BEST-RANGE FLIGHT CONDITIONS FOR CRUISE-CLIMB FLIGHT OF A JET AIRCRAFT

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SUMMARY

The Breguet range equation is developed for cruise-climb flight of a jet aircraft to include the climb angle and is then maximized with respect to the no-wind true airspeed. The expression for the best-range airspeed is a function of the specific fuel consumption and minimum-drag airspeed and indicates that an operational airspeed equal to the fourth root of three times the minimum-drag airspeed introduces range penalties of the order of one percent.

INTRODUCTION

Although there is agreement in the literature as to the fact that a cruise-climb flight program (constant airspeed and constant lift coefficient) yields the maximum range for a given jet aircraft with a given fuel load, particularly at the longer ranges, there is no consensus as to the best-range flight conditions. Nicolai (ref. 1), for example, states that best-range occurs when the lift coefficient is equal to $(C_{DO}/2K)^{1/2}$, whereas Houghton and Brock (ref. 2) specify a lift coefficient equal to $(C_{DO}/3K)^{1/2}$. Miele (ref. 3), on the other hand, agrees with Nicolai at one point but at another implies agreement with Houghton and Brock. Perkins and Hage (ref. 4) mention the desirability of cruise-climb flight but do not identify the best-range flight conditions. Dommasch, Sherby, and Connolly (ref. 5) describe the superiority of cruise-climb flight and imply agreement with Houghton and Brock as to the best-range flight condition.

ANALYSIS

With the assumptions of quasi-steady flight, the thrust aligned with the velocity vector, and a constant flight-path angle γ that is sufficiently small so that its cosine may be assumed zero and its sine equal to the angle itself (expressed in radians), the governing equations of motion of an aircraft can be written as:

$$L = W$$

$$T = D + W\gamma \tag{1}$$

$$\frac{dX}{dt} = V$$

where T is the net installed thrust (N), L is the lift (N), W the gross weight of the aircraft (N), D the drag (N), X the range (km), and V the no-wind true airspeed (km/hr or m/s). The weight balance equation (fuel flow rate) for a jet aircraft can be expressed as:

$$\frac{dW}{dt} = - cT \tag{2}$$

where c is the thrust specific fuel consumption (N/hr/N). With the assumption of a parabolic drag polar (with the lift coefficient at minimum drag taken equal to zero) and with the flight conditions of a constant airspeed and a constant lift coefficient C_L, equations (1) and (2) can be combined and integrated to yield a cruise-climb Breguet range equation in the form:

$$X = \frac{VE}{c(1 + \gamma E)} \ln\left(\frac{1}{1 - \delta}\right) \tag{3}$$

where E is the lift-to-drag ratio (which remains constant) and δ, the cruise-fuel weight fraction, is equal to $\Delta W_f/W_1$, where ΔW_f is the cruise fuel (N) and W_1, is the weight of the aircraft at start of cruise. V is expressed in km/hr so that X is in km.

If both the lift coefficient and the airspeed are to remain constant as the weight of the aircraft decreases, the altitude must increase so as to maintain the ratio W/ρ constant, where ρ is the atmospheric density (kg/m^3). Consequently, the density ratio at the end of cruise, σ_2, can be expressed as

$$\sigma_2 = \sigma_1(1 - \delta) \tag{4}$$

By making use of the exponential approximation of the density ratio variation with altitude, the increase in altitude Δh (m) during cruise is given by

$$\Delta h = 7254 \ln\left(\frac{1}{1 - \delta}\right) \tag{5}$$

Combining equations (5) and (3) produces the following approximate expression for γ in terms of the propulsion and aerodynamic efficiencies:

$$\gamma = \frac{7.254c}{VE} \tag{6}$$

Substitution of equation (6) into equation (3) results in

$$X = \frac{VE}{c\left(1 + \frac{7.254c}{V}\right)} \ln\left(\frac{1}{1 - \delta}\right) \tag{7}$$

