A RAPID METHOD FOR OPTIMIZATION
OF THE ROCKET PROPULSION SYSTEM
FOR SINGLE-STAGE-TO-ORBIT VEHICLES

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A rapid analytical method for the optimization of rocket propulsion systems is presented for a vertical take-off, horizontal landing, single-stage-to-orbit launch vehicle. This method utilizes trade-offs between propulsion characteristics affecting flight performance and engine system mass. The performance results from a point-mass trajectory optimization program are combined with a linearized sizing program to establish vehicle sizing trends caused by propulsion system variations. The linearized sizing technique was developed for the class of vehicle systems studied herein. The specific examples treated in this paper are the optimization of nozzle expansion ratio and lift-off thrust-to-weight ratio to achieve either minimum gross mass or minimum dry mass. Assumed propulsion system characteristics are high chamber pressure, liquid oxygen and liquid hydrogen propellants, conventional bell nozzles, and the same fixed nozzle expansion ratio for all engines on a vehicle.
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SUMMARY

A rapid analytical method for optimization of rocket propulsion systems is presented for a vertical take-off, horizontal landing, single-stage-to-orbit launch vehicle. This method utilizes trade-offs between system characteristics affecting flight performance and engine system mass. The performance results obtained from a point-mass trajectory optimization program are combined with a linearized sizing program to establish vehicle sizing trends caused by propulsion system variations. Although this method is applicable and adaptable to a wide range of flight vehicle systems, the example presented is restricted to a single class of vehicle and propulsion systems. The linearized sizing technique was developed for this class of systems. The resulting sizing program is small enough to be readily adaptable to a desk top programmable calculator and, because of its explicit solution, is fast and precise. The specific example treated in this paper is the optimization of nozzle expansion ratio and lift-off thrust-to-weight ratio to achieve either minimum gross mass or minimum dry mass. Assumed propulsion system characteristics are high chamber pressure, liquid oxygen and liquid hydrogen propellants, conventional bell nozzles, and the same fixed nozzle expansion ratio for all engines on a vehicle. Optimization results show a strong trend toward vehicles having low engine mass rather than high propulsive performance, with optimum values of expansion ratio in the range of 40 to 45 and optimum lift-off thrust-to-weight ratios in the range of 1.20 to 1.35.

INTRODUCTION

Single-stage-to-orbit (SSTO) systems are being studied as potential launch vehicle concepts for the time period following space shuttle. On the basis of projected advancements in technologies and possible changes in mission requirements, such systems are envisioned either as shuttle replacements or in complementary mission roles. If the shuttle system is assumed to have a 15-year operational life, a new replacement system will
be required around 1995. With 8 to 9 years allowed for design and development lead time, the period available for technology preparation is only slightly more than 10 years.

Recent studies at the Langley Research Center have been directed toward defining the necessary and high-yield areas of technology preparation. An integral part of this process is the definition and preliminary design of attractive system concepts, which are then used to measure the effects of technology variations. One of the most crucial technologies for such systems is that of propulsion, which is the principal subject of this paper.

Some method of total system synthesis is required to compare the various propulsion system concepts and to identify many of the trade-offs within a given propulsion concept. The extent of synthesis detail utilized in such studies varies over a wide range, the most detailed methods requiring large computer resources. The method described in this paper emphasizes a simplified synthesis which can reduce the resource requirements for such propulsion system trade studies.

The key to this simplified synthesis is a linearized sizing analysis. Martin, in reference 1, uses a linearized sizing analysis to optimize the phasing points of multiphase SSTO propulsion systems and to compare various types of propulsion concepts on the basis of ideal velocity. The implicit assumption is that the effect of propulsion system variations on the ascent trajectory is small in comparison with the variations due to other factors. In the present study, a similar type of sizing analysis is used in conjunction with actual trajectory performance for analyzing trade-offs between engine systems mass and flight performance for a selected engine concept. The sizing analysis is configured for a vertical take-off, horizontal landing (VTOHL) SSTO with all rocket propulsion. Either gross mass or dry mass may be used as a figure of merit. The fact that the interface between the sizing program and the trajectory program is simple allows a different trajectory program to be substituted if desired.

The example presented herein is the optimization of engine nozzle expansion ratio and lift-off thrust-to-weight ratio to achieve either minimum gross mass or minimum dry mass. The propulsion system uses rocket engines operating at high chamber pressures with liquid hydrogen and liquid oxygen propellants and conventional bell nozzles, all nozzles on a given vehicle having the same fixed expansion ratio. There are other potentially attractive rocket propulsion schemes for VTOHL SSTO application, such as the use of a mixture of fixed expansion ratios and dual-position nozzles; however, these are more complex than the example scheme, both from considerations of vehicle integration and operation and from the standpoint of propulsion system optimization.
SYMBOLS

\[ C_{sd} \]  residual propellant mass coefficient, ratio of residual propellant mass to ascent propellant mass, \( \frac{m_{p, sd}}{m_{p, a}} \)

