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THE PROPER WEIGHTING FUNCTION FOR RETRIEVING TEMPERATURES FROM SATELLITE MEASURED RADIANCES

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THE PROPER WEIGHTING FUNCTION FOR RETRIEVING TEMPERATURES FROM SATELLITE MEASURED RADIANCES

One class of methods for converting satellite measured radiances into atmospheric temperature profiles, involves a linearization of the radiative transfer equation – e.g.,

$$\Delta R = \sum_{i=1}^{s} W_i \Delta T_i , \tag{1}$$

where ΔT_i is the deviation of the temperature in layer i from that of a reference atmosphere, ΔR is the difference in the radiance at satellite altitude from the corresponding radiance for the reference atmosphere, and W_i is the discrete (or vector) form of the T-weighting (i.e., temperature weighting) function W(P), where P is pressure. The top layer of the atmosphere corresponds to i = 1, the bottom layer to i = s - 1, and i = s refers to the surface. Linearization in temperature (or some function of temperature) is at the heart of all linear or matrix methods. The question we raise here is: What is the weighting function that should be used in Eq. (1)?

Methods based upon statistical regression determine W empirically, but those methods that involve direct inversion (see Fleming and Smith, 1972, for an excellent review) use the expression

$$W_i^{(1)} = \left.\frac{dB(T)}{dT}\right|_{T_i} \Delta\tau_i$$

where B(T) is the blackbody radiance at temperature T, $\Delta\tau_i \equiv \tau_{i-1} - \tau_i$, τ_i is the transmission from the top of the atmosphere to the bottom boundary of the ith layer for $i < s$, $\tau_0 \equiv 1$, and $\tau_s \equiv 0$. All quantities are calculated for the reference atmosphere. The continuous form of the atmospheric part of $W_i^{(1)}$ is

$$W^{(1)}(P) = -\left.\frac{dB(T)}{dT}\right|_{T(P)} \frac{d\tau(P)}{d \ln P},$$

where lnP is the height parameter. This expression for the weighting function is correct if $\tau(P)$ is independent of the temperature profile T(P). Otherwise, there is an additional term that comes from differentiating τ with respect to temperature. (In performing the integration, one must note that $\tau(P)$ is a functional of T(P') for $0 \leq P' \leq P$, which, where necessary to be explicit, will be denoted by $\tau\,[T(P')|_0^P\,;\,P]$). The proper linearization of the radiative transfer equation, when τ depends upon the atmospheric temperature profile, leads to the proper T-weighting function

$$W_i = W_i^{(1)} + W_i^{(2)}$$

where

$$W_i^{(2)} = \sum_{j=i}^{S} B(T_j)\,\frac{d\Delta\tau_j}{dT_i}\,. \tag{2}$$

In continuous form, $W(P) = W^{(1)}(P) + W^{(2)}(P)$, where

$$W^{(2)}(P) = +\,B(T_S)\,\frac{d_F\tau\,[T(P)|_0^{P_S};\;P_S]}{[dT(P)\,d\ln P]_F}$$

$$-\int_P^{P_S} B(T(P'))\,\frac{d_F d\tau\,[T(P)|_0^{P'};\;P']}{[dT(P)\,d\ln P]_F\,\ln P'}\,d\ln P', \tag{3}$$

where d_F denotes the functional derivative. The first term contains the functional derivative of the transmission to the surface with respect to T(P); in the integral the transmission has been differentiated twice: first a (logarithmic) derivative with respect to P' and then a functional derivative with respect to T(P). These expressions are derived in the Appendix.

With the T-weighting function properly defined, its physical significance is readily apparent. W(P) is the change in the radiance measured at the top of the atmosphere per unit change of temperature over a unit extent of the height parameter ln P. Thus,

$$\Delta R = W_S\Delta T_S + \int_0^{P_S} W(P)\,\Delta T(P)\,d\ln P,$$

which is the continuous form of Eq. (1). Of course, this equation is strictly correct only in the limit $\Delta T \to 0$ and is practically correct whenever the non-linear terms in the relation between temperature and radiance can be neglected. The

corresponding equation using $W^1(P)$ can be considered practically correct only with the additional limitation that the dependence of transmission on temperature can be neglected.