At this point a cruising airspeed parameter, m, is introduced by writing the cruising airspeed as

$$V = \left[\frac{2(W/S)}{\rho}\right]^{1/2} \left[\frac{mK}{C_{DO}}\right]^{1/4} = m^{1/4} V_{E_{max}} \tag{8}$$

where $V_{E_{max}}$ is the minimum-drag airspeed occurring at E_{max}. The lift coefficient can now be expressed as

$$C_L = \left[\frac{C_{DO}}{mK}\right]^{1/2} \tag{9}$$

where C_{DO} is the minimum-drag coefficient and K the induced drag coefficient. From equation (9) we see that Nicolai calls for an m equal to 2 for best cruise-climb range whereas Houghton and Brock think it should be 3. Miele opts for both values, although 2 is the only value he explicity states.

If V and E are expressed in terms of $V_{E_{max}}$, E_{max}, and m, equation (7) becomes

$$X = \frac{2V_{E_{max}} E_{max}}{c} \left[\frac{m}{(m+1)\left(m^{1/4} + \frac{7.254c}{V_{E_{max}}}\right)}\right] \ln\left(\frac{1}{1-\delta}\right) \tag{10}$$

where $V_{E_{max}}$ and E_{max} are design characteristics of the aircraft. The cruise-climb range is then maximized with respect to the airspeed by setting the first derivative of equation (10) with respect to m equal to zero (with c assumed constant). The resulting condition for the best-range airspeed is

$$m_{br}^{5/4} - 3m_{br}^{1/4} - \frac{29c}{V_{E_{max}}} = 0 \tag{11}$$

whose solution can be approximated with a high degree of accuracy by the expression

$$m_{br} = 3\left[1 + \frac{7.254c}{V_{E_{max}}}\right] \tag{12}$$

so that

$$V_{br} = m_{br}^{1/4} V_{E_{max}} \tag{13}$$

An expression for the associated best-range climb angle can be written as

$$\gamma_{br} = \frac{7.254c}{(VE)_{br}} = \frac{7.254c\ (m_{br} + 1)}{2m_{br}^{3/4}\ (VE)_{E_{max}}} \tag{14}$$

DISCUSSION

The second term in the m_{br} expression of equation (12), $7.254c/V_{E_{max}}$, is normally small with respect to unity as can be seen in figure 1, where it is given the symbol A and represented by constant value lines for various pairings of c and $V_{E_{max}}$. As the minimum-drag airspeed increases and/or the specific fuel consumption decreases, the value of A decreases and the value of m_{br} approaches 3 as a limit so that V_{br} approaches $3^{1/4}$ $V_{E_{max}}$. As an illustration, a $V_{E_{max}}$ of 724 km/hr (450 mph) and a c of 0.5 N/hr/N (0.5 lbf/hr/lbf) yield an A of 0.005, an m_{br} = 3.015, and a V_{br} = 954 km/hr (593 mph). If E_{max} of the aircraft is 18, then $\gamma_{br} = 2.44 \times 10^{-4}$ rad or +0.014 deg.

Relative ranges, normalized with respect to A = 0 (which represents the "level-flight" solution) are shown as a function of both A and m in figure 2. We see that the maximum range occurs when $m = m_{br} = 3(1 + A)$ and that the curves are relatively flat in the vicinity of m_{br}, but drop off sharply as m is decreased below the value of 2. We also see that errors in the actual range introduced by using the level-flight solution are of the order of 1 to 1.5 percent for an m equal to 3 and of the order of 2.5 to 3.3 percent for an m of 2.

As m is increased, so also is the cruise-climb airspeed, as can be seen in figure 3, where again the relative airspeeds are normalized with respect to that for A equal to zero and m = 3. There is an upper limit to the value of m possible for a subsonic aircraft if the best-range airspeed is to remain equal to or less than the drag divergence airspeed. If equal to, m will be 3; if less than, m can be appropriately greater than 3. As a side note, if the best-range airspeed is greater than the drag divergence airspeed, then m must be less than 3 and is no longer a cruising airspeed parameter but rather a wing-loading, cruising-altitude parameter.

