PARAMETER ESTIMATING STATE RECONSTRUCTION

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Office of The Associate Director for Engineering

(NASA-TM-X-73313) PARAMETER ESTIMATING
STATE RECONSTRUCTION (NASA) 99 P HC $5.00
CSCL 12B

June 1976

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama
16. ABSTRACT

The focus of this research is parameter estimation for systems whose entire state cannot be measured. Linear observers are designed to recover the unmeasured states to a sufficient accuracy to permit the estimation process. These systems must be observable. There are three distinct dynamics that must be accommodated in the system design: the dynamics of the plant, the dynamics of the observer, and the system updating of the parameter estimation. The latter two are designed to minimize interaction of the involved systems.

These techniques are extended to weakly nonlinear systems. The application to a simulation of a Space Shuttle POGO system test is of particular interest. A nonlinear simulation of the system is developed, observers designed, and the parameters estimated.
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<tr>
<td>a ≤ b</td>
<td>a less than or equal to b</td>
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<td>control distribution matrix (n x q)</td>
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<td>e(t)</td>
<td>error, x(t) - ̂x(t)</td>
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<tr>
<td>e( )</td>
<td>natural number to ( ) exponent</td>
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<td>F</td>
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<td>h(t,x)</td>
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<tr>
<td>in.²</td>
<td>square inches</td>
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<td>K</td>
<td>observer gain matrix (n x m)</td>
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<tr>
<td>k</td>
<td>kth interval</td>
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<td>$L_D$</td>
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<td>$L_N$</td>
<td>discharge section inertance (in.$^2$/sec)$^{-1}$</td>
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<td>$\ell$</td>
<td>$\ell$th term</td>
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<td>m x n</td>
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<td>revolutions per second</td>
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<td>( S )</td>
<td>Laplace operator</td>
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<td>( S_{O1} )</td>
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<td>sec</td>
<td>seconds</td>
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<td>( T )</td>
<td>sample period (sec)</td>
</tr>
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<td>( t )</td>
<td>time (sec)</td>
</tr>
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<td>( t_0 )</td>
<td>initial time (sec)</td>
</tr>
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<td>( U )</td>
<td>control signal ((q \text{ dimensional}))</td>
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<td>( x )</td>
<td>state vector ((n \text{ dimensional}))</td>
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<td>( \delta(\ ) )</td>
<td>variation of ((\ ))</td>
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<td>( \varepsilon )</td>
<td>an element of, or an arbitrary small number</td>
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<td>$((\cdot):\cdot))$</td>
<td>partitioned matrix</td>
</tr>
<tr>
<td>$(\cdot)$</td>
<td>time derivative</td>
</tr>
<tr>
<td>$(\cdot)^*$</td>
<td>estimate of $(\cdot)$</td>
</tr>
<tr>
<td>$(\cdot)(t)$</td>
<td>time dependence</td>
</tr>
<tr>
<td>$(\cdot)_{ij}$</td>
<td>elements of the matrix $(\cdot)$, i-row, j-column</td>
</tr>
<tr>
<td>$(\cdot)^k$</td>
<td>exponent, kth power</td>
</tr>
<tr>
<td>$(\cdot)^{(i)}$</td>
<td>$i$th time derivative</td>
</tr>
<tr>
<td>$(\cdot)^T$</td>
<td>matrix transpose</td>
</tr>
<tr>
<td>$(\cdot)^{-1}$</td>
<td>matrix inverse</td>
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</table>
The advent of large-scale computing equipment has made these concepts feasible. A logical extension of the technique considered here would be to put an additional loop around the object dynamic system. This loop would determine a model structure with which to estimate parameters. These parameters would permit state determination. This would approach the advanced objective previously mentioned. This research will be confined to those cases for which a rational model structure has been chosen.

To perform a linear analysis, a mathematical model structure must be contrived or selected. Many considerations contribute to the selection of a rational model structure. The first step is to define a response matching criterion and then choose a model structure capable of fulfilling the criterion. This fulfillment, generally checked by simulation, is accomplished by time response comparison, frequency response comparison or, more generally, a combination of both. The inclusion of small signal nonlinearities, such as stiction, windup, hysteresis, and deadband, is dictated by the dynamic effect on the system as ascertained by the response matching criterion. A central feature of model structure selection is system dimensionality truncation. Dimensionality truncations for model selection are of two types: those due to modeling complexity, and those to reduce dimensionality of an already chosen model structure. Modeling complexity is necessarily broken at some level since most complex system models could be made infinite in extent. These truncations are made on the basis of insight, feel, experience, and logistics of the computational equipment available. A rational choice of model structure may simply be the exclusion of dynamic effects in some frequency regime for which the control and/or the plant are nonresponsive. A more complicated scheme consists of including only coupling dynamics. The implementation of the coupling dynamics scheme is straightforward although sometimes computationally difficult. A subcomponent representation is determined first as an isolated system and then compared to the subcomponent representation in the closed-loop system. If the pole-zero representation moves more than some judgmental amount, that subcomponent must be included in the overall system dynamics. If the subcomponent's dynamics may be discarded, the steady-state contribution is accounted for algebraically. A simple state variable criterion is to eliminate those states whose derivatives remain less than some judgmental amount. Many other schemes may be devised as well as combinations of these schemes. The ultimate criterion is the satisfactory working of the finally designed and analyzed system.

The purpose of the research may be summarized as providing the "best" representation of the system linear model, or Jacobian, for a given configuration. Best representation means the best model attainable under a qualitative judgment involving accuracy, measurement inaccuracies, and system disturbance. The principal objective is the assessment of the validity of the mathematical model used to design a subject system.

While there are other techniques providing the same information, the proposed technique reduces the system history required for solution. Most estimation techniques avoid the partially measured state vector because the standard approach is to adjoin the unmeasured states to the parameter matrix. The result is that an n-squared problem has
CHAPTER I
INTRODUCTION

Statement of the Problem and Objectives

An advanced objective of control system theory is to build a learning device so an unknown system is directed to some goal by the device. First the identification of the system is implemented. Based on the identification, the state of the system is determined. An appropriate control stimulus results in a response that satisfies some rational figure of merit. This is presently accomplishable in only basic systems \[1\]: The purpose of this research is to develop and demonstrate a technique, consisting of several concepts, that permits simultaneous calculation of the state and a reasonable facsimile of the plant. These concepts center upon state reconstruction and parameter estimation.

There is a certain amount of literature using these concepts \[2,3\], but only recently have the combined concepts of state reconstruction and parameter estimation been exploited \[4\] to provide information simultaneously of the state and the system representation. The literature is confined to linear autonomous systems, while this research will attempt to extend the developed techniques to nonautonomous and nonlinear systems. The technique of Reference 4 requires a Liapunov function of the unknown system, which is possible for linear systems. The advantage is the synthesis of a globally convergent scheme. The disadvantages are that the Liapunov function may not exist for nonautonomous and nonlinear systems. The method of this research uses a steepest descent of gradient type method. A disadvantage of a gradient method is that initial estimates must be close. However, in practice, the system is reasonably well known and this disadvantage is not overwhelming.

This paper will attempt to apply the combined techniques of parameter estimation and state reconstruction to the measurements of nonlinear physical systems. In practice, linear systems do not exist. However, there are regions of operation on which any system exhibits nearly linear behavior. The limitation may exist that the region of linearity is too small or that expected excitations will drive the system out of its linear region. In any case, a comprehensive control or system analysis begins with a linearization of the subject system. For many applications, the analysis either forms a basis for design or provides a rationale for redesign or alteration of the system. A fundamental, but sometimes unanswered, question is “how good a representation of the system is this linear model?” In many fields and applications, an a posteriori analysis is undertaken to assess the mathematical modeling accuracy. Generally, this consists of a manual iterative assessment until some degree of accuracy is achieved.

Parameter estimating state reconstruction can be applied, as a black box, to a system’s measurements to provide a real-time assessment of the current linear model of that system. By assessing the response from different pieces of hardware, statistics may be compiled as to the spread of that system’s operation. An analysis of extreme conditions provides an assessment of the sensitivity of the system to a real environment. As a result, a quantitative assessment of the system design analysis is possible.
been expanded to an \( n^2 + n \) problem. On the other extreme, the Liapunov approach becomes difficult for systems of greater than single output because a Liapunov function must be contrived for each of the outputs which also are coupled. The proposed technique retains the \( n^2 \) dimensionality of totally measured systems.

Figure 1-1 is a diagram of the system description. A state reconstructor is used to recover the unmeasured states. The state reconstructor is used with the reference model to form an error for the estimation process. Establishing validity of using the state reconstructor output and addressing coupled systems dynamics are the principal concerns to be analyzed.

Chapter Description

Chapter I is a delineation of the area of research with an identification of problem areas and overall objectives. Chapter II will develop the observability theorems for the systems to be considered. This will be followed by developing and presenting the necessary observer or state reconstruction theory to support the research. The treatment will be subdivided to treat linear time invariant, linear time varying, and nonlinear systems. Chapter III will address parameter estimation with state reconstruction. The problem of model structure and model matching criteria will be analyzed.

Chapter IV will develop a dynamic model of the CTL-V Space Shuttle POGO test facility to be analyzed by the technique of this paper. CTL-V is a particularly good example since fully half of the involved parameters are unmeasurable. A nonlinear model will be developed and the linearized equivalent will be analyzed dynamically at the rated power level operating point. Chapter V will design the necessary observers, to permit application of the technique, and present the results of the simulation and analysis. Finally the technique will be summarized considering its advantages, disadvantages, and unique characteristics.
Figure 1-1. Block diagram of the system.
CHAPTER II
OBSERVABILITY AND OBSERVERS

Paramount to any closed-loop control consideration is the measurability and/or observability of the object dynamic system. An unobservable system may be controllable only in an open-loop sense. Therefore, this research will be confined to closed-loop control and thus observable systems. Observability will be dealt with in detail, particularly those aspects pertaining to linear constant coefficient, linear time varying, and nonlinear dynamic systems. Observability is a required condition for the state reconstruction process and is included for completeness.

Many unobservable systems may be recast in a form tractable to the techniques of this research. The procedure consists of partitioning the system into observable and unobservable parts. The partitioned observable part may then be handled as an observable system. If the unobservable partition interacts with the observable partition, the interacting elements may be treated as disturbance inputs to the observable system. The partitioning process can be accomplished by means of a transformation to controllability-observability canonical form [5].

A pedagogic examination of the relationship between controllability and observability will aid in the development of the requirements for observability. If $x$ is an $n$-space representation of the system, $y$ is an $m$-space representation of the measurements of that system, and $x_0$ is the initial state, then the controllability problem may be defined as the existence of a solution from $x_0$ to a desired state, $x_f$. The observability problem is defined as the existence of a unique one-to-one mapping from $x$ to $y$. This has been elucidated by Kalman [5] as the principle of duality. The principle of duality [6] depends on the uniqueness of the solution and the mapping. This principle applies to linear constant coefficient and linear time varying systems. However, for nonlinear systems, existence is not necessarily uniqueness and the principle does not apply [7]. It remains to develop the conditions for assuring a unique one-to-one mapping from $x$ to $y$ for the various dynamical systems.

