AN AIRCRAFT SENSOR PLATFORM	June 1976
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Lamont R. Poole	L-10802
9. Performing Organization Name and Address	10. Work Unit No.
NASA Langley Research Center Hampton, Va. 23665	161-07-02-03
	11. Contract or Grant No.
	13. Type of Report and Period Covered
12. Sponsoring Agency Name and Address	Technical Note
National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The problem considered was transformation of a unidirectional apparent ocean wave spectrum observed from an aircraft sensor platform into the true spectrum that would be observed from a stationary platform. Spectral transformation equations were developed in terms of the linear wave dispersion relationship and the wave group speed. An iterative solution to the equations was outlined and used to transform reference theoretical apparent spectra for several assumed values of average water depth. Results show that changing the average water depth leads to a redistribution of energy density among the various frequency bands of the transformed spectrum. This redistribution is most severe when much of the energy density is expected, a priori, to reside at relatively low true frequencies.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Ocean wave spectra Remote sensing Apparent-spectrum transformation	Unclassified – Unlimited Subject Category 48

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	22	$3.25

*For sale by the National Technical Information Service, Springfield, Virginia 22161

TRANSFORMATION OF APPARENT OCEAN WAVE SPECTRA OBSERVED FROM AN AIRCRAFT SENSOR PLATFORM

Lamont R. Poole
Langley Research Center

SUMMARY

The problem considered herein was the transformation of a unidirectional apparent ocean wave spectrum observed from an aircraft sensor platform into the true spectrum, or the spectrum which would result from observation of the same wave field from a stationary platform. Spectral transformation equations valid for all depth zones were developed in terms of the linear wave dispersion relationship, and an iterative solution for a specified average water depth was outlined. Results obtained by using this iterative solution indicated that different transformed spectra can be obtained by varying the average water depth in the transformation of a single apparent spectrum. The differences were most significant when much of the energy density was expected, a priori, to exist at relatively low true frequencies. Varying the average depth led to a redistribution of energy density among the various frequency bands of the transformed spectrum while the total energy density in the spectrum was preserved. The necessity of seeking an iterative solution to the transformation equations was emphasized in an appendix in which errors resulting from the use of certain approximations were discussed.

INTRODUCTION

With man's increasing usage of the continental shelf regions of the world, the need for techniques to monitor and study broad-scale ocean phenomena such as surface waves and currents has been emphasized. One tool which could potentially play an important role in an overall coastal-zone management program is the wave refraction model, such as that of reference 1. A wave refraction model, in theory, predicts the behavior of ocean surface waves as they cross the continental shelf and impinge on the shoreline. If the model is of a sufficiently large spatial scale that regional differences in wave environment can be highlighted, the process of site selection for offshore and coastal activities is aided.

One must consider, however, the fact that the wave refraction model is a mathematical simplification of a complex physical process. In this light, experimental verification of model results is imperative before the wave refraction model can be used in an operational sense for prediction and decision making. The broad-scale experimental data on the coastal wave environment required for model verification can best be obtained through the use of aircraft carrying instruments such as a laser profilometer or radar altimeter to measure small-scale variations in sea-surface elevation. (See ref. 2, for example.) An important computed product of such surface elevation data is the wave energy density (energy per unit area) spectrum, which in its most common form displays the wave energy density as a function of wave frequency. Such energy density spectra could be obtained for a number of deep-water and nearshore locations along the flight path of the aircraft. The accuracy of the wave refraction model could then be assessed by using the model, as in reference 3, to simulate the variations in wave energy density in discrete bands of the experimentally derived spectra as the wave field propagates from deep-water to nearshore locations.

A problem which must be considered with the use of such remotely sensed surface elevation data is that the observations have been made from a rapidly moving reference frame, the aircraft. A frequency spectrum of wave energy computed from these data accordingly is valid in the moving reference frame but provides a distorted estimate of the true frequency spectrum in a fixed reference frame. The problem of mapping this apparent frequency spectrum into its counterpart in a fixed reference frame was first discussed in a classic paper by St. Denis and Pierson (ref. 4). The basic relationships given in reference 4 were discussed from a more practical computational viewpoint by Cartwright (ref. 5) in a study of the output of a wave recorder on board a moving ship. More recently these relationships have been used in a study of wave generation and growth (ref. 6) performed by using airborne instrumentation in a mode similar to that desired for a refraction model verification study.

In the previously mentioned references concerning transformation of apparent spectra, use has been made of the so-called deep-water approximation to the linear wave dispersion relationship. In general terms, the assumption is made that the water through which the waves are propagating is sufficiently deep to make the wave phase speed a function of wave number alone, rather than a function of both wave number and water depth. Wave spectra to be used for refraction model verification, however, must necessarily be observed in nearshore regions in which the water depth is relatively shallow. For such applications, use of deep-water relationships for transforming apparent spectra observed from an aircraft platform will result in distorted estimates of the true nearshore spectra.

Transformation equations valid for all depth zones are developed in the present paper under the assumption of a unidirectional wave field. The expressions are developed in terms of the linear wave dispersion relationship and the wave group speed. An iterative solution to the transformation equations is outlined for a specified value of average water depth in the area in which the apparent spectrum is observed. Sample cases are presented to illustrate the effect of varying the assumed water depth in the transformation of reference theoretical apparent spectra.

SYMBOLS

c	true wave phase speed, meters/second
c_a	apparent wave phase speed observed from aircraft platform, meters/second
d	local water depth, meters
$\bar{d}$	average water depth in area in which apparent spectrum is observed, meters
ΔE	incremental energy density within a narrow band of the spectrum, meters^2
g	acceleration of gravity, meters/second^2
k	wave number, $\frac{2\pi}{L}$, meter^{-1}
k^*	nondimensional wave number, $k\bar{d}$
L	wavelength, meters
$S(\omega)$	true spectral density, or wave energy density per unit true radian frequency, $\frac{\text{meters}^2}{\text{radian/second}}$
$S_a(\omega_a)$	apparent spectral density, or wave energy density per unit apparent radian frequency, $\frac{\text{meters}^2}{\text{radian/second}}$
$S_{a,r}(\omega_a;W)$	reference apparent spectral density for specified windspeed W, defined by equation (19), $\frac{\text{meters}^2}{\text{radian/second}}$
$S_{PM}(\omega;W)$	Pierson-Moskowitz (ref. 8) spectral density for specified windspeed W, defined by equation (18), $\frac{\text{meters}^2}{\text{radian/second}}$
V	aircraft platform speed, meters/second

W windspeed, meters/second

ζ positive root of quadratic equation (eq. (A7)) defining deep-water approximate frequency transformation, radians/second

ξ root of transcendental frequency transformation equation (eq. (6)), meters^{-1}

ω true radian wave frequency, radians/second

ω^* nondimensional true wave frequency, $\omega\sqrt{\frac{d}{g}}$

ω_a apparent radian wave frequency, radians/second

TRANSFORMATION OF APPARENT SPECTRUM

Developed in this section are equations which relate the apparent wave spectrum observed from an aircraft sensor platform to the spectrum which would result from observation of the same wave field from a stationary platform such as a wave staff. It is assumed that a digitized record of sea-surface elevation measurements made by the airborne sensor is treated by standard time-series analysis methods to produce a spectrum whose ordinate values, when multiplied by (g/2)(Water density), yield an estimate of the distribution of wave energy density over a range of positive apparent frequencies, or frequencies observed from the moving aircraft platform. Such a treatment of the data record inherently assumes that the length of the record is sufficiently short and, thus, the variation in water depth along the aircraft flight path is sufficiently small, to ensure homogeneity of the data sample. For simplicity it is also assumed that the observed wave field is unidirectional (i.e., all waves are traveling in the same direction) and that the aircraft is flying in that same direction at a constant speed which is greater than that of the fastest wave observed.