CONCLUSIONS

Since an aircraft operated in the cruise-climb mode will probably be characterized by an A of the order of 0.01 or less, we conclude that the use of the "level-flight" (A = 0) version of the Breguet range equation will produce range errors of less than one percent and that the "level-flight" value of m can be used operationally without the necessity of calculating a specific m for cruise-climb flight. We also conclude that the Houghton and Brock value of 3 for m_{br} is not only much closer to the actual best-range value than the other published value of 2 but also calls for a cruising airspeed that is 10 percent higher, with a corresponding decrease in the flight time.

REFERENCES

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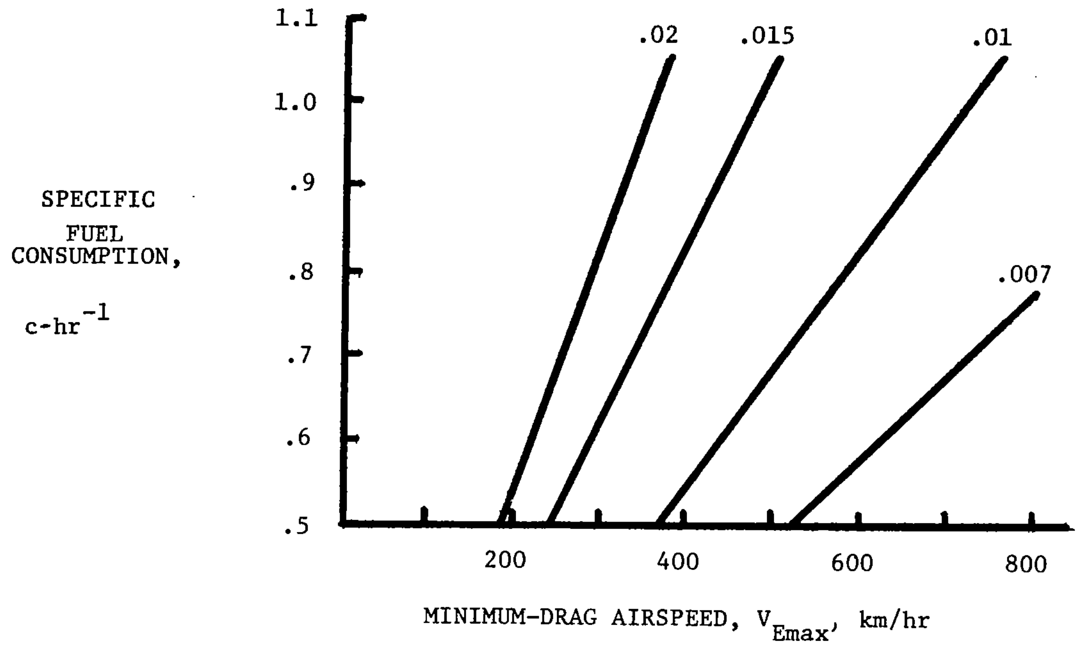

Figure 1.- Constant values of a (7.254 c/V_{Emax}) for various values of c and V_{Emax}.