Observability Theorems

**Theorem: (Linear time invariant)**

The system

\[ x = Ax 
\]

\[ y = C^T x \]

is observable.
where \( x \) is an \( n \) vector, \( y \) is an \( m \) vector \((m \leq n)\), \( A \) is \( n \times n \), and \( C^T \) is \( m \times n \), is completely observable if and only if the composite \( n \times mn \) matrix

\[
[C, A^T C, \ldots, A^{T_{n-1}} C]
\]

is of rank \( n \). The proof of the preceding is given in many texts [6,8].

**Theorem: (Linear time varying)**

The system

\[
\dot{x} = A(t)x, \quad y = C^T(t)x
\]

where the variables are as previously defined, is completely observable on the time interval \( t_0 \leq t \leq t_1 \) if and only if the matrix

\[
M(t_0, t_1) = \int_{t_0}^{t_1} \Phi^T(\tau, t_0) C(\tau) C^T(\tau) \Phi(\tau, t_0) \, d\tau
\]

is nonsingular. The matrix \( \Phi(t, t_0) \) is the unique fundamental matrix satisfying

\[
\frac{d}{dt} \Phi(t, t_0) = A(t) \Phi(t, t_0), \quad \Phi(t_0, t_0) = I_n
\]

For complete observability, the above must hold for every \( t_0 \) and some finite \( t_1 > t_0 \). The proof of the preceding theorem is likewise found in most modern control texts [6,8].

For nonlinear systems, a more precise definition of terms is required because existence and uniqueness are no longer equivalent. The nonlinear system may be represented as
\[ \dot{x} = f(t, x) \]
\[ f: [t_0, t_1] \times \Omega \subseteq E' \times \mathbb{R}^n \rightarrow \mathbb{R}^n \] (2-1)

with measurements

\[ y = h(t, x) \]
\[ h: [t_0, t_1] \times \Omega \subseteq E' \times \mathbb{R}^n \rightarrow \mathbb{R}^m \] (2-2)

The initial state \( x(t_0) \) is in general unknown since \( m \leq n \). Now assume that the \( \ell \)th order derivatives of \( f \) and \( h \) exist for every \( x \in \Omega \) and for every \( t \in [t_0, t_1] \) where \( \ell m \geq n \). Expand \( y(t) \) as a Taylor series

\[ y(t) = y(t_0) + \frac{\partial y}{\partial t}(t_0)(t - t_0) + \ldots + \frac{y^{(\ell)}(t_0)}{\ell!} (t - t_0)^\ell \] (2-3)

where

\[ y(t_0) = h(x(t_0), t_0) - h_0(x(t_0), t_0) \]
\[ \frac{\partial h_0}{\partial x_0} (x(t_0), t_0) + \frac{\partial h_0}{\partial x_0} (x(t_0), t_0) f(x(t_0), t_0) \Delta h_1(x(t_0), t_0) \]

Now define

\[ H(x(t_0)) \]

where

\[ z = \begin{bmatrix} y(t_0) \\ \vdots \\ y^{(\ell-1)}(t_0) \end{bmatrix} \quad H(x(t_0)) = \begin{bmatrix} h_0(x(t_0), t_0) \\ \vdots \\ h_{\ell-1}(x(t_0), t_0) \end{bmatrix} \] (2-4)
The nonlinear map \( H(x(t_0)) \) is called the “observability mapping” of the system. The system described by Eqs. (2-1) and (2-2) is said to be completely observable in \( \Omega_0 \) on the time interval \([t_0, t_1]\) if there exists a one-to-one correspondence between the set \( \Omega_0 \) of initial states and the set of trajectories of the observed output \( y(t) \) for \( t \in [t_0, t_1] \). If the observability map \( H \) is one-to-one \( \Omega_0 \rightarrow H(\Omega_0) \), then knowing \( z \) uniquely determines \( x(t_0) \) so that the system is completely observable. Several publications [7,9] have investigated these conditions for global observability.

**Theorem:**

The system described by Eqs. (2-1) and (2-2) is completely observable in the set \( \Omega_0 \) of initial states on the time interval \([t_0, t_1]\) if

1. \( \mathfrak{m} = n \), where \( n \) is the span of the state, \( m \) is the number of outputs, and \( \mathfrak{f} \) is the \( \ell \)th derivative of \( f \) and \( h \) which are assumed to exist.
2. The observability mapping of this system is differentiable.
3. There exists an \( \varepsilon > 0 \) such that the absolute values of the leading principal minors \( \Delta_1, \Delta_2, \ldots, \Delta_n \) of the system Jacobian satisfy

\[
|\Delta_1| > \varepsilon, \quad \frac{|\Delta_2|}{|\Delta_1|} > \varepsilon, \quad \ldots, \quad \frac{|\Delta_n|}{|\Delta_{n-1}|} > \varepsilon
\]

for all \( x \in E^n \), then \( H \) is one-to-one from \( E^n \) onto \( H(E^n) \). This result is proven in Reference 9.

The development of the conditions for nonlinear observability gives visibility to a minor theorem that can be applied to linear systems. If the observability mapping is related to linear systems, it reduces to the familiar form of the condition for observability. Another use may be made of these results. First, consider the case where there is only one measurement. Now the vector \( z \) is simply

\[
z = \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y(n-1) \end{bmatrix} = \begin{bmatrix} C^T \\ C^T A \\ \vdots \\ C^T A^{n-1} \end{bmatrix} x
\]  

(2-5)
which is the transpose of the observability matrix. This matrix must be of rank \( n \) since the system is observable so that premultiplying by the transpose and taking the inverse, Eq. (2-5) may be written as

\[
x = \begin{bmatrix} \begin{bmatrix} C^T \\ CT_A \\ \vdots \\ CT_A^{n-1} \end{bmatrix} & \begin{bmatrix} C^T \\ CT_A \\ \vdots \\ CT_A^{n-1} \end{bmatrix} \end{bmatrix}^{-1} \begin{bmatrix} C^T \\ CT_A \\ \vdots \\ CT_A^{n-1} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \vdots \\ y(n-1) \end{bmatrix}
\]

(2-6)

Given the output and \( n-1 \) derivatives of the single output, the \( n \)-state vector may be deterministically obtained. This result is of little use because, in practice, it is difficult to differentiate the measurement with great insensitivity or precision.

**Observers**

Observer or state reconstructors have been repeatedly examined in the literature since Luenberger [10,11] quantified the concept. The reason for this interest was the advent of state variable theory [6], which organized a dynamic system in such a fashion that the observations of system are not necessarily the measure of the system. The system may be made up of \( n \) states and observed by \( m \) observations where \( n \) is not necessarily equal to \( m \), but generally is greater than \( m \). The observer fills a need to have \( n \) states from \( m \) observations.

The objective of the observer process is to provide reasonable approximations to those states that are not directly measured. Then these states are available for use in the implementation of a control law or strategy. Observers also find use in system estimation and identification. The conceptual basis for the observer lies in the process of driving an auxiliary dynamic system with the available outputs of the subject dynamic system.

The state reconstructor is an auxiliary dynamic system that deterministically calculates the states using the difference between the real measurements and the measurements from the reconstructor. The state reconstructor is an intriguing mathematical phenomenon because apparently “free information” is acquired. That is, \( m \) measurements are sufficient to determine an \( n \)-vector state. Considerable attention has been focused on “reduced order observers.” If some of the states are directly observed the system may be partitioned to form two related systems. These related systems consist of one measured and the other reconstructed.

One of the most significant problems of observers is knowing initial conditions for the reconstructor system. The initial conditions of the measurements are apparent but these are not necessarily the states. This problem is complicated by errors in the system.
parameters. A solution to this dilemma is the "robust observer" that will be investigated further. These observers have the property of converging to the proper state even though the differential equation coefficients are not accurately known. This is accomplished in a manner analogous to integrating out steady-state error.

Observer Development

An auxiliary dynamic system will almost always serve as an observer in that its state will tend to follow a linear transformation of the subject dynamic systems state. The design of the observer consists of incorporating that linear transformation into the process, thus providing an immediate and direct measure of the state.

Let $D_1$ be a free dynamic system describable by

$$\dot{x}(t) = Ax(t)$$

(2-7)

and $D_2$ will be the auxiliary dynamic system of the form

$$\dot{z}(t) = Fz(t).$$

(2-8)

This auxiliary system will be driven by the outputs of Eq. (2-7)

$$y(t) = CTx(t)$$

(2-9)

so that

$$\dot{z}(t) = Fz(t) + Hx(t)$$

(2-10)

where

$$H = KC^T$$

(2-11)

in which $K$ is a gain matrix selected to achieve some goal. Now

$$\dot{z}(t) - P\dot{x}(t) = Fz(t) + HPx(t) - PAx(t)$$

(2-12)
If

\[ H = PA - FP \]  \hspace{1cm} (2-13) \]

then

\[ \dot{z}(t) - \dot{P}x(t) = F(z(t) - Px(t)) \]  \hspace{1cm} (2-14) \]

which has

\[ z(t) = Px(t) \]  \hspace{1cm} (2-15) \]

as a solution, demonstrating the assertion of the preceding paragraph. Notice that \( D_1 \) and \( D_2 \) need not have the same dimension.

This suggests the "identity observer" where the transformation \( P \) is the identity matrix. For this type of observer, \( D_1 \) and \( D_2 \) must be the same dimension. Note that \( z(t) \) becomes an estimate of \( x(t) \), \( F \) becomes \( A-I \), and Eq. (2-10) may be rewritten as

\[ \dot{z}(t) = \dot{x}(t) = (A - KC^T) \dot{x}(t) + KC^T x(t) \]  \hspace{1cm} (2-16) \]

Let the error between \( x(t) \) and \( \hat{x}(t) \) be defined as \( e(t) \). Now,

\[ e(t) = \dot{x}(t) - \dot{\hat{x}}(t) \]

\[ = Ax(t) - A\hat{x}(t) + KC^T \hat{x}(t) - KC^T x(t) \]

\[ = (A - KC^T)(x(t) - \hat{x}(t)) \]

\[ = (A - KC^T) e(t) \]  \hspace{1cm} (2-17) \]

which expresses the dynamics of Figure 2-1.
If $CT$ and $A$ are real matrices, then the eigenvalues of $A - KC^T$ can be made to correspond to the set of eigenvalues of any $n \times n$ real matrix by suitable choice of $K$ if and only if $(CT, A)$ is completely observable. This has been proven in the literature several times, notably by Gopinath [12]. This implies that $e(t)$ may be driven to zero arbitrarily fast by suitable choice of eigenvalues of the augmented system $A - KC^T$. The response is normally dictated by a trade between accuracy and performance. If the eigenvalues are made extremely large negative, the system tends to act as a differentiator and is highly sensitive to noise and other disturbances. Some of these effects will be demonstrated with a subsequent example.