\[ C_{sv} \]  reserve propellant mass coefficient, ratio of reserve propellant mass to ascent propellant mass, \( \frac{m_{p, sv}}{m_{p, a}} \)

\[ E \]  main engine system mass coefficient, ratio of installed engine system weight to thrust, \( \frac{m_E g \, \text{g}}{T} \)

\[ G \]  glide-reuse systems mass coefficient, ratio of glide-reuse systems mass to entry mass, \( \frac{m_G}{m_e} \), see equation (10)

\[ g \]  standard free-fall acceleration, 9.80665 m/s

\[ I_s \]  engine specific impulse, \( \frac{T}{m_g} \), s

\[ K \]  tankage systems mass coefficient, ratio of tankage systems mass to total main propellant mass, \( \frac{m_K}{m_{p, a} + m_{p, sd} + m_{p, sv}} \)

\[ \dot{m} \]  engine propellant mass flow rate, kg/s

\[ m_d \]  vehicle dry mass, \( m_f + m_K + m_E + m_G \), kg

\[ m_E \]  engine systems mass, kg, see equation (9)

\[ m_e \]  vehicle entry mass, kg, see equation (13)

\[ m_f \]  fixed mass, kg, see equation (7)

\[ m_G \]  glide-reuse systems mass, kg, see equation (10)
\( m_g \) vehicle gross mass, kg, see equation (11)

\( m_K \) tankage systems mass, kg, see equation (8)

\( m_{\text{pay}} \) payload mass, kg

\( m_{p,a} \) mass of ascent propellants, including in-flight losses, kg

\( m_{p,o} \) mass of orbital maneuvering propellants, kg

\( m_{p,r} \) mass of reaction control system (RCS) propellants for entry, kg

\( m_{p,sd} \) mass of main propellant residuals, kg

\( m_{p,sv} \) mass of main propellant performance reserves, kg

\( N \) vehicle system thrust-to-weight ratio, \( \frac{T}{m_g} \)

\( q \) free-stream dynamic pressure, Pa

\( T \) rocket engine thrust, N

\( \alpha \) angle of attack, deg

\( \Gamma_a \) ascent propellant mass ratio, ratio of gross mass to mass at ascent trajectory main engine cut-off, \( \frac{m_g}{m_g - m_{p,a}} \)

\( \Gamma_o \) orbital maneuvering propellant mass ratio, ratio of initial orbital mass to entry mass, \( \frac{m_e + m_{p,o}}{m_e} \)

\( \Gamma_r \) entry RCS propellant mass ratio, ratio of entry mass to landing mass, \( \frac{m_e}{m_e - m_{p,r}} \)
\( \Delta V \) \quad \text{ideal velocity change, m/s}

\( \varepsilon \) \quad \text{engine nozzle expansion ratio}

Subscripts:

- \( a \) \quad \text{main ascent propellant}
- \( o \) \quad \text{orbit maneuvering system (OMS) propellant}
- \( r \) \quad \text{entry reaction control system (RCS) propellant}
- \( \text{sl} \) \quad \text{sea level or lift-off conditions}
- \( \text{vac} \) \quad \text{vacuum}

Abbreviations:

- \( \text{LH}_2 \) \quad \text{liquid hydrogen}
- \( \text{LINSIZ} \) \quad \text{linear sizing computer program}
- \( \text{LOX} \) \quad \text{liquid oxygen}
- \( \text{ODIN} \) \quad \text{optimal design integration, computerized design integration system}
- \( \text{OMS} \) \quad \text{orbital maneuvering system}
- \( \text{POST} \) \quad \text{program to optimize simulated trajectories, trajectory computer program}
- \( \text{RCS} \) \quad \text{reaction control system}
- \( \text{SSME} \) \quad \text{space shuttle main engine}
- \( \text{SSTO} \) \quad \text{single-stage-to-orbit}
- \( \text{TPS} \) \quad \text{thermal protection system}
- \( \text{VTOHL} \) \quad \text{vertical take-off, horizontal landing}
Method of Analysis

Overview

Propulsion system optimization for a vehicle flight system should include such considerations as (1) engine performance and mass characteristics; (2) propellant and tankage requirements; (3) flight trajectory and performance; (4) integration of the propulsion system with the vehicle (installation, aerodynamic effects, center of gravity); and (5) overall system synthesis or sizing. For cases in which the range of propulsion system variables under consideration is restricted in some way, a simplified treatment of one or more of these factors may be appropriate. The method of analysis presented herein applies simplifications to (4) and (5) above. Propulsion system variations are assumed not to impact configuration features such as vehicle shape, aerodynamic coefficients, vehicle balance (center of gravity), etc. Also, system sizing changes are assumed to be adequately represented by a combination of linear relationships. However, for both assumptions, this does not preclude the need for a reference vehicle design which adequately defines and accommodates all the above considerations.

A principal feature of this method of analysis is the use of a simplified sizing program which incorporates considerations (1), (2), and (5); has the form to permit an explicit solution; and is small enough to be run on a desk top programmable calculator. Calibration of this sizing program against the characteristics of a more detailed design assures proper sizing behavior.