Apparent Frequency Transformation

Under the assumption of a unidirectional wave field being observed from an aircraft flying in the direction of wave propagation, the apparent phase speed of a given wave, or the phase speed relative to the airborne sensor, is

$$c_a = c - V \tag{1}$$

where c_a is the apparent phase speed, c is the true phase speed, or phase speed seen from a stationary platform, and V is the aircraft speed. The phase speed can be defined in terms of radian wave frequency and wave number by the equation

$$c = \frac{\omega}{k} \tag{2}$$

where ω is the radian wave frequency and k is the wave number, equal to $2\pi/L$. Since wavelength (and, thus, wave number) is the same in both the stationary and the moving coordinate systems (ref. 4, p. 302), equation (2) relates both true phase speed to true frequency and also apparent phase speed to apparent frequency. Equation (1) can be rewritten then in terms of radian frequencies as

$$\omega_a = \omega - kV \tag{3}$$

where ω is the true wave frequency and ω_a is the apparent wave frequency. Since the aircraft speed is assumed to be greater than the phase speed of any given wave being observed, the apparent frequency (see eq. (1)) is always negative. In order to relate the transformation to positive apparent frequencies which result from time-series analysis of the actual data record from the airborne sensor, it is convenient to redefine the frequency transformation as the negative of equation (3), that is,

$$\omega_a = kV - \omega \tag{4}$$

Frequency and wave number are also uniquely related, according to linear wave theory, by the dispersion relationship

$$\omega^2 = gk \tanh kd \tag{5}$$

where g is the acceleration of gravity and d is the local depth of the water through which the given wave is propagating. By assuming that the local depth in equation (5) can be replaced by an average depth over the observation area and by combining equations (4) and (5), the apparent frequency can be defined in terms of wave number as

$$kV - (gk \tanh k\bar{d})^{1/2} - \omega_a = 0 \tag{6}$$

where $\bar{d}$ is the average water depth in the area in which the apparent spectrum is observed. Equations (5) and (6) in combination, then, relate a given apparent frequency to its corresponding true frequency (or frequency observed from a stationary platform) for specified values of V and $\bar{d}$.

Spectral Density Transformation

The apparent energy density spectrum derived from time-series analysis of the airborne measurements shows the distribution of energy density (energy per unit ocean surface area) with respect to apparent wave frequency. The incremental energy density ΔE within a narrow frequency band of the apparent spectrum can be written as

$$\Delta E = S_a\left(\omega_a|_i\right)\left(\omega_a|_{i+1} - \omega_a|_i\right) \tag{7}$$

where $\omega_a|_i$ and $\omega_a|_{i+1}$ are the limits of the discrete apparent frequency band and $S_a\left(\omega_a|_i\right)$ is the apparent spectral density in this band. The corresponding band in the true spectrum is representative of the same physical waves, viewed from a stationary platform. Thus an equivalent amount of energy density must reside in the corresponding band in the true spectrum, that is,

$$\Delta E = S\left(\omega|_i\right)\left(\omega|_{i+1} - \omega|_i\right) \tag{8}$$

where $\omega|_i$ and $\omega|_{i+1}$ are the transformed values of $\omega_a|_i$ and $\omega_a|_{i+1}$, and $S\left(\omega|_i\right)$ is the true spectral density in the transformed band. Then, setting equation (8) equal to equation (7) results in

$$S\left(\omega|_i\right)\left(\omega|_{i+1} - \omega|_i\right) = S_a\left(\omega_a|_i\right)\left(\omega_a|_{i+1} - \omega_a|_i\right) \tag{9}$$

In the limit as the width of the frequency band approaches zero, equation (9) becomes

$$S(\omega)\, d\omega = S_a(\omega_a)\, d\omega_a$$

or

$$S(\omega) = S_a(\omega_a)\frac{d\omega_a}{d\omega} \tag{10}$$

The derivative $d\omega_a/d\omega$ can be determined directly from equation (4) as

$$\frac{d\omega_a}{d\omega} = V\frac{dk}{d\omega} - 1 = \frac{V}{d\omega/dk} - 1 \tag{11}$$

Equation (10) can then be rewritten as

$$S(\omega) = S_a(\omega_a)\left(\frac{V}{d\omega/dk} - 1\right) \tag{12}$$

In physical terms, the derivative $d\omega/dk$ is known as the wave group speed (ref. 7, p. 187), or the speed at which energy is transmitted by a wave train. Thus the energy density transformation is, by equation (12), a function of the ratio of aircraft speed to the speed at which energy is transmitted by the wave train being observed.

A general expression for $d\omega/dk$ can be found by differentiating the dispersion relationship, equation (5), as

$$\frac{d\omega}{dk} = \frac{g}{2\omega}\left(\tanh kd + kd \operatorname{sech}^2 kd\right) \tag{13}$$

so that the spectral density transformation can be written in final form as

$$S(\omega) = S_a(\omega_a)\left(\frac{2\omega V/g}{\tanh k\bar{d} + k\bar{d} \operatorname{sech}^2 k\bar{d}} - 1\right) \tag{14}$$

where ω, ω_a, and k are functionally related by equations (5) and (6) and $\bar{d}$, again, is the average depth in the area in which the apparent spectrum is observed.

Solution to Transformation Equations

It was found under the assumptions stated in the previous sections that the transformation of an apparent energy density spectrum, observed by an aircraft sensor, into the true energy density spectrum which would result from observation of the same wave field from a stationary platform can be accomplished by use of the following three equations:

$$kV - \left(gk \tanh k\bar{d}\right)^{1/2} - \omega_a = 0 \tag{6}$$

$$S(\omega) = S_a(\omega_a)\left(\frac{2\omega V/g}{\tanh k\bar{d} + k\bar{d} \operatorname{sech}^2 k\bar{d}} - 1\right) \tag{14}$$

$$\omega^2 = gk \tanh k\bar{d} \tag{15}$$

Because of their transcendental nature, a closed-form solution to the transformation equations cannot be found. The studies presented in references 4 to 6 circumvented this problem by using what is known as the deep-water approximation. The assumption is made that the water depth is sufficiently large that for all wave numbers, $\tanh k\bar{d} \approx 1$ and $\mathrm{sech}\ k\bar{d} \approx 0$. With this deep-water assumption the three transformation equations can be reduced to closed-form expressions which are given in the appendix. For application to apparent spectra observed in waters of relatively shallow depth, however, use of these deep-water approximations can result in substantial error. The magnitude of this error, along with the error which could result from certain other approximate solutions to the transformation equations, is discussed in the appendix.