The observer is easily expressed as difference equations. Equation (2-7) becomes

$$x(k+1) = G(T) x(k) + H(T) U(k) \quad (2-18)$$

where $G(T) = e^{AT}$, and Eq. (2-16) becomes

$$\hat{x}(k+1) = G'(T) \hat{x}(k) + L(T) x(k) + L'(T) U(k) \quad (2-19)$$

where $G'(T) = e^{(A-KC^T)T}$

$$L(T) = \int_0^T e^{(A-KC^T)\tau} KC^T \, d\tau \quad (2-20)$$

and

$$L'(T) = \int_0^T e^{(A-KC^T)\tau} B \, d\tau \quad (2-21)$$

This may be represented as in Figure 2-2. If the system is not totally observable, then the ability to place eigenvalues is restricted. In fact, some of the errors may be unbounded. This does not imply a lack of system controllability but rather a lack of adequate control command to the state reconstructor. Only if the system is totally observable can the eigenvalues of the error system be arbitrarily placed. Without total observability, the reconstructor will be uncontrollable.
Let $M$ be the nonsingular transformation that takes the system to observable canonical form:

$$x = Mz \quad (2-22)$$

Rewriting Eq. (2-16)

$$\dot{M\hat{z}} = AM\hat{z} + BU + K(C^TMz - C^T\hat{Mz}) \quad (2-23)$$

$$M\dot{z} = AMz + BU \quad (2-24)$$

$$e = Mz - M\hat{z} \quad (2-2)$$

$$\dot{e} = M\dot{z} - M\hat{z} \quad (2-26)$$

$$\dot{e} = (AM - KCTM) M^{-1}e \quad (2-27)$$

But

$$C^T_M = (C^T : 0_{m \times n-k})$$

where $C^T$ is $m \times n-k$ where $k$ states are unobservable, and $KCTM$ is $n \times n-k$. It is apparent that the last $k$ columns of $AM$ are unaffected by choice of $K$.

Nonlinear observers have been developed for several cases [13,14,15]. In general, these observers are highly system dependent, and are very sensitive to initial conditions and gains. System dependent means that the closed form observer may be developed only for distinct classes of nonlinear systems. Further restrictions are the conditions required for one-to-one mappings which assure observability. More general realizations of observers, characterized by Reference 13, require nonlinear gain schedules for convergence and limitations on initial conditions.

**Observers for Use in Estimation**

The principal problems in applying observers to estimation are isolating the dynamics of the system from the dynamics of the observer and knowing the matrix $A$. The observer existence is based on some knowledge of the matrix $A$. While in many cases
the accuracy of the reconstruction process is independent for small errors of the precision with which $A$ is known, there are cases where the observer will diverge [16]. Due to manufacturing tolerances, material acquisition, sensor accuracy, modeling truncation, and a myriad of other reasons, the $A$ matrix will never be precisely known. The concept of an observer is analogous to a pole-zero canceling compensation so that sensitivity is an inherent design problem.

Observers of rank less than $n$ are known as reduced order observers. The unreconstructed states are obtained directly from measurements and the system is partitioned to separate the directly measured states from the rest. The remaining states are recovered by an observer of order $n$ minus the number of measured states. The minimal observer results when all the measurements are used to identify specific states. However, the minimal observer results in a totally open-loop observer for the unmeasured states. Philosophically, reduced order observers are attractive due to the reduced dimension. In practice the reduced order observer only simplifies the observer design. This simplification is easily outweighed by certain advantages of the identity observer. A minor theorem [17] shows that any identity observer is robust. Battacharyya [17] defines a robust observer as a closed loop system, closed on the error between plant and estimate, and one that possesses redundancy. Measurement redundancy means that, implicitly or explicitly, at least one linear combination of states is contained in the measurements.

These recent studies [16,17] have been directed to the sensitivity of observers. While the emphasis of these studies has been on reduced order observers, the sensitivity results generally apply and will be used as justification for assertions and assumptions of this research. The primary assertion is, if a robust observer is designed, the estimated states will converge asymptotically to the states even if errors exist in the estimate of the plant parameters. The use of identity observers removes the concern that the observer is not robust. Further the observers will be designed so the augmented system eigenvalues are critically damped. This stipulation is of little value for linear systems not disturbed by random inputs. However, for weakly nonlinear and perturbed systems, intuition and experience indicate the critically to highly overdamped roots will behave in a superior manner.
Figure 2-1. Identity observer.

Figure 2-2. Difference equation form of the identity observer.
CHAPTER III
PARAMETER ESTIMATING STATE RECONSTRUCTION

Complex multiloop system analysis and control design are most generally predicted on a linear analysis. Sufficient mathematical modeling is developed to assure a reasonable facsimile of the physical process, at least in the region of interest. Therefore, it is highly desirable to quantify differences between behavior, in the small, of the process and its analysis linear model. This quantification provides a final step in the design cycle, and determines expected variations from the process description used in the system design. These variations may be sufficiently large to dictate redesign of the controls or of the process itself. Unfortunately, all states are generally not available through measurement, so that the process of determining the best linear model is coupled with determining the states on a time history basis. The basic assumptions of this chapter are: that a model structure has been determined, that the system inputs are known, that there is no input disturbance or measurement noise, and that measurement of the state is incomplete.

Estimation of Partially Measured Systems

There are two alternatives available to solve the problem of estimating the parameters of a system whose states are not all available through measurement. The first is to augment the parameter estimation problem with the unmeasured states. The second is to develop in-line estimation algorithms which permit the simultaneous calculation of the desired parameters and states. The former results in an increase in dimensionality of the problem. In general, the parameter estimation problem is an n-squared problem. Augmentation can raise this to as high as n-squared plus n. This causes a long data stream to be required in the calculations and may result in convergence problems due to the age of oldest data. The latter approach is likewise potentially an n-squared plus n problem, but the data stream is the same as for a fully measured estimation problem. There are several methods available for the calculation of the states. The most straightforward approach is the state reconstructor or observer.

Gates [13] points out that the use of standard linear observers causes problems due to the coupling dynamics between the state reconstruction process and the plant estimation process. These problems are of particular concern if the dynamics of the plant are fast. This robs the designer of flexibility in locating system poles and zeroes to better solve the problem. Some of the difficulties of observers under variations have been addressed by Battacharyya [16]. However, if the system is stable the robust observer may be designed. Robust observers are stable and converge to the actual states even if the system differential equations are not accurately known. The reason for this convergence to the actual state is quite analogous to an integrator in the feedback of a control system. The steady-state error is driven to zero.
If the system dynamics are reasonably fast and if the plant is stable, the design approach is straightforward. The approach may be described as a periodic sequential estimation process. This is implemented as an estimation update periodically occurring after the settling time of the state reconstructor has expired. This process may be viewed as a sequence of state reconstruction problems with the error in parameter estimate forming the initial condition guess for the subsequent state reconstruction. The sequence may be designed to provide a set of essentially uncoupled state reconstruction problems, each with a successively better estimate of the differential equation coefficients. For example, if the system has a 0.1 sec settling time, an observer with a settling time of 1 sec is uncoupled from the system and generally four or five iterations provide reasonable convergence. Therefore in this example, the estimation process may be completed in 4 to 5 sec. There is inherent design flexibility to adjust these settling times (of observer and estimator) for a desirable response.

This approach appears more desirable than the nonlinear observer-like system proposed by Gates. The design of Gates' observer is much more system dependent than a system tuning approach. This is undesirable because sufficient information concerning the system may not be available. A further, and more serious, disadvantage of Gates' observer is that a stable configuration may not in fact exist.

The problem is then to keep the $n^2$ estimation problem while developing simultaneously the system states. Consider a robust observer compared with a best system estimate or system model. If the systems are adjusted so that the overall computation cycle is longer than the augmented system settling time, then the states from the first may be used to reestimate the dynamics of the second. The estimation problem is now identical to that of the literature [8,13,18] since the accurate states are now available for the estimation process.

The observer system is released with its initial conditions set to the best guess of the states. After one settling time, accurate states will be available for estimation of the dynamics of the system estimate using the method of Gates [13]. The system estimate is needed to use as a trial horse against the observed system behavior. These systems may be successively stepped to the desired convergence or to follow a slowly time-varying system or a weakly nonlinear system.

Time-varying, linear systems become more interesting and more complex. Now the dynamics of the estimator must be rapid enough to track the system under expected variations to fulfill the response matching criterion. The state reconstruction dynamics are chosen to be faster than the estimator within considerations of physical restrictions and overall stability. For nonlinear systems, one may consider first those that possess only small signal linearities or those that are weakly nonlinear. A well-known artifice of control analysis is to treat weakly nonlinear systems as time-varying linear systems. This approximation is not without risk, since an unstable system may appear to be stable in this type of analysis and vice versa. If the time-varying approximation is permitted, the technique may be applied as with the time-varying case with the additional restriction of defining a region for which the response remains sufficiently weakly nonlinear. The example of Chapters IV and V is of this type.
Stability considerations for these processes are concerned with three distinct elements. The stability of the plant estimate is of primary importance. Parameter variations in the estimation process may move marginally stable eigenvalues into the right half plane. If this occurs special tuning techniques are required to achieve the desired accuracy due to the observer [16] behavior. The second element is the augmented system eigenvalues of $A + KC^T$. In practice this is seldom a difficult task because the elements of $K$ are quite large in the interest of rapid system response. The variations in parameter estimates are normally small, in relation to the elements of $K$; thus, augmented system eigenvalues tend to be insensitive. The last consideration is the stability of the estimation algorithm. Many techniques exist [18] to assure the stability of this process.

System Diagrams and Equation Development

The equations and block diagrams will now be developed to give form to the method. Since most implementation schemes are digital in nature, the equations and algorithm will be developed in discrete form.

The implementation has, as its objectives, the recovery of the unmeasured states and the estimation of the system parameters. The unmeasured and measured states are direct outputs of the observer previously described. These states are used directly in the estimation process developed by Gates [13]. Figure 3-1 is a block diagram of the system estimation process. The plant dynamics are differenced with the dynamics of the reference model to form an error which is used to determine the difference between the reference model and the plant. The reference model is updated with the calculated difference, scaled by an appropriate gain. This gain is chosen for the stability of the estimation process. The process is repeated until the desired convergence is achieved.

Figure 3-2 portrays the combined state reconstruction and parameter estimation process. The plant, the observer, and the reference model are driven by the input $U$. The observer is also driven by the measured output of the plant. An error is formed from the difference of the observer and the reference model. Due to the behavior of the robust observer, the plant estimate $\hat{x}$ will follow the plant $x$. The observer and the reference model use the same estimate of the plant. The previously determined error will be used to calculate the difference between the plant and the reference model. The algorithm performs this function. The reference model and the observer are updated by use of the calculated difference and again scaled by an appropriate gain chosen for estimation stability. The process is repeated until a desired degree of convergence is achieved.