Ascent trajectory performance, which is assumed to be a driving sizing requirement, is computed with a generalized point-mass trajectory optimization program. Trajectory performance is determined for various combinations of the propulsion system characteristics. Because the effects of integrating the propulsion system with the vehicle are assumed not to vary the overall aerodynamic characteristics, it is possible to compute all trajectories by use of the characteristics of a single vehicle (excluding the propulsion system variations of interest). These trajectory results are then used as inputs for the sizing program. In this way, the combined trajectory flight performance and vehicle sizing effects of propulsion system variations are evaluated.

Sizing Analysis

A linearized sizing program was used to determine vehicle sizing for each combination of propulsion system characteristics evaluated for trajectory performance. This program, designated LINSIZ, was implemented on a desk top programmable calculator in about 275 program steps. This linearized method is similar to the approach used in reference 1; however, more details in the propulsion area are provided in the present study. This sizing approach simplifies the overall vehicle system to 11 mass categories:
1 - Main engine ascent propellant
2 - In-flight propellant losses
3 - Ascent propellant reserves
4 - Ascent propellant residuals
5 - OMS (and RCS) orbital propellants
6 - RCS entry propellant
7 - Payload
8 - Fixed items
9 - Tankage systems
10 - Engine systems
11 - Glide-reuse structures and systems (wings, TPS, landing gear, etc.)

Mass groupings that are used in this study are defined as follows:

Dry mass Categories 8, 9, 10, and 11

Entry mass Dry mass plus categories 4, 6, and 7

OMS initial mass Entry mass plus category 5

Burnout mass Entry mass plus categories 3 and 5

Gross mass Burnout mass plus categories 1 and 2 (all categories)

Categories 1 to 7 are self-evident; however, categories 8 to 11 require some explanation. Vehicle fixed mass includes such items as crew, crew provisions (cabin and environmental control), payload bay structure, payload provisions, and avionics, which are independent of propulsion system variations and which are basic to the orbital mission of the vehicle. Tankage systems include basic tanks and associated structure and plumbing, which are sized by the total ascent propellant requirements. Engine systems include engines, plumbing, pumps, gimbals, thrust structure, etc., which are sized by the thrust level and propellant flow rate requirements. The glide-reuse category includes wings, tail, TPS, landing gear, etc., which are sized by the entry mass.

For each mass category, a linear mass relationship appropriate to a VTOHL SSTO vehicle is used:
Ascent propellant, including in-flight losses,

\[ m_{p,a} = \frac{\Gamma_a - 1}{\Gamma_a} m_e \]  

(1)

Reserves and residuals

\[ m_{p,sv} + m_{p,sd} = (C_{sv} + C_{sd}) m_{p,a} \]  

(2)

OMS propellant

\[ m_{p,o} = (\Gamma_0 - 1) m_e \]  

(3)

where

\[ \Gamma_0 = e^{\Delta V_o/g_{I_s,o}} \]  

(4)

RCS entry propellant

\[ m_{p,r} = \frac{\Gamma_r - 1}{\Gamma_r} m_e \]  

(5)

where

\[ \Gamma_r = e^{\Delta V_r/g_{I_s,r}} \]  

(6)

is a useful approximation.

Fixed mass

\[ m_f = F_1 + F_2 m_{pay} \]  

(7)

where \( F_1 \) is a fixed mass constant (in kilograms), \( F_2 \) is a fixed mass coefficient (fraction of payload mass), and \( F_2 m_{pay} \) reflects the mass of payload accommodations to allow the scaling of fixed mass with payload variations. (For the type of enclosed bay with full-length doors assumed in this study, \( F_2 \) appears to have a value of about 0.6.)
However, since payload is not varied in this study, fixed mass is simply $m_f$, and equation (7) is not used.

**Tankage systems**

\[
 m_K = K(m_{p,a} + m_{p,sd} + m_{p,sv})
\]  

(8)

**Engine systems**

\[
 m_E = E_{sl}N_{sl}m_g
\]  

(9)

**Glide-reuse systems**

\[
 m_G = Gm_e
\]  

(10)

The definition of gross mass gives the following expression:

\[
 m_g = m_{p,a} + m_{p,sv} + m_{p,sd} + m_{p,o} + m_{p,r} + m_{pay} + m_f + m_K + m_E + m_G
\]  

(11)

Substitution of equations (2) and (3) gives

\[
 m_g = m_{p,a}(1 + C_{sv}) + m_{p,a}C_{sd} + (\Gamma_0 - 1)m_e + m_{p,r} + m_{pay} + m_f + m_K + m_E + m_G
\]  

(12)

From the definition of entry mass

\[
 m_e = m_{p,a}C_{sd} + m_{p,r} + m_{pay} + m_f + m_K + m_E + m_G
\]  

(13)

equation (12) can be simplified to

\[
 m_g = m_{p,a}(1 + C_{sv}) + \Gamma_0m_e
\]  