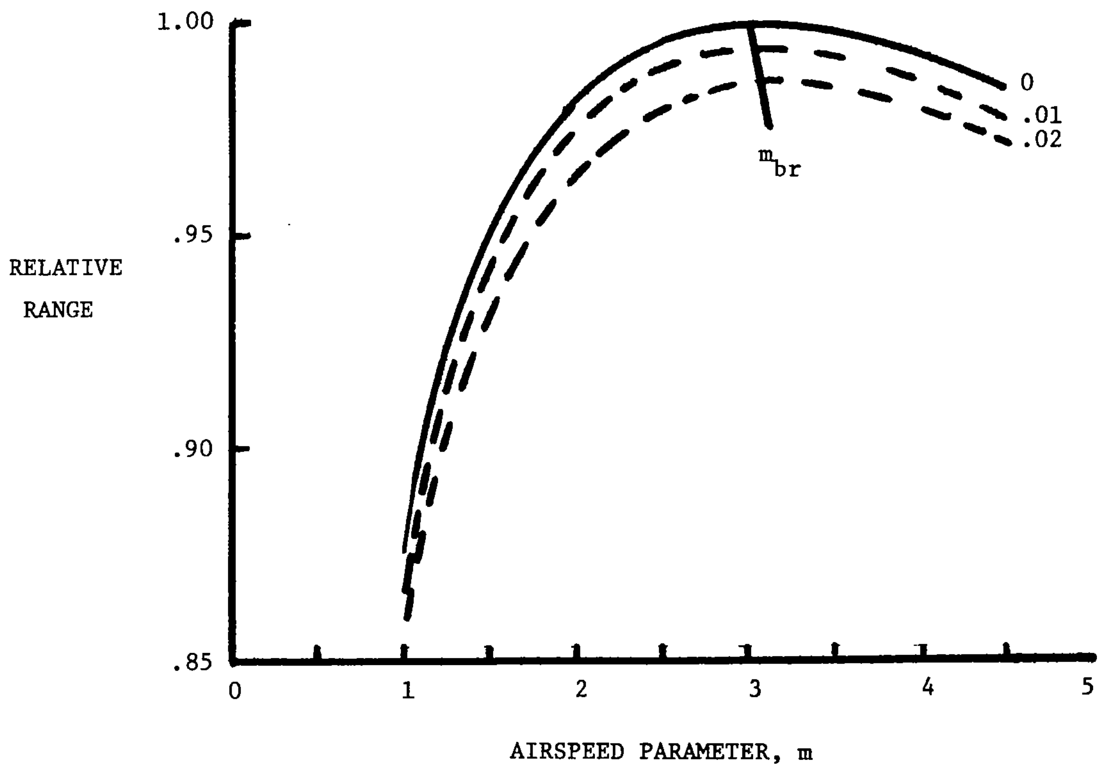

Figure 2.- Relative cruise-climb range as a function of m and A.

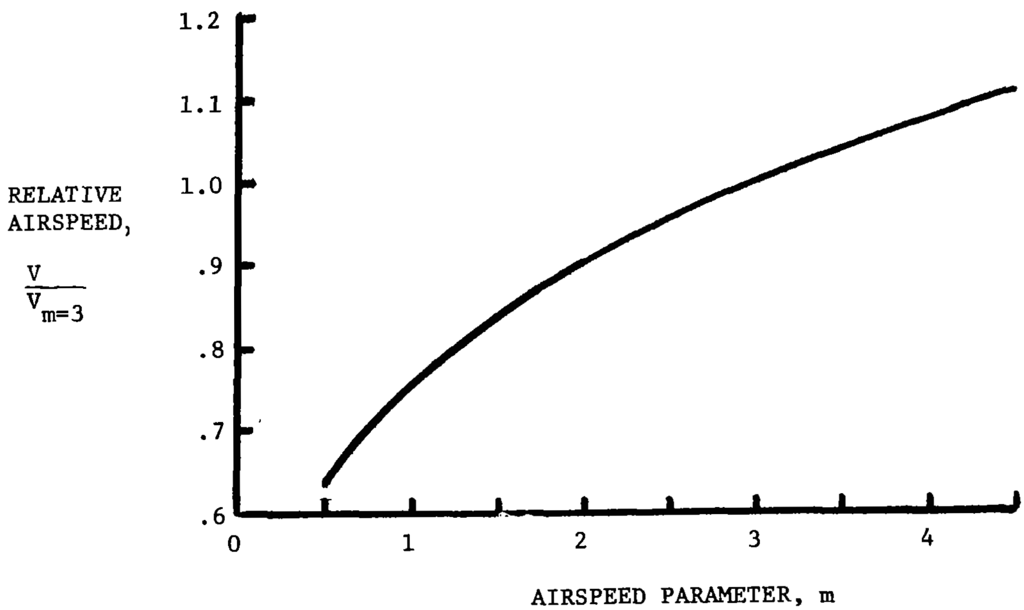

Figure 3.- Relative airspeed as a function of the airspeed parameter m.