A comparison of $W^{(1)}(P)$ with $W(P)$ for typical sounding channels (NIMBUS 6 HIRS channel 5, centered at 717 cm^{-1}, and channel 13, centered at 2244 cm^{-1}) is shown in Fig. 1, calculated for the U.S. Standard Atmosphere (mid-latitude, spring/fall). The differences are significant. For the 717 cm^{-1} channel, the integral of $W^{(1)}(P)$ from 0 to P_S is 1.09 ergs/(cm · sec · str · °K), while the corresponding integral of $W(P)$ is 0.79. Thus, the neglect of $W^{(2)}(P)$ would overestimate by 38% the effect on the radiance of a uniform change in atmospheric temperature. For the 2244 cm^{-1} channel, the overestimate of the radiance due to neglect of $W^{(2)}$ is about half as much, 18%. For non-uniform temperature changes, the error can be larger or smaller.

In addition to reducing the sensitivity of the channel to temperature changes, the additional term also shifts the position of the weighting function. This is illustrated by W_{REN} (dashed curve on left hand side of Fig. 1) which is W renormalized so that its peak value is the same as $W^{(1)}$. The peak of W is shifted downward from $W^{(1)}$ by about 60 mb, it is narrower above the peak, and has a larger tail in the stratosphere.

The effect of the $W^{(2)}$ term on temperature retrievals can best be assessed by considering $W^{(1)}$ to be the T-weighting function incorporated into the retrieval process and corresponding to a channel for which measurements are desired. The actual measurements, however, are different, corresponding to the T-weighting function W. That difference can be treated as a measurement error:

$$\Delta R_E = \sum_{i=1}^{S} W_i^{(2)} \Delta T_i .$$

The error thus depends upon how far the solution is from the initial guess. For a good initial guess, such as that obtainable from forecasts, a typical value of ΔT in the troposphere (below 200 mb) is ~ 2 to 3°K. Under those circumstances,

$$|\Delta R_E| \approx 2.5 \left| \sum_{i=1}^{S} W_i^{(2)} \right| ,$$

which has the value .50 and .0029 ergs/(cm · sec · ster) for the 717 cm^{-1} and 2244 cm^{-1} channels, respectively. At 243°K, the brightness temperature associated with these channels for the standard atmosphere, these radiance errors

correspond to brightness temperature errors of .43°K for the 717 cm^{-1} channel and .21°K for the 2244 cm^{-1} channel.

The above errors, treated as radiance errors, are larger than the instrument noise usually specified for these channels. Furthermore, these errors tend to be consistent over a synoptic scale grid (~500 km), so that averaging the measured radiances within a grid will not reduce this error. Finally, the statistical, or minimum-RMS, method and the closely related minimum information method (reviewed by Fleming and Smith, 1972), which minimize the effects of radiance errors on the retrieved profile, assume that radiance errors are independent of atmospheric temperature, but that is certainly not the case with the error associated with the weighting function definition. Thus, with any of the linear methods – such as the statistical method, the minimum information method, as well as the Backus-Gilbert method (Conrath, 1972), which rely on calculated (as opposed to empirically determined) weighting functions – the neglect of the $W^{(2)}$ term in the weighting function may seriously affect the accuracy of the results.

For weighting functions associated with very narrow spectral intervals, essentially monochromatic, the relative importance of $W^{(2)}$ can be even larger. An example of such a channel, centered at 2386.88 cm^{-1} and 0.3 cm^{-1} wide, is shown in Fig. 2. The effect of $W^{(2)}$, which is usually negative for reasons explained below, is so strong that the proper T-weighting function is actually negative above 400 mb. It simply means that increasing the temperature in the layers above 400 mb will actually reduce the radiance in this channel. The "radiance error" due to the neglect of $W^{(2)}$ would be .014 ergs/(cm · sec · ster) for $\Delta T \approx 2.5$°K. This corresponds to an error of approximately 1°K in brightness temperature at 257°K, the brightness temperature for this channel with a standard atmosphere.