(14)

Substitution of equations (5) and (10) into equation (13) gives

\[
 m_e = \frac{1}{1 - \frac{1}{\Gamma_r} - G}(m_{p,a}C_{sd} + m_{pay} + m_f + m_K + m_E)
\]  

(15)
Then substituting equation (15) into equation (14) gives

\[ m_g = m_{p,a}(1 + C_{SV}) + \frac{\Gamma_o}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G}(m_{p,a}C_{sd} + m_{pay} + m_f + m_K + m_E) \]  \hspace{1cm} (16)

Also, substitution of equations (8) and (9) now gives

\[ m_g = m_{p,a}\left[1 + C_{SV} + \frac{\Gamma_o C_{sd}}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G} + \frac{\Gamma_o K}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G}(1 + C_{SV} + C_{sd})\right] \]
\[ + \frac{\Gamma_o}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G}(m_{pay} + m_f + E_{sl}N_{sl}m_g) \]  \hspace{1cm} (17)

And finally, substituting equation (1) and solving for the gross mass gives

\[ m_g = \frac{(m_{pay} + m_f)\frac{\Gamma_o}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G}}{1 - \frac{\Gamma_o E_{sl}N_{sl}}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G} - \frac{\Gamma_a - 1}{\Gamma_a}\left[1 + C_{SV} + \frac{\Gamma_o C_{sd}}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G} + \frac{\Gamma_o K(1 + C_{SV} + C_{sd})}{1 - \frac{\Gamma_r - 1}{\Gamma_r} - G}\right]} \]
\hspace{1cm} (18)

Equation (18) is the key equation to the explicit sizing solution. Once the gross mass is determined, all the other mass characteristics can be established by using the following sequence:

Ascent propellant, including in-flight losses, by using equation (1)

Reserves and residuals by using equation (2)

Entry mass by rearranging equation (14) to give

\[ m_e = \frac{m_g - m_{p,a}(1 + C_{SV})}{\Gamma_o} \]  \hspace{1cm} (19)
OMS propellant by using equation (3)
RCS propellant by using equation (5)
Tankage systems by using equation (8)
Engine systems by using equation (9)
Glide-reuse systems by using equation (10)

Potentially, any of the inputs to equation (18) could be used as design variables for a sizing study; however, in applying this sizing analysis to the problem of propulsion system optimization, the inputs fall into three general categories:

1. Design assumptions and requirements
   - Payload: \( m_{\text{pay}} \)
   - OMS requirements: \( \Gamma_o \) or \( \Delta V_o \) and \( I_{S,o} \)
   - RCS requirements: \( \Gamma_r \) or \( \Delta V_r \) and \( I_{S,r} \)
   - Propellant coefficients: \( C_{sv}, C_{sd} \)

2. Common mass factors
   - Fixed mass: \( m_f \)
   - Glide-reuse coefficient: \( G \)
   - Tankage coefficient: \( K \)

3. Design variables
   - Engine factors: \( \epsilon, E_{s1}, \text{ and } I_{S,a} \)
   - Thrust-to-weight ratio: \( N_{s1} \)
   - Ascent propellant mass ratio: \( \Gamma_a \)

After the trajectory performance is determined for an assumed combination of \( \epsilon \) and \( N_{s1} \), the resulting ascent mass ratio \( \Gamma_a \), along with the appropriate engine mass factor \( E_{s1} \), is evaluated by the sizing program. The resulting gross mass and dry mass are utilized as the overall indicators of propulsion system optimality.

Within the limitations of the linearized sizing, validity of the results rests primarily with the proper selection of values for the three dry mass coefficients \( E, G, \text{ and } K \) and the fixed mass \( m_f \). In order to establish these coefficient values, a point-design vehicle system is needed for which there are detailed mass information and sizing trend data. It is desirable to duplicate both the absolute sizing level and the sizing trends such as those associated with variations in mass ratio.
Trajectory Analysis

The purpose of the trajectory analysis is to identify the variations in ascent performance, as measured by the ascent mass ratio $\Gamma_a$, which result from variations in characteristics of the rocket propulsion system. To do this, optimized ascent trajectories are computed for each propulsion system configuration by use of a generalized program called POST (program to optimize simulated trajectories) described in reference 2.

POST is a discrete parameter targeting and optimization program having the capability to target and optimize point-mass trajectories while satisfying equality and inequality constraints. Because of the generality of the simulation and optimization routines, POST can be used to solve a wide variety of problems for a powered or unpowered vehicle operating near a rotating oblate planet. These problems can include ascent, reentry, and orbital transfer trajectories. In this study, POST is utilized for powered ascent trajectories which satisfy various in-flight constraints, meet orbital insertion requirements, and optimize the vehicle flight controls in order to minimize the ascent propellant requirement. Typical in-flight constraints include limits on dynamic pressure $q$, product of dynamic pressure and angle of attack $qa$, and vehicle acceleration. Insertion conditions may include altitude, flight-path angle, and velocity.

