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The Recursive Maximum Likelihood Proportion Estimator—User's Guide and Test Results

by

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ABSTRACT:

In this report, we describe our implementation of the recursive maximum likelihood proportion estimator proposed by D. Kazakos in "Recursive Estimation of Prior Probabilities Using the Mixture Approach," (Rice University, ICSA Technical Report #275-025-019). A user's guide to the programs as they currently exist on the IBM 360/67 at LARS, Purdue is included, and test results on LANDSAT data are described. On Hill County data, the algorithm yields results comparable to the standard maximum likelihood proportion estimator.

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1. Introduction:

In this report, we describe our implementation of the recursive maximum likelihood proportion estimator proposed by D. Kazakos in [1]. Numerical results obtained with this algorithm using LANDSAT data are described, and a user's guide for the programs as they currently exist on the IBM 360/67 at LARS (terminal available at NASA-JSC) is included.

Section II contains a description of the algorithm as implemented. Section III serves as a user's guide to the programs available. In section IV, we describe the numerical results we have obtained with this algorithm. An appendix contains listings of the programs.

II. The Algorithm:

Given a set of n-dimensional measurement vectors \{x\} from M normally distributed multivariate pattern classes \(H_j\), \(j = 1, 2, \ldots, M\) the M-1 dimensional recursive maximum likelihood proportion estimate (RMLPE) \(\hat{p}_i^{(1)}\) at the \(i^{\text{th}}\) data vector is given by

\[
p_i = p_{i-1} + \frac{1}{i} L \left[ g \left( p_{i-1}, x_i \right) \right]^{-1} \left( f_1(x_i) - f_M(x_i), f_2(x_i) - f_M(x_i), \ldots, f_{M-1}(x_i) - f_M(x_i) \right)
\]

where \(f_j(x)\) is the density function for the \(j^{\text{th}}\) class;
\[ f_j(x) = (2\pi)^{-\frac{n}{2}} |\mathbf{K}_j|^{-\frac{1}{2}} \exp \left[ -(x - \mu_j)^T \mathbf{K}_j^{-1} (x - \mu_j) \right] \] (2)

where \( \mu_j \) and \( \mathbf{K}_j \) are the mean and covariance matrix, respectively, for the \( j \)th class; \( g(p_i^{-1}, x_i) \) is the mixture distribution estimate, i.e.,

\[ g(p_i^{-1}, x_i) = f_M(x_i) + \sum_{\ell=1}^{m-1} p_{i-1} \left( f_{\ell}(x_i) - f_M(x_i) \right) \] (3)

and \( L \) is a suitably chosen constant in this approximation. The proportion estimate for the \( M \)th class is denoted by \( p_i^m \) and given by

\[ p_i^m = 1 - \sum_{\ell=1}^{m-1} p_{i\ell} \] (4)

In our implementation of this algorithm, we have made several modifications to improve its performance. These include (1) clipping the value of the update (i.e., second) term in eq. (1); (2) renormalizing the \( p_i^k \) at each step so that all \( p_j^i \geq 0 \) and \( \sum_{\ell=1}^{m} p_{i\ell} = 1 \); and (3) introducing an additional damping term in the update term of eq. (1).

The final form of the algorithm is

\[ p_i^k = \text{NORM} \left\{ \varepsilon, \frac{1}{i+n_0} LMT \left[ T, L g(p_i^{-1}, x_i) \cdot \left( f_1(x_i) - f_M(x_i), f_2(x_i) - f_M(x_i), \ldots, f_{M-1}(x_i) - f_M(x_i) \right) \right] \right\} \]

where \( LMT(a, b) \) is the clipping function defined by
LMT(a, b) = \tilde{b}

with \quad \tilde{b}_i = \text{sign}(b_i) \min \left(a, |b_i|\right)

NORM is the renormalizing function defined by

\[ NORM(\varepsilon, y) = \text{the first } M-1 \text{ elements of } \tilde{y} \]

where

\[ \tilde{y}_m = 1 - \sum_{i=1}^{m-1} y_i \]

If

\[ \min(\tilde{y}_i) \geq \varepsilon > 0 \] then finish else

\[ \tilde{y}_i - \tilde{y}_i - \min(\tilde{y}_i) + \varepsilon \quad i = 1, 2, \ldots, M \]

and \( n_0 \) is a positive constant used to damp out early oscillations of the estimate.