It is advisable, then, to seek a general solution to the transformation equations valid for all depth zones. A general solution can be found by first considering equation (6), the apparent frequency transformation in terms of wave number. For specified values of V, ω_a, and $\bar{d}$, standard iteration techniques can be used to find a root $k = \xi$ of equation (6) which is unique under the assumption that the aircraft speed is greater than that of the fastest wave observed. The true frequency corresponding to the specified apparent frequency ω_a can then be found by using equation (15) as

$$\omega(\xi) = \left(g\xi \tanh \xi\bar{d}\right)^{1/2} \tag{16}$$

and the true spectral density can be found by using equation (14) as

$$S\left[\omega(\xi)\right] = S_a(\omega_a)\left[\frac{2\ \omega(\xi)\ V/g}{\tanh \xi\bar{d} + \xi\bar{d}\ \mathrm{sech}^2\ \xi\bar{d}} - 1\right] \tag{17}$$

RESULTS AND DISCUSSION

Sample calculations have been made which illustrate the effect on the transformation of apparent spectra of varying the average depth in the area in which the apparent spectra were observed. The theoretical fully developed wave spectrum form proposed by Pierson and Moskowitz (ref. 8) was used to form a reference apparent spectrum as follows:

(1) The unidirectional wave spectrum as a function of true frequency for a given windspeed was specified as the Pierson-Moskowitz spectrum,

$$S_{PM}(\omega;W) = 8.1 \times 10^{-3}\ \frac{g^2}{\omega^5}\ \exp\left[-0.74\left(\frac{g}{W\omega}\right)^4\right] \tag{18}$$

where W is the speed of the wind generating the fully developed wave field.

(2) A closed-form expression for the reference apparent spectrum for windspeed W was derived by applying the deep-water approximate transformation equations given in the appendix in inverse fashion to equation (18). For a specified aircraft speed V, the reference apparent spectrum for windspeed W was then derived as

$$S_{a,r}(\omega_a;W) = \frac{S_{PM}(\omega;W)}{2\omega V/g - 1} \tag{19}$$

where

$$\omega_a = \frac{\omega^2 V}{g} - \omega \tag{20}$$

as shown in the appendix.

The reference apparent spectrum at windspeed W, given by equations (19) and (20), was then retransformed by using the general solution of the transformation equations (eqs. (6), (14), and (15)) for a specified value of average water depth in the observation area.

Reference apparent spectra for an aircraft speed V = 75 m/sec and various windspeeds (10, 15, and 20 m/sec) were transformed into true spectra for assumed average water depths of 25 m, 50 m, 100 m, and deep water ($\bar{d} = \infty$). Comparison of the transformed spectra for W = 10 m/sec, given in figure 1, indicates that the spectra for water depths of 50 m, 100 m, and deep water are nearly identical. In the 25-m transformed spectrum, there is a smaller value of the maximum spectral density accompanied by a slight broadening of the prominent portion of the spectrum.

True spectra obtained by transforming the reference apparent spectrum for a windspeed W = 15 m/sec are compared in figure 2. The transformed spectra for deep water and $\bar{d}$ = 100 m are again nearly identical. The 50-m transformed spectrum shows a smaller value for the maximum spectral density and a slight broadening in the prominent portion of the spectrum. The 25-m transformed spectrum shows an even smaller value for the maximum spectral density, a more pronounced broadening in the prominent portion of the spectrum, and a downward shift in the frequency at which the maximum spectral density occurs.

True spectra obtained by transforming the reference apparent spectrum for a windspeed W = 20 m/sec are compared in figure 3. Of particular interest in figure 3 is the shift to a significantly lower frequency range of the prominent portion of the 25-m transformed spectrum compared with the location of the prominent portions of the other spectra. In addition, the maximum spectral density is greater in the 25-m spectrum than in the 50-m spectrum, a result contrary to the trend in figures 1 and 2. The disparity among the transformed spectra shown in figure 3 indicates that when the generating windspeed is high and, by inference, much of the energy in the wave field is expected, a priori, to reside at relatively low true frequencies, the use of a proper depth value in transforming an apparent spectrum is vitally important.

It should be reemphasized that, for a particular windspeed, the area under a transformed spectrum, or total energy density, is invariant with assumed water depth (as seen from eqs. (7) to (9)). Thus, while preserving the total energy density of the wave field, use of an incorrect depth value in the transformation of the apparent spectrum of interest can lead to quite distorted estimates of the true spectral distribution of energy density.

CONCLUDING REMARKS

The present paper has considered the transformation of a unidirectional apparent ocean wave spectrum observed from an aircraft sensor platform into the corresponding true spectrum, or the spectrum which would result from observation of the same wave field from a stationary platform. General transformation equations were developed in terms of the linear wave dispersion relationship and the wave group speed. An iterative solution to the transformation equations was outlined for specified values of aircraft speed and average water depth in the area in which the apparent spectrum is observed.

Results obtained by using the iterative solution to transform sample theoretical apparent spectra indicate that significantly different transformed spectra can arise from a single apparent spectrum transformed with different assumed values of average water depth. For experimental cases in which much of the energy in a wave field can be expected, a priori, to reside at relatively low frequencies, use of an incorrect value for water depth in the transformation of the apparent spectra of interest can lead to quite distorted estimates of the true spectral distribution of wave energy.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
May 4, 1976

APPENDIX

APPROXIMATE SOLUTIONS TO TRANSFORMATION EQUATIONS

Approximate solutions to the equations defining the transformation of a unidirectional apparent wave spectrum into its corresponding true spectrum are examined in this appendix. A discussion of the errors associated with these approximate solutions serves to emphasize the need for an iterative solution to the exact transformation equations. The exact equations valid for any depth zone are stated as

$$kV - \omega - \omega_a = 0 \tag{A1}$$

$$S(\omega) = S_a(\omega_a)\left(\frac{V}{d\omega/dk} - 1\right) \tag{A2}$$

where ω and k are related by the linear wave dispersion relationship

$$\omega^2 = gk \tanh k\bar{d} \tag{A3}$$

and $d\omega/dk$, which in physical terms is the wave group speed, is found by differentiating equation (A3) as

$$\frac{d\omega}{dk} = \frac{g}{2\omega}\left(\tanh k\bar{d} + k\bar{d} \operatorname{sech}^2 k\bar{d}\right) \tag{A4}$$