Interface requirements between the sizing program and the trajectory analysis are such that another trajectory program could be substituted for POST if desired. Vehicle simulation inputs such as aerodynamics, planform loading, in-flight constraints, and control modes must be representative of the vehicle being synthesized in the sizing analysis. These inputs are not varied within the trajectory analysis. The inputs that are varied between the different trajectory cases are the propulsion system characteristics. The sizing-trajectory interface requires that the propulsion system weights used in the sizing be consistent with the flight performance characteristics. The principal output of the trajectory analysis is the ascent propellant mass ratio $\Gamma_a$, which is used as a sizing input.

Propulsion system characteristics are input as total vacuum thrust, total mass flow rate, and total nozzle exit area. Effective thrust due to back pressure losses is calculated in the program. In addition to main propellant flows, in-flight propellant losses (due to such things as boiloff and auxiliary power generation) can be simulated by the addition of a zero-thrust, zero-exit-area "engine." The in-flight losses, which are then included in the ascent propellant requirement, have been taken into account in this manner.

EXAMPLE

The example presented in this study is the optimization of the rocket engine nozzle expansion ratio $\epsilon$ and lift-off thrust-to-weight ratio $N_{sl}$ for a VTOHL SSTO vehicle using identical multiple engines with conventional bell nozzles. All nozzles on a given
vehicle have the same fixed expansion ratio. Optimization is on the basis of either minimum gross mass or minimum dry mass. A Langley Research Center (LaRC) in-house preliminary design provided a point-design vehicle for calibration of the linearized sizing model and for the trajectory analyses. This point-design vehicle was based on mission requirements similar to the space shuttle and on assumed 15-year advancements beyond the space shuttle level in two basic technology areas, structures and propulsion. Trajectory and sizing analyses were performed for a matrix of $e$ and $N_{Sl}$ combinations, with $e$ ranging from 30 to 80 and $N_{Sl}$ ranging from 1.15 to 1.50.

**Design Requirements**

Basic space shuttle design requirements were utilized extensively for this study (ref. 3). Although it is unlikely that a post-shuttle vehicle would be designed for the same mission needs as the shuttle, the shuttle offers the best defined set of requirements available at this time. The sizing mission used was the 29,500-kg payload launched from the Kennedy Space Center (KSC) into a 28.5° inclined orbit. A 4.6-m-diameter by 18.3-m-long payload bay is provided. The orbital maneuvering system provided $\Delta V = 229$ m/s, which was sufficient for near-Earth maneuvers and entry deorbit. The OMS provides for all maneuvers beyond the 93-km by 185-km insertion orbit provided by the main propulsion system. The entry cross range requirement is established at the shuttle level of 2037 km. A reaction control system is provided for attitude control during the initial phases of reentry.

**Technology Assumptions**

The assumed development schedule for the SSTO has the advantage of a 15-year advancement in technology beyond the level for the shuttle. Significant gains should result from existing programs in the areas of structures and propulsion.

**Structures.** - The information presented in reference 4 indicates that a 25-percent structural mass reduction from current technology is possible for a vehicle developed in the period from 1985 to 1990. Consequently, a 25-percent structural mass reduction from a space shuttle reference has been assumed in the areas of the body, wing, tail, tanks, and landing gear. No mass reductions are assumed for the propulsion systems or for the supporting subsystems.

**Propulsion.** - The engine characteristics used in this study are based on space shuttle main engine technology, which incorporates a high chamber pressure (20 MPa) and LOX-LH$_2$ propellants. A LOX-LH$_2$ mixture ratio of 7:1, as proposed in reference 5, is assumed instead of the SSME value of 6:1. This variation appears to offer advantages over the SSME for SSTO application because the vehicle sizing advantages due to the increased propellant bulk density (about a 9-percent increase) more than offset the effects
of the somewhat lower specific impulse (about a 1.5-percent decrease). However, the overall differences between these two mixture ratios are not very large.

Engine characteristics used in this study are shown in figures 1 and 2. The variation of specific impulse with expansion ratio $\varepsilon$ is shown in figure 1. This figure is based on the reference point for LOX-LH$_2$ presented in reference 5 (a mixture ratio of 7:1 and $\varepsilon = 200$) and on characteristics of minimum-surface-area nozzles for variations in $\varepsilon$ presented in reference 6. The curve for a mixture ratio of 6:1 with the SSME located at $\varepsilon = 77.5$ is shown for comparison. Figure 2 shows the variation of the engine system mass coefficient $E$ with the expansion ratio for thrust levels at both vacuum and sea level. The sea level mass coefficient $E_{sl}$ is the one used in the sizing program. These mass characteristics were derived from weight reports and trade studies for the space shuttle and reflect the SSME operating at maximum power.