Two other algorithms used in conjunction with this one are
(1) an algorithm to calculate an approximation to \( L \) and (2) an algorithm to scramble all of the data (the RMLPE uses the stochastic approximation, so the data needs to appear in a random order). The first algorithm calculates the following approximation to \( L \)

\[ L = \left( u \cdot \min |K_j|^{\frac{1}{2}} \right)^{-1} \]
where \( u \) is the minimum eigenvalue of \( H \) with \( H = \{ h_{ks} \} \) and

\[
h_{ks} = (2\pi)^{n/2} \int \left( f_k(x) - f_m(x) \right) \left( f_s(x) - f_m(x) \right) dx
\]

\[k, s = 1, 2, \ldots, m\]

The scrambling algorithm employs a procedure described on page 125 of [2].

III. Program Description and Users Guide

Three programs have been written to implement this algorithm: the proportion estimation program, a program to calculate an approximation to \( L \) and a program to scramble data prior to estimating proportions. These programs are described below and listings are provided in the appendix.

Propotion Estimation Program:

This program runs on the IBM 360/67 at LARS. Parameters are read from cards describing characteristics of the data and the statistics, the processing to be performed, and the desired outputs. The same data may be processed with several sets of statistics. The data is assumed to be sixteen channel data (from which any subset of channels may be used) residing on file 11 with one logical record per data vector. The data may be labelled or unlabelled. If labelled, the program will calculate the true proportions and print out the means of the estimates along with the associated variances and mean squared error; if unlabelled, the true proportions are read from cards and the same quantities are then computed. Due to the use of the stochastic approximation in
this algorithm, the data vectors be scrambled before being put on file 11. (Program super AM may be used for this purpose.)

The correspondence between the notation used in the preview section and the variables in the program are:

<table>
<thead>
<tr>
<th>Above</th>
<th>Program</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_i ) ( ^j )</td>
<td>( Q(J, *) )</td>
</tr>
<tr>
<td>( n_o )</td>
<td>( \text{ISTRT} )</td>
</tr>
<tr>
<td>( \varepsilon )</td>
<td>( \text{EPS} )</td>
</tr>
<tr>
<td>( T )</td>
<td>( \text{TLM} )</td>
</tr>
<tr>
<td>( L )</td>
<td>( L )</td>
</tr>
<tr>
<td>( G )</td>
<td>( G )</td>
</tr>
<tr>
<td>( f_i(x) )</td>
<td>( F ) (a function subprogram)</td>
</tr>
<tr>
<td>( \sum_{i} )</td>
<td>( \text{SG} )</td>
</tr>
<tr>
<td>( u_i )</td>
<td>( \text{MU} )</td>
</tr>
</tbody>
</table>

The programs are set up to handle up to 16 channel data from up to 15 classes with as many as 10 different blocking factors. They can treat an unlimited number of data points. The data enters the program in "lines" which contain \( \leq 1500 \) points.

\[
\text{N.B. The total number of points need not be an integral multiple of the points per line, e.g. if there are 5100 total points, we may use } \text{NP (the number of points/line) = 200}
\]
\[
\text{and } \text{NL (the number of lines) } \geq 26
\]
Figure 1
Data Deck Setup for the LANDSAT Version
Figure 1 shows the set-up of the data deck necessary to execute the program. The input parameters and their formats are described below:

1) **HEDNG** - Title to be printed on the output (20A4)

2) **M, MXITER, NK, ISTRT, INQ, OUTPT, L, TLM, EPS, \( K(1), I=1, NK \)**

\( (4 \, I \, 2, \, 2 \, L \, 1, \, 3 \, G \, 10.8, \, 10 \, X, \, 10 \, I \, 3) \)

- **M** - number of classes used
- **MXITER** - number of sets of statistics to use
- **NK** - number of blocking factors to use (set = 1)
- **ISTRT** - starting value of \( n_0 \) in eq. (1)
  (default = 99)
- **INQ** - = F if the initial guess for the proportion estimates (Q's) are to be set = \( \frac{1}{M} \) (then card set (3) are not used)
  = T if the Q's are to be read in (card set (3) is required)
- **OUTPT** - = T if updated Q's are to be printed after each line of data. Otherwise set = F
- **L** - the L value to be used in eq. (1)
- **TLM** - the maximum permissible absolute value for the update quantity for the Q's

\[ \left( \text{i.e.,} \right) \]

\[ L \sum_{i=1}^{K} \frac{f_j(x_j) - f_m(x_s)}{G(p_{i-1}, x_s)} \]

\[ s = K \times (i-1) + L \]
EPS - minimum allowable value for a Q during the estimation procedure ($10^{-2}$ seems to be a good choice)

$K(I), I=1, NK$ - the blocking factors to be used (set $K(1) = 1$)

3) $CSET$

(16 L 1)

$CSET_i = \begin{cases} T & \text{if } i^{th} \text{ channel is to be used} \\ F & \text{otherwise} \end{cases}$