By assuming that the water in the region in which the apparent spectrum is observed is sufficiently deep, so that for all wave numbers, $\tanh k\bar{d} \approx 1$ and $\operatorname{sech} k\bar{d} \approx 0$, equations (A3) and (A4) can be approximated by

$$\omega^2 \approx gk \tag{A5}$$

and

$$\frac{d\omega}{dk} \approx \frac{g}{2\omega} \tag{A6}$$

Substitution of k as defined by equation (A5) into equation (A1) results in a closed-form, quadratic equation for ω in terms of ω_a which can be called the deep-water approximate frequency transformation:

$$\frac{\omega^2 V}{g} - \omega - \omega_a \approx 0 \tag{A7}$$

The positive root $\omega = \zeta$ of equation (A7) can be substituted for ω into equation (A6), and insertion of that expression for $d\omega/dk$ in equation (A2) results in the deep-water approximate spectral density transformation:

$$S(\zeta) \approx S_a(\omega_a)\left(\frac{2\zeta V}{g} - 1\right) \tag{A8}$$

Use of these approximate relationships in the transformation of apparent spectra observed in regions of relatively shallow water can, however, result in substantial error. This error can best be examined by considering nondimensional versions of the dispersion relationship and the wave group speed. By defining $\omega^* = \omega\sqrt{\bar{d}/g}$ and $k^* = k\bar{d}$, the dispersion relationship and group speed given in exact form by equations (A3) and (A4), respectively, can be written in nondimensional form as

$$(\omega^*)^2 = k^* \tanh k^* \tag{A9}$$

and

$$\frac{d\omega^*}{dk^*} = \frac{\tanh k^* + k^* \operatorname{sech}^2 k^*}{2\omega^*} = \frac{\tanh k^* + k^* \operatorname{sech}^2 k^*}{2(k^* \tanh k^*)^{1/2}} \tag{A10}$$

Similarly, the deep-water approximate versions of the dispersion relationship and group speed, given by equations (A5) and (A6), respectively, can be written in nondimensional form as

$$(\omega^*)^2 \approx k^* \tag{A11}$$

and

$$\frac{d\omega^*}{dk^*} \approx \frac{1}{2\omega^*} \approx \frac{1}{2(k^*)^{1/2}} \tag{A12}$$

Plots of the exact and deep-water approximate versions of the nondimensional dispersion relationship (eqs. (A9) and (A11)) as a function of k^* are compared in figure 4, and similar plots of the nondimensional group speed (exact version given by eq. (A10), and deep-water approximate version given by eq. (A12)) are compared in figure 5. It can be seen in these figures that for small values of k^* (less than about 2.0), there is considerable difference between the exact and approximate versions of both the dispersion relationship and the group speed. Thus, the use of the deep-water approximation in the transformation of an apparent-frequency—spectral-density pair when $k^* = k\bar{d}$ is small will result in both a misplacement of the transformed pair with respect to true frequency and a distortion with respect to true spectral density.

In consideration of the disagreement at small values of k^* between the exact dispersion relationship and its deep-water approximation, the temptation exists to proceed to a piecewise-continuous approximation in which distinct functions are defined to approximate the exact dispersion relationship in adjacent ranges of k^*. Candidate functions for such an approximation can be deduced by inspection of the exact nondimensional dispersion relationship shown in figure 4. For small values of k^* $(k^* < 0.5)$, ω^* is seen to vary in a nearly linear fashion with k^*. This follows from equation (A9), since $\tanh(k^*) \approx k^*$ for small values of k^* (shallow water), so that $(\omega^*)^2 \approx (k^*)^2$. For $k^* > 2.0$, the deep-water approximation $(\omega^*)^2 \approx k^*$ can obviously serve as the approximating function. For values of k^* between 0.5 and 2.0, a transition function of the form, for example,

$$(\omega^*)^2 \approx a_0 + a_1(k^* - 0.5)$$

where a_0 and a_1 are constants, could be chosen so as to intersect the so-called shallow-water approximation at $k^* = 0.5$ and the deep-water approximation at $k^* = 2.0$. Shown in figure 6 is this prototype piecewise-continuous approximation, which, stated in equation form, is

$$(\omega^*)^2 \approx \begin{cases} (k^*)^2 & \text{for } k^* < 0.5 \\ 0.25 + 1.167(k^* - 0.5) & \text{for } 0.5 \leqq k^* \leqq 2.0 \\ k^* & \text{for } k^* > 2.0 \end{cases} \tag{A13}$$

The maximum error in frequency which would be incurred by using this approximation is less than 10 percent. However, there would be a more severe problem encountered with the use of the corresponding approximation to the group speed $d\omega^*/dk^*$ which, from equation (A13), is

$$\frac{d\omega^*}{dk^*} \approx \begin{cases} 1 & \text{for } k^* < 0.5 \\ \dfrac{0.5833}{\left[0.25 + 1.167(k^* - 0.5)\right]^{1/2}} & \text{for } 0.5 \leqq k^* \leqq 2.0 \\ \dfrac{1}{2(k^*)^{1/2}} & \text{for } k^* > 2.0 \end{cases} \tag{A14}$$

APPENDIX

A plot of equation (A14) is compared with a plot of the exact nondimensional group speed (given by eq. (A10)) in figure 7. There are substantial differences between the two curves in certain ranges of k^*, in addition to discontinuities in the approximate group speed at the points of intersection of the individual functions in the piecewise-continuous approximation to the dispersion relationship. Sharp changes would thus be induced in the transformed spectra through use of such piecewise-continuous approximations to obtain a closed-form solution to the apparent-spectrum transformation equations.

REFERENCES

1. Goldsmith, Victor; Morris, W. Douglas; Byrne, Robert J.; and Whitlock, Charles H.: Wave Climate Model of the Mid-Atlantic Shelf and Shoreline (Virginian Sea) – Model Development, Shelf Geomorphology, and Preliminary Results. NASA SP-358, VIMS SRAMSOE No. 38, 1974.
2. Ross, Duncan B.; Cardone, Vincent J.; and Conaway, Jack W., Jr.: Laser and Microwave Observations of Sea-Surface Condition for Fetch-Limited 17- to 25-m/s Winds. IEEE Trans. Geosci. Electron., vol. GE-8, no. 4, Oct. 1970, pp. 326-336.
3. Chao, Yung-Yao: Wave Refraction Phenomena Over the Continental Shelf Near the Chesapeake Bay Entrance. Tech. Mem. No. 47, Coastal Eng. Res. Center, U.S. Army Corps Eng., Oct. 1974.
4. St. Denis, Manley; and Pierson, Willard J., Jr.: On the Motions of Ships in Confused Seas. Trans. Soc. Naval Archit. & Mar. Eng., vol. 61, 1953, pp. 280-357.
5. Cartwright, D. E.: The Use of Directional Spectra in Studying the Output of a Wave Recorder on a Moving Ship. Ocean Wave Spectra, Prentice-Hall, Inc., c.1963, pp. 203-218.
6. Barnett, T. P.; and Wilkerson, J. C.: On the Generation of Ocean Wind Waves as Inferred From Airborne Radar Measurements of Fetch-Limited Spectra. J. Mar. Res., vol. 25, no. 3, Sept. 15, 1967, pp. 292-328.
7. Kinsman, Blair: Wind Waves – Their Generation and Propagation on the Ocean Surface. Prentice-Hall, Inc., c.1965.
8. Pierson, Willard J., Jr.; and Moskowitz, Lionel: A Proposed Spectral Form for Fully Developed Wind Seas Based on the Similarity Theory of S. A. Kitaigorodskii. J. Geophys. Res., vol. 69, no. 24, Dec. 15, 1964, pp. 5181-5190.