The equations will now be developed considering known external inputs. The plant is assumed governed by

$$x(k + 1) = G(T) x(k) + H(T) U(k)$$ (3-1)
where

\[ G(T) = e^{AT} \]  \hspace{1cm} (3-2) \]

and

\[ H(T) = \int_{kT}^{(k+1)T} e^{AT} B \, d\tau \]  \hspace{1cm} (3-3) \]

with measurements

\[ y(k) = C^T x(k) \]  \hspace{1cm} (3-4) \]

\( A \) is the system Jacobian, \( U \) is the known input and \( B \) is the control distribution matrix. The observer is described by

\[ \hat{x}(k + 1) = G_1'(T) \hat{x}(k) + H_1(T) U(k) + L_1'(T) y(k) \]  \hspace{1cm} (3-5) \]

where

\[ G_1'(T) = e^{(D+KC^T)T} \]  \hspace{1cm} (3-6) \]

\[ L(T) = \int_{kT}^{(k+1)T} e^{(D+KC^T)\tau} B \, d\tau \]  \hspace{1cm} (3-7) \]

and

\[ L_1' = \int_{kT}^{(k+1)T} e^{(D+KC^T)\tau} K \, d\tau \]  \hspace{1cm} (3-8) \]

\( D \) is the estimate of the system Jacobian, and \( K \) is the observer gain matrix.
The reference model is expressed as

\[ x_1(k + 1) = G_1(T) x_1(k) + H_1(T) U(k) \]  

(3-9)

where

\[ G_1(T) = e^{DT} \]  

(3-10)

and

\[ H_1(T) = \int_{kT}^{(k+1)T} e^{D\tau} B \, d\tau \]  

(3-11)

Now, writing state in terms of the system estimate

\[ x(k + 1) = G(T) x(k) + \delta G(T) x(k) + H_1(T) U(k) + \delta H(T) U(k) \]  

(3-12)

The error between the estimate and the state may be expressed as

\[ e(k + 1) = G_1(T) e(k) + \delta G(T) x(k) + \delta H(T) U(k) \]  

(3-13)

The following matrices are formed

\[ x0 = (x(0), \ldots, x(-n - p + 1)) \]

\[ U0 = (U(0), \ldots, U(-n - p + 1)) \]  

(3-14)

\[ e0 = (e(0), \ldots, e(-n - p + 1)) \]

\[ e1 = (e(1), \ldots, e(-n - p + 2)) \]

where each column is the vector associated with the enumerated time point. Eq. (3-13) can be expressed as a matrix equation that has
\( (\delta G(T): \delta H(k)) = (eI - G_1(T) e0) \begin{bmatrix} x_0 \\ U_0 \end{bmatrix}^{-1} \) \hspace{1cm} (3-15)

as a solution if the inverse of the augmented state and control matrix exists.

The system description is now updated as

\( G_1(T) = G_1(T) + \delta G(T) \) \hspace{1cm} (3-16)

\( H_1(T) = H_1(T) + \delta H(T) \) \hspace{1cm} (3-17)

and

\( G'_1(T) = G'_1(T) + \delta G(T) \) \hspace{1cm} (3-18)

Eq. (3-18) is not exactly correct but, due to the magnitude of elements of \( K \), yields satisfactory results. Several alternatives exist to precisely calculate \( G'_1(T) \). One method is to recover \( \delta D \) from \( \delta G(T) \) and recalculate \( G'_1(T) \) from Eq. (3-6). Another method is to calculate as a continuous system using the numerical integration to discretize the system. The calculation, in this case, yields \( \delta D \). A variation of the last method would be to use the \( \delta D \) to calculate the discrete parameters of Eqs. (3-6), (3-7), (3-8), (3-10), and (3-11). Precise knowledge of \( G'_1(T) \) is not required due to the nature of the robust observer. A topic for future study is the development of a recursive solution to the previously described estimation.

**A Second Order Example**

To demonstrate the procedure the system

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-2 & -3
\end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\] 

(3-19)
with measurement

\[
y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = x_2
\]  

(3-20)

Discretizing this system results in difference equations of the form

\[
\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ -2e^{-T} + 2e^{-2T} & -e^{-T} + 2e^{-2T} \end{bmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix}
\]

(3-21)

where \( T \) is the sampling period.

First the observer dynamics will be developed. The error between the state and the state estimate is dynamically determined by the eigenvalues of \( A + K C T \), or

\[
\begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} + \begin{bmatrix} 0 & K_1 \\ 0 & K_2 \end{bmatrix} = \begin{bmatrix} 0 & K_1 + 1 \\ -2 & K_2 - 3 \end{bmatrix}
\]

(3-22)

which has a characteristic equation of

\[
\lambda^2 + (3 - K_2)\lambda + (2K_1 + 2) = 0
\]

(3-23)

Hence the values for

\[
\lambda = \frac{K_2 - 3 \pm \sqrt{1 + K_2^2 - 6K_2 - 8K_1}}{2}
\]

(3-24)
are the robust observer eigenvalues and may be placed almost arbitrarily. Roots may both be real as $K_2 = -2, K_1 = 2$ which has roots at -2 and -3. Figure 3-3 shows the response of the unmeasured state and state estimate with a perfect model. The settling time is almost 3 sec. Figure 3-4 considers the response for a reasonable estimate of the state. Now the settling time is almost 4 sec. These dynamics are too slow for application of the estimation technique. If $K_1 = 24, K_2 = -12$ which has roots at -5 and -10 then the dynamics are much more suitable to the application of the estimation process. Figure 3-5 presents the response for these observer dynamics and a perfect model. Notice the settling time is of the order of 0.75 sec. Figure 3-6 shows the response for a reasonable estimate of the system dynamics. Again the settling time is less than 1 sec.

Figure 3-7 shows the remarkable property of robust observers to converge to the state even though there are errors in the plant estimate. The settling time is on the order of 3 sec, and this can be improved by adjusting the observer. The observers are all processing data from the plant

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} x$$ (3-25)

while the reasonable estimate was

$$\dot{\hat{x}} = \begin{pmatrix} -0.2 & 0.9 \\ -2.2 & -3.1 \end{pmatrix} \hat{x} - KCT(x - \hat{x})$$ (3-26)

The gross model error is characterized as

$$\dot{\hat{x}} = \begin{pmatrix} -0.5 & 2 \\ -3 & -6 \end{pmatrix} \hat{x} - KCT(x - \hat{x})$$ (3-27)

which is considerably at variance with Eq. (3-25).

The parameter estimation is delayed until reasonable convergence of the observer is achieved. The estimation is then initiated with the error between the trial system and the observer defined to be zero. Figure 3-8 presents a plot of time versus parameter estimate showing the estimation convergence.
Figure 3-1. Block diagram of the estimation process.

Figure 3-2. Block diagram of the combined reconstruction and estimation.
Figure 3-3. Observer with perfect model, slow dynamics ($K_1 = -2, K_2 = 2$).

Figure 3-4. Observer with reasonable estimate, slow dynamics ($K_1 = -2, K_2 = 2$).
Figure 3-5. Observer for perfect model.

Figure 3-6. Observer with reasonable estimate of model.
Figure 3-7. Observer with gross model error.

Figure 3-8. Observer (24, -12) convergence of D to the A matrix.
CHAPTER IV
CTL-V TESTING ANALYSIS

All large space vehicles possess a longitudinal dynamic coupling of structure and propulsion predictably called "POGO." The most benign stage to date is the Saturn IB stage which has nine tanks in a bundle and eight engines providing a maximum of statistical interaction, which results in overall system damping. The Space Shuttle, due to its large single LOX feedline, is expected to be susceptible to POGO. Due to this concern, an accumulator is being designed for the Space Shuttle main engine as a decoupling and suppressive device. A primary qualification test for this device is the CTL-V Test series at the Rockwell International Rocketdyne Division, Santa Susanna Test Facility. These tests will provide assurance as to the dynamic representation of the low pressure oxidizer pump and the effect and effectiveness of the accumulator.

The dynamic head rise characteristics of the low pressure oxidizer pump are nonlinear and not precisely known. They are modeled as nonlinear differential equations whose coefficients are empirically determined parameters. The accumulator characteristics are also ill defined because of the difficulty of obtaining good test results and isolation of the higher frequency effects of the accumulator. The advantages of applying the previously developed techniques are that by matching the time response the frequency response is likewise adjusted. That is, the linearization of the appropriate time response provides a frequency domain representation of the dynamic phenomena. This should be a "best" linear representation at that condition because the estimate is being forced to behave in a fashion similar to the actual system. The method is developed and modified to provide a neighborhood of operation of the low pressure oxidizer pump and of the accumulator.

The pump modeling is defined by the Rocketdyne publication RL00001 [19] defining the Space Shuttle main engine-engine balance and dynamic model. The facility is modeled in a similar fashion. Since the lines and pumps will be essentially chilled to a steady state during a given test, the assumption of incompressibility and thermal steady state is valid. Tests have shown that there is energy trade between temperature and pressure but that these are small effects. The specific objectives of the analysis will be to better define the head rise dynamics of the pump and the parameters of the accumulator. The accumulator parameters are characterized by electromechanical analogy. These parameters consist of a compliance, an inertance, and an equivalent resistance. The reason for an equivalent resistance will be apparent in the equation development. The results of these tests and analyses will be used in the overall Space Shuttle POGO stability analysis to better define system stability before first flight.

CTL-V Equation Development

The system to be tested in CTL-V is that of Figure 4-1. The pump speed \( S_{O1} \) may be assumed constant since the pump is being driven by an extremely high inertia electric motor. The constant pump speed allows analysis of the basic head rise characteristics of the pump uncoupled from available drive torque and torque required which
couple back into flow and pump speed. $P_T$ is the pressure at the feedline inlet. The flow in the feedline $DW_{FL}$ may be represented as

$$DW_{FL} = \frac{1}{L_L} \int_0^t \left[ (P_T - P_O) - R_L DW_{FL}^2 \right] d\tau . \quad (4-1)$$

The "bubble" on the pump has pressure $P_O$:

$$P_O = \frac{1}{C_B} \int_0^t (DW_{FL} - DW_{OS}) d\tau . \quad (4-2)$$

These two elements combine to simulate the 2.5 Hz first resonance of the oxidizer feedline. All damping arises from the resistive term of the feedline flow Eq. (4-2).