Optional 4) \(((Q(I, J), I=1, M-1), J=1, NK)\)

(16 G 5. 3)

The initial guess for the Q's. Used only if INQ on card 2 is $T$

5) $CL(I), (MU(J, I), J=1, 16)$

\((26 X, A 1/(5 X, 5 E 15.8))\) for $I=1, 2, \ldots M$

\((SG (J, I), J=1, 136)\)

\((\cdot X, 5 E 15.8)\)

These cards contain the statistics for the M classes. $CL$ is the class ID, $MU$, the mean vector, and $SG$ is the covariance matrix stored in symmetric storage mode (i.e. upper triangular part stored by columns). Note that there are 33 cards required for each class. Additional sets of statistics follow card set (7).

6) $NP$, $NL$, OUTPP, OUTPX, TRUEP

(2 1 5, 3 L 1)

$NP$ - number of points to use per "line" ($\leq 1500$)

$NL$ - maximum number of "lines" of data
OUTPP - = T the current true proportions are printed after each line (used only if TRUEP = F)
= F do not print these proportions
OUTPX - = T print the data vectors
= F do not print the data vectors
TRUEP - = T if the true proportions are to be read in (card set (7) is then required)
= F the class ID is associated with each data vector and the program will calculate the true proportions (card set (7) not used).

Optional 7) (CLS(J), GT(J), J=1,M)
(8(A 2, G 8.6))

CLS(J) - the class ID for the Jth class
GT(J) - the true proportions for the Jth class

The data vectors should be on file 11 with 1 data vector per logical record in the format
CL, (X(J), J=1, 16)
(8 X, A 1, 6 X, 16 F 4.0, 1 X)
where CL is the class ID (used only if TRUEP on card set 6 = F) and X(J) contains the 16 dimensional data value for a pixel.

The subroutines used in this program are briefly described below:

INSTAT - reads and prints statistics
SUBSET - for LANDSAT data (LD) version, this selects appropriate subsets of the statistics.
F - computes the value of the density function at \( X \)

TP\( \text{POSE} \) - for the pseudo-random \((\text{PR})\) version, transposes the data matrix in situ

GEDATA - obtains or generates a line of data in the required format and order. Also computes the true proportions.

MCHLSK - computes the modified Cholesky decomposition of a covariance matrix stored in symmetric storage mode.

Program to Calculate \( L \):

This program calculates the following approximation to \( L \)

\[
L = \left( h \cdot \min \| \sum_j \right)^{-1} \frac{1}{h}
\]

where \( h = \min (\text{eval} (H)) \)

\[
& H_{ks} = \frac{(2\pi)^n}{2} \int_{\mathbb{E}^n} \left( f_k(x) - f_m(x) \right) \cdot \left( f_s(x) - f_m(x) \right) dx
\]

\[ k, s = 1, 2, \ldots, m \]

All notation is as before.

Input parameters to the program are

\[ \text{CSET, M, N} \]

\[ (16L1, 2X, 12, 2X, 12) \]

\[ [ \text{N.B. Our (limited) experience with the proportion estimation algorithm indicates that a value of } \sim 3 \text{ for } L \text{ appears optimal despite what this program computes.} ] \]
Scrambling Program:

This algorithm scrambles the order of records in a data set and creates a new data set. Two storage arrays are used: one containing the integers 1, 2, ..., N where N is the total number of records and the other containing space for one data record. A temporary direct access data set, which is the same size as the original data set, is used. The algorithm is described below:

1) Set $a_i = i$ for $i = 1, 2, \ldots, N$
2) Scramble the elements of the vector $a$.
   (see e.g. ref. [2]).
3) For $i = 1, 2, \ldots, N$
   a) Read $i^{th}$ record of original data set and store it in vector $d$.
   b) Write $d$ in $a_i^{th}$ record in temporary data set.
4) For $i = 1, 2, \ldots, N$
   a) Read $i^{th}$ record of temporary data set and store it in vector $d$.
   b) Write $d$ on $i^{th}$ record of new data set.
5) Finished.

Note that step 5 may not be necessary if one can use the data from the temporary direct access data set.

IV. Numerical Results:

A variety of numerical experiments were conducted with this program to determine its characteristic. Both pseudo-random and LANDSAT data were used.
The most significant effect of this algorithm is due to the scrambling (i.e., the order in which the data is input). If the data is not scrambled (i.e., blocks of points from single classes appear to the program) unreliable estimates will be produced. Our experience with LANDSAT data indicates that the entire data set, whose proportions are to be estimated, needs to have the individual pixels scrambled. Various scramblings will produce different estimates with a theoretical variance of $L/N$ where $N$ is the total number of pixels.