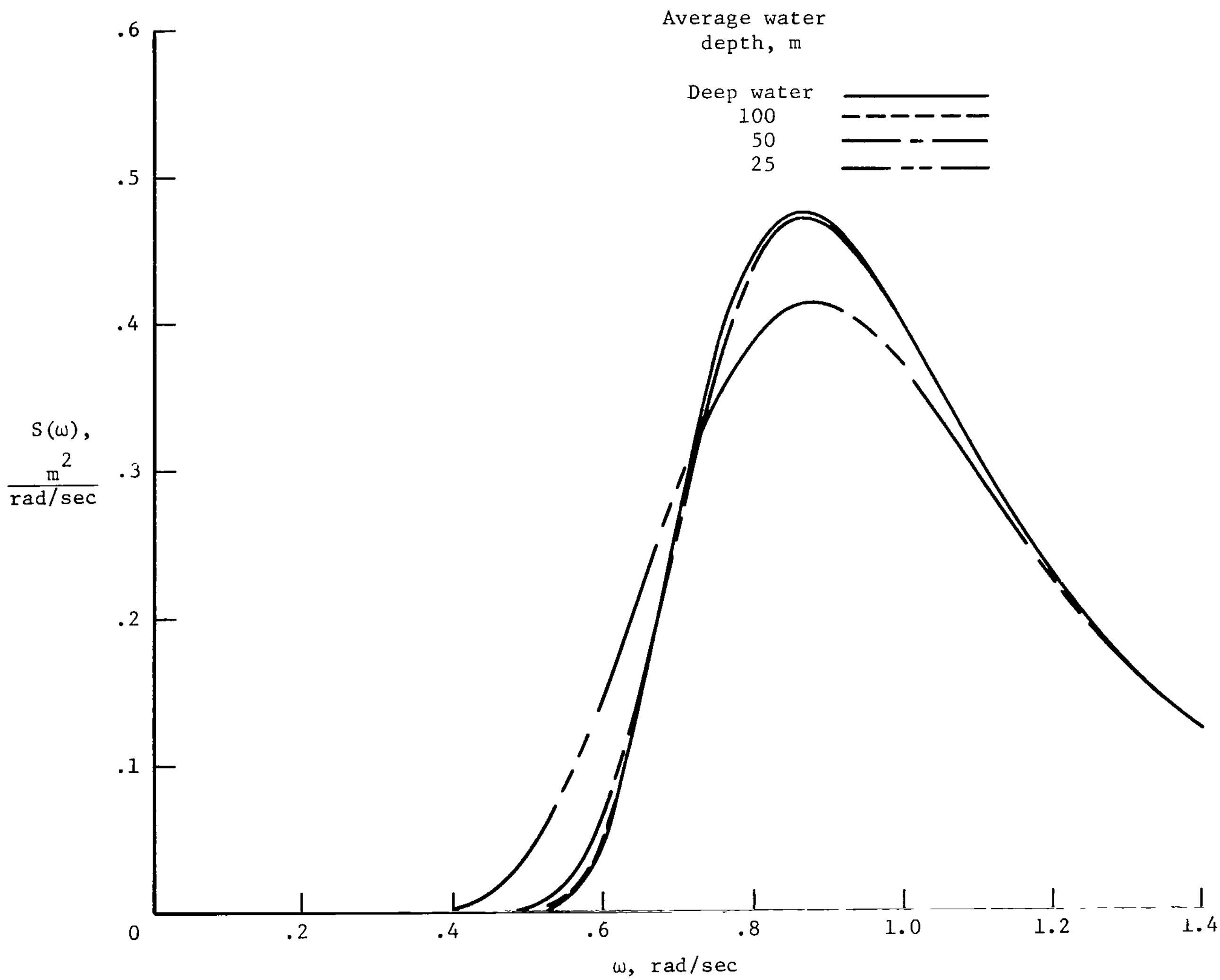

Figure 1.- Transformed spectra for windspeed of 10 m/sec and various average water depths. Aircraft speed $V = 75$ m/sec.

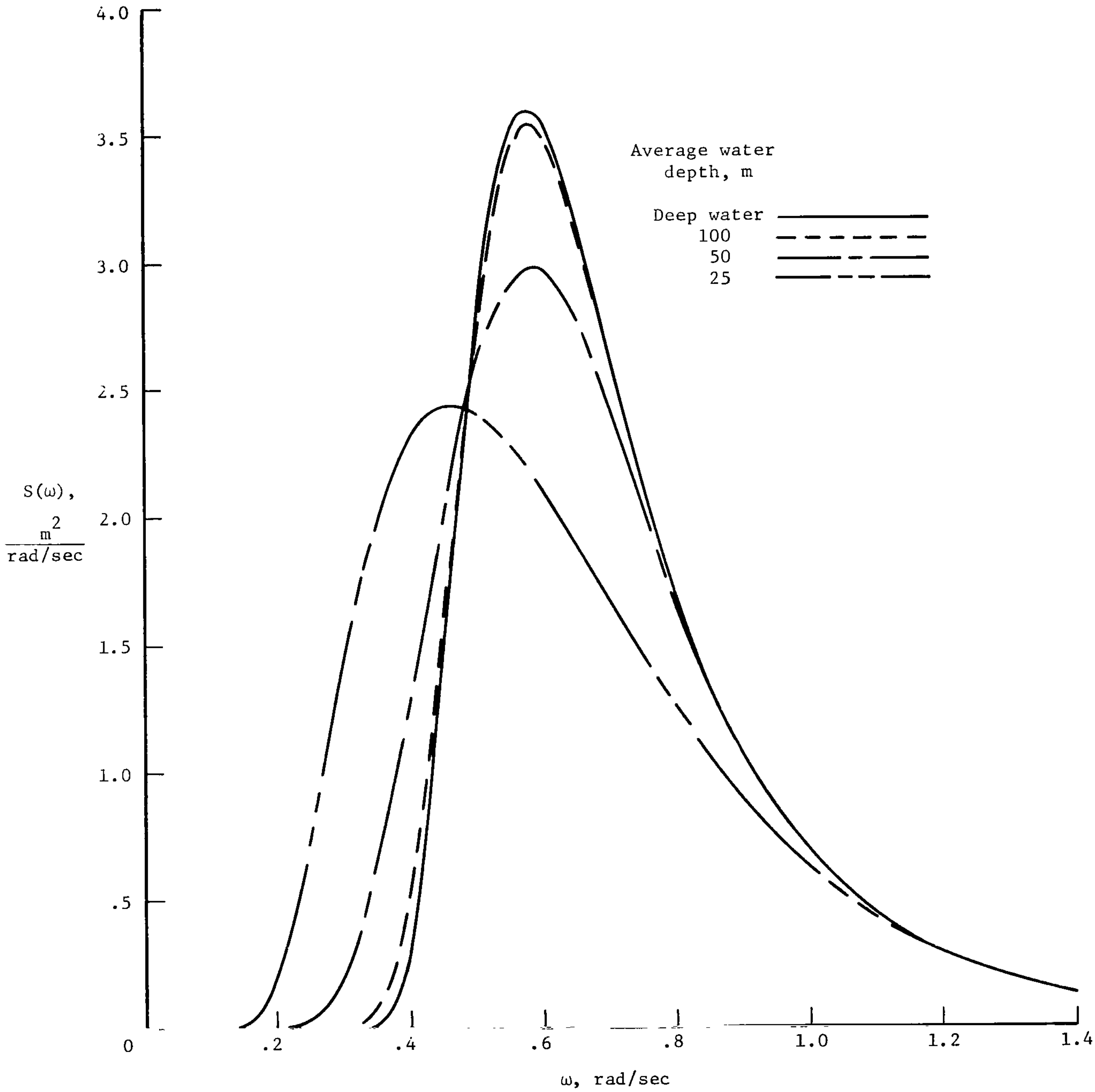

Figure 2.- Transformed spectra for windspeed of 15 m/sec and various average water depths. Aircraft speed V = 75 m/sec.