The factors which determine the sign and magnitude of $W^{(2)}$ (P) can be brought out by inserting into Eq. (3) the general form for transmission

$$\tau(P) \equiv \tau\,[T(P')|_0^P;\ P] = \langle \tau_\nu\,[T(P')|_0^P;\ P] \rangle$$

$$\tau_\nu(P) \equiv \tau_\nu\,[T(P')|_0^P;\ P] = \exp\left[-\int_0^P K_\nu(T(P'),\ P')\, d\ln P'\right]$$

where K_ν — the volume absorption coefficient at wave number ν, times the local scale height — is a point function of pressure P' and the local temperature T(P'). (The angular brackets denote averaging over the spectral response of the channel.) Carrying out the differentiation, with care in taking functional derivatives, leads to the following expression

$$W^{(2)}(P) = \left\langle \frac{dK_\nu(T(P), P)}{dT(P)} \left[B(T(P))\, \tau_\nu(P) - B(T_S)\, \tau_\nu(P_S) + \int_P^{P_S} B(T(P'))\, \frac{d\tau_\nu(P')}{d \ln P'}\, d \ln P' \right] \right\rangle \tag{4}$$

The first term in the square brackets corresponds to the diagonal of the matrix $d\Delta\tau_j/dT_i$ appearing in Eq. (2), which expresses $W^{(2)}$ in vector form. Since $d\tau_\nu/d\ln P$ is negative, it follows that the diagonal term is always of opposite sign to the off-diagonal elements $P' < P$. (The off-diagonal elements on the other side, $P' < P$, are of course zero.) Assuming no temperature discontinuity at the surface, Eq. (4) can be integrated by parts to yield

$$W^{(2)}(P) = -\left\langle \frac{dK_\nu(T(P), P)}{dT(P)} \int_P^{P_S} \tau_\nu(P')\, \frac{dB(T(P'))}{d \ln P'}\, d \ln P' \right\rangle \tag{5}$$

We are now in a position to explain the sign of $W^{(2)}(P)$. For the numerical examples considered here (Figs. 1 and 2), $dK_\nu/dT > 0$ because of the location of these channels on the wings of the CO_2 absorption band; for the dominant absorption lines, the rotational quantum numbers of the lower states involved in the transitions are high and the rotational energies are $> kT$. It then follows that the occupation probabilities for those states increase with increasing temperature, resulting in an increase of absorption with increasing temperature. With $dK_\nu/dT > 0$, the sign of $W^{(2)}(P)$ depends upon the temperature gradient. When $dT(P)/d\ln P > 0$, as is generally the case in the troposphere, $W^{(2)}(P)$ becomes negative.

The magnitude of $W^{(2)}(P)$ is very sensitive to the ratio of the rotational energy E_{rot} to kT. For $E_{rot} >> kT$ (e.g., on the wings of the vibrational-rotational absorption bands) dK_ν/dT is large; but for $E_{rot} \lesssim kT$ (e.g., in the Q-branch) dK_ν/dT is small in magnitude and could be positive or negative. Eq. (5) also reveals that $W^{(2)}(P)$ is approximately proportional to the temperature gradient, and for an isothermal atmosphere, it is zero.

We conclude, therefore, that the contribution of $W^{(2)}(P)$ to the proper weighting function depends upon the spectral position of the channel within the absorption band and upon the temperature gradient. The largest effect is in the troposphere, as in the examples shown here, and one would expect the effect to be smaller in stratospheric channels (in fact, it turns out to be negligible in the Q-branch channels).

It is to be noted that the considerations in this note apply as well to methods which linearize the radiative transfer equation with respect to the Planck function instead of temperature. In that case, the weighting function usually employed is $d\tau(P)/d \ln P$, but the proper weighting function should be

$$W_p(P) = \frac{d\tau(P)}{d \ln P} + \frac{1}{dB/dT} W^{(2)}(P);$$

and the change in radiance is related to the change in Planck radiance $\Delta B(P)$ at each level by

$$\Delta R = \int_0^{P_S} W_p(P)\, \Delta B(P)\, d \ln P.$$

In more general applications — e.g., nonlinear retrieval methods in which the radiative transfer equation is iteratively applied to a temperature profile which undergoes correction at each iteration step — functional derivatives can be used in a Taylor series expansion to relate the transmission function for one temperature profile $T(P)$ to the transmission function for a corrected temperature profile $T(P) + \Delta T(P)$. For example, if the original transmission function is $\tau(P)$ then the transmission for the corrected temperature profile is given by