Another effect that we noticed was that the variance of the estimate for the $M^{th}$ class was always larger than for other classes. This asymmetry, we feel, is due to the fact that the algorithm estimates proportions for the first $M-1$ classes, and the estimate for the $M^{th}$ class is then computed as $1 - \sum_{i=1}^{M-1} p_i$. By reordering the classes and then again estimating proportions, it was determined that the variance of the $M^{th}$ class would decrease from $\sim 10\%$ to $\sim 30\%$, so the effect may not be too harmful. However, the user should be aware of this and assure that the estimate for the $M^{th}$ class is of the least interest.

Detailed tests of this algorithm were run on some Hill County LANDSAT data in order to compare results with those obtained by Coberly and Odell [3] with five other proportion estimation algorithms. Table 1 shows the results obtained from the recursive maximum likelihood estimator (RMLE) for 2600 pixels of the labelled data as compared to the other five estimators. Note that the RMLE and MLE have almost equal variances and mean squared errors.
Table II shows the results obtained from the RMLE for 8400 pixels of the unlabelled data. Here again the variances and mean squared error are approximately the same as those of the MLE.

V. Conclusions:

Our experience with this algorithm indicates several important factors need be taken account of in using this algorithm: (1) all of the data needs to be scrambled point by point, (2) the class of least importance should be used as the last class, and (3) a value of $\sim 3$ for the parameter $L$ appears close to optimal.

Our tests indicate that the recursive maximum likelihood estimator (RMLE) produces results of comparable variance and accuracy as the standard maximum likelihood estimator (MLE) of ref. [3]. The amount of computation involved for the RMLE is equivalent to the first iteration of the MLE plus the scrambling of the data. Also, no additional storage is required by this algorithm to store the density functions for each data point.

Further tests of this algorithm with other LANDSAT data will be necessary to determine the effectiveness of this algorithm in the general situation.
Table 1
Summary of Experiment 1
(Labeled Data, 2600 Pixels)

<table>
<thead>
<tr>
<th></th>
<th>CLASS</th>
<th>ODELL</th>
<th>MLE</th>
<th>RMLE</th>
<th>MIX</th>
<th>MCM</th>
<th>GT</th>
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<tr>
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# Table 2

Summary of Experiment II  
(Total Data Set, 8400 Pixels)

<table>
<thead>
<tr>
<th>CLASS</th>
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<th>MLE</th>
<th>RMLE</th>
<th>MIX</th>
<th>MCM</th>
<th>GT</th>
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</thead>
<tbody>
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<td></td>
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<td>.180347</td>
</tr>
</tbody>
</table>
REFERENCES


APPENDIX

FILE . . . DVR FORTRAN P1

REAL*4 SG(136,30),MU(16,30),O(30,10),LX(16,1500),DET(30),
1 (30,10)/300/N.
REAL*4 DVR(2000).
REAL*4 A(30),G(10).
REAL*4 OS(30,10),OR(30,10)/300/N,
1 0V(30,10)/300/N.
REAL*4 GT(19).
REAL*4 RSS(30).
REAL*4 HEDNG(20).
REAL*4 S,SS.
INTEGER*4 K(I),P(I)10/10=0/.
INTEGER*4 CH2A(16).
INTEGER*2 CL(15).LOGICAL*1 INO,IFNP,INO,OUTPT,END.
LOGICAL*1 CSF(16).LOGICAL*1 RST=TRUE.
COMMON /PASS/ SG,MU,NN.
COMMON /RSET/ INO,
1 GT.
COMMON /NPS/.,.
COMMON /NP/ CL.
COMMON /M/ M, NK,ISTRT,INO,OUTPT,L,TLM, EPS, XMX, MITER,CSET.
MXCHR=16.
MXPTS=1500.
MXCLS=30.

M - NUMBER OF CLASSES USED (0, LE,30).
O - NUMBER OF CHANNELS USED (0, LE,16).
MXITER - NUMBER OF TIMES TO REDO THE RUN WITH DIFFERENT DATA.
NO - NUMBER OF X'S TO BE USED.
ISTRT = INITIAL VALUE OF J IN (J=1) O(J-R) = 1/(J-R)+. (DEF=0).
INO = LOGICAL VARIABLE INDICATING WHETHER TO READ INITIAL.
1 GUESS FOR THE O'S OR NOT.
OUTPT = LOGICAL VARIABLE IF ESTIMATE OF O IS TO BE PRINTED.
L - THE L VALUE USED BY THE ALGORITHM.
TLM = LIMITING VALUE OF LAMS(E-J)-FM/GI (DEF=INFINITY).
MX = LOWER LIMIT ALLOWED FOR THE O'S.
XMX = UPPER LIMIT ALLOWED FOR THE O'S.
CSET = ARRAY INDICATING WHICH OF THE 16 CHANNELS ARE TO BE USED.
K = THE BLOCKING FACTORS TO BE USED (0, LE,10 OF THEM, EACH LE MP).
G = THE ESTIMATES OF THE PRIORS.
SG = COVARIANCE MATRICES STORED IN SYM STORAGE MODE.
MU = MEAN VECTORS.
X = THE DATA VECTORS FOR 1 'LINE' OF DATA.