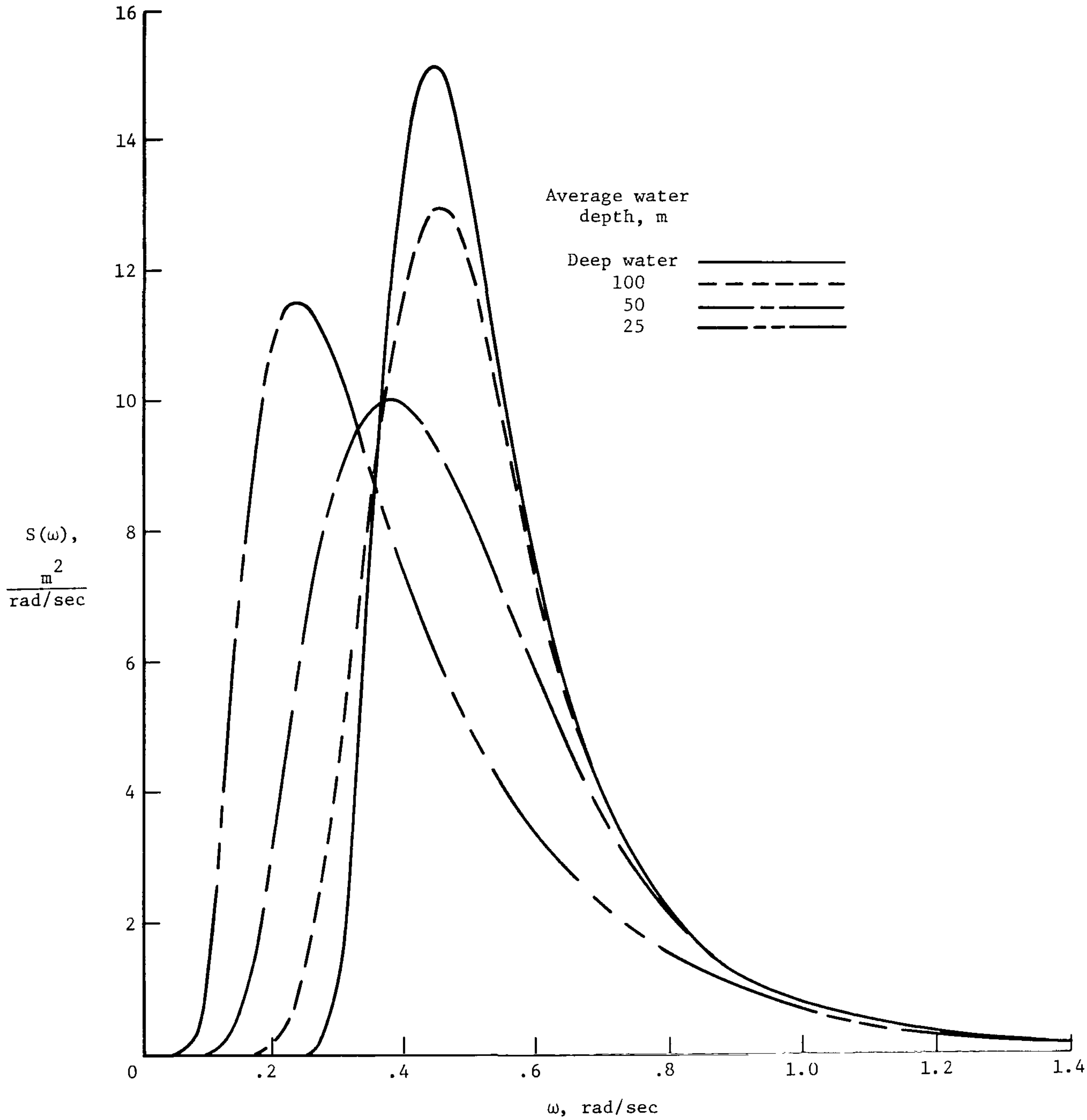

Figure 3.- Transformed spectra for windspeed of 20 m/sec and various average water depths. Aircraft speed V = 75 m/sec.