Point-Design Vehicle

A point-design vehicle derived from LaRC in-house studies of an SSTO system was used for the trajectory evaluation and also to establish coefficients for the linearized sizing program. This design was generated in Langley Research Center studies which utilized the ODIN system (ref. 7). Vehicle mass properties and other characteristics are summarized in table 1 and the physical layout is shown in figure 3. For this vehicle, $\varepsilon$ was 80 and $N_{sl}$ was 1.347. The ascent mass ratio $\Gamma_a$ determined from trajectory analysis was 7.3342. The gross mass was 2.15 Gg, and the reference theoretical wing area was 1087 m$^2$. Design requirements and assumptions for the point-design vehicle were consistent with those previously discussed, with two minor exceptions. First, the OMS mass ratio $\Gamma_o$ was insufficient in the point-design vehicle to give the desired $\Delta V$ of 229 m/s, and second, the value of $I_s$ assumed for the main engines was slightly higher than that assumed for this study, as shown in figure 1. This resulted in a somewhat lower ascent mass ratio $\Gamma_a$ than would be required for the engines with the lower value of $I_s$. Corrections for both these factors are incorporated in the sizing analysis.

Sizing Analysis

Tables 2 and 3 illustrate the use of the point-design vehicle to calibrate the linear sizing program. The mass categories of the point-design vehicle were redistributed according to the linear sizing categories. The fluid categories were all well defined, however, the four categories included in dry mass required some judgment. Assignment of the major vehicle components were straightforward, with two possible exceptions: Basic structure and Growth. Seventy-five percent of the basic structure (item 3.1) was assigned to fixed mass to account for payload accommodations, which include the payload bay, structural supports, and doors. Growth (item 18.0) was proportioned among the four dry-
mass categories based on the mass level without growth. For many of the smaller sub-
systems whose categories were not obvious, category assignments were based on corre-
lations of sizing trends with available trend data. In this current example (and probably
typical in general), the best sizing trend correlation between LINSIZ and ODIN was
achieved by placement of these smaller subsystems in the fixed mass category. The sum-
mation of masses under each category as shown in table 2 provides the information needed
to establish constants and coefficients for the sizing program. On the basis of this mass
distribution, the following input values for the calibration vehicle were established:

\[
N_{sl} = 1.347
\]

\[
\Gamma_a = 7.3342 \quad (\epsilon = 80, \quad I_{s,a,\text{vac}} = 453 \text{ s})
\]

\[
\Gamma_o = 1.03667
\]

\[
\Gamma_r = 1.0168
\]

\[
C_{sv} = 0.002
\]

\[
C_{sq} = 0.007
\]

\[
m_{\text{pay}} = 29,500 \text{ kg}
\]

\[
m_f = 32,600 \text{ kg}
\]

\[
E_{sl} = 0.0248
\]

\[
K = 0.0240
\]

\[
G = 0.296
\]

The resulting calibration vehicle from the LINSIZ program is compared with the
point-design vehicle in table 3. All categories agree to within 1 percent. Figure 4 com-
pares the effect on gross mass of varying the ascent mass ratio as predicted by LINSIZ
and ODIN. The agreement is within 1 percent in the gross mass range of interest for this
study. However, significant errors are generated farther from the calibration point. The
LINSIZ gross mass is higher than the ODIN gross mass at mass ratios above the calibration point and is lower below the calibration point. As discussed in the preceding paragraph, manipulation of the fixed mass category allows a degree of control on these sizing trends, with increases in fixed mass tending to decrease the sensitivity of gross mass to changes in mass ratio.

The baseline vehicle for this sizing study is based on the point-design and calibration vehicles but incorporates the corrections for the OMS velocity change $\Delta V$ and specific impulse $I_S$ of the main engines, as noted earlier. The following changes were made to the LINSIZ inputs:

$$N_{sl} = 1.350$$

$$\Gamma_a = 7.45365 \quad (\epsilon = 80, \quad I_{s, vac} = 449.35)$$

$$\Gamma_0 = 1.05207 \quad (\Delta V_0 = 229 \text{ m/s}, \quad I_{s, o} = 460 \text{ s})$$

The resulting vehicle is about 11 percent heavier than the calibration vehicle and is shown in the last column of table 3. The only sizing inputs varied from the baseline vehicle for the analysis of $\epsilon$ and $N_{sl}$ effects are $N_{sl}$, $\Gamma_a$, and $E_{sl}$.

Trajectory Analysis

A typical ascent trajectory is shown in figure 5. The trajectory simulation was set up to minimize the ascent propellant requirements while satisfying in-flight constraints and orbital insertion requirements. Controls used are pitch angle for the first 30 s and the angle of attack $\alpha$ thereafter. These are implemented as linear functions of time in eight different segments over the ascent trajectory. The in-flight constraints are maximum ascent acceleration, maximum dynamic pressure $q$, and maximum $q\alpha$. Ascent acceleration is limited to 3g by continuous engine throttling after the 3g level is reached. Dynamic pressure is limited to 48 kPa as an inequality constraint. The no-wind value of $q\alpha$ is limited to 120 kPa-deg by controlling the angle of attack in the region of high dynamic pressure. (This $q\alpha$ value was found to be optimal in ref. 8.) Aerodynamic coefficients were obtained from a representative Phase B space shuttle orbiter. Orbital insertion requirements, which are based on the perigee of a 93- by 185-km orbit, are implemented as equality constraints for altitude and flight-path angle. Velocity is the trajectory termination parameter.