PI2=2.3,14159265.
END=FALSE.

READ PARAMETERS

READ (5,1) HEDNG
1 FORMAT (20A6).
WRITE (6,4) HEDNG.
4 FORMAT (20A6).
READ (5,2) M,MXITER,NK,ISTRT,INO,OUTPT,L,TLM,EPS,XMX,(K(I),I=1,NK).
2 FORMAT (212,214,6G10.8,1013).
READ (9,3) CSET.
3 FORMAT (16E1).
IF (ISTRT<6.0) ISTRT=10.
IF (TLM.LT.1.E-2) TLM=1.70.
P=1=1.
IF (INO) GO TO 5.
GO TO 10.

THEN READ IN INITIAL GUESS FOR PRIORS FOR EACH BLOCKING USED.

5 READ (5,7) ((0(I,J),I=1,M),J=1,NK).
7 FORMAT (16E5.3).
GO TO 20.

ELSE SET INITIAL GUESS FOR PRIORS ALL EQUAL.

10 Y=1/M.
20 IF 15 I=1,M.
21 IF 15 J=1,NK.
15 0(I,J)=Y.

C*******************************************************************************
FILE... DUR FORTRAN P1

C
20 CONTINUE
DO 26 I=1, M
DO 26 J=1, MK
26 OS(I,J)=0(I,J)
WRITE (6,10DAT)
TL=TL+J
WRITE (6,21) (K(I),I=1, MK)
21 FORMAT (I, K='1015)
DO 23 I=1, MK
23 WRITE (6,22) (O(I,I),J=1, M)
22 FORMAT (I, INITIAL O='16, 8)

C GET STATS FOR THE CLASSES
C
28 CONTINUE
CALL INSTAT (CSET, CHAN)
IF (IFR,E0,0) P12=P12*(M/2.)
DO 34 J=1, MK
34 P1(J,J)=0
CALL CHOLSK (SG(J), N, X, OET(I))
25 OET(I)=1, DO/(SORT(OET(I))*PI2)

C fetch one line of data
C
60 CALL C-DATA(X,M, CHAN, N, 5100)

C
IF (.NOT.FRST) GO TO 32
FRST=.FALSE.

C LOOP OVER ALL DATA POINTS TO UPDATE ESTIMATE OF PRIORS
C
32 CONTINUE
IFP=.FALSE.
DO 30 I=1, N
IF (I, /0.0) IFP=.TRUE.

C F YIELDS THE VALUE OF THE DENSITY FUNCTION FOR THE CLASS
C
FM=F(X(I), N, SG(I), MU(I), OET(M))

C THE MIXTURE DISTRIBUTION IS SORED IN G

35 G(I)=FM
DO 40 J=1, MK
FJ=F(X(I), N, SG(J), MU(J), OET(J))
A(J)=FJ-FM
AJ=A(J)
DO 45 J=1, MK
45 G(J)=G(J)+O(J,J)*AJ
40 CONTINUE

C LOOP TO UPDATE PRIORS FOR EACH BLOCKING FACTOR

50 CONTINUE
IND=.FALSE.
KL=K(I)
MK=MOD(KL, I)
GL=G(I)
IF (MK, .00, OR. IFP) GO TO 52
GO TO 53

C THEN PREPARE TO UPDATE II-TH PRIORS

52 IK=MINO(MK, KL)
IF (IK, .00) IK=KL
IND=.TRUE.
PI(I)=PI(I)+IK
53 CONTINUE

C compute updated sums

C

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FILE  DOVR FORTRAN P1

S=0.D0 J=1,M1
X(J)=O(J,J1)/G1
T(J,J1)=T(J,J1)+S*SIGN(MIN(ABS(X),TLM),XX)
IF (IND) GO TO 56
GO TO 55

C C

UPDATE THE PRIORS AND RESET

55 XX=O(J,J1)+L*AT(J,J1)/(1P1JII) +ISTRT)
T(J,J1)=O(J,J1)/XX
S=S+XX
CONTINUE
IF (NOT.IND) GO TO 50

C C

RENORMALIZE THE UPDATED ESTIMATES OF THE PRIORS

O(J,J1)=S/O(J,J1)
D0 58 J=2,M
IF (O(J,J1).LT.S) S=O(J,J1)
CONTINUE
IF (S.GT.EPS) GO TO 64
SS=O(J,J1)
D0 62 J=1,M
O(J,J1)=O(J,J1)/SS
CONTINUE
54 CONTINUE
64 CONTINUE
50 CONTINUE
30 CONTINUE