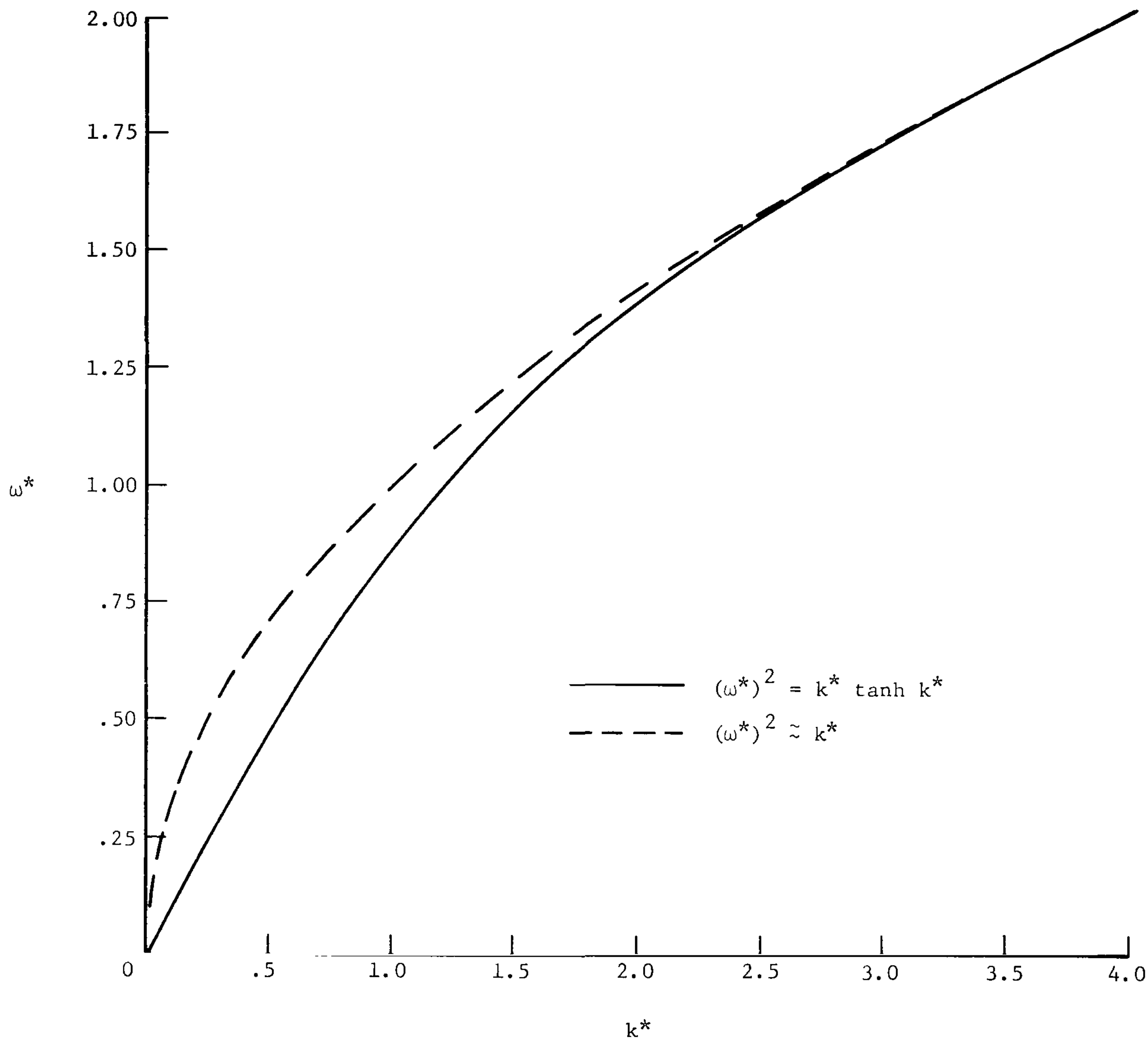

Figure 4.- Exact nondimensional dispersion relationship and deep-water approximation.

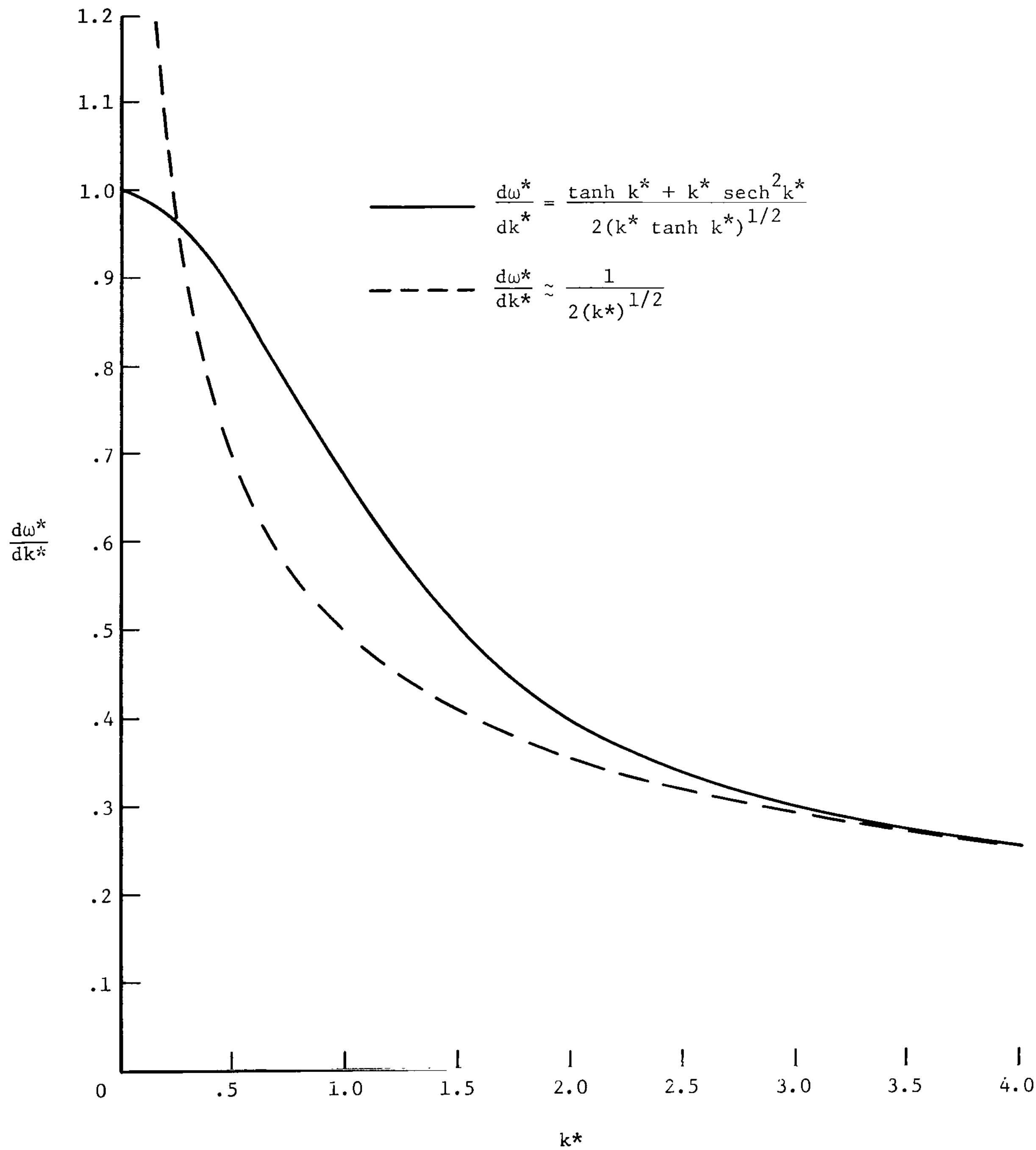

Figure 5.- Exact nondimensional wave group speed $d\omega^*/dk^*$ and deep-water approximation.