In addition to main propellant flows, in-flight propellant losses (due to such things as boiloff and auxiliary power generation) were simulated. This amounted to a total loss
of 0.17 percent of the gross mass expended at a constant rate over the time of the ascent burn. Ascent propellant requirements, which are described by the main propellant mass ratio $\Gamma_a$, include these in-flight losses.

POST iterates the trajectory until convergence criteria which indicate that a satisfactory solution has been found are met. Convergence criteria include both constraint tolerances and measures of optimality. Typical tolerances on converged solutions are 2 m on insertion altitude and 0.004° on insertion flight-path angle.

RESULTS AND DISCUSSION

The method outlined in this paper for the optimization of SSTO rocket propulsion systems is easy and rapid to use once the basic trajectory simulation and sizing program constants are set up. The only input changes required between cases for the trajectory program are derived from the selection of expansion ratio $\epsilon$ and lift-off thrust-to-weight ratio $N_{s1}$. These changes are input as total vacuum thrust, total propellant flow rate, and total nozzle exit area. The input changes required for the sizing program are simply (1) lift-off thrust-to-weight ratio $N_{s1}$, as used in the matching trajectory case; (2) engine mass coefficient $E_{s1}$, corresponding to the selected expansion ratio; and (3) ascent mass ratio $\Gamma_a$, as determined by the trajectory.

Results of the ascent trajectory analysis for the example are shown in figure 6. A matrix of cases was evaluated with the expansion ratio ranging from 30 to 80 and the lift-off thrust-to-weight ratio varying from 1.15 to 1.50. (The expansion ratio of 80 represents an approximate limit for stable nozzle flow at sea level conditions.) The performance map of figure 6 utilizes sliding scales for both $\epsilon$ and $N_{s1}$ in order to have scales for both variables. Significant reductions in propellant requirements are possible at the larger expansion ratios and at the higher thrust-to-weight ratios. However, increases in both expansion ratio and thrust-to-weight ratio are at the expense of increasing engine mass.

Results of the sizing analysis are shown in figures 7 and 8. Figure 7 is a map of the dry-mass change relative to the baseline vehicle ($\epsilon = 80, \ N_{s1} = 1.35$). Minimum dry mass is achieved at $\epsilon = 40$ and $N_{s1} = 1.20$, a 15.9-percent reduction from the baseline vehicle. Details of this vehicle are presented in tables 4 and 5. A comparison of figures 6 and 7 shows nearly a complete inversion of results between trajectory performance and dry mass. The highest trajectory performance (lowest ascent propellant mass ratio) results in the heaviest dry-mass vehicle, whereas the minimum dry mass occurs in the region of lowest trajectory performance. It is apparent that the engine mass penalty required to achieve the low ascent propellant mass ratios more than offsets the benefits of the low mass ratios.
Figure 8 is a map of the gross-mass change relative to the baseline vehicle. In this case, minimum gross mass is achieved at $\epsilon = 45$ and $N_{s1} = 1.35$, a 9.6-percent reduction from the baseline vehicle. Details of this vehicle are presented in tables 4 and 5. This vehicle provides somewhat higher trajectory performance than the minimum-dry-mass vehicle; however, as before, the highest trajectory performance results in the heaviest vehicle. The optimum occurs at a fairly low expansion ratio and a moderate thrust-to-weight ratio; therefore, a strong trade-off between trajectory performance and engine mass is indicated.

Comparison of total propulsion system mass in tables 4 and 5 (that is, the sum of tankage systems and engine systems) indicates that the propulsion system mass is on the order of 50 percent of the dry mass and that it therefore has both strong direct and resizing effects. The propulsion system mass is reduced from 51.7 percent of the dry mass for vehicle 1 to 48.4 percent for vehicle 2 and to 49.1 percent for vehicle 3. As a result of resizing, the propulsion system mass of vehicle 1 is reduced 21 percent for vehicle 2 and 17.5 percent for vehicle 3. Another important measure of vehicle optimality is payload fraction, shown in table 5, based on either dry mass or gross mass. The payload fraction based on vehicle dry mass increased from 11.7 percent for vehicle 1 to 14.0 percent for vehicle 2 and 13.5 percent for vehicle 3.

Results of this example indicate the strong influence of trade-offs between engine system mass and flight trajectory performance for this class of vehicles. These results apply only to the VTOHL SSTO vehicle utilizing the assumed propulsion scheme (all engines on a vehicle have the same fixed expansion ratio). The optimum vehicles tend to accept significant flight performance penalties due to low expansion ratios and low lift-off thrust-to-weight ratios in order to achieve the benefits of low engine mass coefficients. This illustrates the importance of engine mass coefficient $E$ as a technology area for this class of rocket propulsion systems.