Next is the low pressure oxidizer pump. The pump is assumed to have a dynamic gain of one. Mass continuity dictates that the pump be gain one at zero frequency. The pump is characterized simply as a nonlinear head rise device. The pump discharge pressure is

$$P_{OD1} = P_O + H' \quad . \quad (4-3)$$

The head rise $H'$ will be defined by use of a dimensionless parameter

$$\Phi_{OP1} = \frac{A_1}{S_{O1}} DW_{OS} \quad . \quad (4-3a)$$

The head rise itself is given by

$$H' = B_1 S_{O1}^2 \Gamma_{PO1}(\Phi_{OP1}) \quad (4-3b)$$
where $\Gamma_{\text{POP1}}(\Phi_{\text{OP1}})$ is determined from the empirical curve of Figure 4-2. The flow below the pump is the same as the flow existing in the bubble and entering the pump and is defined as

$$DW_{OS} = \frac{1}{L_D} \int_{0}^{t} \left[ (P_{OD1} - P_{OI2}) - R_D(DW_{OS})^2 \right] \, d\tau \quad . \tag{4-4}$$

$L_D$ is the inertance of the fluid in the duct and the pump. $R_D$ is a lumped resistance coefficient combining effects of duct and pump. The pressure upstream of the accumulator is dependent on the compliance, $C_D$, of the duct itself, and has the form

$$P_{OI2} = \frac{1}{C_D} \int_{0}^{t} (DW_{OS} - DW_{OP2} - DW_A) \, d\tau \quad . \tag{4-5}$$

The accumulator is modeled analogously with a pressure change through a compliance and a flow change due to a resistance and delta pressure. The inertance in this case is the mass of that fluid trapped in the standpipe leading to the accumulator. The compliance is a lumped compliance consisting of flexure of the housing and the compressibility of the gas in the accumulator. The gas to be used on the Space Shuttle is GOX in contrast to helium principally used in the past. The GOX is supplied from the tank pressurization heat exchanger and maintains a constant level in the accumulator by use of an overflow port from which GOX is vented back into the feedline above the low pressure pump. That flow is not considered in this analysis. The gas-liquid interface is maintained by four layers of $3/8$ in. Teflon balls which provide internal slosh suppression and help prevent the gas bubble from collapsing in the liquid. The pressure in the accumulator is modeled as

$$P_A = \frac{1}{C_A} \int_{0}^{t} DW_A \, d\tau \quad . \tag{4-6}$$

while the accumulator flow can be represented as
\[ \frac{1}{L_A} \int_0^t \left( (P_{O12} - P_A) - R_A(DW_A)^2 \right) \, d\tau \]

(4-7)

where \( C_A, L_A, \) and \( R_A \) are as previously defined.

\( P_{OP2} \) is, in effect, an output of the system and, because of the orifice, will remain constant once the test conditions are established. With one exception, the equation definition is now complete. The exception is that to represent \( P_{O12} \), the flow downstream of the accumulator must be modeled. This is a small piece of fluid and results in a high frequency root. \( L_N \) represents the inertance of that element and \( R_N \) is its resistance. The flow in this section can then be expressed as

\[ DW_{OP2} = \frac{1}{L_N} \int_0^t \left( (P_{O12} - P_{OP2}) - R_N(DW_{OP2})^2 \right) \, d\tau \]

(4-8)

This completes the equation development required to analyze the test.

These equations may be rewritten as a set of nonlinear differential equations:

\[ DW_{FL} = \frac{1}{L_L} \cdot \frac{P_T}{L_L} - \frac{1}{L_L} \cdot \frac{P_{OS}}{L_L} - \frac{R_L}{L_L} \cdot DW_{FL} \]

(4-9)

\[ \frac{1}{C_B} \frac{1}{C_B} \frac{DW_{FL}}{DW_{OS}} \]

(4-10)

\[ \frac{1}{C_B} \frac{1}{C_B} \frac{DW_{FL}}{DW_{OS}} - A_1 B_1 S_0 \frac{\partial P_{OP1}}{\partial \phi_{OP1}} \frac{R_D}{L_D} \frac{DW_{OS}}{p_{O12}} + A_1 B_1 S_0 \frac{\partial P_{OP1}}{\partial \phi_{OP1}} \frac{L_D}{L_D} \frac{p_{OD1}}{p_{O12}} - A_1 B_1 S_0 \frac{\partial P_{OP1}}{\partial \phi_{OP1}} \frac{L_D}{L_D} \frac{p_{OD1}}{p_{O12}} \]

(4-11)
Linearized Analysis of CTL-V

These equations are in turn linearized to obtain the following set of linear differential equations:

\[
\Delta \dot{p}_{FL} = \frac{1}{L_L} \Delta p_T - \frac{1}{L_L} \Delta p_{OS} - 2 \frac{R_L}{L_L} D_W^{FL} \Delta F_{FL} \quad (4-17)
\]

The term \(2R_LD_W^{FL}\) is normally thought of as being an equivalent damping resistance for an element.

\[
\Delta \dot{p}_{OS} = \frac{1}{C_B} \Delta F_{FL} - \frac{1}{C_B} \Delta F_{OS} \quad (4-18)
\]

\[
\Delta \dot{p}_{OD1} = \frac{1}{C_B} \Delta F_{FL} - D_1 \Delta F_{OS} + D_2 \Delta p_{OD1} - D_2 \Delta p_{O12} \quad (4-19)
\]
where

\[
D'_1 = \frac{1}{C_B} - 2A_1B_1S_{O1} \frac{\partial \Phi_{OP1}}{\partial \Phi_{OP1}} \frac{R_D}{L_D} \Delta W_{OS}
\]

\[
D'_2 = A_1B_1S_{O1} \frac{\partial \Gamma_{POP1}}{\partial \Phi_{OP1}} \frac{1}{L_D}
\]

Completing the equations,

\[
\Delta \dot{F}_{OS} = \frac{1}{L_D} \Delta P_{OD1} - \frac{1}{L_D} \Delta P_{O12} - \frac{2R_D \Delta W_{OS}}{L_D} \Delta F_{OS}
\]

(4-20)

\[
\Delta \dot{P}_{O12} = \frac{1}{C_D} \Delta F_{OS} - \frac{1}{C_D} \Delta F_{OP2} - \frac{1}{C_D} \Delta F_A
\]

(4-21)

\[
\Delta \dot{P}_A = \frac{1}{C_A} \Delta F_A
\]

(4-22)

\[
\Delta \dot{F}_A = \frac{1}{L_A} \Delta P_{O12} - \frac{1}{L_A} \Delta P_A - \frac{2R_A}{L_A} \Delta W_A \Delta F_A
\]

(4-23)

\[
\Delta \dot{F}_{OP2} = \frac{1}{L_N} \Delta P_{O12} - \frac{2R_N}{L_N} \Delta W_{OP2} \Delta F_{OP2}
\]

(4-24)

Notice that \(P_{OP2}\) has dropped out of the linear representation because it is assumed constant due to the orifice. At the beginning of the chapter, the equivalent resistance of the accumulator was discussed. The resistance is equivalent because, to a first-order approximation, it is zero. In Eq. (4-23), the linear term of resistance is \(2R_A/L_A \Delta W_A\). In the steady state \(\Delta W_A\) is zero. Therefore, to first order the resistance effects of the accumulator are indeed zero. However, the resistive effects are important to the analysis so a small nonzero term will be forced in the analysis to assess its effect.
These equations can now be expressed in state variable form as

\[ \dot{x} = Ax + Bu \]

where

\[ x = \begin{bmatrix} \Delta P_{OS} \\ \Delta F_{FL} \\ \Delta P_{OD1} \\ \Delta F_{OS} \\ \Delta P_{OL2} \\ \Delta F_{OL2} \\ \Delta P_A \\ \Delta F_A \end{bmatrix} \]

and

\[ A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{L_B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{L_A} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \]

and

\[ U = P_T \]
In practice, flow is an unmeasurable quantity due to instrumentation difficulties. For this reason the pressures are all that will be measured. The advantages of the scheme presented over most identification schemes are now obvious. Fully half of the state vector is not available for measurement and will be recovered with an observer designed in a fashion described previously.

The measurement vector now becomes

\[
\begin{pmatrix}
PO_S \\
POD_1 \\
POI_2 \\
PA
\end{pmatrix}
= C^T
\begin{pmatrix}
PO_S \\
F_{FL} \\
POD_1 \\
F_{OS} \\
POI_2 \\
F_{OP2} \\
PA \\
F_A
\end{pmatrix}
\] (4-26)

These equations will be discretized using Fourth Order Runge-Kutta Integration, with iteration time sufficiently fast to assure reasonable accuracy. This provides satisfactory precision without cumbersome implementation.

The test series will be operated in two ways, with and without the accumulator. The equations have been arranged to allow partitioning in this manner. Using data that reflect the rated power level test, the eigenvalues of the system without accumulator are as follows:

<table>
<thead>
<tr>
<th>Real</th>
<th>Imaginary</th>
</tr>
</thead>
<tbody>
<tr>
<td>-32.45</td>
<td>-224.4</td>
</tr>
<tr>
<td>-32.45</td>
<td>224.4</td>
</tr>
<tr>
<td>-59.1</td>
<td>71.7</td>
</tr>
<tr>
<td>-59.1</td>
<td>-71.7</td>
</tr>
<tr>
<td>-5.3</td>
<td>0</td>
</tr>
<tr>
<td>-0.01</td>
<td>0</td>
</tr>
</tbody>
</table>
Notice that the line frequency has dropped from 2.5 to 1.2 Hz. The pump and duct combine with critically damped roots, and the orifice segment has a 36 Hz resonance.

Adding the accumulator, the eigenvalues become:

<table>
<thead>
<tr>
<th>Real</th>
<th>Imaginary</th>
</tr>
</thead>
<tbody>
<tr>
<td>-81.8</td>
<td>-423.1</td>
</tr>
<tr>
<td>-81.8</td>
<td>423.1</td>
</tr>
<tr>
<td>-72.7</td>
<td>77.9</td>
</tr>
<tr>
<td>-72.7</td>
<td>-77.9</td>
</tr>
<tr>
<td>-37.5</td>
<td>48.7</td>
</tr>
<tr>
<td>-37.5</td>
<td>-48.7</td>
</tr>
<tr>
<td>-5.35</td>
<td>0</td>
</tr>
<tr>
<td>-0.01</td>
<td>0</td>
</tr>
</tbody>
</table>

The line frequency is essentially unchanged as is the pump and upper duct. The major change is in the lower duct where the duct couples with the accumulator giving the 15 Hz resonance and the accumulator couples with the flow to the orifice which has a 69 Hz resonance.

It remains to locate the eigenvalues of the observer and implement the identification technique.
Figure 4-1. CTL-V test.
Figure 4-2. Low pressure oxidizer pump pressure rise characteristics.
CHAPTER V
CTL-V RESULTS

The problem of higher-order observer design, analysis, and performance will be discussed first, then the actual design will be developed. Results will include those of the CTL-V facility with and without the POGO suppressive accumulator. The principal feature is the demonstration that linear observers can be used in a state-parameter estimation process if sufficient care is taken in the design to insure plant-observer dynamic decoupling. Techniques for desensitizing observer design and the application of parameter estimating state reconstruction will be examined. Sample rate selection and numerical difficulties will be addressed. Finally the curious phenomena of multiple equilibria for fluidic systems of the CTL-V type will be analyzed.