C C

PRINT OUT NEW ESTIMATE OF PRIORS

72 DO 70 J=1,MK
S=O(J,J1)
D0 76 J=1,M1
75 S=S+O(J,J1)
O(J,J1)=S/J0-S
70 WRITE (6,76) NSRT,K(I),O(J,J1),J=1,M
76 FORMAT (/ "UPDATING ESTIMATE OF THE PRIORS FOR LINE 1 , 15 , WITH REL.
1MKING FACTOR K =1.3/4 ( 1 CLASS=1, A2 . 0=1, G15/K, 3X)"
IF (.NOT.IND) GO TO 60

C C

UPDATE MEANS AND VARIANCES OF THE ESTIMATES FOR THIS ITERATION.

ITER=ITER+1
D0 116 J=1,NK
115 O(J,J1)=O(J,J1)+XX
110 C(J,J1)=C(J,J1)+XX*XX
116 CONTINUE

C C

IF (ITER.LT.MXITER) GO TO 2A

C C

FINISHED WITH ALL DATA, PRINT OUT ESTIMATES & STOP

D0 118 J=1,NK
S1=O(J,J1)
D0 119 J=1,M
MSF(J)=O(J,J1)-2.*GT(J)*OR(J,J1))/MXITER+GT(J)*GT(J)
S1=S1+MSF(J)
XX=OR(J,J1)/ITER
OR(J,J1)=XX
XX=O(J,J1)/ITER-XX*XX
119 O(J,J1)=SORT(XX)
WRITE (6,131) K(I),C(J,J1),OR(J,J1),O(J,J1),MSF(J),J=1,M
131 FORMAT (13G16.8,1' MEANS AND SD'S OF THE ESTIMATES & THE MSE'/
1 (A3,G16.8,1 ' +-G16.8,G16.8))
S1=S1/N
WRITE (6,121) S1

(iii)

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FILE... DVR FORTRAN P1

118 CONTINUE
121 FORMAT ('*** MEAN MSE',G16.8)
STOP
CCC FINISHED WITH ALL DATA FOR THIS ITERATION
100 END=TRUE.
GO TO 72
END

FUNCTION :(X,N,L,MU,DET)

COMPUTE THE VALUE OF THE DENSITY FUNCTION AT X
REAL*4 X(1),L(1),MU(1),Y(16)
REAL*8 TF,S

SOLVE L Y=X-MU WHERE L IS THE CHOLESKY DECOMP OF COVAR MATRI
X. DIAG ELEMENTS OF L ARE STORED AS RECIPROCA
LS.
S=X(1)-MU(1)
Y(1)=S
TF=S*S*L(1)
IF (N.EQ.1) GO TO 15
K=1

LOOP TO COMPUTE Y(I)'S)

DO 10 J=2,N
S=X(1)-MU(1)
JJ=J-1
DO 20 J=1,JJ
K=K+1
20 S=S-L(K)*Y(J)
K=K+1
Y(I)=S
TF=TF+S*S*L(K)
10 CONTINUE
15 CONTINUE
IF (TF.LT.325.) GO TO 17
F=0.
RETURN
17 F=EXP(SNGL(-TF/2.))*DET
RETURN
END

ORIGINAL PAGE IS
OF POOR QUALITY
SUBROUTINE GFDATA (X, NP, CHAN, KK, 
REAL 4 SG(136, 30), MU(16, 30)
REAL 4 X(16, 1500), GT(15)
INTEGER 4 CHAN(15)
INTEGER 82 CLS(15), PTs(15)/1590, ITPR/0, SCLS(15)
INTEGER 82 LPTS(15)/1590, IPTS(15)/1590/
LOGICAL 1 FIRST/TRUE,
LOGICAL 1 OUTPUT/OUTPX
LOGICAL 1 TRUE

FOR CURFELY'S DATA
COMMON /PASS/ SG, MU,
COMMON /SET/ LINE, GT
COMMON /DROK/ SCLS
COMMON /GRPTS/ ML
IF (.NOT. FIRST) GO TO 10
FIRST=.FALSE.