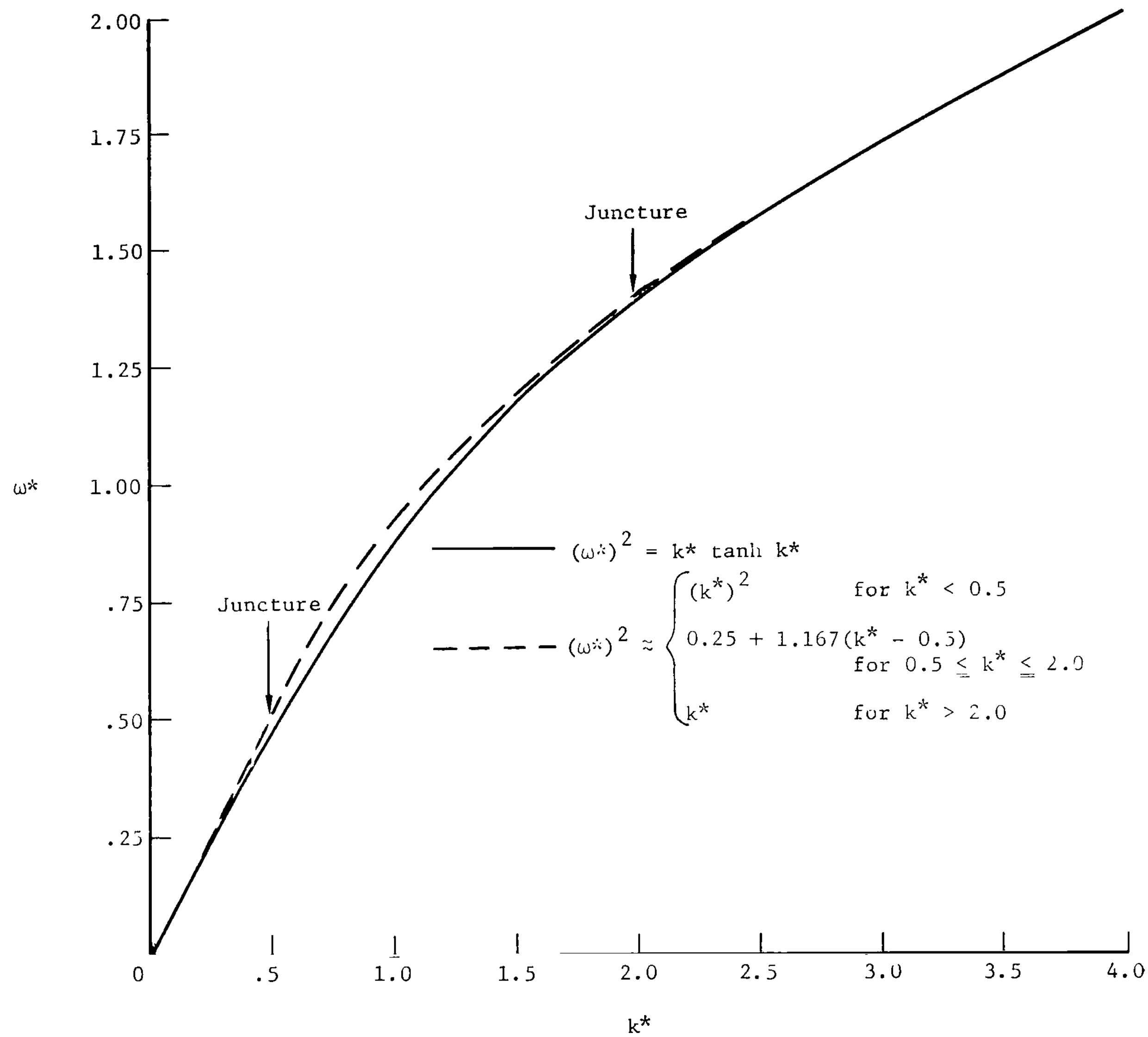

Figure 6.- Exact nondimensional dispersion relationship and piecewise-continuous approximation.

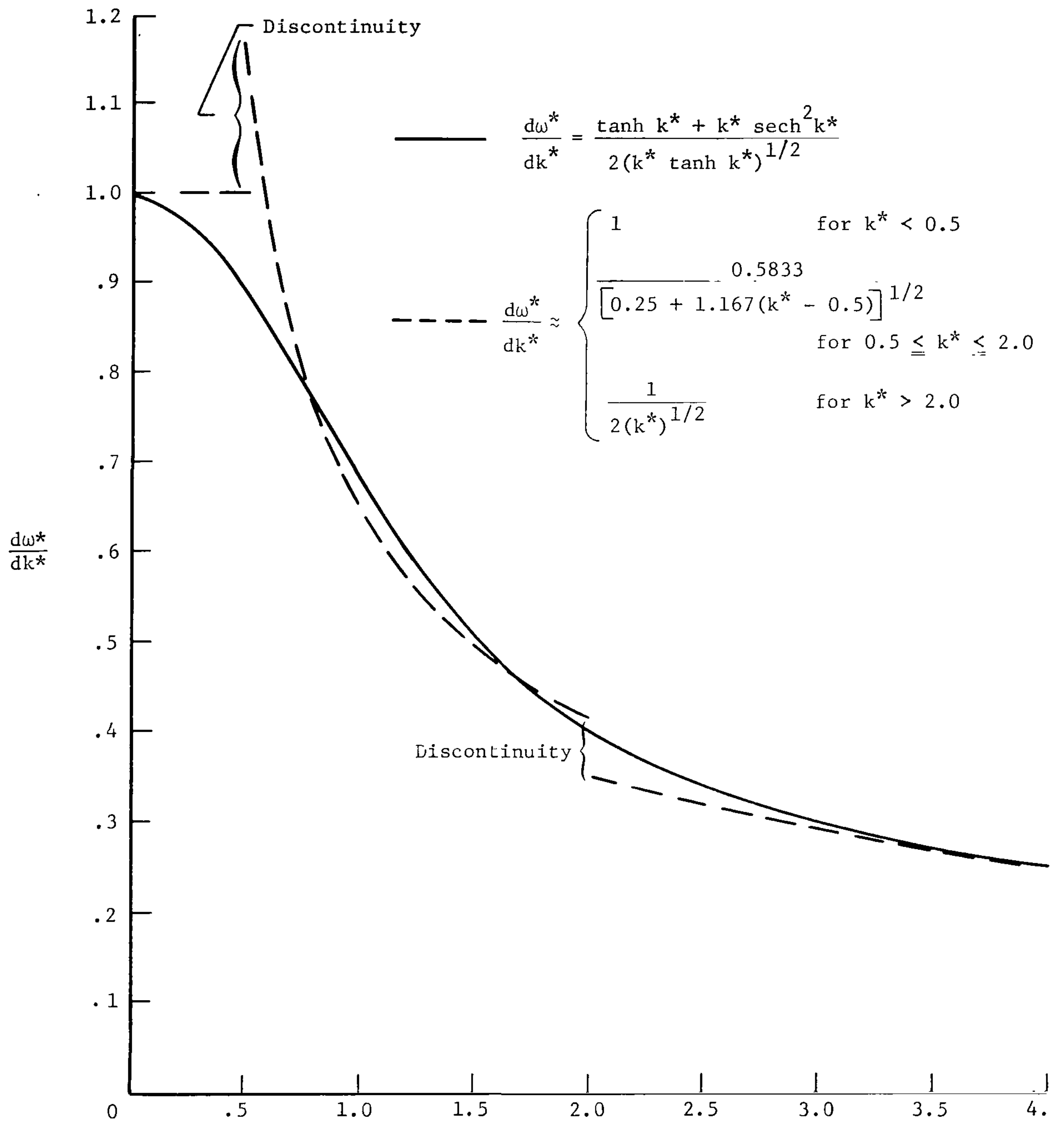

Figure 7.- Exact nondimensional wave group speed and expression derived from piecewise-continuous approximation to dispersion relationship.

NASA-Langley, 1976 L-10802

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons. Also includes conference proceedings with either limited or unlimited distribution.

CONTRACTOR REPORTS: Scientific and technical information generated under a NASA contract or grant and considered an important contribution to existing knowledge.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities. Publications include final reports of major projects, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

TECHNOLOGY UTILIZATION PUBLICATIONS: Information on technology used by NASA that may be of particular interest in commercial and other non-aerospace applications. Publications include Tech Briefs, Technology Utilization Reports and Technology Surveys.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION OFFICE

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546