CTL-V Observer Design

The design of observers for higher-order systems is a topic in itself. The simplest problem is the single input system. If the subject system is observable, there are available \( n \times m \) parameters, of the gain matrix \( K \), to place the \( n \) eigenvalues of the augmented observer system. If there are multiple inputs, the system may be recast as a set of single input systems and treated individually as single input systems. However, while systems designed in this fashion have the desired eigenvalues, the dynamics of the augmented system can be most undesirable because of the location of the system zeros. Undesirable energy trade takes place between the various component single-input systems. The additional degrees of freedom, in the matrix \( K \), may be used to achieve a more desirable overall dynamic response. The term “better dynamic response” must now be quantified. For the purpose of this research, better dynamic response means critically damped with a reasonably fast settling time, while in general the term is dependent on the application and the desires of the designer.

A critically damped response is desired to eliminate or reduce coupling between the plant and the observer. A further precaution is to design the augmented system eigenvalues sufficiently larger than those of the plant. This permits rapid reconstruction of the states and reduces the propensity of the observer dynamics to couple with the plant dynamics. The requirement to critically damp the observer eigenvalues means that some of the desired analysis flexibility has been lost and that the technique is becoming more system dependent. The critical damping also affects the settling time which is another design parameter. The settling time determines how often new estimates of the plant parameters can be determined. Due to the nature of observers, only the directly measured states are known before the reconstruction process. The unmeasured states become available only after the system settling time has passed. This time can also be affected by the size of the error in the initial estimates of the unmeasured states. A settling time is required after each recalculation of the system parameters because the new estimate represents a system discontinuity when it is used in the reconstruction process.
A consideration that would be of low interest to most applications, but is of secondary interest in this application, is the augmented system eigenvalues sensitivity to parameter variations. This interest is of two parts: first the sensitivity of the observer dynamically to deviations in the plant estimate and, second, the stability of the estimated systems eigenvalues due to estimation errors. Due to the nature of the robust observer, the dynamic behavior may be degraded as variations become large. But the system will converge for very large variations in the parameter estimates, as was demonstrated in Chapter III. In fact, the whole concept of parameter estimating state reconstruction is based on that property. But gross excursions can cause divergence of the reconstructor from the plant, and some designs are more or less sensitive to parameter variations. A design procedure then is to verify a low sensitivity to parameter variations.

The second part of the problem is the closed-loop stability of the estimation system. The augmented system may be stable but may perform inadequately for the purposes of this research. If the parameter recalculation causes the observer plant to have unstable eigenvalues, then quite obviously the system will have inappropriate dynamics. Therefore, another design procedure is to analyze the sensitivity of the plant eigenvalues to parameter variations and the overall degree of stability of the object plant. A marginally stable or unstable system is undesirable for analysis by this technique because of observer problems addressed in Chapter II, and the overall system sensitivity. The observer problem is that if the plant is unstable, then the plant must be precisely known and represented in the observer to achieve observer convergence. However, an artifice is available to handle these kinds of difficulties, that is, to synthesize a feedback control that stabilizes or desensitizes the objectionable eigenvalues. This is a straightforward classical technique that provides a system with characteristics that permit analysis by the technique of this research, the control being included in the model structure of the new observer of the altered system.

The eigenvalue placement for observers, as has been noted, is generally overdetermined. There are a variety of ways to choose the elements of the gain matrix $K$. The particular approach for this application was selected because of the manner in which the elements of $K^T$ enter the augmented system matrix $A + K^T$. By choosing $K$ in the form

$$K = \begin{pmatrix} K_{11} & 0 & 0 & 0 & 0 & 0 \\ K_{21} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_{32} & 0 & 0 & 0 \\ 0 & 0 & K_{42} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_{53} & 0 \\ 0 & 0 & 0 & 0 & K_{63} & 0 \end{pmatrix}$$

(5-1)
for the sixth order case, and

\[
KCT = \begin{pmatrix}
K_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & K_{21} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & K_{32} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & K_{42} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & K_{53} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & K_{63} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & K_{74} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & K_{84}
\end{pmatrix}
\] (5-2)

for the eight order case, the augmented system eigenvalues can be less interactively chosen. The system is naturally partitioned to encourage this type of gain selection. There is some interaction, but eigenvalue selection is more independent than if some more-coupled scheme were used. Values for \( K \) and the associated augmented system eigenvalues are shown in Table 5-1. Sensitivity results for sixth order observers are shown in Table 5-2 and for eighth order observers in Table 5-3. The numbers of the \( K \) matrix are in units commensurate with the elements of the \( D \) or \( A \) matrix so that \( A + KCT \) has meaning. If the selected elements of \( K \) are extremely large, errors in the parameter estimates have little effect on the observer dynamics; however, due to the high gain, the observer system becomes very sensitive to noise. If the elements of \( K \) are small the observer becomes more sensitive to parameter estimate errors and the observer response becomes sluggish. These considerations enter the observer system design process.

Physical Interpretation of the Model

The desired output is not simply the linear model, or the matrix \( D \) in the calculations. \( D \) must be interpreted to deduce the parameters of interest, namely the compliances, inertances, and resistances of the CTL-V facility but, most importantly, the slope of the pump curve. The compliance of the bubble on the pump may be determined as

\[ C_B = \frac{1}{\dot{D}_{12}} \] (5-3)
while the feedline inertance is

\[ L_L = \frac{1}{D_{21}} \]  

(5-4)

TABLE 5-1. SIXTH AND EIGHTH ORDER OBSERVER

<table>
<thead>
<tr>
<th></th>
<th>Sixth Order</th>
<th>Eighth Order</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Eigenvalues(^a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(All Real)</td>
</tr>
<tr>
<td>(K_{11})</td>
<td>1200.0</td>
<td>-100.0</td>
</tr>
<tr>
<td>(K_{21})</td>
<td>1998.0</td>
<td>-219.3</td>
</tr>
<tr>
<td>(K_{32})</td>
<td>1477.0</td>
<td>-354.3</td>
</tr>
<tr>
<td>(K_{42})</td>
<td>0.0</td>
<td>-858.3</td>
</tr>
<tr>
<td>(K_{53})</td>
<td>1500.0</td>
<td>-1328.1</td>
</tr>
<tr>
<td>(K_{63})</td>
<td>-1366.0</td>
<td>-1496.4</td>
</tr>
<tr>
<td>(K_{74})</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>(K_{84})</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

\(^a\) Eigenvalues have no order relationship to values.

The line resistance is a little more difficult to recover but may be determined as

\[ R_L = \frac{-D_{22}L_L}{2W_L} \]  

(5-5)

Next the inertance of the duct may be calculated as

\[ L_D = \frac{1}{D_{43}} \]  

(5-6)
<table>
<thead>
<tr>
<th></th>
<th>Value</th>
<th>Eigenvalue&lt;sup&gt;a&lt;/sup&gt;</th>
<th></th>
<th>Value</th>
<th>Eigenvalue&lt;sup&gt;a&lt;/sup&gt;</th>
<th></th>
<th>Value</th>
<th>Eigenvalue&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Real</td>
<td>Imaginary</td>
<td></td>
<td></td>
<td>Real</td>
<td>Imaginary</td>
<td></td>
</tr>
<tr>
<td>K&lt;sub&gt;11&lt;/sub&gt;</td>
<td>1600.0</td>
<td>-97.2</td>
<td>0.0</td>
<td>1600.0</td>
<td>-168.4</td>
<td>87.3</td>
<td>1600.0</td>
<td>-184.9</td>
</tr>
<tr>
<td>K&lt;sub&gt;21&lt;/sub&gt;</td>
<td>1998.0</td>
<td>-207.6</td>
<td>0.0</td>
<td>2498.0</td>
<td>-168.4</td>
<td>-87.3</td>
<td>2498.0</td>
<td>-185.1</td>
</tr>
<tr>
<td>K&lt;sub&gt;32&lt;/sub&gt;</td>
<td>1477.0</td>
<td>-238.8</td>
<td>0.0</td>
<td>1777.0</td>
<td>-198.0</td>
<td>0.0</td>
<td>2477.0</td>
<td>-185.1</td>
</tr>
<tr>
<td>K&lt;sub&gt;42&lt;/sub&gt;</td>
<td>0.0</td>
<td>-1325.9</td>
<td>0.0</td>
<td>0.0</td>
<td>-1337.1</td>
<td>0.0</td>
<td>623.0</td>
<td>-1339.1</td>
</tr>
<tr>
<td>K&lt;sub&gt;53&lt;/sub&gt;</td>
<td>1500.0</td>
<td>-1378.9</td>
<td>0.0</td>
<td>1500.0</td>
<td>-1370.0</td>
<td>0.0</td>
<td>1500.0</td>
<td>-1339.1</td>
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<tr>
<td>K&lt;sub&gt;63&lt;/sub&gt;</td>
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<td>-1508.2</td>
<td>0.0</td>
<td>-1366.0</td>
<td>-1814.7</td>
<td>0.0</td>
<td>-1366.0</td>
<td>-2523.1</td>
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</table>

<sup>a</sup> Eigenvalues have no order relationship to values.
### TABLE 5-3. EIGHTH ORDER SENSITIVITY

<table>
<thead>
<tr>
<th></th>
<th>Eigenvalues&lt;sup&gt;a&lt;/sup&gt;</th>
<th></th>
<th>Eigenvalues&lt;sup&gt;a&lt;/sup&gt;</th>
<th></th>
<th>Eigenvalues&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value</td>
<td>Real</td>
<td>Imaginary</td>
<td>Value</td>
<td>Real</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;11&lt;/sub&gt;</strong></td>
<td>1000.0</td>
<td>-98.6</td>
<td>0.0</td>
<td>1000.0</td>
<td>-94.6</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;21&lt;/sub&gt;</strong></td>
<td>1998.0</td>
<td>-139.5</td>
<td>0.0</td>
<td>1998.0</td>
<td>-146.0</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;32&lt;/sub&gt;</strong></td>
<td>1277.0</td>
<td>-540.6</td>
<td>0.0</td>
<td>727.0</td>
<td>-493.3</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;42&lt;/sub&gt;</strong></td>
<td>0.0</td>
<td>-506.6</td>
<td>-212.4</td>
<td>0.0</td>
<td>-493.3</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;53&lt;/sub&gt;</strong></td>
<td>1500.0</td>
<td>-506.6</td>
<td>212.4</td>
<td>1000.0</td>
<td>-559.8</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;63&lt;/sub&gt;</strong></td>
<td>-1366.0</td>
<td>-860.4</td>
<td>0.0</td>
<td>-1266.0</td>
<td>-559.8</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;74&lt;/sub&gt;</strong></td>
<td>1000.0</td>
<td>-1209.9</td>
<td>0.0</td>
<td>875.0</td>
<td>-778.8</td>
</tr>
<tr>
<td><strong>K&lt;sub&gt;84&lt;/sub&gt;</strong></td>
<td>1500.0</td>
<td>-1294.4</td>
<td>0.0</td>
<td>800.0</td>
<td>-855.8</td>
</tr>
</tbody>
</table>