$$\tau(P) + \int_0^P \frac{d_F \tau\, [T(P')|_0^P;\, P]}{[dT(P')\, d \ln P']_F}\, \Delta T(P')\, d \ln P'$$

$$+ \frac{1}{2} \int_0^P \int_0^P \frac{d_F^2 \tau\, [T(P')|_0^P;\, P]}{[dT(P'')\, d \ln P'']_F\, [dT(P')\, d \ln P']_F}\, \Delta T(P'')\, \Delta T(P')\, d \ln P'\, d \ln P''$$

$$+ \left\{ \begin{matrix} \text{terms with higher order} \\ \text{functional derivatives} \end{matrix} \right\}.$$

In practice, it is not necessary to go beyond the second order term, and even the first order correction is sufficient for most applications; this will be the subject of a paper currently under preparation.

To summarize, we have called attention to a potentially serious omission in the application of linear methods to the retrival of temperatures from satellite measured radiances. The weighting functions usually employed do not properly take into account that atmospheric transmission itself depends upon temperature. We have shown here the form for the proper weighting function. The impact of this omission has been shown to be equivalent to a radiance brightness temperature error of several tenths of a degree for typical tropospheric channels on current sounders and as much as 1°K for very narrow channels that may be employed in high vertical resolution sounders of the future. More generally, we have presented a formalism for correcting transmission functions for changes in the temperature profile; the corrections can be computed to any desired degree of accuracy.

APPENDIX

We can derive the proper T-weighting function by first dividing the atmosphere into isothermal layers and treating the temperature in each layer and at the surface as an independent variable T_j, where $j = 1$ is the topmost layer, $j = s - 1$ is the bottom layer, and s corresponds to the surface. The radiance measured at the top of the atmosphere can then be written

$$R(T_1, T_2, \ldots T_s) = \sum_{j=1}^{s} B(T_j)\, \Delta\tau_j(T_1, T_2, \ldots T_j)$$

where $\Delta\tau_j \equiv \tau_{j-1} - \tau_j$, τ_j is the transmission from the top of the atmosphere to the bottom boundary of the jth layer for $j < s$, $\tau_0 \equiv 1$, and $\tau_s \equiv 0$. The T-weighting function is defined as the change in radiance due to a change in the temperature in layer i, divided by the temperature change, in the limit as the temperature change approaches zero,

$$W_i \equiv \frac{\partial R}{\partial T_i}$$

$$= \frac{dB(T_i)}{dT_i}\, \Delta\tau_i + \sum_{j=i}^{s} B(T_j)\, \frac{d\Delta\tau_j}{dT_i},$$

where the terms in the summation for $j < i$ are dropped because they are necessarily zero.

The above expression is the discrete, or vector form of the T-weighting function. The surface component simplifies to

$$W_s = \frac{dB(T_s)}{dT_s}\, \tau_{s-1}$$

by virtue of the fact that atmospheric transmission is independent of the surface temperature. The atmospheric components can be expressed in continuous form by allowing temperature to be a continuous function of a vertical parameter x

(ranging from 0 at the surface to ∞ at the top of the atmosphere). In that case the transmission will be a continuous point function of x, and at each point x, it will be a continuous functional of T(x') in the range $x \leq x' \leq \infty$, which will be denoted by

$$\tau\,[T(x')|_x^\infty;\ x].$$

The radiation at the top of the atmosphere will also be a functional of T(x),

$$R\,[T(x)|_0^\infty] = B(T_S)\,\tau\,[T(x)|_0^\infty;\ 0]$$

$$+ \int_0^\infty B(T(x'))\,\frac{d}{dx'}\,\tau\,[T(x)|_{x'}^\infty;\ x']\,dx'$$

The T-weighting function is defined in a manner completely analogous to the discrete case, but taking into account the dependence of R on the function T(x). We consider a small positive change in temperature $\Delta T(x)$ which is zero everywhere except in the range $x - 1/2\Delta x \leq x \leq x + 1/2\,\Delta x$, where Δx is a small positive quantity. The T-weighting function is then defined as