These results also indicate the suitability of this method—linearized sizing combined with optimized trajectory performance—for the optimization of propulsion systems when the range of variables is appropriately limited. As illustrated in the example, rocket propulsion systems are characterized by conflicting flight performance and vehicle sizing considerations and therefore require this type of combined performance-sizing analysis for optimization. The linearized sizing analysis provides adequate sizing characterization while also offering the potential for considerable savings in analysis time and/or cost over other sizing analyses. However, because of the linearizing assumptions, care must be taken to provide calibration with a suitable vehicle system and to limit the extent of sizing perturbations.

There are a number of other propulsion schemes and operational modes of potential interest for SSTO application. Other rocket propulsion schemes include mixed expansion
ratios, dual-position nozzles, and dual fuels. The basic assumptions behind this method of analysis are quite general; therefore, all these propulsion-operational schemes are adaptable to this type of analysis—some require only slight modification of the inputs while others may require modifications to the sizing equations. The computation of sensitivity information is another feature of this method associated with the explicit sizing solution. Small perturbations to the sizing inputs can be used to derive sizing sensitivities in order to show the impact of technology changes.

CONCLUDING REMARKS

A method has been developed for the analytical optimization of the rocket propulsion system for a vertical take-off, horizontal landing, single-stage-to-orbit launch system. This method combines the performance results obtained from a point-mass trajectory optimization program with a linearized sizing program to establish vehicle sizing trends. This linearized sizing program, which simplifies the overall system to 11 mass categories (4 dry mass categories), has an explicit solution form and is small enough to offer substantial computational savings over many other sizing programs.

Results are presented for a vehicle system utilizing rocket engines with high chamber pressure, liquid oxygen and liquid hydrogen propellants, conventional bell nozzles, and the same fixed expansion ratio for all engines on a vehicle. Both expansion ratio and lift-off thrust-to-weight ratio were optimized to achieve vehicles with either minimum gross mass or minimum dry mass. Results show that propulsion system mass is a dominant vehicle sizing factor. The trades between engine system mass and trajectory performance show a strong trend toward low engine mass. Optimum values of expansion ratio fell in the range of 40 to 45; optimum values of lift-off thrust-to-weight ratio fell in the range of 1.20 to 1.35.

With proper care in its application, both in establishing the required sizing inputs and in limiting the range of perturbations, this method should give quite satisfactory results. Also, the method is adaptable to many other flight vehicle/propulsion system concepts.

Langley Research Center
National Aeronautics and Space Administration
Hampton, Va. 23665
May 19, 1976
REFERENCES


TABLE 1.- POINT-DESIGN VEHICLE - MASS PROPERTIES

[VTOL SSTO with LOX-LH₂ propellant; reference wing area,  
1087 m²; $N_{sl} = 1.347$; $\epsilon = 80$; and $I_{s,vac} = 453$ s]

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### TABLE 2. - POINT-DESIGN VEHICLE – BREAKDOWN TO LINEAR SIZING CATEGORIES

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TABLE 4.- SUMMARY OF MASS PROPERTIES FOR BASELINE VEHICLE AND TWO OPTIMIZED CONFIGURATIONS

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<td>Vehicle 1 - baseline ($\epsilon = 80; \quad N_{\text{Sl}} = 1.35; \ I_s,\text{vac} = 449.3$ s)</td>
<td>Vehicle 2 - minimum dry mass ($\epsilon = 40; \quad N_{\text{Sl}} = 1.20; \ I_s,\text{vac} = 437.9$ s)</td>
<td>Vehicle 3 - minimum gross mass ($\epsilon = 45; \quad N_{\text{Sl}} = 1.35; \ I_s,\text{vac} = 440.0$ s)</td>
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| Component category | Vehicle 1 - baseline  
(ε = 80; Ns1 = 1.35;  
Iₜₒₓ, vₐₓ = 449.3 s) | Vehicle 2 - minimum dry mass  
(ε = 40; Ns1 = 1.20;  
Iₜₒₓ, vₐₓ = 437.9 s) | Vehicle 3 - minimum gross mass  
(ε = 45; Ns1 = 1.35;  
Iₜₒₓ, vₐₓ = 440.0 s) |
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Figure 1. - Performance of LOX-LH$_2$ high-pressure rocket engine.

Figure 2. - Engine system mass coefficients for a LOX-LH$_2$ mixture ratio of 7:1.
Figure 3 - Point-design VTOHL SSTO vehicle.
Figure 4. - Sizing results from LINSIZ and ODIN. LOX-LH₂ SSTO; 29 500-kg payload.

Figure 5. - SSTO ascent trajectory. \( \epsilon = 40; \ N_{s1} = 1.35 \).
Figure 6.- Effects of expansion ratio and lift-off thrust-to-weight ratio on ascent trajectory performance.
Figure 7.- Effects of expansion ratio and lift-off thrust-to-weight ratio on SSTO vehicle dry mass.
Figure 8.- Effects of expansion ratio and lift-off thrust-to-weight ratio on SSTO vehicle gross mass.
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