<sup>a</sup> Eigenvalues have no order relationship to value.
The slope of the pump curve may now be evaluated as

$$\frac{\partial \Gamma}{\partial \Phi} = \frac{D_{33}L_D}{A_1B_1S_{O_1}}$$  \hfill (5-7)

where $A_1B_1$ is a constant and $S_{O_1}$ is the pump speed. The compliance of the duct is simply

$$C_D = \frac{1}{D_{54}}$$  \hfill (5-8)

The resistance of the duct is

$$R_D = \frac{-D_{44}L_D}{2\bar{W}_D}$$  \hfill (5-9)

The inertance of the small fluid segment between the accumulator and the orifice is calculated as

$$L_N = \frac{1}{D_{65}}$$  \hfill (5-10)

and its equivalent resistance is

$$R_N = \frac{-D_{66}L_N}{2W_N}$$  \hfill (5-11)

The accumulator parameters are determined similarly with

$$C_A = \frac{1}{D_{78}}$$  \hfill (5-12)

$$L_A = \frac{1}{D_{85}}$$  \hfill (5-13)

45
and

\[ R_A = -D_{88} L_A \quad (5-14) \]

The Nonunique Equilibrium of the Sixth-Order Case

The CTL-V test configuration has some unusual properties if the system is tested without the accumulator. The system no longer possesses a unique equilibrium, but is in equilibrium everywhere that the flows become equal. The system is stable with eigenvalues, as reported earlier, but the system has infinite equilibrium conditions. The sixth order system is described as

\[
\begin{pmatrix}
0 & \frac{1}{C_B} & 0 & -\frac{1}{C_B} & 0 & 0 \\
-\frac{1}{L_L} & \frac{2R_L}{L_L} D W_{FL} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{C_B} & D_2 & -D_1 & -D_2 & 0 \\
0 & 0 & \frac{1}{L_D} & -\frac{2R_D}{L_D} D W_{OS} & -\frac{1}{L_D} & 0 \\
0 & 0 & 0 & \frac{1}{C_D} & 0 & -\frac{1}{C_D} \\
0 & 0 & 0 & 0 & \frac{1}{L_N} & -\frac{2R_N}{L_N} D W_{OP2}
\end{pmatrix}
\]

Now clearly

\[
\Delta F_{OS} = \left( \frac{1}{C_B} \right) \Delta F_{FL} - \frac{1}{C_B} \Delta F_{OS} \quad (5-16)
\]
is zero if the two flows become equal, and

\[
\Delta F_{FL} = \left( \frac{1}{L_L} \right) \Delta P_{OS} - \left( \frac{2R_L}{L_L} \right) D W_{FL} \Delta F_{FL} \quad (5-17)
\]

determines the steady-state value of \( \Delta P_{OS} \) for the nonzero \( \Delta F_{FL} \). In the pump equation

\[
\Delta P_{OD1} = \left( \frac{1}{C_B} \right) \Delta F_{FL} + D_2 \Delta P_{OD1} - D_1 \Delta F_{OS} - D_2 \Delta P_{O12} \quad , (5-18)
\]

if \( \Delta P_{OD1} \) is zero, then \( \Delta P_{OD1} \) is related to the other variables as

\[
\Delta P_{OD1} = \left( \frac{1}{D_2} \right) \left( D_1 \Delta F_{OS} - D_2 \Delta P_{O12} - \left( \frac{1}{C_B} \right) \Delta F_{FL} \right) . \quad (5-19)
\]

Proceeding,

\[
\Delta F_{OS} = \left( \frac{1}{L_D} \right) \Delta P_{OD1} - \left( \frac{2R_D}{L_D} \cdot D W_{OS} \right) \Delta F_{OS} - \left( \frac{1}{L_D} \right) \Delta P_{O12} \quad , (5-20)
\]

which, if \( \Delta F_{OS} \) is zero, similarly can be solved for \( \Delta P_{OD1} \) as

\[
\Delta P_{OD1} = L_D \left( \frac{1}{L_D} \right) \Delta P_{O12} + \left( \frac{2R_D}{L_D} \cdot D W_{OS} \right) \Delta F_{OS} \quad . (5-21)
\]

The next equation is

\[
\Delta P_{O12} = \left( \frac{1}{C_D} \right) (\Delta F_{OS} - \Delta F_{OP2}) \quad (5-22)
\]
and finally

\[ \Delta F_{OP2} = \left( \frac{1}{L_N} \right) \Delta F_{OI2} - \left( \frac{2R_N}{L_N} \Delta W_{OP2} \right) \Delta F_{OP2} \]  \hspace{1cm} (5-23)

which yields a steady-state value for \( \Delta F_{OI2} \) when all the flows are equal. Notice that by Eqs. (5-19) and (5-21) there are apparently two definitions of \( \Delta F_{OD1} \). Both are of the similar form

\[ \Delta F_{OD1} = \Delta F_{OI2} + C' \Delta F \]  \hspace{1cm} (5-24)

where in Eq. (5-19)

\[ C' = \left( \frac{D'_1 - \left( \frac{1}{C_B} \right)}{D'_2} \right) \]  \hspace{1cm} (5-25)

and in Eq. (5-21)

\[ C' = 2R_D \Delta W_{OS} \]  \hspace{1cm} (5-26)

Interestingly enough for CTL-V at the operating point corresponding to the 100 percent engine power level, one finds that

\[ \left( \frac{D'_1 - \left( \frac{1}{C_B} \right)}{D'_2} \right) = 2R_D \Delta W_{OS} \]  \hspace{1cm} (5-27)

Therefore, any time that flows become equal, the system will be in equilibrium at that perturbed condition. This property is due to the particular values of the pump compliance, head rise characteristics, steady-state flow, and the duct resistance.
This result has no meaning in the context of the engine because the engine system has a closed fluid path around these elements, thus altering the overall system dynamic characteristics. Further, if the accumulator is added to the CTL-V system, then the system regains a unique stable equilibrium since the flow into the accumulator must go to zero in the steady state. This response poses no real problem to the technique of this research since the system is driven in an oscillatory fashion about the null, as is true of CTL-V itself.

Results and Conclusions

The most significant result is the ability of the observers or state reconstructors to follow the small signal nonlinear signal even though only an estimate of the system is known. Results in Tables 5-4, 5-5, and 5-6 demonstrate that even with estimates that are in error by large amounts, the robust observer provides reasonable estimates of the state for use in the estimation process. The nonlinear small signal values are the oscillations about the system operating point. The linear values are results from an analytic linearization of the nonlinear system equations. The first group of numbers is the linearized system Jacobian. This matrix is the analytic linearization of the nonlinear equations at the system operating point. The second group of numbers is the result of the estimation process at an instant of time, shown as the first number in the third group. The first two matrices may be compared by positions. The first row of eight numbers in the third group are the nonlinear states described in the first line separated by commas. The remaining rows are as described above. The constrained results of Table 5-6 refer to the method of parameter calculation. Parameter recalculations are permitted only for those elements that, due to model structure, are dependent. That is, accumulator parameters are not permitted to be a function of line or pump. The unconstrained estimation allows variations as with any sensitivity technique. These differences may be observed in the tables as the estimate of the system Jacobian. These observers are not simply following the system but are reconverging after each reevaluation of parameters each 0.02 sec. Since the highest observer root is 200 Hz and the discharge has a resonance at 60 Hz, 2 to 4 sec is a very long run time. Either of these strategies work. The constrained method is similar to a steepest descent technique. The feedback gain must be small, for the constrained approach, to maintain computational stability.

Figures 5-1 through 5-9 demonstrate the observer response. Notice that the pressure initial conditions are presumed known while there is error in the flow initial conditions. The observer response quickly eliminates the flow errors before the estimation process begins. This is a consideration in specifying a settling time. Figure 5-5 is a blowup of Figure 5-4 better displaying the observer response. The four second time histories of Figures 5-1 to 5-9 are to demonstrate that the observer response does not diverge over a long time interval, so that reasonable estimates of the unmeasured states are available for long time periods.

Figures 5-10 and 5-11 show that the linear response is different from the small signal nonlinear response. Recall that the observer attempts to follow the small signal nonlinear response.
### TABLE 5-4. OBSERVER PERFORMANCE AFTER 1.01 sec (UNCONSTRAINED)

<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1.72819E5 1 + 153669E4 0 + 301655E8 0 + 147217E8 0 + 157274E8 0 + 121086E8 0 + 8567673E 0 + 20503879E 0</td>
<td>+134659E4 0 + 182376E7 0 + 281742E8 0 + 466865E5 0 + 127692E4 0 + 392725E8 0 + 125045E9 0</td>
<td>+147214E8 0 + 430272E7 0 + 322729E7 + 1151282E2 + 430272E7 + 92981836E 0 + 1573590E 0 + 39354445E 0 + 1939728E 0</td>
</tr>
<tr>
<td>+1.23456E0 1 + 574687E0 0 + 8138475E 0 + 17048140E 0 + 20071532E 0 + 37538289E 0 + 3772780E 0 + 34309047E 0</td>
<td>+134659E4 0 + 182376E7 0 + 281742E8 0 + 466865E5 0 + 127692E4 0 + 392725E8 0 + 125045E9 0</td>
<td>+147214E8 0 + 430272E7 0 + 322729E7 + 1151282E2 + 430272E7 + 92981836E 0 + 1573590E 0 + 39354445E 0 + 1939728E 0</td>
</tr>
<tr>
<td>+1.23456E0 1 + 574687E0 0 + 8138475E 0 + 17048140E 0 + 20071532E 0 + 37538289E 0 + 3772780E 0 + 34309047E 0</td>
<td>+134659E4 0 + 182376E7 0 + 281742E8 0 + 466865E5 0 + 127692E4 0 + 392725E8 0 + 125045E9 0</td>
<td>+147214E8 0 + 430272E7 0 + 322729E7 + 1151282E2 + 430272E7 + 92981836E 0 + 1573590E 0 + 39354445E 0 + 1939728E 0</td>
</tr>
</tbody>
</table>
**TABLE 5-5. OBSERVER PERFORMANCE AFTER 1.36 sec (UNCONSTRAINED)**

<table>
<thead>
<tr>
<th>Linearized System Jacobian</th>
<th>Observer States</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \begin{bmatrix} \cdots &amp; \cdots &amp; \cdots \ \cdots &amp; \cdots &amp; \cdots \ \cdots &amp; \cdots &amp; \cdots \end{bmatrix} )</td>
<td>( \begin{bmatrix} \cdots &amp; \cdots &amp; \cdots \ \cdots &amp; \cdots &amp; \cdots \ \cdots &amp; \cdots &amp; \cdots \end{bmatrix} )</td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>( t )</th>
<th>( \cdots )</th>
<th>( \cdots )</th>
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</thead>
<tbody>
<tr>
<td>( \cdots )</td>
<td>( \cdots )</td>
<td>( \cdots )</td>
</tr>
</tbody>
</table>
### Table 5-6. Observer Performance After 0.81 sec (Constrained)

#### Linearized System Jacobian

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>-13141902E 03</td>
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<tr>
<td>-1458255E 01</td>
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<tr>
<td>-1796282E 01</td>
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#### Estimate of the System Jacobian

<table>
<thead>
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<tbody>
<tr>
<td>00000000E 00</td>
</tr>
<tr>
<td>00000000E 00</td>
</tr>
<tr>
<td>00000000E 00</td>
</tr>
<tr>
<td>00000000E 00</td>
</tr>
<tr>
<td>00000000E 00</td>
</tr>
<tr>
<td>00000000E 00</td>
</tr>
</tbody>
</table>

#### States

- **Linearized States**
  - Observer States
    - 1586978E 01 | 6463368E 00 | 1396384E 00 | 1722220E 03 | 1586624E 00 | 2691993E 00 | 8926115E 01 | 4596173E 00
  - Nonlinear Small Signal States
    - 1580978E 01 | 6589355E 00 | 1357216E 01 | 7366602E 00 | 1571655E 00 | 2666016E 00 | 4908811E 01 | 4535695E 00
  - Linearized States
    - 1796282E 01 | 6939382E 00 | 1468412E 01 | 7658981E 00 | 1669392E 00 | 2997377E 00 | 9824508E 01 | 4512641E 00


- **Linearized States**
  - Observer States
    - 1586978E 01 | 6463368E 00 | 1396384E 00 | 1722220E 03 | 1586624E 00 | 2691993E 00 | 8926115E 01 | 4596173E 00
Figures 5-12 through 5-17 are a sample of the estimation process response by matrix element. For completeness, Figure 5-14 shows the response of a zero element. All these results are for the eighth order, with accumulator case.