$$W(x) = \lim_{\Delta T(x),\ \Delta x \to 0} \frac{\Delta R\,[T(x)|_0^\infty]}{\Delta T(x)\ |\Delta x|}$$

which, by definition, is the functional derivative of R with respect to the function T(x), and which will here be denoted by

$$W(x) = \frac{d_F R\,[T(x)|_0^\infty]}{[dT(x)\,dx]_F}.$$

(See, for example, Evans, 1964, for a mathematical discussion of functionals.) Computing the differentials and then proceeding to the limit, we obtain

$$W(x) = \frac{dB(T)}{dT}\bigg|_{T(x)} \frac{d}{dx}\,[T(x')|_x^\infty;\ x] + B(T_S)\,\frac{d_F\,\tau\,[T(x)|_0^\infty;\ 0]}{[dT(x)\,dx]_F}$$

$$+ \int_0^x B(T(x'))\,\frac{d_F\,d\tau\,[T(x)|_{x'}^\infty;\ x']}{[dT(x)\,dx]_F \cdot dx'}\,dx'$$

where the upper limit of the integral has been adjusted in recognition of the fact that the integrand is zero for $x' > x$.

We carry this expression into the main part of the paper with the substitution $dx = -d \ln P$ for the height parameter.

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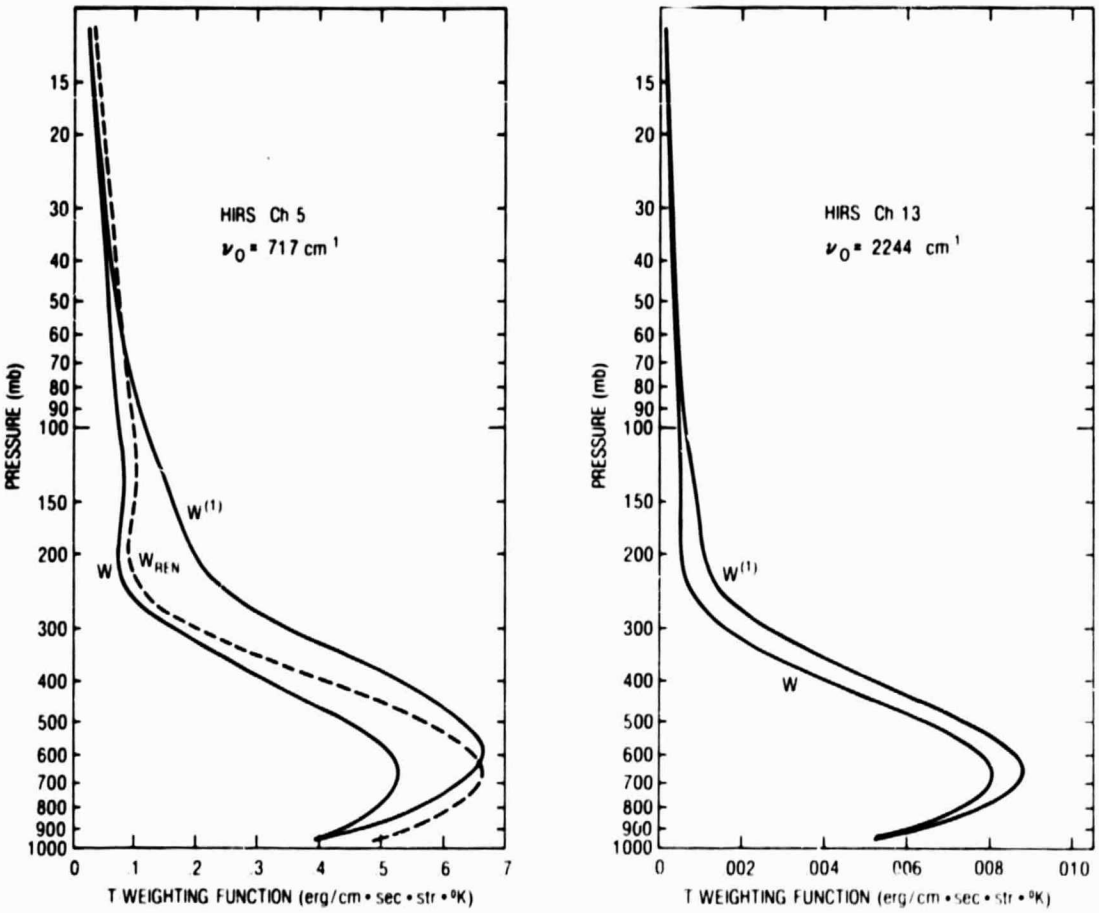