LINE=0

10 READ (5, 9) NP, NL, NUMP, OUTPUT, OUTPUT, TRUE
1 FORMAT (225, ALI)
NUMPS=NP
WRITE (6, 2) NP, NL, OUTPUT, OUTPUT, TRUE
2 FORMAT (9(1 NP=\'i,15', NL=\'i,15', OUTPUT=\'i,15', OUTPUT=\'i,15', TRUE=\'i,15')
1 FORMAT (5(A2,48, A))
10 LINE=LINE+1
NP=NP$;
IF (LINE.LE.NL) GO TO 20
FINISHED WITH THIS PASS OF THE DATA
WRITING 11
10 ITPR=ITPR+1
IF (ITPR.GT.1) RETURN 1

COMPUTE TRUE PROPORTIONS & REARRANGE CLASSES TO THOSE IN STATS
IF (TRUEP) K=K
JJ=0
DO 50 I=1, K
IF (TRUEP) PTS(I)=O
DO 55 J=1, K
IF (CLS(J).EQ.SCLS(I)) GO TO 52
55 CONTINUE
WRITE (6, 53) SCLS(I)
53 FORMAT (9(1 CLASS NOT FOUND, A3)
GO TO 50
52 IF (I.EQ.J) GO TO 50
CLS(I)=CLS(J)
CLS(J)=CLS(I)
IF (TRUEP) PTS(I)=O
PTS(I)=PTS(J)
PTS(J)=PTS(I)
50 CONTINUE
IF (TRUEP) GO TO 57
DO 54 I=1, K
JJ=JJ+PTS(I)
54 CONTINUE

PRINT OUT PROPORTIONS

57 CONTINUE
XJ=JJ
WRITE (6, 51)
51 FORMAT ('1 DATA OBSERVED\'/ CLASS\',T10,\'POINTS\',T20,\'PROPORTIONS\')
DO 60 J=1,K
   IF (TRUEP) GO TO 60
   Y=PTS(J)/X
   P(T1)=X
60 WRITE (6,61)CLS(J),PTS(J),GT(J)
51 FORMAT (A3,T10,T16,T20,616,8)
RETURN
C*******************************************************************************
20 CONTINUE
DO 40 L=1,K
C PLAN OBSERVATION VECTOR
PLAN (11,3),ERR=35,ERR=35) CL, (X(J),J=1,16)
31 FORMAT (A8,A1,6X,16F4,0)
C SELECT SUBSET OF CHANNELS DESIRED
DO 37 L=1,K
37 Y=L EX (CHNL),I)
   IF (TRUEP) GO TO 30
   IF (ITER.GT.1) GO TO 30
   IF (K.GT.1) GO TO 42
   C TALLY FOR COMPUTING TRUE PROPORTIONS
   IF (CLS,F0,CLS(J)) GO TO 45
   40 CONTINUE
   K=K+1
   J=J
   CLS(J)=CL
   PTS(J)=PTS(J)+1
   30 CONTINUE
   IF (ITER.GE.1) RETURN
   64 CONTINUE
   IF (.NOT. TRUEP OR TRUEP) GO TO 87
   C COMPUTE PROPORTIONS OF DATA FOR THIS LINE AND TO DATE
   L=0
   LT=0
   DO 80 J=1,K
     L=PTS(J)
     LPTS(J)=PTS(J)-LPTS(J)
     IPTS(J)=PTS(J)
     80 L=1+LPTS(J)
   X=X+L
   DO 85 J=1,K
     PROP=LPTS(J)/XX
     PPTS(J)=PTS(J)/XX
     85 WRITE (A,66) LINE,CLS(J),PROP,PROP
   C FORMAT ('LINE',I5,' CLASS ',I1,' TRUE PROPORTIONS FOR THIS LINE
   11,16F4,0,' TOTAL TRUE PROP. TO DATE =',16F4,0)
   85 WRITE (A,66) LINE,CLS(J),PROP,PROP
   C WRITE OUT OBSERVATION VECTORS
     WRITE (A,66)
   87 CONTINUE
   IF (TRUEP) RETURN
   WRITE (A,66)
   65 FORMAT ('1 X=',I1)
   DO 70 I=1,NP
     L=I+1
    70 WRITE (A,66) L,X(J),J=1,16
   62 FORMAT (I5,16E5,0)
   RETURN
35 WRITE (A,36) LINE,1
36 FORMAT ('1 END OF DATA ON LINE,15,' AND PIXEL,15)
RETURN
11 NP=I+1
12 LINE=I
   IF (NP.LE.0) GO TO 10
   IF (ITER.LT.1) GO TO 64
RETURN
END