Now examine results of the sixth order, without accumulator, configuration.

Figures 5-18 through 5-23 show the response of the sixth order observer for the varying estimates of the parameters. Again the nonlinear small signal is being followed by a linear observer. Figures 5-24 and 5-25 show the response of the nonlinear small signal states contrasted with the linear response. The estimation response for the sixth order example is demonstrated by Figures 5-26 through 5-29. All of the responses were excited by a 10 Hz perturbation with an amplitude of 27.5 lb/in$^2$.

As was anticipated, the technique is fraught with sensitivity, numerical, and dynamic difficulties. The observer must be properly designed with respect to the observed system. Appropriate time intervals must be chosen to allow different dynamics to settle before the parameter estimation process begins. Sample rates must be chosen so that there is numerically sufficient change in variables, providing well conditioned matrices in the calculation process. Gains must be chosen properly to achieve adequate evaluation stability in the parameter estimation process. These considerations can be formidable, especially for systems possessing a large span in eigenvalues. Trial and error generally provide an adequate performance index to determine the times, gains, and sampling intervals. The technique possesses many shortcomings in terms of implementation and system dependence. However, the system has been demonstrated to work for the CTL-V configuration, and the technique has many advantages in application to systems whose states cannot be directly measured.
Figure 5-1. Eighth order observer response for $\Delta P_{OS}$.

Figure 5-2. Eighth order observer response for $\Delta F_{FL}$. 
Figure 5-3. Eighth order observer response for $\Delta P_{OD1}$.

Figure 5-4. Eighth order observer response for $\Delta F_{OS}$. 
Figure 5-5. Eighth order observer response for $\Delta F_{OS}$ (enlarged).

Figure 5-6. Eighth order observer response for $\Delta P_{O12}$. 
Figure 5-7. Eighth order observer response for $\Delta F_{\text{OP2}}$.

Figure 5-8. Eighth order observer response for $\Delta P_A$. 
Figure 5-9. Eighth order observer response for $\Delta F_A$.

Figure 5-10. Eighth order linear and nonlinear response for $\Delta P_{OS}$. 
Figure 5-11. Eighth order linear and nonlinear response for $\Delta F_{FL}$.

Figure 5-12. Eighth order estimation response for $D_{88}$.

Figure 5-13. Eighth order estimation response for $D_{78}$.

Figure 5-14. Eighth order estimation response for $D_{83}$. 
Figure 5-15. Eighth order estimation response for $D_{43}$.

Figure 5-16. Eighth order estimation response for $D_{33}$.

Figure 5-17. Eighth order estimation response for $D_{21}$.
Figure 5-18. Sixth order observer response for $\Delta P_{OS}$.

Figure 5-19. Sixth order observer response for $\Delta F_{FL}$.
Figure 5-20. Sixth order observer response for $\Delta P_{OD1}$.

Figure 5-21. Sixth order observer response for $\Delta F_{OS}$. 
Figure 5-22. Sixth order observer response for $\Delta P_{O12}$.

Figure 5-23. Sixth order observer response for $\Delta F_{OP2}$. 
Figure 5-24. Sixth order linear and nonlinear response for $\Delta P_{OS}$.

Figure 5-25. Sixth order linear and nonlinear response for $\Delta F_{FL}$. 
Figure 5-26. Sixth order estimation response for $D_{21}$.

Figure 5-27. Sixth order estimation response for $D_{43}$.

Figure 5-28. Sixth order estimation response for $D_{66}$.

Figure 5-29. Sixth order estimation response for $D_{56}$. 
REFERENCES


REFERENCES (Concluded)


BIBLIOGRAPHY


BIBLIOGRAPHY (Concluded)


APPENDIX A

CTL-V SIMULATION WITH CROSS REFERENCE
C CTLV SIMULATION AND PARAMETER ESTIMATION.
C RLN OPTIONS
C
C KL=0=OBESERVER ONLY, I=SHAPE ZEROS UNTAFFECTION ELEMENTS OF CA,
C ILN SET SIGNAL FOR OBSERVER TO FOLLOW FOR LINEAR FOR HAHAN.
C PRINT OPTIONS
C
C IPS=0=PRINTS T,X
C IPS=1=PRINTS T,X,LN,XL,XT
C IPS=2=PRINTS T,X,LN,XL,XT,XY
C IPS=3=PRINTS T,X,XY,XX
C LK=DISCRETIZATION INTERVAL,KA=SETTLING TIME
C
C REAL Kx,MY Nx,Mc,KL,LY,KA,TC,AA,AG,AK
C
C DIMENSION A(8,8),C(8,8),D(8,8),X(8,8),X(8,8),X(8,8),X(8,8),X(8,8),X(8,8)
C
C Comal (8,8),C(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8)
C
C D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8),D(8,8)
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      2 1,8,(X[I],I=1,8),(X[I],I=1,8),((X[I],J),I=1,4),I=1,8)
      3 (C[I],J),I=1,8),I=1,4),X[I],1,4)
      4 ((X[I],J),I=1,8),J=1,8)
      5 FOR A(I=15/I+160/8)
      6
      7 30 continue
      8
      9 C
      10 DO 30 I=1,8
      11 C
      12 DO 40 J=1,8
      13 C
      14 A(I,J)=0.0
      15 C
      16 DO 50 CONTINUE
      17 C
      18 A(I,J)=A(I,J)+A(J,I)
      19 C
      20 WRITE(I,,(A(I,J),I=1,8),I=1,8)
      21 C
      22 FORMAT(/"LINEARIZED SYSTEM JACOBIAN/(B15.8))
      23 C
      24 C
      25 C
      26 C
      27 C
      28 C
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150 151
152

IF (IFS1+RF+1) <= 100 goto 190
IF (IFS1+RF+1) <= 103 goto 111

1, I, (I) GO TO 150

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150 151.000

151 152.000

152
IF (WK*NE+3) G0 TO 39C 153,000
C1(I)=X(I)+Z1(WK+1) 154,000
C2(I)=XL(I)+Z2(WK+1) 155,000
C3(I)=XT(I)+Z3(WK+1) 156,000
C4(I)=X(I)+Z4(WK+1) 157,000
C5=1 158,000
C6=0 TO 36C 159,000
C7(I)=X(I)+Z1(WK+1) 160,000
C8(I)=XL(I)+Z2(WK+1) 161,000
C9(I)=XT(I)+Z3(WK+1) 162,000
C10(I)=X(I)+Z4(WK+1) 163,000
C11=CT/8* 164,000
36C ChAINLE 165,000
37C ChAINLF 166,000
38C GH TO 3IC 167,000
39C IF (WK*NE+1) G0 TO 39C 168,000
W3=WK+1 169,000
39C GH TO 34C 170,000
39D IF (WK*NE+2) G0 TO 39D 171,000
W3=WK+1 172,000
39E GH TO 34C 173,000
39F IF (WK*NE+3) G0 TO 39F 174,000
W3=WK+1 175,000
39G GH TO 39G 176,000
39H IF (WK*NE+4) G0 TO 39H 177,000
W3=WK+1 178,000
39I GH TO 39I 179,000
39J ChAINLE 180,000
39K ChAINLF 181,000
39L GH TO 36C 182,000
39M GH TO 36C 183,000
39N GH TO 36C 184,000
39O GH TO 36C 185,000
39P GH TO 36C 186,000
39Q GH TO 36C 187,000
39R GH TO 36C 188,000
39S GH TO 36C 189,000
39T GH TO 36C 190,000
39U GH TO 36C 191,000
39V GH TO 36C 192,000
39W GH TO 36C 193,000
39X GH TO 36C 194,000
39Y GH TO 36C 195,000
39Z GH TO 36C 196,000
40C ChAINLE 197,000
40D ChAINLF 198,000
40E GH TO 36C 199,000
40F GH TO 36C 200,000
40G GH TO 36C 201,000
40H GH TO 36C 202,000
40I GH TO 36C 203,000
CONTINUE

IF (NL.EQ.0) GO TO 420

DO (X(I),I=IN1)

CONTINUE

IF (X(I).LT.0) GO TO 425

CONTINUE

STOP
357 DA(5,E)*C*
358 DA(5,E)*C*
359 DA(7,E)*C*
360 DB 7586 1*14
361 7585 DA(7,E)*C*
362 DA(7,E)*C*
363 DA(9,E)*C*
364 DB 7586 1*17
365 7586 DA(17)*C*
366 Du 7587 1*14
367 7587 DA(17)*C*
368 DA(6,E)*C*
369 7588 Du 77C 1=100
370 Du 76C_2=100
371 760 D(I,J)=C(I,J)+CA(I,J)*AGA
372 CONTINUE
373 CH T9 758
374 CONTINUE
375 IF (T=GT.TLM) CALL EXIT
376 G9 79 200
377 4 FHR=AT/(1F10*5/(4E16*2))
378 C ZERB MEMORY CALL
379 800 CALL ZAP(K,M)
380 G9 T8 10
381 END
<p>| A    | B   | C    | D    | E    | F    | G    | H    | I    | J    | K    | L    | M    | N    | O    | P    | Q    | R    | S    | T    | U    | V    | W    | X    | Y    | Z    |</p>
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APPENDIX B

SIXTH AND EIGHTH ORDER DATA SETS FOR CTL-V EXAMPLE
PARAMETER ESTIMATING STATE RECONSTRUCTION

By Edwin Bruce George

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

WILLIAM R. MARSHAL
Associate Director for Engineering