Figure 1. The T-weighting functions for two of the channels on the NIMBUS 6 High-resolution Infrared Radiometer Sounder (HIRS). W, the proper T-weighting function, introduced in this paper, is compared with $W^{(1)}$, the T-weighting function usually employed in linear methods of retrieving temperature. W_{REN} (shown only for Ch. 5) is W renormalized so that its peak value is the same as for $W^{(1)}$.

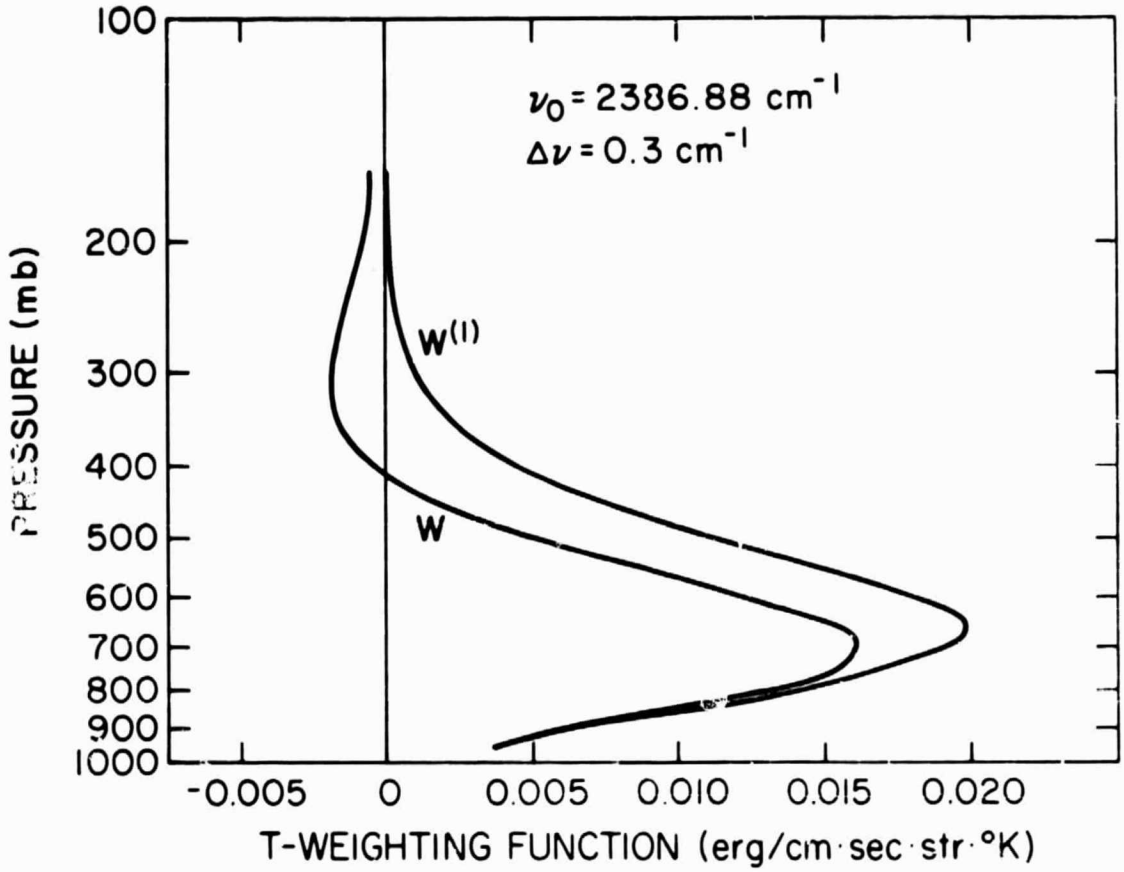

Figure 2. The proper T-weighting function W compared with the usually employed weighting function $W^{(1)}$ for a narrow, practically monochromatic channel mid-way between two absorption lines in the CO_2 4.3 μm band.

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