ORIGINAL PAGE IS
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SUBROUTINE MCMLSK(KK,NV,NM,NF)

C******************************************************************************
C THIS ROUTINE COMPUTES THE MODIFIED CHOLESKY DECOMPOSITION OF
C THE COVARIANCE MATRIX. THE DECOMPOSITIONS OVERLAY THE ELEMENTS OF
C THE COVARIANCE MATRIX.
C
C KK - THE COVARIANCE MATRIX STORED IN SYMMETRIC STORAGE MODE.
C NV - THE NUMBER OF CHANNELS USED.
C NM - A WORK AREA OF SIZE NV-1.
C NF - THE DETERMINANT OF THE COVARIANCE MATRIX.
C
C REAL KK(1,NV),NM(1)
C LOGICAL JF1
C J1=TRUE.
J1=0
JN=0
NF=1.

LOOP OVER ALL CHANNELS

DO 10 J=1,NV
K=J-1
J=J+1
JN=J1+1
JF=KK(J1)
IF (JF1) GO TO 12
K=0

10 CONTINUE

COMPUTE THE DIAGONAL ELEMENTS OF L AND STORE IN KK
TEMPORARILY STORE THE PRODUCT KK(I,1)=KK(J,1) IN NM(1)

DO 15 I=1,KL
R=KK(JN+1)
K1=K1+1
K2=KK(K1+1)
TF=TF-R1*R
NM(I)=1
15 CONTINUE

12 CONTINUE

NF=NF*TF
IF (L.GT.NV) GO TO 10
JN=J1-1

COMPUTE THE R,J-TH ELEMENT OF L USING T1

DO 20 J=1,NV
JTN=JN+J-1
T1=KK(JTN)
IF (JF1) GO TO 16
DO 25 J=1,KL
T1=TI-NUV(J1+1)
16 CONTINUE

18 CONTINUE

25 CONTINUE

JF1=FALSE.
J1=0

10 CONTINUE

STORE THE ELEMENTS OF L IN THIS FORM FOR USE IN SUBROUTINE
CLASS

DO 30 J=1,NV
J1=J1+J
30 KK(J1)= 1./KK(J1)
RETURN
END

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LEVEL 21.8 (JUN 74)  FORTAN II

COMPILER OPTIONS  NAME: MAIN,OPT=GO,LINCELNT=60,SIZE=0000K,
SOURCE,LOC=IC,NCLIST,NOEDIT,NOID,NOREF

THIS PROGRAM SCRAMBLES NP RECORDS (*10000) EACH OF LENGTH 1
*CHDS CONTAINED ON FILE 11 AND PUTS THE RESULTS ON FILE 12

ISN 0002  INT(10000).DAT(1y)
ISN 0003  DEFINE FILE B(*10000,10,0,1J)
ISN 0004  ISEED=134145793
ISN 0005  READ (5,11) NP
ISN 0006  11 FORMAT (15)

ISN 0007  GENERATE THE INTEGERS 1,2,...,NP AND STORE IN ARRAY INT
ISN 0008  DC 10 1=1,NP
ISN 0009  20 INT(11)=1
ISN 0010  J=NP
ISN 0011  GGGBF GENERATES A RANDOM NUMBER FROM U(0,1)
          25 R=99999(1SEED)
          1Y=J*2+1
ISN 0012  IT=INT(Y)
ISN 0013  INT(10)=INT(1x)
ISN 0014  INT(LX)=IT
ISN 0015  J=J+1
ISN 0016  IF (J.GT.1) GO TO 23
ISN 0017  WRITE (6,7)
ISN 0018  7 FORMAT (' SHFLD')

ISN 0019  LCGP TO PUT I-TH RECORD IN INT(11)-TH POSITION ON FILE 3 (A TEMP.
          DIRECT ACCESS FILE)
ISN 0020  DC 30 1=1,NP
ISN 0021  L=INT(1)
ISN 0022  FIND(B*L)
ISN 0023  READ (11) DAT
ISN 0024  1 FORMAT (2I3,2A1,100)
ISN 0025  30 WRITE (B*L) DAT
ISN 0026  8 FORMAT (' IN B')

ISN 0027  COPY FILE 2 TO FILE 12

ISN 0028  DC 40 1=1,NP
ISN 0029  11=1+1
ISN 0030  READ (9*1) DAT
ISN 0031  FIND(B*11)
ISN 0032  IF (I.GT.50) STOP
ISN 0033  3 FORMAT (1D)
ISN 0034  IF (I.GT.50) WRITE (6,3) I
ISN 0035  40 WRITE (12,1) DAT
ISN 0036  STOP
ISN 0037  END

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